

THE DEVELOPMENT OF ELEMENTARY MATHEMATICS TEACHERS'
TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE REGARDING
GEOMETRY THROUGH MATHEMATICS COACHING

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ABSTRACT

THE DEVELOPMENT OF ELEMENTARY MATHEMATICS TEACHERS' TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE REGARDING GEOMETRY THROUGH MATHEMATICS COACHING

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The aim of this study was to investigate the Technological Pedagogical Content Knowledge (TPACK) Development of the in-service mathematics teachers in the field of geometry through participating in mathematics coaching. In order to achieve the purpose of the study, two elementary mathematics teachers working in public schools in Black Sea Region participated in the study. The method of qualitative research was used to get deep and rich information about the development of elementary mathematics teachers' TPACK in the field of geometry. Data was collected from 1) pre-interview, 2) observation of teacher before mathematics coaching, 3) pre-conferences, 4) observation of the teachers during mathematics coaching, 5) post-conference, 6) observation of teacher before mathematics coaching, 7) post-interview, 8) mathematics coaching interview. The progress of teachers' TPACK was analyzed by considering the TPACK Development Model using deductive analysis.

The findings revealed that in-service teachers moved from PCK to TPACK by participating in mathematics coaching. At the beginning of the study, in-service teachers exemplified primarily lower levels of TPACK development. Through the mathematics coaching, they progressed from *lowest levels* to *highest levels* in all the themes and their related descriptors: the curriculum and assessment theme, the learning theme, the teaching theme, and the access theme. Besides, the slowest

progress was observed in the assessment descriptor among all the descriptors in which progress was made during mathematics coaching. The findings also demonstrated that the more-experienced teacher's progression in the TPACK Development Model was faster than that of the less-experienced teacher during mathematics coaching.

Keywords: Technological Pedagogical Content Knowledge, Mathematics Coaching, Professional Development, Geogebra

ÖZ

İLKÖĞRETİM MATEMATİK ÖĞRETMENLERİNİN MATEMATİK KOÇLUĞU SÜRECİNDEKİ GEOMETRİYE İLİŞKİN TEKNOLOJİK PEDAGOJİK ALAN BİLGİLERİNİN GELİŞİMİ

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Bu çalışmanın amacı, ilköğretim matematik öğretmenlerinin matematik koçluğu sürecindeki geometri alanına ait TPAB (Teknolojik Pedagojik Alan Bilgisi) gelişimlerini incelemektir. Çalışmanın amaçlarına ulaşmak için, Karadeniz Bölgesindeki devlet okullarında görev yapan iki ilköğretim matematik öğretmeni çalışmaya katılmıştır. İlköğretim matematik öğretmenlerinin geometri alanındaki TPAB gelişimi hakkında derinlemesine ve zengin bilgi elde etmek için nitel araştırma yöntemi kullanılmıştır. Veriler; 1) ön-görüşme, 2) matematik koçluğu öncesindeki öğretmen gözlemi, 3) ön-konferans, 4) matematik koçluğu sırasında öğretmen gözlemi, 5) son-konferans, 6) matematik koçluğu sonrasındaki öğretmen gözlemi, 7) son-görüşme ve 8) matematik koçluğu hakkındaki görüşme ile toplanmıştır. Öğretmenlerin teknolojik pedagojik alan bilgisi gelişimleri TPAB Gelişim Modeli (Niess et al., 2008) kullanılarak didaktif analiz yöntemi kullanılarak incelenmiştir.

Bulgular öğretmenlerin matemaik koçluğuna katılımı ile birlikte pedagojik alan bilgisinden TPAB'ne doğru ilerlediğini göstermektedir. Çalışmanın başlangıcında, öğretmenlerin TPAB Gelişim Modeline göre düşük oldukları belirlenmiştir. Matematik koçluğu sırasında, öğretmenlerin TPAB Gelişim

Modelindeki bütün temalarda ve temaya ait tanımlayıcılarda (müfredat ve değerlendirme teması, öğrenme teması, öğretme teması ve erişim teması) düşük TPAB seviyelerinden en yüksek seviyelere doğru gelişim gösterdikleri görülmüştür. Bunun yanında, matematik koçluğu sırasında gelişim gösteren tema tanımlayıcıları içerisinde en yavaş gelişim değerlendirmede görülmüştür. Ayrıca bulgular daha deneyimli olan öğretmenin TPAB Gelişim Modelinde daha hızlı ilerlediğini göstermektedir.

Anahtar Kelimeler: Teknolojik Pedagojik Alan Bilgisi, Matematik Koçluğu, Mesleki Gelişim, Geogebra

To My Family

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LIST OF ABBREVIATIONS

ELE	Elementary Mathematics Education
MoNE	Ministry of National Education
METU	Middle East Technical University
NCTM	National Council of Teachers of Mathematics
PCK	Pedagogical Content Knowledge
SSME	Secondary Science and Mathematics Education
TPACK	Technological Pedagogical Content Knowledge

CHAPTER I

INTRODUCTION

Technology has been used for decades in the teaching and learning environment and has led to changes in the nature of education in this information age. The researchers stated that education system must renovate itself with the consideration of rapid social changes (Alkan, 2005; Gülbahar, 2008). Thus, rapid changes in technology imposed changes in the education system in terms of the integration of technology in the classroom. The requirements of the time period we live in should be analyzed and modernized to meet social needs (Akkoyunlu, Altun & Soylu, 2008). The education system should consider the requirements of the society's needs before developing curriculum and instructional strategies. Thus, considering that the students in this timeperiod have grown up in an era of technology, it can be maintained that there is an increasing interest in the application of computers and computer-related technology in the classrooms (Peterson, 2004).

Technology can provide rich opportunities for students to learn curricular concepts in depth (Miller, 2008). Technology helps to increase the quality of mathematical investigations, meaningful mathematical ideas from multiple perspectives, and change traditional ways of teaching mathematics. Additionally, technology, which is one of the school mathematics principles of the National Council of Teachers of Mathematics [NCTM] (2000), is essential in teaching and learning mathematics. NCTM (2000) stated the following:

Electronic technologies—calculators and computers—are essential tools for teaching, learning, and doing mathematics. They furnish visual images of mathematical ideas, they facilitate organizing and analyzing data, and they compute efficiently and accurately....When technological tools are available, students can focus on decision making, reflection, reasoning, and problem solving.(p. 24)

This statement demonstrated the importance of technologies in mathematics education. Technology in mathematics can be used in a variety of ways to improve and enhance the teaching and learning. It helps to facilitate mathematical discovery, understanding, and connections that may be difficult or impossible without its use. In addition, the Association of Mathematics Teacher Educators (AMTE, 2005) described some advantages of technology in the teaching and learning of mathematics, stated as follows:

The computational and graphical capabilities of current technologies enable users to efficiently generate and manipulate a variety of representations of mathematical ideas and processes. Activities that engage students in connecting multiple representations (e.g., graphical, numerical, algebraic and verbal), and those that invite students to analyze or create images, visualizations, and simulations provide wide-ranging opportunities for mathematical exploration and sense making. Instruction that takes full advantage of what technology has to offer can encourage, foster, and support students' construction of mathematical knowledge in a variety of ways. Technology can also improve mathematical communication, facilitate more efficient use of mathematical resources, and raise the quality of mathematical products and presentations. (p.1)

This statement demonstrated that integration of technology in the teaching and learning of mathematics provides environments for students to explore the mathematical concepts by using multiple representations. Furthermore, technology integration is beneficial to improve conceptual understanding. In the light of the advantages of technology, AMTE (2006) stated that technology has become an essential tool for doing mathematics in today's world. Therefore, technology integration is necessary for the teaching and learning of mathematics.

From this point of view, technological facilities of schools have recently increased in Turkey as well. In Turkey, the Ministry of National Education (MoNE) declared its vision statement on Information and Communication Technologies (ICT) in its strategy plan of 2010- 2014 and stated that "to integrate ICT into the education system, support the education system with developments, improve it consistently by assessing it and provide student-centered and project-based learning by using ICT" (MoNE, 2010, p.36). To reach their objectives, MoNE aimed to

equip all schools with Internet access so that new technologies could be used and become more widespread in education, make information technology classes more widespread, rapidly improve the index of the number of students per computer, sustain the support provided for teachers' provision of laptops, and implement campaigns to meet the educational requirements of the information age. (p. 36)

Towards these aims, MoNE has attempted to integrate technology in schools. There are many projects held by MoNE to improve schools technologically by means of such projects as FATİH, e-okul, e-etüt (EĞİTEK, 2011). The most significant educational investment among these projects was "Movement of Enhancing Opportunities and Improving Technology," known as FATİH. This Project aimed to provide equality of opportunity, eliminate the digital gap and enhance the quality of education by means of the 5 components the project:

Accessibility: Offering service any time, anywhere and independent from tools,

Productivity: Providing target oriented and more productive development fields,

Equality (equality of opportunity): Enabling all shareholders' access to the best service.

Measurability: Providing accurate measurement of process and results and giving feedback accordingly for the development to be assessed better.

Quality: Enhancing the quality of whole education in a measurable way.(p.1)

To ensure these components of the Project, "Smart Class" was aimed to be integrated into all schools in preschool education, primary education and secondary education in Turkey. With this project, 42,000 schools and 570,000 classes were aimed be equipped with tablets and interactive white board and other latest information technologies. This project aimed to provide equal opportunities in education and improve the technology in schools to promote more conceptual understanding in the educational process (EĞİTEK, 2011).

Although FATİH Project devotes attention to technological supplements in the classroom, it isn't a project solely based on providing hardware. This project also has an important role in (1) providing hardware and software infrastructure, (2)

providing and managing educational e-content, (3) promoting effective ICT usage in the curriculum, (4) providing in-service training to teachers, and (5) enabling conscious, reliable, manageable and measurable ICT usage. With the FATİH project, Turkey aims to overcome many difficulties regarding the integration of technology, such as lack of teacher training, and lack of knowledge about how to use technological tools. Teachers' knowledge of technology usage is an important component of technology integration in mathematics education and also of the FATİH project in Turkey. Therefore, in-service and pre-service teachers need to have knowledge about how to integrate technology into mathematics education.

Technology Committee for the Association of Mathematics Teacher Educators (AMTE, 2005) declared that technology has become an essential tool for the teaching and learning of mathematics. Pre-service and in-service mathematics teachers should implement appropriate uses of technology in the teaching of K-12 mathematics. Therefore, teacher preparation programs need to focus on strengthening their knowledge of how to incorporate technology to facilitate student learning of mathematics through experiences that:

- allow teacher candidates to explore and learn mathematics using technology in ways that build confidence and understanding of the technology and mathematics;
- model appropriate uses of a variety of established and new applications of technology as tools to develop a deep understanding of mathematics in varied contexts;
- help teacher candidates make informed decisions about appropriate and effective uses of technology in the teaching and learning of mathematics; and
- provide opportunities for teacher candidates to develop and practice teaching lessons that take advantage of the ability of technology to enrich and enhance the learning of mathematics. (AMTE, 2005, p.2)

These components help to develop pre-service knowledge about technology integration for teaching and learning mathematics. Furthermore, in-service teachers also need a professional development program focusing on helping the teachers to enhance their knowledge about how to integrate technology in their classroom.

Specifically, considering that technology improves learning only when it is used appropriately, professional development of teachers regarding technology is crucial. Thus, the increasing importance of technology in education has made technology knowledge in how to use it in the classroom an essential part of teachers' professional knowledge. Teachers play a key role in deciding how to integrate technologies to facilitate and support student learning (Conference Board of Mathematical Sciences [CBMS], 2001; International Society of Technology in Education [ISTE], 2000, 2007). Moreover, NCTM (2000) supports the fact that teachers' knowledge regarding technology integration into the classroom is important in enhancing students' knowledge in mathematics classes. Mishra and Koehler (2006) stated that for effective teaching with technology, teachers should be knowledgeable not only in pedagogy and content but also in use of technology. In the light of this view, educational researchers expanded teachers' knowledge to include the role of technology and combined technology, pedagogy, and content knowledge (Mishra & Koehler, 2006). This teacher knowledge combined with that of technology integration is called Technological Pedagogical Content Knowledge (TPACK). TPACK explains teachers' knowledge about how to understand technology and pedagogical content knowledge, and how to interact with each other for effective teaching via integrating technology.

Considering that teachers are the most important factor in integration of technology in classrooms, it can be asserted that teachers' professional development has a key role in improving teachers' TPACK (Figg & Jaipal, 2012; Gorder, 2008; Harris & Hofer, 2009 ; Harris et al., 2010 ; Niess, 2005a; Voogt et al., 2010). According to Harris and Hofer (2009), professional development provides in-service teachers with opportunities to: (a) select and use learning activities and technologies in a more conscious, strategic, and varied manner; (b) plan instructional activity that is more student-centered, focusing on students' intellectual, rather than affective, engagement; and (c) make deliberate decisions for more judicious educational technology use. Additionally, Sparks and Hirsh (2000) emphasized that effective professional development should entail sustained,

rigorous, and cumulative programs that are directly linked to what teachers do in their classrooms. In professional development, teachers need to design, implement, and reflect on technology-enhanced lessons with a concern for students' understanding (Niess et al., 2009). Thus, professional development programs provide teachers with an environment to collaborate in planning lessons, practice and share new teaching methods to practice solving problems with others (Niess, 2006). Furthermore, Japial-Jamani and Figg (2015) indicated that to improve teachers' TPACK development, effective professional development courses should focus on modeling a technology-enhanced activity as a learning tool to set the context and purpose for tool use.

With the consideration of the components of effective professional development focused on teachers' TPACK, in this study the researcher conducted mathematics coaching as a professional development program to develop in-service teachers' TPACK regarding geometry. Mathematics coaching including three sessions (pre-conference, observation, and post-conference) provides an environment for in-service teachers to enhance their knowledge, learning, and practice to incorporate this new learning into their classrooms. The development of the knowledge regarding how to integrate technology into mathematics education requires a process for teachers. Thus, it is important for teachers to build a TPACK, which should be viewed as a long term trajectory (Fishman & Davis, 2006). In the light of this view, mathematics coaching, which is conducted in this current study, is not a one-day workshop isolated from practice; rather, it emphasizes process in a long-term professional development. Additionally, mathematics coaching helps in-service teachers to combine theory and practice. Through mathematics coaching, in-service teachers can be supported to gain knowledge to integrate technology into the education in their classroom. In the next chapter, mathematics coaching is explained in detail.

1.1 Mathematics Coaching as a Professional Development

Coaching as a professional development firstly proposed by Showers and Joyce (1996) because research studies demonstrated that professional development strategies were not effective to improve teachers' knowledge. Coaching is defined as "the practice of providing deliberative support to another individual to help him/her to clarify and/or to achieve goals" (Bloom et al, 2005, p. 5). The coaching provides support to the teachers as they are learning new teaching practice in their own classroom. According to Loucks-Horsley, Love, Stiles, Mundry, and Hewson (2003) coaching is, in a one-on-one setting to enhance the knowledge, learning, and practice of the teacher who is trying to integrate new learning into her/his classroom practice. Mathematics coaching provides teachers with opportunities to work with coaches in planning, implementing and debriefing to improve teachers' knowledge. Hull, Balka and Miles (2009) stated that coaches work directly with teachers to improve student learning of mathematics in mathematics coaching. Mathematics coaching included the cyclic process of pre-conference, observation, and post-conference (Bay-Williams, McGatha, Kobett, & Wray, 2014). The cyclic process of mathematics coaching is described below in detail (see Figure 1.1).



Figure 1. 1. The Cyclic Process of Mathematics Coaching (Bay-Williams, McGatha, Kobett, & Wray, 2014, p. 6)

In the pre-conference as the first phase of mathematics coaching, teachers and the coach worked collaboratively and designed the lesson together. In addition, the coach can provide the teachers with some good examples to demonstrate how to integrate new practices in mathematics education (West & Staub, 2003). Then, the coach monitored the teachers' practice in their classroom in the observation session. This session of mathematics coaching helps the teacher to put into together theory and practices. Furthermore, the coach collected the data to analyze. In the last part of mathematics coaching (post-conference), the teachers were encouraged to reflect on how to conducted co-planned lesson in terms of its strengths and weaknesses of lesson (West, 2009). Additionally, the teacher and the coach discuss how to improve the teachers' lesson. In addition, the discussion about the implemented lesson provides an opportunity to raise issues, concerns and criticisms related to technology integration and to provide insight into what was important for the teachers and what their knowledge is. Furthermore, in the post-conference the coach can give feedback and suggestions to improve the teachers' knowledge and practice (Bay-Williams, McGatha, Kobett, & Wray, 2014).

In this study, the researcher aims to investigate the teachers' TPACK development during mathematics coaching as a promising professional development. This study has the potential to provide detailed information on mathematics teachers' TPACK development in mathematics coaching regarding geometry.

1.2 Research Questions of the Study

Mathematics coaching as a professional development has a potential to develop teachers' TPACK and support their integration of technology into their mathematics classrooms. Considering the importance of in-service mathematics teachers' TPACK, the purpose of this study is to determine the development of teachers' TPACK in geometry and their progression through the levels of teachers' TPACK in geometry during the mathematics coaching. Furthermore, this study analyzes how

elementary mathematics teachers construct TPACK in geometry before/during/after the mathematics coaching. More explicitly, this study aims to answer the following questions:

1. To what extent does participating in mathematics coaching impact elementary mathematics teachers' technological pedagogical content knowledge in the field of geometry?
 - 1.1. How do elementary mathematics teachers participate in mathematics coaching progress through the levels of Curriculum and Assessment theme in the TPACK Development Model?
 - 1.2. How do elementary mathematics teachers participate in mathematics coaching progress through the levels of Learning theme in the TPACK Development Model?
 - 1.3. How do elementary mathematics teachers participate in mathematics coaching progress through the levels of Teaching theme in the TPACK Development Model?
 - 1.4. How do elementary mathematics teachers participate in mathematics coaching progress through the levels of Access theme in the TPACK Development Model?
2. What are elementary mathematics teachers' perceptions of mathematics coaching in terms of its role in developing technological pedagogical content knowledge?

1.3 Significance of the Study

In parallel to the increasing use of technology in society, the role of technology in mathematics education has gained emphasis. NCTM (2000) stated that technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning. Professional

organizations such as AMTE (2006), ISTE (2007), and NCTM (2000) declared that teachers should learn to become effective users of technology. Teachers need to obtain knowledge of technology combined with pedagogy and content to integrate technology effectively (Mishra & Koehler, 2006; Niess, 2005). According to NCTM (2007), “If teachers are to learn how to create a positive environment that promotes collaborative problem solving, incorporates technology in a meaningful way, invites intellectual exploration, and supports student thinking, they themselves must experience learning in such an environment” (p. 119). Therefore, teachers should be provided with an environment to experience how to integrate technology in mathematics education. Considering the importance of teachers’ knowledge, it can be stated that more research is needed in the area of TPACK and teaching mathematics with technology.

In recent times, the Turkish Ministry of National Education (MoNE) has placed great emphasis on technology integration in schools. Moreover, the increasing numbers of schools equipped with technology (such as smartboards) help to understand the importance of technology knowledge. There are many projects, such as FATİH, e-okul, e-etüt in Turkey (EGİTEK, 2011). Although these projects show that the Turkish Ministry of National Education focuses on technology integration in the classroom, there is a need for more qualified teachers in terms of TPACK. For more effective teaching with technology, mere technological knowledge is not sufficient. Teachers should have a complex mixture of technology, pedagogy and content knowledge (Koehler & Mishra, 2009, Mishra & Koehler, 2006). To improve teachers’ TPACK, teachers should be trained in how to use technology in the classroom. From this perspective, teacher professional development is important for the improvement of TPACK.

Professional development for teachers should be well prepared to empower them in integrating technology in their classes. However, traditional approaches to professional development are inadequate to improve teachers’ knowledge and ability (Knight, 2007). The reason of inefficiency of traditional professional development is that they are kept independent of and disconnected from the

classroom (Ball & Cohen, 1999; Knight, 2007). For effective professional development, teachers are helped to build their content and pedagogical content knowledge and to put it into practice. According to Loucks-Horsley et al. (2003), coaching is an effective professional development strategy that provides one-on-one learning opportunities for teachers that focus on improving their knowledge by means of self-reflection and implementation of their reflection into classroom practice. However, coaching is a new strategy for professional development and there are not many research studies on coaching especially in the field of mathematics teacher professional development. This study meets that need of research on coaching as a professional development strategy. Furthermore, in the coaching program, in-service teachers are provided with an opportunity to learn and practice how to integrate technology. Thus, the coaching program can help teachers to improve their TPACK. This study also investigates how in-service teachers develop their TPACK by participating in mathematics coaching as a professional development. From this point of view, this study has the potential to provide information on teachers' TPACK development in professional development regarding geometry.

The aim of this study was to investigate the TPACK Development of the in-service mathematics teachers' technological pedagogical content knowledge in the field of geometry through participating in mathematics coaching. In addition, this study is to analyze the development of elementary mathematics teachers' TPACK in geometry according to themes and levels in the TPACK Development before/during/after mathematics coaching. By focusing on the TPACK Development Model (Niess, Ronau et al., 2009), it is believed that exploration of the TPACK of geometry teachers will make a major contribution to literature because this TPACK Development Model is proposed to describe teachers' progression in developing the specialized knowledge for mathematics teachers in detail. The TPACK model of Mishra and Koehler (2006) is general and not discipline specific, and their components of TPACK are not clear. On the other hand, the TPACK Development Model is subject-specific and includes the teacher's

knowledge of incorporating technology into teaching mathematics under four themes and five levels. Hence, this model is used in this study to identify teachers' TPACK levels under these four themes: curriculum and assessment, learning, teaching and access. Because of the detailed structure of the TPACK Development Model, it is beneficial to look into the in-service teachers' knowledge from the perspective of this development model.

Additionally, literature reviews show that studies on TPACK of teachers are not sufficient (especially in Turkey) despite the importance attached to technology integration. Because of the lack of research, the results of this study can make a significant contribution to the field regarding how elementary mathematics teachers perceive TPACK and its dimensions and the relationships among the TPACK components. Literature review revealed that most of the studies on TPACK were conducted with pre-service teachers. There is a lack of studies on in-service teachers. Thus, there is a need for more research studies on in-service teachers' TPACK. Additionally, research studies on TPACK encountered in the related literature are not subject specific. By focusing on elementary mathematics teachers' knowledge of technology integration in geometry, this study will contribute to the literature by filling in this gap. In this study, in-service mathematics teachers designed a technology-enhanced lesson, applied the lesson and reflected on their implementation of integrating Geogebra as technological tool in the polygon units in geometry. In other words, this study revealed how in-service teachers integrated technology (specifically Geogebra) in geometry. Teachers implemented Geogebra in their geometry lesson during the current study. How the in-service teachers integrated Geogebra as a learning and teaching tool in their lesson and how they progressed through the Geogebra-enhanced lessons were investigated. In addition, most of the studies on TPACK development merely emphasized the graphing calculator as an instructional technology. Thus, there is a need to conduct more studies on TPACK via different instructional technologies, such as Geogebra, Cabri3D, and Tinkerplots. Therefore, the aims of this research study have been to

investigate in-service mathematics teachers' TPACK Development by integrating Geogebra in geometry through mathematics coaching.

1.4 Definitions of Important Terms

Elementary mathematics teachers: An elementary mathematics teacher is one who provides education to students in grades 6, 7 and 8 in the educational institution they are working at. Elementary mathematics teachers refer to mathematics teachers in public middle school as a part of primary education system. They teach 7th grade students in public schools located in the Black Sea region in Turkey.

Technological Pedagogical Content Knowledge: Technological pedagogical content knowledge is “the total package required for integrating technology, pedagogy, and content knowledge in the design of instruction for thinking and learning mathematics with digital technologies” (Niess et al., 2009, p.7). In this study TPACK refers to the effective integration of technology, pedagogy, and content knowledge via Geogebra in geometry education.

Geogebra: GeoGebra is a free dynamic geometry software that “brings together geometry, algebra, spreadsheets, graphing, statistics, and calculus” (GeoGebra Tube, 2015). In this study, Geogebra is used a technological tool which the teachers integrated in their lesson.

The TPACK Development Model: The TPACK development model is proposed by Niess, Sadri, and Lee (2007) to describe a development of TPACK for teaching mathematics. In addition, this model consisted of four themes (the curriculum and assessment theme, the learning theme, the teaching theme, and the access theme). In this study, it refers to in-service mathematics teachers' development of TPACK in their Geogebra-enhanced geometry lesson.

Curriculum and Assessment Theme: The Curriculum and Assessment theme includes two descriptors: curriculum and assessment (Niess et al., 2009). Whereas the curriculum descriptor focuses on the treatment of the subject matter, the assessment descriptor emphasizes assessing the students' understandings. In this study, it refers to in-service mathematics teachers' knowledge of how to integrate Geogebra and curricular materials, and how to assess students' understandings via Geogebra.

Learning Theme: The learning theme includes two descriptors: mathematics learning and conceptions of students thinking (Niess et al., 2009). Whereas the mathematics learning descriptor emphasizes whether teachers integrate technology which emerged as a potential tool in learning mathematics, the conceptions of students thinking descriptor demonstrates the development of students' thinking skills via technology. In this study, it refers to in-service mathematics teachers' knowledge of how to integrate Geogebra as a learning tool, and implement technology with consideration of the development of students' thinking skills.

Teaching Theme: The teaching theme includes four descriptors: mathematics learning, instructional, environment, and professional development. The mathematics learning descriptor focuses on the views of teachers regarding technology (Niess et al., 2009). The instructional descriptor indicates teachers' knowledge of instructional strategies by incorporating technology, and the environment descriptor demonstrates mathematical environments to incorporate technology as a learning tool for supporting students' learning. The last descriptor (the Professional descriptor) focuses on teachers' tendency to professional development. In this study, it refers to in-service teachers' views about technology integration in geometry and their knowledge about instructional strategies and mathematical environments by integrating

Geogebra. Furthermore, it refers to in-service mathematics teachers' tendency to professional development.

Access Theme: The access theme includes three descriptors: usage, barriers, and availability (Niess et al., 2009). Usage descriptor focuses on whether or not students are allowed to use technology. The Barrier descriptor emphasizes how teachers address barriers to technology integration. The Availability descriptor indicates how technology makes more and higher levels of mathematics available for students to investigate. In this study, it refers to whether or not in-service teachers let students use Geogebra. Furthermore, it also refers to how in-service teachers overcome the challenges of Geogebra integration in their geometry lesson and how they integrate Geogebra to promote students' higher level thinking.

The level of Technological Pedagogical Content Knowledge: The TPACK Development Model has five stages, which is sequential process, namely recognizing, accepting, adapting, exploring, and advancing stages. In this study, the level of Technological Pedagogical Content Knowledge refers to the recognizing, accepting, adapting, exploring, and advancing levels in the development of TPACK for teaching mathematics, which is developed by Niess, Sadri, and Lee (2007).

The Recognizing level: In this level teachers might use the technology and recognize the alignment of its use in mathematics, but they cannot integrate technology effectively in teaching and learning of mathematics (Niess et al., 2010).

The Accepting level: In this level teachers have challenges in finding ways to integrate the technology effectively in their mathematics curriculum. In addition, they had a favorable or unfavorable attitude toward teaching and learning mathematics with an appropriate technology (Niess et al., 2009).

The *Adapting* level: In this level they start to realize some advantages of using appropriate technologies as teaching and learning tools. They might decide whether to adopt or reject the technology (Niess et al., 2009).

The *Exploring* level: In this level teachers modify and design activities to align with their curriculum. Furthermore, they search innovative ways for teaching the content by using technologies as learning tools (Niess et al., 2010).

In the *Advancing* level: In this level teachers evaluate the integration of appropriate technology by making changes in the mathematics curriculum (Niess et al., 2010).

Mathematics Coaching: In a one-on-one setting, the coaching promotes teachers in the teaching context to enhance the knowledge, learning, and practice of the teacher to incorporate this new learning into classroom practice (Loucks- Horsley et al., 2003). Mathematics coaching, which is a kind of professional development, includes cyclic process of pre-conference, observation, and post-conference. In this study, mathematics coaching was conducted to improve in-service mathematics teachers' TPACK in geometry.

Pre-conferences: In pre-conference, the coach and the teacher design or redesigned the lesson by “developing a shared view of the understanding, strategies, concepts, and skills” (West & Staub, 2003, p. 4). In this current study, in the pre-conferences the coach gained insight of the teachers' TPACK and co-designed the lesson with teachers

Observations: In the observation, the coach monitors the teachers' implementation of co-designed lesson (West & Staub, 2003). In this study, the coach observed the mathematics teachers' Geogebra-enhanced lessons. In

addition the coach collected data how to integrate Geogebra in their geometry lesson.

Post-conferences: The coach and the teacher reflected their ideas about the teacher's implementation and discussed the weaknesses and strengths of the observed lesson (West & Staub, 2003). In this study, the teachers reflected their ideas about how to integrate Geogebra in their geometry lesson.

CHAPTER II

LITERATURE REVIEW

The purpose of this study was to investigate the TPACK Development of the in-service mathematics teachers' technological pedagogical content knowledge in the field of geometry through participating in mathematics coaching. The theoretical background of the present study and relevant studies has been referred to throughout the chapter.

To provide more clarity, this chapter is divided into the following subsections: a) Teacher knowledge, b) Technological Pedagogical Knowledge (TPACK), c) Research studies on TPACK, d) Mathematics Coaching as Professional Development and e) Research studies on mathematics coaching. At the end of the chapter, a summary of the literature review is provided.

2.1 Teacher Knowledge

In this section, theoretical frameworks of teacher knowledge have been explained in detail. As the present study was aimed to analyze the development of the in-service teachers' TPACK through mathematics coaching, frameworks of teacher knowledge, the moving of PCK (pedagogical content knowledge) into TPACK, and TPACK frameworks have been explicated.

The knowledge of teachers has been investigated for decades through various studies (Ball, 1990a, 1990b, 2000; Ball, Thames, & Phelps, 2008; Grossman, 1990; Hill, Ball, & Schilling, 2008; Shulman, 1986, 1987). According to Ball, Thames, and Phelps, (2008), teaching is professional work including content knowledge for teaching. Shulman (1986) indicated that there was a sharp distinction between content knowledge and pedagogy. He stated that in previous centuries sometimes it was pedagogy that was ignored, or sometimes it was the content that

was neglected. The pendulum swung in the areas of research and policy. Shulman realized that there was a “missing paradigm” in the research on teaching and teacher knowledge. In order to find the “missing paradigm,” Shulman posed the following questions: “Where do teachers’ explanations come from? How do teachers decide what to teach, how to present it, how to question students, and how to deal with problems of misunderstanding?” (Shulman, 1986, p. 8). Within this scope, Shulman proposed that a more coherent theoretical framework was needed. Shulman (1986, 1987) divided content knowledge of teachers into three categories, namely subject matter knowledge (SMK), pedagogical content knowledge (PCK), and curricular knowledge. In this framework, subject matter knowledge is the knowledge that students need to learn. Curricular knowledge is the knowledge of the full range of programs designed for the teaching of a specific subject. Pedagogical content knowledge (PCK), which links content knowledge and pedagogical knowledge, is teachers’ understanding of how to help students to understand the subject matter.

In 1987, Shulman enhanced a teacher knowledge framework. He identified seven major categories of teacher knowledge: 1) content knowledge, 2) general pedagogical knowledge, 3) curriculum knowledge, 4) pedagogical content knowledge, 5) knowledge of the learners and their characteristics, 6) knowledge of educational contexts, 7) knowledge of educational ends, purposes, and values, and their philosophical and historical grounds.

Among these categories, PCK is of special interest because it indicates the distinctive bodies of knowledge for teaching. Shulman (1987) defined PCK as “special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding.” (p. 8). In addition, Shulman (1987) expressed the reason of PCK’s getting special among seven categories as;

It identifies the distinctive bodies of knowledge for teaching. It represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction. Pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue. (p.8)

Furthermore, Shulman stated that the intersection between the two components pedagogical knowledge and content knowledge- constitute pedagogical content knowledge. Shulman (1986) defines pedagogical content knowledge as

The most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations including an understanding of what makes the learning of specific concepts easy or difficult: the concepts and preconceptions that students of different ages and backgrounds bring with them to the learning (p. 9).

Thus, PCK emphasizes an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction. Therefore, PCK is “the category most likely to distinguish the understanding of the content specialist from that of the pedagogue” (Shulman, 1987, p.8).

Shulman (1986, 1987) presented the most dominant framework of teachers' professional knowledge for all disciplines. After Shulman's theoretical framework, many researchers explored teachers' knowledge via using Shulman's categorization of teachers' knowledge such as Ball (1990a; 1990b; 2000), Grossman (1989, 1990, and 1991), Hill, Ball, and Schilling (2008), and Tamir (1988).

After Shulman's framework about teachers' knowledge, Grossman (1989, 1990, 1991) researched teachers' PCK based on Shulman's model. He focused on the description and understanding of the teacher knowledge. He classified the development of teachers' pedagogical content knowledge into four components:

1. An overarching conception of what it means to teach a particular subject
2. Knowledge of instructional strategies and representations for teaching particular topics
3. Knowledge of students' understandings, thinking, and learning in the subject
4. Knowledge of curriculum and curriculum materials with learning in the content areas

The first component is related to the knowledge and beliefs about the purposes for teaching a particular subject at different grade levels. The second component includes the knowledge of teaching a particular subject using a variety of instructional strategies and representations to increase students' understanding. The third component focuses on students' understanding, conceptions, and misconceptions of particular topics. The fourth component emphasizes the knowledge of the organization of topics in a specific grade level. Furthermore, it refers to the knowledge of which books and instructional materials are appropriate to effectively teach a particular subject. According to Grossman (1990), teachers should possess rich repertoires of metaphors, experiments, activities, or explanations that are particularly effective for teaching a particular topic. Also, Grossman's four components of pedagogical content knowledge can be used as a framework for analyzing a teacher's knowledge in the presentation of a particular lesson (Borko, 1991). Furthermore, the main difference of Grossman's teacher knowledge and Shulman's teacher knowledge is that Grossman's framework focuses on teacher's knowledge in a particular subject.

The frameworks proposed by Shulman (1986) and Grossman (1990) emphasize teachers' knowledge, but they were general, not subject specific. However, Ball and her colleagues enlarged and elaborated on the theoretical framework of Shulman's pedagogical content knowledge and combined the knowledge of teaching and mathematics content (Ball & Bass, 2000; Ball & Bass, 2003; Hill, Schilling, & Ball, 2004). Ball and his colleagues studied with mathematics teachers, and presented a framework of mathematical knowledge for teaching to analyze the content knowledge for teaching mathematics. In this framework, teaching refers to what teachers need to do in teaching mathematics. The mathematical knowledge for teaching consists of two dimensions: subject matter knowledge and pedagogical content knowledge (see Figure 2.1). Ball et al. (2008) subdivided content knowledge into three types: common content knowledge (CCK), specialized content knowledge (SCK), and horizon knowledge in the light of Shulman's subject matter knowledge. Common content knowledge is defined as

mathematical knowledge and skills which every person has whether s/he is a mathematics teacher or not, whereas specialized content knowledge is the unique knowledge of the teacher who teaches mathematics. Finally, Horizon knowledge refers to “an awareness of how mathematical topics are related over the span of mathematics included in the curriculum” (Ball et al., 2008, p. 403).

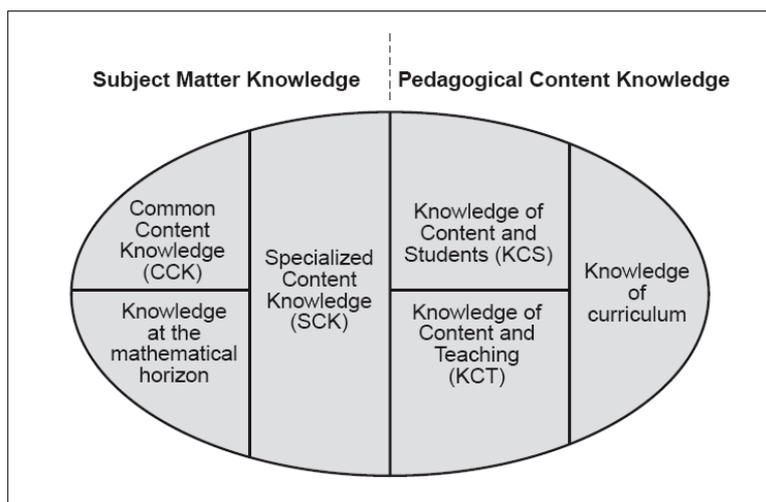


Figure 2. 1 Mathematics teachers' knowledge for teaching (Ball et al., 2008, p. 403)

In the model proposed by Ball et al. (2008), pedagogical content knowledge has been divided into three parts: (a) knowledge of content and students (KCS), (b) knowledge of content and teaching (KCT), and (c) knowledge of curriculum. KCS consists of knowledge about students and knowledge about mathematics, whereas KCT is described as the combination of knowledge about teaching and knowledge about mathematics (Ball et al., 2008). KCS emphasizes the knowledge of how students think, know, or learn specific content. KCS involves knowledge of what students may think and what they will find confusing, interesting and motivating, and to interpret students' spoken words and written work. KCT includes the knowledge of determining the best instructional method, choosing examples that are appropriate for students. The last component of pedagogical content knowledge is knowledge of curriculum, which is related to the knowledge of the characteristics of the curriculum designed for teaching particular subjects at a specific level.

Considering this model, Ball et al. (2008) stated that the four components of teachers' knowledge - CCK, SCK, KCS, and KCT- have strong relationship each other. Ball et al. (2008) described this relationship as

Recognizing a wrong answer is CCK, whereas sizing up the nature of an mistake, especially an unfamiliar mistake, typically requires nimbleness in thinking about numbers, attention to patterns, and flexible thinking about meaning in ways that are distinctive of SCK. In contrast, familiarity with common mistakes and deciding which of several mistakes students are most likely to make are examples of KCS. (p.401)

As considering this statement, in the Ball et al. (2008) teachers' knowledge intersect each other. This means that one of the parts of teachers' knowledge (such as SCK) is necessary for other part of teachers' knowledge (such as CCK). Therefore, the domains of the teachers' knowledge are interrelated to each other.

As stated in previous frameworks, Shulman (1986, 1987), Grossman (1990), and Ball et al. (2008) discussed teacher knowledge without consideration of technology integration. According to Mishra and Koehler (2006), they did not focus on technology and its relationships to pedagogy and content because the modern computer technologies had not affected the field of education until the 1990s. With the shift toward the effective use of technology in education, researchers realized that there was a need for new knowledge so that teachers could effectively integrate technology into their educational practices. With the light of the teacher knowledge theory proposed by Shulman (1986), researchers began to discuss the combination of technology and pedagogical content knowledge. The next section emphasizes the knowledge needed for teaching with technology in an assigned subject or grade level.

2.2 Technological Pedagogical Content Knowledge

Technology changes our world and our daily life, and education is no exception. The degree of technological changes in schools and schools of education is increasing. Indeed, technologies such as graph calculators and computer software have been integrated into the classrooms (Kaput, 1992). The use of technology in the classroom introduces a new set of variables into teacher knowledge, and creates complexity in the rapidly-changing nature of classrooms (Koehler & Mishra, 2008). Thus, recognizing the importance of a broader perspective of teaching via technology, researchers focused on the integration of technology, content, and pedagogy. Mishra and Koehler (2006) developed a theoretical framework of technological knowledge by building upon Shulman's pedagogical content knowledge; it combines pedagogical knowledge and content knowledge. This new framework is called Technological Pedagogical Content Knowledge (TPACK) (see Figure 2. 2).

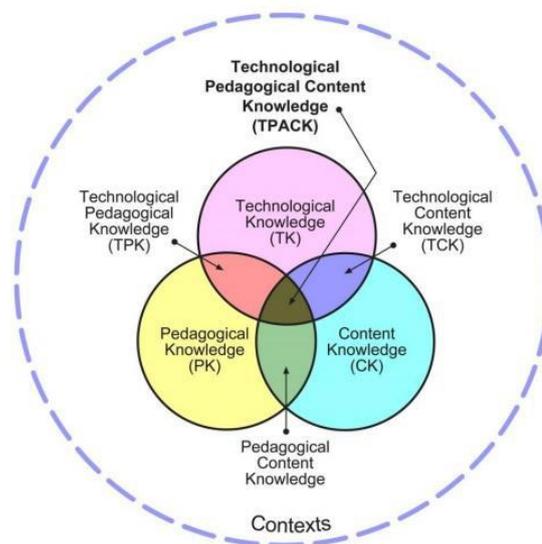


Figure 2. 2 Technological Pedagogical Content Knowledge Framework (Koehler & Mishra, 2009, p. 63)

This framework, TPACK, explains how teachers understand technology and pedagogical content knowledge, and how they interact with each other for effective

teaching with the use of technology. On the other hand, TPACK is not just an awareness of technology, pedagogy and content, but also an awareness of the connections, interactions, affordances and constraints between and among the three components (Mishra & Koehler, 2006). In this approach, technology integration in the classroom goes beyond isolated knowledge of specific hardware or software. In other words, technology integration “causes the representation of new concepts and requires developing sensitivity to the dynamic, transactional relationship between all three components” (Koehler & Mishra, 2005a, p. 134). Koehler et al. (2013) stated that effective teaching with technology does not mean to simply add a new piece of technology upon existing structures. Thus, for effective teaching with technology teachers’ knowledge requires a shift in existing pedagogical and content domains.

As can be seen in Figure 1.1, the interaction among the three components - content, pedagogy and technology- bring about seven different subsets: Technological Knowledge (TK), Pedagogical Knowledge (PK) and Content Knowledge (CK), Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK), Pedagogical Content Knowledge (PCK), and Technological Pedagogical Content Knowledge (TPACK). The relationships or interactions between these different subsets are also important in the TPACK framework.

Content Knowledge (CK), one of the three main areas, is knowledge about the subject matter that is to be taught or learned. Content knowledge is an important component of the TPACK framework because it focuses on discipline-specific modes of thinking unique to each field.

Pedagogical Knowledge (PK), another main area, refers to knowledge about the processes and practices or methods of teaching and learning and how it encompasses, among other things, overall educational purposes, values, and aims. Koehler et al. (2013) defined PK as

“... the set of skills that teachers must develop in order to manage and organize teaching and learning activities for intended learning outcomes. This knowledge involves, but is not limited to, an understanding of classroom management activities, the role of student motivation, lesson

planning, and assessment of learning. PK may also describe knowledge of different teaching methods, such as knowing how to organize activities in a way conducive to students' constructive building of knowledge. (p.3)

Technological Knowledge, another outer region of the Venn diagram, is defined as knowledge about standard technologies (books, chalk and blackboard, etc.), and more advanced technologies (the Internet and digital video, etc.) and the skills required to operate particular technologies. TK is related to knowledge of how to use computer software and hardware, presentation tools such as document presenters and projects, and other technologies used in educational contexts. Additionally, it focuses on the ability to adapt to and learn new technologies due to the rapid rate of change in technology (Mishra, Koehler & Kereluik, 2009). For instance, dynamic software has quickly become obsolete. Hence, because of the protean nature of technology, teachers should be open-minded to the technology (Koehler & Mishra, 2008). For instance, computers can be used in areas of research, communication, media consumption and in the creation of pedagogical tasks.

Pedagogical Content Knowledge (PCK), consistent with and similar to Shulman's PCK, is based on the assertion that effective teaching requires more than understanding content and pedagogy separately. It is knowledge of pedagogy that is applicable to the teaching of specific content. PCK emphasizes the core business of teaching, learning, curriculum, assessment and to enhance students' learning and the links among curriculum, assessment, and pedagogy. According to Koehler and Mishra (2009), it includes

“An awareness of common misconceptions and ways of looking at them, the importance of forging connections among different content-based ideas, students' prior knowledge, alternative teaching strategies, and the flexibility that comes from exploring alternative ways of looking at the same idea or problem are all essential for effective teaching” (p. 64).

Technological Pedagogical Knowledge (TPK) refers to the knowledge of the existence, components, and capabilities of various technologies used in teaching and

learning settings, and an understanding of how teaching and learning changes when particular technologies are used. Some examples of TPCK are expressed by Koehler et al., (2013) as follows:

Collaborative writing can take place with Google Docs or Google Hangouts instead of face-to-face meetings, extending collaborative activities over distances. Also, the advent of online learning and more recently, massively open online courses (MOOCs) require teachers to develop new pedagogical approaches that are appropriate for the tools at hand. (p.4)

Technological Content Knowledge is an understanding of the impact of technology on the practices and knowledge of a given discipline. For instance, students can learn about the relationship between geometric shapes by playing around with the properties of geometric concepts on portable devices.

Technological Pedagogical Content Knowledge (TPACK) is an understanding that emerges from an intersection of content, pedagogy and technology knowledge. For the teacher with TPACK, knowledge of technology, pedagogy, and content is synthesized and put to use for the design of learning experiences to enhance students' knowledge. TPACK represents

...an understanding of the representation of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; knowledge of students' prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge and to develop new epistemologies or strengthen old ones. (Koehler & Mishra, 2008, p. 17–18)

This framework proposes tackling all of the variables for creating effective teaching with technology. The TPACK framework functions as theoretical and a conceptual lens for researchers and educators to measure pre-service and in-service teachers' readiness to teach effectively with technology. Thus, researchers have developed a range of quantitative and qualitative instruments to measure TPACK (Schmidt, et al., 2009).

The description of TPACK provided by Mishra and Koehler (2006) is generic, not discipline or subject specific. In mathematics education, Grandgenett (2008) made a contribution to TPACK for in-service mathematics teachers. He proposes that mathematics teachers with strong TPACK have six main characteristics. One of the main characteristics of mathematics teachers with strong backgrounds in TPACK is that they would probably have a relative openness to experimentation with the ever-evolving technological tools available to them in the mathematics classroom (Grandgenett, 2008). In addition, in the second main characteristic stated that they would stay on task and focus on the mathematics concepts, while still taking advantage of the instructional opportunities offered by technology (Grandgenett, 2008). As indicated in third characteristic (Grandgenett, 2008), they approach their mathematics instruction with clear and systematic pedagogical strategies in mind by knowing where students are academically, what students need to know, and how the lesson should be taught. The fourth characteristic of mathematics teachers with strong backgrounds in TPACK is that they help students understand why a particular technology is useful for instructing a particular mathematics topic. Furthermore, they embrace the administrative capabilities of technology to help guide their mathematics instruction for teaching, assessment, and classroom management as indicated in fifth characteristic. The last characteristic demonstrates that these teachers are comfortable and optimistic about changes in technology (Grandgenett, 2008). These characteristics help to understand the dimensions of mathematics teachers' TPACK.

Grandgenett (2008) discussed the TPACK in mathematics as subject specific. In addition, Niess (2005) proposed a subject specific framework for describing the outcomes for TPACK development in a teacher preparation program. Niess (2005) adapted the four components of PCK developed by Grossman (1988, 1989) by taking into consideration the teacher's knowledge of incorporating technology into mathematics instruction. According to this model, mathematics teachers' TPACK related to the knowledge and beliefs demonstrate

- An overarching conception about the purposes for incorporating technology in teaching mathematics;
- Knowledge of students' understandings, thinking, and learning of mathematics with technology;
- Knowledge of curriculum and curricular materials that integrate technology in learning and teaching mathematics;
- Knowledge of instructional strategies and representations for teaching and learning mathematics with technologies. (Niess et. al., 2009, p.8)

These components of TPACK in mathematics is consistent with Mishra and Koehler's (2006) description of TPACK in general as

...an understanding of the representations of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; knowledge of students' prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge and to develop new epistemologies or strengthen old ones. (p. 1029)

With the perspective of four components of TPACK, mathematics teacher preparation programs should provide the means for the development of teachers' knowledge and thinking considering the development of an overarching conception of teaching with technology, pedagogy, and content (Niess, 2005). Besides, this framework guides professional practice that supports the improvement of mathematics teaching and learning. Additionally, in the light of this component, Niess, et al. (2009) framed the themes of the Mathematics Teacher TPACK Development Model.

Niess, Sadri, and Lee (2007) proposed a model to describe a development of TPACK for teaching mathematics. There are four major themes in the TPACK development model: curriculum and assessment, learning, teaching, and access. Curriculum and assessment theme is related to the treatment of the subject matter and assessing the students' understandings. Rather than being curriculum and assessment separately, they were grouped to highlight the connection between the curricular and assessment decision-making process. The learning theme emphasizes

the demonstration of conceptions of how students learn and the development of students' thinking skills. The teaching theme is related to instructional approaches, classroom environment and professional development. The access theme focuses on whether or not students are allowed to use technology, and how teachers address barriers to technology integration. Additionally, this theme is related to how technology makes more mathematics available for investigation of mathematics.

In addition to the themes of the TPACK Development Model, this developmental model of TPACK has a sequential process that progresses through the stages of recognizing, accepting, adapting, exploring, and advancing level. Mathematics teachers' progress through the TPACK Development Model as their level of TPACK develops. Teachers' level of TPACK develops as teachers, who begin with a developed PCK, move through the stages of recognizing, accepting, adapting, exploring, and advancing for each of the four major themes. As teachers progress along this developmental model, the intersection of the constructs of technology with pedagogy and content knowledge forms and expands (Niess et al., 2009).

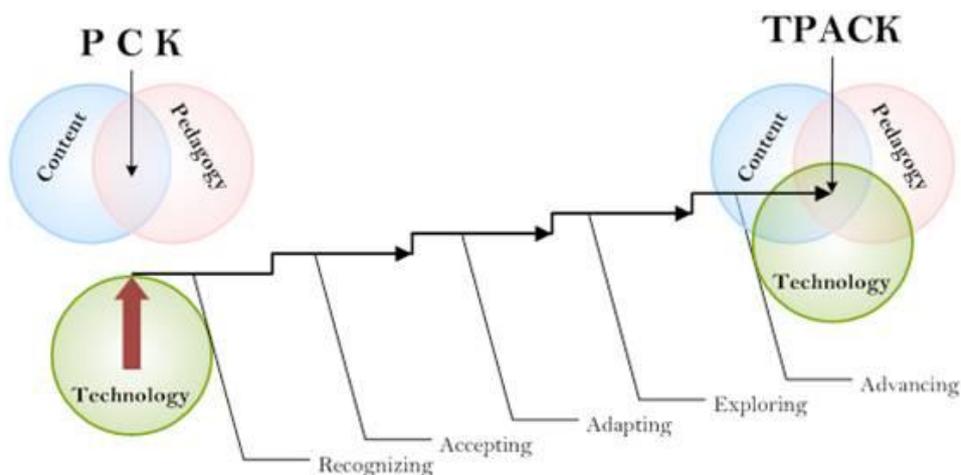


Figure 2. 3 The Teachers' TPACK Development Model (Niess, Ronau et al., 2009)

As illustrated in Figure 2. 3, the constructs of content, pedagogy, and technology intersect through progression of the stages of the TPACK developmental model.

In the *Recognizing* (knowledge) level, teachers are able to use the technology and recognize the alignment of its use in mathematics, but they cannot integrate the technology in teaching and learning of mathematics (Niess et al., 2009, p.9).

In the *Accepting* (persuasion) level, teachers form a favorable or unfavorable attitude toward teaching and learning mathematics with an appropriate technology (Niess et al., 2009, p.9). They have difficulties in finding ways to integrate the technology effectively in their mathematics curriculum.

In the *Adapting* (decision) level, they start to understand some advantages of using appropriate technologies as teaching tools. They can determine whether to adopt or reject the technology (Niess et al., 2009).

In the *Exploring* (implementation) level, teachers actively integrate teaching and learning of mathematics with an appropriate technology where they rephrase and design activities to align with their curriculum (Niess et al., 2009, p.9). Furthermore, they search different ways for teaching the content by using technologies as learning tools.

In the *Advancing* (confirmation) level, teachers evaluate the results of the decision to integrate appropriate technology into teaching and learning mathematics by making changes in the curriculum to take advantage of technology affordances (Niess et al., 2010, p.9).

The TPACK Development model for mathematics teachers was proposed to describe teachers' progression in developing this specialized knowledge (Niess et al., 2009). Therefore, in this study the TPACK Development Model (Niess et al., 2009) has been used because teachers' progress in developing TPACK in mathematics is described in detail. A description of TPACK provided by Mishra and Koehler (2006) is general, and the components of TPACK in Mishra and Koehler's (2006) framework are not well-defined. Furthermore, the definition for TPACK made by Mishra and Koehler (2006) is more generic (not discipline

specific), whereas the TPACK Development Model includes the teacher's knowledge of incorporating technology into teaching mathematics. Hence, this model is used in this study to identify teachers' TPACK level.

2.3 Geometry and TPACK

This study investigates the TPACK Development of the in-service mathematics teachers participating in mathematics coaching in terms of geometry. Thus, in this section the relationship between geometry and TPACK is illustrated. Geometry is a field that started with people's observations in an environment and then became an intellectual and a more abstract field (Laborde, Kynigos, Hollebrands and Strasser, 2006). According to NCTM (2000),

Geometry is a natural area of mathematics for the development of students' reasoning and justification skills that build across the grades. As the study of the relationships among shapes and their properties becomes more abstract, students should come to understand the role of definitions and theorems and be able to construct their own proofs. (p.41)

While dealing with relationships between objects in geometry and the practice of reasoning, students think more critically in everyday situations (Lukac, 2010). Moreover, the use of technology provides an environment in which students can explore geometric relationships and make and test conjectures. Jones (2002) stated that interacting with technology such as the dynamic geometry environment can help students to explore, conjecture, construct and explain geometrical relationships. In a dynamic geometry environment, students can create on the screen geometrical objects which can be manipulated and dragged. When an element of geometrical objects is dragged by means of the mouse, this object is modified while all the geometric relations used in its construction are preserved (Laborde et al., 2006). In other words, it isn't modified according to the wishes of the user; it is modified according to the geometry of its constructions. In this environment, the user can easily understand constructs and relations of geometric objects. To conclude,

technology environments facilitate students' conceptual understanding. Teaching geometry in computer environments, which are ideal media, facilitates the visualization of geometrical concepts (Clements, Sarama, & Battista, 1998). Consequently, students should be provided with an opportunity to use such technology in geometry education.

Considering the importance of technology in geometry education, in-service teachers should adapt their teaching to take advantage of technology in geometry. Coffland and Strickland (2004) examined teachers' use of technology in geometry classrooms. This study revealed that professional development on technology integration could increase the use of technology in teaching geometry. In addition, in a study by Roberts and Stephens (1999) it was indicated that technology had taken over the role of an instructor, possibly because the teacher did not know how to utilize technology more effectively for the teaching and learning of geometry. Therefore, their research implied for a need for TPACK development. In addition, studies on technology integration in teaching geometry demonstrated that teachers who incorporated technology into their geometry lesson encounter many challenges (Coffland & Strickland, 2004; Roberts & Stephens, 1999). To overcome teachers' challenge in teaching geometry by utilizing technology, teachers' need professional development to improve TPACK development in geometry. In this study, the researcher aims to investigate the teachers' TPACK development in geometry by means of mathematics coaching.

2.4 Research studies on TPACK

The available research regarding TPACK and the development of TPACK is limited related to pre-service teachers and in-service teachers (Ozgun-Koca, Meagher, Edwards, 2009,2010; Niess, 2005). However, few researches are available to give insight into teachers and their TPACK. Some research studies investigated the (in-service/pre-service) teachers' TPACK in mathematics, others focused on (in-service/pre-service) teachers' development in TPACK in mathematics. Examples of

these TPACK studies have been expressed in below. Firstly, the researcher studies on teachers' TPACK have been demonstrated, and then the researcher studies on teachers' TPACK development through professional development have been represented.

Stoilescu (2011) investigated the TPACK for in-service mathematics secondary teachers. In order to describe the process of integrating technology, the researcher used the four components proposed by Niess (200): 1) An overarching conception of what it means to teach mathematics that integrates technology, 2) Knowledge of instructional strategies and representation for teaching mathematics with technology, 3) Knowledge of students' understanding, thinking, and learning with technology in mathematics, and 4) Knowledge of curriculum and curriculum materials that integrate technology in mathematics (p. 511). Participants selected for this study were three secondary mathematics teachers from an urban public school, who had over 10 years of experience in teaching mathematics. The in-service teachers were observed in their classrooms and interviewed about their experiences of teaching mathematics and integrating technology, such as Graphic Calculator, Mathematical Software, SMART Board, and manipulatives. This research study demonstrated that teachers had specific approaches to integrate computer technology in order to a) provide students with opportunities to learn and experiment with their mathematical knowledge; b) help them transfer the content to the students in the process of teaching mathematics; and c) assess students' work, and give them feedback. They had difficulties in integrating new technologies as these required time, preparation, and dedication. Furthermore, they had some difficulties in motivating the students to use computers in a significant way to learn mathematics conceptually. According to Stoilescu (2011), in-service teachers should have opportunities to update their computer and pedagogical skills, a long term perspective in integrating technology in mathematics education, and professional and technical support.

In another research study, Taylor (2011) used the TPACK Development Model to assess technology-related practices of in-service mathematics teachers. This qualitative research study explored the beliefs and practices as regards the integration of instructional technologies, such as graphing calculators, non-graphing calculators, computer software, and the TI-Navigator System. The researcher conducted an initial interview, a classroom observation, and a follow-up interview with seven secondary mathematics teachers. In addition, the teachers submitted sample lessons and completed a TPACK Development Model Self-Report Survey. In this research, the findings demonstrated that the in-service teachers were largely unable to differentiate between instructional technologies and non-instructional technologies. For instance, they erroneously reported presentation tools, such as LCD projectors, as instructional technology. According to the result of this study, the in-service teachers considered themselves to have much higher TPACK levels. Another important result of this study was that the lack of pedagogical content knowledge (PCK) in in-service teachers resulted in low TPACK, which indicated that their teaching practices limited technology integration. In addition, most of the participants did not integrate technology as a tool for learning mathematics. Instead, many participants felt that technology posed a threat to the learning process.

Similar to the study of Taylor (2011), McBroom (2012) utilized the TPACK Development Model to describe in-service teachers' technology integration and to identify their TPACK development levels. The researcher investigated TPACK and their integration of dynamic geometry in the classroom instruction. This study focused on how four secondary mathematics teachers developed and used their knowledge in teaching geometry with technology. The sources of data were an initial interview, observations, documents, a closing interview, a survey, implementation questionnaires, professional development attendance records and the researcher's log. The data were analyzed according to the TPACK Development Model. This study indicated that the in-service mathematics teachers with the least teaching experience demonstrated the highest levels of TPACK. Furthermore, the teachers with the most teaching experience with dynamic geometry showed the

widest range from the recognizing level to exploring level in the TPACK Development Model. The in-service teachers claimed the curriculum and standardized testing to be the main barriers to increase technology use in their classroom. Another result of this study was that an ongoing professional development and easy access to computers did not result in frequent and effective incorporation of dynamic geometry in teaching and learning. Thus, the researcher proposed a professional development model designed for teachers interested in integrating dynamic geometry in the classroom instruction. The researcher's suggestion was that participants develop their TPACK through attending professional development workshops and implementing what they learned in the classroom instruction.

In addition to the research studies on in-service mathematics teachers' TPACK, in the literature there are also research studies on pre-service mathematics teachers' TPACK. Balgalmış (2013) investigated the pre-service teachers' TPACK with the contribution of field experience and the reflection-on-practice process. Three pre-service teachers participated in this study and they designed and implemented three technology-enhanced lessons via Geogebra. In order to analyze the data, the researcher used the Lyublinska and Tournaki (2011) rubric based on the five level of TPACK Development Model. According to the findings, pre-service teachers viewed the educational technology as a tool that assisted them to enhance students' learning and improve their motivation. Furthermore, the pre-service teachers' TPACK level progressed throughout the study. In addition, the researcher indicated that the teaching experience and reflection-on-action is necessary to improve pre-service teachers' TPACK. Like Balgalmış (2013), Meagher, Ozgun-Koca and Edwards (2011) conducted a research to examine pre-service teachers' TPACK. They analyzed pre-service teachers' TPACK evolving relationships with advanced digital technologies in their teaching, examined through the lens of their TPACK (Koehler & Mishra, 2005; Niess 2005, 2006, 2007). The participants of the study were 22 pre-service teachers enrolled in a mathematics teaching methods course. The method course included two separate field

experiences in school. In addition, the methods focused on the use of advanced digital technologies in the teaching and learning of mathematics - especially the use of the TI-Nspire but also SmartBoard, websites, and Geometer's Sketchpad. In this study, if technology use was required of pre-service teachers, they tended to use technology; when technology use was optional, they preferred not to use the technology. As a result of this study, it was found that they needed the experience of exemplary practice to convince them of the benefits of incorporating technology in their own teaching. Like Balgalmış (2013), Meagher, Ozgun-Koca and Edwards (2011) also suggested teaching practice to improve the TPACK.

These research studies demonstrated that developing knowledge of TPACK is an important aspect in teacher training because pre-service and in-service teachers need to have opportunities to develop their experience and knowledge of how to integrate technology. For instance, Niess (2006) emphasized the importance of TPACK development and he stated that "TPACK is an important body of knowledge for teaching mathematics, for the importance of integrating its development within the coursework in teaching and learning, as well as within the coursework directed at developing knowledge of mathematics" (p.198). Niess suggested that all in-service and pre-service teachers must obtain knowledge and experience for technology integration in teaching and learning mathematics.

Niess (2005) conducted a qualitative study that examined the TPACK of pre-service teachers in a science and mathematics teacher preparation program that integrated teaching and learning with technology throughout the program. Although 22 pre-service teachers had enrolled in the program, there were five case studies being conducted in documenting and describing successes and difficulties of each pre-service teacher as their TPACK progressed. In this study, the four components of TPACK adapted from Grossman's (1988, 1989) pedagogical content knowledge definition were used to analyze the pre-service teachers' TPACK development. The teacher preparation program for this study was designed as a 1-year program which focused on equipping science and mathematics teachers with the skills to integrate technology. This 1-year program included technology courses, microteaching, and

pedagogy courses. In the technology course, problem-based science and mathematics activities were used to guide the pre-service teachers in learning about (a) various technologies, (b) pedagogical considerations with these technologies, and (c) teaching and learning with these technologies (Niess, 2005, p.513). In the microteaching courses, pre-service teachers gained teaching experience in four specific instructional methods: demonstrations, hands-on practices/ laboratories, and inductive versus deductive modes. Then, the remaining 6 months of the program – the pedagogy course- provided extended practical experiences that required the student teachers to plan, teach, and reflect on teaching hands-on lessons with technology. At the end of the courses, it was observed that 14 of the 22 students had made a significant improvement in their TPACK for using technologies to engage students in learning science and mathematics. The remaining eight students still needed more work toward TPACK. Niess (2005) described five case studies to reveal the differences in their development of TPACK. These five cases were coded as Denise, Marissa, Terry, Karen, Dianne.

In the first component of TPACK, which was “an overarching conception of teaching science/mathematics with technology”, pre-service teachers revealed important considerations regarding the interaction of the content of science/mathematics and the content of the specific technology. Terry extended his lessons to have his students investigate the concepts and their relationship, whereas Karen resisted using class time to explore concepts in the design of the technology. Additionally, thinking of the technology as a tool to do science rather than a tool embodying science, Denise simply rejected the consideration of the science of the technology. Only some of the pre-service teachers realized the interplay of technology and mathematics/science despite the emphasis laid on it throughout the program. Niess (2005) stated that teacher preparation programs need to guide pre-service teachers in enhancing their understandings of the interactions of the knowledge of technology and the knowledge of their subject area.

In light of the second component of TPACK, the “Instructional strategies and representations for teaching with technologies”, many pre-service teachers had

had limited experience in instructional strategies and representations of their subject within a technology framework at the beginning of study. The program provided an environment to focus on instructional strategies and representations of the content with technology. Thus, the pre-services teachers considered instructional strategies that incorporated technology.

The third component of TPACK was “Students’ understandings, thinking, and learning in a subject with technology”. In the beginning, pre-service teachers were naturally focused on their own teaching and less likely to think about their students’ understandings, thinking, and learning. With the help of the pedagogy courses, which emphasized how students’ interpreted the concepts when technology was part of the instruction, they considered how the students understood the concepts in technology-enhanced instructional activities. According to Niess (2005), the pre-service teachers emphasized students’ understandings in their lesson because their concern for their own actions as a teacher had shifted to a concern for what the students were learning as a result of the teaching.

In the light of the last and fourth component of TPACK, which was related to “Curriculum and curricular materials”, the pre-service teachers were challenged to consider the curriculum more broadly at the beginning. They had been experiencing difficulties in considering how technology supported the national standards. At the end of this study, Niess (2005) recommended the teacher educators and researchers to ask questions about the component of TPACK, such as: “What program models support teachers in gaining the skills, knowledge, and beliefs that support teaching different subjects with technology?”, “How does TPACK change for different content areas? What experiences are essential in building a TPACK?” and “What Technologies are important?” (p. 522)

Other research has been conducted on pre-service teachers and TPACK. Cavin (2007) conducted a study that examined the development of TPACK in pre-service teachers as they participated in a study on a microteaching lesson. Participants were six pre-service teachers enrolled in the microteaching lesson study and they worked in small groups to refine lessons that incorporated technology

through repetitive cycles of teaching, reflecting, and modifying a group lesson. Instructional technologies included graphing calculators, Excel, Geometer's Sketchpad, TI-Interactive, and various online tools in this study. In the process of the study on the microteaching lesson, pre-service teachers taught the lesson to discuss and make adjustments. As a result of this study, this teacher preparation program with technology integration provided the teachers with an opportunity to develop their TPACK. They developed an awareness of the importance of teaching with technology in a student-centered learning environment instead of traditional teaching methods, such as sequencing, pacing and written directions. Furthermore, pre-service teachers also improved their views on mathematical knowledge. At the beginning of the study, they integrated technology at a procedural level to do the math faster. After the microteaching lesson study, they developed a more conceptual view of technology enhanced mathematics.

In another research on lesson study, Lee and Hollebrands (2008) prepared examples of materials focused on statistics and probability for a teacher education program by means of a project called Preparing to Teach Mathematics with Technology (PTMT). The PTMT project was developed using an approach that integrally develops teachers' understandings of content, technology, and pedagogy to prepare them to teach data analysis and probability topics using specific technology tools, based on the components of TPACK (Koehler & Mishra, 2005; Niess, 2005) and recommendations from the Association of Teachers of Mathematics Educators (AMTE, 2009). In this study, pre-service teachers participated as learners in a mathematics technology task, they reflected on how students might think, and then they watched a video case designed to highlight the student thinking through the same task. Then, they analyzed student thinking in group discussions. The researchers indicated that while analyzing the task, the pre-service teachers engaged in reasoning, which was at the intersection of technology, pedagogy, and content. In PTMT, pre-service teachers focused on and predicted student thinking in a technology-enhanced task, observed and reflected on a

technology-enhanced lesson, and discussed with their peers, which all facilitated the pre-service teachers' TPACK reasoning.

Another research study on TPACK development through lesson study was conducted in Turkey. Kurt (2016) focused on pre-service teachers' TPACK development in statistics teaching with virtual manipulatives. Within the scope of a micro teaching lesson study, two groups of pre-service teachers prepared one lesson plan including a virtual manipulative for the instruction of a specific statistical concept. Data were collected from three consecutive group discussions for each group through microteaching lesson study. As a result of the study, it was found that pre-service mathematics teachers' TPACK progressed through microteaching lesson study. They were observed to have significant developments regarding TPACK in statistical content knowledge, statistical pedagogical knowledge and technological content knowledge.

Like Niess (2005) and Cavin (2007), Mudzimiri (2012) conducted a study to develop the pre-service teachers' TPACK. In this study, the pre-service teachers participated in three courses that were offered in collaboration: (1) a mathematics teaching methods course, (2) a technology-intensive content-rich mathematical modeling course, and (3) a practicum course. These courses aimed to study the development of the connections between technology, content and pedagogy. In this multiple case study, five pre-service teachers were analyzed in terms of the changes in their TPACK during a period of about 15 weeks. The pre-service teachers designed technology-enhanced lessons and implemented them in the classroom. The data sources were a TPACK survey, teaching philosophy statements, lesson plans, student teaching episodes, and weekly instructor meeting notes. This study indicated that the development of pre-service teachers' mathematics TPACK is complex, and there are a number of factors that are at play, such as the pre-service teachers' prior experiences with technology, their mathematical backgrounds and their beliefs about the use of technology in mathematics instruction. Furthermore, in this study the available model for mathematics teachers' TPACK was developed based on the findings obtained from the observations of in-service mathematics teachers,

although assessing the development of TPACK in pre-service teachers is a complicated process.

In addition to research studies on pre-service mathematics teachers' TPACK development, few researchers focused on in-service mathematics teachers' TPACK development. Riales (2011) examined the technological, pedagogical, and content knowledge (TPACK) of a group of in-service secondary mathematics teachers as they participated in a technology-based lesson study. This lesson study emphasized the use of TI-84 graphing calculators and the TI-Navigator system. The researcher conducted qualitative research study using a layered case study. In this study, not only interactions of the group members during lesson study meetings but also individual case studies of four of the six participants were analyzed. The data sources were initial surveys, initial and post-interviews, initial and post-classroom observations, writing prompts, and transcriptions of lesson study group meetings. The theoretical framework of the study was the TPACK Development Model (Niess, 2009), like the studies of Taylor (2011), and Riales (2011). The data were analyzed to determine the TPACK development levels for different themes during the lesson study. The result of this study demonstrated that the design and purpose of a technology-based lesson study helped the in-service teachers to practice actions from the higher levels of the TPACK Development Model during the lesson study. This study revealed that the in-service mathematics teachers who had less experience with technology in their educational backgrounds demonstrated greater positive changes in the TPACK Development Model.

Based on the literature, teachers' TPACK as an area of study is quite new. Therefore, it has not been explored intensively. The studies in the literature demonstrated that more research is needed to analyze in-service and pre-service teachers' TPACK and their TPACK development in professional development. In addition, the literature revealed that most of the TPACK studies were conducted with pre-service teachers, as compared to in-service teachers. Thus, there is a need for more research studies on in-service teachers' TPACK. As indicated in the

literature, the in-service teachers' TPACK is at lower level to integrate technology in their mathematics classroom. To improve teachers' TPACK, teachers need professional development about how to use technology in their classroom. From this perspective, teacher professional development is important for improvement of the in-service teachers' TPACK. Furthermore, most of the studies focused on graphing calculator as an instructional technology. This indicated that there is a need to conduct more studies on TPACK via different instructional technologies, such as Geogebra, Cabri3D, and Tinkerplots. Besides, the studies on TPACK did not focus on specific subjects. Therefore, the aims of this research study have been to investigate in-service mathematics teachers' TPACK Development regarding geometry through mathematics coaching.

2.5 Mathematics Coaching as a Professional Development Strategy

Teacher professional development is an indispensable mechanism for enhancing teachers' understanding, content knowledge and improving their teaching practices (Bybee and Loucks-Horsley 2000; Cohen & Hill, 2001; Garet, Porter, Desimone, Birman, & Yoon, 2001). Professional development is aimed at helping teachers in acquiring, refining, and transferring knowledge and skills into practice. However, traditional approaches to professional development are insufficient to help teachers learn what they need to know (Knight, 2007). Traditionally, teacher professional development occurs in workshops that were separated and disconnected from the classroom and curriculum and were fragmented occurrences (Ball & Cohen, 1999). Professional development should not be seen as additive, which means adding more things for teachers to do and learn, but instead as transformative (Loucks-Horsley, et al., 2003). On the other hand, some studies have shown that effective professional development programs influence teacher practice (Coleman & Goldenberg, 2010; Huston & Weaver, 2007; Joyce & Showers, 1982; Knight, 2010). For effective professional development, Loucks-Horsley, Love, Stiles, Mundry and Hewson (2003) state that it needs to be driven by a well-defined

image of effective classroom learning and teaching, needs to provide opportunities for teachers to build their content and pedagogical content knowledge and knowledge about practice, needs to be research-based and engage teachers as adult learners, should use practices that will also be used with students, should be based on student learning data, and should provide opportunities for collaboration with colleagues. Moreover, time for reflection on both past and current actions is an important property of this effective professional development. According to Driscoll (2008), effective professional development especially for mathematics teachers should (1) improve teachers' content knowledge, (2) actuate research-based instructional strategies in classrooms, (3) build teachers' capacity to use a variety of assessments to monitor student understanding and achievement, (4) engage teachers in taking an inquiry approach to teaching whereby they come to understand; question; and where appropriate, shift their attitudes and beliefs. These demonstrates what professional development focus on to improve teachers' knowledge.

According to Loucks-Horsley, et al. (2003), there are 16 different professional development strategies that cluster around six categories: “immersion in content, standards, and research; examining teaching and learning; aligning and implementing curriculum, professional development structure” (p.28). One of an alternative approach to professional development, coaching, is a model that is a reproduction of the corporate coaches who provide support and guidance as their trainees incorporate their suggestions and practice the new strategies or approaches (Joyce and Showers, 1982). Showers and Joyce (1996) first purposed the idea of coaching because research showed that typical professional development strategies were not effective to lead teachers to change or to implement new ideas. According to Loucks-Horsley et al. (2003), coaching is a professional development strategy that provides one-on-one learning opportunities for teachers to focus on improving mathematics and science teaching by reflecting on one's own and or another's practice to enhance the knowledge, learning, and practice of the teacher to incorporate this new learning into classroom practice. Coaching is described as a job-embedded form of professional development with the purpose of improving

professional practice and the purpose of improving student learning (Brandt, 1987, Knight, 2007, Loucks-Horsley, et al., 2003, Showers, 1985). The coaching model attaches importance to the content and methods introduced to the teachers, and the coach provides support to the teachers when teachers try new strategies in their own classrooms.

Coaching, which is a school-based professional development, has different forms with different purposes and different techniques, such as peer coaching, cognitive coaching, content coaching and technical coaching. Knight (2009) indicates that these different coaching strategies have common features including the fact that coaching is job-embedded, requires an equal partnership between the coach and the teacher, is a long lasting, intensive relationship, engages in dialogue and reflection, is confidential and non-evaluative, and requires thoughtful communication. Additionally, despite the numerous types of coaching and responsibilities of a coach, all incorporate a traditional coaching model that pays attention to classroom observation and use a preconference-observation-postconference cycle (Loucks-Horsley, et al., 2003)

Content-focused coaching, one of the strategies of coaching focuses on helping teachers improve instruction in a particular academic discipline, such as literacy and mathematics (Neufeld & Roper, 2003). West and Staub (2003) refer to the idea of content-focused coaching as a mathematics teacher professional development tool focusing on the mathematics content and pedagogy. In this type of mathematics coaching session, the teacher and coach work together to implement and analyze mathematical strategies and tasks intended to increase student learning and achievement. Mathematics Coaching sessions which are grounded in an actual lesson allow content-focused coaches to meet with individual teachers or small groups of teachers to plan for, teach, and debrief a lesson (West & Staub, 2003). According to content-focused coaching, there are three phases in a coaching session: pre-conferencing, the lesson itself (observation), and post-conferencing. Pre-conferencing enables the coach to better recognize the teacher's planning habits, content knowledge, and pedagogical knowledge about best practices in mathematics

instruction and students' learning experiences (West & Staub, 2003). During this time, the coach meets with the teacher to assist the teacher in identifying lesson objectives, specifying the important mathematical concepts in the lesson, and identifying appropriate instructional strategies. Furthermore, the teacher and coach may discuss potential misconceptions of students and how they will be overcome during the lesson.

With respect to the teaching of the lesson, the role of the coach can be differentiated prior to the lesson: the coach may teach the lesson, the coach and teacher may coteach, or the teacher can teach (West, 2009). The coach may teach the entire lesson together with the teacher to help the teacher to observe a specific instructional practice or the talk and actions of students. The teacher and coach may co-teach the lesson either as a team, or the teacher may teach the entire lesson while the coach observes the teacher. During the lesson, the coach has to have the knowledge and experience to highlight a moment that the teacher and coach can unpack during the post-conference dialog (West, 2009).

In the post-conferencing session, the teacher is asked to reflect upon the success of the lesson, while identifying areas that need to be revisited (West, 2009). The coach and teacher engage in an instrumental discussion in determining the following steps for instruction, including the interventions around and extensions of the content material (West & Staub, 2003). The most important goal of the post-conference is to consider the focus of the next lesson in light of their reflections, particularly their examination of student work and comments.

Providing teachers with opportunities to work with coaches in planning, implementing and debriefing increases levels of mathematics content knowledge and pedagogy. In mathematics coaching, coaches work directly with classroom teachers to improve student learning of mathematics (Hull, Balka and Miles, 2009). Mathematics coaches are responsible for working with teachers to improve mathematics achievement, managing and controlling curriculum and instructional materials, managing and regulating professional development, monitoring program implementation, building the mathematics program by using its strengths and

reducing its weaknesses, maintaining and sharing best-practice research, building collaborative teams and networks, and gathering, analyzing, and interpreting data, such as from assessments and benchmark tests, to inform instruction.

This study was intended to contribute to mathematics education research by studying the progress of elementary mathematics teachers' TPACK in geometry through participation in mathematics coaching.

2.6 Research studies on Mathematics Coaching

Most of the research studies on coaching are related to creating and sustaining a coaching program. But there is a limited number of studies on mathematics coaching. Research on mathematics coaching is just beginning to be regarded as an area of study. Although their research is not related to mathematics coaching in particular, Knight (2007), Showers and Joyce (1996), and Black, Molseed, and Sayler (2003) indicated that participation in coaching programs helps the teacher to change in their practice. Knight's (2007) indicated that teachers who work with coaches are four times more likely to implement new practices as compared to teachers who do not work with coaches. Supportively, Showers and Joyce's (1996) studies indicated that the level of implementing new practices rose amongst teachers using coaching. Furthermore, a research study by Black, Molseed and Sayler (2003) demonstrated that teachers participating in coaching had more profound knowledge about how they might improve their own practice.

In the literature, there are few studies on mathematics coaching indicating that coaching can have a positive impact on teachers' instructional practice (Becker, 2001; McGatha, 2008; Campbell and Malkus, 2010). Becker (2001) investigated the efficacy of coaching in improving instruction in elementary mathematics classrooms. The researcher identified three different styles in coaching: the coach as a collaborator, the coach as a model, and the coach as a leader. Although there were differences in coaching, all the coachings aimed to develop teachers' practice and improve mathematics instruction through cooperative lesson planning, modeling of

instruction, or guiding the teacher to deeper reflection. The researcher indicated that teachers focused more on conceptual understanding, and problem-solving over skills development.

McGatha (2008) examined two coaches to establish effective coaching relationships. The researcher collected data during a seven-month period and included: (a) the coaches' reflective journals; (b) pre- and post-surveys from each coach and teacher; (c) audio-recorded post-interviews with each teacher; and (d) audio-recordings of meetings with the coaches, the teachers, and the researcher. McGatha observed positive changes in teachers' instructional practice. And he indicated that there is a lack of research on coaches' interactions with teachers in coaching situations and how they can support teacher professional growth (McGatha, 2008). In another research study, Campbell and Malkus (2010) revealed that trained elementary mathematics coaches had a positive impact on teachers' beliefs and involvement in professional development activities.

In addition to the research studies which analyzed the effectiveness of mathematics coaching, Campbell and Malkus (2011) investigated the impact of elementary mathematics coaches on students' achievement. In this study, the researcher worked with two cohorts of coaches who were participating in a funded teacher-enhancement effort addressing the development and refinement of mathematics content, pedagogy, and leadership courses for the coaches. In this 3-year randomized control study, the researcher found that over time coaches positively affected student achievement in grades 3, 4, and 5. In addition, Valente (2013) investigated the difference between students' achievement levels before and after a mathematics coaching. The results of the study indicated a significant difference in students' mathematics scores prior to and after coaching.

Nicometi (2011) examined teacher perceptions regarding the impact of mathematics coaching. Data were collected from 45 mathematics teachers of Grades 3, 4, and 5. The researcher conducted the survey consisting of two parts. The first part was composed of 41 questions based on a 5-point Likert-type scale. The second part consisted of three open-ended questions. These results of the study indicated

that teachers were highly satisfied with the mathematics coaches in terms of their performance, collaboration, environment, and attitude. Furthermore, the teachers commented that their coach had taught them to be reflective thinkers through observation and feedback.

Larsen (2012) investigated elementary mathematics coaching from the point of view of two coaches, five elementary classroom teachers, and two principals. Qualitative data were collected through observations of both teachers and coaches and interviews with the coach, teacher, and principal. According to Larsen (2012), this study had six major findings. Teacher, coaches, and principals expressed that participating in mathematics coaching brought about change in the teachers' classroom practices. Furthermore, they indicated that trusting and collaborative relationship between teachers and coaches is one of the most important properties of coaching. According to teacher, coaches, and principals, time is the major barrier to in the coaching program. The researcher suggested that coaching programs that include an emphasis on collaboration through reflective discussion are likely to bring about changes in teacher practice. Thus, the school should motivate the teachers to attend the mathematics coaching. Another research study also investigated the perception of teachers regarding mathematics coaching. McLaughlin (2012) examined teachers' views on the coaching model which focused on the teachers' mathematics subject matter knowledge, pedagogical content knowledge, and self-efficacy. As a result of this study, the teachers and the coach believed that mathematics coaching is effective in several aspects, including unpacking new standards, collaboratively planning, incorporating new instructional strategies and manipulatives, collecting, reporting, and analyzing multiple sources of student data, and developing interventions for struggling students.

Like McLaughlin (2012), Drust (2013) also analyzed the elementary mathematics teachers' perceptions by focusing on the effectiveness of the coaching program in order to justify its cost. The participants in the study consisted of 12 certified teachers who were involved in the year-long mathematics coaching. The data were collected from questionnaires, interviews, and archival data. The findings

of the study revealed five major themes reflecting teachers' perceptions about what contributes to a successful program: creating instructional change, providing support, building confidence, navigating challenges, and increasing student achievement. Furthermore, the teachers thought that the coach's feedback is supportive and positively impactful of their instructional practices. These studies demonstrated that mathematics teachers believed that they made progress in their performance through mathematics coaching as a professional development (Drust, 2013; Larsen, 2012; Mclaughin, 2012; Nicometi, 2011).

The studies on mathematics coaching demonstrated that teachers participated in this kind of a professional development strategy made changes in their instruction and belief. Although coaching has become an increasingly utilized professional development strategy, little research has been conducted on the nature of mathematics coaching and its effects. Most of the studies focused on teachers' perspective to mathematics coaching. McGatha (2009) stated that "the empirical and anecdotal evidence suggest that coaching is a promising professional development that can lead to improved teaching and learning. However, we need to continue to pursue research that can support these initial findings." (p.2) In addition, McDougall (1997) emphasized the importance of having a mentor or a coach as part of professional development to enhance teachers' technical and mathematical expertise. Some TPACK studies on mathematics pre-service and in-service teachers have integrated components of lesson study as a professional development (Cavin, 2007; Riales, 2011; Kurt, 2016). Although there is little research on developing teachers' TPACK through lesson study, in the literature there are not many research studies analyzing teachers' development of TPACK through mathematics coaching. Therefore, this study is important to bridge the gap in the literature.

2.7 The Summary of Literature Review

The literature demonstrates that teachers did not effectively integrate technology in their mathematics classroom. Specifically, in-service teachers should

be provided an environment to improve their TPACK. In other words, they need experience how to integrate technology in mathematics education (McBroom, 2012; Stoilescu, 2011; Taylor, 2011). As considering the importance teachers' TPACK, more research is needed to improve teachers' technology integration. This research study meets that need of research about the in-service mathematics teachers' TPACK Development.

In addition, professional development is an important to enhance teachers' TPACK. At this point, the literature indicated that mathematics coaching is an effective professional development (Knight, 2007; Showers & Joyce, 1996; Black, Molseed, & Sayler, 2003). Although the effectiveness of coaching in the development teachers knowledge and practice, in the literature there is a few reseach study on mathematics coaching (Drust, 2013; Larsen, 2012; Mclaughin, 2012; Nicometi, 2011). Thus, there is a need analyzing the teachers' knowledge in the mathematics coaching. From this point of view, this study has the potential to provide information on in-service mathematics teachers' TPACK development in mathematics coaching regarding geometry.

CHAPTER III

METHODOLOGY

This chapter, which presents the method of the study, includes five sections, namely The Design of the Study, Sampling and Selection of the Participants, Implementation, Data Collection, Data Analysis, and Trustworthiness.

The purpose of this study was to examine the changes that occurred in elementary mathematics teachers' Technological Pedagogical Content Knowledge (TPACK) in geometry during the coaching session, and how mathematics teachers progressed through the stages of the TPACK development model before, during and after mathematics coaching. The perceptions of elementary mathematics about mathematics coaching in terms of technology, pedagogy, and content was one of the concerns of this study.

To this end, this qualitative study sought to answer the following research questions:

1. To what extent does participating in mathematics coaching impact elementary mathematics teachers' technological pedagogical content knowledge in the field of geometry?
 - 1.1. How do elementary mathematics teachers participate in mathematics coaching progress through the levels of Curriculum and Assessment theme in the TPACK Development Model?
 - 1.2. How do elementary mathematics teachers participate in mathematics coaching progress through the levels of Learning theme in the TPACK Development Model?
 - 1.3. How do elementary mathematics teachers participate in mathematics coaching progress through the levels of Teaching theme in the TPACK Development Model?

- 1.4. How do elementary mathematics teachers participate in mathematics coaching progress through the levels of Access theme in the TPACK Development Model?
2. What are elementary mathematics teachers' perceptions of mathematics coaching in terms of its role in developing technological pedagogical content knowledge?

3.1 The Design of the Study

The present study aimed to capture the a big and in-depth picture of the development of the technological pedagogical content knowledge of elementary mathematics teachers and their level of technological pedagogical content knowledge regarding geometry before, during and after mathematics coaching. In line with the nature and the purpose of this research, the present study used the method of qualitative research to reveal the development that occurred in elementary mathematics teachers' TPACK in the field of geometry. Qualitative research is defined by Creswell (1998) as an inquiry process of understanding based on distinct methodological traditions of inquiry exploring a social or human problem. Denzin and Lincoln (2005) define qualitative research as follows:

Qualitative research is a situated activity that locates the observer in the world. It consists of a set of interpretive, material practices that make the world visible. These practices transform the world. They turn the world into series of representations, including field notes, interviews, conversations, photographs, recordings and memos to the self. At this level, qualitative research involves an interpretive, naturalistic approach to the world. This means that qualitative research study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of the meanings people bring to them. (p.3)

The qualitative research design provides an opportunity for the researcher to observe the type of processes and strategies engaged in by participants, to understand what is happening in these processes, and to observe participants in their natural setting without a treatment (Fraenkel & Wallen, 2005; Merriam, 1988; Patton, 1990).

Creswell (2007) stated five approaches under the umbrella of qualitative research: narrative research, phenomenological research, grounded theory research, ethnographic research, and case study research. In this study, the case study was used as the qualitative research methodology. In this research methodology, the bounded system, or a case, is explored in detail through multiple, rich, and in-depth data collection and reports.

Yin (2003) defined the case study as “an empirical inquiry that investigates a contemporary phenomenon within its real life context, especially when the boundaries between the phenomenon and the context are not clearly evident” (p.13). Furthermore, he stated that

...[the case study research methodology] copes with the technically distinctive situation in which there will be many more variables of interest than data points, and as one result relies on multiple sources of evidence, with data needing to converge in a triangulating fashion, and as another result benefits from the prior development of theoretical propositions to guide data collection and analysis (p. 13-14).

From the definitions, it can be concluded that the most important characteristic of the case study is the object of the study or case. Additionally, the object of the study is that it is specific, unique and is a bounded system where some components of the study are within the case (Yin, 2003). Researchers should define carefully the case and its context because the case is a bounded system. Merriam (1998) also believed that bounds of the case are very important to define the case. In addition, the unit of analysis or the object of the study is one of the important components of case studies. The unit of study is related to the way the researcher has defined his/her initial research question (Yin, 2003).

The current study was characterized through Creswell and Yin's views of the case study. The aim was to facilitate an in-depth understanding of the problem being studied. In order to investigate the development of elementary mathematics teachers' TPACK in geometry, elementary mathematics teachers from public primary school in one of city in the Black Sea Region were selected. The cases were two mathematics teachers in public primary school. The cases were bounded by both 7th grade level that teachers taught; and their teaching experience which was less than 10 years.

In addition Yin (2003, 2009) categorized into four case studies based on 2x2 matrix; single-case holistic (Type 1) and single-case embedded design (Type 2), multiple-case holistic designs (Type 3), and and multiple-case embedded designs (Type 4). This matrix demonstrates that single case and multiple case reflect different design situations within two variants: unitary unit or multiple units of analysis. Thus, the single-case design is a common design in case studies where it involves one or more than one unit of analysis. The Single case (holistic) model of case study analyzes contextual conditions in relations to the case, and dotted lines between two demonstrate that the boundaries between caase and the context are not sharp. The model for single case (holistic) design is given in Figure 3. 1.

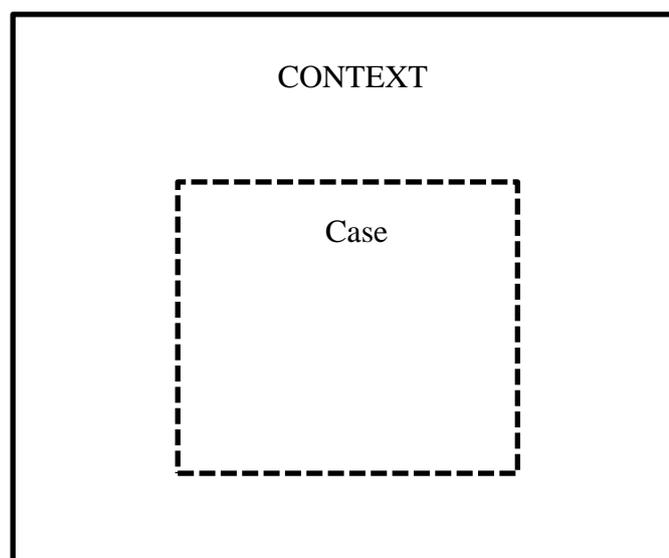


Figure 3. 1 Single Case (Holistic) Design (single- unit of analysis) (Yin, 2003, p. 40)

The research design of this research was a single-case (holistic) design (Yin, 2003). The case was two novice elementary mathematics teachers, and TPACK of elementary mathematics teachers constitute the unit of analysis. In addition, the context of the study is public primary schools in Black Sea Region. The model of this research with respect to single case (holistic) design (single-unit analysis) is given in Figure 3. 2.

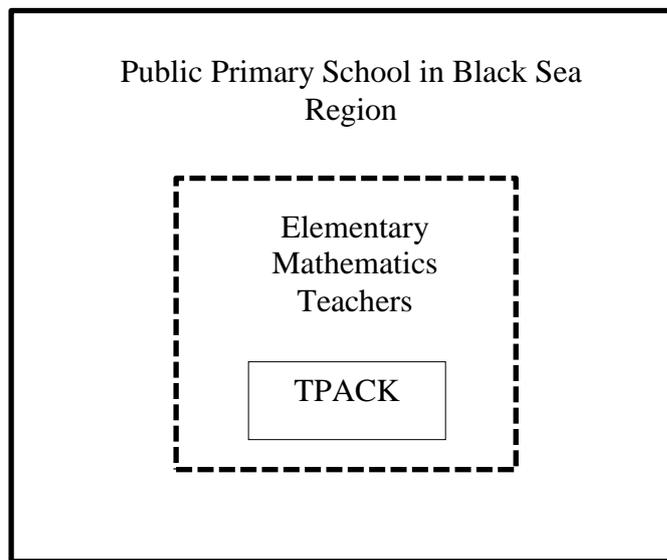


Figure 3. 2 Single Case Design (single- unit of analysis)

3.2 The Context of the Study

According to Marshall and Roseman (2006), humans' behaviors vary from context to context. Therefore, researchers should pay attention to the context while conducting a case study. The study was carried out with elementary mathematics teachers in public primary schools. To understand the context of the study, Turkish elementary mathematics teachers and primary education institutions are described in detail.

The age interval for compulsory primary school in Turkey is between the ages of 6 and 14. The eight-year compulsory education for all students is free in public schools. The primary education includes primary education lasting the four

years and the four years lower secondary education which give opportunity to allow between different programmes lasting the four years. In Turkey, there are 17,019 elementary schools, 280,804 elementary mathematics teachers and 5,478,399 students at lower secondary education levels (MoNE, 2014). The main goal of primary education is to ensure that “every Turkish child acquires the necessary knowledge, skills, behavior and habits to become a good citizen and is raised in accordance with the concept of national morals and that he/she is prepared for life and for the next level of education in accordance with his/her interests, talents and capabilities” (MoNE, 2014, p.13).

There have been some attempts to develop and improve the Turkish education system in the last decades; new curricula for primary and secondary schools have been developed and implemented since 2004. This curricular reform is aimed to transform teacher-centered curricula to learner centered curricula and base their pedagogies on constructivism rather than behaviorism (Bulut, 2007). As part of the curriculum reform, the primary mathematics curriculum (including middle schools) was mainly based on the constructivist approach and enriched with teaching activities with manipulative usage, technology usage and multiple assessment methods and techniques. In addition there is a spiral approach for each of the five learning areas: Numbers, algebra, geometry and measurement, data, and probability (MoNE, 2013). The aim of the middle school mathematics curriculum is to equip students with the knowledge, skills and attitudes in mathematics that they may need in their daily lives and their future education. After the reform, the Board of Education (BoE) (2013) have listed the basic objectives of the mathematics curriculum in Turkey as follows:

Students will be able to

1. understand mathematical concepts, establish a relationship between these concepts, and use these concepts and relationships in their daily lives and other disciplines;
2. learn the essential mathematical knowledge and skills in order to receive advanced education in areas related to mathematics;
3. express their own thoughts and reasonings in problem-solving processes;

4. use mathematical language and terminologies accurately to explain and share their mathematical thoughts logically;
 5. effectively use their prediction and mental computation skills;
 6. develop problem solving strategies and use these in solving problems in their daily lives;
 7. express concepts in different ways of representation;
 8. develop self-confidence and a positive attitude toward mathematics;
 9. develop characteristics of being systematic, careful, patient and responsible; and
 10. develop research skills, and skills of knowledge production and use.
- (p.8)

In conclusion, the curriculum not only encourages conceptual learning, fluency in computations, and communication via their mathematical knowledge, but also emphasizes the development of a positive attitude toward mathematics and the development of problem solving skills. In addition, it attaches importance to students' deriving mathematical meaning from their concrete experiences and to their establishment of abstractions and relationships. Furthermore, it also aims at enabling students not only to learn the fundamental concepts and problem solving skills in mathematics, but also to think mathematically, understand problem solving strategies and become aware that mathematics is an important means in real life.

3.3 Sampling and Participants

The purposeful sampling method was used in this study to provide rich information about the research questions. Purposeful sampling is based on the assumption that "the investigator wants to discover, understand, and gain insight and therefore must select a sample from which the most can be learned" (Merriam, 1998, p. 61). Purposeful sampling requires "the researcher [to] establish in advance a set of criteria or a list of attributes that the units for study must possess" (Patton, 1990, p. 69). Thus, it is essential for the researcher to set up some bases/criteria to choose participants who match those criteria. The current study had five criteria for the sampling procedure. The first criterion was related to long term attendance in the study so that the researcher could get richer and deeper information. In this study it

is aimed to investigate the development of teachers' TPACK before/during/after mathematics coaching. One-time study or short-time study was not enough to analyze the teachers' progress in TPACK Development Model before/during/after mathematics coaching. The long period of the study was necessary to get better information about their development. Therefore, the teachers who were able to attend this study at least 2 months period were selected. The second criterion was related to teachers' willingness for technology integration. This study analyzed how the teachers are moving through the levels of TPACK Development Model. Therefore, the teachers who were open to integrate technology in their classroom were selected. The third criterion of the sampling procedure was being novice teachers who had a potential to provide rich data. According to Webb and Staub (2003), novice teachers may have more willingness to new methods than veteran teachers - who have ten or more years of teaching experience in the classroom (National Center for Educational Statistics, 2009). Furthermore, veteran teachers may resist changes such as technology integration in the classroom. The other criterion was related to the grade level the teachers taught. It was essential for the teachers to be teaching 7th graders because the polygon unit as a geometry concept is a 7th grade topic in the national curriculum (MoNE, 2013). The last criterion issue in selecting the participants was the context in which they work, because the context might influence how teachers teach (Park & Oliver 2008). The teachers working in the same or similar context should be selected. In this study two elementary mathematics teachers from public primary schools in the Black Sea Region were picked as participants to eliminate the context's manipulation.

In consideration of the criteria, the sample of this study included two novice elementary mathematics teachers in a public school in the Black Sea Region. Each teacher volunteered to participate in the research study. Each teacher indicated their willingness to develop professionally through mathematics coaching in respect to technological pedagogical knowledge. The detailed information about the teachers was presented below.

One of the participants, Murat (Pseudonym), had been teaching in public schools for 3 years and he was a master student of mathematics education. The average number of students in his mathematics class was 21. In his mathematics classroom there was just a projector and one computer for teacher usage. Murat used technology in his daily life effectively and also used social network to communicate with people and his students. He knew how to use computer software such as Geogebra and Cabri3D. Besides, Murat usually used virtual manipulatives (from eba.gov.tr) and educational programs such as Vitamin. Murat's technical knowledge about how to use Geogebra was average as he said, "when I see a good example of Geogebra, I feel that I don't know anything." He wanted to improve himself to effectively integrate technology into their lessons. He said that he used technology for teacher demonstration tool in their mathematics lessons. In addition, the instructions via technology were teacher-directed. He believed that technology is useful for students because technology is error-free and faultless. And he said that students understand more easily by means of technology.

The second participant, Esen (Pseudonym), was in her sixth year of teaching, and she had graduated from elementary mathematics education. In her mathematics class there were 28 students. Like in Murat's class, there was a computer for the teacher and a projector. Esen also used technology in daily life but not very much. She did not integrate effectively virtual manipulative or any other program in her classroom. Esen's technical knowledge in using Geogebra was basic level. She knew that how to construct polygon and measure the angles or length, but she did not know high level access. She stated that she implemented technology-enhanced activities and PowerPoint presentations prepared by pre-service teachers from teaching practice lessons. Additionally, Esen felt insufficient when she saw technology-enhanced activities. She was willing to improve herself for technology integration into her lessons, like Murat. Esen indicated that she implemented technology to draw students' attention and to make the concepts more concrete. In addition, both the participants believed that every geometry topic could be presented

via technology. In Table 3. 1, the teachers' characteristic feature and perceptions regarding use of technology can be observed.

Table 3. 1 Participants' characteristic features and perception of technology

Participants	Murat	Esen
Gender	Male	Female
Graduate	MS (2 nd year)	Bachelor
Number of Students	21	28
School's Technological Devices	Projector, computer (just for teacher)	Projector, computer (just for teacher)
Technology usage in daily life	Effectively	Normal
Proficiency of Technology	Sufficient but should improve	Sufficient but should improve
Virtual Manipulatives	Usually (Eba.gov.tr, Vitamin)	Never
Computer Software	Cabri3D, Geogebra, Logo, Derive, Inspiration	Cabri3D, Geogebra,
The technical knowledge about Geogebra	Medium	Basic
The advantage of technology in geometry education	Useful for learning, error-free and faultless	Take an attention, make more concrete
The role of teacher	Teacher-directed	Teacher-directed

This study was carried out in the spring semester of the 2013-2014 academic year and lasted 16 hours for each teacher. In order to analyze the development of TPACK, researcher conducted mathematics coaching as a professional development.

3.4. Mathematics Coaching as a Professional Development

Mathematics coaching as a professional development was carried out by researcher. This professional development was undertaken with a focus on enhancing the teachers' knowledge of the nature of technology, assessment strategies via Geogebra, how to apply curricular ideas in a technological environment, how to use Geogebra as a learning and teaching tool for effective teaching of technology.

The cyclic process of this study was designed by researcher in the light of literature of mathematics coaching and TPACK. According to literature, mathematics coaching should include pre-conference, observation and post-conference (West & Staub, 2003). Therefore the researcher identified the parts of mathematics coaching as pre-conference, observation and post-conference. Then, with the help of the TPACK and mathematics coaching literature the researcher designed components of this cyclic process. For instance, According to Jaipal-Jamani and Figg (2015), teachers should work with exemplary activities to improve their TPACK. Therefore, the researcher determined “demonstrating exemplary technology-enhanced activities” as a component of pre-conference. In addition, According to Bay-Williams, McGatha, Kobett, and Wray (2014), the teacher should reflect their ideas about their implementation in mathematics coaching. In the light of this notion, the researcher labeled “reflecting on implementation” as a component of post-conference. The cyclic process of mathematics coaching is described below in detail (see Figure 3. 3).

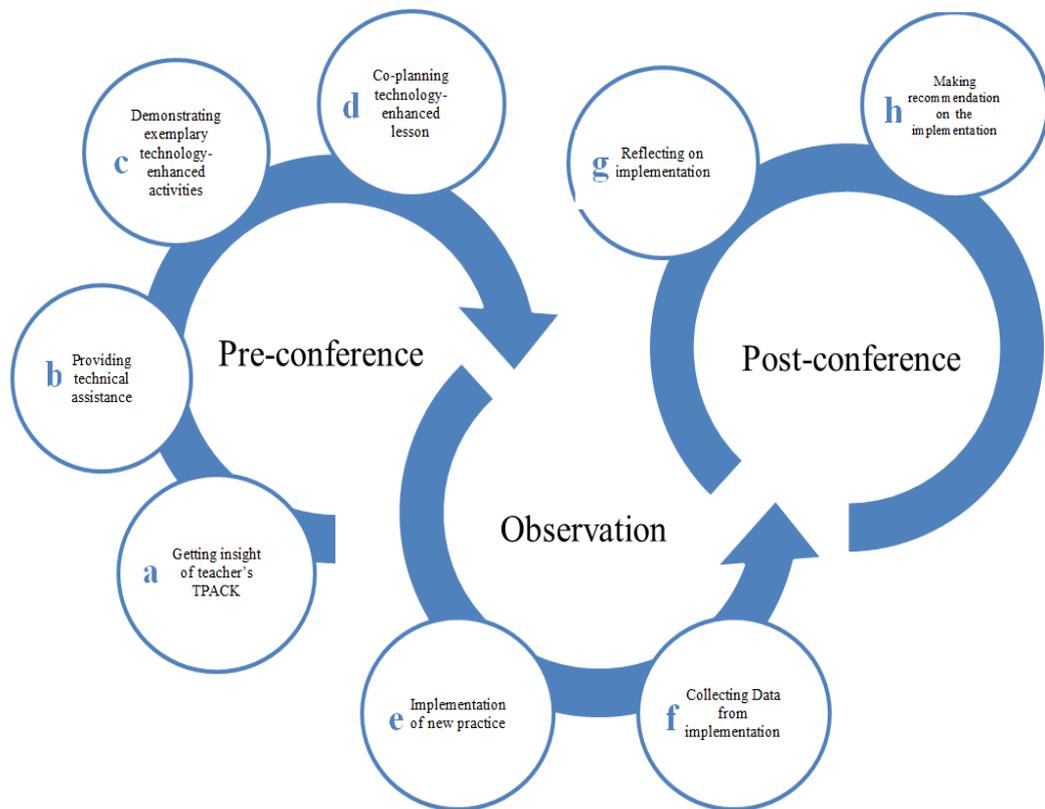


Figure 3. 3 The Cyclic Process of Mathematics Coaching

3.4.1. Pre-conference

The first part of mathematics coaching cycle was one-on-one pre-conference with teachers. The face to face pre-conferences lasted at least one hour. The pre-conference, which focused on lesson planning, provided the researcher as a coach with the opportunity to understand the teachers' TPACK and planning habits.

Before the conference, the teachers designed mathematics lessons on polygons utilizing Geogebra. With the assistance of the conversation between the coach and the teachers during the pre-conference, the lesson plans were reconstructed. But the coach firstly should gain insight into mathematics teachers' TPACK during the pre-conference. The coach should focus on the teachers' existing knowledge -such as how curriculum/curricular materials and technology are integrated in the teaching and learning process or how the teachers assess students'

understandings with the integration of technology. This knowledge helps the conversation during pre-conference to be specific rather than general. In the pre-conference, the coach gets the opportunity to understand the teachers' TPACK in detail. In the pre-conference, the researcher concentrated on the teacher's attention on mathematics teaching and learning with technology and asked for deliberate technological decisions designed to foster students' understanding. This pre-conference helped to shed light on the teachers' TPACK and needs.

The pre-conference facilitated the assessment of how elementary mathematics teachers construct TPACK regarding geometry in general, and polygons in particular. Pre-conferencing promoted the teachers to take an analytical stance toward the mathematics curriculum and encourage the teachers to modify curriculum with the consideration the affordance of Geogebra. The teachers were encouraged to facilitate their mathematical experiences regarding polygon units via Geogebra-enhanced activities, which advanced students' creativity and higher order thinking skills.

The coach demonstrated some exemplary technology-enhanced activities on Geogebra and made suggestions to improve teachers' TPACK (see appendix K). For instance, in second mathematics coaching, the researcher demonstrated already-prepared Geogebra activities on the properties of quadrilateral to teachers (see Figure 3. 4). In the rectangle activity, the parallelogram provided an environment to make a generalization about the properties of diagonals of a rectangle by using Geogebra. In this activity, the students dragged the vertex, they could see the changes in sides, length of diagonal and angles. In the one-on-one pre-conference, Murat and researcher as a coach decided to rearrange the activity, such as adding show/hide checkboxes for the sides, angles and diagonals. In addition, researcher demonstrated the similar Geogebra activity for square, Murat did not to prefer use it because he wanted to construct square in his classroom to discuss the properties of square. On the other hand, Esen wanted to integrate this square activity. The teacher and the researcher as a coach modified the activity with using the check boxes like parallelogram activity.

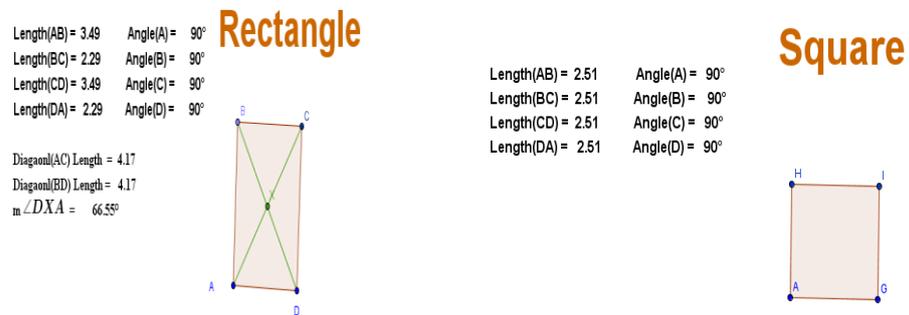


Figure 3. 4 The Rectangle and Square Geogebra Activities

In the pre-conference, the teachers should feel free to reject or accept the recommendations of the coach. The teacher and the coach were able to modify the recommended activities such as rectangle activity or they did not want to integrate their mathematics lesson such as Murat rejected to integrate the square activity. For instance, in the MCC3, the researcher suggested the integration of Pick's Theorem Geogebra activity in the teachers' lesson (see Figure 3. 5). This Geogebra activity helped to explore the formula of area of irregular shapes. The formula of area of polygon is calculating in terms of the number i of lattice points in the interior located in the polygon and the number b of lattice points on the boundary placed on the polygon's perimeter. The formula is area of the polygon: $A = i + (b/2) - 1$. In the activity the sides' numbers were able to change. In addition after dragging the value of the interior lattice points, boundary points were recorded in spreadsheet. Murat and Esen did not want to integrate the Pick's Theorem activity because it was outside of the curriculum. The teacher preferences in pre-conference helped to analyze their TPACK level.

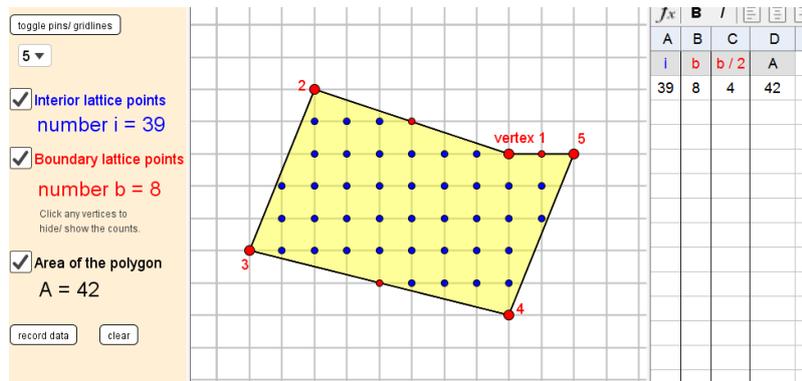


Figure 3. 5 The Pick's Theorem Geogebra Activity.

In addition to curricular ideas, during the pre-conferencing time was spared for discussion on effective strategies in delivering instruction. Furthermore, pre-conferencing promoted the teacher to serve not as a director but as a guide to student learning via technology. Teachers were encouraged to provide an environment of discourse not only among students but also between the teacher and his/her students. Furthermore, they were assisted in helping students to be responsible for their own learning in a Geogebra-enhanced environment. Furthermore, the researcher gave advice about how to integrate this activity for active learning environment. For example, both of the teachers (Murat and Esen) had the tendency to make a control students' action in Geogebra-enhanced activities. The researcher suggested that they should have allotted time for self-exploration and they should have served as a guide rather than the director of students' exploration. The teacher and the coach discussed these suggestions and they restructured the lesson plan taking into consideration this discussion. Finally, it provided the coach and the teachers with a common focus of technology integration in their mathematics classroom for observation of mathematics coaching.

In the pre-conference, the coach provided technical assistance when necessary. For instance, the researcher showed Murat how to add sliders in Geogebra. Furthermore, the coach supported the teacher by showing how to search on the Internet for good ideas regarding technology integration and Geogebra. For instance, the coach introduced "www.geogebraTube.com" to the teachers. To sum

up, pre-conferencing provided the coach with insight into the teachers' thought processes and needs. Besides, pre-conferencing enabled the coach to give feedback about the lesson during the post-conference based on the shared lesson plan and its implementation.

3.4.2 Observation

Observation of the teachers' lesson is another part of the mathematics coaching. Every mathematics coaching cycles, the coach observed the teachers' lessons for two or four hours. In the observation session, the teachers implemented Geogebra enhanced lesson which they had discussed in the pre-conference. The coach took notes of what was happening in the classroom to unpack during the post-conference dialog. Observing the implementation of the coplanned lesson was a source of information that allowed the teachers and the coach to reflect on how they integrated technology as learning and teaching tool during the post-conference. Furthermore, the coach collected students' work on Geogebra and their dialogs in relation to polygons in order to determine the following steps of mathematics coaching.

The observation of the teachers' lesson provided information about the teachers' level in TPACK Development Model. For example, the teachers' integration of Geogebra as a static tool indicated lower TPACK level. Besides, the teachers' integration of Geogebra with considering the dynamic properties demonstrated higher TPACK level. Thus, the observation of the lesson provided an evidence of teachers' level in TPACK Development Model.

3.4.3 Post-conference

After the observation of the lesson, the coach and the teachers spent time together discussing the lesson in relation to integration of technology. During the post-conference, the coach encouraged each teacher to express his/her thoughts on the lesson in order to foster their competence. During the post-conference, teachers reflected on their performance in terms of technology, pedagogy and content. For

instance in mathematics coaching cycle 1, Murat reflected that his lesson needed to improve in terms of technology integration. He realized that he implemented Geogebra as a demonstration tool. Therefore, the teacher and the coach discussed how the lesson could improve. In addition, giving opportunity to raise issues, concerns and critics related to the observed lesson provided insight into what was important for the teachers and what their TPACK was.. Furthermore, in the post-conference the coach can give feedback and suggestions to improve the teachers' TPACK and practice. The coach shared information from her observation of the lesson and the notes she had taken and provided feedback. For example, in the first mathematics coaching cycle, Esen demonstrated the Geogebra activities about the quadrilaterals on the board. She did not let the students to use Geogebra although they had a computer in their group. After the students made generalizations about the properties of the quadrilaterals, she motivated the students to use Geogebra. The lesson was teacher-directed. In the post-conference of the same mathematics coaching cycle, the researcher as a coach gave advice about students-centered lesson. The coach motivated the teacher to gave a time to students' self-exploration. After giving feedback, the coach and the teachers reached an agreement to try on a Geogebra with a new lens is what promotes growth. To sum up, the post-conference provided the opportunity to assess learning, inform instruction and adjust educational plans. The most important goal of the post-conference was to assist the following lessons in light of the reflections of the teachers and the coach.

3.5 The Data Collection Process

To analyze the development of elementary mathematics teachers' TPACK, the researcher designed the implementation in three sections: before mathematics coaching, during mathematics coaching, and after mathematics coaching. This research process consisted of (1) pre-observation and pre-interview before mathematics coaching, (2) the four cyclic process of pre-conference, observation, and post-conference during mathematics coaching and (3) post-observation, post-

interview and mathematics coaching interview after mathematics coaching. The process of study is depicted in Figure 3.6.

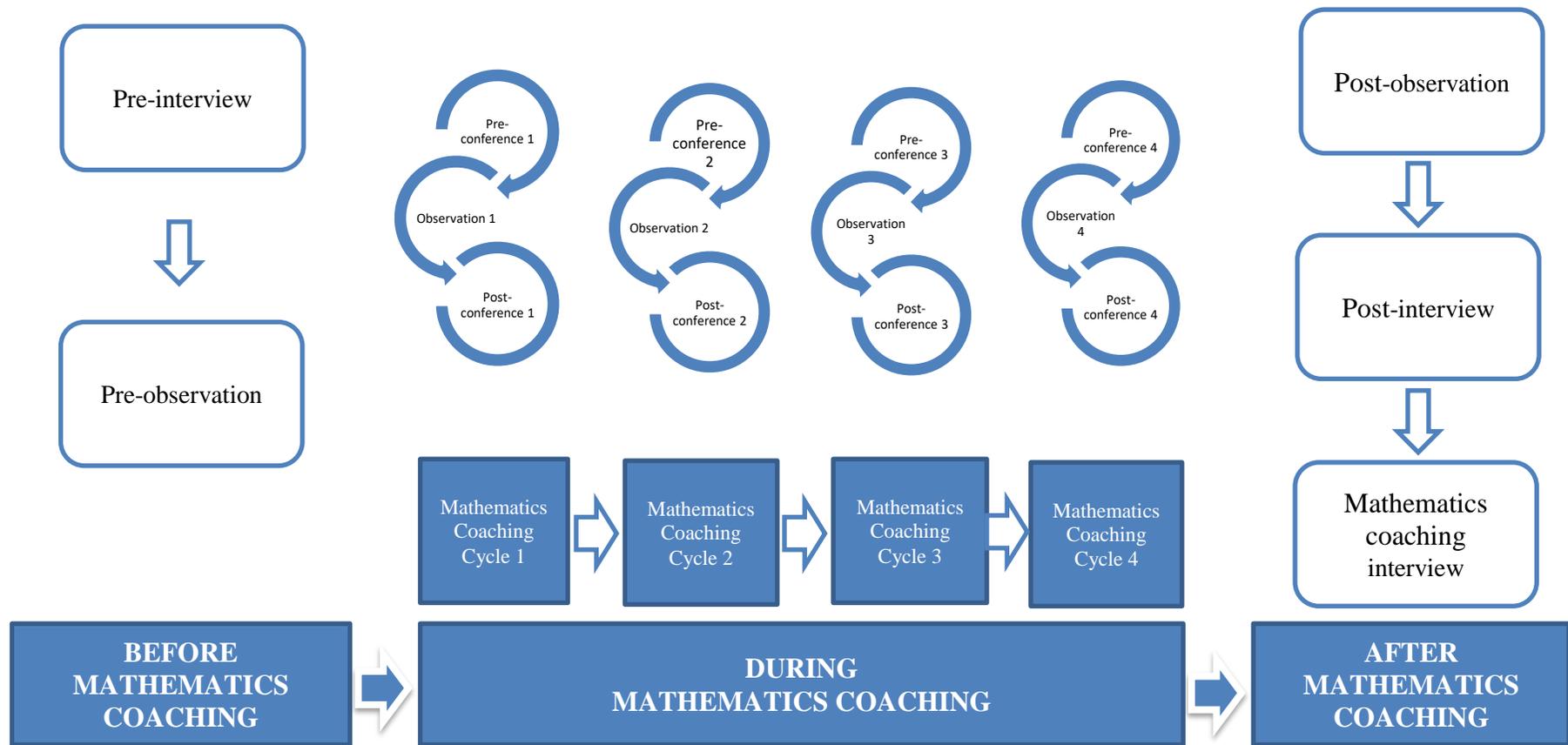


Figure 3. 6 The process of Study

Before mathematics coaching the researcher started to conduct pre-interview about TPACK and pre-observation for two hours. The pre-interview and pre-observation helped to analyze the teachers' level in the TPACK Development Model before mathematics coaching. Then, the researcher carried out mathematics coaching cycles as a professional development. The cyclic process of mathematics coaching was repeated four times in this study and was labeled as Mathematics Coaching Cycle 1 (MCC1), Mathematics Coaching Cycle 2 (MCC2), Mathematics Coaching Cycle 3 (MCC3), and Mathematics Coaching Cycle 4 (MCC4). The researcher determined the teachers' TPACK level in the cycles during mathematics coaching to identify the development of teachers' TPACK. Each of mathematics coaching cycles included three parts: pre-conference, observation, and post conference as explained in the previous part. After mathematics coaching, the researcher observed the teacher for two hours and conducted post-interview and mathematics coaching interview. In addition, the researcher determined final TPACK level of the teachers to reveal their development in TPACK Model.

In the 7th grade spring semester, the polygons unit, which included 11 objectives, was in the national mathematics curriculum (MoNE, 2013). Before the conducting the study, the teachers and researcher meet up one by one and determined hours and objectives of the lessons in the study. The observation of lessons lasted sixteen hours and labeled as the pre-observation, observation 1, observation 2, observation 3, observation 4, and post-observation totally. Pre-observation of the lesson was conducted before mathematics coaching. Observation 1, observation 2, observation 3, and observation 4 were implemented as a part of coaching cycles during mathematics coaching. Post-observation was occurred after mathematics coaching. The objectives and hours of observation of the lessons can be seen in Table 3. 2.

Table 3. 2 The objectives and hours of lessons

	Objectives	Hours
Pre-observation	-To determine the interior and exterior angles and diagonals of polygons	2 hours
MCC1	-To calculate the sum of the interior angles of polygons.	2 hours
MCC2	-To identify the properties of quadrilaterals' side, angle and diagonal properties.	4 hours
MCC3	-To compare polygons and determine whether or not they are congruent and draw polygons congruent to another polygon -To compare polygons and determine whether they are similar and draw polygons similar to another polygon -To predict the area of quadrilateral figures by using a strategy. -To construct the formula for the area of a parallelogram figure.	4 hours
MCC4	-To construct the formulas for the area of the rhombus shape. -To construct the formula for the area of the trapezoid shape.	2 hours
Post-observation	-To explain the relationship between the side length and the area. -To explain the relationship between the perimeter and the area.	2 hours

(Mathematics Coaching Cycle1 [MCC1], Mathematics Coaching Cycle [MCC2], Mathematics Coaching Cycle1 [MCC3], Mathematics Coaching Cycle4 [MCC4])

Frankel and Wallen (2006) stated that there exist three basic ways; interview, observation and document in order to offer detailed description of the phenomenon studied in qualitative research. Merriam (1998) states that interviewing is the best data collection technique for conducting case studies. Interview provides an environment for researcher to hear participants' voice. Interview reveals

participants' attitude, their values, and what they think they do. Observations provide the researcher with the opportunity to observe participants' behavior in the natural setting (Merriam, 1998). The third data source in qualitative research is documents which provide data related to institutional, organizational, and personal vision, plan, and action (Patton, 2002).

To obtain deep information related to the development of mathematics teachers' TPACK in geometry through mathematics coaching, the data were gathered through interviews, observations, and field notes. In this study, the researcher conducted interviews before, during and after mathematics coaching: pre-interview, pre-conferences, post-conferences, post-interview, and interview on mathematics coaching. Additionally, observations were conducted before, during and after mathematics coaching. The researcher monitored the teachers to reveal how they conducted the designed lessons. Furthermore, the researcher recorded field notes from each inservice teacher's classroom observation.

Data collection from two elementary mathematics teachers of 7th grade mathematics classrooms in the Black Sea Region was conducted in the spring semester from February 2014 to June 2014. The schedule of events conducted for data collection is presented in Table 3. 3.

Table 3. 3 The schedule of events

Date	Events
01.07.2013-30.01.2014	Development of data collection tools (TPACK questionnaire, pre-conference protocol, observation protocol, post-conference protocol, and interview protocol)
10.02.2014-14.03.2014	Obtaining permission from a Provincial Directorate for National Education Pilot of study (One inservice teacher (Erdogan))
14.03.2014-27.03.2014	Revision of the implementation of study in the light of the pilot study
27.04.2014-05.06.2014	Data collection and Implementation of study (Two inservice teacher (Murat and Esen))

The time schedule for the data collection from the teachers in the main study is presented in Table 3. 4.

Table 3. 4 Timeline of data collection of the main study

	Murat	Akgül
Before MC	06.05.2014-07.05.2014	27.03.2014-28.03.2014
MCC1	09.05.2014-13.05.2014	02.04.2014-05.04.2014
MCC2	14.05.2014-22.05.2014	06.04.2014-10.04.2014
MCC3	26.05.2014-29.05.2014	11.04.2014-18.04.2014
MCC4	02.06.2014-03.06.2014	19.04.2014-22.04.2014
After MC	04.06.2014- 05.06.2014	25.04.2014-05.05.2014

(MC is indicating Mathematics Coaching)

3.5.1 Data Collection Tools

In this study, multiple forms of data from before, during, and after mathematics coaching were collected to answer the research questions. Data collection procedures included 1) pre-interview on TPACK before mathematics coaching, 2) observation of teacher before mathematics coaching, 3) one-on-one pre-conference with mathematics teachers during mathematics coaching, 4) observation of the teachers during mathematics coaching, 5) one-on-one post-conference with mathematics teachers during mathematics coaching, 6) post-interview on TPACK after mathematics coaching, and 7) mathematics coaching interview after mathematics coaching.

3.5.1.1 Data Collection Tools Before Mathematics Coaching

Before mathematics coaching, researcher collected data through pre-interview and pre-observation to determine two elementary mathematics teachers' TPACK in the TPACK development model. The details of data collection before mathematics coaching is presented in Table 3. 5.

Table 3. 5 The details of data collection tools before mathematics coaching

	Descripton of Purposes	Method	Lenght
Pre-interview	-To determine teachers' TPACK level in terms of the four themes of the TPACK development model. -It provided data for first research question and sub-research questions.	It was audio-taped.	About 90 minutes
Pre-observation	-To determine the teachers' TPACK level in terms of the four themes of the TPACK development model. -It provided data for first research question and sub-research questios	It was video-recorded.	Two lesson hours.

3.5.1.2 Pre- interview Before Mathematics Coaching

Before mathematics coaching the researcher conducted the TPACK semi-structured interview protocol as a pre-interview (see appendix A). Pre-interview were audio-taped via a digital voice recorder with the permission of the participants. In addition, each participant was interviewed individually during approximately 90 minutes. The reason of the use of semi-structured interview was that it allowed asking further questions when it was essential and appropriate to clarify in more detail some of the responses of the interviewees. Furthermore, semi-structured interview generally has a framework of themes to be explored, like this study.

The TPACK interview protocol in this study included open-ended questions, which helped the researcher to determine teachers' knowledge about technology integration in the 7th grade geometry topics in terms of the TPACK Development Model. The teachers' answers on the TPACK interview protocol helped to determine their TPACK level before mathematics coaching. Additionally, it provided data for first research problem and sub- research problems.

In order to understand the TPACK of in-service elementary mathematics teachers on geometry and their TPACK level, the TPACK interview protocol was developed by the researcher based on the Major Themes and their eleven descriptors in the Mathematics Teacher TPACK Development Model (Niess, Ronau et al., 2009). The themes and the related descriptors are presented in

Table 3. 6 Themes and Descriptors in the Mathematics Teacher TPACK Development Model

Themes	Descriptors
Curriculum and Assessment	<ul style="list-style-type: none"> • Curriculum • Assessment
Learning	<ul style="list-style-type: none"> • Mathematics learning • Conceptions of student thinking
Teaching	<ul style="list-style-type: none"> • Mathematics learning • Instructional • Environment • Professional development
Access	<ul style="list-style-type: none"> • Usage • Barriers • Availability

The curriculum and assessment theme of the Mathematics Teacher TPACK Development model, which includes the (1) curriculum descriptor, and (2) assessment descriptor, focuses on how curriculum and curricular materials and technology are integrated in the teaching and learning process and how they assess students' understandings with the integration of technology. The examples of questions based on curriculum and assessment theme in the TPACK interview protocol are presented below:

- How would you integrate technology into the “polygons” unit in the 7th grade mathematics curriculum?
- To what extent would you keep to the school curriculum and the textbook while implementing the technology-enhanced “polygons” unit? What kinds of changes would you make? Why?
- Could you explain how you would integrate technology into your lessons during the instruction of the “polygons” unit?
- What are the measurement and evaluation techniques that you would recommend in assessing students' knowledge on the concepts in the “polygons” unit?
- How would you make use of technology while assessing the concepts in this topic? What is the purpose of using technology as a assessment tool?

The learning themes of the Mathematics Teacher TPACK Development model, which includes the descriptors of (1) Mathematics learning, and (2)

Conceptions of student thinking, concentrates on whether teachers integrate technology which emerged as a potential tool in learning mathematics and the development of students' thinking skills via technology. The related questions about this theme are presented below:

- For what purpose would you use technology in enabling students to learn the “polygons” unit? Why?
- What is the role of technology in students' discovery of a concept? Why? If so, how? Could you explain in detail?
- Which specific examples would be useful in increasing students' understanding of this unit? Why?
- How would you use technology to deepen students' understanding of the unit? Could you give an example?
- How would you make use of technology to eliminate your students' misconceptions or the difficulties they face in understanding concepts? Could you provide detailed explanations for each of the following areas: the features of polygons, the classification of polygons, congruence and similarity of polygons, and the area of quadrilaterals?

The teaching theme of the Mathematics Teacher TPACK Development model, which includes (1) Mathematics learning, (2) Instructional, (3) Environment, (4) Professional development, focuses on the views of teachers about technology, teachers' knowledge of instructional strategies by incorporating technology, mathematical environments to incorporate technology as a learning tool for supporting students' individual and collaborative mathematical learning and creativity, and their tendency to professional development. The related questions about this theme are stated below:

- Which features of technology would you take into consideration while presenting the technology-enhanced “polygon” unit? Could you explain in detail how you would use technology as a means of instruction?
- What are the instructional strategies, methods and techniques that you would recommend using in the instruction of the “polygons” unit?
- What kind of an environment would you establish during the instruction of “polygons” unit?”
- What is the role of technology in this course? What is the purpose of using technology in the instruction of this unit?

- What is the role of the student in this course? Do you enable students to actively participate into the lesson? How?
- What do you think of Professional development, such as in-service training and seminars? Would you like to participate in a technology related seminar, workshop or any other professional development event? Why?
- What kind of a training on technology would you like to have?

The Access theme of the Mathematics Teacher TPACK Development model, which includes (1) Usage, (2) Barriers, (3) Availability, focuses on whether or not students are allowed to use technology, how teachers address barriers to technology integration, and how technology makes more and higher levels of mathematics available for investigation for students. The some of the related questions about this theme are presented below:

- When do you provide students with the opportunity to use technology in the “polygons” unit? Why?
- What kinds of opportunities and limitations does technology possess in achieving the learning outcomes?
- How do you think technology contributes to this course?
- What kinds of difficulties can be encountered with the integration of technology in the “polygons” unit?
- What difficulties can students face in the use of technology? What would you do to eliminate these difficulties?
- Do you have concerns with respect to experiencing difficulties in class management? Why? If you do, what would you do to overcome these difficulties? Could you explain in detail?
- How would you use technology while using multiple representations? What impact would technology have on multiple representations?

The questions in the TPACK interview protocol were checked by mathematics educators who specialized in technology to determine the content validity. The reviewers came to an agreement. Turkish and English versions of the TPACK interview protocol are presented in Appendix A and B.

3.5.1.3 Pre- observation Before Mathematics Coaching

Researcher observed the mathematics teachers in the real context to make a deep analysis of data. Creswell (2003) stated that a researcher’s role in observations

varied from a complete participant to a complete observer. In this study the researcher's role can be described as a nonparticipant observer. Thus, the researcher did not engage in the lesson in order not to affect teachers' behaviours while implementing technology-enhanced activities.

Researcher conducted the pre-observation before mathematics coaching to reveal how the teachers conducted the designed lessons and how they integrated technology in their lesson. The pre-observation helped the researcher to determine teachers' TPACK level before mathematics coaching. The pre-observation was video recorded to triangulate the data for the trustworthiness of study. In addition, the Observation Protocol (see Appendix E) was used for classroom observations. The observation protocol included three parts: Basic information about the lesson (objectives, classroom size, etc.), the evaluation of TPACK and additional information about technology integration. Basic information parts included objectives, classroom size, and technological devices. The evaluation of TPACK part developed by modifying the statements of the TPACK Development Model Self-Report Survey (Taylor, 2011). Taylor (2011)'s TPACK Development Model Self-Report Survey included 55 statements that pertained to eleven descriptors of the four themes identified by the TPACK Development Model. Researcher changed the subjects of each of 55 statements to modify the observation protocol. In addition, researcher leaved a blank to note events related to four themes and their eleven descriptors of the TPACK Development Model in the observation protocol. In the additional information part, researcher took notes aout the advantages and disadvantages of observed lesson to give an idea for determining the level of teacher. Observation protocol was filled after every observation of mathematics teachersby the researcher and co-coder.

Merriam (1998) suggests that the researcher take field notes consisting of a verbal description of the setting and the participants being observed. The researcher can also take direct quotations or the substance of what students say or do. The field notes can include the researcher's reflections regarding his/her feelings, ideas, hunches, impressions, or problems in the classroom. Accordingly, researcher

recorded field notes from each inservice teacher’s classroom practice. The field notes were used as supplements to the data gathered via other sources such as observations, interviews and questionnaires (Bogdan & Biklen, 2007).

3.5.1.4 Data Collection Tools During Mathematics Coaching

Researcher collected data through pre-conferences, observaitons, and post-conferences during four mathematics coaching cylces to determine two elementary mathematics teachers’ TPACK levels. The detail of data collection during mathematics coaching is presented in Table 3. 7.

Table 3. 7 Data Collection Tools During Mathematics Coaching

	Descripton of Purposes	Method	Lenght
Pre-conferences (Pre-conference 1, Pre-conference 2 Pre-conference 3 Pre-conference 4)	-To determine the teachers’ TPACK level in terms of the four themes of the TPACK development model. -It provided data for first research problem and sub- research problems.	It was audio-taped.	About 90 minutes
Observations (Observation 1, Observation 2 Observation 3 Observation 4)	-To determine the teachers’ TPACK level in terms of the four themes of the TPACK development model. -It provided data for first research problem and sub- research problems	It was video-recorded.	2+4+4+2 lesson hours.
Post-conferences (Post-conference 1, Post-conference 2 Post-conference 3 Post -conference 4)	-To determine the development of teachers’ TPACK level in terms of the four themes of the TPACK development model. -It provided data for first research problem and sub- research problems		About 40 minutes

3.5.1.5 Pre-conferences During Mathematics Coaching

The researcher conducted four pre-conferences as a part of mathematics coaching. In the pre-conferences, teachers designed mathematics lessons with the assistance of the conversation held between him/herself and the coach. This conversation should be focus specific rather than general. The pre-conference focused on lesson planning to provide the researcher as a coach with the opportunity to understand the teachers' TPACK and his/her level in the Mathematics Teacher TPACK Development Model. The pre-conference protocol was designed according to this TPACK model and its themes and descriptors (see appendix C and D). The questions in the pre-conference protocol were modified according to teachers' needs in mathematics coaching process. It included two parts: (1) Reviewing the lesson plan and (2) Reconstructing the lesson plan.

In reviewing the lesson plan, the researcher asked questions to reveal the teachers' knowledge about technology integration and their habits about designing technology enhanced lessons. During the pre-conference, the researcher asked the following questions in terms of four major themes of TPACK Development Model.

Table 3. 8 The examples of questions in the reviewing the lesson part

Curriculum and Assessment Theme
<ul style="list-style-type: none"> • Which resources did you refer to while preparing the lesson plan? • Have you prepared this technology-enhanced lesson plan in accordance with the way the unit is covered in the curriculum and textbook? • To what extent did you keep to the textbook while preparing the activities to be used in the lesson? What kinds of changes did you make? Why? • Did you use technology in the assessment of the lesson? Why? How did you use technology in the assessment? What are your opinions regarding this issue?
Learning Theme
<ul style="list-style-type: none"> • Does technology have a role in students' discovery of the concept stated in the learning? Why? If so, how? • This lesson, what is the role of technology in enhancing students' mathematical thinking skill? For what purpose are the students going to use technology? Could you please explain? • What do you think is the impact of technology on eliminating the difficulties and misconceptions that students may experience? How will you benefit from technology to overcome these difficulties?
Teaching Theme
<ul style="list-style-type: none"> • How did you integrate technology into the method and strategies related to the stated learning outcome? • What is the role of the teacher in this lesson? What are you as a teacher making use of technology? What are your duties? Could you please explain? • What is the role of the students in this lesson? Are the students making effective use of technology? Are the students actively participating in the lesson? How?
Access Theme
<ul style="list-style-type: none"> • What are the obstacles – difficulties – that you may face regarding the integration of technology into the lesson? Could you please explain? • Considering the technology that you will be using, how would you organize the classroom? Which difficulties that may arise during class management did you take into consideration while preparing this plan?

After reviewing the lesson plan, the researcher asked questions to restructured teachers' lesson plan to support technology integration in their mathematics classroom. The question related to the restructuring of the lesson plan provided an

environment for the teachers to make progress in the TPACK development Model.

The related questions about this section are as follows:

Table 3. 9 The questions in the restructuring of the lesson plan

<p>Emphasizing the good ideas in teacher’s lesson plan. Advicing him to develop.</p> <ul style="list-style-type: none"> this is well thought of; well, can you add an activity such as.... ? What do you think? Will it be useful?
<p>Discussing the activities which be less effective in students’ learning of the related concept.</p> <ul style="list-style-type: none"> Do you think this technology-enhanced activity that you prepared will develop your students’ understanding of the unit? What other activities could there be? Do you think such an activity as would be beneficial for the students? Why?
<p>Showing the reference activities and discussing.</p> <ul style="list-style-type: none"> What do you think about this activity.....? Does it develop the students’ understanding?
<p>Advicing about technological pedagogical content knowledge.</p> <ul style="list-style-type: none"> What do you think about these activities: (activities that are suitable for deduction and those for induction are shown)? Can you implement them?

3.5.1.6 Observations During Mathematics Coaching

The researcher observed the mathematics teacher as part of mathematics coaching cycles: MCC1, MCC2, MCC3, MCC4. In the observation1, observation 2, observation 3, and observation 4, the researcher monitored the teachers to reveal how they conducted the codesigned lessons. Furthermore the researcher conducted observation protocol and took field notes to determine teachers’ TPACK level.

3.4.1.7 Post-conference Protocol

The post-conference protocol was conducted to provide an environment for teachers and the researcher to reflect on themselves about the related lesson observation in terms of TPACK (see Appendix F and G). At the beginning, the

researcher asked questions to prioritize the teacher's reflections about the strengths and weaknesses of related lesson in terms of technology integration. Then, the researcher demonstrated examples, anecdotes and videos from the related lesson to analyze the related lesson. Furthermore, the teacher made suggestions about the lesson to progress the teachers' TPACK. The post-conference protocol was designed according to the TPACK Development Model and its themes and descriptors. The sample of questions in the post-conference are presented below.

Table 3. 10 The questions in the post-conference protocol

<p>General Questions</p> <ul style="list-style-type: none"> • Which parts have proceeded successfully as regards the integration of technology into the lesson? Why? How else can it be improved? • Were there any unsuccessful parts in the integration of technology into the lesson? Why? What kinds of changes should be made to make it more successful? • What would be remain the same if you had chance to teach again, what would be changed as well? Why? (Is there anything you want to add to lesson or remove from also?) Do you explain?
<p>Curriculum and Assessment Theme</p> <ul style="list-style-type: none"> • What do you think about lesson's suitability to mathematic curriculum? In your opinion, how must be the technology used in mathematic curriculum and textbook? • How did you use technology in this lesson with respect to assessment? How do you think your assessment was here [shows a part of the video recording of the lesson]? How can it be improved? What can be done to use technology more effectively during the assessment stage? Could it have been better to it in ... way? What are your opinions regarding this issue? (suggestions)
<p>Learning Theme</p> <ul style="list-style-type: none"> • At what stage in the lesson and for what purpose did you use technology as a means of teaching? Could you please explain? Do you think technology was useful? For what other purposes could technology have been used? For what purpose did you use technology here [<i>shows a part of the video recording of the lesson</i>]? What do you think is its impact upon the students? In what other way could technology have been used to enhance students' understanding of the unit? Would it have been better to use it in this way: (<i>suggestions</i>)? Why? What questions did you ask while teaching students

Table 3.10 (continued)

mathematics in this technology-enhanced lesson? Which ones were worthwhile, which one weren't? What other kinds of questions could you have asked? Why? You have asked students a question like ... [shows a part of the video recording of the lesson]. If you had asked the students a question like ... what kinds of answers do you think you would have received? What are your opinions regarding this issue? (*suggestions*)

Teaching Theme

- Which instructional strategy was effective in the use of technology? Why? Did you deductive or inductive strategies? Did technology have an impact? What are the strengths and weaknesses of these strategies? You have answered a student's question by saying ... [shows a part of the video recording of the lesson]? What other strategy could you have used to teach students? Would it have been better here to do it in way? What are your opinions regarding this issue? (suggestions) the role of teacher and student in lesson? What is the effect of technology on there roles?

Access Theme

- What difficulties did you experience in integrating technology into the lesson? Did you experience difficulties in class management when using technology? How did you overcome them? In this section [shows a part of the video recording of the lesson], it seems that you are having difficulties with respect to ...? What do you think is the reason? How could you have solved this problem? Do you think it would have been better to do it in way? What are your opinions regarding this issue? (suggestions)

3.5.1.8 Data Collection Tools After Mathematics Coaching

Researcher collected data through post-observation, post-interview and mathematics coaching interview after mathematics coaching to determine two elementary mathematics teachers' TPACK levels and reveals their perceptions about mathematics coaching. The details of data collection after mathematics coaching is presented in Table 3. 11

Table 3. 11 The details of data collection tools before mathematics coaching

	Descripton of Purposes	Method	Lenght
Post-observation	-To determine teachers' TPACK level in terms of the four themes of the TPACK development model. -It provided data for first research question and sub-research questions.	It was audio-taped.	About 90 minutes
Post-interview	-To determine the teachers' TPACK level in terms of the four themes of the TPACK development model. -It provided data for first research question and sub-research questios	It was video-recorded.	Two lesson hours.
Mathematics Coaching Interview	-To determine the teachers' perception about mathematics coaching -It provided data for second research question	It was audio-taped.	About 30 minutes

3.5.1.8.1 Post-Observation After Mathematics Coaching

The researcher conducted observation to define the teachers' final TPACK level after mathematics coaching. The researcher used observation protocol and took field notes during post-observation. The researcher did not help the teachers to design the lesson. Thus, post-observation of teachers technology implementation revealed what TPACK level he/she reach by him/herself.

3.5.1.8.2 Post-Interview After Mathematics Coaching

The researcher interviewed the teacher individually and used TPACK interview protocol as a post-interview. Post-interview helped the researcher to find teachers' final TPACK level.

3.5.1.8.3 Mathematics Coaching Interview After Mathematics Coaching

In the mathematics coaching interview open-ended questions were asked to gather as much data as possible on the in-service teachers' perceptions about the mathematics coaching process in terms of TPACK (see Appendix H and I). Mathematics coaching interview provided data for second research question. The researcher investigated what teachers thought about the benefits of a coaching program, and what the challenges or drawbacks of a coaching program were. The questions posed in the interview are stated below.

- How can you evaluate the mathematics coaching?
- Which aspect of the training provided did you like the most? Why?
- Was the mathematics coaching effective with respect to integrating technology into the mathematics lesson? Could you please explain?
- How do you think you made use of the technology, and your knowledge of content and pedagogy throughout the mathematics coaching?
- What are the strengths of the mathematics coaching with respect to the development of the use of technology in mathematics lessons? Could you please explain?
- What are the weaknesses of the mathematics coaching with respect to the development of the use of technology in mathematics lessons? Could you please explain?
- Has the training provided meet your expectations? Why?
- What would you like to be changed and what would you like to be kept in the mathematics coaching process? Why?
- What do you think can be done to improve the mathematics coaching?
- How did this training you received contribute to you professionally?
- Is there anything else you would like to add?

3.6 The pilot study

The administration of a pilot case study was the final preparation for data collection (Yin, 2009). The pilot case study helped to refine the data collection considering the content of the data and the procedures followed. For these purposes, the researcher conducted a pilot study with a novice elementary mathematics teacher (Erdogan) in a public school. He had two years of mathematics teaching

experience and his class consisted of 8 students. His school was in the country side of the Black sea region and was a school with high achievement rates in the national exams. In his school there was a projector and just one computer for the teacher. He effectively used technology and social media in his daily life. In addition, he declared that he knew the dynamic geometry environments, such Geogebra, and Cabri 3D. He was also willing to improve himself in terms of technology integration. He voluntarily participated in this pilot case study. For the analysis of the progress of the teacher's TPACK before, during, and after mathematics coaching, the data collection tools (explained in detail in the data collection section), namely pre-interview, pre-observation, pre-conferences, observations, post-conferences, post-observation, post-interview and mathematics coaching interview, were conducted. After conducting the pilot case study, the researcher increased the number of mathematics coaching cycles from three to four in mathematics to better elicit participants' progress in teacher TPACK. Increasing the number of cycles in the mathematics coaching was the most significant changes were made in the study. The teacher in the pilot study agreed with the usefulness of the data collection tools. Furthermore, the pilot study assisted in assessing the researcher's role as a coach during pre-conference, observation and post-conference. During mathematics coaching, the reseacher experienced how to support the development of teacher's TPACK. For example, one of the most important properties of mathematics coaching, the teachers should feel free to say their ideas and to reject or accept the suggestions in each parts of mathematics coaching. The pilot of study provided an opportunity to internalize the role of coach. In addition it helped to the coordination time and the data sources needed to conduct the main study.

3.7 Data Analysis

In qualitative research study, data collection and analysis are simultaneous activities and are reflective and ongoing processes (Merriam, 1998). Patton (2002) stated that the first data analysis begins with data collection. According to Bogdan

and Biklen (1992, p.153), “data analysis is the process of systematically searching and arranging the interview transcripts, field notes, and other materials that you accumulate to increase your own understanding of them and to enable you to present what you have discovered to others.”

In order to produce an in-depth description of the development of in-service mathematics teachers’ TPACK on the polygons unit as a geometry concept, questionnaires, pre-conferences, observations, field notes, post-conferences and semi-structured interviews were used as the basis for the analysis of this study. All the audio recordings of the pre/ post -interview, pre-conferences, post-conferences, and interviews, and the videos of the classroom observations were transcribed. All information about the teachers was organized and used together in a case study data base.

To address the first research question and its sub-questions: “To what extent does participating in mathematics coaching impact elementary mathematics teachers’ technological pedagogical content knowledge in the field of geometry?”, the transcripts of the pre/post-interviews, pre-conferences and post-conferences, video recordings and the observations before, during, and after mathematics coaching, and the field notes of these observations were analyzed. The progress of teachers’ TPACK was determined by considering the TPACK Development Model using deductive analysis. Deductive analysis is appropriate “where the data are analyzed according to an existing framework” (Patton, 2002, p. 453). In the deductive analysis, themes and codes are pre-selected based on previous literature, theories or the specifics of the research question. The first step into data analysis was to transcribe the audiotaped interviews and videotaped observations. Then, the researcher read the transcripts and watched and listened the records to check for accuracy. The researcher highlighted the relevant quotes and noted the TPACK development level (i.e., recognizing, accepting, adapting, exploring, advancing) and the theme (curriculum and assessment, learning, teaching, access) by using TPACK Development Model (see Appendix J). In this model there were 11 descriptors and their examples to help clarify of TPACK levels. The transcripts of the pre-interview

and pre-observation and the video recordings and the transcripts of pre-observation were used to determine the teachers' TPACK level before mathematics coaching. The transcripts and recordings of the pre-conferences and observations and post-conferences were analyzed to define the teachers' TPACK level in the related mathematics coaching cycle. Then, the transcripts of the questionnaire conducted after the mathematics coaching (post-questionnaire) and the video recordings of the post-observation helped to determine the teachers' TPACK level after mathematics coaching. The descriptors of the TPACK Development Model (except the professional development of teaching theme) were analyzed from the data gathered before, during and after mathematics coaching. The level of the professional development of teaching theme was defined from the questionnaire conducted before and after mathematics coaching.

Every descriptors of theme in the sequential TPACK Development Model was described in details. For examples, the assessment descriptor of curriculum and assessment theme is given below:

Recognizing level: Resists idea of technology use in assessment indicating that technology interferes with determining students' understanding of mathematics.

Example: Does not allow calculator use when assessing students' understanding of solving linear equations.

Accepting level: Acknowledges that it might be appropriate to allow technology use as part of assessment but has a limited view of its use (i.e., use of technology on a section of an exam).

Example: Attends and participates in a mathematics assessment professional development to consider ideas for assessing students' understanding of solving systems of linear functions using the calculator as a tool. Mimics the assessment idea to explain the use of the calculator for solving systems of linear functions by using the trace function to identify the intersection. Often retests technology questions with paper and pencil questions to be sure that the concept was learned the 'right' way.

Adapting level: Understands that if technology is allowed during assessments that different questions/items must be posed (i.e., conceptual vs. procedural understandings).

Example: Allows use of calculator in an assessment but designs the assessment to focus on gathering students' conceptual understanding of

solving systems of linear functions in addition to their procedural understanding.

Exploring level: Actively investigates use of different types of technology-based assessment items and questions (e.g., technology active, inactive, neutral or passive).

Example: Designs assessments where students are expected to show their understanding of mathematical ideas using an appropriate technology that extends beyond paper and pencil type questions.

Advancing level: Reflects on and adapts assessment practices that examine students' conceptual understandings of the subject matter in ways that demand full use of technology.

Example: Develops innovative assessments to capture students' understandings of the mathematics embedded in the particular technology. (Niess et. al., p. 20-21)

As it is seen, there were explanations and examples for each TPACK level for assessment descriptor of curriculum and assessment theme. For example, Geogebra provides an environment to assess the students' knowledge. The researcher checked whether the teacher integrated Geogebra as assessment tool or not. If the teacher did not allow to use Geogebra in the assessment, it is indicated that he/she is in the recognizing level. In addition second specialist in mathematics education coded the data. If there were inconsistencies in the coding, the researcher and second coder discussed and consensus was reached at the end of the coding process.

To answer the second research question, "What are elementary mathematics teachers' perceptions of mathematics coaching in terms of TPACK", the researcher analyzed the recordings of the semi-structured interview. In order to analyse the interview data, the inductive process of open coding was administered. The transcripts of the interview were analyzed in the sequence of organizing the data, generating categories, themes and patterns, coding the data, testing the emergent understandings, searching for alternative explanations and writing reports (Marshall & Rossman, 2011). There were four themes: (1) benefits of the mathematics coaching, (2) differences between mathematics coaching and regular in-service training, (3) challenges of mathematics coaching, and (4) suggestions. The benefits of mathematics coaching includes improving their TPACK, improving instructional practice, increasing student achievement, providing scaffolding/collaborating, and

building confidence. A difference between mathematics coaching and regular in-service training themes has two codes which are focusing on process, and putting together practice and theory. Challenges of mathematics coaching theme includes preparing lesson, and being stressful. The last theme (suggestion) has five codes: should be common in turkey, should be semester-long, should be conducted in different subject, should be used with different mathematics software, and should be in group.

3.8 Trustworthiness

Merriam (1998) stated that “data do not speak for themselves; there is always an interpreter, or a translator. One cannot observe or measure a phenomenon or event without changing it, even in physics where reality is no longer considered to be single-faceted. Numbers, equations, and words are all abstract, symbolic representations of reality, but not reality itself” (p. 202). Therefore, the researcher can achieve interpretation of the reality, which is closer to the reality through observations and interviews. Validity and reliability are important issues for all researchers when designing a study, analyzing the data and judging the quality of the study (Patton, 2002). Validity determines whether the findings are accurate from the point of the view of the researcher, the participant or the reader of a study in qualitative research (Creswell & Miller, 2000). Lincoln and Guba (1985) prefer to use alternative terms for the concepts of internal validity, external validity, reliability and objectivity: credibility, transferability, dependability and confirmability, respectively. They believe that these terminologies are more suitable for the nature of the qualitative research. In the following part, evidences for trustworthiness of the study were given.

3.8.1 Credibility

Credibility in qualitative research is related to the congruence of the research findings and the reality (Merriam, 1998). Merriam suggested the six strategies for

increasing credibility of qualitative research: triangulation, member checks, long-term observation, peer-examination or peer debriefing, participatory or collaborative modes of research and clarifying researcher's biases. In this study, triangulation, member checks, peer examination and long-term observation were utilized to ensure credibility.

The first procedure for credibility was data triangulation which means that "Researchers make use of multiple and different sources, methods, investigators and theories to provide collaborating evidence (Creswell, 2007, p. 208). In this study, data triangulation and investigator triangulation were used. The obtained multiple evidence from different sources, namely pre-conferences, classroom observations, post-conferences, interviews, and researcher's fieldnote, were conducted in the present qualitative study. In addition, the researcher worked with 2 mathematics teachers more than one individual. In order to ensure investigator triangulation, second coder checked the data analysis and coded the data and the codes were examined.

The second procedure was a member check for credibility, which necessitated the reporting of the data, analyses, interpretations, and conclusions back to the participants so that they could judge the accuracy and credibility of the account (Creswell, 2007). During the questionnaire, pre-conferences, post-conferences, and the interview, the researcher and teachers held discussions on their responses to interpret the teachers' responses correctly. In addition, in this study, the transcripts of the study were presented to the teachers to seek their representation.

The third procedure was peer examination which means that "asking colleagues to comment on the findings as they emerge." (Meriam, 1998, p.204). My colleagues in mathematics education participated in the analysis of data in this study.

The last procedure was prolonged engagement and persistent observation in the field. This procedure includes "building trust with participants, learning the culture, and checking for misinformation that stems from distortions introduced by the researcher or informants" (Creswell, 2007, p. 201). Thus, observations were

performed in trustable and comfortable environments. In addition, before the study the researcher became closely connected with the teachers' mathematics classroom to obtain the trust and comfort zone of both the teachers and students. As for the duration of the study, it lasted 2 months for each teacher.

3.8.2 Dependability

Dependability which is the another criteria to ensure the trustworthiness of the qualitative research refers to reliability in quantitative research. To ensure dependability during data analysis, a second researcher, a doctorate in mathematics education, examined the coding process and acted as second coder. Inter-rater reliability between the two coders was calculated according the formula of Miles and Huberman (1994):

$$Reliability = \frac{Number\ of\ agreements}{total\ number\ of\ agreements + disagreements} \times 100$$

The inter-reliability of two coders was calculated to be 95% by using this formula. The inconsistencies in the coding were discussed and consensus was reached at the end of the coding process.

3.8.3 Transferability

A thick and rich description allows readers to make decisions regarding transferability referring to external validity. When descriptions of participants and settings are thick and rich, the information can be transferred to other settings and help to determine whether the findings can be transferred because of shared characteristics (Creswell, 1998, p. 203). Hence, in the present study, the researcher made a thick and rich description of the participants and settings. The context of the study and and participants were described in detail. Furhertmore, the study was conducted with two elementary mathematics teachers.

3.8.4 Confirmability

Confirmability is “concerned with establishing that data and interpretations of the findings are not figments of the inquirer’s imagination, but are clearly derived from the data” (Tobin & Begley, 2004, p. 392). Lincoln and Guba (1985) suggest that to ensure confirmability of a qualitative study, an audit trail, reflexive journal and triangulation ought to be used. According to Bowen (2009) an “audit trail offers visible evidence—from process and product—that the researcher did not simply find what he or she set out to find” (p. 307). For audit trail, the inquiry process, the raw data, interview and observation notes, documents and recordings collected from the field were kept for crosschecking. Furthermore, in this study the researcher gave direct quotations in order to increase confirmability.

3.9 Role of the researcher

The researcher as a coach collaborated with individual teachers through coplanning, reflecting and assessing. The researcher developed positive and supportive relationships with the teachers. In addition, the researcher planned, collaborated and reflected together with the teachers to promote growth in their TPACK. The role of the researcher was an active listener and encourager as the teachers elaborated and explained their perspective of technology integration. The researcher demonstrated best practices of Geogebra in mathematics instruction to the teachers who acted as active participants to improve mathematics achievement. The role of the researcher as a coach monitored program implementation to elaborate on the strengths and weaknesses of technology-enhanced lessons. After gathering data about teachers’ TPACK, the researcher analyzed and interpreted them to inform teachers about how to combine pedagogy, mathematics and technology. Furthermore, the researcher prepared the pre-conference, observation and post-conference protocol to collect data about teachers’ TPACK, and other questions were asked for more detailed explanations from the teachers. Open-ended questions were asked to gather as much data as possible on how elementary

mathematics teachers progressed through the TPACK Development Model regarding geometry. Furthermore, because of the deficiency of technological devices in the teachers' classroom, the researcher provided every 3 or 4 students with a computer.

In addition to researcher role as coach, in qualitative research the researcher has crucial role as collecting and analyzing the data (Merriam, 1998). According to Johnson (1997), qualitative research is open-ended and less structured than quantitative research. Therefore, researcher has a potential threat to validity. In this study, I had a long-term involvement in the context to researcher bias. I meet up the participants before the beginning of the study. In addition, I involved the participants' lesson before the study to help them get used to communicate with me.

CHAPTER IV

FINDINGS

The main goal of this study was to examine the progress of elementary mathematics teachers' Technological Pedagogical Content Knowledge (TPACK) in geometry through participation in mathematics coaching. The TPACK Development Model (Niess et al., 2009) was used as a theoretical framework to analyze the data gathered in this case study.

This chapter summarizes the findings of the research study under two main sections and related sub-sections. In the first section, the progress of TPACK of two elementary mathematics teachers in the field of geometry through participation in mathematics coaching was analyzed under the TPACK Development Model. More specifically, the results addressing the first question and its sub-questions of the study are presented in the first section under the following headings: Curriculum and Assessment Theme, Learning Theme, Teaching Theme and Access Theme. The second section addressing the second research question of the study presents the teachers' perceptions of mathematics coaching in terms of TPACK.

4.1 The TPACK Development of Mathematics Teachers

This section depicts how the teachers -Murat and Esen- combined and put Geogebra, geometry and pedagogy into practice during their lessons. It sheds light onto the TPACK development of Murat and Esen as elementary mathematics teachers in geometry before/during/after attending mathematics coaching session. Their performances were evaluated based on the four themes (curriculum and assessment, learning, teaching, and Access) and related descriptors (curriculum, assessment, learning- mathematics learning, conception of student thinking,

teaching- mathematics learning, instructional, environment, Professional development, usage, barrier, and availability) of the TPACK development model (Niess et al., 2009).

4.1.1 The Curriculum and Assessment Theme

The curriculum and assessment theme included two descriptors: curriculum, and assessment descriptors. The curriculum descriptor focuses on the treatment of the subject matter and how curriculum and curricular materials and technology are integrated in the teaching and learning process. The assessment descriptor emphasizes teachers' decision-making processes regarding assessment and how they assess students' understandings with the integration of technology in. In the subsequent sections, Murat's and Esen's TPACK is analyzed according to the TPACK Development Model's curriculum and assessment theme.

4.1.1.1 Mathematics Teachers' TPACK Level for Curriculum and Assessment Theme Before Mathematics Coaching

The lessons of Murat and Esen were observed for two hours to determine their TPACK level before the mathematics coaching process (weekly pre-conference/observation/post-conference). All quotes and dialogs in the following paragraphs were derived from the TPACK pre-interview and the pre-observation lesson prior to the mathematics coaching.

4.1.1.1.1 Murat's TPACK Level for Curriculum and Assessment Theme before Mathematics Coaching

Murat's performance before mathematics coaching indicated *the accepting level* of TPACK for the curriculum descriptor whereas *the recognizing level* of TPACK for the assessment descriptor in the curriculum and assessment theme.

At the beginning of the lesson Murat sought to motivate his students and attract their attention by asking them where they saw polygons in their everyday

environment. These questions provided students with the opportunity to connect the subject with the real world context. Then they discussed the properties of polygons, such as the minimum number of sides polygons have, by using the polygon examples provided by the students. The whole group discussion helped students to remember the concept of polygon. Then he projected on the board some google images of polygon examples. The whole class examined each image to see whether or not they were polygons. These warm-up activities were strongly linked with the main lesson whose objective was “to determine the interior and exterior angles and diagonals of polygons” (MoNE, 2013). Students were able to revise the important knowledge about polygons in that polygons have at least 3 sides and these sides cannot be a curve. In addition, the use of these activities helped students to associate polygons as a mathematics concept with their daily life. They were aligned with the curriculum goals to reinforce the concept of polygon. Furthermore, searching for polygons as google images was an innovative way to revise the properties of polygons. Despite the advantages of these warm-up activities, Murat did not use Geogebra as a technological tool to support students’ education in mathematics.

After the warm-up activities, Murat distributed the “Introducing polygons” activity worksheet which he had created to the students groups. This technology-supported activity provided the small student groups with the opportunity to investigate concave and convex polygons via Geogebra. Murat opened a new Geogebra file and projected it on the blackboard. He constructed polygons similar to those in the activity worksheet to show how polygons are constructed in Geogebra. Then the students were asked to construct three different polygons, similar to those in the activity worksheet by using the polygon tools in Geogebra (see in Figure 4. 1 The “Introducing polygons” activity).

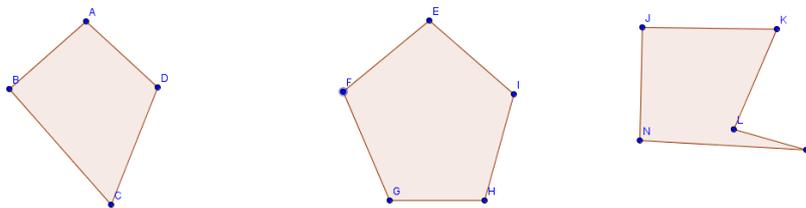


Figure 4. 1 The “Introducing polygons” activity

After constructing these concave and convex polygons, Murat constructed segments between two vertices in the first polygon to lead students to carry out the next instruction in the activity worksheets. According to this next instruction, students were required to draw segments between nonconsecutive vertices of polygons by using segment tools between two points in Geogebra. After students constructed every segment of these three polygons, Murat directed them to discover the properties of diagonals by asking questions. This Geogebra-enhanced activity helped students to define diagonal as a line segment joining two nonconsecutive vertices of a polygon. Then the whole class discussed the position of these diagonals which lead to discovering the concave and convex polygon. The concave and convex polygon was a new concept for the students. Murat preferred to use Geogebra for the new geometric concept of concave and convex polygon instead of reinforcing the already-learned concepts. However, technology was mostly used for the teacher’s presentations and the students followed their teacher on their computer via Geogebra.

After the “Introducing polygons” activity worksheet, Murat distributed the “Finding the number of diagonals” activity worksheet as a last activity of the day (see Figure 4. 2). In this activity, students investigated the relationship between the number of sides of a polygon and the total number of diagonals. Constructing diagonals from a single vertex in each polygon was asked for in the first instruction of this activity.

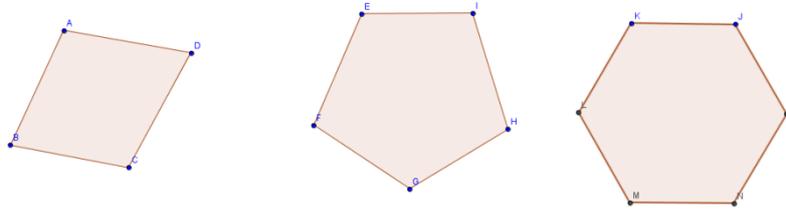


Figure 4. 2 The “Finding the number of diagonals” activity

Then he wanted to complete the table on the top of the Activity Worksheet (Table 4. 1) according to the findings relating to the number of sides of a polygon to the number of diagonals from a single vertex. Creating the polygons and their diagonals from a single vertex via Geogebra helped students to be able to complete the first second columns of the chart. By using the patterns that the students found, they were able to discover the relationship between the side number and diagonal numbers in a single vertex.

Table 4. 1 The number of diagonals

Kenar Sayısı	Köşegen Sayısı	<i>bir köşeden çizilen köşegen sayısı</i>	<i>Toplam köşegen sayısı</i>
4			
5			
6			

Then Murat wanted the students to construct all the diagonals of a quadrilateral, a pentagon and a hexagon respectively via Geogebra. Then he forwarded a question about the number of diagonals of a pentagon. The dialogue below occurred as students answered the question.

M: So how many diagonals are there in a hexagon?

S1: 5

M: Emre says there are five; let's try to draw them.

S1: 10,

S2: 5,

S3: 8

M: Raise your hand.

S4-S5: 5,

S6-S7: 10,

M: Let's draw them together.

This dialogue indicated that technology is used as a verification tool by graphical representation. Then he allowed the students some time to construct the diagonals of a hexagon via Geogebra. Murat wanted the students to fill in the table on the top of the Activity Worksheet (Table 4. 1) according to the total number of diagonals. With the help of the teacher's guiding questions, the students extended the chart down to the heptagon. Then a whole class discussion was conducted so that students could share their results. This technology-enhanced activity helped students to look for a pattern and write an equation to find the number of diagonals for a polygon of any number of sides. This Geogebra-enhanced activity indicated that the teacher had gained awareness of some of the advantages of technology in the teaching and learning process of polygons, such as verification.

Murat created his own activity worksheets, such as the "Introducing polygons" and "Finding number of diagonals" activities to help students investigate concave/convex polygons and the number of diagonals for polygons by means of technology. These technology-enhanced activities demonstrated that Murat integrated Geogebra into his classroom practices to explore the new concepts instead of reinforcing the already known concepts in the mathematics classroom. It showed that Murat "understands some benefits of incorporating appropriate technologies as tools for teaching and learning the mathematics curriculum" (Niess et al., 2009) at *the adapting level*. Additionally, during the pre-interview where he was asked how to integrate Geogebra into the curriculum, Murat said:

"What does the student do with Geogebra – the child finds the answer him/herself and this enables him/her to internalize the concepts better. I do not give it directly to the child. I enable the child to find it somehow. For example, while moving on from the square to a rhombus, I aim to cut. Let's say I take a rectangular paper and cut its sides so that it becomes a rhombus. Then I transmit to area concept. Thanks to dynamic geometry, we enable the students to find this directly on the computer without having them striving in this way. I can do this but I haven't done this yet. But it can be done. I can also have students find the relationship between the length of the sides and the area by having them write the formula of the variables directly onto the computer because I do an operation each time but actually when finding the area I can I mean perhaps I can use this..... My aim, let's say it is to discover. There

is discovery in the lesson activities. Somehow the child concentrates more in the lesson. Let's say the child is more engaged when seeking for a general concepts or constructing the formula for the area. S/he feels that s/he has found the answer him/herself. By finding it him/herself, the child internalizes it. S/he says, oh, it's like this. Perhaps it reinforces his/her learning process. I don't show it directly on the board; instead, it enables the child to realize that the multiplication of these two sides yield this, or that in a rhombus when explaining a.h, why I am taking the height from there."

This statement demonstrated that he focused on the exploration of a new concept when he integrated Geogebra into the curriculum. Murat's aim was for students to investigate the topics related to polygons via Geogebra. In fact, Murat incorporated Geogebra into almost every phase of his lesson (except for the warm-up and assessment activities). He was aware of some benefits of incorporating Geogebra as a tool for the implementation of the mathematics curriculum. But he used Geogebra as a replacement for the non-technology based tasks within a traditional curriculum approach. This implied that he was not able to develop innovative ways to use technology to develop mathematical thinking in students via Geogebra as indicated at higher level of TPACK Development Model. He did not take the dynamic properties of Geogebra into consideration. He could not modify the mathematics curriculum to efficiently and effectively incorporate Geogebra as a teaching and learning tool. His lesson was strongly aligned with the curriculum goals. In the pre-interview, he explained how he prepared his lessons,

"First for example, I look at the objectives when preparing the polygons subject so that I don't go digress from the objectives because there are things that need to be and shouldn't be taught regarding polygons. Now it says convex and concave polygons are to be mentioned but details should not be given. So I prepare taking this into consideration. I also looked at the activities in the textbook and designed [my activities] accordingly."

This statement revealed that Murat did not advance curriculum to take advantage of technology as a tool for the teaching and learning process. Furthermore, he replaced non-technology based tasks in the traditional curriculum

without considering the dynamic properties of Geogebra as *at the accepting level*. Thus, technology was not extended beyond the paper and pencil, such as dragging to develop geometric concepts. That is, most of the time during his pre-observation lesson, technology was used for visualization of geometric concepts. In fact, during his pre-interview, he made a point about the visualization properties of Geogebra:

“I can use technology for the sake of having them visualize it more vividly. Then especially when classifying polygons. For example, when explaining the concept of congruence I explain by drawing [figures] on the board. Their side, side lengths, and angles are the same. For the child this is a little abstract. I tell them to draw a polygon. I say we are going to draw a congruent polygon. After we draw it, we look at its angles. Their angles and their side lengths turn out to be the same. Then we say, these are the same. Their angles are the same and so are their sides. What can we call these? Congruent polygons.”

As understood from this statement, he integrated Geogebra to enhance the students’ understanding. However, he did not use the dynamic properties of Geogebra. Geogebra activities were adapted to the non-technology curriculum. On the other hand, Geogebra sometimes was used as a verification tool to explore mathematics during the technology-enhanced activities. This implied that the primary goal for employing Geogebra was to demonstrate and verify geometric ideas. Hence, this lesson provided evidence that Murat was at the *accepting level* for the curriculum descriptor although he focused on students’ exploration of polygon via Geogebra. He “expressed his desire but demonstrated difficulty in identifying topics in his own curriculum for including technology as a tool for learning.”(Niess et al., 2009, p. 20). Murat viewed the use of technology as a demonstration tool for adequate performance in a lesson that was more in alignment with their curriculum.

In addition to curriculum descriptor of Curriculum and Assessment Theme, Murat’s performance before mathematics coaching indicated *the recognizing level* of TPACK Development Model for the assessment descriptor. *At the recognizing level*, teachers have suspense for using appropriate technology as an assessment tool. Murat implemented a technology-enhanced activity, which consisted of a worksheet on “Introducing polygons” to maximize students’ learning of concave

and convex polygons. He incorporated knowledge of all students' understanding, thinking and learning of concave and convex polygons via Geogebra. After using technology as an educational tool, Murat projected the students' mathematics book on the blackboard to assess the students' understanding of concave and convex polygons. He asked students to identify whether the projected polygons (Figure 4. 3) were concave or convex polygons. Then he called students to the blackboard to solve questions. Volunteer students drew diagonals of different polygons on the blackboard to identify whether they were concave or convex without using Geogebra. Murat did not integrate technology when assessing concave and convex polygons taught with the use of technology. This indicated that Murat had hesitations regarding the application of Geogebra to facilitate an effective assessment strategy.

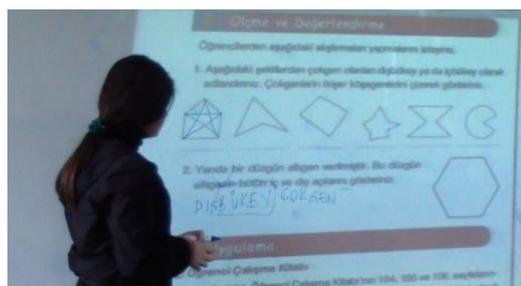


Figure 4. 3 The assessment of pre-observation

In another assessment activity, students were asked to find the number of diagonals in heptagons and decagons. Murat wrote these two questions on the board: “How many diagonals can be drawn on a heptagon?” and “How many diagonals can be drawn on a decagon?” Then he gave time to solve these questions and walked among the students to respond to any questions they might have had. However, he did not allow students to use Geogebra when they tried to find a solution. Some students used a formula as an algebraic representation and some of them drew polygons using a paper and pencil to find diagonals. Then the students who volunteered displayed their algebraic solutions on the board.

These two assessment activities implied that Murat did not understand the role technology played in assessment. He did not allow Geogebra to be used when

assessing students' understanding of concave/convex polygon and the number of diagonals of polygon although he used Geogebra when these concepts were taught. Additionally, in the pre-interview when he was asked how to integrate Geogebra into the assessment process, he stated,

“Generally I give homework to the students telling them that there may be a similar task in class. For instance, in the previous classes there were some students who were eager to use the program. Dynamic software...Some of them came and showed me what they did. There are some nice outputs. They do some things in finding the area. I also assigned it as homework, as performance work.”

This statement demonstrates that he incorporated technology as an add-on assessment activity for willing students. He had a limited understanding of assessing students' knowledge via Geogebra. Furthermore, he talked about how to use Geogebra in assessment, saying

“Somehow I design an activity in the last phase of the activities I do in class. I ask one question and end the class. I ask them a question related to this formula I have found or related to the definition. For example, a written exam can be prepared on questions that can measure whether the teaching objectives are achieved. In this aspect Geogebra can be used.”

It can be understood from this quotation that he used Geogebra to prepare a paper-based assessment activity. Then he added,

“They found the general terminology, and then placed it in the formula to see if it met the ... I ask them to compare their answers. In this way, they check their own answers. I thought they could think about whether they did it correctly or incorrectly, perhaps that kind of thing.”

He admitted that he might incorporate Geogebra to verify their understanding of polygons. The pre-observation indicated that Murat had a limited view of the usage of Geogebra in assessment. Besides, he did not incorporate Geogebra to assess students' knowledge of polygons. So, Murat's pre-observation performance was aligned with the following statement: “Resists idea of technology use in assessment indicating that technology interferes with determining students' understanding of mathematics.” in *the recognizing level* (Niess et al., 2009). He

was concerned about the idea of using technology in assessment to determine students' understanding of geometry.

4.1.1.1.2 Esen's TPACK Level for Curriculum and Assessment Theme before Mathematics Coaching

Esen's performance before Mathematics Coaching demonstrated *the recognizing level* of TPACK for the curriculum descriptor of the curriculum and assessment theme. At the beginning of the lesson, she asked the students to give polygon examples in their daily environment to motivate and attract their attention. Then she projected a powerpoint file related to the polygon unit (seeFigure 4. 4) to show examples in daily life. She wanted students to find the polygons and their names. In addition, she supported them to find different polygons such as convex and concave polygons. But they did not give a name and they just recognized the convex and concave polygons. The use of this activity provided the students with an environment in which they could link polygons as a mathematics concept with their daily life and recognize different polygons in daily life. In addition, the activities were aligned with the curriculum goals to support the concept of polygon. Like Murat, she did not integrate technology at the beginning of the lesson to support students' knowledge.



Figure 4. 4 The figures in polygon unit

After the warm-up activity, Esen distributed a geometry board as concrete manipulative for students groups to improve students' knowledge of concave and convex polygons (seeFigure 4. 4). At first, she asked them to construct a triangle by using elastic bands and they discussed its properties such as including three vertices and three segments. Then, the students constructed quadrilaterals and showed their

quadrilateral. At first the students drew convex polygons such as a square, rectangle and scalene quadrilaterals. Then Esen motivated them to draw different quadrilaterals and shared one of the students' examples on the geometry board, which was a concave polygon. Then the students analyzed the properties of this convex quadrilateral. Subsequently, the students constructed a pentagon, hexagon and heptagon, respectively. Then the teacher summarized the properties of these polygons by saying, for example, "they are made of straight lines" and "the shape should be closed". She asked students to draw diagonals on the geometry board after her demonstration of the diagonals of convex and concave quadrilaterals. Then the students built diagonals in concave and convex polygons. Furthermore, Esen distributed toothpicks to the students for them to construct polygons, such as triangle, quadrilaterals and pentagon. They constructed the diagonals of polygons to find the difference between a convex and concave. They realized that at least one of the diagonals of concave polygons lies outside. In these activities, Esen did not integrate Geogebra as a technology tool for the students to discover convex and concave polygons as new concepts for students.

After using the geometry board and toothpicks, Esen draw a polygon on paper to discuss the interior and exterior angles of polygons (see Figure 4. 5). She explained that when the interior angle and exterior angle are added, they get a straight line, which is 180° . Then she allowed time for the students to draw a different polygon and its interior and exterior angles. Students explored that the interior and exterior angles are supplementary.



Figure 4. 5 The activities with concrete materials

At the end of the lesson, Esen projected a blank Geogebra file on the board. She did not allow her students to use Geogebra because there was just one computer for the teacher's usage. She drew a hexagon and built its diagonals using segment tool in Geogebra (see Figure 4. 6). The students said that it was a concave because the diagonal was outside of the polygon. Then she drew another hexagon and built every diagonal of this hexagon. The students explained that every diagonal of convex polygons should be inside. She used Geogebra as a demonstration tool after teaching concave and convex polygons as new concepts by using concrete manipulatives such as a geometry board. In addition, after using Geogebra, she integrated Geogebra to demonstrate the interior angle of different polygons. This technology integration indicated that she integrated these activities as supplements to instruction after teaching geometry concepts. Furthermore, she used Geogebra for the visual representation as indicated in the *recognizing level*.

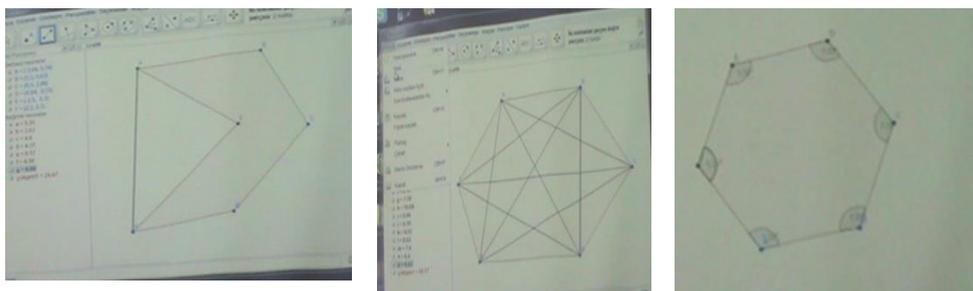


Figure 4. 6 The screenshots of Geogebra activities

She integrated Geogebra in her lesson to reinforce the concepts by using concrete manipulatives in the mathematics classroom in place of exploring new concepts in the curriculum. This indicated that she was at a lower level than Murat in the TPACK Development Model because Murat implemented Geogebra to help students to discover concave and convex polygons as new concepts. In parallel to Esen's lesson, in the pre-interview she stated,

“After showing the drawings, we will show the drawings again on electronic media. They will see them with further proof. During the explanation that a side of a polygon is equal, the students will be able to see clearly by measuring. There will be further proof. Yes, we are drawing and giving them ready to the children but instead of measuring

them in cm, they will see them on the program. We will observe the benefits of this.”

This statement provided evidence that she viewed Geogebra as a demonstration tool to reinforce concepts. She “acknowledged that mathematical ideas displayed with the technologies can be useful for making sense of topics addressed in the curriculum” (Niess et al., 2009, p. 20), which shows that she was at *the recognizing level* for the curriculum descriptor. She conceived that Geogebra helped to visualize concave/ convex polygons and the interior of polygons but she was unsure of how this might help the students learn these concepts. So, she implemented Geogebra after these concepts were learned. In addition, her lesson was strongly aligned with the curriculum and textbooks. Because of this, she integrated Geogebra as a static tool without considering the dynamic nature of Geogebra. This usage was similar to Murat’s lesson before mathematics coaching. But the main difference between them was that Esen implemented Geogebra to supplement instruction at the end of the lesson after teaching the concepts.

Esen’s performance for the assessment descriptor was the lowest level of the TPACK Development Model, as with the curriculum descriptor. She did not integrate Geogebra to assess the students’ knowledge on polygons. She just projected some figures and asked the students to identify what kind of a polygon they were, such as concave or convex polygon (see Figure 4. 7). These assessment questions were similar to Murat’s assessment questions before mathematics coaching. Both of them hesitated to implement Geogebra as an assessment tool. Furthermore, in the pre-interview Esen declared that “I don’t think it will provide many benefits [in assessment]. I believe that it’s an area to which it will provide the least benefits. I don’t use it much.” This demonstrated that she did not consider the advantages of Geogebra as an assessment tool. So she did not have students use Geogebra when assessing their understanding of polygons. This strongly coincided with the description that goes as follows: “Resists idea of technology use in assessment indicating that technology interferes with determining students’

understanding of mathematics” (Niess et al., 2009, p.20) just as at *the recognizing level* for the curriculum descriptor in the curriculum and assessment theme.

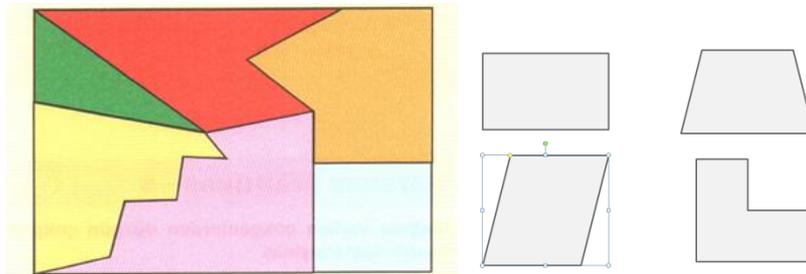


Figure 4. 7 The screenshots of assessment activities

4.1.1.2 Mathematics Teachers’ TPACK Level for Curriculum and Assessment Theme in the MCC1

The lessons of Murat and Esen were observed for two hours to determine their TPACK level in the first coaching cycle during the mathematics coaching process. All the quotations and dialogs in the following paragraphs were derived from the pre-conferences/post-conferences and the mathematics lesson from the mathematics coaching cycle 1.

4.1.1.2.1 Murat’s TPACK Level for Curriculum and Assessment Theme in the MCC1

Murat’s MCC1 performance revealed *the accepting level* of TPACK for the curriculum descriptor whereasthe *recognizing level* of TPACK for the assessment descriptor in the curriculum and assessment theme.

At *the accepting level* for curriculum descriptor, the teachers were willing to try out some activities in the classroom that typically mimicked the activities in their own tradition curriculum. In MCC1 lesson, Murat started to ask questions about concave and convex polygons to revise the previous lesson as students got their assigned computers. Then he distributed “the Angles of Polygons” activity worksheet which he had created to the students groups. In this activity, students

constructed a quadrilateral by using the line tool in Geogebra. Murat demonstrated how students could construct an interior angle of polygon. Then they found the interior angles in a polygon by using the angles tool. Every student group expressed the value of the interior angle of the quadrilateral. Subsequently, the students found the exterior angle of the quadrilateral and shared their results (see Figure 4. 8). This activity helped students to notice how the interior and exterior angles always added to 180° . In this activity, each student group drew a different scalene quadrilateral to investigate the relationship between the interior and exterior angle. This technology-enhanced activity provided students with the opportunity to see that the interior and exterior angles were supplementary. In this activity, the relationship between the interior and exterior angles was a new concept for students. Murat preferred to integrate Geogebra into the lesson to teach the interior angle and exterior angle as a new geometric concept instead of reinforcing the already-learned concepts. However, technology was mostly used for the teacher's presentations.

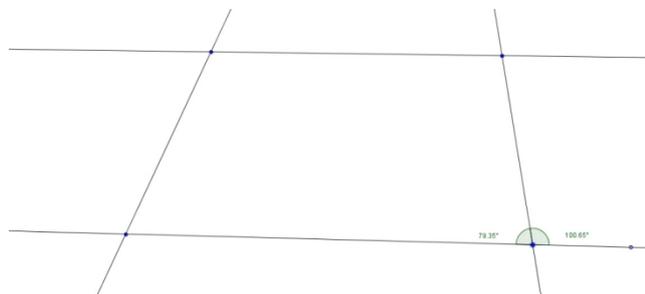


Figure 4. 8 The Angles of Polygons Activity

After the “Angles of Polygons” activity worksheet, Murat distributed the “Finding the sum of interior angle of polygons” activity worksheet. Then, the students opened the polygons.ggb file, which has been prepared previously (see Figure 4. 9).

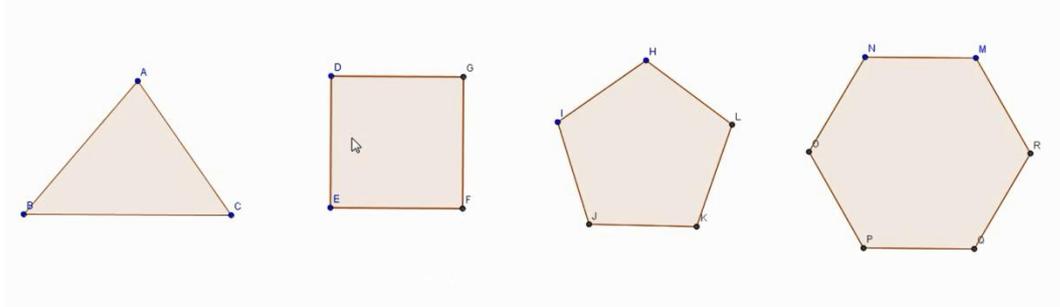


Figure 4. 9 The “Finding the sum of interior angle of polygons” activity

In this activity, students investigated the sum of the interior angles of a polygon by using the triangle via Geogebra. Murat asked students to analyze their first shape which was a triangle and fill the table in their activity worksheet (see Table 4. 1). Then he asked one student to read the next instruction in the activity sheet aloud. According to the instruction, the students were required to construct diagonals from any one of the vertices to others in the quadrilateral via Geogebra. At this step, the groups chose different vertices of the quadrilateral, and then they realized that all the quadrilateral form two triangles from any vertex. Then they tried to find how many triangles a pentagon contained by constructing diagonals via Geogebra. They noticed that a pentagon had 5 sides and 3 triangles formed by connecting the vertices. Consequently, they investigated the sum of the interior angles of a pentagon as $3 \times 180 = 540$ by using Geogebra. After the students filled the related spaces in Table 4. 2, they created diagonals of the hexagon to find out how many triangles there were in a hexagon. With the teacher’s guidance, students realized the pattern in the Table 4. 1, that the number of triangles is 2 less than the number of sides. They were able to discover the relationship between the side numbers and the sum of the interior angles of a polygon, which is $180(n - 2)$ since the number of sides of a polygon was formed as n . Although Murat helped students to realize the relationship between the side and the sum of interior angles with this technology-enhanced activity, he could not effectively utilize technology as a dynamic tool.

Table 4. 2 The sum of the interior angles of polygon

The number of Sides	The number of triangle	The sum of interior angles
3		
4		
5		
6		

These Geogebra-enhanced activities demonstrated that Murat preferred to use Geogebra for a new geometric concept instead of reinforcing the already-learned concepts. However, technology was mostly used for the teacher's presentations as in the pre-observation lesson. Additionally, the students used Geogebra to investigate geometrical relations statically rather than dynamically. This indicated that although he was aware of some of the benefits of technology, he was unable to incorporate technology appropriately as an educational tool.

Murat's MCC1-lesson activities -"Angles of Polygons" and the "Finding the sum of interior angle of polygons"- revealed his desire to use technology in general and Geogebra in particular as an educational tool. He incorporated technology to discover the new concepts of interior and exterior angles and the sum of the interior angles in a polygon. He preferred to use technology in the learning and teaching process of new geometric concepts instead of reinforcing the already-learned concepts. In the pre-conference, he stated, "I want them to write their own sentences. Make your sentence and if it is correct, that is the best. I don't want to spoon-feed them. Let them discover Geogebra on their own". This statement clarified the aim of his lesson during MCC1 to integrate Geogebra to enable the exploration of new concepts. Furthermore, Murat incorporated Geogebra into almost every phase of his lesson (except the introduction and assessment parts of the lesson). This demonstrated that he did not use technology as adds-on to the standard approach of the curriculum. Besides, these technology-enhanced activities were aligned with curriculum goals. He said, "I am also making use of the course textbook." However, these activities were adapted by the non-technology

curriculum so technology did not actually extend beyond the paper and pencil. He used Geogebra for the visual effect as a static tool. He did not use the dynamic properties of Geogebra to develop geometric concepts. He said, “Not using Geogebra for a long time, I have forgotten some of its properties.” Because of his lack of technical knowledge, he could not effectively incorporate Geogebra into his lesson. Briefly, the teacher is more prone to use the static properties rather than the dynamic properties like his previous lesson before mathematics coaching. In conclusion, he was not able to modify the mathematics curriculum in innovative ways to use technology as an educational - tool. Therefore, these provided evidence that Murat was at *the accepting level* for the curriculum descriptor. He “expresses desire but demonstrates difficulty in identifying topics in own curriculum for including technology as a tool for learning” (Niess et al., 2009, p.20).

For assessment the descriptor of the curriculum and assessment theme, Murat’s MCC1 performance demonstrated *the recognizing level* of TPACK. Murat incorporated technology to teach interior and exterior angles of polygons. After teaching these concepts, he asked students to solve the assessment questions in an activity sheet. The first question was “What is the sum of the interior angles of a decagon?” Then he warned the students not to draw a decagon in Geogebra. After giving some time, one of the volunteer students solved the question by using the formula $-180.(n-2)-$ which they had learned previously. The other questions were similar to the first question, such as “What is one of the interior angles of a regular pentagon?” and “What is the exterior angle of a regular pentagon?” One of the questions is illustrated in Figure 4. 10

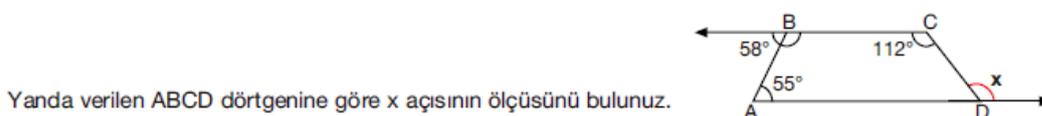


Figure 4. 10 The assessment of MCC1

Among these questions there was a question asking for the exterior angle of a quadrilateral. These questions showed that he preferred procedural questions in the assessment session. However, he did not allow the use of Geogebra as an

assessment tool while he used it as an educational tool, like previous lesson of Murat before mathematics coaching. Most of the time, he integrated Geogebra into his classroom as a tool for presentation and statistical representation. After teaching interior and exterior angles of polygons via Geogebra, he thought that it was not necessary to use Geogebra as a means of presentation because they had learned algebraic representation, which becomes more abstract with these technology-enhanced activities. He told the students not to use Geogebra in the assessment session. This indicated that Murat had a resistance towards using Geogebra. During pre-conference he focused on enhancing students' knowledge via Geogebra and he was not willing to use Geogebra in assessment. In addition, in the post-conference he stated,

“Perhaps assessment could have been done via Geogebra. But I couldn't take the risk. ... I mean if I had used it I could have solved a maximum of two questions then. The situation is this; you know it's the end of the term and I have to complete the topic and there is one more unit I have to teach. I may not even have enough time to complete that unit.”

He was concerned about the idea of using technology in the assessment process to determine students' understanding of geometry. It is aligned with the description, “Resists idea of technology use in assessment indicating that technology interferes with determining students' understanding of mathematics” in *the recognizing level* (Niess et al., 2009).

4.1.1.2.2 Esen's TPACK Level for Curriculum and Assessment Theme in the MCC1

Esen's performance in the MCC1 during mathematics coaching indicated *the accepting level* of TPACK for the curriculum descriptor as opposed to the *recognizing level* of TPACK for the assessment descriptor in the curriculum and assessment theme.

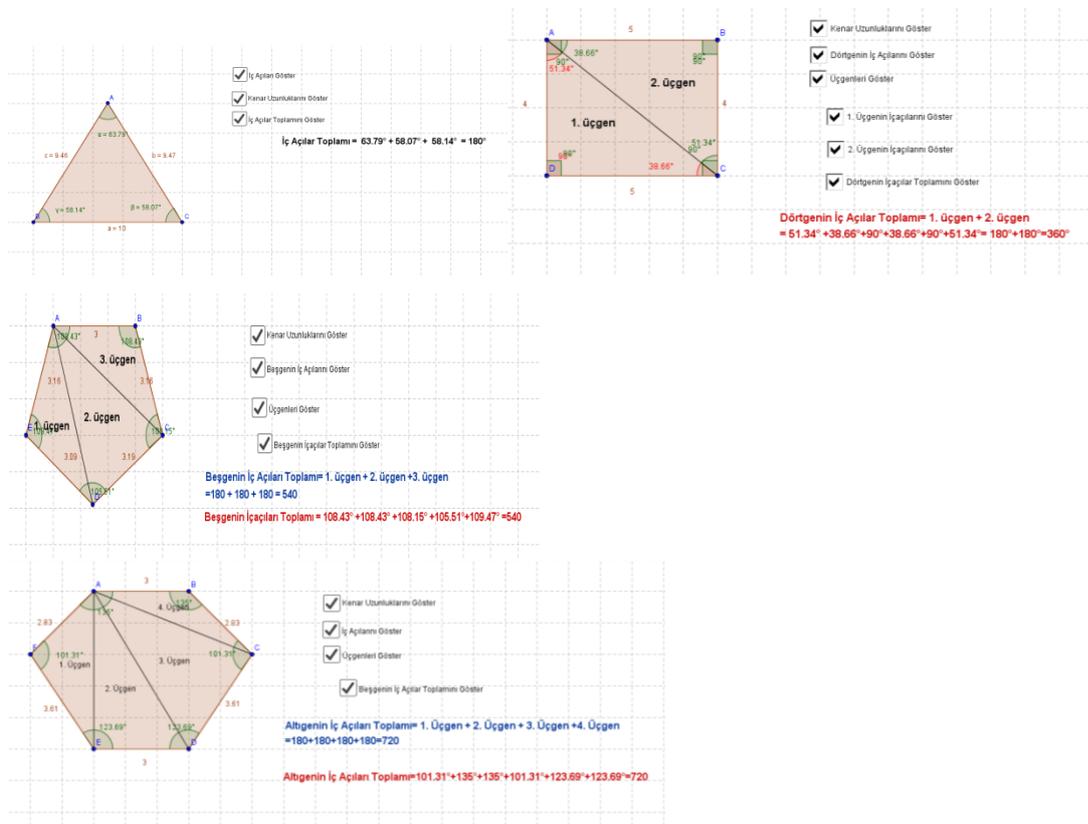


Figure 4. 11 The sum of interior angles activities

In the MCC1 Esen implemented an already-prepared Geogebra files for the students to discover the relationship between the sum of interior angles and the side number of polygons (see Figure 4. 11). Initially she opened the first Geogebra activity to discuss the triangle's properties, such as the sum of angles. At this point, she did not allow the students to use Geogebra although they had a computer in their group. She demonstrated the first activities on triangles on the board and she clicked the check boxes indicating "show interior angles", "show sides", and "show the sum of the interior angles." Then, she opened the quadrilateral file and clicked on the check boxes of "show interior angles" and "show sides". She asked the students to identify how many triangles there were in the rectangle. She separated two triangles by using the "show triangle" check box. She wanted the students to fill the table (see Table 4.3) to realize the pattern of the sides and the number of triangles in polygons. After the quadrilateral activity, she opened the pentagon activity on Geogebra, and asked the students find the sum of interior angles in a

similar way to the previous quadrilateral activity. Then Esen showed three triangles in a pentagon by using the check box and asked the students to fill in the table. Finally, she demonstrated the hexagon Geogebra activity on the board. By using check boxes, she provided an environment to discuss the pattern related to the sides, the triangles in a polygon and the sum of interior angles. Furthermore, filling the table helped the students to discover this relationship. The students realized that the sum of interior angles is $180(n - 2)$ where n is the number of sides. After finding this generalization, she allowed time for the students to use Geogebra. The students dragged the vertices of polygons to see whether their generalization is valid or not. Furthermore, Esen mostly integrated Geogebra as a static tool. These activities indicated that Esen began to use Geogebra to discover new concepts unlike the previous lesson (before mathematics coaching). In addition, she viewed Geogebra not just as add-on activities. Thus, she moved through higher level in the TPACK Development Model for curriculum descriptor. Although these activities related to the sum of interior angles were dynamic, she did not use the dynamic properties of these activities at first. For instance, she discussed with the students the properties of a projected rectangle; she did not drag the vertex of a rectangle to observe the changes. She used Geogebra as a drawing tool to demonstrate these polygons (triangle, quadrilateral, pentagon and hexagon). This usage was similar to that of Murat in the MCC1. Although the activities were suitable for dynamic usage, she did not prefer to extend beyond the paper and pencil. Thus, she mimicked the activities in their own traditional curriculum without considering the affordance of Geogebra such as dragging.

Table 4. 3 The table of the sum of interior angles

Çokgenlerin İç Açıları

Çokgenler	Kenar Sayısı	Üçgen Sayısı	İç Açılar Toplamı
Üçgen 			
Dörtgen 			
Beşgen 			
Altıgen 			
Yedigen 			
...
N gen			

- 1) Bir sekizgenin içaçılar toplamı kaçtır?
- 2) Bir yüzgenin içaçılar toplamı nedir?

After the sum of interior angles activities, she opened the already prepared Geogebra activity on exterior and interior angles of polygons (see Figure 4. 12). Students summed the exterior angles of these polygons which were a triangle, quadrilateral, pentagon, and hexagon. The students made a generalization that the sum of exterior angles of polygons is 360° . Like the previous activity, the teacher implemented Geogebra as a demonstration tool without using its dynamic properties. Furthermore, this technology-enhanced activity was aligned with the curriculum without considering the dynamic nature of Geogebra.

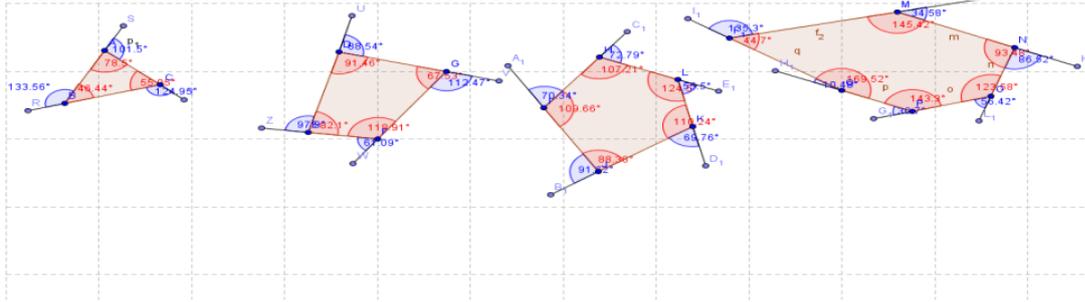


Figure 4. 12 The interior and exterior angles of polygons activity

After the sum of interior angles activities, she drew regular polygons - equilateral triangle, square, regular pentagon, and regular hexagon - by using a regular polygon tool in Geogebra. Then she asked them to find one interior angle of the triangle by using the angle tool. Besides, she asked the students to fill in the table about one of the interior and exterior angles of these polygons (see Table 4.

43). The students measured the angles of regular polygons to make generalizations about one of the interior and exterior angles. The students discovered that the formula of one of the interior angles is $(180(n-2))/n$ and the formula of one of the exterior angles is $360/n$ as n is the number of the sides. Like other activities in MCC1, the students did not use Geogebra, such as its dragging tool, effectively. They integrated Geogebra as a static tool.

Table 4. 4 The table of the interior and exterior angles

Çokgenler	Üçgen Sayısı	İç Açılar Toplamı	Bir İç Açısı	Bir Dış Açısı	Dış Açılar Toplamı
Üçgen 					
Dörtgen 					
Beşgen 					
Altıgen 					
Yedigen 					
...					
N gen					

Esen's MCC1 performance indicated that she implemented Geogebra to teach new concepts such as the sum of interior angles. This was the biggest difference from the previous lesson where Geogebra as a technological tool was used for reinforcing the already-known concepts with concrete materials. In addition, Esen implemented these technology-enhanced activities without considering the dynamic nature of Geogebra. In parallel to Murat's performance in MCC1, she integrated Geogebra mostly for the visual effect as a static tool. In addition, in the pre-conference, she said, "I am making use of the activities in the book." So, she mimicked the non-technology textbook activities. Consequently, she used Geogebra as a static tool for demonstration even though these activities were suitable for dynamic usage. These indicated that she was at *the accepting level* in the TPACK Development Model for the curriculum level. Thus, she "desire[s] but demonstrates difficulty in identifying topics in [her] own curriculum for including technology as a tool for learning" (Niess et al., 2009, p.20).

Esen's performance regarding the assessment descriptor was at *the recognizing level* of the TPACK Development Model. She did not integrate Geogebra to assess the students' knowledge. In the pre-conference, she stated, "I did not use Geogebra during the assessment process because they have to do it individually on the board. That will take up too much of our time. A worksheet is possible if we have time." In the MCC1 lesson, she asked the students questions, such as "What is the sum of the interior angles of a octagon?" "What is the sum of the interior angles of a shape with a hundred sides?", "What is the measure of an exterior angle of a regular octagon?", "What is the sum of the exterior angles of a regular octagon?". These questions revealed that she preferred procedural questions in the assessment. Furthermore, most of the time the students did not use Geogebra. The teacher wanted them to use the related formulas related to interior and exterior angles. Only once, the students used Geogebra to check her solution regarding one of the interior angles of a regular octagon. These indicated that she had resistance towards using Geogebra as an assessment tool, like Murat in MCC1. Supportively, in the post- conference she stated, "This did not benefit us all that much. We just solved questions. We could have done them in all kinds of different ways. I don't think it will be very effective in assessment." And she added, "But for assessment and evaluation, well, it is not something that I can use and benefit from presently. I don't think there will be a difference between writing and solving two questions on the board and showing 1 or 2 questions on Geogebra. ". These indicated that she did not deliberate that Geogebra could be used beyond the paper and pencil assessment activities. She did not believe in the advantages of Geogebra as an assessment tool. Thus, she resisted the idea of technology use in assessment as *the recognizing level* of TPACK Development Model for the assessment descriptor.

4.1.1.3 Mathematics Teachers' TPACK Level for Curriculum and Assessment Theme in the MCC2

The lessons of Murat and Esen were observed for four hours to determine their TPACK level in the second coaching cycle during the mathematics coaching

process. All the quotations and dialogs in the following paragraphs were derived from one-on-one the pre-conferences/post-conferences and the mathematics lesson from the mathematics coaching cycle 2.

4.1.1.3.1 Murat's TPACK Level for Curriculum and Assessment Theme in the MCC2

Murat's MCC2 performance indicated *the exploring level* of TPACK for the curriculum descriptor whereas *the accepting level* of the assessment descriptor of the curriculum and assessment theme in TPACK.

In the MCC2 lesson, Murat integrated Geogebra as an educational tool to improve students' knowledge about the properties of quadrilaterals (square, rectangle, parallelogram, rhombus, and trapezoid) in 4 hours. Murat showed a short video about quadrilaterals from Vitamin. In this video, there was a room where quadrilaterals could be found. Students were able to recognize different quadrilaterals in the room, such as a parallelogram, square, and rectangle. This warm-up activity motivated students towards these geometric topics.

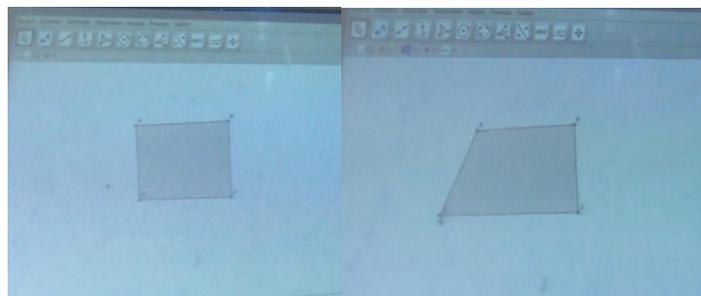


Figure 4. 13The Screenshot of square activity

After the warm-up activities, Murat asked students to open a new blank Geogebra file. He asked the students how many sides there were on a square to fill the side of polygon input in Geogebra. Then he demonstrated how to draw a square by using the Regular Polygon tool. After the teacher's demonstration of how to draw a square, the students drew a square in Geogebra (see). He walked among the students to help them construct a square. One of the student-groups used polygon tool to draw a square. Then Murat drew a similar square on his computer to project

it. He posed a question about whether or not the square's properties were maintained when it was moved from a vertex of this shape. He dragged one of the vertices of the quadrilateral and the final shape of the quadrilateral was no longer a square. Then the students said that the 4-sided regular polygon tool should be used to maintain the square.

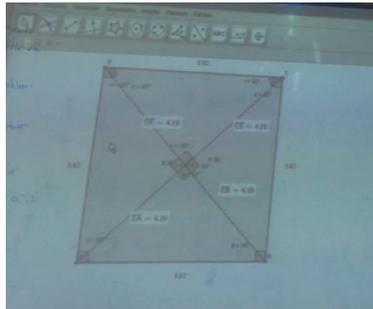


Figure 4. 14 The Students' Screenshot in the square activity

After every student group constructed a square, they started to analyze the properties. Firstly, by using the angle tool they found that its interior angles were congruent and 90° . Then Murat wanted them to find the relationship between the sides of the square. They discovered that all four sides of the square were congruent. Murat asked the students whether the congruence of all the sides of the square would remain the same after dragging any vertex of the constructed square. After dragging, students said that the sides of the square had different values from the previous ones but still remained equal to each other. Then they constructed diagonals to observe the relationship between them. The students measured the diagonals of the square and found the angles between the diagonals. They made a generalization before dragging. Then Murat asked them to see whether their generalization was true or not by dragging. The students analyzed the shape and observed that the diagonals of a square were congruent and bisected each other at a right angle. Then they moved the vertex of the square to prove this for all the squares. After every conjecture, Murat wrote these properties of a square on the blackboard. He asked the students to find the angles between the diagonals and sides. They measured the angles via Geogebra. They observed that the angles between the diagonals and sides were 45° . This technology-enhanced activity helped

students to make a conjecture about the diagonals and sides of a square. Subsequently, Murat asked the students to fill in the table he had given them about the properties of quadrilaterals. Murat asked the whole class whether or not each property was true or not and they put a tick if it was true. These properties were as follows: “the opposing sides are equal to each other, the length of the opposing sides are equal, all the sides are equal to each other, all the diagonals are equal, all the angles are right angles, the diagonals center each other, the diagonals are perpendicular to each other, the opposing angles are equal to each other.”,

Murat wanted the students to open a `dikdortgen.ggb` (rectangle) file which was already prepared (see Figure 4. 15). At first sight, based on the picture of a rectangle, the students said that the interior angles were 90° and the opposite sides were equal. Then he guided them to click the “show interior angles” and “show sides” check boxes. Students were able to observe that their judgments were correct. In this situation, Geogebra was used as a verification tool. Then the students dragged the vertex of the rectangle to observe the relationship between the sides. Then they approved that opposite sides of a rectangle were equal and wrote this down. Then they discussed the diagonals of a rectangle by clicking on the “show diagonals” check boxes. After clicking, the checkboxes of “show diagonals”, “show the length of diagonals 1 / 2” and “show the angles of diagonals” checkboxes appeared. Thus, the students were able to make a generalization about the properties of diagonals of a rectangle by using Geogebra. As in the square activity, in this activity students made a conjecture from a rectangle in a static way, and then they moved a vertex of rectangle to see whether or not the conjectures still remained. After exploring the properties of the rectangle, Murat asked the students what the relationship between a square and a rectangle was. Students transformed the rectangle to a square by dragging a vertex. They realized that every square is a rectangle. In addition, while dragging they observed different positions of a rectangle, not just one prototype of it.

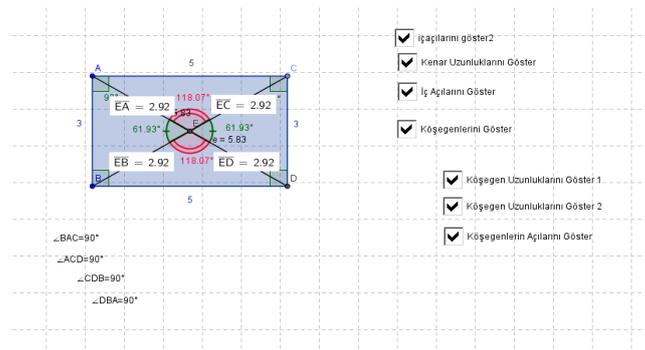


Figure 4. 15 The screenshot of rectangle.ggb

During the second two hours, which was on a different day, Murat taught the parallelogram, rhombus and trapezoid via Geogebra (see Figure 4. 16). In these already-prepared Geogebra activities, students were able to discover the properties of these quadrilaterals by using check boxes. For instance, students checked the “show the sides” box in the parallelogram activity, and then they made a conjecture that their opposite side was equal. Then the teacher motivated them to drag the vertex. They realized that this relationship was valid for all parallelograms. Using such a method of exploration, students discovered the properties of the parallelogram, rhombus and trapezoid, such as interior angles, length of diagonals, and angles between diagonals, by using check box and dragging. Furthermore he asked the students whether or not the opposite sides of these quadrilaterals were parallel. Thus, students used the parallel tool in Geogebra activity to investigate this relationship. To illustrate, in the trapezoid activity (see Figure 4. 16), they built a parallel line of one of the sides, so they were able to say that the trapezoid always has at least one pair of parallel sides. In these Geogebra-enhanced activities, Murat also provided the students with the opportunity to discuss the relationship between quadrilaterals. For example, he transformed a rhombus to a parallelogram by dragging the vertices and asked whether or not the rhombus retained its properties. With these guiding questions, the students made a generalization that the rhombus is always a parallelogram. In these activities, Murat used technology in general and Geogebra in particular for students’ exploration of geometric properties by dragging, unlike MCC1.

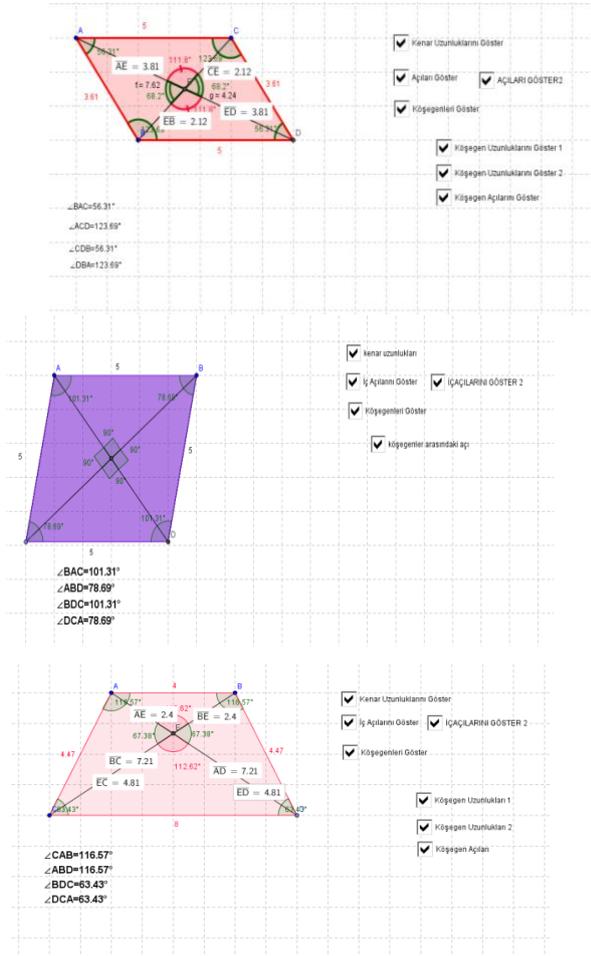


Figure 4. 16 The screenshot of parallelogram, rhombus, and trapezoid activities

In the MCC2 lesson, Murat provided a technologically supported environment for students to make conjectures about quadrilaterals based on their exploration.

During the pre-conference Murat said

“They know that the internal angles of a square are right angles. What else, that the sides are equal. Perhaps they will hesitate in the parallel structure. They wouldn’t know the diagonals. So I will have them draw diagonals and then mark the intersecting point. They I will have them measure them separately. First I will have them measure the diagonals. Then in this activity I will have them discover that the diagonal dissects the angles into 45 and 45 degrees.”

This statement also demonstrated that he concentrated on students’ exploration of the quadrilateral. Furthermore, Murat adapted ideas from the current curriculum and developed his own ideas by using Geogebra to enhance students’

comprehension of the subject. The usages of the dynamic nature of these activities, such as dragging, differed from the activities in the previous lessons-MCC1. By means of dragging, students could see the different positions of quadrilaterals and make generalizations about them. For instance, students could investigate the position of diagonals in different quadrilaterals by means of dragging. They could observe that diagonals of a square bisected each other at right angles, while those of a parallelogram did not. The curriculum of the school required students to analyze properties from just one prototype of a quadrilateral and memorized their properties. However, in these activities students observed the relationship between different quadrilaterals with the dynamic nature of the Geogebra. In the post-conference Murat admitted the advantages of the dynamic nature of technology and added “using Geogebra dynamically facilitated students’ learning process”. Furthermore, the incorporation of technology in the lessons was not a part of the mathematics curriculum of the school, so Murat integrated Geogebra into his lessons and developed technology-enhanced activities. These provided evidence that he “developed his ideas in using technology to enhance the current curriculum” (Niess et al., 2009,p.21) at *the exploring level* of the curriculum descriptor of the curriculum and assessment theme. In addition, the usage of Geogebra in almost the entire lesson (except during warm-up activities) indicated that technology had a more integral role for the development of the mathematics that students were learning. Consequently, Murat’s MCC2 indicated that he was at *the exploring level* for the curriculum descriptor.

Murat’s MMC2-observation performance indicated *the accepting level* of the assessment descriptor of the curriculum and assessment theme in TPACK. After the integration of Geogebra as a teaching and learning tool, Murat wanted to assess students’ knowledge on quadrilaterals. Firstly he asked his students to open the dortgenler.ggp (quadrilateral) file. In this assessment activity, there were 20 different quadrilaterals and students could color by choosing the type of quadrilaterals –square, rectangle, rhombus, parallelogram, and trapezoid, which

were on the right side (see Figure 4. 17). Also, there was a ruler to measure the sides of quadrilaterals.

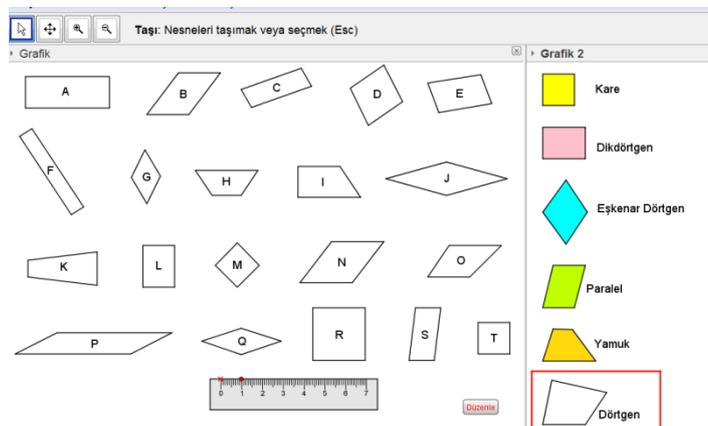


Figure 4. 17 The first assessment of MCC2

He asked students for each of twenty quadrilaterals is square which first choice in right side or not. Then the whole class answered it; they moved to the other quadrilateral, which was a rectangle. The students did not use Geogebra individually or in group. He did not allocate time to solve this activity in their small group. For instance, he wanted to them to find parallelograms in this activity, so he asked whether quadrilateral A or B was a parallelogram. He colored the quadrilaterals which were parallelograms according to the students' answers. In the last part of this activity, he asked the students to find trapezoids. Then students replied that all the quadrilaterals in the activity were trapezoids and the teacher colored all the quadrilaterals. In this Geogebra activity, he integrated technology as a visualization tool to motivate students. Furthermore, students did not assume an active role.

After this assessment activity, he wanted students to open another Geogebra file which was related to classifying quadrilaterals (see Figure 4. 18). But he had 5 minutes left. So he did not spare time for students to solve this assessment activity. He explained the activity stating that a trapezoid is a subset of a quadrilateral, and a parallelogram is a subset of a trapezoid. Students should have constructed proper quadrilaterals according to this classification by dragging. Additionally, when students built quadrilaterals properly, feedback was given as "Congratulations".

This helped students to verify whether their construction was correct or not. All this indicated that he was aware of some of the advantages of Geogebra in the assessment process. But he just summarized this activity because of limited time.

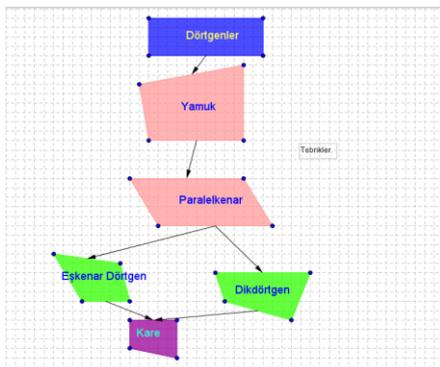


Figure 4. 18 The second assessment of MCC2

In the pre-conference, he designed another Geogebra assessment activity where the students were asked to cluster the nine quadrilaterals according to three different questions (Figure 4. 19). In the first question, the students had to classify quadrilaterals into rhombuses and rectangles by dragging them and dropping into clusters. In the second question, these quadrilaterals had to be clustered in terms of the labels, “opposite sides are parallel” and “sides are perpendicular”. In the last question, two clusters were “opposite sides are equal” and “diagonals are equal” and the students grouped them according to these clusters. In this assessment activity, a check box was used to hide and show questions. And dragging was to be used for moving to cluster. This activity was not one which went beyond the paper and pencil activity.

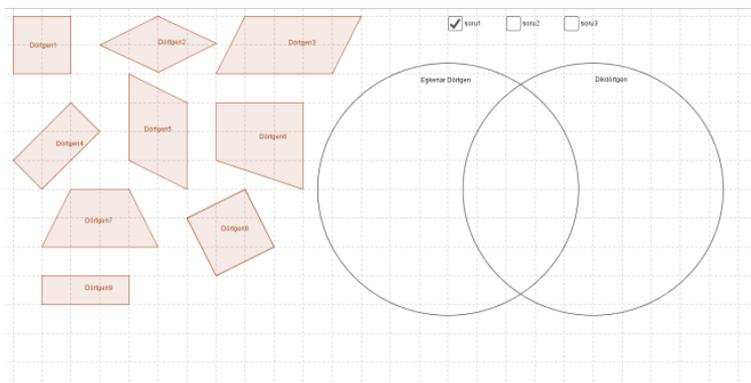


Figure 4. 19 The cluster of quadrilateral

These three assessment activities indicated Murat's awareness that Geogebra might be used as assessment tool, so he incorporated Geogebra as an assessment tool into MCC2. In the second activity, he used feedback in Geogebra to verify their construction. When Murat was asked in the post-conference what he needed to pay attention to in using Geogebra in assessment, he said, "I think I need to give feedback carefully because in the last phase no mistakes should be made. Mistakes persist." This statement demonstrates that he integrated Geogebra as a verification tool in the assessment process. However, he could not integrate technology that extended beyond the paper and pencil in the assessment process. In these assessment activities students did not take actions on quadrilaterals as objects to realize the mathematically meaningful consequences of those actions. For instance, in the first activity students just chose and colored quadrilaterals according to their type. This did not differ from fingering quadrilaterals. This provided evidence that he used technology for visualization. Additionally, in the post-conference he stated, "In the assessment I moved the rectangle the square, the triangle. I used the things that appealed to visual memory." This statement supported that his main focus on integration of technology in the assessment process was visualization. However, even though every student group had these activities on their computer, he did not allow them to solve these Geogebra-enhanced activities by themselves. He projected these activities and, students who volunteered answered them. This situation indicates that he had a limited view of the usage of Geogebra in assessment. This session included the elements of *the accepting level* of the assessment descriptor of the curriculum and assessment theme. Murat "Acknowledges that it might be appropriate to allow technology use as part of assessment but has a limited view of its use" (Niess et al., 2009, p.20).

4.1.1.3.2 Esen's TPACK Level for Curriculum and Assessment Theme in the MCC2

Esen's performance in MCC2 during mathematics coaching demonstrated *the adapting level* of TPACK for the curriculum descriptor and the assessment descriptor in the curriculum and assessment theme.

In the MCC2 lesson, Esen started the lesson by reviewing the previous lesson on the interior and exterior angles. Then she asked the students to give examples of quadrilaterals in students' daily life. She projected the quadrilateral examples in the PowerPoint presentation to find their several characteristics in common (see Figure 4. 20). This warm-up activity motivated the students to recognize different quadrilaterals in daily life. But she did not integrate Geogebra in warm-up activities.

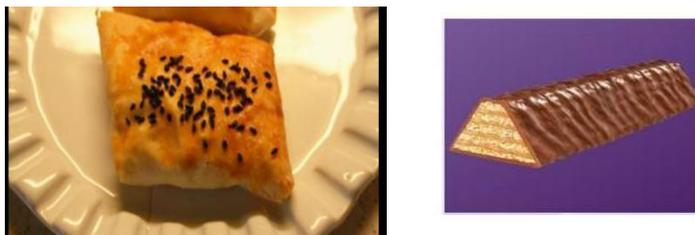


Figure 4. 20 The quadrilateral examples in the PowerPoint

After this warm-up activity, she asked the students to open an already-prepared Geogebra file on the properties of the square (see Figure 4. 21). In this Geogebra activity, there were check boxes - "show sides", "show interior angles", "show diagonals", "show the length of diagonals $1 / 2$ " and "show the angles of diagonals" - to reveal various properties of the square. Firstly, the teachers wanted her students to watch the board. She provided the students with an environment to discuss with the whole class the properties of the square. For example, she asked the students to find the relationship between the diagonals of the square. By clicking on the "show diagonals", the students discovered the equality between diagonals. Then she clicked on the "show the length of diagonals $1 / 2$ " and "show the angles of diagonals". The students explored that they bisected each other at a right angle. After these explorations, she allowed time to the student groups for self-exploration. She walked amongst the student groups to help them. In addition, the teacher

motivated them to drag the vertices of the square. She asked the students to find out whether their generalizations were true or not after dragging. The students moved the vertex of the square to prove their generalizations such as “all the sides are equal to each other”, “all the angles are right angles”, “all the diagonals are equal”, “the diagonals center each other”, and “the diagonals are perpendicular to each other”. Furthermore, this Geogebra-enhanced activity helped the students to see the non-prototype of the square by dragging. This usage of dragging indicated that she implemented Geogebra as a dynamic tool, unlike her previous lesson in MCC1. She began to understand the dynamic nature of Geogebra in the teaching and learning of polygon concepts.

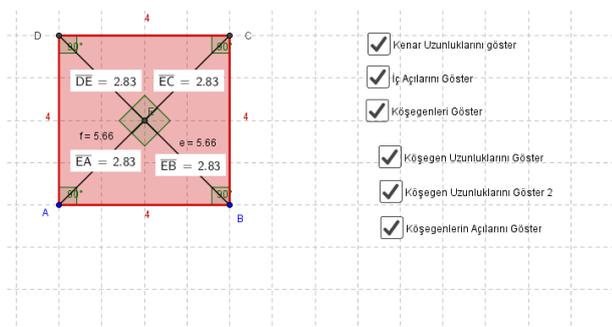


Figure 4. 21 The properties of square Geogebra activity

After the square activity, Esen worked with other quadrilateral Geogebra activities (rectangle, parallelogram, rhombus, and trapezoid, respectively) during the MCC2 lesson. These Geogebra-enhanced activities were similar to the square activity. In addition, she provided the students with an environment to explore the properties of these quadrilaterals by using checkboxes. For instance, in the rhombus activity, she clicked on the “show diagonals angles” check box. The students discovered that the diagonals are perpendicular. After the students’ generalization regarding the rhombus, she allowed time to the student groups and motivated them to drag the vertex of the rhombus to see whether or not the generalizations still remained. She implemented Geogebra in this way during MCC2. Furthermore, the usage of Geogebra was similar to Murat’s in MCC2. At first, both of them integrated Geogebra as a static tool, but then they used Geogebra dynamically.

While the students' discovered the properties of quadrilaterals by using Geogebra, Esen asked questions about the relationship between quadrilaterals such as the rectangle and square, the rectangle and parallelogram, the parallelogram and trapezoid. But she did not integrate Geogebra effectively to discuss these relationships. For instance, by analyzing whether “the square is a rectangle” or not, she asked the students to compare the properties by using the table on the properties of quadrilaterals. Then the students realized that the properties of the rectangle were valid for the square as well. They discovered that the square was a subset of the rectangle. Furthermore this discussion helped to explore the relationship between quadrilaterals. She did not integrate Geogebra for this exploration. For example, Murat integrated Geogebra to enable students to discover the relationship between the rhombus and the parallelogram by transforming a rhombus to a parallelogram via dragging vertices. This indicated that she was at a lower level than Murat in the TPACK Development Model for the curriculum descriptor. Furthermore, after integrating Geogebra as learning and teaching tool, she used concrete manipulatives such as a ruler and wooden quadrilaterals to summarize the properties of quadrilaterals (see Figure 4. 22). For instance, she demonstrated the wooden rectangle and square to sum up their properties. In addition, she asked the students what the relationship between a square and a rectangle was by showing the wooden rectangle and square. The students said that the square is a special rectangle. Furthermore, she asked the students to explain the relationship between a square and a rhombus. In order to help the students, she pushed the vertex of the square and transformed it into a rhombus. Although she was able to do this transformation in Geogebra, she did not prefer to use Geogebra. Besides, she summarized the lesson with concrete manipulatives as non-technology tools after using Geogebra. These indicated that she had a limited understanding of integrating Geogebra into the mathematics curriculum.



Figure 4. 22 The concrete materials in MCC2

In the MCC2 lesson, Esen implemented Geogebra-enhanced activities for the students' exploration of quadrilaterals. Unlike in MCC1, in this lesson, she viewed Geogebra as a dynamic tool. This indicated that she understood the benefits of Geogebra as a tool for teaching and learning the mathematics curriculum. However, at first she provided an environment for the students to discover the properties of quadrilaterals by using Geogebra as a picture tool. After the students' exploration based on static quadrilaterals in Geogebra, the students dragged their vertices to verify their exploration. This provided evidence for *the adapting level* of the TPACK Development model for the curriculum descriptor. Furthermore, she preferred concrete materials instead of Geogebra to summarize the students' knowledge on quadrilaterals. This indicated that she had limited knowledge in integrating Geogebra as a learning and teaching tool into the mathematics curriculum. Supportively, in the post-conference she said, "I think that only this much is possible in these circumstances. Nothing more is possible. ... Apart from this, I believe we have gained positive benefits from Geogebra." This speech demonstrated that she accepted the advantages of Geogebra but still had limited knowledge. To conclude, she "Understands some benefits of incorporating appropriate technologies as tools for teaching and learning the mathematics curriculum" (Niess et al., 2009, p.20), which shows that she is at *the adapting level* of the curriculum descriptor.

Esen's MCC2 performance indicated *the adapting level* of the TPACK Development Model. She firstly integrated Geogebra to assess the students' knowledge in her class. This demonstrated that she moved through the TPACK Development Model for the assessment descriptor. She conducted two Geogebra

activities to assess the students' knowledge on quadrilaterals. In the first one she wanted the students to color the different quadrilaterals according to their type such as square, rectangle, rhombus, parallelogram, and trapezoid. This activity was similar to Murat's assessment activity in MCC2. By comparison with Murat's activity, she gave more time to the student groups for self-exploration. Besides, both of them used Geogebra as a visualization tool in this assessment. In addition, Esen assessed their procedural knowledge on quadrilaterals. For example, if a square was a rectangle, they would choose the square by color type.

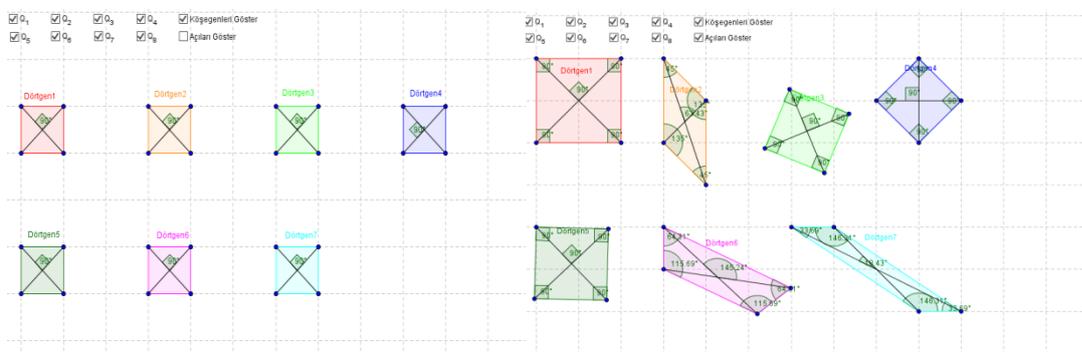


Figure 4. 23 The assessment activity in MCC2

In the second assessment activity, she asked the students to identify the type of quadrilaterals in Geogebra. At first, all of the seven quadrilaterals were square-shaped (see Figure 4. 23). She motivated them to drag the vertices of these seven quadrilaterals. She wanted the students to analyze the properties of these quadrilaterals and identify what type of quadrilateral - parallelogram, trapezoid, or rhombus – they were. The students clicked on the check boxes “show diagonals” and “show angles” to observe their properties. By dragging, the students analyzed which properties did not change. This activity provided a dynamic environment for the students to assess their conceptual understanding of quadrilaterals. The dynamic properties of the second assessment tool indicated that Esen was at a higher level than Murat in the TPACK Development Model for the assessment descriptor. While Murat integrated Geogebra as just a visualization tool in assessment, she implemented the Geogebra activity in assessment which was the beyond paper and pencil. In addition, Esen's activities in MCC2 helped to assess the students'

conceptual understanding in addition to their procedural understanding of quadrilaterals. These provided evidence that she “understood that if technology is allowed during assessments that different questions/items must be posed (i.e., conceptual vs. procedural understandings)” (Niess et al., 2009, p. 20), which is an indication of *adapting level* in the TPACK Development Model for the assessment descriptor.

4.1.1.4 Mathematics Teachers’ TPACK Level for Curriculum and Assessment Theme in the MCC3

In the third coaching cycle during the mathematics coaching process, the lessons of Murat and Esen were observed for four hours to determine their level in TPACK Development Model. All the quotations and dialogs in the following paragraphs were derived from the pre-conferences/post-conferences and the mathematics lesson from the mathematics coaching cycle 3.

4.1.1.4.1 Murat’s TPACK Level for Curriculum and Assessment Theme in the MCC3

Murat’s MCC3 performance demonstrated *the exploring level* of TPACK for the curriculum descriptor and *the accepting level* of the assessment descriptor of the curriculum and assessment theme in TPACK.

In the MCC3 lesson, Murat implemented Geogebra as learning and teaching tool in 4 hours to improve students’ knowledge of the congruence and similarity of polygons, the area of irregular an polygon and the area of the parallelogram. To motivate the students, at the beginning of the lesson, Murat asked where students saw congruence polygons in their everyday environment. With this question, students started to give examples of congruent polygons that they observed in their everyday environment and discussed the properties of congruence polygons. Then Murat asked students to open the already-prepared Geogebra file (which was similar to the popular game of candy crush). In this Geogebra activity, students found and brought close together congruent shapes of candies. This warm-up activity was

highly motivating for students. Furthermore it provided an opportunity to discuss the relationship of the side length and angles of congruent polygons. This Geogebra-enhanced warm-up activity was closely linked with the curriculum objectives. Furthermore, Murat integrated Geogebra as a technological tool to enhance students' learning process during the presentation stage of the lesson.

After the warm-up activity, Murat asked students to open the Geogebra activity which was related to congruence and similar triangles. Firstly Murat introduced the activity by explaining, for example, how AB, DE, α , and β slider worked. At the beginning of the activity there were just line segments (see Figure 4. 24); then students constructed congruent triangles by using sliders. Students realized that the A and D, B and E, and C and F angles were the same. Students observed that “the corresponding interior angles were the same” and “the corresponding sides of each were congruent.” Then students used sliders to change the sides and angles of a triangle. The triangles remained similar to each other despite the change in their size. Furthermore, students dragged the vertex of the triangles, which helped them to discover that the triangle ABC and DEF still remained congruent even though their position changed. Then they discovered that the ratio of the corresponding sides was always 1, even if the position of the triangles changed.

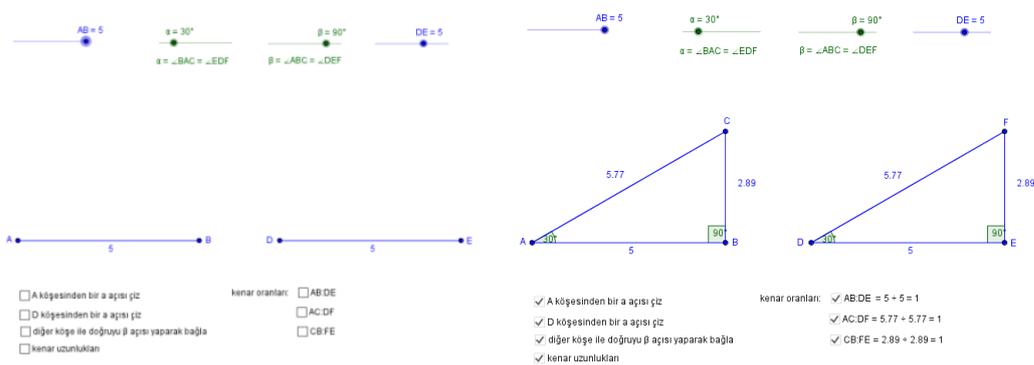


Figure 4. 24 The screenshot of congruence and similarity activity in MCC3

Then, the students dragged the AB and DE sliders to change the sides of the triangles. Students realized that the corresponding interior angles were the same but

the sides were different in size. So they discussed the relationship between the corresponding sides. They observed that these similar triangles were two polygons with the same shape, but not necessarily the same size. When an AB segment was 4 and DE was 2, the ratio of the corresponding sides was 2. They investigated that the scale factor was 2 for these triangles. Then they dragged the sliders to change the scale factor and observed the differences. They made a generalization that “similar polygons have corresponding angles that are congruent, and corresponding sides that are proportional.” After dragging the vertex, they observed that they were still similar in spite of the different direction of the shapes.

After the triangle activity, they opened a regular polygon tool in Geogebra. In this activity, the slider was able to change the number of sides of a polygon (see Figure 4. 25). If students wanted to make a square, the slider had to be 4 indicating the side number. Furthermore, students were able to click on “show the scale ratio” and “show the ratio of perimeter” to analyze the relationship between any pair of regular polygons with the same number of sides. With the dynamic nature of Geogebra, they noticed that “all equilateral triangles were similar” and “all squares were similar.” They added that regular polygons with the same number of sides were similar. Furthermore, they realized that the scale ratio was equal to the ratio of the perimeter. They were able to explore the congruence and similarity of polygons as geometric concepts by using sliders and dragging in a dynamic environment. Like the school curriculum, there was no integration of technology in the mathematics textbook as well, so Murat integrated Geogebra into his lesson by using the dynamic properties of Geogebra.

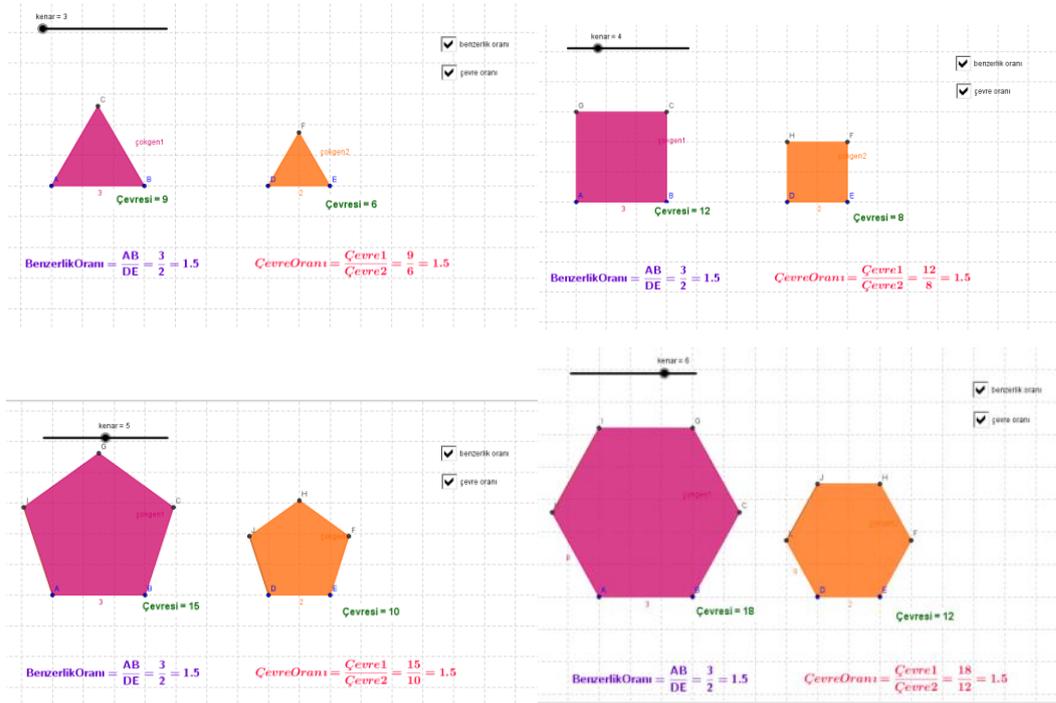


Figure 4. 25 The regular polygon activity

After the first two hours, Murat taught the area of irregular polygons and parallelogram with Geogebra-enhanced activities. Murat asked students to open home.ggb in Geogebra and to find the area of the front surface of a house (see Figure 4. 26). Students discussed how to find the area of that irregular shape. They suggested that it could be broken down into a rectangle and triangle by drawing diagonals from one of the vertices. Then they found the length of the front surface of the home by choosing “show the length” check box. Their approach was to break the polygon up into pieces, the area of which they were able to solve. Using this Geogebra-enhanced activity helped students to transfer the topic of area of polygons as a geometric concept into their daily life. Furthermore, it motivated the students and drew their attention to the lesson. They began to make strategies in how to find irregular polygons by means of this Geogebra-enhanced activity.

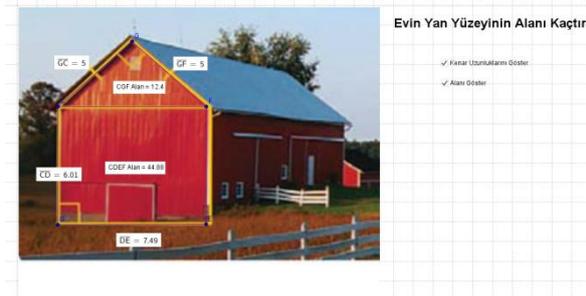


Figure 4. 26 The Home activity

After the warm-up activity, students opened `basketsahasi.ggb` (basketballcourt) in Geogebra (see Figure 4. 27). In this activity, students had to find the area of a shaded region in a basketball court. Murat gave time to students to solve this question and walked around the classroom among the students. He assisted them by questioning and facilitating their learning with the integration of Geogebra. Students made different strategies by using the segment tool, such as dividing it into two triangles, or a triangle and a rectangle, or into square units. Then the students shared the solution they had come up with in this activity with the class. Murat projected one of students' solutions on the board. Then he used the area tool to verify the solution. He provided an environment for students to explore and discover geometric concepts and verify their solution. After the basketball court activity, students worked on a similar activity, which required them to find the area of a shaded region. Murat gave students time to solve the problem and assumed the role of a guide in forming their solution strategies. They approached these activities either by cutting the shape into smaller pieces and adding their areas, or by starting with a larger shape and subtracting pieces from it via Geogebra. Students expanded their mathematical ideas on the basis of technology explorations with these problem solving tasks by using Geogebra. This was an element of *the exploring level* of TPACK for the curriculum descriptor of the curriculum and assessment theme.

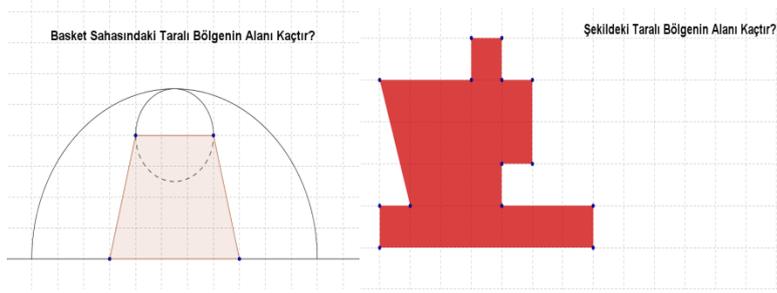


Figure 4. 27 The basketball court and finding of shaded area

After the implementation of the area of irregular polygons, Murat asked the students to open another already-prepared Geogebra file which was related to the area of parallelogram (see Figure 4. 28). With the aid of the dynamic properties of Geogebra, students transformed a parallelogram to a rectangle. In this activity, the right triangle on the left side was cut and moved to the other side by using a slider. Students realized that this transformation changed the parallelogram to a rectangle, while it did not change the area. This Geogebra-enhanced activity helped students to realize that the area formula for a rectangle was also the area formula for parallelograms. Then, they dragged the vertex of the parallelogram to move and reshape the parallelogram. They observed the results on the spreadsheet while dragging and realized that the area of parallelograms was related to the base and height as in a rectangle. So they made a conjecture that the area formula of a parallelogram is $A=b.h$. This technologically supported environment helped students to make conjectures about the area of the parallelogram based on their exploration. Thus, Murat integrated Geogebra into MMC3 lesson to enhance the non-technological mathematics curriculum.



Figure 4. 28 The area of parallelogram activity

After the finding the area of a parallelogram activity, students opened another Geogebra file which was about finding the area of a parallelogram from a different base. Students realized that using the horizontal base is not just one way to find the area of a parallelogram. They dragged and calculated the area of a parallelogram. With the spreadsheet of Geogebra, they realized that the area of a parallelogram is the multiplication of the base and the corresponding height.

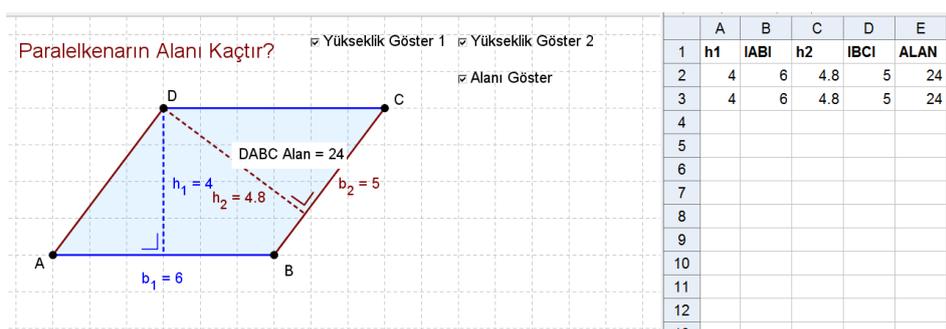


Figure 4. 29 The finding of area parallelogram via different height

The MCC3 lesson demonstrated that Murat implemented the technology to develop the mathematics that her students would be learning such as the congruence and similarity of polygon, the area of irregular polygon and the area of parallelogram. During the first two hours, students explored the concepts of similarity and congruence by using dragging in Geogebra-enhanced activities. Additionally, in the activities of finding the area of irregular shapes, students were asked to expand their mathematical ideas about the area of polygons by means of technology-aided problem solving tasks. These indicated that the usage of technology was appropriate for the curriculum topics and technology was aligned with curriculum goals. Furthermore, in the area of parallelogram activities students were able to discover the formula of the parallelogram with the aid of the dynamic properties of Geogebra. This indicated that Murat had integrated Geogebra into the current curriculum by using technology beyond the paper and pencil. Additionally, in the post-conference he stated that “the last activity I did (area of the parallelogram) was dynamic, that I pulled it to the left and right enabled the children to learn the concepts better.” That is, he realized the advantages of using technology beyond the paper and pencil. Furthermore, he said,

“I will definitely do the lesson in the same way (technology supported) again because in 6th grade classes I started the perimeter in polygons. I explained it in the standard way; I did not use software. I explained it orally. Children get bored. It seems like it is a repetition of the same things. But using the computer, using technology adds a different perspective. It enables them to see different things. They discover them themselves. It enables them to see them not only verbally or numerically but also by means of multiple presentations.”

Thus, by considering the advantages of Geogebra he enhanced the pre-existing activities in the curriculum. These provided evidence that Murat was at *the exploring level* for the curriculum descriptor. He “investigated the use of topics in own curriculum for including technology as a tool for learning” as at *the exploring level* (Niess et al., 2009,p.20). In the pre-conference, the researcher as a mathematics coach suggested that he could integrate Geogebra for Pick’s theorem to find the area of irregular polygon but the teacher did not want to use it. This showed that he did not integrate technology beyond the curriculum. In *the advancing level* which is the last level, teachers develop innovative ways to use technology and modify and advance the curriculum. Murat did not reach that level. However, he altered pre-existing activities and created new activities for the current curriculum. Furthermore, the integration of Geogebra into the entire lesson (including warm-up activities) indicated that he investigated ideas in how to implement technology in a more integral way for the development of the mathematics that the students were learning. Consequently, Murat’s MCC3 demonstrated *the exploring level* for the curriculum descriptor.

Murat’s MMC3-observation performance indicated *the adapting level* of TPACK for the assessment descriptor of the curriculum and assessment theme. In the pre-conference, he designed an assessment activity on similarity and congruence but he did not implement it because of lack of time (see Figure 4. 30). In this activity, students had to find and bring close together congruent and similar polygons via dragging. The teacher had assessment considerations of students’ outcomes when finding congruent and similar polygons. In this activity, the teacher

wanted to assess students' procedural knowledge about similarity and congruence without using the dynamic nature of Geogebra.

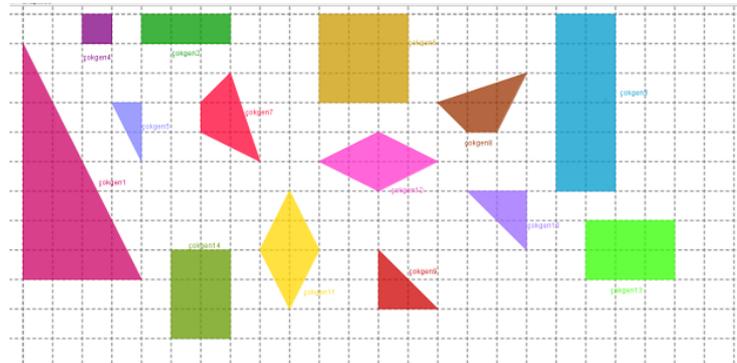


Figure 4. 30 The first assessment of MCC3

In the second two hours he conducted three assessment activities in the area of parallelogram. In the first activity, he asked students to calculate the height of IBDI in the parallelogram of ABCD (see Figure 4. 31). Additionally, the students wanted to find the height of IBDI after constructing different parallelograms by dragging the vertex. The students dragged the vertexes and recalculated the height. This assessment activity assessed students' procedural understanding of the area of parallelograms. Furthermore, this activity was similar to the assessment activities in textbooks.

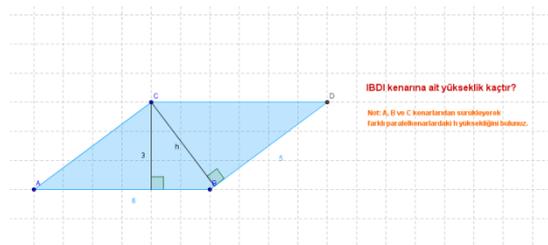


Figure 4. 31 The first assessment of MCC3

In the second assessment activity, students were asked to observe the area of parallelogram ABCD by dragging A (see Figure 4. 32). The area of parallelogram did not change while the position of point A was changed by dragging. The teacher asked the students to find the reason of the situation. Students noticed that the base of a parallelogram was stable while dragging the vertex of the parallelogram. Besides, the students were able to see the value of height and area in a parallelogram

by clicking on “show height” and “show area” checkboxes. With the help of dragging, which is an element of the dynamic nature of GeoGebra, students realized that the area of parallelogram did not change because the height was fixed. This indicated that he allowed technology in the assessment process to gather students’ conceptual understanding of the area of parallelograms. Furthermore, this activity demonstrated that Murat integrated technology as an assessment tool that went beyond the paper and pencil type of activity. In contrast to the assessment in MCC2, students used Geogebra actively on their computer. Additionally, the teacher gave time to the students to find the solution themselves.

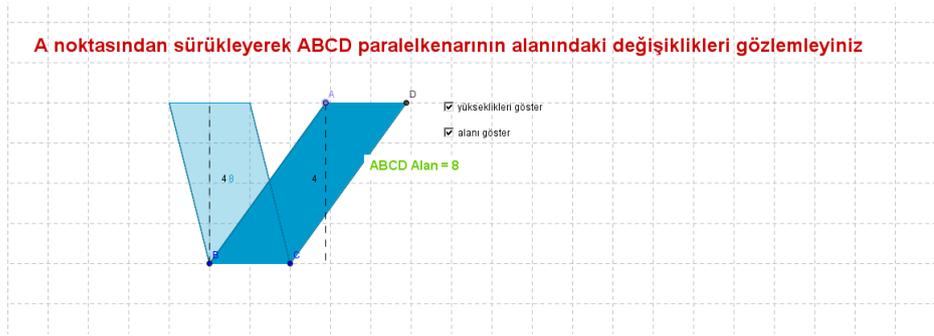


Figure 4. 32 The second assessment of MCC3

After this assessment activity, Murat wanted to open another Geogebra activity about the area of parallelograms. In this activity, there were two congruence parallelograms whose sides were 4 and 2 (see Figure 4. 33). He asked the students to explain what happened to the areas of these parallelograms as the vertex was moved. In the above parallelogram, the units of the sides were fixed to 4 and 2. When the vertex of the above parallelogram was moved, the height changed while the units of the sides did not. The students observed the change in the area with the help of Geogebra. In the below parallelogram, when the vertex was dragged, the sides changed but the area of the parallelogram did not. The students made actions on parallelograms as objects to realize the mathematically meaningful consequences of those actions. This assessment activity indicated that the teacher focused on understanding students’ conceptual understanding of the area of parallelograms with the help of the dynamic nature of Geogebra.

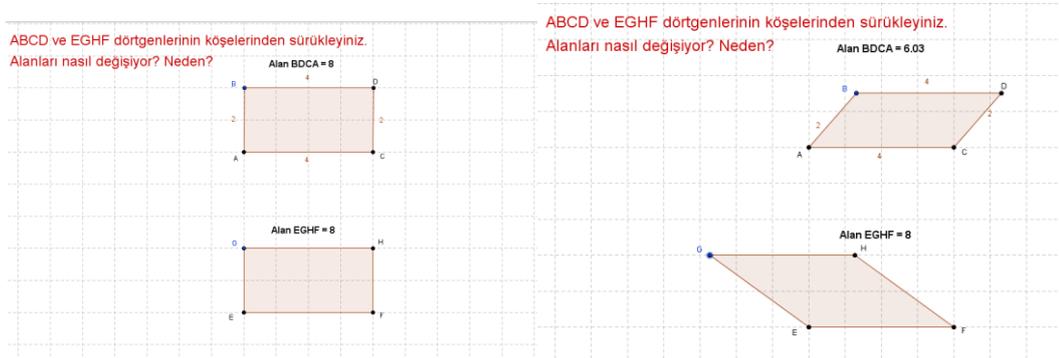


Figure 4. 33 The second assessment of MCC3

Murat’s overall MCC3 lesson demonstrated that he started to understand how to use technology in the assessment process. He let the students engage freely in Geogebra-enhanced activities unlike in MCC2. Furthermore, the teacher designed assessments to reveal students’ understanding of geometrical ideas using an appropriate technology that sometimes extended beyond the paper and pencil based questions. Two of the activities on area of parallelograms provided a dynamic environment to assess students’ conceptual knowledge. On the other hand, the other activities were designed to gather students’ procedural knowledge. Besides, these assessment activities were similar to those in textbooks. In the post-conference, the teacher stated,

“I use technology in assessment, but when I adapt it to the textbook I am worried about whether they can solve them. Now they can do these assessment activities on Geogebra, but the questions in the textbook are related more to operational skills. I need to focus on whether they can adapt them. I am hesitant but other than that the assessment was good.”

This statement revealed that he waver about how students solved the textbooks assessment activities. Because of this consideration, he wanted to align with the textbook questions. Thus, he gave attention to gather students’ conceptual knowledge in addition to their procedural knowledge via Geogebra. Murat’s MCC3 lesson session included elements of *the adapting level* of the assessment descriptor of the curriculum and assessment theme. He “Understands that if technology is allowed during assessments that different questions/items must be posed (i.e., conceptual vs. procedural understandings)” as in *the adapting level* (Niess et al., 2009, p.20).

4.1.1.4.2 Esen's TPACK Level for Curriculum and Assessment Theme in the MCC3

Esen's performance in MCC3 during mathematics coaching indicated *the exploring level* for the curriculum descriptor and *the adapting level* for the assessment descriptor in the curriculum and assessment theme of the TPACK Development Model.

In the MCC3 lesson, Esen integrated Geogebra as learning and teaching tool to improve students' knowledge on the congruence and similarity of polygons, the area of irregular polygons and the area of the parallelogram by enhancing the current curriculum. Esen asked the students to give an example of congruence from their daily environment. Then, Esen demonstrated pictures on the board to provide students with examples of congruence and similarities from daily life, such as twins, the ball used in football, and miniaturk. These examples motivated the students in identifying congruence and similarity. Then she asked the student groups to open the already-prepared Geogebra file on congruence. In this activity, there were congruent windows in an apartment. She motivated the students to measure the angles and sides of these windows by using the measurement tools in Geogebra. This warm-up activity was provided as an opportunity to explore the relationship of the side lengths and the angles of the congruent shapes. In addition, this activity was closely linked with the curriculum objectives. After the congruence activity, she asked the students to open another Geogebra file (see Figure 4. 34). In this Geogebra-enhanced activity, the students constructed two similar rectangles by using sliders. The teacher motivated them to minimize or maximize the rectangle by changing the sides via sliders. In this activity the students recognized that the corresponding sides of the rectangle were proportional. Then she asked the students to open the Geogebra activity about the Russian nested doll. The students found the relationship between the sides of the dolls. They realized that there was a constant ratio between each of the two dolls. These activities indicated that Esen implemented Geogebra as learning and teaching tool to improve students' learning via enhancing the curriculum.

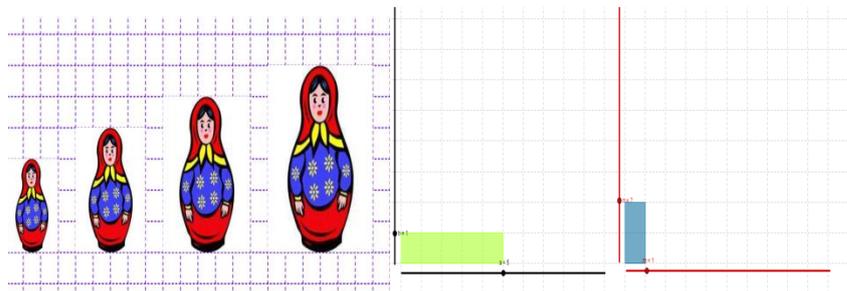


Figure 4. 34 The Geogebra activities in MCC3

After the Russian nested dolls activity, Esen asked the students to open an already-prepared Geogebra activity on the congruence and similarity of polygons (Figure 4. 35Figure 4. 34). She allowed time for the students to explore the relationship among three quadrilaterals. In this activity, the students dragged the vertices of polygons to discover changes between the quadrilaterals. The students realized that “the corresponding interior angles were the same” via dragging. Furthermore, they realized that two of them were congruent even though their directions were opposite. In addition, they explored the ratio between quadrilaterals by using the slider. The students recognized that two of the polygons remained similar to each other despite the change in their size. This Geogebra-enhanced activity provided an environment for the students to discover that similar polygons have a constant ratio, which was a scale ratio. Then she asked the students to open the Geogebra file on the regular hexagon (see Figure 4. 35). Esen motivated her students to drag the vertices of the hexagon in order to change the size and position of polygons. In addition, Esen allowed time for the students to find the relationship between the ratio of the perimeter and the scale ratio. By dragging, they explored that the scale ratio was equal to the ratio of the perimeter. In these activities on congruence and similarity, the students integrated Geogebra to explore the congruence and similarity of polygons as geometric concepts by using sliders and dragging in a dynamic environment. Furthermore, Esen used Geogebra as learning and teaching tool by considering the affordance of Geogebra. Thus, she began to alter the pre-existing non-technology based in the textbook in order to go beyond the paper and pencil. In addition, the students effectively used the properties of

Geogebra, such as dragging and sliders, to discover the concepts of congruence and similarity. Esen’s integration of technology into the mathematics curriculum was similar to Murat’s in the MCC3 lesson. Both of them generated ideas within the mathematics curriculum to implement technology in a more integral way for the development of the mathematics that the students were learning.

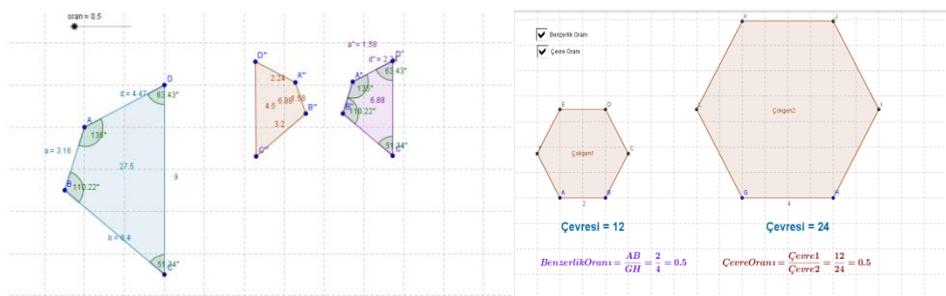


Figure 4. 35 The congruence and similarity activities in MCC3

In the following lesson after the congruence and similarity activities, Esen asked the students to open an already- prepared Geogebra file on the area of the polygon (see Figure 4. 36). The students tried to find the solution of the following problem: “Ahmet Bey's living room is like the shape on the side. Wanting to cover the floor of the living room with carpet, how many square center meters of carpet should Ahmet Bey use?” The students used the segment tool and the polygon tool in Geogebra to divide the already-known shape to find the area. In the meantime, Esen reviewed the area concepts, such as the area of the square, the rectangle and the triangle, which were taught in the previous class. Esen walked among the student groups and assisted them by asking questions. In addition, Esen encouraged the students to conduct different solution strategies to find the area of this irregular shape. After the students came up with solutions, the teacher wanted them to find another way to solve it. After this activity, Esen wanted to open another Geogebra activity to estimate the surface area of Ankara, the capital of Turkey. The students applied different solutions, such as cutting the shape into smaller pieces, subtracting pieces from a larger shape and dividing units via Geogebra. The students explored their mathematical ideas and developed different solutions in these problem solving activities by using Geogebra. These indicated that Esen was *at the exploring level*

of TPACK for the curriculum descriptor of the curriculum and assessment theme. However, in the pre-conference, she did not want to use Pick's theorems in Geogebra to find the area of irregular shapes. In this activity, the students would find the area formula of a simple lattice polygon by using the number of interior lattice points and the number of boundary lattice points. Furthermore, by dragging the vertex of a polygon, they would explore the relationship among the area of polygon, the number of interior lattice points, and the number of boundary lattice points. She refused to integrate this Pick's theorem activity into her class because she thought that it was an extra-curricular activity. These indicated that she had not yet modified or advanced the curriculum to take advantage of technology as a tool for teaching and learning as described in *the advancing level* for the curriculum descriptor.

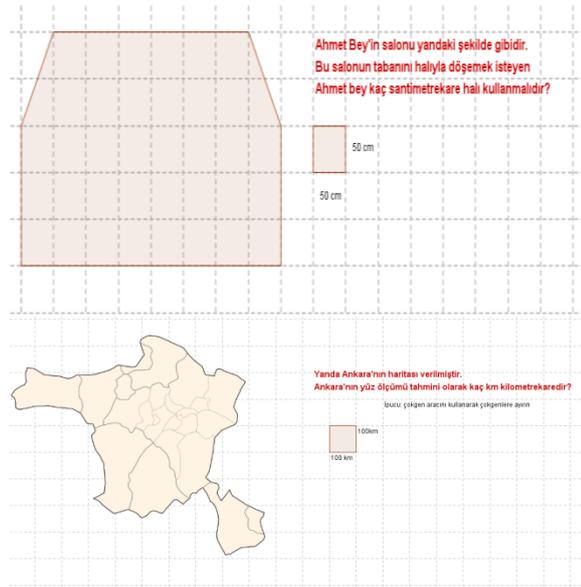


Figure 4. 36 The area of irregular shapes activities

After the area of irregular polygons, the students worked on an activity regarding the area of the parallelogram in Geogebra. They transformed the parallelogram into a rectangle by using the slider (see Figure 4. 37). Furthermore, the students dragged the vertices of the parallelogram to see the relationship between height, base and area. This activity provided an environment for the

students to discover the area formula of the parallelogram via Geogebra, which was used beyond the paper and pencil.

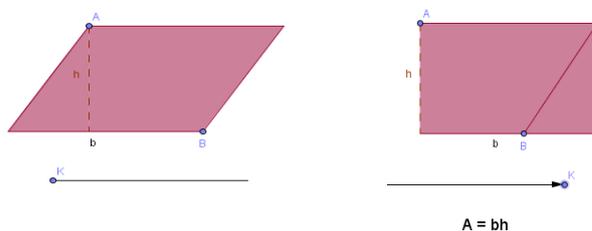


Figure 4.37 The area of parallelogram activity

After the first activity on the area of the parallelogram, Esen asked the students to open the Geogebra file on the area of the parallelogram and rectangle (see Figure 4.38). The students dragged point C in the rectangle to see the changes in the area of the parallelogram and the rectangle. While dragging point C, the height of the rectangle and parallelogram changed. This movement helped the students to discover the relationship between the height and the area of the parallelogram and the rectangle. In addition, the students dragged vertex J of the parallelogram to change the slope of the parallelogram. The students discovered that the height and side of the parallelogram remained the same in spite of the change in the lateral sides, and that the area did not change. The students effectively integrated Geogebra to discover concepts regarding the area of the parallelogram and the rectangle. After this activity, the students opened up another already-prepared Geogebra file on the area of the parallelogram. The students rotated the parallelogram via the slider to find the area of the parallelogram with a different base and height. They dragged and calculated the area of a parallelogram from different bases. They realized that they were able to calculate the area of the parallelogram by using any of the sides and corresponding heights of the parallelogram. Furthermore, in another activity Esen provided the students with an environment in which they could find the area using any of the heights within or outside of the parallelogram. In these activities, the teacher motivated her students to use Geogebra actively by using dragging in order to explore the area of the parallelogram.

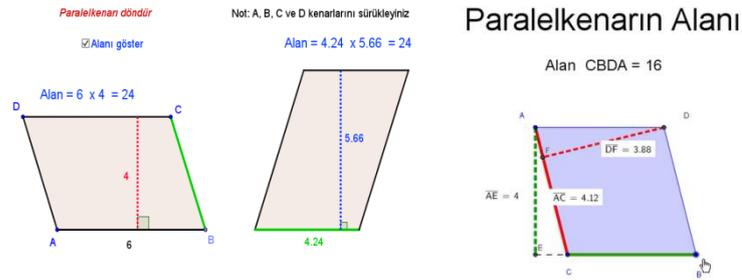
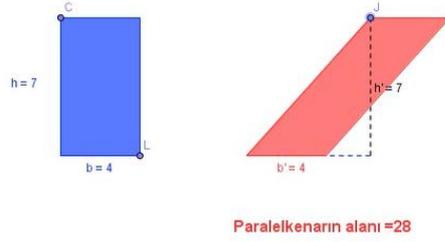


Figure 4. 38 The area of parallelogram activities in MCC3

Esen's MCC3 activities indicated that she moved through the TPACK Development Model for the curriculum descriptor. In the previous MCC2 lesson, she had difficulties in integrating Geogebra into the mathematics curriculum although she recognized the advantages of Geogebra. She had integrated Geogebra as a dynamics tool after exploring the concepts by using Geogebra as a static tool. But in MCC3, she integrated Geogebra by considering the dynamic properties to explore the concepts of congruence and similarity of polygons, the area of irregular polygons and the area of parallelograms. The activities in MCC3 extended beyond the paper and pencil to discover these geometrical concepts. The students effectively used the dynamic properties, such dragging and using sliders for transformation. In the post- conference, Esen said, "it was very nice to see that they saw no change in enlarging and minimizing the angles in the congruency and similarity activities. They constantly used changeable things and saw that when a side changed, the other side also changed at the same ratio. In this sense, it was highly beneficial." This statement indicated that she realized the advantages of the dynamic properties of Geogebra in her activities. In addition, in the pre-conference she started to design her lesson with the help of the examples on the site at

www.geogebra.org. Thus, she began to develop her ideas about using technology to enhance the current curriculum. On the contrary, like Murat, Esen did not want to modify the curriculum with extracurricular ideas, such as Pick's Theorem activity, in the MCC3 lesson. Neither of them reached the top level of the TPACK Development model for the curriculum descriptor in MCC3. These provided an evidence that Esen "sought ideas and strategies for implementing technology with a more integral role for the development of the mathematics that students are learning" (Niess et al., 2009, p.20) and "adapted the mathematics curriculum to include technology" (Niess et al., 2009, p.20) as in *the exploring level* of TPACK Development Model for curriculum descriptor.

Esen's MCC3 performance demonstrated *the adapting level* of the TPACK Development Model. She effectively integrated Geogebra as learning and teaching tool to explore geometrical concepts, such as the congruence and similarity and the area of polygons. Although she used Geogebra which extended beyond the paper and pencil to teach those concepts, she did not effectively implement Geogebra as a dynamic tool in assessment. In the first assessment activity, she wanted the students to find the scale ratio between the two triangles (see Figure 4. 39). The students did not effectively implement Geogebra with the consideration of affordance. Furthermore, the students' procedural knowledge on similarity was assessed. In the second assessment activity in MCC3, the teacher asked the students to draw different parallelograms in the Geoboard activity (see Figure 4. 39). She allowed time for the student groups to construct different parallelograms and to calculate their areas. This activity was like the previous activity in MCC3. In these activities Geogebra was integrated as a static tool. In addition, she was at a lower level than Murat in MCC3 for the assessment descriptor because Murat integrated appropriate Geogebra activities, which extended beyond the paper and pencil type of questions to assess the knowledge of students.

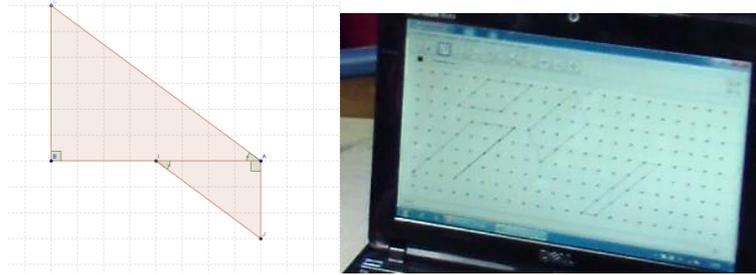


Figure 4. 39 The assessment activities in MCC3

The third assessment activity of Esen was similar to Murat' assessment activity in MCC3. The students were asked to reveal the change in the area of parallelogram ABCD with a stable base and height by dragging the vertex of the parallelogram. In this activity, the students had to integrate the element of the dynamic nature of Geogebra in trying to solve this activity. But the students did not pay attention to this activity because time was running out. Although this activity was not conducted properly, it went the beyond the paper and pencil type in assessing the students' conceptual understanding of the area of parallelograms. In the post-conference, she stated that if she had had more time, she would have believed that the last assessment activity was more effective in assessing the students' knowledge. The overall performance of Esen in MCC3 indicated that she "Understands that if technology is allowed during assessments ... different questions/items must be posed (i.e., conceptual vs. procedural understandings)." (Niess et al., 2009, p.20), which demonstrates *adapting level* in the TPACK Development Model for the assessment descriptor.

4.1.1.5 Mathematics Teachers' TPACK Level for Curriculum and Assessment Theme in the MCC4

In the fourth coaching cycle during the mathematics coaching process, Murat and Esen were observed for two hours to determine their level in TPACK Development Model. All the quotations and dialogs in the following paragraphs were derived from the pre-conferences/post-conferences and the mathematics lesson from the mathematics coaching cycle 4.

4.1.1.5.1 Murat's TPACK Level for Curriculum and Assessment Theme in the MCC4

Murat's MCC4 performance demonstrated *the exploring level* of TPACK for the curriculum descriptor and *the adapting level* of TPACK Development Model for assessment.

Murat asked students to open an already-prepared Geogebra file on the area of rhombus. In this activity, students explored the relationship between the area of a rectangle and a rhombus to arrive at a generalization about the area of the rhombus. The diagonals of a rhombus are perpendicular and form four right triangles through their intersection. By using sliders four right triangles in the rhombus were replicated, rotated and transformed to a rectangular shape (see Figure 4. 40). The rectangle surrounded the rhombus in the last figure. The students tried to find the area of the rhombus. They observed that the sides of the rectangle surrounding the rhombus were parallel and perpendicular to the diagonals of the rhombus. Since the width was indicated with the symbol e and the length with the symbol f , it meant that the rectangle had an area of exf . Thus, they made a conjecture that the area formula of rhombus was $A=exf/2$. Then, they dragged the vertex of the rhombus to move and resize the rhombus. As the size of the rhombus changed, the area was recalculated. In addition, they clicked on "show the area of rhombus" to see the area. They observed the results on the spreadsheet on the right side while dragging. They conceptualized the area of the rhombus by dragging and using the sliders. This technologically supported environment helped students to make a generalization about the area of the rhombus based on their exploration. In addition, Murat integrated Geogebra into MMC4 lesson to advance non-technological mathematics curriculum by considering the dynamic properties of Geogebra.

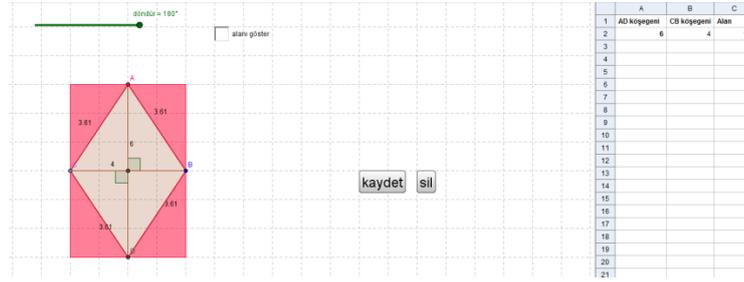


Figure 4. 40 The area of rhombus activity

After the rhombus activity, Murat asked the students to open the Geogebra file which was related to the area of a trapezoid (see Figure 4. 41). In this Geogebra-enhanced activity, the students learned how to find the area of a trapezoid by composing a parallelogram. By using the slider, the trapezoid could be copied, rotated, and connected to itself creating a parallelogram whose base was the sum of the two bases of the original trapezoid. The teacher provided an environment to find the formula for the area of a trapezoid by relating it to the known area of a parallelogram. In this activity they realized that the area of a trapezoid was half of the area of the parallelogram. The base of the parallelogram was equal to the sum of the bases ($b_1 + b_2$) of the trapezoid. Therefore, the area of the parallelogram is $((b_1 + b_2) \times \text{height})/2$. They dragged the vertex of the rhombus to make a random-size trapezoid. As the size of the trapezoid changed, the area was recalculated. They realized the relationship between the area, height and bases by dragging. They conceptualized the area of the rhombus by dragging and using the sliders. This technologically supported environment helped students to find the formula of the area of the trapezoid. In addition, Murat integrated Geogebra into the MMC4 lesson to modify the non-technological mathematics curriculum by considering the dynamic properties of Geogebra, like MCC3.

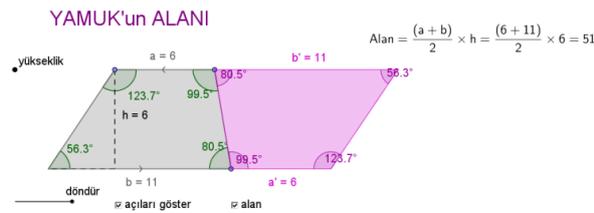


Figure 4. 41 The area of trapezoid activity

The MCC4 lesson indicated that Murat implemented technology to develop the mathematics that his students were learning with respect to the area of the rhombus and trapezoid. In both of the activities, students copied, rotated, and joined the shapes to create a new shape which they already knew by using Geogebra. Students were able to discover the formulas for the areas of the rhombus and trapezoid with the aid of the dynamic properties of Geogebra. In the pre-conference he stated,

“They don’t remember the formula. Normally I explain it as it is to be memorized. I can’t have them prove it in that activity... At the end I find $exf/2$. They immediately want to solve problems. I move on from there. But I think this is more reinforcing. Especially when I prepare the activities in a more dynamic way, the area of a rhombus, the area of a trapezoid are more reinforced.”

This statement indicates that the teacher is aware of the benefits of technology and has the desire to integrate technology into the lesson by taking into consideration especially its dynamic structure. In addition, in the post-conference after the lesson, he evaluated the lesson by saying,

“In terms of area, there are things that students are seeing for the first time. They know what a trapezoid is. They immediately say it is a quadrilateral with all sides being equal but here area is something that constrains me. I experience difficulty when explaining that point. I mean formula when I say general term. But what happened in class? They found it themselves. We started to free ourselves from memorization. Well, they think like this. Mathematics is memorization. We have taken them out of this mood a little. For instance that activity we did on the trapezoid. I mean dividing it by 2 has ended. It instantly appeared there. It was easy. It facilitated me. I mean I didn’t need to make much effort.”

Thus, it can be said that Murat integrated Geogebra to enhance students’ knowledge in the area of the rhombus and trapezoid by using technology beyond the paper and pencil. He modified the pre-existing activities in the curriculum by considering the dynamic nature of Geogebra. The teacher advanced curriculum to take advantage of technology as an educational tool such as using Geogebra to explore the formula of the area of the rhombus and trapezoid. Hence, these provided evidence that Murat was at *the advancing level* for the curriculum descriptor. Thus,

he “understood that sustained innovation in modifying own curriculum to efficiently and effectively incorporate technology as a teaching and learning tool is essential” as in *the advancing level* (Niess et al., 2009).

Murat’s MMC4 performance demonstrated *the adapting level* of TPACK Development Model for assessment. Murat assessed students’ knowledge on the area of the rhombus and trapezoid via Geogebra in lesson MCC4. In the first assessment activity, students were asked to find the relationship between the rhombus and the square (see Figure 4. 42). Murat asked students when the area of the rhombus had the highest value. Students realized that when the rhombus turned into a square by dragging from vertex, the area was of a maximum value because of the height. In this assessment activity, Murat determined students’ conceptual understanding of the rhombus. Furthermore, he used the dynamic nature of Geogebra to assess students’ understanding of the area of rhombus.

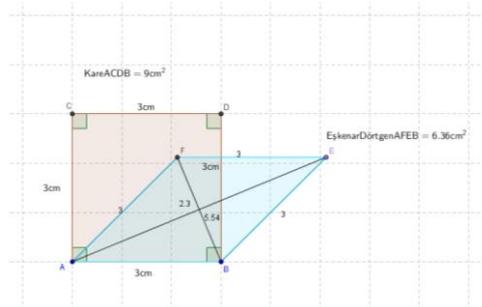


Figure 4. 42 The first assessment of MCC4

In another activity, students had to solve the problem posed in Geogebra. The problem was as follows: “Eren Bey wants to have his car painted. The measurements of Eren Bey’s car doors are $IABI=2$ m, $IBCI=1$ m, $IDEI=1,5$ m, $IEFI=0,5$ m, $IBEI=1$ m. The car painter charges 50 TL for an area of $1m^2$. How much does Eren Bey have to pay to have the four doors of his car painted?” The student had to find the area of the trapezoid-shaped car door. This activity was parallel to the assessment questions in regular textbooks. Students made calculations without using the dynamic nature of Geogebra. Furthermore, the teacher aimed to assess students’ procedural knowledge. This provided an evidence that he was at *the adapting level* for assessment descriptor

Eren Bey arabasının kapılarını boyatmak istiyor. Eren Bey'in arabasının kapılarının boyutları $|AB|=2$ m, $|BC|=1$ m, $|DE|=1,5$ m, $|EF|=0,5$ m, $|BE|=1$ m'dir. Boyacı, 1 m^2 lik alan için 50 TL ücret istiyor. Arabanın dört kapısının da boyanması için Eren Bey'in toplam kaç TL ödemesi gerekir?

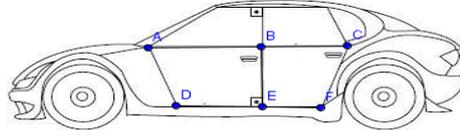


Figure 4. 43 The second assessment of MCC4

In another assessment activity, he asked the problem situation about the area of a rhombus. The problem was as follows: “A kite of a quadrilateral shape will be made using the two sticks given. How much paper does Ceren need to cover the front surface of her kite?” Firstly, students had to move the stick of the kite to make a rhombus by dragging. Then, they calculated the area of the rhombus. This activity was related to procedural knowledge regarding the area of the rhombus. Like the previous activity, it was aligned with the regular textbooks activities.

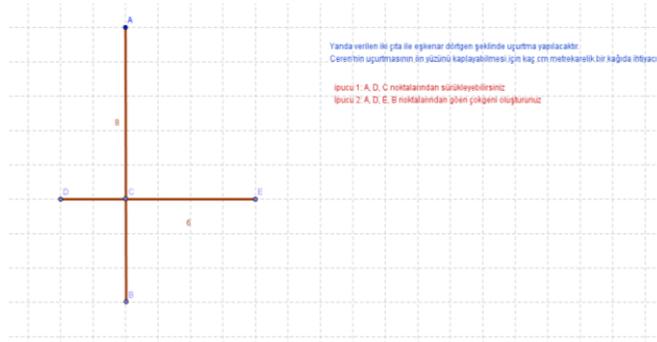


Figure 4. 44The third assessment of MCC4

The overall of Murat's MCC4 indicated *the adapting level* of the TPACK Development Model for the assessment descriptor because he let the students use Geogebra as an assessment tool and designed the assessment process to gather the students' conceptual understanding in addition to their procedural understanding. In the pre-conference, he stated that he wanted to align the some of the assessment with the textbook question because of the education system and exam. Consequently, the dynamic nature of Geogebra was not used actively in some of the activities such as the car activity. But in the post-conference he criticized himself, saying

“I could have done it more effectively; I am doing self-evaluation here. Uumm I could have pushed students further to make assessment more parallel to the constructivist approach. There was not much of a dynamic structure.”

He realized the importance of the dynamic nature of Geogebra in the assessment process. In his lesson, the students used the technology freely in the assessment activities like in MCC3. In the post-conference, he stated “the assessment stage was good in many ways. I think students were free. There were no problems. It proceeded rather well.” To conclude, he “Understands that if technology is allowed during assessments that different questions/items must be posed (i.e., conceptual vs. procedural understandings)” as in *the adapting level* (Niess et al., 2009, p.20).

4.1.1.5.2 Esen’s TPACK Level for Curriculum and Assessment Theme in MCC4

Esen’s performance in MCC4 during the mathematics coaching indicated *the adapting level* for the curriculum descriptor and *the exploring level* for the assessment descriptor in the curriculum and assessment theme of the TPACK Development Model.

In the MCC4 lesson, Esen asked the students to open Geogebra files on the area of the rhombus. In this activity the rhombus was divided into four congruent triangles and two of these triangles were transformed via the slider. Thus, the students transformed the rhombus into a rectangle by using sliders. This activity provided an environment for the students to discover the relationship between the area of the rhombus and the rectangle. They realized the relationship between the rhombus diagonals and the sides of the rectangle. One of the sides of the rectangle was equal to one of the diagonals indicated as f . The other side was half of another diagonal which was indicated as e . Thus, they found that the area formula of the rhombus was half of the multiplication of the diagonals: $A = ef/2$. In this activity, the dynamic nature of Geogebra was effectively used for the students’ exploration of the area formula of the rhombus.

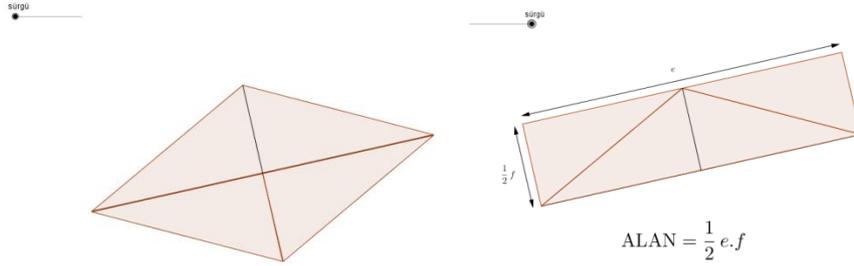


Figure 4. 45 The first area of rhombus activity

After the first rhombus activity, the students worked on another Geogebra activity. This activity was similar to Murat's activity in MCC4 (see Figure 4. 46). The student used four sliders to replicate and rotate the related triangle in the rhombus. After replicating and rotating the triangles, the rhombus shape was transformed into a rectangle. The students discovered that the area of the rectangle was twice that of the rhombus. Besides, the sides of the rectangle were equal to the diagonals of the rhombus which were indicated as e and f. After all, the students made a conjecture that the area formula of the rhombus was $A=ef/2$. This activity provided another perspective regarding the area of the rhombus. The students were motivated to make a conjecture about the area of the rhombus with these two Geogebra activities. In addition, like Murat, Esen provided an environment for the students to drag and explore the changes via the spreadsheet. These Geogebra supported activities helped students to enhance their exploration of the area of the rhombus by advancing the non-technological mathematics curriculum by considering the dynamic properties of Geogebra.

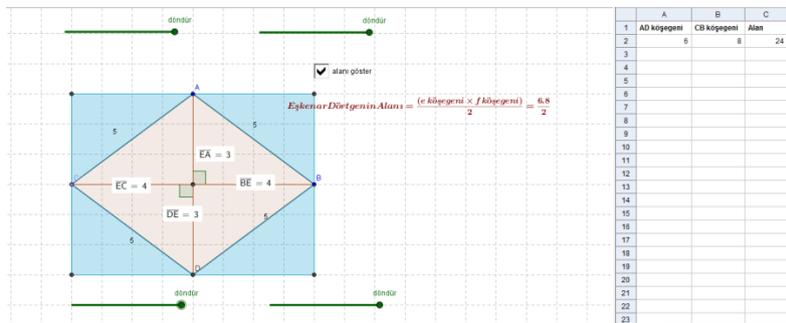


Figure 4. 46 The second area of rhombus activity 2

After the activities on the area of the rhombus, Esen asked the students to open up the Geogebra file on the area of the trapezoid, which was similar to Murat's activity in MCC3. In this activity, the students replicated and rotated the trapezoid by using the slider. With this transformation, the students obtained a parallelogram shape which included two congruence trapezoids. The students realized that the base of a parallelogram was the sum of the bases of a trapezoid. Then they defined the area of the parallelogram as a $(b_1+b_2) \times h$ where b_1 and b_2 are the bases of the trapezoid and h is the height. Hence, they explored that the area formula of the trapezoid is $((b_1+b_2) \times h)/2$. In addition, the students dragged the vertex of a trapezoid and observed the changes in the spreadsheet. Thus, they conceptualized the area of the trapezoid by using the sliders, by dragging, and observing the changes in the spreadsheet.

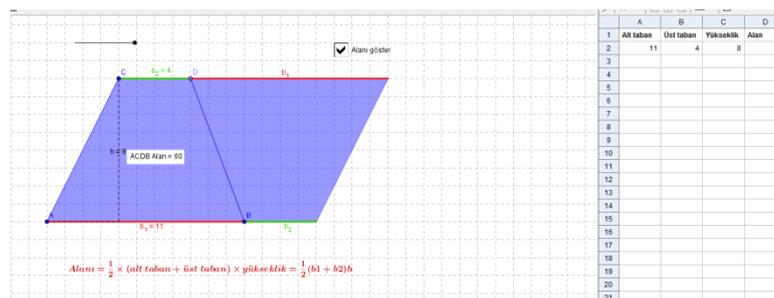


Figure 4. 47 The first area of trapezoid activity

After the first trapezoid activity, the students opened up another Geogebra file on the area of trapezoid. The students manipulated the bases of the trapezoid and the height by using the sliders. This activity helped the students to discover the area of a non- prototypical trapezoid with a different scale. This activity helped the students to explore how the sides and height affect the area of a trapezoid. These trapezoid activities revealed that Esen integrated Geogebra to develop students' conceptual knowledge on the area of the trapezoid by considering the dynamic nature of Geogebra.

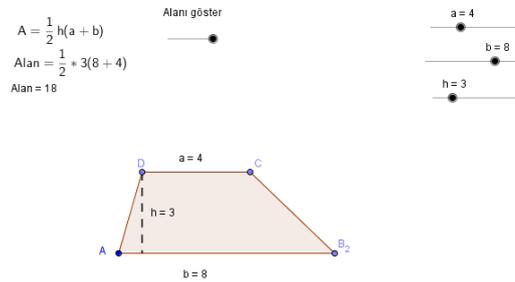


Figure 4. 48 The second area of trapezoid activity

Esen's MCC4 activities indicated that she moved through the TPACK Development Model for the curriculum descriptor when compared to the previous activities in MCC3. She was at *the advancing level* for the curriculum descriptor, like Murat in MCC4. In MCC4, Murat and Esen integrated Geogebra into their classroom with the consideration of the dynamic nature of Geogebra and advanced the non-technology based mathematics curriculum. In the pre-conference, Esen suggested a lot of Geogebra activities about the area of the rhombus and the parallelogram. She searched different sources based on technology, such as www.GeogebraTube. These indicated that she was interested in an innovative way to use technology to develop students' thinking process. She did not just enhance the pre-existing non-technology based activities in the textbook. In addition, in the post-conference she stated, "They used it in every section of the lesson. We used it in the presentation, practice and evaluation stages. The lesson that we had in the most effective way was this one." This statement demonstrated that she was also satisfied with the integration of technology as a tool for teaching and learning. For this satisfaction, she modified the curriculum to take advantage of Geogebra as a teaching and learning tool to have students explore the area of the rhombus and the trapezoid. Thus, she "Understands that sustained innovation in modifying [one's] own curriculum to efficiently and effectively incorporate technology as a teaching and learning tool is essential" as stated in *the advancing level* for the curriculum descriptor (Niess et al., 2009, p.40).

In MCC4, Esen was at *the advancing level* of the TPACK Development Model for the assessment descriptor. She integrated Geogebra not only to use it as a

tool to teach and enable students to discover the area of the rhombus and the trapezoid but also to assess their knowledge via the dynamic environment. Different from the assessment activities in MCC3, the students integrated Geogebra more dynamically when assessing their knowledge. In the first assessment activity, the teacher asked them to open the already-prepared Geogebra file on the rhombus (see Figure 4. 49). The teacher wanted the students to find the area of the rhombus and to explain the changes in the area by using the slider to change the interior angle. The students realized that the area did not change while the lengths of the diagonals were stable. Besides, the students used the “show triangle” checkboxes to reveal the triangles in the rhombus. With the help of the demonstration of the triangles, the students analyzed the relationship between the triangle and the rhombus. In addition, they dragged the vertices of the rhombus and moved the slider to change the length of the side to observe the area of the rhombus. This activity provided the students with an environment to assess their conceptual knowledge on the area of the rhombus by using the dynamic properties of Geogebra, such as dragging the vertices, hiding /showing triangles, and using the sliders to change the interior angles and sides.

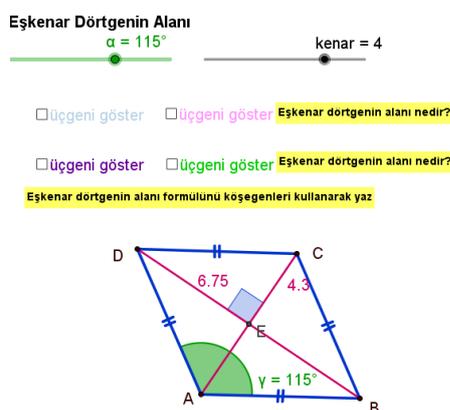


Figure 4. 49 The assessment activity about the area of rhombus

After this assessment activity, Esen asked the students to draw a different sized rhombus in the Geogebra file (see Figure 4. 50). After drawing the rhombus, the students calculated its area. In this activity the technology was not actively involved. However, after this activity, the students actively

integrated Geogebra, which extended beyond the paper and pencil by analyzing the relationship between the area of the square and the rhombus. This assessment activity was similar to Murat' assessment activity in MCC4. In this activity, the students had to find the maximum area of the rhombus. By dragging the vertices of the rhombus, the students observed that the area had the highest value when it was transformed into a square. They analyzed that the diagonals of the rhombus was maximum when the rhombus was square shaped. These assessment activities indicated that Esen implemented different types of Geogebra-based assessment questions, such as technology active/inactive, and procedural/conceptual understanding.

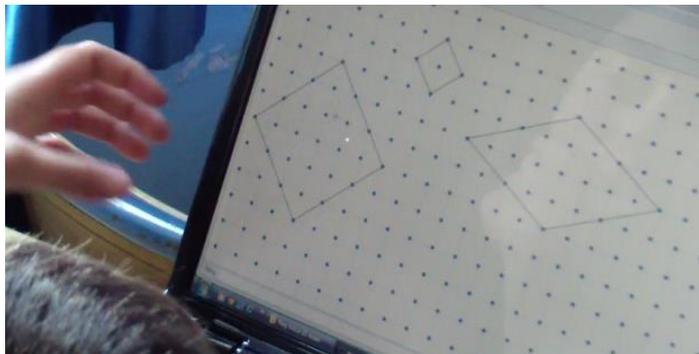


Figure 4. 50 The screenshot of the students' rhombus in Geoboard.ggb

In the another assessment activity in MCC4, the teacher asked the students to construct quadrilaterals and match their related properties and the area formula (seeFigure 4. 51). The students dragged the vertices of the polygons to draw a trapezoid, parallelogram, rectangle, rhombus, and square. Then they moved the related properties such that “interior angles was 90°”, “the sides were equal” and the area formula such as “area = height x base”. This activity assessed the students' procedural knowledge. This activity was prepared in line with the textbooks and the technology used was inactive.

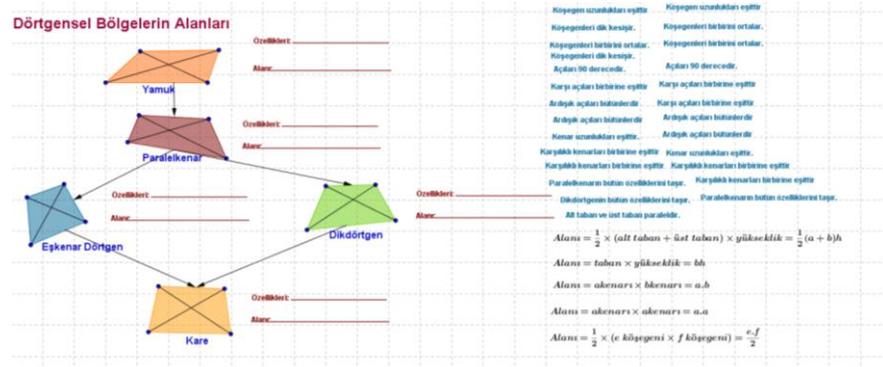


Figure 4. 51 The last assessment activity

Compared to the previous assessment activities in MCC3, Esen integrated Geogebra much more dynamically. Nevertheless, her overall performance in MCC4 was at *the adapting level* for the curriculum descriptor. She designed the assessment activities to reveal the students' conceptual and procedural understanding. In the previous lessons, she had a problem with timing and she did not devote proper attention to the assessment activities. In the post-conference, she declared that “We didn’t have time for assessment and evaluation in the other lessons. The first lessons were not very good as regards the students and my becoming accustomed. This lesson was good.” This speech demonstrated that she started to understand how to conduct Geogebra as an assessment tool. In the post-conference, she said, “I omitted one of the activities to spare time for assessment and evaluation. It’s beneficial. Enables them to practice and to use what they learned.” This statement indicated that she realized the advantages of Geogebra in assessing the students’ knowledge. Additionally, she said, “assessment went well. It benefitted us substantially. . It facilitated our work. We saw the drawings as well. What the child could envision was displayed to him/her.” In this lesson, she conducted the assessment activities to reveal their conceptual and procedural knowledge on area concepts, like Murat in MCC4. Furthermore, she allowed the use of Geogebra in order to make use of its dynamic and static properties in assessment. These indicated that Esen “understood that if technology is allowed during assessments that different questions/items must be posed (i.e., conceptual vs.

procedural understandings)” (Niess et al., 2009, p.20), which indicates *the adapting level* for the curriculum descriptor.

4.1.1.6 Mathematics Teachers’ TPACK Level Curriculum and Assessment Theme After Mathematics Coaching

After the mathematics coaching process, The lessons of Murat and Esen were observed for two hours to determine their TPACK level. All quotes and dialogs in the following paragraphs were derived from the TPACK post-interview and the post-observation lesson after the mathematics coaching.

4.1.1.6.1 Murat’s TPACK Level for Curriculum and Assessment Theme After Mathematics Coaching

Murat’s post-observation performance indicated *the advancing level* for the curriculum descriptor and *the exploring level* for the assessment descriptor in the TPACK Development Model of curriculum and assessment theme.



Figure 4. 52 The pattern of area of rectangle activity

In the MCC4 lesson, Murat integrated Geogebra as an educational tool in 2 hours to enhance students’ knowledge on the relationship between area and perimeter. At the beginning of the lesson, Murat asked the students to open an already-prepared Geogebra file to see the differences between area and perimeter (see Figure 4. 52). In this lesson, with the use of sliders, students discovered through analysis that the area and perimeter of a rectangle when wide of the rectangle is increasing. At first, the sides of the rectangle were 1x1, and the students

calculated the area and perimeter. The students were able to record these values in a spreadsheet on the right side of Geogebra. Then, they moved the slider to “2”; now the rectangle sides were 1x2. The students manipulated the rectangular shape to create shapes and to see the increase in the area of the shapes. Then the students recalculated the area and the perimeter again. While the width of the rectangle was increasing by using sliders, the students tried to see patterns of the area and the perimeter of the rectangle. The students found a pattern and wrote an equation of the area and the perimeter of a rectangle as n and $2n+2$, respectively. In this activity, the students expanded their experiences with measurement concepts by using Geogebra.

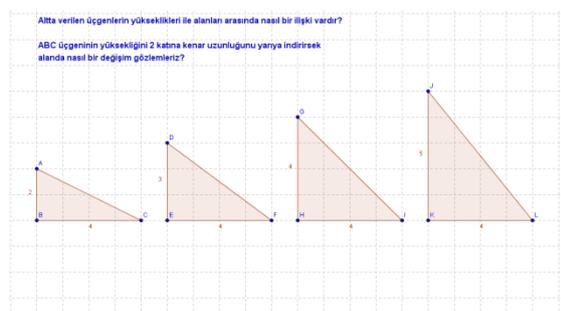


Figure 4. 53The area of triangle

After this activity, Murat wanted to open up a triangle activity to compare the area of triangles (see Figure 4. 53). The students had to find the area of these four triangles whose bases were the same, whereas the height was increasing one by one. The students groups calculated the area of the triangles and described the pattern which occurred between the areas of the triangles. The students began to make conjectures based on the patterns they observed in this situation. Then, in the second step of the problem they increased the height of the ABC triangle and decreased the base of it by dragging. The students realized that the area of the ABC triangle still remained the same. In the first part of the activity, Geogebra was used as a demonstration tool, but in second part of it the technical properties of Geogebra were used.

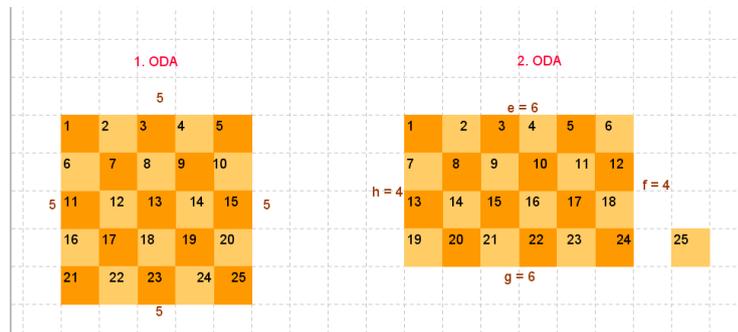


Figure 4. 54 The area of room activity

In another Geogebra activity, Murat showed two different rooms whose perimeters were 20 meters (seeFigure 4. 54). This activity was like a warm-up activity of the next activity. While the perimeters of these two rooms were the same, their areas were different. The students discussed the differences between these two rooms. The students might have had a misconception about the area and perimeter, such as “the perimeter and area has a linear relationship”. This activity provided a hint about the fact that when the perimeter was fixed, the area could have different values. After this activity, the students worked on a “fixed perimeter” Geogebra activity (seeFigure 4. 55). Murat asked for the biggest area of the rectangle whose perimeter was 24 units. The slider helped to change the sides of the rectangle. The students moved the slider to observe the changes in the lengths of the sides, perimeter and area. By using the “save” button, they were able to record the sides of the rectangle, the area and perimeter. Murat asked his students to explain what they had observed. Using sliders provided an environment to see the relationship between the area and perimeter more easily. With the aid of the dynamic structure of Geogebra, students made a conjecture that when the side lengths were brought closer together, the area became larger. The teacher implemented this activity to explore the relationship between the area and perimeter by using Geogebra as a problem-solving tool. Murat made innovative changes in the curriculum that promoted problem solving by taking advantage of Geogebra affordances.

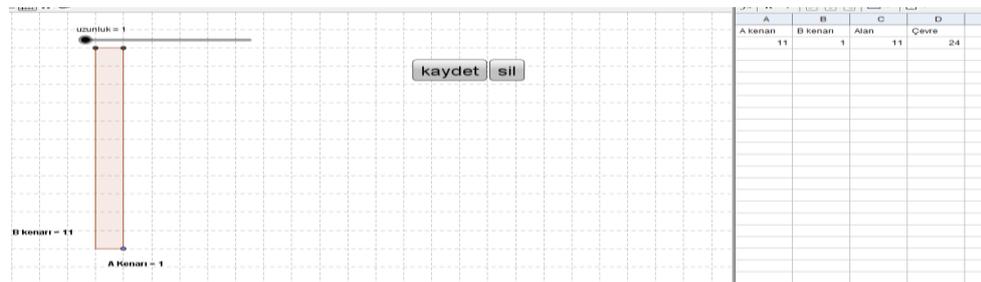


Figure 4. 55 The area of fixed perimeter activity

The post-observation lesson indicated that Murat implemented the technology in general and Geogebra in particular to enhance students' knowledge on the relationship between area and perimeter. Especially in the last activity, the teacher integrated technology to modify the current curriculum by considering the affordances of Geogebra. He designed the activity to reveal the dynamic nature of Geogebra in problem solving. In addition, in the first activity the students discovered the pattern and made a generalization about the area and perimeter for that situation by using the slider. In general, he incorporated Geogebra as a dynamic tool to encourage students' explorations and experimentation. Furthermore, he declined that he modified the lesson in the post-interview were

“Our textbooks seem to have given dominant place to the constructivist approach but perhaps people have not had complete faith in this. This study that I am doing is probably above their level. It really is a study that I embraced. I kept to the learning objectives but produced a more unique outcome. I did not abide by the textbook very much.”

Additionally, as thinking previous lesson he said, “(In the problems requiring calculating the area during MCC3) I, for example, should have specifically used the pick theorem. I could have developed a different perspective in students. But when I consider the exams that are administered, because I thought of the current system, I did not do it.” This statement showed that he had had hesitations regarding the integration of Geogebra in an innovative way. However, he evaluated the results of his lessons to incorporate the teaching and learning of the polygons unit with Geogebra and he willingly made a modification in the curriculum. In addition, he said,

“I should as a program ... no longer stick to structural things. I mean instead of giving the definitions, children should be given the opportunity to find the definitions, use their own expressions, be completely active in his/her discovery process; the teacher should actually remain playing the role of a guide. Of course I may not be in the position to question the proficiency of the program; however, considering this unit, it can be developed a little further. It can be integrated into a dynamic structure.”

He admitted that he became aware of the advantages of integrating technology into the curriculum, saying,

“When I integrate technology into the classroom, different views of students emerged. In other words, I did not just think over the figures I drew; towards the last weeks, I understood more clearly. If you integrate the constructivist approach completely into the system (curriculum), the child becomes enlightened when s/he finds the answer him/herself.”

His statements coincided with his post-observation lesson in terms of using Geogebra to encourage students’ explorations and experimentation in polygons. During the post-observation, to design the lesson, Murat searched Geogebra activities and technology- enhanced tasks on the Internet. He said,

“To prepare an activity using Geogebra requires a lot of preparation. You have to think and put effort into it because there is among the programs, there is also Geogebra. However, you have make slight changes. You have to adapt it to the level of the class. Or you have to adapt it to our country’s conditions or the learning objective. It requires some effort. I mean it’s clear in the textbook. I project the textbook onto the board. The book is there. I do and finish the activity there. It is fast and easy.”

Besides, the fact that designing technology enhanced activity needs more effort, he specified that it should be modified not just mimicked. These provided evidence that he willingly considered using Geogebra in building geometric concepts and especially concepts of polygons by advancing in the curriculum to take advantage of its dynamic nature. Thus, he exemplified elements of *the advancing level* of TPACK for the curriculum descriptor of the curriculum and assessment. He “modified and advanced curriculum to take advantage of technology as a tool for teaching and learning” as in *the advancing level* (Niess et al., 2009, p.21).

Murat’s post-observation performance demonstrated *the exploring level* of the TPACK Development Model for assessment descriptor. To assess students’

comprehension in area-perimeter, the teacher first started with an activity related to constant area (see Figure 4. 56). In this Geogebra activity, students were asked to find the largest possible perimeter of a shape with an area of 12 unit squares. Students could change the side lengths of rectangles having the same area by using the slider in this activity. By using the slider they calculated the perimeter of different rectangles and found the biggest rectangle. By inserting different values into the “number” box, the relation between rectangles with different constant areas can also be examined. While the student groups were solving this problem, Murat walked around the class and answered students’ questions. The teacher asked the students to find the features of the rectangle that had the greatest perimeter. The students responded by claiming that the wider the difference between the side lengths of a rectangle with a constant area was, the bigger the perimeter was. With the dynamic structure of Geogebra, the students found the area-perimeter relation of a constant area. With this activity Murat assessed students’ conceptual knowledge by using technology and by changing and modifying the assessment activities in the curriculum. He developed innovative assessments to reveal students’ understandings of the area and perimeter with Geogebra. This activity indicated that he reached the higher level of TPACK Development of assessment descriptor.



Figure 4. 56 The first assessment of Post-observation

In another assessment activity of Murat, a problem case in which a constant perimeter that was associated with daily life was given (see Figure 4. 57). The problem was as follows: “The principal of your school wants you to draw the figure of a sand pool to be built in the school yard. The sides of the sand pool will be built with timber. You are asked to use four pieces of timber of 3, 8, 10 and 11 meters to

design a quadrangle-shaped sand pool with the greatest area. Explain at which lengths you would use your timber.” In this activity, the students were asked to find the greatest area of the shape having a constant perimeter of 32 meters. In this activity, the students raised their hands to explain their responses. However, the students did not use Geogebra; they solved the problem in their notebooks. In this assessment activity Geogebra was solely used as a means of presentation; none of its dynamic features were made use of. Teacher B put students’ solutions on the board without using Geogebra. This activity was prepared in line with the textbooks and technology was merely used as a means of picture display such as Geogebra paint. Only the problem case was presented on Geogebra; Geogebra was not used to solve the problem.



Figure 4. 57 The second assessment of Post-observation

To sum up, he was at *the exploring level* of the TPACK development model. Going beyond the pen and paper, he prepared assessment activities using appropriate technology to measure students’ conceptual knowledge. Moreover, the teacher enabled their students to actively participate in the assessment process. In the post-interview, he said that his ideas about the usage of Geogebra in assessment changed and added,

“I thought that I would use technology only in the developmental stage... I realized that the dynamic structure is more suitable for assessments that reflect the constructive approach to assessment. I saw that the template in my mind could be used not just in the developmental stage but throughout the entire process.If I am undertaking this job, I should do assessment supported with technology. In all its phases, the introduction, the development... if I need to evaluate this course, undertaking such an assessment could be perfect. ”

He admitted that at the beginning of the study he had refused to use Geogebra as an assessment tool. But by the end of the study, he had incorporated Geogebra to gain insight into students' understandings of geometry by considering the dynamic features of Geogebra. Furthermore, when asked what the difference of a technology-aided assessment was, he stated,

“Well, the fact that children internalize what they see that moment. What is that? There are general formulas or things related to the area that the child finds in the exercises he/she does in these activities. The child needs to see that the area to be calculated is that of a trapezoid; he/she needs to see it and say ‘yes, that’s it.’ S/he needs to feel the need to take the dynamic structure I used in class and use it during assessment. S/he will draw the trapezoid there, move it, and use it. S/he will say “it worked’ or ‘no it didn’t work.’ S/he will draw something different. They are actually answering their own questions. Something is always intriguing their minds.”

Additionally, he gave an example as

“(when the area of an equilateral quadrangle and that of a square are affected) I questioned in what situations their areas would be the same. I didn’t experience any problems. Maybe I would have experienced problems while explaining it there, but they moved them and dragged them and saw it themselves. Their using the dynamic structure there made it easy for them.”

These statements indicated that he focused on using an appropriate technology that extended beyond the paper and pencil based questions when designing assessment activities to gain insight into students' understanding of geometrical ideas. On the other hand, Murat used technology in his second activity only as a means of presentation; he did not make use of the advantages of Geogebra in solving the problem. This diversity of Geogebra-enhanced assessment activity provided evidence that Murat was at *the exploring level* for the assessment descriptor. He “Actively investigates use of different types of technology-based assessment items and questions (e.g., technology active, inactive, neutral or passive)” as in *the exploring level* (Niess et al., 2009, p.20).

4.1.1.6.2 Esen’s TPACK Level Curriculum and Assessment Theme After Mathematics Coaching

Esen’s performance after the mathematics coaching indicated *the advancing level* for the curriculum and the assessment descriptors in the curriculum and assessment theme of the TPACK Development Model.

In the mathematics lesson after mathematics coaching, Esen integrated Geogebra as learning and teaching tool to enhance the students’ knowledge on the relationship between the area and perimeter. During the initial activities, the students opened up a Geogebra activity file on the area and perimeter of triangles (seeFigure 4. 59). They dragged the vertices of triangles to observe the changes in their area and perimeter. In addition, they worked with another Geogebra file on the area and perimeter of the rectangle (seeFigure 4. 59). These activities helped the students to understand the relationship among the sides, area and the perimeter. As the length of the sides increased, the area and perimeter also increased.

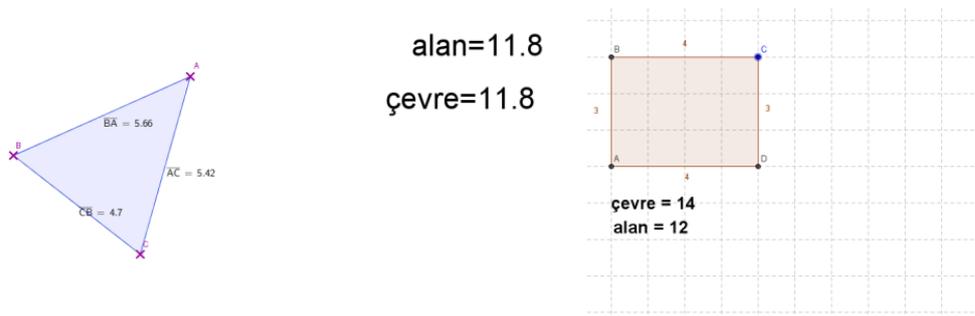


Figure 4. 58 The area and perimeter activities

After these warm-up activities, the teacher asked the students to open up a Geogebra file on the fixed perimeter (seeFigure 4. 58). In this activity, the students dragged the red point to increase the height of a rectangle while decreasing the length of its base. By dragging, the students constructed a new rectangle. So the old rectangle turned into the color red. At that point, some questions appeared on the Geogebra file: “Did the perimeter change? Did the area become smaller or bigger?” By clicking on the “see the difference” button, the students observed the differences between areas of the old and new rectangles. Then, the question of “When does the area have the highest value?” appeared. The students searched the

answer of this question in this Geogebra activity. After the students discovered the difference between the area of the old and new rectangle, they clicked on the “new area” button to fill the new rectangle with the color red. Then, the students increased the height again to explore the area with a fixed perimeter. With the help of this Geogebra activity, the students realized that as the side lengths were brought closer together, the area became larger. The dynamic nature of Geogebra was effectively involved in this Geogebra activity. Furthermore, this activity revealed that Esen developed innovative ways for the integration of Geogebra as a learning and teaching tool to support students’ exploration as indicated in *the advancing level* for the curriculum descriptor.

The screenshot shows a Geogebra activity interface. At the top, there is a text prompt: "Yüksekliği 1 birim atırıp tabanı 1 birim azalttığınızda alanda ne gibi bir değişiklik olur? (İpucu=kırmızı noktayı sürükleyin)". Below this is a rectangle with a red dot at its top-right corner. To the right is a spreadsheet with the following data:

A	B	C	D
taban	yükseklik	çevre	alan
9	1	20	9

Below the spreadsheet are two question prompts. The first one asks: "Çevresi nasıl değişti? Alan arttı mı azaldı mı? 'Farkı görün'e tıklayın". Below it is a button labeled "Farkı görün". The second one asks: "Çevresi nasıl değişti? Alan arttı mı azaldı mı? 'Farkı görün'e tıklayın'. Ne zaman alan en büyük değerini alır? Yeni problemler için 'yeni alan'a tıklayın.". Below it are buttons labeled "Geri" and "Yeni alan".

Figure 4. 59 The fixed perimeter activity

After the fixed perimeter activity, the students opened up another Geogebra file on the area and perimeter of the rectangle (see Figure 4. 59). The students increased the base of the rectangle one by one by using the slider. Furthermore, the students recorded the value of the sides, perimeter, and area in the spreadsheet via the “save” button. With the help of the spreadsheet, the students explored the

pattern between the area and perimeter. This activity helped the students to make a conjecture about the area and perimeter by using Geogebra in an innovative way.

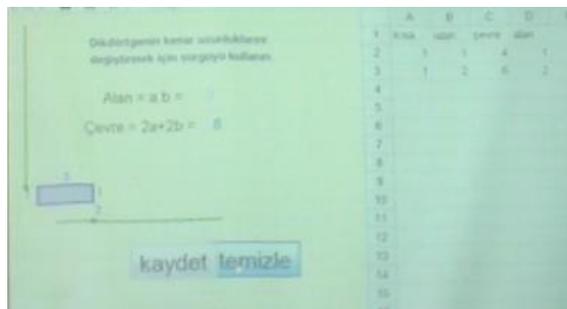


Figure 4. 60 The screenshot of area and perimeter of rectangle activity

After the mathematics coaching, Esen integrated Geogebra to enhance the students' knowledge on the relationship between area and perimeter. In the post-interview, she said that she implemented Geogebra as an exploration tool and added, "For example, regarding the area of the trapezoid, we got used to resembling the side of a parallelogram. They rotated it. Then they realized that a trapezoid had two parallel sides and then saw that its formula transformed." This speech revealed that she used Geogebra to support the students' knowledge as of the mathematics coaching session. In addition, she declared that she searched different sources such as the internet and geogebra tube to design her lesson. In addition, she stated, "I do not keep to the textbook very closely. That's why I try not to use the textbook in class. Well, I made use of supplementary resources and prepare most of Geogebra myself. I did not keep to the textbook very much." This speech was parallel to her lesson because she modified and advanced the curriculum to benefit from Geogebra as learning and teaching tool. In addition, Murat also modified the curriculum by integrating a problem solving activity on the relationship between the area and perimeter. Both of them developed an innovative way to use technology to develop students' mathematical thinking skills. Thus, Esen "Understood that sustained innovation in modifying [one's] own curriculum to efficiently and effectively incorporate technology as a teaching and learning tool is essential,"(Niess et al., 2009, p.20) which indicates *the advancing level* for the curriculum descriptor.

In another activity, Esen asked the students to open up an already-prepared Geogebra file which was related to maximum area. The problem was “What is the biggest possible rectangular garden enclosed within a 300-meter fence?” Students had to find the maximum area of a fixed perimeter. In this problem case, Teacher A constructed a connection with the real world and drew students’ attention. To ensure that each student group understood the activity, Teacher A walked around in class. Then she asked the students to move the slider and explain what they observed. In this activity, the students could use the slider and change the base length of the rectangular shape and, as a result, could easily observe the measurement of the height and the changes in the area. In addition, by using the “save” button, they could save the base, height and area values on the spreadsheet calculated on the right. The spreadsheet could enable the students to see the relation between the area and perimeter more easily. The students said that a garden with a perimeter of 300 meters had the greatest area when it was square shaped. Then the teacher asked what needed to be done for a garden to have the largest area when a certain perimeter was given. As the slider was getting closer to the middle, the students realized that the area was larger. So, the students responded by claiming that the side lengths should be brought closer together. In this activity, the students could see the results of their mathematically meaningful actions on Geogebra and arrive at related generalizations and conclusions. The students actively participated in this assessment activity supported with Geogebra and discovered the relation between area and perimeter themselves. In this activity, Esen went beyond the paper and pencil and used Geogebra to assess students’ knowledge of the relationship between area and perimeter. Moreover, in this assessment activity, the teacher focused more on conceptual knowledge rather than procedural knowledge and asked the students to make generalizations. In this way, she redesigned a subject in the curriculum by taking into consideration the features and advantages of Geogebra.

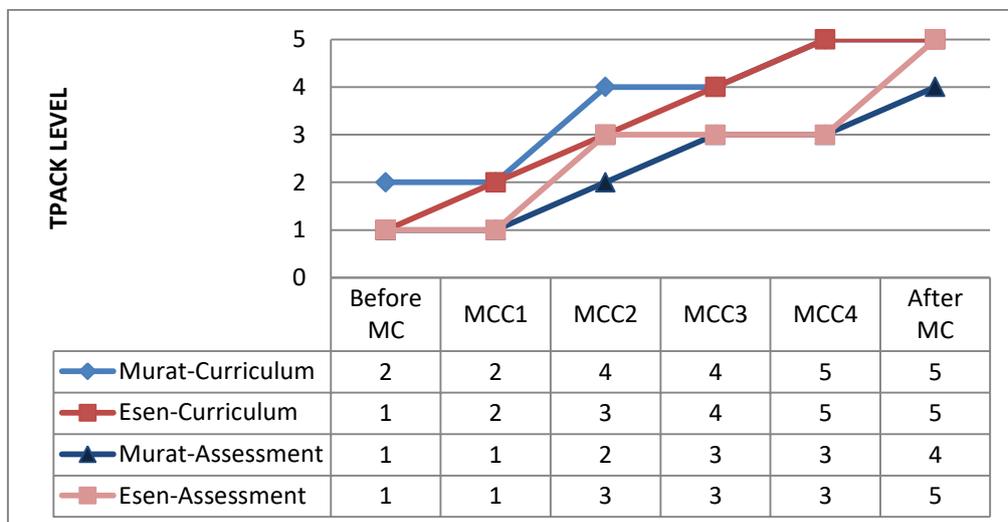
Esen’s overall lesson demonstrated that she designed assessments to reveal students’ understanding of geometrical ideas using an appropriate technology that extended beyond paper and pencil type of questions. Both area and perimeter

activities assess students' conceptual knowledge instead of procedural knowledge. Furthermore, she modified the assessment activities that investigated students' understandings of area and perimeter with the consideration of the advantages of Geogebra. In the post interview, she stated, "I think it was good. They understood it. And we could easily do the assessment and evaluation. In fact, I focused specifically on assessment and evaluation. I said we should keep the activities short and do more of assessment and evaluation.." This statement revealed that she gave attention to Geogebra as an assessment tool. Furthermore, she said that her view about the usage of Geogebra in assessment changed and added, "because I realized that technology could be used in assessment and evaluation as well. In fact, I am thinking of using it in grade 5." At the beginning of the study, she did not consider Geogebra as an assessment tool. This speech revealed that she made progress in the integration of Geogebra as an assessment tool. In addition to this progress, she implemented Geogebra in innovative ways, such as for a problem solving activity, and focused on the students' conceptual understanding. She implemented Geogebra activities to assess the students' conceptual understandings of the relationship between area and perimeter in ways that demanded full use of technology. As a result, this assessment activity shows us that Esen is at *the advancing level* according to the TPACK development model. She "developed innovative assessments to capture students' understandings of the mathematics embedded in the particular technology," which indicates *the advancing level* for the assessment descriptor (Niess et al., 2009, p.21).

4.1.1.7 The progress of Murat and Esen's TPACK of Curriculum and Assessment Theme through Mathematics Coaching

The performances of Murat and Esen in the six technology-enhanced lessons before/during/after mathematics coaching are presented in Table 4. 5. The teachers demonstrated progress in the TPACK development model for the curriculum, and assessment descriptors.

Table 4. 5 The progress of teachers for curriculum and assessment descriptors



The curriculum descriptor demonstrated how to integrate technology into the curriculum as an educational tool in this study. At the beginning of the study, Esen was at a lower level than Murat in the TPACK Development Model for the curriculum descriptor. Because, Murat implemented the Geogebra-enhanced activities to advance students’ knowledge on polygons, whereas Esen used Geogebra to reinforce the knowledge which taught with concrete material. Besides, both of them did not take into account the advantage of Geogebra affordances such as its dynamic nature. The teachers replaced the activities in the textbook to meet curricular needs without going beyond the paper and pencil. The teacher struggled to conceive how Geogebra could be incorporated into the mathematics lesson with taking advantages of dynamic nature. The main teachers’ usage of Geogebra in the first two lessons (pre-observation and MCC1) was as a tool for presentation in the teaching and learning of the concepts related to the polygon. During the mathematics coaching the teachers moved to a higher level of the TPACK development model for the curriculum descriptor. The teacher started to understand and investigate some of the benefits of integrating Geogebra appropriately as a tool into the mathematics curriculum. Although this understanding of the benefits of Geogebra, Esen conducted concrete materials instead of Geogebra to summarize the

students' knowledge in MCC2. Thus, she had still a limited knowledge about the integration of Geogebra as learning and teaching tool into mathematics curriculum. She had made slower progress than Murat in the TPACK Development Model for the curriculum descriptor from the beginning of the study to MCC3. In the MCC3, Esen and Murat was at the same level (*the exploring level*). Both of them sought for curricular ideas to provide students with stronger conceptual understandings. Towards the end of the study, the teacher generated a vision on how to incorporate Geogebra into their lesson and modify the current curriculum. While moving into *the advancing level*, they searched for different curricular ideas than those in the textbook from various sources such as GeogebraTube and internet. Furthermore, they incorporated some of the main features of Geogebra as dynamic geometry software, such as dragging and using sliders. Especially in the last two lesson – MCC4, and after MCC- the teachers innovatively made changes in the curriculum as advancing in the relationship area and perimeter by using Geogebra as a problem solving tool. These demonstrated that the teachers reached the highest TPACK level at the end of the study. Finally, they discerned that modifying and advancing the curriculum in an innovative way is crucial, efficient and effective integration of technology in general and Geogebra in particular.

For the assessment descriptor the teachers also demonstrated growth in the TPACK development model for the assessment descriptor demonstrating how to integrate technology as an assessment tool in this study. Initially, Murat and Esen resisted the idea of using Geogebra in the assessment process indicating that technology is not necessary to determine students' understanding of geometry. Both of them did not let their students use Geogebra in the assessment process even though they integrated Geogebra in the learning and teaching process. Then they started to integrate Geogebra as a visualization tool to gain insight into students' understanding of geometry. Additionally, they did not take into account the dynamic features of Geogebra. During the mathematics coaching the teachers moved to a higher level of the TPACK development model for the assessment

descriptor. In the MCC2, Esen provided a dynamic environment for the students to assess her students' understanding while Murat integrated Geogebra as just a visualization tool in assessment. Thus, Esen was at a higher level than Murat in the TPACK Development Model for the assessment descriptor in MCC2. In the next coaching cycle- MCC3- Murat reached Esen's level in TPACK Development Model for assessment descriptor. Both of them implemented the Geogebra activity in assessment which was the beyond paper and pencil in MCC3. The teacher investigated some of the benefits of integrating Geogebra extending beyond the paper and pencil into the assessment process. But in some activities the dynamic nature of Geogebra were not used effectively. Murathad concerns related to the Turkish education system and exams. So he wanted some of the assessments to be aligned with the textbook question focusing on students' procedural knowledge like Esen. Besides, they prepared Geogebra-enhanced assessment activities to understand not only the students' conceptual understanding but also their procedural understanding. While moving into the end of the study, they realized that they should give attention to the dynamic nature of Geogebra to capture students' conceptual understanding. Especially in the post-observation after mathematic coaching, they designed a variety of Geogebra-enhanced activities by considering the dynamic features of it to assess the students' comprehension level of geometric concepts. Moreover, the teachers provided an environment for the students to actively engage in the assessment process. They made actions and observed the consequences of their action in Geogebra to solve the assessment activity. Furthermore, after mathematics coaching Esen implemented Geogebra in innovative ways via a problem solving activity to focus on the students' conceptual understanding. Thus, she modified the curriculum to assess the students' conceptual understandings in ways that demanded full use of Geogebra.

4.1.2 The Learning Theme

The learning theme included two descriptors: mathematics learning descriptors, and the conception of student thinking. The mathematics learning

descriptor of learning theme concentrates on whether teachers integrate technology emerged as a potential tool into learning mathematics. The conception of student thinking descriptor of learning theme concentrates on the development of students' thinking skills with technology. In the subsequent sections, Murat's and Esen's TPACK is analyzed according to the TPACK Development Model's learning theme, mathematics learning descriptor and the conception of student thinking descriptor.

4.1.2.1 Mathematics Teachers' TPACK Level for the Learning Theme Before Mathematics Coaching

Before the mathematics coaching process, lessons of Murat and Esen were observed for two hours to determine their TPACK level for mathematics descriptor of learning theme on concave and convex polygon. All quotes and dialogs in the following paragraphs come from teachers' TPACK pre-interviews and the pre-observation lessons before mathematics coaching.

4.1.2.1.1 Murat's TPACK Level for the Learning Theme before Mathematics Coaching

Murat's performance before mathematics coaching indicated *the adapting level* of TPACK for mathematics learning descriptor, and conception of student thinking descriptor in the learning theme.

In the "Introducing polygons" activity students investigated diagonals of polygons and concave /convex polygons with the help of the position of diagonals via Geogebra. Students were asked to construct three different polygons including 4-sided convex, 5-sided convex and 6-sided concave, respectively. Then they drew segments between two vertices in polygons, which led students to analyze the properties of these segments. After students constructed every segment of these three polygons, they discussed the common properties of line segments. This Geogebra-enhanced activity provided students an opportunity to define and explore diagonal as a line segment joining two nonconsecutive vertices of a polygon.

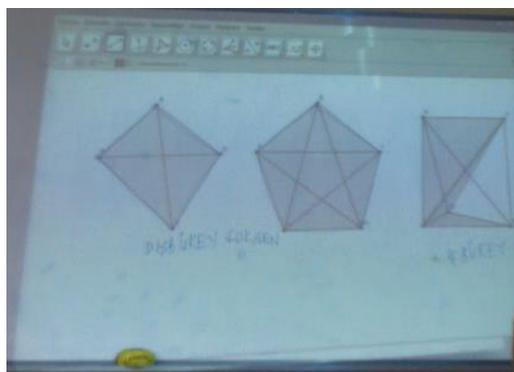


Figure 4. 62 The screenshot of board

After defining diagonals, Murat asked the questions in the worksheet about the position of these diagonals which lead to discover concave and convex polygon (see Figure 4. 62). The students and teacher were analyzing the differences among the positions of diagonals in different polygons to define the type of polygons. With the guidance of teacher, the students realized that “if some of the diagonals of polygon lie outside the polygon, it called concave polygon” and “all of the diagonal lie inside of the polygon, it called convex polygon”. This Geogebra-enhanced activity helped students with their thinking and understanding of concave and convex polygons and their learning was able to improve through Geogebra-enhanced activities. Students explored and discovered concave and convex polygons as geometric concepts thanks to the whole class conservation via Geogebra.

The “Finding number of diagonals” activity allowed students to explore the relationship between the number of sides and the total number of diagonals in polygons. Students were directed to construct all diagonals of quadrilateral, pentagon and hexagons respectively via Geogebra in this activity (Figure 4. 63).

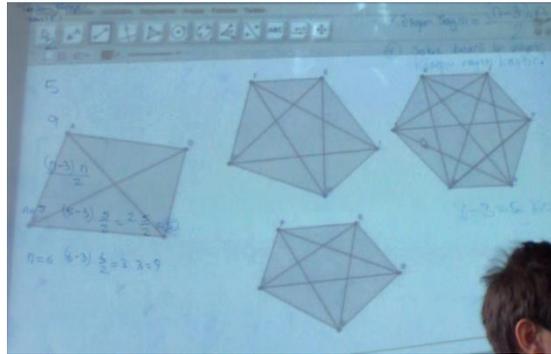


Figure 4. 63 The screenshot of diagonals of polygons

Under the teacher’s guidance, students completed the table about side number and total number of diagonals. Using the pattern among diagonals and sides, students were able to discover relationship between side number and total number of diagonals in polygons. This whole class discussion helped the students to share their results with the teacher’s support. Murat summarized all of the conjectures that students came up with during a whole-class discussion to notice the pattern of the total number of diagonals from one polygon to another. Students were able to generalize the relationship between the number of diagonals and sides of polygons. This technology-enhanced activity led the students to make a conjecture and find an equation of the number of diagonals for any number sided polygons. Students made inductive reasoning to formulate a conjecture about the number of diagonals of an n -sided polygon as a $\frac{n(n - 3)}{2}$. With the help of this Geogebra-enhanced activity, students used numeric and geometric patterns to make generalizations about geometric properties including the number of diagonals for any number sided polygons.

Murat helped students to make a conjecture and explore the concepts about polygons at “Introducing polygons” and “Finding number of diagonals” activity worksheets via Geogebra. Based on explorations via Geogebra, students were able to make generalize and test their conjectures about polygons and their component parts in the pre-observation lesson. Students constructed and justified statements about polygons as geometric figures and their properties. Furthermore, he said that he prefer to integrate Geogebra as a learning tool not a teaching tool. He added

“Thanks to technology, we don’t give direct information. We don’t use like a book. Specially the dynamic software. How should I say, we don’t go like this is there, you should find this here, we make the child discover. We are here to guide them like a reference. Different things can turn out. They can be diverted to other things but with the question right below, we try to pull them to back to the same side. What do we do here? While helping them learn themselves, they also start learning themselves what to do, what there should be... When it is at the last stage the gain which was given at the beginning is somehow reached. Hence this is a learning tool. It is a tool, not to teach the kid here but a learning tool to find out themselves and be involved.”

Thus, it can be said that Murat is at the adapting level of TPACK for the learning theme, mathematics learning descriptor. Teachers at the adapting level for learning theme begin to experiment and use the technology as a learning tool. In this level, students can explore some mathematics topics using technology as Murat did. This situation emphasizes the adapting level of TPACK for the learning theme, mathematics learning descriptor.

Murat’s performance before mathematics coaching also demonstrated *the adapting level* of TPACK for the learning theme, conception of student thinking descriptor. In the warm-up section, Murat started the lesson by asking where students see polygons in everyday environment to attract their attention. Then they discussed the examples and properties of polygons such as at least how many sides a polygon has. These kinds of questions are important to remind the concept of polygon. Then, Murat searched polygons’ pictures at google to give other examples of polygons especially in daily life (see Figure 4. 64). The whole class discussed each image to understand whether or not it is a polygon. These warm-up activities help students not only draw into the lesson but also give the opportunity to remind the concept of polygons. Students revised some important knowledge about polygons such as a polygon has at least 3 sides and these sides cannot be curve. These warm-up activities oriented them towards the context of this lesson which is diagonals of polygons and convex/concave polygons. Additionally, using these activities provided students an opportunity to integrate polygons as a mathematics concept into their daily life. Despite of advantages of these warm-up activities,

Murat didn't use Geogebra as a technological tool to enhance students' learning about polygons at the beginning of lesson. At *the adapting level* for students' conceptions descriptor, teachers begin exploring the technology to develop appropriate mathematical thinking skills. Furthermore, teachers at *the adapting level* are still not sure about how students develop appropriate mathematical thinking skills via Geogebra as a learning tool. Consequently, technology is used for many topics, while introduction remains without using technology like Murat's lesson (Niess et al., 2009).



Figure 4. 64 The screenshot of board

In the “Introducing polygons” activity worksheets small student groups investigated concave and convex polygons by using diagonals of polygons via Geogebra. To overcome possible students' confusion over concave and convex polygons, teacher emphasized the properties and characteristics of concave and convex polygons by a whole class discussion via Geogebra. In this activity, Murat asked students to draw similar polygons in the activity sheet via Geogebra - a 4-sided convex, 5-sided convex and 6-sided concave, respectively -. Firstly, he directed the discussion to remind general properties of diagonals such as segments connecting two nonconsecutive vertices. Then, he made a whole group discussion about the position of different polygons' diagonals to provide students with an opportunity to discover concave and convex polygons. Students were encouraged to realize that a convex polygon has no diagonal in the exterior of the polygon whereas a concave polygon has at least one diagonal in the exterior of the polygon with this Geogebra activity. Murat paid close attention to students' own language. He

provided students with the opportunity to demonstrate and explain their specific terms so that Murat overcame possible misconceptions. On the other hand, he asked to students to draw exactly the same polygon as on the board. He did not encourage them to draw different polygons; consequently, the students analyzed the concave and convex properties in non-prototype polygons.

In the “finding number of diagonals” activity worksheets, conceptual understanding was emphasized rather than formula memorization to overcome children’s difficulties finding the sum of interior angles of a polygon. The students were asked to construct a quadrilateral, a pentagon and a hexagon, respectively to find diagonals from a single vertex in each polygon. Teacher asked some guiding questions such as “Do you notice any relationship?” and “What is the relationship between the number of sides and the number of diagonals in a single vertex?”. These questions helped students to make a conjecture and discover the relationships between side and diagonals. Then, he asked students to find all diagonals of the quadrilateral. After he drew all diagonals of the quadrilateral, he wanted students to find the number of diagonals of pentagons and hexagons by constructing diagonals of these polygons one by one via Geogebra. The teacher provided an opportunity for students to discuss their findings and thoughts about the number of diagonals of pentagons and hexagons. Murat wanted students to fill in the table about diagonal numbers in the Activity Worksheet and to share their final results with the whole class under the teacher’s guidance. Murat helped students to make inductively reasoning of the number of diagonals of an n -sided polygon as $n(n - 3) / 2$. He created a Geogebra-enhanced environment to challenge students’ understanding and broaden their generalizations. Furthermore, he supported children to move to higher levels of thinking by using guiding questions.

The teacher did not include Geogebra in the warm-up activities. In the pre-interview, he stated that

“i bring three dimensional materials to the introduction part. Otherwise, i show them the pictures and shapes i find on the internet and use them somehow ant the introduction part. I am not sure if Geogebra can be used at the introduction. In terms of gathering attention, maybe if there

is something previously prepared, how should I say maybe an event can be designed at home. When something is done regarding this, when we open it directly to the children, something like this can be done but; I mean at introduction, besides that I don't think it is appropriate, maybe during development stage. During the discovery of new term maybe."

This explanation supports that he had a limited usage of Geogebra as learning and teaching tool. Although it might indicated that Murat was at *the accepting level* of TPACK for the learning theme, mathematics learning descriptor,

Murat did not use technology just to make computations, reinforce and enhance concepts previously taught without technology, or to verify solutions which indicate *the accepting level* of TPACK development for the learning theme, conception of student thinking descriptor. In Murat's pre-observation performance, Technology was used to improve students thinking rather than practicing and memorizing. Murat drew attention to students' thinking of mathematics while students were using technology to make sense concepts related to polygons. Besides, he took into consideration how to overcome the students' possible difficulties and misconceptions. In the pre-interview, he stated that

"while find the number of diagonals, finding the total of internal angles we have a difficulty here. During the stage of going for a generalization they need to be steered into the right direction with the right questions. The child doesn't understand what you are trying to say at that moment. They make different sentences. I also don't understand what they are saying. I have problems while making generalizations."

And added that "however, when they find it themselves, when they do it on Geogebra the problem is less." On the other hand, this Geogebra-enhanced lesson did not provide an opportunity for the teacher to manage to direct student engagement and self-direction in learning mathematics. Even if technology is used for students' independent explorations, teacher-led works with technology is common. This situation provides evidence that he started to implement Geogebra for the development of students' knowledge. Furthermore, in the pre-interview, he admitted that "what can I say I go over it with technology after I teach the subject. So in reality I don't use it that much. I actually want to use. Of course it makes more

sense for the children to discover the concepts themselves there and find it with the technology.” In addition, this explanation indicates that Murat “began developing appropriate mathematical thinking skills when technology is used as a tool for learning”, which is at *the adapting level* of TPACK for the learning theme, conception of student thinking descriptor.

4.1.2.1.2 Esen’s TPACK Level for the Learning Theme before Mathematics Coaching

Esen’s performance before mathematics coaching indicated the *recognizing level* of TPACK for both the mathematics learning descriptor and the conception of student descriptor in the learning theme.

Esen integrated concrete materials to provide an environment for the students to explore the concave and convex polygons. Firstly, the students were asked to construct polygons of various sides (triangle, quadrilateral, pentagons, hexagon, and heptagon) via using the geometry board and elastic bands. The teacher motivated her students to create convex polygons and shared the students’ examples on the geometry board, which were concave polygons (see Figure 4. 64). The students started to make generalizations about concave and convex polygons based on their shapes, for example “concave polygons were like battered.” Subsequently, the students worked with toothpicks to discover concave and convex polygons. The students constructed concave and convex polygons and found their diagonals. They explored that all diagonals of a convex polygon should be in the interior of the polygon, whereas at least one of the diagonals of a concave polygon should lie outside the polygon.

After exploring the concave and convex polygons by integrating concrete materials, Esen implemented Geogebra to reinforce the students’ knowledge. She drew concave and convex hexagons and built the diagonals of these hexagons. With this activity, the students focused on the diagonals of polygons once again to define the type of the polygon. The teacher used Geogebra as a supplement to instruction after using concrete materials. This indicated that she did not integrate Geogebra to

provide students with an environment in which they could explore new concepts, as described in the recognizing the level of mathematics learning.

In the pre-interview, Esen stated, “The new generation is in the computer age. It draws their attention. I believe that it is 100 times more interesting than direct instruction. It may not be as effective as an activity prepared with concrete materials but I think technology can also be effective.” This statement was consistent with her lesson before mathematics coaching. At first she integrated the concrete materials (geometry board and tooth sticks) to support the students’ exploration and then she used Geogebra as a supplementary tool. To justify her preference of concrete materials, she said, “In the activity he/she engages in hands-on activity That’s why I thought that way.” That is, she believed that concrete materials were more effective than technology supported activities because concrete materials were more presentative. In addition, in the pre-interview she stated “Technology enables him/her to see more things by solving more sample questions.” This speech supported the usage of Geogebra in the lesson before mathematics coaching. She implemented Geogebra to demonstrate examples of concave and convex polygons, which was explored already via concrete materials. Thus, it can be said that Esen is at *the recognizing level* of TPACK for mathematics learning descriptor of the learning theme. As indicated in *the recognizing level*, in her mathematics lesson “Mathematical exploration with technology [is] rarely seen” (Niess et al., 2009, p. 21). Her level in the mathematics learning descriptor was lower than Murat’s because Murat integrated Geogebra to help students’ exploration even though he used Geogebra as a static tool.

Esen’s performance for the descriptor on the conception of student thinking was at the lowest level of the TPACK Development Model, as with the mathematics learning descriptor. In the warm-up stage of the lesson, she demonstrated examples of polygons by means of a PowerPoint presentation without using Geogebra. As in the warm-up stage, she did not integrate Geogebra during the assessment stage. Furthermore, she did not use Geogebra as an exploration tool during the lesson. In the lesson she preferred to use concrete materials to explore concepts, such as

concave and convex polygons and interior and exterior angles. The activities with concrete materials helped students to discover the relationships between diagonals and the type of polygons and to make a conjecture. In addition, in the paper activity, the students drew two polygons by dividing the paper. This activity helped the students to make a generalization that the interior and exterior angles are supplementary to each other. These activities provided an environment in which the students' misconceptions and difficulties could be overcome. While using concrete materials for improving the students' knowledge, she integrated Geogebra to give examples of concave and convex polygons and of interior and exterior angles. In the pre-interview she stated, "We are going to have them do many drawings via technology to eradicate misconceptions of concepts. On geogebra we can have them do drawings to show that the distance between parallel lines never decrease, that is we can take two points on parallel sides and show them that the distance between these two points never change." This statement demonstrated that she provided examples of polygons in Geogebra to overcome students' difficulties regarding concave and convex polygons. However, she used Geogebra to replicate classroom activities after exploring geometry concepts, the usage of which was not sufficient to overcome the students' misconceptions. Furthermore, she said, "I'm using technology because it draws attention...well you know we're in technology age and paper and pencil don't attract much attention. If they set up the program there, I mean in their homes, it would be more interesting for them to engage in something like this on the computer rather than studying at their desks." This speech indicated that technology was used outside of normal classroom activities, such as reviewing the lesson at home as indicated in the recognizing level.

Esen did not let the students use technology before mathematics coaching. She did not want to distribute computers to her students. Besides, unlike Murat, she implemented Geogebra as a teaching tool rather than a learning tool in her lesson before mathematics coaching. Murat used Geogebra as a learning tool to develop students' appropriate mathematical thinking skills while "introducing polygons" and during activities in which students had to "find the number of diagonals.". In the

pre-interview, Esen declared, “In other words, if everyone had a computer it would be learning; if there is one, it is teaching. In my case at present, it is teaching.” This statement was consistent with how she used it in this lesson. In the pre-interview she explained how she integrated Geogebra as a teaching tool as follows:

“I will continuously do some practice. I mean mostly sample questions. I will draw there many samples of squares and draw parallelograms. I will do what we do with normal materials in a virtual environment. It will be transferring concrete materials to technology. The advantage is that it will attract attention. But not much will change. I think it would be more effective for the student to learn hands-on with concrete materials. Concrete materials would be more effective. For example, fixing a rubber band onto the Geometry board or being able to change a movable parallelogram will be more effective than using technology I think. But when he/she goes home, it may be technology that is effective.”

These explanations demonstrated that Esen was concerned about the fact that students did not develop appropriate mathematical thinking skills when technology was used especially when compared with concrete materials. But she believed that technology was more useful outside of the classroom, such as at home. In addition, she was “more apt to accept the technology as a teaching tool rather than a learning tool” (Niess et al., 2009, p. 21), as described in *the recognizing level* of the descriptor for the conception of students thinking.

4.1.2.2 Mathematics Teachers’ TPACK Level for the Learning Theme in the MCC1

During the mathematics coaching process, the lessons of Murat and Esen were observed for two hours to determine their TPACK level for mathematics descriptor of learning theme on polygon. All quotes and dialogs in the following paragraphs came from teachers’ TPACK one-on-one pre-conference 1, post-conference and lessons during mathematics coaching cycle 1 .

4.1.2.2.1 Murat's TPACK Level for the Learning Theme in the MCC1

Murat's performance in MCC1 demonstrated *the adapting level* of TPACK for both the mathematics learning descriptor and the conception of student descriptor in the learning theme.

At the adapting level, teachers begin to allow his/her students to explore mathematics topics using technology. Although technology is mostly used for teaching and learning, assessment mostly remains technology-free as Murat did (Niess et al., 2009). At the beginning of MCC1 lesson, Murat didn't use Geogebra to help students revise geometric concepts. He just asked some questions about concave and convex polygons taught at the previous lesson such as; "How do we classify polygons?", "What are polygons classified according to?" and "What is the generalization about diagonals from a single vertex in a polygon?". After the revision, he distributed the "Angles of Polygons" activity worksheet to teach the new geometric concept via Geogebra. This technology-enhanced activity aims to help students understand better the concept of interior and exterior angles at one vertex of polygon. Students drew different quadrilaterals and found interior and exterior angles at one vertex of polygon via Geogebra. Then they shared their findings as in the classroom. This helped students to notice that the sum of the interior and exterior angles at one vertex is always 180° even in different polygons. Each student group drew a different scalene quadrilateral. Although they didn't make a dragging in Geogebra, they were able to make a generalization about the interior and exterior angles by means of the dialogue. They made a conclusion that the interior angle is always supplementary to an exterior angle. This technology-enhanced activity improved students' thinking and understanding of this topic, but the teacher did not use dynamic properties of technology. It demonstrated the elements of adapting level of TPACK development for the learning theme in the beginning to use Geogebra as a mathematics learning tool.

In the "Finding the sum of interior angles of polygons" activity, students made a conjecture about the sum of interior angles of polygons using triangle via Geogebra. Every student group found how many triangles there are in a triangle,

quadrilateral, pentagon and hexagon, respectively and then they filled in the Table 3.2. This table helped students to explore the relationship between sides and interior angles of polygons. In the pre-conference he stated that “I don’t use it during the introduction. Since I prepare activities, I don’t usually design the rest with technology. During the introduction I asked them to give examples from daily life” Thus, he did not prefer using Geogebra in the introduction of lesson. However, he gave attention to improve students’ knowledge via Geogebra. Under the teacher’s guidance, students found numeric patterns to make generalizations about sides and the sum of interior angles of polygons. They developed the formula $180(n - 2)$ in which n represents the number of sides of a polygon. They made a generalization about the sum of interior angles for any number sided polygon. In addition, Murat provided a technology-enhanced environment to make inductive reasoning easier for students. These dialogues above showed that Murat applied technologies to help with students’ exploration of the new geometric concepts such as interior and exterior angles and to maximize students’ learning. Furthermore, there weren’t any limitations of using Geogebra for students during the instruction. This indicated that Murat did not have any concern about students’ attention being diverted from learning of geometric concepts to a focus on the technology as at *accepting level*. He facilitated technology-enhanced activities to encourage all students to develop a generalization by promoting the whole class discourse. However, he did not use technology dynamically so it shows that he started to explore experiment and practice of integrating technologies. This use of Geogebra demonstrates *the adapting level* of TPACK for the learning theme, mathematics learning descriptor.

Murat’s MCC1 performance also emphasized *the adapting level* of TPACK for the learning theme, conception of student thinking descriptor. At *the adapting level*, teachers begin exploring technology to improve appropriate mathematical thinking skills. Furthermore technology is used for many topics, while assessment remains without using technology like Murat did at the MCC1 lesson (Niess et al., 2009). In the “Angles of Polygons” activity, Murat helped students to find interior and exterior angles in a polygon by using angles tool in Geogebra. Student groups

drew a different scalene quadrilateral for investigation of the relationship between interior and exterior angles. Then, he motivated students to share their findings about the interior and exterior angles of quadrilaterals. This allowed the students to discover that interior and exterior angles always add to 180° . By using Geogebra, he supported the students' explorations not their memorization to overcome children's possible difficulties. In the other technology-enhance activity, students made a conjecture about the sides and the sum of interior angles. Students made inductively reasoning that the sum of interior angles of an n-sided polygon is $180(n - 2)$. This Geogebra-enhanced environment improved students' understanding and developed their generalizations. These activities did not repeat without technology to be certain that students really learned exterior and interior angles like at accepting stage. It shows that Murat began using technology to improve appropriate thinking, make conjecture, and overcome possible difficulties. In the post-conference he stated that "I gave them the total of the internal angles. $N-2 \times 180$ i. Even Asli said $n-2 \times 180$ and put it in place and found the question. So I saw that they took what we gave. They know what they need to do when they try to do it. We have no problem there." Additionally, Murat supported students to use their own language and to explain their ideas in order to improve students' thinking rather than practicing and memorizing. On the other hand, instead of students' independent explorations, teacher-led works occurred with technology. These Geogebra-enhanced lessons indicates that Murat "began developing appropriate mathematical thinking skills when technology is used as a tool for learning", which is at adapting level of TPACK for the learning theme, conception of student thinking descriptor.

4.1.2.2.2 Esen's TPACK Level for Learning Theme in Mathematics Coaching in the MCC1

Esen's performance in MCC1 demonstrated *the adapting level* of TPACK for both the mathematics learning descriptor and the conception of student descriptor in the learning theme.

Esen integrated Geogebra to improve students' understanding of geometry, unlike in the previous lessons prior to mathematics coaching. In MCC1, Esen started to use Geogebra as a tool for learning and teaching. In this lesson she did not prefer to use concrete materials first and then technology. Unlike in the previous lesson, Geogebra activities were used to improve students' exploration of geometry concepts, such as the sum of the interior angles and the exterior angles. Esen distributed to the student groups already-prepared Geogebra activities on interior angles. In these activities the students made generalizations about the sum of interior angles via using Geogebra. The students worked on triangle, quadrilateral, pentagon and hexagon activities to find a pattern. They used checkboxes to divide polygons into triangles, and to see the interior angles of every single triangle. In addition, she asked the students to fill a table on the interior angles of polygons. This table helped the students to discover the pattern of the sides and the number of triangles in polygons. Finally, the students found the formula of the sum of interior angles as $180(n - 2)$ (n represents the number of sides of a polygon). After the activity on the sum of interior angles, the students worked with the exterior angles of polygons in Geogebra. They measured the interior and exterior angles to see the relationship between them. They realized that the sum of interior and exterior angles is 180° . After this generalization, the teacher asked them to find the pattern between one of the exterior angles of a regular polygon and the number of sides in polygons. To find this relationship, the students constructed regular polygons and measured their interior and exterior angles. Like the sum of interior angles, the students filled in a table to explore the pattern. Finally, they realized that one of the exterior angles of a regular formula is $360/n$ where n represents the number of sides. This activity helped the students to discover the relationship between geometry concepts like the sum of interior angles. In addition, Esen's technology integration in MCC1 was similar to Murat's. Both of them implemented Geogebra-enhanced activities to motivate all the students to develop a generalization. Yet, they did not effectively use technology as a dynamic tool. For instance, Esen did not motivate the students to take actions, such as dragging the vertex of regular polygons in order to realize

the mathematically meaningful consequences of those actions. The students used Geogebra as a picture tool. Thus, these provided evidence that she was at *the adapting level* of TPACK for the learning theme, mathematics learning descriptor.

In the MCC1 observed lesson, Esen integrated Geogebra to improve students' exploration in geometry concepts: the sum of interior angles and exterior angles. Supportively, in pre-conference 1, she stated, "I believe that technology is bound to be beneficial in their discovery [learning]. We are going to draw the shapes on Geogebra where they will engage in discovery." She added "They will find how many triangles there are in a polygon and then pass onto their interior angles. From the interior angles of a triangle, they will pass onto the interior angles of a polygon." This speech indicated that in MCC1 "Students explored some mathematics topics using technology" as in *the adapting level* (Niess et al., 2009, p.21). Furthermore, in post-conference 1, she declared, "We did not apply the ones with concrete material. We could have reinforced them a little more. But they learned the topic." This speech indicated that she still wanted to integrate concrete materials to improve students' knowledge, but she also believed that technology was useful for students' learning. In conclusion, as indicated in *the adapting level* TPACK for the learning theme, mathematics learning descriptor, she "Began to explore, experiment and practice integrating technologies as mathematics learning tools." (Niess et al., 2009, p.21).

Esen's level of TPACK for the conception of student descriptor in the learning theme was also at *the adapting level*. Like Murat in MCC1, Esen used Geogebra to overcome possible misconceptions and difficulties about the interior and exterior angles of polygons. The students did not just memorize the formulas in this unit. They engaged in inductive reasoning to find the formulas of the sum of interior angles of an n-sided polygon ($180(n - 2)$) and one of the exterior angles of an n-sided regular polygon ($360/n$). In addition, she said, "as regards interior angles, I primarily used inductive reasoning" in post-conference 1. In pre-conference 1, Esen said, "I believe that they will no longer hold many misconceptions because when the students pass on from the triangle to the rectangle, from the pentagon to the

hexagon, they themselves will see the changes in the increase of the interior angles.” This speech demonstrated that the activities in MCC1 were designed to overcome the misconceptions. In addition, in pre-conference 1, she said, “I will use Geogebra as a learning tool.”. This demonstrated that she started to use Geogebra as a learning tool, unlike in the previous lesson prior to mathematics coaching. Thus, she “Begins developing appropriate mathematical thinking skills when technology is used as a tool for learning” (Niess et al., 2009, p.21) as described in *the adapting level* for students’ conception descriptor. Furthermore, although she integrated Geogebra to enable the students to understand more effectively, assessing student thinking remains mostly technology free as indicated in *the adapting level*.

4.1.2.3 Mathematics Teachers’ TPACK Level for the Learning Theme in the MCC2

During the mathematics coaching cycle 2, the lessons of Murat and Esen were observed for four hours to determine their TPACK level for mathematics descriptor of learning theme on polygon. All quotes and dialogs in the following paragraphs came from teachers’ TPACK one-on-one pre-conference 2, post-conference 2 and lessons during mathematics coaching cycle 2.

4.1.2.3.1 Murat’s TPACK Level for the Learning Theme in the MCC2

Murat’s performance in MCC2 demonstrated *the exploring level* of TPACK for both the mathematics learning descriptor and the conception of student descriptor in the learning theme.

Murat started to the lesson with demonstration of a short video from Vitamin. Students tried to find quadrilaterals in the room showed in the video (see Figure 4.65).



Figure 4. 65 The screenshot of Vitamin

This video helped to motivate students and attract their attention to quadrilaterals. In addition, finding quadrilaterals in the room provided an opportunity to connect the lesson with the real world context. Despite of the advantages of this warm-up activity, Murat did not use Geogebra as a technological tool to support students' mathematics learning in the beginning of the lesson. It proves that he had a limited usage of Geogebra as a learning tool during the introduction of lesson.

After the warm-up activity, he asked students to construct a square via Geogebra. In this activity students created dynamic squares by using regular polygon tool in Geogebra to analyze properties of a square. By using measurement tool, students were able to discover that all four sides of a square are equal. After this discovery, they dragged the vertex to see whether it was true for all squares. Students realized that sides of a square have a different value from the previous one, but still remain equal to each other. Firstly, they made a generalization, then dragged squares to prove that these properties are true for all squares. The students observed the relationship between diagonals in a square. Besides, there was an example of a non- prototypical square. The discussion about this non-prototypical square helped students to overcome possible difficulties about squares. After this dialogue, by dragging the vertex of the square in Geogebra, they made a generalization that "diagonals in a square cross at right angles". In this way, students found out that in a square "All sides are of equal length", "The diagonals bisect each other at right angles" and "The diagonals bisect the angles" via

dragging. Furthermore, students constructed parallel lines to determine if the opposite sides are parallel or not.

After constructing squares in Geogebra to analyze the properties, students opened already-prepared Geogebra activities which were related to rectangle, parallelogram, rhombus and trapezoid. Students explored the properties of quadrilaterals by using checkbox which showed angles, sides and diagonals. After student discussed the properties of quadrilaterals, they dragged the vertex to see the differences. By this way, they observed whether their ideas were true or not. For example, they changed their ideas about diagonals of a parallelogram after dragging. See the following dialogue.

E: (for rectangle) let's look at its diagonals. Let's get the diagonals drawn.

-sir the diagonals have equal lengths

E: let's hold from the corner, is it equal now

-it is not

E: so what shall we say, the lengths of diagonals are not always equal (said it after dragging)

In this example, students made a generalization about diagonals using just one parallelogram. By means of dynamic properties of Geogebra, students were able to realize that the diagonals of a parallelogram don't have to equal each other after dragging. Dragging helped students to see their ideas were mistaken. Furthermore, they were able to observe non- prototypical quadrilaterals. This dynamic environment made easier to make a conjecture about quadrilaterals. Another example of this generalization is shown in Figure 4. 66. By using the parallel line tool in Geogebra, it was investigated that the opposite side of trapezoid is not always parallel. This helped students to make conceptual understanding not just memorizing.

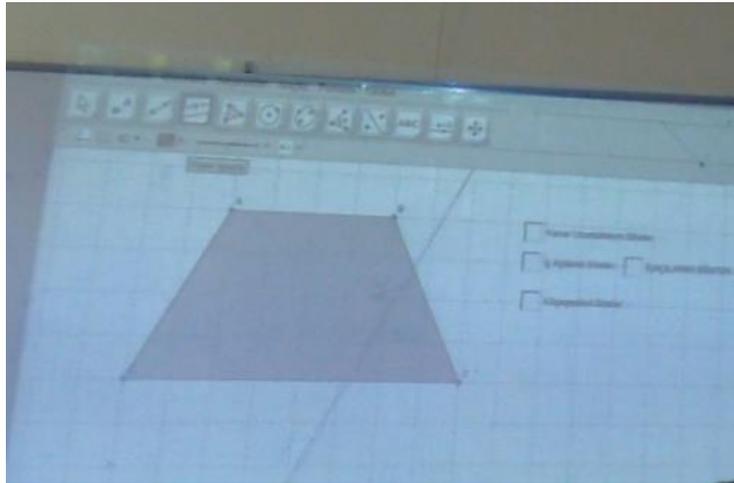


Figure 4. 66 The screenshot of parallelogram

In the MCC2 Murat did not use Geogebra in the introduction of lesson. In the post-conference, he said that

“(at introduction) can be done with examples from daily life verbally maybe but I think it is ok even if there is no Geogebra because the child does not discover anything there. They already have it mentally, we just need to show them. I use daily life examples like where do we see rectangles. At that moment they already have that in their minds. We need to form something to bring that. If an activity is designed, it can be used during the introduction part.”

This speech shows that he did not prefer to use Geogebra in the introduction, but aimed to improve students' knowledge via Geogebra. The overall usage of technology in Murat's MCC2 indicated that he integrated all students' understandings, thinking, and learning of quadrilaterals with technology. Geogebra-enhanced activities provided an environment for students to take mathematically meaningful actions on quadrilaterals as objects. With the help of the teacher, students were able to see the mathematically meaningful consequences of those actions. According to the observation, the goal of the lesson was to come up with different conjectures and generalizations related to quadrilaterals via Geogebra. This indicated that Murat utilized Geogebra to enhance students' thinking and understanding of quadrilaterals. In the post-conference, he said that

“I made constructivist approach happen with what we did. Q and A's, the child's discovery, having that settle mentally, them telling me the definitions themselves, on correcting the other one's definition. I show

them here that mathematics is not a doctrine and everyone can figure something out.”

Besides this, because it uses the dynamic structure more compared to the previous class, he said the class was more effective and added

“When I use the dynamic structure, the children understand it better when they see that this situation does not happen under only that condition but works in every other condition as well. Only here I don’t just explain the properties and move on, I stay on each and every properties, make them see and discover, and then you told me already but I didn’t put much thought on it to begin with. Holding the diagonal of the polygon and dragging it for example, them seeing that diagonals are with right angle or I don’t know always intersecting with diagonals right in the middle, them seeing this and it working when we are dragging too.”

This statement points out that he started to realize the advantages of dynamic nature of Geogebra when he used technology to support students’ explorations. Therefore, Murat preferred Geogebra “to facilitate the learning of specific topics in the mathematics curriculum” (Niess et al., 2009). This use of Geogebra in MCC2 lesson demonstrates *the exploring level* of TPACK for the learning theme, mathematics learning descriptor.

Murat’s MCC2-observation performance indicated *the exploring level* of TPACK for the learning theme, conception of student thinking descriptor. In the MCC2 lesson, assessing student thinking with technology demonstrated that Murat was over *adapting level* of this descriptor. He was more certain how students develop appropriate mathematical thinking skills via Geogebra as a learning tool. In the MCC2 lesson, Murat focused on non-prototype quadrilaterals with the help of Geogebra activities to overcome possible children’s difficulties. Students often encountered with difficulties when the quadrilaterals were not in the prototypical shape and in the prototypical orientation. In MCC2 lesson, by dragging students observed different positions of quadrilaterals, not just one prototype of them. With the help of dynamic properties of Geogebra, they were able to realize that the

properties of quadrilaterals did not change as their positions changed. In addition, they discussed the properties of quadrilaterals from different positions of quadrilaterals, not just one prototype of quadrilaterals. This indicates that Murat used Geogebra to overcome students' misconceptions whose source was prototypical quadrilaterals.

Murat emphasized the categorization of quadrilaterals and their properties. For example, the students were confused by the fact that a square is also rectangle. The following dialogue demonstrates students' difficulties in the relationship between square and rectangle.

S1:sir can I say something? When you squeeze the sides of the rectangle it becomes square.

M:ok then is every square a rectangle?

S1:Every square is not a rectangle

M: what if it has all the characteristics.

S2:yes

M: now think about it. Think about the definition of rectangle and definition of square.

S1:The rectangle has equal internal angles the diagonals are equal too

S3:sir even if some characteristics are not the same there are a lot of characteristics and also if we connect from square we form a rectangle.

With the dynamic properties of Geogebra, students realized that there is a relationship between square and rectangle. They did not think anymore that those two quadrilaterals are two separate polygons. The students discovered that every square is a rectangle. Moreover, they discussed that every rectangle is a parallelogram, every rhombus is parallelogram and every parallelogram is trapezoid by dragging in Geogebra activities. Additionally, in the pre-conference the teacher stated that "kids know the quadrilaterals separately so when you say square is a rectangle they will object. So they don't know that the square is a special type of rectangle. For example they don't know that rectangles are a special type of parallelogram." This speech demonstrates that the teacher recognized the possible students' misconceptions and difficulties about quadrilaterals and he tried to overcome these misconceptions and difficulties in his lesson. For example, students

constructed a parallelogram with dragging in rhombus activities. This helped students to classify quadrilaterals in different ways and use different names. Besides, in the assessment session students chose all these different quadrilaterals as a trapezoid in classification activities. In other words, Murat integrated Geogebra into his lesson to overcome students' difficulties and enhance conceptual understanding about quadrilaterals. In addition, in the post-conference he admitted that technology is useful and effective to overcome students' difficulties and stated that

“They expanded on the template they had in their minds. Or if they a different idea they got it settled. So we managed to give the logic of square is also a rectangle. I think it settled more now. Because normally that is very difficult to explain. Last year we explained and gave it and moved on. We had problems when we came to rhombus. When we said square is also a rhombus we had a lot of problems there. We could only move until certain steps. Square, rectangle, parallelogram. In the last activity when we coloured, find the squares and colour. At first I thought it wasn't going to happen. It ended well. At the last stem when we said trapezoid, they said we will colour sir. They understood it now.”

Thus, the Geogebra activities in MCC2 lesson were implemented and evaluated with respect to student learning of mathematics and student attitudes towards quadrilaterals by considering students' misconceptions. Furthermore, students were able to direct a conversation like this dialogue. While dragging the rectangle, students realized that it was a square and the conversation way changed. This demonstrates that Murat managed the technology-enhanced activities by directing student engagement and self-direction in learning mathematics as *in the exploring level* of TPACK for the learning theme, conception of student thinking descriptor.

4.1.2.3.2 Esen's TPACK Level in Mathematics in the MCC2

Esen's performance in MCC2 indicated *the adapting level* of TPACK for both the mathematics learning descriptor and the conception of student descriptor in the learning theme.

Esen implemented Geogebra to provide an environment for students' exploration in MCC2, just as in MCC1. The students explored the properties of the square, rectangle, parallelogram, rhombus and trapezoid via Geogebra. For instance, the students made generalizations about the diagonals of a parallelogram bisecting each other via using the checkbox to show the length of a diagonal. After the length of diagonals appeared, the students realized the relationship between the diagonals of a parallelogram. Then, they dragged the vertex of the parallelogram to see whether or not the generalizations still remained. This usage was similar to Murat's usage in MCC2. Both of them implemented Geogebra as a picture tool, and then used it as a dynamic tool. In the post-conference, she stated, "Technology supports students' conceptual learning. They themselves discovered the rhombus. They also discovered the trapezoid. It went well." She also explained how to use Geogebra for students' learning by saying, "Initially we used technology for practice. Now it's used for discovery." These statements demonstrate that she provided the students with an environment in which they could explore geometry concepts with respect to quadrilaterals. However, she was at a lower level than Murat in the mathematics learning descriptor because after the students discovered the concepts in relation to the properties of the quadrilateral, she conducted hands-on activities to summarize the properties. This usage indicated that she still explored how to use Geogebra to improve students' understanding. Thus, she "Began to explore, experiment and practice integrating technologies as mathematics learning tools" as described in the adapting level for the mathematics learning descriptor in the learning theme (Niess et al., 2009, p.21).

Esen's performance in MCC2 indicated that she was also at *the adapting level* of TPACK for the conception of student descriptor in the learning theme. In pre-

conference 2, she said, “From now on I will teach from Geogebra when integrating technology. From now on the entire lesson will be taught via technology. However, we will begin as of this lesson. Last lesson I wasn’t sure. But it didn’t turn out to be as I had feared. ...I will use technology as a learning tool.” This speech indicated that she wanted to develop students’ geometrical skills by using Geogebra as a learning tool. In fact, she started to use Geogebra as a dynamic tool in MCC2. Like Murat, she emphasized non-prototype quadrilaterals by means of dragging Geogebra activities related to quadrilaterals to overcome children’s possible difficulties. She wanted the students to drag the vertex of quadrilaterals and asked them to find whether or not the quadrilaterals still retained their properties. With this guidance question, students encountered quadrilaterals in their non-prototypical orientation. Furthermore, she asked them to find the relationship between squares and rectangles. Finding the relationship between squares and rectangles was important to overcome their misconception that the quadrilaterals are independent of each other. However, she did not effectively use Geogebra to reveal the relationship between the square and the rectangle. The students discovered that the square was a subset of the rectangle when the properties of the square and the rectangle were compared by means of the table on the properties of quadrilaterals. In addition, to reveal the relationship between the rhombus and the square, she used square shaped sticks as concrete materials. She pushed the vertex of the square to transform it into a rhombus. The students were able to explore the relationship between the square and the rhombus via using dragging in Geogebra. But she did not prefer to use Geogebra for this exploration. In conclusion, she “Begins developing appropriate mathematical thinking skills when technology is used as a tool for learning” (Niess et al., 2009, p.21). These provided evidence that she was at *the adapting level* for the conception of the student thinking descriptor in the learning theme.

4.1.2.4 Mathematics Teachers' TPACK Level for the Learning Theme in the MCC3

During the mathematics coaching cycle 3, the lessons of Murat and Esen were observed for four hours to determine their TPACK level for mathematics descriptor and the student thinking descriptor in the learning theme. All quotes and dialogs in the following paragraphs came from teachers' TPACK one-on-one pre-conference 3, post-conference 3 and lessons during mathematics coaching cycle 3.

4.1.2.4.1 Murat's TPACK Level for the Learning Theme in the MCC3

Murat's performance in MCC2 demonstrated *the exploring level* of TPACK for both the mathematics learning descriptor and the conception of student descriptor in the learning theme.

At warm-up session, students were supposed to find and bring close together congruent shapes of candies in the already-prepared Geogebra activity. Furthermore, this activity was similar to candy crush which is a popular game. It helped to attract students' attention to congruent polygons via famous game connection. Additionally, finding congruence candies provided an opportunity to discuss the relationship between side length and angles of a congruent polygon. Students categorized the pairs of figures using the relationship with a congruent polygon. This warm-up activity demonstrates that Murat started to use Geogebra as a technological tool to support students' mathematics learning in the beginning of the lesson. Furthermore, technology usage particularly during the introduction of MCC3 lesson confirmed that he did not have a limited perspective of Geogebra as a learning tool.

After the warm-up activity in MCC3, with the help of Geogebra-enhanced activities students explored the relationship between congruent and similar figures. While using Geogebra-enhanced activities, Murat assisted students by asking explorative questions such as: "Is there a relationship among the sides of a triangle?" and "Is there a relationship among the angles of a triangle?". By means of

these questions, students explored that when two figures are congruent, their all corresponding sides and angles are also congruent. After dragging, students matched the sides of the triangle and discovered that these two triangles were identical by the teacher's guiding questions. Furthermore, students explored the ratio of congruent figures. Students made a generalization that the ratio of length of corresponding sides is 1: 1. After this generalization, students changed the sides and angles by using sliders to prove this generalization for all figures. Then, thanks to the Geogebra-enhanced activity, students investigated properties of similar figures. Murat guided students to make a generalization about similar polygons using sliders and dragging in Geogebra activity. Students realized that the figures which have the same shape, but not necessarily the same size are similar. Additionally, he asked students some questions to clarify and reinforce concepts related to corresponding angles and sides, ratios and proportions. Students dragged sliders and observed the differences between the sides of figures. They developed a conjecture that similar polygons have corresponding angles which are congruent and corresponding sides which are proportional. After this conjecture, Murat assisted students to find the relationship between perimeter and similar figures. Students observed the difference between scale factor and perimeter by using slider changing the scale factor. With Murat's support, students made a conjecture that the ratio of the perimeters is the same as the scale factor relating to the lengths. Besides, students were able to overcome possible difficulties and misconceptions about the relationship between similar shapes and perimeter. Moreover, by the congruent and similar figures in Geogebra activities students developed another conjecture which was related to regular polygons and similarity. Students analyzed similarity in different regular polygons such as equilateral triangle, square, and regular pentagon. They generalized that regular polygons which have the same number of sides are similar. This extension task was used for further explorations of the similarity and congruence concepts. These indicate that he provided an environment for students to explore similarity and congruence concepts using technology, sometimes ranging outside curriculum.

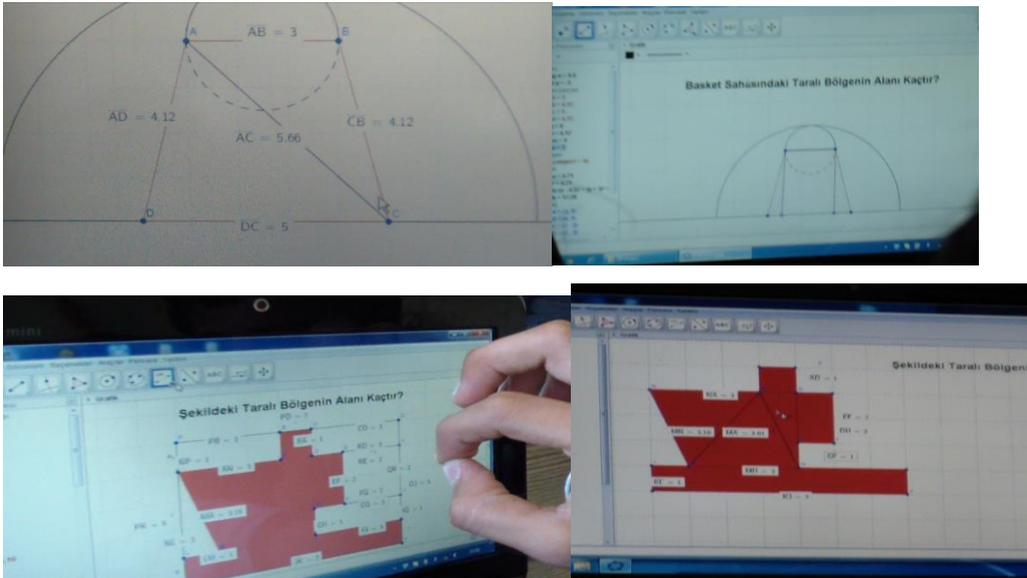


Figure 4. 67 The students solution to find area of irregular polygons

In other activities (see Figure 4. 67), the students worked with area of irregular polygons with Geogebra after congruence and similarity activities . They tried to find area of irregular polygons and had different solutions. Murat assisted them through questioning and facilitating their learning. Firstly, students clumsily divided into small pieces such as parallelogram which they did not know how to find the area. Then, Murat motivated them to turn into familiar polygons which they knew how to find the area. Finally, students used different solution strategies such as cutting the shape into smaller pieces and adding their areas, and by starting with a larger shape and subtracting pieces from it via Geogebra (see Figure 4. 67). Murat promoted students to solve problems in multiple ways contributing to the development of their creativity and critical thinking by using Geogebra.

In the area of parallelogram activity, students explored the formula of parallelograms' area with Geogebra. This Geogebra activity helped the students to use the area of a rectangle formula deriving from a corresponding formula for parallelograms. The strategy was related to the concept of the conservation of area. Parallelogram was cut into right triangle and repositioned at the opposite end of the

parallelogram to form a rectangle. According to the conservation of area, the rectangle area did not change at this equivalent. Hence, students discovered that the area of this equivalent rectangle is multiplication of the same perpendicular height h and length of base b as the original parallelogram. Thus, they developed a conjecture that the area formula of a parallelogram is $A=b.h$. Students did not just memorize the formula, but explored the formula of area of a parallelogram via Geogebra. Also, the connection of area of a rectangle as an already- known concept helped with students' explorations. This technologically supported environment allowed students to make conjectures about the area of parallelogram based on their exploration. Besides, students overcame their possible difficulties about finding area of a parallelogram from different height in another Geogebra activity. Students realized the alternative methods to calculate the area of a parallelogram. Furthermore, they became aware that using the horizontal base is not only way to find the area of a parallelogram. They dragged and calculated the area of parallelogram. With the spreadsheet of Geogebra, they realized that the area of a parallelogram is the multiplication of base and corresponding height.

The usage of technology in Murat's MCC3 demonstrates that he implemented technology to improve knowledge of all students' understanding, thinking, and learning of congruence and similarity of polygons and area of irregular polygons and parallelograms. In the post-conference, he stated that

“The other day we asked in congruent and similarity, we asked what is congruent. They are giving their own definitions. So whatever they say here, their answers are not memorized and they say their own definitions. I started liking this now. “

Murat used Geogebra to facilitate mathematical experiences that encourage all students to develop higher order thinking skills and to help students explore the concepts of congruence, similarity of polygons and area of irregular polygons and parallelograms. Furthermore, he promoted discourse among students as well as between teacher and students in Geogebra activities. In the Geogebra enhanced environment, students brought different solutions to the posed problems under the

guidance of Murat. They realized the value of different solution strategies in the process of problem solving. Thus, Murat focused on enhancing students' knowledge on learning similarity of polygons and area of irregular polygons and parallelograms. The use of Geogebra in MCC lesson indicates *the exploring level* of TPACK for the learning theme, mathematics learning descriptor.

Murat's MCC3-observation performance demonstrated *the exploring level* of TPACK for the learning theme, conception of student thinking descriptor. This question helped students to realize that congruent shapes can be matched by placing one a top the other. By dragging vertex of triangle in the Geogebra activity, students explored that the position of figures does not change the congruence of figures (see Figure 4. 68).

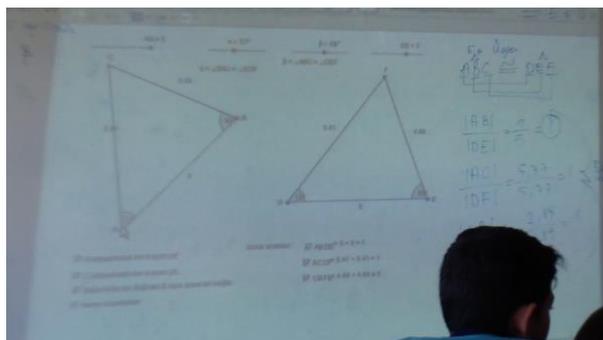


Figure 4. 68 The screenshot of board

In another similarity and congruence activity, students worked with regular polygons via Geogebra. The teacher focused on students' knowledge on the relationship between perimeter and scale ratio in this activity. By dragging, students explored that ratio of the perimeters is the same as the scale factor relating the lengths. This activity helped students to overcome possible difficulties and misconceptions about the relationship between similar shapes and perimeter. These activities indicated that Murat integrated knowledge of all students' understandings, thinking, and learning of congruence and similarity with technology

In addition, Murat implemented Geogebra to enhance students' conceptualization about the area of parallelograms. In the Geogebra-enhanced parallelogram activity, he posed some question to guide students' exploration such

as; “How did the shape change when the parallelogram was turned into a rectangle”, “What is the relationship between the rectangle and parallelogram? and “Has the area changed?”. Students answered these questions using sliders which help to rearrange the shape as a rectangle. With the help of Murat, students investigated the formula of parallelogram using dynamic properties of Geogebra. Furthermore, students might have had difficulties in the formula of parallelogram. By using Geogebra, they explored the formula of the area of a parallelogram via Geogebra instead of memorizing the formula. In another activity, Murat said that students might have not understood how to find the area of a parallelogram using different heights. Thus, he implemented Geogebra to overcome students’ difficulties in the multiplication of base and corresponding height. Students had an inclination to use the horizontal base for calculating the area of a parallelogram. With the spreadsheet of Geogebra, they realized that the horizontal base is not the only way to find the area of a parallelogram. They investigated that corresponding height can be used to find the area of a parallelogram.

During these Geogebra activities, Murat directed his students to overcome their difficulties in geometry concepts. Additionally, he assessed students’ knowledge about the area of a parallelogram. Hence, he implemented and evaluated with respect to students’ learning of congruence and similarity, and areas of an irregular polygon and a parallelogram. He gave an example of how he integrated Geogebra into the post-conference:

“In finding the area of parallelogram, in the height we brought on the long side, the height we brought on the short side, we showed the children that both areas are the same. They saw that it doesn’t change when we move the corner. When we were explaining there I told them verbally. $ax \times h$ or $bx \times h$. But they say sir these are different sides. We had a problem there. Now they saw it easier in Geogebra with the heights we brought from different sides. We somehow proved it. They saw that the area doesn’t change”

Furthermore, he said that Geogebra as a learning tool was effective and added

“I had troubles on this topic last year when I taught it because I draw the shape, cut them out of cardboard and show them. Activities are like that anyway, they want us to cut with scissors and do it. There weren’t that many bringing it. Now when we are making them count the unit squares and showing them the height in a mobile way, we can explain to them better that the height doesn’t change. I mean at least with dragging we showed better.”

Additionally, he managed Geogebra-enhanced activities to direct students’ engagement in learning geometry. This demonstrates that Murat “planned, implemented, and reflected on teaching and learning with the concern to guide the students through an understanding” in *the exploring level* of TPACK for the learning theme, conception of student thinking descriptor.

4.1.2.4.2 Esen’s TPACK Level in Mathematics for the Learning Theme in the MCC3

Esen’s performance in MCC3 indicated *the exploring level* of TPACK for both the mathematics learning descriptor and the conception of student descriptor in the learning theme.

Esen integrated Geogebra dynamically to provide the students with an environment exploration in MCC3, just as in MCC2. She used Geogebra to improve the students’ understanding of the congruence and similarity of polygons, the area of irregular shapes and the area of parallelograms. Firstly, she integrated Geogebra to explore similarity in daily life. In this Geogebra activity, the students measured the angles and sides of windows to realize the relationship between them. With this warm-up activity, she first implemented Geogebra in the introduction of the lesson. This indicated that she went beyond the lower level of the mathematics learning descriptor where the teacher has a limited usage of Geogebra, particularly during the introduction of key topics.

After the warm-up activity, Esen provided an environment to discover the relationship between similar polygons via using sliders in the rectangle activity on

Geogebra. They realized that corresponding sides of the rectangle were proportional. In another congruence and similarity activity, there were three quadrilaterals. Esen motivated the students to drag the vertices of the polygons to see the relationship between them. Furthermore, she asked them to find the relationship of the angles and sides among polygons. They discovered that their corresponding angles were similar although the positions of the polygons were different. Additionally, they discovered the ratio between quadrilaterals by using the slider. The students made a conjecture that the corresponding sides of similar polygons were proportional and there was a scale ratio. In the activities on the area of irregular polygons, the students also used Geogebra to find the area via different strategies. In the activities on the area of parallelograms, the students used Geogebra to explore the area formula of parallelograms by means of dragging. This activity demonstrated that Esen considered the dynamic properties of Geogebra by providing an environment for students' exploration. When compared to MCC2, Esen did not use concrete materials to enhance students' exploration. Esen took consideration of the advantages of Geogebra to provide an environment for students' exploration. Supportively, in the post-conference, she stated, "The students understood the concepts by engaging in hands-on activities. You know how they were continuously dragging from certain points, well, we wouldn't have been able to do that under normal conditions. Even if I had presented them on power point slides, it wouldn't have been the same. The students see it there. They see that when the big or small shape is enlarged, the others become larger too. They observed this from direct experience." In this speech, Esen focused on the actions and their mathematical consequence. Like Murat in MCC3, Esen motivated her students to engage in actions such as using sliders to transform the parallelogram into a rectangle. In addition, as consequences of the actions in Geogebra, the students made a conjecture as regards the area formula of the parallelogram: $\text{area} = \text{base} \times \text{height}$. To sum up, Esen's performance in MCC3 provided evidence to the fact that she was at the exploring level for the mathematics learning descriptor in the learning theme. She "used technologies as tools to facilitate the learning of specific

topics in the mathematics curriculum” (Niess et al., 2009, p.21) as described in the exploring level.

Esen was also at *the adapting level* of TPACK for the conception of the student descriptor in the learning theme. In the pre-conference, she stated, “Sometimes they cannot make the transition between whether similar polygons are equivalent, or whether equivalent polygons are always similar. Or sometimes they can say that they are equal, thinking that the sides are equal. They forget that they need to consider the angles as well. In problems something else happens. They cannot make connections between the angles when they are given polygons placed within each other or adjacent to each other; that is, they cannot do the proportioning of the side corresponding to the angle, the sides corresponding to equal angles.” She added that she designed the lessons in MCC3 to overcome these possible misconceptions. This demonstrated that she planned Geogebra-enhanced activities with the concern to guide students in their learning process. Furthermore, in the post-conference she said, “Technology was useful in overcoming misconceptions during the lesson and the students saw this.” This speech supported the fact that she had integrated Geogebra with the consideration of fostering students’ understanding of the topic and overcoming their difficulties. In addition, unlike in MCC2, Esen motivated the students towards Geogebra activities in MCC3 to improve students’ engagement and self-direction in learning mathematics. For instance, Esen provided the students with an environment to develop different methods of solutions to finding the area of irregular polygons. These indicated that Esen “Plans, implements, and reflects on teaching and learning with concern for guiding students in understanding” (Niess et al., 2009, p.21) as indicated in *the exploring level* for the conception of students thinking descriptor.

4.1.2.5 Mathematics Teachers' TPACK Level for the Learning Theme in the MCC4

During the mathematics coaching cycle 4, the lessons of Murat and Esen were observed for two hours to determine their TPACK level for mathematics descriptor and the student thinking descriptor in the learning theme. All quotes and dialogs in the following paragraphs came from teachers' TPACK one-on-one pre-conference 4, post-conference 4 and lessons during mathematics coaching cycle 4.

4.1.2.5.1 Murat's TPACK Level for the Learning Theme in the MCC4

Murat's performance in MCC4 indicated *the exploring level* of TPACK for both the mathematics learning descriptor and the conception of student descriptor in the learning theme.

In the MCC4 lesson, Murat used Geogebra as a learning and teaching tool to facilitate the students' learning of the area of rhombus and trapezoid. In the area of a rhombus activity, students were helped to learn how to find the area of a rhombus by duplicating and composing into a rectangle via Geogebra. In this Geogebra-enhanced activity, a rhombus was cut into right triangle and replicated/ repositioned at the side of the rhombus to form a rectangle by using the slider. The teacher provided an environment to find the area of a rhombus by relating it to the already-known area of a rectangle. Students realized that the area of a rectangle is twice of the area of a rhombus with the help of Geogebra. Furthermore, they dragged the vertex of rhombus to see whether this generalization is valid for all rhombuses. After this verbal generalization, they developed a conjecture that the area formula of a rhombus is $A=e.f/2$ as e and f diagonals of rhombus. In this activity, the students did not just memorize the formula, but discovered the formula of area of a rhombus by using dynamic nature of Geogebra.

After the rhombus activity, the students worked with Geogebra file which was related to the area of a trapezoid. In this activity, students analyzed how to derive the formula for the area of a trapezoid by creating a parallelogram from two

congruent trapezoids. With using slider in Geogebra-enhanced activity, students replicated and rotated the trapezoid to create a parallelogram. Therefore, it helped students to understand the formula. According to this formula, the area of a trapezoid is half of the area of the parallelogram. Students recognized the relationship between the area formulas for a trapezoid and a parallelogram. This dialogue illustrates that students made a conjecture about the area of a trapezoid in this activity as rhombus activity. In both Geogebra-enhanced activities, students used Geogebra as reasoning and learning tool to develop in-depth understanding of the area of a rhombus and a trapezoid.

In the MCC4, Murat integrated Geogebra into the whole lesson to improve students' understandings, thinking, and learning of knowledge of the area of rhombus and trapezoid.

These Geogebra-enhanced activities provided an environment for students to take mathematically meaningful actions on rhombus and trapezoid as objects. Before MCC4 lesson, students already knew the formula for area of a rectangle, parallelogram and triangle. To find the area of a rhombus and a trapezoid, the students transformed the rhombus and trapezoid to already -known area formulas of shapes. With the help of this dynamic transformation, the students explored the area of rhombus and trapezoid. For instance, in the trapezoid activity, the students were able to identify that the combined figure was a parallelogram and that the combined base length is $b_1 + b_2$ via using slider. Finally, students wrote the formula for area of a trapezoid. The activities in MCC4 allowed the dimensions of the trapezoid and rhombus to be changed. Besides, students dragged the vertex to test their generalization about whether the areas of rhombus and trapezoid were true or not. The areas were hidden so that the students could compute the area on their own. Additionally, they expressed algebraically their verbal generalization. Furthermore, in the pre-conference, Murat stated that

“Up to now (in area calculation) they never used the multiplication of diagonals. They will only do it from rhombus. I normally have a very hard time there. So far I haven't explained it in a different way. Maximum thing I did was the activities in the book. But I think this way

(the applied way) will be better. They will notice something visually. They will understand something. It will be clarified.”

He expected that using already-known area formulas of shapes helped the students to discover the new formulas via Geogebra. In addition, in the post-conference he said that these Geogebra-enhanced activities were useful to improve students' understanding. In brief, Murat integrated Geogebra into advanced students' mathematical experiences to improve higher order thinking skills and exploring the concept of areas of rhombus and trapezoid, not just memorizing. Murat's MCC4 performance expresses *the exploring level* of TPACK for the learning theme, mathematics learning descriptor, because he preferred Geogebra “to facilitate the learning of specific topics in the mathematics curriculum” (Niess et al., 2009).

Murat's MCC4 performance also demonstrated *the exploring level* of TPACK for the learning theme, conception of student thinking descriptor. In the MCC4 lesson, Murat took into consideration students' possible difficulties and errors about the area of rhombus and parallelogram. For instance, in the pre-conference he stated that “they have a hard time in trapezoid. I mean bottom base plus top base. They never saw something like that so far. They say base times height. To explain it better it would be nice to change it into parallelogram. I mean it would be very effective to explain it.”. In the Geogebra-enhanced rhombus and trapezoid activities, the teacher provided an environment with the students to overcome their misconceptions and difficulties. In both activities, the students transformed the shapes to a quadrilateral which they know how to find its area. For example, to explore the area formula of a rhombus, the students turned the rhombus into a rectangle using sliders in Geogebra activity. The students were able to recognize that the diagonals ought to be used to formulate the area of a rhombus. Then, the students discovered that the area of a rhombus is the half of the area of a rectangle. They found the formula of the area of a rhombus as a half of the multiplying of diagonals of the rhombus. In this activity, the students explored the formula of the rhombus using dynamic nature of Geogebra rather than memorizing. Similarly, in the trapezoid activity the students used the parallelogram to find the trapezoid via Geogebra. As dragging, the students became aware of the relationship

between the areas of a parallelogram and a trapezoid. In the post-conference, the teacher stated that

“They tell that a rhombus is a quadrilateral with all equal sides just like that but area here is what makes it a little difficult for me. I have a hard time explaining it to them. When saying general statement or giving formula it is difficult. But what happened there, they found it themselves. We started no longer memorizing. I mean they already think that mathematics is memorization. We got them a little bit more out of that mindset. That activity we did on trapezoid, division by two was over. It was out there just like that. It was very comfortable. It made it easier for me and stopped me from making too much effort”

Thus, the teacher indicated that students had difficulties finding the area of rhombus and trapezoid. He implemented Geogebra to overcome their possible difficulties and misconceptions. Furthermore, the teacher used Geogebra to improve students’ explorations not memorizing. Therefore, Murat “planned, implemented, and reflected on teaching and learning with the concern to guide the students through an understanding” in *the exploring level* of TPACK for the learning theme, conception of student thinking descriptor.

4.1.2.5.2 Esen’s TPACK Level for the Learning Theme in the MCC4

Esen’s performance in MCC4 indicated *the advancing level* of TPACK for the mathematics learning descriptor as opposed to *the exploring level* of TPACK for the conception of student descriptor in the learning theme.

In MCC4, Esen used Geogebra to improve students’ learning about the area of the rhombus and the trapezoid. To improve students’ understanding of the area of the rhombus, she integrated two different Geogebra activities. At first, the students divided the rhombus from its diagonals and added the pieces of the rhombus on top of the rhombus. In this activity, the students transformed the rhombus into a rectangle via using the slider. They realized that the sides of a rectangle were equal to one of the diagonals of the rhombus and half of the other diagonals; therefore, the area of the rhombus should equal the area of the rectangle, which is $e \times \frac{1}{2}f$ (e and f

are the diagonals of the rhombus). In the second activity, the students divided the rhombus into four triangles and replicated these triangles. Then, they combined the original rhombus and these triangles to make bigger rectangles via the slider. In this activity, the students discovered the relationship between the area of the rectangle and the rhombus. They made a conjecture that the area of the rhombus was half of the area of the rectangle, which is $e \times f$. These two Geogebra activities provided the students with an environment in which they could explore the area of the rhombus from different perspectives, unlike Murat in MCC4. This demonstrated that Esen used Geogebra to enhance student thinking and understanding of the mathematics through the integration of various Geogebra activities *at advancing level*. In parallel to the Geogebra activities on the area of the rhombus, Esen used different Geogebra activities to explore the area of the trapezoid from different perspectives. In the first activity, the students replicated and rotated the trapezoid to make a parallelogram via the slider. Then, they discovered that the area of the trapezoid was half of the area of the parallelogram whose sides were equal to the sum of the bases of the trapezoid. Finally, they found the formula to be $((b_1+b_2) \times h)/2$ (b_1 and b_2 are the bases of trapezoid and h is the height of the trapezoid). In the second Geogebra activity, the students were able to change the bases and the height of the trapezoid via the sliders. They explored the function of the bases and the height of the trapezoid while calculating the area of the trapezoid. The activities in MCC4 demonstrated that the students explored the concepts of the area of the rhombus and the trapezoid by integrating various Geogebra activities in attempts to better understand these concepts as *at the advancing level*. This technology usage was different from that of Murat's in MCC4. In addition, she integrated Geogebra as a learning tool within the whole lesson. In the post-conference she stated, "The first lessons did not go so well for the students and for me as I had to get used to the class, but this lesson went well. We used it in all the stages while seeking the formula. We used it in various problems and activities." This statement demonstrated that she was above the lower level of the mathematics descriptor where the teacher did not integrate technology especially in the introduction and

assessment stages. This indicated that she satisfied of using Geogebra as a learning tool to explore geometrical concepts. In conclusion, she “Plans, implements, and reflects on teaching and learning with concern and personal conviction for student thinking and understanding of the mathematics to be enhanced through integration of the various technologies” (Niess et al., 2009, p.21) as described in *the advancing level* for the mathematics learning descriptor.

Esen was at *the exploring level* of TPACK for the conception of student descriptor in the learning theme. In the pre-conference, she stated, “The formula of the trapezoid will be more challenging when compared to the others.” To overcome the possible misconceptions of the area of the trapezoid, she integrated Geogebra activities on the trapezoid. These demonstrated that she used Geogebra with the consideration of students’ knowledge. In the post-conference she said, “There should no longer be any misconceptions. There were no misconceptions in relation to the trapezoid but they experienced some difficulty in understanding the formula. But it was an effective activity. It enables the students to establish a relationship with the parallelogram.” This statement indicated that the transformations of the trapezoid into a parallelogram helped the students to explore the area formula and overcome the possible misconception regarding the area of the parallelogram. In addition, in the post-conference she stated, “If the students experience difficulties, well you know I don’t tell them directly. I lead them to discover via Geogebra.” Her perspective to Geogebra demonstrated that Esen implemented Geogebra in activities related to the area of the rhombus and the trapezoid to enhance and assess students’ understanding of the concepts. Furthermore, Esen guided her students to discover the relationship between the area of the rectangle and the rhombus and the relationship between the area of the parallelogram and the trapezoid. She did not directly tell the students the formula; instead, she had them explore the formula. These provided evidence that she “Plans, implements, and reflects on teaching and learning with concern for guiding students in understanding” (Niess et al., 2009, p.21) as indicated in *the exploring level* for the conception of students thinking descriptor. Furthermore, in the post-conference, she stated, “They used it in all the stages of the

lesson. We used it in the introduction, practice and the assessment stages. It was this lesson in which we could do a meaningful lesson. We didn't have time for assessment in the other lessons." This statement demonstrated that she was above the lower level of the mathematics descriptor where the teacher did not integrate technology especially in the introduction and assessment stages. Besides, she implemented and evaluated student learning of mathematics and students' attitudes toward mathematics as indicated in *the exploring level* for the conception of students' thinking descriptor.

4.1.2.6 Mathematics Teachers' TPACK Level for the Learning Theme After Mathematics Coaching

After mathematics coaching, the lessons of Murat and Esen were observed for two hours to determine their TPACK level for mathematics descriptor and the student thinking descriptor in the learning theme. All quotes and dialogs in the following paragraphs came from teachers' TPACK one-on-one post-observation and post-interview after mathematics coaching.

4.1.2.6.1 Murat's TPACK Level for the Learning Theme After Mathematics Coaching

Murat's performance after mathematics coaching indicated *the advancing level* of TPACK for both the mathematics learning descriptor and the conception of student descriptor in the learning theme.

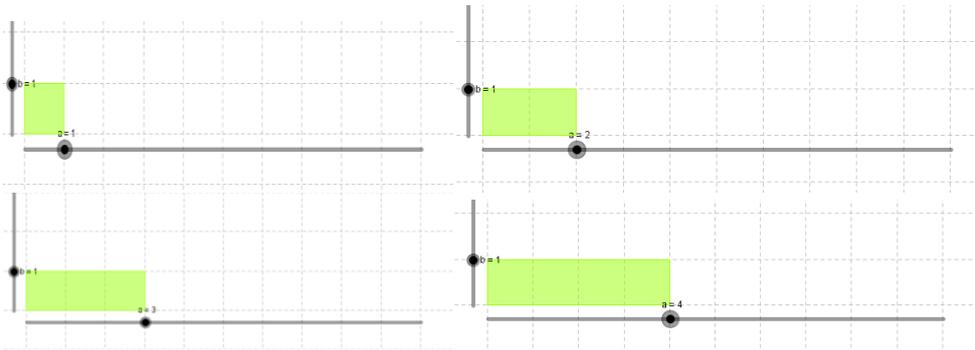


Figure 4. 69 The students dragging to find the relationship between area and perimeter

In the post-observation lesson, Murat used Geogebra as a learning and teaching tool in 2 hours to enhance students' knowledge about the relationship between area and perimeter. In the first activity in the post-observation lesson, the teacher provided an environment to find pattern between sides and perimeter, and sides and area. In this activity, the students increased the width of rectangle one by one using slider. Through the observation of pattern between sides and perimeter, and sides and area, the students began to identify some properties of the pattern. The students developed an understanding of the concept of relationship among sides, perimeter and area, using pattern via Geogebra. The pattern was investigated so that the students developed an understanding of how it grew numerically as well as geometrically (see Figure 4. 69). The students engaged in a higher level thinking process while solving the problem. The whole class conversation was occurred as below:

M: let's see long side is 2 units and short side is 1 unit total 3. Then 3×2 equals 6 our perimeter is 6. We will try to find the connection between these. Now let's make a 3. What is our area?

S1:3

M: what is our perimeter

S2:8

M: lets increase a one more time. What is our area? From short side multiplied by long side

S1:it's 4. Okay perimeter

S2:10

E: long side is 4 short side is 1. 5×2 equals 10. Ok let's find the relationship between this area and perimeter.

S3:Sir I multiplied 2 and 1

M: you mean you multiplied the area by 2
S3:yes then I added 2

The students found a relationship between the area and perimeter of rectangle as $2n+2$ (n refers the area of rectangle). Then they tested their solution and tried to find different solutions. This conservation indicates that the students explored new concepts such as relationship between perimeter and area, developing ideas, making mathematical conjectures, and justifying results. This activity provided an environment for students to reason mathematically as the teacher encouraged students to explain and justify their mathematical thinking, and to consider and evaluate their ideas. Furthermore, the teacher asked the students to think and analyze their equation in different conditions. The teacher provided an opportunity for the students to focus on the equation deeply and to analyze it when it works. The students recognized that in different conditions the pattern ought to be changed and there is not a constant relationship between area and perimeter. Through the activity, the students were motivated to make conjectures based on the patterns seen in one situation and to test their conjecture to see if it is applied to additional situations. With this Geogebra-enhanced activity, the students engaged in inquiry and developed their deep understanding of the relationship between area and perimeter. To conclude, in this activity the teacher supported the students to develop conceptual understanding by attending to the relationship between area and perimeter. Additionally, the students extended their mathematical thinking by finding solutions, trying to find alternative ways to solve and justify their solutions.

After this activity, the teacher posed the problem “what is the biggest area of the rectangle whose perimeter is 24 units”. The students tried to find the solution using sliders changing the sides of the rectangle in Geogebra. As moving the slider, the students observed the changes in the lengths of the sides, perimeter and area in the spreadsheet which was in right side. The students made a generalization to find the solution. This Geogebra-enhanced activity helped students extend their understanding of the relationships between area and perimeter. The students explored that the area becomes larger as the side lengths are brought closer together.

This activity provided a rich problem solving context that allows students to develop their understanding of the relationship between area and perimeter.

In the post-observation lesson, Murat incorporated Geogebra in the whole lesson to enhance students' understandings, thinking, and learning of knowledge of the relationship between area and perimeter. The students explored the relationship between area and perimeter as a mathematics topic, integrating Geogebra as a learning tool in attempts to understand better mathematical concepts in the post-interview. In addition, Murat expressed that

“technology allows the children to do themselves. Allows them to move step by step. I make the child discover. Instead of the chalk or pen work on board, I directed them to an area which the children themselves can be wanting. For example the number of diagonals drawn from a corner... I asked them a question about that written too. What is the total of internal angles of a decagon? they didn't think in the general form for example. But they used this sir remember we drew a table 1st step 2nd step I looked, they reached the general formula themselves. So the children found something there themselves without memorizing, $n-2 \times 180$ there 8×180 . They put it there. Which means, when you go step by step and make the children discover, even if they forget the general formula there, they can find it for that question and once they find it they solve it. What did we do here? It was really beneficial that the dynamic structure was moving step by step and we had it drawn one by one and moved it to the spreadsheet on the side. We learned permanent information.”

In parallel with the post-observation lesson, this speech demonstrates that he used Geogebra as a learning tool for students to explore mathematical ideas. Besides, he admitted that he was surprised by the effectiveness of Geogebra on enhancing students' mathematical knowledge. He stated that

“ at first I thought I would use it only at the discovery stage but I did not know it would be this effective. I thought probably I will have to tell them at the end, I thought they couldn't reach it, they couldn't use it. I was in that mood. But my thoughts have changed. For example in the last class, at finding the relationship between perimeter and area. What did we do? We wrote it one by one and showed. Then before I knew it they started saying it themselves just like that.”

And he added “I always say those students finding the answer just like that, getting excited, getting to a result themselves after a generalization it changes their facial expressions.” These expressions support that he facilitated Geogebra-enhanced geometrical experiences that foster the students’ knowledge and encourage all students to develop higher order thinking skills while promoting mathematics exploration. Thus, he exemplified elements of *the advancing level* of TPACK for the learning theme, mathematics learning descriptor. He “planned, implemented, and reflected on teaching and learning with concern and personal conviction for student thinking and understanding of the mathematics to be enhanced through integration of various technologies” as in the *advancing level* (Niess et al., 2009).

Murat’s performance after mathematics coaching indicated *the advancing level* of TPACK for the learning theme, conception of student thinking descriptor. In the post-observation lesson, Murat used Geogebra as a learning and teaching tool in 2 hours to improve students’ knowledge about the relationship between area and perimeter. In the first activity, the students investigated the pattern between area and perimeter. The students made a generalization on area and perimeter while increasing the wide of rectangle one by one using slider. The formula for perimeter is $2n+2$ where n represents the area. Then, the teacher asked the students to find the area of another rectangle in Geogebra. At first, they applied the formula to the rectangle. He helped them to recognize the formula is valid for the height which is 1 unit. The students tended to apply the formula to everywhere. The teacher provided an environment with the students to overcome their overgeneralization.

In another activity, the teacher asked the students a question by using Geogebra as follows “what is the biggest area of the rectangle whose perimeter is 24 units”. The students tried to find the solution changing the sides by using sliders. This activity helped to overcome the misconception that “increasing the perimeter of a shape will always increase the shape’s area and vice versa”. In this activity the students realized that the relationship between area and perimeter is not in the direct proportion. The students explored that the area could increase or decrease while

changing the sides in a fixed perimeter by using sliders. This problem solving activity helped the students to learn the relationship between area and perimeter by using technology as a learning tool.

In the post-observation lesson, the integration of Geogebra was integral rather than in addition to the development of the students' mathematical knowledge. This provided an evidence that the teacher was in *the advancing level* of TPACK for the learning theme, conception of student thinking descriptor. Furthermore, Murat incorporated Geogebra to develop advanced levels of understanding of the relationship between area and perimeter as mathematical concepts. The students explored the relationship between area and perimeter, integrating Geogebra as a learning tool into the attempts to understand better these mathematical concepts and to overcome their difficulties and misconceptions. In the post-interview Murat said that

“I can say that I used technology as a tool in learning. Maybe it was the most important thing to be done in the 21st century. We try to get a permanent information by using technology. Technology is not only a tool we use on our daily lives or social lives but also a tool we use in the field of education so it should be within the education”

and he added that

“In the subject of area, us not giving them the formulas directly, the children discovering it themselves, finding it and the dynamic medium with that dragging was amazing. It caused the information to be permanent and not be just a memorization.”

This speech indicates that he emphasized the Geogebra as a learning tool to improve students' explorations. Furthermore, the activities in the post-observation provided evidence that the teacher led the students to “engage in high-level thinking activities (such as project-based, , problem solving and decision making activities) for learning mathematics using the technology as a learning tool” as in *the advancing level* of TPACK for the learning theme, conception of student thinking descriptor.

4.1.2.6.2 Esen's TPACK Level for the Learning Theme After Mathematics Coaching

Esen's performance after mathematics coaching indicated *the advancing level* of TPACK for both the mathematics learning descriptor and the conception of student descriptor in the learning theme.

As in the previous lesson (MCC4), Esen implemented Geogebra to improve students' learning of such concepts as the relationship between the area and the perimeter via various activities. In the introduction of the lesson, she integrated Geogebra to enable students to see the area and the perimeter of triangles and rectangles in different activities. The students dragged the vertices of polygons to see the changes in the area and perimeter. Then students worked on a Geogebra activity, which helped the students' to understand the relationship between the area and the perimeter. The students dragged the vertex of a rectangle to get the biggest value of the area. They made a conjecture that the more the side lengths were brought closer together, the larger the area became via dragging. In another activity, the students tried to find a pattern between the area and the perimeter. Every student group increased the base of the rectangle one by one by using the slider of the rectangle and observed the consequences of their actions. These activities indicated that Esen provided an environment for the students to explore the relationship between the area and the perimeter by implementing various activities from different perspectives such as the biggest area in a fixed perimeter, and the pattern between the area and perimeter, which are attempts to enable students to better understand these concepts as indicated in *the advancing level*. In the post-interview, she stated, "Geogebra offers students a great sample of discovery. They can try however many times as they want. They can see what changes and what don't. I think it's mostly beneficial in discovery. They used especially the dragging to play around with values. They saw what changed when the values changed, for instance in the area of the parallelogram. What effect it has, what are multiplied and how the area is produced. The students themselves see the answers to such questions as what

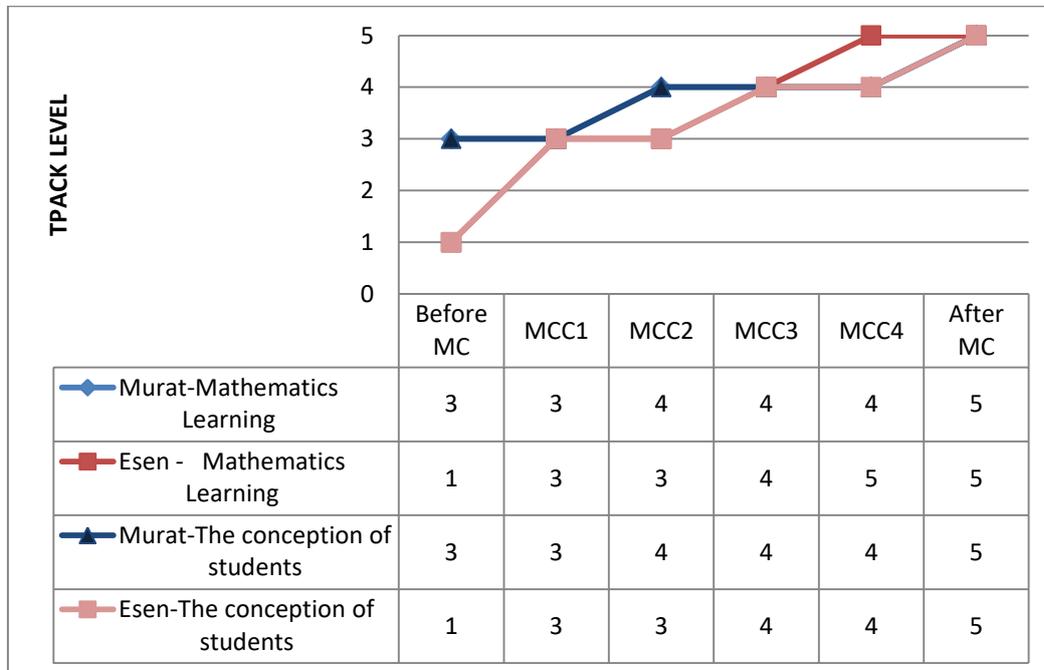
happened since they do the dragging themselves. They can clearly see how it changes and which operation they used. This speech indicated that Esen focused on the dynamic nature of Geogebra to improve students' exploration. Furthermore she added, "With concrete materials you can try out something several times. But in this [Geogebra] you can try it out infinite number of times. It broadens the vision of the students...for example, we could have prepared concrete materials in class. We could have drawn rectangles with the same perimeters. In other words, what are their areas, we could have expressed in unit squares but this requires cutting and pasting. It would have taken much of our time. But in Geogebra, the children realized this by means of a shorter way, by dragging, and by means of more examples." This statement demonstrated that her ideas about Geogebra changed because at the beginning of mathematics coaching, she preferred the concrete materials because she believed that concrete materials was more useful to explore the relationship. This change also demonstrated that Esen moved to a higher level in TPACK. To sum up, Esen "Plans, implements, and reflects on teaching and learning with concern and personal conviction for student thinking and understanding of the mathematics to be enhanced through integration of the various technologies" (Niess et al., 2009, p.22) as indicated in *the advancing level* for the mathematics learning descriptor.

Esen was *the advancing level* of TPACK for the conception of student descriptor in the learning theme. She integrated Geogebra in the whole lesson to improve the students' understanding. In the post-interview, she stated, "My aim in using technology is initially to lead the students to discovery and subsequently for measurement and evaluation. After discovery, they do the application anyway. By preparing questions we do measurement and evaluation as well. For example, in calculating the area of a trapezoid, we got used to resembling a trapezoid to a parallelogram. They turned it around and then discovered that the trapezoid consists of two parallelograms and thus that its formula was converted. After discovery, we passed onto their applications and calculations. Then in measurement and evaluation, with the questions we placed [on Geogebra] we tried to measure their

learning.” This indicated that Esen integrated Geogebra to enhance students’ exploration during the lesson and assesment. Furthermore, The integration of Geogebra into the lesson after mathematics coaching was integral (rather than additional) to the development of the mathematics students’ learning. This provided evidence that she was at *the advancing level* for the conception of student descriptor in the learning theme. Like Murat, Esen implemented Geogebra as a learning tool to engage students in high-level thinking activities such as problem solving activities. Finding the biggest area in the fixed perimeter activitiy was an example for innovative Geogebra activities for advanced levels of understanding the relationship between the area and the perimeter. In addition, she emphasized the role of Geogebra as a learning tool in the observed lesson. The students directed the lesson and their understanding. In the post-observation, she stated, “I used Geogebra as a learning tool.. On Geogebra, the students continuously tried out and discovered things on his own. They could even invented some things.” She then added, “By playing around (dragging) in area and perimeter activities, they were able to see that the closest sides yielded the largest area. I used it in my other classes as well. I played with the slider. It functioned well. It was useful for students’ discovery learning.” These statements demonstrated that Esen implemented Geogebra as a learning tool to improve the students’ exploration. To sum up, she “Engages students in high-level thinking activities (such as project-based and problem solving and decision making activities) for learning mathematics using the technology as a learning tool.” (Niess et al., 2009, p.22) as indicated in *the advancing level of TPACK* for the conception of student descriptor in the learning theme.

4.1.2.7 The progress of Murat and Esen’s TPACK of Learning Theme through Mathematics Coaching

Table 4. 6 The progress of teachers for Learning Theme



The performances of Murat and Esen for the learning theme in the six Geogebra-enhanced lessons before/during/after mathematics coaching are presented in Table 4. 6. The teachers demonstrated the growth into *advancing level* in the TPACK development model for mathematics learning descriptor and the conception of students in the learning theme.

The mathematics learning descriptor focuses on learning mathematics as integrating with technology. Esen was lower level than Murat for mathematics learning descriptor before mathematics coaching. She used Geogebra as supplements to instruction after using concrete materials. Furthermore, Murat integrated Geogebra to help students’ exploration even though he used Geogebra as a static tool at the beginning of the study. He implemented Geogebra to encourage the students to develop a generalization while promoting a whole class discourse, but he did not consider the dynamic nature of Geogebra. The use of Geogebra

demonstrated that he did not have any hesitation about students' attention being diverted from learning mathematical concepts to a focus on the technology in the activities. In the MCC1, Esen reached the Murat's TPACK level. They started to experiment it to determine effectiveness of technology. The teachers had a limited usage of Geogebra as a learning and teaching tool; he did not use technology during the introduction and assessment. At the beginning, the students were provided with an environment to use Geogebra for most topics, but the assessment and introduction remained without using technology. The teacher started to move to a higher level of the TPACK development model for mathematics learning descriptor during mathematics coaching. The teachers began to realize the advantages of dynamic nature of Geogebra as integrating technology to support students' exploration unlike the beginning of the study. Towards the end of the study, the teachers provided an environment for students to explore geometric topics by integrating Geogebra into attempts to understand better mathematical concepts. In addition, the teachers recognized that technology not only provides an instructional tool for his lesson but also potentially interferes with learning geometrical ideas. At the last lesson, the teachers planned, implemented, and reflected teaching and learning to guide students to understanding mathematics using the Geogebra as a tool for learning. He reached the highest level of TPACK Development Model for the learning theme, mathematics learning descriptor.

The teachers made a progress in the TPACK development model for the conception of student thinking descriptor. Esen had concerned that students do not develop appropriate mathematical thinking skills when the technology was used especially comparing concrete materials at the beginning of the study. She implemented Geogebra as teaching tool rather than learning tool unlike Murat in lesson before mathematics coaching. In the Murat's lesson before mathematics coaching, the students used Geogebra to make a generalization, but the teacher did not assess their thinking by using Geogebra, which demonstrates *the adapting level*. He also made an attempt to overcome the students' possible misconceptions and difficulties. Moreover, he provided an environment for the students to use their own

language and explain their ideas in order to enhance their mathematical knowledge. However, Geogebra was mostly used for demonstrations or teacher-led student-follow work. In the MCC1 Esen reached the Murat's level of the conception of students descriptor. Through the mathematics coaching, Both of them planned, integrated, and evaluated with respect to student learning of polygons as mathematics concepts and student attitudes towards these concepts. That is to say, the focus on the use of Geogebra in the middle of the study was to enhance and assess students' understanding of the polygon concepts. They also utilized Geogebra as a learning tool to improve students' conceptual understanding. In the last lesson, the students engaged in high-level thinking activities such as problem solving activities in Geogebra. In addition, the integration of Geogebra was integral rather than in addition to developing *advancing levels* of understanding of polygons concepts. Thus, they reached the highest level of TPACK Development Model for the learning theme, conception of student thinking descriptor.

4.1.3 Teaching theme

Teaching theme focuses on the views of teachers about technology, teachers' knowledge of instructional strategies incorporating technology and mathematical environments to incorporate technology as a learning tool for supporting students' individual and collaborative mathematical learning and creativity. The teaching theme included three descriptors: mathematics learning descriptor, instructional descriptor, and environment descriptor. In the subsequent sections, TPACK of Murat and Esen is analyzed according to the TPACK Development Model's teaching theme

4.1.3.1 Mathematics Teachers' TPACK Level for the Teaching Theme Before Mathematics Coaching

Before the mathematics coaching process, lessons of Murat and Esen were observed for two hours to determine their TPACK level in teaching theme. All

quotes and dialogs in the following paragraphs come from teachers' TPACK pre-interviews and the pre-observation lessons before mathematics coaching.

4.1.3.1.1 Murat's TPACK Level for the Teaching Theme Before Mathematics Coaching

Murat's pre-observation performance demonstrated *the adapting level* of TPACK for the teaching theme, mathematics learning descriptor and environment descriptor, while *the accepting level* of TPACK for the teaching theme, instructional descriptor, and professional development descriptor.

Murat used Geogebra to enhance students' thinking rather than engage at the end of units for "days off" extra-curricular activities which indicated at *the accepting level* for teaching theme. Murat focused on improving students' mathematics conceptual understanding via Geogebra-enhanced activities such as the "Introducing polygons" activity and the "Finding the number of diagonals" activity. In the "Introducing polygons" activity, he incorporate Geogebra to explore convex polygon which has no diagonal in the exterior of the polygon and concave polygon which has at least one diagonal in the exterior of the polygon by drawing diagonals of polygons. Additionally, in the "finding number of diagonals" activity, he helped students to generalize and discover the relationships between side and diagonals on a single vertex by drawing diagonals of a single vertex via Geogebra. Firstly, students found the formula which is $n-3$ with drawing diagonals of different-sided polygons in Geogebra-enhanced activity. Then, they explored the relationships between side and diagonals of polygons and formulate the number of diagonals of an n -sided polygon as a $n(n - 3) / 2$. Thus, Murat incorporated Geogebra his mathematics class to make inductively reasoning at the number of diagonals of an n -sided polygon. He provided Geogebra-enhanced environment to make generalizations about geometric properties of polygons with using numeric patterns between sides and diagonals of polygons. Supportively, in the pre-interview he stated thatv"Generally I use inductive reasoning. By means of instructions, I enable the students to reach the desired level." This indicated that he was over *the*

recognizing level for instructional descriptor that teacher “Does not use technology to develop mathematical concepts”. In addition, this provided evidence that he believed the benefits of Geogebra with providing his students to explore the properties of concave and convex polygons and discover relationships of sides and diagonals of polygons instead of just telling them what they were. Supportively, in the pre-interview he stated that “Students’ being able to construct shapes, their being able to do it themselves, its non-passive structure, my being able to have them draw on the spot and the visuals’ appearance to them all enable me to act flexibly because it doesn’t require me to explain it to them.” Furthermore, he did not use technology as add-on activities or as drill and practice. He did not think technology take away time from teaching mathematics. In the pre-interview when asked about having any concerns about teaching new concepts via technology, he declared that “No, I go prepared anyway to prevent any possible wrong learning. I do a mental planning of what students could ask in order not to fall into error. Or when they ask something different, I think about whether or not there is such a thing. That’s why I go to class prepared and try to eliminate that problem.” This speech indicated that he integrate Geogebra to teach new concepts without any hesitations. However, Murat did not integrate Geogebra to enhance students’ higher-level thinking (such as project-based and problem solving and decision making activities) for learning mathematics using the technology which indicated in *exploring level*. Besides, he did not focus on using Geogebra as a learning tool. Briefly, these placed him at *the adapting level* for teaching theme, the mathematics learning descriptor.

Although he motivated students to reasoning inductively, student monitored teacher what he did on the projected Geogebra, and then they follow the teachers’ instruction which was parallel to activities’ handout. He did not provide an opportunity to manage towards directing student engagement and self-direction in learning mathematics. Although, every students group had laptops and handouts of activities, most of time students did not independently explore the concepts about polygons with using Geogebra. In the “Introducing polygons” activity, he drew

segments between two vertices in first polygon to discover diagonals, then he let students groups to draw segments of other polygons. He firstly demonstrated every first step of activity then students followed him. In addition, he asked the students to read aloud the instructions on the activity sheets rather than to work collaboratively in groups. The dialogue that proceeded between Murat and his students are as follows:

M: Now, stop working on your computers and listen to me. Read the 5th item.

S1: Does any of the line segments you drew here lie outside the shape?

S2: Yes.

M: Write the answer then. Has everybody written the answer? So what is the answer *Student 1*?

S1: Some of the diagonals of the third polygon lie outside.

M: What did your group write *Student 2*?

S2: In the third shape, the kl and lj line segments are outside.

M: Okay then read the 6th question.

S2: Teacher, in the third polygon, half of the kl line segment is inside and the other half outside.

M: One part is lies outside. For us the important thing is whether or not there are any remaining outside. Is there any part remaining outside of the Kl polygon?

S2: Yes there is.

M: *Student 3*, 6th item.

S3: If all the line segments you draw are inside the polygon, what can you call this polygon?

This dialogue showed that he was a director of the exploration rather than guide. Students did not free to use Geogebra for exploring geometric concepts with communicating, discussing, providing arguments, explaining reasoning. He did not incorporate Geogebra to promote student-directed inquiry and collaborative work with technology-enhanced activities through teacher guidance to draw their own conclusions. Mostly the students did not involve actively in learning in technology supported environment to reach their own insights by individual or collaborative work which demonstrated lower level of teaching theme. On the other hand, most of time students did not work independently with Geogebra; sometimes he wanted them to explore concepts such as the relationship between side and diagonals on

their own while he was walking around. For instance, in the “finding number of diagonals” activity, he gave a time to construct diagonals of pentagon and hexagon to make a conjecture after drawing diagonals of quadrilateral. This indicated that he attempts to adapt lessons for his mathematics classes as at *adapting level* for this descriptor (Niess et al., 2009).

In addition, in the pre-interview the teacher declared that
Actually the teacher’s role should be to somehow guide the students. We are not trying to give the concept but to identify the concept. I don’t say the concept is such and such. I make the students discover the concept. I tell them name it. I ask them what its name could be. At that point, the students make some definitions in their own way. If there are some students who have studied a little, they directly say it. Of course if there are students who are competent in their prior knowledge, they can deduce the concept for us. I guide the students while they are doing the activities. Sometimes we don’t proceed in accordance with the constructive teaching method. In the question-answer or presentation format...And if we can, we try to direct the lesson towards discovery learning if we can. Our idealized lesson is not this of course.”

This statement is parallel to pre-observation lesson performance. Because his activities were coherent to constructivism although he taught lesson with teacher demonstration. He explained the reason of teacher demonstration is that

“Our insufficiencies in terms of technology. That studies conducted in this area are not shown as reference. I mean the teacher needs to be pre-prepared and the school needs to be sufficiently equipped to do this with the students. When there are many insufficiencies, inevitably the teacher chooses the easy way.”

With this statement, the teacher attributes the reason of a teacher-centered lesson to insufficiencies in technology and the long-lasting preparation process. Additionally, this statement revealed his desire to use already-prepared mathematics activities for incorporating without any adaptation as in *accepting level* of TPACK for the teaching theme, instructional descriptor.

In the pre-interview, Murat indicated that he wanted to attend a professional development course. Furthermore, he said,

“It would be very nice to have it [professional development]. It was during my undergraduate years when I last took a course related to technology. It’s been quite some time. Since then I can’t say I followed up with technology. Perhaps I am involved as I’m pursuing my master’s degree. And if I weren’t I would probably be indifferent to the problem. I mean that there is in-service training, to which we, together with our friends, are participating, and that we are dealing with and using technology within the classroom will be beneficial for us. We don’t have teachers who read a lot. How many of us read articles?; how many of us read journals? There is a problem in this sense. That is, perhaps thanks to in-service training and by encouraging teachers a little we can engage them in the issue.”

This statement revealed that he sought a professional development course related to technology integration. Furthermore, he realized that in-service teachers needed to attend this kind of a course. He recognized the need to participate in a technology - related professional development course as indicated in *the accepting level* for the descriptor of professional development. In addition, he indicated the properties of a technology-related professional development course as follows:

“This training could be not in the form of a one-to-one presentation, but in a way by which I can also produce something. Another type of training could be where we are given something to focus on and we are included in the presentation too. That kind of a thing can be established. It could be a magazine. We could share it with our friends. We did this; we produced this. We can make progress by sharing things.”

This statement demonstrated that he sought a professional development that would make him a more productive and active learner. In the light of these statements, he “seek[s] out technology-related professional development, workshops that are directed at developing the technology in the learning of mathematic” as indicated in *the accepting level* for the professional development descriptor in the teaching theme (Niess et al., 2009, p. 22).

4.1.3.1.2 Esen's TPACK Level for the Teaching Theme Before Mathematics Coaching

Esen's performance before mathematics coaching demonstrated *the accepting level* of TPACK for the teaching theme, mathematics learning descriptor, while *the recognizing level* of TPACK for the teaching theme, instructional descriptor, environment descriptor, and professional development descriptor.

Esen implemented Geogebra after teaching new concepts via concrete materials such as the geometry board. In the pre-interview, she stated, "I don't think it's inconvenient to use Geogebra in teaching concepts. It will definitely have a positive impact because I think that the more activities we add to the lesson, the more effective teaching can become. I think we should add everything to maths. You should continuously develop yourself. Visuals, materials. For example, Geogebra is perfect in this sense." This speech indicated that she was not concerned about the fact that Geogebra would take up time from teaching mathematics as indicated in *the accepting level* of the mathematics learning descriptor in the teaching theme. In addition, the teacher in *the accepting level* uses technology as presented. But this speech demonstrated that Esen thought that Geogebra could contribute to students' learning. Therefore, she was above *the accepting level*. Besides, she did not integrate Geogebra to teach new concepts such as concave and convex polygons. She implemented Geogebra activities as a supplement that offered some advantages during the 15 minutes at the end of unit. She repeated the activities using concrete materials at the end of the lesson via Geogebra. For instance, the students explored the differences between concave and convex polygons via the geometry board. Then she opened Geogebra and demonstrated the properties of the concave and convex polygons by creating diagonals on Geogebra. She repeated the non-technology activities via Geogebra. She used Geogebra to foster representations while teaching mathematics and did not view technology as an essential component of the mathematics classroom. Thus, "Technology-enhanced activities are not used for topics that require more advanced technology skills" as indicated in *the*

accepting level for mathematics learning in the teaching theme (Niess et al., 2009, p. 22).

In the pre-interview, Esen declared, “Students can repeat what we did in class with Geogebra. Or technology can enable them to solve more problems and thus see more sample questions.” This statement was consistent with her usage of Geogebra in the observed lesson before mathematics coaching. She used Geogebra for menial or rote activities such as showing the type of polygon after teaching. She did not let the students use technology to learn or access mathematics or explore new concepts with Geogebra. Thus, she “did not use technology to develop mathematical concepts” as indicated in *the recognizing level* for the instructional descriptor in the teaching theme (Niess et al., 2009, p. 22).

The observed lesson before the mathematics lesson was teacher-led student-follow. This emphasized the lower level of the instructional descriptor. The students were not offered opportunities to make decisions about how to proceed. Furthermore, the teacher stated, “My role in the lesson; I reflect them onto the board and the students again come to the board and solve them. But it is under my control.” Besides, the teacher declared the role of technology in her classroom as follows: “We can regard their role after implementing the activities as activities in a technological environment. They are what can be used in the practice stage of the lesson. They can be used in the reinforcement stage. ...To enable them to learn more effectively. Enable students to learn a second time.” This statement demonstrated that she defined the role of the technology as enhancing the already-known concepts. Thus, she “Uses technology to reinforce concepts taught without technology” as indicated in *the recognizing level* for the environment descriptor in the teaching theme (Niess et al., 2009, p. 22).

In the pre-interview, Esen stated that attending a professional development course might be useful to improve herself in terms of technology. In addition, she said,

“I have a simple computer certificate. There are a lot of things that I can later do with technology. It is useful for us while preparing questions. I prepare questions based on what I have done in class. It would be good

if there were an advanced level computer training program... Of course I would really want to attend a course in use of technology in mathematics education. As I said, it would be most useful with respect to exams. Of course it would be in other areas too I think. There are new things emerging every day.”

This statement revealed that she considered attending a professional development course to learn more about technologies. This statement also provides evidence that she wanted to “attend local workshops that focus on gaining skills with the technology; context of the learning activities is mathematics” as indicated in *the recognizing level* for the professional development descriptor in the teaching theme (Niess et al., 2009, p. 22).

4.1.3.2 Mathematics Teachers’ TPACK Level for the Teaching Theme in the MCC1

In the MCC1 during the mathematics coaching process, the lessons of Murat and Esen were observed for two hours to determine their TPACK level for mathematics descriptor of learning theme on polygon. All quotes and dialogs in the following paragraphs came from teachers’ TPACK one-on-one pre-conference 1, post-conference and lessons during mathematics coaching cycle 1.

4.1.3.2.1 Murat’s TPACK Level for the Teaching Theme in the MCC1

Murat’s MCC1 performance indicated *the adapting level* of TPACK for the teaching theme, mathematics learning descriptor and environment descriptor, and *the accepting level* of TPACK for the teaching theme, instructional descriptor.

Murat integrated Geogebra to enhance students’ thinking rather than to reinforce already-learned topics, like the previous lesson which was pre-observation. He did not use technology as add-on activities or as drill and practice which indicated at *the accepting level* for mathematics learning descriptor. He provided a Geogebra-enhanced environment for students to learn the relationship interior and exterior angles in a polygon by using angles tool. Furthermore, they

came up with conjecture about the sum of interior angle, an n -sided polygon as a $180(n - 2)$ via Geogebra. These concepts were new for the students. Based on the observed lesson, Murat used Geogebra to improve students' mathematics conceptual understanding. Instead of writing formulas/relationship in the classroom and accepting them, his students discovered the relationships, made conjectures, and actually proved it by using Geogebra. Although the teacher integrated Geogebra to make a generalization about interior angles of polygons as a new concept, he did not consider the dynamic nature of Geogebra. In the post-conference, he admitted that he should use Geogebra dynamically to teach polygons as a new concept. To conclude, it suggested him at *the adapting level* for teaching theme, the mathematics learning.

In the Murat's observed lesson, every students group had laptops and handouts of activities - "The Angles of Polygons" and "the Finding the sum of interior angle of polygons". He showed first step of activities, then he asked students to follow him. Most of time students did not work independently to explore the related concepts with using Geogebra like previous lesson pre-observation. For example, in "the Angles of Polygons" activity, he constructed quadrilateral and interior angles of one vertex, and then students followed him. He warned students to give up computer and listen to him for several times. Furthermore, he asked students read aloud the following instructions in the activity sheet. These indicated that he was a director of the exploration rather than guide. Although in the pre-conference he was advised to support students' exploration by teacher guidance, he was tendency to control students' action in Geogebra-enhanced activities. He did not provide a Geogebra-enhanced environment to promote student-directed inquiry and work collaboratively through teacher guidance. This indicated that his Instructional strategies with technologies are teacher-directed in order to maintain control of the how the activity progresses which is at *the adapting level* of environment descriptor.

Although his instructional purpose was a director of learning of polygons, students were offered opportunities to make a generalization about interior and

exterior angles of polygons and the sum of interior angles of polygon due to the prescribed nature of the task.

Furthermore, he designed these activities from textbook without consideration of the dynamic nature of Geogebra. Additionally, as asked to how to teach these concepts without technology he said that “Let’s say I am going to proceed from the triangle. ...after finding the angle we were going to make a generalization. Then we were going to draw it on the board. We were going to draw a triangle, a quadrilateral, a pentagon, a hexagon and then make a generalization.” This statement demonstrated that technology did not change instructional ideas. Thus he just applied technology to differentiate the environment without taking into account to affordance of Geogebra. This provided evidence that he merely mimiced the mathematics curricular ideas for incorporating the technologies as in *the accepting level* of TPACK for the teaching theme, instructional descriptor. (Niess et al., 2009).

4.1.3.2.2 Esen’s TPACK Level for the Teaching Theme in the MCC1

Esen’s performance in MCC1 demonstrated *the adapting level* of TPACK for the teaching theme, mathematics learning descriptor and environment descriptor, while she was at *the accepting level* of TPACK for the teaching theme, instructional descriptor.

The students explored the concepts of the interior and exterior angles by using technology. She did not use Geogebra as a day-off activity like in the previous lesson. In addition, she viewed Geogebra to improve students’ knowledge like Murat in MCC1. But both of them did not provide an environment for the students to engage in high-level thinking activities when using Geogebra such as project-based activities, and problem solving and decision making activities. Thus, Esen was at a lower level than *the exploring level* for the mathematics learning descriptor of the teaching theme. In addition, in pre-conference 1, she declared, “Since we are in a technology age, I think it will draw students’ attention more than it paper and pencil would.” This statement emphasized that she used technology to draw students’ attention. These indicated that Esen Used technology to enhance or

reinforce mathematics ideas as indicated in *the adapting level* of TPACK for the teaching theme, mathematics learning descriptor.

The students made generalizations about the sum of the interior angles and one exterior angle of the polygon due to the prescribed nature of the task. She started to use Geogebra to develop geometrical concepts unlike in the previous lesson before mathematics coaching. Although she motivated the students to discover, she did not consider the dynamic nature of Geogebra, just like Murat in the MCC1. That is, she integrated Geogebra as a picture tool. These indicated that Esen mimicked the mathematics curricular ideas for incorporating technologies as described in *the accepting level* of TPACK for the teaching theme, instructional descriptor (Niess et al., 2009).

In MCC1, every student group had access to the Geogebra from the moment the lesson started. The students used the technology for instructional purposes in a structured and teacher-directed environment. In the pre-conference, she stated, “In this case, we will use it for demonstrating and making them do things. I will do something first and then have a volunteer to do it.” This statement demonstrated that she wanted to direct students’ understanding through the lesson. In the observed lesson, she projected the activities and the students discussed the concepts on the blackboard. Then, Esen let the students use Geogebra on their computer. Geogebra was utilized to help control the classroom environment as Esen demonstrated the mathematics to the students, like Murat in MCC1. In MCC1, instructional strategies with technologies are teacher-directed in order to maintain control of how the activity progresses as in *the adapting level* of TPACK for the teaching theme, environment descriptor (Niess et al., 2009).

4.1.3.3 Mathematics Teachers’ TPACK Level for the Teaching Theme in the MCC2

In the MCC2 during the mathematics coaching process, the lessons of Murat and Esen were observed for four hours to determine their TPACK level for mathematics descriptor of learning theme on polygon. All quotes and dialogs in the

following paragraphs came from teachers' TPACK one-on-one pre-conference 2, post-conference 2 and lessons during mathematics coaching cycle 2 .

4.1.3.3.1 Murat's TPACK Level for the Teaching Theme in the MCC2

Murat's MCC2 performance demonstrated *the exploring level* of TPACK for the teaching theme, mathematics learning descriptor and *the adapting level* for environment descriptor and instructional descriptor.

Murat implemented Geogebra to improve students' conceptual understanding about quadrilaterals instead of enhancing already-learned topics. He integrated technology to classify quadrilaterals in different ways and use different names (e.g. a square is also a rectangle and a trapezoid) with the help of dynamic environment of Geogebra. Murat helped students to develop an adequate understanding of hierarchical relationships with the Geogebra-enhanced activities. Furthermore, the students were able to identify non- prototypical quadrilaterals in the Geogebra-enhanced activities with the help of dragging. In the Geogebra-enhanced activities, Murat also provided an opportunity to discuss the properties of quadrilaterals and the relationship between them. For example he moved the rhombus by dragging the vertices and asked which properties did not change. With this guiding question, the students made conjectures about rhombus such as diagonals bisect at right angle. In the post-conference, he explained the purposes of the technology which are to

“Enabling the students to be in the position to learn the concept themselves. I don't give them the concept here; they discover it themselves. I see that they learn better when I tell them only in the end that diagonals intersect in the middle. And when I ask them whether diagonals intersect at right angle, they can click on it and see for themselves. I have started applying it in grade 5 as well, especially since your study. I'm using Geogebra in 5th grades. The lessons are more enjoyable. The students are asking questions. Then they try to find the answers there. They ask, “Can we do it like this?” We do it and it works. I mean they see their discovery, their generalizations. ... I enable them to develop themselves in terms of cognition.”

This speech proved that he engaged students in high-level thinking activities (defining, finding relationship and make conjectures) for learning mathematics using the technology as a learning tool at *the exploring level* of TPACK for the teaching theme, mathematics learning descriptor.

In the Murat's MCC2 lesson, the students discovered the properties of quadrilaterals by using the dynamic nature of Geogebra. His first usage of the dynamic nature of Geogebra unlike previous lessons, such as dragging, indicated that he attempted to adapt lessons for his mathematics class as in *the adapting level* for instructional descriptor. In addition, with using Geogebra dynamically the students could see the different positions of quadrilaterals and make generalizations about their properties. For instance, students checked the "show the angles" box in the parallelogram activity, and then they made a generalization that the opposing angles are equal to each other by whole-class discussion. By teacher motivation the students dragged the vertex to see whether their generalization was valid for all parallelograms or not. Thus these provide an evidence that he incorporated the technologies to take care of students' need while attempting to adapt lessons for his mathematics classes as in *adapting level* of TPACK for the teaching theme, instructional descriptor. (Niess et al., 2009). In the post- conference as comparing previous lesson, he stated that "Now I'm in the position of asking questions all the time. I mean I'm not in the position of lecturing. Establishing a question mark in their minds, guiding them through the question marks, leading them to the correct path somehow if they are going the wrong way. I'm making them talk more." This speech demonstrated that he wanted serving as guide - whereas he was director in MMC2. Besides, he added that "The students are highly motivated. The lesson does not proceed in a routine manner. They are continuously trying to answer questions. This is what I want actually. I want them to participate, to be motivated." This speech demonstrated that students involved actively in MCC2 lesson. On the other hand, students' first generalization was based on the board where projected by teacher. Then the teacher suggested the students to use Geogebra. In the post-conference he explained the reason of his attitude was that "I

used to fear that I wouldn't be able to sum up, to complete the lesson in time. Timing." Because of anxiety about timing, at first he controlled the students activities in Geogebra activities then he gave a time to self-exploration. This indicated that he still wanted to make a control in instructional activities as in *the adapting level* for environment descriptor.

4.1.3.3.2 Esen's TPACK Level for the Teaching Theme in the MCC2

Esen's MCC2 performance demonstrated *the exploring level* of TPACK mathematics learning descriptor, instructional descriptor and environment descriptor in the teaching theme.

Like Murat in MCC2, Esen integrated Geogebra for the students to explore new geometric concepts, such as the properties of quadrilaterals, in the whole lesson. In addition, she started to take the advantage of the dynamic nature of Geogebra in order to improve the students' thinking process. The students made conjecture about the properties of quadrilaterals via dragging the vertices of quadrilaterals. For instance, the students explored that the diagonals of the rhombus bisect each other and are perpendicular by dragging the vertices of the rhombus. Thus, the students engaged in high-level thinking activities to learn mathematics using the technology as a learning tool as indicated in *the exploring level* of TPACK mathematics learning descriptor in the teaching theme.

In addition, Esen's instructional purposes for the students were clear; she engaged students in explorations using Geogebra where students took control of their learning. Esen walked around the classroom and answered any questions the students had in order to guide them. In the pre-conference she stated, "The students will be completely active from the beginning of the lesson from now on." This statement demonstrated that she did not direct their every move and took control of their learning. Furthermore, in the post-conference she said, "Since they can no do some of the things, they begun to assume most of my roles. But after some point, you need to take control because you need to sum up the topic. Teachers are never

free of their guiding role.” This speech emphasized that Esen acted as a guide to improve the students’ exploration. Supportively, the students used Geogebra to explore new concepts as Esen functioned mostly as a guide in the lesson of MCC2. At this point Esen made a difference in guiding the lesson as compared to Murat’s lesson in MCC2 because Murat directed the students in the Geogebra activities. To sum up, Esen “Engaged students in explorations of mathematics with technology where the teacher is in role of guide rather than director of the exploration” as indicated in *the exploring level* for the instructional descriptor in the teaching theme (Niess et al., 2009, p. 22). Furthermore, Esen’s role of guidance in MCC2 emphasized that she was above the adapting level of the environment descriptor in the teaching theme. The teacher in *the adapting level* (like Murat in MCC2) directed the lesson to maintain control of how the activity progressed. Besides, in *the adapting level* instructional strategies with technologies should be primarily deductive. In Esen’s lesson the students mostly used inductive strategies while engaging in Geogebra activities. In the pre-conference, Esen stated, “In this lesson I will generally use induction this time. Questions and answers all the time because they will discover the features all on their own.” This statement emphasized that she wanted to try inductive strategies to improve students’ thinking. To sum up, she “Explored various instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics” as indicated in *the exploring level* of TPACK for the teaching theme, environment descriptor (Niess et al., 2009, p.22).

4.1.3.4 Mathematics Teachers’ TPACK Level for the Teaching Theme in the MCC3

In the MCC3 during the mathematics coaching process, the lessons of Murat and Esen were observed for four hours to determine their TPACK level for mathematics descriptor of learning theme on polygon. All quotes and dialogs in the following paragraphs came from teachers’ TPACK one-on-one pre-conference 3, post-conference 3 and lessons during mathematics coaching cycle 3.

4.1.3.4.1 Murat's TPACK Level for the Teaching Theme in the MCC3

Murat's MCC3 performance indicated *the advancing level* of TPACK for the teaching theme, mathematics learning descriptor and *the exploring level* for environment descriptor and instructional descriptor.

Murat's third lesson on similarity and congruence, finding area of the irregular polygons and the area of parallelogram help students to enhance their knowledge via Geogebra. In the post-interview he indicated the role of the technology in his classroom that "It was the dynamic nature. For example, the dynamic nature of the last activity we did (the area of the parallelogram), dragging the shape here and there, enables the students to better understand the concepts." this speech demonstrated that the focus of his MCC3 lesson was on improving his students' understanding with the consideration of advantages of Geogebra such as dragging. He also indicated that

"Now they know this rectangle and they know the topic of area. But it is not conceptually clear in their mind; they express themselves hesitantly. Now I see that they are giving me firm answers. The other day I asked them a congruence question in the topic of similarity. I asked what congruence was. They gave their own definitions. I mean, here they give conceptually firmer answers. The answers and definitions are unique; they are not out of memorization. I started to like this."

Based on the MCC3 lessons and his speech, Murat integrated Geogebra as a learning tool for higher-level thinking activities (exploring, finding relationship and make conjectures) suggesting him at the exploring level for the teaching theme, mathematics learning descriptor. Moreover, he stated that "I mean we had a command of technology in the whole lesson. We used it in the introduction by making it count the square units... then in the discovery stage we again used the slider. We used it to transfer from the parallelogram to the rectangle. In the last stage, the assessment stage, we again had questions. We used it in this stage too. We used it actively in all the stages." This statement demonstrated that he did not view Geogebra as a supplement that offers some advantages in mathematics learning. He developed a vision of using Geogebra in every component of the mathematics

teaching such as assessing and exploring the concepts. Thus, he viewed technology as an essential component of the MCC3 lesson. Furthermore, his acceptance of Geogebra as a tool for learning was active and consistent which was an indicator of the advancing level. In view of the properties of Geogebra, he developed new ideas for learning mathematics. For instance, to find the area of parallelogram students transformed the parallelogram to rectangle via using slider. To sum up, his “active, consistent acceptance of technologies as tools for learning and teaching mathematics in ways that accurately translate mathematical concepts and processes into forms understandable by students” as at *the advancing level* of TPACK for the teaching theme, mathematics learning descriptor.

The students discovered new geometrical concepts such as the congruence and similarity, the area of irregular polygon and parallelogram by using the dynamic nature of Geogebra. The students started to use effectively Geogebra with the considering the dynamic nature of Geogebra. He started to guide the students’ exploration unlike the direct them in the MCC2. He started to give up the teacher-led student follow lesson in the MCC3. He engaged students in explorations using Geogebra where students started to took control of their learning, and he was in the role of guide. He probed students with questions and answered questions from his students. Thus, he “engaged students in explorations of mathematics with technology where the teacher is in role of guide rather than director of the exploration.” as indicating in *the exploring level* for instructional descriptor in the teaching theme (Niess et al., 2009, p. 22). Murat was over *the adapting level* for the environment descriptor in the MCC3. Because, He adapted instructional approaches that allow students opportunities to explore with technology in the whole lessons. Murat in the MCC2 directed the lesson to maintain control of the how the activity progresses. But he provided an environment for the students to direct themselves in thinking about the mathematics. Furthermore, He supported the inductive and deductive reasoning in the MCC3. This provided evidence that he was at exploring level for environment descriptor. He “Explored various instructional strategies (including both deductive and inductive strategies) with

technologies to engage students in thinking about the mathematics.” as indicating *the exploring level* of TPACK for the teaching theme, environment descriptor (Niess et al., 2009, p.23).

4.1.3.4.2 Esen’s TPACK Level for the Teaching Theme in the MCC3

Esen’s MCC3 performance demonstrated *the advancing level* of TPACK mathematics learning descriptor and environment descriptor in the teaching theme, while *the exploring level* for instructional descriptor.

In MCC3, Esen provided the students with an environment to explore the new concepts, such as congruence and similarity, the area of irregular polygons and the area of the parallelogram. The students engaged in high-level thinking activities (problem solving and making conjecture) to learn mathematics using the technology as a learning tool. Like Murat, she viewed Geogebra as a novel idea in helping students learn mathematics with technology as indicated in *the advancing level* for the mathematics learning descriptor. For instance, in the area of the parallelogram and rectangle, the students dragged the vertex of parallelogram to see the change in the area of the parallelogram. Finally, the students explored that the area did not change in spite of the change in the lateral sides. This activity was an innovative idea for the students in terms of learning the area of the parallelogram. In addition, this activity provided evidence that Esen integrated Geogebra activities on transformation to make geometrical concepts more understandable. To sum up, Esen displayed “Active, consistent acceptance of technologies as tools for learning and teaching mathematics in ways that accurately translate mathematical concepts and processes into forms understandable by students” as indicated in *the exploring level* of TPACK for the teaching theme, environment descriptor (Niess et al., 2009, p.23).

In MCC3, Esen engaged students in explorations of these new geometrical concepts using Geogebra where the students took control of their learning. She functioned mostly as a guide like in MCC2. In the post-conference she stated, “We no longer use presentation and then practice. The students discover for themselves.

We use induction. We can say they explore themselves through the method of discovery.” This statement demonstrated that Esen gave up the teacher-led method in which students follow. Thus, she “engaged students in explorations of mathematics with technology where the teacher is in role of guide rather than director of the exploration” as indicated in *the exploring level* for the instructional descriptor in the teaching theme (Niess et al., 2009, p. 22).

Esen realized the benefits of the students’ explorations of geometric concepts and relationships via Geogebra instead of just telling them what they were. In order to support the students’ exploration, she created an environment where students were engaged and self-directed in learning. In this self-directed environment, the students were measuring, observing, exploring, finding relationships, and finding multiple solutions. For instance, the students followed multiple strategies to find the irregular shape by means of self-direction. Furthermore, after finding the activity, the teacher asked to open another Geogebra activity related to finding the area of irregular shapes. Thus, Esen “Manages technology-enhanced activities in ways that maintains student engagement and self-direction in learning the mathematics” as indicated in *the advancing level* for the environment descriptor in the teaching theme (Niess et al., 2009, p. 23).

4.1.3.5 Mathematics Teachers’ TPACK Level for the Teaching Theme in the MCC4

In the MCC4 during the mathematics coaching process, the lessons of Murat and Esen were observed for two hours to determine their TPACK level for mathematics descriptor of learning theme on polygon. All quotes and dialogs in the following paragraphs came from teachers’ TPACK one-on-one pre-conference 4, post-conference 4 and lessons during mathematics coaching cycle 4.

4.1.3.5.1 Murat's TPACK Level for the Teaching Theme in the MCC4

Murat's MCC4 performance demonstrated *the advancing level* of TPACK for the teaching theme, mathematics learning descriptor and *the exploring level* for environment descriptor and instructional descriptor.

Murat's last lesson during mathematics coaching on the area of rhombus and trapezoid provided an environment to learn these mathematical concepts using Geogebra as a learning tool. The students learned the area of rhombus and trapezoid as new concepts by using Geogebra. The Geogebra-enhanced activities in MCC4 were designed to become more understandable by students. For example, the teacher stated that the area of rhombus were hard to understand for students, because of using the diagonals in the formula. Therefore, he integrated Geogebra to discover the formula of the area of rhombus by transforming the rhombus into rectangle. Besides, the teacher viewed Geogebra as a resource for novel ideas to help students learn the area of rhombus and trapezoid. Furthermore, he accepted the Geogebra as a learning and teaching tool. He stated he learnt it to provide mathematics classroom for students' free engagement with using Geogebra. These provided an evidence that he was at advancing level of TPACK for the teaching theme, mathematics learning descriptor, because "active, consistent acceptance of technologies as tools for learning and teaching mathematics in ways that accurately translate mathematical concepts and processes into forms understandable by students"

Like previous lesson (MCC3) Murat supported the students to explore new geometrical concepts such as the area of rhombus and trapezoid by using the dynamic nature of Geogebra. He served mostly guide to improve the students' exploration. Thus, he "engaged students in explorations of mathematics with technology where the teacher is in role of guide rather than director of the exploration." as indicating in *the exploring level* for instructional descriptor in the teaching theme (Niess et al., 2009, p. 22). Furthermore, He incorporated a variety of technologies for these topics in the MCC4 as indicating *the exploring level* for environment descriptor in the teaching theme. Additionally, He supported the

inductive and deductive reasoning. Thus, He “Explored various instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics.” as indicating the exploring level of TPACK for the teaching theme, environment descriptor (Niess et al., 2009, p.23).

4.1.3.5.2 Esen’s TPACK Level for the Teaching Theme in MCC4

Esen’s MCC4 performance demonstrated *the advancing level* of TPACK mathematics learning descriptor, instructional descriptor and environment descriptor.

Like MCC3, Esen used Geogebra as a resource for novel ideas to help students learn the geometrical concepts. She integrated innovative activities regarding the areas of the rhombus and the trapezoid to explore the areas of the rhombus and the trapezoid by considering the dynamic nature of Geogebra. In the post-conference she stated, “I tried to make students discover the concepts.” This statement supported the fact that she integrated Geogebra to improve the students’ exploration. In addition, the observed lesson demonstrated that Esen implemented Geogebra activities on transformation to make the concept more understandable, like Murat. For instance, in the trapezoid activity, the students transformed the trapezoid into a parallelogram whose formula they know. This transformation helped the students to explore the area of the trapezoid. Thus, Esen displayed an “Active, consistent acceptance of technologies as tools for learning and teaching mathematics in ways that accurately translate mathematical concepts and processes into forms understandable by students” as indicated in the advancing level for mathematics learning descriptor in the teaching theme (Niess et al., 2009, p. 23).

The role of Esen was to provide guidance to promote the students’ exploration, like in the previous MCC3 lesson. In the post-conference she stated, “My role has completely changed. I left the final process of discovery to the students. I wrapped up at appropriate points. I acted as a guide.” This statement supported the role of Esen in MCC4. Furthermore, she integrated different

Geogebra activities to promote the students' exploration from different perspectives. For instance, the students worked with two different Geogebra activities to find the formula for the area of the rhombus. This provided evidence that she was at *the advancing level* for the instructional descriptor. Esen "helped students move fluently from one tool to another while demonstrating a focus on and a joy of deeply understanding mathematical topics" (Niess et al., 2009, p. 23).

In addition, she provided the students with an environment to engage and self-direct in learning the area of the rhombus and the trapezoid. She "managed technology-enhanced activities in ways that maintains student engagement and self-direction in learning the mathematics" as indicated in in *the advancing level* for environment descriptor in the teaching theme (Niess et al., 2009, p. 23).

4.1.3.6 Mathematics Teachers' TPACK Level for the Teaching Theme After Mathematics Coaching

After mathematics coaching, the lessons of Murat and Esen were observed for two hours to determine their TPACK level in the learning theme. All quotes and dialogs in the following paragraphs came from teachers' TPACK one-on-one post-observation and post-interview after mathematics coaching.

4.1.3.6.1 Murat's TPACK Level for the Teaching Theme After Mathematics Coaching

Murat's performance after mathematics coaching demonstrated *the advancing level* of TPACK mathematics learning descriptor, instructional descriptor, environment descriptor, and professional development.

In the post-observation lesson Geogebra was used as a learning tool to discover the relationship between area and perimeter. In the post-interview he stated that

"Geogebra provided the students with a different perspective. A different perspective emerged. Towards the final weeks, I got a clearer picture. I mean, not just having students draw shapes and think about

them, but integrating it into the constructivist system where the students themselves engage in discovery enables them to become enlightened.”

This indicated that he developed a vision of using Geogebra in teaching polygon concepts. Additionally, the focus of his post-observation provided an environment to facilitate the students’ high level thinking with technology. For instance, the student worked with Geogebra –enhanced problem solving activity (Fixed perimeter activity). In this activity, the students used Geogebra to solve the following problem: “What is the biggest area of the rectangle whose perimeter was 24 units?” Furthermore, this activity was a novel idea to enhance students’ knowledge via Geogebra. Besides, in the whole lesson Geogebra as a learning tool was used actively and consistently. These provided evidence that he was *at the advancing level* of TPACK for the teaching theme, mathematics learning descriptor. His “ active, consistent acceptance of technologies as tools for learning and teaching mathematics in ways that accurately translate mathematical concepts and processes into forms understandable by students.” as indicating in *the advancing level* for mathematics learning descriptor in the teaching theme (Niess et al., 2009, p. 23). Murat adapted the various activities into Geogebra to to engage students in thinking about the mathematics. He integrated different Geogebra activities to explore the concepts in different perspective. In addition, He motivated the spreadsheet in Geogebra to expand the fluency in the representation. These provided an evidence that he was *at the advancing level* for instructional descriptor. Murat helped “students move fluently from one tool to another while demonstrating a focus on and a joy of deeply understanding mathematical topics.” as indicating in *the advancing level* for instructional descriptor in the teaching theme (Niess et al., 2009, p. 23). Furthermore, Murat created an environment where his students were engaged and self-directed to explore the relationship between area and perimeter via Geogebra. Thus, he “managed technology-enhanced activities in ways that maintains student engagement and self-direction in learning the mathematics” as indicating in as indicating in *the advancing level* for environment descriptor in the teaching theme (Niess et al., 2009, p. 23).

In the post-interview, he stated that he wanted to engage in professional development focused on technology integration in mathematics education. Furthermore, he described the properties of professional development courses as follows:

“As I said, we are no longer students.. We are gaining some experience. Perhaps its not much but at least we have some foundation. If we hold different opinions, then we can hold workshops during which we can produce things. Together we can produce things. For example, by taking into consideration grade 6 objectives, one person can do something related to numbers, another can take up geometry, or decimal representation; everyone can do or produce something different. We can come and present to each other and notice our weaknesses. Well, here is a wonderful resource. There needs to be some time, a long period of time to for this. Cooperation. Not with individual but with group work...I am member of geogebra Turkey. I follow it in terms of resources. I especially wanted to attend this summer. I examined it the other night to see what was there.”

This speech revealed that he sought in-service teachers education courses which emphasized the process of teachers’ development and motivated the teachers to work collaboratively. Thus, he “Seeks ongoing PD to continue to learn to incorporate emerging technologies. Continues to learn and explore ideas for teaching and learning mathematics with multiple technologies to enhance access to mathematics” as indicated in *the advancing level* for the professional development descriptor in the teaching theme (Niess et al., 2009, p. 22).

4.1.3.6.2 Esen’s TPACK Level for the Teaching Theme after Mathematics Coaching

Esen’s performance after mathematics coaching demonstrated *the advancing level* of TPACK mathematics learning descriptor, instructional descriptor, environment descriptor, professional development descriptor.

Like MCC4 lesson, Esen integrated innovative Geogebra activities to improve the students’ understanding. For instance, in the fixed perimeter area the students discovered the relationship between the relationship of the area and the

perimeter. In this activity, the students dragged the vertex of a rectangle to create a new one and then compared the area of the old and the new rectangle via clicking on the “new area” button. This activity provided novel ideas in helping students learn mathematics with Geogebra. In addition, the teacher implemented Geogebra in the whole lesson. This indicated that she accepted Geogebra consistently to learn and teach the relationship between the area and the perimeter. This indicated that Esen displayed an “Active, consistent acceptance of technologies as tools for learning and teaching mathematics in ways that accurately translate mathematical concepts and processes into forms understandable by students” as indicated in *the advancing level* for mathematics learning descriptor in the teaching theme (Niess et al., 2009, p. 23).

Esen implemented Geogebra activities to explore the relationship between the area and the perimeter. She wandered around the classroom to help their exploration of the relationship between the area and the perimeter. In the fixed perimeter Geogebra activity, the students used the clicks to create a new rectangle and to compare the area of the old and new one. Additionally, they added the value of the new and old rectangle in the Geogebra’s spreadsheet. These tools helped them to gain an in-depth understanding of the relationship between the area and the perimeter. In the post-interview she said, “I just walked among the groups. It is more difficult for me, but enjoyable for the students. I guided them through it. I left the entire discovery process to them. When they asked me something, I replied, but they found most of the formulas themselves.” This statement indicated that the Geogebra activities provided the students with an environment to explore the concepts in an enjoyable way. As indicated in *the advancing level* for mathematics learning, Esen “helped the students move fluently from one tool to another while demonstrating a focus on and a joy of deeply understanding mathematical topics” (Niess et al., 2009, p. 23).

In the observed lesson after mathematics coaching, the students directed their learning regarding the relationship between the area and the perimeter. Additionally, the fixed perimeter activity motivated the students after every action

via asking questions. For instance, the question of “Did the perimeter change? Did the area become smaller or bigger? ” appeared after clicking on new area. Then the students clicked on the “see the difference” button, and the question of “When does the area have the highest value?” appeared. This activity guided the students to find the answer of these questions. This indicated that the students were responsible for their exploration. Furthermore, in the post-interview, she stated, “The students have the role of discovering something. They are the knowledge seekers...they are to reach knowledge through trial, practice, experience and by playing around with the slider.” This statement supported the role of the students in the observed lesson after mathematics coaching. She “managed technology-enhanced activities in ways that maintains student engagement and self-direction in learning the mathematics” as indicated in *the advancing level* for the environment descriptor in the teaching theme (Niess et al., 2009, p. 23).

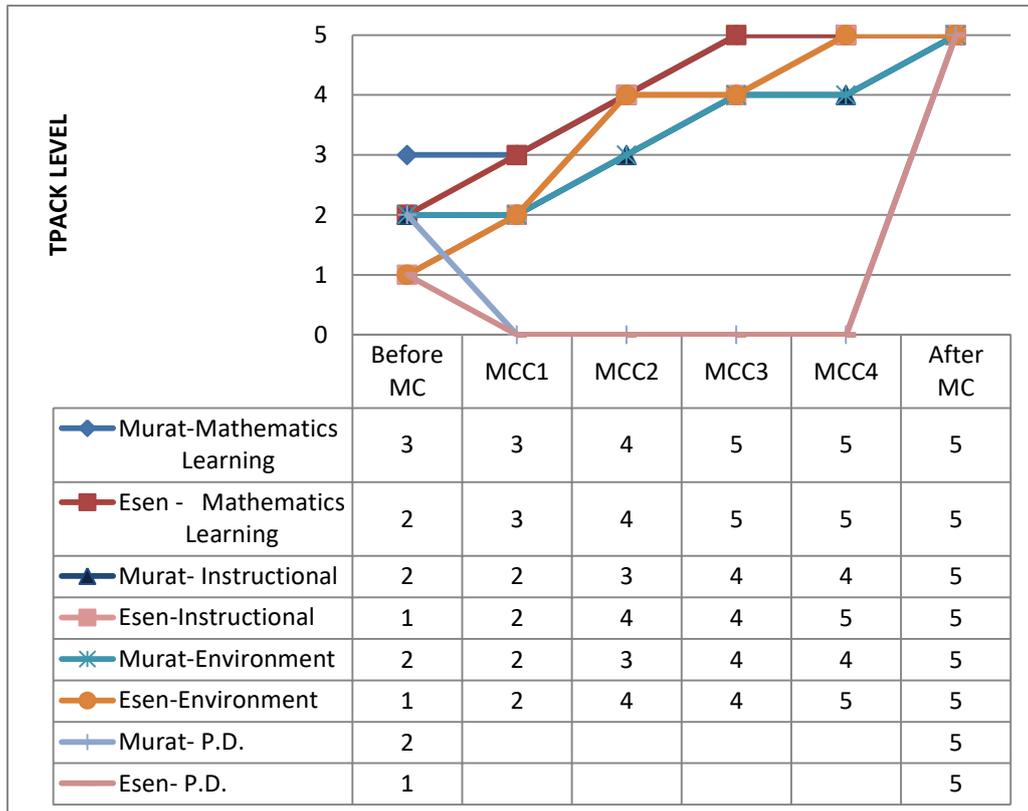
In the post-interview, Esen declared that she wanted to engage in a professional development course to enhance her knowledge of technology integration. Furthermore, she added,

“Of course I would like to attend more courses. This training had a positive impact on me. I would have liked to be able to use these programs and others professionally, be able to produce other more different things and have come to a point at which we could prepare things ourselves. In general I didn’t have a negative approach. At the moment I have become more positive.”

This statement revealed that she wanted to engage in a professional development course on other software in mathematics education. Thus, she “Seeks ongoing PD to continue to learn to incorporate emerging technologies. Continues to learn and explore ideas for teaching and learning mathematics with multiple technologies to enhance access to mathematics” as indicated in *the advancing level* for the professional development descriptor in the teaching theme (Niess et al., 2009, p. 22).

4.1.3.7 The progress of Murat and Esen’s TPACK of Learning Theme through Mathematics Coaching

Table 4. 7 The Progress Of Teachers For Teaching Theme



The performances of Murat and Esen for the teaching theme in the six Geogebra-enhanced lessons before/during/after mathematics coaching are presented in Table 4. 7. The teachers made growth into advancing level in the TPACK development model for mathematics learning descriptor and the conception of students in the teaching theme

Esen’s performance for mathematics learning, instructional, and environment descriptor in the teaching theme was lower than Murat at the beginning of the study. But both of them illustrated the growth into advancing level in the TPACK development model for teaching theme. The mathematics descriptor in teaching theme focuses on what the teacher’s views about technology are. At the

beginning, Esen implemented Geogebra after teaching new concepts via concrete materials such as geometry board. She used Geogebra activities as a supplement that offers some advantages such as demonstration at the end of unit. Thus, she was the lowest level of the mathematics descriptor. On the other hand, Murat integrated Geogebra without the consideration of dynamic properties to enhance the students' knowledge at the beginning of the study. In the MCC1, Esen reached the Murat's level of mathematics descriptor. Their usage of Geogebra indicated that they did not have any concerns about the technology would take away time from teaching geometry. Additionally, they did not view Geogebra-enhanced activities as add-on or drill and practice. Both of them implemented technology to improve students' exploration about new geometrical concepts. In the middle of study, they realized the dynamic environment of Geogebra, and he incorporated Geogebra into classroom instruction by engaging his students in high-level thinking activities. Towards the end of the study, they used Geogebra to transform geometrical concepts into more understandable by students. By using Geogebra as a learning tool, they got the novel idea to improve students' learning. Furthermore, their acceptance of Geogebra was active and consistent as indicator of advancing level of TPACK Development Model for the teaching theme, mathematics learning descriptor. Thus, both of them reached the highest level after the MCC3 lesson.

At the beginning of the study, Esen used Geogebra for menial or rote activities such as showing the type of polygon after teaching whereas Murat integrated to improve the students' exploration. Therefore, she was at lower level of instructional descriptor. In addition, Esen did not let the students to use technology to learn or access mathematics or explore new concepts with Geogebra. In the MCC1, Esen reached the Murat's level and used of Geogebra for the students' exploration. They merely mimiced the mathematics curricular ideas for incorporating the technologies without the consideration of dynamic nature of Geogebra. In the MCC2 Esen passed the Murat's TPACK level of instructional descriptor. Esen served mostly as a guide rather than director of the exploration like Murat. Both of them made a progress through mathematics coaching in terms of

instructional descriptor inteaching theme. At the end of the study, they reached the highest level of TPACK in this descriptor. They she integrated different Geogebra activities to promote the student' exploration with different perspective. They "helped students move fluently from one tool to another while demonstrating a focus on and a joy of deeply understanding mathematical topics" (Niess et al., 2009, p. 23).

Like instructional descriptor, Esen was in the lowest level of TPACK of environment descriptor. Esen used technology to reinforce concepts taught without technology whereas Murat integrated Geogebra to teach new geometrical concepts. Nevertheless, The Murat's lesson was directed and step-by-step process. Then, they made a progress through mathematics coaching. Murat started to adapt instructional approaches that allow students opportunities to explore with Geogebra in the MCC2. But his lesson was still teacher-directed in order to maintain control of the how the activity progresses. Besides, in the MCC2 Esen explored various instructional strategies with Geogebra to engage students in thinking about the mathematics. The student was more self-directed. At the end of the study, both of them reached highest level of TPACK for environment descriptor. They managed Geogebra-enhanced activities in ways that maintains student engagement and self-direction in learning the mathematics.

4.1.4 Access Theme

The access theme included three descriptors: usage descriptor, barriers descriptor, and availability descriptor. The usage descriptor focuses on whether the teacher lets the students use Geogebra or not. The barriers descriptor emphasizes how the teacher overcomes the problems related to technology integration. Additionally, the availability descriptor addresses how technology makes higher levels of and more mathematics available for students' investigation. In the subsequent sections, TPACK of Murat and Esen is analyzed according to the TPACK Development Model's access theme.

4.1.4.1 Mathematics Teachers' TPACK Level for the Access Theme Before Mathematics Coaching

Before the mathematics coaching process, the lessons of Murat and Esen were observed for two hours to determine their TPACK level in the access theme. All quotes and dialogs in the following paragraphs come from teachers' TPACK pre-interviews and the pre-observation lessons before mathematics coaching.

4.1.4.1.1 Murat's TPACK Level for the Access Theme before Mathematics Coaching

Murat's performance before mathematics coaching demonstrated *the adapting level* of TPACK for the access themes, usage descriptor, while *the accepting level* of TPACK for the teaching theme, barrier and availability descriptor.

Murat's performance indicated *the adapting level* of TPACK for the access theme, usage descriptor. At *the adapting level*, teachers "permit students to use technology in specifically designed units" (Niess et al., 2009, p.24). In the pre-observation lesson Murat assigned students to groups according to students' desire. There were 7 small student groups (2 or 3 students), and every student group had a computer to use Geogebra. Murat also projected his computer onto the blackboard. Students were able to use their computer for approximately the whole lesson except during warm-up activities and assessment sessions. The pre-observation lesson indicated *the adapting level* for the usage descriptor because Murat incorporated Geogebra into the mathematics lesson only in specific ways. For instance, he did not use Geogebra to reinforce and remind the related concepts of polygons and to assess students' knowledge of polygons. On the contrary, the students were able to access Geogebra. Additionally, Geogebra was available for the exploration of geometric concepts which were new for the students, such as concave and convex polygons and the number of diagonals of a polygon. Besides, the students' exploration was usually done by means of the teacher's demonstration. In the pre-interview, he stated,

“Technology and computers are what students most want and mostly use actively. Our active use of these in all aspects enables us to lead the class more effectively. Therefore, when we have access during the introduction, practice and assessment stages, we see that the students are more active. They are more motivated. They are eager to attend the following lesson. I mean even if this is the mathematics course, they look forward to it. I think it should be used in every stage of the lesson.”

This statement indicated that he allowed the students to use Geogebra in the whole lesson to motivate the students and to enable them to become engaged in the lesson. To sum up, these suggested that Murat was at *the adapting level* of TPACK for the access theme, usage descriptor. In the lesson before mathematics coaching “Access to and use of technology is available for exploration of new topics, usually with the teacher’s demonstration” (Niess et al., 2009, p.24).

Murat’s pre-observation performance suggested *the accepting level* of TPACK for the access theme, barrier descriptor. In the pre-observation lesson, Murat walked around the classroom to help students when they struggled in Geogebra. For instance, at the beginning of lesson students should have drawn a pentagon and a hexagon to investigate convex and concave polygons. Some groups had struggled to construct a pentagon and a hexagon. He showed students how to construct these polygons. But his support was related to technical issues such as how to draw a segment. In addition, he first constructed a polygon which was quadrilateral in Geogebra on his computer; then the students followed him. Furthermore, although he gave out a handout to every student group, he asked one of the students to read aloud the instructions in the activity sheets. He wanted to take control of how the activity progressed and what the students did. He did not provide the students with an independent environment in which students had responsible for their own learning. Furthermore, he was sensitive to noise, and he did not allow much time to the students to work freely and to hold discussions with each other. He did not want the students to do anything but follow his instructions, so he asked them to quit using the computer and to listen to him. Moreover, he said that they couldn’t follow the instruction if they did not monitor him. In the pre-interview, he

stated, “I’m not sure whether or not I should dominate the class while using technology. I mean the balance should be maintained carefully. The teacher should interrupt the students’ use of the computer at some stages and direct them.” This speech demonstrated that he had conflicting views in relation to management issues. In addition, this speech supported his usage of the computer in the pre-observation, such as asking the students not to use the computer. These indicated that he “worried about management issues with respect to incorporating technology in the classroom” as indicated in *the accepting level* of TPACK for the access theme, barrier descriptor.

Murat’s pre-observation performance suggested the accepting level of TPACK for the access theme, availability descriptor. At *the accepting level*, teachers provide the students with an opportunity to explore patterns and functions by using technology. In the “Introducing polygons” activity, Murat asked the students to construct different polygons to investigate concave and convex polygons via Geogebra. Then students drew segments between two vertices in polygons to discover the position of diagonals. Although this activity led students to define, explore and analyze concave and convex polygons, Murat did not provide the students with a dynamic environment that can be manipulated by dragging free objects. The students constructed a quadrilateral, a pentagon and a hexagon to find the number of diagonals of n-sided polygons in the “finding number of diagonals” activity. Students drew diagonals of these different sided polygons and discussed their results with the whole class. However, technology was used as a graphic tool, such as paint. If students had dragged the vertex of polygons, they would have seen that the number of diagonals of all polygons with the same number of sides was equal to each other. In this way, students would have had the opportunity to investigate geometrical relations dynamically rather than statically. It indicated that Murat couldn’t reach *the adapting level*, at which concepts are taught differently since technology provides access to connections formerly out of reach.

Murat provided a visual representation for various polygons, such as concave/convex polygons and different sided polygons to explore the geometric concepts related to polygons via Geogebra. He also used table to help students to explore and to gain a conceptual understanding of the relationship between sides and diagonals of polygons in the “finding number of diagonals” activity. However, he drew and filled in the table (see Table 4.1) on the blackboard rather than on a Geogebra Spreadsheet. At the beginning of the lesson, Murat helped the students to connect polygons as a mathematics concept to their daily life by asking questions, such as “Can you give me some examples of polygons from daily life?” and “What kind of polygon do you see in the surrounding?” Furthermore he searched for polygons on google pictures to see examples of polygons in daily life. But he did not use Geogebra as a representation for real life situations. Briefly, Murat did not use multiple representations (table, written symbols, real life situation) via Geogebra to enhance students’ high level thinking as indicated in the higher level of the availability descriptor. He used Geogebra for visualization of mathematical concepts to help students investigate patterns. It suggested that Murat was at *the accepting level* of TPACK for the access theme, availability descriptor.

4.1.4.1.2 Esen’s TPACK Level for the Access Theme before Mathematics Coaching

Esen’s performance before mathematics coaching demonstrated *the recognizing level* TPACK for the access themes, usage descriptor, barrier descriptor, and availability descriptor.

In Esen’s class there was only one computer for the teacher’s usage. She did not prefer to assign a computer for the students. Additionally, she did not let the students use Geogebra. Esen used concrete materials for students’ discovery of the new concepts such as concave and convex polygons. Then, she implemented Geogebra after teaching these concepts via geometry board and toothpicks as concrete materials. This usage was different from Murat’s usage in the lesson before mathematics coaching. Murat permitted the students to use Geogebra to explore new

concepts. Therefore, Esen was at a lower level than Murat in the usage descriptor in the access theme. In the pre-interview, she stated, “The use of technology will draw attention; it will be a review for the students.” This speech indicated that she used Geogebra after the concepts were taught and learned and she used it to attract the students’ attention. Thus, she “permitted students to use technology ‘only’ after mastering certain concepts”(Niess et al., 2009, p.23), as indicated at *the recognizing level* of TPACK for the access theme, usage descriptor.

In the lesson before mathematics coaching, Esen implemented Geogebra as a supplement to regular classroom activities. This usage demonstrated that she had concerns about the usage of Geogebra. In the pre-interview, Esen declared, “It seems like I will experience difficulty as it is not individualized. Since we are not able to have each student do it and since we do not have that much time, again those who are active in class will learn it and those who are not won’t. To ensure that balance, we need to provide the students who aren’t active with support by giving them more chance come to the board.” This statement indicates that the teacher has concerns about the fact that some students will fall behind. Furthermore, she stated that, “Difficulty in class management, well that is my greatest fear; noise in the classroom. It is disturbing to have noise come from the back of the classroom while I’m dealing with the computer. I don’t what to do when that happens. Perhaps a warning can be made.” This statement demonstrated that she had a concern about management issues while integrating Geogebra. To control students’ behavior, she wants to access and use Geogebra within limited time period after concave and convex polygons are learnt. Thus, in the lesson before mathematics coaching, “Student access to technology is limited to ‘after’ they have learned the given concepts using paper and pencil procedures and only for rote activities” (Niess et al., 2009, p.23), as indicated in *the recognizing level* of TPACK for the access theme, barriers descriptor.

Like Murat, she did not integrate Geogebra to incorporate multiple representations of the concepts and their connections. She gave examples of polygons from daily life, used concrete materials while integrating multiple

representations but the whole representation was not in the technological environment. In the pre-interview, she stated, “I can’t say much in terms of using technology for multiple representations. Well, we can just take and post their photos” This speech indicated that Geogebra was a limited tool for multiple representation. Furthermore, she did not implement Geogebra to investigate patterns and functions. Therefore, she was at a lower level than Murat because in Murat’s lesson the students integrated Geogebra to explore patterns regarding the sum of diagonals number of polygons as indicated in *the accepting level*. Thus, she was at *the recognizing level* of TPACK for the access theme, availability descriptor.

4.1.4.2 Mathematics Teachers’ TPACK Level for the Access Theme in MCC1

In MCC1 during the mathematics coaching process, the lessons of Murat and Esen were observed for two hours to determine their TPACK level of the access theme regarding polygons. All quotes and dialogs in the following paragraphs came from teachers’ TPACK one-on-one pre-conference 1, post-conference and lessons during mathematics coaching cycle 1.

4.1.4.2.1 Murat’s TPACK Level for the Access Theme in MCC1

Murat’s performance in MCC1 demonstrated *the adapting level* of TPACK for the access themes, usage descriptor, while *the accepting level* of TPACK for the teaching theme, barrier and availability descriptor.

In Murat’s observed lesson, students were assigned to 7 groups according to the students’ desire. They had a computer to use Geogebra. As in the previous lesson before mathematics coaching, the students used Geogebra in approximately the whole lesson except during the warm-up activities and assessment sessions. He permitted his students to use Geogebra in “The Angles of Polygons” and “the Finding the sum of interior angle of polygons” activities. Students had access to Geogebra for exploring the relationship of interior and exterior angles and the sum of interior angles of polygons, which were new concepts for the students. However,

during the technology-enhanced activity, the students' exploration was usually through teacher demonstration. In addition, he asked the students not to use Geogebra unless told to do so. He wanted them to follow his actions in Geogebra. These provided evidence that Murat was at *the adapting level* of TPACK for the access theme, usage descriptor. In the MCC1, "Access to and use of technology is available for exploration of new topics, usually with the teacher's demonstration." (Niess et al., 2009, p.24).

As in the previous lesson before mathematics coaching, Murat wanted to have control over the students while using Geogebra. He distributed handouts that included Geogebra activities. He asked one of the students to read aloud each instruction on the handout. The student followed the teacher and did what the teacher did on the blackboard. This step-by-step process indicated that he had concerns about the management. During the lesson, he told the classroom, "I'm showing you on the blackboard. Why? Because I don't want you to make mistakes. So that we can proceed fast." This statement demonstrated that his concern about time issues related to access to and management of the technology presented a barrier to his Geogebra usage. In addition, in the post-conference he stated, "Let's not distribute the worksheet. Let's first look because there is a computer, paper and the teacher. I think the students got confused as to what to pay attention to." These statements revealed that he had difficulties about management during MCC1 as indicated in the accepting level of TPACK for the access theme, barriers descriptor. He was "worried about access and management issues with respect to incorporating technology in the classroom." (Niess et al., 2009, p.23).

Murat's MCC1 performance suggested *the accepting level* of TPACK for the access theme, availability descriptor. As in the previous lesson before mathematics coaching, Murat provided the students with an opportunity to explore patterns such as the sum of interior angles of polygons using Geogebra, as indicated in *the accepting level*. However, he was at lower level than at the adapting level because at the adapting level, students use dynamic geometry software to investigate and make connections between mathematical concepts. However, Murat in the MCC1 did not

integrate Geogebra with the consideration of dynamic nature. As in the previous lesson, Geogebra was used as a graphic tool, such as paint. For instance, the students drew the diagonal to find the number of triangles in polygons. Then they discovered the formula of the sum of the interior angles of polygons, but they did not use Geogebra its dynamic features, such as dragging. It indicated that Murat couldn't reach *the adapting level*, at which concepts are taught differently since technology provides access to connections formerly out of reach. Furthermore, he did not integrate Geogebra to incorporate multiple representations, such as the spreadsheet, the written symbols. He just provided a visual representation for polygons to find a pattern of the number of sides and the sum of the interior angles. These suggested that Murat was at *the accepting level* of TPACK for the access theme, availability descriptor.

4.1.4.2.2 Esen's TPACK Level for the Access Theme in the MCC1

Esen's performance in MCC1 indicated *the adapting level* of TPACK for the access theme, usage descriptor, while *the accepting level* of TPACK for the access theme, barrier and availability descriptor.

In MCC1 Esen gave each student group a computer for them to use Geogebra. Thus, every small student group had a computer and access to and use of technology, unlike in the previous lesson before mathematics coaching. In addition, she implemented Geogebra to explore new concepts, such as the sum of interior angles, and one of the exterior angles of regular polygons. Esen asked them to open the already prepared Geogebra activities on these concepts. Although every student group had these Geogebra activities on their computer, Esen did not let them use it without her instruction. Like Murat, she projected Geogebra activities on the board, and motivated them to explore these new concepts. Then the students worked on Geogebra activities on their computer. Thus, Esen was at *the adapting level* of TPACK for the access theme, usage descriptor. In MCC1, "access to and use of technology is available for exploration of new topics, usually with the teacher's demonstration" (Niess et al., 2009, p.24).

In the pre-conference, she stated that “Classroom management should not disappear. When there is noisy environment, the students cannot learn.” This statement demonstrated that she had a concern about management. Parallel to this speech, in the lesson of MCC1 she wanted to take control and the students did not feel free to use Geogebra. They should follow the teacher what she did. This usage was similar to Murat’s usage in MCC1. She was at *the accepting level* of TPACK for the access theme, barriers descriptor, like Murat in MCC1. She “worried about access and management issues with respect to incorporating technology in the classroom” (Niess et al., 2009, p.23).

Esen motivated the students to explore patterns, such as the sum of interior angles of polygons and one of the exterior angles of regular polygons by using Geogebra as indicated in *the accepting level*. She implemented Geogebra to explore the new concepts instead of reinforcing the already-known concepts in the previous lesson. However, the students investigated and made conjecture about the interior and exterior angles; they did not use the dynamic properties of Geogebra effectively. In addition, Esen did not use the multiple representations via Geogebra (except the pictures), indicating the lower level of the availability level. She just used Geogebra as a visual representation for polygons to find patterns regarding the interior and exterior angles of polygons. Therefore, her TPACK level for availability was lower than *the adapting level*. To sum up, these suggested that Esen was at *the accepting level* of TPACK for the access theme, availability descriptor.

4.1.4.3 Mathematics Teachers’ TPACK Level for the Access Theme in MCC2

In MCC2 during the mathematics coaching process, the lessons of Murat and Esen were observed for four hours to determine their TPACK level in the access theme as regards polygons. All quotes and dialogs in the following paragraphs came from teachers’ TPACK one-on-one pre-conference 2, post-conference 2 and lessons during mathematics coaching cycle 2.

4.1.4.3.1 Murat's TPACK Level for the Access Theme in the MCC2

Murat's MCC2 performance demonstrated *the exploring level* of TPACK for the access themes, usage descriptor, while he was at *the adapting level* of TPACK for the teaching theme, barrier and availability descriptor.

Murat let the students use Geogebra to investigate the properties of quadrilaterals. In the post-conference, when he was asked when he had permitted the students to use Geogebra, he replied, "They used it during applications; they used it actively especially during the discovery phase...This time they used it in evaluation." This speech demonstrated that he permitted the students to use technology to explore the properties of quadrilaterals, as indicated in *the exploring level*. In addition, he started to allow time for the students' exploration to use Geogebra, unlike in MCC1. Therefore, he was at a higher level than the adapting level, which demonstrated that access to technology was available for exploration of new topics, usually with the teacher's demonstration. In MCC2, Murat motivated the students to explore these concepts during the entire class time. Thus, in MCC2 "Access to and use of technology is available and encouraged for mathematics exploration during most class times," as indicated in *the exploring level* of TPACK for the access theme, usage descriptor" (Niess et al., 2009, p.24).

In the post- conference, Murat stated, "The preparation stage is a little tiring.. I want to do some things but I get less tired in class. I feel that they learn from what they say, from the sentences they construct." This statement demonstrated that the new scope and sequence with technology integration required more time to be spent on planning lessons. In addition, he said that,

"At first I thought like this. Will the students be able to understand this? There was the fear that the student might become adapted to the computer and, thus, not pay attention to the lesson. Apart from this, your guidance developed my vision. Whether I would do it right or wrong, our interaction with each other enabled me to think that I would find the correct way with the help of your feedback."

This speech indicated that at the beginning of the study he had had worries about access and management issues. But his view about technology integration

changed throughout the study. So he was moving through the TPACK Development Model for the barrier descriptor. In MCC2, he started to spare time for the students' exploration but he still wanted to take control of the process. Furthermore, he declared, "I made a difference in this lesson. Thanks to the new activities we created or designed, we started to use the time more effectively. We hadn't given place to students' use during the applications, but now we do." This statement indicated that integrating Geogebra-enhanced activities as a new approach to mathematics helped to overcome possible difficulties in time and management. Thus, it can be understood that Murat "Uses technology as a tool to enhance mathematics lessons in order to provide students a new way to approach mathematics," as indicated in *the adapting level* of TPACK for the access theme, barriers descriptor.

In MCC2, Murat provided a technologically supported environment for the students to make conjectures about quadrilaterals based on their exploration with the consideration of the dynamic nature of Geogebra. The students used Geogebra as a dynamic tool to investigate the properties of quadrilaterals and the relationship between quadrilaterals as indicated in *the adapting level* of availability descriptor. For instance, as compared to the square and rectangle, the students dragged the vertex of the rectangle and transformed it into a square. The dragging of the vertex helped the students to understand that the square was the subset of the rectangle. This example provided evidence that Geogebra helped to teach differently with the consideration of the dynamic nature of Geogebra. Thus, "Concepts are taught differently since technology provides access to connections formerly out of reach," as indicated in *the adapting level* of TPACK for the access theme, availability descriptor.

4.1.4.3.2 Esen's TPACK Level for the Access Theme in MCC2

Esen's MCC2 performance demonstrated *the exploring level* of TPACK for the access themes, usage descriptor, barrier descriptor, while demonstrating *the adapting level* of TPACK for the access theme, availability descriptor.

Like Murat in MCC2, Esen provided an environment for students' exploration of the properties of quadrilaterals. The student group worked with Geogebra-enhanced activities under Esen's guidance. Furthermore, Esen integrated Geogebra into approximately the entire time, including assessment. Thus, in MCC2 "Access to and use of technology is available and encouraged for mathematics exploration during most class times," as indicated in *the exploring level* of TPACK for the access theme, usage descriptor (Niess et al., 2009, p.24).

In the post- conference, she stated,

"The students have become used to it; they use Geogebra well. I mean for the time being, they like it a lot. In fact, one student said that our lessons had started to be enjoyable. Actually I had classroom management problems with this class. So it has been useful in that sense. I did not receive any negative reaction from the students."

This statement indicated that she started to solve management issues via Geogebra because the students were interested in Geogebra as a new approach. Furthermore, while the students worked with Geogebra-enhanced activities on quadrilaterals, she walked around the classroom to keep in communication with the students. While walking among the student groups, she responded to the students' questions about the activities and guided them to explore the properties of Geogebra, unlike Murat in MCC2. Murat wanted to take the students under his control during the MCC2 lesson. For instance, if the student group did not effectively use Geogebra as a dynamic tool, she motivated them to drag the vertex of polygons to the properties of quadrilaterals. This demonstrated that she sought for ways to obtain technology for classroom use and began creating methods for technology management issues. Furthermore, in MCC2, Geogebra was used extensively in assessments with the consideration of dynamic properties, as indicated in *the exploring level* for the barrier descriptor. To sum up, she "recognized challenges for teaching mathematics with technologies, but explores strategies and ideas to minimize the impact of those challenges" (Niess et al., 2009, p.24).

Like Murat in MCC2, Esen supported the students to use Geogebra as a dynamic tool in order to investigate and make connections between the properties of quadrilaterals. For instance, she motivated the students to drag the vertex of quadrilaterals and explore their properties, such that diagonals bisect each other for all parallelograms. Thus, she taught the properties of quadrilaterals as geometric concepts differently, and they discovered these concepts instead of memorizing them throughout the lecture. Besides, she did not use Geogebra for incorporating multiple representations, as indicated in the exploring level of availability descriptor. She stated, “From multiple representations, there were just shapes. A spreadsheet could actually have been used; it would have been nice. We can do it in the other lesson as well.” This speech demonstrated that she did not incorporate multiple representation expect for the picture in the Geogebra-enhanced activities. So she could not reach the exploring level. To sum up, “Concepts are taught differently since technology provides access to connections formerly out of reach” in MCC2, as indicated in *the adapting level* of TPACK for the access theme, availability descriptor.

4.1.4.4 Mathematics Teachers’ TPACK Level for the Access Theme in the MCC3

In MCC3 during the mathematics coaching process, the lessons of Murat and Esen were observed for four hours to determine their TPACK level for access theme. All quotes and dialogs in the following paragraphs came from teachers’ TPACK one-on-one pre-conference 3, post-conference 3 and lessons during mathematics coaching cycle 3.

4.1.4.4.1 Murat’s TPACK Level for the Access Theme in the MCC3

Murat’s MCC3 performance demonstrated *the advancing level* of TPACK for the access themes, usage descriptor, while it displayed *the exploring level* of TPACK for the access theme, barrier descriptor, and availability descriptor.

In MCC3, the students accessed and used Geogebra and were motivated in geometrical exploration in relation to congruence and similarity, the area of irregular polygons and the area of parallelograms during most class times. Murat implemented Geogebra in every aspect of the mathematics class. For instance, the students accessed Geogebra in assessment by using the dynamic nature of technology. In addition, the students were responsible for their learning of the geometrical concepts with the Murat's guidance. In MCC3, Geogebra was seen as an opportunity to challenge notions of what mathematics students can master, as indicated in the advancing level because the teacher implemented Geogebra dynamically beyond the paper and pencil. Thus, he "permitted students to use technology in every aspect of mathematics class," as indicated in *the advancing level* of TPACK for the access theme, usage descriptor (Niess et al., 2009, p.24).

In the post-conference, he stated, "I was trying to be dominant in the classroom to maintain class management but it was good to let students work on the computer, to spare time for this. The students were more active while carrying out the activities; I realized that became more motivated. They weren't able to use the computer effectively because I was trying to dominate the class and I was trying to proceed fast. So we weren't getting the effectiveness we desired." This statement demonstrated that he started to solve management issues via Geogebra by guiding the students. In addition, he realized that if the students acted freely during the Geogebra-enhanced activity, they were more interested in the lesson. In this way, he tried to solve the challenges in order to teach geometry by means of Geogebra by changing the role of the students and the teacher in MCC3. Namely, he sought for ways to obtain Geogebra for classroom use and began creating methods for management issues. Thus, he "recognized challenges for teaching mathematics with technologies, but explores strategies and ideas to minimize the impact of those challenges," as indicated in *the exploring level* for barrier descriptor (Niess et al., 2009, p.24).

Murat integrated Geogebra by using multiple representations: real life situations, pictures, written symbols, and tables in spreadsheet, unlike in the previous lesson in MCC2. For instance, he asked the students to find the area of a basketball court, which was a real life context. He added the written symbols onto the geometrical concepts in Geogebra, such as the area formula of the parallelogram, which appeared by clicking on the “show the formula” button. Furthermore, he motivated the students to use spreadsheet in the Geogebra-enhanced activity on the area of parallelograms to discover the pattern in the area, height and base. In the post-conference, he stated, “We used multiple representation effectively in class; we used it better than we did in the previous lessons. First there were drawings as visuals. During this lesson, apart from the drawings, we presented it on an excel table. We created its numerical expression.” This speech demonstrated that he implemented Geogebra effectively by using multiple representations, such as tables, symbols, and pictures. Thus, in MCC3 “Through the use of technology, key topics are explored, applied, and assessed incorporating multiple representations of the concepts and their connections,” as indicated in *the exploring level* for the availability descriptor (Niess et al., 2009, p.24).

4.1.4.4.2 Esen’s TPACK Level for the Access Theme in MCC3

Esen’s MCC3 performance demonstrated *the advancing level* of TPACK for the usage descriptor, *the exploring level* of TPACK for the barrier descriptor, and *the adapting level* for the availability descriptor in the access themes.

Like Murat in MCC3, the students were encouraged to explore geometrical concepts regarding congruence and similarity, the area of irregular polygons and the area of parallelograms during the entire class time. The students integrated Geogebra in every aspect of the mathematics class dynamically or by assessing the students’ knowledge, or by introducing and investigating the new concepts. Furthermore, the students were free to access the program during the exploration of the geometrical concepts under the teacher’s guidance. Esen provided an environment for the students to master the geometrical concepts with the dynamic

nature of Geogebra and used technology as an opportunity to challenge the notions of what mathematics students can learn. Therefore, she “permitted students to use technology in every aspect of mathematics class,” as indicated in *the advancing level* of TPACK for the access theme, usage descriptor (Niess et al., 2009, p.24).

In MCC3, Esen guided the students while they carried out Geogebra-enhanced activities. She wandered around the classroom to respond to the students’ questions about the activities, like in MCC2. The teacher’s walking around the class helped to eliminate possible miscommunications deriving from use of technology. To attract the students’ attention, Geogebra was used dynamically, beyond the paper and pencil. She stated that when students’ attention is drawn, classroom management becomes easier. This statement demonstrated that she created methods for technology management issues. Thus, she “recognized challenges for teaching mathematics with technologies, but explores strategies and ideas to minimize the impact of those challenges” (Niess et al., 2009, p.24).

Like in the previous lesson in MCC2, Esen integrated Geogebra to help the students to learn differently since Geogebra provided access to connections formerly out of reach. The students used made effective use of Geogebra as a dynamic tool in order to investigate concepts, such as congruence and similarity. She used Geogebra to incorporate some representations, such as pictures, daily-life situations and written symbols during MCC3. But she did not effectively integrate multiple representations, such as the use of spreadsheet to explore patterns, as indicated in *the exploring level* of the availability descriptor. Hence, she was at a lower level than the exploring level. To sum up, “Concepts are taught differently since technology provides access to connections formerly out of reach” in MCC2, as indicated in *the adapting level* of TPACK for the access theme, availability descriptor.

4.1.4.5 Mathematics Teachers’ TPACK Level for the Access Theme in MCC4

In the MCC4 during the mathematics coaching process, the lessons of Murat and Esen were observed for two hours to determine their TPACK level for

mathematics descriptor of learning theme on polygon. All quotes and dialogs in the following paragraphs came from teachers' TPACK one-on-one pre-conference 4, post-conference 4 and lessons during mathematics coaching cycle 4.

4.1.4.5.1 Murat's TPACK Level for the Access Theme in MCC4

Murat's MCC4 performance demonstrated *the advancing level* of TPACK for the access themes, usage descriptor, and barrier descriptor, while demonstrating *the exploring level* of TPACK for the access theme, availability descriptor.

As in the previous lesson in MCC3, the students accessed and made use of Geogebra to explore geometrical concepts, such as the area of parallelograms and trapezoids during most of the class time. The students used Geogebra in every aspect of the mathematics class, such as the dynamic tool, or the assessment tool under Murat's guidance. These provided evidence for the fact that he was at *the advancing level*. Furthermore, the students used Geogebra dynamically beyond the paper and pencil to become specialized. For instance, the students transformed the trapezoid into a parallelogram to investigate the area of the trapezoid. Thus, for Murat, "Technology is seen as an opportunity to challenge notions of what mathematics students can master," as indicated in *the advancing level* of TPACK for the access theme, usage descriptor (Niess et al., 2009, p.24). In addition, Geogebra was used to expand mathematics concepts, such as the area of the rhombus and the trapezoid, which can be accessed by students, as indicated in *the advancing level* of TPACK for the barrier theme. The use of Geogebra in an innovative way (like the area of the rhombus and trapezoid activity) was seen as an opportunity for the students to master geometrical concepts. This helped in management issues. As in the previous lesson, the students acted freely while doing the Geogebra-enhanced activity, and they were more interested in the lesson. By guiding the students, Murat tried to solve the challenges existent in teaching geometry via Geogebra. Thus, he "Recognizes challenges in teaching with technology and resolves the challenges through extended planning and preparation

for maximizing the use of available resources and tools,” as indicated in *the advancing level* for the barrier descriptor (Niess et al., 2009, p.24).

During MCC4, Murat incorporated multiple representations, such as real life situations, pictures, written symbols, and tables in spreadsheet, like in the previous lesson in MCC3. In the post-conference, he stated, “For example there was the car activity with which I made use of multiple representations. We painted the doors. There I made each door look like a trapezoid. We made them become aware of the trapezoid while looking at the shapes. We had them find the area using the trapezoid. After having them find the area, there was one more door. We had them multiply the area by two. So it was an example of the real life context. How much would he pay if he had such a square meter painted for such a price? That activity was an exact adaptation of real life.” As indicated in this statement, he posed a problem consisting of a daily life context in the assessment activity. Furthermore, in another activity, the students used spreadsheet to explore the area of a trapezoid. The spreadsheet helped the students to find a pattern between the area of the rhombus and the diagonals of the rhombus. In addition, the area formula of the trapezoid and the rhombus appeared as a written symbol by clicking on the check boxes. Thus, in MCC3 “Through the use of technology, key topics are explored, applied, and assessed incorporating multiple representations of the concepts and their connections,” as indicated in *the exploring level* for the availability descriptor (Niess et al., 2009, p.24).

4.1.4.5.2 Esen’s TPACK Level for the Access Theme in MCC4

Esen’s MCC4 performance demonstrated *the advancing level* of TPACK for the access themes, usage descriptor, and barrier descriptor, while demonstrating *the exploring level* of TPACK for the access theme, availability descriptor.

The students accessed and made use of Geogebra to explore geometrical concepts, such as the area of the parallelogram and the trapezoid with the consideration of the dynamic nature of Geogebra during most of the class time. The teacher permitted the students to use Geogebra in every aspect of mathematics class,

as a dynamic tool, or an assessment tool, as indicated in the advancing level for the usage descriptor. The students used Geogebra dynamically in an innovative way beyond the paper and pencil to master the area of the trapezoid and the rhombus. For instance, the students transformed a rhombus into a rectangle in two different ways to investigate the area of a rhombus from a different perspective. Thus, for Murat “Technology is seen as an opportunity to challenge notions of what mathematics students can master,” as indicated at *the advancing level* of TPACK for the access theme, usage descriptor (Niess et al., 2009, p.24). In addition, Geogebra was used to expand mathematics concepts, such as the area of the rhombus and the trapezoid that can be accessed by students as indicated in *the advancing level* of TPACK for the barrier theme.

In the post-conference, the teacher stated, “In terms of class management, everything is positive except for the noise. And I am happy to be using the program.” This speech demonstrated that she recognized challenges in teaching via Geogebra. By walking around in class and guiding the students, she tried to solve the challenges in teaching geometry via Geogebra. In addition, she emphasized preparing and planning the MCC4 lesson to maximize the use of available resources and tools, such as using different activities related to the rhombus from different perspectives. Furthermore, Geogebra was used to expand the geometrical concepts about the area of the rhombus and the trapezoid that can be accessed by students, as indicated in *the advancing level* for the barrier descriptor, like Murat in MCC4. To sum up, she “recognizes challenges in teaching with technology and resolves the challenges through extended planning and preparation for maximizing the use of available resources and tools,” as indicated in *the advancing level* for the barrier descriptor (Niess et al., 2009, p.24).

In the post-conference, she stated, “Of course multiple representation had a positive impact on students’ learning process. For instance, in spreadsheet, students saw the changes in the values in activities related to calculating areas and thus, how the area changed.” This speech demonstrated that she was pleased to incorporate multiple representations of real life situations, pictures, written symbols, and

especially tables in spreadsheet during MCC4. For instance, the spreadsheet in the trapezoid activity helped the students to find the pattern between the area, bases and height. Furthermore, she added written symbols, which appeared by clicking on check boxes in every Geogebra activity. In MCC4 “Through the use of technology, key topics are explored, applied, and assessed incorporating multiple representations of the concepts and their connections,” as indicated in *the exploring level* for the availability descriptor (Niess et al., 2009, p.24).

4.1.4.6 Mathematics Teachers’ TPACK Level for the Access Theme After Mathematics Coaching

After mathematics coaching, the lessons of Murat and Esen were observed for two hours to determine their TPACK level in the access theme. All quotes and dialogs in the following paragraphs came from teachers’ TPACK one-on-one post-observation and post-interview after mathematics coaching.

4.1.4.6.1 Murat’s TPACK Level for the Access Theme After Mathematics Coaching

Murat’s MCC4 performance demonstrated *the advancing level* of TPACK for the access themes, usage descriptor, and barrier descriptor, while demonstrating *the exploring level* of TPACK for the access theme, availability descriptor.

Like MMC4, the students accessed and made use of Geogebra to explore geometrical concepts, such as the relationship between the area and perimeter during the whole lesson after mathematics coaching. In the post-interview, he stated,

“It was very good to use this, this dragging. Enabling them to see that the structures we built weren’t static and that some things, some values did not change when they were moved was very good. I mean, for example that a parallelogram remains as it is drawn, even when we turn it to another direction it remains the same, or I don’t know, that the diagonals of a square intersect. I mean, it was highly beneficial to use this dynamic structure actively. We did multiple representations. These were also useful. Then you know, you helped us by adding different buttons here as thanks to those buttons we saved time at some points. At the point of making generalizations, we quickly reached them with a

click. While explaining it in class I was going to draw a table and wait for them to fill in each value. So it was useful.”

This indicated that the students used Geogebra in every aspect of mathematics class as a dynamic tool, or as multiple representations under Murat’s guidance. In addition, the students accessed Geogebra beyond the paper and pencil to help the students to gain an in-depth understanding of these concepts. For instance, in the fixed area Geogebra-enhanced activity, the students found the largest possible perimeter of a shape with an area of 12 unit squares via the slider. This innovative assessments activity helped the students to reveal their understanding of the area and perimeter with Geogebra. Thus, for Murat “Technology is seen as an opportunity to challenge notions of what mathematics students can master,” as indicated in *the advancing level* of TPACK for the access theme, usage descriptor (Niess et al., 2009, p.24).

In the post-interview, He said that

“Let me tell you the difficulty I faced from my perspective; I experienced great difficulty in classroom management. Or I experienced difficulty in guiding all the students. I couldn’t have a command of all the groups. But recently I have become more competent. I maintained silence by wandering around the classroom and sparing time. I enabled them to think about the topic. Such a convenience was brought about. There are those hesitations when something new is initiated; there are things that lack of experience brings. After some experience and observation, you become aware; you come to realize certain things. The issue becomes more positive.”

This statement revealed that he had experienced difficulty as regards management issues. However, he created a method to overcome this challenge such as changing the teacher’s role. In addition, he planned and prepared Geogebra-activities by taking into consideration the students’ motivation and to provide an environment for their learning. Thus, he “Recognizes challenges in teaching with technology and resolves the challenges through extended planning and preparation for maximizing the use of available resources and tools,” as indicated in *the advancing level* for the barrier descriptor (Niess et al., 2009, p.24). Furthermore, the students used multiple representations to explore the relationship between area

and perimeter. Thus, in MCC3 “Through the use of technology, key topics are explored, applied, and assessed incorporating multiple representations of the concepts and their connections,” as indicated in *the exploring level* for the availability descriptor (Niess et al., 2009, p.24).

4.1.4.6.2 Esen’s TPACK Level for the Access Theme after Mathematics Coaching

Esen’s performance after mathematics coaching demonstrated *the advancing level* of TPACK for the access themes, usage descriptor, and barrier descriptor, while demonstrating *the exploring level* of TPACK for the access theme, availability descriptor.

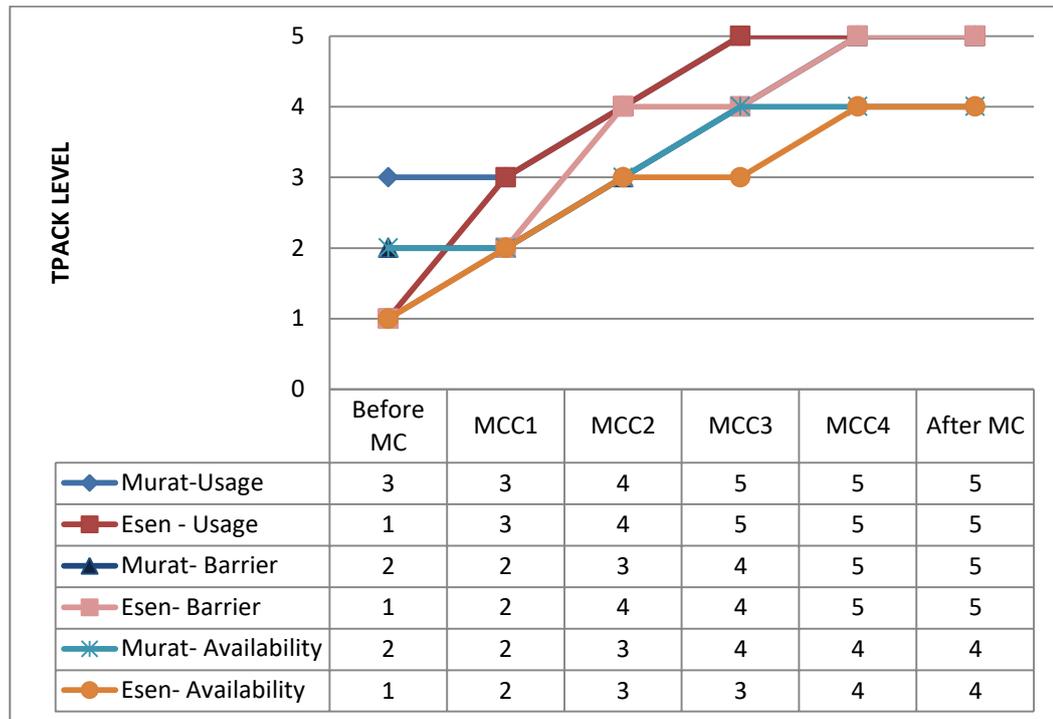
Like Murat, the students accessed and made use of Geogebra to investigate geometrical concepts, such as the relationship between area and perimeter in the whole lesson. The Geogebra activities in this lesson were seen as innovative ways to explore the concepts and assess the students’ knowledge. For instance, in the fixed perimeter activity, the students dragged the red point to increase the height of a rectangle with a fixed perimeter. Thus, the students constructed a new rectangle. So the old rectangle turned into the color red. Then the “see the difference” button appeared. The students observed the differences between areas of the old and new rectangles by clicking this button. While repeating this motion, in this Geogebra activity the students searched the answer of “When does the area have the highest value?” Furthermore, they recorded the values of the rectangle, such as their sides, perimeter and area in the spreadsheet. The dynamic nature of Geogebra was effectively made use of in this Geogebra activity to explore the relationship between area and perimeter. Furthermore, this activity revealed that Esen developed innovative ways for the integration of Geogebra as a learning and teaching tool to support students’ exploration as indicated in *the advancing level* for the usage descriptor. These activities demonstrated that she emphasized preparing and planning lesson to expand the use of available resources and tools, such as using different activities related to the relationship between area and perimeter by

different perspectives as indicated in *the advancing level* for the barrier descriptor. Thus, she “recognized challenges in teaching with technology and resolves the challenges through extended planning and preparation for maximizing the use of available resources and tools,” (Niess et al., 2009, p.24).

In the post-interview, “If I evaluate the lesson in terms of multiple representations, I think that we have been able to use them all. To begin with, we used algebraic expression. In our last lesson, we opened up Excel (spreadsheet) documents. We gave samples from daily life. We converted Turkish into mathematics. Having Excel documents was useful too as we wrote the values there. In fact, one of our assessment activities was related to this.” The students used multiple representations to explore the relationship between the area and the perimeter. Thus, in MCC3 “Through the use of technology, key topics are explored, applied, and assessed incorporating multiple representations of the concepts and their connections,” as indicated in the exploring level for availability descriptor (Niess et al., 2009, p.24).

4.1.4.7 The progress of Murat and Esen’s TPACK of Access Theme through Mathematics Coaching

Table 4. 8 The Progress Of Teachers For Access Theme



The performances of Murat and Esen for the access theme in the six Geogebra-enhanced lessons before/during/after mathematics coaching are presented in Table 4. 8. The teachers made growth into higher level in the TPACK development model for usage, barrier and availability descriptor in the access theme. Esen’s performance for usage, barrier and availability descriptor in the access themedescrptor in the teaching theme was lower than Murat at the beginning of the study. But both of them illustrated the growth in the TPACK development model for access theme.

The usage descriptor in access theme focuses on how the teacher use technology and when the teacher lets the students to use technology. At the beginning, Esenpermittedthe students to use Geogebra after mastering geometry concepts such as concave and convex polygons via concrete materails. Thus, she was the lowest level of the usage descriptor. On the other hand, in the Murat’s

lesson before mathematics coaching, the students accessed to and used of Geogebra for exploration of concave and convex polygons with usually with the teacher's demonstration. Thus, he was at *the adapting level* at the beginning of the study. In the MCC1, Esen reached the Murat's level of usage descriptor. Both of them permitted the students to use Geogebra via teacher demonstration. In the middle of study, they provided an environment to encourage for geometrical exploration by accessing to and using of Geogebra In the MCC3, they reached the highest level of usage descriptor. They provided an environment for the students to master the geometrical concepts with the dynamic nature of Geogebra and used technology as an opportunity to challenge the notions of what mathematics students can learn. They permitted the students to use technology in every aspect of mathematics class such as using dynamic nature or improving students' exploration. Through the end of the study, they integrated Geogebra in every aspect and let the students to use Geogebra.

The barriers descriptor emphasizes how the teacher overcomes the possible problems related to technology integration. At the beginning of the study, Esen integrated Geogebra as a supplement to regular classroom activities because she had concerns about the usage of Geogebra. In her lesson, the students accessed to Geogebra in a limited way after they had learned the geometrical concepts using paper and pencil procedures and concrete materials and only for rote activities. Besides, Murat implemented Geogebra to enhance the student exploration. On the contrary, the concepts were not taught differently although Geogebra provided access to connections formerly out of reach. Thus he had worries about access and management issues with respect to incorporating Geogebra in the classroom as at the accepting level. In the MCC1, Esen reached the Murat's level and used of Geogebra for the students' exploration with worries about the management issues. In the MCC2, Murat implemented Geogebra-enhanced activities as a new approach to mathematics helped to overcome possible difficulties in time and management as indicating in *the adapting level*. On the other hand, Esen sought for ways to obtain

technology for classroom use and began creating methods for technology management issues. Unlike Murat, she responded to the students' questions about the activities and guided them to explore the properties of Geogebra as walking among the student groups. In the MCC2, Murat wanted to take the students under his control during the MCC2 lesson. Thus, Esen was at higher level for the barrier descriptor in the MCC2. Through the mathematics coaching, both of them made a progress in the TPACK Development Model for barrier descriptor. In the MCC4, Both of them reached the highest level. They viewed Geogebra as an opportunity for the students to master geometrical concepts in an innovative way. This situation helped management issues. Thus, they "recognizes challenges in teaching with technology and resolves the challenges through extended planning and preparation for maximizing the use of available resources and tools," as indicated in *the advancing level* for the barrier descriptor (Niess et al., 2009, p.24).

Like usage and barrier descriptor, Esen was in the lowest level of TPACK of availability descriptor before mathematics coaching. She did not integrate Geogebra to incorporate multiple representations of the concepts and their connections. Furthermore, she did not use Geogebra to investigate patterns and functions. Therefore, she was at a lower level than Murat because in Murat's lesson the students explored the patterns regarding the sum of diagonals number of polygons via Geogebra as indicated in *the accepting level*. In the MCC1 she reached the Murat's level for availability level. But both of them used Geogebra as a visual representation for polygons to find patterns regarding the interior angles of polygons. In the MCC2, the students use Geogebra to investigate and make connections. Furthermore, the geometrical concepts were taught differently since technology provided access to connections formerly out of reach as indicating in the *adapting level* of TPACK for availability descriptor. In the MCC3, Murat implemented Geogebra effectively by using multiple representations, such as tables, symbols, and pictures. In the next lesson- the MCC4-, Esen reached the Murat's TPACK level in availability descriptor. Thus, "Through the use of

technology, key topics are explored, applied, and assessed incorporating multiple representations of the concepts and their connections,” as indicated in *the exploring level* for the availability descriptor (Niess et al., 2009, p.24).

4.2 Mathematics teachers’ perceptions of mathematics coaching

This section addressing the second research question of the study presents the teachers’ perceptions of mathematics coaching in terms of TPACK. All quotes in the following paragraphs come from “mathematics coaching interview” after mathematics coaching.

The purpose of this section was to investigate the in-service teacher perspective about the mathematic coaching which focuses to improve their TPACK in polygon unit. The researcher identified four major themes: (1) benefits of the mathematics coaching, (2) differences between mathematics coaching and regular in-service training, (3) challenges of mathematics coaching, and (4) suggestions. A summary of the four themes and related codes are presented in Table 4. 9.

Table 4. 9 Themes and Codes in the mathematics coaching interview

Themes	Codes
Benefits	Improving their TPACK Improving instructional practice Increasing student achievement Providing scaffolding/collaborating Building confidence
Differences	Focusing process Putting together practice and theory
Challenges	Preparing lesson Being stressful
Suggestions	Should be common in turkey Should be semester-long Should be conducted in different subject Should be used with different mathematics software Should be in group

4.2.1 Benefits of mathematics coaching

Murat and Esen's perspective about their experience engaging in mathematics coaching was positive. They expressed that mathematics coaching help to improve them. In addition they said that mathematics coaching came up to their expectations. For instance, Murat stated that

“The mathematics coaching definitely met my expectations. Let me tell you frankly that I wasn't expecting this much. I was expecting that you were going to prepare and then follow up, that's what I was expecting. Or that you were going to tell me my mistakes. But, especially our doing the pre-conferences broadened my vision. In this respect, it made me gain different things.”

Additionally, Esen stated that “The teachers, of course, need to learn new things every day. This was something very new to me. And it is very useful too.” Both of them depicted that they provided benefits from mathematics coaching.

They expressed that one of most important strengths of the mathematics was *improving their TPACK*. They realized that they changed their view about technology integration during the mathematics coaching. Esen revealed her ideas about the effect of mathematics coaching process in terms of technology integration as; “this process changed my view to technology in a positive sense. I had never considered it negatively. I came to consider it more positively” and she added that “It made my view towards technology integration more positive. I saw that we could do many things. I had some concerns about assessment but it also changed my concerns in a positive sense.”

Furthermore, both of them reflected that their TPACK were positively impacted by their participation in the mathematics coaching. They agreed that their instructional planning and instructional approach were positively impacted by their participation in the mathematics coaching. Thus, they stated that one of the benefits of mathematics coaching is *improving instructional practices*. They expressed that mathematics coaching Murat stated that

“This process certainly had an impact on my use of technology, pedagogy and content knowledge. I need to be competent in content knowledge. I need to guide students well with the questions I ask them.

Pedagogically, let me word it this way; I become aware of the mistakes I do in the lesson. I asked the coach some questions such as “Is this sufficient?” or “ Shall we stop here?”I gave feedback, which has its pluses and minuses in the post-conference When think pedagogically, it enabled us to see the mistakes I did in terms of the method I used. I was speaking fast and trying to get feedback fast in the lesson. During the initial weeks, I did not let the students use the computer effectively. It [mathematics coaching]) enable me to fix these. The structure [of mathematics coaching] necessitates this. In direct teaching the teacher needs to be active but in constructivism it is exactly the opposite.”

This statement revealed that the parts of mathematics coaching (pre-conference, observation and post-conference) provided an environment to change his TPACK. He noticed his fault in the technology integrated lesson via mathematics coaching. For instance, at the beginning of the lesson he directed the students’ behavior and did not motivate the student to use Geogebra effectively. Mathematics coaching helped him to realize his usage and fix it. Furthermore, both of them reflected that they realized that they ought to integrate Geogebra to improve the students’ exploration during the mathematics coaching. They felt the mathematics coaching benefited the student directly and indirectly because they realized the importance of the supporting the students exploration with Geogebra. Esen stated that “The lessons in the mathematics coaching effectively equipped the students with the skill of discovery. It enabled them to discover. It was very useful in this sense. It made me realize this.”This statement demonstrated that the other advantage of mathematics coaching was *increasing the students’ achievement*.

They expressed that their perspective about classroom environment was also changed during the mathematics coaching. Murat stated that

“Generally teachers have the tendency to just teach their lessons and leave the class. Let me say that this training [mathematics coaching] enabled them to see the issue from a different perspective. They shouldn’t look at the issue from one perspective. I explained the lesson, okay, to what extent did the student learn or not learn. At this point I saw this. The study we did made the student a more active. It made them motivated. The outcome we received. The feedback they gave is more pleasant, more effective. Towards mathematics, no matter how much we

[the mathematics teachers] try to make them like mathematics, they always feel the coldness of mathematics. Well, they gained a different perspective.”

According to this statement, Murat applied teacher-led students-follow lesson, through the mathematics coaching he started to guide his students to explore the mathematical concepts. Thus, the mathematics coaching helped to change his view about classroom environment.

According to Murat and Esen, *providing counseling/support* in the mathematics coaching was helpful to improve their knowledge. Murat said that

“I always asked the things I didn’t know to you. I received my answers somehow and learned different things. I learned that I could use the program [Geogebra] in different ways. I learned what it was useful for. I learned how effective it was on the students.”

This statement expressed that the coach supported the teacher in many aspect such as; how to use Geogebra, how to integrate Geogebra as a learning tool. Furthermore, Esen stated that

“Now we mathematics teachers are consulting each other within our group [zümre]. Thank god we don’t have any social problems ... but we are always communicating in groups. Mathematics coaching resembles this.”

This statement revealed that the mathematics coaching resembled the mathematics teachers group in the school in terms of counseling according to Esen. In the mathematics teachers group they share their ideas about teaching mathematics and give good examples of mathematics activities.

In addition, they believed that the pre-conferences helped to realize how to integrate Geogebra with discussing the lesson plan. They reflected that the main focal point of the pre-conferences was designing the lesson with the scaffolding of the coach and it helped to improve their TPACK. Esen stated that

“It [pre-conference] was highly beneficial. It enabled me to learn many things too. You taught me some things. We prepared the plans together. That was very good. If I had been on my own, I wouldn’t have been able to do it this much. It was very very useful for me.”

And she added

“I don’t think the pre-conference had any negative aspects. The pre-conferences were something like establishing a mathematics community (zümre). We exchange ideas. Making a plan about what we can do, how we can do integrate Geogebra.”

This statement revealed that Esen benefited from pre-conference as designing the lesson in terms of technology integration. Like Esen, Murat also believed that the most important part of pre-conference was preparing lesson together via getting feedback. He stated that

“... you know first I liked the thing we did, the pre-conference.. I mean what we can do. What product we can put forward. What we can design. This was good because it makes me think about the unit, about the objectives. Seeking things at home, then receiving feedback as preparing the lesson plan enables me to see my weaknesses and not to make the same mistakes in the future. The things we did; they enabled me to see what we could do better. I mean, it was productive in terms of my job.”

This statement revealed that the scaffolding at the planning session helped to think how to improve the lesson with integrating Geogebra.

In the interviews the teachers reflected that they were also supported emotionally such as *building confidence* during the mathematics coaching. Murat stated that “At first I gain self-confidence during mathematics coaching. The guidance is what I need to do” and he added

“However, at the beginning I experienced a slight lack of self-confidence. I had some questions in my mind as to how to do it. But with your support and guidance, you’re seeing my weaknesses and telling me somehow developed my self-confidence. It instilled in me the feeling that I could do it. Then when I saw the products that were put forward, products here are the children’s gains...the knowledge they achieved...when see these I felt that we did something good. Well, the scores they got in the written exam that I did, they all reinforced it.”

Furthermore, Esen declared that

“In the beginning I was afraid that I wouldn’t be able to do it. You [the coach] have enabled me to overcome these fears. You have been helpful. I learned most of the technical things from you. I had no concerns left towards the end. It was a very nice feeling.”

This speech revealed that the coach helped the teacher to overcome her feeling.

4.2.2 Differences between Mathematics Coaching and Regular In-Service Training

In the interview the coach asked the teachers to compare the mathematics coaching and regular in-service training. Murat and Esen stated that they preferred the mathematics coaching to improve their knowledge as part of professional development. Esen asserted that

“It was better to do it this way within a process rather than as in-service seminars. We learned everything during the process. In the beginning I couldn’t spare much time for assessment, but I learned through practice and experience. I also learned what we are not supposed to do. Then towards the end, everything started to proceed as I wanted.”

In this statement Esen emphasized the importance of *process* for professional development. Supportively, Murat reflected that

“When I first started teaching at an elementary school, we were subject to in-service training. They were gathering approximately 150 people in one room. It continued from 8:30 in the morning to 14:30 in the afternoon. Well, for it to be effective, the trainer needs to make me feel this. The normal presentation method was used. There were slides; the trainer would present and then leave. 150 teachers were there. They explained what I had already learned while studying for the KPSS exam.... The conferences held every week during mathematics coaching and this process were what I wanted.”

This statement revealed the insufficiency of regular teacher training to enhance teachers’ knowledge. Furthermore, whereas in-service training is separated and disconnected from the classroom practices, the mathematics coaching *put together theory and practice*. Murat stated that

“Teachers are bored of filling out forms and ticking things in studies that are being conducted. If they do different things, they can reflect on themselves. What’s education? What am I doing? Because what I do here [in the mathematics coaching] I am receiving theoretical education. Additionally, I am receiving it [theoretical knowledge] without being

aware of it and applying it. When both are done, the process ends....It is what we do put together the theory and the practices. It somehow showed me what I need to do. Now I see them more clearly.”

This statement revealed that mathematics coaching helped the teachers in acquiring, refining, and transferring theoretical knowledge and skills into practice. Besides, both of them believed that they improved themselves in terms of professional development. Murat declared that

“In terms of Professional development... well, remember I told you, I wasn't competent in some things during the first stage; when we arranged the polygons on top of each other, I didn't even feel competent. Perhaps it made me realize. Well, I came to realize thanks to your feedback. I became a more professional. I started to become a more competent in my field. I mean when, instead of giving the formulas directly, supporting the students to discover them by activating the dynamic structure, it made me consider the issue differently. I developed myself. I didn't think it would have such an effect upon the students.”

4.2.3 Challenges of mathematics coaching

The teachers expressed that the mathematics coach did not have any important weakness. Esen stated that “There was no weakness of the coaching. I saw the positive impacts. We [The teacher and coach] have been in rapport with each other.” Although Murat and Esen did not see any disadvantages of mathematics coaching, they indicated some challenges in mathematics coaching such as; *the preparing the lesson*. Preparing the lesson was hard for them and took time and. But also they thought that it was essential for their professional development. Esen expressed that “This process [mathematics coaching] was positive but I also experienced stress as to whether I could do it. At first I experienced many difficulties. But later I got used to it too.” This statement revealed that at the beginning the teachers can be *stressful*, but they can overcome their feeling during the mathematics coaching.

4.2.4 Suggestions for mathematics coaching

In addition to the challenges, they had some suggestions about mathematics coaching. Murat and Esen thought that mathematics coaching should be given as a

part of regular in-service teacher education courses. Furthermore, it should be *common in Turkey*. Murat stated that

“I recommend mathematics coaching to my friends. I explained it to them already. They are curious about it in that they are eager to participate in it if there is something as such in other areas because teachers proceed in the same way. They do not initiate anything new [during the seminar]. Yes, there are seminars at the end of the semester or year but we do not interact with anyone there. We don’t share our knowledge with others. This study drew people’s attention. I told my friends about it. Especially colleagues who now keep up with technology question where they are within the education program, where they have left it, whether they are sufficient enough. If there is a life coach in daily life, why shouldn’t there be one in the field of education? Perhaps it is one of the areas where we need it most.”

This statement revealed that the mathematics coaching can satisfy the need for professional development of mathematics and other subject teachers. In addition, he thought that mathematics coaching should be widespread in mathematics education and *other subjects*. In addition, he said that

“For mathematics coaching let me say this: perhaps in the future it will turn into something like counseling. It will gradually turn it that. Because teacher education course always proceeds in a static way; the same things are done over and over again. Or the smart board is installed but teachers don’t use them. Students don’t use their tablets competently. So it [mathematics coaching] is necessary”

This statement indicated that traditional approaches to professional development are insufficient to help teachers. But teacher learn what they need to know via mathematics coaching. Thus, he believed that in the future the professional development course change direction towards to mathematics coaching.

In this study, the researcher conducted the mathematics coaching to improve mathematics teachers’ TPACK on polygon unit via Geogebra. So, the teacher suggested that mathematics coaching can conduct in *other unit* via *other dynamic software*. Murat said that “Of course we just did mathematics coaching in the topic

of polygons. I think there may be a need in other areas too.” Additionally, Esen stated that

“Mathematics coaching can be spread out into the semester. There can be trainings on different software. I’m sure there are many teachers who don’t know Geogebra either. People are still unaware that the education program has changed. It’s a very bad situation...”

These statement demonstrated that the mathematics coaching can be helpful the teacher in the other unit such as algebra, statistic with other software Tinkerplots, Cabri.

The other suggestion about the *mathematics coaching in group*. Murat believed that if the teachers and the coach work together, it may be more helpful. He stated that

“Mathematics coaching could have been done in groups. When social group that is in interaction is formed, different opinions emerge. In other words, every individual can put forward different things with different perspective. Perhaps interaction can produce something more effective. Maybe it can accelerate the process because addressing 4-5 people at once instead of one. And one of them can make everyone active. Or someone may have a different experience. How can I say it? Maybe s/he can have a misconception; maybe s/he can share it with them. Or s/he can say, ‘I did it this way, but I don’t advise you to do it.’ In this way, I would avoid making the same mistake. Feedback, pre-conferences, post-conferences can be useful.”

This statement demonstrated that if mathematics coaching is in group, the teachers can share their experience to provide awareness.

CHAPTER V

CONCLUSION, DISCUSSION, AND IMPLICATION

In the present study, it has been aimed to investigate the TPACK development of the mathematics teachers' technological pedagogical content knowledge in the field of geometry by participating in mathematics coaching. This chapter presents the discussion of the findings based on the research questions. The discussions are related to mathematics teachers' TPACK Development based on Niess et al. (2009)'s TPACK Development Model for mathematics teachers, and teachers' perceptions about mathematics coaching. Then, it continues with the implications for educational practices, and the mathematics coaching model for TPACK Development, and concludes with recommendations for future research studies.

5.1 Discussion on the TPACK Development of Mathematics Teachers Participating in Mathematics Coaching

A detailed analysis of the findings presented in Chapter 4 demonstrates how elementary mathematics teachers participate in mathematics coaching progress through the levels in the TPACK Development Model in terms of four main themes: curriculum and assessment, learning, teaching, and access. In addition, this study examined the impact of participating in mathematics coaching on the TPACK development of elementary mathematics teachers regarding geometry. The findings indicate that teachers demonstrated growth in the TPACK development model for each theme and their related descriptors. The overall performances of mathematics teachers -Murat and Esen- in the six technology-enhanced lessons before/during/after mathematics coaching are presented in Table 5.1. The recognizing, accepting, adapting, exploring, and advancing levels of TPACK Developments Model are coded respectively as 1, 2, 3, 4, and 5.

Tablo 5. 1 The progress of teachers in four themes and their related descriptors

Themes	Descriptors	Teachers	During Mathematics Coaching					After MC
			Before MC	MCC 1	MCC 2	MCC 3	MCC 4	
Curriculum & Assessment Theme	Curriculum Descriptor	Murat	2	2	4	4	5	5
		Esen	1	2	3	4	5	5
	Assessment Descriptor	Murat	1	1	2	3	3	4
		Esen	1	1	3	3	3	5
Learning Theme	Mathematics Learning Descriptor	Murat	3	3	4	4	4	5
		Esen	1	3	3	4	5	5
	Conception of Student Thinking Descriptor	Murat	3	3	4	4	4	5
		Esen	1	3	3	4	4	5
Teaching Theme	Mathematics Learning Descriptor	Murat	3	3	4	5	5	5
		Esen	2	3	4	5	5	5
	Instructional Descriptor	Murat	2	2	3	4	4	5
		Esen	1	2	4	4	5	5
Environment Descriptor	Murat	2	2	3	4	4	5	
	Esen	1	2	4	4	5	5	
Access Theme	Professional Development Descriptor	Murat	2	-	-	-	-	5
		Esen	1	-	-	-	-	5
	Usage Descriptor	Murat	3	3	4	5	5	5
		Esen	1	3	4	5	5	5
Barrier Descriptor	Murat	2	2	3	4	5	5	
	Esen	1	2	4	4	5	5	
Availability Descriptor	Murat	2	2	3	4	4	4	
	Esen	1	2	3	3	4	4	

5.1.1 Before Mathematics Coaching

The findings of this study demonstrated that in-service teachers exemplified primarily lower levels of TPACK development (recognizing, accepting, and adapting) before mathematics coaching, as seen from Table 5.1.

In the curriculum and assessment theme for the curriculum descriptor, Murat was at a higher level than Esen in the TPACK Development Model, although Murat and Esen's performances indicated a lower TPACK level. Murat integrated the Geogebra-enhanced activities to enhance students' knowledge on polygons but he did not advance the curriculum to take advantage of technology, such as its dynamic nature, as a tool for the teaching and learning process. Besides, he replaced non-technology based tasks in the traditional curriculum as indicated in the lower level of TPACK (*the accepting level*). Murat implemented these tasks to teach new concepts for students, whereas Esen used Geogebra to reinforce the students' knowledge on polygons, which were taught with the use of concrete material. Thus, she viewed Geogebra as add-on to display mathematical ideas addressed in the curriculum as indicated in *the recognizing level*. Research studies on technology integration indicated that teachers focused more on the technology rather than how to integrate technology in their curriculum in innovative way (Herrington et al., 2009; Hilton et al., 2010). In parallel to these studies, the teachers in this current study did not use Geogebra as a technological tool to modify curriculum innovatively as indicated in highest TPACK levels. They just focused on using Geogebra and replaced the activities in the curriculum. The reason of this usage might be that the teachers in the current study (especially Esen) were techno-centric, which means they focused on how to use the tool rather than how to teach with the tool to meet curriculum goals (Harris, 2005; Harris & Hofer, 2009; Niess, 2005b). Furthermore, in the present study the teachers did not integrate Geogebra to assess the students' knowledge on polygons. They resisted the idea of using Geogebra in the assessment process because of the thought that technology is not necessary or useful to determine students' understanding of geometry (*the recognizing level*).

Neither of them let their students use Geogebra during assessment even though they implemented Geogebra in the learning and teaching process. This demonstrated that the teachers in this study had limited vision to integrate Geogebra as a technological tool in curriculum. The reason underlying their limited vision of Geogebra as an assessment tool might be related to the fact that they could not find any good examples of Geogebra-enhanced activities to assess the students' knowledge on geometry. Thus, they might not know how to integrate Geogebra effectively in the assessment because of the lack of well-designed Geogebra-enhanced assessment activities.

Before mathematics coaching, in the learning theme Murat's performance indicated *the adapting level* of TPACK for the descriptors of both mathematics learning and the conception of student thinking, whereas Esen's performance was at the lowest level (*the recognizing level*) of the TPACK Development Model. Like the curriculum descriptor, Murat was at a higher level than Esen was. Murat helped students to make a conjecture and explore the concepts related to polygons by means of Geogebra-enhanced activities. Based on explorations via Geogebra, students were able to make generalizations and test their conjectures about polygons and their component parts. His students constructed and justified statements regarding polygons as geometric figures and their properties. Furthermore, in his lesson technology was used to improve students' thinking skills rather than practicing and memorizing. In addition, via Geogebra, he provided the students with an environment in which they could overcome possible misconceptions and difficulties. Moreover, he supported the students to use their own language and explain their ideas in order to enhance their mathematical knowledge. However, although Geogebra was used to enhance students' knowledge, he implemented Geogebra as a static tool. Geogebra was mostly used for demonstrations or teacher-led student-follow work. These situations provide evidence that he started to explore, experiment and practice integrating Geogebra as a mathematics learning tools (*the adapting level*). Murat's level in the learning level might be related the content-centric approach. According to Koehler and Mishra (2005), content-centric

approach focuses on the rich connections between technology, the subject matter, and the means of teaching it which means pedagogy. Considering Murat's usage, it can be said that he did not reach the content-centric approach. As a consequence this situation, he might not integrate Geogebra to enhance the students' knowledge with considering of advantages of technology. He used Geogebra as a demonstration tool in statics way, although he provided an environment for his students to explore geometrical concepts. He did not implement a Geogebra-enhanced lesson according to the content-centric approach. Therefore, he might not reach the highest TPACK level in the learning theme. On the other hand, Esen implemented Geogebra to demonstrate examples of concave and convex polygons rather than to provide students with an environment in which they could explore geometrical concepts. She integrated the concrete materials (geometry board and tooth sticks) to support the students' exploration process and then she used Geogebra as a supplementary tool. In her mathematics lesson, geometrical exploration with technology was rarely seen. According to Harris and Hofer (2009), techno-centric lesson rarely help students to learn the concepts in curriculum. In that respect, Esen's lesson before mathematics coaching was parallel to the techno-centric approach. Because of her techno-centric lense, she might not provide the students with an environment to explore the polygon concepts in her lesson before mathematics coaching. Besides, she was more apt to accept Geogebra as a teaching tool rather than a learning tool in line with the techno-centric approach. Therefore, she might be at the lowest TPACK level in the learning theme.

In the teaching theme Esen's performance for mathematics learning, instructional, and environment descriptor was lower than that of Murat at the beginning of the study, before mathematics coaching. Esen integrated Geogebra to repeat the non-technology activities (such as geometry board) via Geogebra. She used Geogebra to foster visualization and did not view technology as an essential component of the mathematics classroom. In her geometry lesson, Geogebra-enhanced activities were not used for concepts that required more advanced skills as indicated in *the recognizing level*. She implemented Geogebra for menial or rote

activities, such as showing the type of polygon after teaching via concrete materials. In addition, she did not let the students use technology to learn or access mathematics or explore new concepts via Geogebra. On the other hand, Murat integrated Geogebra (without the consideration of dynamic properties) to enhance the students' knowledge (*the adapting level*). He integrated Geogebra to teach new geometrical concepts without any hesitations. However, Geogebra was not used to enhance students' higher-level thinking (such as project-based and problem solving and decision making activities) in learning mathematics by using the technology indicated in higher levels (exploring and advancing levels). Supportively, other studies have also found that as teachers integrated technology in their lesson, they tended to apply didactic strategies that focused on lower-level thinking skills and basic knowledge (Becker & Ravitz, 2000; Mann et al., 2009). In addition, their lesson was teacher-directed and followed a step-by-step process. Niess, van Zee, and Gillow-Wiles (2011) found that most of the teachers exhibited teacher-centered instructional strategies via using technology at the beginning of their study. In parallel with the literature, the teachers in this current study (Murat and Esen) implemented teachers-directed students-followed lesson via Geogebra. These indicated that Murat and Esen displayed weak knowledge of Geogebra in the teaching theme, like in the other themes. They did not use student-centered strategies via Geogebra-enhanced lessons. For instance, Murat acted as a director of the students' exploration process rather than a guide. His students were not free to use Geogebra to discover geometric concepts. During the Geogebra-enhanced activities, most of the time the students were not actively involved in learning to reach their own insights through individual or collaborative work. These serve as evidence to indicate the lower level of the teaching theme. In the Niess, van Zee and Gillow-Wiles (2010)' study, teachers' classroom with using spreadsheet as a technological tool was in alignment with the teacher-centered classroom pedagogy, where was appropriate control of the technology to the students. The researchers indicated that refocusing from teacher to student was outside of teachers' comfort zone. In the light of the study by Niess, van Zee and Gillow-Wiles (2010), the

teachers in this current study might not want to implement Geogebra outside of their comfort zone. Thus, they did not integrate Geogebra that maintained student engagement and self-direction in learning polygons. The teacher-centered classroom might be comfortable for the teachers in this study than the student-centered classroom because of their desire to control students' actions in Geogebra-enhanced activities. In addition, this situation might cause underlying their not being at a higher TPACK level in the teaching theme before mathematics coaching.

In the access theme, Esen's performance for usage, barrier, and availability descriptors was lower than that of Murat at the beginning of the study. Murat assigned the students into groups and every student group had a computer to use Geogebra. He also projected his computer onto the blackboard. His students were able to use the computer for approximately the whole lesson. Access to and use of technology was available for exploration of polygons as a new concept, usually with the teacher's demonstration (*the adapting level*). On the other hand, Esen integrated technology solely for the teacher's usage. In Esen's class there was only one computer and she did not let the students use Geogebra. She permitted the use of and access to Geogebra 'only' after mastering polygon concepts via the geometry board and toothpicks as concrete materials as indicated at *the recognizing level*. Furthermore, to control students' behavior, she wanted to integrate Geogebra within a limited time period. This demonstrated that she was at the lowest level in terms of finding solution for management issues. As compared to Murat, Esen was at a lower level. However, Murat did not provide the students with an independent environment for them to take responsibility for their own learning. Furthermore, he warned the students not to deviate from his instructions, and he did not allow much time for the students to work freely. Thus, he had conflicting views in relation to management issues. He worried about management issues with respect to the integration of Geogebra into his geometry classroom. These results indicated that the teachers in this current study had a barrier in management issues. According to Jaipal-Jamani and Figg (2015), teachers ought to adapt their knowledge of general pedagogical strategies for technology-enhanced lesson. In addition, the management

strategies in technology-enhanced lesson may vary from regular classroom management strategies. In the light of this notion, the teachers in this current study might not adapt their management strategies while integrating Geogebra. Therefore, they might have challenges in teaching mathematics with Geogebra in their classroom. They might want to control students' actions and their access to Geogebra to overcome the challenges in management issues. Furthermore, they might not provide students with an environment to improve their self-exploration because of these challenges in management issues. In addition, neither Murat nor Esen integrated Geogebra to incorporate multiple representations of the concepts and their connections. Furthermore, they did not use the dynamic nature of Geogebra in their lessons. According to Kushner Benson, Ward, and Liang (2015), limited knowledge and implementation skills in the management caused lack of interactive/dynamic and diverse media to attract student interest in technology-enhanced lesson. In parallel to this notion, the reason of teachers' lack of integrating multiple representations and dynamic nature of technology in the Geogebra-enhanced lesson in this current study might be related to their limited knowledge in the area of management. In other words, the teachers in this study were strict in management to provide an environment for students' engagement in exploration of polygons' concepts. Therefore, they might not be flexible to supports students in order to make connections between different representations and realize the mathematically meaningful consequences of the dynamic actions. As a consequence of this situation, the teachers' TPACK level in the access theme might be at a lower level.

To sum up the teachers' performance before mathematics coaching, Murat was at *the recognizing level* for one descriptor (assessment descriptor in the curriculum and assessment theme), at *the accepting level* for five descriptors (the curriculum descriptor in the curriculum and assessment theme; the instructional and environment descriptors in the teaching theme; and the barrier and availability descriptors in the access theme), and *the adapting level* for the other five descriptors (the mathematics learning and conception of student thinking descriptors in the

learning theme, the mathematics learning and professional development descriptors in the teaching theme; and the usage descriptor in the access theme). On the other hand, Esen was at *the recognizing level* for nine descriptors (the assessment and curriculum descriptors in the curriculum and assessment theme; the mathematics learning and conception of student thinking descriptors in the learning theme; the instructional and environment descriptors in the teaching theme; the usage, barrier, and availability descriptors in the access theme), at *the accepting level* for one descriptor (the mathematics learning in the teaching theme), and *the adapting level* for one descriptor the (professional development descriptor in the teaching theme). This indicated that the teachers in the current study who did not participate in professional development course to enhance technology implemented Geogebra in their mathematics lesson were at a lower TPACK Development level. A study by Niess, van Zee, and Gillow-Wiles (2011) supports these findings. Before they conducted an online course to engage mathematics and science teachers in learning about spreadsheets, they analyzed how teachers implemented technology as a learning and teaching tool. All the teachers were identified to be at *the recognizing level*. The reason of the lower TPACK levels of teachers in this study might be that the teachers did not see any effective usage of Geogebra in the mathematics classroom. Teacher education program and traditional professional development emphasizes technical skill development which is out of context, separated from classroom practice and far away giving examples about the utility and worth of technology (Jaipal-Jamani & Candace Figg, 2015; McKenzie, 2001). Both of the teachers in this current study were graduates of an elementary mathematics teacher education program in Turkey. In the light of the study of the studies by Jaipal-Jamani and Candace Figg (2015) and McKenzie (2001), the separation of elementary mathematics teacher education programs in Turkey from classroom practice in which Geogebra is used might not help to improve teachers' TPACK. The teachers in this current study did not experience how to effectively incorporate technology Geogebra in particular into the mathematics classroom during their enrollment in the teacher education program in Turkey. This situation might be the

reason underlying the teachers' being at a lower TPACK level in this current study at the beginning of the study. In addition, after their graduation they did not involve professional development course which focusing of the how to integrate technology in the mathematics classroom. In Turkey, professional development project (such as FATİH project) mostly focused on technical support although one of their missions was promoting effective ICT usage in the curriculum (EĞİTEK, 2011). Angeli and Valanides (2009) stated that proficiency in technology as a tool does not mean effective usage of technology in daily instructional practices. In line with the study of Angeli and Valanides (2009), the professional development which emphasizes technical support and technological proficiency might not help teachers to reach the highest TPACK level. In other words, as considering the lack of professional support about how to effectively integrate technology, TPACK of the teachers in this current study was at lower level.

Before the mathematics coaching, the teachers were not involved in any professional development course to develop their TPACK. Although both of them demonstrated a lower degree of technology integration in their geometry classroom, Esen was at a lower level than Murat in the TPACK Development Model for the nine descriptors of four themes - curriculum, mathematics learning, conception of student thinking, instructional, environment, usage, barrier, and availability. This indicated that Murat implemented Geogebra more effectively in the classroom than Esen did to teach geometrical concepts at the beginning of the study. In addition, Murat integrated Geogebra in approximately the whole lesson time except during warm-up activities and assessment sessions, whereas Esen implemented Geogebra in the limited time subsequent to the mastering of geometrical concepts. Taking into consideration Murat's being less experienced than Esen, this finding supports earlier research studies (Baek, Jong & Kim, 2008; Inan & Lowther, 2010; National Center for Education Statistics, 2000). The U.S. National Center for Educational Statistics (2000) stated that teachers with less teaching experience are more likely to integrate technology into their classroom than teachers with more teaching experience. According to the report, teachers with up to three years of teaching experience spent

more time on utilizing computers in their lesson than teachers with a teaching experience between 4 and 9 years. This report is in agreement with our findings that Murat (with a teaching experience of 2 years) used Geogebra as technological tool during the whole lesson, whereas Esen (with a teaching experience of 6 years) integrated Geogebra in a limited period of time. The discrepancy of the amount of time spent on the use of Geogebra in this study can be attributed to the differences between Murat and Esen's length of experience. According to the National Center for Educational Statistics (2000), the reason of this discrepancy can be that novice teachers have more experience in using the technology in their daily life. In that respect, Murat as a less-experienced teacher in this current study used technology in his daily life effectively and also used social network every day. Besides, Esen as a more-experienced teacher did use technology very much in her daily life. Considering this situation, one of the reasons of difference between the Murat and Esen's TPACK might be their usage of technology in their daily life. Furthermore, Baek, Jong, and Kim (2008) concluded that experienced teachers have a lower level of readiness to integrate technology into the classroom than less experienced teachers. Similarly, Inan and Lowther (2010) reported that a teacher's level of experience in teaching has negative effects on both their computer proficiency and technology integration. In that respect, Murat as a less-experienced teacher indicated that his proficiency of Geogebra was medium, whereas Esen as a more-experienced teacher expressed that she knew just basic technical knowledge about Geogebra. In parallel to literature, the difference between teachers' experience in this current study might affect that their technology integration in their mathematics classroom. For instance, being a more experienced teacher, Esen did not integrate Geogebra to teach new geometry concepts, while Murat, who was a less-experienced teacher, did. Esen preferred to use Geogebra after mastering the concepts via concrete materials. Furthermore, Esen incorporated Geogebra for a limited time period, whereas Murat used Geogebra approximately during the whole lesson. Thus, despite being a more-experienced teacher, Esen was at a lower TPACK level than Murat, who was a less-experienced teacher. This situation

demonstrated that the difference between their TPACK levels might be related to the discrepancy between teacher experience.

5.1.2 During Mathematics Coaching

During mathematics coaching, mathematics teachers demonstrated progress to higher levels in the TPACK development Model in all themes: the curriculum and assessment theme, the learning theme, the teaching theme, and the access theme, as seen in Table 5.1.

In curriculum and assessment theme, the teachers progressed from *the accepting level* to *the advancing level* for the curriculum descriptor during mathematics coaching. At the beginning of the mathematics coaching, implemented as a professional development programme, the teachers integrated Geogebra as a tool for presentation in the teaching and learning of the concepts related to the polygon. Thus, they understood some of the advantages of integrating Geogebra as a tool into the mathematics curriculum, but they experienced difficulty in the integration of Geogebra into the mathematics curriculum with the aim of implementing it as a tool for learning. In fact, Esen used concrete materials instead of Geogebra at the end of lesson to summarize the lesson. Through mathematics coaching, both Esen and Murat sought ideas about how to incorporate Geogebra into their lesson and to modify/advance the current curriculum. At the end of the mathematics coaching, they developed innovative ways to use Geogebra to provide students with stronger conceptual understandings by taking into consideration the dynamic nature of technology. In the a content-centric approach, teachers emphasizes how to teach with the tool to meet curriculum goals rather than how to use the tool as techno-centric (Harris, 2005; Niess, 2005). In the light of this situation, it can be said that the teachers in this current study moved from techno-centric to content-centric approach during mathematics coaching. At the first coaching cycle, they had tendency to focus on technical skills as a part of techno-centric. Then, they realized the importance of modifying the curriculum with the consideration of properties of Geogebra as indicated in techno-centric approach.

Furthermore, it can be said that mathematics coaching help the teachers to promote content-centric approach. Furthermore, the teachers also demonstrated growth in the TPACK development model for the assessment descriptor. At the beginning of the study, they did not allow Geogebra to be used in the assessment of students' knowledge of polygons. Thus, they resisted the idea of using Geogebra in the assessment process as indicated in the *recognizing level*. Through mathematics coaching, they started to integrate Geogebra as a visualization tool without any modification in textbook assessments. They did not take into account the dynamic features of Geogebra in assessment. Through the mathematics coaching, they started to use Geogebra appropriately that extended beyond the paper and pencil type of questions. However, they still prepared Geogebra-enhanced assessment activities in a static way, which was aligned with the textbook questions focusing on students' procedural knowledge. They wanted to implement Geogebra during assessments, so different activities were implemented (such as conceptual vs. procedural understandings) as indicated in *the adapting level*. Although the teachers progressed in the assessment descriptor of the TPACK Development Model, their final level in mathematics coaching was at *the adapting level* during mathematics coaching. Among all the descriptors in which progress was made, the slowest progress was observed in the assessment descriptor. Murat stated that he had concerns related to the Turkish education system and national exams, so he preferred some of the assessments to be aligned with the textbook questions related to procedural knowledge. The reason underlying the finding that the slowest progress was observed in the assessment descriptor might be attributed to the Turkish education system and national exams, such as TEOG (the exam for Transition from Elementary to Secondary Education). Hew and Brush (2007) noted that the pressures of higher standards and high scores on standardized tests can be a major barrier to technology integration. This view was supported by Butzin (2004), who stated that the pressure of standardized test creates a daunting challenge for any teacher in technology integration. In Turkey, Ministry of National Education (MoNE) administers centralized exams such as the TEOG exam for elementary

school students. The scores obtained from the TEOG exam are being employed in the process of students' transition to secondary school (MoNE, 2013). In addition, the scores of students in the TEOG is also determining the success rating of elementary school. Therefore, this exam cause to intensive pressure on school administrators, teachers, and students. In the light of the literature, the pressure of TEOG might create intensive challenges on integrating Geogebra as an assessment tool. Thus, as a consequence of this pressure, the teachers in this current study might make the slowest progress in the assessment descriptor. Furthermore, Taylor (2011) indicated that high-stakes testing tended to motivate mathematics teachers to use technology more for rote computations or to verify solutions rather than for exploration. In addition, Çopur, Yekrek, Özgeldi (2016) stated that the mathematics questions in TEOG do not focus on the highest stage in Bloom's Revised Taxonomy (Creating and Evaluating stages). Considering the Taylor (2011) notion, this tendency to focus on the lower levels in Bloom's Taxonomy in TEOG might the reason why teachers emphasize procedural knowledge in Geogebra-enhanced assessment activities. To reach a higher level in the assessment descriptor, teachers should focus on examining students' conceptual understandings while integrating technology. In this study, moving from the lowest level to the highest level took time possibly because the teachers were emphasizing procedural knowledge rather than conceptual knowledge in the Geogebra-enhanced assessment activities. To sum up, the teachers' slowest progress might be observed in the assessment descriptor because of the pressure of national exams in Turkey and the national exams' focus on procedural knowledge.

In the learning theme, the teachers progressed from *the adapting level* to the *exploring and advancing level* for the descriptor of mathematics learning and conception of students' thinking during mathematics coaching. At the beginning of mathematics coaching, they started to explore the integration of Geogebra as a mathematics learning tool. However, they had limited usage of Geogebra as a learning and teaching tool; he did not implement Geogebra during the introduction and assessment stages of the lessons. Through the mathematics coaching, they

implemented Geogebra dynamically, as a tool to facilitate the learning of polygons. They realized the benefits of the dynamic nature of Geogebra, specifically integrating technology to develop students' exploration, unlike at the beginning of the study. They encouraged the students to explore geometric topics by integrating Geogebra into attempts to better understand mathematical concepts. Furthermore, Geogebra-enhanced activities were implemented and evaluated with respect to students' learning of mathematics and students' attitudes toward polygons. In the last cycle of mathematics coaching, Esen reached the highest level in the TPACK Development Model for the mathematics learning descriptor in the learning theme. She planned, implemented, and reflected on teaching and learning to guide students to understand mathematics by using the Geogebra as a tool for learning as indicated in the *advancing level*. To sum up, mathematics coaching provided an environment for the teachers to promote their TPACK in learning theme. According to Özgün-Koca, Meagher, and Edwards (2011), teacher ought to be supported in terms of not only how to use technological tool, but also how to integrate technology to help students' learning. In mathematics coaching, the teachers were supported by their coach to improve their knowledge about how to implement Geogebra to enhance students' knowledge about polygon as a geometry concept. This supports might cause that the developments of teachers' TPACK in learning theme during mathematics coaching. In mathematics coaching, the coach made suggestions about how to integrate Geogebra to overcome possible students' misconceptions and difficulties. For instance, the coach suggested that the teacher transform the parallelogram to rectangle to find the area of parallelograms via Geogebra because students might experience difficulties in finding the area of parallelograms. This suggestion helped the teacher to integrate Geogebra to improve student understanding regarding the area of parallelograms by exploring, not just by memorizing. In addition, the coach's suggestions might help the teachers to incorporate Geogebra to improve students' higher level thinking process, such as problem solving. For instance, the coach supported the teachers to integrate Geogebra to improve student learning in finding the area of irregular shapes via

problem solving activities. During the mathematics coaching, the teachers were supported by the coach to focus on student understanding in Geogebra-enhanced activities. To sum up, these supports might have helped the teachers to reach the highest TPACK level in the learning theme.

In the teaching theme, teachers progressed from the *accepting* and *adapting level* to the *exploring* and *advancing level* for the mathematics learning and instructional and environment descriptors during the mathematics coaching. At the beginning of mathematics coaching, Esen explained her concern about how technology would take up the time from teaching polygons. Besides, Murat and Esen's lessons were implemented utilizing a directed and step-by-step process. In the second mathematics coaching cycle, Esen served mostly as a guide rather than the director of the exploration process. On the other hand, Murat still wanted to direct the students' exploration process. However, Murat reached Esen's TPACK level (*exploring level*) in the next coaching cycle (MCC3). Furthermore, by the end of the mathematics coaching, Murat and Esen provided the students with an environment in which they could engage in high-level thinking activities (such as problem solving and decision making activities) in order to learn polygons as geometry concepts via using Geogebra as a learning tool. To be specific, Esen moved from integrating adds-on Geogebra activities to high-level thinking Geogebra activities. Murat and Esen incorporated Geogebra to transform geometrical concepts so that they would become more understandable for the students. They created novel ideas to help students learn mathematics via technology. Furthermore, Esen managed Geogebra-enhanced activities in ways that maintained student engagement and self-direction in learning mathematics as indicated in *the advancing level*. Thus, mathematics coaching helps the teachers to progress through TPACK development model in teaching theme. According to Kushner Benson, Ward, and Liang (2015), teachers need professional development course to improve their instructional strategies and technology implementation skill. In this current study, the teachers moved from teacher-centered to student-centered in Geogebra-enhanced learning environments. Furthermore, they provided an

environment for their students to explore concepts in higher-level thinking activities. To sum up, mathematics coaching might satisfy the teachers' need the knowledge about how to improve their instructional strategies and learning environment via integrating Geogebra as technological tool. During the mathematics coaching, the teachers were supported by the coach to improve their TPACK in the teaching theme. For instance, the coach made suggestions to the teachers to focus on student-centered lesson, such as giving opportunity for students' exploration and giving time for students to direct themselves in learning geometry. This support might have helped the teachers to improve their TPACK in the teaching theme. Furthermore, according to Richardson (1997), teachers supporting constructivism organize student-centered activities to enhance their learning, and implementing student meaning-making activities to focus the learning process. In the light of this notion, one of the teachers (Murat) stated that mathematics coaching helped him to deeply understand constructivism. He added that he started to understanding the importance of student-centered lesson. The teachers realized how to integrate Geogebra in the teaching theme aspect via their classroom experiences during mathematics coaching. While moving away from the teacher-centered lesson toward the student-centered lesson, the teachers realized that the students constructed the geometry concepts in a student-centered Geogebra-enhanced lesson and internalized the concepts in the student-centered learning environment. This awareness might have helped them to focus on students' self-direction in the learning environment to learn the geometry concepts. To sum up, mathematics coaching might have helped teachers to provide the students with a student-centered learning environment for them to construct the polygon concepts via Geogebra.

In the access theme, teachers progressed from *accepting* and *adapting level* to *exploring* and *advancing level* for the descriptors of usage, barrier and availability during mathematics coaching. Esen moved to a higher TPACK level in the access theme during the first cycle of mathematics coaching, and reached Murat's level. At the beginning of the mathematics coaching, both of them permitted the students to use and access Geogebra via teacher-led student follow type of interaction.

Although they motivated the students to use Geogebra, they had worries about management issues with respect to incorporating technology into the classroom. Besides, they used Geogebra solely as a visual representation of polygons to find patterns. They did not integrate Geogebra to facilitate multiple representations and the dynamic nature of Geogebra. Through mathematics coaching, the teachers encouraged the students to explore geometry by accessing and using Geogebra. However, Esen recognized the challenges involved in teaching geometry via Geogebra, and sought for ways to obtain technology for classroom use (*exploring level*). To control management issues in Geogebra-enhanced activities, she guided the students to explore the properties of Geogebra by walking among the student groups. Besides, Murat implemented Geogebra as a tool to enhance the mathematics lessons in order to provide students with a new way to approach mathematics. However, he wanted to take the students under his control (*adapting level*). Thus, he was at a lower level than Esen in the middle of mathematics coaching (MCC2). In addition, Murat and Esen started to teach geometrical concepts differently since Geogebra provided the students with an environment in which they could make connections that were formerly out of reach. They motivated the students to use Geogebra dynamically in order to investigate and make connections between polygon concepts. By the end of mathematics coaching, they had reached the highest levels of the access theme. They permitted students to use Geogebra in every aspect of mathematics class. They viewed Geogebra as a dynamic environment in which the students could master the geometrical concepts in an innovative way, and used it to challenge the notions of what mathematics students could learn. They also implemented Geogebra effectively by using multiple representations, such as tables, symbols, and pictures. In addition, they extended planning and preparation for maximizing the use of available resources in order to resolve the challenges in technology integration (*advancing level*). According to Shreiter and Ammon (1989), the challenges in teaching such as potential changes in classroom environment needs engagement in the process of assimilation and accommodation to reconstruct of their experience in teaching mathematics.

Furthermore, Niess (2015) stated that the teachers' engagement in teaching with new ideas via technology needs time to transform their thinking about teaching their content with the technology. In the light of the studies by Shreiter and Ammon (1989) and Niess (2015), during the process of mathematics coaching, teachers practiced and experienced Geogebra-enhanced lessons. During this process, the teachers faced the challenges that emerged in their implementation of a Geogebra-enhanced lesson, such as classroom management, and then they resolved these challenges. Mathematics coaching, which emphasizes the process of teachers' implementations (rather than one-shot implementations), might have helped the teachers to face and realize their strengths and weaknesses in their practice. In other words, mathematics coaching might have helped them to transform their thinking via Geogebra and gave time to practice it during their classroom implementation. This might help them to move highest TPACK level in access theme.

In summary, Murat and Esen progressed to higher levels in the TPACK development Model in all the themes and their descriptors – the curriculum and assessment theme, the learning theme, the teaching theme, and the access theme during mathematics coaching. Besides, when the overall performances of teachers during mathematics coaching are considered, it is seen that Esen reached a higher level than Murat seven times, whereas Murat reached a higher level than Esen only four times. This situation demonstrated that Esen's progression in the TPACK Development Model was faster than Murat's, although her TPACK level was primarily at the lowest level at the beginning of the study. Especially in the teaching theme, Esen reached and passed Murat's TPACK level. Thus, Esen as a more-experienced teacher moved faster than Murat as a less-experienced teacher through TPACK development model. In support of this finding of this current study, Munoz-Catalan, Carillo Yanez, and Climent Rodriguez (2010) reported that it took time for the novice teacher in their study to improve her knowledge in a collaborative environment as a professional development. Recognizing the importance of dissenting opinions about her teaching to improve her practice and applying alternative ideas took time in their study. In the parallel to the study of Munoz-

Catalan, Carillo Yanez, and Climent Rodriguez (2010), Murat as less-experienced teacher in this current study might have had difficulty in recognizing the importance of suggestions of the coach about how to integrate Geogebra in his geometry lesson. For instance, he stated that he recognized the necessities of students-centered lesson as integrating Geogebra at the end of the study. Taking into consideration the fact that Esen was more experienced in teaching, it can be concluded that Esen had rich experience in teaching and classroom management, which made it easier for her to realize her deficiencies in teaching via Geogebra and overcome these deficiencies. Thus, this finding could primarily be attributed to the discrepancy in their teaching experience. In addition, Esen's experience in teaching might have helped her in identifying her strengths and weaknesses and improving herself by integrating Geogebra into her lesson.

Some research studies also indicated that experienced teachers integrated technology more effectively than less-experienced teachers (Jang & Tsai, 2012; Lau & Sim, 2008; Russell, Bebell, O'Dwyer & O'Connor, 2003). On the other hand, Russell, Bebell, O'Dwyer, & O'Connor (2003) stated that new teachers did not incorporate ICT in their teaching although they were highly skilled with technology more than experienced teachers. The reason was attributed to the fact that new teachers focused on how to use ICT instead of on how to integrate ICT into their teaching. In parallel to the study by Russell, Bebell, O'Dwyer, & O'Connor (2003), despite being a less-experienced teacher, Murat emphasized the use of Geogebra in his lesson. The reason underlying his slower progress than that of Esen in the TPACK development Model might have been due to his focus on technical properties of Geogebra instead of the incorporation of Geogebra to improve students' understanding in a student-directed environment. . In addition, the researchers stated that new teachers experienced some challenges in teaching and classroom management (Russell, Bebell, O'Dwyer, & O'Connor, 2003). In another research study, Lau and Sim (2008) stated that experienced teachers had many more opportunities in teaching and classroom management, which made it easier for them to incorporate technology into their teaching. Similarly, the results of a study by of

Jang and Tsai (2012) showed that teachers who had more years of teaching experience demonstrated significantly higher TPACK than did teachers who had fewer years of teaching experience. As considering the result of these studies, Esen as a more-experienced teacher might have easily accommodated to integrate Geogebra in her geometry classroom because of her experience in teaching and classroom management. To sum up, the research studies support the finding in this study that Esen, as a more experienced teacher, progressed faster in the TPACK Development Model. The reason of this finding might be attributed to the fact that Esen, who was a more experienced teacher, had more opportunities and experiences in teaching and applying various teaching strategies. Therefore, she could easily adapt to integrate technology into her teaching practices by identifying her strengths and weaknesses during the mathematics coaching. Furthermore, Murat as a less-experienced teacher emphasized the properties of Geogebra such dynamic nature and the usage of Geogebra in his geometry classroom. Therefore, it might take time for him to realize and resolve the difficulties emerging in his classroom environment during mathematics coaching.

5.1.3 After Mathematics Coaching

After mathematics coaching, mathematics teachers reached the highest level in the TPACK development Model in all the themes: the curriculum and assessment theme, the learning theme, the teaching theme, and the access theme, as seen in Table 5.1. When the periods before and after mathematics coaching were compared, it could be observed that the teachers progressed from *lowest levels* to *highest levels* in all the themes and their related descriptors: the curriculum and assessment theme, the learning theme, the teaching theme, and the access theme. Through the mathematics coaching process, the teachers engaged in all four themes and their related eleven descriptors of TPACK Development Model, as they developed the integration of Geogebra in their geometry lesson. In conclusion, mathematics coaching –which was focusing TPACK Development Model- helped them to adopt specific approaches to incorporate Geogebra in their lessons:

- a) to modify and advance the curriculum to take advantage of Geogebra as a teaching and learning tool,
- b) to design assessment activities to reveal students' understanding of geometrical ideas via Geogebra that extended beyond paper and pencil,
- c) to provide the students with an opportunity to explore geometrical concepts by integrating various technologies in attempts for better understanding,
- d) to motivate the students to engage in high-level thinking activities (such as problem solving and decision making activities) via Geogebra as a learning tool,
- e) to accept actively and consistently Geogebra as a learning and teaching tool with novel ideas which help students to translate geometrical concepts into more understandable ,
- f) to guide students rather than direct them while engaging in explorations of geometrical concepts with Geogebra-enhanced activities by using a breadth of instructional strategies (including both deductive and inductive strategies),
- g) to manage student engagement and self-direction in learning geometry via Geogebra-enhanced activities,
- h) to be disposed to attend other professional development courses to explore ideas for teaching and learning mathematics with multiple technologies,
- i) to permit students to use and access Geogebra which enables the students to master geometric concepts,
- j) to resolve challenges in teaching Geogebra as a technological tool by means of extended planning and preparation,
- k) to incorporate multiple representations of the concepts and their connections to explore and assess the geometric concepts via Geogebra

as indicated at the highest levels in the four themes and their related eleven descriptors of Niess et al (2009, p.20-22)'s the TPACK Development Model.

This research study demonstrates that mathematics coaching shows a promise for developing mathematics teachers' TPACK and practice. The more advanced levels of TPACK development involve practices, such as planning,

implementing, and reflecting on technology lessons with a possible concern for guiding students in understanding mathematics (Niess et al., 2009). These practices are the main components of mathematics coaching in this current study with a focus on improving teachers' TPACK. Thus, with the help of the main component of mathematics coaching (planning, implementing and reflecting) the teachers reached advancing level of TPACK in this current study. Furthermore, some research studies echo the finding in this study that coaching, in particular mathematics coaching, leads to changes in teacher knowledge. Although their research is not on mathematics coaching in particular, Black, Molseed and Sayler (2003) indicated that teachers participating in coaching had a deeper knowledge of how to improve their own practice than teachers not participating in coaching. Furthermore, a research study by Knight (2007) indicated that teachers engaging in coaching for professional development are four times more likely to implement new practices as teachers who do not. Thus, mathematics coaching in this current study might support the teachers to implement Geogebra-enhanced activities as new practices. They changed their own practice by considering the properties of Geogebra, and then they reached the highest TPACK level. They made great progress in their knowledge of how to integrate Geogebra in their geometry lesson. The reason of this progress might be the impressiveness of mathematics coaching as stated in the studies of Black, Molseed and Sayler (2003) and Knight (2007). Furthermore, Race, Ho, and Bower (2002) stated that mathematics teachers participating in mathematics coaching changed their instructional practice by providing students with activities for self-engagement to build students' conceptual understanding. In addition, the researcher claimed that without the support of coaches, teacher would not make a change in their instructional practices. In parallel to a study by Race, Ho, and Bower (2002), after the mathematics coaching in this current study, the teachers focused on students' self-engagement in Geogebra-enhanced activities. The teachers stopped controlling the students' exploration and guided them in discovering new geometrical concepts. In addition, they provided the students with an environment in which the students took responsibility for their own learning. The students

directed their learning via Geogebra. The teachers in this current study managed learning environments so that the students could engage in self-directed learning in geometry. Thus, mathematics coaching might help to teachers' change in their instructional practices by using Geogebra. Becker (2001) also indicated that teachers participating coaching focused on the big ideas of mathematics rather than just following the textbook. They were more concerned with improving students' understanding of mathematics, and discussed more processes such as problem solving. In addition, Bruce and Ross (2008) found that teachers in mathematics coaching conducted more open-ended tasks in their classroom. Similar to these studies, the teachers in this current study emphasized the students' understanding by using Geogebra after mathematics coaching. Thus, mathematics coaching help the teachers to focus on improving the students' understanding about geometry by using Geogebra. At the beginning of the study, the teacher mostly focused on procedural knowledge. By the end of the study, they integrated Geogebra to enhance students' high order thinking level via open-ended and problem-solving activities. Research studies support the result of this study that mathematics coaching helped to develop teachers' knowledge (in particular TPACK) and their practices. Mathematics coaching in this study provided the teachers with an environment to design, implement and reflect on their geometry lessons via Geogebra in the pre-conferences, observations and post-conferences. These components of the mathematics coaching might have helped the teachers in this study to improve their TPACK and their practices. For instance, the pre-conferences in mathematics coaching helped the coach to gain insight into teachers' TPACK regarding geometry in this current study. According to Kushner-Benson, Ward, and Liang (2015), creating individual TPACK profiles can be useful tools for promoting teachers' TPACK. As considering the study of Kushner-Benson, Ward, and Liang (2015), gaining insight into teachers in mathematics coaching might have made a contribution of teachers' progress in TPACK. As stated in this study, gaining insight into teachers' TPACK in pre-conferences might have helped the teachers to realize their knowledge about the integration of Geogebra and improve themselves in this

respect because collecting data about teachers' TPACK shed light on their weaknesses and strengths in technology integration. Determining the teachers' knowledge in the pre-conferences might be the first step towards teachers' TPACK development because without knowing teachers' knowledge and their weaknesses and strengths, the coach might not be able to support teachers in their enhancement of their knowledge. Furthermore, According to Jaipal-Jamani and Figg (2015), the facilitator of professional development course should instruct the teachers in the technical skills required to use technology in their lesson. Developing activity-specific technical skills through short tool demonstrations help the teachers to improve themselves in technology integration. In this current study, the coach supported the teachers to develop their technical skills on using Geogebra in the pre-conference. For instance, the coach demonstrated how to construct square in Geogebra or how to add sliders in Geogebra. As considering the study of Jaipal-Jamani and Figg (2015), providing technical supports as a component of the mathematics coaching in this current study might help the teachers' TPACK development.

As another component of pre-conference in mathematics coaching, the teachers were supported by demonstrating to them effective Geogebra-enhanced geometry activities, in particular polygon units. Some research studies supports that exemplary curriculum materials help teachers to understand the innovations in curriculum and improve their knowledge in teaching (Van den Akker, 1988; Ottevanger, 2001). In the light of the literature, As a result of this study, demonstrating good examples in teaching geometry via Geogebra might help the teachers to understand how to modify the curriculum with integrating technology. Furthermore, Ferrini-Mundy and Breaux (2008) stated that "in the absence of professional development on instructional technology and curriculum materials that integrate technology use into the lesson content, teachers are not particularly likely to embed technology-based or technology-rich activities into their courses" (p. 437). Niess et al., (2009) revealed that accessing technology without knowledge of mathematics curriculum materials did not support teachers in integrating the

technology in their mathematics classroom. As considering the literature, in this current study mathematics coaching focusing on Geogebra-enhanced curriculum materials might have contributed that teachers effectively integrated Geogebra in their geometry lesson. According to Jaipal-Jamani and Figg (2015), teachers' TPACK includes knowledge about a repertoire of technology-enhanced activity types which appropriate to teach the content. Furthermore, developing a repertoire of technology-enhanced activities is necessary for teachers' development of TPACK (Hofer & Harris, 2010). In light of these studies, giving good examples of Geogebra integration in this study might have satisfied the need of teachers. By demonstrating exemplary Geogebra-enhanced activities in the pre-conferences, the teachers got the chance to understand how to use them to teach geometry in the classroom. Therefore, this property of mathematics coaching might help the teachers to progress through TPACK Development Model regarding geometry.

In mathematics coaching, the teachers and coach worked collaboratively to design lesson in pre-conference. Yopp et al. (2011) emphasized the value of collaborating, reflecting, communicating, and examining teachers' own content knowledge and practice in professional development. Furthermore, Harris and Hofer (2009) stated that working on instructional planning facilitate discussion about technology, pedagogy, and content. Therefore, planning technology-enhanced lesson supports the growth of TPACK in a dynamic way. When considered from this point of view, the co-planning in mathematics coaching in this study might have contributed to teachers' TPACK development. Costa and Garmston (2002) argued that coaching has the most effective support function such as co-planning that can contribute to transforming teacher practice. According to scaffolding changed the teaching practices and beliefs with regard to design and use of technology in the lessons to support mathematics teaching (So & Kim, 2009). In parallel to these studies, co-planning Geogebra-enhanced lessons in the pre-conferences might support the teachers' practice via Geogebra. In addition, Polly and Hannafin (2011) stated that co-planning with more knowledgeable individuals, such as in mathematics coaching, help the teachers in teaching practice via technology.

Supportively, they indicated that teachers may not enact fully their TPACK until they are adequately supported in lesson planning by considering Tharp and Gallimore (1988)'s neo-Vygotskian construct of teaching as assisted performance. In the context of supporting teachers' learning, Tharp and Gallimore redesigned Vygotsky (1978)'s idea of the Zone of Proximal Development (ZPD) defined as the distance between what learners can accomplish independently and what that same learner can accomplish while working with more knowledgeable people. In the light of ZPD, Tharp and Gallimore (1988) stated that teachers need scaffolding especially in co-planning lessons with more knowledgeable individuals to be able to implement effective lessons. In addition, the research studies revealed that teachers' co-planning with more knowledgeable individuals, such as coaches, is useful for teachers' professional development (Polly, 2008b; Polly & Hannafin, 2011; Stein & Brown, 1997). In this current study the coach as more knowledgeable person provided scaffolding for the teachers to design Geogebra-enhanced geometry lessons. In parallel to literature, this scaffolding, which includes a social interaction with the coach as a more knowledgeable person, might help teachers to maximize their development of TPACK. In other words, the scaffolding of the coach might help the teachers to enact their highest TPACK in the ZPD. In addition, co-planning with the coach as a more knowledgeable person might help the teachers in their implementation via Geogebra. Furthermore, the researchers highlight the importance of scaffolding and collaborative relationships in bringing about teacher change (Attard, 2007; Blank, 2010; Day, 1999; Fullen & Hargreaves, 1992; Hargreaves & Shirley, 2009, Mclaughlin, 2012). Furthermore, collaborating with a more knowledgeable person is also important while teachers are integrating computer technology into their lesson (Evoh, 2007). In parallel to the literature, the teachers' TPACK in this current study might have enhanced through mathematics coaching as teachers worked collaboratively with their coach to respond to student needs via Geogebra. Scaffolding and working collaboratively provided social interaction between the coach and the teachers to improve the teachers' technology integration. As stated in the literature, scaffolding in this social interaction might be

a crucial component of teacher development and increases teachers' knowledge about technology integration in their geometry lesson. In the light of these, the coach in this study might provide scaffolding for the teachers to improve knowledge about how to integrate Geogebra in their teaching strategies, and how to use Geogebra to facilitate student learning of geometry. By means of a collaborative method, such as scaffolding, teachers might establish connections among pedagogy, content, and technology. Thus, co-planning technology-enhanced lessons by scaffolding of the coach might have contributed that teachers progressed through TPACK Development Model in all themes and their related descriptors.

Observation of teachers as a part of mathematics coaching is also emphasized in this study. In the observations session in this current study, the teachers implemented the co-planned lesson via Geogebra to develop students' understanding about polygon unit. According to Jaipal-Jamani and Figg (2015), teachers should support to implement technology-enhanced lesson to practice new technical skills. In addition, this teaching practice can help the teachers to promote their TPACK. Penuel et al. (2007) stated that professional development should take place at the school or district levels in order to facilitate a change in teacher practices. In the light of these studies (Jaipal-Jamani & Figg, 2015; Penuel et al., 2007), teachers' implementation of Geogebra-enhanced lesson at their schools might contribute to teachers' TPACK development regards geometry in this current study. Besides, the teachers' implementation of Geogebra-enhanced lesson in mathematics lesson might help them to put together theory and practice. Furthermore, Kushner Benson, Ward, and Liang (2015) stated that professional development embedded in teachers' real-world work is most effective to develop teachers' knowledge. Thus, like this current study providing professional development with teachers' implementation Geogebra-enhanced lesson at their school might help improving the teachers' knowledge about how to integrate Geogebra as a learning and teaching tool. In addition, Desimone (2009) reported that traditional professional development such as one-shot workshops and conferences do not address the needs of teachers. The teacher should be observed in

their classroom to identify their strengths and weaknesses in practice. From this perspective, coaching has been specified as a highly promising professional development programme (Desimone 2011; Jeanpierre et al., 2005; Johnson et al., 2007; Penuel et al., 2007). In this current study, teachers were observed to collect data from their implementation by coach. The collecting data from their implementation in this mathematics coaching process helped to reveal teachers' TPACK regarding Geogebra. Thus, teachers' implementation of Geogebra-enhanced lessons might provide information about their strengths and weaknesses in the practice of technology integration to improve the teachers' TPACK. Furthermore, in the light of the literature, focusing on teachers' implementation via Geogebra in this current study might have helped to respond to the teachers' needs with respect to combining theory and practice. Implementation of coaching at the school or district levels might contribute to the development of teachers' knowledge about how to integrate Geogebra into their geometry classrooms.

In another component of mathematics coaching in this study (post-conference), teachers were encouraged to reflect on their performance in terms of technology, pedagogy and content. Teachers' reflections as a component of post-conference might provide an environment for the teachers to improve their TPACK and practice in this current study. Munoz-Catalan, Carillo-Yanez, and Climent-Rodriguez (2010) stated that reflection plays a crucial role to develop teachers' sufficient competency. In addition, teachers' reflection on the difficulties encountered in their practices is essential for their improvement (Goodell, 2006). In parallel to literature, after the last cycle of mathematic coaching, the teachers also made progress in the TPACK Development Model. Compared with MCC4 performance, Murat's TPACK level progressed in the assessment descriptor in the curriculum and assessment theme, mathematics learning, and conception of student thinking descriptors in the learning theme, and the instructional and environment descriptors in the teaching theme; on the other hand, Esen's TPACK level growth in the assessment descriptor in the curriculum and assessment theme, and the conception of student thinking descriptor in the learning theme. This progress

provides evidence that the components of post-conference might help the teachers to improve their teaching knowledge with technology because in the last lesson in this study -which was after mathematics coaching- the teachers did not receive any support to design and implement the lesson via Geogebra. The last stage of the professional development was the post-conference in MCC4. The post-conference in this current study might have helped the teachers to realize their weaknesses and strengths in incorporating technology as a learning and teaching tool. The post-conference, enabled them to make progress in the TPACK Development Model from the lesson in MCC4 to the lesson after mathematics coaching. Thus, the reflection and the suggestions on teachers' implementation in the post-conference help the teachers to develop their knowledge about technology-integration in geometry lesson. Furthermore, Özgün-Koca, Meagher, and Edwards (2011) emphasized the importance of reflection on learning experiences in professional development. Thus, the teachers need to reflect on how their technological capabilities might help their students' learning. In this current study provided an environment for the teachers to reflect their ideas about how they use Geogebra in their geometry lesson and to take suggestions about their usage in the post-conference. This environment might have helped the teachers to see their weaknesses and strengths from the perspective of the themes and descriptors of the TPACK Development Model and analyze how to improve themselves with the help of the suggestions made by the coach. Thus, post-conferences which include reflections and recommendations on teachers' implementation might have made a contribution to teachers' TPACK development.

Considering the effectiveness of mathematics coaching in the development of teachers' TPACK, the components of mathematics coaching (pre-conference, observation, post-conference) have a critical role in developing teachers' TPACK. To sum up, the properties of the pre-conferences, observation, and post-conferences in the mathematics coaching (such as; gaining insight into the teacher's TPACK, providing technical assistance, demonstrating exemplary technology-enhanced activities, co-planning technology-enhanced lessons, implementing the new

practice, collecting data from the implementation, reflecting on the implementation, and making recommendations on the implementation) made contributions to the in-service teachers' moving from PCK to TPACK regarding geometry.

5.2 Discussion on Teachers' Perceptions of Mathematics Coaching

The teachers' perspectives about the mathematic coaching, which focus on improving their TPACK in the polygon unit, concentrate on five major themes: (1) benefits of the mathematics coaching, (2) differences between mathematics coaching and regular in-service training, (3) challenges of mathematics coaching, and (4) suggestions for mathematics coaching.

The teachers' views about the benefits of mathematics coaching include *improving TPACK, positive instructional change, increasing students' achievement, providing scaffolding/collaborating, and building confidence*. The teachers expressed that one of the most important benefits of mathematics coaching was *improving their TPACK*. Research studies have also found evidence of coaching leading to changes in teachers' knowledge and practice (Black, Molseed & Saylor, 2003; Costa & Garmston, 2004; Olson & Barret, 2004). Furthermore, the teacher participating in mathematics coaching in a study by Larsen (2012) stated that coaching helps them to improve their knowledge and practice. These research studies demonstrate that mathematics coaching is an effective professional development to improve teachers' knowledge. These research studies provide evidence that mathematics coaching assists teachers in their development of knowledge in technology integration into mathematics lesson. Besides, perception of teachers participating in this current study also advocated that mathematics coaching helps improve teachers' TPACK. Furthermore, they thought that the pre-conferences, observations, and post-conferences as components of mathematics coaching help them to change their TPACK positively. Thus, the teachers thought that mathematics coaching is effective professional development to improve their TPACK.

According to teachers in this study, improving *instructional practice* is another important benefit of mathematics coaching. They expressed their views of how participating in mathematics coaching created instructional change in their Geogebra-enhanced mathematics lesson. One of the teachers stated that mathematics coaching helps to notice their mistake in the technology integration, such as not using Geogebra dynamically to improve students' understanding. Furthermore, they implemented a teacher-led students-follow lesson at the beginning of the study. Through the mathematics coaching, they guided their students to explore the mathematical concepts. They stated that the mathematics coaching helped to change their views about their instructional practice. Some research studies revealed that mathematics coaching has potential to positively influence teachers' instructional practices in their classroom (Baldinger, 2014; Campbell, 1996; Race, Ho, & Bower, 2002). Supportively, Bradley (2007) reported that teachers participating in coaching expressed their views that the changes in their instruction is as an outcome of coaching. Teachers also reported that they integrated models, such as manipulatives and representations for the establishment of connections among mathematics ideas and emphasized students' learning. In addition to these findings, mathematics teachers thought that mathematics coaching created instructional changes in their lessons (Drust, 2013). In the parallel to these studies, the teachers also expressed that mathematics coaching help them to improve their instruction by using Geogebra. They also stated that with the help of mathematics coaching, they incorporated new ideas into their teaching. Thus, mathematics coaching might make a contribution in changing their instructional practices.

Mathematics teachers in this study held the opinion that mathematics coaching helps to increase *students' achievement*. This perception demonstrates another benefit of mathematics coaching. Teachers improved their knowledge of how to integrate Geogebra to improve students' exploration during mathematics coaching. They stated that mathematics coaching established an environment in which students could potentially perform higher. Thus, mathematics coaching

helped the students directly and indirectly as a consequence of the improvement in the teachers' knowledge of how to develop students' understanding. Supportively, Desimone (2009) stated that professional development can cause to increase student achievement in an indirect manner. In addition, some research studies found increases in students' standardized test scores in the school which participated after coaching (Blount and Singleton, 2008; Brady, 2007; and Miles-Grant and Davenport, 2009). Furthermore, in particular to mathematics coaching, there are some research studies which revealed the positive influence of mathematics coaching as an effective professional development on students' mathematics achievement (Balfanz, Mac Iver, & Byrnes, 2006; Brosnan & Erchick, 2010; Campbell & Malkus, 2011, Foster & Noyce, 2004; Harbour, Karp, & Adelson, 2016). In parallel to the literature, mathematics coaching as a professional development in this current study might provide an environment for students to improve their knowledge and achievement in geometry as their teachers improved their knowledge about integrating Geogebra as learning and teaching tool.

The teachers in this study expressed that *providing scaffolding/collaborating* especially in the pre-conferences was one of the crucial benefits of mathematics coaching. They indicated that scaffolding especially during lesson planning was beneficial for them. Furthermore, one of the teachers stated that without co-planning, s/he would not be able to incorporate Geogebra effectively in the lesson. Scaffolding helped the teachers to become aware of their weaknesses and strengths in technology integration. Furthermore, previous research studies advocated that teachers emphasize the importance of the coach's scaffolding and collaboration in mathematics coaching (Drust, 2013; Larsen, 2012). In a study by Drust (2013), mathematics teachers' opinion was that the coach's support in lesson planning established an environment in which students performed higher. Additionally, Larsen (2012) stated that a collaborative relationship between teachers and coaches is essential for teacher engagement in mathematics coaching. The teachers in a study by Larsen (2012) indicated that if the coach did not support them, continuing mathematics coaching was meaningless for them. To sum up, the results of this

study, along with those of previous research, provide evidence of the view held by teachers that providing scaffolding to teachers is a key element necessary for professional development. Thus, mathematics coaching in this study might be one of promising professional development by providing scaffolding for teachers about how to integrate Geogebra in their geometry lesson.

The teachers in this study also reflected that they were also supported emotionally such as *building confidence* during the mathematics coaching. They stated that they gained self-confidence in teaching with Geogebra with the help of their coach. Supportively, in a study by Drust (2013), the teachers participating in the mathematics coaching expressed that they felt more confident because they were able to acquire new knowledge and new strategies and ideas for teaching mathematics with the support of the coach. The results of the current study, along with those reported in previous research by Drust (2016), provide evidence of the positive influence that mathematics coaching has on teachers' confidence. The teachers learned new ideas regarding the integration of Geogebra into their mathematics lesson and it might boost their confidence in teaching via Geogebra-enhanced activities.

The teachers' opinion about the differences between mathematics coaching and regular in-service teacher training emphasizes *focusing process* and *putting together theory and practice* in mathematics coaching. The teachers expressed that mathematics coaching is not a one-day workshop isolated from practice; rather, it focuses on *process* in long-term professional development. Traditional in-service teacher professional development occurs in short-term workshops that are separated and disconnected from practice (Ball & Cohen, 1999). However, research studies indicated that in-service and pre-service teachers need more time to develop their knowledge and practice new approaches in their lessons (Agyei & Voogt, 2012; Ball & Cohen, 1999; Fishman & Davis, 2006; So & Kim, 2009). Furthermore, in-service teacher training should be long-term, not a one-day workshop which is separated and disconnected from the classroom and curriculum (Ball & Cohen, 1999). Professional development should be seen as process-driven instead of fragmented

occurrences. In addition to the focus on process in mathematics coaching, teachers expressed that mathematics coaching differs from traditional in-service teachers training in terms of *putting together theory and practice*. The teachers were supported to establish alignment between theory and practice during mathematics coaching in this current study. The study of Wei et al (2009) indicated that much of the traditional professional development is not useful in improving themselves. According to Loucks-Horsley et al. (2003), the reason underlying this situation can be that teachers' learning is disconnected from their practice. The practice-based professional development, such as coaching, focuses on teachers' practice in a real world context (Ball & Cohen, 1999). This professional development helps teachers in acquiring, refining, and transferring theoretical knowledge and skills into practice. In parallel to Ball and Cohen (1999)'s study, in this current study mathematics coaching might help the teachers to put ito together theory and practice, because they designed and implement the Geogebra-enhanced lesson. Designing lesson might provide an environment for teachers to recognize the theoretical knowledge, and then implemented the designed lesson might support them to focus on their practice.

The teachers' view about the challenges of mathematics coaching includes *preparing the lesson*, and *being stressful*. The teachers stated that the mathematics coaching did not have any disadvantages or deficiency. They believed that mathematics coaching had positive impacts on their knowledge. Despite this situation, they stated that preparing lessons was tiring and took time although it was necessary for their improvement. Thus, Teachers' intensive effort in preparing lesson might cause the difficulties. In addition, one of the teachers expressed that she was stressful at the beginning of mathematics coaching because of lack of confidence. However, through mathematics coaching the teacher overcame this feeling. The reason of teachers' emotion might be the lack of the confidence in integrating Geogebra. Through at the end of the study she learned how to integrate Geogebra in her lesson. Thus, it might help the teachers to overcome her stress.

The teachers' opinions regarding suggestions for mathematics include *should be common in Turkey, should be semester-long, should be conducted different subject, should be used with different mathematics software, and should be in group coaching*. The teachers indicated that mathematics coaching should be a part of regular in-service teacher training programmes and should be common in Turkey. Professional development in Turkey does not content-centric and they separated from practice. Thus the teachers did not any opportunities to focus on how to integrate Geogebra in polygon as geometry concepts in this current study. Furthermore, this current study focused on teachers' practice on technology integration. This situation might be the reason of their view that mathematics coaching should be prevalent in Turkey. Furthermore, considering its advantages, they expressed that mathematics coaching should be conducted during the whole semester. They suggested that mathematics coaching can be conducted to improve teachers' knowledge in the other unit via other software. Mathematics coaching can also provide an environment for teachers to enhance their knowledge in algebra, probability and statistics, number and operation via using Tinkerplots, Cabri and Geogebra. In addition to these suggestions, one of the teachers indicated that if mathematics coaching were conducted in groups, not one-on-one, it might be more helpful to teachers because when they are in a group, teachers can share their ideas with each other to analyze their weaknesses and strengths in their lessons. The reason of his suggestion that he was novice teachers and he might want to explore other teachers' experience.

5.3 Summary of Discussion

The data collected from two mathematics teachers in this study before, during and after mathematics coaching yielded eleven major results:

- 1) Mathematics teachers moved from PCK to TPACK by participating in mathematics coaching as indicated in TPACK Development Model.

- 2) Mathematics teachers exemplified primarily lower levels of TPACK development (recognizing, accepting, and adapting) in four themes and their related descriptors before mathematics coaching.
- 3) The more experienced teacher was at a lower level than the less experienced teacher in the TPACK Development Model before mathematics coaching.
- 4) Mathematics teachers demonstrated progress into a higher level in the TPACK development Model in all themes and their related descriptors during mathematics coaching.
- 5) The slowest progress was observed in the assessment descriptor among all the descriptors in which progress was made during mathematics coaching.
- 6) The more-experienced teacher's progression in the TPACK Development Model was faster than that of the less-experienced teacher during mathematics coaching.
- 7) Mathematics teachers reached the highest level in the TPACK development Model in all themes and the related descriptors after mathematics coaching.
- 8) The teachers' views about the benefits of mathematics coaching include progress in TPACK, positive instructional change, increase in students' achievement, provision of scaffolding/collaboration, and increase in confidence.
- 9) The teachers' opinions about the differences between mathematics coaching and regular in-service teachers training emphasize focusing process, and putting together theory and practice in mathematics coaching.
- 10) The teachers' views about the challenges of mathematics coaching include preparing lesson plans and being stressful.
- 11) The teachers' opinions as regards suggestions for mathematics include being common in Turkey, being semester-long, being different subject, being different mathematics software, and being in group coaching.

5.3 Implications of the Study

In this current study, there are two main implications of the study: implications for TPACK Development Model, and implications for mathematics coaching

5.3.1 Implications for TPACK Development Model

In this current study, the teachers' knowledge about technology integration was analyzed according to the TPACK Development Model (Niess et al., 2009). This model helps to identify the in-service teachers' TPACK in detail according to the four themes and eleven descriptors. Considering that the TPACK Development Model provides detailed information about teachers' knowledge on technology integration, this model is beneficial to reveal teachers' TPACK.

The frameworks of TPACK in the literature, such as Mishra and Koehler's (2006) TPACK framework, are general and not well-defined. In the light of this, TPACK Development Model becomes prominent because it provides detailed information about teachers' knowledge. In addition, the frameworks of TPACK are more generic rather than discipline specific. At this point, the TPACK Development Model (Niess et al., 2009) has advantages for revealing mathematics teachers' TPACK because it is subject-specific and focuses on the teacher's knowledge of incorporating technology into teaching mathematics. Furthermore, the TPACK Development Model is proposed to describe mathematics teachers' progression in developing this specialized knowledge by using five TPACK levels (Niess et al., 2009). Therefore, the TPACK Development Model (Niess et al., 2009) can be used to reveal the in-service teachers' development through a professional development course. To sum up, this TPACK Model can provide rich information about in-service mathematics teachers in terms of not only the four themes and their eleven descriptors but also the five TPACK levels. However, this model might not be appropriate for pre-service teachers because it needs intensive data to evaluate four themes and eleven descriptors. In addition, this model mostly focuses on how to integrate technology into the mathematics lessons to improve the students' knowledge. In other words, it emphasizes the implementation of technology in teachers' actual instructional practices in their mathematics classrooms. Therefore, it might not be effective to analyze the pre-service mathematics teachers' TPACK.

With respect to the TPACK Development Model, this study indicated that the teacher's TPACK level can vary across the different themes and descriptors. For instance, before mathematics coaching, Murat was at the adapting level in the conception of students thinking descriptor in the learning theme in TPACK, whereas he was at the recognizing level for the assessment descriptor in the curriculum and assessment themes. Therefore, it is hard to identify the teachers' TPACK level in general. Furthermore, the TPACK Development Model needs extensive study and different data sources as in this current study (pre-interviews, pre-conferences, observations, post-conferences, and post-interviews) to provide evidence about teachers' TPACK level for the themes and descriptors. Despite the collection of intensive data, it can be difficult to identify teachers' TPACK levels in the themes and descriptors because some descriptors of the themes are intertwined. In addition, some of the descriptors are not well-defined and tacit.

The curriculum and assessment theme and its related descriptors (curriculum, and assessment descriptors) were clear and well-defined according to the analysis of intensive data of this study. However, there are some challenges in identifying the teachers' knowledge of technology integration in curriculum descriptors. For instance, before mathematics coaching, Murat integrated Geogebra to improve students' understanding via exploring the new concepts instead of reinforcing the already known concepts in the mathematics classroom. It can be indicated that Murat "understands some benefits of incorporating appropriate technologies as tools for teaching and learning the mathematics curriculum" (Niess et al., 2009, p.20) at *the adapting level*. However, he did not integrate Geogebra beyond the paper and pencil, such as by dragging, to develop geometric concepts. He replaced non-technology based tasks in the traditional curriculum without considering the dynamic properties of Geogebra. Therefore, he should have been at a lower level than *the adapting level*. In addition, the explanation of *the accepting level* of the curriculum descriptor is as follows: "expresses desire but demonstrates difficulty in identifying topics in own curriculum for including technology as a tool for learning." (Niess et al., 2009, p.20). However, Murat's situation did not

perfectly match this description. *The accepting level* of the curriculum descriptor should be adjusted to match the data of Murat. Thus, it can be modified as “expresses desire but demonstrates difficulty in enhancing/modifying topics in own curriculum for including technology as a tool for learning by considering the nature of technology.”

The levels of the assessment descriptor perfectly matched the data of this current study. On the other hand, there is lack of information about how to integrate technology according to its affordances in the assessment. For instance, when the teachers in this study were at the adapting level, they assessed the students’ conceptual and procedural knowledge, but most of time they did not use Geogebra dynamically in assessment. Therefore, the description of *the adapting level* of the assessment descriptor can be modified as “understands that if technology is allowed during assessments that different questions/items must be posed mostly without considering the affordance technology (i.e., conceptual vs. procedural understandings)” (Niess et al., 2009).

The learning theme and its related descriptors (mathematics learning descriptor, and conception of the students descriptor) were intertwined. For instance, both mathematics learning and conception of the students descriptors focus on using technology as a learning tools. In the adapting level, the mathematics descriptors are stated as “Begins to explore, experiment and practice integrating technologies as mathematics learning tools.” (Niess et al., 2009, p.21), whereas the descriptor for the conception of mathematics students is stated as “Begins developing appropriate mathematical thinking skills when technology is used as a tool for learning.” (Niess et al., 2009, p.21). As can be seen in these description, there is not a clear distinction between them. In order to make a clear distinction, the statements of these descriptors can be modified. For instance, mathematics learning descriptors might mostly focus on students’ exploration with technology as a learning tool. In addition, the conception of the students descriptor might emphasize how to integrate technology to overcome students’ misconceptions and to improve

their higher level thinking. For instance, in the MCC2 Murat emphasized non-prototype quadrilaterals to overcome possible children's difficulties via Geogebra. Students have possible difficulties as the quadrilaterals were not in the prototypical shape and in the prototypical orientation. Murat provided an environment for students to realize that the properties of quadrilaterals did not change as their positions changed via dragging of vertex of quadrilaterals. But the exploring level of the conception of mathematics students does not include any statement about students' misconception. Therefore, the description of *the exploring level* can be modified as "plans, implements, and reflects on teaching and learning with concern for guiding students in understanding and overcoming possible students' misconception and difficulties" (Niess et al., 2009).

The teaching theme and its related descriptors (mathematics learning descriptor, instructional, environment and professional development descriptor) were also intertwined. There are some challenges in identifying the teachers' knowledge of technology integration in mathematics learning descriptors in the teaching theme. For instance, Murat used Geogebra to enhance students' thinking, which indicated that he was above *the accepting level* before mathematics coaching because he did not use Geogebra to engage students at the end of the units for "days off" extra-curricular activities. In addition, he did not incorporate Geogebra to enhance students' higher-level thinking (such as problem solving and decision-making activities) for learning mathematics using the technology, which indicated *the exploring level*. This situation placed him at *the adapting level* of the mathematics learning descriptor in the teaching theme. The phrase of *the adapting level* in the mathematics learning descriptor is as follows: "Uses technology to enhance or reinforce mathematics ideas that students have learned previously." (Niess et al., 2009, p. 22). This did not fit Murat's situation. Therefore, this description might need to be modified as "Uses technology to enhance mathematics ideas *as new concepts* or reinforce mathematics ideas that students have learned previously." Besides, the differences between instructional and environment descriptors are not well-defined. For instance, the exploring level of the

environment descriptor is as follows: “Explores various instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics” (Niess et al., 2009, p. 23), whereas the advancing level of the instructional descriptor is as such: “adapts from a breadth of instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics” (Niess et al., 2009, p. 23). As can be seen, both of these descriptors emphasize instructional strategies. Therefore, these descriptors should be modified to reveal the differences between them, or these descriptors should be combined into just one descriptor.

The access theme and its related descriptors (usage, barrier and availability descriptors) were clear and well-defined according to the intensive data of this study. The notions of these descriptors were effective to identify the teachers’ TPACK level in terms of the data of this study. On the other hand, the availability descriptors might strongly emphasize multiple representations in its related description. For instance, *the adapting level* of the availability descriptor is as follows: “Concepts are taught differently since technology provides access to connections formerly out of reach” (Niess et al., 2009, p. 23). In this description, there is a lack of emphasis on multiple representations such as table, picture, and written symbols to improve the connections of geometri concepts. Therefore, this description could be modified as “Concepts are taught differently since technology provides access to connections formerly out of reach, and supply multiple representations” (Niess et al., 2009, p. 23).

To sum up, the TPACK Development Model (Niess et al., 2009) is a useful framework for detailed analysis. However, considering the data of this research study, this model needs small modifications.

5.3.2 Implications for Mathematics Coaching

The results of this study have suggested some implications that need to be taken into consideration by teacher educators and the researchers who are interested

in in-service teachers' TPACK development. The current study demonstrated that in-service mathematics teachers progressed to higher levels in the TPACK development Model regarding geometry during mathematics coaching. In fact, their TPACK reached the highest TPACK level in all the themes –curriculum and assessment, learning, teaching and access– although they had started at the lowest level. Thus, considering this development, it can be said that mathematics coaching is an effective professional development model to improve in-service mathematics teachers' TPACK. In addition, the teachers agreed that mathematics coaching helps them to improve their TPACK and practice in technology integration. Considering the effectiveness of mathematics coaching in the development of teachers' TPACK, it can be said that the components of mathematics coaching (pre-conference, observation, post-conference) have a critical role in developing teachers' TPACK. The result of this study indicated that the properties of the mathematics coaching help the teachers moving from PCK to TPACK regarding geometry. Thus, teachers educators and the researcher interesting in professional development should consider these properties of the mathematics coaching in this study: gaining insight into the teacher's TPACK, providing technical assistance, demonstrating exemplary technology-enhanced activities, co-planning technology-enhanced lessons, implementing the new practice, collecting data from the implementation, reflecting on the implementation, and making recommendations on the implementation. Furthermore, the literature also supports these properties of mathematics coaching to improve teachers' knowledge. To sum up, this study claimed that mathematics coaching helped the teachers' development in TPACK. To facilitate a change in teachers' knowledge and practices, professional development should take place within the classroom (Penuel et al., 2007). In addition, teachers need to be involved in practices, such as planning, implementing, and reflecting on technology lessons to improve their TPACK. In the light of the literature and the results of the current study, mathematics coaching might be an effective professional development model to improve teachers' knowledge (in particular technology integration).

5.4 Recommendations for Further Research

Based on the results of the current study, the researcher proposed a TPACK-focused Mathematics Coaching as a professional development model. In future studies, this model can be conducted to improve teachers' TPACK. In addition, further research studies can implement this model to evaluate its effectiveness.

Before/during/after the mathematics coaching, the development of teachers' TPACK in geometry and their progression through the levels of teachers' TPACK in geometry were analyzed in this study. According to the results of the current study, by participating in mathematics coaching, the in-service teachers improved their knowledge of how to integrate Geogebra into their geometry lesson by combining their knowledge of pedagogy, content and technology. In further research, mathematics coaching can be conducted in other learning areas also (such as algebra, measurement, data analysis and probability) or other dynamic software (such as Tinkerplots, Fathom, and Cabri3D).

In the light of the current research and literature, mathematics coaching can be made more widespread in teacher education in Turkey. Professional development necessitates combining theory and practice in Turkey. The traditional professional development courses are kept independent of and disconnected from classroom practice. This approach to professional development is insufficient to help teachers to enhance their knowledge especially in technology integration. Considering the results of the current study, it can be asserted that mathematics coaching provides an effective alternative in teachers' professional development. It is recommended that specialists involved in teacher education in Turkey consider mathematics coaching to develop in-service and pre-service teachers' knowledge.

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APPENDICES

APPENDIX A

TPACK INTERVIEW PROTOCOL (IN ENGLISH)

1. How would you integrate technology into the “polygons” unit in the 7th grade mathematics syllabus?
 - To what extent would you keep to the school curriculum and the textbook while implementing the technology-enhanced “polygons” unit? What kinds of changes would you make? Why?
2. Could you explain how you would integrate technology into your lessons during the instruction of the “polygons” unit?
 - What are the technological tools that you would use in the instruction of this unit? Why?
 - Could you give a detailed example of when and how you would use technology in the “polygons” unit?
 - What are the reasons of using these tools? Please explain.
3. For what purpose would you use technology in enabling students to learn the “polygons” unit? Why?
 - Do you see technology as a means for teaching or learning? Why?
 - At what stage of the lesson would you use technological devices? What role does technology play on students?
 - What is the role of technology in students’ discovery of a concept? Why? If so, how? Could you explain in detail?
4. Could you explain in detail how you would use technology as a means of learning in the “polygons” unit?
 - What are the important concepts in the “polygons” unit?
 - Which specific technologies would be useful in increasing students’ understanding of this unit? Why?
 - What is the impact of the use of technology upon students’ understanding of the “polygons” unit?
 - How would you use technology to deepen students’ understanding of the unit? Could you give an example?

5. What are the probable misconceptions and difficulties that students might have regarding the “polygons” unit in the mathematics syllabus?
 - How would you make use of technology to eliminate your students’ misconceptions or the difficulties they face in understanding concepts? Could you provide detailed explanations for each of the following areas: the features of polygons, the classification of polygons, congruence and similarity of polygons, and the area of quadrilaterals?
6. Which features of technology would you take into consideration while presenting the technology-enhanced “polygon” unit?
 - Could you explain in detail how you would use technology as a means of instruction?
 - How do you think technology would contribute to this unit?
 - Would you use technology to teach a new concept to your students? Why? Could you give an example?
7. What are the instructional strategies, methods and techniques that you would recommend using in the instruction of the “polygons” unit?
 - Which instructional strategy would be beneficial in the integration of technology? How can technology be made use of during this process?
 - Do you use deductive and/or inductive strategies? Why? Could you explain by giving examples?
 - What is the role of learning in this course? How would you as a teacher use technology? Could you explain in detail?
8. What kind of an environment would you establish during the instruction of “polygons” unit?“
 - What is the role of technology in this course? What is the purpose of using technology in the instruction of this unit?
 - What do you pay attention to while integrating technology into the course? Why?
 - What is the role of the student in this course? Do you enable students to actively participate into the lesson? How?
 - Do students work in groups or individually?
 - When and how do students use technology? Could you explain in detail?
9. When do you provide students with the opportunity to use technology in the “polygons” unit? Why?

- What do you pay attention to regarding the integration of technology? Why?
 - What is the difference between explaining the “polygons” unit with and without the use of technology? What kinds of changes occur when technology is used?
 - What kinds of opportunities and limitations does technology possess in achieving the learning outcomes?
 - How do you think technology contributes to this course?
 - What is the purpose of using technology? Could you explain in detail?
10. What kinds of difficulties can be encountered with the integration of technology in the “polygons” unit?
- What difficulties can students face in the use of technology?
 - What would you do to eliminate these difficulties?
 - Do you have concerns with respect to experiencing difficulties in class management? Why? If you do, what would you do to overcome these difficulties? Could you explain in detail?
11. What are the multiple representations that can be used to foster students’ understanding of the “polygons” unit in the mathematics syllabus?
- How would you use technology while using multiple representations?
 - What impact would technology have on multiple representations?
 - How are you going to use technology in the transitions between the representations?
12. What are the measurement and evaluation techniques that you would recommend in assessing students’ knowledge on the concepts in the “polygons” unit?
- How would you make use of technology while evaluating the concepts in this topic?
 - What is the purpose of using technology during this stage?
13. What do you think of Professional development, such as in-service training and seminars?
- Would you like to participate in a technology related seminar, workshop or any other professional development event? Why?
 - What kind of a training on technology would you like to have?

APPENDIX B

TPACK INTERVIEW PROTOCOL (IN TURKISH)

1. 7. Sınıf matematik öğretim programında yer alan “çokgenler” ünitesine teknolojiyi nasıl entegre edersin?
 - a. “Çokgenler” ünitesini teknoloji destekli işlerken okul müfredatına ve ders kitabına hangi düzeyde bağlı kalırsın? Ne tür değişiklikler yaparsın? Neden?
2. “Çokgenler” ünitesinin öğretiminde teknolojiyi derse nasıl entegre edeceğini açıklar mısın?
 - a. Bu konunun öğretiminde kullanacağın teknolojik araçlar nelerdir? Neden?
 - b. “Çokgenler” ünitesinde teknolojinin ne zaman ve nasıl kullanılacağı ile ilgili ayrıntılı bir örnek verebilir misin?
 - c. Bu araçları kullanma nedenlerin nedir? Açıklayın.
3. “Çokgenler” ünitesinin öğrenci tarafından öğrenilmesinde teknolojiyi hangi amaçla kullanırsın? Neden?
 - a. Teknolojiyi bir öğretim aracı olarak mı öğrenme aracı olarak mı görüyorsun? Neden?
 - b. Teknolojik araçları dersin hangi aşamasında kullanırsın? Teknolojinin öğrenci üzerindeki rolü nedir?
 - c. Öğrencilerin bir kavramı keşfetmesinde teknolojinin rolü var mıdır? Neden? Eğer varsa nasıl? Ayrıntılı olarak açıklayabilir misin?
4. “Çokgenler” ünitesinde bir öğrenme aracı olarak teknolojiyi nasıl kullanacağını ayrıntılı olarak açıklayabilir misin?
 - a. “Çokgenler” ünitesindeki önemli kavramlar nelerdir?
 - b. Bu konudaki öğrenci anlamalarını artırmak için hangi özel teknolojiler yararlı olabilir? Neden?
 - c. “Çokgenler” ünitesinin öğretiminde teknoloji kullanımının öğrencilerin anlamasına etkisi nedir?
 - d. Öğrenci anlamalarını derinleştirmek için teknolojiyi nasıl kullanırsın? Örnek verir misin?

5. Matematik öğretim programında yer alan “çokgenler” ünitesinde öğrencilerin sahip olabileceği olası kavram yanılgıları ve zorlukları nelerdir?
 - a. Öğrencilerinizin anlamakta zorlandıkları kavramlar ya da kavram yanılgılarını gidermek için teknolojiden nasıl yararlanırsın? Çokgenlerin özellikleri, dörtgenlerin sınıflandırılması, çokgenlerde eşlik ve benzerlik ve dörtgenlerin alanları konusunda her biri için ayrı olarak ayrıntılı bir şekilde açıklayabilir misin ?
6. Teknoloji destekli “Çokgenler” ünitesini anlatırken teknolojinin hangi özelliklerini göz önünde bulundursun?
 - a. Bir öğretme aracı olarak teknolojinin nasıl kullanacağını ayrıntılı olarak açıklayabilir misin?
 - b. Teknoloji bu konuya nasıl katkı sağlayacağını düşünüyorsun?
 - c. Öğrencilerin yeni bir kavram öğretirken teknolojiyi kullanır mısın? Neden? örnek verebilir misin?
7. “Çokgenler” ünitesinin öğretiminde kullanılmasını önerdiğiniz öğretim strateji, yöntem ve teknikleri nelerdir?
 - a. Teknoloji entegrasyonunda hangi öğretim stratejisi faydalı olur? Bu süreçte teknolojiden nasıl yararlanılabilir?
 - b. Tümdengelim ve ya tümevarım stratejilerini kullanır mısın? Neden? Örnek vererek açıklayabilir misin?
 - c. Bu derste öğretmenin rolü nedir? Öğretmen olarak teknolojiyi nasıl kullanırsın? Ayrıntılı olarak açıklar mısın?
8. “Çokgenler” ünitesinin öğretiminde nasıl bir öğrenme ortamı hazırlarsın?
 - a. Teknolojinin bu dersteki rolü nedir? Bu konunun öğretiminde teknoloji kullanımının amacı nedir?
 - b. Derse teknolojiyi entegre ederken dikkat ettiğin şeyler neler? Neden?
 - c. Bu derste öğrencinin rolü nedir? Öğrenciler aktif olarak derse katılmalarını sağlar mısın? Nasıl?
 - d. Öğrenciler grup olarak mı yoksa bireysel olarak mı çalışırlar?
 - e. Öğrenciler teknolojiyi nasıl ve ne zaman kullanırlar? Ayrıntılı olarak açıklar mısın?
9. “Çokgenler” ünitesinde öğrencilerin teknoloji kullanmalarına ne zaman fırsat verirsin? Neden?
 - a. Teknoloji entegrasyonunda nelere dikkat edersin? Neden?

- b. “Çokgenler” ünitesini teknoloji kullanarak anlatma ile teknoloji kullanmayarak anlatma arasında nasıl bir fark var? Teknoloji kullanıldığında ne tür değişiklikler oluyor?
 - c. Belirlenen kazanımlara ulaşılmasında teknoloji ne tür imkân ve kısıtlamalara sahip?
 - d. Teknolojinin bu derse nasıl katkı sağlayacağını düşünüyorsun?
 - e. Teknoloji kullanımının amacı nedir? Ayrıntılı olarak açıklayabilir misin?
10. “Çokgenler” ünitesinde teknoloji entegrasyonunda karşılaşılabilecek zorluklar nelerdir?
 - a. Öğrencinin teknoloji kullanımında karşılaşılabileceği zorluklar nelerdir? Bu zorlukların giderilmesi için ne yaparsın?
 - b. Sınıf yönetiminde zorluk yaşama konusunda endişelerin var mı? Neden? Varsa bu zorlukları aşmak için ne yaparsın? Ayrıntılı olarak açıklar mısın?
11. Matematik öğretim programında yer alan “çokgenler” ünitesinde öğrencilerin anlamalarını güçlendirmek için kullanılabilecek çoklu temsiller nelerdir?
 - a. Çoklu temsilleri kullanırken teknolojiyi nasıl kullanırsın?
 - b. Teknolojinin çoklu temsiller üzerine etkisi nasıl olur?
 - c. Temsiller arası geçişte teknolojiyi nasıl kullanacaksınız?
12. Öğrencilerin “çokgenler” ünitesi kapsamındaki kavramlara ilişkin bilgilerini değerlendirirken kullanılmasını önerdiğiniz ölçme ve değerlendirme teknikleri nelerdir?
 - a. Bu konu kapsamındaki kavramları değerlendirirken teknolojiden nasıl yararlanırsınız?
 - b. Bu süreçte teknoloji kullanımının amacı nedir?
13. Hizmet için eğitim, seminer gibi profesyonel gelişim hakkında ne düşünüyorsun?
 - a. Teknoloji ile ilgili seminer, atölye veya herhangi bir profesyonel gelişim etkinliğine katılmak ister misin? Neden?
 - b. Teknoloji konusunda nasıl bir eğitim almak istersin?

APPENDIX C

PRE- CONFERENCE PROTOCOL (IN ENGLISH)

Section 1: Reviewing the lesson plan

1. What are the learning outcomes of the lesson you will be teaching?
2. Which resources did you refer to while preparing the lesson plan?
3. Have you prepared this technology-enhanced lesson plan in accordance with the way the unit is covered in the syllabus and textbook? To what extent did you keep to the textbook while preparing the activities to be used in the lesson? What kinds of changes did you make? Why?
4. What is the role of technology in this lesson? What is your purpose in using technology in this lesson? How did you integrate it into your lesson?
5. In which stage of the lesson are you going to use technology? For what purpose are you going to use technology? What will the role of technology be on students?
6. What impact does technology have on students' understanding of the unit in the lesson plan you prepared? Why? How have you made use of technology to enable students to gain a more in-depth understanding of the unit?
7. Does technology have a role in students' discovery of the concept stated in the learning outcome? Why? If so, how?
8. How do you use technology as a means of learning? In this lesson, what is the role of technology in enhancing students' mathematical thinking skill? For what purpose are the students going to use technology? Could you please explain?
9. What are the difficulties and misconceptions that students will have in relation to the topic you will explain? What do you think is the impact of technology on eliminating the difficulties and misconceptions that students may experience? How will you benefit from technology to overcome these difficulties?
10. Where and for what purpose have you used technology as a means of teaching in this lesson? Could you please explain?

11. What is the place of technology in teaching a new concept? Why? In this lesson which new concepts will your students be learning? And how did you integrate technology to teach these concepts?
12. Which questions will you be asking when teaching mathematics to your students in this technology-enhanced lesson? Why?
13. Which method and strategies are you using this lesson? How did you integrate technology into the method and strategies related to the stated learning outcome?
14. Do you use deductive or inductive strategies in the lesson? How? What is the impact of technology? How did you make use of technology in these strategies?
15. What is the role of the teacher in this lesson? What are you as a teacher making use of technology? What are your duties? Could you please explain?
16. What is the role of the students in this lesson? Are the students making effective use of technology? Are the students actively participating in the lesson? How?
17. Could you describe the learning environment in this lesson? What is the place of technology in this environment? What is the purpose of using technology in teaching this mathematical knowledge?
18. What do you pay attention to when integrating technology into the lesson? Why?
19. When in the lesson do you allow your students to use technology? Why?
20. How does technology affect the lesson you prepared? If you didn't use technology, what would be different? What difference does the use of technology create?
21. What opportunities and limitations does technology possess in achieving the stated learning outcome?
22. Which features of technology did you take into consideration while preparing the lesson? How do you think technology will contribute to this lesson?
23. What are the obstacles – difficulties – that you may face regarding the integration of technology into the lesson? Could you please explain?

24. Considering the technology that you will be using, how would you organize the classroom? Which difficulties that may arise during class management did you take into consideration while preparing this plan?
25. Do you think students will be experiencing difficulties as regards the technology you will be using in class? From which aspects? What kinds of measures did you take to overcome these difficulties?
26. Are you going to use multiple representations in the lesson? How? What is the impact of technology in using multiple representations during the explanation of the concept? Why? How will you be using technology during the transitions between the representations?
27. Did you use technology in the assessment stage of the lesson? Why? How did you use technology in the assessment stage? What are your opinions regarding this issue?
28. What did you think about professional development while preparing this lesson? What are your views regarding professional development activities? Would you like to participate in a technology related professional development activity such as a seminar and/or in-service training? Why?

Section 2: Reconstructing the Lesson Plan

- The effective aspects of the teacher's lesson plan are emphasized. Recommendations are made to develop them further.
 1. this is well thought of; well, can you add an activity such as.... ?
What do you think? Will it be useful?
- The activities that can remain weak in enabling students to learn the concept are discussed.
 2. Do you think this technology-enhanced activity that you prepared will develop your students' understanding of the unit? What other activities could there be? Do you think such an activity as would be beneficial for the students? Why?
- Sample activities are shown and discussed.
 3. What do you think of this activity: ...? Do you think they can enhance students' understanding of the unit?
- Suggestions related to technological pedagogical content knowledge are made.

4. What will happen when we use technology in way to eliminate students' misconceptions? Do you think it will be effective? What are your opinions regarding this issue?
5. (Tümdengelim ve tümevarım stratejilerine uygun etkinlikler gösterilip) What do you think about these activities: (activities that are suitable for deduction and those for induction are shown)? Can you implement them?
6. Could we implement multiple representations in way? What benefits will be achieved?
7. Do you think we could ask a question like to assess students' preliminary knowledge? What are your opinions?

III. Additional Comments

How was technology integrated into mathematics curriculum? To what extent was the curriculum adhered to?

What role does technology play on these students' understanding of the unit? Does technology play a role on students' discovery process ? How?

How was technology used in eliminating students' misconceptions? How was technology used in enhancing students' understanding of the unit?

How did the teacher use technology in the teaching process? How did technology contribute to mathematics instruction?

Which instructional strategy was used in the integration of technology? How? What is the role of the teacher in the lesson?

How was the learning environment? Did the students participate actively? How did the teacher manage the class while teaching the lesson?

What is the purpose of the use of technology in the lesson? When was permission given to use technology?

What are the difficulties faced by the teacher in the integration of technology?

Which multiple representations did the teacher use? How? How did the teacher make use of technology in the multiple representations?

How did the teacher assess students' understanding of the unit? What was the role of the teacher during the assessment process?

Was the teacher able to implement the lesson in accordance with the lesson plan?

Was there anything that proceeded differently from the lesson plan?

What are the advantages of using technology? Were any disadvantages observed in the use of technology? What are they?

APPENDIX D

PRE-CONFERENCE PROTOCOL (IN TURKISH)

Bölüm 1: Ders planını gözden geçirme

29. Anlatacağın dersin kazanımları nedir?
30. Ders planını hazırlarken hangi kaynaklara başvurdu?
31. Teknoloji destekli hazırladığın bu ders planını programdaki ve ders kitabındaki işleniş şekline göre mi düzenledin? Derste kullanılacak etkinlikleri hazırlarken ders kitabına hangi düzeyde bağlı kaldın? Ne tür değişiklikler yaptın? Neden?
32. Teknolojinin bu derste rolü nedir? Bu derste teknolojiyi kullanma amacın ne? Derse nasıl entegre ettin?
33. Teknolojiyi dersin hangi aşamasında kullanacaksın? Teknolojiyi hangi amaçla kullanacaksın? Teknolojinin öğrenci üzerindeki rolü ne olacak?
34. Hazırladığın ders planında teknolojinin öğrenci anlamalarına nasıl bir etkisi var? Neden? Öğrencilerin daha derinlemesine anlamaları için teknolojiyi nasıl kullandın?
35. Öğrencilerin kazanımda belirtilen kavramı keşfetmesinde teknolojinin rolü var mı? Neden? Eğer varsa nasıl?
36. Bir öğrenme aracı olarak teknolojiyi nasıl kullanıyorsun? Bu derste öğrencilerin matematiksel düşüncelerini geliştirmede teknolojinin rolü nedir? Teknolojiyi öğrenciler hangi amaçla kullanacaklar? Açıklar mısın?
37. Anlatacağın kavrama yönelik öğrenci zorlukları ve kavram yanılgıları nelerdir? Sence öğrenci zorluklarının ve kavram yanılgılarının giderilmesinde teknolojinin etkisi nedir? Bu zorlukları gidermek için teknolojiden nasıl faydalanacaksın?
38. Bir öğretme aracı olarak teknolojiyi bu derste nerede ve hangi amaçla kullandın? Açıklar mısın?

39. Yeni bir kavramın öğretilmesinde teknolojinin yeri nedir? Neden? Öğrencilerin bu derste hangi yeni kavramları öğrenecekler? Peki teknolojiyi bu kavramların öğretilmesinde nasıl entegre ettin?
40. Teknoloji destekli bu derste öğrenciler matematiği öğretirken hangi soruları soracaksınız? Neden?
41. Derste hangi yöntem ve stratejileri kullanıyorsunuz? Belirlenen kazanıma yönelik yöntem ve stratejilere teknolojiyi nasıl entegre ettin?
42. Derste tümdengelim ve ya tümevarım stratejilerini mi kullanıyorsunuz? Nasıl? Teknolojinin etkisi ne? Teknoloji bu stratejilerde nasıl ele aldın?
43. Bu derste öğretmenin rolü ne? Teknolojiyi kullanımında öğretmen olarak neler yapıyorsunuz? Görevleriniz neler? Açıklar mısınız?
44. Bu derste öğrencinin rolü ne? Öğrenciler teknolojiyi etkin şekilde kullanıyor mu? Öğrenciler derse aktif olarak katılıyor mu? Nasıl?
45. Bu derste öğrenme ortamını açıklayabilir misin? Bu öğrenme ortamında teknolojinin yeri ne? Öğrencilere bu matematiksel bilgiyi öğretirken teknoloji kullanmanın amacı nedir?
46. Derse teknolojiyi entegre ederken dikkat ettiğin şeyler neler? Neden?
47. Bu derste öğrencilerin teknoloji kullanmasına ne zaman izin veriyorsun? Neden?
48. Hazırladığın dersti teknoloji nasıl etkiliyor? Eğer teknoloji kullanmasaydın neler değişirdi? Teknoloji kullanımının farkı ne?
49. Belirlenen kazanıma ulaşılmasında teknoloji ne tür imkân ve kısıtlamalara sahip?
50. Dersi hazırlarken teknolojinin hangi özelliklerini göz önünde bulundurdun? Teknolojinin bu derse nasıl katkı sağlayacağını düşünüyorsun?
51. Derse teknoloji entegrasyonunda karşılaşılabileceğin engeller –zorluklar-nelerdir? Açıklayabilir misin?

52. Kullanacağın teknolojiyi düşünerek, sınıfı nasıl organize ettin? Bu planlamayı yaparken, sınıf yönetiminde çıkabilecek hangi zorlukları göz önüne aldın?
53. Derste kullanacağınız teknolojiye yönelik öğrencilerin zorluk yaşayacağını düşünüyor musun? Hangi noktalarda? Bu zorlukların aşılması için nasıl tedbir aldınız?
54. Derste çoklu temsilleri kullanacak mısın? Nasıl? Anlatacağın kavramda çoklu temsilleri kullanırken teknolojinin etkisi nedir? Neden? Temsiller arası geçişte teknolojiyi nasıl kullanacaksın?
55. Bu derste değerlendirme sürecinde teknolojiyi kullandın mı? Neden? Değerlendirme sürecinde teknoloji nasıl kullanılmalı? Bu konu hakkında ne düşünüyorsun?
56. Bu dersi hazırlarken profesyonel gelişim hakkında ne düşündünüz? Profesyonel gelişim etkinlikleri hakkında ne düşünüyorsun? Teknoloji ile ilgili bir seminere, hizmetiçi eğitim gibi bir profesyonel gelişim etkinliğine katılmak ister misin? Neden?

Bölüm 2: Ders planını yeniden oluşturma

- Öğretmenin ders planında bulunan iyi fikirler vurgulanır. Geliştirmesi için tavsiyelerde bulunulur.
 8. güzel düşünmüşsün peki şeklinde bir etkinlik ekleyebilir misin? Ne düşünüyorsun? Faydalı olur mu?
- Öğrencilerin kavramı öğrenmesinde zayıf kalacak etkinlikleri tartışılır.
 9. Sence hazırladığın bu teknoloji destekli etkinlik öğrencilerin anlamalarını geliştirecek mi? Daha farklı nasıl olabilirdi? şeklinde bir etkinlik öğrenci için faydalı olur mu? Neden?
- Örnek etkinlikler gösterilir ve tartışılır.
 10. ... bu etkinlik hakkında ne düşünüyorsun? Öğrencilerin anlamalarını geliştirir mi?
- Teknolojik pedagojik alan bilgisi ile ilgili tavsiyelerde bulunulur.
 11. Öğrencilerin kavram yanılgılarını gidermek için teknolojiyi şeklinde kullandığımız da ne olur? Etkili olur mu sence? N düşünüyorsun bu konuda?
 12. (Tümdengelim ve tümevarım stratejilerine uygun etkinlikler gösterilip) bu etkinlikler hakkında ne düşünüyorsun? Uygulayabilir misin?

13. Çoklu temsilleri şeklinde uygulayabilir miyiz? Ne gibi yararı olur sence?
14. Öğrencilerin ön bilgilerini değerlendirmek için Gibi bir soru sorabilir miyiz? Ne düşünüyorsun?

III. Ek yorumlar

Teknoloji matematik müfredatına nasıl entegre edildi? Müfredata ne düzeyde bağlı kalındı?

Teknolojinin bu öğrencinin anlaması üzerindeki rolü ne? Öğrencilerin keşfetmesinde teknolojinin rolü var mı? Nasıl?

Teknolojinin öğrencilerin kavram yanılgılarının giderilmesinde nasıl kullanıldı?

Teknoloji öğrencilerin anlamalarını güçlendirecek şekilde nasıl kullanıldı?

Öğretmen öğretim sürecinde teknolojiyi nasıl kullandı? Teknoloji matematik öğretime nasıl katkı sağladı?

Teknoloji entegrasyonunda hangi öğretim stratejisi kullanıldı? Nasıl? Derste öğretmenin rolü nedir?

Öğrenme ortamı nasıldı? Öğrenciler aktif olarak katıldı mı? Öğretmen ders anlatımı esnasında sınıfı nasıl yönetti?

Derste teknoloji kullanımının amacı nedir? Teknoloji kullanımına ne zaman izin verildi?

Öğretmenin teknoloji entegrasyonunda karşılaştığı güçlükler nelerdir?

Öğretmen hangi çoklu temsilleri kullandı? Nasıl? Çoklu temsillerde teknoloji nasıl kullanıldı?

Öğretmen öğrenci anlamalarını nasıl değerlendirdi? Teknolojinin bu değerlendirme sürecindeki rolü nedir?

Öğretmen ders planına uygun bir ders işleyebildi mi? Ders planından farklı şekilde ilerleyen bir olay gerçekleşti mi?

Teknoloji kullanımının avantajları nelerdir? Teknolojinin kullanmanın dezavantajı gözlemlendi mi? Nedir?

APPENDIX E

TPACK OBSERVATION FORM

I. Temel Bilgiler

Öğretmen adayı:

Gözlem tarihi ve süresi:

Öğrenme alanı ve konu başlığı:

Öğretim programı kazanımları:

Sınıf Mevcudu:

Fiziki Durum: (sizin yeriniz, öğrencilerin yeri, teknolojik araçlar)

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II. TPAB değerlendirmesi

		Gösterge
CURRICULUM & ASSESSMENT-C: Curriculum descriptor		
	Acknowledges that mathematical ideas displayed with the technologies can be useful for making sense of topics addressed in the curriculum.	
	Expresses desire but demonstrates difficulty in identifying topics in own curriculum for including technology as a tool for learning	
	Understands some benefits of incorporating appropriate technologies as tools for teaching and learning the mathematics curriculum.	
	Investigates the use of topics in own curriculum for including technology as a tool for learning; seeks ideas and strategies for implementing technology in a more integral role for the development of the mathematics that students are learning	
	Understands that sustained innovation in modifying own curriculum to efficiently and effectively incorporate technology as a teaching and learning tool is essential.	

<i>Ekleme istedikleriniz</i>		
CURRICULUM & ASSESSMENT -A: Assessment descriptor		
	Resists idea of technology use in assessment indicating that technology interferes with determining students' understanding of mathematics.	
	Acknowledges that it might be appropriate to allow technology use as part of assessment but has a limited view of its use (i.e., use of technology on a section of an exam).	
	Understands that if technology is allowed during assessments that different questions/items must be posed (i.e., conceptual vs. procedural understandings).	
	Actively investigates use of different types of technology-based assessment items and questions (e.g., technology active, inactive, neutral or passive).	
	Reflects on and adapts assessment practices that examine students' conceptual understandings of the subject matter in ways that demand full use of technology.	
<i>Ekleme istedikleriniz</i>		
LEARNING-M: Mathematics learning descriptor		
	Views mathematics as being learned in specific ways and that technology often gets in the way of learning.	
	Has concerns about students' attention being diverted from learning of appropriate mathematics to a focus on the technology in the activities.	
	Begins to explore, experiment and practice integrating technologies as mathematics learning tools.	
	Uses technologies as tools to facilitate the learning of specific topics in the mathematics curriculum.	
	Plans, implements, and reflects on teaching and learning with concern and personal conviction for student thinking and understanding of the mathematics to be enhanced through integration	

	of the various technologies.	
<i>Ekleme istedikleriniz</i>		
LEARNING -C: Conception of student thinking descriptor		
	More apt to accept the technology as a teaching tool rather than a learning tool.	
	Is concerned that students do not develop appropriate mathematical thinking skills when the technology is used as a verification tool for exploring the mathematics.	
	Begins developing appropriate mathematical thinking skills when technology is used as a tool for learning.	
	Plans, implements, and reflects on teaching and learning with concern for guiding students in understanding.	
	Technology-integration is integral (rather than in addition) to development of the mathematics students are learning.	
<i>Ekleme istedikleriniz</i>		
TEACHING -M: Mathematics learning descriptor		
	Concerned that the need to teach about the technology will take away time from teaching mathematics	
	Uses technology activities at the end of units, for “days off,” or for activities peripheral to classroom instruction.	
	Uses technology to enhance or reinforce mathematics ideas that students have learned previously.	
	Engages students in high-level thinking activities (such as project-based and problem solving and decision making activities) for learning mathematics using the technology as a learning tool.	
	Active, consistent acceptance of technologies as tools for learning and teaching mathematics in ways that accurately translate mathematical concepts and processes into forms understandable by students.	

<i>Ekleme istedikleriniz</i>		
TEACHING -I: Instructional descriptor		
	Does not use technology to develop mathematical concepts.	
	Merely mimics the simplest professional development mathematics curricular ideas for incorporating the technologies.	
	Mimics the simplest professional development activities with the technologies but attempts to adapt lessons for his/her mathematics classes.	
	Engages students in explorations of mathematics with technology where the teacher is in role of guide rather than director of the exploration.	
	Adapts from a breadth of instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics.	
<i>Ekleme istedikleriniz</i>		
TEACHING -E: Environment descriptor		
	Uses technology to reinforce concepts taught without technology.	
	Tightly manages and orchestrates instruction using technology.	
	Instructional strategies with technologies are primarily deductive, teacher-directed in order to maintain control of the how the activity progresses.	
	Explores various instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics.	
	Manages technology-enhanced activities in ways that maintains student engagement and self-direction in learning the mathematics	
<i>Ekleme istedikleriniz</i>		
TEACHING - PD: Professional development descriptor		
	Considers attending local professional	

	development to learn more about technologies	
	Recognizes the need to participate in technology related PD.	
	Continues to learn and explore ideas for teaching and learning mathematics using only one type of technology (such as spreadsheets).	
	Seeks out and works with others who are engaged in incorporating technology in mathematics.	
	Seeks ongoing PD to continue to learn to incorporate emerging technologies. Continues to learn and explore ideas for teaching and learning mathematics with multiple technologies to enhance access to mathematics.	
<i>Ekleme istedikleriniz</i>		
ACCESS -U: Usage descriptor		
	Permits students to use technology „only“ after mastering certain concepts	
	Students use technology in limited ways during regular instructional periods.	
	Permits students to use technology in specifically designed units.	
	Permits students to use technology for exploring specific mathematical topics.	
	Permit students to use technology in every aspect of mathematics class.	
<i>Ekleme istedikleriniz</i>		
ACCESS -B: Barrier descriptor		
	Resists consideration of changes in content taught although it becomes accessible to more students through technology.	
	Worries about access and management issues with respect to incorporating technology in the classroom.	
	Uses technology as a tool to enhance mathematics lessons in order to provide students a new way to approach mathematics.	
	Recognizes challenges for teaching mathematics with technologies, but explores strategies and ideas to minimize the impact of those challenges	

	Recognizes challenges in teaching with technology and resolves the challenges through extended planning and preparation for maximizing the use of available resources and tools.	
<i>Ekleme istedikleriniz</i>		
ACCESS - A: Availability descriptor		
	Notices that authentic problems are more likely to involve „unfriendly numbers“ and may be more easily solved if students had calculators.	
	Calculators permit greater number of examples to be explored by students.	
	Concepts are taught differently since technology provides access to connections formerly out of reach.	
	Through the use of technology, key topics are explored, applied, and assessed incorporating multiple representations of the concepts and their connections.	
	Students are taught and permitted to explore more complex mathematics topics or mathematical connections as part of their normal learning experience	
<i>Ekleme istedikleriniz</i>		

III. Ek yorumlar

Teknoloji matematik müfredatına nasıl entegre edildi? Müfredata ne düzeyde bağlı kalındı?

Teknolojinin bu öğrencinin anlaması üzerindeki rolü ne? Öğrencilerin keşfetmesinde teknolojinin rolü var mı? Nasıl?

Teknolojinin öğrencilerin kavram yanlışlarının giderilmesinde nasıl kullanıldı?
Teknoloji öğrencilerin anlamalarını güçlendirecek şekilde nasıl kullanıldı?

Öğretmen öğretim sürecinde teknolojiyi nasıl kullandı? Teknoloji matematik öğretime nasıl katkı sağladı?

Teknoloji entegrasyonunda hangi öğretim stratejisi kullanıldı? Nasıl? Derste öğretmenin rolü nedir?

Öğrenme ortamı nasıldı? Öğrenciler aktif olarak katıldı mı? Öğretmen ders anlatımı esnasında sınıfı nasıl yönetti?

Derste teknoloji kullanımının amacı nedir? Teknoloji kullanımına ne zaman izin verildi?

Öğretmenin teknoloji entegrasyonunda karşılaştığı güçlükler nelerdir?

Öğretmen hangi çoklu temsilleri kullandı? Nasıl? Çoklu temsillerde teknoloji nasıl kullanıldı?

Öğretmen öğrenci anlamalarını nasıl değerlendirdi? Teknolojinin bu değerlendirme sürecindeki rolü nedir?

Öğretmen ders planına uygun bir ders işleyebildi mi? Ders planından farklı şekilde ilerleyen bir olay gerçekleşti mi?

Teknoloji kullanımının avantajları nelerdir? Teknolojinin kullanmanın dezavantajı gözlemlendi mi? Nedir?

APPENDIX F

POST- CONFERENCE PROTOCOL (IN ENGLISH)

1. How do you think the lesson proceeded? Could you briefly summarize the lesson?
2. How do you evaluate the lesson in general?
3. Which parts have proceeded successfully as regards the integration of technology into the lesson? Why? How else can it be improved?
4. Were there any unsuccessful parts in the integration of technology into the lesson? Why? What kinds of changes should be made to make it more successful?
5. What do you think about the conformity between the lesson and the mathematics curriculum? How do you think technology should be used in the mathematics curriculum and the textbook?
6. Do you think the use of technology in today's lesson helped students to understand the lesson? How? Why?
7. At which stage of the lesson did you use technology and for what purpose did you use it? What role does it play on the students? What are its pluses and minuses? How can students' understanding in this section be developed [*A part of the lesson is shown from the video recording*]? Were you able to facilitate students' discovery process? What else can be done? Do you think a technology-enhanced activity such as... could be more effective? Why?
8. What do you think about the role of technology during the implementation of the lesson and the use of technology in mathematics education? How did you use technology in the development of mathematical thought? Could you please explain? What would you like to be changed and what to be kept as it is?
9. What was the impact of technology on the elimination of students' misconceptions and the difficulties they faced? What do you think are the strengths and weaknesses of technology in the elimination of students'

misconceptions and the difficulties they faced? Do you think you have been able to eliminate the student's misconception here [*shows a part of a video recording of the lesson*]? In what other way could you have used technology? Do you think you could have caused students to possess a misconception here [*shows another part of the video recording of the lesson*]? Why? What do you think should have been done? How could you have used technology? What kinds of questions could you have asked? Do you think a technology-enhanced activity such as ... could have eliminated these misconceptions? (*suggestions*)

10. How did you use technology in enabling students to discover a concept? Was it effective? What else could have been done?
11. At what stage in the lesson and for what purpose did you use technology as a means of teaching? Could you please explain? Do you think technology was useful? For what other purposes could technology have been used? For what purpose did you use technology here [*shows a part of the video recording of the lesson*]? What do you think is its impact upon the students? In what other way could technology have been used to enhance students' understanding of the unit? Would it have been better to use it in this way: (*suggestions*)? Why? What questions did you ask while teaching students mathematics in this technology-enhanced lesson? Which ones were worthwhile, which one weren't? What other kinds of questions could you have asked? Why? You have asked students a question like ... [*shows a part of the video recording of the lesson*]. If you had asked the students a question like ... what kinds of answers do you think you would you have received? What are your opinions regarding this issue? (*suggestions*)
12. Which instructional strategy was effective in the use of technology? Why? Did you deductive or inductive strategies? Did technology have an impact? What are the strengths and weaknesses of these strategies? You have answered a student's question by saying ... [*shows a part of the video recording of the lesson*]? What other strategy could you have used to teach students? Would it have been better here to do it in way? What are your opinions regarding this issue? (*suggestions*)
13. How was the learning environment in the lesson? What was the place of technology in this learning environment? What is the purpose of using technology in teaching students this mathematical knowledge? What can be done to improve the learning environment?

14. What were the roles of the teacher and the students in the lesson? What is the impact of technology on these roles?
15. How do you think technology impacted the lesson? If you hadn't used technology, what would have been different? What difference does the use of technology create? What opportunities and limitations do you think technology possesses in achieving the stated learning outcome?
16. When did you allow the students to use technology? Could it have been done in a different way?
17. What difficulties did you experience in integrating technology into the lesson? Did you experience difficulties in class management when using technology? How did you overcome them? In this section [*shows a part of the video recording of the lesson*], it seems that you are having difficulties with respect to ...? What do you think is the reason? How could you have solved this problem? Do you think it would have been better to do it in way? What are your opinions regarding this issue? (*suggestions*)
18. How did you use technology in your lesson with respect to multiple representations? What are the strengths and weaknesses of technology on multiple representations? In this section [*shows a part of the video recording of the lesson*] you are using technology in multiple representations. How do you think it contributes to technology? Could it have been better to do it in ...way? What are your opinions regarding this issue? (*suggestions*)
19. How did you use technology in this lesson with respect to assessment? How do you think your assessment was here [*shows a part of the video recording of the lesson*]? How can it be improved? What can be done to use technology more effectively during the assessment stage? Could it have been better to it in ... way? What are your opinions regarding this issue? (*suggestions*)
20. If you were to repeat the lesson, what would stay the same and what would you change? Why? (Is there anything that you would want to add or omit from the lesson plan?) Could you please explain?
21. Did you experience any difficulties during the implementation? Which parts were they?

22. Were there moments when you didn't abide to the lesson plan? Why?
23. Did you experience any unexpected event in the lesson? If so, how did you deal with it?
24. How were the students' reactions to the lesson? Were they as you expected?
25. How did this lesson contribute to your Professional development? Would you like to participate in technology related professional development activities? Why?

APPENDIX G

POST- CONFERENCE PROTOCOL (IN TURKISH)

1. Sizce ders nasıl geçti? Dersi kısaca özetleyebilir misiniz?
2. Dersi genel olarak nasıl değerlendiriyorsunuz?
3. Derse teknoloji entegrasyonunda başarılı geçen kısımları nelerdir? Neden? Daha nasıl geliştirilebilir?
4. Derse teknoloji entegrasyonunda başarısız olan kısımları var mıydı? Neden? Daha başarılı olması için ne gibi değişiklikler yapılmalı?
5. Dersin matematik müfredatına uygunluğu hakkında ne düşünüyorsun? Sence teknoloji matematik müfredatında ve ders kitabında nasıl kullanılmalı?
6. Sence bugünkü derste teknoloji kullanımı öğrencilerin anlamalarına yardımcı oldu mu? Nasıl? Neden?
7. Teknolojiyi dersin hangi aşamasında ve hangi amaçla kullandın? Öğrenci üzerindeki rolü neydi? Artı ve eksileri nelerdir? *[Ders anlatımından bir video bölümü gösterilerek]* sence bu bölümde öğrenci anlamaları nasıl geliştirilebilirdi? Öğrencinin keşfetmesini sağlayabildin mi? Başka ne yapılabilir? ... şeklinde teknoloji destekli etkinlik sence bu kavramı keşfetmesi için daha etkili olabilir mi? Neden?
8. Dersin işlenişi sırasında teknolojinin rolü ve teknolojinin matematik eğitimindeki kullanımını hakkında ne düşünüyorsun? Matematiksel düşüncenin gelişiminde teknolojiyi nasıl kullandın? Açıklayabilir misin? Nelerin değişmesini nelerin aynı kalmasını istersin?
9. Öğrencilerin kavram yanlışlarının ve zorluklarının giderilmesinde teknolojinin etkisi neydi? Sence teknolojinin kavram yanlışlarının ve zorluklarının giderilmesindeki güçlü ve zayıf yönleri nelerdir? *[Ders anlatımından bir video bölümü gösterilerek]* burada sence öğrencinin kavram yanlışsını giderebildin mi? Teknolojiyi başka nasıl kullanabilirdin? *[Ders anlatımından başka bir video bölümü gösterilerek]* peki burada öğrencide bir kavram yanlışsı oluşturmuş olabilir misin? Neden? Ne yapılması gerekiyordu sence? Teknolojiyi nasıl kullanabilirdin? Nasıl sorular sorabilirdin? ... şeklinde teknoloji destekli etkinlik sence bu kavram yanlışları giderilebilir miydi? *(öneriler)*
10. Öğrencilerin bir kavramı keşfetmesinde teknolojiyi nasıl kullandın? Etkili miydi? Başka neler yapılabilirdi?
11. Bir öğretme aracı olarak teknolojiyi bu derste nerede ve hangi amaçla kullandın? Açıklar mısın? Teknoloji sence faydalı mıydı? Teknoloji başka

- hangi amaçlarla kullanılabilir? [*Ders anlatımından bir video bölümü gösterilerek*] burada teknolojiyi hangi amaçla kullandın? Öğrenciler üzerindeki etkisi sence nasıl? Öğrencilerin anlamalarını artırmak için teknoloji başka nasıl kullanılabilir? şeklinde kullansan daha iyi olabilir miydi? Neden? (*öneriler*) Teknoloji destekli bu derste öğrenciler matematiği öğretirken hangi soruları sordun? Hangi sorular işe yaradı hangileri işe yaramadı? Başka nasıl sorular sorabilirdin? Neden? [*Ders anlatımından bir video bölümü gösterilerek*] öğrencilere ... şeklinde soru sormuşsun? Burada öğrencilere Şeklinde soru sorsan sence nasıl yanıtlar alırdın? Ne düşünüyorsun bu konuda? (*öneriler*)
12. Teknoloji kullanımında hangi öğretim stratejisi etkiliydi? Neden? tündengelim ve ya tüme varım stratejilerini kullandın mı? Teknolojinin etkisi oldu mu? Bu stratejilerin güçlü ve zayıf yönleri nelerdir? [*Ders anlatımından bir video bölümü gösterilerek*] öğrencinin sorusuna ... şeklinde cevap vermişsin? Öğrenciye başka hangi stratejiyi kullanarak anlatabilirdin? Burada şekilde yapsan daha iyi olabilir miydi? Ne düşünüyorsun bu konuda? (*öneriler*)
13. Dersteki öğrenme ortamını nasıldı? Bu öğrenme ortamında teknolojinin yeri neydi? Öğrencilere bu matematiksel bilgiyi öğretirken teknoloji kullanmanın amacı nedir? Öğrenme ortamının iyileştirilmesi için ne yapılabilir?
14. Derste öğretmenin ve öğrencinin rolü neydi? Teknolojinin bu rollere etkisi nedir?
15. Sence dersi teknoloji nasıl etkiledi? Eğer teknoloji kullanmasaydın neler değişirdi? Teknoloji kullanımının farkı ne? Belirlenen kazanıma ulaşılmasında teknoloji ne tür imkân ve kısıtlamalara sahip olduğunu düşünüyorsun?
16. Öğrencilerin teknoloji kullanmalarına ne zaman izin verdin? Daha farklı olabilir miydi?
17. Derse teknoloji entegrasyonunda karşılaştığın zorluklar nelerdi? Teknolojiyi kullanırken sınıf yönetiminde zorluk çektin mi? Nasıl üstesinden geldin? [*Ders anlatımından bir video bölümü gösterilerek*] bu bölümde konusunda zorlandığın görülüyor. Sence sebebi ne? Bu sorunu nasıl çözebilirsin? Burada şekilde yapsan daha iyi olabilir miydi? Ne düşünüyorsun bu konuda? (*öneriler*)
18. İşlediğin derste teknolojiyi çoklu temsiller açısından nasıl kullandın? Bu derste teknolojinin çoklu temsiller üzerindeki güçlü ve zayıf yönleri nelerdir? [*Ders anlatımından bir video bölümü gösterilerek*] bu bölümde teknolojiyi çoklu temsillerde kullanımın var. Sence öğrencilere katkısı nedir?

Burada şekilde yapsan daha iyi olabilir miydi? Ne düşünüyorsun bu konuda? (öneriler)

19. Bu derste değerlendirme sürecinde teknolojiyi nasıl kullandın? [*Ders anlatımından bir video bölümü gösterilerek*] sence burada nasıl bir değerlendirme yaptın? Nasıl geliştirilebilir? Değerlendirme sürecinde teknolojiyi daha etkin kullanmak için neler yapılabilir? şekilde yapsan daha iyi olabilir miydi? Ne düşünüyorsun bu konuda? (öneriler)
20. Bu dersi tekrardan işleyecek olsan neler aynı kalırdı, neleri değiştirdin? Neden? (Derse eklemek ya da dersten çıkarmak istediğin bir şey var mı?) Açıklar mısın?
21. Uygulama esnasında zorlandığımız kısımlar oldu mu? Nereler?
22. Ders planına uymadığın oldu mu? Neden?
23. Derste beklenmedik bir olay ile karşılaştın mı? Olduysa bu durumu nasıl karşıladın?
24. Öğrencilerin derse tepkisi nasıldı? Beklediğin şekilde miydi?
25. Bu ders size profesyonel gelişim hakkında nasıl sağladı? Teknoloji ile ilgili profesyonel gelişim etkinliklerine katılmak ister misin? Neden?

APPENDIX H

MATHEMATICS COACHING INTERVIEW (IN ENGLISH)

1. How can you evaluate the mathematics coaching?
2. Which aspect of the training provided did you like the most? Why?
3. Was the mathematics coaching effective with respect to integrating technology into the mathematics lesson? Could you please explain?
4. How do you think you made use of the technology, and your knowledge of content and pedagogy throughout the mathematics coaching?
5. What are the strengths of the mathematics coaching with respect to the development of the use of technology in mathematics lessons? Could you please explain?
6. What are the weaknesses of the mathematics coaching with respect to the development of the use of technology in mathematics lessons? Could you please explain?
7. Has the training provided meet your expectations? Why?
8. What would you like to be changed and what would you like to be kept in the mathematics coaching process? Why?
9. What do you think can be done to improve the mathematics coaching?
10. How did this training you received contribute to you professionally?
11. Is there anything else you would like to add?

APPENDIX H

MATHEMATICS COACHING INTERVIEW (IN TURKISH)

1. Matematik koçluğu sürecini nasıl değerlendiriyorsun?
2. Verilen eğitimin en çok hangi yönünü beğendin? Neden?
3. Matematik koçluğu teknolojiyi matematik dersine entegre etme açısından etkili miydi? Açıklayabilir misin?
4. Matematik koçluğu süresince sahip olduğunuz teknoloji, pedagoji ve alan bilginizi nasıl kullandığınızı düşünüyorsunuz?
5. Matematik koçluğunun matematik dersinde teknoloji kullanımının gelişimi açısından güçlü yönleri nelerdir? Açıklayabilir misin?
6. Matematik koçluğunun matematik dersinde teknoloji kullanımının gelişimi açısından zayıf yönleri nelerdir? Açıklayabilir misin?
7. Verilen eğitimin beklentilerinizi karşıladı mı? Neden?
8. Matematik koçluğu sürecinde nelerin aynı kalmasını nelerin değişmesini istedin? Neden?
9. Matematik koçluğunun daha iyi olması için sence ne yapılabilir?
10. Aldığımız bu eğitim mesleki açıdan size nasıl bir katkı sağladı?
11. Eklemek istediğiniz başka bir şey var mı?

APPENDIX J

MATHEMATICS TEACHER TPACK DEVELOPMENT MODEL

CURRICULUM & ASSESSMENT THEME

C: Curriculum descriptor Ex: Mathematics Example

Recognizing

C: Acknowledges that mathematical ideas displayed with the technologies can be useful for making sense of topics addressed in the curriculum.

Ex: Creates graphs of multiple linear functions using graphing calculators to provide a visual representation for varying slopes. Considers these visuals as making sense of the idea of slope but is unsure of how this might help students learn the basic concept.

Accepting

C: Expresses desire but demonstrates difficulty in identifying topics in own curriculum for including technology as a tool for learning.

Ex: Attends and participates in mathematics dynamic geometry system workshop to identify curricular ideas for incorporating the technologies as learning tools. Mimics the incorporation of a dynamic geometry system idea from the workshop to display measuring the sum of the angles of a triangle that upon multiple changes of the triangle suggests that the sum of the angles of any triangle is 180 degrees.

Adapting

C: Understands some benefits of incorporating appropriate technologies as tools for teaching and learning the mathematics curriculum.

Ex: Targets key topics students investigate with technology. Develops lessons to demonstrate mathematics concepts with technology and activities for students to use technology to verify or reinforce those concepts. After students have learned to create graphs of specific linear functions, students are challenged to use the spreadsheet to verify the graphical representation of the ordered pairs.

Exploring

C: Investigates the use of topics in own curriculum for including technology as a tool for learning; seeks ideas and strategies for implementing technology in a more integral role for the development of the mathematics that students are learning.

Ex: Adapts own previous mathematics lesson to include technology.

Ex: Develops own ideas about using technology to enhance current curriculum; thus, begins altering preexisting activities or creating new activities for current curriculum.

Advancing

C: Understands that sustained innovation in modifying own curriculum to efficiently and effectively incorporate technology as a teaching and learning tool is essential.

Ex: Develops innovative ways to use technology to develop mathematical thinking in students such as using virtual algebra tiles to extend ideas of handheld manipulatives to focus on variables in algebraic expressions.

Ex: Modifies and advances curriculum to take advantage of technology as a tool for teaching and learning such as using CAS to explore more complex algebraic expressions.

CURRICULUM & ASSESSMENT THEME

A: Assessment descriptor Ex: Mathematics Example

Recognizing

A: Resists idea of technology use in assessment indicating that technology interferes with determining students' understanding of mathematics.

Ex: Does not allow calculator use when assessing students' understanding of solving linear equations.

Accepting

A: Acknowledges that it might be appropriate to allow technology use as part of assessment but has a limited view of its use (i.e., use of technology on a section of an exam).

Ex: Attends and participates in a mathematics assessment professional development to consider ideas for assessing students' understanding of solving systems of linear functions using the calculator as a tool. Mimics the assessment idea to explain the use of the calculator for solving systems of linear functions by using the trace function to identify the intersection. Often retests technology questions with paper and pencil questions to be sure that the concept was learned the 'right' way.

Adapting

A: Understands that if technology is allowed during assessments that different questions/items must be posed (i.e., conceptual vs. procedural understandings).

Ex: Allows use of calculator in an assessment but designs the assessment to focus on gathering students' conceptual understanding of solving systems of linear functions in addition to their procedural understanding.

Exploring

A: Actively investigates use of different types of technology-based assessment items and questions (e.g., technology active, inactive, neutral or passive).

Ex: Designs assessments where students are expected to show their understanding of mathematical ideas using an appropriate technology that extends beyond paper and pencil type questions.

Advancing

A: Reflects on and adapts assessment practices that examine students' conceptual understandings of the subject matter in ways that demand full use of technology.

Ex: Develops innovative assessments to capture students' understandings of the mathematics embedded in the particular technology.

LEARNING THEME

M: Mathematics learning descriptor Ex: Mathematics example

Recognizing

M: Views mathematics as being learned in specific ways and that technology often gets in the way of learning.

Ex: Mathematical exploration with technology rarely seen.

Accepting

M: Has concerns about students' attention being diverted from learning of appropriate mathematics to a focus on the technology in the activities.

Ex: Limits student technology use, particularly during the introduction and development of key topics.

Adapting

M: Begins to explore, experiment and practice integrating technologies as mathematics learning tools.

Ex: Students explore some mathematics topics using technology.

Exploring

M: Uses technologies as tools to facilitate the learning of specific topics in the mathematics curriculum.

Ex: Students explore numerous topics using technology, sometimes ranging outside the topic at hand.

Advancing

M: Plans, implements, and reflects on teaching and learning with concern and personal conviction for student thinking and understanding of the mathematics to be enhanced through integration of the various technologies.

Ex: Students explore mathematics topics, integrating various technologies in attempts to better understand mathematical concepts.

LEARNING THEME

C: Conception of student thinking descriptor Ex: Mathematics example

Recognizing

C: More apt to accept the technology as a teaching tool rather than a learning tool.

Ex: Technology is used only outside of normal classroom activities, such as checking homework, calculating large numbers, etc.

Accepting

C: Is concerned that students do not develop appropriate mathematical thinking skills when the technology is used as a verification tool for exploring the mathematics.

Ex: Activities that use technology are almost always redone without technology to be certain students really learned the particular concept.

Adapting

C: Begins developing appropriate mathematical thinking skills when technology is used as a tool for learning.

Ex: Although students use technology for most topics, assessing student thinking remains mostly technology free.

Exploring

C: Plans, implements, and reflects on teaching and learning with concern for guiding students in understanding.

Ex: Technology activities are implemented and evaluated with respect to student learning of mathematics and student attitudes toward mathematics.

Ex: Manages technology-enhanced activities towards directing student engagement and self-direction in learning mathematics.

Advancing

C: Technology-integration is integral (rather than in addition) to development of the mathematics students are learning.

Ex: Engages students in high-level thinking activities (such as project-based and problem solving and decision making activities) for learning mathematics using the technology as a learning tool.

Ex: Technology is used to develop advanced levels of understanding of mathematical concepts.

TEACHING THEME

M: Mathematics learning descriptor Ex: Mathematics example

Recognizing

M: Concerned that the need to teach about the technology will take away time from teaching mathematics.

Ex: Students use technology on their own and little or no instruction with technology is present.

Accepting

M: Uses technology activities at the end of units, for “days off,” or for activities peripheral to classroom instruction.

Ex: Technology-enhanced activities are not used for topics that require more advanced technology skills.

Adapting

M: Uses technology to enhance or reinforce mathematics ideas that students have learned previously.

Ex: Students use technology to reinforce previously teacher-taught concepts.

Exploring

M: Engages students in high-level thinking activities (such as project-based and problem solving and decision making activities) for learning mathematics using the technology as a learning tool.

Ex: Teachers share classroom-tested, technology-based lessons, ideas, and successes with peers.

Advancing

M: Active, consistent acceptance of technologies as tools for learning and teaching mathematics in ways that accurately translate mathematical concepts and processes into forms understandable by students.

Ex: Teacher is seen as a resource as novel ideas for helping students learn mathematics with technology.

TEACHING THEME

I: Instructional descriptor Ex: Mathematics example

Recognizing

I: Does not use technology to develop mathematical concepts.

Ex: Technology, if used in class, is used for menial or rote activities.

Accepting

I: Merely mimics the simplest professional development mathematics curricular ideas for incorporating the technologies.

Ex: Introduces the Pythagorean Theorem algorithmically; teacher use of dynamic geometry to verify the Pythagorean Theorem; students find solutions to example problems using paper and pencil.

Adapting

I: Mimics the simplest professional development activities with the technologies but attempts to adapt lessons for his/her mathematics classes.

Ex: Technology-based lessons are incorporated that are tailored to students' needs.

Exploring

I: Engages students in explorations of mathematics with technology where the teacher is in role of guide rather than director of the exploration.

Ex: Students use technology to explore new concepts as the teacher serves mostly as a guide.

Advancing

I: Adapts from a breadth of instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics.

Ex: The teacher helps students move fluently from one tool to another while demonstrating a focus on and a joy of deeply understanding mathematical topics.

TEACHING THEME

E: Environment descriptor Ex: Mathematics example

Recognizing

E: Uses technology to reinforce concepts taught without technology.

Ex: Focus on linear functions where students practice creating graphs by hand to explore different functions. After students have demonstrated competence with linear functions, summarize the knowledge, with a spreadsheet example or a graphing calculator example.

Accepting

E: Tightly manages and orchestrates instruction using technology.

Ex: Technology is directed, in a tightly sequenced, step-by-step process. Skill-based, non-exploratory technology use.

Adapting

E: Instructional strategies with technologies are primarily deductive, teacher-directed in order to maintain control of the how the activity progresses.

Ex: Begins to adapt instructional approaches that allow students opportunities to explore with technology for part of lessons.

Exploring

E: Explores various instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics.

Ex: The teacher incorporates a variety of technologies for numerous topics.

Advancing

E: Manages technology-enhanced activities in ways that maintains student engagement and self-direction in learning the mathematics.

Ex: The teacher forms and reforms learning groups where individual and group learning is valued and encouraged.

TEACHING THEME

PD: Professional development descriptor Ex: Mathematics example

Recognizing

PD: Considers attending local professional development to learn more about technologies.

Ex: Attends local workshops that focus on gaining skills with the technology; context of the learning activities is mathematics.

Accepting

PD: Recognizes the need to participate in technology related PD.

Ex: Seeks out technology-related professional development, workshops that are directed at developing the technology in the learning of mathematics.

Adapting

PD: Continues to learn and explore ideas for teaching and learning mathematics using only one type of technology (such as spreadsheets).

Ex: Shares ideas from professional development with other mathematics teachers in the building.

Exploring

PD: Seeks out and works with others who are engaged in incorporating technology in mathematics.

Ex: Organizes teachers of similar mathematics and grade level in investigating the mathematics curriculum to integrate appropriate technologies.

Advancing

PD: Seeks ongoing PD to continue to learn to incorporate emerging technologies. Continues to learn and explore ideas for teaching and learning mathematics with multiple technologies to enhance access to mathematics.

Ex: Engages teachers in the district in evaluating and revising the mathematics curriculum to more seamlessly integrate technology throughout the grades, adjusting the curriculum for a 21st century mathematics curriculum with appropriate technologies.

ACCESS

U: Usage descriptor Ex: Mathematics example

Recognizing

U: Permits students to use technology ‘only’ after mastering certain concepts.

Ex: Mathematical exploration with technology tools is challenged by beliefs about how students need to learn mathematics.

Accepting

U: Students use technology in limited ways during regular instructional periods.

Ex: Student activities with technology are limited to brief tightly controlled situations.

Adapting

U: Permits students to use technology in specifically designed units.

Ex: Access to and use of technology is available for exploration of new topics, usually with the teacher's demonstration.

Exploring

U: Permits students to use technology for exploring specific mathematical topics.

Ex: Access to and use of technology is available and encouraged for mathematics exploration during most class times.

Advancing

U: Permit students to use technology in every aspect of mathematics class.

Ex: Technology is seen as an opportunity to challenge notions of what mathematics students can master.

ACCESS THEME

B: Barrier descriptor Ex: Mathematics example

Recognizing

B: Resists consideration of changes in content taught although it becomes accessible to more students through technology.

Ex: Student access to technology is limited to 'after' they have learned the given concepts using paper and pencil procedures and only for rote activities.

Accepting

B: Worries about access and management issues with respect to incorporating technology in the classroom.

Ex: Students can only use technology in isolated situations or non-important learning situations.

Adapting

B: Uses technology as a tool to enhance mathematics lessons in order to provide students a new way to approach mathematics.

Ex: Concepts learned with technology are not assessed with technology.

Exploring

B: Recognizes challenges for teaching mathematics with technologies, but explores strategies and ideas to minimize the impact of those challenges.

Ex: Technology is used extensively in assessments. Seeks out ways to obtain technology for classroom use and begins creating methods for technology management issues.

Advancing

B: Recognizes challenges in teaching with technology and resolves the challenges through extended planning and preparation for maximizing the use of available resources and tools.

Ex: Technology is used to expand the mathematics concepts that can be accessed by students.

ACCESS THEME

A: Availability descriptor Ex: Mathematics example

Recognizing

A: Notices that authentic problems are more likely to involve ‘unfriendly numbers’ and may be more easily solved if students had calculators.

Ex: Assigns some mathematics problems using school and community data but saves them for “extra credit” work if students have calculators.

Accepting

A: Calculators permit greater number of examples to be explored by students.

Ex: Student use calculators to investigate patterns and functions.

Adapting

A: Concepts are taught differently since technology provides access to connections formerly out of reach.

Ex: Students use dynamic geometry software to investigate and make connections between trigonometry functions.

Exploring

A: Through the use of technology, key topics are explored, applied, and assessed incorporating multiple representations of the concepts and their connections.

Ex: Simultaneous equations are developed from an authentic situation, solved, and interpreted using graphs, tables, symbols and data.

Advancing

A: Students are taught and permitted to explore more complex mathematics topics or mathematical connections as part of their normal learning experience.

Ex: Using the Internet to find interesting mathematical problems, students investigate the role that technologies can play in finding solutions to the problems.

APPENDIX K

INDEX OF EXEMPLARY GEOGEBRA ACTIVITIES IN THE MCC3

These activities were derived from www.geogebra.org

1) Finding the similarities between polygons activity

File Edit View Options Tools Window Help

SideRatio = $\frac{AB}{A_1B_1} = \frac{5.68}{4.35} = 1.31$

Perimeter of = 28.39 Perimeter of = 21.73

PerimeterRatio = $\frac{poly1}{poly2} = \frac{28.39}{21.73} = 1.31$

Side Ratio
 Perimeter ratio

2) Making the congruence and similarities between triangle activity

File Edit View Options Tools Window Help

AB = 5 $\alpha = 39^\circ$ $\beta = 91^\circ$ DE = 5

$\alpha = \angle BAC = \angle EDF$ $\beta = \angle ABC = \angle DEF$

Side ratios: $AB/DE = 5/5 = 1$
 $AC/DF = 6.53/6.53 = 1$
 $BC/FE = 4.11/4.11 = 1$

Line from A at angle α
 Line from D at angle α
 Connecting lines at angle β
 Side lengths

1) Finding the similarities between polygons activity

1) Finding the similarities between polygons activity

3) Finding the similarities between polygons activity

File Edit View Options Tools Window Help

Move
Drag or select objects

SIMILAR POLYGONS

Criteria:

- The ratio of corresponding side lengths must be the same, or is constant (increasing by a specific scale factor).
- Must contain congruent corresponding angles.

Congruent Corresponding Angles

Corresponding Side Lengths

The ratio of corresponding side lengths is 1:2 from the inner polygon to the middle polygon. The ratio of the inner polygon to the outer polygon is 1:3. The middle to outer polygon ratio is 2:3. This tells us that the polygons increase at a constant scale factor.

Polygon AFBCDE $\times 2$ equals Polygon PQHILJ
Polygon AFBCDE $\times 3$ equals Polygon MNVOPR

Measurements increase by a constant scale factor. From the innermost polygon, to the outer polygon, the side lengths increase at a constant rate. The outer polygon is a dilation of the inner polygon, being 2 times larger than the original. Because they follow the necessary criteria, we can conclude that these polygons are similar.

Dilations and Similar Polygons
The triangular figure demonstrates similar polygons, because it is a dilation. A dilation is a figure in which is either scaled up or down in size by a constant factor. This contains similar corresponding angles, and congruent corresponding sides. As you move point V, you can see the ratio of sides change when attached to the point of dilation, which is point W.

4) Finding the similarities between polygons activity

File Edit View Options Tools Window Help

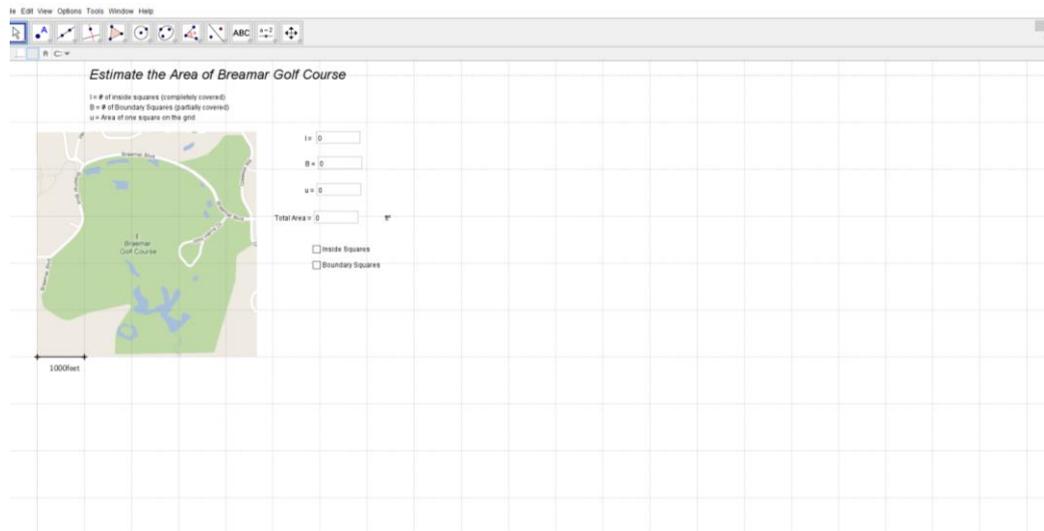
Move
Drag or select objects

Move the slider to solve the problem.

Measure the 6 angles in the triangles.

$HJK = 84^\circ$	$NPQ = 73^\circ$
$JKH = 43^\circ$	$PQN = 40^\circ$
$KHJ = 53^\circ$	$QNP = 68^\circ$

6) Finding the area of golf course activity



7) Finding the area of irregular shape by using Pick's theorem

Pick's Theorem

In a lattice polygon, whose boundary consists of a sequence of connected, non intersecting straight-line segments:

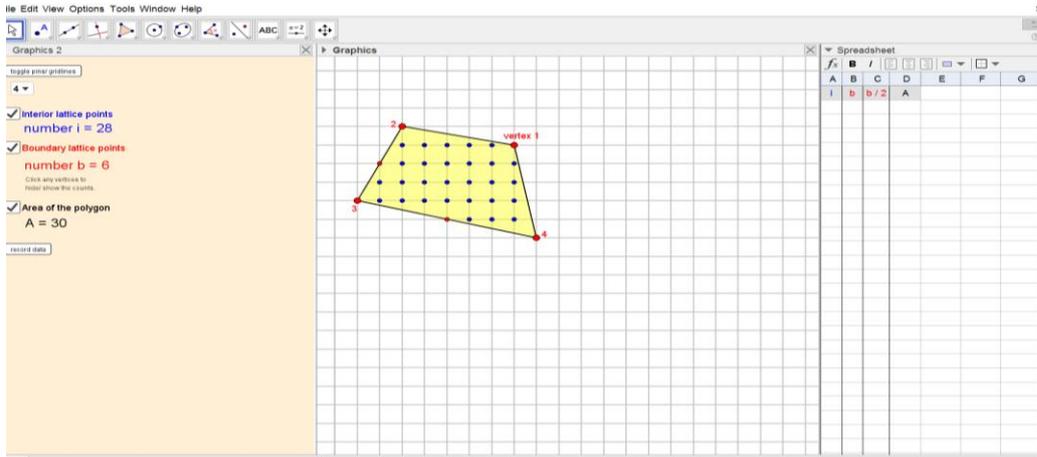
$$\text{Area} = I + \frac{B}{2} - 1$$

I = number of interior lattice points
 B = number of boundary lattice points

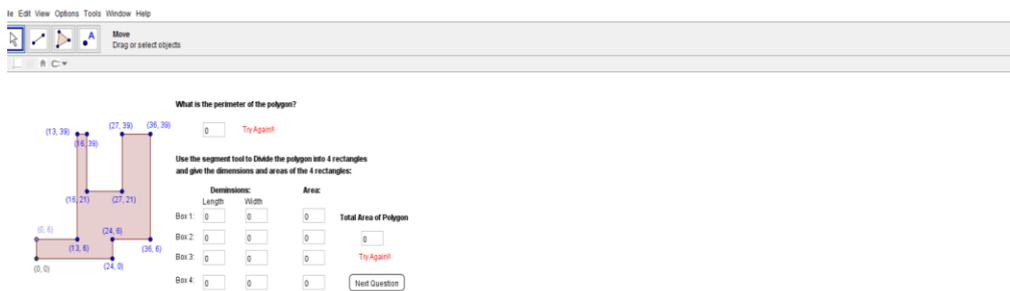
I = 35 B = 5
 Area = $35 + \frac{5}{2} - 1 = 36.5$

Drag the polygon's vertices : A, B, C, D, E

8) Finding the area of irregular shape by using Pick's theorem



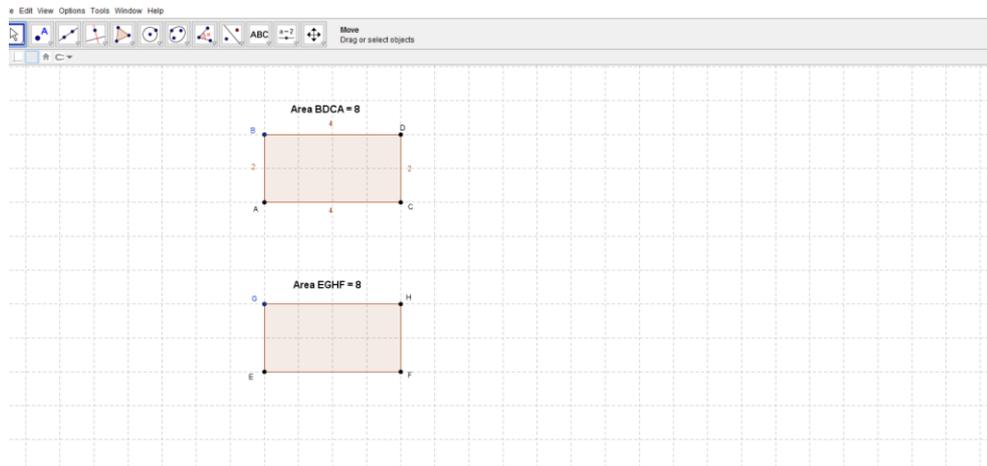
9) Finding the area of irregular shape



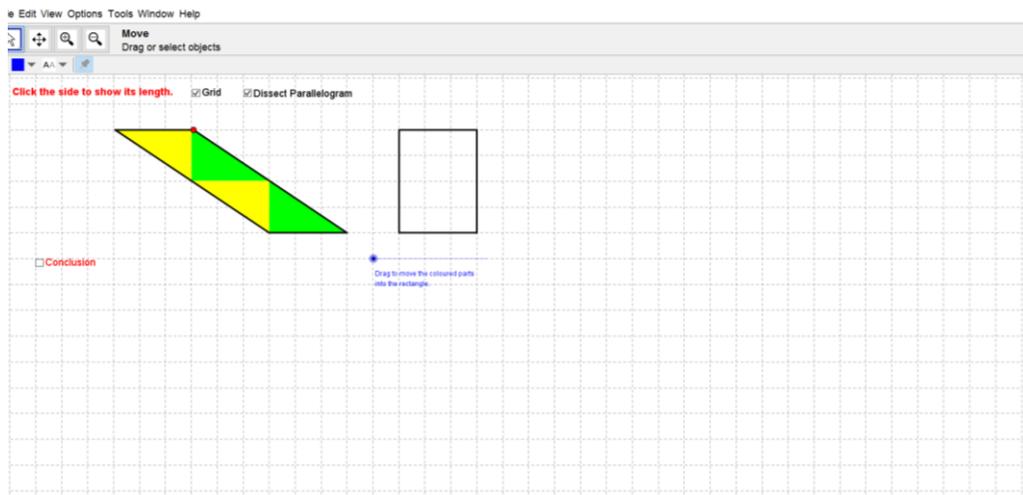
10) Finding the area of parallelogram



11) Comparing the area of different parallelgrom via dragging



12) Finding the area of parallelgrom



APPENDIX L

CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: Aygün, Berna
Nationality: Turkish (TC)
Date and Place of Birth: 12 April 1985 , Kayseri
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EDUCATION

Degree	Institution	Year of Graduation
MS	Hacettepe University, Elementary Education	2010
BS	Baskent University, Computer Education and Instrcuitonal Education	2009
BS	Baskent University, Elementary Mathematics Education	2008
High School	Kayseri High School of Science	2003

WORK EXPERIENCE

Year	Place	Enrollment
2010- Present	Recep Tayyip Erdogan University, Education Faculty	Research Assistant

FOREIGN LANGUAGES

English

PUBLICATIONS

1. Baran-Bulut, D., AYGÜN, B., İpek, A. S. (2016) İlköğretim Matematik Dersi Sorularının Math Taksonomiye Göre Analizi. Türk Bilgisayar ve Matematik Eğitimi Dergisi
2. Aygün B., Uzun, N. ve Atasoy, E., (2016) Öğretmen Adaylarının Teknopedagojik Eğitim Yeterliliklerinin İncelenmesi., Türk Bilgisayar ve Matematik Eğitimi Dergisi

3.Yılmaz-Kođar, E., & Aygun, B. (2015). Temel eđitimden orta ođretime geiř sınavı (TEOG)'nın matematik temel alanına ait testlerin kapsam geerliđinin incelenmesi. Pegem Eđitim ve Ođretim Dergisi, 5(5), 667-680, <http://dx.doi.org/10.14527/pegegog.2015.036>.

4. Atasoy, E., Uzun, N. ve Aygün B., 2015. Dinamik Matematik Yazılımları ile Desteklenmiř Ođrenme Ortamında Ođretmen Adaylarının Teknolojik Pedagojik Alan Bilgilerinin İncelenmesi, Bartın University Journal of Faculty of Education, 4(2), s.611–633.

HOBBIES

Trekking, Movies, Painting

APPENDIX M

PERMISSION OF THE STUDY



T.C.
RİZE VALİLİĞİ
İl Millî Eğitim Müdürlüğü

Sayı : 96972123-604/202518
Konu : Proje İzni

15/01/2014

VALİLİK MAKAMINA

İlgi: Recep Tayyip Erdoğan Üniversitesi Eğitim Fakültesi Dekanlığının 07.01.2014 tarih ve 72940495-604/14 sayılı yazısı.

Recep Tayyip Erdoğan Üniversitesi Eğitim Fakültesi Dekanlığı ilgi yazılarında; İlköğretim Bölümü İlköğretim Matematik Eğitimi Anabilim Dalı öğretim elemanı Arş. Gör. Berna AYGÜN'ün "İlköğretim Matematik Öğretmenlerinin Geometriye İlişkin Teknolojik Pedagojik Alan Bilgilerinin Matematik Koçluğu Sürecindeki Gelişimi" adlı tez çalışması yapmak istediğini belirtmektedir.

Müdürlüğümüzce, il genelindeki ortaokullarda görev yapan matematik öğretmenleri ile İlköğretim Matematik Öğretmenlerinin Geometriye İlişkin Teknolojik Pedagojik Alan Bilgilerinin Matematik Koçluğu Sürecindeki Gelişim konulu tez çalışmanın yürütülmesi uygun görülmektedir.

Makamlarınızca da uygun görüldüğü takdirde gereğini olurlarınıza arz ederim.

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Müdür a.
Millî Eğitim Müdür Yardımcısı

OLUR
15/01/2014

Mustafa KALENDER
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Millî Eğitim Müdür Vekili


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APPENDIX N

TURKISH SUMMARY/ TÜRKÇE ÖZET

İLKÖĞRETİM MATEMATİK ÖĞRETMENLERİNİN MATEMATİK KOÇLUĞU SÜRECİNDEKİ GEOMETRİYE İLİŞKİN TEKNOLOJİK PEDAGOJİK ALAN BİLGİLERİNİN GELİŞİMİ

1. Giriş

Teknolojinin içinde bulunduğumuz bilgi çağı uzun yıllardır kullanılması öğretim ve öğrenim ortamlarının niteliklerinde değişikliklere neden olmuştur. Teknoloji kullanımı öğrencilerin müfredatta bulunan kavramları derinlemesine öğrenmelerini sağlamaktadır (Miller, 2008). Aynı zamanda teknoloji, matematik alanındaki araştırmaların daha nitelikli olmasına, farklı bakış açıları yansıtan anlamlı matematiksel düşüncelerin oluşmasına ve matematik öğretimindeki geleneksel yöntemlerin değişmesine yardımcı olur. Amerika'nın Ulusal Matematik Öğretmenler Birliği'nin (NCTM) (2000) okullara ilişkin belirlediği matematik ilkelerinden biri olan teknoloji, matematik öğretim ve öğreniminde gerekli olarak görülmekte olup matematik eğitiminde teknoloji ile ilgili aşağıdaki ifadeyi belirtilmiştir:

Elektronik teknolojiler – hesap makineleri ve bilgisayarlar – matematiğin öğretimi, öğrenimi ve uygulanmasında gerekli olan araçlardır. Bu teknolojiler, matematiksel düşüncelerin görsel görüntülerini sağlar, verilerin düzenlenip analiz edilmelerine yardımcı olur ve etkin ve doğru biçimde hesaplama yapar. ...Teknolojik araçlar mevcut olduğunda, öğrenciler karar verme, derinlemesine düşünme, muhakeme etme ve problem çözme üzerine odaklanabilir. (s. 24)

Bu ifade, teknolojinin matematik eğitimindeki önemini göstermektedir. Matematikte teknoloji entegrasyonu öğretim ve öğrenimi iyileştirme ve geliştirme

gibi çeşitli biçimlerde kullanılabilir. Bununla birlikte teknoloji, matematiksel kavramları keşfetmede ve kavramlar arası bağlantıları kurmada yardımcı olmaktadır. Teknolojinin sunduğu bu avantajlar ışığında AMTE (2006), günümüzde teknolojinin matematik için gerekli bir gereç olduğunu ifade etmektedir. Bu nedenle matematik öğretim ve öğreniminde teknolojinin entegrasyonunun gerekli olduğu görülmektedir.

Bu açıdan bakıldığında Türkiye’deki okullarda yer alan teknolojik olanaklar da artmıştır. Türkiye’de Milli Eğitim Bakanlığı (MEB), 2010-2014 strateji planında Bilgi ve İletişim Teknolojilerine (BİT) ilişkin vizyon metnini yayınlamış ve vizyonunu şöyle ifade etmiştir: “eğitim sistemine BİT’i entegre etmek, eğitim sistemini gelişmelerle desteklemek, değerlendirmeler yaparak onu sürekli olarak geliştirmek, ve BİT kullanarak öğrenci-merkezli ve proje tabanlı öğrenim sağlamak” (MEB, 2010, s. 36). Hedeflerine ulaşmak için MEB, yeni teknolojilerin kullanılabilirliği ve eğitimde yaygınlaştırılmaları için bütün okullara internet erişiminin sağlanmasını amaçlamıştır. Bu amaçlar doğrultusunda MEB, okullara teknolojiyi entegre etme girişimlerinde bulunmuştur. Okullardaki teknoloji kullanımını geliştirmek için MEB, FATİH, e-okul ve e-etüt gibi birçok proje yürütmektedir (EĞİTEK, 2011). Bu projelerin arasında eğitime yapılan en önemli yatırım FATİH olarak bilinen “Fırsatları Artırma ve Teknolojiyi İyileştirme Hareketi” projesidir. Bu Proje, fırsat eşitliği yaratmayı, dijital alandaki boşluğu gidermeyi ve eğitimin kalitesini artırmayı amaçlamıştır. Ayrıca Türkiye’de, okul öncesi, ilköğretim ve ortaöğretim düzeyindeki tüm okullara akıllı sınıfın entegre edilmesi amaçlanmıştır. Bu proje ile, 42.000 okul ve 570.000 sınıfın tablet, interaktif beyaz tahta ve en yeni bilgi Teknolojileri ile donatılmaları amaçlanmıştır. Bu proje, eğitim sürecinde daha iyi bir kavramsal anlayışı desteklemek için eğitimde fırsat eşitliği sağlamayı ve okullardaki teknolojiyi geliştirmeyi amaçlamıştır (EĞİTEK, 2011).

Öğretmenlerin teknoloji kullanımı konusundaki bilgileri, matematik eğitiminde teknoloji entegrasyonunun ve Türkiye’deki FATİH projesinin önemli bir unsurudur. Bu nedenle, hizmet içi ve hizmet öncesi öğretmenlerinin teknolojiyi

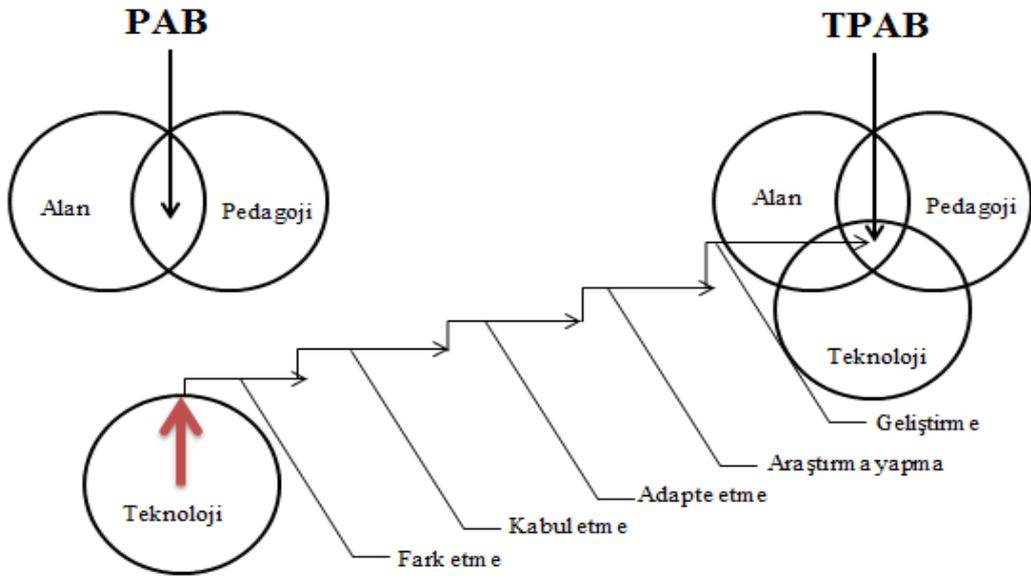
matematik eğitimine nasıl entegre etmeleri gerektiğine ilişkin bilgi sahibi olmaları gerekir. Diğer bir deyişle, hizmet öncesi ve hizmet içi matematik öğretmenleri, okul öncesinden ortaöğretime kadar matematik öğretiminde teknolojiyi uygun şekilde kullanmalıdırlar. Ayrıca, sınıflarında teknolojiyi nasıl entegre edebileceklerine ilişkin bilgilerini geliştirmede yardımcı olmak için hizmet içi öğretmenlerinin mesleki geliştirme programlarına ihtiyaçları vardır. Özellikle teknolojinin ancak doğru bir şekilde kullanıldığında öğrenimi etkinleştirdiği düşünülürse, öğretmenlerin teknoloji konusundaki mesleki eğitiminin önemli olduğunu söylenebilir. Teknolojinin eğitimdeki öneminin artması ile birlikte teknolojinin sınıfta nasıl kullanılabileceğine ilişkin bilgi, öğretmenlerin mesleki bilgilerinin gerekli bir bileşeni haline gelmiştir. Öğretmenler, öğrencilerin öğrenimini kolaylaştırmak ve desteklemek için teknolojilerin nasıl entegre edileceğine karar vermede önemli rol oynamaktadırlar (Matematiksel Bilimler Konferans Kurulu [CBMS], 2001; Uluslararası Eğitim Teknolojileri Derneği [ISTE], 2000, 2007). Ayrıca NCTM (2000), teknolojinin sınıfa entegrasyonuna ilişkin öğretmenlerin sahip olduğu bilginin, öğrencilerin matematik derslerindeki bilgilerinin artırılmasında önemli olduğunu desteklemektedir.

1.1 TPAB Gelişim Modeli

Niess, Sadri ve Lee (2007), matematik öğretiminde Teknolojik Pedagojik Alan Bilgisi (TPAB) gelişimini tanımlayan bir model geliştirmişlerdir. Bu TPAB gelişim modelinde başlıca dört tema bulunmaktadır: müfredat ve değerlendirme, öğrenim, öğretim ve erişim. Müfredat ve değerlendirme teması, ders konusunun ele alınış şekli ve öğrencilerin kavrayışlarını değerlendirmeye ilişkindir. Müfredat ve değerlendirme teması, müfredat ve değerlendirme ile ilgili kararlar arasındaki ilişkiyi vurgulamak amacıyla aynı gruba dahil edilmişlerdir. Öğrenim teması, öğrencilerin nasıl öğrendikleri ve öğrencilerin düşünme becerilerine ilişkin anlayışları göstermeyi amaçlamaktadır. Öğretim teması ise öğretim yaklaşımları, sınıf ortamı ve mesleki gelişim ile ilgilidir. Erişim teması, öğrencilerin teknolojiyi

kullanmalarına izin verilip verilmediği ve öğretmenlerin teknolojiyi entegre etme konusunda karşılaştıkları engelleri nasıl ele aldıkları ile ilgilidir.

TPAB Gelişim Model’indeki temalara ek olarak bu gelişim modeli, farkında olma, kabul etme, uyum sağlama, araştırma ve gelişme aşamaları şeklinde sıralı bir ilerleme sürecine sahiptir. Matematik öğretmenleri, kendi TPAB seviyeleri geliştikçe TPAB Gelişim Modeli’nde ilerleme kaydetmektedirler. Gelişmiş bir PAB düzeyi ile başlayan öğretmenler, dört temanın her birinde fark etme, kabul etme, adapte etme, araştırma yapma ve geliştirme aşamalarında ilerledikçe TPAB seviyeleri gelişir. Öğretmenler bu gelişim modelinde ilerledikçe, teknoloji, pedagoji ve alan bilgisinin kesiştiği alanlar oluşur ve genişler (Niess ve ark., 2009).



Şekil 1: TPAB Gelişim Modeli

Şekil 1’de gösterildiği gibi, alan, pedagoji ve teknoloji bilgisi, TPAB gelişim modelinin aşamalarında ilerledikçe genişler.

Fark etme (bilgi) aşamasında öğretmenler, teknolojiyi kullanabilir ve matematikteki kullanım şeklini fark edebilir. Ancak teknolojiyi matematiğin öğrenim ve öğretim sürecine entegre edemezler (Niess ve ark., 2010).

Kabul etme (ikna) aşamasında öğretmenler, matematiğin uygun bir teknoloji aracılığıyla öğretimi ve öğrenimine yönelik olumlu ya da olumsuz bir tutum oluştururlar (Niess et al., 2009). Teknolojiyi etkin bir şekilde matematik müfredatına entegre etme konusunda zorlanırlar.

Adapte etme (karar verme) aşamasında öğretmenler, uygun teknolojileri öğretim araçları olarak kullanmanın bazı avantajlarını görebilirler. Teknolojiyi kullanıp kullanmama kararını verebilirler (Niess ve ark., 2009).

Araştırma yapma(uygulama) aşamasında öğretmenler, aktif bir şekilde uygun olan teknolojiyi müfredata uygun düşmesi için etkinlikleri tekrar yapılandırıp tasarlayarak matematiğin öğretim-öğrenim sürecine entegre edebilirler. Ayrıca teknolojileri öğrenim araçları olarak kullanarak ders konusunu farklı şekillerde öğretme arayışı içinde olurlar (Niess ve ark., 2010).

Geliştirme (teyit) aşamasında öğretmenler, teknolojinin sağladığı kolaylıklardan yararlanmak için müfredatta değişiklikler yaparak matematik öğretim-öğrenim sürecine uygun olan teknolojiyi entegre etme kararlarının sonuçlarını değerlendirirler (Niess ve ark., 2010).

Matematik öğretmenlerine yönelik olan TPAB Gelişim Modeli, bu uzmanlık bilgisinin gelişiminde öğretmenlerin ilerleme düzeylerini tanımlamak için geliştirilmiştir (Niess ve ark., 2009). Bu nedenle, bu çalışmada öğretmenlerin matematik alanında TPAB gelişimindeki ilerlemeyi detaylı bir şekilde incelemek için TPAB Gelişim Modeli kullanılmıştır (Niess ve ark., 2009). Mishra ve Koehler (2006) tarafından geliştirilen TPAB tanımı geneldir ve Mishra ve Koehler'in (2006) TPAB modelindeki bileşenler açık tanımlanmamıştır. Ayrıca Mishra ve Koehler (2006) tarafından yapılan TPAB tanımı daha geneldir (belirli bir alana özgü değildir). TPAB Gelişim Modeli ise öğretmenlerin matematik öğretiminde teknolojiyi entegre etmeye ilişkin bilgisini de dahil eder. Bu nedenle bu çalışmada öğretmenleri TPAB seviyelerini belirlemek için TPAB Gelişim Modeli kullanılmıştır.

1.5 Mesleki Gelişim Stratejisi olarak Matematik Koçluğu

Öğretmenlikte mesleki gelişim, öğretmenlerin kavrayışlarını, alan bilgilerini ve öğretim uygulamalarını geliştirmede vazgeçilmez bir mekanizmadır (Bybee ve Loucks-Horsley 2000; Cohen & Hill, 2001; Garet, Porter, Desimone, Birman, & Yoon, 2001). Mesleki eğitim, öğretmenlere bilgi ve beceriler edinme, onları düzenleme ve uygulamaya aktarmada yardımcı olur. Ancak mesleki gelişime yönelik geleneksel yaklaşımlar, öğretmenlerin ihtiyacı olan konularda yardımcı olmada yetersiz kalır (Knight, 2007). Geleneksel mesleki gelişim programları, sınıf ve müfredattan bağımsız ve ayrı olan çalıştaylar oluşmaktadır (Ball & Cohen, 1999). Bunun yanında mesleki gelişim programları, öğretmenlerin gelişiminde ilave bir unsur olarak görülmektedir. Diğer bir deyişle, mesleki gelişim programları, öğretmenlere daha fazla yapılacak ve öğrenilecek şeylerin eklenmesi anlamına gelmez. Daha ziyade öğretmenleri dönüştürücü role sahip olmalıdır (Loucks-Horsley, ve ark., 2003). Diğer yandan bazı çalışmalar, etkin bir mesleki gelişim programları için öğretmen uygulamalarının bu programlara dahil edilmesi gerektiğini göstermiştir (Coleman & Goldenberg, 2010; Huston & Weaver, 2007; Joyce & Showers, 1982; Knight, 2010).

Loucks-Horsley ve arkadaşlarına (2003) göre mesleki gelişim programlarına alternatif yaklaşımlardan biri de koçluktur. Koçluk mesleki gelişim programı, yeni strateji ve yaklaşımları uygulamada destek sağlayan ve rehberlik eden şirketlerdeki koçluk sisteminin benzeridir (Joyce ve Showers, 1982). İlk olarak koçluk fikrini Showers ve Joyce (1966) ileri sürmüştür. Çünkü literatürdeki araştırmalar mevcut mesleki gelişim stratejilerinin öğretmenlerin değişim ve yeni düşünceler uygulamaları açısından etkin olmadığını göstermiştir (Showers & Joyce, 1966). Loucks-Horsley ve arkadaşlarına (2003) göre koçluk, öğretmenlere, kendi ve başkalarının uygulamaları üzerinde düşünerek matematik ve fen bilgisinin öğretimi üzerinde yoğunlaşma ve böylelikle kendi bilgisi, öğrenimi ve uygulamalarını geliştirerek sınıf içi uygulamalarına entegre etme konusunda birebir öğrenme fırsatları sağlayan bir mesleki gelişim stratejisidir. Koçluk, mesleki uygulamaları ve

öğrenci öğrenimini artırma amacıyla olan uygulama ile bütünleşik bir yapıya sahip kişisel gelişim programı olarak tanımlanmaktadır (Brandt, 1987, Knight, 2007, Loucks-Horsley, ve ark., 2003, Showers, 1985). Koçluk modelinde, öğretmenlere sunulan içerik ve yöntemlere önem verilmektedir ve öğretmenler sınıflarında yeni stratejiler denedikleri zaman, koç onlara destek sağlamaktadır.

Okul-tabanlı bir mesleki gelişim yöntemi olan koçluğun farklı biçimleri, amaçları ve teknikleri vardır; örneğin, akran koçluğu, bilişsel koçluk, içerik koçluğu ve teknik koçluğu. Knight (2009), bu farklı koçluk stratejilerinin ortak bazı özellikleri olduğunu belirtir. Bunlar, koç ve öğretmen arasındaki ilişkinin eşit derecede katkı sağlaması ilkesine dayanması, uzun süreli olması, yoğun bir ilişkinin olması, karşılıklı konuşma ve konu üzerinde düşünmeye dayalı olması, sadece söz konusu öğretmene özel olup öğretmenin her hangi bir değerlendirmeye maruz kalmaması ve saygı çerçevesinde bir iletişim biçimi gerektirmesidir. Ayrıca, çeşitli koçluk türleri ve koçun farklı sorumluluklarına rağmen hepsinde geleneksel bir koçluk modeli vardır ve hepsi, sınıf içi gözleme önem verip ön-konferans/gözlem/son-konferans döngüsünü takip eder (Loucks-Horsley, ve ark., 2003).

Koçluk stratejilerinden biri olan içerik tabanlı koçluk, öğretmenlere, okuma yazma ve matematik gibi belirli bir akademik disiplinde öğretimlerini geliştirmede yardımcı olur (Neufeld & Roper, 2003). West ve Staub (2003), içerik odaklı koçluğu, matematik alan bilgisi ve pedagoji odaklı matematik öğretmeni mesleki eğitim gelişim aracı olarak tanımlar. Bu tür matematik koçluğunda, öğrencilerin öğrenimlerini ve kazanımlarını artırmaya yönelik matematik strateji ve etkinlikleri analiz etme ve uygulamada öğretmen ve koç birlikte çalışırlar. Gerçek bir derse dayalı olan Matematik Koçluk oturumlarında, içerik odaklı koçlar, her bir öğretmen veya küçük öğretmen grupları ile buluşup bir dersin planlanma, uygulanma ve değerlendirilme sürecinde rol alır (West & Staub, 2003). İçerik odaklı koçluk oturumlarında üç aşama vardır: ön-konferans, gözlem ve son-konferans. Ön-konferans koça, öğretmenin planlama alışkanlıklarını, içerik bilgisini, ve matematik öğretiminde ve öğrencilerin öğrenim deneyimleri alanlarındaki en iyi uygulamalar

hakkındaki pedagojik bilgiyi daha iyi bilgi edinme fırsatı verir (West & Staub, 2003). Bu süreçte koç, dersin objektiflerini, derste önemli matematiksel kavramları ve uygun öğretim stratejilerini belirlemede öğretmene yardımcı olmak için onunla buluşur. Ayrıca öğretmen ve koç, öğrencilerin potansiyel yanılgılarını ve derste bunların üstesinden nasıl gelinebileceğini tartışabilirler. Dersin öğretimi açısından koçun rolü, dersten önce farklılaştırılabilir: dersi, tek başına koç, koç ve öğretmenle birlikte veya tek başına öğretmen sunabilir (West, 2009). Koç, öğretmenin belirli bir öğretim uygulamasını veya konuşmasını veya öğrencilerin eylemlerini gözlemlemesinde yardımcı olmak için tüm dersi öğretmenle birlikte sunabilir. Öğretmen ve koç, dersi bir takım olarak birlikte öğretebilir veya koçun gözlemi altında tüm dersi öğretmen kendi sunabilir.

Son-konferansta ise öğretmenden dersin başarısı üzerine düşünmesi ve yorum yapması ve bunu yaparken geliştirilmesi gereken alanların tespit edilmesi istenir (West, 2009). Koç ve öğretmen, öğretimde takip eden süreçlerin ve ders içeriği materyalleri konusunda atılacak adımların neler olacağını tespit etmek üzere etkili bir konuşma gerçekleştirirler (West & Staub, 2003). Son-konferansın en önemli amacı, ders üzerine olan düşüncelerin, özellikle öğrenci çalışmaları ve yorumlarının, ışığında bir sonraki dersin odak noktasını belirlemektir.

Öğretmenlere, planlama, uygulama ve değerlendirmede koçla birlikte çalışma olanağı tanımak, onların matematik alan ve pedagoji bilgi düzeylerini artırmaktadır. Matematik koçluğunda öğrencilerin matematik öğrenimini geliştirmek için koçlar, doğrudan sınıf öğretmenleri ile çalışırlar (Hull, Balka ve Miles, 2009). Matematik koçlarının sorumlulukları şunlardır: matematikte başarıyı artırmak için öğretmenlerle birlikte çalışmak, müfredat ve öğretim materyallerini idare etmek ve kontrol etmek, mesleki gelişimi idare etmek ve düzenlemek, programın uygulanmasını takip etmek, güçlü yönleri kullanıp zayıf yönleri azaltarak matematik programını oluşturmak, en iyi uygulamalar üzerine yapılan araştırmaları sürdürmek ve paylaşmak, işbirlikçi grup ve sosyal ağlar oluşturmak, ve öğretime yön vermek açısından değerlendirme ve temel ölçütler gibi araçlardan veri toplama, analiz etme ve yorumlamak. Bu çalışma, ilköğretim matematik öğretmenlerinin

matematik koçluęuna katılımlarıyla geometri alanında TPAB seviyelerindeki ilerlemeyi inceleyerek matematik eęitimi alan yazınına katkı saęlamayı amaçlamıştır.

1.1 Çalışmanın Araştırma Soruları

Matematik öğretmenlerinin TPAB seviyelerinin önemli olduęu kabul edilerek, bu çalışmanın amacı, matematik koçluęu süresince öğretmenlerin geometrideki TPAB gelişimlerini ve öğretmenlerin geometrideki TPAB seviyelerindeki ilerlemelerini belirlemektir. Bu çalışmada aşağıdaki araştırma sorularına yanıt aranmıştır:

1. Matematik koçluęu mesleki gelişim programına katılmak, ilköğretim matematik öğretmenlerinin geometri alanındaki teknolojik pedagojik alan bilgilerini ne ölçüde etkilemektedir?
 - 1.1. Matematik koçluęu mesleki gelişim programına katılan ilköğretim matematik öğretmenleri, TPAB Gelişim Modeli'ndeki Müfredat ve Deęerlendirme temasının seviyelerinde nasıl bir gelişim göstermektedirler?
 - 1.2. Matematik koçluęu mesleki gelişim programına katılan ilköğretim matematik öğretmenleri, TPAB Gelişim Modeli'ndeki Öğrenim temasının seviyelerinde nasıl bir gelişim göstermektedirler?
 - 1.3. Matematik koçluęu mesleki gelişim programına katılan ilköğretim matematik öğretmenleri, TPAB Gelişim Modeli'ndeki Öğretim temasının seviyelerinde nasıl bir gelişim göstermektedirler?
 - 1.4. Matematik koçluęu mesleki gelişim programına katılan ilköğretim matematik öğretmenleri, TPAB Gelişim Modeli'ndeki Erişim temasının seviyelerinde nasıl bir gelişim göstermektedirler?

2. İlköğretim matematik öğretmenlerinin matematik koçluğunun teknolojik pedagojik alan bilgisinin gelişiminde aldığı roller açısından algıları nelerdir?

1.2 Çalışmanın Önemi

Teknolojinin toplumda artan kullanımına paralel olarak teknolojinin matematik eğitimindeki rolü önem kazanmıştır. NCTM (2000), teknolojinin matematik öğretim ve öğreniminde teknolojinin gerekli olduğunu ifade etmiş ve öğrencilerin matematik öğrenimlerini artırdığını belirtmişlerdir. AMTE (2006), ISTE (2007), ve NCTM (2000) gibi mesleki kuruluşlar, öğretmenlerin teknolojiyi etkin biçimde kullanmayı öğrenmeleri gerektiğini belirtmişlerdir. Teknolojiyi etkin biçimde entegre edebilmeleri için öğretmenlerin pedagoji ve alan bilgisi ile birlikte teknoloji bilgisi de edinmeleri gerekmektedir (Mishra & Koehler, 2006; Niess, 2005). NCTM'ye (2007) göre, eğer öğretmenler, işbirlikli problem çözmeyi destekleyen pozitif ortamları nasıl oluşturacaklarını, teknolojiyi anlamlı biçimde nasıl entegre edeceklerini, kavramların keşfedilmesini nasıl sağlayacaklarını ve öğrencilerin düşünmelerini nasıl destekleyeceklerini öğreneceklerse, kendilerinin de bizzat bu tür bir ortamları deneyimlemeleri gerekmektedir. Bu nedenle öğretmenlere, teknolojinin matematik eğitime nasıl entegre edileceğini deneyimleyecekleri bir ortam sağlamak gerekmektedir. Öğretmenlerin bilgi düzeylerinin önemli olduğuna dayanarak TPAB alanında ve teknoloji ile matematik öğretimi konusunda daha fazla araştırmaya gereksinim olduğu söylenebilir.

Son zamanlarda T.C. Milli Eğitim Bakanlığı (MEB), okullarda teknoloji entegrasyonuna büyük önem vermiştir. Ayrıca (akıllı tahtalar gibi) teknoloji ile donatılan okulların sayısındaki artış, teknoloji bilgisine verilen önemi göstermektedir. Türkiye'de FATİH, e-okul, ve e-etüt gibi birçok proje vardır (EGİTEK, 2011). Bu projeler, T.C. Milli Eğitim Bakanlığı'nın teknolojinin derse entegrasyonu konusuna önem verdiğini gösterse de, TPAB açısından daha nitelikli öğretmenlere ihtiyaç vardır. Teknoloji ile daha etkin öğretim için sadece teknoloji bilgisi yeterli değildir (Koehler & Mishra, 2009, Mishra & Koehler, 2006).

Öğretmenlerin teknoloji, pedagoji ve alan bilgilerinin birlikte olması gerekmektedir (Koehler & Mishra, 2009, Mishra & Koehler, 2006). Öğretmenlerin TPAB seviyelerini geliştirmek için, sınıf içinde teknolojiyi nasıl kullanabilecekleri konusunda onlara eğitim sunulması gerekmektedir. Bu açıdan bakıldığında TPAB alanında gelişim için öğretmenin mesleki gelişim programlarına katılması önemlidir.

Öğretmenler için mesleki gelişim programları, sınıflarına teknolojiyi entegre etme konusunda öğretmenleri yetkin kılmayı amaçlamalıdır. Ancak mesleki gelişimde geleneksel yaklaşımlar öğretmenlerin bilgi ve becerilerini geliştirmede yetersiz kalmaktadır (Knight, 2007). Geleneksel mesleki gelişim yaklaşımlarının yetersizliklerinin sebebi, öğretmenlerin sınıftan bağımsız ve uzak bırakılmaları olarak belirtilmektedir (Ball & Cohen, 1999, Knight, 2007). Etkin mesleki gelişim programları, öğretmenlere alan ve pedagojik alan bilgilerinin oluşumu ve uygulamaya konulması konusunda yardımcı olmaktadır. Loucks-Horsley ve arkadaşlarına (2003) göre koçluk, öğretmenlerin öğrendiklerini sınıf pratiğine uygulayarak bilgilerini geliştirmeyi amaçlayan ve öğretmenlere birebir öğrenim fırsatları sağlayan etkin bir mesleki gelişim stratejisidir. Ancak koçluk, mesleki gelişim için yeni bir stratejidir ve özellikle matematik öğretmenlerinin mesleki gelişiminde koçluk ile ilgili çok fazla araştırma yapılmamıştır (Drust, 2013; Larsen, 2012; McLaughlin, 2012; Nicometi, 2011). Bu çalışma mesleki gelişim stratejisi olarak koçluk programına duyulan araştırma ihtiyacını karşılamaktadır. Ayrıca, koçluk programında, hizmet içi öğretmenlere teknolojinin nasıl entegre edileceğine dair öğrenim ve uygulama fırsatı sunulmaktadır. Bu nedenle, koçluk programı TPAB seviyelerini geliştirme konusunda öğretmenlere yardımcı olmaktadır. Bu çalışma, mesleki gelişim programı olan matematik koçluğuna katılan hizmet içi öğretmenlerinin TPAB seviyelerini nasıl geliştirdiklerini incelemektedir. Bu açıdan bakıldığında bu çalışma, geometri alanında öğretmenlerin mesleki gelişim açısından TPAB seviyelerindeki gelişim konusunda bilgi sağlama potansiyeline sahiptir.

Bu çalışmanın amacı, matematik koçluğu mesleki gelişim programına katılan matematik öğretmenlerinin geometri alanına ait teknolojik pedagojik alan

bilgilerindeki gelişimini incelemektir. Ayrıca bu çalışma, ilköğretim matematik öğretmenlerinin, koçluk öncesinde/süresince/sonrasında, geometri alanında TPAB seviyelerindeki gelişimlerini, TPAB tema ve seviyelerine göre incelemeyi amaçlamaktadır. TPAB Gelişim Modeli (Niess, Ronau ve ark., 2009) üzerinde durularak öğretmenlerinin geometrideki TPAB seviyelerinin araştırılmasının alan yazına önemli katkı sağlayacağına inanılmaktadır. Çünkü bu TPAB Gelişim Modeli (Niess, Ronau ve ark., 2009), matematik öğretmenlerine yönelik uzmanlık bilgisinin oluşturulmasında öğretmenlerin gelişimini betimlemek için geliştirilmiştir. Mishra ve Koehler (2006) tarafından sunulan TPAB modeli genel olmakla birlikte herhangi bir alana özgü değildir. Diğer yandan TPAB Gelişim Modeli (Niess, Ronau ve ark., 2009) alana özgüdür ve teknolojinin matematik öğretimine entegrasyonunu dört tema ve beş seviye kapsamında öğretmen bilgisine dahil eder. Dolayısıyla, bu çalışmada TPAB Gelişim Modeli öğretmenlerin TPAB seviyelerini dört tema (müfredat ve değerlendirme, öğrenim, öğretim ve erişim) kapsamında belirlemek için kullanılmıştır. TPAB Gelişim Modeli (Niess, Ronau ve ark., 2009)'nin detaylı yapısından dolayı öğretmenlerin bilgi düzeylerine bu gelişim modeli açısından bakmak yararlı olacaktır.

Ayrıca alan yazın, teknoloji entegrasyonuna verilen öneme rağmen öğretmenlerin TPAB seviyelerinin (özellikle Türkiye'de) yetersiz olduğunu göstermektedir. Bu çalışmanın sonuçları, ilköğretim matematik öğretmenlerinin TPAB bileşenleri ve seviyeleri hakkında bilgi sağladığından dolayı alana önemli katkı sağlayabilecektir. Alan yazın taramasında görülmüştür ki TPAB konusundaki çoğu araştırma, hizmet öncesi öğretmenleri ile yapılmıştır. Hizmet içi öğretmenlerin TPAB konusunda çok fazla araştırma yoktur (McBroom, 2012; Stoilescu, 2011; Taylor, 2011). Bu nedenle hizmet içi öğretmenlerinin TPAB seviyeleri konusunda daha fazla araştırmaya gereksinim vardır. Ayrıca alan yazındaki TPAB üzerine olan araştırmalar alana özgü değildir. Geometri dersine teknolojinin entegrasyonuna ilişkin ilköğretim matematik öğretmenlerinin bilgi düzeyi üzerinde durarak bu çalışma, bu açığı kapatarak alanyazına katkı sağlayacaktır. Bu çalışmada hizmet içi matematik öğretmenleri, geometride çokgen ünitelerinde teknoloji aracı olarak

Geogebra'ya derse entegre ederek teknoloji destekli bir ders planlamış, uygulamış ve uygulama sonrası üzerinde düşünmüşlerdir. Diğer bir deyişle bu çalışma, hizmet içi öğretmenlerinin teknolojiyi (özellikle Geogebra'ya) geometri dersine nasıl entegre ettiklerini göstermiştir. Bu çalışma kapsamında öğretmenler, geometri derslerinde Geogebra'ya uygulamışlardır. Hizmet içi öğretmenlerinin Geogebra'ya bir öğrenim ve öğretim aracı olarak derslerine nasıl entegre ettikleri ve Geogebra-destekli derslerini nasıl yürüttükleri incelenmiştir. Ayrıca, TPAB gelişimi üzerine olan alan yazındaki çoğu araştırma, öğretim teknolojisi olarak sadece grafik hesap makinesi üzerinde durmuştur. Bu nedenle, Geogebra, Cabri3D ve Tinkerplots gibi farklı öğretim teknolojilerinin kullanıldığı TPAB çalışmalarına ihtiyaç vardır. Dolayısıyla bu çalışma, matematik koçluğu aracılığıyla geometri derslerine Geogebra'ya entegre eden hizmet içi matematik öğretmenlerinin TPAB gelişimini incelemeyi amaçlamıştır.

2. Yöntem

Bu çalışma, matematik koçluğu öncesinde, süresince ve sonrasında ilköğretim matematik öğretmenlerinin teknolojik pedagojik alan bilgilerinin gelişimlerinin incelenmesi amaçlamıştır. Buna ek olarak geometri alanında teknolojik pedagojik alan bilgilerine ilişkin derinlemesine ve kapsamlı bir araştırma yapılmasını hedeflenmiştir. Bu araştırmanın doğası ve amacına uygun olarak, bu çalışmada ilköğretim matematik öğretmenlerinin geometri alanında TPAB seviyelerindeki gelişimi ortaya çıkarmak amacıyla nitel araştırma yöntemi kullanılmıştır. Bu çalışma, Creswell (2007) ve Yin (2003)'in durum çalışması ile nitelendirilmiştir. İlköğretim matematik öğretmenlerinin geometri alanındaki TPAB seviyelerindeki gelişimi araştırmak amacıyla Karadeniz bölgesinde bulunan ilköğretim okullarından öğretmenler seçilmiştir. Bu doğrultuda çalışmaya iki matematik öğretmeni katılmıştır. Çalışma 2013-2014 akademik yılının bahar döneminde yapılmış olup ve her bir öğretmen için 16 saat sürmüştür. TPAB

gelişimlerini analiz etmek için araştırmacı, bir mesleki gelişim olarak matematik koçluğu yapmıştır.

2.1 Mesleki Gelişim olarak Matematik Koçluğu

Matematik koçluğu, bir mesleki gelişim olarak araştırmacı tarafından sunulmuştur. Bu mesleki gelişim, öğretmenlerin teknolojinin niteliklerine ilişkin bilgilerini, Geogebra yoluyla değerlendirme stratejilerini, müfredata ilişkin fikirlerini teknolojik ortamda nasıl uygulayabileceklerini ve bir öğrenim ve öğretim aracı olarak Geogebra'yı teknolojinin etkin öğretimi için nasıl kullanabileceklerini geliştirmeyi amaçlamıştır. Matematik koçluğu, ön-konferans, gözlem ve son-konferans şeklinde döngüsel bir süreç içermektedir. Bu çalışmada ön-konferans/gözlem/ son-konferans'tan oluşan matematik koçluğu döngüsü dört kez tekrar edilmiş ve matematik koçluğu döngüsü 1 (MKG1), matematik koçluğu döngüsü 2 (MKG22), matematik koçluğu döngüsü 3 (MKG3), matematik koçluğu döngüsü 4 (MKG4) olarak isimlendirilmiştir.

2.1.1 Ön-konferans

Matematik koçluğu döngüsünün ilk kısmını öğretmenlerle olan birebir ön-konferans oluşturmaktadır. Yüz yüze olan bu görüşmeler en az bir saat sürmekte ve dersin planlanması üzerinde yoğunlaşmaktadır. Aynı zamanda bir koç olarak araştırmacıya öğretmenlerin TPAB ve planlama alışkanlarını anlama olanağı vermiştir.

Ön-konferansta öğretmenler, Geogebra kullanarak çokgenler konusunda matematik dersleri planlamışlardır. Ön-konferansta yapılan görüşme esnasında koç ve öğretmenler arasında geçen konuşmalar yardımıyla ders planları tekrar yapılandırılmıştır. Aynı zamanda ön-konferansta araştırmacı, öğretmenin teknoloji ile matematik öğretim ve öğrenimine olan ilgisi üzerinde yoğunlaşmış ve öğretmenden, öğrencilerin dersi daha iyi anlamalarını destekleyen teknolojiye ilişkin yapılan maksatlı kararları anlatmasını istemektedir. Bu ön-konferansta, öğretmenlerin ihtiyaçlarına ve TPAB seviyelerin ışık tutmuş ve bu işbirlikli

planlama oturumu ile ders planının bazı bölümlerine, özellikle çokgen ünitesine ilişkin amaçlar üzerinde yoğunlaşmıştır.

Ön-konferansta, ilköğretim matematik öğretmenlerinin genelde geometri, özelde çokgenler, açısından nasıl TPAB oluşturduklarını değerlendirmede yardımcı olmuştur. Ön-konferansta, matematik müfredatına yönelik daha analitik bir duruş edinmeleri ve Geogebra'nın sunduğu kolaylıkları düşünerek müfredat üzerinde değişiklik yapmaları için teşvik etmiştir. Ayrıca öğretmenler, öğrencilerin yaratıcılık ve daha üst düzey düşünme becerilerini geliştiren Geogebra destekli etkinlikler kullanarak çokgen ünitelerine ilişkin matematik deneyimlerini kolaylaştırmaları konusunda teşvik edilmişlerdir. Buna ek olarak koç, Geogebra üzerinde bazı etkinlikleri göstermiş ve öğretmenlerin TPAB seviyelerini iyileştirmek için bazı önerilerde bulunmuştur. Örneğin, her iki öğretmenin de (Murat ve Esen) Geogebra destekli aktivitelerde öğrencilerin çalışmalarını kontrol etme eğilimleri vardı. Araştırmacı öğretmenlere, öğrencilere kendi kendilerine keşfetmeleri için zaman vermeleri ve öğrencilerin keşif süreçlerini yönetmekten ziyade onlara rehberlik etme önerisinde bulundu. Öğretmen ve koç, bu önerileri tartıştılar ve bu tartışmayı göz önünde bulundurarak ders planı tekrar yapılandıldılar. Nihayetinde ön-konferansa, koç ve öğretmenlere, matematik koçluğunun gözlem aşaması için teknolojinin matematik dersine entegrasyonu şeklinde ortak bir odak noktası sağlamıştır.

Ön-konferansta koç gerektiğinde teknik yardım da sağlamıştır. Örneğin araştırmacı, Murat'a Geogebra'da nasıl sürgüleri ekleyebileceğini göstermiştir. Ayrıca koç, öğretmene teknolojinin entegrasyonu ve Geogebra'ya ilişkin fikirler için İnternette nasıl araştırma yapabileceğini göstermiştir. Örneğin koç, öğretmenlere “www.geogebra.com” adresindeki siteyi tanıtmıştır. Özet olarak ön-konferans, öğretmenlerin düşünce süreçleri ve ihtiyaçlarına yönelik koça içgörü sağlamıştır. Ayrıca bu ön-konferans, koçun öğretmene üzerinde konuşulan ders planı hakkında geri bildirimde bulunmasına olanak sağlamıştır.

2.1.2 Gözlem

Matematik koçluğunun bir başka kısmı, öğretmenlerin derslerinin gözlenmesidir. Her matematik koçluğu döngüsünde koç, öğretmenlerin derslerini iki

veya dört saat izlemiştir. Gözlem sırasında öğretmenler, ön görüşmede tartışılan Geogebra destekli dersi uygulamışlardır. Koç gözlem sonrası yapılan görüşmede üzerine konuşabilmek için derste olanlar ile ilgili not tutmuştur. Birlikte planlanan dersin gözlemlenmesi, öğretmenler ve koçun, teknolojiyi bir öğretim ve öğrenim aracı olarak nasıl entegre ettikleri üzerine gözlem sonrası görüşmede düşünüp konuşmalarını sağlayan bilgi kaynağı olmuştur. Ayrıca, matematik koçluğun takip eden adımlarını belirleyebilmek için koç, öğrencilerin Geogebra üzerinde yaptıkları çalışmaları ve çokgenler üzerine yaptıkları diyalogları derlemiştir.

Öğretmenlerin derslerini gözlemlemek, onların TPAB Gelişim Modeli'ndeki seviyeleri hakkında bilgi vermiştir. Örneğin öğretmenlerin Geogebra'yı statik bir araç olarak entegre etmeleri, onların daha düşük TPAB seviyesinde olduklarını göstermektedir. Onların Geogebra'nın dinamik özelliklerini dikkate alarak teknolojiyi entegre etmeleri, daha üst düzey TPAB seviyesine işaret etmiştir. Bu nedenle dersin gözlenmesi, öğretmenlerin TPAB Gelişim Model'indeki seviyeleri hakkında kanıt niteliğinde bilgi sağlamıştır.

2.1.3 Son-konferans

Dersin gözlemlenmesinden sonra koç ve öğretmenler, teknolojinin entegrasyonunu temel alarak dersi tartışmak için birlikte zaman geçirmişlerdir. Son-konferans esnasında öğretmenler, teknoloji, pedagoji ve içerik bakımından performansları üzerine derinlemesine düşünmüşlerdir. Örneğin, birinci matematik koçluğu döngüsünde Murat, teknoloji entegrasyonu bakımından dersinin iyileştirilmesi gerektiğini düşünmüştür. Geogebra'yı bir sunum aracı olarak uyguladığını fark etmiş ve bu durumu koç ile birlikte dersin nasıl iyileştirilebilmesi için tartışmışlardır. Ayrıca izlenen dersle ilgili bazı konuların, kaygıların ve eleştirilerin gündeme gelmelerine olanak sağlama, öğretmenler için nelerin önemli olduğu ve onların TPAB seviyeleri hakkında bilgi vermiştir. Koç, izlediği ders ve aldığı notlarla ilgili bilgi paylaşmış ve geri bildirimde bulunmuştur. Örneğin, ilk matematik koçluğu döngüsünde Esen, dörtgenlerle ilgili Geogebra aktivitelerini tahtada göstermiştir. Her öğrenci grubunda bilgisayar olmasına rağmen Esen,

öğrencilere Geogebra'yı kullandırtmamıştır. Öğrenciler dörtgenlerin özellikleriyle ilgili genellemeler yaptıktan sonra Esen, Geogebra'yı kullanmaları için öğrencileri teşvik etmiştir. Ders öğretmen merkezli olup son-konferansta, koç öğrenci merkezli ders hakkında önerilerde bulunmuştur. Koç, öğrencilerin kendi keşifleri için onlara zaman vermesi konusunda öğretmeni teşvik etmiştir. Geri bildirimden sonra koç ve öğretmenler, Geogebra'ya yeni bir bakış açısıyla bakmaya çalışmanın daha yararlı olacağı konusunda uzlaşmışlardır. Özet olarak son-konferansta, öğrenmeyi değerlendirme, ders hakkında bilgi verme ve eğitimsel planları uyarlama fırsatı sağlanmıştır. Gözlem-sonrası görüşmenin en önemli amacı, koç ve öğretmenlerin düşünceleri ışığında takip eden derslere destek sağlamaktır.

2.2 Veri Toplama Araçları

Bu çalışmada araştırma sorularına yanıt verebilmek için matematik koçluğu öncesinde, sürecinde ve sonrasında çeşitli biçimlerde veri toplanmıştır. Veri toplama süreçleri şunlardır: 1) matematik koçluğu öncesinde TPAB üzerine ön-görüşme, 2) matematik koçluğu öncesinde öğretmenin gözlenmesi, 3) matematik koçluğu sürecinde matematik öğretmenleri ile birebir ön-konferans, 4) matematik koçluğu sürecinde öğretmenlerin gözlemlenmeleri, 5) matematik koçluğu sürecinde matematik öğretmenleri ile birebir son-konferans, 6) matematik koçluğu sonrasında TPAB üzerinde gözlem sonrası görüşme ve 7) matematik koçluğu sonrasında matematik koçluğu görüşmesi.

2.3 Veri Analizi

Nitel araştırmalarda veri toplama ve analizi aynı anda gerçekleşen etkinliklerdir ve yansıtıcı ve süreklilik arz eden süreçlerdir (Merriam, 1998). “Matematik koçluğu mesleki gelişim programına katılmak, ilköğretim matematik öğretmenlerinin geometri alanındaki teknolojik pedagojik alan bilgilerini ne ölçüde etkilemektedir?” şeklindeki ilk araştırma sorusu ve alt soruları için matematik koçluğun çeşitli evrelerinde gerçekleşen ön-konferans ve son-konferanstaki

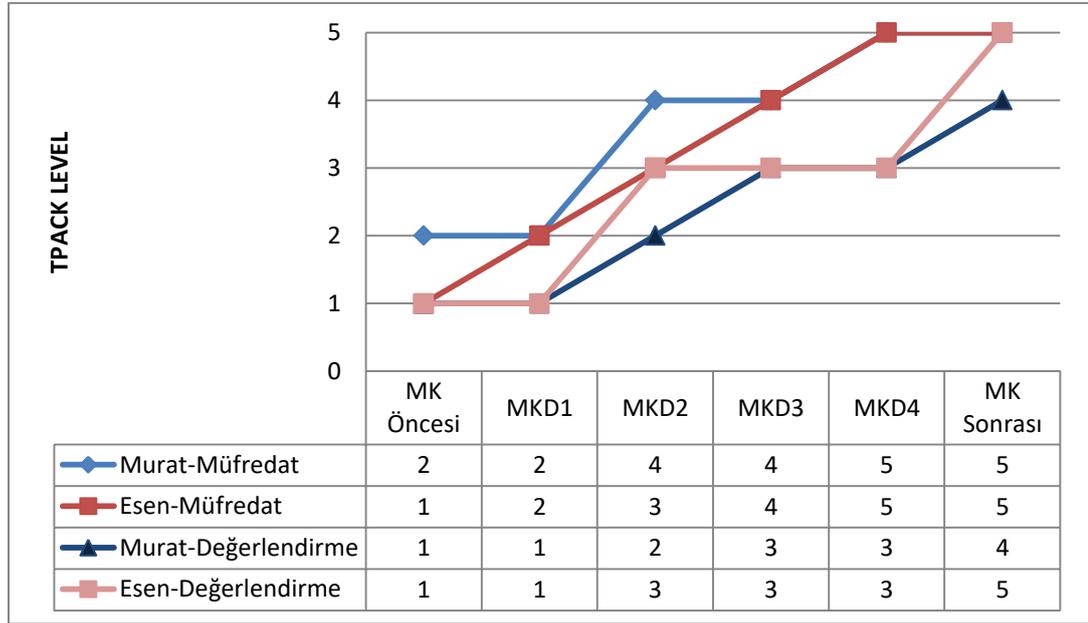
mülakatlar ile transkripsiyonları, video kayıtları, gözlemler ve bu gözlemlere ilişkin alan notları analiz edilmiştir. Öğretmenlerin TPAB seviyelerindeki gelişim, tündengelim analiz yöntemi kullanılarak TPAB Gelişim Modeli aracılığıyla tespit edilmiştir. Verilerin mevcut bir yapıya göre analiz edildiği durumlarda tündengelim analizi uygulanmıştır (Patton, 2002). Tündengelim analizinde temalar ve kodlar, alanyazın, kuramlar veya araştırma sorularının ayrıntıları temel alınarak önceden seçilmiştir. Araştırmacı, ilgili alıntıları belirleyip ve TPAB Gelişim Modeli'ni (Ek J'ye bakınız) kullanarak TPAB gelişim seviyesi (fark etme, kabul etme, adapte etme, araştırma yapma, geliştirme) ve tema (müfredat ve değerlendirme, öğrenim, öğretim, erişim) hakkında not düşmüştür. Bu modelde TPAB seviyelerin anlaşılmasını sağlayan 11 tane betimleyici ifade ve örnekleri bulunmaktadır.

“İlköğretim matematik öğretmenlerinin matematik koçluğunun teknolojik pedagojik alan bilgisinin gelişiminde aldığı roller açısından alguları nelerdir?” olan ikinci araştırma sorusu için araştırmacı, yarı-yapılandırılmış mülakatların kayıtlarını analiz etmiştir. Mülakat verilerinin analizi için tümevarım sürecinin benimsendiği açık kodlama yöntemi uygulanmıştır. Mülakatların transkripsiyonları analiz edilmiştir ve analiz edilirken şu sıra takip edilmiştir: verilen düzenlenmesi, kategorilerin, temaların ve örüntülerin oluşturulması, verilen kodlanması, ortaya çıkan anlayışların test edilmeleri, alternatif açıklamaların araştırılması ve raporların yazılması (Marshall & Rossman, 2011). Matematik koçluğunun faydaları, matematik koçluğunu diğer mesleki gelişim programlarından ayıran farklılıklar, matematik koçluğunun zorlukları ve öneriler olmak üzere dört tema açığa çıkmıştır.

3. Bulgular

3.1 Matematik Koçluğunda Murat ve Esen'in Müfredat ve Değerlendirme Temasına İlişkin TPAB Seviyelerinde Gösterdikleri Gelişim

Tablo 1. Öğretmenlerin Müfredat ve Değerlendirme Temasına İlişkin TPAB Seviyeleri



Murat ve Esen'in matematik koçluğu öncesinde, sürecinde ve sonrasında altı teknoloji destekli derslerindeki müfredat ve değerlendirme alanındaki performansları Tablo 1'de sunulmuştur. Öğretmenler, TPAB gelişim modelinde müfredat ve değerlendirme alanında gelişim göstermişlerdir.

Bu çalışmada müfredata ilişkin tanım, bir eğitim aracı olarak teknolojinin müfredata nasıl entegre edileceğini tarif etmektedir. Çalışmanın başında, Murat'la kıyaslandığında Esen, müfredat alanında TPAB Gelişim Modeli'nde daha alt seviyede bulunmaktadır. Çünkü Murat, Geogebra destekli etkinliklerini öğrencilerin çokgenle ilgili bilgilerini geliştirmek için kullanırken, Esen Geogebra'yı elle tutulur materyallerle öğretilen bilgiyi pekiştirmek için kullanmıştır. Ancak ne Murat, ne

Esen Geogebra'nın dinamik özelliği gibi sunduğu avantajları dikkate almıştır. Öğretmenler, müfredatın gerekliliklerini yerine getirmek için ders kitabındaki etkinlikleri değiştirmişlerdir, ancak yine kalem-kağıt kullanılarak yapılan etkinliklerin ötesine geçmemişlerdir. Öğretmenler, Geogebra'nın dinamik doğasının sunduğu avantajları dikkate alarak onun matematik dersine nasıl entegre edileceğini anlamada güçlük çekmişlerdir. Öğretmenlerin çalışmanın başında (matematik koçluğu öncesi ve MKD1) Geogebra, çokgenlere ilişkin kavramların öğretimi ve öğreniminde sunum aracı olarak kullanılmıştır. Matematik koçluğu sürecinde öğretmenler, TPAB gelişim modelinin müfredat alanında daha üst düzeye geçmişlerdir. Öğretmenler, Geogebra'yı bir teknoloji aracı olarak matematik müfredatına entegre etmenin bazı yararlarını anlamaya ve araştırmaya başlamışlardır. Ancak, Geogebra'nın faydalarını anlamasına rağmen Esen, öğrencilerin MKD2 bilgilerini özetlemek için Geogebra yerine somut materyaller kullanmıştır. Bu nedenle Esen'in Geogebra'yı bir öğrenim ve öğretim aracı olarak matematik müfredatına entegre etmeye ilişkin bilgisi hala sınırlı olarak değerlendirilebilir. Çalışmanın başı ve MKD3 arasında Esen, müfredat alanında TPAB Gelişim Modeli'nde Murat'tan daha yavaş gelişim kaydetmiştir. MKD3'te ise Esen ve Murat aynı seviyede bulunmaktadır (*araştırma yapma seviyesi*). Her ikisi de öğrencilerin kavramları daha güçlü biçimde kavramaları için farklı müfredatla ilgili farklı fikirler araştırmışlardır. Çalışmanın sonunda doğru öğretmenler Geogebra'yı derslerine nasıl entegre edeceklerine ilişkin bir vizyon geliştirmiş ve mevcut müfredatta değişiklikler yapmışlardır. *Geliştirme seviyesine* ilerlerken öğretmenler, ders kitabından ayrı, GeogebraTube ve İnternet gibi değişik kaynaklardan müfredatla ilgili farklı fikir arayışında olmuşlardır. Ayrıca öğretmenler, sürükleme ve kaydırma gibi dinamik geometri yazılımı olan Geogebra'nın ana özelliklerinden bazılarını entegre etmişlerdir. Özellikle son iki derste – MCC4 ve MC sonrasında – öğretmenler, Geogebra'yı problem çözme aracı olarak kullanarak alan ve çevre arasındaki ilişkiyi genişletmek yenilikçi bir yaklaşımla müfredatta değişiklikler yapmışlardır. Bunlar, öğretmenlerin çalışma sonunda en üst TPAB seviyesine ulaştıklarını göstermektedir. Son olarak

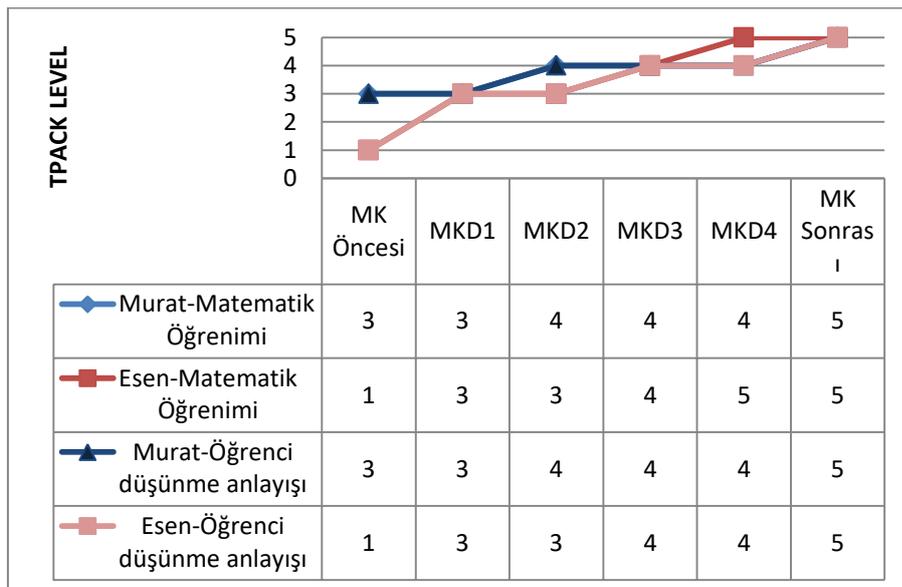
öğretmenler, müfredatta yenilikçi bir yaklaşımla değişiklik yapmanın ve onu geliştirmenin genelde teknolojinin, özelde Geogebra'nın, verimli ve etkin entegrasyonu için önemli olduğunu anlamışlardır.

Bu çalışmada öğretmenler, teknolojinin bir değerlendirme aracı olarak nasıl entegre edileceğini tanımlayan TPAB gelişim modelindeki değerlendirme alanında da gelişim göstermişlerdir. Önce Murat ve Esen Geogebra'yı değerlendirme sürecinde kullanma fikrine direnmişlerdir. Öğrencilerin geometriyi anlayıp anlamadıklarını tespit etmede teknolojinin gereksiz olduğunu ifade etmişlerdir. Murat ve Esen Geogebra'yı öğrenim ve öğretim sürecine dahil etmiş olsalar da ne Murat ne de Esen, değerlendirme sürecinde öğrencilerinin Geogebra'yı kullanmalarına izin vermiştir. Daha sonra her ikisi de, öğrencilerinin geometriyi ne derece kavradıklarını anlamak için Geogebra'yı görselleştirme aracı olarak entegre etmeye başlamışlardır. Ancak Geogebra'nın dinamik özelliklerini dikkate almamışlardır. Matematik koçluğu sürecinde öğretmenler, TPAB gelişim modelinin değerlendirme alanında daha üst seviyeye ilerlemişlerdir. MKD2'de Esen, öğrencilerinin öğrenim düzeylerini anlamak için öğrencilere dinamik bir ortam sunmuştur. Diğer yandan Murat, değerlendirme alanında Geogebra'yı sadece görselleştirme aracı olarak kullanmıştır. Bundan dolayı Murat'a kıyasla Esen, MKD2'de TPAB Gelişim Modeli'nin değerlendirme alanında daha üst seviyede olarak değerlendirilebilir. Bir sonraki koçluk döngüsünde – MKD3– Murat, Esen'in TPAB Gelişim Modeli'ndeki değerlendirme alanındaki seviyesine ulaşmıştır. Her ikisi de MCC3'te kağıt ve kalemin ötesine geçip değerlendirmede bir Geogebra aktivitesi kullanmıştır. Öğretmenler, kalem ve kağıdın ötesine geçerek değerlendirme alanına entegre ettikleri Geogebra'nın bazı faydalarını araştırmışlardır. Ancak bazı aktivitelerde Geogebra'nın dinamik doğası etkin bir şekilde kullanılmamıştır. Murat'ın Türk eğitim sistemi ve sınavlarına ilişkin endişeleri bulunmaktadır. Bu nedenle Esen gibi Murat da, bazı değerlendirme sorularının öğrencilerin işlemsel bilgilerini temel alan ders kitabı sorularıyla benzerlik göstermelerini istemiştir. Öğrencilerin sadece kavramsal değil, yöntemsel bilgilerini de anlamak için ikisi de Geogebra destekli değerlendirme etkinlikleri

hazırlamışlardır. Çalışmanın sonuna doğru her ikisi de, öğrencilerin kavramsal bilgilerini artırmak için Geogebra'nın dinamik yapısına önem vermeleri gerektiğini anlamışlardır. Özellikle matematik koçluğundan sonraki gözlem sonrası sürecinde öğretmenler, öğrencilerin geometri kavramlarını anlama düzeylerini değerlendirmek için Geogebra'nın dinamik özelliklerini dikkate alarak çeşitli Geogebra destekli etkinlikler geliştirmişlerdir. Ayrıca öğretmenler, öğrencilere aktif biçimde değerlendirme sürecine katılmalarına olanak sağlayan ortam sağlamışlardır. Değerlendirme aktivitesini çözümlmek için Geogebra'da bazı faaliyetlerde bulunmuşlardır ve bunların sonuçlarını gözlemlemişlerdir. Ayrıca matematik koçluğundan sonra Esen, öğrencilerin kavramsal bilgilerini değerlendirmek için yaptığı bir problem çözme etkinliği aracılığıyla Geogebra'yı yenilikçi biçimlerde uygulamıştır. Böylece Esen, öğrencilerin kavramsal bilgilerini değerlendirmek için müfredatta değişiklikler yapmış ve bu da Geogebra'nın en geniş boyutta kullanımını gerektirmiştir.

3.2 Matematik Koçluğunda Murat Ve Esen'in Öğrenim Temasına İlişkin TPAB Seviyelerinde Gösterdikleri Gelişim

Tablo 2. Öğretmenlerin Öğrenim Temasına İlişkin TPAB Seviyeleri



Murat ve Esen'in matematik koçluğu öncesinde, sürecinde ve sonrasında altı teknoloji destekli derslerindeki öğrenme teması alanındaki performansları Tablo 2'de sunulmuştur. Öğretmenler, TPAB gelişim modelinde öğrencilerin öğrenme temasındaki matematik öğrenimi alanında *geliştirme seviyesine* doğru bir ilerleme göstermişlerdir.

Matematik öğrenimi alanı, teknoloji dersteki matematik öğrenimi üzerinde durmaktadır. Matematik koçluğundan önce, Murat'a kıyasla Esen, matematik öğrenimi alanında daha düşük seviyededir. Elle tutulur materyaller kullandıktan sonra Esen, Geogebra'yı öğretimi destekleyici bir işlev ile kullanmıştır. Murat, Geogebra'yı çalışmanın başlangıcında statik bir araç olarak kullanmasına rağmen daha sonra onu, öğrencilerin araştırma yapması için entegre etmiştir. Murat, öğrencilerin genellemeler geliştirmeleri teşvik etmek ve tüm sınıfın dahil olduğu bir sınıf diyalogunu desteklemek için Geogebra'yı uygulamıştır, ancak Geogebra'nın dinamik yapısını dikkate almamıştır. Murat'ın Geogebra'yı kullanması, öğrencilerin matematiksel kavramlar yerine etkinliklerdeki teknolojiye odaklanıp dikkatlerinin dağılmasından endişe duymadığını göstermektedir. MCC1'de Esen, Murat'ın TPAB seviyesine ulaşmıştır. Hem Esen hem Murat, teknolojinin etkinliğini belirlemek için onu kullanmayı denemeye başlamıştır. Öğretmenler, bir öğrenim ve öğretim aracı olarak Geogebra'yı sınırlı biçimde kullanmış, buna ek olarak Murat, giriş ve değerlendirme esnasında teknolojiyi kullanmamıştır. Başlangıçta öğrencilere çoğu konuda Geogebra'yı kullanma ortamı sağlanmasına rağmen değerlendirme ve giriş kısımlarında teknoloji kullanılmamıştır. Öğretmenler, matematik koçluğu sürecinde TPAB gelişim modelindeki matematik öğrenimi alanında daha üst seviyeye ilerlemeye başlamışlardır. Çalışmanın başlangıcından farklı olarak öğretmenler, öğrencilerin araştırma yapmalarını desteklemek için teknolojiyi entegre ederken Geogebra'nın dinamik yapısının avantajlarını farketmeye başlamışlardır. Çalışmanın sonuna doğru ise öğretmenler, matematik kavramlarını daha iyi anlamaları için Geogebra'yı entegre ederek öğrencilerin geometri konularını araştırmalarına olanak tanıyan ortamlar sağlamışlardır. Ayrıca öğretmenler, teknolojinin derslerde hem bir öğretim aracı olarak kullanılabildiğini hem de

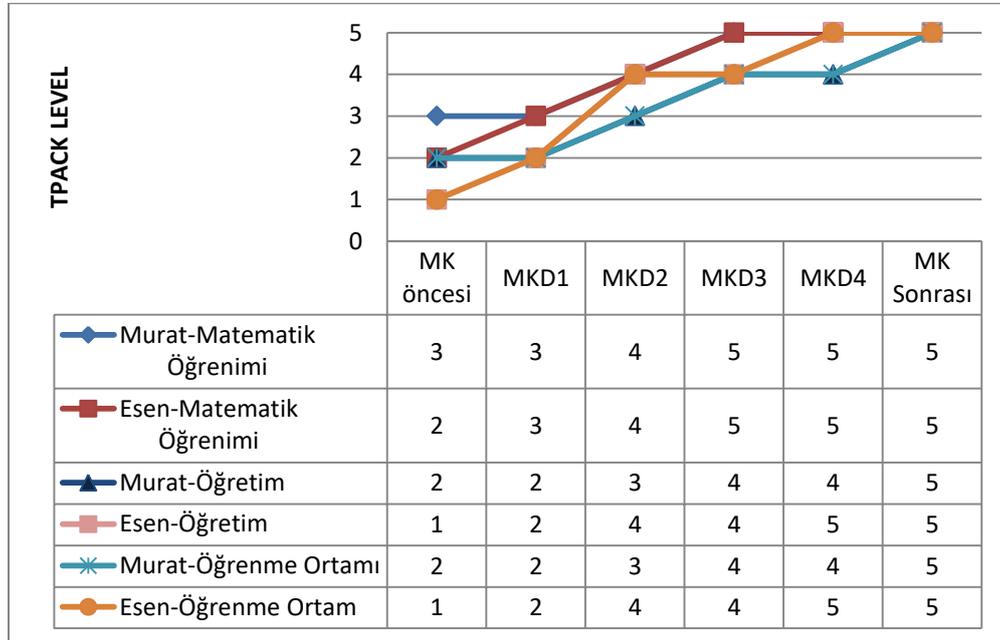
geometri alanındaki potansiyel düşünceler üzerinde etki ettiğini farketmişlerdir. Son derste öğretmenler, öğrencilerin matematiği anlamaları için öğrenme aracı olarak Geogebra'yı kullanmalarında rehberlik etmişlerdir. Dersi, bu doğrultuda planlamışlar, uygulamışlar ve üzerine düşünmüşlerdir. Murat, TPAB Gelişim Modeli'nin öğrenim teması kapsamındaki matematik öğrenimi alanında en üst seviyeye ulaşmıştır.

Öğretmenler TPAB gelişim modelinin öğrenci düşünme anlayışı alanında ilerleme kaydetmişlerdir. Çalışmanın başında, özellikle elle tutulur materyaller ile karşılaştırma yapılırken, Esen'in teknoloji kullanımı ile ilgili endişeleri bulunmaktaydı. Teknoloji kullanıldığında öğrencilerin uygun matematiksel düşünme becerileri geliştirip geliştirmediklerinden şüphe duymaktadır. Matematik koçluğundan önce Murat'tan farklı olarak Esen, Geogebra'yı öğrenim aracı olarak değil, öğretim aracı olarak uygulamaktaydı. Matematik koçluğundan önce Murat'ın dersinde öğrenciler genelleme yapabilmek için Geogebra'yı kullanmış, ancak Murat, öğrencilerin düşünme becerilerini değerlendirmek için Geogebra'yı derse entegre etmemiştir. Bu durum onun *adapte etme seviyesinde* olduğunu göstermektedir. Ayrıca Murat, öğrencilerin olası yanlışlarını ve zorluklarını aşma gidermek ve öğrencilerin matematik bilgilerini geliştirmek için onların kendi dillerini kullanmaları ve düşüncelerini açıklamalarını sağlayan uygun bir ortam sağlamıştır. Ancak Geogebra-destekli etkinlikleri, daha çok gösteri yaptırma ve öğretmenin yönlendirip öğrencilerin takip ettiği çalışmalar için kullanılmaktadır. MKD1'de Esen, öğrenci düşünme anlayışı alanında Murat'ın seviyesine ulaşmıştır. Matematik koçluğu kanalıyla her ikisi de öğrencilerin, matematiksel kavramlar olarak çokgenleri öğrenme süreçlerini ve bu kavramlara yönelik öğrenci tutumlarını planlamış, entegre etmiş ve değerlendirmişlerdir. Diğer bir deyişle, çalışmanın ortasında Geogebra'nın kullanım amacı, öğrencilerin çokgen kavramları ne ölçüde anladıklarını değerlendirmek ve daha iyi anlamalarını sağlamaya yönelik olmuştur. Ayrıca bir öğrenme aracı olarak Geogebra'yı, öğrencilerin kavramsal bilgilerini geliştirmek için kullanmışlardır. Son derste ise öğrenciler, Geogebra'da problem çözme etkinlikleri gibi üst düzey düşünme aktiviteleri uygulamışlardır. Ayrıca

öğrencileri çokgenlere ilişkin kavramları anlamalarında üst düzeylere getirmek için Geogebra, derse ek olarak değil, ayrılmaz bir parçası şeklinde entegre edilmiştir. Böylece öğretmenler, TPAB Gelişim Model’indeki öğrenme teması kapsamındaki öğrenci düşünme anlayışı alanında en üst seviyeye ulaşmışlardır.

3.3 Matematik Koçluğunda Murat ve Esen’in Öğretim Temasına İlişkin TPAB Seviyelerinde Gösterdikleri Gelişim

Tablo 3. Öğretmenlerin Öğretim Temasına İlişkin TPAB Seviyeleri



Murat ve Esen’in matematik koçluğu öncesinde, sürecinde ve sonrasında altı teknoloji destekli derslerindeki öğrenme teması alanındaki performansları Tablo 3’te sunulmuştur. Öğretmenler, TPAB gelişim modelindeki öğretim temasına ilişkin matematik öğrenimi ve öğrencilerin kavrayışları alanlarında öğretmenler, *geliştirme seviyesine* doğru ilerleme göstermişlerdir.

Çalışmanın başında Esen’in öğretim temasındaki matematik öğrenimi, öğretim ve öğrenim ortamı alanlarındaki performansı, Murat’inkinden düşük olarak değerlendirilmiştir. Ancak her ikisi de, öğretim temasında TPAB gelişim modelinde

ilerleme göstermiştir. Öğretim temasındaki matematik öğrenimi alanı, öğretmenin teknoloji konusundaki görüşleri üzerinde durmaktadır. Başta Esen, yeni kavramları geometri tahtası gibi somut materyaller kullanarak öğrettikten sonra Geogebra'yı kullanmıştır. Ünitinin sonunda Geogebra'yı, gösterim aracı olarak derse ek olarak entegre etmiştir. Bu nedenle matematik öğrenimi alanında en düşük seviyede olarak değerlendirilmiştir. Diğer yandan çalışmanın başında Murat, öğrencilerin bilgilerini geliştirmek için Geogebra'yı onun dinamik özelliklerini dikkate almadan derse entegre etmiştir. MKD1'de ise Esen, Murat'ın matematik alanında bulunduğu seviyesine ulaşmıştır. Geogebra'yı kullanmaları, onların geometri öğretirken teknolojinin zaman kaybına yol açtığına dair herhangi bir endişeye sahip olmadıklarını göstermektedir. Ayrıca öğretmenler Geogebra destekli etkinlikleri, derse ek veya alıştırmaları olarak derse entegre etmemişlerdir. Çalışmanın ortasında her ikisi de Geogebra'nın dinamik ortamını fark etmiş ve Geogebra'yı derse dahil ederek öğrencilerine üst düzey düşünme etkinlikleri yapmışlardır. Çalışmanın sonunda doğru her iki öğretmen de, Geogebra'yı öğrencilere geometrik kavramları daha anlaşılır kılmak için kullanmışlardır. Geogebra'yı öğrenim aracı olarak kullanarak öğretmenler, öğrencilerin öğrenimlerini geliştirme konusunda yeni fikirler edinmişlerdir. Ayrıca Geogebra'yı kabul edişlerinde, öğretim temasındaki matematik öğrenim alanının TPAB Gelişim Model'indeki *geliştirme seviyesinde* tanımlandığı gibi aktif ve tutarlı olup, MKD3 sonra her ikisi de en üst seviyeye ulaşmıştır.

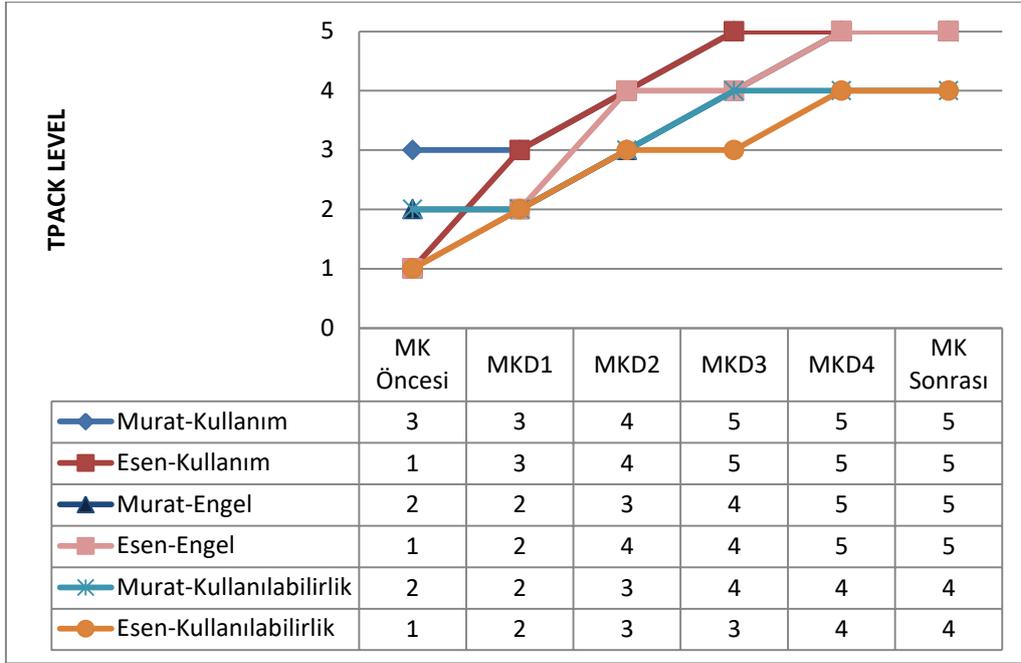
Çalışmanın başında Esen, Geogebra'yı çokgenlerin türlerini onları öğrettikten sonra onları göstermesi gibi sıradan veya ezbere dayalı etkinlikler için kullanmıştır. Murat ise Geogebra'yı öğrencilerin araştırma yapma becerilerini geliştirmek için entegre etmiştir. Bu nedenle Esen, öğretim alanının en düşük seviyesinde bulunmaktadır. Ayrıca Esen, öğrencilerinin teknoloji kullanarak matematiği öğrenmelerine veya matematiğe erişmelerine ya da Geogebra ile yeni kavramları araştırmalarına fırsat vermemiştir. MKD1'de Esen, Murat'ın seviyesine ulaşmış ve Geogebra'yı öğrencilerin araştırma yapmaları için kullanmıştır. Her iki öğretmen, teknolojiyi derse entegre ederken sadece matematik müfredatındaki

düşünceleri yansıtmış ve Geogebra'nın dinamik yapısını dikkate almamışlardır. MKD2'de Esen, Murat'ın bulunduğu öğretimsel alanındaki TPAB seviyesini geçmiş ve Murat gibi Esen de öğrencilerin kavramı keşfetme sürecinde yönetici rolü değil, daha çok rehber rolü üstlenmiştir. Her ikisi de matematik koçluğu sayesinde öğretim temasındaki öğretim alanında ilerleme kaydetmişlerdir. Çalışmanın sonunda her ikisi de bu alanda TPAB'nin en üst seviyesine ulaşmış olup öğrencilerin farklı bakış açılarıyla araştırma yapmalarını desteklemek için Geogebra etkinlikleri entegre etmişlerdir. Öğrencilere matematik konularını derinlemesine işleyip onları anlama zevkini yaşatırken her iki öğretmen, öğrencilere bir araçtan diğerine kolay bir şekilde geçmelerine yardımcı olmuşlardır (Niess ve ark., 2009).

Öğretim alandaki gibi Esen, öğrenim ortamı alanında da TPAB'nin en düşük seviyesinde olarak değerlendirilmiştir. Esen, teknolojiyi teknoloji kullanılmadan öğretilen kavramları pekiştirmek için kullanmıştır. Murat ise Geogebra'yı yeni geometri kavramlarını öğretmek için entegre etmiştir. Buna rağmen Murat'ın dersi, adım adım ilerleyen ve öğretmen-odaklı bir sürece sahip olmaktadır. Daha sonra ise her iki öğretmen de matematik koçluğu sürecinde ilerleme kaydetmişlerdir. Murat, MKD2'de öğrencilere Geogebra ile araştırma yapma fırsatları sunacak şekilde adapte etmeye başlamıştır. Ancak etkinliklerin gidişatını kontrol altında tutma amacıyla dersi hala öğretmen merkezli olmaktadır. Ayrıca MKD2'de Esen, öğrencilerinin matematik üzerine düşüncelerini sağlamak için Geogebra ile çeşitli öğretim stratejileri araştırmış ve öğrencilerin kendi öğrenimlerinden sorumlu olacakları Geogebra-destekli etkinlikleri derse entegre etmiştir. Çalışmanın sonunda her iki öğretmen de öğrenim ortamı alanında TPAB'nin en üst seviyesine ulaşmış olup matematik öğreniminde öğrencilerin katılımları ve öz-yönlendirmeci olmalarını sağlayacak şekillerde Geogebra destekli etkinlikleri kullanmışlardır.

4.4. Matematik Koçluğunda Murat ve Esen'in Erişim Temasına İlişkin TPAB Seviyelerinde Gösterdikleri Gelişim

Tablo 4. Öğretmenlerin Öğretim Temasına İlişkin TPAB Seviyeleri



Murat ve Esen'in matematik koçluğu öncesinde, sürecinde ve sonrasında altı teknoloji destekli derslerindeki erişim teması alanındaki performansları Tablo 4'te sunulmuştur. Öğretmenler, TPAB gelişim modelinde öğrencilerin erişim temasındaki kullanım, engel ve kullanılabilirlik alanlarında daha üst seviyeye doğru ilerleme göstermişlerdir. Çalışmanın başında Esen'in erişim temasındaki kullanım, engel ve kullanılabilirlik alanlarındaki seviyesi, Murat'inkinden düşük olmasına rağmen her ikisi de TPAB gelişim modelindeki erişim temasında gelişme göstermiştir.

Erişim temasındaki kullanım alanı, öğretmenin teknolojiyi nasıl kullandığı ve öğrencilere teknolojiyi ne zaman kullandırdıkları ile ilişkilendirilmektedir. Başlangıçta Esen, öğrencilerine Geogebra'yı kullanmalarına ancak somut materyaller aracılığıyla içbükey ve dışbükey çokgenler gibi geometri kavramlarını öğrenmelerinden sonra izin vermektedir. Bu nedenle Esen, kullanım alanının en

düşük seviyesinde bulunmaktadır. Diğer yandan matematik koçluğundan önce Murat'ın dersinde öğrencilerin, genellikle öğretmenin gösterimiyle, içbükey ve dışbükey çokgenleri araştırmak için Geogebra'ya erişimleri ve onu kullanma olanakları bulunmaktadır. Bu nedenle çalışmanın başında Murat, *adapte etme* seviyesinde yer almaktadır. MKD1'de Esen, Murat'ın kullanım alanında bulunduğu seviyeye ulaşmış olup her ikisi de öğretmenin gösterimiyle öğrencilerine Geogebra'yı kullanmaları için izin vermiştir. Çalışmanın ortasında her ikisi de, Geogebra'ya erişim ve onun kullanımıyla geometrik kavramları keşfetmeye teşvik için bir ortam sağlamıştır. Bununla birlikte öğretmenler MKD3'te kullanım alanında en üst seviyeye ulaşmışlardır. Öğrencilere, Geogebra'nın dinamik yapısını kullanarak geometri kavramlarını derinlemesine öğrenmeleri için ortam sağlamış ve teknolojiyi, matematik öğrencilerinin öğrenmelerine katkıda bulunmak için bir fırsat olarak değerlendirmişlerdir. Öğrencilerinin teknolojiyi matematik dersinin her aşamasında kullanmalarına izin vermişlerdir.

Engel alanında, öğretmenlerin teknoloji entegrasyonundaki olası sorunları nasıl çözdükleri ile ilgilidir. Çalışmanın başında Esen, Geogebra'nın kullanımına ilişkin endişeleri olduğu için Geogebra'yı olağan sınıf etkinliklerine ek olarak entegre etmiştir. Öğrenciler, somut materyaller kullanarak geometri kavramlarını öğrendikten sonra Geogebra'ya sınırlı bir şekilde erişebilmekteydiler. Ayrıca Murat, Geogebra'yı öğrencilerin araştırma yapmaları için de kullanmasına rağmen geometrik kavramları kağıt-kalemle farklı biçimde entegre etmemiştir. Murat'ın *kabul etme* seviyesinde tanımlandığı gibi Geogebra'yı sınıfa entegre etmede ve erişim ve yönetim alanlarında endişeleri olduğu görülmüştür. MKD1'de ise Esen, Murat'ın seviyesine ulaşmış ve yönetimle ilgili endişeleri olmasına rağmen Geogebra'yı öğrencilerin yeni kavramları öğrenmeleri için kullanmıştır. MKD2'de Murat, matematiğe yeni bir yaklaşım olarak Geogebra destekli etkinlikleri uygulamış olup böylece, *uyum sağlama* seviyesinde tanımlandığı gibi, Geogebra'nın zaman ve yönetim alanlarında olası zorlukların üstesinden gelinmesinde yardımcı olacağı ortam hazırlamıştır. Murat'tan farklı olarak Esen ise, öğrencilerin etkinliklerle ilgili sorularına yanıt vermiş ve öğrenci grupları arasında dolaşarak

öğrencileri Geogebra'nın özelliklerini arařtırmaları konusunda yönlendirmiřtir. MKD2 esnasında Murat, öğrencileri kontrolü altına almak istemiřtir. Bu nedenle Esen, MKD2'de engel alanında Murat'tan daha üst seviyede bulunmaktadır. Matematik koçluęu sürecinde her ikisi de, TPAB Geliřim Modeli'nin engel alanında ilerleme kaydetmiřtir. MKD4'te her ikisi de en üst seviyeye ulařmıřtır. Geogebra'yı, öğrencilerin geometri kavramlarını yenilikçi bir yaklařımla kavramaları için bir fırsat olarak görmüřlerdir. Bu durum, sınıf yönetimiyle ilgili konularda da yardımcı olmuřtur. Böylece engel alanında *geliřtirme seviyesinde* belirtildięi gibi her iki öğretmen, teknoloji destekli öğretimde yer alan zorlukların farkında olup ve elde edilebilir kaynak ve araçların azami seviyede kullanımları için detaylı bir planlama ve hazırlık yaparak zorlukların üstesinden gelmiřlerdir (Niess ve ark., 2009).

Kullanım ve engel alanlarındaki gibi Esen, matematik koçluęundan önce kullanılabilirlik alanında da TPAB'nin en düşük seviyesinde bulunmaktadır. Geogebra'yı, kavramlar arası baęlantılar ve çoklu gösterimleri entegre etmek için kullanmamıřtır. Ayrıca Geogebra'yı örüntü ve kavramlar arası iliřkileri arařtırmak için kullanmamıřtır. Bu nedenle Esen, Murat'tan daha alt seviyede bulunmaktadır. Çünkü Murat'ın dersinde öğrenciler, *kabul etme* seviyesinde tanımlandıęı gibi Geogebra aracılıęıyla çokgenlerdeki köřegen sayısı gibi örüntüleri arařtırmıřlardır. MKD1'de Esen, Murat'ın kullanılabilirlik alanında bulunduęu seviyeye ulařmıř ancak her ikisi de, çokgenler konusunda Geogebra'yı, çokgenlerin iç açılarına iliřkin örüntüleri bulmak amacıyla görsel gösterim olarak kullanmıřlardır. MKD2'de ise öğrenciler, Geogebra'yı arařtırma yapmak ve iliřkiler kurmak için kullanmıřlardır. Ayrıca geometrik kavramları kalem kâğıttan farklı olacak řekilde öğretilmiř olup kullanılabilirlik alanı için TPAB'nin *adapte etme seviyesinde* tanımlandıęı gibi, teknoloji, daha önce eriřilemeyen alanlara eriřim saęlamıřtır. MKD3'te Murat, tablolar, semboller ve resimler gibi çoklu gösterimler kullanarak Geogebra'yı etkin bir řekilde kullanmıřtır. Bir sonraki derste –MKD4'te– Esen, Murat'ın kullanılabilirlik alanında TPAB seviyesine ulařmıřtır. Böylece, kullanılabilirlik alanının *arařtırma yapma seviyesinde* tanımlandıęı gibi,

kavramların ve ilgili uzantılarının çoklu gösterimleri entegre edilerek teknolojinin kullanılması ile anahtar konular araştırıp ve uygulayıp değerlendirmişlerdir (Niess ve ark., 2009).

5. Sonuç

Matematik koçluğundan sonra matematik öğretmenleri, TPAB gelişim modelindeki tüm temalarda (müfredat ve değerlendirme, öğrenim, öğretim, erişim) en üst seviyeye ulaşmışlardır. Matematik koçluğu öncesi ve sonrası dönemler karşılaştırıldıklarında öğretmenlerinin tüm temalarda ve ilgili alanlarda *en düşük seviyelerden en yüksek seviyelere* ilerledikleri gözlemlenmiştir. Bu çalışma kapsamında iki matematik öğretmeninden matematik koçluğu öncesinde, sürecinde ve sonrasında elde edilen verilerden başlıca şu sonuçlar elde edilmiştir:

- 1) Matematik koçluğu sürecine katılan öğretmenler, PAB'den TPAB'ne doğru hareket etmişlerdir.
- 2) Matematik koçluğu öncesinde öğretmenler, dört tema ve ilgili alanlarında daha düşük TPAB seviyeleri (fark etme, kabul etme ve uyum sağlama) sergilemişlerdir.
- 3) Matematik koçluğu öncesinde daha deneyimli olan öğretmen, TPAB Gelişim Modeli'nde daha az deneyimli olan öğretmenden daha düşük seviyede bulunmaktadır.
- 4) Matematik koçluğu sürecinde matematik öğretmenleri, TPAB Gelişim Modeli'nin tüm temalarında ve ilgili alanlarında daha üst seviyeye doğru bir gelişim sergilemişlerdir
- 5) Matematik koçluğu sürecinde ilerleme kaydedilen tüm alanlar arasında En yavaş gelişim değerlendirme alanında görülmüştür.
- 6) Matematik koçluğu sürecinde daha deneyimli olan öğretmen TPAB Gelişim Modeli'ndeki ilerleyişi, daha az deneyimli olan öğretminkinden daha hızlı olarak değerlendirilmiştir.

- 7) Matematik koçluğu sonrasında hizmet içi matematik öğretmenleri, TPAB Gelişim Modeli'ndeki tüm temalar ve ilgili alanlarında en üst seviyeye ulaşmıştır.
- 8) Matematik koçluğunun yararlarına ilişkin öğretmenler TPAB seviyesinde gelişim, öğretimde olumlu değişim, öğrencilerin başarılarında artış, işbirliği ve desteğin sağlanması ve özgüvende artış konularında görüşleri belirtmişlerdir.
- 9) Öğretmenlerin matematik koçluğu ve olağan hizmet içi öğretmen yetiştirme programları arasındaki farklara ilişkin görüşlerinde, matematik koçluğunda süreç üzerinde durulmasına ve teori ile pratiğin birleştirilmesine vurgu yapılmıştır.
- 10) Matematik koçluğunda yer alan zorluklara ilişkin öğretmenlerin görüşleri arasında ders planlarının hazırlanması ve stres faktörü yer almaktadır.
- 11) Öğretmenlerin matematik koçluğuna ilişkin önerileri arasında, Türkiye'de yaygınlaştırılması, dönem boyunca olması, farklı alanlarda uygulanması, farklı matematik yazılımları üzerine olması ve grup koçluğunun yapılması yer almıştır.

APPENDIX O

TEZ FOTOKOPİSİ İZİN FORMU

ENSTİTÜ

Fen Bilimleri Enstitüsü	<input type="checkbox"/>
Sosyal Bilimler Enstitüsü	<input checked="" type="checkbox"/>
Uygulamalı Matematik Enstitüsü	<input type="checkbox"/>
Enformatik Enstitüsü	<input type="checkbox"/>
Deniz Bilimleri Enstitüsü	<input type="checkbox"/>

YAZARIN

Soyadı : Aygün
Adı : Berna
Bölümü : İlköğretim

TEZİN ADI (İngilizce) : The Development Of Elementary Mathematics Teachers' Technological Pedagogical Content Knowledge Regarding Geometry Through Mathematics Coaching

TEZİN TÜRÜ : Yüksek Lisans Doktora

1. Tezimin tamamından kaynak gösterilmek şartıyla fotokopi alınabilir.
2. Tezimin içindekiler sayfası, özet, indeks sayfalarından ve/veya bir bölümünden kaynak gösterilmek şartıyla fotokopi alınabilir.
3. Tezimden bir bir (1) yıl süreyle fotokopi alınamaz.

TEZİN KÜTÜPHANEYE TESLİM TARİHİ: