AN ANALYSIS OF PROSPECTIVE MIDDLE SCHOOL MATHEMATICS TEACHERS' ARGUMENTATION STRUCTURES IN TECHNOLOGY AND PAPER-PENCIL ENVIRONMENTS

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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ABSTRACT

AN ANALYSIS OF PROSPECTIVE MIDDLE SCHOOL MATHEMATICS TEACHERS' ARGUMENTATION STRUCTURES IN TECHNOLOGY AND PAPER-PENCIL ENVIRONMENTS

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The purpose of the current study was to investigate the nature of argumentation structures of prospective middle school mathematics teachers while solving geometry tasks within the GeoGebra and Paper-Pencil groups. The study employed qualitative case study design and the data of which were collected from 16 prospective middle school mathematics teachers during the fall semester of the 2013-2014 academic year. Data were based on the video recordings of the implementations, the focus group interviews and documents.

The findings revealed five types of global argumentation structures, two of which emerged from the present study: Line-structure and Independent-Arguments structure. In addition, the participants employed 9 types of local arguments while three of them were most frequently preferred. Finally, the local argumentations were examined by focusing on the justifications of the participants. This analysis revealed the characteristics of the local argumentations that prospective middle school mathematics teachers use while solving geometry tasks in GeoGebra and Paper-Pencil groups.

Keywords: Argumentation Structures, Prospective Middle School Mathematics Teachers, Technology, Global Argumentation, Local Argumentation

ÖZ

ORTAOKUL MATEMATİK ÖĞRETMEN ADAYLARININ ARGÜMANTASYON YAPILARININ TEKNOLOJİ VE KAĞIT-KALEM ORTAMLARINDA İNCELENMESİ

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Bu çalışmanın amacı, GeoGebra ve Kağıt-Kalem gruplarında geometri soruları çözen ortaokul matematik öğretmen adaylarının argümantasyon yapılarının doğasını incelemektir. Bu çalışmada nitel durum çalışması deseni kullanılmış ve veriler 2013-2014 akademik yılının sonbahar döneminde 16 ortaokul matematik öğretmen adayından toplanmıştır. Veri kaynaklarını uygulamaların video kayıtları, odak grup görüşmeleri ve belgeler oluşturmaktadır.

Bulgularda beş çeşit global argümantasyon yapısı ortaya çıkmıştır. Bunların ikisi Çizgi/Hat yapı ve Bağımsız-Argümanlar yapısı bu çalışmada ortaya çıkmıştır. Ayrıca katılımcılar 9 çeşit lokal argüman yapılarından üçünü çok sık kullanmayı tercih etmişlerdir. Son olarak, katılımcıların gerekçelendirmelerine odaklanarak lokal argümantasyonları incelenmiştir. Bu analiz, ortaokul matematik öğretmen adaylarının GeoGebra ve Kağıt-Kalem gruplarında geometri soruları çözerken kullandıkları lokal argümantasyonların niteliğini ortaya çıkarmıştır.

Anahtar Kelimeler: Argümantasyon Yapıları, Ortaokul Matematik Öğretmen Adayları, Teknoloji, Global Argümantasyon, Lokal Argümantasyon

To *Beren ERKEK*, my sweet dearie daughter

&

To *Erhan ERKEK*, my dear husband for their endless patience, support and love.

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NCTM	National Council of Teachers of Mathematics
DCW	Data-Claim-Warrant
DWC	Data-Warrant-Claim
CDW	Claim-Data-Warrant
WDC	Warrant-Data-Claim
CD	Claim-Data
DC	Data-Claim
CW	Claim-Warrant
WC	Warrant-Claim
C	Claim
W	Warrant
D	Data
B	Backing
Q	Modal Qualifier
R	Rebuttal
DGE	Dynamic Geometry Environment
DGS	Dynamic Geometry Software
GG	GeoGebra Group
PPG	Paper-Pencil Group
AS	Argumentation Stream
GT	Geometry Task

CHAPTER I

INTRODUCTION

Mathematics education researchers have been conducting research studies in order to increase mathematical learning of students for years. From time to time new trends in the teaching and learning of mathematics are proposed by different organizations. One of the largest organizations which has a significant impact on mathematics teaching all over the world is the National Council of Teachers of Mathematics [NCTM], which makes publications by considering the changing needs and new developments in the world. NCTM (1991) emphasized the importance of some skills which were necessary in mathematics education: complex problem solving, high level reasoning, making connections across mathematical domains, and communicating. In addition, instead of the transmission of factual knowledge from the teacher to the student, the active participation of students in discussing ideas, making convincing arguments, making reflection and clarifying their thoughts (which are the requirements of argumentation) have been promoted and expected for a better mathematical understanding (Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Hufferd-Ackles, Fuson, & Sherin, 2004; Krummheuer, 2000; Stein, Engle, Smith, & Hughes, 2008). Similarly, NCTM (2000) emphasized the necessity of the integration of *reasoning and proof* in classroom discussions of all topics with the statement, “Reasoning and proof are not special activities reserved for special topics in the curriculum but should be a natural, ongoing part of classroom discussions, no matter what topic is being studied” (p.342). At this point, the importance of argumentation in mathematics emerged since it was claimed by researchers that reasoning and argumentation were closely related to each other (Conner, Singletary, Smith, Wagner, & Francisco, 2014b). They defended this idea by saying, “When an individual is creating an argument, he or she is reasoning, and when an individual is reasoning, he or she is creating an argument; thus we contend that, when considered as individual activities, argumentation and reasoning refer to the same process in mathematics" (p. 183). On the other hand, there are researchers who identified the subtle distinction between argumentation and reasoning by saying that argumentation requires an attempt of convincing an active or passive audience of a claim, while reasoning requires careful consideration to come to a justified claim. Despite this subtle distinction, argumentation and reasoning were accepted to refer to the same process and became the main concern of the present study.

Another current trend in mathematics education was analyzing the process of teaching and learning in accordance with the socio-cultural perspective (Recio & Godino, 2001). Mathematical knowledge was asserted to be developed within institutions so that it could be accepted as a socio-cultural product (Godino & Batanero, 1998). In the past, the Australian Education Council took the attention of the researchers to the argumentation based actions to do mathematics in 1991 by stating that:

> the systematic and formal way in which mathematics is often presented conveys an image of mathematics which is at odds with the way it actually develops. Mathematical discoveries, conjectures, generalizations, counter-examples, refutations and proofs are all part of what it means to do mathematics. School mathematics should show the intuitive and creative nature of the process, and also the false starts and blind alleys, the erroneous conceptions and errors of reasoning which tends to be a part of mathematics. (p. 14; as cited in Vincent, Chick & McCrae, 2005)

As understood from the paragraph, the issues (discoveries, conjectures, generalizations, counter-examples, refutations and proofs), which were expressed as closely related parts for doing mathematics, refer to the actions which were essential in argumentation. In addition, it was emphasized that these actions should be applied in school mathematics naturally to encourage mathematics learning in a socio-cultural way. That is to say, researchers started to give importance to the process of learning knowledge through social interaction by means of the argumentation activities such as conjecturing refuting, generalizing, and providing counter-examples. Thus, the context of the present study was defined by considering the tendency in learning with social interaction in mathematics education.

The model of argumentation was firstly introduced in 1958 by Toulmin to describe the non-mathematical arguments in the field of science. In fact, Toulmin wrote his book 'The Uses of Argument' in 1958 to criticize the formal logic (Pease, Smaill, Colton & Lee, 2008) and that book allowed researchers of a wide variety of domains to analyze arguments (Conner et al., 2014b; Erduran, Simon, & Osborne, 2004; Forman et al., 1998; Hoyles & Küchemann, 2002; Jiménez-Aleixandre, Rodríquez, & Duschl, 2000; Krummheuer, 1995; Lavy, 2006; Osborne, Erduran, & Simon, 2004; Pedemonte, 2007; Yackel, 2002). Moreover, Toulmin's model was asserted to be viewing argumentation from a practical perspective instead of a pure logico-mathematical viewpoint (Hollebrands, Conner, & Smith, 2010). In the framework, Toulmin (1958) proposed a layout to enable discussion analysis by reconstructing arguments in different fields so his layout has become a popular model across disciplines (Knipping, 2008). In this layout, there were argument components which were *claim*, *data*, *warrant*, *backing*, *modal qualifier* and *rebuttal*. These components were explained in literature part but here the relation between them was expressed with an example in Figure 1.1 which was offered by Toulmin (1958, 2003).

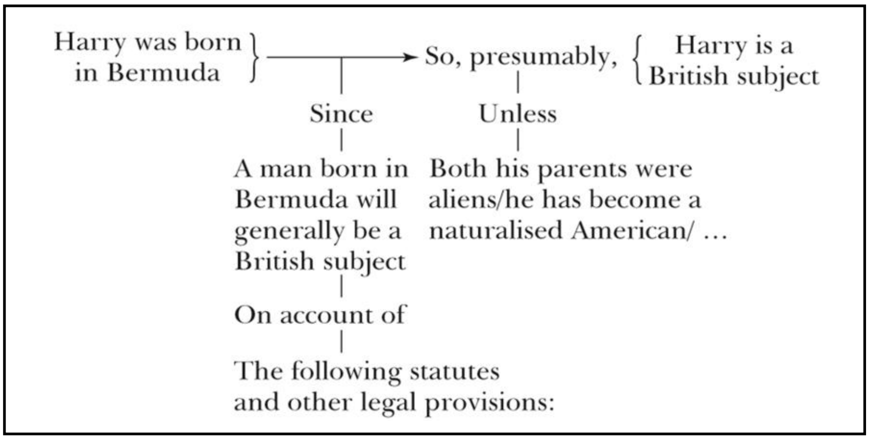


Figure 1.1 Toulmin's sample for an argument (2003, p. 97)

In his book, Toulmin (2003) presented a sample claim which was ‘Harry is a British subject’. In order to support this claim, he stated a fact as a datum: ‘Harry was born in Bermuda’. Then, the connection between the datum and the claim was expressed as a warrant: ‘A man born in Bermuda will generally be a British subject’. However, the warrant needed additional support of the backing, which was related to “the dates of enactment of the Parliament Acts and other legal provisions which regulate the people’s nationality in British colonies” (Toulmin, 2003, p. 97). Moreover, the force of the warrant is not sufficient to express the certainty of the argument, so the qualifier ‘presumably’ was stated. Lastly, there are possible rebuttals to the argument, such as ‘when both Harry’s parents were aliens’ or ‘he has become a naturalized American’.

The given example above belonged to a verbal statement but Toulmin’s (1958) argumentation model could be used in other fields because of its field independency nature, such as in mathematics. That is, the researchers advocated that the model was field independent and, thus, could be used in a variety of contexts, but the validity of the argument elements were stated to be field dependent. More clearly, the arguments in science, mathematics or philosophy may have the same argument elements (data, claim, warrant ...etc) but the content of these elements and their validity were stated to be field dependent. In addition, Toulmin’s (1958) model of argumentation was stated to be useful in terms of focusing on various aspects of arguments such as warrants, qualifiers and rebuttals. The present study also aimed to investigate the argumentation structures of the prospective middle school mathematics teachers. Thus, Toulmin’s (1958) argumentation model, which was used to analyze argument structures in the literature (Walter & Barros, 2011), is suitable in determining and analyzing argument structures in mathematics field, so it was selected for the data analysis of the present study.

The importance of social interaction and argumentation in achievement was recognized by many researchers in the literature (Cross, 2009; Inagaki, Hatano, & Morita, 1998; Kosko, Rougee, & Herbst, 2014; Sfard, 2008; Walter & Barros, 2011). For instance, Inagaki, Hatano and Morita (1998) mentioned that making contribution to discussions, asking questions, having their ideas evaluated, and receiving immediate feedback were some of the effective strategies for knowledge construction. Likewise, Sfard (2008) suggested taking part in specific forms of discourse and communicational activities in order to learn mathematics more effectively. In addition to these, Cross (2009) found that students who engaged in activities which were based on cognitive and socio-cultural views of knowledge obtained higher achievement. Specifically, she investigated the specific scaffolds for facilitating argumentation of 9th grade students claimed that students who were consistently prompted to justify and explain their reasoning had significant gains when compared to the ones who did not receive such scaffolding. Thus, she concluded that engagement in mathematical argumentation and justification had a positive impact on mathematics achievement (Cross, 2009). Similarly, Walter and Barros (2011) asserted that students' being active in the production of substantial arguments and working on different solution approaches to reach a consensus contributed to the development of higher-order mathematical thinking (Kosko et al., 2014) and reflective mathematical reasoning. Considering conducted studies, it can be concluded that using argumentation in mathematics class would be beneficial not only for students' achievement but also for teachers in terms of understanding their students' mathematical concept development. In this way, teachers would make decisions about the necessary contributions for providing collective argumentation and would assume the suitable role in order to elicit such contributions (Yackel, 2002).

1.1 Problem statement and purpose of the study

Arguing includes skills such as justifying, challenging, counterchallenging and conceding (Schwarz, 2009). According to the NCTM (2000), learning to argue, which means acquiring the mentioned skills, was stated in many curricula all over the world. Moreover, taking part in a productive form of mathematical argument, conjecturing and justifying their reasoning, was suggested by mathematics educators for students of all grade levels from prekindergarten through grade 12

(Ball & Bass, 2003; Hanna & De Villiers, 2008; NCTM, 2000). This means people from all ages need to learn and use arguing on the concepts and issues for meaningful learning. Arguing has a crucial place in argumentation and researchers mentioned the benefits of argumentation for students in many studies (Cross, 2009; Manoucheri & St John, 2006; McCrone, 2005; Wood, Williams, & McNeal, 2006). For instance, the students who took part in an argumentation were said to have the chance of hearing others' ideas, detecting misconceptions, confirming their own thinking (Cross, 2009) and so they could enhance their reasoning, explanations and justification (Manoucheri & St John, 2006; McCrone, 2005; Wood, Williams, & McNeal, 2006). In addition to these benefits, when they confront a situation of conflict during the discussion, they will conjecture and make explorations, thus acquiring the opportunity to enhance their conceptual knowledge and generate new knowledge (Cross, 2009). As an arguing-based method, argumentation was asserted to require higher-order thinking skills for both students and teachers. Therefore, not only students but also teachers should practice argumentation and improve their higher-order thinking skills in order to be able to facilitate argumentation in their classes. Based on the literature, it is well-known that students at different school levels had difficulties in justification, argumentation and proof (Ellis, 2007; Harel & Sowder, 1998; Healy & Hoyles, 1998; Reiss, Klieme, & Heinze, 2001; Selden & Selden, 2003; Walter & Barros, 2011), which is also a valid situation for Turkish students. This was our starting point for the present study and we considered arguing and argumentation as an issue in which there were important research questions remaining to be addressed.

In Turkey, middle school mathematics curriculum was revised in 2013 (MoNE, 2013), and the intense contents of each grade level were reduced. When the goals and objectives of the last curriculum were examined, it was seen that there were sentences regarding reasoning, which we accept to be the same as argumentation. However, in other parts of the curriculum, there were no detailed explanations or expectations from students regarding argumentation, so it could be said that argumentation was a tacit objective which was expected from students to perform in the recent curriculum. In this regard, the teachers who implement the new curriculum in class have an important role and are, thus, regarded as an important constituent in the process. Especially in argumentation, teacher facilitation was found to be of prime importance. It was found in empirical studies that small children can also do deductive reasoning, but they might not do this naturally without the help of a facilitator (Stylianides, 2007). At this point the importance of the teacher who orchestrates the argumentation emerges for productive argumentation. Then, a question ‘what are the actions that a teacher should perform in argumentation class?’ arise. Common Core State Standards Initiative [CCSSI] (2010) outlined the opportunities that teachers should provide students with in argumentation environment: giving opportunities for conjecturing, providing time to students for constructing arguments, recognizing and using counterexamples, and making plausible arguments. However, there is no course related to implementation of argumentation in class in middle school mathematics teacher education programs so mathematics teachers in Turkey graduate from university without having the necessary skills regarding argumentation. It is argued that a teacher should be able to formulate strong mathematical arguments and proofs in order to respond to students’ arguments and explanations in classroom for high level mathematical learning (Rice, 2012). Accordingly, the answer of the question ‘How far can teachers argue and develop arguments?’ is an issue of concern of the current study. Thus, understanding the nature of prospective teachers’ knowledge regarding argumentation became an important point to be investigated for future developments regarding argumentation in middle school mathematics teacher education programs.

Recently, analyzing argumentation in terms of argument structures has been suggested by researchers in the literature (Walter & Barros, 2011). For example, Walter and Barros (2011) offered two grounded theories for the argumentation structure of calculus students who were building mathematical arguments in their study. One of them was related to the collaborative development of mathematical methods. The other one was related to students’ choices for using warrants, clarification and convincement of others for the validity of their conjectures. In short, they analyzed mathematical argumentation at a qualitative level to reveal the meaning making, communicating and problem solving accuracy of the calculus students. They emphasized the importance of analyzing argument structure as follows:

> Careful analysis of structural elements in students' substantial arguments provides important details with respect to how students can reason about and make sense of problem situations to build and refine representations and understandings of mathematical ideas that they did not previously know or need in problem solving. (Walter & Barros, 2011, p. 340)

As understood, the analysis of argument structures in the problem solving process would give precious information about how students make sense and reason in detail. In this regard, it is believed that analysis of argumentation structures of prospective middle school mathematics teachers will provide invaluable information about their reasoning and learning in an argumentation environment. Ultimately, based on the information gathered from the argumentation structure analysis studies, the researchers could obtain information about teachers' future performances in their mathematics classes. In the present study, prospective middle school mathematics teachers' argumentation structures will be analyzed in terms of global and local argumentation and this will provide information about their reasoning and so future performances.

The other important concern of the current study was the environment in which argumentation would take place. Since argumentation was a popular topic in mathematics education, there are many issues to be investigated related to argumentation in this field. One of them could be the technology enhanced argumentation environment. Technology use in geometry was encouraged by many researchers since it was believed that it allows users to perform various geometrical activities ranging from constructing accurate diagrams to visualizing abstract relationships among concepts (Hollebrands, 2007; Laborde, Kynigos, Hollebrands, & Sträβer, 2006; NCTM, 2000). Although there were few studies investigating argumentation in the dynamic geometry environment (Hewit, 2010; Hollebrands et al., 2010; Inglis, Mejia-Ramos & Simpson, 2007; Prusak, Hershkowitz & Schwarz, 2012), the findings of these studies include signs of positive effects of technology in argumentation. It was claimed that individuals who used technology had the chance of engaging in in-depth thinking in their investigation, so they could be able to notice the relationships that could not be discovered by the students using paper and pencil. Argumentation was also known as being a method requiring higher-order thinking (Hewit, 2010; Lin & Mintzes, 2010). At the beginning of the present study, I was curious about how technology use would support argumentation structures of prospective middle school mathematics teachers and prepared two groups, GeoGebra group and Paper-Pencil group, who will solve the same geometry tasks with argumentative classroom. Thus, it is believed that the present study will provide comparable results regarding the argumentation structures of prospective middle school mathematics teachers in technology and paper-pencil environments. In addition, it will be revealed whether or not technology has positive effects on argument structures of participants.

With the expectation of meeting the needs in the issues mentioned above, the purpose of this study was to investigate the nature of argumentation structures of prospective middle school mathematics teachers while solving geometry tasks in technology and paper-pencil environments. The following questions were formulated for this study:

1. What is the nature of argumentation structures of prospective middle school mathematics teachers while solving geometry tasks in GeoGebra and Paper-Pencil groups?
2. What are the characteristics of the local arguments within the global argumentation structures?
    - What are the characteristics of local arguments based on the flow of argument components (claim, data, warrant) that prospective middle school mathematics teachers express while solving geometry tasks in GeoGebra and Paper-Pencil groups?
3. What are the characteristics of local argumentations that prospective middle school mathematics teachers utilize while solving geometry tasks in GeoGebra and Paper-Pencil groups?

1.2 Significance of the study

The main issue of concern of the current study was argumentation structures utilized by prospective middle school mathematics teachers. The most significant aspects of this study are clarified in the following paragraphs.

In the field of science, argumentation was defined as “the evaluation of knowledge claims in the light of available evidence” (Jiménez-Aleixandre & Erduran, 2008, p. 478) and it was seen as an epistemic practice (Jiménez-Aleixandre, 2014). Likewise, in a mathematics study, argumentation was defined as statements including rhetoric means, the goal of which was persuading someone of the truth of falsehood of a statement (Antonini & Martignone, 2011). In the literature, argumentation was accepted by many researchers as a collective discourse technique rather than the sole individual action (Krummheuer, 1995; Walter & Johnson, 2007). There are specific actions to be performed by the participants during the argumentation. Hewit (2010) listed the actions taken in argumentation as generating hypothesis, justifying a position, synthesizing problems, challenging others’ views, comparing different perspectives, and evaluating the hypothesis consistency by using empirical evidences. These were not simple actions because they require critical thinking and reasoning, which require higher order thinking. In addition, in one of the studies in science, Lin and Mintzes (2010) stated that rebutting an argument is a higher-order thinking skill and it is quite difficult cognitive task for most of the students. The reason for this difficulty was that the student should consider both an argument and the opposing argument before constructing rebuttals (Lin & Mintzes, 2010). Considering such thinking processes and related studies, it was believed that mathematical argumentation which was characterized by such actions as “sharing, explaining, and justifying … mathematical ideas” (Cross, 2009, p. 908), had a positive impact on mathematical learning (Hoyles & Küchemann, 2002; Hufferd-Ackles et al., 2004; Krummheuer, 2000; Stein et al., 2008).

At this point, the implementation of argumentation in mathematics education emerged as an important issue. The *teacher* is the main element who orchestrates argumentation in mathematics classes. It was claimed that a significant problem about argumentation-based science classes rises to be lack of pedagogical knowledge of teachers to design the lesson (Duschl, 2008). Although middle school mathematics teachers in Turkey are familiar with inquiry-based teaching approach, the same situation is also valid for mathematics teachers since argumentation has not been common in middle school mathematics curriculum yet. Researchers alleged that there is a need of training for teachers to orchestrate argumentation in class so they suggested teachers to first experience argumentation as learners themselves to be able to facilitate argumentation effectively (Prusak et al., 2012). Hence, it would be beneficial for prospective teachers to gain experience in developing arguments and facilitating an argumentation class before starting their professional life. However, in Turkey, there are no elective or must courses which include information related to the argumentation method and its implementation in middle school mathematics teacher education programs. Therefore, prospective middle school mathematics teachers graduate from university without any information regarding argumentation. Prospective teachers, who will guide the argumentation of the future students, have an important role in productive argumentation. Specifically, teacher performance in formulating strong mathematical arguments and proofs play an ultimate role in mathematical learning since teachers are the ones who must respond to students' claims or explanations in the classroom (Rice, 2012). Thus, prospective middle school mathematics teachers' own argumentation needed to be looked into in detail. This need may be met by means of the current study with the path that it will open by providing the initial necessary information regarding the nature of prospective middle school mathematics teachers' argumentation, and, thus, may help to lead subsequent studies aiming to develop teacher education programs. That is, the present study could provide invaluable implications for mathematics educators and policy makers in designing the course contents of middle school mathematics teacher education programs.

Argumentation was studied in the field of science in terms of quite many aspects and its benefits regarding scientific reasoning and conceptual understanding have been revealed with many research studies (Lawson, 2010; Yeşiloğlu, 2007). However, argumentation was a relatively new topic to be studied in the field of mathematics education so the benefits of argumentation in mathematics learning of different topics and how the argumentation to be integrated in these topics in different environments is an issue of concern. As the argumentation is not a method to be taught in middle school mathematics teacher education programs, little is known about how the argumentation-based teaching affect conceptual learning and reasoning of students in mathematics field.

Considering the studies in the literature, there were many aspects to be considered in mathematical argumentation. In this regard, another significance of the present study is the *integration of technology* in the argumentation. In the related literature some researchers have been interested in examining argumentation in a technology environment (Hollebrands et al., 2010; Inglis et al., 2007; Prusak et al., 2012). In some studies existent in the literature, the argument structures of college geometry students (Hollebrands et al., 2010), modal qualifier use and warrant types of postgraduate mathematics students (Inglis et al., 2007), and peer unguided argumentation of preservice teachers (Prusak et al., 2012) were investigated. As understood, technology use in argumentation has been a popular topic but an area that lacked in-depth study. One of the important aspects to be studied is dynamic geometry programs which are known to provide students with the opportunity to construct accurate diagrams, which enable students to realize the relationships between abstract and general properties of geometry. Therefore, researchers wonder whether it assists or hinders the development of reasoning (Vincent, Chick & McCrae, 2002). That is, how the technology guide the argumentation was not clear in the literature and there was no study confronted in literature regarding this issue within the Turkish context. Thus, it is believed that the findings of the current study related to the prospective middle school mathematics teachers' argumentation structures in technology and paper-pencil environment will reflect the implications regarding the relationship between technology use and argumentation.

Furthermore, it is expected that findings will contribute to a large extent to the technology integration approach in the Turkish education system. As a tool promoting social interaction, GeoGebra was expected to supply productive argumentation among prospective middle school mathematics teachers. Ultimately, current study focused on prospective middle school mathematics teachers' own argumentation structures in a technology environment, supported with the GeoGebra dynamic geometry program, and paper-pencil environment. Moreover, in which aspects the use of GeoGebra affected the argumentation structures of prospective middle school mathematics teachers was an important issue for the present study. Two groups were established (GeoGebra group and Paper-Pencil group) to make comparisons about argumentation structures and to investigate whether or not GeoGebra use make changes in global/local argumentation structures of prospective middle school mathematics teachers. Considering the findings, teachers can make decisions in which situations/topics they should integrate technology into their argumentation classes by considering the context explained in the present study.

The findings of the present study related to prospective middle school mathematics teachers' *argumentation structures* has a great importance in the field of mathematics education since teachers' own practices would signify their future intentions regarding their teaching preferences. In the literature, Knipping (2008) also conducted a proof study with argumentation structures and emphasized the need for further research regarding the argumentation structures. The present study revealed prospective middle school mathematics teachers' reasoning style by examining their global argumentation structures and flow of argument components (claim, data, and warrant). Moreover, local argumentations of the participants were examined within the geometry context in order to contribute to the existing classification developed by Knipping (2008). In this way, patterns related to the prospective middle school mathematics teachers' argument construction process and reasoning were added to the argumentation literature in geometry context. For instance, by means of the current study, the most frequently used local argumentation types (visual argumentation or conceptual argumentation) were investigated in two groups and the ones which were valued by prospective middle school mathematics teachers were revealed. The differences in local argumentation preferences of GeoGebra group and Paper-Pencil group illustrate the effect of technology use in prospective middle school mathematics teachers' justification characteristics. This information is invaluable since it includes hints related to possible justification preferences for future teachers while planning their argumentation classes. For instance, they should know that the students in technology supported environment might present visual argumentation more, so they should encourage them to support their arguments with theoretical justifications if they need justification with conceptual argumentation. Thus, the present study will open a new door to studies regarding argumentation structures in technology enhanced or paper-pencil environments.

To conclude, the present study revealed the aspects of prospective middle school mathematics teachers' argumentation within a technologically enhanced geometry context. There has been no argumentation related course in mathematics teacher education programs up to now, so it is believed that this study will draw the attention of the teacher educators and policy makers to this issue and will raise their awareness regarding argumentation in mathematics education. Thus, the use of argumentation will be encouraged in the mathematics field and teaching of future students would be fostered.

1.3 Definition of the important terms

The research questions consist of several terms that need to be constitutively and operationally defined.

Prospective middle school mathematics teachers:

These were the participants of the study, and they were the senior students majoring in middle school mathematics education. Being in their fourth year of undergraduate teacher education program, they had taken all the courses regarding teacher education. In addition, they were the candidates who would teach mathematics from fourth grade to eighth grade in primary and middle schools after their graduation.

Argument:

Argument was defined by Krummheuer (1995, p.231) as "The intentional explication of the reasoning of a solution during its development or after it". In the current study, each statement made by students consisting of a judgement followed by a conclusion and justification (if exists) was accepted as an argument. In order to identify the arguments, firstly the conclusions that the participants reached were detected, and then the justification and the data they had were sought in the transcriptions.

Argumentation:

The accepted definition for argumentation in the current study was "A process of logically connected mathematical discourse" (Vincent, 2002, p. 11). In the current study, argumentation included both pair-discussions and class discussions in which the participants presented a conclusion or a standpoint they had and then justified their conclusion by considering the data they had. In addition, the major aim was to convince other participants about their conclusion with the support of their justifications. This convincing action occurred between pairs during pair-work, between the participants who were at the board and as whole-class in class discussions. This means that the entire geometry problem solving process of the study was accepted as an argumentation.

Argument Components (claim, data, warrant):

Argument components were the elements that were defined by Toulmin (1958) in his book 'The Uses of Argument'. These components were claim, data and warrant. In the present study 'claim' refers to the conclusions of the participants' arguments. The data are the facts that participants appealed to for support of the claim. Finally, the warrant refers to the statements justifying the connection between data and claim.

Global Argumentation Structures:

Knipping (2008) defined global argumentation as "layout of the structure of the argument as a whole (the anatomical structure)" (p. 430). In the present study, the layouts of the argumentation of each geometry task in two groups were drawn in order to see the whole picture of the argumentation. These general schemas were named as global argumentation structures and these structures were analyzed.

Local Arguments:

Knipping (2008) defined the local arguments as “single out distinct arguments” (p. 430). The present study also refers to single arguments with the components of the claim, data and warrant (all component may not exist) as a local argument.

Local Argumentations:

Knipping (2008) analyzed the *warrant* components of the local arguments and classified the warrants according the notions of figural and conceptual aspects of reasoning developed by Fischbein (Fischbein 1993; Mariotti & Fischbein 1997). Visual field of justification and conceptual field of justification were the two main local argumentations she classified. She explained these two levels as “Reasoning based on the figural aspect concerns perceptions of space. Reasoning based on the conceptual aspect concerns abstract and theoretical knowledge” (Knipping, 2008, p. 436). In the present study, Knipping’s (2008) classification was used, and the local argumentations were analyzed by evaluating the warrant components of the local arguments.

Dynamic Geometry Environment:

Dynamic geometry environment is a general term which was used for the computer microworld with Euclidean geometry as the embedded infrasutructure and students have the opportunity to interact with geometrical figures (Lopez-Real & Leung, 2007) via dynamic geometry softwares such as Cabri, GeoGebra and Geometer’s Sketchpad. Dynamic geometry programs refer to computer programs which can be used in geometry interactively. In the present study, the dynamic geometry environment refers to the computers laboratory in which the participants have the opportunity to use GeoGebra dynamic geometry software while solving geometry tasks.

GeoGebra:

GeoGebra is a dynamic geometry program which can be used in the teaching and learning process by a large age group ranging from middle school to university level (Hohenwarter & Preiner, 2007). The program has a range of tools to be able to construct geometric objects. These tools range from primitive objects, such as point, line and segments to classical constructions, such as midpoint, perpendicular and parallel line (Jones, 2002). Moreover, there are transformations and measurements which help users to see the relations. GeoGebra also provides teachers with opportunities to share their materials online and with other teachers and students for free. GeoGebra has a graphic window and an algebra window, and any change in the graphic window can be seen on the algebra window simultaneously.

CHAPTER II

REVIEW OF LITERATURE

The purpose of this study was to investigate the nature of the argumentation structures of prospective middle school mathematics teachers while solving geometry tasks in technology and paper-pencil environments. In addition, local arguments (core arguments including claim, data and warrant components) and local argumentations (only warrant components of the local arguments) were analyzed. More specifically, the kinds of global argumentation structures the prospective elementary mathematics teachers employed, the kinds of local arguments they expressed based on the flow of argument components, and the kinds of local argumentations they utilized to justify their arguments were investigated.

The theoretical background related to the argumentation framework, Toulmin's argumentation model, the uses of Toulmin's argumentation model in mathematics education research, technology support and argumentation, and teacher responsibilities in argumentation process are presented throughout the chapter in order to provide insight and a better understanding of the scope of the study.

2.1 Theoretical background

Developments in mathematics learning have highlighted the importance of social and cultural processes in learning (Cobb & Bauersfeld, 1995; Yackel, Ramussen, & King, 2000). Specifically, it was claimed that learning contains both individual and social components which are critical for academic achievement (Cobb, Yackel, Wood, Nicholls, Wheatley, & Trigatti, 1991; Lesh, Doerr, Carmone, & Hjalmarson, 2003; Schoenfeld, 1992). The few theoretical perspectives that were claimed to have contributed to mathematics education by means of argumentation are social interaction, socio-mathematical norms, sociocultural influences, and social negotiation of roles (Walter & Barros, 2011). Each of these perspectives put forward different suggestions to educators regarding the teaching and social learning of mathematics. With an awareness of the importance of social learning, the main interest of the present study can be considered as social interaction in an argumentation environment. Argumentation is accepted as closely related concept with proof by many researchers (Conner, 2007b; Hemmi, Lepik, & Viholainen, 2013; Stylianides & Al-Murani, 2010) so the relationship between these two terms were explained in the next section.

2.1.1 Argumentation and proof

The key concepts addressed in the present study were *argument* and *argumentation*. Krummheuer (1995) defined *argument* as "The final sequence of statements accepted by all participants, which are more or less completely reconstructable by the participants or by an observer as well" (p. 247), while *argumentation* was defined by Antonini and Martignone (2011) as the statements consisting of rhetoric means, the goal of which is to convince individuals of the truth or the falsehood of a statement. It can be inferred from these definitions that argumentation is regarded as a process of logically connected mathematical discourse (Vincent, 2002), while an argument is referred to as the end-product of the argumentation.

Antonini and Martignone (2011) defined mathematical proof as a statement consisting of a logical sequence of propositions regarding the validity of the statement. Another definition was stated as "the argumentative process that mathematicians develop to justify the truth of mathematical propositions, which is essentially a logical process" (Recio & Godino, 2001, p. 94). In the literature, researchers specified five functions of proofs, which were *verification, explanation, systematisation, discovery* and *communication* (Hanna, 2000; De Villiers, 1999). Viholainen (2011) advocated that although argumentation has a broader meaning than the term proof, it has the same functions mentioned for proofs. Hanna (2000) claimed that the main function of proof in mathematics education is providing explanations as well as justifications and verifications. She also stated that the explanatory proofs could be in the form of calculations, visual demonstrations, a guided discussion, a pre-formal proof, an informal proof, or a proof conforming to strict norms of rigor.

In the literature, some of the researchers accepted proof as a special type of argumentation (Conner, 2007b; Hemmi et al., 2013). For example, Stylianides (2007) was one of the researchers who accepted proof as a mathematical argument when it possessed the following characteristics:

> - Proof is a mathematical argument, a connected sequence of assertions for or against a mathematical claim, with the following characteristics:
> - It uses statements accepted by the classroom community (set of accepted statements) that are true and available without further justifications;
> - It employs forms of reasoning (modes of argumentation) that are valid and known to, or within the conceptual reach of, the classroom community;
> - It is communicated with forms of expression (modes of argument representation) that are appropriate and known to, or within the conceptual reach of, the classroom community (Stylianides, 2007, p. 107).

As understood, the proof expressions that include taken-as-shared statements, which do not need further justifications, can be accepted as mathematical arguments when various forms of reasoning are used, and these opinions are expressed to the classroom by means of argument representation modes. Likewise, in another study, Stylianides and Al-Murani (2010) clarified the criteria for an argument to be regarded as proof:

> 1. It can be used to convince not only myself or a friend but also a sceptic. It should not require someone to make a leap of faith (e.g., ‘This is how it is’ or ‘You need to believe me that this pattern will go on forever.’)
> 2. It should help someone understand why a statement is true (e.g., why a pattern works the way it does).
> 3. It should use ideas that our class knows already or is able to understand (e.g., equations, pictures, diagrams).
> 4. It should contain no errors (e.g., in calculations)
> 5. It should be clearly presented. (p. 312)

It can be deduced that a proof needs to be sceptic and convincing, and explanatory to be accepted as an argument. In addition, it should include a reason for why a statement is true, have correct calculations and be clearly presented. Considering these properties, it can be understood that proof and argumentation are closely related concepts. Therefore, it is not surprising that there are many studies related to argumentation and proof which were explained in the section explaining the uses of argumentation in mathematics education research. However, the Toulmin's Argumentation model was primarily explained in the next section in order to make argumentation studies explained in subsequent sections more meaningful to the reader.

2.1.2 Toulmin's argumentation model

Toulmin, a British philosopher and logician, wrote a book 'The Uses of Argument' in 1958 and proposed an argumentation model. In his book, Toulmin presented a structure for rational arguments and interrelated the components/elements − claim, data, and warrant − of the argument. He also mentioned three auxiliary components, which were modal qualifier, backing and rebuttal. Rumsey (2012) stated that these auxiliary elements were not essential components but could be present in arguments. 'The Uses of Argument', emphasizes the following points listed by Hitchcock and Verheij (2005):

1. Reasoning and argument involve not only support for points of view, but also attack against them.
2. Reasoning can have qualified conclusions.
3. There are other good types of argument than those of standard formal logic.
4. Unstated assumptions linking premisses to a conclusion are better thought of as inference licenses than as implicit premisses.
5. Standards of reasoning can be field-dependent, and can themselves be the subject of argumentation (p. 255)

Each item above represents each component of Toulmin's (1958) argumentation model (Hitchcock & Verheij, 2005). The first one is the rebuttal, the second one is the modal qualifier, and the other three items represent the warrant and backing components of the model (Hitchcock & Verheij, 2005). In addition to the components listed above, one other component, which is the main component that each argument has to possess, is the claim/conclusion [C]. The emergence of these components during the argumentation is not so straightforward. For instance, a statement which was proposed as data in one argument can have the function of a claim or warrant in following arguments (Conner, Singletary, Smith, Wagner & Francisco, 2014b; Forman et al., 1998). In order to support the claim or conclusion of any argument, there should be some facts, information or other statements which refer to the data [D] (Yackel, 2002). Then the following question may come to mind 'Are the data valid for the claim?' Then, several data or separate arguments can be presented as data to the argument (Yackel, 2002). Subsequently, the explanatory relevance of the data to the claim is questioned, and the legitimacy of the data will be stated with the help of the warrant [W] (Yackel, 2002). In some cases, further supports, such as general theories, beliefs, and primary strategies, will be needed for the warrant which corresponds to the backing [B] component (Yackel, 2002). Furthermore, to be able to express the degree of confidence, modal qualifiers [Q] are needed. Finally, exceptional situations, if any, where the claim is not valid, can be added as rebuttal [R] to the argument. Toulmin (1958) reveals the relationship between these components with a specific layout or schema displayed in *Figure 2.1*.

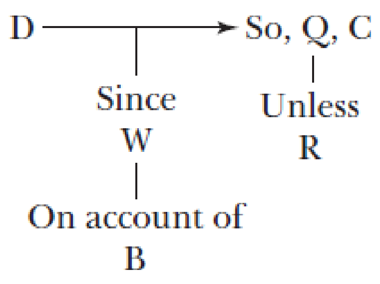


Figure 2.1 Toulmin's (1958) argument layout (p. 97)

In a majority of studies, researchers have defined the argument components in various ways, and some of these definitions are listed in the following table.

Table 2.1 Argument components and definitions

Component	Definitions
Claim	Conclusion of the argument (Toulmin, 1958, p. 101) The statement of the speaker (Pedemonte, 2007, p. 27) The statement that the argument is meant to establish (Walter & Johnson, 2007, p. 708) Assertion about an issue (Lin & Mintzes, 2010) Statements whose validity is being established (Conner, Singletary, Smith, Wagner, & Francisco, 2014a, p. 404) The statement of which the arguer wishes to convince an audience (Nardi, Biza & Zachariadez, 2012, p. 159)
Data	Consists of facts that support the claim (Conner, Singletary, Smith, Wagner, & Francisco, 2014a, p. 404; Verheij, 2005) Facts we appeal to as the foundation of the claim, or minor premise (Toulmin, 1958, p. 101) Specific facts relied on to support a given claim (Sekiguchi, 2002) The facts that serve as the basis for the conclusion (Walter & Johnson, 2007, p. 708) Foundations on which the argument is based; this includes evidence relevant to the claim being made (Nardi, Biza & Zachariadez, 2012, p. 159)
Warrant	The statement authorising the move from the data to the claim, or major premise (Toulmin, 1958, p. 101) The inference rule that allows data to be connected to the claim (Pedemonte, 2007, p. 27) The statement that explain or authorize why data establish the conclusion acceptably (Walter & Johnson, 2007, p. 708) Statements that connect data with claims (Conner, Singletary, Smith, Wagner, & Francisco, 2014a, p. 404) Justifies the connection between data and conclusion; warrants include appealing to a definition, a rule, an example, or an analogy (Nardi, Biza & Zachariadez, 2012, p. 159)
Backing	Further reason to believe the warrant (Toulmin, 1958, p. 101) Additional support for the warrant (Pedemonte & Reid, 2011) The statement that attempts to establish the authority of the warrant (Walter & Johnson, 2007, p. 708) Usually unstated, dealing with the field in which the argument occurs (Conner, Singletary, Smith, Wagner, & Francisco, 2014a, p. 404) Further evidence, justifications or reasons (Nardi, Biza & Zachariadez, 2012, p. 159)

Table 2.1 (continued)

Component	Definitions
Modal Qualifier	The statements which express the force of the claim (Toulmin, 1958, p. 101) The statement that express the strength of the argument (Pedemonte & Reid, 2011) Statements describing the certainity whith which a claim is made (Conner, Singletary, Smith, Wagner, & Francisco, 2014a, p. 404) Qualifies the conclusion by expressing the degrees of the arguer's confidence (Nardi, Biza & Zachariadez, 2012, p. 159)
Rebuttal	The exceptional conditions which might be capable of defeating or rebutting the warranted conclusion (Toulmin, 1958, p. 101) Introduces counter-argument (Pedemonte & Reid, 2011) A valid rejection of a warrant that is in support of a counterargument (Lin & Mintzes, 2010) Statements describing circumstances under which the warrants would not be valid (Conner, Singletary, Smith, Wagner, & Francisco, 2014a, p. 404) Potential refutations of the conclusions;rebuttals include exceptions to the conclusion or citing the conditions under which the conclusion would not hold (Nardi, Biza & Zachariadez, 2012, p. 159)

Toulmin's argumentation model was revealed to be used in a variety of contexts, so it can be said that the model is field independent. However, some of the researchers have asserted that the backing component of the model is field-specific, which means that it is field dependent since the source of authority is related to the nature of the argument (Hollebrands et al., 2010; Verheij, 2005; Vincent, 2002). Similarly, Hollebrands et al. (2010) assert that the reason why backing is specifically field dependent is that backings are accepted and well-comprehended in the field in which the argument is constructed. In other words, an argument may have the same components in science, law, philosophy and mathematics, but the valid data, warrant and especially backing are said to be different based on the field in which the argument is made (Hollebrands et al., 2010).

Another point regarding argument components is their presence in student discussions. That is, Nardi, Biza and Zachariadez (2012) emphasized that not all components of the arguments are explicitly presented by individuals in all arguments. In some situations, warrants may be implicit (Voss, 2005), so the modal qualifiers and exceptions support the warrants (Cramer, 2011). In such a situation, the teacher can ask students to justify their reasoning so that they can clearly state the warrant of the argument.

Lastly, a possible stituation that can be encountered in argumentation studies is that the argumentation process may include argumentation structures which entail one or more steps. That is, when more than one step exist, the claims of the initial arguments, which are accepted by the other participants as taken-as-shared, can become the data or warrant of the later arguments (Cramer, 2011). In addition, Cramer (2011) states that there are situations in which participants question the data, warrant or backing of the argument. In these situations, the questioned part is asserted to be examined in a separate argumentation process before considering it in the primal argument. These separate argumentation processes were named as 'lines of argumentation' by Krummheuer and Brandt (2001) (as cited in Cramer, 2011). After giving a detailed explanation regarding the Toulmin's argumentation model, the studies in mathematics education research related to argumentation were explained in the next sections.

2.2 Uses of Toulmin's argumentation model in mathematics education research

Initially, Toulmin's (1958) work was not accepted among philosophers and logicians, but the model was gradually adopted by communication theorists and became the keystone of the study of argumentation (Aberdein, 2008). The model was first introduced to describe non-mathematical arguments (Toulmin, 1958), but later Toulmin, Richard and Allan (1979) applied the theorem in order to prove Theaetetus's proof, which says that 'there are exactly five platonic solids'. There has been a growing interest among mathematics researchers in practices related to argumentation in recent years (Inglis et al., 2007). Numerous researchers have used

Toulmin's (1958) model of argumentation to examine the argument structures and proof structures in the mathematics field (Giannakoulias, Mastorides, Potari, & Zachariades, 2010; Krummheuer, 2007; Pedemonte, 2007; Pedemonte & Reid, 2011). Moreover, the model was used by mathematics education researchers to analyze classroom discussions (Forman et al., 1998; Krummheuer, 1995, 2007; Moore-Russo, Conner, & Rugg, 2011; Pedemonte & Reid, 2011; Yackel, 2001), interview data of students (Hollebrands, Conner, & Smith, 2010; Inglis et al., 2007), interview data of teachers (Nardi, Biza, & Zachariades, 2012; Steele, 2005), and quality of mathematical arguments (Inglis & Mejia-Ramos, 2008; Pedemonte, 2007). This section summarizes earlier studies (Forman et al., 1998; Krummheuer, 1995; McClain, 2009; O'Connor, 1998; Pedemonte, 2007; Pedemonte & Reid, 2011; Wood, 1999; Yackel, 2002) regarding argumentation and mathematics.

The author of the book entitled 'The Ethnography of Argumentation', Krummheuer (1995) was the first researcher to adapt and use Toulmin's (1958) argumentation model to conduct research in mathematics education. The significant point of this study was that Krummheuer (1995) used the restricted version of the model, which means the arguments included only the claim, data and warrant components. Moreover, he did not view argumentation as a singular activity in which an individual tries to persuade others of his/her claim. Instead, he analyzed students' argumentation in a social environment and propounded the term *collective argumentation,* which was defined as "a social phenomenon, when cooperating individuals tried to adjust their intentions and interpretations by verbally presenting the rationale of their actions (Krummheuer, 1995, p. 229). Similarly, collective argumentation was stated to be concerned with a group's reaching consensus and differed from the Aristotelian argumentation which is based on an individual's endeavor to convince a group (Conner, 2007b). While there some researchers analyzed individuals' construction of arguments (Hollebrands, Conner, & Smith, 2010; Inglis et al., 2007), many other research studies in the literature focused on collective argumentation in mathematics education (Conner, Singletary, Smith, Wagner, & Francisco, 2014a; Krummheuer, 2007; Ramussen & Stephan, 2008; Yackel, 2002). For instance, Conner et al. (2014a) focused on teacher support in collective argumentation with secondary mathematics students. Particularly, they investigated direct teacher contributions to arguments, teachers' question types and other supportive actions and then revealed ways of analyzing conversations of collective argumentation in terms of mathematical aspects by using the framework. Ultimately, they admitted that the framework was useful in the examination of how teachers support reasoning and argumentation of the students. In another study, a methodology to document collective activity was presented by Ramussen and Stephan (2008). They proposed a three-phase methodological approach in analyzing collective activity of students. This approach was also based on Toulmin's argumentation model schema, but they chose to use core arguments like one that was previously used by Krummheuer (1995). They asserted that in the first phase the claims made by the students or the teacher existing in the transcriptions were identified. Then, in the second phase, they took the argumentation log as data and examined the whole class sessions to determine the mathematical ideas expressed in the arguments which were accepted as part of the group's way of normative reasoning. For this reason, they prepared a three-column table for each day:

> a column for the ideas that now function as if shared, (b) a column of the mathematical ideas that were discussed and that we want to keep an eye on to see if they function subsequently as if they were shared, (c) a column of additional comments, both practical and theoretical, or connections to related strands of literature" (Ramussen & Stephan, 2008, p. 200).

Finally in the third phase, the charts of phase two were taken and the ideas from "as-if-shared" column were listed to organize them around common mathematical activities. Considering the analyses of collective argumentation, Yackel (2002) asserted that it was not sufficient to analyze the sequence of the statements that were made, so the researchers could analyze the functions of the statements in the interactions of participants in order to make sense of the collective argumentation. It can be inferred that the analysis of the collective argumentation was a critical issue and changed based on the nature of the data in each study.

Following Krummheuer (1995), who was the first scholar to use Toulmin's argumentation model in mathematics education, many researchers used the restricted version of the model, i.e. the core of the argument, which included only the claim, data and warrant components (Conner et al., 2014b; Forman et al., 1998; Hoyles & Küchemann, 2002; Krummheuer, 1995; Lavy, 2006; Pedemonte, 2007; Yackel, 2002). However, some researchers defended the importance of using Toulmin's (1958) model as a whole in mathematics education (Aberdein, 2005; Hollebrands, Conner & Smith, 2010; Inglis et al., 2007; Jahnke, 2008). For example, Inglis et al. (2007) conducted a study with highly talented postgraduate mathematics students and analyzed the data of their task-based interviews. In this analysis, they used the full scheme of Toulmin (1958), including backing, rebuttal and qualifier components. It was emphasized that the role of modal qualifiers in mathematical argumentation was underestimated (Inglis et al., 2007). Ultimately, the researchers proposed that developing students' abilities to matching warrant-types with modal qualifiers should be one of the main goals of instruction (Inglis et al. 2007).

In literature, researchers examined argumentation from various perspectives in mathematics. For instance, some researchers contributed to the literature by analyzing reasoning types (Conner et al. 2014b; Pease & Aberdein, 2011; Pierce, 1960). Some of them took arguments as a whole and classified argument types (Aberdein, 2005; Viholainen, 2011), while others investigated the inner part (warrant component) and classified warrant types (Inglis et al., 2007; Nardi, Biza, & Zachariades, 2012; Walter & Barros, 2011). In the following sections, studies related to each perspective are explained.

2.2.1 Argumentation studies related to reasoning types

Pierce's (1960) study was considered as a corner stone among the studies analyzing reasoning types. In this study, Pierce (1960) defined three types of reasoning: abduction, deduction and induction. Abduction was used to explain facts, while deduction was used to produce testable results. To make it clear, Pierce (1960) defined abduction as follows:

> abduction looks at facts and looks for a theory to explain them, but it can only say “might be”, because it has a probabilistic nature. The general form of an abduction is: a fact A is observed; if C was true, then A would certainly be true; so, it is reasonable to assume C is true. (p. 372)

As can be deduced, after the observation of fact A, the statements of the person who is talking abductively would not be accurate since he/she could not test C, so there was no certainity of fact A.

The other reasoning type, induction, which refers to reasoning from specific cases to general rules, was used to compare predictions and observed behaviors (Pease & Aberdein, 2011). The most frequently used reasoning type in proof studies was deduction, which refers to reasoning from general rules to specific cases (Pease & Aberdein, 2011). Although Pierce believed that only deduction could be used in mathematics, Pease and Aberdein (2011) stated that abduction was a useful and applicable reasoning type in mathematical thinking as well.

Another study which focused on reasoning types in collective argumentation was a study by Conner et al. (2014b). They combined Toulmin’s argumentation model with Pierce’s reasoning classification mentioned above. At the end of the study, they recommended that teachers use this combination to detecting and support students’ different kinds of reasonings (deductive, inductive, abductive, analogical reasoning). Conner et al. (2014b) characterized deductive reasoning as the only reasoning type enabling individuals to arrive at a conclusion with certainty; furthermore, it was defined as a reasoning type including the logical consequence of aforementioned assumptions to arrive at conclusions. They also explained inductive reasoning as drawing abstractions and generalizations using individual observations (Conner et al., 2014b). Another definition proposed for inductive reasoning was reasoning proceeding from specific to general (Reid & Knipping, 2010). The other reasoning type, abductive reasoning, was defined as “an inference which allows the construction of a claim starting from an observed fact” (Pedemonte, 2007, p. 29). Conner et al. (2014b) stated that abduction was observed in a mathematics classroom when students’ first come across the result and then have to guess which particular rule and case afforded such a result. Finally, analogical reasoning was characterized with the requirement of developing a claim based on the similarities among related cases (Reid & Knipping, 2010).

2.2.2 Argumentation studies related to different types of arguments

As mentioned, some researchers focused on different types of arguments in their studies (Aberdein, 2005; Baccaglini-Frank & Mariotti, 2010; Viholainen, 2011). One of these studies was conducted by Viholainen (2011). Viholainen (2011) mentioned two types of arguments, *formal arguments* and *informal arguments*, in his paper presented at the 7th Congress of European Research in Mathematics Education (CERME 7). Formal arguments were described as those arguments having warrants which were based on definitions, axioms and previously proven theorems. On the other hand, the arguments whose warrants were based on the concrete interpretations of mathematical concepts were accepted as informal arguments (Viholainen, 2011). He clarified the difference between constructing formal and informal arguments as follows:

> Construction of formal arguments often requires exact and detailed analytical reasoning based on symbolic representations and procedural skills to carry out calculations and other technical procedures. However, informal arguments may reveal holistic features and wider trends, which, yet, may also be very important in the construction process of the argument, by simplifying and concretising the problem situation (Viholainen, 2011, p. 5).

As can be understood, the process in constructing formal arguments is more challenging than that of informal arguments. In formal arguments, the evidence and its presentation via symbolic representations after analytical reasoning is important, while reasoning in informal arguments is holistic in nature. In the differentiation of formal and informal arguments, Viholainen (2011) suggested categorization of arguments based not on the reasoning process but on the final forms of arguments. The reason was explained by Viholainen (2011) with the following example: For instance, when a person used visualization as an aid of thinking in reasoning, this could not guarantee that the argument was an informal argument. On the other hand, during the informal argument construction process, a person could use formal definitions (as backing) to justify visual or physical interpretations, but this did not make the argument formal.

Another classification of different types of arguments was proposed in a study on proof by Aberdein (2005). He presented how the argumentation model can be used in both *regular arguments* and *critical arguments*. According to Toulmin, Richard, and Allan (1979, p. 247), regular arguments were the ones "conducted within or as applications of a scientific theory", while critical arguments were the ones "challenging a prevailing theory or seek[ing] to motivate an alternative" (Toulmin, Richard, & Allan, 1979, p. 247). Aberdein (2005) particularly focused on regular argumentation in his study since mathematical proof corresponds to regular argumentation.

Lastly, another argument type, *instrumented argument*, was proposed in a study with the use of technology (Baccaglini-Frank & Mariotti, 2010). In their study, Baccaglini-Frank and Mariotti (2010) defined the *instrumented argument* as an argument, the warrant of which includes the tools of a dynamic geometry program, such as the dragging tool. They claimed that the persuasion of the participant came from the use of the dragging tool and its intrinsic logic in instrumented arguments.

In the present study, the different types of arguments were examined in terms of the order of the statements of the argument components (claim, data and warrant), which had not been investigated in earlier studies. Therefore, this study will contribute to the literature by presenting a pattern of argument construction which was used by the prospective elementary mathematics teachers in the geometry context.

2.2.3 Argumentation studies related to different types of warrants

Some of the research related to argumentation focused on the analysis of the warrant components of arguments (Inglis et al., 2007; Knipping, 2008; Nardi, Biza, & Zachariades, 2012; Walter & Barros, 2011) since justification had an important place in argumentation studies. For instance, Inglis et al. (2007) conducted a study related to the classification of mathematical reasoning but focused particularly on the warrants. They especially examined the modal qualifier component of Toulmin's argumentation model, stressed the importance of modal qualifiers in mathematical arguments and proposed three types of warrant developed by postgraduate mathematics students: *the inductive warrant*, *the structural-intuitive warrant* and *deductive warrant*. The definition of the inductive warrant type was explained based on the inductive proof schema of Harel and Sowder (1998), which was "when students ascertain for themselves and persuade others about the truth of a conjecture in one or more specific cases" (p. 252). They mentioned that a similar strategy was used to decrease uncertainty in the claim of an argument in the inductive type of warrant. In the structural-intuitive type of warrant, the individual persuades others by using observations, experiments, some kind of mental structure and visual things (Inglis et al. 2007). The last type of warrant, the deductive warrant, corresponds to the use of formal mathematical justifications, such as axioms, algebraic manipulations and counterexamples, to justify the claim. Ultimately, Inglis et al. (2007) emphasized that the inductive and structural-intuitive types of warrant played an important role in mathematical argumentation.

Similarly, Nardi, Biza and Zachariades (2012) focused on the warrant components of secondary mathematics teachers' arguments in their study. The data were collected through written responses and interviews after teachers were engaged with the classroom scenarios prepared from the mathematical areas of analysis and algebra. The researchers adapted Toulmin's argumentation model and Freeman's classification, which differentiates between epistemological and pedagogical a priori warrants, professional and personal empirical warrants, epistemological and curricular institutional warrants and evaluative warrants (Nardi, Biza, & Zachariades, 2012). At the end of the study, they proposed a classification of warrants which included four types: a priori, empirical, institutional and evaluative warrants. They asserted that teachers' arguments should not only be analyzed in terms of accuracy but should also be evaluated in the light of other teacher considerations and priorities since teacher arguments were shaped by different sources of teacher knowledge (Kennedy, 2002; Shulman, 1987).

Other researchers who focused on warrant types were Walter and Barros (2011). They mentioned another warrant type in their study, namely *semantic warrants*. Their study was emphasizing the importance of collaboration in mathematical learning and they examined the linguistic invention and semantic warrant production of elementary teachers within mathematical discourse on graphs. They defined semantic warrants as “personally meaningful instantiations which ground developmental reasoning and support mathematical inferences” (Walter & Barros, 2011, p. 325). They asserted that learners construct mathematical meaning in the process of creating *semantic warrants* since learners reconstruct, evolve, and deepen their mathematical reasoning in this process. Besides, they stated that semantic warrants were prefered by the individuals purposefully to convince both themselves and others about the correctness of a constructed mathematics. In this respect, they believed that the semantic warrants contained clues to the learners’ reasoning processes in the development of a mathematical truth and deepened their mathematical reasoning. In addition, they concluded that students’ understanding of general mathematical concepts can increase if their personal talks in conventional language were supported by semantic warrants.

Lastly, Knipping (2008) examined local argumentations to analyze warrants of arguments and proposed a classification by considering the nature of warrants used in proof context. According to her classification, local argumentations were divided into two: *conceptual argumentation* and *visual argumentation*. In addition, she looked into visual argumentations in detail and revealed two types of visual argumentations, namely *empirical-visual argumentation* and *conceptual-visual argumentation*. The details of these local argumentations are explained in detail in the method section since this classification was used in the data analysis of the current study.

2.2.4 Argumentation studies based on an adapted version of Toulmin's models

Although the literature includes many studies based on Toulmin's argumentation model, there were situations in which the model was not sufficient to present the argumentation which took place in various studies. Therefore, in such studies, the researchers adapted the model and/or added new components to the model (Conner et al., 2014a; Prusak et al., 2012; Voss, 2005; Walter & Johnson, 2007) or combined Toulmin's (1958) model with other models (Conner et al., 2014b). For instance, Voss (2005) studied ill-structured problems and analyzed the data using Toulmin's model. He proposed six extensions, or generalizations, which were as follows:

1. Claim of an argument can serve as datum for a second argument
2. Backing can be an argument itself
3. Implicit warrant exists in every argument
4. A rebuttal can have a backing
5. A rebuttal can be an argument
6. Qualifiers can be arguments (p. 326)

These extensions emerged from that study and they contributed to the argumentation model as modifications. For instance, based on the second extention, a backing component can be presented as an argument which has a claim, data and warrant as displayed in the *Figure 2.2*.

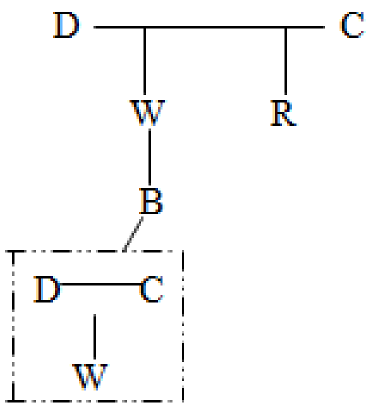


Figure 2.2 Modification example based on extention 2 of Voss (2005)

Similarly, Conner et al. (2014b) determined an insufficiency in Toulmin's (1958) argumentation model (with claim, data and warrant) in differentiating the nuanced evolution of reasoning in arguments and, thus, combined the model with Pierce's (1956) rule, case and result model. In Reid and Knipping's (2010) study which was based on a reinterpretation of Pierce's work, the *case*, *rule* and *result* were defined. According to this study, *case* was "a specific observation that a condition holds" (Reid & Knipping, 2010, p. 83) and the example for a case was presented as "2 is a natural number" and the condition in this example was being a natural number. The other concept, *rule*, was defined as "a general proposition that states that if one condition occurs then another one will also occur" (Reid & Knipping, 2010, p. 83). The sample for the rule was "Natural numbers are integers" (Reid & Knipping, 2010, p. 83) and it could be said that the two linked conditions were 'being a natural number' and 'being an integer'. Finally the *result* concept was defined by Reid and Knipping (2010) as being similar to the *case* concept, but it was not only a specific observation (like in the case) but also required a condition to hold. Conner et al. (2014b) used Toulmin's model and Pierce's rule and the following diagrams emerged and started to be used in reasoning analyses in argumentation (See *Figure 2.3*).

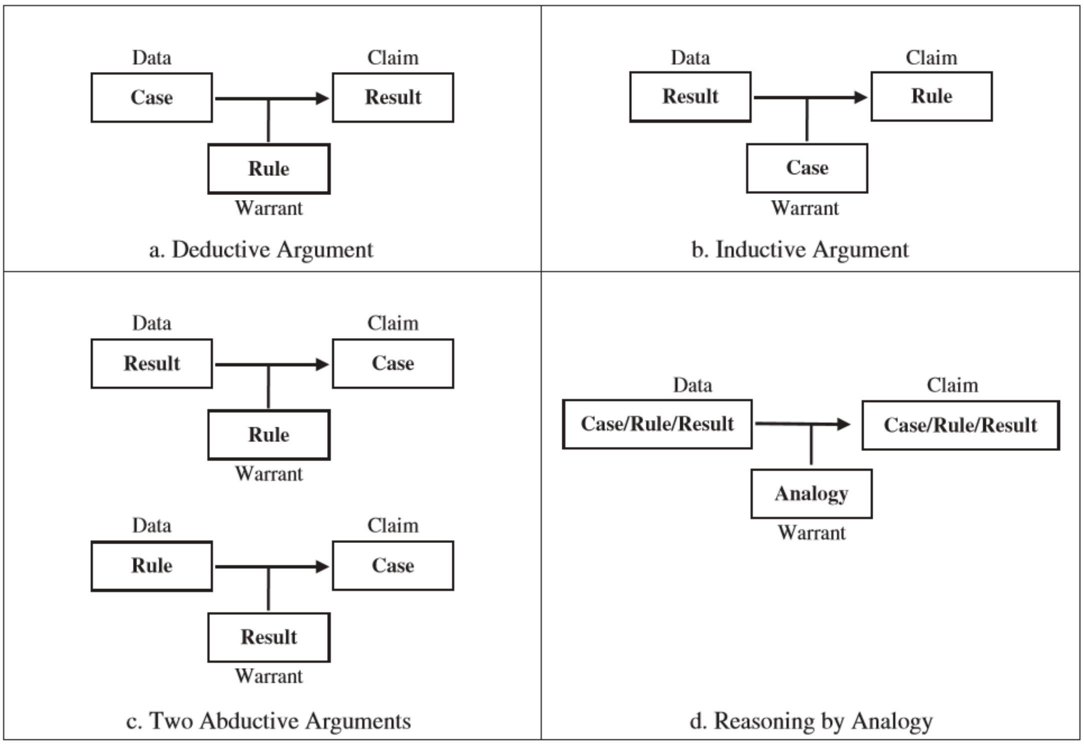


Figure 2.3 Toulmin-style diagrams of arguments reflecting different kinds of reasonings (Conner et al., 2014b, p. 186)

At the end of the study, they asserted that analyzing the core of the argument was sufficient in determining the different types of reasonings of the participants (Conner et al., 2014b). Moreover, they recommended teachers to use this adaptation in detecting and supporting students' different kinds of reasonings (deductive, inductive, abductive, analogical reasoning).

Up to now, the studies focusing on Toulmin's argumentation model and its components were explained in detail. In the next section, the proof-related argumentation studies were addressed.

2.2.5 Studies related to proof and argumentation

The proof studies in the literature focused on investigating the proof schemes of students (Harel & Sowder, 1998; Housman & Porter, 2003), examining student difficulties in providing proof (Chazan, 1993; Moore, 1994; Weber, 2001), criticizing the dichotomy/continuum between argumentation and proof (Balacheff, 1991; Boero, 2007; Douek, 1999; Garuti, Boero, Lemut, & Mariotti, 1996; Mariotti, Bartolini Bussi, Boero, Ferri, & Garuti, 1997; Mariotti, 2006; Pedemonte, 2007; Raman, 2002), seeking for the relationship between technology and learning proof in geometry (Laborde, 2000; Leung & Lopez-Real, 2002; Mariotti, 2006), examining proof perceptions (Recio & Godino, 2001) and analyzing the validation of texts as proofs (Selden & Selden, 2003; Stylianides, Stylianides, & Philippou, 2004).

Initially, the proof schemas framework was propounded by Harel and Sowder (1998). In their study, the participants developed three types of arguments, which were *external arguments* (e.g. an authority figure, meaningless symbolic manipulation), *empirical arguments* (e.g. numerical substitution, measurements, visio-spatial images), and *deductive arguments* (e.g. generic examples, examples depending on the axiomatic system). It was asserted that these three types of arguments may be valid in different mathematical content areas since students developed the same type of arguments in different proof tasks. Researchers investigated which types of arguments were used by participants of various grade levels (Healy & Hoyles, 1998; Klieme, Reiss & Heinze, 2003; Lin, 2000; Reiss, Heinze, Renkl & Groβ, 2008; Reiss, Hellmich & Reiss, 2002). For instance, Healy and Hoyles (1998) administered a survey to 10th grade high-achieving students and identified difficulties that many of students exprienced in implicating, so they used empirical-inductive arguments. In the same way, the findings of the studies conducted by Reiss, Hellmich and Reiss (2002) and Klieme, Reiss and Heinze (2003) illustrated that middle school (grade 7 and 8) and high school (grade 12 and 13) students, respectively, prefered to seek empirical evidences, such as analyzing one or two examples, measuring angles and lines in geometry, in proof production. Reiss, Heinze, Renkl and Groβ (2008) asserted that the students of western countries might have the deficit of using empirical arguments like generalizing from a few examples as a proof. On the other hand, students of Asian countries were known to be encouraged to use deductive arguments from the beginning of the proving task (Lin, 2000). Reiss et al. (2008) argued that the reason why Asian students use deductive arguments might be their teachers' viewpoint regarding the distinction of argumentation and proof. It was claimed that teachers' own beliefs also affect their teaching performance in many aspects (Conner, 2007b). A proof study supporting this idea was conducted with high school geometry teachers by Conner (2007b). Specifically, Conner (2007b) investigated the argumentation produced in a high school geometry class and provided evidence of the relationship between a student teacher's proof perception and her facilitation in collective classroom argumentation. She concluded that a student teacher's proof perception and her argumentation support in classroom had parallel aspects. This suggests that the proof conception of the teacher had the potential to be a crucial factor for him/her in determining how to facilitate the argumentation process in the classroom. Moreover, she claimed that Toulmin's (1958) model was efficient in analyzing teacher support in argumentation and constituencies within the classroom, which supported argumentation in detail. Another study which investigated teachers' refutations and their preference in refuting in argumentation was conducted by Giannakuolias, Mastorides, Potari and Zachariades (2010). In this argumentation and proof study, Giannakuolias et al. (2010) focused on teachers' refutations of students' invalid algebraic claims. To be more specific, they analyzed the content of teachers' argumentation, teachers' argumentation structure, the underlying reasoning, and the different types of counterexamples they generated. They concluded that teachers considered refutation by theorems to be providing stronger and more general conclusions when compared to refuting by counterexamples. It was inferred that refutation of invalid claims with counterexamples was undervalued by teachers and they used counterexamples when they could not use an appropriate theorem or as a complementary support to the refutation by theorems. It is highly probable that these teachers would appreciate the refutations their students made based on an appropriate theorem, but would undervalue refutations via counterexamples. In the current study, the prospective teachers were selected as participants since their performance in argumentation and their argumentation structures would include indications of their future teaching practices based on argumentation. In addition, Toulmin's model was selected to be used in data analysis since it was claimed that the model was effective in the analysis of teacher support in argumentation.

Another issue which was most frequently dwell upon in proof studies was its relation with argumentation. That is, the dichotomy (Balacheff, 1991; Douek, 1999; Mariotti, 2006; Pedemonte, 2007) and the continuum between argumentation and proof (Boero, 2007; Douek, 1999; Garuti, Boero, Lemut, & Mariotti, 1996; Mariotti et al., 1997; Raman, 2002) was discussed by many researchers in the literature. For instance, Balacheff (1991) maintained that argumentation and mathematical proof were different in terms of social point of view by stating that:

> The aim of argumentation is to obtain the agreement of the partner of the interaction, but not in the first place to establish the truth of some statement. As a social behavior it is an open process, in other words it allows the use of any kind of means; whereas, for mathematical proofs, we have to fit the requirement for the use of knowledge taken from a body of knowledge on which people (mathematician) agree (p. 188).

This expression asserts that the main focus of proof is establishing the truth of some statement, while in argumentation it is to convince others, so these terms were regarded to be different in this respect. Balacheff (1991) also expressed the difference between argumentation and proof in terms of the actions specific to argumentation, such as intuition, experimental methods and everyday practices external to a mathematical theory. Duval was the other researcher who emphasized the gap between argumentation and proof in terms of cognitive and logical point of view (as cited in Barrier, Mathe, & Durand-Guerrier, 2010). He asserted that the discursive process of argumentation acted against a valid reasoning process in ordinary language, so it could result in misunderstandings and obstacles as regards the meaning of proof (as cited in Barrier, Mathe, & Durand-Guerrier, 2010). In addition, Duval explained the distinction between deductive reasoning (used in proof) and argumentation as follows:

> Deductive reasoning holds two characteristics which oppose it to argumentation. First, it is based on the operational value of statements and not on their epistemic value (the belief which may be attached to them). Second, the development of a deductive reasoning relies on the possibility

> of chaning the elementary deductive steps, whereas argumentation relies on the reinterpretation or the accumulation of arguments from different points of view (Duval, 1991 as cited in Barrier, Mathe, & Durand-Guerrier, 2010, p. 193).

As can be understood, unlike in argumentation, deductive reasoning, which is the base of proof, is operation-based and relies on the possibility of chaining the elementary deductive steps, whereas in argumentation, arguments are reinterpreted from various perspectives. Although Duval's arguments were strong, the distinction between argumentation and proof was still debated since some of the researchers asserted that the structural continuity between argumentation and proof could be constructed. To be more precise, when the processes of argumentation and proof generation were investigated, some researchers highlighted the continuity between argumentation and proof by proposing the framework of *Cognitive Unity* (Boero, 2007; Garuti, Boero, Lemut, & Mariotti, 1996; Mariotti, et al., 1997; Raman, 2002). These researchers focused on argument production in problem solving, experimentation and exploration context and expected the constructed arguments to be organized logically in the formation of mathematical proof (Hanna & de Villiers, 2008). In addition, these studies postulated that in open-ended problems there may or may not be a continuity between argumentation and the related mathematical proof. It was asserted that the crucial thing was identification of the factors favoring continuities and the factors leading to the gap between argumentation and proof (Antonini & Martignone, 2011). Boero et al. (1996) claimed that "it [reasoning] allows students to consciously explore different alternatives, to progressively specify the statement [of the conjecture] and to justify, the plausibility of the produced conjecture" (p. 118). Based on this idea, Boero, Douek, Morselli and Pedemonte (2010) asserted that structural continuity could be satisfied if inferences in argumentation and proof were connected through the same reasoning structure (abduction, induction or deduction) since he believed that reasoning taking place in argumentation had a crucial role in the proof which was produced at the end. Boero et al. (1996) introduced the notion of cognitive unity as follows:

> During the production of the conjecture, the student progressively works out his/her statement through an intense argumentative activity functionally intermingled with the justification of the plausibility of his/her choices: during the subsequent proving stage, student links up with his process in a coherent way, organizing some of the justifications (arguments) produced during the construction of the statement according to a logical chain (p. 113).

Based on the notion of Boero et al. (1996), it can be deduced that when the reasoning in the initial argumentative process and the reasoning in the constructed proof at the end are in line with each other, the cognitive unity between argumentation and proof is obtained. Similarly, cognitive unity was also expressed as the the continuity of reasoning from conjecture producing process to proof construction (Mariotti et al. 1997). In other words, when there is continuity between the argumentative activity occuring in the conjecturing phase and the process of formal justification occuring during the proving phase, it can be said that the cognitive unity is established (Baccaglini & Frank, 2010). According to results reported in experimental studies, proof was more accessible to students when cognitive unity between argumentation and proof was established (Boero, Garuti, & Mariotti, 1996; Garuti, Boero, Lemut, & Mariotti, 1996; Garuti, Boero, & Lemut, 1998).

The distance between argumentation and proof was stated to be inevitable in the deductive proof production process since the structure of argumentation was usually not deductive (Pedemonte, 2007; Pedemonte & Buchbinder, 2011). Pedemonte and Buchbinder (2011) conducted a study regarding argumentation and proof and investigated the role of examples in the proving process. They conducted a study with 17-18 year old secondary school students who were trying to find a general rule for triangular numbers. It was claimed that cognitive unity did not cover all aspects of the relationship between argumentation and proof (Pedemonte & Buchbinder, 2011). They proposed the necessity of *structural continuity*, which is considering the structural difference between argumentation and proof as well as providing *cognitive unity* for the cognitive analysis. Pedemonte (2007) defined structure as a logical cognitive connection between statements which correspond to different types of reasoning. That is, the structure of proof was usually deductive while *abduction*, which means looking at facts and looking for a theory to explain reasoning (Pierce, 1960), and *induction* were asserted to be the most frequently used reasoning types in argumentation (Hemmi et al., 2013; Pedemonte, 2007; Pedemonte & Buchbinder, 2011). Therefore, it was asserted that argumentation and proof should have the same logical structure in order to provide structural continuity (Pedemonte, 2007). To make it clear, the results of the Pedemonte's (2007) study could be referred to. She focused on the structure of arguments developed by students for two mathematical problems to analyze the quality of arguments. She concluded that there were structural continuities and structural distances between argumentation which supported a conjecture and its proof. Moreover, it was observed in her study that students were not able to construct a deductive proof while solving geometry problems since they could not transform their abductive argumentation (naturally used in geometry) into deductive proofs (Pedemonte, 2007; Martinez & Pedemonte, 2014). Thus, it was suggested that researchers or teachers should consider cognitive unity and select appropriate examples which would be effective for proof construction (Pedemonte & Buchbinder, 2011). On the other hand, Pedemonte (2007) believed that in the Geometry context, the distance between argumentation and proof was removed because discovering and conjecturing processes were often characterized by abductive argumentation. Likewise, Arzarello (2008) maintained that by means of various modalities, such as the "dragging" option of dynamic geometry programs, the shift from inquiry to proving could be stimulated within a rich argumentation. In addition, researchers revealed that dynamic geometry tools were helpful in engaging students in informal mathematics actions and bridging these informal efforts with formal ones (Prusak et al., 2012). As noted by Mariotti (2006), the students should change the abductive structure into deductive structure while constructing proof. Otherwise, the structural continuity between argumentation and proof could be broken.

Although the relationship between argumentation and proof was not the focus of the current study, it was an important issue to be considered in preparing the geometry tasks of the present study. In order to provide cognitive unity, the researcher paid special attention to select questions which did not only require deductive reasoning with the algebraic operations. In addition, geometry was selected as a context since it was claimed that the discovering and conjecturing processes requiring abductive reasoning would be helpful in removing the structural difference between argumentation and proof (Pedemonte, 2007). Lastly, argumentation in the dynamic geometry environment was also prepared in order to engage students in an inquiry which would help enrich argumentation (Arzarello, 2008).

2.3 Technology support and argumentation

Literature review illustrated that dynamic geometry softwares (DGS) were advised to be used in geometry teaching by many mathematics researchers and organizations (Hollebrands, 2007; Laborde, Kynigos, Hollebrands, & Sträβer, 2006; NCTM, 2000). One of the reasons was that these programs gave the opportunity to construct accurate diagrams for students to recognize the relationships between general abstract properties of geometry (Jones, 2002). In addition, dynamic geometry softwares allow users to do a wide range of geometrical activity to solve various types of questions by exploring, conjecturing and explaining the geometric relationships (Jones, 2002).

2.3.1 Technology use and its benefits

Many researchers argued that use of DGS promoted achievement in geometry (Hollebrands, 2007; Laborde, Kynigos, Hollebrands, & Sträβer, 2006; NCTM, 2000). For instance, Laborde et al. (2006) found that the use of DGS increased secondary students' understanding of geometry concepts. Similarly, in the research study conducted with high school students, Hollebrands (2007) revealed a bidirectional relationship between use of DGS and geometric understanding.

In the literature, the benefits of using DGS were emphasized by many researchers (González & Herbst, 2009; Lampert, 1993; Ruthven, Hennessy, & Deaney, 2005; Scher, 1999; Vincent, 2002). One of the benefits was that the use of

DGS was regarded by teachers as a source of motivation for students and a tool that could illustrated many examples simultaneously, thus leading to improvements in the classroom environment (Lampert, 1993; Ruthven, Hennessy, & Deaney, 2005) and encouragement in geometric reasoning (Vincent, 2002). Another benefit mentioned was that the DGS users could interact with various geometrical objects and relationships which enabled them to construct and manipulate new objects and relations (Healy & Hoyles, 2000). In addition to these benefits, the studies focusing on justification and reasoning also revealed the benefits of the use of DGS in geometry. To illustrate, it was revealed that students could make conjectures, prove properties for a given geometric figure and model, and investigate a dynamic physical situation to detect the effect of changing various parameters (Vincent, 2002). Likewise, González and Herbst (2009) indicated that students who used the tools of DGS had the opportunity to engage in in-depth thinking in their investigation; in this way, they were able to notice the relationships that could not be discovered by the students using the paper and pencil. In the same vein, Scher (1999) asserted that proving a theorem or solving a problem via paper-pencil prevented students from exploring new relationships since they were dealing with static drawings, so they recommended the use of DGS for the visualization of the relationships. As it is clear in the literature, the justification is a keystone in argumentation studies. Regarding justification via DGS in proof, Mariotti (1997) asserted that users' justifications for their constructions could be accepted as proving a theorem since they were explaining why it worked and forsaw that it would function. Thus, the applicability of DGS in argumentation studies was open to research since there were few studies focusing on justification using technology.

Another important benefit of using DGS in argumentation was asserted to be promoting peer interaction, which was important for argumentation studies (Vincent, Chick & McCrae, 2005). Vincent, Chick and McCrae (2005) stressed the importance of the level of peer interaction in argumentation in their study which was conducted with 8th grade students in Australia. They administered three types of conjecturing/proving tasks: pencil-and-paper proofs, computer-based (using Cabri Geometry II) tasks and geometry tasks including the investigation of appropriate mechanical linkages. They concluded that argumentation was a social process and the quality of the benefit obtained from this social interaction was affected by the level of peer interaction in all tasks (Vincent, Chick, & McCrae, 2005). Likewise, Hewit (2010) emphasized the importance of social interaction and time for productive argumentation in computer environment. Hewit (2010) asserted that learners who lacked domain knowledge would be unable to construct convincing warrants for their claims or analyze other participants' arguments. Thus, Hewit (2010) suggested giving sufficient lead-time to students before class discussion in order for students to develop a reasonable and deep understanding of the domain to interpret the validity of the other positions.

The last benefit of DGS in argumentation was the *dragging* option, which is one of the most important options. Dynamic geometry softwares are used in research studies since they provide users with several opportunities that cannot be performed without technology. Dragging is performed by grabbing the elements of a geometrical figure via the computer mouse, changing the place of those elements on the screen and observing the responses of various other parts of the figure dynamically. This action enables users to see the preserved properties of geometrical objects. Moreover, users can observe infinite examples to support their claim and have the opportunity to detect the counterexamples to a statement (Hanna & de Villiers, 2008). Researchers see dragging as a beneficial option in terms of many aspects. For instance, Lopez-Real and Leung (2007) believe that dragging facilitates the theoretical concept formation (Hanna & de Villiers, 2008) and should not be accepted solely as a confirmation or exploration tool. Similarly, Arzarello, Olivero, Paola and Robutti (2002) referred to the contribution of the dragging option to the proof and conjecturing process since dragging provided feedback to the discovering phase and thus supported proof explanations. Therefore, checking the construction by dragging was advised by researchers in order to observe the necessity of relevant geometric facts (Hoyles & Jones, 1998).

As previously mentioned, evidence has a crucial role in argumentation studies. In many studies, dynamic figures were considered to provide students with strong evidence that a property was true since *dragging* seemed sufficient to guarantee the truth of the observed property (Arzarello et al., 2002; Chazan, 1993; De Villiers, 2003; Healy & Hoyles, 2000). On the other hand, Heinze and Reiss (2007) stated that empirical arguments, which include concrete geometrical objects based on the observation, could be accepted as validation in the geometry context although experimentally generated results seemed not to offer explanations (justifications) for the observed relations. Likewise, González and Herbst (2009) criticized using measurements (empirical evidence) calculated by dynamic geometry software to provide evidence for the generalizations. Noss and Hoyles (1996) claimed that the students who used dynamic geometry software were prone to attribute their results to measurements instead of theoretical considerations. Thus, Chazan and Houde (1989) advised teachers to avoid using dynamic geometry software measurements as the basis of statements in a geometry class. Moreover, teachers were advised to construct and sequence the tasks carefully while using technology in order to motivate students in creating formal proof (Hoyles & Healy, 1999). Another suggestion to teachers who used dynamic geometry software in their mathematics classes was motivating students to find out *why* a conjecture is true (Arzarello et al., 2002). It was asserted that when the teacher made the role of proof in justifying explicit, the students would be motivated to prove *why* a certain proposition is true after seeing that it is true within the dynamic geometry environment (Arzarello et al., 2002). As an important factor in many research studies, technology was also a matter of discussion for some argumentation studies. These studies are explained in the following section.

2.3.2 Studies related to argumentation via technology

The argumentation application in a dynamic geometry environment was investigated in several studies (Hewit, 2010; Hollebrands, Conner & Smith, 2010; Inglis, Mejia-Ramos & Simpson, 2007; Prusak et al., 2012). For instance, Hollebrands, Conner and Smith (2010) used Toulmin's argumentation model in their study with college geometry students with access to technology. They conducted a task-based interview with students while they were solving hyperbolic geometry tasks. In the end, they revealed the themes regarding the structure of students' arguments. These themes were related to the *explicitness of warrants*, *technology use*, and *task types*. According to the results, they claimed that students who were solving tasks related to justification and proof did not use technology but provided explicit warrants. On the other hand, they found that students who did not use explicit warrants had used technology. They claimed that this indirect relation was due to the students' lack of familiarity with the use of technology in a formal mathematical environment. To be more precise, the students did not need to explain how the objects on the screen led to the claims and accepted the appearance of the relations on the screen as a sufficient warrant (Hollebrands, Conner, & Smith, 2010). Similar to the results reported in the study conducted by Inglis et al. (2007), Hollebrands et al. (2010) also emphasized the importance of the use of qualifiers in the use of technology-aided environment. They stated that the students used the technology when they were uncertain about the claim. According to the findings, when technology use confirmed the claim, they accepted that as evidence; otherwise, they changed their claims (Hollebrands, Conner, & Smith, 2010).

Another study focusing on peer interaction of argumentation with DGS was conducted by Prusak et al. (2012). They used Toulmin's argumentation model and examined peer interaction of two preservice teachers by creating a conflict situation, creating a collaborative situation and providing a device (DGS) for checking hypotheses/conjectures. They also aimed to investigate the reasoning processes in peer-unguided argumentation. The core arguments -arguments including claim, data, warrant and backing elements- of Toulmin's argumentation model were analyzed. The design enabling the shift from intuitive/visual argumentation to logical-deductive considerations was presented in the findings. The researchers claimed that the three design principles, which were creating a situation of conflict, a situation of collaboration and providing tools to raise and check hypotheses, promoted productive argumentation. In addition, the adaptation of Toulmin's argumentation model was claimed to be useful in tracing the dynamic changes in collective argumentation through dyad interaction. In the end, they elaborated on the methodology to identify learning in peer argumentation.

There are also various studies in the literature supporting the use of DGS and in which the proof and reasoning of the participants were analyzed (Baccaglini-Frank & Mariotti, 2009; Hoyles & Healy, 1999; Olivero & Robutti, 2001; Mariotti, 2006; Vincent, 2002). In some of these studies, the researchers claimed that dynamic geometry environments were beneficial in opening new frontiers which link informal argumentation with formal proofs (Hoyles & Healy, 1999; Olivero & Robutti, 2001; Mariotti, 2006; Vincent, 2002). For instance, Vincent (2002) investigated the role of mechanical linkages which correspond to the devices based on systems of hinged rods and dynamic geometry software in bridging empirical justification and deductive reasoning. She used Toulmin's argumentation model to analyze the structure of geometry proofs developed by 12-14-year-old students. Vincent (2002) used mechanical linkages and dynamic geometry software to provide the context of conjecturing, argumentation and deductive reasoning. At the end of the study, she concluded that all students, including the ones who had a lower level of geometric understanding, developed an understanding of deductive proofs and made significant progress in understanding geometric properties. Moreover, the mechanical linkages and dynamic geometry software were found to be highly suitable to bridge empirical and deductive reasoning. Similarly, Christou, Mousoulides, Pittalis and Pitta-Pantazi (2004) also encouraged the use of dynamic geometry software and appropriate questions, which motivate students to justify their conjectures in geometric proofs in order to bridge inductive exploration and deductive proof.

The Cabri dynamic geometry program was used in another research study to investigate the conjecturing and proving processes of students working on open problems in Euclidean geometry (Baccaglini-Frank & Mariotti, 2009). The researchers analyzed the process rather than the product, and they conceived a model related to the dragging schemes and the processes occuring while students were conjecturing and proving. It was claimed that students were be able to make dynamic conjectures in a Dynamic Geometry Environment [DGE] rather than the static-conjectures developed in paper-pencil environment (Baccaglini-Frank & Mariotti, 2009). Likewise, Mariotti (2006) asserted that DGE contributed to the reasoning and proving processes of individuals who were solving open problems by making conjectures. Similarly, some cognitive difficulties that students confronted during conjecturing and proving in geometry were stated to be overcome with the encouragement of DGE on learners' constructions and ways of thinking (Noss & Hoyles, 1996; De Villiers, 2004).

In another study, Leung and Lopez-Real (2002) analyzed the proofs of secondary school students in the Cabri environment in Hong Kong. They focused on how the students' constructions motivated their visual-cognitive scheme on seeing proof in a dynamic geometry environment and how this scheme could fit into cognitive unity, which was proposed by Boero, Garuti and Mariotti (1996). Moreover, Leung and Lopez-Real (2002) proposed a framework related to the theorem acquisition and justification in DGE as follows:

> Theorem acquisition and justification in DGE is a schematic cognitive-visual dual process potent with structured conjecture forming activities, in which dynamic visual explorations through different dragging modalities are applied on geometric entities. These activities stimulate argumentative/transformational reasoning, which enables the process to converge towards integrated figural concepts that could bring about formal mathematical proofs, hence producing a cognitive unity in acquiring and proving geometrical theorems (p. 9).

The framework above emphasized the importance of the knowledge producing process instead of the rational proof produced at the end of the process. That is, the process was seen more important than the product. In addition, it was deduced that technology use contributed to obtaining the cognitive unity between argumentation and proof by stimulating argumentative reasoning with the help of integrated figural concepts.

On the other hand, there were also a few studies in which ynamic Geometry Software [DGS] was not beneficial in proof (Hoyles & Healy, 1999). For instance, Hoyles and Healy (1999) conducted a study regarding the use of the Cabri dynamic geometry software in proof production. Specifically, they analyzed the relationship between students' visual reasoning in Cabri and their motivation in using empirical conjectures in formal proof. They concluded that the students' perceptions related to the Cabri construction differed from the Euclidean proof they constructed. That is, Cabri was asserted to be beneficial in defining and identifying geometrical properties but not beneficial in proving them (Hoyles & Healy, 1999).

In the literature, researchers suggested planning the instructions and activities of studies on DGS and argumentation carefully. The reason for this was that students may not always follow instructions and may experiment and notice unexpected relations which was called *play paradox* (Healy & Hoyles, 2000). In addition, it was asserted that DGS itself did not guarantee the participants to transit from empirical to generic objects, so the importance of teacher role in guiding students to theoretical thinking was emphasized (Jones, 2002; Leikin & Grossman, 2013). Likewise, Hölzl (2001) emphasized the importance of the way DGS is used and suggesting not using DGS only for verification.

To sum up, in this section, the opportunities that DGS provided the researchers with, such as solving various types of questions by exploring, conjecturing and explaining the geometric relationships have been presented. In addition, DGS was claimed to enable peer interaction in mathematics studies, so DGS is considered to be suitable and beneficial for argumentation studies. In short, literature indicated that the crucial role that technology plays in the teaching and learning of mathematics by means of argumentation was approved by the researchers, theorists, educators.

2.4 Teacher responsibilities in argumentation process

Argumentation is considered to be a social activity by some scholars as can be understood in the following definition of argumentation, which emphasizes the characteristics and the social and rational aspects of argumentation.

> Argumentation is a verbal activity that can be performed orally as well as in writing. It is also a social activity: In advancing argumentation, one directs oneself by definition to others. In addition, argumentation is a rational activity that is aimed at defending a standpoint in such a way that it becomes acceptable to a critic who takes a reasonable attitude (van Eemeren, Grootendorst, & Henkemans, 2002, p. xi).

In the social environments, there could be some shared and accepted information during the interactions. In particular, it was claimed that mathematical justification includes the *taken-as-shared* knowledge of the participants (Simon & Blume, 1996). For a result or statement to be taken-as-shared knowledge, all participants in a class should accept the truth of that result with prior justifications. At this point, the participants accept the result without needing further justification in subsequent discussions (Simon & Blume, 1996; Yackel, 2002). In their study, Yackel, Ramussen and King (2000) emphasized the importance of taken-as-shared knowledge in the argumentation process, and argued that the argumentation process was reflexively related to the taken-as-shared basis for communication. Thus, taken-as-shared knowledge could be considered as an important issue in analyzing discussions of argumentation studies.

Yackel and Cobb (1996) are prominent researchers who emphasized the close relationship between reasoning/making sense and interactive constitution of taken-as-shared mathematical meanings. They believe that learning is a social process and that meaning making is formed in and through the process of interpreting and interacting with others. Many researchers maintain that the more students are encouraged to join in argumentation and justification, the higher the quality of their reasoning, justification and explanation will be (Manoucheri & St John, 2006; McCrone, 2005; Wood, Williams & McNeal, 2006). Although children have the potential to develop persuasive and defensible arguments, their arguments rely on untested presumptions of shared knowledge and may not have sufficient evidence (Anderson, Chinn, Chang, Waggoner, & Yi, 1997); thus, they still need teacher assistance and facilitation to be able to integrate many sources of information (Strom, Kemeny, Lehrer, & Forman, 2001) and to be effectively engaged in mathematical argumentation (Cobb, Stephan, McClain, & Gravemeijer, 2001; Conner, 2007a; Forman et al. 1998; Hunter, 2007; Yackel & Cobb, 1996). Research studies emphasized the importance of teacher role in establishing the mathematical quality of the class and the norms for mathematical aspects of student actions (Heinze & Reiss, 2007; Yackel & Cobb, 1996). For a productive argumentation, the teacher should have some necessary skills to orchestrate the argumentation and to provide suitable environment for argumentative environment.

Suggestions for suitable teaching environment for argumentation were provided in the literature. For instance, the teaching environment for argumentation should be arranged in such a way that students will not hesitate and will not feel any risk and pressure while talking about their ideas and answers (Lin & Mintzes, 2010). In this way, students will be aware of their own ideas and listen to the ideas of others more attentively (Lin & Mintzes, 2010). In order to provide such an environment and to engage more students in the mathematical discourse, teachers can encourage students to exchange their thoughts with their classmates and prompt students' answers (Hunter, 2007; Kosko et al., 2014) instead of being satisfied with their correct answers to the questions and judging the students' responses directly. In this way, teachers will be encouraging students to justify and support their answers (Wood & McNeal, 2003).

In argumentation, which is an inquiry based teaching method, the supportive role of the teacher is of great importance. For supporting the whole class inquiry in secondary school, Staples (2007) clarified teacher actions as "guiding the mathematics, establishing and monitoring a common ground, and supporting students in making contributions" (p. 172). Likewise, Yackel and Cobb (1996) clarified teacher roles in inquiry classrooms as facilitating mathematical discussions, acting as a participant to legitimizing certain aspects of students' activities, and making sense of children's wide range of solutions. Yackel (2002) also maintained that the teacher should initiate collective argumentation, support interaction among students and raise awareness regarding omitted or implicitly stated argument elements in arguments. By the same token, McClain and Cobb (2001) emphasize that the teacher's role should be to facilitate not to transmit knowledge to the students. How can teachers facilitate discussions? According to Cross (2009), teachers should ensure that students follow the route and make sense of the questions. Moreover, teachers should follow each group by listening to their conversations to ensure that all students participate (Cross, 2009). The other important roles of the teacher are to encourage students to provide justifications to their statements and to pose questions to the students in order to engage them in in-depth thinking (Cross, 2009). In addition, the teacher could provide students with clues and suggestions when they get stuck at some point (Cross, 2009). Finally, the facilitator teacher should not use evaluative statements to respond to students since statements, such as ‘That is correct/right answer’ are believed to impede student discourse (Cross, Taasoobshirazi, Hendricks, & Hickey, 2008). Another obstacle to be confronted when the teacher uses evaluative statements is students’ reluctance to talk since the student would feel fear of being judged regarding his/her thoughts in the exploratory stage (Mercer, 2000). When the teacher responds to students’ arguments without indicating her/his position, the teacher would direct all the participants by thinking carefully, and in this way, students would have the chance to examine the presented idea and develop mathematical backing to agree/disagree the conjecture (Hunter, 2014).

One of the beneficial methods for promoting productive argumentation has been asserted as teachers’ encouragement of students to convince other participants. Specifically, after a student provides justification for his/her claim, the teacher should ask that student to convince other students about the claim in the argumentation (Martino & Maher, 1999). Sample questions of promoting justification are as follows: “How did you reach that conclusion?, Could you explain to me what you did? and Can you convince the rest of us that your method works?” (Martino & Maher, 1999, p. 57). In addition, the questions helping students to focus on other students’ ideas have been listed as “Did anyone have the same answer but a different approach?, Is there anything about your solution that’s the same as your classmate’s? and “Can you explain what your classmate has done?” (Martino & Maher, 1999, p. 57). Finally, “Have you ever worked on a problem like this one before?” (Martino & Maher, 1999, p. 57) is a question that leads to making generalizations and mathematical connections. Ultimately, the teachers who want to apply argumentation in their classes should make effort to use such questions in order to provide interactive argumentation environment for better conceptual learning.

Research related to argumentation illustrates that another skill that a teacher should have to engage students in mathematical argumentation is *questioning* (Kosko et al., 2014). Questioning is not only used for assessing student knowledge but also for challenging students' conceptual frames and obtaining knowledge related to students' thinking processes and development of mathematical ideas (Martino & Maher, 1999). However, the quality of the questions is of great significance in this respect. Martino and Maher (1999) stress that the main benefit of skillfully questioning students is that the teacher is provided with the necessary knowledge related to students' mathematical concept development. However, Kosko et al. (2014) state that every question does not have the potential to encourage justification and student engagement in mathematical argumentation. For instance, in a study by Temple and Doerr (2012), it is reported that 10th grade classrooms were observed and it was found that in some lessons, the teacher encouraged students to provide an explanation of their thinking process and add to each others' contributions, while in some other lessons the questioning of the teacher was more concerned with accuracy, rather than exploration, and, thus, encouraged students to provide precise explanations. Temple and Doerr (2012) assert that leading questions can be used in stimulating conversations about previously learned content but not effective in encouraging justification in mathematical argumentation. On the other hand, probing questions enable students to make connections between various ways of representing the mathematics and establishes a more productive environment for argumentation since these questions often include requests for justification (Temple & Doerr, 2012). Similarly, Boero (1999) emphasizes the need for strong teacher mediation in Toulmin-Type argumentation and name these interventions as 'warrant-prompts'. A sample question given for warrant-prompts is "Why do you say that?". This question directs students to think about their claims/conjectures (Vincent, 2002). Similarly, Wood (2003) mentions prompting questions as "How are the two things the same? Does this make sense? Does it always work? Why does this happen?" (p. 440). By the same token, Owens (2005) states that in order to promote student justification, teachers could ask the following questions: "Would you tell us what you thought?

How did you decide this? Are there patterns? Is there a different way you can do this?" (p. 34). The argumentative talk prompts, such as 'Can you explain X?, Articulate X with your own words, What do you think about the issue?, Could you add anything else about X?', were defined as *ground rules* by Mercer (2000), and the researcher claims that these prompts foster interaction among students and enable students to inter-think (Mercer, 2000). Ultimately, it can be concluded that the quality of questions is of great importance for students to construct arguments, so the teachers' facilitation and scaffolding are needed for effective mathematical argumentation (Kosko et al., 2014).

Another essential method which should be used by teachers to make students engaged in collective argumentation is stated as *revoicing* (Chapin, O'Connor & Anderson, 2003; O'Connor & Michaels, 1996). The goals of revoicing are stated as clarifying/amplifying the content, explaining the reasoning further, intoducing particular ideas or redirecting the discussion (Forman et al., 1998). The main benefit of revoicing is clarifying and making a participant's contribution to the discussion more comprehensible to all the other participants in class (Conner et al., 2014a). Moreover, revoicing is claimed to make students take a specific stance in a dialogue and, thus, develop inquiry skills and mathematical argumentation while defending their ideas (O'Connor & Michaels, 1996). Forman et al. (1998) claim that reported speech could be used to align students with argumentative positions. Moreover, teachers' repetitions in pointing out important aspects of the arguments are also believed to be beneficial in orchestrating argumentation (Forman, et al., 1998). That is, Forman et al. (1998) postulate that the mentioned roles of the teacher are necessary and highly important since the teacher's framing is the main factor affecting the convincingness of students' arguments.

To sum up, the role of the teacher is of vital importance in the process of fostering productive argumentation; to this end, they should be the facilitator, not the knowledge transmitter in the class. As for methods to orchestrate argumentation in mathematics, the key methods can be listed as questioning, revoicing and encouraging students to convince other students. Prusak et al. (2012) have stated that there is a need for teacher training in the area of fostering collective argumentation since there is no training in teacher education programs related to orchestrating argumentation. Specifically, Prusak et al. (2012) have advised that teachers should first experience argumentation as learners themselves in order to facilitate argumentation in class effectively. Thus, the focus of the current study is on the prospective middle school mathematics teachers since they will facilitate argumentation in their mathematics classes in the future. In the next section, some research studies related to argumentation in mathematics education are explained in detail.

2.5 Summary of the literature

To sum up, theoretical background and Toulmin's argumentation model was introduced and the layout prepared by Toulmin was explained in detail in this chapter. Subsequently, argumentation studies in mathematics education research were listed in detail to provide a rationale for the current study. Afterwards, the use of technology and technology supported argumentation studies were explained. Lastly, the teacher responsibilities in argumentation process were explained.

Literature review illustrated that, Toulmin's argumentation model was used to examine argument structures, proof structures, reasoning types, argument types and warrant types in numerous studies. The focus of the proof studies were on proof schemas of students, student difficulties in proving, criticism of the relationship between argumentation and proof, proof perceptions, validation of texts as proofs and technology use in proof. On the other hand, the technology related studies mostly focused on the benefits of using technology in terms of various aspects. One of the most important benefits of technology was asserted as promoting the peer interaction which was critical in argumentation studies (Vincent, Chick & McCrae, 2005). In addition to these, the primary factor in argumentation, teacher, and the responsibilities of the teachers in argumentation classes were investigated in many studies. However, as clarified in significance and literature parts, there were few studies focusing on argumentation structures of prospective teachers who will be the future teachers facilitating argumentation classes. Especially, in Turkey, there is no such and investigation on prospective middle school mathematics teachers since argumentation is a quite new method in mathematics field. Thus, the current study focused on the investigation of prospective middle school mathematics teachers' argumentation structures in a technology integrated environment and paper-pencil environment. It is believed that this attempt will give valuable insights to both policy makers and mathematics educators to improve middle school mathematics teacher education programs by integrating argumentation into mathematics field.

CHAPTER III

METHODOLOGY

The purpose of this study was to investigate the nature of argumentation structures of prospective middle school mathematics teachers while solving geometry tasks in technology and paper-pencil environments. More specifically, the argumentation structures of the prospective middle school mathematics teachers, local arguments (core arguments including claim, data and warrant components) in the global argumentation structures, and the local argumentations (only warrant components of the local arguments) were analyzed in detail. More specifically, the kinds of global argumentation structures they produced, the kinds of local arguments they used, and the kinds of local argumentations they utilized to justify their arguments were investigated.

In this chapter, the research questions, design of the study, procedure, pilot study, main study, trustworthiness of the study, researcher role and bias, and limitations are described. In short, the method of inquiry is depicted in detail.

3.1 Research questions

The following major questions and sub-questions were formulated for this qualitative case study:

1. What is the nature of argumentation structures of prospective middle school mathematics teachers while solving geometry tasks in GeoGebra and Paper-Pencil groups?

2. What are the characteristics of the local arguments in the global argumentation structures?

- What are the characteristics of local arguments based on the flow of argument components (claim, data, warrant) that prospective middle school mathematics teachers express while solving geometry tasks in GeoGebra and Paper-Pencil groups?

3. What are the characteristics of local argumentations that prospective middle school mathematics teachers utilize while solving geometry tasks in GeoGebra and Paper-Pencil groups?

3.2 Design of the study

The present study employs qualitative research techniques. Fraenkel and Wallen (2012) state that some researchers' interests are based on the quality of a particular activity rather than on its frequency. When the quality of relationships, activities, situations or materials is the issue, the preferred research type is qualitative research (Fraenkel & Wallen, 2012).

How people interpret their lives and how they construct their meanings from their experiences have been stated to be the main interest of qualitative researchers (Merriam, 2009). Similarly, Denzin and Lincoln (2005) emphasize the importance of meaning derived by the individual him/herself in qualitative research by stating that "qualitative researchers study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of the meanings people bring to them" (p. 3). Therefore, qualitative researchers are disposed to collect data directly within the natural setting in which participants experience the problem or the phenomena (Creswell, 2007). In qualitative research, descriptive data is collected and they are presented in terms of words and pictures instead of numbers (Bogdan & Biklen, 2007). The other crucial issue in qualitative research is collecting multiple sources of data such as observations, interviews, documents and audiovisual materials (Creswell, 2007) in order to ensure the validity and reliability of the findings. Lastly, the preferred method in data analysis in qualitative studies is inductive data analysis instead of testing initially formulated hypotheses (Bogdan & Biklen, 2007). Creswell (2007) categorized qualitative approaches under five headings: narrative research, phenomenology, grounded theory, ethnography and case studies. The design utilized in the present study was a case study.

Various definitions of the qualitative case study can be encountered in the related literature. To illustrate, Stake (2005) states that "qualitative case study is characterized by the researcher spending extended time on site, personally in contact with activities and operations of the case, reflecting, and revising descriptions and meanings of what is going on" (p. 450). Another definition is made by Yin (2003), who defines the scope of the case study as follows: "A case study is an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident" (p. 13). He also differentiates the case study from other methods by comparing the characteristics of the methodologies.

> Case study inquiry copes with the technically distinctive situation in which there will be many more variables of interest than data points, and as one result relies on multiple sources of evidence with data needing to converge in a triangulating fashion, and as another result benefits from the prior development of theoretical propositions to guide data collection and analysis (Yin, 2003, p. 13).

Merriam (2009) also defines the qualitative case study by referring to as "an intense holistic description and analysis of a bounded phenomenon such as a program, an institution, a person, a process, or a social unit" (p. x). This definition entitles the case as an entity or a unit and emphasizes the importance of the boundaries of the case. Similarly, Creswell (2007) defined the case study as "a qualitative approach in which the investigator explores a bounded system (a case) or multiple bounded systems (cases) over time, through detailed, in-depth data collection involving multiple sources of information, and reports a case description and case-based themes" (p. 73). The way case study was employed in the current research study is in accordance with Merriam and Creswell's perspective since the aim was to present an in-depth description of the argumentation of prospective middle school mathematics teachers in GeoGebra and Paper-Pencil environments.

Creswell (2007) and Stake (1995) categorize case studies into three based on their intents: the single instrumental case study, the collective or multiple case study, and intrinsic case study. The instrumental case study is a case study in which the researcher stays focused on the issue or concern and then selects one bounded case to express that issue (Stake, 1995). As for the multiple-case study, the researcher again selects the issue but studies several cases jointly in order to examine this issue (Creswell, 2007). In the last type, intrinsic case study, the researcher focuses on the case itself (such as evaluating a program or studying a student who has difficulty) since the case is an unusual or unique situation (Creswell, 2007). Accordingly, the present study can be specified as a multiple case study since the argumentation of the prospective middle school teachers was examined in two cases: GeoGebra and Paper-Pencil.

In the qualitative case study, the unit of analysis can be an event, a program, or an activity (Creswell, 2007). Similarly, Yin (2003) asserts that the unit of analysis can be not only an individual and a group but also an event, an implementation process or entity. He proposed four basic types of case study designs based on the unit of analysis, which are single-case design with single unit of analysis (Holistic), single-case design with multiple units of analysis (Embedded), multiple-case design with single unit of analysis (Holistic), and multiple-case design with multiple units of analysis (Embedded) (Yin, 2003). In addition, Yin (2003) modelled these basic case study designs as illustrated in Figure 3.1.

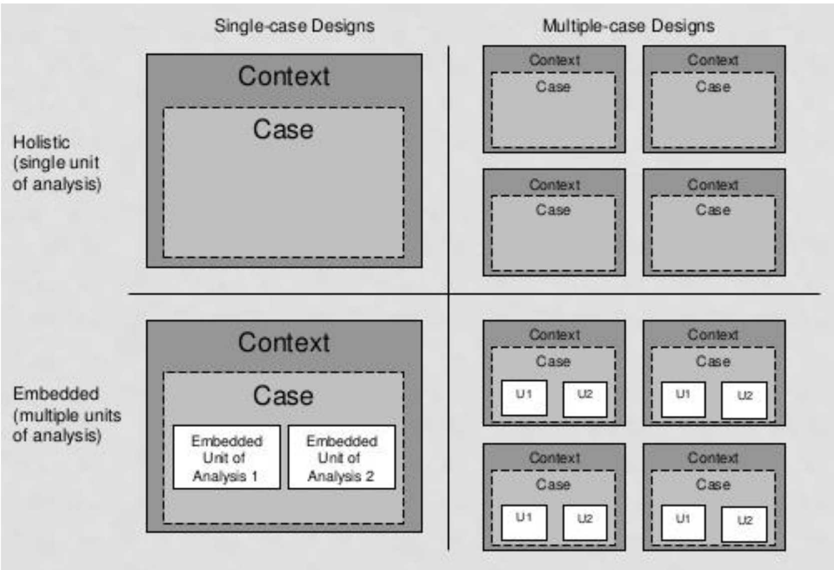


Figure 3.1 Basic types of designs for case studies (Yin, 2003, p. 40)

In the present study, the unit of analysis was the argumentation structures of prospective middle school mathematics teachers and the cases were two working groups, GeoGebra and Paper-Pencil, in the context of the Middle School Mathematics Teacher Education Program. Thus, the model of the analysis of the present study was holistic multiple case study design and can be summarized as in Figure 3.2.

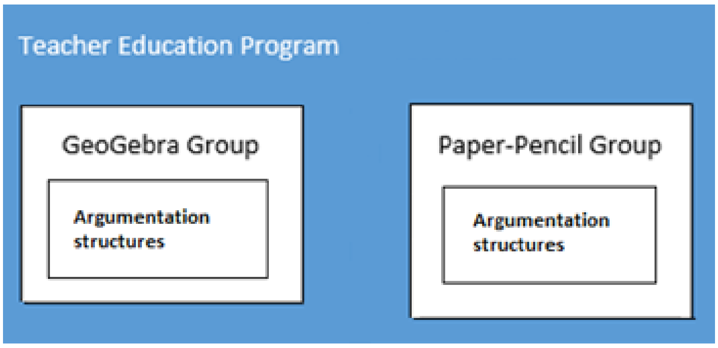


Figure 3.2 Holistic multiple case study, design of the present study

The following section provides detailed information about the data collection procedure pursued in the current study.

3.3 Procedure

The current study entailed a pilot study and a main study. Thus, initially, the participants and the data collection procedure of the pilot study is explained; subsequently, detailed information about the main study is presented in the following sections.

3.4 Pilot Study

Yin (2003) proposed a pilot test prior to the main implementation in order to refine data collection plans and develop a relevant line of questions. Accordingly, in the present study data for the pilot study was collected in the 2012-2013 spring semester. The specific purposes of conducting the pilot study were to check whether the geometry tasks were suitable for argumentation, to estimate the necessary time for each geometry task, to decide on the number of tasks to be applied each week, to be sure about the clarity of the statements in the geometry tasks and to decide on the number of people in each group.

3.4.1 Participants of the pilot study

The participants were selected through convenience sampling from a Middle School Mathematics Education undergraduate program of one of the public universities in Ankara. They were 4th grade students (9 females) who had volunteered to participate in the study. Based on their level of GeoGebra knowledge, they were assigned to either the GeoGebra group or the Paper-Pencil group. In addition, they were randomly divided into subgroups of two or three. The dates, the number of groups, the number of people in each group and the number of computers used in the GeoGebra group for three weeks are presented in Table 3.1.

Table 3.1 Pilot study dates and details about the groups

Date	Group	# of groups	# of people in small groups	# of computers in small groups
09.05.2013	GeoGebra	2	3 and 2	1
11.05.2013	Paper-Pencil	2	2 and 2	-
17.05.2013	GeoGebra	2	2 and 2	2
18.05.2013	Paper-Pencil	2	2 and 2	-
30.05.2013	GeoGebra	2	2 and 2	1
25.05.2013	Paper-Pencil	2	2 and 2	-

Computers and GeoGebra, a dynamic geometry program, were provided to the participants in the GeoGebra group to solve the geometry tasks. On the other hand, paper, pencil, a protractor, a ruler and a compass were provided to the participants in the Paper-Pencil group.

3.4.2 Data collection tools of the pilot study

In the next sections, the geometry tasks, interviews and reflection papers of the pilot study are explained in detail.

3.4.2.1 Geometry tasks in the pilot study

Initially, 10 geometry tasks were prepared for the pilot study. Four of them were based on triangles, three of them on quadrilaterals and three of them on circles. During the preparation of the tasks, challenging questions to accompany the geometrical figures were sought. Moreover, they needed to be questions that could be solved via both GeoGebra and paper-pencil. One other characteristic sought in the questions was to ensure that they had multiple solutions so that argumentation could emerge. After the pilot study, some of the tasks were adapted, while some of them were omitted. The details of the geometry tasks have been explained in the data collection part of the main study.

3.4.2.2 Interviews and reflection papers in the pilot study

Pilot interviews are crucial to figure out which questions are confusing, need rewording, or yield useless data (Merriam, 1998). There were pilot interviews with 4 prospective middle school mathematics teachers, 2 of whom were from the GeoGebra group and 2 of whom were from the Paper-Pencil group. All the interviews were videotaped, recorded and transcribed. The purposes of doing pilot interviews were to detect the possible difficulties which might be faced during the interviewing process of the main study, to check whether the questions were clear, and to determine the approximate time needed for the interview.

The participants of the pilot interview were asked whether or not the interview questions were clear to them, and their suggestions were taken into account to modify the interview protocol for the main study. Specifically, there were questions about the geometry background of the participants, the difficulty of the tasks, the use of materials provided, suggestions for the application process to promote argumentation, and the clarity of the statements in the worksheets.

The reflection paper also consisted of the same questions to collect the opinions of all the participants. However, the participants' answers to the reflection paper questions were highly superficial. In the interviews, the researcher collected more detailed information from the participants by questioning their answers. As previously indicated, the interviews were more beneficial when they were held just after the application.

3.4.3 Data collection procedure

There were three administrations and the administrations were implemented once a week for each group (GeoGebra and Paper-Pencil Groups). The application was recorded with a camera and audio recorders.

In the first administration, the participants solved 4 geometry tasks related to triangles. Initially, they studied in small groups, and then voluntary pairs presented their solutions on the board and discussed their arguments with the whole class. Meanwhile, the researcher guided the discussion by promoting the participants to develop arguments. To illustrate, the researcher asked such questions as 'How do you know that is true?, What does that mean to you?, Can you tell me more about your thinking process…?, Why do you think so?, and Are there any other ideas?' after discussing two geometry tasks, the participants were tired and the researcher gave a break. Following the break, the participants continued discussing the other two geometry tasks. Immediately after the first application, the researcher conducted an interview about the application with one pair of voluntary students, one from the GeoGebra group and one from the Paper-Pencil group. The other remaining pairs were asked to write a reflection paper, which they sent to the researcher on the application day till midnight by e-mail.

The second administration was applied one week after the first administration with 8 participants, rather than 9, because one of the participants in the GeoGebra group decided to give up participating in the study. During the application, 3 geometry tasks related to the quadrilaterals were solved. The same application procedure was followed in that week.

During the third and the last week, the participants solved three geometry tasks related to circles following the same procedure. Different geometric figures were used in the tasks of different administrations (e.g. circles, triangles) to offset the possibility that the students might not have been able to produce arguments related to the geometric figures they were not competent at. To illustrate, a student may not have been able to create an argument for the tasks on circles, but s/he could be more competent in other geometrical figures, and thus be able to create arguments for tasks on, for example triangles and/or quadrilaterals. If the researcher had prepared all of the tasks based solely on circles, not all the students may have been able to create arguments, so the researcher would have missed out on some data. Thus, the researcher decided to prepare the tasks on three different geometrical figures in order to ensure that arguments were collected from all the participants. In this way, the researcher was able to collect arguments of all the participants.

3.4.4 Conclusions of the pilot study

The video recordings of the applications in the pilot study were analyzed in terms of argumentation. In addition, the reflection papers and interview recordings were analyzed in order to make inferences for the arrangement of the main study.

In the first week, the researcher prepared 4 geometry tasks on triangles. However, 4 activities were found to be too tiring for the students. The students experienced difficulties in discussing the questions towards the end of the application. Thus, the researcher decided to reduce the number of tasks and discuss the questions in detail in each application in the main study.

Since the researcher was not sure whether the prepared tasks were suitable for argumentation, the decision was made to solve 3 tasks in each implementation of the pilot study and then to omit the tasks that did not work prior to the main study. In the first week, two of the four tasks related to triangles were easily solved in a short time by the participants, so the class discussion was not rich in terms of argument development. Thus, those tasks were omitted from the study. In the second week, the researcher prepared 3 geometry tasks on quadrilaterals. However, these 3 tasks related to quadrilaterals were omitted from the study specifically because in the first task, there were no accurate solutions for some of the questions. The participants in the GeoGebra group held effective discussions, but sometimes they did not use GeoGebra to arrive at the solution. Thus, that task was omitted from the study. Besides, the other two quadrilateral tasks were omitted since they led participants to use only algebraic expressions rather than GeoGebra to solve the problem. In the third week, the researcher prepared 3 geometry tasks on circles. In the class of the GeoGebra group the 1st and 2nd tasks were conductive to the use of GeoGebra and argumentation. However, in the 3rd task, the students did not hold effective discussions because there was only one answer and it was easy to show it via a GeoGebra file. That is, the solution was easy after drawing the required drawing figure on GeoGebra. Similarly, in the class of the Paper-Pencil group, there was a short discussion based on their drawings. Thus, it was concluded that the third task was not suitable for argumentation, so it was omitted from the study.

In the GeoGebra group, there were 5 people. The two small groups were generated with 3 people in one group and 2 people in the other group. In this way, the researcher had the chance of examining the arguments developed in small groups of two and three. The interview results indicated that in the group of two people, the participants expressed more opinions, and the discussion was more effective, while in the group of three people, one of the participants did not join the discussion most of the time. After analyzing the audio recordings in terms of arguments, the researcher decided that the ideal number of people in the small groups should be 2 for the main study.

In the GeoGebra group, each pair had only one computer and one activity sheet to work on the tasks. In the second week the researcher tried to provide two computers for each pair to work on the tasks. In the meantime, each participant in a pair worked on his/her own computer and shared his/her thoughts less frequently with his/her partner. That is, the communication between the pairs was minimal when they had their own computer. Thus, the researcher decided to provide one computer and one activity sheet to each pair in the main study in order to encourage discussion and argument development.

In the Paper-Pencil group, there were 4 people in the first week. The two groups were generated with 2 people. A ruler, protractor and a pair of compasses were provided to each group. Most of the time, they first produced a rough draft of their solutions to the questions and then they checked their solutions by using the materials they were provided with. Similarly, the people in the GeoGebra group used the GeoGebra program to check their claims and solutions in some of the tasks. That is, they were drawing the shape with paper and pencil and then checking their solution via GeoGebra afterwards in some of the tasks so they were using GeoGebra not through the whole solution process. Considering this, the researcher decided to encourage the GeoGebra group to use the GeoGebra program more frequently in main study.

In the first week, the researcher allowed less time for the discussion in small groups and started the class discussion. However, the students did not efficiently criticize the opinions of the student who was explaining her/his solution on the board because they were anxious about their own solutions and did not carefully listen to the students at the board. Thus, the researcher decided to spare more time for pair work. In the following weeks, more time was spared for pair work so the class discussions were more effective.

In addition to the above arrangements, the reflection paper and the interview questions of the pilot study were improved to collect more detailed information in the main study. In the first week, there was no time for the interview just after the application in the GeoGebra group. Then it was observed that the participants had forgotten the details of the implementation process, so their answers to the interview questions were not satisfying. For this reason, the researcher decided to make the interviewees skim the video of the implementation quickly just before asking the interview questions in the main study. Moreover, the questions asking for the participants' suggestions regarding the application were omitted because the purpose of asking those questions was to improve the quality of the main study.

3.5 Main Study

In the following sections, the details regarding the participants, the data collection tools, data collection procedure and the data analysis were explained.

3.5.1 Participants of the main study

In quantitative research studies, large representative samples are selected randomly but qualitative research studies include relatively small samples that are selected purposefully. According to Patton (2002), purposeful sampling is powerful because the participants who can provide rich information are selected to obtain in-depth information. In addition, it is crucial to determine the selection criteria according to the purpose of the study. In the related literature, argumentation was said to be a dialogical event which was done among two or more individuals (Duschl & Osborne, 2002). Therefore, the number of participants should not be too few in order to provide a highly argumentative environment when the findings of the pilot study was considered. Moreover, senior students are considered to be suitable as participants since they have taken most of the courses related to the teaching profession and can analyze the geometry tasks in a multidimensional way.

Thus the participants of the current study were 16 senior undergraduate students who were enrolled in an Middle School Mathematics Education undergraduate program at a public university in Ankara. Participants from only one public university were chosen since the time and classrooms for the administration of the study had to be arranged at the hours that did not overlap with each participant's weekly schedule. The participants in the GeoGebra group (8 students) were the students who took the course 'Exploring geometry with dynamic geometry applications' and knew how to use GeoGebra - a dynamic geometry program. The rationale underlying this criterion was to eliminate having to teach the participants how to use GeoGebra; this was advantageous as it saved time. The participants in the Paper-Pencil group were again 8 senior students who were selected from among the volunteers without considering their GeoGebra knowledge because they did not need to use GeoGebra during the study.

In the present study, pseudonyms were used instead of the participants' names. Some demographic information about the participants in the GeoGebra and Paper-Pencil groups, such as gender, number of pairs and grade level are presented in Table 3.2.

Table 3.2 Gender, number of discussion pairs and grade levels of the participants

	Gender		# of discussion pairs	Grade level of participants
	Female	Male		
GeoGebra Group	7	1	4	4
Paper-Pencil Group	6	2	4	4

Effort was made to include a similar number of females and males in the GeoGebra and Paper-Pencil groups. Moreover, their personal characteristics such as talkativeness, enthusiasm, predisposition towards technology, shyness, and disinterestedness were taken into consideration, while arranging the pairs in each group. The researcher did not experience any difficulty in selecting and arranging the participants in groups since the researcher knew the participants well as she was the assistant of their courses related to middle school mathematics teaching and had attended their courses from the beginning to the end of each semester to become closely familiarized with the whole class.

3.5.2 Data collection of the main study

Data were collected at the end of the fall semester of the 2013-2014 academic year. In order to examine the argumentation process of the participants, an in-depth analysis was needed where multiple sources of evidence were necessary to ensure the accuracy of the results. For this reason, multiple sources of information were collected in this study. These sources were the recordings of the implementation of geometry tasks (transcriptions of audiotapes and videotapes of pair-works and group discussions), interview recordings and documents (reflection papers), which are explained in detail below.

3.5.2.1 Data collection tools

In this section, the data collection tools of the main study are explained in detail. More precisely, the geometry tasks, interviews and documents are addressed respectively.

3.5.2.1.1 Geometry tasks

Subsequent to the pilot study, the best working 4 tasks were selected to be applied in the main study. That is, the tasks that were most suitable to the nature of the argumentation process were kept in the main study. Two tasks were related to triangles, while the other two tasks were related to circles. The tasks were prepared and adapted in such a way that they included challenging open-ended questions so that the participants could develop arguments and support their opinions. In addition, the questions had multiple solutions and the participants discussed their own answers initially with peers and then with the whole class. Moreover, the tasks were arguable and solvable with both GeoGebra and Paper-Pencil.

The first geometry task was taken from a master's thesis of Ceylan (2012), who studied the proof types of 2nd year pre-service teachers while using GeoGebra. This task was selected since it requires hypothesizing and testing conjectures, which are crucial in argumentation process. In addition, it is also solvable by both GeoGebra and paper-pencil. After receiving permission, the task was edited to make it conductive to justification and argumentation as illustrated in *Figure 3.3*.

GEOMETRY TASK 1

ABC is a triangle. The midpoints of sides |AB| and |AC| are points D and E, respectively. F and G points are placed on the side |BC| so as to be |BG|=|CF|.
The segments |DG| and |EF| intersects at point H.

When does |AH| become the angle bisector of ∠A? (Think about all types of triangles). Explain your reasoning and justify your solutions.

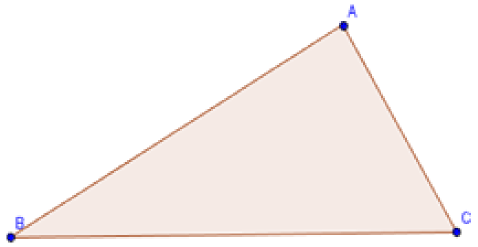


Figure 3.3 Geometry task 1

The focus of the present study was on their reasoning, so in addition to the original questions of the task, the researcher asked them to justify their answers all the time. In geometry task 1, it was expected from participants to think all possible drawings while placing poins F and G on segment |BC|. In this way, they should find alternative solutions to the questions. They were also expected to justify their interpretations for different solutions they found.

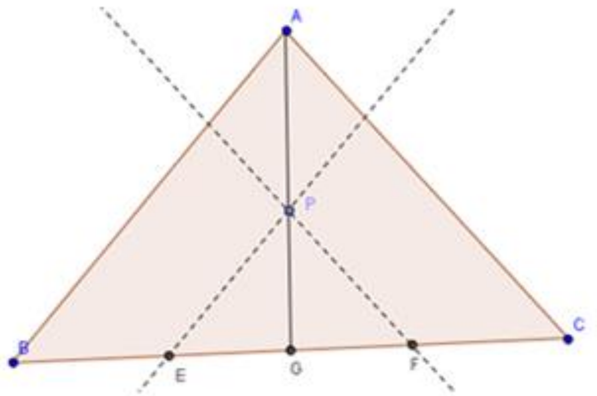
The second geometry task (*see Figure 3.4*), which was adapted from Iranzo-Domènech's doctoral dissertation (2009), has three sub-questions. The first question asks the relation between |EG| and |GF| when the given conditions are ensured. The second question asks the same thing when the triangle ABC is equilateral or isosceles triangle. The third question in geometry task 2 is different since it asks the specific position of a dynamic point P.

GEOMETRY TASK 2

Let P be any point on the median of |AG| of a triangle ABC. Let m and n parallel lines through P to the sides |AB| and |AC|. of the triangle.

1. What relation is there between the segments |EG| and |GF|? Explain your reasoning.

2. What if the triangle ABC is equilateral or isosceles triangle? Can any generalization be made for the relation between the segments |EG| and |GF|? Explain your reasoning.



3. Where must be the point P positioned such that |BE|=|EF|=|FC|. What if the triangle ABC is equilateral or isosceles triangle? Justify your solution.

Figure 3.4 Geometry task 2

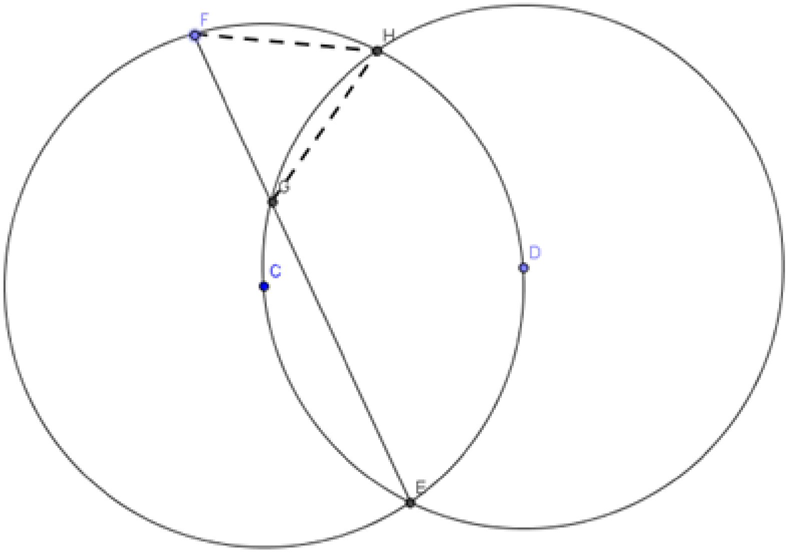
This task was also suitable for argumentation by both the GeoGebra and the Paper-Pencil groups in the pilot study. It was expected from participants to be able to notice the dynamic structure of point P and place point P to the right position to provide |BE|=|EF|=|FC|. Providing justifications for their solutions were also expected via theoretical and visual supports in both groups.

The third and fourth geometry tasks were taken from the book entitled ‘Challenging problems in Geometry’, written by Posamentier and Salkind (1988), from whom the necessary permission was taken. The book includes many challenging geometry problems, their solutions and hints. The following figures illustrate the geometry tasks taken from the book. Both tasks were based on circles.

GEOMETRY TASK 3

Two circles each of which passes through the center of the other, intersect at points H and E. A line from E intersects circles at F and G.

1. If |FG|= 6, compute the area of the triangle FGH? Justify your solution.



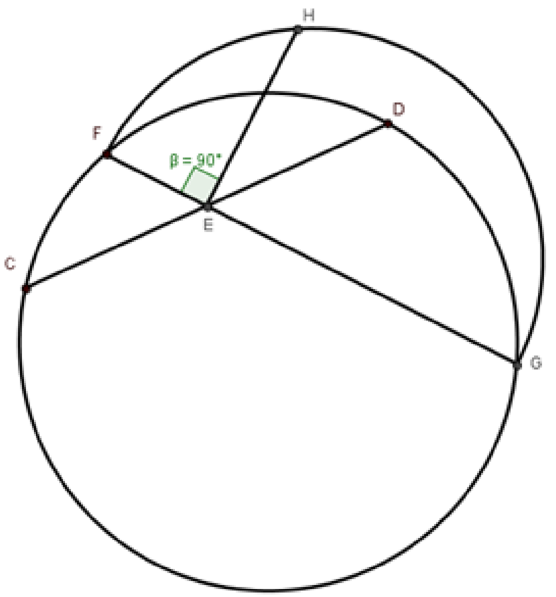
2. If r is the measure of the radius of each circle, find the minimum value and maximum value of the area of triangle FGH. Justify your solution.

Figure 3.5 Geometry task 3 (Posamentier & Salkind, 1988, p. 26)

As it can be seen in *Figure 3.5*, the Geometry Task 3 entailed two questions related to circles. In the first question, the participants were asked to find the area of triangle FGH. This question could be solved with knowledge based on the properties of circle. Therefore, it was expected from participants to justify their arguments based on the relationships between angles and arcs of the circles. However, solving the second question required mental imagination and calculations or a dynamic geometry program. The participants were expected to ensure the given conditions of the geometry task 3 after dragging point F while they were finding the minimum and maximum area of triangle FGH.

GEOMETRY TASK 4

1. |CD| is a chord of a circle, and bisected by another chord |FG| at point E. A semicircle is drawn with diameter |FG|. |EH|, perpendicular to |FG|, meets this semicircle at point H. Prove |EH|=|CE| and justify your reasoning.



2. Show whether the theorem is trivial if chord |FG| is a diameter of the first circle, or if |FG| coincides with |CD|. Justify your reasoning.

Figure 3.6 Geometry task 4 (Posamentier & Salkind, 1988, p. 17)

Geometry Task 4, which was based on circles, is presented in *Figure 3.6*. There were two questions. The first one asked the participants to prove that |EH|=|CE| in the given condition and justify their answers. The second question required imagining the semicircle’s movement on the circle. The participants were asked to interpret the relationship between |EH| and |CE| in the new conditions given in the second question. They were expected to imagine the new positions of the points and segments mentally after dragging to be able to make interpretations in both groups. Even the participants in GeoGebra group needed the mental imagination of the points for their drawings to be dynamic, and for providing the given conditions of the task after dragging.

3.5.2.1.2 Interviews

According to Patton (2002), interviewing makes the researcher enter into another person's perspective so the interviewing process requires attention and effort on the part of the researcher. Interviews are categorized into three by Yin (2003) as open-ended interviews, focused interviews and structured interviews. In the present study, focused (semi-structured) interview was conducted since this kind of interview collects detailed information in a style that is somewhat conversational. The semi-structured interview protocol was prepared, and the interviews were recorded by a camera and an audio recorder.

After the pilot study, the interview questions were refined by considering their applicability and suitability for the main study. Subsequently, two mathematics educators were asked to check the face validity of the interview questions. The purpose of checking face validity was to determine whether the interview questions matched with the research questions, and whether they were in agreement with the goal of the study. In addition, biased and leading questions were detected and changed within this process.

The aim of implementing an interview in the main study was to clarify the arguments of the participants. Some components of the arguments were missing in the discussion when the discussion flowed to different directions. In some situations, the participants did not explain their reasoning but implied it somehow in the main study. In the interview, the researcher asked for clarification of these parts to the participants by discussing the tasks. Thus, the interview questions were different each week since the questions related to the arguments developed during each administration were different (See Appendix A). At the end of each interview, the interviewees were asked to think about some arguments related to the task of the week and justify or refute them in order to understand whether or not they got the main points in the application.

3.5.2.1.3 Documents

In this study, documents include the worksheets on which the participants took notes for each geometry task and the reflection papers. All the participants were required to write a reflection paper at the end of the application. In fact, reflection paper questions were prepared from the interview questions of the focus group which were executed after each application. The researcher selected the questions which were not specific to the arguments developed in the focus groups and then organized them to gather information from all the participants (see Appendix B). Specifically, there were questions about mathematical background of the participants, the use of materials, the difficulties while using materials (GeoGebra / ruler, protractor, compass), the difficulty of the tasks, the justification preferences of participants, and two arguments to be supported or refuted. The reason for asking the participants to justify or refute these arguments was for triangulation purposes; they were required to write anything that they had forgotten to say during the argumentation. The reflection paper questions were the same for both the GeoGebra group and the Paper-Pencil group, except for the questions related to the material used (GeoGebra / Compass, Protractor and Ruler) since they had to answer the question according to the materials they had used. The researcher sent each student the reflection paper questions with his/her own activity sheets of the four geometry tasks to make them remember what they had done. They answered the reflection paper questions and sent their answers to the researcher by e-mail.

3.5.2.2 Data collection procedure

The researcher administered the geometry tasks by herself and she was not able to take notes about the process; thus, she was a participant-observer in this study. According to Yin (2003) participant-observers may not raise questions about the process from a different perspective as a good observer may not have enough time to take notes. In order to solve this problem, the entire administrations were recorded with video cameras, and the audio recorders were placed on the desks of each pair of group members. The number of cameras was arranged in a way that enabled the whole class to be observed from different perspectives. These recordings were used after the administration. They were transcribed and the researcher took notes and coded them. These recordings were beneficial data because the researcher did not miss any part or situation related to the argumentation. In this way, the process was analyzed holistically in order to be able to understand the whole process.

As soon as the 16 voluntary prospective middle school mathematics teachers were selected for the main study, the procedure which was determined with the help of the pilot study was administered. Two groups were arranged one of which was the GeoGebra Group (GG) and the other one was the Paper-Pencil Group (PPG). Then, two applications with each group were carried out. The dates and the number of pairs in each group and the number of people in each group are presented in Table 3.3.

Table 3.3 Main study dates and detail about groups

Date	Group	# of pairs in each group	# of people in each group	# of computer for each pair in GeoGebra group
12.11.2013	Paper-Pencil	4	8	-
15.11.2013	GeoGebra	4	8	1
19.11.2013	Paper-Pencil	4	8	-
22.11.2013	GeoGebra	4	8	1

The administrations of the GG were in a computer laboratory while the administrations of the PPG were in an ordinary classroom on different days. At the beginning of the first administration, necessary information about the argumentation process was given and the participants were informed about what they were expected to do during the administration. Then, the worksheet of the first geometry task was distributed to the pairs. The organization of the class of the main study for the GG is illustrated in the *Figure 3.7* below.

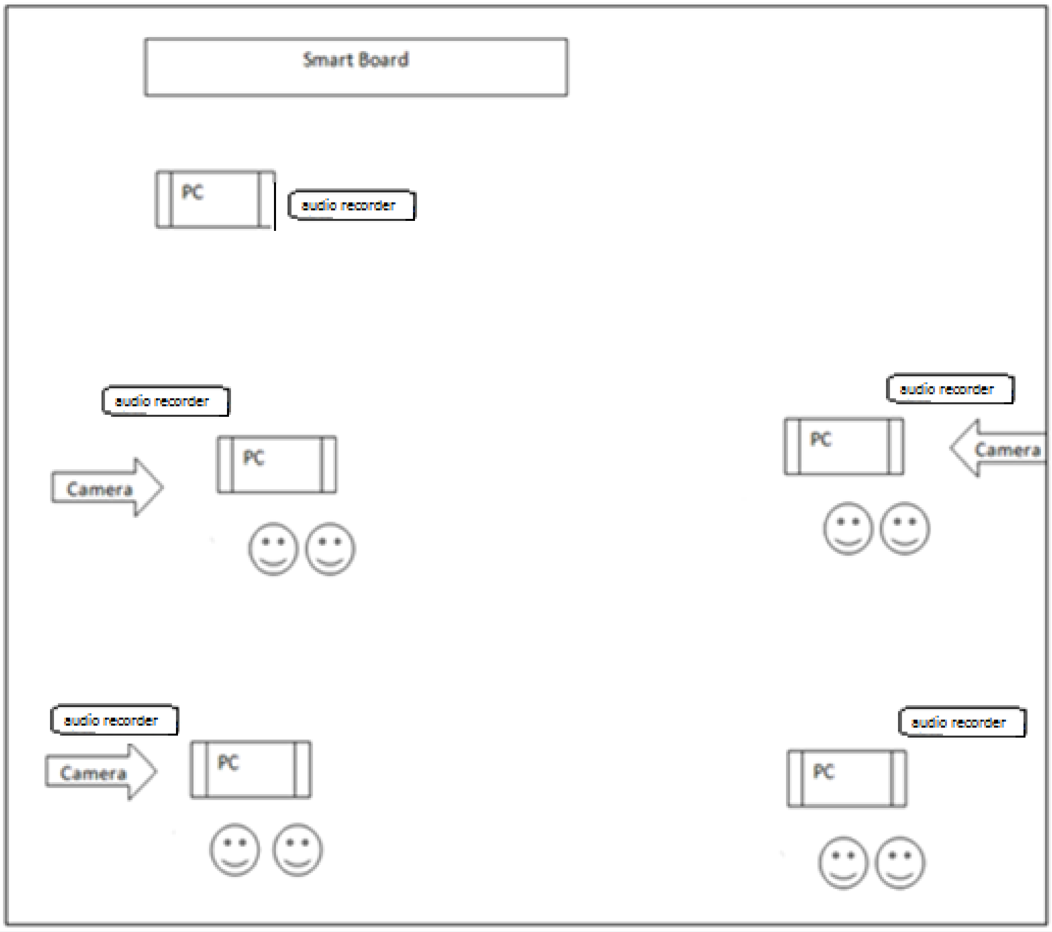


Figure 3.7 Organization of the GeoGebra group in the main study

In the GeoGebra group, the participants worked on the given geometry tasks in pairs initially with one computer and one worksheet (see Table 3.3). Then, they discussed their solutions with the whole class. After the class discussion, the worksheet of geometry task 2 was distributed to the pairs. In the same way, a pair discussion and a class discussion were executed consecutively. Just after the first week's administration, the researcher watched the pair-work video recording of the focus pair and the class discussion to prepare the interview questions for that week. One day after each administration, the researcher held an interview with the focus pair. At the beginning of the interview, the researcher made the interviewees skim the video of their pair-work and class discussion quickly in order to remind them the administration. They also looked at their own activity sheets. Then, the researcher asked interview questions and recorded the interview process via a camera and an audio recorder.

Except for the use of computer to solve geometry tasks, the same procedure was valid for the PPG. Instead of a computer, there were one ruler, one protractor and one compass on the desk of each pair. The organization of the class of the main study for PPG is illustrated below in *Figure 3.8*.

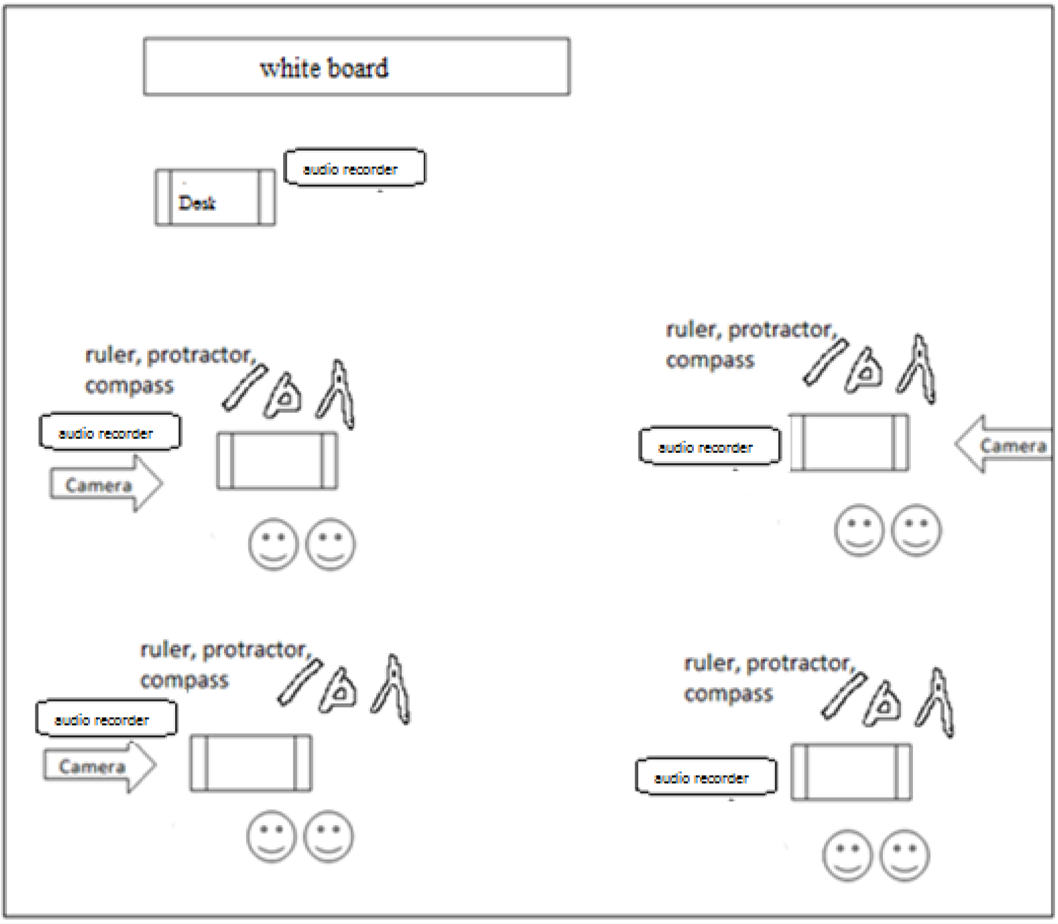


Figure 3.8 Organization of the Paper-Pencil group in the main study

In the second week, the participants solved geometry tasks 3 and 4 following the same procedure explained above. After the last administrations, the reflection paper questions (see Appendix B) were prepared and sent to the participants via e-mail. They were given a deadline to answer and send their answers back again via e-mail.

3.5.2.3 Data analysis

In case studies, data collection and analyzing processes occur simultaneously since the main purpose is to provide intensive and holistic description of the case (Merriam, 1998; Yin, 2003). According to Yin (2003), the researcher should examine, categorize, tabulate, test or otherwise recombine both qualitative and quantitative evidence to analyze data to refer to initial propositions. Creswell (2009) also explained the data analysis in qualitative research studies in such a way that it requires “preparing the data for analysis, conducting different analyses, moving deeper and deeper into understanding the data, representing the data, and making an interpretation of the larger meaning of the data (p. 183). An informative schema, which is illustrated in *Figure 3.9*, was also provided by Creswell (2009) for qualitative researchers.

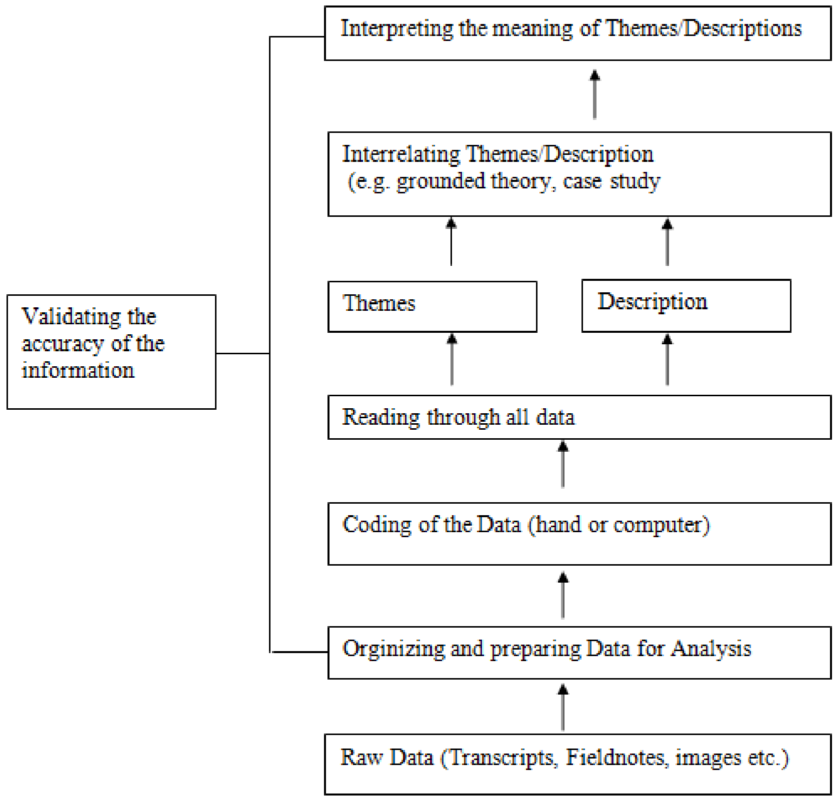


Figure 3.9 Data analysis in qualitative research

Thus, within the data analysis procedures, initially the collected data were organized and prepared for data analysis. For this purpose, the first step, which took a long time for the researcher, was to transcribe all the videotaped and audio recorded data. These transcriptions and all other data sources were organized and stored as computer files. The spoken language was Turkish throughout all the transcriptions. The researcher translated only the necessary parts of the transcriptions into English for the results chapter. The researcher had the chance of engaging in the whole data since she watched the videotapes and audio recordings several times. In addition, the translated data were compared to the original data in terms of their grammatical, syntactic and linguistic aspects in order to ensure the accuracy of the transcription.

The second step of the data analysis was coding of the data in order to identify themes and categories. Merriam (2009) maintained that data analysis in qualitative studies was a complex process which includes moving back and forth between the parts of data, and between inductive and deductive reasoning. Similarly, Yin (2003) emphasized the difficulty of data analysis in case studies since there were no clearly defined strategies and techniques. In this study, the data (transcriptions of the interviews and observation videos and documents) were reviewed repeatedly to make sense of the data, and notes were taken to identify the arguments in the data. The researcher decided to work with an intercoder while determining the arguments in the discussions. The intercoder was a doctoral student studying mathematics education in the Elementary Education program of Middle East Technical University (METU). In addition, she was knowledgeable in the qualitative research method and the nature of argumentation in mathematics. The reason for this intercoder application was to have consensus about the arguments and their elements (claim, data, warrant, backing…) in the data, and reduce researcher bias. The researcher provided the intercoder with information from literature about the elements of the arguments. The researcher and intercoder determined the local arguments in 25% of the transcriptions of the applications separately. Subsequently, they got together. The interrater reliability was calculated as 76% before discussion. Then, they discussed the differences between their arguments and arrived at a 100

% consensus for all of the arguments. This work was beneficial for the researcher since the argument elements became clearer in her mind. After the intercoder application, the researcher determined the arguments of the remaining data based on the intercoder application results.

Lastly, the argument schemas of the four geometry tasks of the GG and the PPG were drawn according to the argumentation model of Toulmin (1958). Subsequently, the researcher did another intercoder application with two doctoral students who were studying mathematics education. This time, the researcher gave the inter-coders 25% of the drawn argument schemas and the information about the argumentation model of Toulmin (1958). Then, the researcher asked them to read the transcriptions, look at the *argument schemas,* and take notes about whether they agreed or disagreed with the arguments and their components. In this intercoder application, the process was in the reverse order of the previous inter-coder application. The researcher and the two inter-coders worked separately with the data; then they came together to discuss the differences they detected. Before discussion there was 60% agreement between the researcher and the intercoders. In the end, they discussed their ideas by watching the observation videos and built consensus for all the arguments with 100% agreement. The researcher did these two inter-coder applications to be sure about the arguments of the study.

In order to answer the first research question which sought for the nature of the argumentation structures of prospective middle school mathematics teachers while solving geometry tasks using dynamic geometry software or using paper and pencil, the global argumentation structures for each geometry task were generated. The framework for this analysis was developed by Knipping and Reid (2013) in the proof context with 9th grade students from Germany and Canada. They suggested a method to analyze and reconstruct the complex argumentations in proving processes. Firstly, they used the argumentation model of Toulmin (1958) to determine developed arguments, while 9th grade students were working on the proof of the Pythagorean Theorem. Then, they analyzed the general structure of the process which they called as *global argumentation structure*. In the present study, the global argumentation structures in the geometry context were analyzed.

Before explaining Knipping's (2008) classification of global argumentation structures, it would be of benefit to clarify the terms *argumentation steps*, *argumentation stream* and *parallel arguments*. Knipping (2008) used the term argumentation step as for distinct arguments. That is, a single argument with one conclusion and other elements (data, warrant, backing), if exists, can be defined as an argumentation step. It has the same meaning with local arguments. The other term argumentation stream was defined as "a chain of argumentation steps by which a target conclusion is justified" (Knipping, 2008, p. 434). In the present study, the series of arguments which were connected to each other to justify the target conclusions were defined as argumentation stream. The last term to be defined was parallel arguments, which refers to different arguments supporting the same conclusion in an argumentation stream, and this happens when the participants develop substantially different arguments for the same conclusion (Knipping, 2008).

The schematic representation which was developed by Knipping (2008) was used in the present study. In order for the representation to be more meaningful the sample argumentation stream is illustrated in *Figure 3.10*.

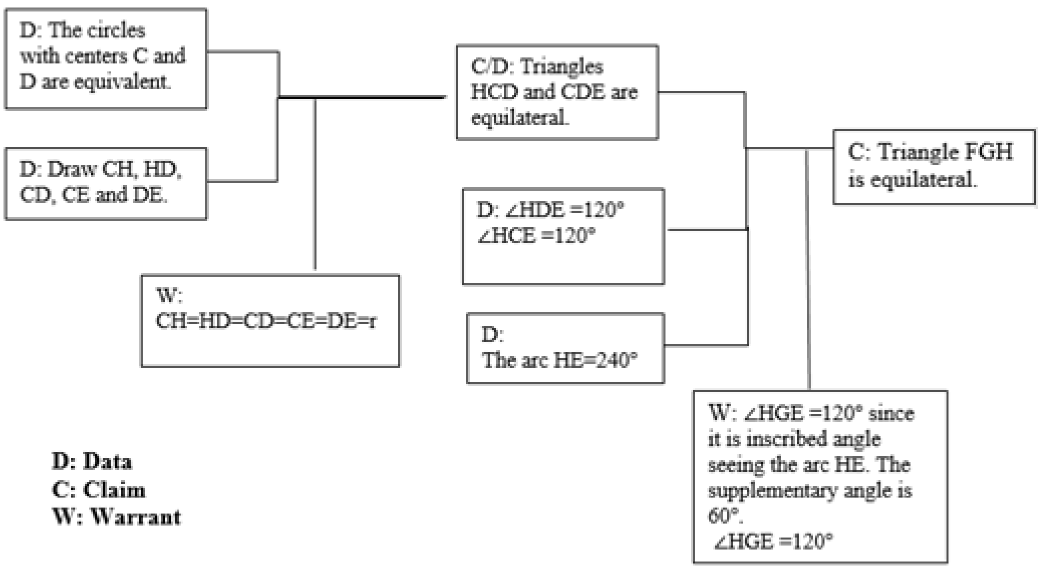


Figure 3.10 Sample argumentation stream

In Figure 3.10, the letters at the beginning of the sentences convey a meaning. ‘D’ means *data*, ‘W’ represents *warrant,* while ‘C’ stands for *claim* of the argument. In this argumentation stream the target conclusion was ‘Triangle FGH is equilateral’. The participant first presented the argument which concludes that ‘Triangles HCD and CDE are equilateral’ and then used this information as datum for the target conclusion by also using other data, which were ‘∠HDE =120°, ∠HCE =120°’ and ‘The arc HE=240°’. The schematic representation of this based on the method prosed by Knipping (2008) is presented in *Figure 3.11*.

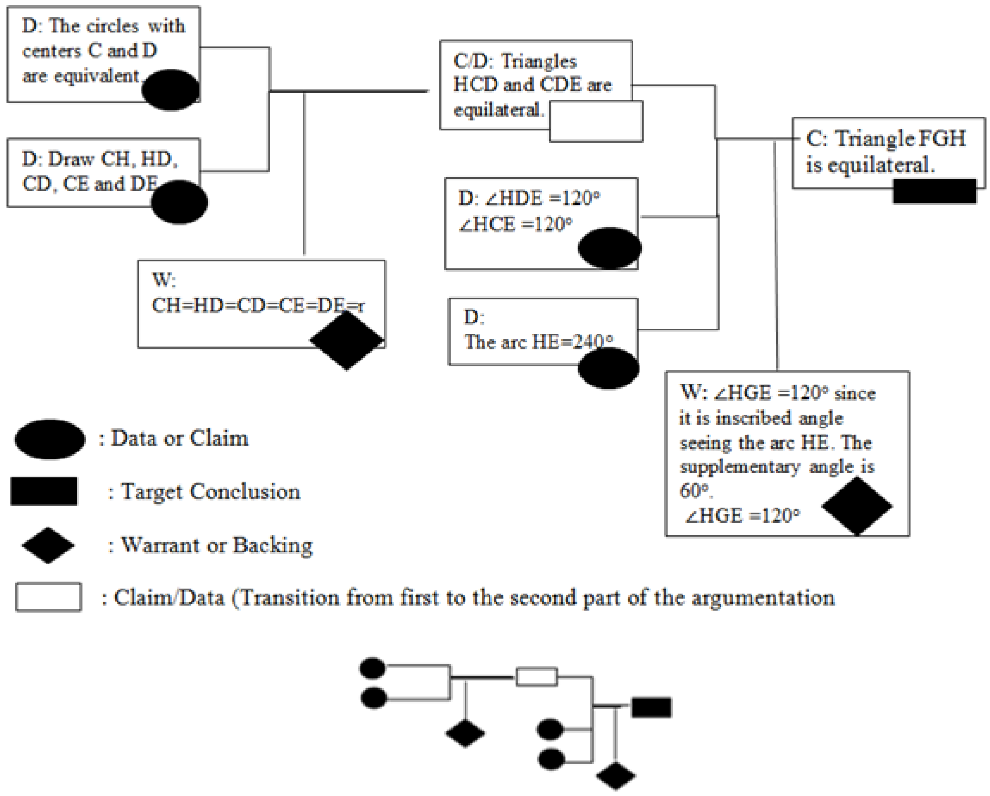


Figure 3.11 The schematic representation of argumentation stream in Figure 3.10 based on the method of Knipping (2008)

As it can be inferred from the *Figure 3.11*, Knipping (2008) represented data or the claim of the argument with black circles, while she used a black rectangle for the target conclusion of the global argumentation structure. In addition, black rhombus represents the warrant, while the white rectangle stands for the claim/data in this schematic representation. Claim/Data means that the claim of an argument is used as a datum for the following argument and functions like a transition component between these two arguments. In the global argumentation structure analysis, the schemas were like the miniature representation under the argumentation stream in *Figure 3.11*. In the following paragraphs, each global argumentation structure was explained in detail with schematic representations.

The global argumentation structures developed by Knipping (2008) and Reid and Knipping (2010, 2013) were source-structure, reservoir-structure, spiral-structure and gathering-structure. In the present study, there was no global argumentation structure which was suitable to the gathering structure, so the gathering structure was not used in the adapted classification of the global argumentation structures. Moreover, some of the argumentation structures which emerged in the present study possessed different properties and, thus, did not fit into any one of these four global argumentation structures. For this reason, the researcher categorized those structures under new categories; line-structure and independent arguments-structure. Therefore, the model used in the present study included the following global argumentation structures, each of which is explained in detail: Source-structure, Reservoir-structure, Spiral-structure, Line-structure, and Independent arguments-structure.

In *source-structure*, the ideas and arguments flow as if arising from a variety of origins (Reid & Knipping, 2010). That is, arguments are like water welling up from many springs (Reid & Knipping, 2010). This structure has the following characteristics:

> - Argumentation streams that do not connect to the main structure
> - Parallel arguments for the same conclusion
> - Argumentation steps that have more than one datum, each of which is the conclusion of an argumentation stream.
> - The presence of refutations in the argumentation structure (Reid & Knipping, 2010, p. 180).

In addition to these characteristics, Reid and Knipping (2010) have stated that source-structure can lack explicit warrant and data. Moreover, a funneling effect, which refers to the fact that many arguments are considered at the beginning of the discussion and connecting to only to one concluding statement at the end, is asserted to be apparent in source structure (Reid & Knipping, 2010). During the discussions, there may be situations that a false conjecture is constructed. In such situations, false conjectures can be disproved but are valued at the same time (Knipping, 2008). The source-structure, taken from a study by Knipping (2008), is illustrated in *Figure 3.12.*

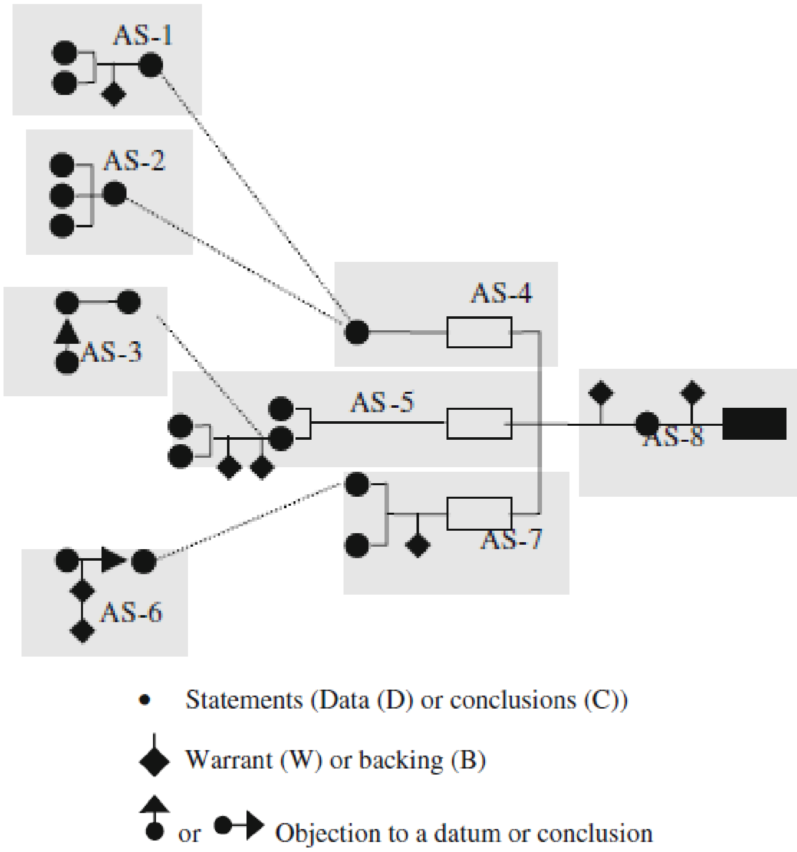


Figure 3.12 Source-structure schema (Knipping, 2008, p. 437)

As displayed in *Figure 3.12*, the arrows in the schema represent the objection to a datum or conclusion. In present study, black rounded rectangles were used to represent the rebuttals instead of arrows. Knipping (2008) stated that AS-6 (Argumentation stream-6) was a refuted argument by the teacher and other participants, so it was not connected to the main structure. In addition, AS-4 was supported by more than one justification of a statement (AS-1 and AS-2). AS-1, and AS-2 were parallel arguments for the same conclusion. AS-8 had more than one datum, each of which was the conclusion of an argumentation stream. AS-3 and AS-6 had refutations in the schema.

Another global argumentation structure that emerged from the data was *reservoir-structure*. The reservoir-structures had a flow towards intermediate target conclusions, which were distinct and self-contained. These intermediate target conclusions were described like reservoirs holding and purifying water before allowing it to pass to the next stage (Reid & Knipping, 2010). In this type, argumentation steps lacked explicit warrants or data, like in the source-structure, but it was less frequent when compared to source-structure. The differentiating characteristic of the reservoir-structure is that the reasoning occasionally moved backwards and then forward again in order to provide further support by the data. When this need was satisfied, the deductions that followed led to the final conclusion (Reid & Knipping, 2010). That is, this process included more in-depth discussion than the other structures since students thought about the arguments repeatedly to provide additional supports and data by moving back and forth.

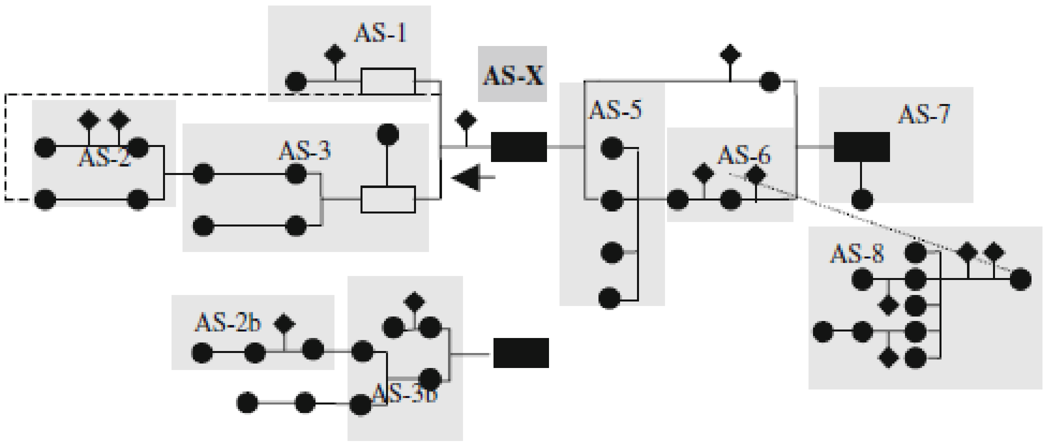


Figure 3.13 Reservoir-structure schema (Knipping, 2008, p. 437)

In *Figure 3.13*, the arrow shows the differentiating property of the reservoir-structure, which is reasoning backwards, and the dotted line shows where the reasoning goes back to. Specifically, after constructing AS-1, AS-2 and AS-3, the reasoning moved backwards to AS-2 again. Then, AS-2b and AS-3b parts were discussed. Until AS-X, the target conclusion was self-contained. The second part (AS-5, AS-6 and AS-7) was also a closed structure but reasoning went only forward in that part (Reid & Knipping, 2010). Reid and Knipping (2010) stated that at the end of the discussion, a further justification was requested, so the AS-8 was discussed.

The third global argumentation structure that emerged in the current study was *spiral-structure*. Reid and Knipping (2010) stated that spiral-structure had the same four characteristics with source-structure which were as follows:

> - Argumentation streams that do not connect to the main structure
> - Parallel arguments for the same conclusion
> - Argumentation steps that have more than one datum, each of which is the conclusion of an argumentation stream.
> -The presence of refutations in the argumentation structure (Reid & Knipping, 2010, p. 187).

As it can be inferred, the properties of the spiral structure seemed to be the same with those of source-structure. However, they were different in terms of some properties. Specifically, the main difference between spiral-structure and source-structure was the place of the parallel argumentation structures within the global argumentation (Reid & Knipping, 2010). Parallel argumentation streams were located at the beginning of the discussion in source-structure, while they were located at the end of the discussion in spiral-structure (Reid & Knipping, 2010). That is, in spiral-structure the target of the parallel argumentation streams was the final conclusion (Reid & Knipping, 2010), whereas in source-structure the target of parallel argumentation streams was the claim/data that emerged during the argumentation. The other difference between source-structure and spiral-structure was the frequency of the emergence of the explicit warrants or data in the process. Reid and Knipping (2010) asserted that the lack of explicit warrant or data was observed less often in spiral-structure when compared to source-structure.

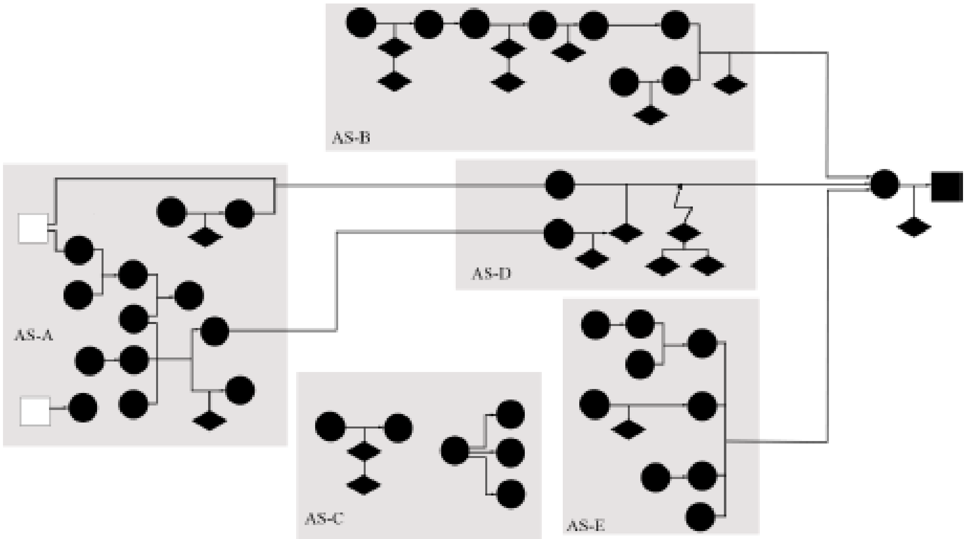


Figure 3.14 Spiral-structure schema (Reid & Knipping, 2010, p. 437)

In *Figure 3.14*, AS-C is the argumentation stream that is not connected to the main structure. In addition, there are three parallel argumentation streams, AS-B, AS-D and AS-E, which lead directly to the target conclusion. The argumentation steps which possess more than one datum, each of which is the conclusion of another argumentation stream can be seen in AS-A and within the final conclusions of AS-B and AS-E (Reid & Knipping, 2010). There was only one refutation in AS-D in this sample.

The fourth global argumentation structure that emerged from the data was *line structure.*

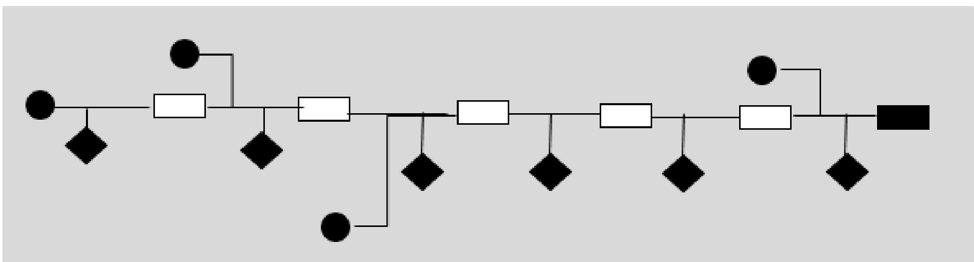


Figure 3.15 Line-structure schema

As it can be seen in *Figure 3.15*, line structure is different from the other structures since it flows like a line; the transitions are provided with claim/data components and the argumentation ends with the target conclusion. The claims which became the data for the subsequent argument were confronted very often. There are explicit warrants and refutations but these refutations did not end the flow of the discussion. There were no parallel argumentation streams which differentiated the line-structure from source-structure and spiral-structure. Moreover, in line structure, there was no reasoning going backwards then forwards which was the characteristic property of reservoir-structure.

The last global argumentation structure that emerged in the present study was *independent arguments-structure*. The sample shape of the structure is illustrated in *Figure 3.16*.

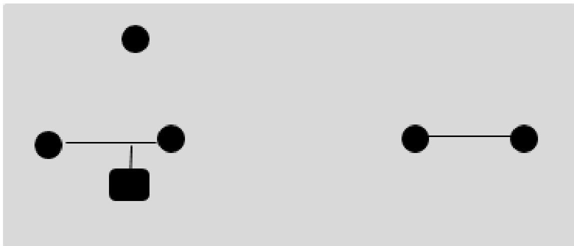


Figure 3.16 Independent arguments-structure schema

The independent arguments-structure schema emerged when the participants could not solve the problem, but expressed an opinion about the solution. In addition, these single arguments were made during the class discussion as if the participant were thinking loudly. Sometimes the participant expressed the argument and then refuted it herself/himself. Another situation in which independent argument-structure emerged was when the participant constructed an argument and then made another argument which was not related to the previous one. Yet, another situation was that the participants sometimes solved a part of the problem with only one argument and then continued with another part of the problem. Therefore, that argument was not connected to the other arguments. When all these situations were examined, it was concluded that there was no connection between those arguments, so the global structure included distinct arguments. In addition, independent arguments-structure was different from disconnected argumentation streams, which a term used by Reid and Knipping (2010) in the properties of the spiral structure and source structure. Knipping and Reid (2013) defined disconnected argumentation streams as “the contributions that do not lead to the conclusion result” (p. 136). However, in the present study it was not possible to talk about such argumentation streams since each local argument was independent of each other. Thus, that global argumentation structure was entitled with a new term *independent arguments-structure.*

In the data analysis process, initially, the global argumentation structures of each geometry task were drawn for both groups based on the adapted model of Reid and Knipping (2010). At this point, another intercoder application was needed and, thus, conducted in order to arrive at a consensus in the identification of the argumentation structure types and to reduce researcher bias. The researcher recruited another doctoral student who was knowledgeable in the qualitative research method and the nature of argumentation in mathematics and who had already taken part in the previous intercoder study done to determine the arguments and argument components. The researcher gave intercoder 50% of the global argument structure schemas of the present study and the necessary information from literature about the global argumentation structures and then asked the intercoder to decide in which category each global argumentation structure fit. Subsequently, the researcher and the intercoder came together and discussed all global argumentation structures in order to arrive at a 100 % consensus. After the intercoder application, the researcher categorized all the global argumentation structures of the present study. Then, the researchers analyzed the global argumentation structures and compared each geometry task separately. In addition, the global argumentation structures produced by the GeoGebra and Paper-Pencil groups were compared. Finally, the global argumentation structures were compared on the basis of the mathematical contents (triangle tasks / circle tasks) in order to reveal any significant pattern or theme.

The second research question, ‘What are the characteristics of the local arguments in the global argumentation structures?’, necessitated the investigation of the characteristics of local arguments based on the flow of the argument components (claim, data, warrant) that prospective middle school mathematics teachers use while solving geometry tasks in the GeoGebra and Paper-Pencil groups. The researcher wondered whether or not there was a pattern in the flow of argument construction in geometry. That is, the prospective middle school mathematics teachers' arguments were analyzed in the order the argument components were stated. In other words, the researcher examined the order in which the participants stated the components of local arguments, namely the claim, data and warrant during the discussion. For this reason, the researcher read through the transcriptions and numbered the argument components to find the most frequently used patterns. Thus, the present study contributed to the related literature by identifying 9 different local argument types in the geometry context. The next step was to compare the local argument types on the basis of groups, mathematical contents (triangle tasks / circle tasks), and task by task in order to find any emerging theme.

The third research question was related to the characteristics of the local argumentations that prospective middle school mathematics teachers utilized to justify their arguments while solving geometry tasks in GeoGebra and Paper-Pencil groups. Knipping (2008) developed this classification in her study which was within the context of proof. In order to analyze local argumentations, she examined the types of warrants (and backings) that were employed by students and teachers to identify the field of justification that applied in that classroom. In the current study, the researcher also examined the warrants and backings of each local argument to answer this sub-question. The schema of the classification by Knipping (2008) is presented in *Figure 3.17* and the researcher used this classification to analyze local argumentations of the prospective middle school mathematics teachers in the geometry context.

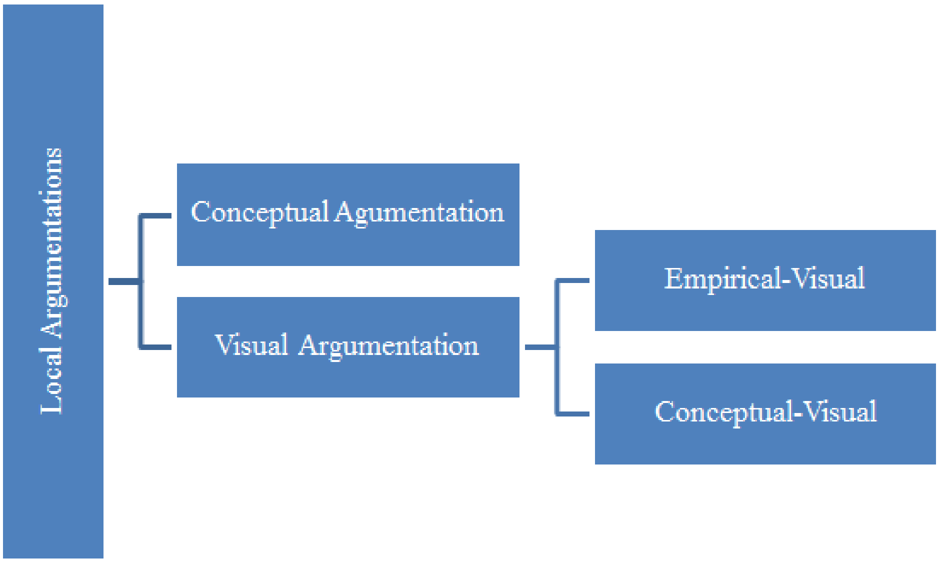


Figure 3.17 Local argumentation classification developed by Knipping (2008)

Knipping (2008) asserted that conceptual argumentation can be categorized in the deductive conceptual field of justification, which requires using concepts and general conceptual principles as warrants to justify the conclusions. Specifically, Knipping (2008) pointed out that the warrants of conceptual argumentations were composed of mathematical concepts, mathematical relations between concepts, and references to theorems, definitions, axioms and rules of logic. She also emphasized the language used while stating conceptual argumentations as the use of conjunctions such as 'as' and 'since', and a statement like 'because one knows that …', which shows a generally accepted status of the conceptual warrant. On the other hand, the reference to figures as warrants came to the forefront in visual argumentation. For instance, a reference to the figure or diagram on the board as justification can be a visual argumentation since conclusions are drawn from the figure rather than the results of the individual steps in the argumentation (Knipping, 2008). Visual argumentation is divided into two levels: empirical-visual level and conceptual-visual level (Knipping, 2008). The empirical-visual level entails an argument based on a concrete diagram and the relations among its components, which can be accepted as the justification for the claim in the diagram. More specifically, Knipping (2008) states that properties and relations in mathematics can be perceived through the senses as they are bound to concrete figures. However, in the conceptual-visual level, the diagram can be accepted as the representation of the idea (concept) (Knipping, 2008). That is, the generalization can be made by using the conceptual-visual level argumentation.

In the related literature, no example presented for local argumentation levels (empirical-visual, conceptual-visual and conceptual) were encountered so the researcher consulted David Reid and Christine Knipping via e-mail in order to ask whether or not they could provide any examples. The following example (see in *Figure 3.18*) was sent to the researcher:

Argument: The sum of the first n natural numbers is equal to n(n+1)/2.

Conceptual argumentation: The sum can be written 1+2+3…+(n-1)+n and this can be added to the same sum in reverse: (1+n)+(2+n-1)+…+(n-1+2)+(n+1).This new sum has n terms, each on equal to n+1, and so the sum is equal to n(n+1). It is two times 1+2+…+(n-1)+n, so 1+2+…+(n-1)+n = n(n+1)/2. No reference to a diagram is made.

Empirical visual argumentation: The sum can be represented with a picture:

O
OO
OOO
OOOO
OOOOO

This can be added to the same picture in reverse:

O OOOOO
OO OOOO
OOO OOO
OOOO OO
OOOOOO

There are 5 rows, each with 5+1 tokens. So in this case, we see that 1+2+3+4+5 = 5(5+1)/2.

Conceptual-visual argumentation: In addition to the diagram given in empirical-visual argumentation, the following extension can be made:

In general, there will be n rows, each with n+1 tokens. So 1+2+…+(n-1)+n = n(n+1)/2. This statement (if it is understood) prompts us to see the diagram in a more general way, transforming the argument into a conceptual-visual argument.

Figure 3.18 Example argument for local argumentation levels

After making the distinction of the local argumentation levels clear, the data were analyzed according to the classification presented by Knipping (2008). First of all, the warrants and backings of all arguments were classified according to the classification in the current study. In order to arrive at a consensus regarding the classification and reduce researcher bias, the researcher did an intercoder application with another researcher who was competent in argumentation and mathematics education and working as an instructor in the Middle School Mathematics Education department of the Faculty of Education at METU. The researcher and the intercoder read 25% of the arguments together and assigned each argument's warrant and backing to one of the categories given in Figure 3.17. Before discussion the researcher and the intercoder agreed on 28 arguments out of 36 arguments which corresponds to 77% agreement. They discussed until they persuaded each other and they built full consensus on all arguments in the end. Subsequently, the researcher classified the remaining 75% arguments by taking into consideration the consensus arrived at with the inter-coder. Finally, she compared the local argumentations of the participants on the basis of tasks and mathematical contents (triangle tasks / circle tasks), by considering each group (the GeoGebra group and the Paper-Pencil group) to generate a conclusion.

3.6 Trustworthiness of the study

Two important issues to be considered in scientific studies are validity and reliability. Fraenkel and Wallen (2012) defined validity as "…the appropriateness, correctness, meaningfulness, and usefulness of the specific inferences researchers make based on the data they collect" (p. 151) and reliability as "the consistency of the scores obtained how consistent they are for each individual from one administration of an instrument to another and from one set of items to another" (p. 157). These two concepts were asserted to be considered by any researcher while designing a study, analyzing results, and judging the study's quality (Patton, 2002). In qualitative studies, the term trustworthiness is used to refer both validity and reliability. Moreover, the concepts of validity and reliability are perceived and named differently by different qualitative researchers (Creswell & Miller, 2000; Lincoln & Guba, 1985; Patton, 2002; Shenton, 2004, Yin, 2003). For instance, in order to judge the quality of qualitative case study designs, Yin (2003) mentioned four design tests, which are construct validity, internal validity, external validity and reliability. As indicators of trustworthiness of qualitative studies, other terminologies were proposed by Lincoln and Guba (1985), namely credibility, transferability, dependability and confirmability. In the present study, the term 'trustworthiness' is preferred to be used instead of validity and reliability, and the terms mentioned by Lincoln and Guba (1985) were addressed in order to assure the trustworthiness of the present study.

One of the most important criteria to assure the trustworthiness of a qualitative study is credibility, which refers to internal validity (Lincoln & Guba, 1985). Merriam (2009) mentioned the questions of concern to establish credibility as "How congruent are the findings with reality? Do the findings capture what is really there? Are investigators observing or measuring what they think they are measuring?" (p. 213). Although it is quite hard to capture an objective "truth" and "reality" for qualitative researchers, there are some strategies offered by different researchers to increase the credibility of the studies (Merriam, 2009). For instance, Creswell (2007) suggested the following eight strategies for credibility: prolonged engagement and persistent observation, triangulation, peer review or debriefing, negative case analysis, clarifying researcher bias, member checking, thick description and external audit. In addition to the suggestions offered by Creswell (2007), Merriam (2009) suggested six basic strategies to increase credibility: triangulation, member checks, engagement in data collection adequately, reflexivity and peer examination.

One of the strategies that was used in the present study to ensure credibility was triangulation, defined as "a validity procedure where researchers look for convergence among multiple and different sources of information to form themes or categories in a study" (Creswell & Miller, 2000, p. 126). Another way of defining triangulation is its being "a process of using multiple perceptions to clarify meaning, verifying the repeatability of an observation or interpretation" (Stake, 2000, p. 443). In fact, four different types of triangulation in the literature on qualitative research are mentioned. These are data triangulation, investigator triangulation, methodological triangulation and theory triangulation (Creswell & Miller, 2000; Creswell, 2007; Patton, 2002). In the present study, data triangulation, investigator triangulation and methodological triangulation were used to establish credibility. To be more precise, 16 prospective middle school mathematics teachers were used as a source of data (data triangulation). Moreover, multiple sources of data (methodological triangulation), which were observation, interview and documents, were collected. The other strategy for credibility was counteracting researcher bias by explaining the researcher's initial beliefs and perspectives towards the current study in detail. In addition, all the procedures and the administrations were managed and followed by the researcher to assure the prolonged involvement. Moreover, the findings of the previous research were investigated, a thick description was made and debriefing sessions with the supervisor and the thesis committee members were conducted to benefit from their feedbacks.

The other important criterion to assure the trustworthiness of a qualitative study is transferability, which refers to external validity. The concern of transferability is the generalizability of the findings of a research study. Although the purpose in qualitative studies is not to make inferences from a small sample and then to generalize them to a larger population, it is possible to ensure transferability when the sufficient data is provided (Merriam, 1998). For instance, it was suggested that providing thick descriptions for the readers can be one method to achieve transferability (Miles & Huberman, 1994). Similarly, Lincoln and Guba (1985) give the responsibility of providing adequate contextual information about the fieldwork site to the researcher. In this way, readers will have the chance to compare the results of that study with the findings of their own study to make inferences. Thus, as indicated by Shenton (2004), "…it is the responsibility of the investigator to ensure that sufficient contextual information about the fieldwork sites is provided to enable the reader to make such a transfer" (p. 69).

The aim of the researcher of the present study was to gain an in-depth understanding of the argumentation of prospective middle school mathematics teachers in the geometry context; thus, the generalization of the findings to all prospective middle school mathematics teachers was not the concern of this study. However, our findings can be shared with the instructors of other universities which have similar characteristics. In order to ensure the transferability of the findings, the context of the study was tried to be explained in detail. Specifically, the following contextual information was presented in this study: the selection criteria of the participants, the findings of the pilot study, the number of participants in the pilot and main studies, the data collection tools, data collection methods that were employed, and the data collection procedures in detail in the method section. Ultimately, a thick description of the study was provided for the readers for the sake of the transferability of the findings.

The third criterion for the trustworthiness of qualitative studies is dependability, which corresponds to reliability in quantitative studies. Merriam (1998) defined reliability as "the extent to which research findings can be replicated" (p. 220). However, the concern is not whether the same results are obtained by other researchers in qualitative studies. It is whether the results of the study are dependable and consistent with the data (Merriam, 1998). According to Yin (2003), the aim of reliability in qualitative studies is to minimize the errors and biases in a study. In the related literature, researchers suggested some methods to ensure dependability. For instance, Shenton (2004) advised researchers to describe the research design, implementation procedure, and data collection procedure in detail and then evaluate the effectiveness of the process to provide dependability. In addition, he asserted that another method is to establish credibility, which helps to ensure dependability. Another researcher who suggested techniques to ensure dependability was Patton (2002). He suggested explaining an investigator's position, triangulating the data and doing audit trail to establish dependability. Lastly, Creswell (2007) stated that getting detailed field notes and *obtaining intercoder agreement* are other methods to ensure reliability in qualitative studies.

In order to address dependability of the present study to a certain extent, the researcher described the research design, implementation and data collection procedure in detail as Shenton (2004) advised. Furthermore, in the present study, as Creswell (2007) suggested, intercoder agreements from another doctoral student and an instructor were obtained in several parts of the data analysis process for dependability. The researcher initially discussed the codes with her advisor and then coded the data with the intercoder. Specifically, the researcher and the second coder coded the data individually and then they came together to discuss their codes. They compared their initial codes until they reached a total consensus. Furthermore, the researcher ensured the dependability of her study by establishing the credibility of the study as suggested by Shenton (2004).

The last criterion to ensure trustworthiness in qualitative studies is confirmability. Confirmability corresponds to objectivity in quantitative studies. Shenton (2004) asserted that it should be ensured that the findings are the results of the experiences and ideas of the participants and they are independent of the characteristics and preferences of the researcher. Shenton (2004) and Lincoln and Guba (1985) suggested triangulating the data to reduce the effects of researcher bias to assure confirmability. Likewise, Miles and Huberman (1994) emphasized the importance of the researcher's admissions for his/her bias for confirmability.

The methods utilized to ensure confirmability of the present study were triangulation of the data, in-depth description of the methodology, and the admission of bias. Triangulation was assured as mentioned above by collecting data from different sources to see whether they converged to the same findings (Lincoln & Guba, 1985; Shenton, 2004). Additionally, a detailed description of the methodology was helpful in ensuring confirmability. Lastly, the researcher bias was reduced in order to assure confirmability. In the present study, the entire data collection procedure was conducted by the researcher herself, so she was in active interaction with all participants. For this purpose, the researcher studied and improved her argumentation facilitating skills and interview skills to a considerable extent in the pilot study. In addition, she repeatedly examined the video recordings of the application and looked into whether or not she had interfered in anything.

Thus, the researcher believes that she was honest and objective as much as possible throughout the whole process and that her study did not yield biased findings.

3.7 Researcher role and bias

In qualitative studies, which are open-ended and less structured, the researcher can be considered as the key instrument for collecting and analyzing data (Merriam, 1998). The researcher's bias causes the researcher to find what she/he wants to find unconsciously in her/his study since her/his views and beliefs will affect her/his interpretations. In order to lessen and control the researcher's bias, reflexivity, which refers to a researcher's active engagement in critical self-reflection on her/his bias, was proposed (Johnson, 1997).

During the study, the researcher was working as a teaching assistant at the university at which the participants of the present study were studying. Therefore, she had a strong relationship with prospective middle school mathematics teachers. In addition to being a teaching assistant, she was also the supervisor of some of the senior prospective middle school mathematics teachers who took part in the pilot study. Since the pilot study of the present study was conducted in the spring semester and then the main study was conducted in the following fall semester, the participants of the pilot study had graduated. Thus, the researcher had no participant who was in a supervisor-student relationship with the researcher in the main study. However, they knew the researcher from the other courses that they had previously taken. This was advantageous for the researcher because as soon as she explained the purpose of her study, they accepted to participate in the study and share their knowledge voluntarily. Moreover, she knew all of them personally and had the chance to select the most suitable participants for the study by means of purposeful sampling. In this way, she was able to work with the participants who could provide rich and in-depth information for the study.

In order to lessen the effect of the research assistant-student relationship, the researcher followed several strategies. After giving information about the application, she told them that they could participate in the study on a voluntary basis which meant that their participation was not compulsory. In addition, the researcher ensured the confidentiality of their answers and dialogs. This meant that the researcher was the only person who had the access to the data. Furthermore, the researcher analyzed the data using pseudonym names for the participants in order to eliminate any bias and or favoritism. Therefore, their grades in the program were not affected by any professor or teaching assistant since they did not see any of the data.

The researcher was involved in the process as a teacher who guided the discussions of the two groups and who held the interviews with the focus groups during the study. She wanted to obtain information about the reasoning of the participants as much as possible since the justification of the ideas has a great importance in argumentation theory. Her aim was to provide an argumentative environment and identify the arguments of the participants. Therefore, she placed cameras and audio recorders onto the tables of all the groups by taking their permissions in order not to miss any conversation. In general, the individuals tended to express their conclusion (claim) by stating the information they had (data) without stating their justification (warrant) since they knew the reason themselves. However, the researcher needed to hear their reasoning and warrants to analyze their argumentation. Thus, at the beginning of the application, she asked all the participants to talk about their thoughts all the time. That is, she requested them to think loudly all the time. She also reminded the participants that what was important about the study was their justifications and how their thinking processes, not the accuracy of their solutions. Moreover, she asked them to listen to the responses of the participants who were showing their solution(s) on the board and to contribute to the discussion by supporting or refuting their solution(s) in order to increase interaction among the students. Meanwhile, the researcher asked probing questions to make them think deeper about their solutions and justifications. In addition, she asked them to find more than one solution to the questions if possible. In order to increase the interaction between the participants in pairs, she gave only one computer and one worksheet to each pair in the GeoGebra group, and only one worksheet, one protractor, one ruler and one compass in the Paper-Pencil group.

The researcher gave them sufficient time for pair-work discussions and class discussions by asking them whether they needed extra time to discuss further. The role of the instructor was important to supply a good argumentative environment. Therefore, she always asked questions which encouraged students to justify their answers since they sometimes did not need to express warrants for their claims. Some of these questions were as follows: 'How do you know that it is true?, What does that mean to you?, Can you tell me more about your thinking process…?, Why do you think so?, Are there any other ideas?, Is there anyone who doesn't agree with this idea?'. Additionally, the researcher did not judge the participants for the accuracy or inaccuracy of their answers while they were presenting their solutions. Instead, she questioned and made other participants think about the solution to find the correct answer together. In this way, she guided the discussion by making the participants reveal their reasoning.

After the applications, the researcher also held an interview with the focus pairs, one from the GeoGebra group and one from the Paper-Pencil group. She provided them with a place where they would feel comfortable in terms of place and timing, so they would not feel in a rush while answering the questions.

Argumentation is a topic that has been studied within the science context for a long time and it is a new area to be studied in mathematics education. Determining the arguments and their elements, classifying the warrant types and argumentation structure issues were challenging for the researcher, who studies argumentation in mathematics. During the data analysis process she sometimes needed an intercoder in order to decrease researcher bias because she was undecided in determining the arguments and their elements, and classifying the argumentation structures and warrant types. Thus, she obtained intercoder agreement of two doctoral students and an instructor at different stages of the data analysis process.

Lastly, being affected from the findings of the literature was another type of researcher bias which the researcher needed to pay attention to. This means that a researcher who is influenced by the theories in the related literature would try to reach similar findings to be in line with the literature. However, the researcher of the present study was not influenced by the literature and carried out the present study objectively. She did not try to arrive at findings similar to those reported in the literature. In fact, the researcher implemented a new data analysis method to seek an answer to the second research question in order to contribute to the literature with new findings.

3.8 Limitations

The primary limitation of the present study is representativeness since the purposefully selected participants of the present study were 16 prospective middle school mathematics teachers who were 4^{th} grade students in the teacher education program at one public university in Ankara. Moreover, the content of the geometry tasks implemented were two triangle and two circle tasks so the content of the geometry tasks was also a limited. Therefore, the findings were limited to the answers of these 16 prospective middle school mathematics teachers' and the two mathematical concepts, triangles and circles. Thus, the readers should evaluate the findings by considering the limitations.

The researcher efficiency in facilitating argumentation was the other limitation of the present study. After reading the essential teacher actions in argumentation from the literature, the researcher tried to orchestrate the argumentation in this study for the first time. Although the researcher conducted a pilot study in order to reduce the effect of this limitation, the researcher could still have some deficiencies in following all the arguments during the applications. Some of the arguments did not have some components such as warrant and data components. The reason could be that the researcher did not question the argument for justification effectively enough or missed the argument in a collective discussion.

CHAPTER IV

RESULTS

The findings of the present study are summarized in this chapter under three main sections, each of which addresses one of the research questions of the study. The first research question, which is taken up in the first section, was 'What is the nature of argumentation structures of prospective middle school mathematics teachers while solving geometry tasks in the GeoGebra and Paper-Pencil groups?'. In the second section, the characteristics of the local arguments (core arguments including claim, data and warrant components) were analyzed. Specifically, the second part addresses the characteristics of local arguments based on the flow of argument components (claim, data, warrant) that prospective middle school mathematics teachers express while solving geometry tasks in the GeoGebra and Paper-Pencil groups. In the third section, local argumentations (only warrant components of the local arguments) within the global argumentation structures are analyzed. That is, the third part dwells on the characteristics of local argumentations that prospective middle school mathematics teachers utilize while solving geometry tasks in the GeoGebra and Paper-Pencil groups.

4.1 Nature of argumentation structures developed in the geometry context

As mentioned above, what this study intended to reveal was the nature of argumentation structures of prospective middle school mathematics teachers while solving geometry tasks in the GeoGebra and Paper-pencil groups. In order to determine the general pattern of the argumentation, all arguments and the relationships among them were examined. For this purpose, a discussion for each geometry task was analyzed as a whole with a schematic representation of the overall argumentative structure. This layout of the structure of the argumentation as a whole was defined as the global argumentation structure (Knipping, 2008). Categorization for the global argumentation structures proposed by Reid and Knipping (2010) was adapted in the current study. Then, the global argumentation structures developed in the geometry context were identified based on the adapted categorization. These global argumentation structures are presented in detail with sample conversations in this section.

In this study, 4 geometry tasks (GT) were analyzed both for the GeoGebra group and the Paper-Pencil group. Two of the tasks were triangle activities while the other two were circle activities. The analysis revealed five main global argumentation structures: Source-Structure, Reservoir-Structure, Spiral-Structure, Line-Structure and Independent Arguments. While source-structure, reservoir-structure, and spiral-structure were obtained from a study by Reid and Knipping (2010), line-structure and independent arguments-structure did not exist in the literature and emerged from the data of the present study. Table 4.1 illustrates the global argumentation structures and the number of times they emerged in the discussions of each geometry task.

Table 4.1 Global argumentation structures that emerged in the working groups for each geometry task

	GeoGebra	**Paper-Pencil**
Geometry Task 1	1 Reservoir-structure 1 Spiral-structure	1 Reservoir-structure 1 Line-structure 8 Independent arguments
Geometry Task 2	3 Spiral-structure 2 Independent arguments	2 Spiral-structure 2 Independent arguments 1 Source-structure
Geometry Task 3	1 Reservoir-structure 1 Line-structure 4 Independent arguments	1 Reservoir-structure 1 Line-structure 6 Independent arguments 1 Source- structure
Geometry Task 4	2 Spiral-structure 5 Independent arguments 1 Source-structure 1 Reservoir-structure	3 Spiral-structure 5 Independent arguments

As revealed in Table 4.1, the source-structure type of argumentation emerged three times in the discussions throughout the entire application. Moreover, the reservoir-structure was observed five times, while the spiral-structure was used eleven times. The global argumentation structure which was used most frequently – eleven times - within both the GeoGebra and Paper-Pencil groups was the spiral-structure. The other global argumentation structure which emerged in the present study was the *line-structure* and it was used three times throughout the application. Finally, there were *independent arguments* which also emerged in the study and it was frequently used by both groups. It emerged thirty-two times in total.

When the global argumentation structure distribution is considered task by task, some similarities and differences between the GeoGebra and Paper-Pencil groups can be observed. In geometry task 1, the discussion of both groups included one reservoir-structure. In addition, while one spiral-structure emerged in the GeoGebra group, there were one line structure and eight independent arguments emerging in the Paper-Pencil group. In geometry task 2, the GeoGebra group used three spiral-structures and two independent arguments. Similarly, the Paper-Pencil group used two spiral-structure and 2 independent arguments. Additionally, one source-structure emerged in the Paper-Pencil group. The argumentation for the geometry task 3 was similar for both groups. The GeoGebra group used one reservoir-structure, one line-structure and four independent arguments. Similarly, the Paper-Pencil group used one reservoir-structure and 6 independent arguments. Unlike the GeoGebra group, one source-structure was used in the Paper-Pencil group in geometry task 3. In the last geometry task, the GeoGebra group used variable global argumentation structures. They used two spiral-structure, one source-structure, one reservoir-structure and five independent arguments. On the other hand, three spiral-structure and five independent arguments were used in the Paper-Pencil group.

In order to make these structures clearer, sample conversations and argumentation structure schemas for each global argumentation structure are explained in detail below.

4.1.1 The source-structure argumentation

In the present study, the source-structure type of argumentation was used twice. One of them emerged during the discussion on geometry task 4 within the GeoGebra group and the other one emerged during the discussion on geometry task 3 within the Paper-Pencil group. The source-structure example could be given from geometry task 3 in the Paper-Pencil group. In geometry task 3, two circles, each of which passes through the center of the other circle, was given as illustrated in *Figure 4.1(a)*. The circles intersect at points H and E, and a line from point E intersects the circles at points F and G. The questions asked were as follows:

If |FG|= 6, compute the area of the triangle FGH? Justify your solution.

If r is the measure of the radius of each circle, find the least value and greatest value of the area of triangle FGH. Justify your solution.

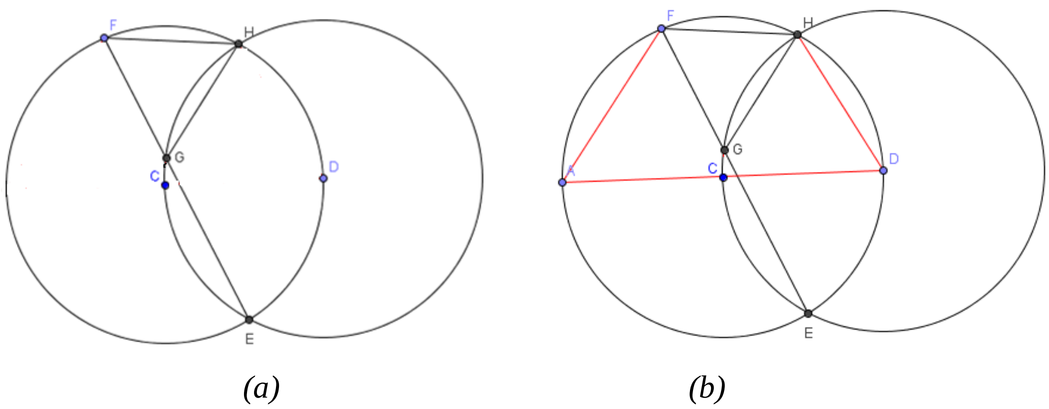


Figure 4.1 The shape of geometry task 3 and Gözde's additional drawings (red segments)

The red segments on the shape in *Figure 4.1(b)* were the additional segments which were drawn by Gözde while she was explaining her solution on the whiteboard.

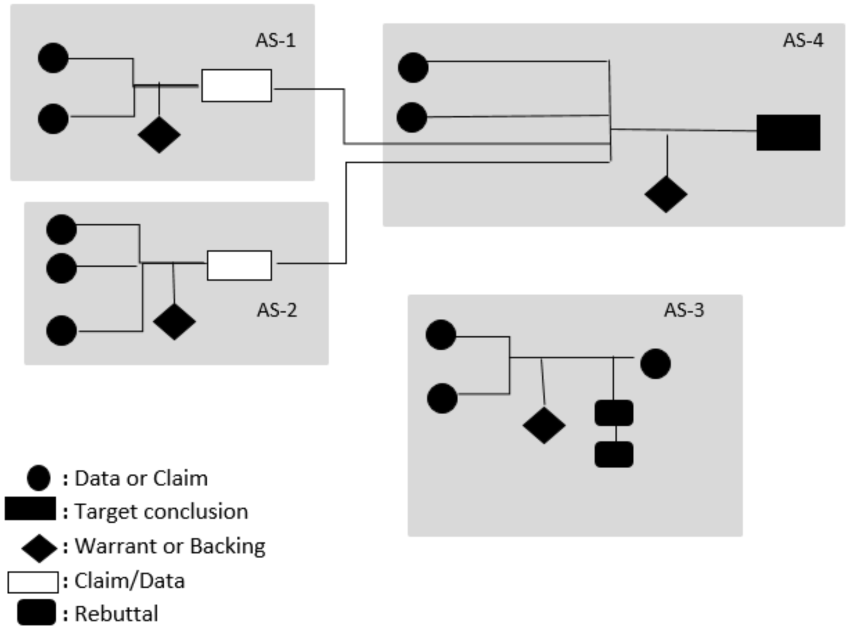


Figure 4.2 A source-structure example from geometry task 3 in the PPG

The overall global argumentation structure was like in *Figure 4.2*. In AS-1, Gözde drew the radiuses |CH|, |HD|, |CD|, |CE| and |DE| and claimed that 'The triangles HCD and CDE are equilateral'. Then, she used these equilateral triangles and the arcs of the circle to conclude that ∠HFG=60° in AS-2. At this point, Gözde drew the red lines as in *Figure 4.1(b)* and claimed that arc FH=60°, which was refuted by the other participants and the teacher. The details of this refutation are given within the conversation below. Then, she used these two claims - AS-1 and AS-2 - as data for AS-4 with additional data '∠HDE=120° , ∠HCE=120°' and 'arc HE=240°' and claimed that 'triangle FGH was an equilateral triangle in AS-4'.

The typical characteristics of source-structure argumentation and the argumentation streams including those characteristics are demonstrated below:

- The argumentation streams which were not connected to the main structure (AS-3)
- Parallel arguments for the same conclusion (AS-1 and AS-2)
- Argumentation steps which have more than one datum and each of these data is the conclusion of an argumentation stream (AS-1, AS-2, AS-3 and AS-4)

- Refutations refuting mostly data in arguments (AS-3)

The parallel arguments in this structure were AS-1 and AS-2, which were both leading to the conclusion that the triangles HCD and CDE were equilateral. All argumentation streams had more than one datum, which were presented with circles as in *Figure 4.2*. Thus, it can be concluded that this global argumentation structure was suitable for source-structure.

AS-3 includes the refutation which was not connected to the main structure. Therefore, presenting some details about this argument and how it was refuted could be of benefit. The discussion of AS-3 occurred as in the following conversation. The teacher asked the students to explain their solution to the class and Gözde came to the board voluntarily.

Gözde	: We said ∠HCD is equal to 60°.
Teacher	: Yes
Gözde	: │AD│ is the diameter of the circle (**D47**). Since arc HD is 60° we can say that arc AH is equal to 120° (**W47**).
Teacher	: Yes, true.
Gözde	: From alternate-interior angles ∠ACF sees the arc which is 60°. We know that arc AH is equal to 120° so arc FH will be 60° (**C47**).
Erhan	: Ok but I did not understand why arc AF is equal to 60^0. How did you find that triangle ACF is equilateral? (**R47**)
Gözde	: From alternate-interior angles. It is as if a cross is drawn from the center C with chords │AD│ and │FE│.
Teacher	: But you still do not have a connection with triangle FGH.
Bahar	: But the drawing on the board does not resemble our drawing on our worksheet.
Teacher	: I drew the initial shape like this. You have drawn the chord EF as if it crosses from the center C. But in the original shape it does not cross from the center C. (**R47**)
Bahar	: You are right. We thought that we obtained an isosceles trapezoid AFHD but we made a mistake in our drawing.

In this argumentation, Gözde drew the red lines indicated in *Figure 4.1* and talked as if chord │FE│ crossed over point C and conjectured that 'arc │FH│= 60°'. The teacher and another student refuted the data of this argument by showing that ∠ACF and ∠ECD were not alternate-interior angles and chord |FE| did not cross over point C, so arc |FH| could not be 60°. Thus, the discussion, in terms of the Toulmin model, can be drawn as illustrated in *Figure 4.3* below.

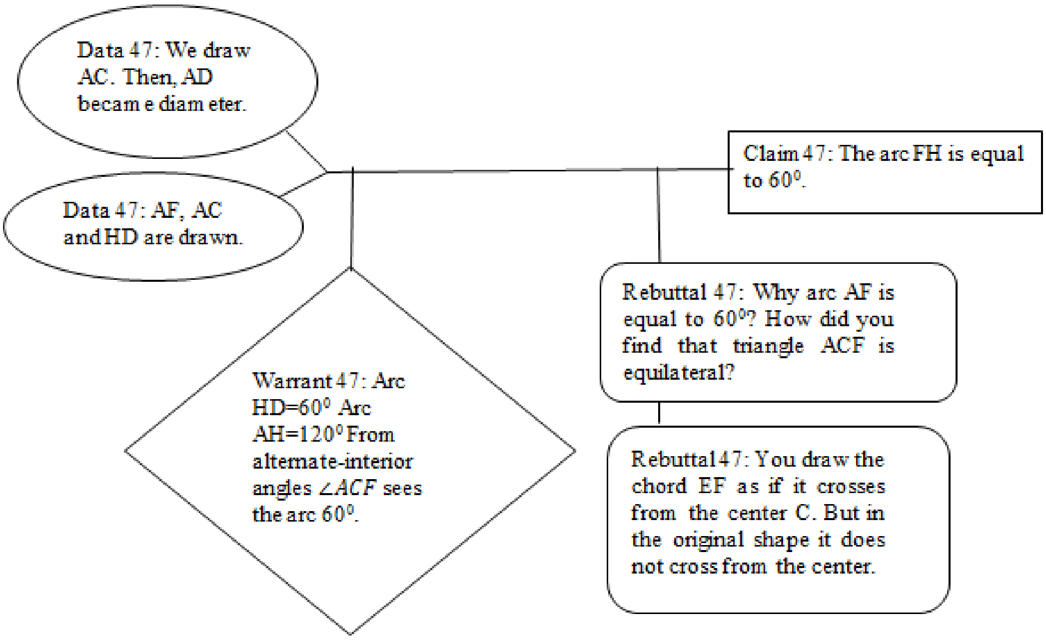


Figure 4.3 Toulmin's schema drawn for AS-3

4.1.2 Reservoir-structure argumentation

The reservoir-structure emerged 5 times in the current study. Three of them emerged in the GeoGebra group in geometry tasks 1, 3 and 4, while two of them emerged in the Paper-Pencil group in geometry tasks 1 and 3. The following example, which is an example of the reservoir-structure, comes from the discourse of the GeoGebra group regarding geometry task 1. The task was presented in the worksheet in the following way:

> 'ABC is a triangle. The midpoints of sides |AB| and |AC| are points D and E, respectively. F and G points are placed on the side |BC| so as to be |BG|=|CF|. The segments |DG| and |EF| intersects at point H.
> When does |AH| become the angle bisector of ∠A? (Think about all types of triangles). Explain your reasoning and justify your solutions.'

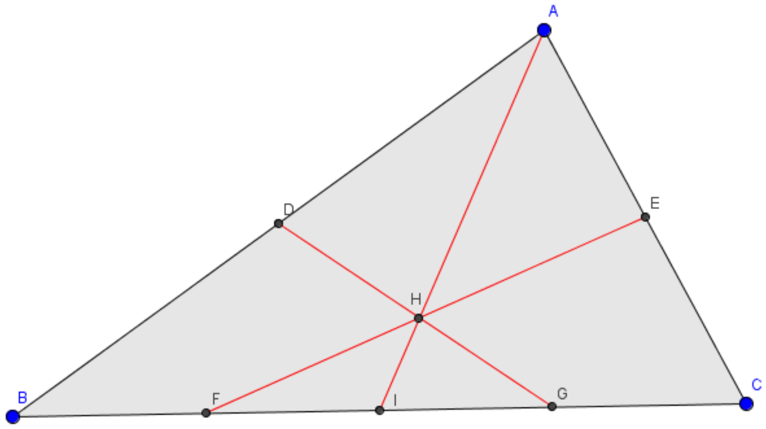


Figure 4.4 The shape of geometry task 1 and students' drawings (red segments)

The participants discussed among themselves to find the triangle types in which |AH| became an angle bisector when the givens (see Figure 4.4) were satisfied.

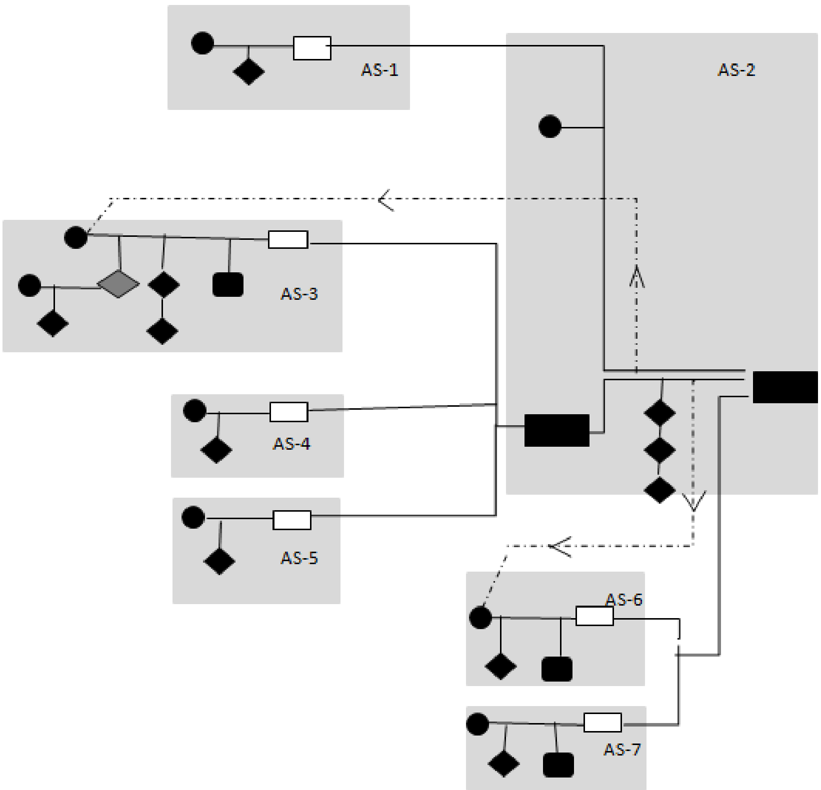
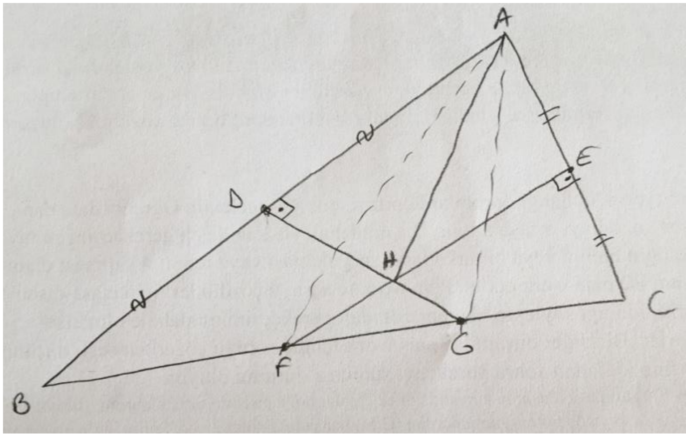


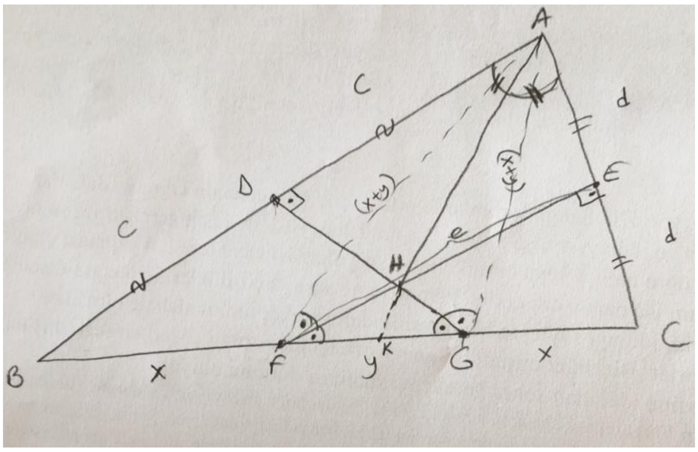
Figure 4.5 A reservoir-structure example from geometry task 1 in the GG

Firstly, the students decided how to place the points F and G on segment │BC│ in AS-1 illustrated in Figure 4.5. In AS-1, they marked F to the left, G to the right on segment │BC│ and drew the shape based on the givens in the task. Then, after a dragging move on GeoGebra concluded that │AH│ was an angle bisector when triangle ABC was isosceles and equilateral triangles in AS-2. Then, the teacher asked why they had placed points F and G in that order on segment │BC│. She asked what if points F and G were switched. Upon this question, which required the participants to provide a more detailed solution, the participants thought about it again, which meant reasoning moved backwards to AS3, AS4 and AS5 (a dashed line shows the direction of reasoning). They claimed that when they placed points F and G reversely, point H would again be on the line passing through │AI│ in equilateral triangles (AS-3), in isosceles triangles (AS-4), and in scalene triangles (AS-5). They explained their additional justification, clarifying their reasoning sufficiently and concluding with the intermediate target conclusion which was 'In any triangle, when point F and G are inversely placed, point H will change place on the line passing through │AI│'. Afterwards, they solved the task again when triangle ABC was an isosceles and an equilateral triangle separately in AS-6 and AS-7, respectively. They asserted that 'When triangle ABC is isosceles, │AH│ will be an angle bisector' (AS-6) and 'when triangle ABC is equilateral, │AH│ will be an angle bisector'. This was also again a reasoning that moved backwards, which is a characteristic of a reservoir-structure. In this type of global argumentation structure, there can also be refutations as in AS-3, AS-6 and AS-7, although all refutations were not successful in refuting the claim. As it can be inferred, reasoning moved backwards to give more detail about their justification, and the existence of the intermediate target conclusion were the properties of the reservoir-structure, so this argumentation structure could be categorized within the reservoir-structure.

The second example for the reservoir-structure is from the Paper-Pencil group's geometry task 1. The geometry task has already been explained in the previous example. The participants discussed among themselves to identify the triangle types in which │AH│ became an angle bisector when the givens of geometry task 1 were satisfied. Güler came to the board and started solving the task by drawing the shape. She selected any triangle ABC and placed F and G in such a way that F was on the left, G was on the right to segment |BC| (see Figure 4.6(a)).



(a)



(b)

Figure 4.6 The shape drawn on the board by Güler for geometry task 1

In the givens, points D and E were the midpoints of segments |AB| and |AC|. Güler assumed that F and G were placed on |BC| in such a way that '|FE| ⊥ |AC|' and '|GD| ⊥ |AB|'. Then she drew |AF| and |AG|. The teacher asked whether or not she could take both of them perpendicular at the same time. This point remained as a question in the minds of all students and Güler continued to present her solution.

The global argumentation structure of this solution is illustrated in Figure 4.7 below.

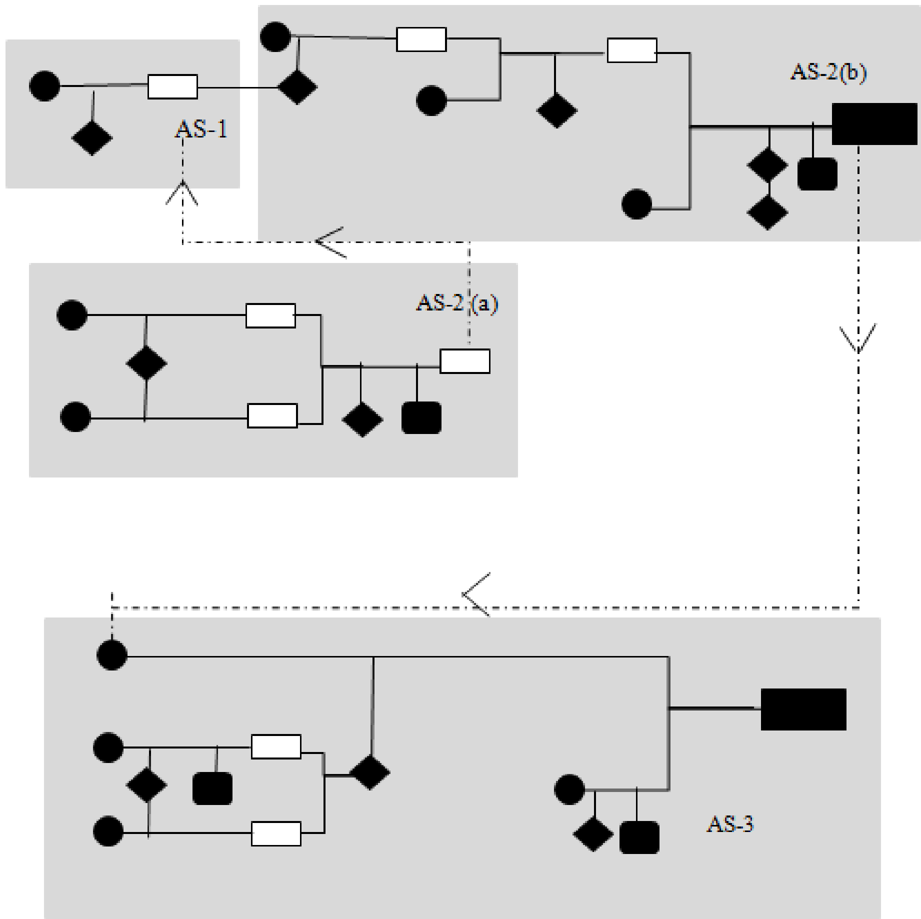


Figure 4.7 A reservoir-structure example from geometry task 1 in the PPG

In AS-1, Güler conjectured that the triangle AFG was an isosceles triangle by using the isosceles triangle properties of triangles AFC and AGB to find the lengths of |AF|=|AG|= x+y. Then, she used the isosceles triangle properties again and conjectured that |GD| was the angle-bisector of ∠AFG and in the same way |FE| was the angle-bisector of ∠AFG. Thus, she expressed an intermediate conclusion of AS-2(a), which was '|AH| is also an angle-bisector of ∠BAC'. This argument was refuted by other participants since AH was the angle-bisector of ∠FAG, not ∠BAC. Therefore, the reasoning moved backwards to AS-1 and continued with AS-2(b). Güler stretched |AH| to segment |BC| and claimed that '|AK|⊥ |BC|' and segment

|FK|=|KG|=y/2. Then, an angle-bisector ratio was written for triangle ABC for segment |AK|. That is, $\frac{2c}{x+(\frac{y}{2})} = \frac{2d}{x+(\frac{y}{2})}$. Then, they concluded that segment |AH| was an angle-bisector when c=d, which meant that triangle ABC was an isosceles triangle. Then, Okan said that three perpendicular segments and three angle-bisectors coincided at one point, so we could say that triangle ABC was an equilateral triangle in AS-2(b), which was an intermediate target conclusion. However, another participant, Bahar, noticed that all three perpendicular segments did not belong to the same triangle ABC. That is, |GD| was the perpendicular segment for triangle AGB, |FE| was the perpendicular segment for triangle AFC while |AK| was the perpendicular segment for triangle AFG (see *Figure 4.6(b)*). Similarly, these three segments were not the angle-bisectors of triangle ABC either. Thus, the argument claiming that triangle ABC was an equilateral triangle was refuted. Again the students' reasoning moved backwards and they questioned the solution again. In AS-3, İnci came to the board and overviewed the assumptions and the solution and concluded that |AH| was the angle-bisector of triangle ABC when triangle ABC was an equilateral and isosceles triangle, not a scalene triangle. The justification was the angle-bisector ratio explained above, and the intersection of the three perpendicular segments were not related to the solution. Furthermore, the only assumption was the one which Güler had stated at the beginning of the solution while placing points F and G on segment |BC| in such a way that '|FE| ⊥|AC|' and '|GD|⊥|AB|' at the same time. Ultimately, it was deduced that this argumentation was suitable for the reservoir-structure since the students moved back to some parts of the solution again and again in order to give more detailed justifications, there were refutations in AS-2(a), AS-2(b) and AS-3, and there was an intermediate target conclusion during the process before reaching the target conclusion.

4.1.3 Spiral-structure argumentation

The spiral-structure was one of the most frequently used structures (11 times) in the current study. In the GeoGebra group it emerged once in geometry task 1, 3 times in geometry task 2, and 2 times in geometry task 4. On the other hand, in the Paper-Pencil group, it was seen twice in geometry task 2, and 3 times in geometry task 4. The following example comes from the GeoGebra group's argumentation in geometry task 4 to represent the spiral-structure. Figure 4.8 illustrates the given shape. In the task, the circle with center A and the semi-circle with diameter $|AG|$ were given. The chord $|CD|$ was bisected by $|FG|$ at point E. Moreover, $|HE|$ was perpendicular to $|FG|$ as given. The tasks of the students were:

Prove that $|CE| = |HE|$ and justify your reasoning.

Show whether the theorem is trivial if chord $|FG|$ is a diameter of the first circle, or if $|FG|$ coincides with $|CD|$. Justify your reasoning.

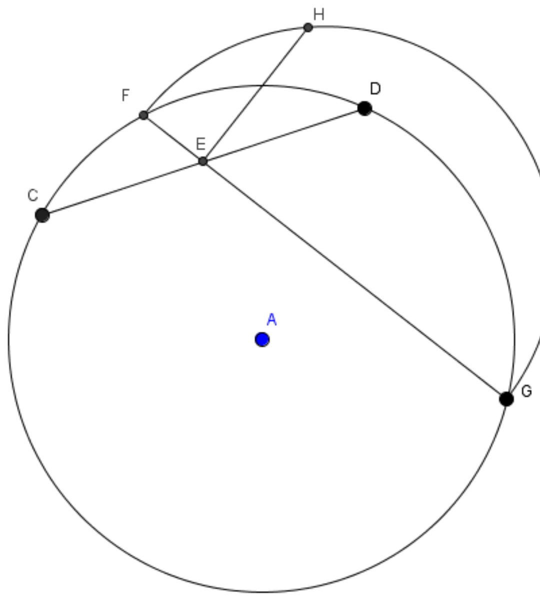


Figure 4.8 The shape of geometry task 4

The overall argumentation structure was drawn as in *Figure 4.9* for spiral-structure. The typical characteristics and the argumentation streams including those characteristics were revealed as indicated below:

- The argumentation streams which were not connected to the main structure (AS-C)
- Parallel arguments for the same conclusion (AS-A, AS-B and AS-D)

- Argumentation steps which have more than one datum and each of these data is the conclusion of an argumentation stream. (AS-A)
- Refutations refuting mostly data in the arguments (AS-C)

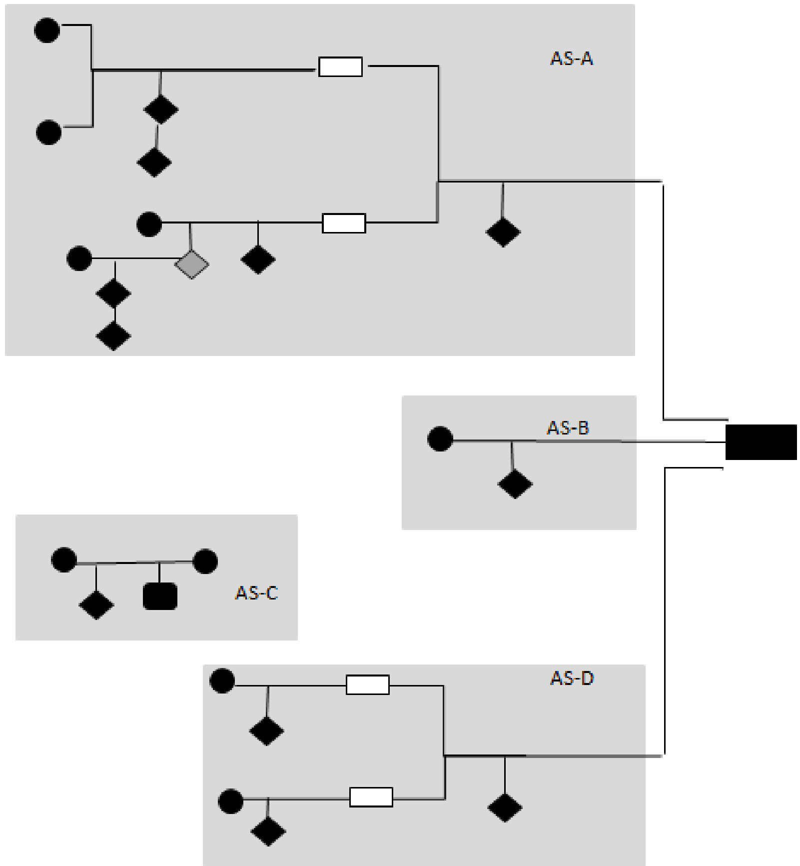


Figure 4.9 A spiral-structure example from geometry task 4 in the GG

The target conclusion of the global argumentation was $|CE| = |HE|$. In AS-A, the students initially used the Euclid's theorem and found $|HE|^2 = |FE|.|EG|$, and then they used angle-angle-angle triangle similarity and found $|CE|^2 = |FE|.|EG|$. As it can be seen, these two equations were equal to each other so the students concluded that $|CE| = |HE|$ The details of this argumentation stream are given below. In AS-B, students empirically discovered the measurement and dragging options of the dynamic geometry program GeoGebra that $|CE| = |HE|$. Specifically, they measured the lengths of $|CE|$ and $|HE|$, and then dragged the shape to see the relationship between these lengths. There was an argument stream which was not connected to the main structure, AS-C, since it was related to the second question of the task. In this argument, Özer claimed that when │FG│ was the diameter of the first circle with center A, │CE│ and │HE│ would not be equal since the place of │HE│ moved to the center of the semicircle and it did not fit the givens of the task. Other participants refuted this claim by showing their GeoGebra drawing and justifying the dynamic property of segment │HE│. Finally, in AS-D, the students produced the equality │CE│^{2}= a^2-x^2 from the intersecting chords theorem where │OG│= a , │EO│= x and point 'O' was the center of the semicircle. Then, they produced the equality │HE│^{2}= a^2-x^2 from the Pythagoras' theorem in the right-angled-triangle HEO. These two equations were equal to each other so they claimed that │CE│=│HE│which was again the target conclusion of the global argumentation structure. This argumentation structure fitted the spiral-structure since the three parallel arguments, AS-A, AS-B and AS-D lead to the target conclusion that the two segments │CE│and │HE│were equal in the end. That is, the parallel argumentation streams were at the end of the global argumentation structure. Moreover, each parallel argumentation stream reached the target conclusion separately and was self-contained.

The conversation of one of the parallel argumentation streams AS-A is given below in detail with Toulmin's (1958) argument schema illustrated in *Figure 4.10*.

Beren : Firstly, we drew │FH│ and │HG│(**D55**). Then we saw that ∠FHG sees the diameter of the semicircle, so it is a right angle (**W55**).

Teacher : You mean the inscribed angle which sees the diameter?

Beren : Yes. │HE│⊥│FG│, so we could use the Euclid's theorem in this triangle (**W55**) and found the equality │HE│^{2}=│FE│.│EG│(**C55**).

Teacher : Ok. Then?

Beren : We drew chords │CF│ and │DG│ (**D56**) and saw that the inscribed angles ∠CFE and ∠EDG sees the same arc so these angles are equal to each other (**W56**).

Teacher : You are saying that they see the same arcs. Ok.

Beren : Similarly the angles ∠FCE and ∠DGE are equal to each other. And the alternate-interior angles ∠FEC and ∠DEG are equal to each other (**W56**). Therefore there is an angle-

angle-angle similarity between these triangles (**C56**). We can write the similarity ratio $\frac{CE}{EG} = \frac{FE}{ED}$. In the givens of the task, it says that |CE|=|ED|. Therefore the equality turns out to be $|CE|^2=|FE|.|EG|$ (**C57**).

Teacher : So you found another equation.

Beren : These two equations are equal to each other. Therefore, $|HE|^2=|CE|^2$ (**W58**). Finally, we can say that |HE|=|CE| since their squares are equal to each other (**C58**).

Teacher : Is there anybody who wants to add something to this solution? Do you all agree with Beren?

Aslı : We can find the same equality by applying the intersecting chord theorem to chords |CD| and |FG| (**W57**).

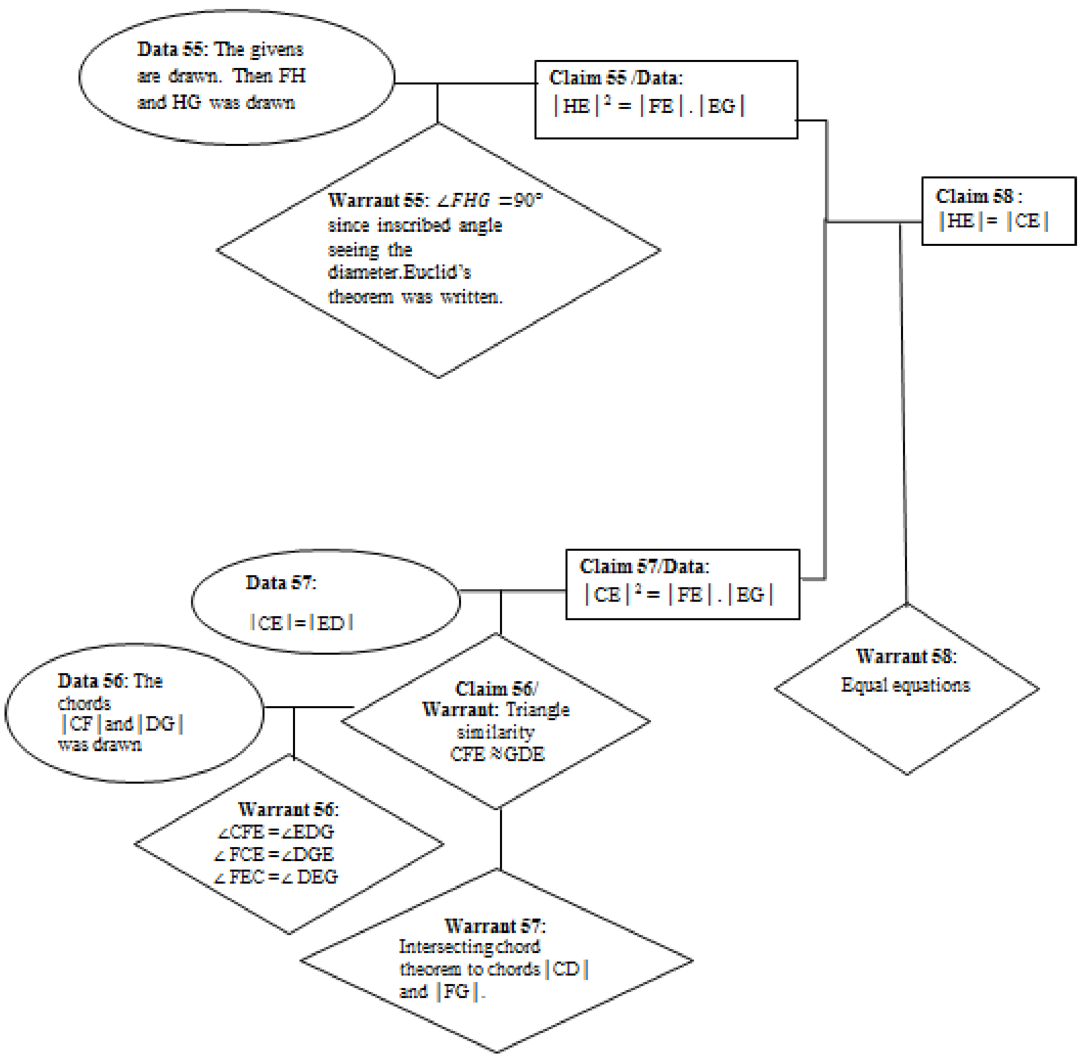


Figure 4.10 Toulmin's schema drawn for AS-A

In AS-A, the shape of the task was drawn and chords $|FH|$ and $|HG|$ were drawn to obtain the right triangle. Afterwards, the Euclid's theorem was used in triangle FHG to form the equality $|HE|^2=|FE|.|EG|$. Then, chords $|CF|$ and $|DG|$ were drawn and similar triangles CFE and GDE were obtained by using the angle-angle-angle similarity, which became a warrant for the subsequent argument. Beren wrote the similarity equation to get $|CE|^2=|FE|.|EG|$. Then, in AS-C, the equality of these two equations resulted in the equality of $|HE|=|CE|$, which was the answer of the task. Finally, Aslı added another warrant to the equation $|CE|^2=|FE|.|EG|$, which was based on the intersecting chords theorem.

4.1.4 Line-structure argumentation

In the present study, the line-structure emerged 3 times. One of them was found to emerge in geometry task 1 in the Paper-Pencil group, another one was in geometry task 3 in the GeoGebra group, while the third one was again in geometry task 3 in the Paper-Pencil group. The example for line structure comes from the discourse of geometry task 3 in the GeoGebra group. The geometrical shape used in geometry task 3 was as displayed in *Figure 4.11* and the details of the task were previously given in the source-structure section since the example of the source-structure was also from geometry task 3.

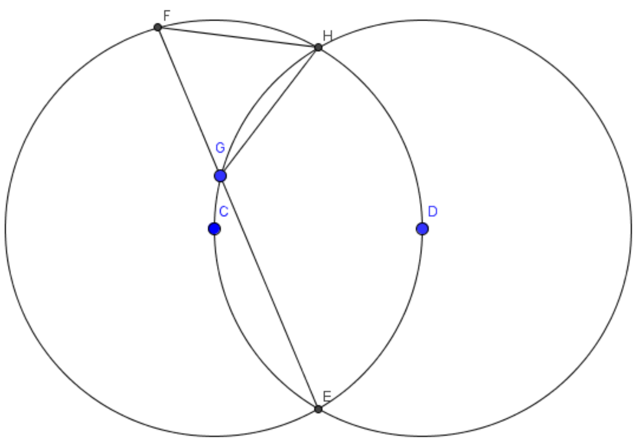


Figure 4.11 The shape of geometry task 3

The overall structure of the line-structure argumentation was drawn as displayed in *Figure 4.12*. The remarkable property of this structure is the existence of claim/data components through the global argumentation structure until the end of target conclusion. Each claim is used as data for subsequent argument steps, so it functions as a transition between them. There may be additional data to support arguments beside claim/data. In addition to that, the line-structure had no parallel argumentation streams from the beginning to the end of the global argumentation structure. The shape of the whole argumentation seems like a line, so this type of argumentation structure was defined as the line-structure.

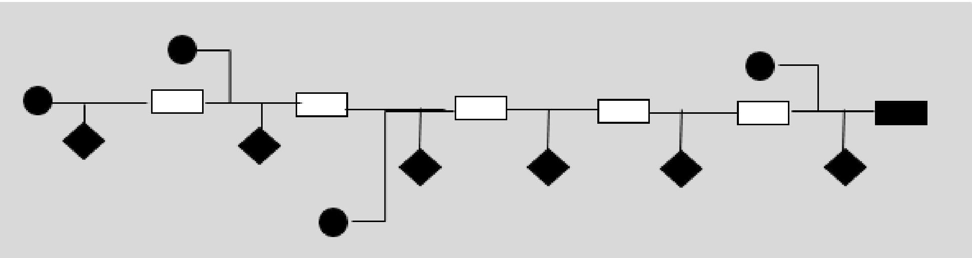


Figure 4.12 The line-structure example from geometry task 3 in the GG

In this geometry task, the students were trying to find the area of triangle FGH when the length of │FG│ was 6 units. The claims, data, and warrants were numbered from 34 to 39 in the original data of the GeoGebra group. Their contents are presented within the conversation below which took place during the application of geometry task 3:

Özde : Firstly we draw │HE│(**D34**). Then we looked at angle ∠FEH. This angle is an inscribed angle and sees chord │FH│ on the circle with center C and chord │GH│ on the circle with center D. Therefore, these chords are equal to each other (**W34**). Then we see that the arcs that ∠FEH see are equal to each other (**W34**), so │FH│=│GH│(**C34**).

Teacher : Yes. Do you all agree with Özde? Okay. But this is true since the circles are identical, aren't they? (**B34**)

Özde : Yes. Then I know that CD is the radius of both circles. We drew │CD│, │CH│ and │HD│(**D35**). These are all equal to each other and they are the radius of the circles (**W35**). Therefore, triangles CHD and CED are equilateral triangles (**C35**).

Teacher : Yes.

Özde : Similarly, we drew │CE│ and │DE│ which are the sides of an equilateral triangle CED (**D36**). Let's look at arc HDE. The central angle HCE the measure of which is equal to 120^0 sees arc HDE (**W36**). Thus, arc HDE=120^0 (**C36/D37**).

Teacher : Okay.

Özde : Let's look at angle ∠HFE. It sees arc HDE and it is an inscribed angle (**W37**), so its measure is 60^0 (**C37/D38**).

Teacher : Okay.

Özde : As we know that |FH|=|HG|, and the measure of ∠HGF is 60^0 (**W38**), I can say that FGH is an equilateral triangle (**C38**). Thus, the area of an equilateral triangle with one of the sides equal to 6 (**D39**) can be found with the formula $a^2\sqrt{3}/4$ (**W39**). The answer is $9\sqrt{3}$ (**C39**).

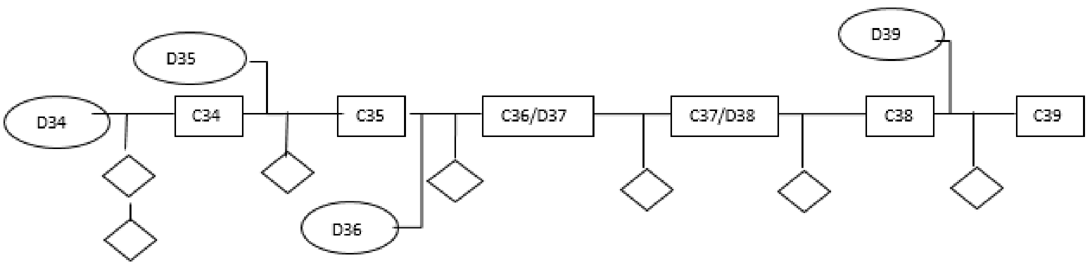


Figure 4.13 Toulmin's schema drawn for line-structure

As it can be inferred from Toulmin's schema drawn for line structure in *Figure 4.13*, the main characteristic of this structure included many 'claim/data' elements and, therefore, the arguments are connected to each other and formed a shape like a line. Moreover, there was no parallel argumentation streams and disconnected argumentation streams. Thus, this discussion was labelled as line-structure.

In C34, Özde found that chords │FH│ and │GH│ were equal by drawing │HE│ and by using one of the circle property which was 'the inscribed angles seeing the chords which have the same length are equal to each other'. Then Özde drew segments │CD│, │CH│ and │HD│, which were all the radius of the circle and concluded that triangles CHD and CED were equilateral in C35. There was no interruption or refutations made by the other students in the classroom. Then, Özde continued drawing │CE│ and │DE│, using the circle property ‘The measure of the arc is equal to the central angle seeing that arc’ as a warrant, and concluding that arc HDE=120^0 in C36. Subsequently, using the circle property ‘The measure of the inscribed angle is the half of the measure of the arc that it sees’ Özde claimed that ∠HFE=60^0 (C37). This information helped Özde to see that triangle FGH is equilateral (C38) since she first found the equality $|FH| = |GH|$ in C34 and identified one of the base angles of an isosceles triangle FGH as 60^0. Finally the area formula of the equilateral triangle was used to find the area of triangle FGH since one side of the triangle was given as 6 units in the task. As it can be inferred, the reasoning in line-structure flows quickly without interruptions and ends with the target conclusion. The claims were the main transition elements since they were used as data for the subsequent arguments. The characteristics of the line-structure were noticed in this conversation, so this global argumentation structure was labelled as the line-structure.

4.1.5 Argumentation based on independent arguments-structure

In the current study, the most frequently used global argumentation structure was the independent arguments-structure. It emerged in the GeoGebra group 11 times: twice in geometry task 2, 4 times in geometry task 3, and 5 times in geometry task 4. In the Paper-Pencil group, it emerged 21 times: 8 times in geometry task 1, twice in geometry task 2, 6 times in geometry task 3, and 5 times in geometry task 4. The sample shape of the structure was as illustrated in *Figure 4.14*.

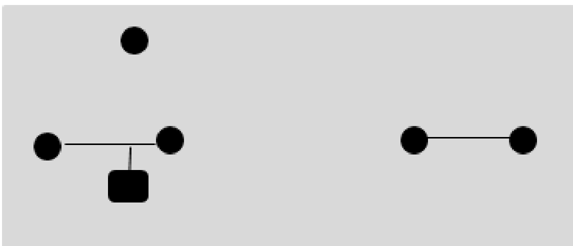


Figure 4.14 Independent arguments-structure from geometry task 1 in the PPG

The example above comes from the discourse produced in the Paper-Pencil group for geometry task 1 (see the shape used in the task in *Figure 4.4*) to represent the independent arguments-structure argumentations. The details of geometry task 1 were explained in the reservoir-structure section. The task of the students was to identify the triangle types in which |AH| became the angle bisector when the givens were satisfied. The arguments were not connected to each other. In C1, the participant was claiming that points F and G could be interchanged. After a few minutes, the other participant claimed that |AH| was an angle bisector when triangle ABC was an isosceles triangle, but she was refuted by her friend who said that 'Why do you think so? This is not an isosceles triangle'. However, she did not present a justification for her claim. After thinking on the problem for a while, they decided to work on the equilateral triangle (see *Figure 4.15*) and they drew the givens on the equilateral triangle. Then, they realized and claimed that |HI| also became an angle bisector of triangle FHG (C3), but again they did not justify their claim.

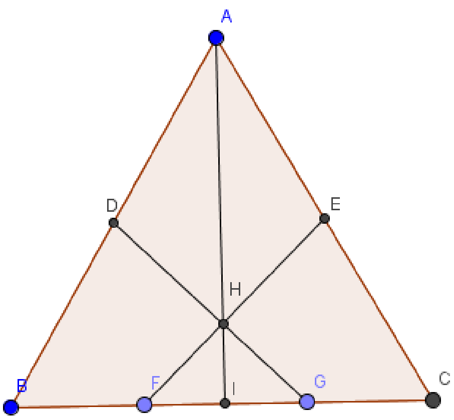


Figure 4.15 The shape drawn by the participants for geometry task 1

As it can be inferred from this discussion, the arguments were independent of each other, and they were stated as if the participants were thinking loudly. Therefore, the global argumentation structure includes single arguments without connections to other arguments.

4.2 The characteristics of the local arguments

While the previous section examined the global argumentation structures of the prospective middle school mathematics teachers, this section focuses on their local arguments within these structures. The second research question of the current study necessitated revealing the characteristics of the local arguments, based on the flow of argument components (claim, data, warrant), in the argumentation structures developed by prospective middle school mathematics teachers while discussing the geometry tasks in technology or paper-pencil environment. For this purpose, the researcher read through all the transcriptions of 4 geometry tasks of the GeoGebra and Paper-Pencil groups and numbered the argument components (claim, data, warrant) based on the order they were stated by the participants during the discussion. Then, the similar arguments were grouped to identify the categories of the argument types based on the flow of the components. The results revealed nine main types of arguments. These argument types emerged from the current study, not obtained from the literature. Each argument type is explained below in detail with sample arguments below in the following sections.

4.2.1 Local argument type 1: Data-Claim-Warrant (DCW)

In this type of argument, the participants first talked about the data they had, and then stated their claim. Then they justified their reasoning by stating the warrant. Thus, the order of flow was Data, Claim and Warrant respectively. The component flow of the DCW type argument is presented in *Figure 4.16*.

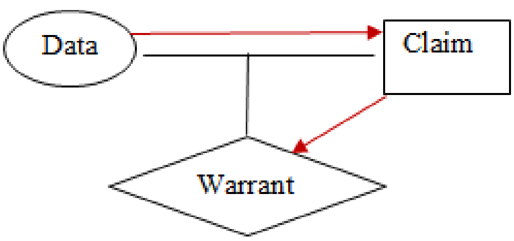


Figure 4.16 Component flow in DCW type of argument

The red arrows in *Figure 4.16* show the direction of the statement of the components. When the red arrows were examined, it was concluded that the statements of the components proceeded from the data to the claim and then to the warrant. To make it clearer, the following sample local argument from geometry task 2 in the GeoGebra group and the Toulmin schema (see *Figure 4.18*) was given for the DCW type of argument:

Geometry task 2 was related to triangles and it was presented in the worksheet as presented below with the shape displayed in *Figure 4.17*:

'Let P be any point on the median of |AG| of a triangle ABC. Let parallel lines m and n proceed through P to sides |AB| and |AC| of the triangle.

1. What relation is there between segments |EG| and |GF|? Explain your reasoning.

2. What if triangle ABC is equilateral or isosceles triangle? Can any generalization be made for the relationship between segments |EG| and |GF|? Explain your reasoning.

3. Where must point P be positioned, such that |BE|=|EF|=|FC|. What if triangle ABC is equilateral or isosceles triangle? Justify your solution.'

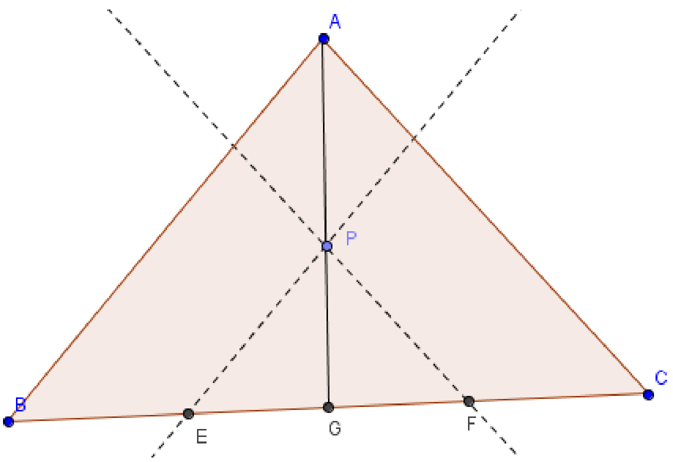


Figure 4.17 Shape given in the worksheet for GT 2

In the GeoGebra group, the following conversation occurred as an example for the DCW type of local argument:

Elif : Now, we draw an equilateral triangle. We first measured |BE|, |EF| and |FC|. The task says that these segments should be equal to each other, so there is a ratio here 1:2. We looked at this ratio and said it is the centroid of the equilateral triangle. Then, we talked about isosceles triangle since it is also valid here. However, it is not valid for a scalene triangle.

Teacher : How did you conclude that it is the centroid?

Elif : In order to make |BE|, |EF| and |FC| equal, we can say units to these segments. Let |BE|=|FC|=2a, |EG|=|GF|=a. Then, |GE|:|EB|=1:2=|PG|:|AG|. This ratio is valid for both equilateral and isosceles triangles and it is the centroid.

Teacher : Okay that is good.

Bade : Why isn't it valid for scalene triangles?

Aslı : It is also valid for scalene triangles.

Elif : Can you say that it is also the centroid for a scalene triangle?

Bade : If you say that |AG| is a median, it should also be valid for scalene triangles.

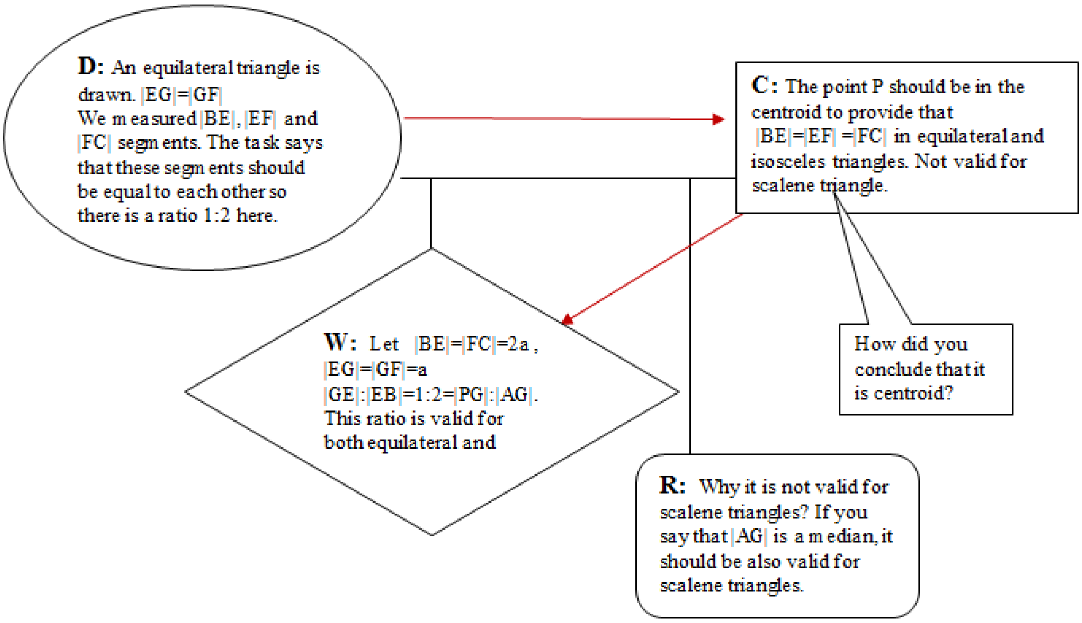


Figure 4.18 A sample local argument for a DCW type of argument

As it can be seen in the conversation, Elif talked about the data they had and the claim they deduced, and then the teacher interrupted by asking 'How did you conclude that it is the centroid?'. Then, Elif explained her justification by assigning the unit 'a' to the sides |EG| and |GF|, '2a' to sides |BE| and |FC| and showing the ratio |GE|:|EB|=1:2=|PG|:|AG| as a warrant. Afterwards, Bade and Aslı interrupted to modify the claim since the conclusion was also valid for all types of triangles.

4.2.2 Local argument type 2: Data-Warrant-Claim (DWC)

In this type of argument, the participants first talked about the data they had, and then stated their warrant about their claim. Afterwards, they expressed their claim. That is, they used the data they had and stated their conclusion after presenting their warrant. Thus, the order of flow of the argument components was Data, Warrant, and Claim, respectively. The component flow of the DWC type of argument is presented in *Figure 4.19*.

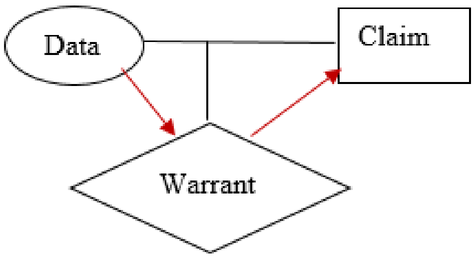
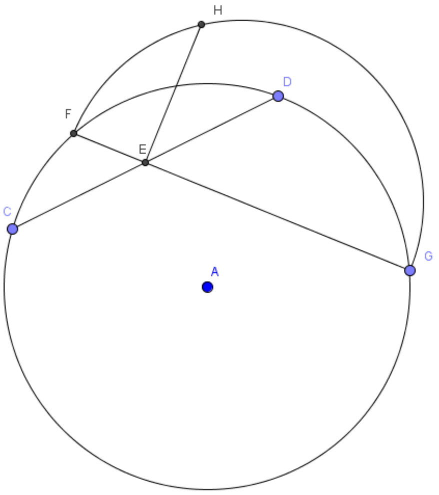
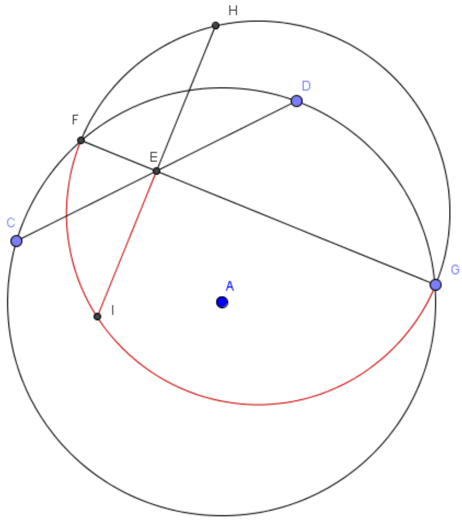


Figure 4.19 Component flow in the DWC type of local argument

When the red arrows in *Figure 4.19* are examined, it can be concluded that the direction of the statement order of the components was from the data to the warrant and then to the claim. A sample local argument for the DWC type of argument comes from geometry task 4 in the Paper-Pencil group. In the task, the circle with center 'A' and the semi-circle with diameter |AG| were given (see *Figure 4.20(a)*). Chord |CD| was bisected by |FG| at point E. Moreover, |HE| was perpendicular to |FG| as given. The task of the students was to prove that |CE| = |HE| and to justify their reasoning. A part of the conversation which was related to the sample local argument is given below, and Pelin's drawings on the board are represented with the color red in *Figure 4.20(b)*. Then, the Toulmin schema (see *Figure 4.21*) was drawn for the type of DWC local argument.



(a)



(b)

Figure 4.20 The shape of GT 4 (a) and the additional drawings of Pelin (b)

Pelin : Now let me write ‘x’ for |CE| and |ED|. Then, ‘a’ and ‘b’ for segments |FE | and |EG | respectively (**D71**). We can write the equation of the intersecting chord theorem for these chords. x.x=a.b so we obtained the first equation ‘x^2=a.b’ (**W71**).

Teacher : Okay.

Pelin : For the second equation, we first drew arc FG and segment |EI| by drawing the symmetry of |HE| with respect to |FG| (**D71**). Then I remembered that |HE|=|EI| since |FG| is the diameter dividing chord |HI| into two. Thus, I can label |HE| and |EI| with ‘c’. Then I can write the intersecting chord theorem equation again with c.c=a.b . Then we found ‘c^2=a.b’ (**W71**).

Teacher : Yes.

Pelin : Thus we had two equations ‘c^2=a.b’ and ‘x^2=a.b’ which were equal to each other (**W71**). Then c=x which means |HE|=|CE| (**C71**).

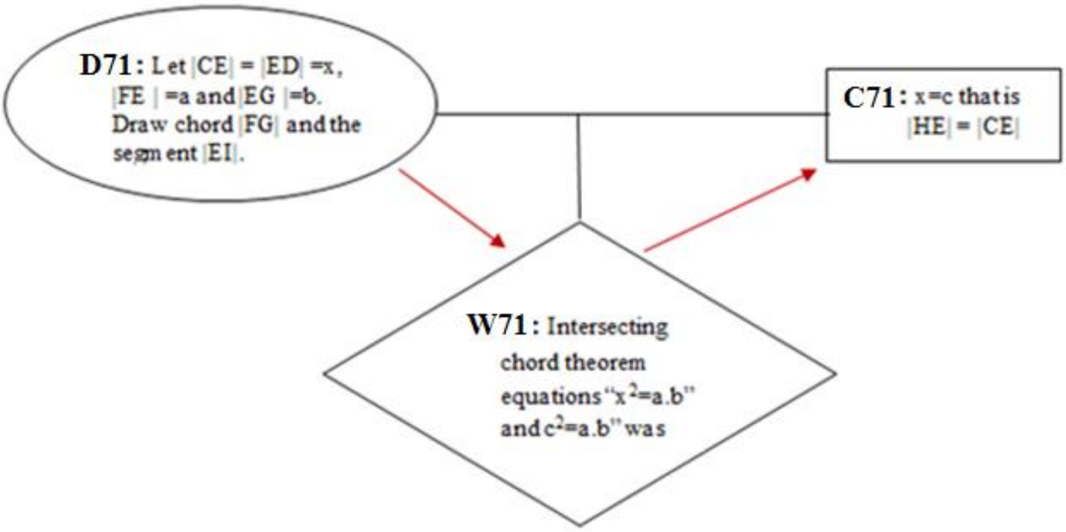


Figure 4.21 A sample local argument for the DWC type of argument

As it can be seen in the conversation, Pelin talked about the data they had by labeling the segments on the shape and by doing additional drawings, using the color red, upon the original shape. Then she stated the warrant, which was the use of the intersecting chord theorem to conclude that |HE|=|CE|.

4.2.3 Local argument type 3: Claim-Data-Warrant (CDW)

In this type of argument, the participants first stated their claim, and then they talked about the data they had, and subsequently stated their justification about their claim. Thus, the order of the flow of the argument components was Claim, Data, and Warrant, respectively. The component flow of the CDW type of argument is presented in *Figure 4.22*.

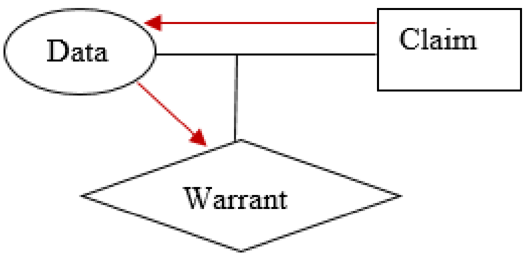
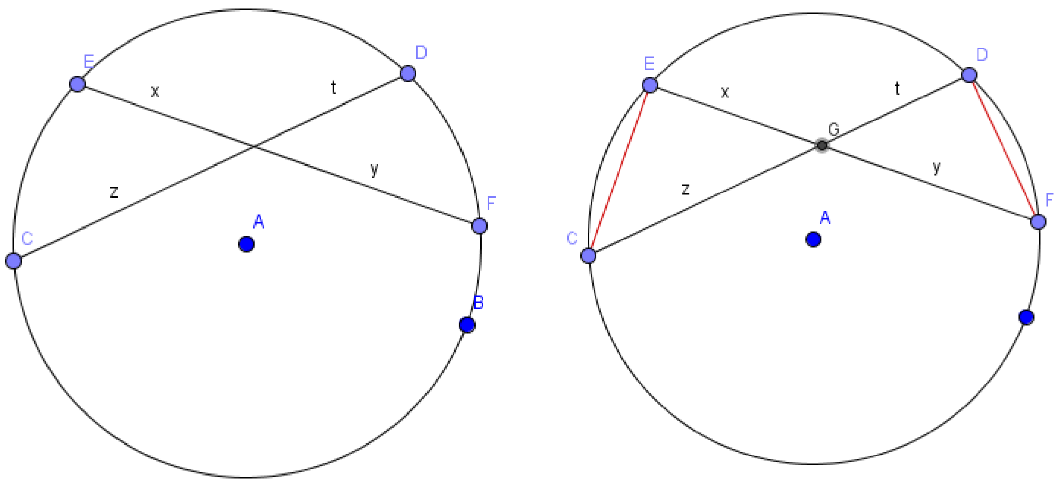


Figure 4.22 Component flow in the CDW type of local argument

When the red arrows in *Figure 4.22* are examined, it can be understood that the direction of the statement order of the components was from the claim to the data and then to the warrant. A sample local argument for argument type 3 comes from geometry task 4 in the Paper-Pencil group. The details of the task are given in the DWC type of argument section and illustrated in *Figure 4.20(a)*. The task was to prove that |CE|=|HE|. In some parts of the discussion, one of the participants talked about the intersecting chords theorem. Then, the teacher asked the participants where the equation of the intersecting chords derived from. Afterwards, the sample argument for the CDW type of argument emerged since they knew the claim 'x.y=z.t' and tried to justify this rule. The conversation of the argument and the shape drawn on the board by the participants (see *Figure 4.23*) are presented below. Finally, the Toulmin schema (see *Figure 4.24*) was drawn for the CDW type of local argument:



(a) **(b)**

Figure 4.23 Shapes drawn on the board by instructor (a) and the participants (b)

Teacher	: So where does the intersecting chords theorem derive from?
Erhan	: Okay I found it. One moment. Let |EG|=x, |GF|=y, |CG|=z and |GD|=t. (He draws EC and DF) (**D70**). These arcs are the same, aren't they?
Okan	: I said the same, yes. But are they equal?
Gözde	: Yes, equal.
Okan	: Yes, but as far as I know to be able to express that equality,

	the intersection point G should be in the center of the circle (**R70**).
Gözde	: No it is an inscribed angle. You do not need a central angle (**W70**).
Erhan	: Then, let ∠ECG and ∠DFG be alpha, (**W70**)
Teacher	: You mean both angles see the same arc?
Erhan	: Yes. So let ∠GEC and ∠GDF be beta. Finally, let $\angle EGC$ and ∠DGF be teta (**W70**). Then the intersecting chord rule derives from triangle similarity. Angle-Angle-Angle triangle similarity (**W70**).
Gözde	: Let's write the similarity.
Erhan	: |EG|:|GF| = |CG|:|GD|= |EC|:|DF|. Then x.y=z.t derives from this similarity (**C70**).

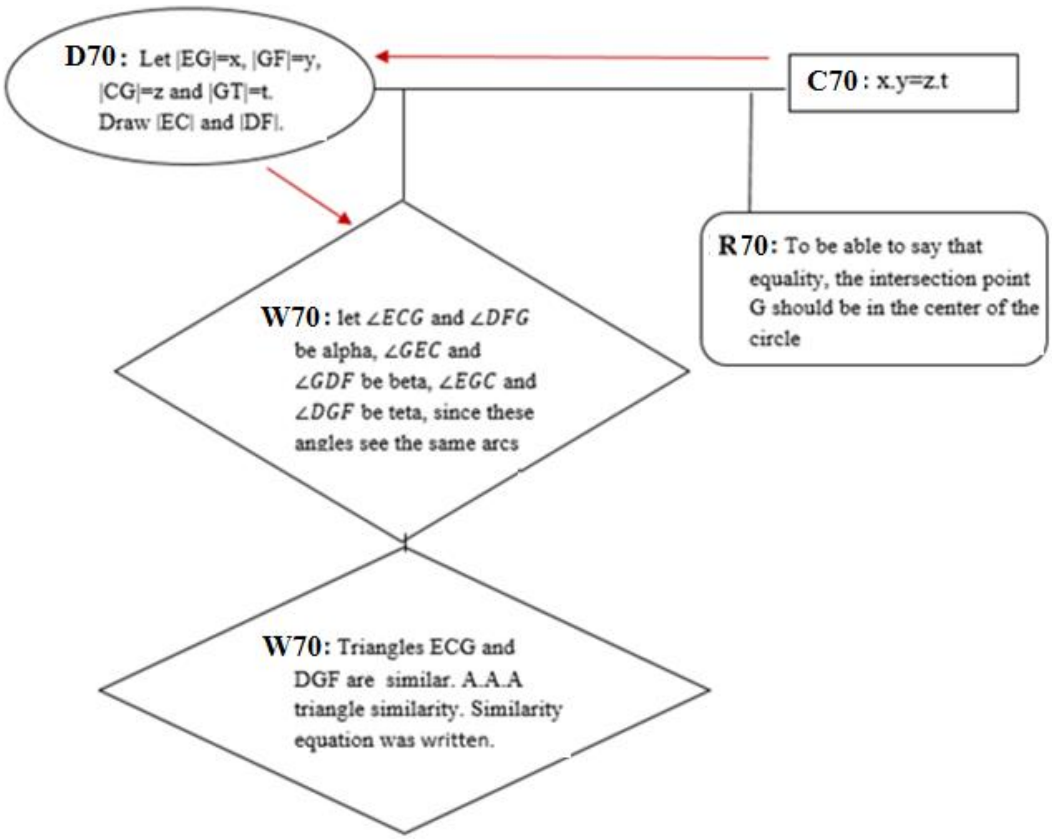


Figure 4.24 A sample local argument for the CDW type of argument

As can be seen in the conversation, first the claim 'x.y=z.t' was made during the discussion by the participant since the students knew the intersecting chords theorem as a taken-as-shared knowledge. However, the teacher probed participants to think more on this theorem and asked 'Where does the equation derive from?'.

Then, Erhan drew a circle, labeled the segments and angles, and then tried to find the justification for the intersecting chords theorem. In the end, he found the angle-angle-angle triangle similarity as a warrant.

4.2.4 Local argument type 4: Warrant-Data-Claim (WDC)

In this type of argument, the participants first stated the justification, and then they talked about the data they had, and finally stated their claim. Thus, the order of the flow of argument components was Warrant, Data, and Claim, respectively. The component flow of the WDC type of local argument is presented in *Figure 4.25*.

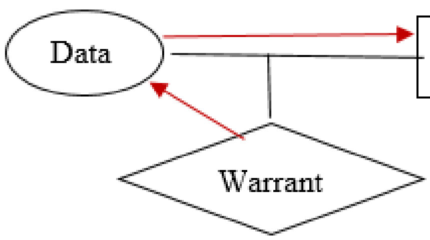


Figure 4.25 Component flow in the WDC type of local argument

When the red arrows in *Figure 4.25* are examined, it can be concluded that the direction of the statement order of the components was from the warrant to the data and then to the claim. A sample local argument for the WDC type of argument comes from geometry task 1 in the Paper-Pencil group. The task was presented in the worksheet in the following way:

> 'ABC is a triangle. The midpoints of the sides |AB| and |AC| are points D and E respectively. Points F and G are placed on side |BC| so as to be |BG|=|CF|. The segments |DG| and |EF| intersect at point H.
> When does |AH| become the angle bisector of ∠A? (Think about all types of triangles). Explain your reasoning and justify your solutions.'

Güler was solving the task on the board. She drew the shape given in *Figure 4.26* and used the triangle property that 'If a segment of a triangle is both an altitude and a median, then it has to be an angle bisector also' as a warrant for an argument. Then the following local argument emerged as a sample for the WDC type of argument. In this type, the warrant was first stated. The reason for the warrant to be stated before the claim and the data was its being a taken-as-shared rule. The conversation below occurred while a sample argument for argument type 4 was produced.

Güler	: We assumed that |DG| ⊥|AB| and |FE|⊥ |AC| at the same time.
Teacher	: Okay let's assume that both are perpendicular.
Güler	: We drew |AG and |AF|. The triangle AFG becomes an isosceles triangle because a side-angle-side similarity exists (**D12**).
İnci	: Yes.
Güler	: Now look at triangle AGB. |DG| is the altitude and the median of the triangle, so it becomes the angle bisector at the same time (**W12/13**).
Teacher	: You mean the triangle property that 'when a segment is both the altitude and the median, it also has to be an angle bisector'.
Güler	: Yes. Similarly, we can conclude with the same rule that in triangle AFC, FE is the altitude and the median, and thus it will be an angle bisector too (**C13**).

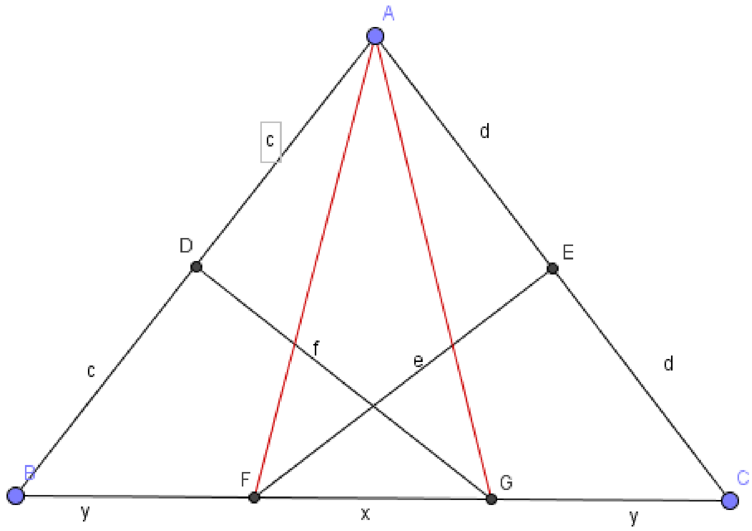


Figure 4.26 The shape Güler drew on the board

As it can be seen in the conversation, Güler first talked about how she found that |GD| is the angle bisector of ∠AGB with a justification including the rule 'When a segment is both the altitude and the median of a triangle, it also has to be an angle bisector'. This warrant became a taken as shared rule at that moment. Then she talked about triangle AFC. Argument 13 in *Figure 4.27* is the sample local argument for the WDC type of argument since the warrant was stated first. Then, Güler talked about the data of the argument, followed by the claim of the argument as presented with the red arrows in *Figure 4.27*.

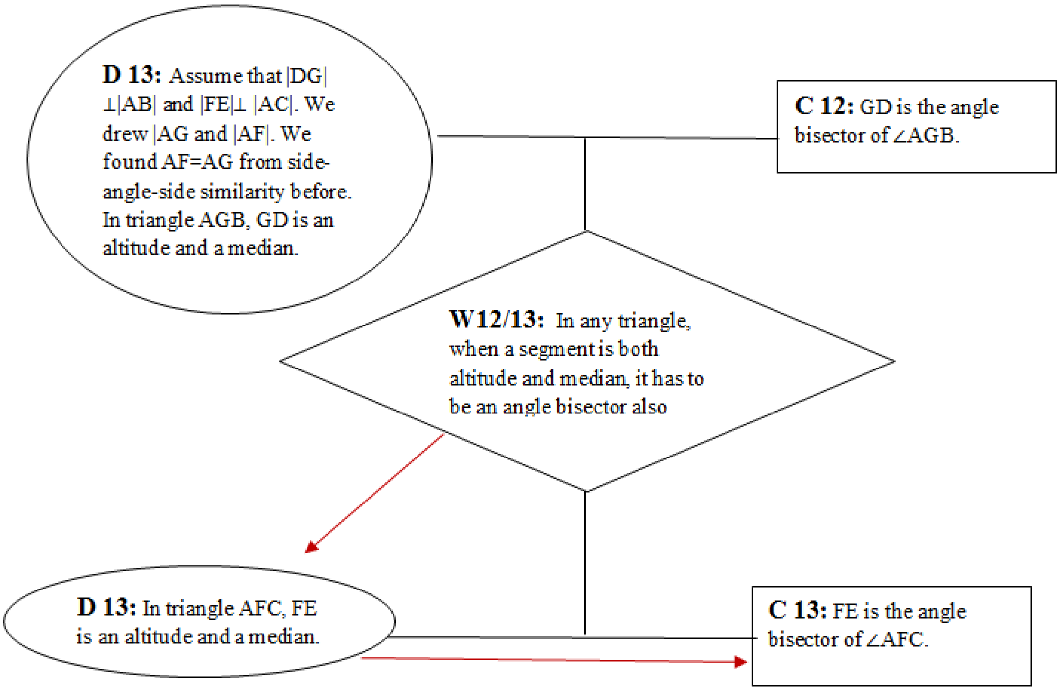


Figure 4.27 A sample local argument for the WDC type of argument

4.2.5 Local argument type 5: Claim-Data (CD)

In this type of argument, the participants first stated their conclusions and then stated the data of their argument but they did not justify their claims. Thus, the order of the flow of the argument components was Claim and Data, respectively. The component flow of the CD type of local argument is presented in *Figure 4.28*.

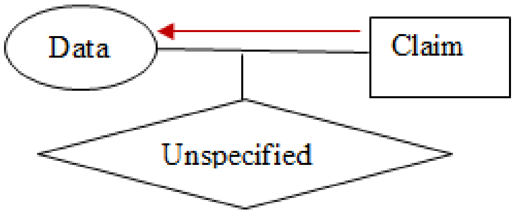


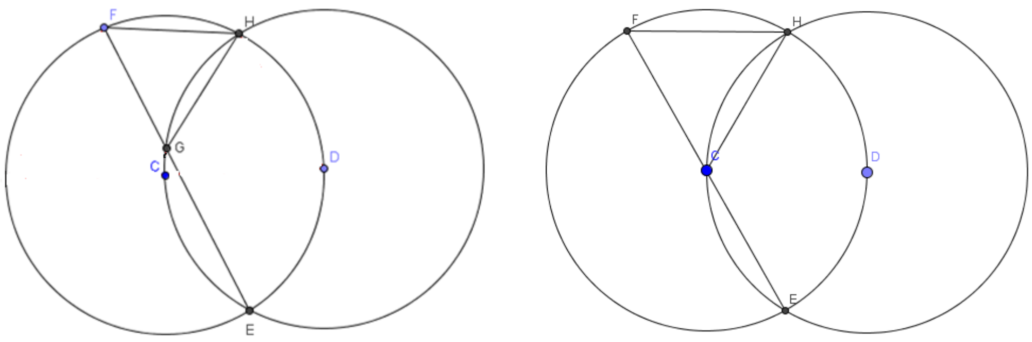
Figure 4.28 Component flow in the CD type of local argument

When the red arrow in *Figure 4.28* is examined, it can be understood that the direction of the statement order of the components was from the claim to the data and the warrant was not stated. A sample local argument for the CD type of argument comes from geometry task 3 in the Paper-Pencil group. In geometry task 3, two circles, each of which passes through the center of the other circle, were given as in *Figure 4.29(a)*. The circles intersect at points H and E, and a line from E intersects the circles at points F and G. The questions that were asked were as follows:

> 'If |FG|= 6, compute the area of the triangle FGH? Justify your solution.
> If r is the measure of the radius of each circle, find the minimum value and maximum value of the area of triangle FGH. Justify your solution.'

The task was to find the triangle with maximum area when the givens were satisfied, as can be understood from the following conversation:

Teacher	: What do you think about the second question? What is the maximum area of triangle FGH?
Gözde	: In my opinion, chord |EF| should pass from point C for the maximum area (**C49**). I mean one side of the triangle FGH should be the radius of the circle (**D49**).
Okan	: No. I think you can drag the chord |EF| beyond point C. You can pass center C and the triangle becomes larger (**R49**).



(a) *(b)*

Figure 4.29 The shape of geometry task 3 (a), and Gözde's drawing (b)

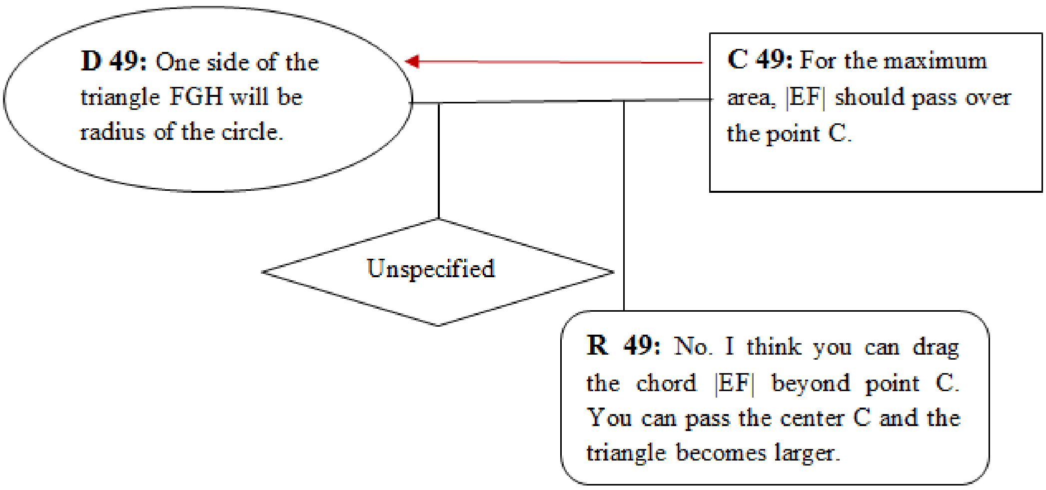


Figure 4.30 A sample local argument for the CD type of argument

As can be seen in the conversation, Gözde thinks that maximum area for triangle FGH could be obtained when chord |EF| passes through center C. However, Okan opposed her idea by saying that chord |EF| can be drawn beyond point C to obtain a larger triangle FGH. At this moment, the discussion stopped and Gözde turned to her worksheet and thought about it. Thus, the argument flow was from the claim to the data, but there was no justification as presented in *Figure 4.30*.

4.2.6 Local argument type 6: Data-Claim (DC)

In this type of argument, the participants firstly stated the data they had, and then they talked about their conclusion without justification. Thus, the order of the flow of the argument components was Data and Claim, respectively. The component flow of the DC type of local argument is presented in *Figure 4.31*.

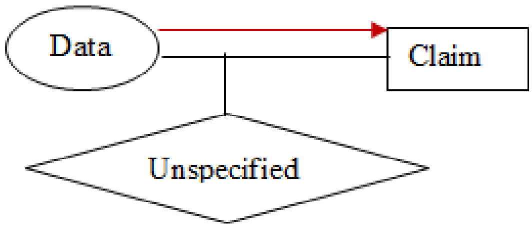
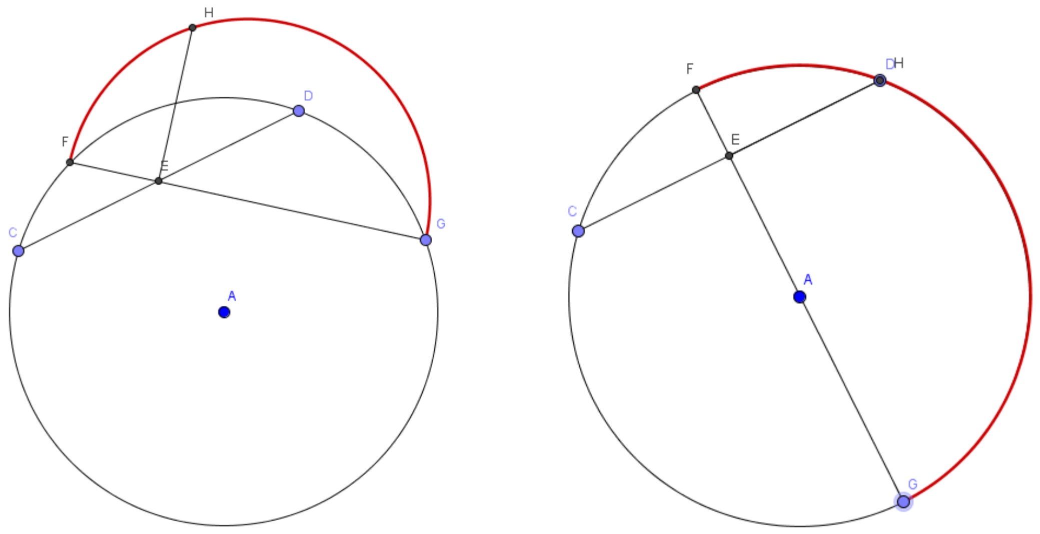


Figure 4.31 Component flow in the DC type of local argument

When the red arrow in *Figure 4.31* is examined, it can be understood that the direction of the statement order of the components was from the data to the claim. A sample local argument for the DC type of argument comes from geometry task 4 in the GeoGebra group. The details of geometry task 4 has already been explained in the section on the DWC type of argument, in *Figure 4.20(a)*. The task was to find whether |CE|=|HE| when the chord |FG| becomes the diameter of the big circle. Bade was showing her solution to the class by using the GeoGebra program. The shape of the task prior to dragging is given in *Figure 4.32(a)*, while the shape subsequent to dragging is presented in *Figure 4.32(b)*. Bade's utterances while dragging point G are as follows:

Bade : I dragged point G in order to make chord FG the diameter of the big circle (**D61**). Here, we can see that |CE|=|HE| (**C61**).



(a) **(b)**

Figure 4.32 The shape Bade dragged via GeoGebra

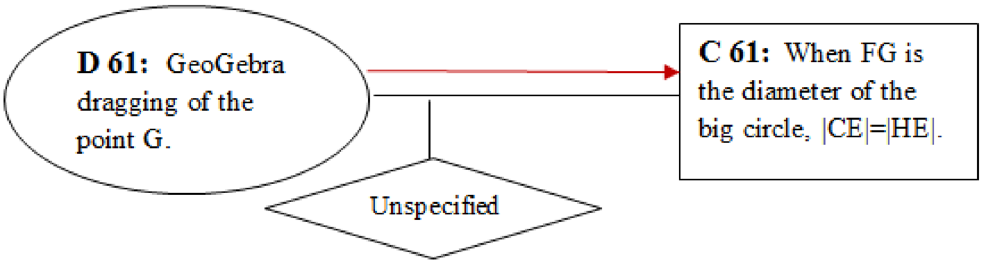


Figure 4.33 A sample local argument for the DC type of argument

As can be seen in Bade's talk, she did the dragging move by using the dynamic geometry program GeoGebra to show her claim which was 'When FG is the diameter of the big circle, |CE|=|HE|'. Then, she passed onto the other part of the task, which was asking whether |CE|=|HE| when chord |FG| overlapped with chord |CD|. Bade dragged the shape quickly to show the answer of the other part of the task. Participants were satisfied with the GeoGebra dragging, so nobody in class asked for her justification. Thus, this local argument became an example for the DC type of argument.

4.2.7 Local argument type 7: Claim-Warrant (CW)

In this type of local argument, the participants first stated their conclusion, and then talked about their justification. Thus, the order of the flow of the argument components was Claim and Warrant respectively. The component flow of the CW type of local argument is presented in *Figure 4.34*.

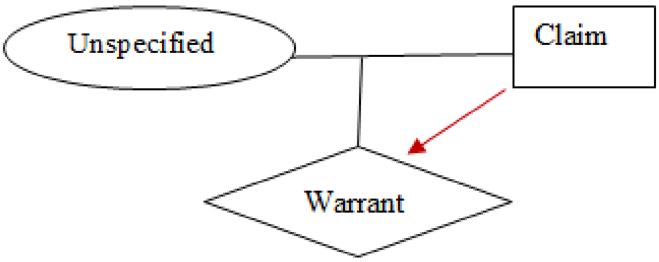
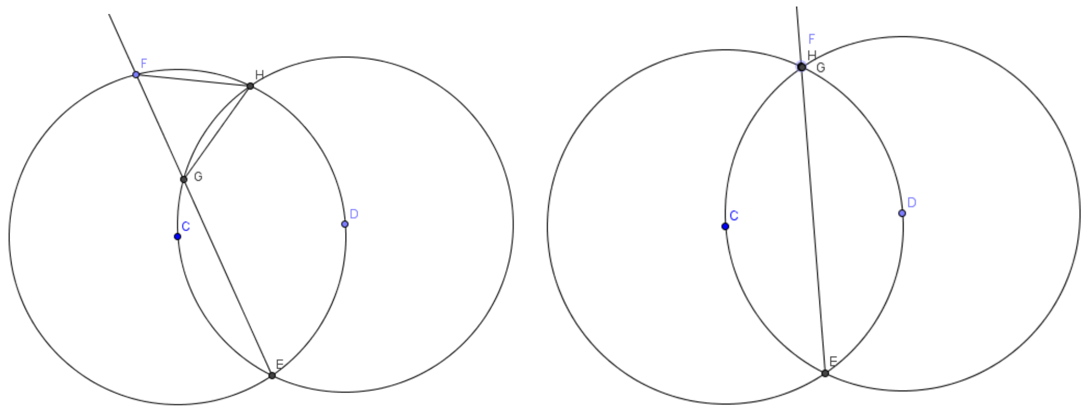


Figure 4.34 Component flow in the CW type of local argument

When the red arrow in *Figure 4.34* is examined, it can be concluded that the direction of the statement order of the components was from the claim to the warrant, and the data was not stated. A sample local argument for the CW type of argument comes from geometry task 3 in the Paper-Pencil group. The details of geometry task 3 were given in the section on the CD type of local argument with the shape in *Figure 4.29(a)*. The task of the conversation was to find the minimum area of triangle FGH. Bahar was at the board to show the triangle FGH with the minimum area. She drew chord |EF| to point H (see *Figure 4.35(b)*). Then she said, 'The minimum area can be zero. When I drag point F to point H, the triangle disappears. Thus, there is no area and the minimum area is zero'.



(a) *(b)*

Figure 4.35 The shape of geometry task 3 (a) and the shape dragged by Bahar (b)

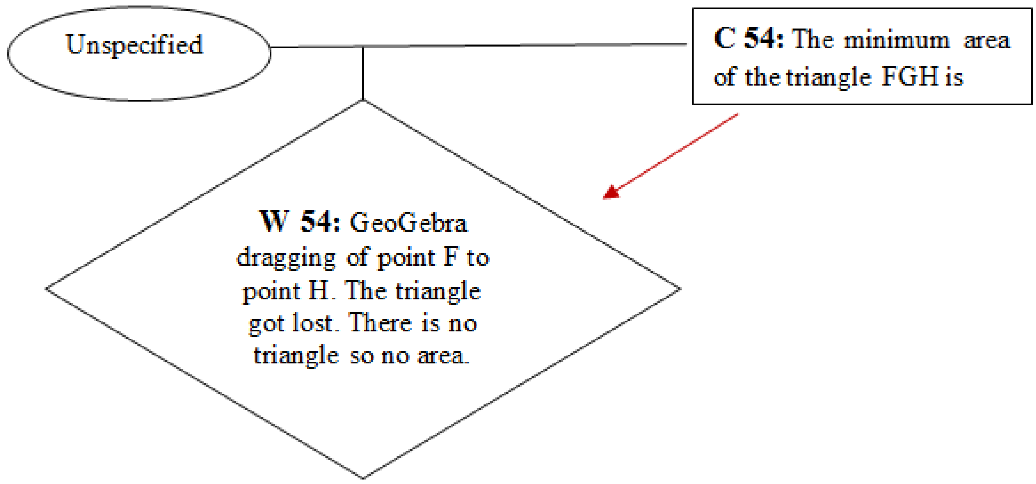


Figure 4.36 Sample local argument for the CW type of argument

As can be seen in the conversation, Bahar claimed that the minimum area of triangle FGH was zero, and then she justified her conclusion by using GeoGebra dragging. The participants were satisfied with the dragging, so the argumentation stopped at that point.

4.2.8 Local argument type 8: Warrant-Claim (WC)

In this type of local argument, the participants first stated the justification, and then they talked about their claim. Thus, the order of flow of the argument components was Warrant and Claim, respectively. The component flow of the WC type of local argument is presented in *Figure 4.37*.

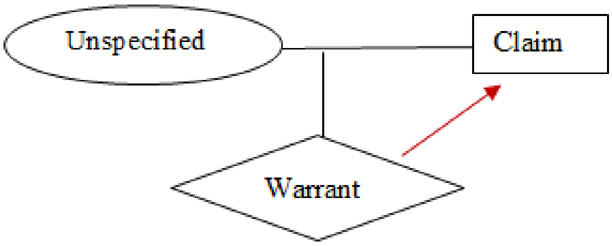


Figure 4.37 Component flow in the WC type of local argument

When the red arrow in *Figure 4.37* is examined, it can be understood that the direction of the order of the components was from the warrant to the claim. A sample local argument for the WC type of argument comes from geometry task 1 in the GeoGebra group. The task was related to triangles and it was asked in the worksheet in the following way:

> ‘ABC is a triangle. The midpoints of sides |AB| and |AC| are points D and E, respectively. Points F and G are placed on side |BC| so as to be |BG|=|CF|. Segments |DG| and |EF| intersect at point H.
>
> When does |AH| become the angle bisector of ∠A? (Think about all types of triangle). Explain your reasoning and justify your solutions.’

The task was to identify the triangle types in which segment |AH| becomes an angle bisector. The shape of the task was previously drawn via GeoGebra. Özer came to the computer to show his solution. He drew a circle from the midpoint of |BC| in order to make points F and G dynamic (see in *Figure 4.38*). Then he measured ∠BAH and ∠HAC. Subsequently, he made an observation by dragging point A and by changing the shape of triangle ABC.

Özer	: I drew the circle with center T in order the points F and G be dynamic. So I can see all the different situations.
Aslı	: We can first think of drawing the angle bisector, altitude and median and then drag the shape to make an inference.
Özer	: I was already looking at |AH| to be an angle bisector by

dragging point A and changing the triangle (**W11**). Then, I saw that when |AH| is an angle bisector, it crosses over the center of the circle all the time (**W11**). I found that |AH| became the altitude of the triangle at the same time. Then I concluded that triangle ABC is an isosceles triangle (**C11**) since |AH| is both an angle bisector and an altitude.

Teacher : Can you say that AH should be an altitude for all isosceles triangles?

Bade : Yes, we can say that.

Özer : Yes all the time.

Teacher : What about the other triangles? When you change the triangle into a scalene triangle by dragging, what can you say?

Bade : In that case, |AH| cannot be an altitude.

Özer : In that case, it cannot be an angle bisector either. I saw that whenever it is angle bisector, it crossed over the midpoint of |BC| so it was also the median.

Bade : So |AH| cannot be an altitude in scalene triangles (**R11**).

Özer : I conclude this because of segment |AH| became both an angle bisector and the median and thus, the altitude.

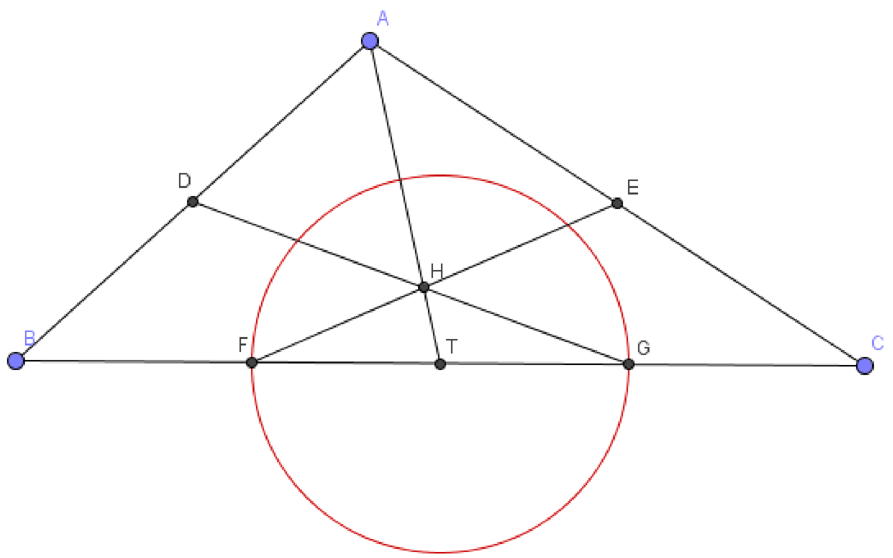


Figure 4.38 The shape Özer dragged via GeoGebra

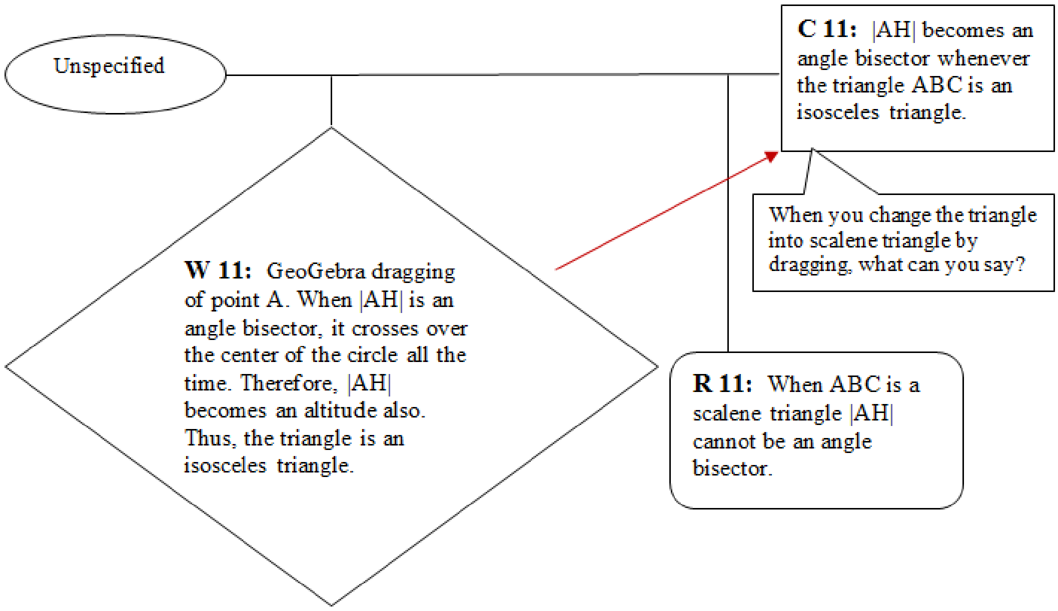


Figure 4.39 A sample local argument for the WC type of argument

As can be seen in the sample local argument, Özer first dragged the shape and showed his justification, and then he talked about his conclusion which was '|AH| becomes an angle bisector whenever triangle ABC is an isosceles triangle'. That is, he initially talked about his warrant and then he stated his claim. Finally, the teacher promoted further thinking about scalene triangles by asking a question. In this way an exceptional condition emerged as a rebuttal to the argument.

4.2.9 Local argument type 9: Claim (C)

In this type of local argument, the participants only stated their conclusion and then said nothing related to the data they had or the justification they had in mind. Thus, the sole component of the argument was the Claim as presented in *Figure 4.40*.

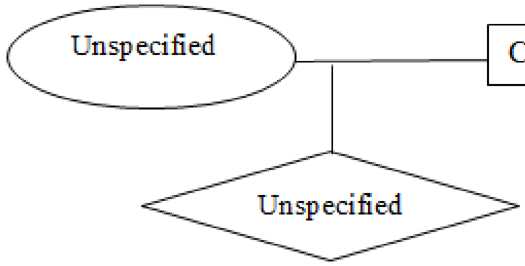


Figure 4.40 C type of local argument

Arguments of this type emerged when the participants were thinking aloud or when it was a shared idea, so there was no need to justify the claim. A sample local argument comes from the Paper-Pencil group's geometry task 1. While solving the task, different groups placed points F and G in the same order on side |BC|. The teacher asked the students why they all placed F and G in that order? Then, Güler said, 'Friends, we will conclude at the end of the discussion that when points F and G are dragged and switched, their places |DG| and |EF| will always intersect on the line passing through |AT|' (see *Figure 4.38*). That is, point H will move up and down on the line passing through |AT| when you move points F and G dynamically. The argument schema for this conversation is presented in *Figure 4.41*.

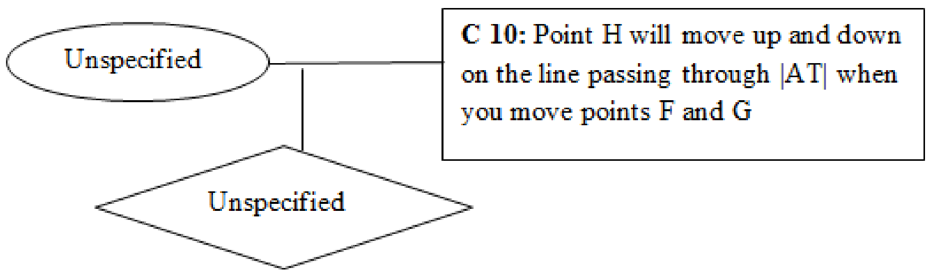


Figure 4.41 A sample local argument for the C type of argument

4.2.10 The analysis results of the different types of local arguments based on the flow of the argument components

So far, each type of local argument has been introduced with sample local arguments and conversations from the current study. In this section, the numbers of local arguments in each type are analyzed. The total number of local arguments in each group is presented in Table 4.2.

Table 4.2 The total number of local arguments in each type of argument in the GeoGebra and the Paper-Pencil groups

	GeoGebra	**Paper Pencil**
1. *DCW*	**11**	**10**
2. *DWC*	**29**	**38**
3. *CDW*	1	4
4. *WDC*	2	2
5. *CD*	1	2
6. *DC*	2	3
7. *CW*	**10**	**8**
8. *WC*	4	3
9. *C*	3	4

In conclusion, DWC, DCW and CW were the most frequently confronted types of local arguments in both the GeoGebra and the Paper-Pencil groups. Especially the DWC type of argument was the most frequently used argument by prospective middle school mathematics teachers. That is, the prospective middle school mathematics teachers were disposed to initially state the data they had, and then express their justification and finally to state what they concluded. Sometimes, they changed the places of the warrant and the claim, thus stating the data they had first, and then the claim of their argument and finally the warrant they used to justify as in DCW type of local argument. In the third most frequently used argument type, they did not mention the data they had but stated their argument components in the order of claim and warrant. In this type, the data became taken-as-shared in the class discussion, so they avoided repetition.

The frequencies of the other types of local arguments were close to each other. When the groups are examined, it can be seen that the number of local arguments in both groups were close to each other. That is, a similar pattern occurred in the two groups in terms of the flow of argument components which were developed by prospective middle school mathematics teachers in the geometry context. That means, the existence of the dynamic geometry program, GeoGebra, did not change the course of reasoning of the prospective teachers and they behaved similarly in the argument construction process in terms of the flow of argument components.

After analyzing the different types of local arguments between the groups, the researcher analyzed the geometry tasks separately. The numbers of different types of local arguments by geometry task are listed in Table 4.3.

Table 4.3 Number of arguments for each type of local argument by geometry task

	GeoGebra				**Paper Pencil**			
	GT 1	**GT 2**	**GT 3**	**GT 4**	**GT 1**	**GT 2**	**GT 3**	**GT 4**
1. DCW	2	1	3	**5**	3	2	2	3
2. DWC	**3**	**10**	**7**	**9**	**8**	**11**	**8**	**11**
3. CDW	0	1	0	0	1	0	1	2
4. WDC	1	1	0	0	2	0	0	0
5. CD	0	0	0	1	0	0	2	0
6. DC	1	0	0	1	2	0	0	1
7. CW	**5**	1	**4**	0	**4**	1	**3**	0
8. WC	3	0	1	0	2	1	0	0
9. C	1	0	0	2	3	0	0	1

When the distribution of the local arguments is listed in terms of geometry task (see Table 4.3), it can be inferred that the DWC type of local argument was the most confronted type in all geometry tasks in both the GeoGebra and the Paper-Pencil groups, except for GeoGebra group’s geometry task 1. This means that the use of GeoGebra did not make any difference in prospective middle school mathematics teachers' preferences regarding the type of local argument. However, in geometry task 1, the GeoGebra drawing of the givens shortened the solution process and they did not need to talk while they were drawing the shape. Their drawing process was in fact the data they possessed, but they did not state their actions while drawing the shape, so the type of local argument they used most frequently turned out to be the CW type of argument in geometry task 1 in the GeoGebra group. Upon seeing this, the researcher encouraged the participants in the GeoGebra group in the subsequent tasks to talk about every action they did during the solution process, just as they would in thinking aloud, to catch the data components of their arguments. Nevertheless, it can be seen in Table 4.3 that the frequency of the CW type of local argument was again high in both groups in geometry task 3.

As it can be seen in Table 4.3, the DCW type of local argument emerged in all the geometry tasks in both groups. In addition, the number of the DCW local argument was a little high in GeoGebra group's geometry task 4. When those arguments were examined, the researcher noticed that after the participant stated the data and the claim, the researcher or another participant asked for his/her justification, so the warrant came later in the process.

In the present study, two geometry tasks were related to triangles while the other two were related to circles. Therefore, the different types of local argument in terms of the flow of argument components are compared in terms of these two mathematical contents in Table 4.4.

Table 4.4 Number of local arguments in each argument type by mathematical content

	GeoGebra		Paper Pencil	
	Triangle	**Circle**	**Triangle**	**Circle**
1. *DCW*	**3**	**8**	**5**	**5**
2. *DWC*	**13**	**16**	**19**	**19**
3. *CDW*	1	0	1	3
4. *WDC*	2	0	2	0
5. *CD*	0	1	0	2
6. *DC*	1	1	2	1
7. *CW*	**6**	**4**	**5**	**3**
8. *WC*	3	1	3	0
9. *C*	1	2	3	1

The distribution of the local arguments in different types of argument in triangle tasks, which were geometry task 1 and 2, had a similar pattern in both the GeoGebra and Paper-Pencil groups. The most frequently emerging local argument was found to be the DWC type of local argument. In addition, in the GeoGebra group, the CW type of local argument also had a high frequency in the triangle tasks. On the other hand, in the Paper-Pencil group, the DCW and CW types of local argument had a high frequency in the triangle tasks. Similarly, in the circle tasks, which were geometry tasks 3 and 4, the distribution of the local arguments was similar in the GeoGebra and Paper-Pencil groups. Similar to triangle tasks, the highest number of local arguments can be seen in the DWC type of local argument in the circle tasks. Moreover, in the circle tasks, the DCW type of local argument also had a high number of local arguments when compared to the other types in both groups.

In summary, both groups showed a similar pattern in their argument construction process. That is, the flow of argument components was similar in both groups. Moreover, the most frequently used three types of local arguments were DWC, DCW and CW in both groups when the patterns were investigated by group, by geometry task and by mathematical content.

4.3 Local argumentation analysis

In the previous sections, the global argumentation structures developed by prospective middle school mathematics teachers were analyzed for the purpose of responding to the first research question. Subsequently, the different types of local arguments developed by prospective middle school mathematics teachers based on the flow of argument components were analyzed to respond to the second research question. This section seeks to respond to the third research question, 'What are the characteristics of local argumentations that prospective middle school mathematics teachers utilize while solving geometry tasks in the GeoGebra and the Paper-Pencil groups?'. The local arguments were analyzed based on the classification developed by Knipping (2008), which is presented in *Figure 4.42*.

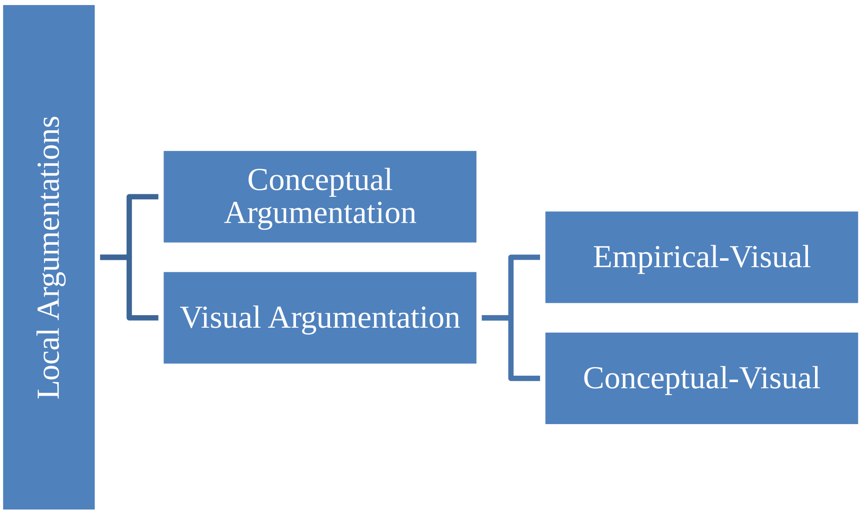


Figure 4.42 Local argumentation classification developed by Knipping (2008)

Knipping (2008) classified local argumentations into two basic levels: conceptual argumentation and visual argumentation. Visual argumentation was divided further into two levels, namely empirical-visual and conceptual-visual argumentations (Knipping, 2008). The method of classification of Knipping (2008) was based on the examination of the *warrants* and *backings* of the local arguments. During the data analysis of local argumentations in the present study, the warrants and backings of the local arguments were classified according to the classification developed by Knipping (2008). Then, the local arguments were analyzed on the basis of the tasks (GT1, GT2, GT3, GT4) and the mathematical content (triangle, circle) by considering each group (GeoGebra group, Paper-Pencil group). Some of the local arguments did not have any warrant or backing so they could not be placed in any of the groupings within the classification. Those local arguments were listed in tables in the last sections under the heading 'Arguments which don't have a warrant'. Moreover, some of the warrants did not fit into any of the groupings in the classification, so the researcher explained their special conditions within each analysis under the heading 'new condition'. Initially, the sample local arguments for each local argumentation level are presented and then each type of analysis is explained under different headings in the following sections.

The sample local argument for *conceptual argumentation* is from geometry task 2 in the GeoGebra group. Beren labelled the segments |EG|=a, |GF|=c, |BE|=b, |FC|=d (see *Figure 4.43*).

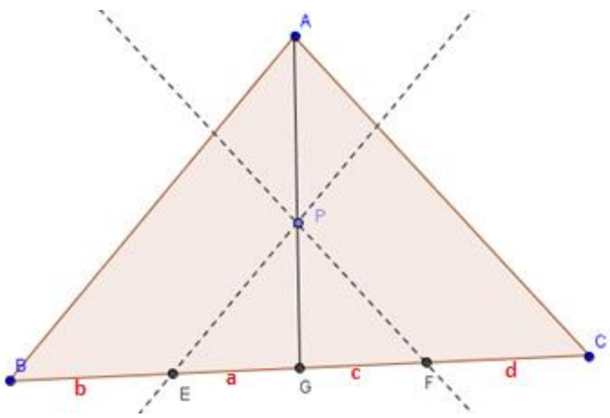


Figure 4.43 Diagram drawn for GT 2 as an example of conceptual argumentation

Then, she said that b+a=c+d since |AG| is the median. Afterwards, she used the angle-angle-angle triangle similarity and wrote a similarity ratio for the triangles PGF≈AGC and for the triangles PGE≈AGB as a warrant. In the end, she found the conclusion a=c, which means |EG|=|GF|. The Toulmin schema for this local argument was as follows:

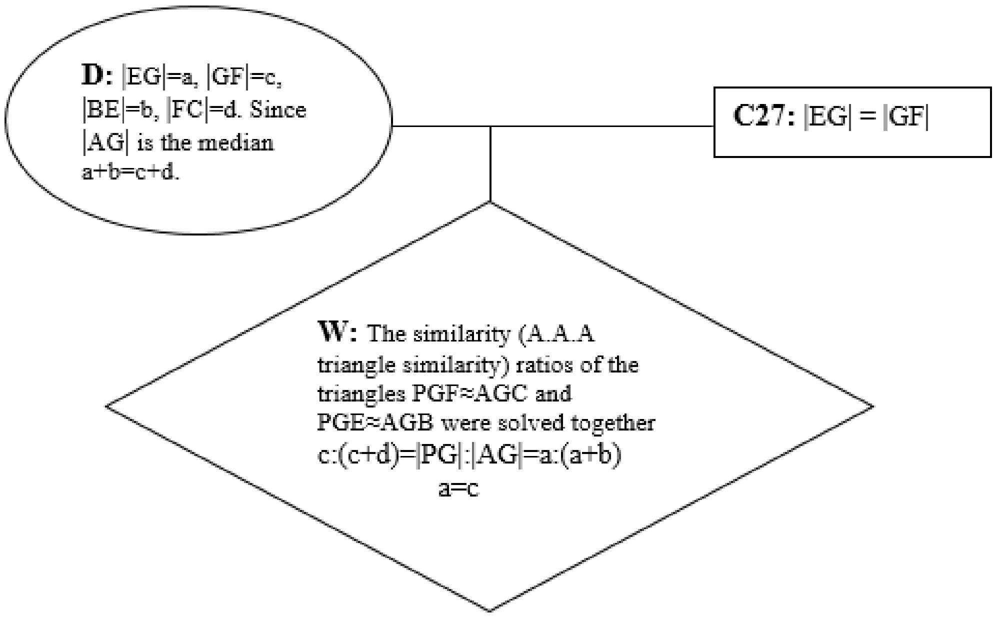


Figure 4.44 A Toulmin schema for a sample local argument of conceptual argumentation from the GeoGebra group

The warrant of the local argument in *Figure 4.44* was directly a reference to the triangle similarity theorem. Moreover, Beren worked with such general units 'a, b, c, d', not with specific values. Thus, it can be concluded that the local argument fits the conceptual argumentation level.

Another example for conceptual argumentation was from geometry task 1 of the Paper-Pencil group. Similarly, Erhan labelled the segments in *Figure 4.45* using general units.

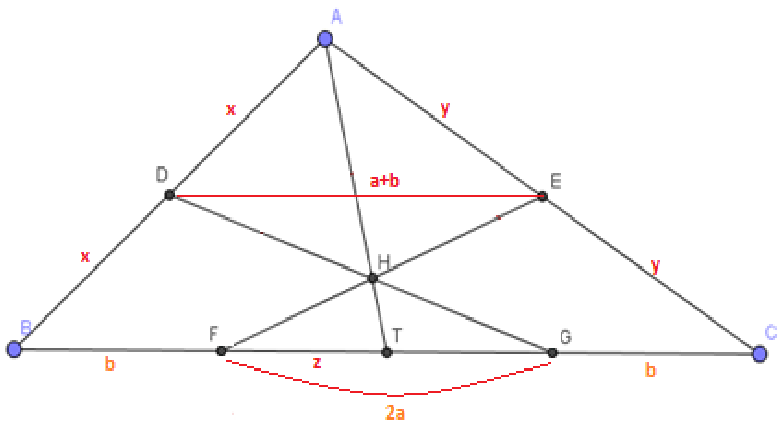


Figure 4.45 Labeling of Erhan for GT 1 as an example for conceptual argumentation

Then, he used the Menelaus' theorem from both sides of the triangle as the warrant. Afterwards, he solved two equations obtained from the Menelaus theorem and found that a=z, which means |AT| was a median of the triangle ABC as a claim. The Toulmin schema for this local argument is given in *Figure 4.46*.

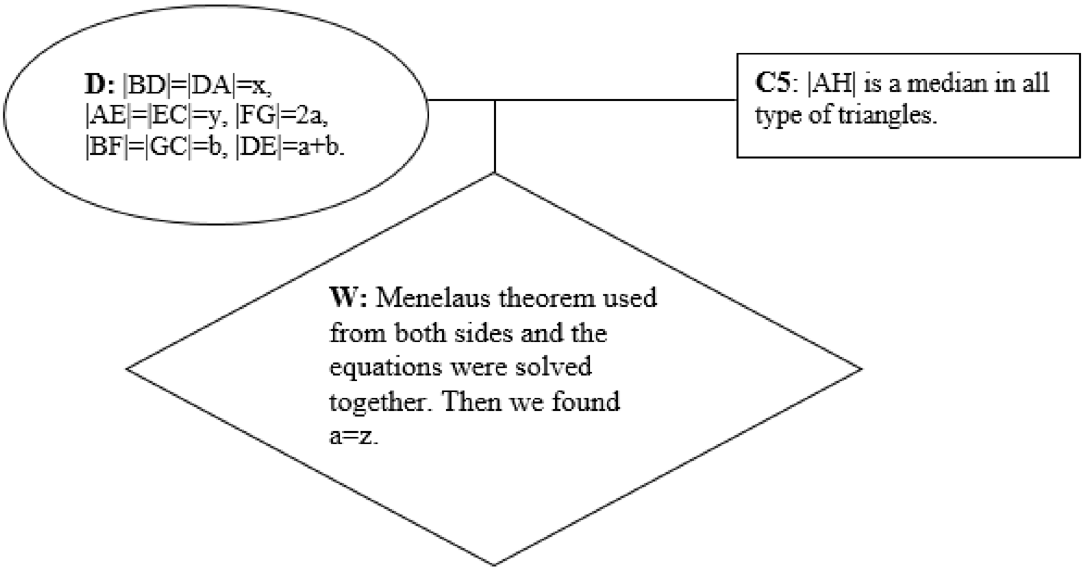


Figure 4.46 A Toulmin schema for a sample local argument of conceptual argumentation from the Paper-Pencil group

Similar to the local argument in *Figure 4.44*, the warrant of the argument in *Figure 4.46* was also directly a reference to the Menelaus theorem. Moreover, Erhan worked with general units 'x, y, z, a, b', not with specific values. Thus, it can be concluded that the local argument fits the conceptual argumentation level.

The second local argumentation level which was proposed by Knipping (2008) was visual argumentation, which has two levels: empirical-visual and conceptual-visual. The sample local argument for *empirical-visual level* is again from geometry task 1 in the GeoGebra group. Özer was trying to find the triangle types in which |AH| was an angle-bisector. He drew the givens of the task using the GeoGebra dynamic geometry program and dragged point A to make |AH| an angle-bisector. He realized that when |AH| became an angle-bisector, it was also a median of side |BC|. He showed this on various triangle figures by dragging point A. Subsequently, he noticed that |AH| also became an altitude when |AH| became an angle-bisector. Özer said that being an angle-bisector, median and the altitude at the same time was the property of isosceles triangles as the justification of his claim, which was '|AH| is a median in isosceles triangles'. The Toulmin schema of the local argument was as illustrated in *Figure 4.47*.

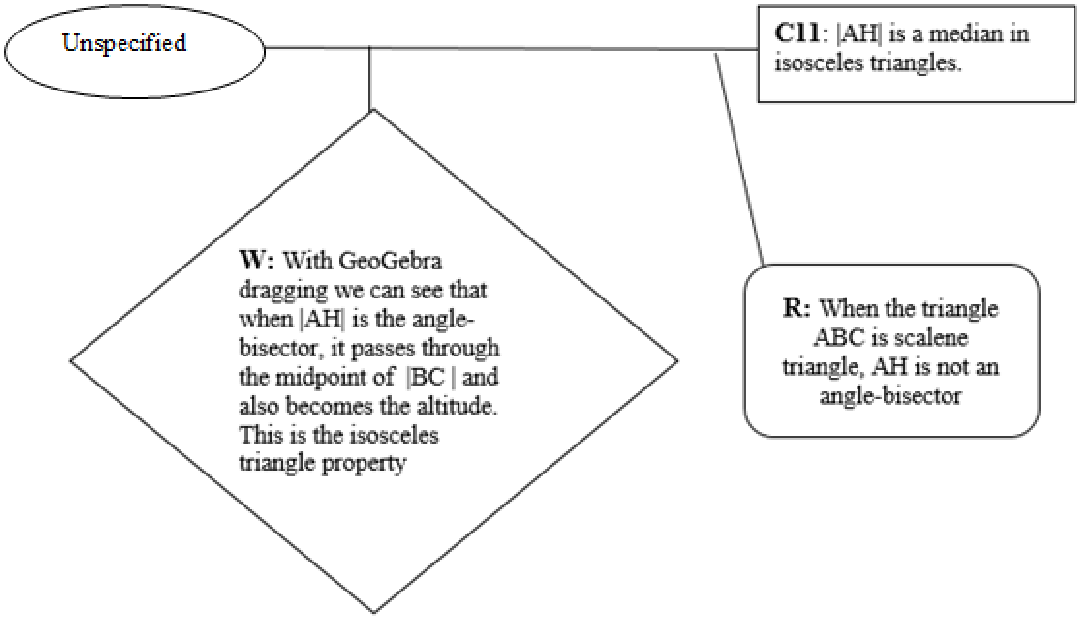


Figure 4.47 A Toulmin schema for a sample local argument of empirical-visual argumentation from the GeoGebra group

The teacher asked Özer in which cases this relationship was valid. He answered by saying that this was valid for isosceles and equilateral triangles but not valid for scalene triangles, which corresponded to the rebuttal as an exception of the situation. This sample local argument fitted the empirical-visual level since Özer showed the relationship and his warrant on a visual figure, not based on a concept, a theorem, or a rule etc. That is, his justification was the conclusion of his trial and error in dragging of a diagram on GeoGebra.

Another example for empirical-visual argumentation was from the Paper-Pencil group's geometry task 2. İnci, together with Erhan, labelled the diagram on their worksheet as illustrated *Figure 4.48.*

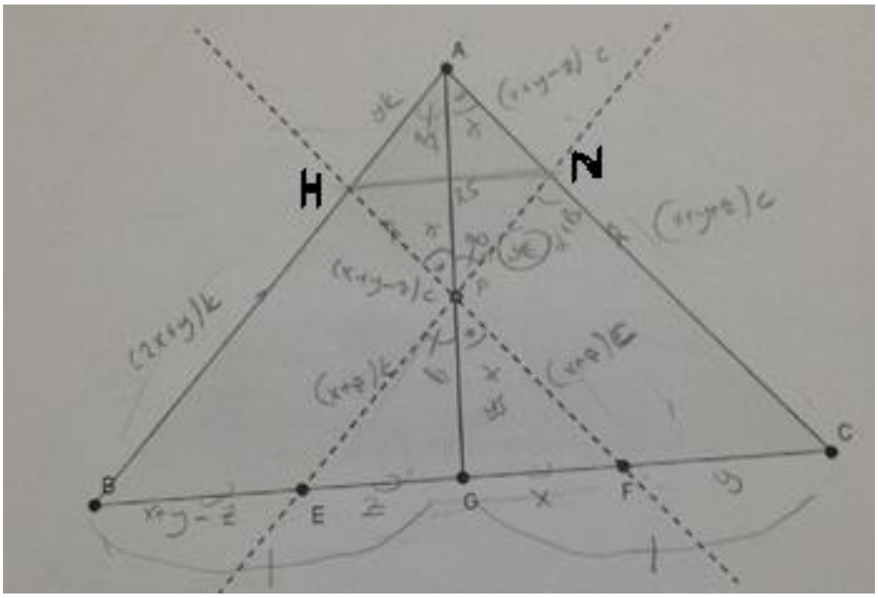


Figure 4.48 Labeling of İnci on the worksheet for GT 2 as example for empirical-visual argumentation

İnci claimed that |AN|=|HP| and similarly |AH|=|NP|. Erhan asked, 'How did you conclude that?' and İnci justified her claim by showing the parallelism of |AC|//|HF| and |AB|//|NE| on the diagram. She also added that 'The quadrilateral ANPD became a parallelogram and in parallelogram the opposite sides have equal lengths'. Therefore, it can be concluded that the local argument fits the empirical-visual argumentation level. The Toulmin schema of the local argument is presented in *Figure 4.49*. This sample local argument fitted the empirical-visual level since İnci showed the relationship and his warrant on a visual figure, not based on a concept, a theorem, or a rule etc.

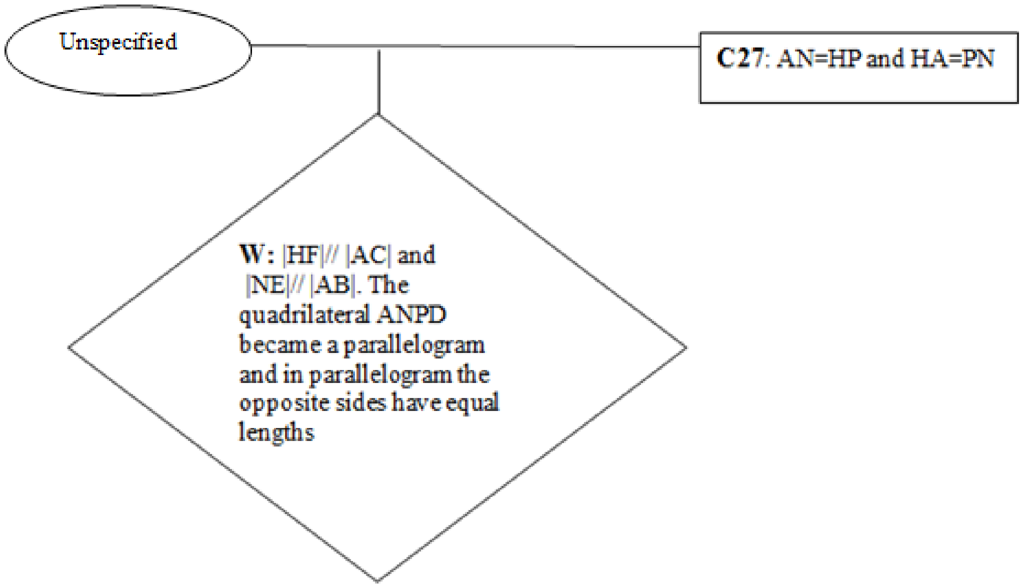


Figure 4.49 A Toulmin schema for a sample local argument of empirical-visual argumentation from the Paper-Pencil group

The last local argumentation level proposed by Knipping (2008) was the conceptual-visual level for local argumentation. The sample local argument for this level comes from the GeoGebra group's geometry task 1. The diagram drawn by Özer using the GeoGebra program was as illustrated in *Figure 4.50*.

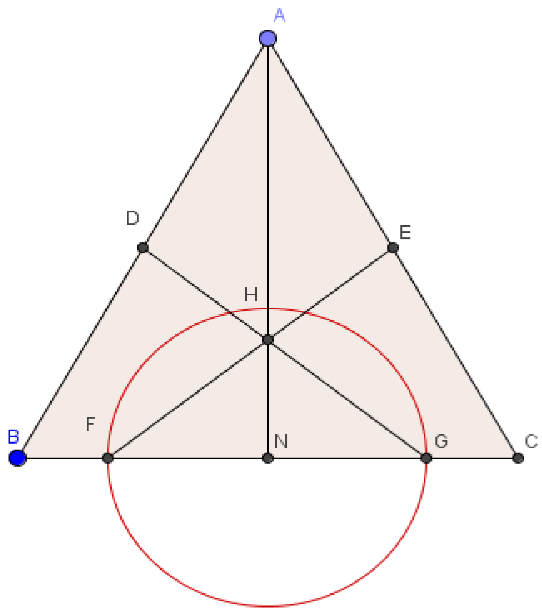


Figure 4.50 The diagram of GT 1 drawn by Özer using the GeoGebra program

Özer was discussing geometry task 1 with his partner Aslı. He drew the triangle ABC. Then, he found the midpoints of the sides |AB| and |AC|, which were points D and E, respectively. The task asked to place points F and G as equidistant to vertices B and C to find the intersection point H. Özer decided to find the midpoint of segment |BC| and labelled it with N. He drew a circle with center N to find the points F and G on segment |BC|. Drawing segments |DG| and |EF|, he complemented the givens in the task. Then, he dragged the points to look at the changes in the shape and to find the triangle types in which |AH| became an angle-bisector. He noticed that when |AH| became an angle-bisector, its extension passed through point N, which was the midpoint of side |BC|. He said that segment |AN| was an angle-bisector when triangle ABC was an isosceles and an equilateral triangle. He used this information as a warrant to his claim, which was '|AH| became an angle-bisector in isosceles and equilateral triangles'. His justification was the triangle property that 'When a segment is both an angle-bisector and a median in a triangle, this means that the segment is also an altitude and the triangle is isosceles or an equilateral triangle.' The Toulmin schema was drawn for Özer's local argument as displayed in *Figure 4.51*.

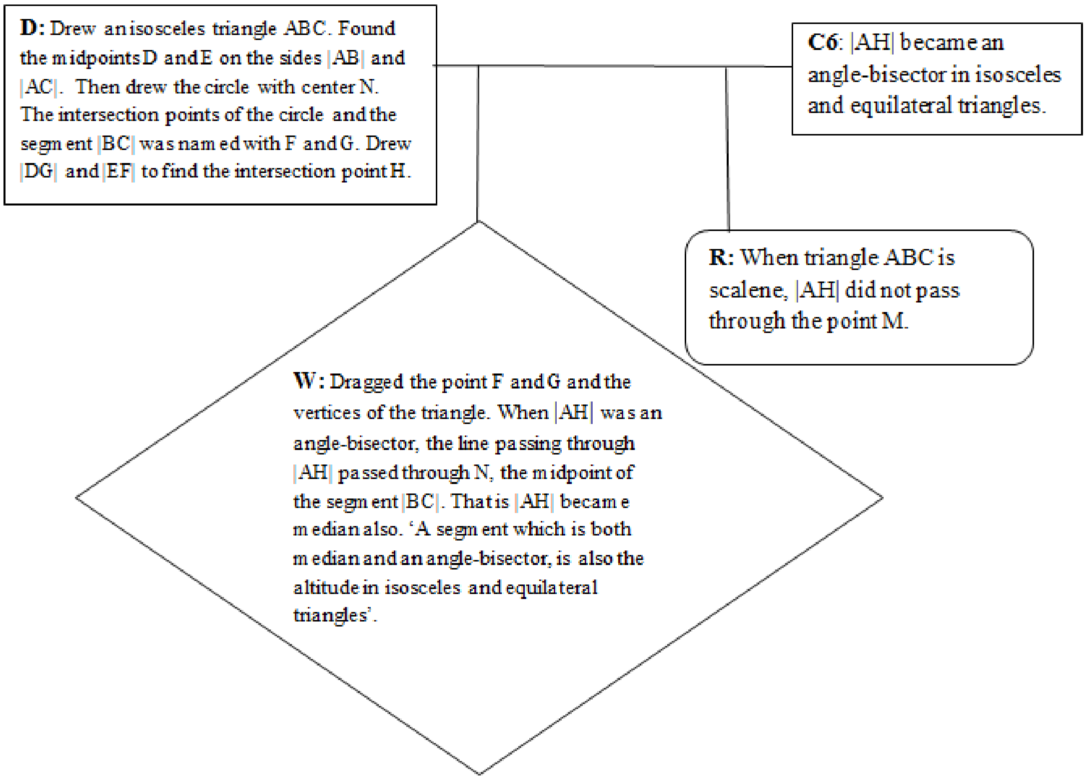
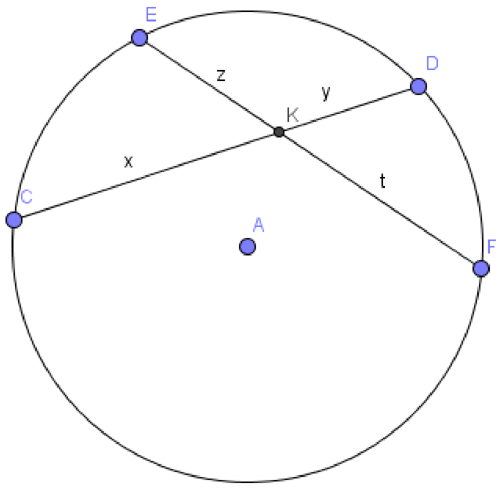
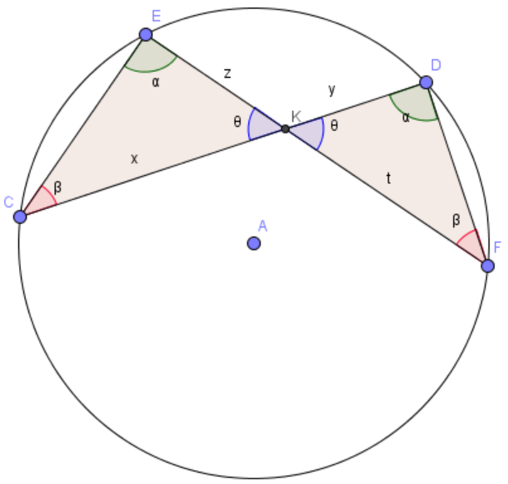


Figure 4.51 A Toulmin schema for a sample local argument of conceptual-visual level from the GeoGebra group

This local argument fitted the conceptual-visual level argumentation since Özer used a visual diagram drawn via GeoGebra to justify his claim. To make a generalization, he also used the conceptual triangle property that 'When a segment is both an angle-bisector and a median in a triangle, this means that the segment is also an altitude and the triangle is isosceles or an equilateral triangle'. That is, his warrant enabled him to generalize his finding to the isosceles and equilateral triangles. Thus, he could be able to say that "|AH| became an angle-bisector in isosceles and equilateral triangles".

Another sample local argument for conceptual-visual level was from the Paper-Pencil group's geometry task 4. While solving the circle task, Bade used the intersecting chords theorem. Then, the teacher asked the whole class where the intersecting chords theorem had derived from. Following her question, the teacher drew the diagram in *Figure 4.52(a)* and wrote "x.y=z.t" on the board. She asked the students to justify this claim.





(a) *(b)*

Figure 4.52 Teacher's drawing (a) and Erhan's solution (b) as a sample local argument for conceptual-visual level.

Erhan came to the board and drew segments |EC| and |DF|. Then, he noticed the inscribed angles seeing the same arcs in the circle. He labelled the equal angles with symbols as illustrated in *Figure 4.52(b)*. At that moment Bade asked, 'Isn't it necessary for point K to be in the center of the circle in order to label those angles?' Erhan answered by emphasizing that he was dealing with the inscribed angles, so he did not need point K to be in the center of the circle. Afterwards, he used the angle-angle-angle triangle similarity as a warrant for his claim. Using the triangle similarity equation, he found that x.y=z.t, as presented in Toulmin's schema in *Figure 4.53.*

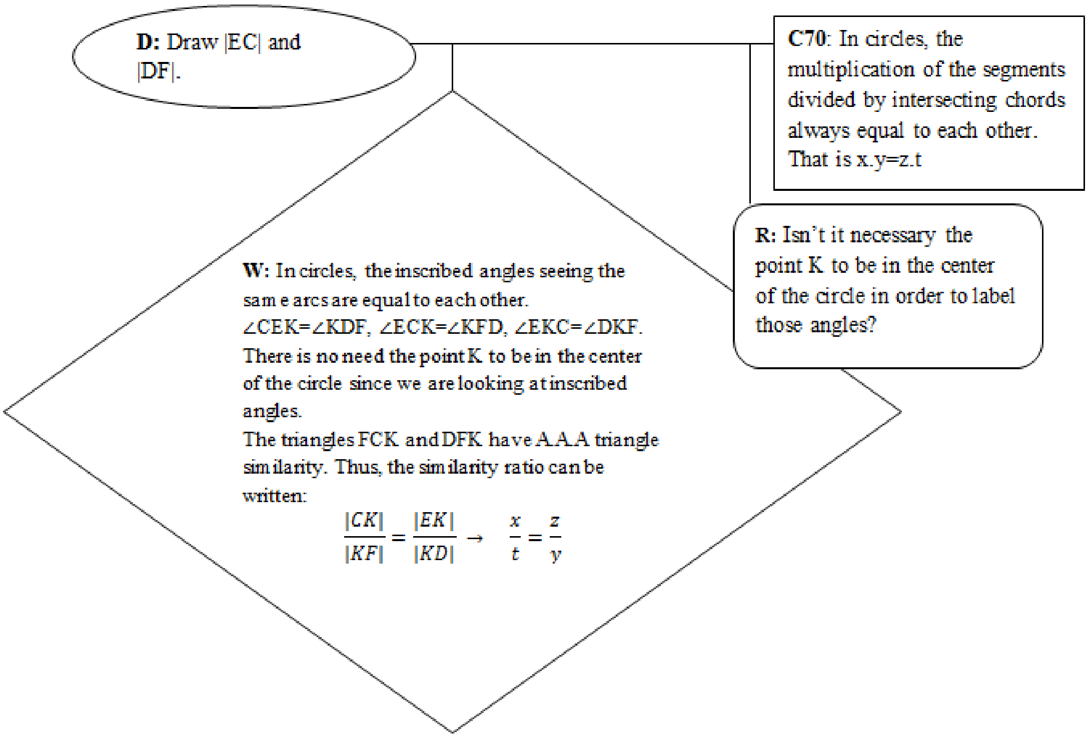


Figure 4.53 A Toulmin schema for a sample local argument of conceptual-visual level from the Paper-Pencil group

This local argument was accepted as a conceptual-visual local argument since Erhan showed his solution on the diagram by using the circle properties and the A.A.A triangle similarity theorem to be able to justify the intersecting chords theorem. He could be able to make generalizations with the relationships he showed on the diagram.

In the current study, sometimes the participants did not state the warrant, which means their warrant was implicit in some of their local arguments. In the local argumentation analysis of Knipping (2008) the differentiating component was warrant (and backing) since the local argumentation type was decided by looking at the characteristics of the warrants. Thus, the local arguments which did not have a warrant component could not be classified into one of the local argumentation levels developed by Knipping (2008). Those arguments were listed in the following tables under the heading '*Arguments which doesn't have a warrant*'. The sample local argument comes from the GeoGebra group's geometry task 4, which has already been explained in the spiral structure argumentation section (see *Figure*

4.8). It was a circle task and required dragging the shape to solve the question. Bade was explaining her solution to the question, ‘Show whether the theorem is trivial if chord |FG| is a diameter of the first circle, or if |FG| coincides with |CD|. Justify your reasoning’.

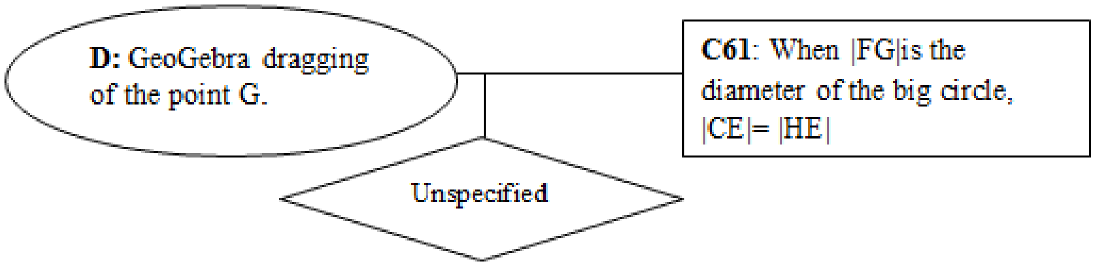


Figure 4.54 A Toulmin schema of the sample local argument for the DC type of argument

Bade responded to the first part of the question -‘if chord |FG| is a diameter of the first circle’- by dragging the shape and then moved to the second part –‘if |FG| coincides with |CD|’. The other participants did not question Bade for her warrant. Thus, this local argument (see in *Figure 4.54*) did not have a warrant and could not be classified into one of the groups in Knipping’s (2008) classification.

Finally, in the present study, some local arguments did not completely fit the levels of Knipping’s (2008) classification. These arguments were the following local arguments*: the givens as justification*, *GeoGebra measurement as justification, GeoGebra actions as justification.*

The sample local argument for *the givens as justification* in the task comes from the Paper-Pencil group’s geometry task 1. Erhan claimed that points F and G should be placed on segment |BC| in such a way that F should be close to vertex B, and point G should be close to vertex C. He drew the givens on the triangle ABC and stated his warrant as ‘In the givens it says that segments |DG| and |EF| should coincide to create point H’. The Toulmin schema was as illustrated in *Figure 4.55*.

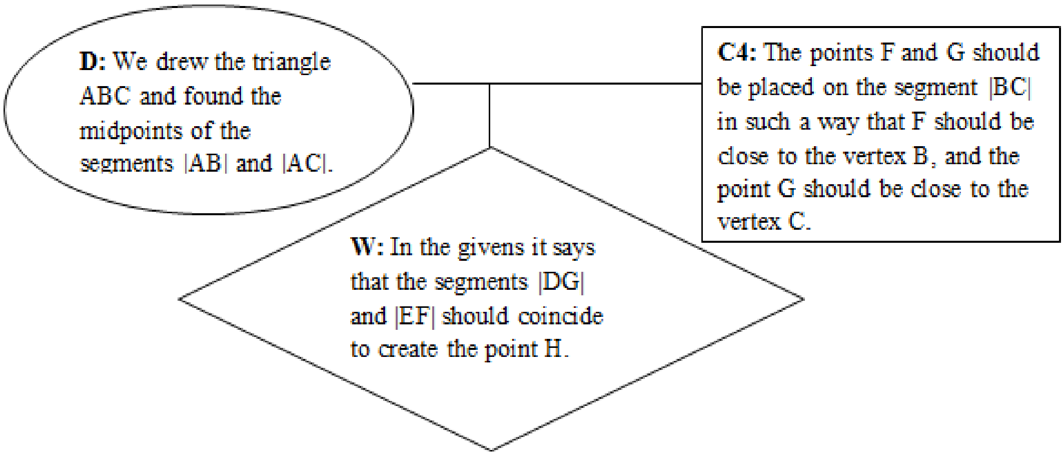


Figure 4.55 A Toulmin schema of a sample local argument for a new type: the givens as justification

The local argument in *Figure 4.55* did not fit the conceptual argumentation since it did not have a theorem, rule, axiom etc. as a warrant. Moreover, it did not fit empirical-visual or conceptual visual levels since it did not include the justification on the diagram. The warrant only includes the givens in geometry task 1. Thus, this argumentation type was named as the givens as justification.

The second local argument which did not fit Knipping's (2008) classification is presented with a sample from GeoGebra group's geometry task 3. In this argument, Özer claimed that 'Triangle FGH is an equilateral triangle'. He concluded by drawing the givens in the task (see *Figure 4.56*) and justifying his claim by only measuring the sides of triangle FGH in the diagram. The Toulmin schema of this local argument was as illustrated in *Figure 4.57*.

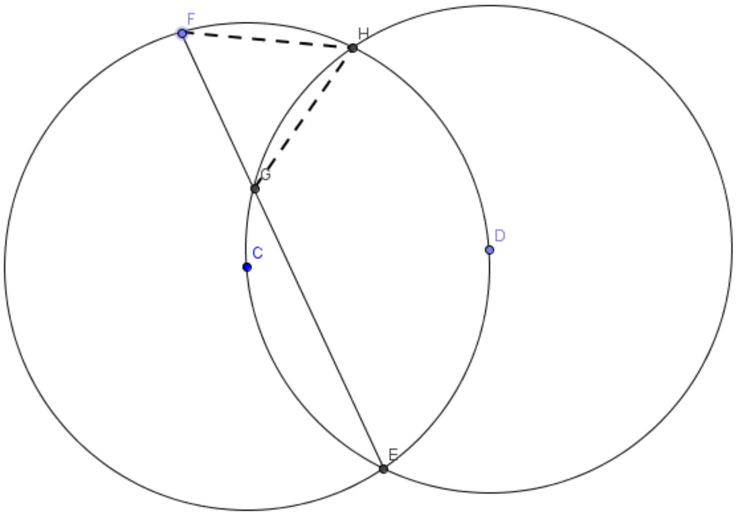


Figure 4.56 Özer's drawing for GT 3 as a sample for new type: GeoGebra measurement as justification

This argument did not fit any of the local argumentation levels of Knipping (2008) since its warrant was not a theorem, a rule, an axiom, a representation on a diagram, or a representation of a concept on a diagram. The warrant was only an act of measurement. Thus, this new type of justification was named with the name GeoGebra measurement as justification.

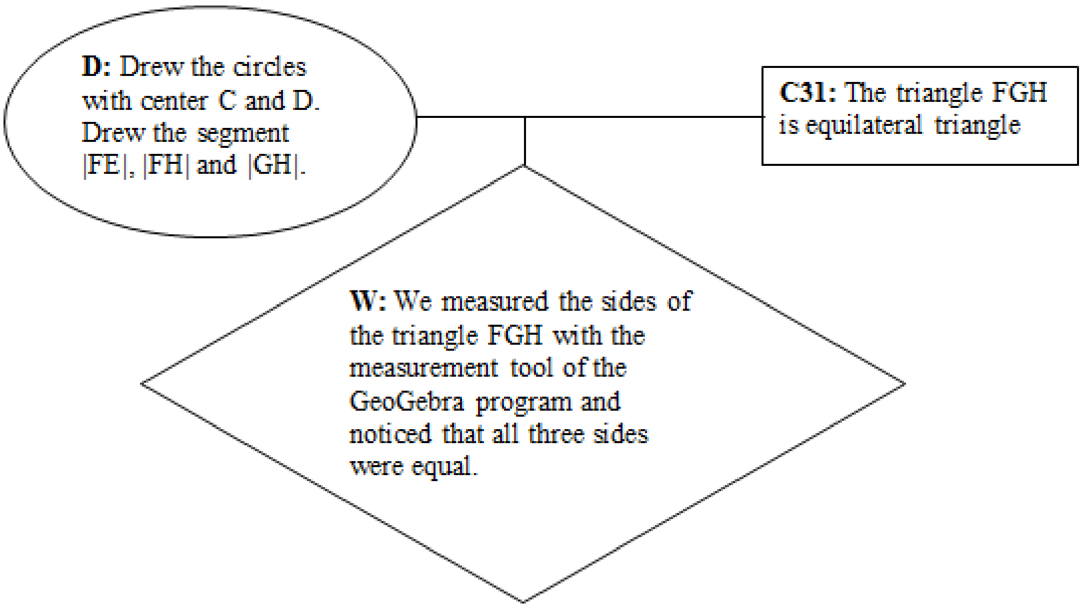


Figure 4.57 A Toulmin schema of a sample local argument for new type: GeoGebra measurement as justification

The last local argument which did not fit the classification of Knipping (2008) was presented with a sample from the GeoGebra group's geometry task 3. Bade came to the board and drew the diagram based on the givens in geometry task 3. Then, she dragged point G in order to obtain triangle FGH with the minimum area. When point G coincided with point E, the triangle disappeared and the area value was 'undefined' in the GeoGebra program since the program could not calculate the area of a point. Then, Bade claimed that the minimum area of triangle FGH was undefined and she justified her claim by saying, 'The GeoGebra program showed it as such'. The Toulmin schema of this local argument was as displayed in *Figure 4.58*.

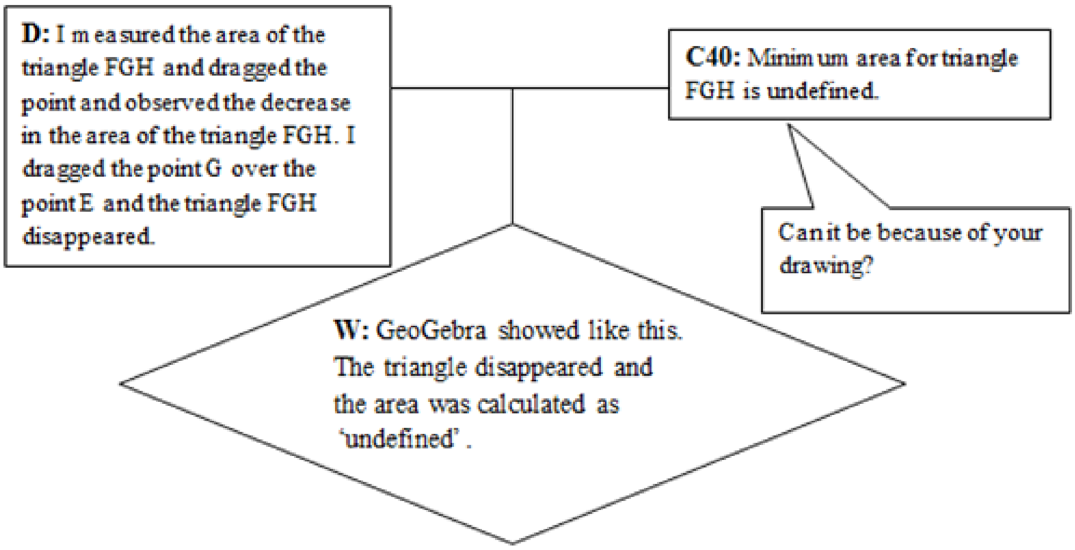


Figure 4.58 A Toulmin schema of a sample local argument for a new type: GeoGebra actions as justification

Similar to the previous two arguments, this argument did not fit any of the local argumentation levels of Knipping (2008) either since its warrant was not a theorem, a rule, an axiom, a representation on a diagram, or a representation of a concept on a diagram. The warrant was only the reaction of the GeoGebra program to the user's drawing and dragging. Thus, it was named as GeoGebra actions as justification.

4.3.1 The characteristics of the local argumentations

The local argumentations were classified according to the warrants and backings of local arguments. There were 4 geometry tasks in the application, and the argumentation types were listed for each geometry task in the following tables, respectively. The warrants of the arguments were numbered with the number of their claims. For instance, '1' in the table refers to the warrant of the first claim while '5' refers to the warrant of the fifth claim. Table 4.5 reveals the distribution of the warrants of geometry task 1 in each local argumentation level.

Table 4.5 Distribution of local arguments of GT 1 across local argumentation levels

Local Argumentation		**Visual Argumentation**		**Conceptual Argumentation**	**Arguments which don't have a warrant**	**New Condition**
		Empirical-visual	**Conceptual-visual**			
Argument numbers	**GeoGebra group**	2, 5, 7,9, 10, 11,12, 13, 14, 16	6	8, 15	1, 3	4
	Paper-Pencil group	9, 14, 18, 20, 23, 24	----	5, 6, 7, 8, 11, 12, 13, 15, 16, 17, 19, 21, 22	1, 2, 3, 10, 25	4

Geometry task 1 entailed a triangle task. As it can be inferred from Table 4.5, the GeoGebra group mostly used empirical-visual warrants, while they were working with the GeoGebra dynamic geometry program. They asserted the conceptual-visual warrant which was stated in the 6^{th} argument and the conceptual warrants which were stated in the 8^{th} and 15^{th} arguments. Moreover, they did not justify arguments 1 and 3. In addition, there was one new condition which included the *givens as justification* to their claim in argument 4. On the other hand, in the Paper-Pencil group, the participants mostly used the conceptual type of argumentation as local argumentation. They also alleged six empirical-visual warrants: 9, 14, 18, 20, 23, and 24. In addition, the arguments in which they did not include any justification (warrant) were arguments 1, 2, 3, 10, and 25. Similar to the GeoGebra group, there was one new condition which included the *givens as justification* to their claim in argument 4. The difference between the GeoGebra group and the Paper-Pencil group was obvious in this task in terms of local argumentation. The use of GeoGebra directed the participants in the GeoGebra group to justify their claims by talking about the dynamic figure they drew, while the participants in the Paper-Pencil group used the concepts, theorems, rules, and mathematical relations to justify their reasoning. Specifically, the participants in the Paper-Pencil group were inclined to use theorems, such as Menelaus' theorem and Angle-bisector theorem and mathematical relations, such as triangle similarity and triangle properties to show that they drew the givens of the task appropriately and to show the situations in which segment |AH| was an angle-bisector.

Geometry task 2 was also based on triangles. Table 4.6 presents the argument numbers of geometry task 2 in each local argumentation level.

Table 4.6 Distribution of local arguments of GT 2 within local argumentation levels

Local Argumentation		**Visual Argumentation**		**Conceptual Argumentation**
		Empirical-visual	**Conceptual-visual**	
Argument numbers	**GeoGebra group**	19, 20, 22, 23	17, 24, 28, 29, 30	18, 21, 25, 26, 27
	Paper-Pencil group	27, 30, 33, 39	32, 35, 36	26, 28, 29, 31, 34, 37, 38, 40

As it can be observed in Table 4.6, the participants in GeoGebra group mostly used visual warrants (empirical-visual and conceptual-visual). They also used five conceptual warrants which were stated in arguments 18, 21, 25, 26 and 27. On the other hand, in the Paper-Pencil group, the number of local argumentations was equally distributed across the visual and conceptual levels. It can be seen that there was no argument without justification and there was no new condition in the discussion of GT 2 in both groups. The difference between GT 1 and GT 2 was that the participants decided how to place points F and G and drew the dynamic figure. Then, they interpreted the dynamic relations in GT 1, but in GT 2, the places of the points were apparent, they were only interpreting the dynamic relations of the figure. This could lead to a longer discussion among the participants in the Paper-Pencil group on the relationships in the figure besides the conceptual arguments in GT 2 when compared to GT 1.

Geometry task 3 was based on the circles. Table 4.7 presents the argument numbers of geometry task 3 in each local argumentation level.

Table 4.7 Distribution of local arguments of GT 3 within local argumentation levels

Local Argumentation		**Visual Argumentation**		**Conceptual Argumentation**	**Arguments which don't have a warrant**	**New Condition**
		Empirical-visual	**Conceptual-visual**			
Argument numbers	**GeoGebra group**	32, 33, 34, 35, 36, 41, 42, 45	37, 38, 43, 44	39	---	31, 40
	Paper-Pencil group	42, 44, 48, 52, 45, 47, 53, 54	41, 43, 50, 46	51, 56	49, 55	---

As it can be inferred from Table 4.7, it was obvious that the participants in both the GeoGebra and the Paper-Pencil groups mostly used visual warrants in their local arguments in GT 3. Only one conceptual warrant was presented in the GeoGebra group in the 39th argument. Finally, two new condition warrants were presented in the GeoGebra group. The warrant in the 31st argument was similar to the warrants of the empirical-visual level argumentation, but it provided an opportunity for generalization with the help of the measurement tools of the GeoGebra dynamic geometry program, so it did not completely fit the empirical-visual argumentation. The other local argument which fit the new condition was stated in the 40th argument. It was asserting the actions of the GeoGebra program as justification to their claim. In the Paper-Pencil group, there were two conceptual argumentations in the 51st and 56th arguments. In addition, there were two local arguments (49 and 55) which were not justified by the participants in the Paper-Pencil group. In GT 3, the participants used mathematical relations, such as circle properties in the first question by referring to the figure and using visual argumentations. Similarly, the solution of the second question completely required the interpretation of the dynamic figure, so the participants' interpretations included the references to the dynamic figure which resulted in an increase in the number of the visual argumentations in both the GeoGebra and Paper-Pencil groups.

Geometry task 4 was also based on circles. Table 4.8 presents the argument numbers of GT 4 in each local argumentation level.

Table 4.8 Distribution of local arguments of GT 4 within local argumentation levels

Local Argumentation		**Visual Argumentation**		**Conceptual Argumentation**	**Arguments which don't have a warrant**
		Empirical-visual	**Conceptual-visual**		
Argument numbers	**GeoGebra group**	51, 52, 53, 54, 59, 62	55, 56, 63	47, 48, 57, 58, 60,	46, 49, 50, 61
	Paper-Pencil group	57	64, 65, 69, 70, 71, 73, 74	59, 60, 62, 63, 66, 67, 68, 72	58, 61

As it can be observed in Table 4.8, the participants in the GeoGebra group mostly used visual argumentation. There were four arguments (46, 49, 50, and 61) which were not justified in the GeoGebra group. Finally, there was no 'new condition' in both groups. In the Paper-Pencil group, the distribution of the warrants in the visual and conceptual argumentations was approximately equal. Only one empirical-visual argumentation (57) was used in the Paper-Pencil group. There were two arguments (58 and 61) which were not justified in the Paper-Pencil group. In GT 4, the first question was one that asked for proof (prove that …) and the second question was a dynamic figure question which required the dragging of the points to see mathematical relations. In the GeoGebra group, the participants used both the dragging option of the GeoGebra and the theorems and rules to justify their reasoning. In the Paper-Pencil group, the participants did not have a tool to check their interpretations regarding the dynamic figure in their imagination, so they once again resorted to conceptual rules to justify their claims.

After analyzing the different types of local argumentations task by task, the researcher decided to examine them in terms of mathematical content since there were two triangle tasks (GT1 and GT2) and two circle tasks (GT3 and GT4).

In the triangle tasks, it was apparent that the participants in the GeoGebra group preferred to use mostly visual argumentation, especially empirical-visual argumentation. On the other hand, the participants in the Paper-Pencil group preferred conceptual argumentation. This was an expected result since the use of GeoGebra had the potential to direct the participants to think and talk about the figure they drew and dragged, so they talked about specific examples while solving the geometry tasks, referring to empirical-visual argumentation. As for the Paper-Pencil group, it was also an expected result since the participants used their conceptual knowledge to justify their drawings and inferences.

In the circle tasks, the most preferred local argumentation by the participants of the GeoGebra group was again visual argumentation, especially empirical-visual argumentation. This meant that the mathematical content did not make a difference in the use of the various types of local argumentation for the participants in the GeoGebra group. However, the situation was different for the Paper-Pencil group. At the time, the participants in the Paper-Pencil group mostly resorted to visual argumentation in the circle tasks. In the circle tasks, the numbers of local arguments in the empirical-visual and the conceptual-visual argumentation were nearly equal in the Paper-Pencil group. This was an interesting finding since the participants in the Paper-Pencil group could imagine the difference in the figures after dragging the points correctly and made interpretations mostly by referring to the figures they drew rather than using theorems and rules as a conceptual argumentation.

CHAPTER V

DISCUSSION, CONCLUSION AND IMPLICATIONS

The aim of the present study was to investigate the nature of the argumentation structures employed by prospective middle school mathematics teachers while solving geometry tasks in GeoGebra and Paper-Pencil groups. In addition, the prospective middle school mathematics teachers' local arguments in the global argumentation structures were analyzed in detail. More specifically, the kinds of global argumentation structures the prospective middle school mathematics teachers employed, the kinds of local arguments they expressed based on the flow of argument components, and the kinds of local argumentations they utilized to justify their arguments were investigated.

This chapter addresses the discussions based on the findings, the conclusions and implications, and recommendations for further research studies. That is, the striking points of the results of the study are reviewed and discussed by referring to the related literature. The chapter is organized in three main sections based on the three research questions of the study. The first section presents the discussion regarding the global argumentation structures of the prospective middle school mathematics teachers with reference to previous studies. In the second section, the local arguments based on the flow of argument components are discussed. Finally, in the third section, the local argumentations based on the warrant and backings of the arguments are discussed in detail.

5.1 Prospective middle school mathematics teachers' global argumentation structures

In an argumentation environment, the participants constructed arguments collectively. Some of these arguments were interconnected, meaning the claim of one argument stood for a data or warrant for subsequent arguments. In addition, there were some arguments which were independent of the other arguments. That is, the processes involved in the discussions within each classroom were based on rationales peculiar to themselves, and thus there was a need to reconstruct and analyze complex argumentative structures in classrooms where argumentation took place (Knipping & Reid, 2013). Moreover, based on the assertion that formal mathematical logic is insufficient to present the rationale in argumentation (Knipping & Reid, 2013), the present study sought to gain insight into the whole picture of the discourse of the geometry tasks in order to interpret the general situation occurring in the classroom, and to understand the rationale and contextual constraints shaping the argumentations. In addition, it was believed that analyzing this general structure would be helpful in improving the efforts made to teach geometry within a technology-enabled environment. More specifically, the researchers would gain insight into the quality of the argumentations taking place in the classroom when they analyze the big picture. It was expected that those students who were taught in a well-directed argumentation environment would display the concept relationships in their minds. They would even reveal the misconceptions they inadvertently possessed. In this way, the instructors would have the chance to gain information about their students' knowledge and they would be able to evaluate their students' abilities in reasoning, justification and performance in the topic that was covered via argumentation. In the literature, Knipping (2008) detected this need and defined *global argumentation* as an anatomical structure, which means considering the layout of argument structures as a whole. She also considered the single arguments, schematized via Toulmin's model, and called them *local arguments,* which are discussed in the following sections.

In the present study, Knipping's (2008) global argumentation structures were considered, but the model was revised and developed since there were structures among the data of the present study which did not fit any of these global argumentation structures. Therefore, names were created for these structures. Thus, data analysis revealed five main global argumentation structures in a geometry context: Source-Structure, Reservoir-Structure, Spiral-Structure, Line-Structure and Independent Arguments-Structure.

When the global argumentation structures of the GeoGebra group and the Paper-Pencil group were compared task by task, it was seen that there were some similarities and differences between the two groups. The first difference identified between the groups was that in geometry task 1, there were one reservoir-structure and one spiral structure in the GeoGebra group, while there were one reservoir-structure, one line structure and eight independent arguments in the Paper-Pencil group. When the data were examined in detail, the high number of independent arguments in the Paper-Pencil group drew attention. The participants in the Paper-Pencil group drew the figures with measurement tools, such as the ruler, protractor and compass, so their drawings were not precise as in the GeoGebra drawings. Therefore, they were not sure about some of the interpretations they made while they were solving the geometry task. As a result, they stated some disconnected facts while they were discussing among each other. Thus, the reason underlying the use of many independent arguments in the Paper-Pencil group could be due to the absence of a reliable tool like GeoGebra to justify their drawings. In addition, one of the participants (Güler) mentioned several assumptions at the beginning of her solution for geometry task 1, which could not be justified till the end of the solution. These assumptions were solely stated since they could not be checked via a tool such as GeoGebra in the Paper-Pencil group. Thus, another reason for the high number of independent arguments could be attributed to the assumptions of Güler in her solution since other participants talked about her assumptions frequently and those arguments were not connected to the whole structure and remained as independent arguments. The second difference between the groups was the existence of the spiral structure, which included parallel argumentation streams, in the GeoGebra group. The existence of parallel argumentation streams could be attributed to teacher facilitation since the teacher asked the other pairs in the group to explain their solutions, so the task was solved repeatedly, which led to the emergence of parallel argumentation streams. The conclusion of these parallel argumentation streams was the target conclusion, so the global argumentation structure had the characteristics of the spiral structure. The third difference was the existence of the line structure in the Paper-Pencil group. This argumentation structure existed at the beginning of the discussion and included arguments related to the drawing of the shape in the task. Therefore, the claims and subsequent data (claim/data) were connected to each other like a line and the line structure was constructed. However, in the GeoGebra group, the drawing process was connected to the reservoir-structure since the participants discussed the solution by drawing and dragging the shape simultaneously. As for the similarity between the two groups, it was revealed that in both groups the reservoir-structure was existent since both groups moved backwards in their reasoning process, which is a characteristic of the reservoir-structure. The reason could be rooted in the detailed solution of the tasks and turning back to clarify some missing parts which were previously discussed in both groups with the help of the teacher's facilitation.

The global argumentation structures of the GeoGebra and the Paper-Pencil groups were similar when the global argumentation structures of geometry task 2 were compared. That is, in the Geogebra group, three spiral structures and two independent arguments were observed, while in the Paper-Pencil group, two spiral structures, two independent arguments, and one source-structure emerged. The spiral structures in both groups emerged either when the teacher asked the other groups for another solution to the task or when the nature of the task encouraged solving the problem again from the beginning by changing one of the properties. For instance, the first question related to triangles asked the relationship between segments |EG| and |GF|, while the second question asked the relationship between segments |EG| and |GF| when triangle ABC was an isosceles or an equilateral triangle. In the first question, the participants found a general relation that $|EG| = |GF|$, and then they solved the question again by drawing an isosceles triangle ABC. Subsequently, they proceeded with the solution by drawing an equilateral triangle ABC. In the end, they found the same relationship, and the spiral structure was formed during the argumentation. The only difference between the global argumentations of the two groups was the emergence of the source structure in the Paper-Pencil group. When the data was examined, it was seen that the participants in the Paper-Pencil group had used Menelaus's theorem and similarity in triangles to justify their claim that |EG| = |GF|. Specifically, different participants proposed different parts of the solution by using theorems and mathematical relationships, and then they reached the target conclusion. Thus, the arguments were flowing from different sources. As a result, the parallel arguments occurred at the beginning of the solution, which was a characteristic of the source structure. The reason for the emergence of a source-structure in the Paper-Pencil group could be the use of algebraic solutions instead of dynamic drawing in the Paper-Pencil group. More specifically, when the shape of the argumentation was examined, it was seen that the opinions of participants were flowing as if arising from a variety of origins and this led to a funneling effect which was said to be apparent in source-structure argumentations (Reid & Knipping, 2010). As a result, the argumentation structure was named as source structure in the Paper-Pencil group.

The third geometry task was a circle task and the global argumentation structures were again similar to those in geometry task 2. In both groups, the reservoir-structure, the line-structure and independent arguments were observed, but the only single difference was the existence of one source-structure in the Paper-Pencil group. The reason for this was the existence of the parallel argumentation streams at the beginning of the solution via collective argumentation. That is, the initial properties were stated by different individuals simultaneously, and then the target claim of the structure was stated collectively in the argumentation. In the Paper-Pencil group, the solution of the geometry task was justified by theorems or relations proposed by different individuals, while in the GeoGebra group, the participants also considered the options provided by the GeoGebra dynamic program. That is, when the participants focused on the actions of the GeoGebra program, they rarely interrupted the participant solving the task on the board until the end of the solution. Therefore, in the GeoGebra group, most of the time parallel argumentations were not placed at the beginning of the argumentation (parallel arguments at the beginning of the solution is a characteristic of a source structure). An interesting finding was that both groups used line-structure to answer the same part of the question, which asked for the area of triangle FGH. This could be accounted for with the nature of geometry task 3 and the properties of the circle. In geometry task 3, the part in which the participants were trying to find the area of triangle FGH, the solution required step by step connected arguments based on the properties of the circle. That is, they found the measure of the length of an arc, and then they used that value to find another arc or length. In this way, the claims of the arguments became the data of the subsequent arguments (claim/data) repeatedly until the target solution was reached. This led the participants of both groups to use the line structure in this part. Another finding was the existence of many independent arguments in both groups. In both groups, the researcher encouraged the participants to express their thought all the time during the argumentation. Therefore, there were many independent arguments which were not connected to the solution directly but were the bases of the target conclusion. Thus, this finding could be linked to the thinking aloud of the participants most of the time with the encouragement of the instructor to express their opinions.

The last geometry task was again a circle task which required dragging the segments. There were again both similarities and differences between the global argumentation structures emerging in the GeoGebra group and the Paper-Pencil group. Two spiral-structures, five independent arguments, one source-structure and one reservoir-structure emerged in the GeoGebra group, while three spiral-structures and five independent arguments emerged in the Paper-Pencil group. In this task, different solutions were encouraged by the teacher, so the spiral-structure was frequently used in both groups. In addition, the task was a bit difficult, so students proposed some ideas (although they were not sure that the idea would work) which were not related to the solution but seemed to contribute to the solution. Hence, the number of independent arguments in both groups was high. On the other hand, the differences between the groups were the emergence of the source-structure and the reservoir-structure in the GeoGebra group. The participants in the Geogebra group were brainstorming by dragging the segments and they analyzed the shape in detail with the help of the GeoGebra. Therefore, the reasoning moved backwards in the logical structure and then forward again, leading to the emergence of the reservoir-structure. In addition, the ideas were stated by different participants, while an individual was drawing the shape on the board, and the parallel arguments were placed at the beginning of the solution process. Thus, it was observed that a source structure was produced within the argumentation of the GeoGebra group. Ultimately, it could be deduced that the existence of GeoGebra contributed to the emergence of the source-structure and the reservoir-structure in the GeoGebra group in geometry task 4 since the task required changing the shape by dragging and interpreting the new shape based on the given properties. That is, it was necessary to drag the shape and then return to the previous arguments and then move forward in order to interpret the new shape correctly, which led to the emergence of the reservoir-structure.

After examining the findings task by task, a general inference was made by considering the reasoning in all the geometry tasks and the studies in the literature. There were four global argumentation structures which were proposed in the context of proof in the literature (Knipping, 2008; Knipping & Reid, 2013; Reid & Knipping, 2010): source-structure, reservoir-structure, spiral-structure and gathering-structure. However, within the argumentations of the present study only three of these global argumentation structures emerged. It was believed that examining global argumentation structures would facilitate the understanding of the rationales and contextual limitations of participants' argumentations, which, in turn, would provide researchers with insight into the features of the proving processes utilized in the classrooms. Ultimately, teaching proof could be improved (Reid & Knipping, 2010). In addition, the analysis of argumentation structures was believed to uncover different classroom cultures and approaches of teaching (Reid & Knipping, 2010). They mentioned that their findings were not stable and strict as they could be affected by the cultures in which they were implemented, the nature of the mathematics content and the application of teachers' goals (Reid & Knipping, 2010). The findings of the current study were interpreted taking all these into consideration.

An interesting finding of the present study was the emergence of new global argumentation structures that did not exist in the literature in the geometry context, namely the line-structure and independent arguments-structure. These new structures clearly demonstrated the prospective middle school mathematics teachers' nature of argumentation in geometry in the Turkish context. Specifically, these new global argumentation structures did not have a structure like a complex net of connected arguments. Instead, the line-structure was a simple flowing structure which had arguments connected to the claim/data. As can be inferred from its name, independent arguments entailed disconnected facts, which were proposed by the participants when they were thinking aloud at any time of the argumentation. The interesting observation was the high number of independent arguments in almost all the geometry tasks. These single arguments indicated that the prospective middle school mathematics teachers could not see the relationships among concepts and, thus, could not reach the target solution. That is, the independent arguments stood alone during the collective argumentation as disconnected facts and the bonds between those arguments could not be constructed effectively. Although the independent arguments could be regarded as natural since the participants were thinking aloud, the connections of these independent arguments were not constructed even at later stages during the argumentation. That is, it could be deduced that both in the GeoGebra group and the Paper-Pencil group, the prospective middle school mathematics teachers were prone to construct simple logical relations (not complex structures) and the complex structures rarely emerged only with the support, facilitation and questioning of the instructor. At this point, the quality of the mathematical reasoning that prospective middle school mathematics teachers presented should be considered. In the literature, one of the crucial aspects of learning and doing mathematics was stated as mathematical reasoning (Conner et al. 2014b) and it was defined as "purposeful inference about mathematical entities or relationships" (Conner et al., 2014b, p.183). As understood from the definition, mathematical reasoning is based on seeing relationships and it is a crucial characteristic that each mathematics teacher should have in order to do mathematics with students. When the simplicity of most of the argumentation structures of the participants is considered, it is clear that the prospective middle school mathematics teachers' mathematical reasoning could be regarded as weak since they did not notice and present the complex mathematical relationships in their arguments. The reason for the low level mathematical reasoning of the prospective middle school mathematics teachers could be their unfamiliarity with argumentation in their previous educational experiences. They were not used to learning mathematics and geometry through argumentative activities during their education. Another reason for poor mathematical reasoning of the participants might be the national examination system and the curriculum applied in Turkey. In any country, the examination system directs the education system and the applied curriculum at schools. In Turkey, students are subjected to multiple-choice exams several times throughout their education. Therefore, study of mathematics is based on memorizing the rules and doing drill questions most of the time in Turkish schools. This leads to the situation where students do not engage in high-level mathematical reasoning and proof, which are said to be essential in student learning (NCTM, 1991, 2000). Moreover, students in Turkey rarely engage in classroom discussions which require student interaction and social learning (Hewit, 2010; Prusak et al., 2012; Vincent, Chick & McCrae, 2005; Yackel, Ramussen & King, 2000). The participants of the present study were also educated with the mentioned curriculum and practices, without high-level reasoning experiences. Although, they obtained the university education, they still prone to behave according to their past habits of background education. Considering all of these background experiences, it could be deduced that their low-level mathematical reasoning could be the source of the high number of independent arguments in their argumentation.

Another important finding was the relatively frequent use of the spiral-structure among the global argumentation structures existent in the literature. Consistent with the characteristics of the spiral-structure, the parallel argumentation streams ended with the target conclusion, which means the parallel argumentations were located at the end of the discussion (Knipping & Reid, 2013). In particular, the target conclusion of the discussion was reached by means of different solutions and each solution corresponded to one of the parallel argumentation streams. The reason for the existence of the spiral structure more than the other structures (source-structure and reservoir-structure) could be attributed to teacher facilitation. The teacher promoted alternative solutions by asking 'Is there any other different solution?' all the time and invited each pair to the board to present their solution. Thus, the task was solved again and each solution corresponded to one of the argumentation streams. This finding revealed the importance of teacher facilitation in argumentation (Conner, 2007a; Forman et al. 1998; Heinze & Reiss, 2007; Hunter, 2007; Yackel & Cobb, 1996). If the instructor had not asked for another solution, there would not have been parallel argumentation streams and so the global argumentation structure could be like a line structure which had only one solution. In the present study, the instructor supported student interaction, and raised the students' attention to hear all the argument elements by asking for justification all the time (Cross, 2009). Other teacher actions performed in the present study were following up on each group in the discussion to encourage student participation, providing hints when students could not proceed further and avoiding the use of evaluative statements which had the potential to end the discussion (Cross, 2009). Furthermore, the questioning (Kosko et al., 2014; Vincent, Chick, & McCrae, 2005; Wood, 2003) and revoicing (Chapin, O'Connor & Anderson, 2003; Conner, Singletary, Smith, Wagner, & Francisco, 2014a; O'Connor & Michaels, 1996) methods were used all the time in order to promote argumentation. Thus, the researcher's facilitation for alternative solutions and using argumentation promoting actions might have contributed to the high number of the spiral-structure in both groups in the present study.

5.2 Local arguments of prospective middle school mathematics teachers

The findings related to the local argument types are important since they reveal how the prospective middle school mathematics teachers reason and what they mostly pay attention to during the construction of local arguments. As mentioned in the literature section, the different types of arguments were examined in some studies (Aberdein, 2005; Baccaglini-Frank & Mariotti, 2010; Viholainen, 2011). The researchers proposed formal arguments and informal arguments (Viholainen, 2011), regular arguments and critical arguments (Aberdein, 2005), and instrumented arguments (Baccaglini-Frank & Mariotti, 2010). In the present study, the analysis of the flow of argument components was conducted in order to seek any existing pattern in the arguments constructed by prospective middle school mathematics teachers in the geometry context. Thus, in this section, the findings of the second research question related to the local arguments based on the flow of argument components are discussed.

The findings revealed nine types of local arguments constructed by prospective middle school mathematics teachers: DCW, DWC, CDW, WDC, CD, DC, CW, WC, C. Letter *D* represents *data*, letter *C* represents *claim* and *W* represents the *warrant* of the argument. The names of the local argument types were given by considering the order of the argument component's emergence during the discussion. For instance, the Data-Claim-Warrant (DCW) type of argument was constructed in such a way that the *data* was stated first, and then the *claim*, and finally the *warrant* of the argument were stated. After the identification of the local argument type of all the arguments, the number of times each type emerged were compared. The most frequently used local argument type was Data-Warrant-Claim (DWC). This was an expected result since the participants mostly stated the data of the argument first while they were drawing the geometric shape, and then they justified their argument to show how the data led to the claim of the argument and lastly they stated their claim. The second and third mostly used local argument types were DCW and CW, respectively (their frequency was quite close to each other). In DCW, the participants started by mentioning the data, and then they stated their claim and finally justified their reasoning. In CW, which was similar to the previous one (DCW), the participant made a claim and then justified his/her claim but did not state the data most probably since the data seemed taken-as-shared to him/her during the argumentation. As mentioned in the literature, taken-as-shared knowledge exists in class discussions (Simon & Blume, 1996; Yackel & Cobb, 1996; Yackel, Ramussen, & King, 2000). Similarly, in the present study, the participants continued their discourse without stating some parts of the arguments. In the other local argument types, which were Claim-Data (CD), Data-Claim (DC), Warrant-Claim (WC) and Claim (C), a similar situation could be mentioned. Consistent with the *taken-as-shared* knowledge definition proposed by Yackel

(2002), the participants could have thought that the warrant and/or data of the argument were mentioned and were understood with prior justifications by the rest of the participants, so they may not have felt the need to restate it while discussing. This was a striking finding to be considered since teachers could confront such situations during the argumentation in their future classes. When they do so, they should not immediately think that the student does not know the related concept and has memorized the method of solution when se/he does not mention the warrant or data of the argument. The student could think that it was taken-as-shared so there is no need to restate that information. Thus, prospective middle school mathematics teachers in the present study might know the absent components of the arguments in CD, DC, WC and C types of local arguments but think that it was taken-as-shared, so they might not have felt the need to restate them. In order to request detailed information about missing argument components, some sample questions to be asked were suggested in the literature: “Why do you say that?” (Vincent, 2002, p. 147), “How are the two things the same? Does this make sense? ... Does it always work? Why does this happen?” (Wood, 2003, p. 440), and “Would you tell us what you thought? How did you decide this? Are there patterns? Is there a different way you can do this?” (Vincent, Chick & McCrae, 2005, p. 284). If I had noticed these absent components in the current study and asked the mentioned questions to the participants for additional information at the right time (as soon as the argument was stated), I would have been sure about their knowledge regarding those arguments.

The remaining local argument types (CDW and WDC) were observed a few times in the current study. Based on the flow of collective argumentation, the place of the components changed unexpectedly, leading to the emergence of these local argument types. For instance, in WDC the warrant was stated first, and then followed the data and the claim. However, these types could not be accepted as the characteristic to the local arguments produced by prospective middle school mathematics teachers within a technology-aided environment since they were stated rarely. Instead, it could be deduced that the prospective middle school mathematics teachers generally use DWC, DCW and CW local argument types in a geometry context in a technology-aided environment. The researcher requested from the participants to present justifications to their claims all the time at the beginning of the implementation. Therefore, the participants were aware of the fact that they should present evidence (warrant) to their arguments. However, they could have forgotten to mention their warrant in some arguments since they were not familiar to argumentation in their ordinary lessons. At this point, the researcher encouraged them to justify their conclusion. Thus, sometimes warrant was presented before their claim (DWC), and sometimes after the claim (DCW). Moreover, they could think that the data was taken-as-shared in some situations. At those times, they might not state the data of their argument, so the local argument type they used became CW.

When the GeoGebra and Paper-Pencil groups were compared, it was observed that the frequency of local arguments in each local argument type was similar. Therefore, it was concluded that use of technology did not make a significant change in the flow of argument components in the argumentation of the GeoGebra group. Moreover, the local arguments produced by both groups were analyzed for each geometry task. The findings revealed that the two groups' local arguments were similar in each geometry task also. The only difference was in geometry task 1 where the most frequently used local argument type in the GeoGebra group was CW, while it was DWC in the Paper-Pencil group. The GeoGebra group participants did not talk about the *data* of their arguments while they were drawing and dragging the shape of the first geometry task, which could be attributed to the interaction with the GeoGebra screen. Even though they were presenting an explanation to the class, they were inclined to remain quiet while they were drawing the shape since they were face to face with a computer screen. On the other hand, the reason for stating the *claim* component first might be due to the degree of difficulty of geometry task 1. More specifically, the dragging option of the GeoGebra program facilitates solving geometric problems. In the literature it was asserted that dragging allows individuals to notice geometrical relationships hidden in static diagrams (González & Herbst, 2009). As geometry task 1 was relatively easy for the dynamic geometry group, they could present their *claim* immediately after drawing the shape of the task. Afterwards, the participants searched for relationships by dragging the shape. Subsequently, the justification was required by the instructor (if the participants hadn't stated a *warrant*). Thus, the local argument type most frequently used in geometry task 1 was CW in the GeoGebra group. The paper-pencil group participants also discussed the drawing process of the task. Therefore, they emphasized the data component in their arguments. Thus, their argumentation was ordinary like in the other three geometry tasks.

Lastly, the local arguments were examined based on the geometry contents, which were triangles and circles. It was concluded that again each group had a similar distribution for different types of local argument, which means each group frequently used DWC, then DCW and finally CW types of local arguments. Thus, the existence of technology in one group did not make a significant difference between the types of local arguments used by prospective middle school mathematics teachers when the content of the geometry tasks were considered. The flow of argument types might be related to the reasoning process of the participants. More specifically, the participants possess a habit in expressing their opinions which they obtained throughout their entire education. For instance, they did not learn by an inquiry based method like argumentation as it was not part of the Turkish middle school mathematics curriculum. Therefore, although the environment in which they were solving the geometry tasks were not the same (GeoGebra and Paper-Pencil), their approach to the tasks might have been similar. Therefore, their expressions of local arguments were overall similar. In addition, the order of the argument components might change because of the questions asked by the instructor and the other participants. That is the instructor's directions could affect the presentation of the ideas of the participants. For instance, if the instructor asks the proof of a rule, the rule will be claim of the argument (stated first) and the participants will search for the *data* and *warrant* of that argument. In this case, the local argument type could be CDW, CW or CD. In the literature, there was no study investigating the pattern about the flow of argument components, so this study could be regarded as a preliminary study focusing on this issue in argumentation in a technology-aided environment. Therefore, it could open the door to further studies in different areas of topic and with participants from different grade levels.

5.3 Local argumentations of prospective middle school mathematics teachers

The last issue to be discussed is the third research question related to the local argumentations of the prospective middle school mathematics teachers. Since argument is a product while argumentation is a process (Krummheuer, 1995; Vincent, 2002), local argument is considered different from the local argumentations. In local arguments types mentioned in the previous section, the local arguments were interpreted as a whole. However, in local argumentation analysis, the characteristics of the *warrant (and backing)* component, in other words the justification part of the arguments, were analyzed. As asserted, justification is the indispensable effort of argumentation (Cross, 2009), and the present study seeks to find out whether the prospective middle school mathematics teachers prefer a specific kind of justification or not. In order to investigate this issue, the justification of the participants were analyzed based on Knipping's (2008) classification, which divides local argumentations into two: visual argumentation (empirical-visual and conceptual-visual) and conceptual argumentation. As previously mentioned, in visual justification, the warrants include a reference to the figure or diagram and the conclusions are drawn from that figure (Knipping, 2008). On the other hand, in conceptual argumentation, the warrants are formed by means of mathematical concepts, mathematical relations and references to theorems/definitions/axioms/rules of logic (Knipping, 2008). In addition, visual argumentation is divided into two as empirical-visual level (argument is based on a concrete diagram and the relations can be perceived through senses of the individual) and conceptual-visual level (diagram can be accepted as the representation of the idea and generalization can be made).

Initially, the local argumentations of the prospective middle school mathematics teachers were compared task by task. In geometry task 1 and 2 (GT1 and GT2), which were triangle tasks, the local argumentations of the groups differed. The participants in GeoGebra group mostly used visual argumentation (empirical-visual in GT1, conceptual-visual in GT2), while those in the Paper-Pencil group used conceptual argumentation. On the other hand, in GT3 and GT4, the groups performed similar local argumentations. Specifically, both groups preferred visual argumentation in the circle tasks.

In GT1, the participants in the Paper-Pencil group used concepts, theorems (Menelaus' theorem and Angle-bisector theorem), rules and mathematical relations (triangle similarity and triangle properties) to justify their arguments while the ones in the GeoGebra group focused on their drawings and dragging with GeoGebra to make justifications. The use of dynamic geometry might have naturally encouraged the participants in the GeoGebra group to use empirical conjectures rather than theoretical ones (Hoyles & Healy, 1999). In a proof study by Healy (2000), it was stated that the use of CABRI helped the students to identify geometrical properties but did not contribute to their proof since they believed that the connection between the empirical and theoretical cognitive domain could not be constituted via experimental actions. Contrary to this idea, in the current study, the participants in the GeoGebra group accepted their empirical actions as justifications to their arguments. Therefore, it can be deduced that the participants in the GeoGebra group could have the opinion that exploratory activities in which the theorem can be experimentally verified can be accepted as evidence (Chazan, 1993). Thus, the participants who determined the mathematical relations via their drawings on GeoGebra thought that they proved the relationship but in fact they remained at the empirical-visual level of argumentation. Ultimately, it could be deduced that the difference in justifications of the two groups when the warrants and local argumentations were considered in GT1 might have derived from the existence of the GeoGebra tool in the classroom. Similarly, in GT2, the participants in the Paper-Pencil group focused on the mathematical relations to be able to draw the best figure to solve the task, so their local argumentation was mostly conceptual. In contrast, the GeoGebra group dealt with dynamic relations with the help of GeoGebra, so their argumentation was mostly visual. As expected, the participants in the GeoGebra group were directed to think and talk about the figure they drew and dragged by means of GeoGebra, but this time, they could sometimes make generalizations from their drawings by presenting conceptual-visual warrants to their claims. In the literature, it was asserted that the students might not be familiar in how to present formal proof and justification in a technology-aided environment and they might primarily use technology for explorations via dynamic geometry programs (Chazan, 1993; Harel & Sowder, 1998; Hollebrands, Conner, & Smith, 2010). Therefore, the reason why the justifications of the GeoGebra group were based on visual argumentation could be their unfamiliarity in justifying at conceptual level in a technology environment. However, in the Paper-Pencil group, the situation was different. They used both visual and conceptual argumentation although most of their arguments were conceptual. It was an expected result since the participants in the Paper-Pencil group were familiar in providing conceptual justifications by using paper and pencil since they had the experience of providing proof using paper and pencil in their ordinary classes where they used paper and pencil.

When the local argumentations in the circle tasks (GT3 and GT4) were analyzed, it was concluded that both groups showed similar performance, which was an unexpected finding. The circle tasks required more dragging and it was not so easy for the students to visualize the final shape to appear after dragging. Nevertheless, the participants in the Paper-Pencil group were as successful as the participants in the GeoGebra group in these dynamic problems. Since the Paper-Pencil group did not have a tool to check their drawings after dragging, they also used conceptual argumentation as much as visual argumentation in GT4. This finding was quite interesting since the Paper-Pencil group could visualize the differences in the shape after drawing the desired changes via paper and pencil. This finding could be accounted with the participants' own ability, such as dynamic visualization ability, which was defined in the literature as forming moving pictures in the mind (Harel & Sowder, 1998). In the literature, it was claimed that individuals who have a high level of dynamic visualization ability could reason about the fundamental properties of moving, shrinking or rotating figures in their minds (Harel & Sowder, 1998). Thus, it is highly likely that the participants in the

Paper-Pencil could visualize the drawings after the dragging occurred in their minds and they drew the correct figures because of their high dynamic visualization ability. Another possible reason could be that the geometry task was not so difficult for the participants in the Paper-Pencil group, so they could predict the necessary changes on the shape without the help of GeoGebra.

In summary, when the local argumentations of the participants were examined in general, it was concluded that the prospective middle school mathematics teachers in the GeoGebra group used empirical-visual argumentation most frequently while those in the Paper-Pencil group were prone to use conceptual argumentation. One of the reasons underlying the use of empirical-visual warrants by the GeoGebra participants could be the use of a dragging option of the dynamic geometry program. With the help of dragging, they could make interpretations based on the specific shapes that they constructed. That is, they talked about the measurements of the lengths, angles and arcs that they drew via GeoGebra. Therefore, they sometimes did not need to explain the relationships they explored with theoretical support (Chazan, 1993). Another reason for underlying the use of empirical-visual warrants by the participants of GeoGebra group might be their unfamiliarity in providing theoretical evidence in a technology environment (Chazan, 1993; Harel & Sowder, 1998; Hollebrands, Conner, & Smith, 2010). On the other hand, the participants in the Paper-Pencil group needed to explain the relationships that they noticed with theoretical support, which corresponds to conceptual argumentation. Specifically, they defended their opinions by using axioms, theorems, rules and mathematical properties that they knew in order to generalize their solutions since they did not have a tool like GeoGebra to show the precise drawings. As indicated in the literature, the use of empirical evidence, such as dragging and measuring for justification, is a critical issue for the mathematics educators (Arzarello et al., 2002; Chazan, 1993; De Villiers, 2003; Healy & Hoyles, 2000). Although Hoyles and Healy (1999) asserted that DGS was not beneficial in proving theorems since it promoted empirical conjectures in formal proof, many researchers argued that DGS was useful in proving (Christou et al., 2004; Heinze & Reiss, 2007; Vincent, 2002). For instance, Vincent (2002) claimed that dynamic geometry software was highly suitable in order to bridge empirical and deductive reasoning. Similarly, Christou et al. (2004) stated that DGS motivated students and bridged inductive exploration and deductive proof. In addition, Heinze and Reiss (2007) accepted empirical justifications as validation in geometry context although experimentally generated results did not seem to offer explanation for the observed relations. In line with the studies in the literature, it could be deduced that GeoGebra was useful in argumentation in the geometry context but using solely empirical evidence, such as dragging and measurement, should not be accepted sufficient for the solution. As Arzarello et al. (2002) asserted, students could be motivated to prove *why* a certain proposition is true after seeing that it is true with DGS. Then a theoretical support could also be desired from students for better conceptual understanding.

Finally, the findings of the present study revealed local argumentations which did not fit into Knipping's (2008) classification of local argumentations; thus, they were named as *the givens as justification*, *GeoGebra measurement as justification*, and *GeoGebra actions as justification*. As can be understood from its name, in arguments named as *the givens as justification*, the participant showed what was given in the geometry task as evidence. In some other arguments, the participants showed a measurement of an angle, side or any length via GeoGebra as evidence, which was accepted as a new condition and called *GeoGebra measurement as justification*. Similarly, if the participant provided an evidence of a dragging, tracing, or some other actions of GeoGebra, it was a new condition called *GeoGebra actions as justification*. These were the justifications that the prospective middle school mathematics teachers used as evidence to their claims in some of their arguments. In fact, these justifications were not so strong to support the claims of the arguments since they were based on the givens in the geometry task or the actions and measurements of the GeoGebra without any logical reasoning and theoretical support (Chazan, 1993). The reason underlying the presentation of such justifications could be that the participants who had the GeoGebra tool did not need to provide conceptual explanations in these rare situations when they saw the relationship with the help of GeoGebra (Chazan, 1993). Instead, they demonstrated

GeoGebra measurements and the givens of the tasks as authority and evidence for their arguments. These new justifications were rare when the number of all arguments were considered. Namely, it could be deduced that these few justifications did not represent the general tendency in the justifications of prospective middle school mathematics teachers in technology enhanced argumentation. However, this finding could be taken into consideration in further studies, if encountered by other researchers.

5.4 Implications

As indicated above the prospective middle school mathematics teachers did not use high level of mathematical reasoning in argumentation. Literature review illustrated that mathematical reasoning could be developed with computer-based applications, educational plays, concrete manipulatives, daily life examples, interactive argumentation and dealing with open-ended problems (Erdem, 2015). Thus, these findings recommend the urgent need for the reconstruction of the courses offered in the teacher education programs to develop prospective middle school mathematics teachers' mathematical reasoning and to provide them with argumentation skills. In Turkey, argumentation is taught in the courses offered in science and technology teacher education programs but there is limited information regarding argumentation in the content of courses offered in the middle school mathematics teacher education program. Ultimately, argumentation, which requires higher order thinking, should be prevalent in middle school mathematics teacher education programs like in the science teacher education programs in Turkey. For this purpose, educators in middle school mathematics education program should offer both must and elective courses including the applications to bring in the argumentation skills to prospective middle school mathematics teachers. That is, prospective middle school mathematics teachers should practice argumentation in technology based applications, concrete manipulative applications, interactive argumentation applications and open-ended problem solutions (Erdem, 2015) and develop the necessary skills to be able to orchestrate an argumentation class. These skills were mentioned in the literature as encouraging students to participate in argumentation (Cross, 2009; Staples, 2007), providing challenging questions to be discussed for in-depth thinking (Cross, 2009), not using evaluative statements to student responses so as not to make them feel fear of being judged (Mercer, 2000), encouraging students to convince others about their claims (Martino & Maher, 1999), asking the key questions/warrant-prompts to promote justification all the time (Boero, 1999; Kosko et al., 2014; Martino & Maher, 1999; Owens, 2005), revoicing to clarify the content, to explain the reasoning further or to redirect the argumentation (Forman et al., 1998). If the undergraduate course contents were revised in such a way that the mentioned skills were practiced by the prospective middle school mathematics teachers, the future teachers would be equipped with the necessary skills to provide an argumentative environment to their students to develop higher level of mathematical reasoning and, thus, a high level of achievement in mathematics.

Another possible solution to improve mathematical reasoning of teachers may be changing the national examination system. The national exams may be redesigned to ask students open-ended problems to promote higher order thinking (Hmelo & Ferrari, 1997). This would naturally result in reforms in school programs. In this way, the mathematics curriculums could be revised in such a way that students would find the opportunity to take place in argumentative environments in which challenging problems are discussed collectively in order to engage in higher order thinking practices while preparing for the national exams.

Based on the conclusions, the prospective middle school mathematics teachers appeared to be using the givens of the task, the GeoGebra measurement and GeoGebra actions in order to give evidence to their arguments. Based on the literature, these kinds of justifications were criticized by the researchers and were not regarded as satisfying a justification (Chazan & Houde, 1989; González & Herbst, 2009; Noss & Hoyles, 1996). Therefore, the prospective middle school mathematics teachers could be informed by their instructors that it is not enough to use GeoGebra measurement, the givens of the tasks and geogebra actions as justification in argumentation classes. They should be aware of that they should also support their claims with not only via visual evidences (visual argumentation) but also with theoretical evidences (conceptual argumentation). At this point, the role of instructor is important. More specifically, when the prospective middle school mathematics teachers use such visual argumentations, the instructors should question the thoughts in their minds by asking the questions mentioned above, such as ‘Why do you think so?’ (Arzarello et al., 2002). In this way, they would have the chance to think about their claim again to produce more valid evidences. Another possible way to make participants in GeoGebra group to share their ideas conceptually could be preparing a step-by-step worksheet directing them to search for theoretical evidences after presenting their visual argumentation with GeoGebra illustrations. Thus, critical questions could be prepared to make them think deeper on their visual drawings.

One more suggestion could be the limitation regarding the use of GeoGebra buttons. More specifically, the buttons such as ‘drawing a rectangle, drawing a midpoint, drawing an equilateral triangle’ which enables the basic drawings could be inhibited during the solution of the geometry tasks. In this way, the participants have to think deeply about the properties of the shapes in order to draw the correct figures. By this way, prospective middle school mathematics teachers could be encouraged to use logical reasoning to provide conceptual arguments besides visual argumentation structures.

5.5 Recommendations for further research studies

The current study focused on the nature of argumentation structures of prospective middle school mathematics teachers in technology and paper-pencil environments. Specifically, their global argumentations, local argument types and local argumentations were investigated in two groups: GeoGebra group and Paper-Pencil group. In the view of the results, the offered recommendations for further studies are explained in the following paragraphs.

This study contributed to the literature with the new global argumentation structures (line-structure and independent-arguments structure) in geometry context.

Therefore, the validity of these structures in mathematical argumentation is an issue to be tested with new models. These structures could also be searched in other mathematics topics in order to talk about its generalizability. Thus, new research studies could be conducted in order to test these global argumentation structures.

The results of the present study were limited to the data collected from one public university in Ankara. However, the argumentation structures of the individuals might show variation from one university to another and even from culture to culture. Argumentation method is based on convincing, supporting and even refuting others. Therefore, its application in different cultures and contexts might lead to different results (Reid & Knipping, 2010). For instance, in the Japanese culture, people communicate to provide harmony, and disagreements are believed to be a threat to the harmony (Sekiguchi, 2000). Thus, Sekiguchi (2000) claimed that Japanese people avoid explicit disagreement statements in public. Therefore, it is believed that implementation of the argumentation method for teaching mathematics in the Japanese culture would be difficult (Sekiguchi, 2000). Similar situations could be confronted in other cultures too. Therefore, further research might be conducted to investigate the applicability of the argumentation method at the international level.

In the present study, the contents of the argument components were not analyzed. A more comprehensive study might be conducted to analyze the contents of warrants, data and rebuttals. In this way, the type of knowledge prospective middle school mathematics teachers use in order to justify their claims, to provide base for their arguments, and to rebut the constructed arguments might be revealed. A research study from this aspect could be beneficial for comprehending the rationale that shapes the argumentation of prospective middle school mathematics teachers, and thus, could help improve the geometry teaching in technology supported and paper-pencil based argumentation environments.

Lastly, the present study was based on geometry tasks entailing two topics: triangles and circles. Hence, the content of the findings was limited to these topics. Moreover, there were only 4 geometry tasks. In order to further explore various aspects of argumentation in the geometry context, geometry tasks related to other concepts such as quadrilaterals and polygons could be prepared in technology and paper-pencil environments. Moreover, the number of tasks could be increased in order to address more aspects of the topics comprehensively.

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APPENDICES

A: INTERVIEW QUESTIONS OF THE MAIN STUDY

Interview questions prepared for the focus groups (GeoGebra group)

For triangle activities (GeoGebra group)

1. Which high school did you graduate from?

2. Do you think that you are successful in Geometry? What are the grades you got from the geometry courses you took from the department of Mathematics? (Calculus, Analytical Geometry, Elementary geometry).

3. What was your grade point average (GPA) in the last semester?

4. Was the GeoGebra helpful while you were solving the two triangle tasks? If yes, in which aspect was the GeoGebra helpful? (Did you use the GeoGebra program to check your solution or to solve the problem?)

5. Did GeoGebra help you to discover the things that you were not able to see in the drawings that you made with paper and pencil? (Hint: In geometry task 1, you did not notice that point H could be still on the line passing through the angle bisector of angle A when it was outside triangle ABC.)

6. Have you encountered any difficulties when you were using the GeoGebra program? If yes, in which situations? (Did you have difficulties while drawing the givens of the tasks? Explain the situations in which you had difficulties by giving specific examples from the implementation.

7. In Activity 1, you drew a circle, the center of which was the midpoint of the segment BC. Afterwards, you placed F and G points on segment BC. What was the reason for drawing the circle from that point? If you suppose that F and G points are dynamic, how does the place of point H change when you drag the points F and G?

8. Discuss the following arguments. Do you think they are true? How can you support or refute these arguments?

- When the givens were drawn and the segment AH was stretched, the line divides the segment FG into two equal segments in all kinds of triangles.

- AH is the angle-bisector in all the triangles whose median is also its angle-bisector.

9. Throughout the implementation, you provided justifications to your answers, defended your ideas, evaluated your friends' opinions and criticized others' ideas. What were the things you used when you were defending your ideas (and answering the questions such as 'Why do you think so? ...how did you know that it is so?' (Rule, theorem, past experiences, GeoGebra visual drawings).

10. In activity 1, Özer drew two circles the diameters of which were the segments |DG| and |EF| on an equilateral triangle. But he could not remember to explain why |AH| became the angle-bisector of the triangle ABC. Can you explain it now?

11. In activity 2, Aslı said that 'We solved it by using Menelaus' theorem' but you did not explain it. Can we solve this geometry task by using Menelaus's theorem? How?

For Circle Activities (GeoGebra group)

1. Was the GeoGebra helpful while you were solving the two circle tasks? If yes, in which aspect was the GeoGebra helpful? (Did you use the GeoGebra program to check your solution or to solve the problem?)

2. Have you encountered any difficulties when you were using the GeoGebra program? If yes, in which situations? (Did you have difficulties while drawing the givens of the tasks? Explain the situations in which you had difficulties by giving specific examples from the implementation.

3. Throughout the implementation, you provided justifications to your answers, defended your ideas, evaluated your friends' opinions and criticized others' ideas. What were the things you used when you were defending your ideas (and answering the questions such as 'Why do you think so? ...how did you know that it is so?' (Rule, theorem, past experiences, GeoGebra visual drawings).

4. In geometry task 3, you could not find the solution in pair-work. Why did not you see that triangle FGH was an equilateral triangle? What was the reason of your mistake? In which part did you make a mistake?

5. In the second question of geometry task 4, Özer was confused since the place of point H changed as the shape was dragged. He thought that the situation in which the segment HE passed through the center of the circle was false. Do you still think the same Özer? Aslı, what do you think about this idea? (If Aslı thinks the opposite: Can you convince Özer?)

For triangle activities (Paper-Pencil group)

1. Which high school did you graduate from?

2. Do you think that you are successful in Geometry? What are the grades you got from the geometry courses you took from the department of Mathematics? (Calculus, Analytical Geometry, Elementary geometry).

3. What was your grade point average (GPA) in the last semester?

4. Were the materials helpful while you were solving the two triangle tasks? If yes, in which aspects were the materials helpful? Give specific examples.

5. Were there any situations that you had difficulties while using the materials you had? If yes, in which situations? Explain the situations in which you had difficulties by giving specific examples.

6. Did you need any tool in order to solve the triangle tasks? If yes, which materials? For what did you need that tool?

7. In geometry task 1, you placed points F and G on segment |BC|. According to what did you place those points? Think that the points F and G were dynamic. Make interpretations about the place of point H when you move the points F and G.

8. Discuss the following arguments. Do you think they are true? How can you support or refute these arguments?

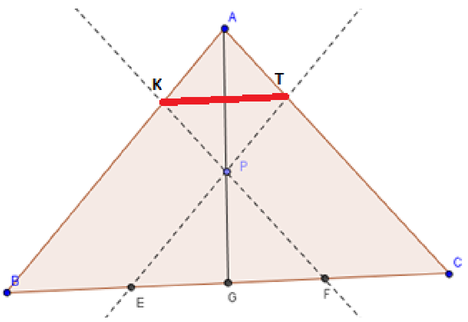
- When the givens were drawn and the segment AH was stretched, the line divides the segment FG into two equal segments in all kinds of triangles.

- AH is the angle-bisector in all the triangles whose median is also its angle-bisector.

9. You showed that |AH| was angle-bisector in isosceles and equilateral triangles. Okan asked the situation for scalene triangles. İnci said that ‘In scalene triangles, the angle-bisector cannot pass through the point H. Therefore, it is not valid for scalene triangles’. Do you agree with İnci? Why?

10. Throughout the implementation, you provided justifications to your answers, defended your ideas, evaluated your friends’ opinions and criticized others’ ideas. What were the things you used when you were defending your ideas (and answering the questions such as ‘Why do you think so? ...how did you know that it is so?’ (Rule, theorem, past experiences, GeoGebra visual drawings).

11. In geometry task 2, we discussed whether or not the segment |KT| was parallel to the segment |BC|. What do you think about this? Explain with reasons.



For circle activities (Paper-Pencil group)

1. Were the materials helpful while you were solving the two circle tasks? If yes, in which aspects were the materials helpful? Give specific examples.

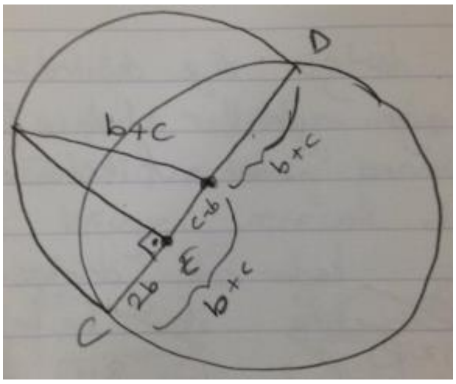
2. In geometry task 3, you said that the task was quite easy just like the ones in university entrance examinations. But you had difficulties in the argumentation. At the end, you could not solve the task. What was your mistake?

3. Have you encountered any difficulties when you were using the materials you had? If yes, in which situations? Explain the situations in which you had difficulties by giving specific examples.

4. Did you need any tool in order to solve the circle tasks? If yes, which materials? For what did you need that tool?

5. In geometry task 4, you did not discuss the first part of the last question. That is 'Show whether the theorem is trivial if chord |FG| is a diameter of the first circle'. Can you discuss and solve this question right now?

6. In geometry task 4, you drew the situation in which the |FG| coincides with |CD| like in the figure below. Is this drawing true or false? Why?



B: REFLECTION PAPERS

REFLECTION – Geogebra Group

1. Which high school did you graduated from?

2. Do you think that you are successful in Geometry? What are the grades you obtained from the geometry courses you took from the department of Mathematics? (Calculus, Analytical Geometry, Elementary geometry).

3. What was your grade point average (GPA) in the last semester?

4. Was the GeoGebra helpful while you were solving the geometry tasks? If yes, in which aspects weas the GeoGebra helpful? (Did you use the GeoGebra program to check your solution or to solve the problem?) Give specific examples.

5. Did GeoGebra program help you while you were solving the geometry tasks? If yes, in which aspects was the program helpful? Explain with specific examples.

6. Have you encountered any difficulties when you were using the GeoGebra program? If yes, in which situations? (Did you have difficulties while drawing the givens of the tasks? Explain the situations in which you had difficulties by giving specific examples from the implementation.

7. Throughout the implementation, you provided justifications to your answers, defended your ideas, evaluated your friends' opinions and criticized others' ideas. What were the things you used when you were defending your ideas (and answering the questions such as 'Why do you think so? ...how did you know that it is so?' (Rule, theorem, past experiences, GeoGebra visual drawings).

8. Discuss the following arguments. Do you think they are true? How can you support or refute these arguments?

- When the givens were drawn and the segment AH was stretched, the line divides the segment FG into two equal segments in all kinds of triangles.

- AH is the angle-bisector in the triangles whose median is also its angle-bisector.

REFLECTION –Paper-Pencil group

1. Which high school did you graduated from?

2. Do you think that you are successful in Geometry? What are the grades you obtained from the geometry courses you took from the department of Mathematics? (Calculus, Analytical Geometry, Elementary geometry).

3. What was your grade point average (GPA) in the last semester?

4. Were the materials helpful while you were solving the geometry tasks? If yes, in which aspects were the materials helpful? Give specific examples.

5. Were there any situations that you had difficulties while using the materials you had? If yes, in which situations? Explain the situations in which you had difficulties by giving specific examples.

6. Did you need any tool in order to solve the geometry tasks? If yes, which materials? For what did you need that tool?

7. Throughout the implementation, you provided justifications to your answers, defended your ideas, evaluated your friends' opinions and criticized others' ideas. What were the things you used when you were defending your ideas (and answering the questions such as 'Why do you think so? ...how did you know that it is so?' (Rule, theorem, past experiences, GeoGebra visual drawings).

8. Discuss the following arguments. Do you think they are true? How can you support or refute these arguments?

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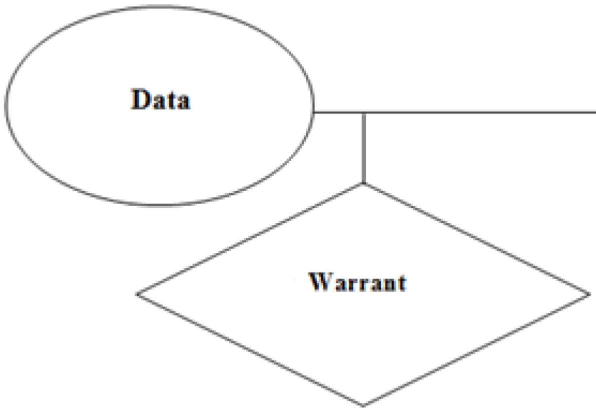
C: ARGUMENTS OF THE GEOGEBRA GROUP

The arguments in this study have been analyzed by means of schematization based on Toulmin's argument layout as in the following sample argument.



In the table below, the contents of the argument components put forward by the participants in the GeoGebra group are presented. There are some argument components that do not exist in some of the arguments. The explanation/contents box for these components are left blank in the table.

Argument No:	The Contents of the Argument Components
1	***Data***: 'F and G points placed on the side |BC| so as to be |BG|=|CF|' Given statement in the task. ***Warrant: -*** ***Rebuttal: -*** ***Claim:*** F and G points can be placed on the side |BC| separately or they can be placed on the midpoint of side |BC| together.
2	***Data:*** It says 'the point that |DG| and |EF| intersects" in the givens. F and G cannot be the midpoint of the side side |BC|. ***Warrant:*** I made a mistake by selecting the midpoint. F and G should be dynamic in order to construct the intersection of |DG| and |EF|. ***Rebuttal:*** Do we have to divide the segment side |BC| into three segments? We can also divide it into four segments. ***Claim:*** We should divide the segment |BC| into three segments while placing the points F and G.
3	***Data: -*** ***Warrant: -*** ***Rebuttal: -*** ***Claim:*** We can divide |BC| into four.

4	***Data: -*** ***Warrant:*** But we should consider the order given in the task. Therefore, we should place the points F and G as they are presented in the task. ***Rebuttal: -*** ***Claim:*** We should place the points F and G on the segment |BC| in the order of 'B, G, F, C'.
5	***Data: -*** ***Warrant:*** Since it satisfies |BG|=|FC|. When we draw |DG| and |EF| , the intersection point H is constructed. ***Rebuttal: -*** ***Claim:*** We can also place the points F and G on the segment |BC| in the order B, F, G, C.
6	***Data:*** Draw any triangle ABC. The midpoints of |AB| and |AC| are D and E respectively. Let's the midpoint of |BC| be 'M'. I draw a circle with center M. The intersection points of the circle and the segment |BC| are F and G. We can find the intersection point H by drawing the segments |DG| and |EF|. ***Warrant:*** (Observation by dragging the radious of the circle with center M) Can you see that the line passing through |AH| and the center of the circle M intersects. Namely, |AH| becomes both angle-bisector and median at the same time. That is the property of isosceler triangle and equilateral triangle. ***Rebuttal:*** This situation is not valid for scalene triangle ABC. ***Claim:*** |AH| is angle-bisector in isosceles and equilateral triangles.
7	***Data: -*** ***Warrant:*** |BD|=|DA| and |CE|=|EA|. If |BG|=|CF| then |GM|=|MF|. So |GH|=|FH| and |HD|=|HE| . Therefore triangle FGH becomes an isosceles triangle. ***Rebuttal: -*** ***Claim:*** |AH| is angle-bisector when triangle ABC is an isosceles or equilateral triangle.
8	***Data: -*** ***Warrant:*** Since triangle FHG is isosceles triangle, |GH|=|HF|. |HM| is both median, angle-bisector and altitude of this triangle. Since ABC is an isosceles triangle, |AD|=|AE| and |DB|=|EC|. ***Rebuttal:*** But we did not prove that |AH | and |HM| are on the same line (linear). ***Claim:*** The angle-bisector of the triangle FHG is also the angle-bisector of the triangle BAC.
9	***Data:*** Let's draw any triangle ABC. The midpoints of |AB| and |AC| are D and E respectively. ***Warrant:*** I would like |DG | ve |EF | to intersect to construct point H. If the radious of the circle can be negative, it could be vice versa. ***Rebuttal: -*** ***Claim:*** We should place the points F and G on the segment |BC| in the order B, F, G, C.

10	***Data:*** I drew the givens of the task with GeoGebra. I used slider and triangle in order to make points F and G dynamic. ***Warrant:*** I measured the angles BAH and HAC with GeoGebra. The measures of these angles can be observed while dragging the vertices of the triangle until the two angles become equal to each other. I saw that whenever |AH| became an angle-bisector, it passes through the point M. (The midpoint of the segment |BC|). Since |BF|=|CG|, |FM|=|GM|, |BD|=|DA|, |AE|=|EC|, |DG|=|EF| the triangle FGH is isosceles triangle. If triangle FGH is isosceles, triangle ABC should be isosceles triangle too. ***Rebuttal: -*** ***Claim:*** |AH| is angle-bisector when triangle ABC is an isosceles or equilateral triangle.
11	***Data: -*** ***Warrant:*** Observation is made by changing the shape of the triangle with GeoGebra. When |AH| is the angle-bisector, it passes through the middle point of |BC|. |AH| becomes both the bisector and the median. At the same time it becomes the height. So it will be isosceles triangle. ***Rebuttal:*** When triangle ABC is a scalene triangle, |AH| is not the angle bisector. ***Claim:*** |AH| is angle-bisector when triangle ABC is an isosceles triangle.
12	***Data:*** It is made drawing on an equilateral triangle with GeoGebra. Points F and G are taken as the midpoint of |BC| as if they are the same point. In this case, the same point becomes H point. |AH| is drawn. ***Warrant:*** Angles are measured and it is seen that |AH| is the angle bisector. When we make dynamic by reflecting the points F and G according to the midpoint of |BC|, point H is moving on |AF| and nothing changes. ***Rebuttal:*** However, we have made this only on equilateral triangle, not on isosceles triangle. ***Claim:*** |AH| is angle-bisector when triangle ABC is an equilateral triangle.
13	***Data: -*** ***Warrant:*** With Geogebra, the movement of point H can be observed by dragging the points F and G. Because there is no point H, there is no such thing as |AH| is the angle bisector. ***Rebuttal:*** There is also a situation where |DG| and |EF| do not intersect and there is no point H. When it is |DG|//|EF|. ***Claim:*** If points F and G are replaced on the equilateral triangle, H point again moves on the line | AH |.
14	***Data:*** The circles the radius of which are | DG | and | EF | are drawn. The line passing through the intersection points of these circles is drawn. It can be seen that it passes through the angle bisector of angle A. ***Warrant:*** Since | DG | = | EF |, I have drawn these radius circles.

	Rebuttal: - ***Claim:*** The line passing through the intersection points of the circles on equilateral triangular is the bisector of angle A.
15	***Data:*** The line passing through the intersection points of the circles on equilateral triangular is the bisector of angle A.(*Claim 14*) ***Warrant:*** An equilateral triangle is also an isosceles triangle. That is why the same is true for the isosceles triangle. ***Rebuttal: -*** ***Claim:*** When the triangle ABC is the isosceles triangle, | DG | and | EF| intersect on the bisector line.
16	***Data: -*** ***Warrant:*** With the counter example on a scalene triangle, we can show that there is not an angle bisector. ***Rebuttal: -*** ***Claim:*** On scalene triangles, AH is not angle bisector.
17	***Data:*** Any ABC triangle is drawn. A median drawn from the vertex A and a point P on it are drawn. Lines paralel to edges | AB | and | AC | from point P are drawn. The point P is moved on the median. The lengths | BE | and | FC | are measured. ***Warrant:*** The median of the triangle BAC is also the median of the triangle EPF. | BE | = | FC | . Since they are the equivalent triangle, this is also the median of the small triangle. There is a A.A.A similarity. ***Rebuttal: -*** ***Claim***: |EG|=|GF|
18	***Data:*** **(**Claim 17) ***Warrant:*** We have already found that |EG|=|GF| on a general triangle. This is true for all triangles. ***Rebuttal: -*** ***Claim:*** If the triangle ABC were a isosceles triangle or equilateral triangle, it would be |EG|=|GF|.
19	***Data:*** Let triangle ABC be isosceles triangle, let's draw it like that. ***Warrant:*** |AG| will be both the median and the height. Since the height of BAC will also be the height of EPF, |EF|will be divided into two equal parts. ***Rebuttal: -*** ***Claim:*** When the triangle ABC is isosceles triangle, it is |EG|=|GF|
20	***Data***: The lines drawn form P are parallel tothe edges |AC| and |AB|. The position of point P when |EF|:|FC|=1:2 is asked in the task. ***Warrant:*** It is |EF|:|FC|=1:2=|PF|:|HC|=|PG|:|AP|. From smilairty. ***Rebuttal: -*** ***Claim:*** In order to be |BE|=|EF|=|FC|, the location of the point P must be as |AP|:|PG|=2:1
21	***Data:*** ABC≈PEF. | AG | is the also the median of PEF. The intersection point of medians is found. Point P is dragged onto this point and | BE |, | EF |, | FC | lengths are examined.

	Warrant: EFP≈ECT . |EF|:|FC|=|PF|:|AC|=|PG|: |AP| . Since we show on a general trianglei it is true for all triangles. ***Rebuttal: -*** ***Claim:*** In all triangles, in order to be |BE|=|EF|=|FC|, point P must be at the circumcenter.
22	***Data:*** Any ABC triangle is drawn. |AG| is the median of the triangle ABC. The lines drawn form P are parallel to the edges of the triangle ABC. The triangle PEF is drawn. ***Warrant:*** Because of parallelism. ***Rebuttal: -*** ***Claim:*** BAC≈EPF
23	***Data:*** BAC≈EPF (Claim 22) ***Warrant:*** The edge |AG| is also the median of the triangle EPF. ***Rebuttal: -*** ***Claim:*** |EG|=|GF|
24	***Data:*** Let ABC be isosceles or equilateral triangle. |EG|=|GF| (*Claim 23*) ***Warrant:*** As we show this equality on a scalene triangle, it is already true for other triangles. We can see that it is also valid in other triangles by making the triangle move by dragging it. ***Rebuttal: -*** ***İddia:*** |EG|=|GF|
25	***Data:*** Any ABC triangle is drawn. A median drawn from A and a P point on it are drawn. Lines paralel to edges | AB | and | AC | from point P are drawn. |EP| is stretched, the point T is found. |FP| is stretched and the point Y is found. ∠ABG=∠PEG=α. ∠ACG= ∠PFG= β. Let it be ∠BAC= ∠EPF=∠BYF=∠ETC=c Let it be |EG|=a, |BG|=b, |GF|=c, |FC|=d. We know that a+b=c+d ***Warrant:*** When I applied a similarity to the BYF ≈ETC triangle, I had found that b and d are equal, but now I could not. ***Rebuttal: -*** ***Claim:*** a=c → |EG|=|GF|
26	***Data:*** The shape is drawn on the white board. (Data 25) ***Warrant:*** ETC ≈ BYF are similar triangles. There is A.A.A similarity. ***Rebuttal: -*** ***Claim:*** |PF| /|TC| = |PE| / |YB|
27	***Data:*** |EG|=a, |GF|=c, |BE|=b, |FC|=d. When |AG| is the median, a+b=c+d. ***Warrant:*** PGF≈AGC are similar triangles. PGE≈AGB are similar triangles. Let's write similarity ratio: c:(c+d)= |PG|:|AG|=a:(a+b) . a=c ***Rebuttal: -*** ***Claim:*** |EG|=|GF|
28	***Data***: An equilateral triangle and the givens are drawn. |BE|, |EF| and |FC| are measured. The point P is dragged and observed. There is a ratio 1:2 here. (Claim 23 and 27)

	Warrant: |BE|=2a=|FC|. |EG|=a=|GF|. There is the ratio |GE|:|EB|=1:2 =|PG|:|AG| This ratio on equilateral triangle is also valid for the isosceles triangle. Since |AG| was a median, circumcenter will be on this segment. ***Rebuttal:*** Why is not it true for a scalene triangle? It is. If you say that |AG| is the median, it should be also valid for scalene triangles. ***Claim:*** In equilateral triangle, in order to be |BE|=|EF|=|FC|, point P must be at the circumcenter. It is not valid in scalene triangles.
29	***Data:*** In any triangle, if a parallel is drawn from the circumcenter to the edges of the triangles, it divides the edges by 2:1 ***Warrant:*** If we draw a paralel to| AB | and | AC | and write the ratio 2:1 , the edge | BC | is divided into 3 equal parts. A triangle is drawn with Geogebra. Two medians are drawn and the circumcenter is found from their intersection. Parallel lines are drawn from the circumcenter and | BE |, | EF |, | FC | lengths are measured. ***Rebuttal: -*** ***Claim:*** In all triangles, in order to be |BE|=|EF|=|FC|, the point P must be at the circumcenter.
30	***Data:*** Any triangle is drawn onto the white board. The point P and line parallel to the edges are drawn. Let |BE|=|EF|=|FC| =a. ***Warrant:*** From EFP ≈ ECT similarity →It is |PF|:|TC|=k:2k From AGC≈PGF similarity →It is |PG|:|AG|=|PF|:|TC|=1:2 |PG|=m, |AG|=2m. Since |AG| is median |GF|=a/2. From AGC≈PGF , |GF|:|GC|=a/2: 3a/2=|PG|:|AG| . ***Rebuttal: -*** ***Claim:*** In order to be |BE|=|EF|=|FC|, the point P must be on |AG| in the ratio of 2:1.
31	***Data:*** Any two circles that have equal radius and pass through the center of each other are drawn. The intersection points of the circles are H and E. A chord is drawn in such a way that the two circles of E are cut off to form an FGE chord. | FH | and | HG | are drawn and the FGH triangle is created. ***Warrant:*** We measured the sides of the triangle FGH with GeoGebra and noticed that all three sides were equal. ***Rebuttal: -*** ***Claim:*** The triangle FGH is an equalateral triangle.
32	***Data:*** Any two circles that have equal radius and pass through the center of each other are drawn. The intersection points of the circles are H and E. A chord is drawn in such a way that it intersects with the two circles to form an FGE chord. | FH | and | HG | are drawn and the FGH triangle is created. ***Warrant:*** The G point is dragged and the edge lengths of the triangle are observed. The edge length of the equilateral triangle is always increases when we move the point G upon to the point K. When it passes over the point K, it is starting to decrease. At the point K, one edge of the triangles becomes as the diameter. ***Rebuttal: -***

	Claim: One edge of the possible largest FGH triangle becomes 2R long.
33	***Data:* -** ***Warrant:*** By combining TE we can show from |HF|//|TE|. Is it true? ***Rebuttal:*** No, you can not know that they are equal. For example, it can be |FG|=4, |GT|=4, |HG|=2, |GE|=6 ***Claim***: |GT|=|GE|
34	***Data:*** Let's draw |HE|. Let the ∠FEH be alpha. These are the arcs that the alpha sees in the two circles. ***Warrant:*** The arcs of the Alpha angle seen on the two circles are equal to each other. Therefore, the chords they see are equal to each other. ***Rebuttal:* -** ***Claim:*** |FH|=|GH|
35	***Data:*** |CH| and |HD| are drawn. |CD| is drawn. CE| and |DE| are drawn. ***Warrant:*** Since |CD|, |CH|, |HD|, |CE| and |DE|, they are all equal to the radius of the triangle, they are all equal to each other. ***Rebuttal:* -** ***Claim:*** CHD and CED are equilateral triangles.
36	***Data:*** CHD and CED are equilateral triangles. (*Claim 35*). Because ∠HCD = ∠DCE= 60° , it is ∠HCE=120° ***Warrant:*** Since HDE is the arc which was seen by the central angle HCE. ***Rebuttal:* -** ***Claim:*** arc HDE =120°
37	***Data:*** arc HDE =120° (*Claim 36*) ***Warrant:*** Since it is inscribed angle, the measurement of the angle ∠HFE is equal to the half of the chord it sees. ***Rebuttal:* -** ***Claim:***It is ∠HFE = 60°.
38	***Data:*** |FH|=|GH| (*Claim 34*), ∠HFE = 60° (*Claim 37*) ***Warrant:*** Since FGH is an isosceles triangle and its one base angle is 60 degrees, The other angles are also 60°. ***Rebuttal:* -** ***Claim:*** The triangle FGH is an equalateral triangle.
39	***Data:*** The triangle FGH is an equilateral triangle. (*Claim 38*). |FG|=6 unit. ***Warrant:*** On equilateral triangle if the area formula $A=a^2\sqrt{3}:4$ is applied. ***Rebuttal:* -** ***Claim:*** The area of the triangle FGH is $9\sqrt{3}$
40	***Data:*** Point G is dragged up to point E. The triangle disappears. ***Warrant:*** When we calculate the area, GeoGebra showed like this. Triangle disappeared and area was calculated as 'undefined'. ***Rebuttal:* -** ***Claim:*** When the point G is dragged onto the point E, the area of the triangle becomes undefined so minimum area is undefined.

41	***Data:* -** ***Warrant:*** The point F is dragged upto the point H. The area disappeared. ***Rebuttal:*** Is the area zero? The area can not be zero. The minimum area is larger than zero. ***Claim:*** The area of the triangle FGH is minimum zero.
42	***Data:* -** ***Warrant***: The point G is dragged to the point E in GeoGebra. ***Rebuttal:* -** ***Claim:*** The maximum area fort he FGH triangle is when the chord FGE is the tangent to the circle that is D centered.
43	***Data:* -** ***Warrant:*** Because of the arc that the angles sees, It is always equilateral triangle according to the solution way that Özde shows. ***Rebuttal:* -** ***Claim:*** FGH triangle is an equilateral triangle in all cases.
44	***Data:*** (*Claim 42 and Claim 43*). |CF|, |CG| and |CH| are combined. These lenghts are equal to R. ***Warrant:*** It is ∠FCG=∠FCH=∠HCG=120°. In 30-30-120 triangle, the isosceles are R, and the long edge is R√3. One edge of the FGH triangle is R√3. If we put it in the triangle area Formula. ***Rebuttal:* -** ***Claim:*** The area of the triangle FGH is maximum ($3R^2\sqrt{3}$):4
45	***Data:*** With GeoGebra dragging, FGH triangle is drawn as its one edge is 2R. ***Warrant:*** For example, when I draw a chord from E, when I take the extension, it cuts that triangle outside, then G is out there. ***Rebuttal:*** But if we do the drawing in that way, we change the question. The question says the chord that is drawn from E. Is it true that we should do it with the line that passes from | FE |? Here the chord is divided into two. The chord is the line segment. G point can cut the D centered circle at maximum E point. When we continue, G point is not on the chord | EF | . But would that be the extension of the chord? The chord is formed by joining two points on the circle. If the G point continues, it is not on the chord | EF |. There becomes two chords, | EF | and | EG | In fact, there is a line passing through the circles, not a chord. The question talks about a chord drawn from E. That's not all. ***Claim:*** The largest FGH triangle forms when its one edge is 2R.
46	***Data:*** |FH| and | HO| are drawn. ∠HOF = α. The arc FH is α olur. Arc HG =180-α.. ∠HFO = 90-(α/2). ***Warrant:* -** ***Rebuttal:*** It is not equal.∠FHE =α/2 but ∠EHO = 90-α ***Claim:*** |FH|=|HO|

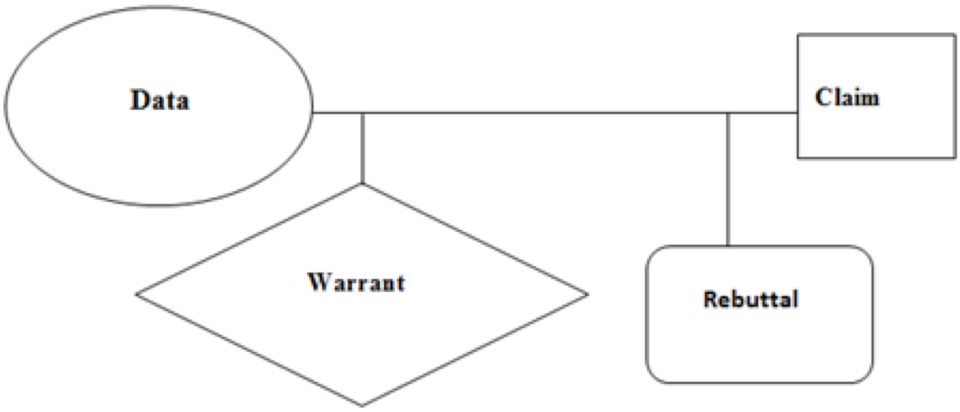
47	***Data:*** Let's say |FE|=x , |EG|=y. |CE|=|ED|=a. |FH| and |HG| are drawn. ***Warrant:*** The triangle FGH is a right angled triangle. And, an altitude was drawn to the hypotenuse. Euclidean formula can be applied. $\|HE\|^2$=x.y ***Rebuttal: -*** ***Claim:*** |HE|=√x.y
48	***Data:*** |HE|=√x.y *(Claim47)* ***Warrant:*** In circle there is the intersecting chords theorem. If x.y=a.a, a^2=$\|CA\|^2$=x.y ***Rebuttal: -*** ***Claim:*** |CA| =√x.y.
49	***Data: -*** ***Warrant: -*** ***Rebuttal: -*** ***Claim:*** If |FG| and |CD| overlaps, there is no intersection of |HE| and |CD|.
50	***Data: -*** ***Warrant: -*** ***Rebuttal: -*** ***Claim:*** When |FG | and |CD | overlaps, |HE| and |CE| can not be equal.
51	***Data:*** The shape given is drawn with GeoGebra. |FG| is moved by being dragged and overlapped with |CD|. ***Warrant:*** |EH| and |CE| are both equal to the radius of the hemicyle. ***Rebuttal: -*** ***Claim:*** When FGE overlaps with |CD|, |EH|=|CE| again.
52	***Data:*** The shape is drawn with GeoGebra. |CD| and |FG| is dragged as in the figure. ***Warrant:*** If this angle is 90 degrees, that angle is also 90 degrees… Ohh, It is 90 degrees. When we bring this here as if it is the diameter… Look, this is |FG|. ***Rebuttal:*** |CE| here is equal with what on the screen? The letters are different.When it is diameter, it is not equal. ***Claim:*** If FEG is the diameter of the large circle, |EH|=|CE|.
53	***Data:*** The segment FEG is dragged until it overlaps the diameter of the big circle. ***Warrant:*** ∠HEF =90. |EH| and |CE| becomes the radius of the big circle. ***Rebuttal: -*** ***Claim:*** When segment FEG overlaps the diameter of the big circle, |EH|=|CE|.
54	***Data:*** Geogebra dragging. ***Warrant:*** In the givens |HE| was on the left side of the center. However, after dragging FEG, it passed through the center of the circle. I think |HE| should not glide. Geogebra begaves according to

	our drawing choices. Therefore, there is a problem in our drawing. We should immobilize some points. After dragging, |HE| should stay on the left side of the circle. ***Rebuttal:*** But |HE| is still perpendicular to FEG. I think there is no problem in the gliding of the |HE| to the center. ***Claim:*** |EH|≠|CE| when FEG coincides with |CD|.
55	***Data:*** The givens of the task is drawn. We draw |FH| and |GE|. ***Warrant:*** ∠FHG =90° since it is inscribe angle which sees the diameter. Euclidean formula can be applied in right triangle. ***Rebuttal:*** **-** ***Claim:*** $\|HE\|^2 = \|FE\|.\|EG\|$
56	***Data:*** We drew the chords |CF| and |DG| . ***Warrant:*** Let the measure of ∠CEF and ∠GED be α. alternate-interior angles. ∠CFE is equal to ∠GDE since they both see the same arc and they are the inscribed angles. The third angles of the triangles are also equal. ***Rebuttal:*** **-** ***Claim:*** The triangles CFE and GDE are similar triangles.
57	***Data:*** │CE│=│ED│ ***Warrant:*** The triangles CFE and GDE are similar triangles. (*Claim 56*). We can use the intersecting chords theorem in circles and write this equation. ***Rebuttal:*** **-** ***Claim:*** $│CE│^2 = │FE│.│EG│$
58	***Data:*** $\|HE\|^2 = \|FE\|.\|EG\|$ *(Claim 55* $│CE│^2 = │FE│.│EG│$ (*Claim 57)* ***Warrant:*** $│HE│^2 =│CE│^2$ If the squares of two number is equal, these numbers will be equal to each other. ***Rebuttal:*** **-** ***Claim:*** │CE│=│HE│
59	***Data:*** Drawing the shape with GeoGebra. ***Warrant:*** We measured the |CE| and |HE| with the measure tool of GeoGebra. By dragging the shape, it is seen that in almost all cases the segments are equal to each other. ***Rebuttal:*** **-** ***Claim:*** │CE│=│HE│
60	***Data:*** |OG|=a (radius), |EO|=x, |FE|= a-x . |CE|=|ED|=y. Let ∠HEG =90 °. ***Warrant:*** By using the intersecting chords theorem in circles, $a^2-x^2=y^2 =\|CE\|^2$. And then a Pytagorean formula in right triangle is applied to triangle HEO → $\|HE\|^2= a^2-x^2$. Since $│CE│^2=│HE│^2$ ***Rebuttal:*** **-** ***Claim:*** │CE│=│HE│

61	***Data:*** The segment FEG is dragged until it becomes the diameter of the big circle. ***Warrant: -*** ***Rebuttal: -*** ***Claim:*** When FEG is the diameter of the big circle │CE│=│HE│.
62	***Data:*** The segment FEG is dragged until it overlap on the segment |CD|. ***Warrant:*** |CE| = |HE| since they are both radius of the same circle. At the beginning of the solution we did not know the lenght of FEG. After dragging, the lenghts of the segments changed. In the givens, it says that |HE| passes through the midpoint of the chord |CD|. But when we dragged, The midpoint of the |CD| became the center of the circle. In the givens, it says that |HE| divides the chord |CD| into two and is perpendicular to the diameter of the half circle. The only segment can be drawn from the center of the circle in such a situation. ***Rebuttal:*** In the given shape, |HE| does not pass through the center of the circle. It should not pass through the center after dragging the shape. I think |CD|=FEG at the beginning in otder to overlap after dragging. Thus, |HE| should pass on the left side of the cnter of the half circle. . ***Claim:*** When |CD| overlapsthe segment FEG, |CE|=|HE| .
63	***Data:*** FEG is dragged until it becomes the diameter of the big circle. We know that |CE| and |ED| are radius. Then, this line will be perpendicular to |CD|. ***Warrant:*** If a segment divides the chord into two and it passes from the diameter of the circle, it should be perpendicular to the diameter. Then this segment is also a radius. Both segments are the radius of the big circle so they are equal to each other. ***Rebuttal: -*** ***Claim:*** When FEG is the diameter of the big circle |CE|=|HE|.

D: ARGUMENTS OF THE PAPER-PENCIL GROUP

The arguments in this study have been analyzed by means of schematization based on Toulmin's argument layout as in the following sample argument.



In the table below, the contents of the argument components put forward by the participants in the Paper-Pencil group are presented. There are some argument components that do not exist in some of the arguments. The explanation/contents box for these components are left blank in the table.

Argument No:	Contents of the Argument Components
1	***Data:*** - ***Warrant:*** - ***Rebuttal:*** - ***Claim:*** The points F anf G can be placed vice versa. The places of them can be changed.
2	***Data:*** The midpoints of the |AC| and |AB| is drawn. The points F and G are placed on the segment |BC| in such a way that |DG| and |EF| intersects at point H. |AH| is drawn. Let |FG|=a, |BF|=b=|CG| ***Warrant:*** - ***Rebuttal:*** Why? It is not an isosceles triangle. ***Claim:*** Then |AH| becomes an angle-bisector when the triangle ABC is an isosceles triangle.
3	***Data:*** An equilateral triangle, the side of which is 6a unit is drawn. The givens of the task are drawn. It is assumed that |AH| is an angle-bisector. ***Warrant:*** - ***Rebuttal:*** -

	Claim: Then, |HI| also became an angle-bisector.
4	***Data:*** A triangle is drawn . D and E points are placed. ***Warrant:*** In the givens it says that the segments |DG| and |EF| should coincide to create the point H. ***Rebuttal:*** - ***Claim:*** The points F and G should be placed on the segment |BC| in the order of 'B-F-G-C'.
5	***Data:*** Assume that|BD|=|DA|= x , |AE|=|EC|=y. |BG|=|CF| so |FG|=2a, |BF|=|GC|=b. |DE| =a+b is drawn. Two equations are written by using Menelaus's theorem and the similarity ratio between parallel lines. ***Warrant:*** The Melanous's theorem is applied two times from different directions and they were solved together to find a=z. ***Rebuttal:*** - ***Claim:*** |AH| is a median in all type of triangles.
6	***Data:*** *(Claim5)* ***Warrant:*** In all triangles, median is not angle-bisector at the same time. ***Rebuttal:*** - ***Claim:*** |AH| is not angle-bisector in all triangles.
7	***Data:*** *(Claim5 and Claim 6)* ***Warrant:*** The angle-bisector ratios are written on the sides of the triangle. 2x/a+b=2y/a+b . In order the equality to be satisfies x should be equal to y. That means |AB|=|AC|. ***Rebuttal:*** In scalene triangles, x will not be equal to y so the equality will not be satisfied. ***Claim:*** |AH| is angle-bisector in isosceles and equilateral triangles.
8	***Data:*** Let the midpoint of the segment |BC| be Z. |DE|, |DZ| and |EZ| are drawn. The ADZE parallelogram is formed. ***Warrant:*** In parallelogram the opposite edges and angles are equal to each other. If |DE|=|AZ| in ADZE paralleolgram. ***Rebuttal:*** We cannot know whether or not the extension of |AH| will pass through the intersection point of the intermediate bases. ***Claim:*** The extension of |AH| is the point Z. Namely, Z is the intersection point of the intermediate bases of |DZ| and |EZ|.
9	***Data:*** - ***Warrant:*** In order the |DG| and |EF| intersect and form the point H. ***Rebuttal:*** - ***Claim:*** The points F and G should be placed on the segment |BC| in the order of 'B-F-G-C'.
10	***Data:*** - ***Warrant:*** - ***Rebuttal:*** - ***Claim:*** When points F and G move (dragged), point H will move on the line passing through AT.
11	***Data:*** We combined |AF| and |AG|. Asssume that |DG| and |EF| are perpendicular to the sides |AB| and

	|AC| respectively. ***Warrant:*** When we look at the triangles ECF and EFA. Because of the Side-Angle-Side similarity in triangles |FA|=|FC| = x+y In the same way, because of the similarity in triangles DGB and DGA, |AG|=|GB|= x+y. ***Rebuttal:* -** ***Claim:*** Triangle AFG is an isosceles triangle.
12	***Data:*** |DG| is perpendicular to |AB| and |AD|=|DB|. We found AF=AG from side-angle-side similarity before. In triangle AGB, GD is an altitude and a median. ***Warrant:*** If a segment is both median and an altitude, it will also be an angle-bisector in any triangle. ***Rebuttal:* -** ***Claim:*** |GD| is the angle-bisector of ∠AGB.
13	***Data:*** |FE| is perpendicular to |AC| so it is altitude and |AE|=|EC|. ***Warrant:*** If a segment is both median and an altitude, it will also be an angle-bisector in any triangle. ***Rebuttal: -*** ***Claim:*** |FE| is the angle-bisector of ∠AFC.
14	***Data:*** |GD| is the angle-bisector of ∠AGB (*Claim12*). |FE| is the angle-bisector of ∠AFC. (*Claim13*). ***Warrant:*** But |AH| is the same line. Isn't it the angle-bisector of ∠BAC? ***Rebuttal:*** |AH| becomes the angle-bisector of ∠FAG. Not the angle-bisector of the big triangle (ABC). You are assuming that ∠BAF=∠GAC . We cannot know that. ***Claim:*** |AH| is the angle-bisector of ∠BAC.
15	***Data:* -** ***Warrant:*** Previously we found AFG as isosceles triangle. |AG|=|AF| ***Rebuttal:* -** ***Claim:*** ∠AFG = ∠AGF.
16	***Data:*** *(Claim 15)* ***Warrant: Triangle*** AFG is isosceles and |AH| is the angle-bisector of ∠BAC. ***Rebuttal:* -** ***Claim:*** The line passing through |AH| is perpendicular to |BC|.
17	***Data:*** *(Claim 12, Claim 13, Claim 15 and Claim 16).* |AH| is perpendicular to |BC|. |AH| divides |FG| into two equal segments as y/2 and y/2 . ***Warrant:*** In triangle ABC üçgeninde |AZ| is angle-bisector since it is both median and altitude. It suits also the angle-bisector relation. 2c/(x+(y/2)) = 2d/(x+(y/2)) ***Rebuttal:*** But we assumed that |FE| and |GD| are perpendicular to the segments |AC| and |AB| at the same time, respectively. Is such a situation can not exist, this solution is wrong. You said that equilateral triangle but in the figure |FE| and |DG| are not

	the medians of triangle ABC. In the same way, point H is not the intersection point of the altitudes in triangle ABC. ***Claim:*** In equilateral triangles |AH| is the angle-bisector of ∠BAC.
18	***Data:*** The extension of |AH| divides |FG| into two equal segments. ***Warrant:*** The angle-bisector is drawn but it will not pass through |AH|. It will not satisfy the givens of the task. |AH| is drawn in such a way that it divides |FG| into two equal segments. ***Rebuttal:* -** ***Claim:*** In scalene triangles |AH| cannot be an angle-bisector.
19	***Data:*** *(Claim17)* ***Warrant:*** Equilateral triangles are isosceles triangles at the same time. ***Rebuttal:*** In the drawing, point H is not the intersection point of the altitudes or medians of the triangle ABC. ***Claim:*** If |AH| becomes angle-bisector in isosceles triangles, it will be angle-bisector of the equilaterat triangles also.
20	***Data:*** *(Claim 11* and *Claim 15).* The only assumption is that the perpendicular segments being at the same time. |AF|=|AG|. |AH| is the angle-bisector of ∠FAG. ***Warrant:*** |DG|=|FE| (*Claim21*). |BD|=|EC| , c=d (*Claim22*). ***Rebuttal:*** Right now, the 90 degree is also our assumption to be proved. ***Claim:*** In both equilateral and isosceles triangles, |AH| will be the angle-bisector of ∠BAC.
21	***Data:* -** ***Warrant:*** A.A.A smilarity. BDG≈ CEF triangles are identical. You can say that the edges are equal to 90 degrees. ***Rebuttal:*** The angles are the same but the triangles are not similar ***Claim:*** |DG|=|FE|
22	***Data:* -** ***Warrant:*** A.A.A smilarity. BDG≈ CEF triangles are identical. You can say that the edges are equal to 90 degrees. *(Warrant 21)* ***Rebuttal:* -** ***Claim:*** |BD|=|EC| . That is, c=d.
23	***Data:*** ABC Isosceles triangle. |AE|=|EC| ***Warrant:*** ABC isosceles triangle | BE | is perpendicuar to | AC | hence, FE could not be perpendicular to | AC | at the same time. ***Rebuttal:*** How do you know that height pass through the middle of the |AC|? The height you draw from corner B can be parallel with| FE |. To be so, the ABC must be an equilateral triangle. You are coming off the side of the isosceles triangle hill, not the height. So it does not come to E point. You need to download from A ***Claim:*** |FE| can not be perpendicular to |AC|
24	***Data:* -** ***Warrant:*** Because, in our first drawing, perpendicular bisectors were drawn. ***Rebuttal:*** Then the result would be wrong for the isosceles triangle.

	Something that is not right at the equilateral triangle is not even in the isosceles triangle. ***Claim:*** In an equilateral triangle |FE| and|GD| could be perpendicular to sides of |AC| ve |AB| respectively.
25	***Data:*** **-** ***Warrant:*** **-** ***Rebuttal:*** **-** ***Claim:*** The result is the same when F and G are replaced.
26	***Data:*** |BG|=|GC|. |GF|=x, |FC|= y, |EG|=z, |BE|=x+y-z Since |NE|//|AB| then |HA|=yk, |HB|=(2x+y)k Since |HF|//|AC| then |NA|=(x+y-z)c, |NC|= (x+y+z)c ***Warrant:*** If we apply the Menelaus theorem***:*** (x/2xy).(2xy/y).|AP|/|PG|=1 Since |PG|.x=|AP|.y |AP|/|PG|=x/y is found. There is a similarity in triangles.BCA ≈ BFH ve CBA ≈ CEN ***Rebuttal:*** **-** ***Claim:*** |PG|=ys , |AP|=xs So |PG| is multiple of y,|AP| is multiple of x.
27	***Data:*** **-** ***Warrant:*** |HF|//|AC| ve |NE|//|AB|. ANPH quadrilateral is a parallelogram. ***Rebuttal:*** **-** ***Claim:*** |AN|=|HP| and |HA|=|PN|
28	***Data:*** *(Claim26)* ***Warrant:*** FEP≈FBH are similar triangles, |PE|// |HB|. ***Rebuttal: -*** ***Claim:*** |EP|=(x+z)k
29	***Data:*** *(Claim 25 and Claim 26)* ***Warrant:*** EPF≈ENC are similar triangles |PE|// |HB| ***Rebuttal: -*** ***Claim:*** |FP|=(x+z)c
30	***Data:*** **-** ***Warrant:*** Since |AB|//|EN| ve |AC|//|HF| , would not the third be parallel? ***Rebuttal: -*** ***Claim:*** |BC|//|HN|
31	***Data:*** *(Claim26, Claim28 and Claim29).* |HN| is drawn. Then it is assumed that |HN|//|BC|. ***Warrant:*** HNP≈FEP are similar triangles. KP:PE = HP:PF . yk: (x+z)k= (x+y-z)c : (x+z)c → ***Rebuttal:*** We need to prove that |HN|//|BC| in order to say it is true. ***Claim:*** |EG|=|GF|. z=x
32	***Data:*** *(Claim 31),* The ABC triangle is an isosceles triangle or an equilateral triangle. ***Warrant:*** We can generalize to all the triangles because we make the solution over a scalene triangle.

	Rebuttal: - ***Claim:*** |EG|=|GF|
33	***Data: :*** |AP|=xs, |PG|=ys. The shape is drawn. In the first drawing, y=|FG|. Take |BE|=2x, |EG|=x=|GF|, |FC|=2x y=2x . z=x (*Claim 31)* ***Warrant:*** |BE|=|EF|=|FC|, x+y-z=z+x=y. If we put x instead of z , 2x=y. We said that |AP|:|PG|. And xs : ys. |AP|:|PG|= x:2x. ***Rebuttal: -*** ***Claim:*** If |BE|=|EF|=|FC| is, P point must be in proportion to |AP|:|PG|=1:2.
34	***Data:*** ABC triangle is any triangle, Take |EP|//|AB| and |FP|//|AC|. |EG|=a , |GF|=b, |BG|=|GC|=x ***Warrant:*** GEP≈GBA and GPF≈GAC are similar triangles. Common edges of those are| AG |. When the similarity ratio is written: It is |GP|:|AG|=ak:xk=b:x (x and k are simplified). a=b. ***Rebuttal: -*** ***Claim:*** In all triangles |EG|=|GF| → a=b
35	***Data:*** Assume ABC is isoscelene triangle . |AB|=|AC|. The measure of angle B is same with that of angle C. ***Warrant:*** |AG| is perpendicular to |BC| and bisector. |AG| is also bisector of angle of EPF. Due to parallelism triangle EPF is isoscelene triangle. Hence |EG|=|GF| ***Rebuttal: -*** ***Claim:*** |EG|=|GF|
36	***Data:*** Assume ABC is equileteral triangle. |AG| is bisector, altitude and median. |AG| divides angle A into two with 30-30 degrees.. ***Warrant:*** |EP|//|AB| ve |FP|//|AC| olduğundan |EP|=|FP| olur açılardan. In the EPF triangle, altitude drawn from the top is bisector and also median. Is not it obvious that a triangle similar to a big triangle is formed because of parallelism? ***Rebuttal: -*** ***Claim:*** |EG|=|GF|
37	***Data****: (Claim 34),* Assume |EF|=|FG|=a and |BE|=|FC|=2a ***Warrant:*** Since GEP ≈ GBA triangles are similar a:3a=|PG|:|AG|= 1:3. ***Rebuttal: -*** ***Claim:*** The position of the P point should be such that | AP |: | PG | = 2:1 for any other triangle. This point is the center of gravity in equilateral triangle.
38	***Data:*** Assume |EG|=z, |GF|=x, |FC|=y ve |BE|=x+y-z. ***Warrant:*** |EP|//|AB|, |FP|//|AC|. FPE≈FKB ve ECT≈EFP triangles are similar. Because of this similarity, the multiples are written on the edges and the Melanous theorem is applied. x:(2x+y) * (2x+y)k:yk * |AP|:|PG|=1 ***Rebuttal: -***

	Claim: There is a ratio that |AP|:|PG| = x:y
39	***Data:*** FEP≈FBK . Since |EP|//|AB|, |FP|//|AC|, |KP|=(x+y-z)c and |PT|=yk. ***Warrant:*** |KT| is combined (which is parallel to |BC|). |KT|//|BC|. Again due to proportionality of | KP |: | PF | = | TP |: | PE | it was found that x = z. ***Rebuttal: -*** ***Claim:*** |EG|=|GF| , z=x
40	***Data:*** *(Claim 39).* |BE|=|EF|=|FC| is desired. ***Warrant:*** Let's solve it according to the letters we give such as x + y-z = z + x = y. We know that z = x. Here 2x = y. We found that| AP | = xs and | PG | = ys. Since at base, |BE|=2x and |EG|=x, due to similarities of triangles of AGB≈PGE, it will be found the ratio of | AP |: | PG | = 2: 1. We wrote 'xs' and 'ys' are in the wrong place. ***Rebuttal:*** But you found the opposite. It's not the center of gravity then. ***Claim:*** |AP|:|PG|=1:2
41	***Data:*** |HD|=|DE|=|EH|=|EC|=|CD|=R HCD and ECD are equilateral triangles. The radiuses of the circles are equal to each other. |GD| is drawn. ***Warrant:*** GDFH becomes deltoid. Triangle FGH is equilateral. We can apply the area formula of equilateral triangles $A^2\sqrt{3}/4$. ***Rebuttal: -*** ***Claim:*** The area of triangle FGH is 9√3.
42	***Data:*** Assume that ∠EDG = 2α, ∠GDH = 2^{β} . ***Warrant:*** ∠GFH and ∠ GHF are the same $(90-\alpha+^{\beta})/2$. |GH|=|GF| ***Rebuttal:*** My theorem did not work. I wrote wrong letter. The arc |DH| cannot be 90-beta since the angle is not in the center of the circle. ***Claim:*** FGH is an isosceles triangle.
43	***Data:*** |CD|, |DE|, |EC|, |HD|, |GD|, |CH| are drawn. |CD|=|DE|=|EC|=|HD|=|CH|=r CDE and CHD are equilateral triangles. ***Warrant:*** Since ∠HCD= 60 º , the arc HD will also be 60 º. In the same way, ∠DCE =60 º so the arc DE = 60 º. ∠HFG is the inscribed angle which sees the arc measured 120 º. ∠HFG =120:2=60 º. ***Rebuttal: -*** ***Claim:*** ∠HFG = 60º.
44	***Data:*** |CD|=|DF|=|DH|=R. ***Warrant:*** Three sides of the rectangle are equal to the radius. Isn’t the fourth side be equal to the radius? ***Rebuttal:*** Does it have to be radius? But we do not know the angle. ***Claim:*** |HF| = R.
45	***Data:*** The circles with centers C and D are identical. |CH|, |HD|, |CD|, |CE| and |DE| are drawn. ***Warrant:*** |CH|=|HD|=|CD|=|CE|=|DE| = r Since all of them are

	radius. ***Rebuttal: -*** ***Claim:*** Triangles HCD and CDE are equilateral.
46	***Data:*** HCD = DCE=60º HCE = 120 º and it is central angle. The arc HDE = 120 º . ***Warrant:*** ∠HFG is the inscribed angle which sees the arc measured 120 º . ***Rebuttal: -*** ***Claim:*** ∠HFG = 60 º.
47	***Data:*** |CD| is extended anf it intersected with the circle (C centered) at point A. |AD| becomes the diameter. |AF| and |FC| are drawn. ***Warrant:*** Arc HD=60 º . Arc AF=60 º . |FE| and |AD| formed alternate-interior angles at the center of the circle the center of which is C. ∠ACF =60 º. |FH| will be 180-(60+60)=60 º. ***Rebuttal:*** How does the alternate-interior angle be 60 º? I am drawing the whole shape in a smaller shape again. You extended |CE| and combined it with point F. That part of the drawing is not linear. You draw the chord FE as if it crosses from the center point C. But in the original shape it does not cross from the center. ***Claim:*** Arc |FH| =60 º.
48	***Data:*** *(Claim45* and *Claim46).* Since arc HDE = 120 º then arc HCE yayı will be 120 º also. The major arc of HE=360-120=240 º. ***Warrant:*** ∠HGE sees the arc of 240 º so its measure is 120 º. 60 º remained to ∠HGF. So ∠FHG =60 º. ***Rebuttal: -*** ***Claim:*** Triangle FGH is an equilateral.
49	***Data:*** Let the chord drawn from point E pass through point C. One side of the triangle becomes radius. ***Warrant: -*** ***Rebuttal:*** No. I think you can drag the chord |EF| beyond point C. You can pass the center C and the triangle becomes larger. ***Claim:*** For the triangle FGH with maximum area, the chord drawn from point E should pass through point C.
50	***Data: -*** ***Warrant:***We did not use the 6unit while solving the question. We used the arcs. ***Rebuttal: -*** ***Claim:*** When the chord |EF| is dragged, the triangle FGH is observed as equilateral triangle all the time.
51	***Data:*** ***(****Claim50****)***, Sinus formula of an area is (½).a.b.Sinα. ***Warrant:*** The edge of a maximum of a triangle becomes R. The formula of (½).x^2.Sin60 is derived. Derivative of the formula: 2X√3/4 For the an edge of the triangle R is put instead of X. Area becomes A=2R√3/4= R√3/2 ***Rebuttal:*** Why did you use the derivative? You found the growth rate.

	Claim: The maximum area of the FGH triangle becomes R√3/2.
52	***Data:*** It bocomes when the chord drawn from point E passes through point H, isn't it? ***Warrant: -*** ***Rebuttal: -*** ***Claim:*** The maximum area of the FGH triangle becomes zero.
53	***Data:*** When the same procedure is applied to triangle FGH it will be equilateral triangle again. Radius of the circles is R. ***Warrant:*** When we dragged the point F to the right, triangle expands, when we shift the point F to the left it becomes smaller. The biggest triangle becomes when the chord of |FE| passes through the center of point C. Then the triangle don't pass through the center. Still we don't know whether it is equilateral triangle or not.. ***Rebuttal:*** why doesn't it become true when the chord of |EF| is dragged even more? Why it becomes the greatest triangle when passing through the center? According to Hande's solution the angle of HGE becomes again 120^{o}, the angle of GFH becomes again 60^{o}. So the equilateral triangle is not distorted. Still providing even if passes the center (point C) ***Claim:*** The maximum area of the FGH triangle becomes $(R^2\sqrt{3})/4$.
54	***Data: -*** ***Warrant:*** The smallest triangle becomes when point F overlaps with point H. Triangle got lost. There is no triangle so no area. ***Rebuttal: -*** ***Claim:*** The area of FGH triangle becomes minimum 0.
55	***Data:*** The chord |FE| is drawn in such a way that it will be tangent to the circle with D centered. C-centered circle becomes the circumcircle of FGH triangle. One edge of the triangle is R√3. ***Warrant: -*** ***Rebuttal: -*** ***Claim:*** The largest FGH triangle becomes, when the chord | FE | is tangent to D centered circle at point E. Area becomes $(3R^2\sqrt{3})/4$
56	***Data:*** (Claim53 and Claim55) ***Warrant:*** If I enlarge further, the chord | EF | does not cut the second circle and triangle does not occur. But then we do not mean to draw a chord. Instead two chords occured. Chord is a segment which cuts the circle at two point. Therefore, the situation that we drew the radius of circumcenter is true. ***Rebuttal:*** Is chord a line? Here it is. It cuts circle at points F and G. ***Claim:*** The largest FGH triangle occurs, when the chord | FE | is tangent to the D centered circle at E point and area becomes (3R2√3)/4
57	***Data:*** Point E was a midpoint. It says |FG| is diameter. The line passing through EH is perpendicular to diameter |FG| and cuts the radius at point H. ***Warrant:*** Angles are different

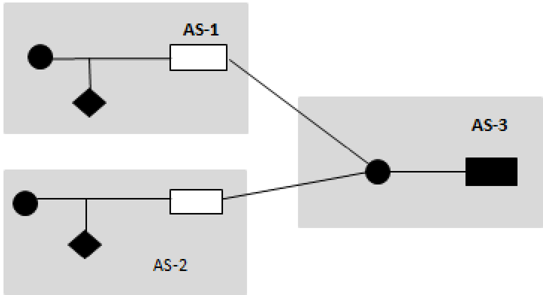
	Rebuttal: But angles are the same since teacher mentioned it in the question ***Claim:*** |HE| is not equal to |CE|
58	***Data:*** - ***Warrant:*** - ***Rebuttal:*** - ***Claim:*** |FE|. |EG| = |CE|. |ED|
59	***Data:*** |CE|=|ED|=a and |FE|=2b, |EG|=2c . ***Warrant:*** |CE|.|ED|=a.a=2b.2c =4bc. ***Rebuttal:*** - ***Claim:*** |CE|=√4bc
60	***Data:*** Let the center of the half circle be 'O' |OG|=b+c and |EO|=c-b. Let it |HE|=t and |HO| is drawn. ***Warrant:*** $t^2=(b+c)^2-(c-b)^2$ then $t^2=4bc$. ***Rebuttal:*** - ***Claim:*** |HE|=√4bc
61	***Data:*** |CE|=√4bc (Claim59), |HE|=√4bc (Claim60), T^2 equals to A^2. ***Warrant:*** - ***Rebuttal:*** - ***Claim:*** |HE|=|CE|
62	***Data:*** If we draw the|HF| and |HG| ***Warrant:*** because it is the inscribed angle viewing the diameter ***Rebuttal:*** - ***Claim:*** ∠FHG= 90 º .
63	***Data:*** ∠FHG= 90 º (Claim62) ***Warrant:*** Euclid ***Rebuttal:*** - ***Claim:*** T^2 = 2b.2c
64	***Data:*** |CD| and FEG overlaps. |HE| is drown left side of the center. |HC| and |HD| are drown. The midpoint of the |CD| is E, ,isn't it? |CE|=a. ***Warrant:*** |HO| is drawn and it is found by pythagorean theorem as well.There is an Euclidean theorem in the right triangle HCD. |CD| is diameter. Because the ∠CHD is inscribed angle viewing the diameter, it is 90 degree. Euclidean theorem can be used there. |HE|2=|CE|. |ED| $\| HE\|^2=a.a=a^2$ ***Rebuttal:*** But here |HE|=√4b and |CE|= 2b. They are not equal to each other ***Claim:*** When |CD| and FEG overlaps, still |HE|=|CE| .
65	***Data:*** |HF |and |FG| are drawn ***Warrant:*** |CD| is diameter of half circle. Inscribed angle viewing the diameter it is 90 degree. ***Rebuttal:*** - ***Claim:*** ∠FHG =90º.
66	***Data:*** ∠FHG =90º (Claim 65). Let the center of the half circle be

	point 'O'. |FE|=a, |EO|=b, |OG|=a+b. And |HE|=y . ***Warrant:*** We applied the Euclidean formula in right triangle FHG $\|HE\|^2=y^2=a(a+2b)$. ***Rebuttal: -*** ***Claim:*** $\|HE\|^2=y^2=a(a+2b)$
67	***Data:*** |CE|=|ED|=x. Let the center of the half circle be point 'O'. |FE|=a, |EO|=b, |OG|=a+b. ***Warrant:*** Inner force in a circle is used |CE|.|ED|=|FE|.|EG|. ***Rebuttal: -*** ***Claim:*** $x^2=a(a+2b)$
68	***Data:*** $\|HE\|^2=y^2=a(a+2b)$ (Claim 66) ve $x^2=a(a+2b)$ (Claim 67) ***Warrant:*** the lenghts cannot be negative. ***Rebuttal: -*** ***Claim:*** |HE|=|CE| , y=x.
69	***Data:*** The end points of the chords joined together. The segments were named as x, y, z and t. ***Warrant:*** The chord lenghts of opposite (alternate) angles are written based on the Cosinus theorem. X^2+T^2-2XT. ***Rebuttal:*** There is no parallelism, so there cannot be similarity. Only one angles of triangles are equal so similarity cannot exist. ***Claim:*** x.y=z.t (intersecting chords theorem)
70	***Data:*** The end points of the chords joined together. ***Warrant:*** The inscribed angles which see the same lenght are equal to each other in circles so the angles are found to be equal. ∠ECG=∠DFG ∠GEC= ∠GDF, and ∠EGC=∠DGF Then A.A.A similarity exists. x.y=z.t . No it is an inscribed angle. You do not need a central angle. ***Rebuttal:*** Does not G have to be in the center? ***Claim:*** The intersecting chords relation in circle is calculated by x.y=z.t.
71	***Data:*** |CE|=|ED|=x, |FE|=a, |EG|=b, |HE|=c. Half circle is completed to the whole circle. FEG is the diameter of that circle. ***Warrant:*** |HE| is extended and intersected with circle at point K. If I named |HE|= c, |EK|= c ince this is the diameter. We can apply intersecting chords theorem in both circles: $c^2=a.b$ and $x^2=a.b$ so x=c. ***Rebuttal: -*** ***Claim:*** |HE|=|CE|
72	***Data:*** Let the center of the half circle be 'O'. |HO| is drawn. Let the diameter of the half circle be 2b+2c. |FE|=2b, |EO|=c-b, |OG|=b+c, |HO|=b+c. ***Warrant:*** From the intersecting chords teorem in circle x.x=2b.2c |CE|=x=√4bc. HEO is a right triangle. Using the Pythagorean formula $\|HE\|^2=(b+c)^2-(c-b)^2$ → |HE|=√4bc. ***Rebuttal: -***

	Claim: |HE|=|CE|
73	***Data:*** FEG is drawn so as it would be the diameter of the big circle. Then, the half circle and the big circle overlaps. |CD| is drawn. |CE|=|ED|=x, |FE|=a, |EG|=b. Point H is formed by drawing a perpendicular to the diameter at point E. |HE| is extended and intersected with circle at point K. ***Warrant:*** |HE| is equal to |EK| since it is perpendicular to the diameter. |HE|=|EK|=c. From the power of point relation in circles, x^2=a.b and c^2=a.b. → x=c Since we found x=c that means |CD| is perpendicular to the diameter.|CD| and |HK| became the same chord. ***Rebuttal: -*** ***Claim:*** When FEG becomes the diameter of the big circle |HE|=|CE| .
74	***Data:*** Half circle is drawn in such a way that its diameter be |CD|. Point H is formed by drawing a perpendicular line to the diameter of the half circle. |CE|=|ED|=r, |HE| → It is perpendicular to |CD| . |CH| and |HD| are drawn. ***Warrant:*** ∠CHD = 90 º since it sees the diameter of the circle. Euclidean formula can be applied in triangle. $\|HE\|^2$=r.r. |HE|= r=|CE|. |HE| is also the radius of the half circle at the same time. We proved that. Both of them are radius so they are equal. ***Rebuttal: -*** ***Claim:*** When segment FEG and |CD| overlaps, |HE|=|CE| .

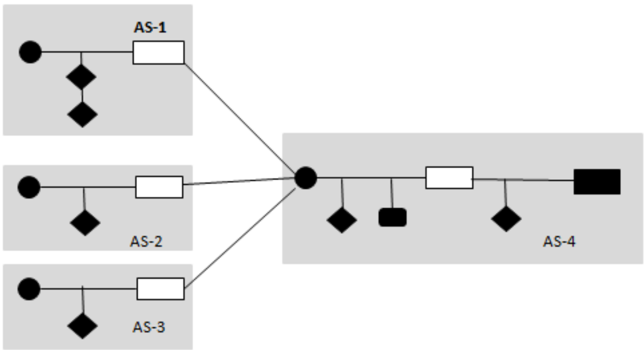
E: GLOBAL ARGUMENTATION STRUCTURES

Source-Structure argumentations of GeoGebra group



From GT 4

Source-Structure argumentations of Paper-Pencil group



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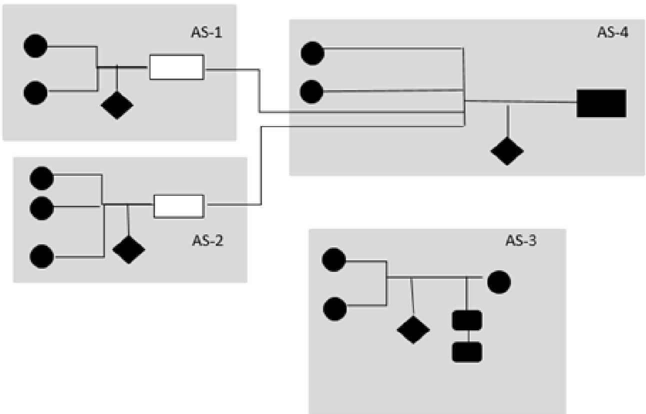

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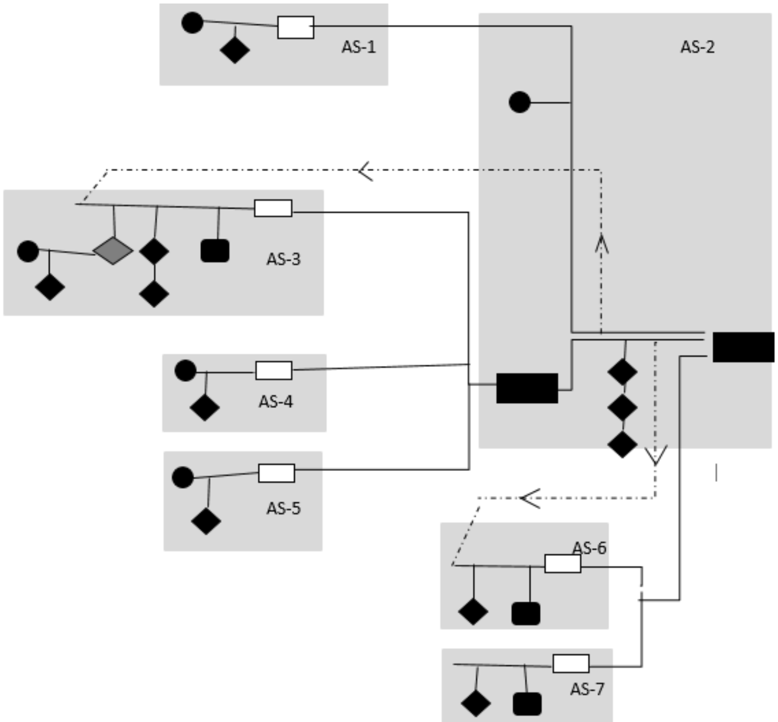
Reservoir-Structure argumentations of GeoGebra group



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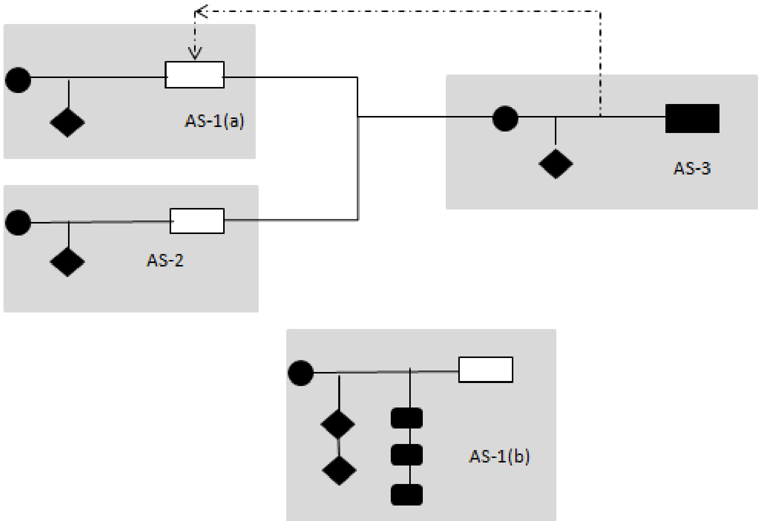

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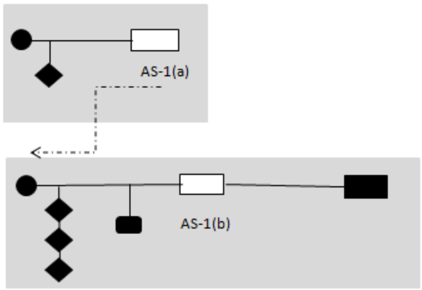

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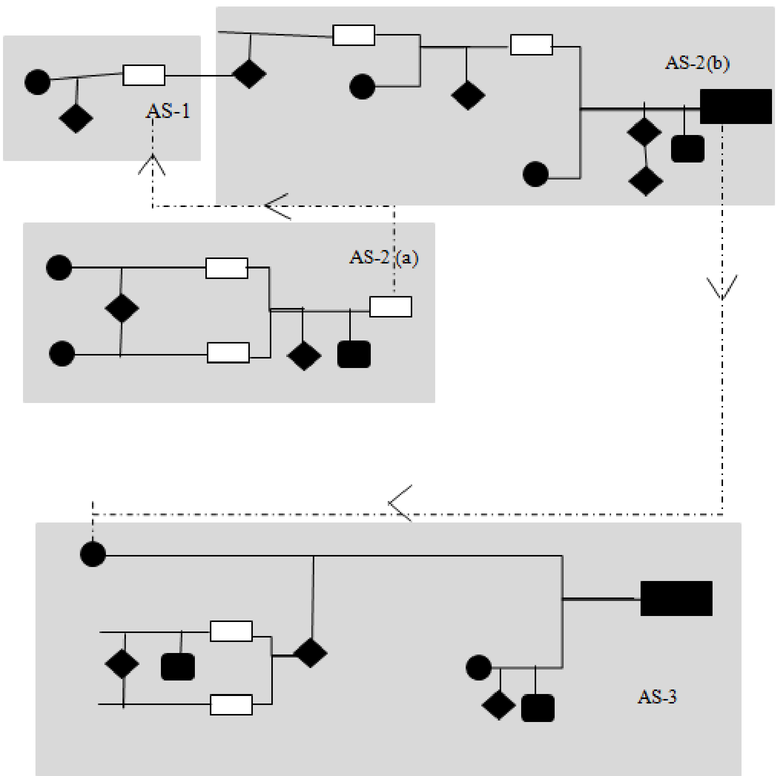

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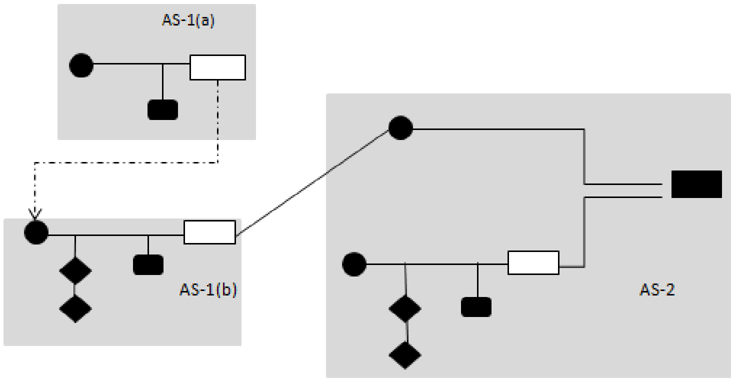

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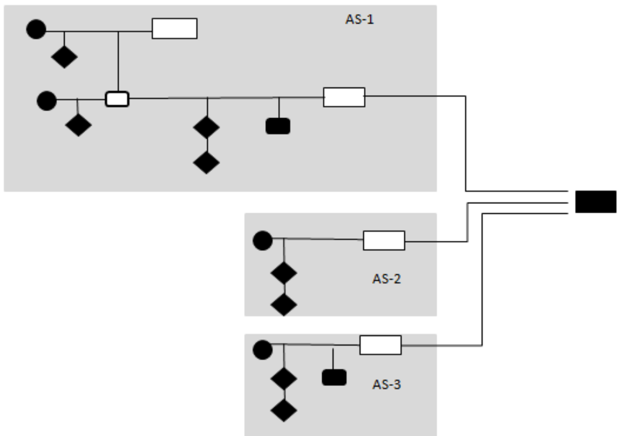
Spiral-Structure argumentations of GeoGebra group



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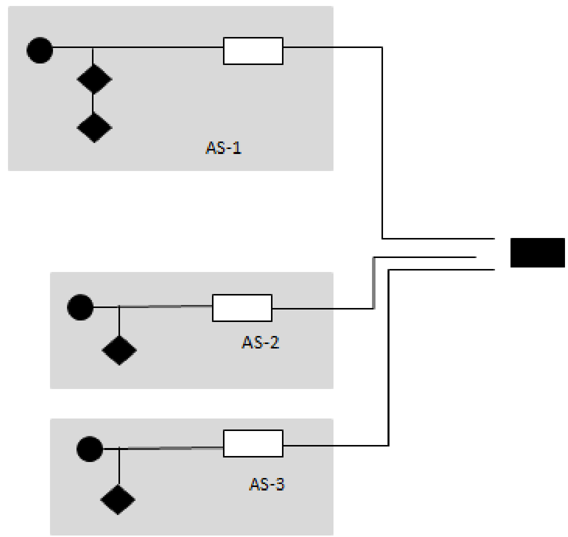

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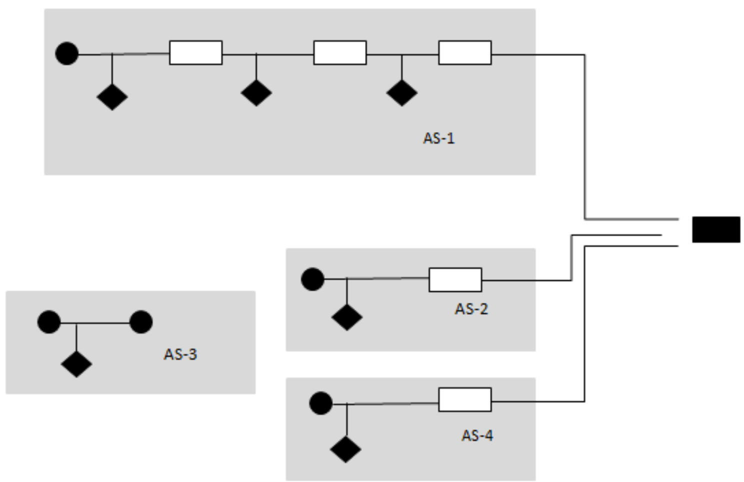

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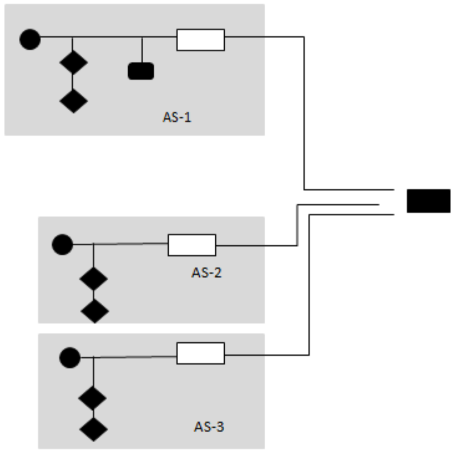

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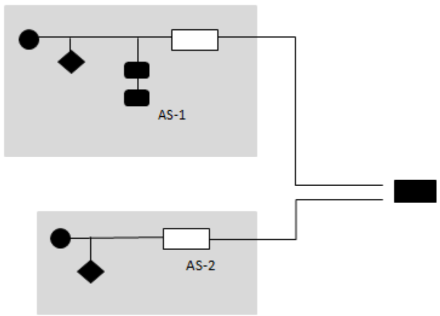

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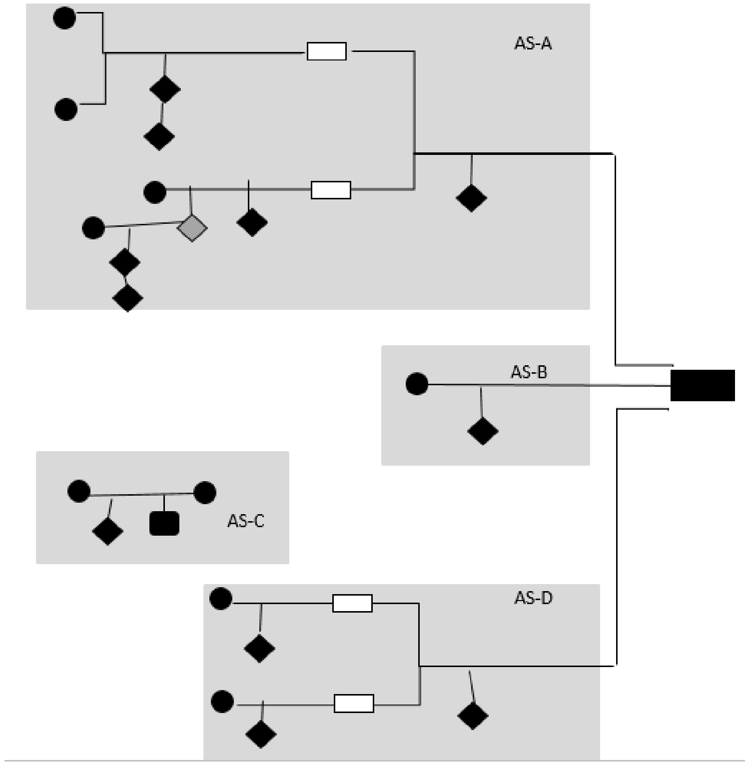

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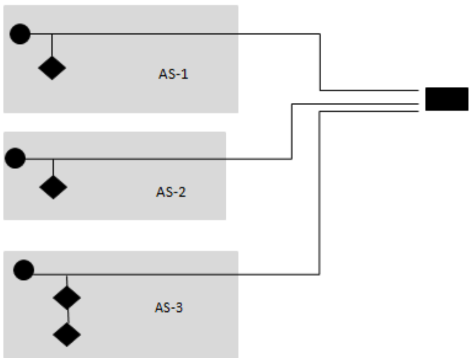

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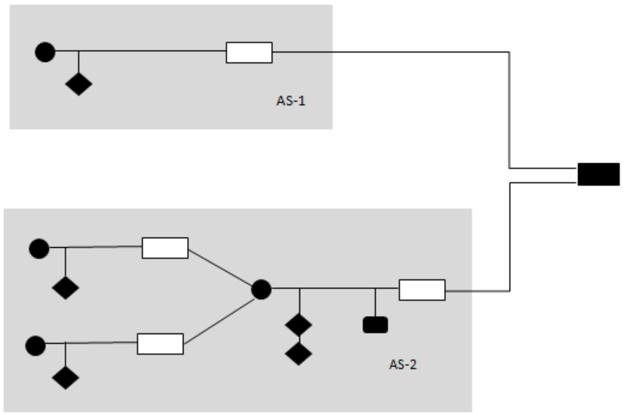

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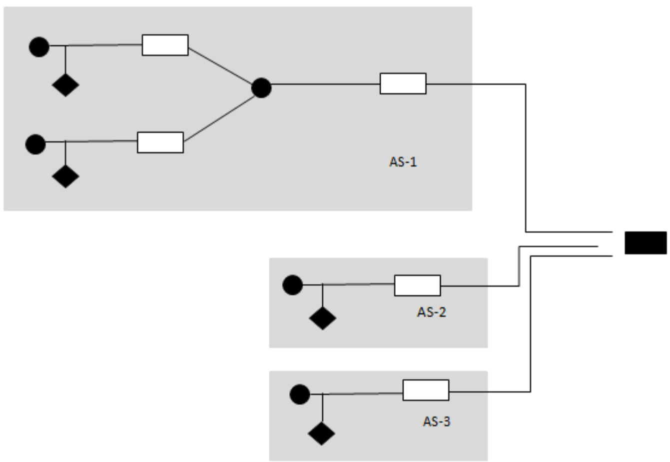

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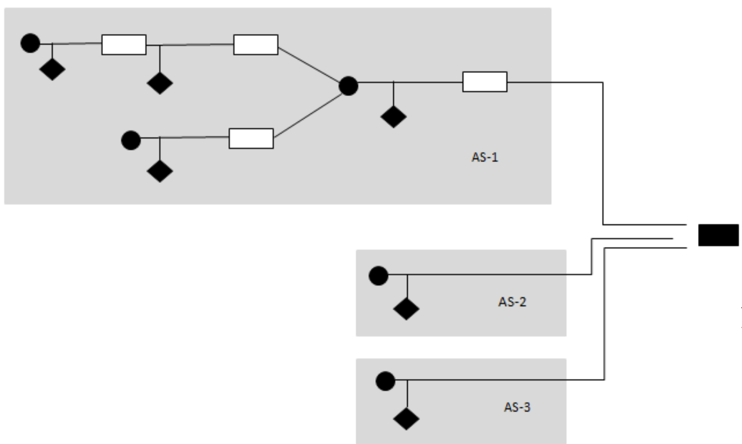

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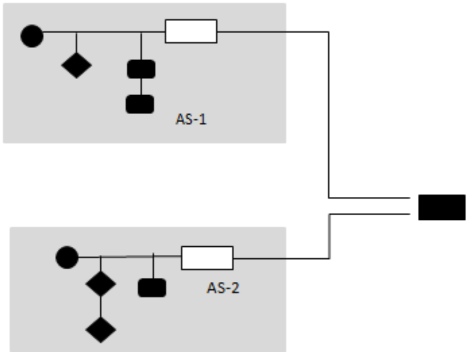

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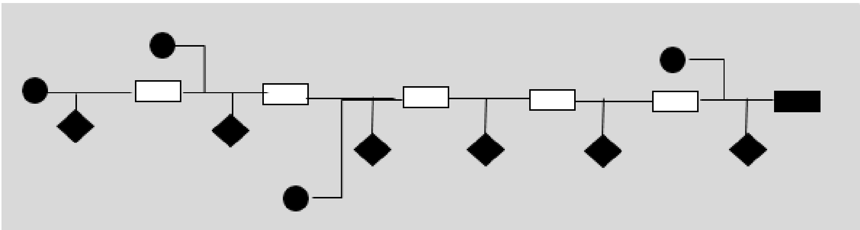
Line-Structure Argumentations of GeoGebra Group



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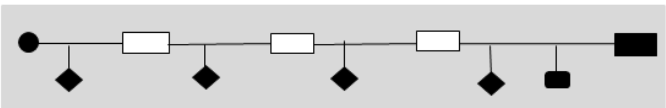

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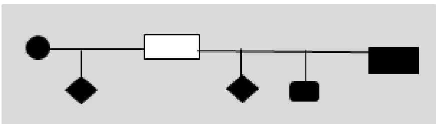

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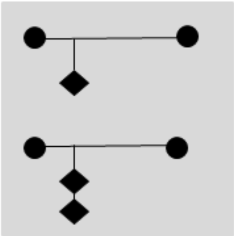
Independent Arguments Structures of GeoGebra Group



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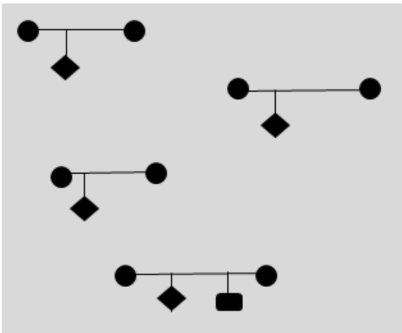

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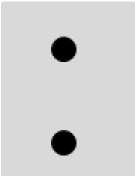

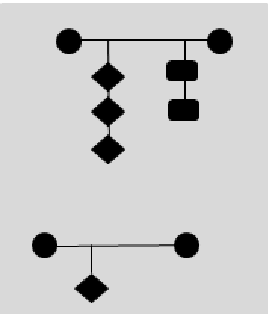

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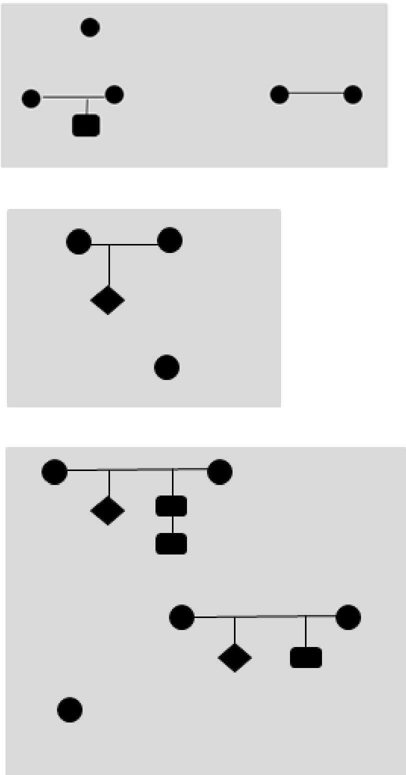
Independent Arguments Structures of Paper-Pencil Group



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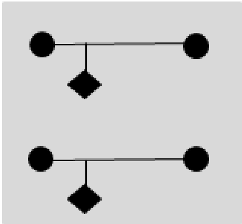

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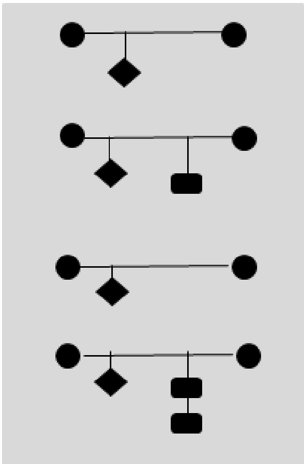

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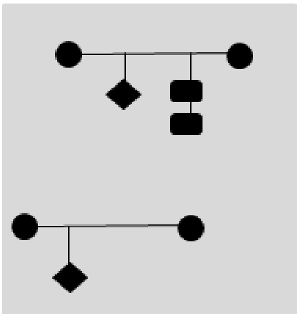

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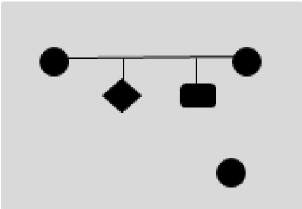

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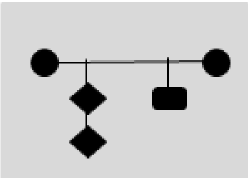

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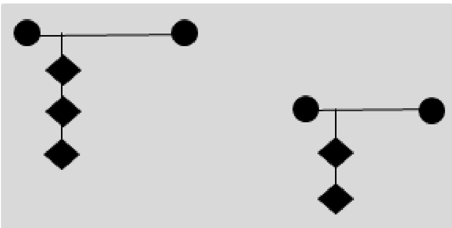

From GT4

F: LOCAL ARGUMENTS

In the tables below, the numbers represent the argument numbers. For instance, '2' refers to the second argument while '41' refers to the 41st argument.

Local Arguments of GeoGebra Group

Local Argument Type	**Argument number**
1. *DCW*	2, 9, 28, 31, 40, 44, 63, 51, 52, 53, 54,
2. *DWC*	6, 10, 12, 17, 18, 19, 20, 22, 23, 24, 25, 29, 30, 32, 34, 35, 36, 37, 38, 39, 47, 48, 55, 56, 57, 58, 59, 60, 62
3. *CDW*	21
4. *WDC*	14, 27
5. *CD*	46
6. *DC*	1, 61
7. *CW*	4, 7, 13, 15, 16, 26, 33, 41, 43, 45
8. *WC*	5, 8, 11, 42
9. *C*	3, 49, 50

Local Arguments of Paper-Pencil group

Local Argument Type	**Argument number**
1. *DCW*	6, 14, 19, 29, 32, 44, 56, 57, 62, 69
2. *DWC*	4, 5, 7, 11, 12, 17, 20, 23, 26, 28, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 46, 47, 48, 51, 53, 59, 60, 63, 65, 66, 67, 68, 71, 72, 73, 74
3. *CDW*	8, 45, 64, 70
4. *WDC*	13, 18
5. *CD*	49, 55
6. *DC*	2, 3, 61
7. *CW*	9, 16, 21, 24, 27, 50, 52, 54
8. *WC*	15, 22, 30,
9. *C*	1, 10, 25, 58

G: ETHICS PERMISSION

Approval of the Ethics Committe of METU Applied Ethics Research Center

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

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08 Mayıs 2013

Gönderilen: Doç.Dr. Mine IŞIKSAL
İlköğretim Bölümü

Gönderen : Prof. Dr. Canan Özgen
IAK Başkanı

İlgi : Etik Onayı

Danışmanlığını yapmış olduğunuz İlköğretim Bölümü Doktora öğrencisi Özlem Erkek'in "İlköğretim Matematik Öğretmen Adaylarının Argümantasyon Süreçlerinin Analizi: Dinamik Geometri Programı Kullanılarak Geometri Soruları Çözümü" isimli araştırması "İnsan Araştırmaları Komitesi" tarafından uygun görülerek gerekli onay verilmiştir.

Bilgilerinize saygılarımla sunarım.

Etik Komite Onayı

Uygundur

08/05/2013

Prof.Dr. Canan ÖZGEN
Uygulamalı Etik Araştırma Merkezi
(UEAM) Başkanı
ODTÜ 06531 ANKARA

H: TURKISH SUMMARY (TÜRKÇE ÖZET)

ORTAOKUL MATEMATİK ÖĞRETMEN ADAYLARININ ARGÜMANTASYON YAPILARININ TEKNOLOJİ VE KAĞIT-KALEM ORTAMLARINDA İNCELENMESİ

Giriş

1.1 Matematik eğitiminde argümantasyon

Matematikte derinlemesine öğrenmenin anahtarları şüphesiz ki muhakeme ve ispattır. Alanyazındaki birçok çalışmada adı ispat ile birlikte anılan argümantasyon, matematik öğrenme konusunda oldukça önemsenmesi gereken bir yöntemdir (Conner, 2007b). Argümantasyon, bireylerin bilimsel iddialarını deneysel veya kuramsal delillerle destekledikleri ve değerlendirdikleri bilimsel tartışma ve sosyal etkileşim sürecine verilen addır (Jimenéz-Aleixandre & Erduran, 2008). Bireyler argümantasyon sürecinde argüman oluşturur, argümanlarının gerekçelerini sorgular, farklı bakış açılarıyla sunulmuş argümanları değerlendirir ve bilimsel anlamda kaliteli açıklamalara ulaşırlar (Driver, Newton ve Osborne, 2000). Conner ve diğerleri (2014b) argümantasyonun ve muhakemenin eş süreçler olduklarını savunmuşlardır. Ulusal Matematik Öğretmenleri Konseyi (NCTM, 2000), muhakeme ve ispatın belli başlı konularda uygulanması gereken bir yöntem olmadığını, okullardaki bütün konularda gerçekleştirilecek sınıf tartışmalarına dahil edilmesi gerektiğini vurgulayarak matematik eğitiminde argümantasyonun önemine dikkat çekmiştir.

Toulmin (1958)'in 'The Uses of Argument' adlı kitabıyla alanyazına kazandırdığı argümantasyon yöntemi, öğrenmede sonuçtan çok sürecin ve sosyal öğrenmenin önem kazanmaya başlamasıyla (Reiss, Heinze, Renkl & Gross, 2008) birçok araştrmacının dikkatini çekmiştir. İlk olarak fen alanında alanyazına kazandırılan ve üzerine birçok çalışma yapılan bu yöntem, matematik alanında yeni

yeni kullanılmaya başlamıştır. Alanyazındaki birçok çalışmada argümantasyon ve sosyal etkileşimin başarı üzerindeki olumlu etkisinden bahsedilmiştir (Cross, 2009; Inagaki, Hatano, & Morita, 1998; Kosko, Rougee, & Herbst, 2014; Sfard, 2008; Walter & Barros (2011). Örneğin Sfard (2008), kişilerin matematikte başarılı olmak için belirli tartışma biçimlerine ve iletişimsel etkinliklere katılmalarını önermiştir. Benzer şekilde, Walter ve Barros (2011) öğrencilerin daha derin matematiksel düşünmelerinin sağlanması için onların argüman geliştirmede aktif olmalarını ve bir sonuca varmak için farklı çözüm yaklaşımları üzerinde çalışmalarını önermişlerdir. Dolayısıyla argümantasyonun matematik alanında uygulanmasına yönelik çalışmalar, matematik başarısının arttırılması için araştırılmaya değer görülmektedir.

Tartışma-tabanlı öğretim yöntemi olan argümantasyon, hem öğrenciler, hem de tartışmayı yöneten öğretmen açısından üst düzey düşünme gerektiren bir yöntemdir. Alanyazındaki çalışmalara göre farklı sınıf seviyelerindeki öğrencilerin gerekçe sunma, argümantasyon ve ispat konularında zorlandıkları bilinmektedir (Ellis, 2007; Harel & Sowder, 1998; Healy & Hoyles, 1998; Reiss, Klieme, & Heinze, 2001; Selden & Selden, 2003; Walter & Barros, 2011). Bu durum Türkiye'deki öğrenciler için de geçerli olduğundan argümantasyon, içerisinde cevaplanması gereken birçok soru barındıran bir konu olarak karşımıza çıkmaktadır. Türkiye'de ortaokul matematik müfredatı en son 2013 yılında revize edilmiştir. Müfredatın ana hedeflerinde öğrencilere "araştırma ve sorgulama yapabilecekleri, iletişim kurabilecekleri, eleştirel düşünebilecekleri, gerekçelendirme yapabilecekleri, fikirlerini rahatlıkla paylaşabilecekleri ve farklı çözüm yöntemlerini sunabilecekleri" (MEB, 2013, s. I) sınıf ortamlarının sağlanması yer almaktadır. Bu beceriler argümantasyon yönteminde yer aldığından, argümantasyonun etkili bir biçimde uygulanmasının yeni müfredatta örtük kazanım olarak yer aldığı görülmektedir. Bu yöntemi uygulayacak olan öğretmenlerin bu konudaki becerisi, araştırılması gereken bir durumdur. Peki öğretmenler ne kadar sorgulama yapıyor / nasıl argümanlar üretebiliyor? Bu açıdan bakıldığında öğretmenlerin argümantasyon yapılarının incelenmesi araştırılması gereken önemli bir konu olarak karşımıza çıkmaktadır.

Argümantasyon yapısının incelenmesi araştırmacılara argümanı ileri süren kişinin nasıl anladığı ve nasıl muhakeme ettiği ile ilgili detaylı bilgi sunmaktadır. Bu nedenle alanyazında bazı araştırmacılar argümantasyon yapılarının incelenmesini tavsiye etmiştir (Walter & Barros, 2011). Öğretmen adaylarının gelecekteki öğretim şekilleri ile ilgili çıkarımlarda bulunabilmek için bu çalışmanın odağı argümantasyon yapılarının detaylı incelenmesi olarak belirlenmiştir.

Alanyazındaki çalışmalar incelendiğinde teknoloji destekli ortamlarda gerçekleşen argümantasyon çalışmalarının sayısının az olduğu sonucuna varılmıştır (Hewit, 2010; Hollebrands, Conner & Smith, 2010; Inglis, Mejia-Ramos & Simpson, 2007; Prusak, Hershkowitz & Schwarz, 2012). Mevcut çalışmalarda teknoloji kullanımının argümantasyona olumlu etkileri olduğundan bahsedilmiştir. Ayrıca teknolojinin derinlemesine düşünmeye ve kağıt kalemle yapılan çözümlerde farkedilemeyecek ilişkileri keşfetmeye yardımcı olduğu iddia edilmektedir. Argümantasyonun ileri seviyede düşünmeyi gerektirdiği dikkate alındığında teknolojinin ortaokul matematik öğretmen adaylarının geliştireceği argüman yapılarını nasıl değiştireceği merak edilmektedir.

Yukarıda bahsedilen konulardan yola çıkılarak bu çalışmada ortaokul matematik öğretmen adaylarının argümantasyon yapılarının teknoloji ve kağıt kalem ortamlarında karşılaştırmalı olarak incelenmesine karar verilmiştir ve aşağıdaki araştırma sorularına cevap aranmıştır:

1. GeoGebra ve Kağıt-Kalem gruplarında geometri problemleri çözen ortaokul matematik öğretmen adaylarının kullandıkları argümantasyon yapılarının doğası nasıldır?
2. Global argümantasyon yapılarının içindeki lokal argümanların özellikleri nasıldır?
    - GeoGebra ve Kağıt-Kalem gruplarında geometri problemleri çözen ortaokul matematik öğretmen adaylarının kullandıkları lokal argümanların, argüman elemanı (iddia, veri, gerekçe) akış sırasına göre çeşitleri nelerdir?

3. GeoGebra ve Kağıt-Kalem gruplarında geometri problemleri çözen ortaokul matematik öğretmen adaylarının kullandıkları lokal argümantasyon özellikleri nelerdir?

1.2 Çalışmanın önemi

Argümantasyon, hipotez kurma, kendi görüşünü gerekçelendirme, problem sentezleme, başkalarının görüşlerine meydan okuma, farklı bakış açılarını karşılaştırma, deliller kullanarak hipotez tutarsızlıklarını değerlendirme gibi becerileri içerdiğinden (Hewit, 2010) ileri seviyede ve eleştirel düşünme gerektiren bir yöntemdir. Bu becerilerin öğrencilere kazandırılması için argümantasyonu yöneten kişi olan öğretmen önemli bir faktördür. Argümantasyonu yönetmeden önce öğretmenin argüman üretmeyi ve sınıfı argümantasyona uygun şekilde yönetmeyi deneyimlemesi gerekmektedir (Prusak et al., 2012). Türkiye'deki matematik öğretmenliği eğitimi programı incelendiğinde öğretmen adaylarının hiçbir dersi kapsamında argümantasyon uygulaması ile karşılaşmadığı görülmüştür. Ayrıca matematik eğitimi alanında bu konuyu içeren herhangi bir seçmeli derse de rastlanmamıştır. Dolayısıyla ortaokul matematik öğretmen adaylarının argümantasyon yöntemi uygulamasından bihaber olarak mezun oldukları sonucuna varılmıştır. Geleceğin öğrencilerine argümantasyon-tabanlı matematik dersi anlatabilecek öğretmen adaylarının yetiştirilebilmesi için bu çalışmanın bulgularının gelecekte yapılacak araştırmalara yol göstereceğine inanılmaktadır. Ayrıca bu çalışma ile program geliştiricilerin dikkatinin argümantasyon yöntemine çekilmesi amaçlanmıştır. Böylelikle öğretmen yetiştirme programlarında bu açıdan düzenleme yapılması konusunda eğitimcilerin ve politikacıların bilgilendirilmesi sağlanacaktır.

Fen alanında çeşitli açılardan incelenen argümantasyon yönteminin, matematik alanında çok yeni olması sebebiyle araştırılması gereken birçok yönü vardır. Bunlardan birisi de teknoloji ortamında gerçekleşen argüman yapılarıdır. Bu çalışmanın bulguları hem teknoloji ortamında hem de kağıt kalem ortamında kullanılan argümantasyon yapılarının karşılaştırılmasına olanak sağlaması açısından önemlidir. Bu çalışmada dinamik geometri programının kullanılması doğal olarak geometri problemlerinin argümantasyon yöntemiyle çözülmesine olanak sağlamıştır. Adı birçok bilimsel çalışmada ispat ile birlikte anılan argümantasyonun, geometri alanında incelenmesi ile bu açıdan da alanyazına katkıda bulunulacaktır.

Bu çalışmanın matematik eğitimi alanyazınına bir diğer katkısı da ortaokul matematik öğretmen adaylarının *argümantasyon yapılarını* incelemesidir. Alanyazında ispat konusunda yaptığı çalışmada Knipping (2008) argümantasyon yapılarının analizinin önemini vurgulamıştır. Bu çalışmada öğretmen adaylarının argümantasyon yapıları geometri alanında incelenerek bu alandaki muhakeme süreçleri, tercih ettikleri lokal argüman ve lokal argümantasyon çeşitleri hakkında bilgiler sunulacaktır. Bu bilgiler, geleceğin öğretmenlerinin argümantasyon becerileri ve muhakeme süreçleri hakkında ipuçları verecek ve onların argümantasyon yöntemini uygularken neleri önemseyeceğini gözler önüne serecektir. Böylelikle bu çalışmanın sonuçları, gelecekte teknoloji veya kağıt kalem ortamlarında yapılacak olan argümantasyon çalışmalarına argümantasyon yapısının incelenmesi açısından ışık tutacaktır.

2. ALANYAZIN TARAMASI

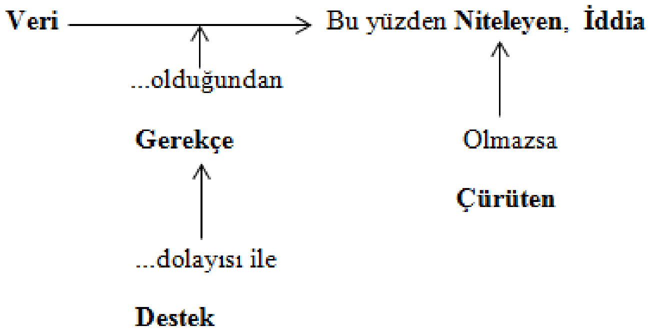
2.1 Teorik çerçeve

Öğrenmenin hem bireysel hem sosyal boyutları vardır ve bu boyutlar akademik başarı için büyük öneme sahiptirler (Cobb, Yackel, Wood, Nicholls, Wheatley, & Trigatti, 1991; Lesh, Doerr, Carmone, & Hjalmarson, 2003, Schoenfeld, 1992). Bu çalışmanın odak noktası öğrenci merkezli öğrenmeyi ön planda tutan, argümantasyona dayalı öğrenme yaklaşımıdır.

Krummheuer'in (1995) tanımına göre *argüman*, katılımcılar veya gözlemciler tarafından kısmen ya da tamamen yeniden yapılandırılabilen, tartışmanın sonunda tüm katılımcılar tarafından kabul edilen ifade dizisidir. *Argümantasyon* ise Antonini ve Martignone (2011) tarafından amacı bireyleri bir ifadenin doğruluğuna veya yanlışlığına inandırmak olan söylemler içeren tartışma süreci olarak tanımlanmıştır.

Bu tanımlara bakılarak argümantasyonun bir süreç, argümanın ise bir ürün olduğu sonucuna varılabilir.

Argümantasyon yaklaşımının bilimsel olarak incelenmesi Toulmin'in 1958 de yayınladığı 'The Uses of Argument' isimli kitaba dayanmaktadır. Toulmin (1958) bu kitabında, birbiriyle ilişkili üç ana üç yardımcı elemandan oluşan rasyonel argüman yapısı sunmuştur. Ana elemanlar iddia (claim), veri (data), ve gerekçe (warrant) iken yardımcı elemanlar destek (backing), niteleyen (qualifier) ve çürütendir (rebuttal). Rumsey (2012) yardımcı elemanların olmazsa olmaz elemanlar olmadıklarını fakat argümanlarda bulunabileceklerini söylemiştir. İddia (claim) tartışmacının diğerlerini ikna etmeyi düşündüğü ifade, sonuç veya görüştür (Nardi, Biza & Zachariadez, 2012, s.159). İddiayı destekleyen gerçekler veya kanıtlar veri (data) olarak tanımlanmıştır (Conner, Singletary, Smith, Wagner, & Francisco, 2014a, s.404). Gerekçe (warrant) ise veriden iddiaya nasıl ulaşıldığını ileri süren genel ifadelerle kurulan bir köprüdür (Toulmin, 1958, s.101). Yani gerekçe, verilerden iddiaya ulaşmayı sağlayan varsayımlar olarak tanımlanabilir. Modelin yardımcı elemanlarından destek (backing), gerekçe için ek destek olarak tanımlanabilir (Pedemonte & Reid, 2011). Destek elemanı, gerekçe kabul edilmediğinde veya yetersiz kaldığında onun otoritesini arttırmak amacıyla ileri sürülebilir. Destek, verileri ve iddia-veri arasındaki ilişkiyi destekleyen her türlü bilgi olabilir. Bir diğer yardımcı eleman olan niteleyen (qualifier) iddianın gücünü, kesinliğini ifade eder (Toulmin, 1958, s.101). Genellikle, nadiren, kesinlikle ve sıklıkla gibi gelimeler niteleyene örnek olabilir. Son olarak çürüten (rebuttal) elemanı, gerekçe sunulmuş iddiayı yenme/çürütme kapasitesine sahip istisnai durumlar olarak tanımlanmıştır (Toulmin, 1958, s.101). Başka bir deyişle iddianın geçerli olmadığı koşullar çürüten olabilir. Toulmin'in ileri sürdüğü argümantasyon modelinde argüman elemanlarının arasındaki ilişki Toulmin tarafından Şekil 2.1'deki gibi gösterilmiştir.



Şekil 2.1 Toulmin'in (1958) argüman şeması (s. 97)

Toulmin modeli alandan bağımsız olarak ileri sürüldüğünden alanyazında ekonomi (Cho & Jonassen, 2002), fen eğitimi (Erduran, Simon, & Osborne, 2004; Jiménez-Aleixandre, Rodriguez & Duschl, 2000; Osborne , Erduran & Simon, 2004; Walker & Sampson, 2013) ve matematik (Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Krummheuer, 1995) gibi birçok alanda kullanılmıştır.

2.2 Argümantasyonda öğretmen sorumluluklarına dair çalışmalar

Sosyal öğrenme ortamlarında paylaşılan (taken-as-shared) bilginin matematiksel gerekçelendirmede önemi büyüktür. Paylaşılan bilgi sınıftaki bütün öğrenciler tarafından doğru olarak kabul edildiğinden tartışmanın ileri safhalarında tekrar sorgulanmaz ve doğru kabul edilerek tartışmaya devam edilir. Bu nedenle paylaşılan bilgi gerekçe olduğunda bazı argümanlarda gerekçe elemanı bulunmayabilir. Dolayısıyla argümantasyon analizinde paylaşılan (taken-as-shared) bilgi dikkat edilmesi gereken bir husus olarak karşımıza çıkmaktadır. Yackel ve Cobb (1996) muhakeme ile paylaşılan matematiksel bilgi arasındaki ilişkiye dikkat çeken önde gelen araştırmacılardır. Argümantasyon ortamında sosyal etkileşimi sağlayacak olan ve argümantasyonu yönetecek olan kişi öğretmendir. Dolayısıyla öğretmenlerin bu konularda kendilerini yetiştirmiş olması önemli bir durum olarak karşımıza çıkmaktadır.

Argümantasyonun uygulanabilmesi için öğretmenler, öğrencilerin fikirlerini söyleme konusunda tereddüte düşmeyeceği bir sosyal ortam sağlamalıdırlar. Yani öğrenciler fikrini söylediğinde diğer arkadaşları veya öğretmeni tarafından yargılanmayacağından ve aşağılanmayacağından emin olmalıdırlar (Shu-Sheng, Mintzes, 2010). Bunun yanında, Yackel (2002) öğretmenlerin toplu argümantasyonu başlatma, öğrenciler arasındaki etkileşimi teşvik etme, argüman elemanlarında eksik kalan kısımların farkedilmesini sağlama gibi konularda kendilerini yetiştirmeleri gerektiğini savunmaktadır. Diğer yandan, argümantasyon yöntemini uygulayan öğretmenin sahip olması gereken iki önemli özellik; öğrencileri gerekçe sunmaya teşvik etme ve öğrencilerin derinlemesine düşünmesini sağlayan sorular sorma olarak ileri sürülmüştür (Cross, 2009). Bunların dışında, öğretmenin yargılayıcı yorumlardan kaçınması ve sınıftaki akranların da yargılayıcı konuşmasını engellemesi, öğrencilerin varsayımlarını uygun matematiksel ifadelerle savunmalarını sağlayacaktır (Hunter, 2014).

2.3 Toulmin'in argümantasyon modelinin matematik eğitimi araştırmalarında kullanımı

Toulmin'in (1958) argümanyasyon modeli ilk olarak matematiksel olmayan tartışmalarda kullanılmıştır. Daha sonra Toulmin, Richard ve Allan (1979) ile birlikte bu modeli, Theaetetus'un 'Kesinlikle 5 adet platonik katı cisim bulunmaktadır' iddiasını ispat etmek için kullanmıştır. Matematik eğitiminde ise ilk olarak Krummheuer (1995) tarafından toplu (collective) argümantasyon analizinde kullanılan bu model, daha sonra matematik araştırmacıları arasında ilgi görmeye başlamıştır. Toulmin'in argümantasyon modeli matematikte argüman yapılarını ve ispat yapılarını incelemede (Giannakoulias, Mastorides, Potari, & Zachariades, 2010; Krummheuer, 2007; Pedemonte, 2007; Pedemonte & Reid, 2011), matematik eğitiminde sınıf tartışmalarının analizinde (Forman et al., 1998; Krummheuer, 1995, 2007; Moore-Russo, Conner, & Rugg, 2011; Pedemonte & Reid, 2011; Yackel, 2001), öğrencilerle yapılan görüşme verilerinin analizinde (Nardi, Biza, & Zachariades, 2012; Steele, 2005) ve matematiksel argümanların kalitesinin incelenmesinde (Inglis & Mejia-Ramos, 2008; Pedemonte, 2007) kullanılmıştır.

Alanyazıdaki çalışmalar incelendiğinde argümantasyona farklı açılardan odaklanıldığı görülmektedir. Bazı araştırmacılar muhakeme çeşitlerini (Tümevarım, tümdengelim vb.) inceleyerek argümantasyon alanyazınına katkıda bulunmuşlardır (Conner et al. 2014b; Pease & Aberdein, 2011; Pierce, 1960). Örneğin, Conner ve arkadaşları (2014b) Toulmin'in modelini Pierce'nin (1960) muhakeme sınıflandırması ile birleştirerek argümanları muhakeme çeşidine göre sınıflandırmışlardır. Bu sınıflandırmada *deductive, inductive, abductive* ve *reasoning by analogy* olmak üzere dört çeşit argüman ileri sürmüşlerdir. Argümantasyonu farklı açıdan inceleyen kimi araştırmacılar da argümanı bir bütün olarak ele alıp argüman çeşitlerini incelemişlerdir (Aberdein, 2005; Viholainen, 2011). Örneğin, Viholainen (2011) çalışmasında resmi (formal) ve resmi olmayan (informal) argüman çeşitlerinden söz etmiştir. Gerekçe olarak *tanım, aksiyom* ve *teoremlerin* sunulduğu argümanlar resmi argüman olarak tanımlanmıştır (Viholainen, 2011). Somut matematiksel yorumların ve matematiksel kavramların gerekçe olarak sunulduğu argümanlar ise resmi olmayan argüman olarak tanımlanmıştır (Viholainen, 2011). Bunların dışında argümanın gerekçe (warrant) elemanına odaklanarak gerekçe çeşitlerini inceleyen çalışmalar da mevcuttur (Inglis ve diğerleri, 2007; Nardi, Biza, & Zachariades, 2012; Walter & Barros, 2011). Örneğin, Knipping (2008), ispat konusunda yaptığı çalışmasında, argüman yapılarını gerekçelerine odaklanarak incelemiştir. Knipping'in (2008) ileri sürdüğü sınıflandırmada argümantasyon, gerekçelerin yapısına göre *kavramsal* ve *görsel* olmak üzere iki çeşittir. Görsel argümantasyon ise *ampirik-görsel* ve *kavramsal-görsel* olmak üzere ikiye ayrılmaktadır.

Alanyazın incelendiğinde Toulmin modelini kullanan araştırmacılardan bazılarının bu modeli kendi çalışmasına göre adapte ettiği görülmektedir. Daha açık söylemek gerekirse, kimi araştırmacılar modele ek elemanlar eklemiş (Conner, Singletary, Smith, Wagner & Francisco, 2014a; Prusak, Hershkowitz, & Schwarz, 2012; Voss, 2005; Walter & Johnson, 2007), kimileri modeli başka modellerle birleştirerek kullanmışlardır (Conner, Singletary, Smith, Wagner & Francisco, 2014b). Yapılan her bir argümantasyon çalışması kendine özgü tartışma süreçleri içereceğinden veri analizinde araştırmacıların bu tür düzenlemeler yapması beklenen bir durumdur.

2.4 İspat ve argümantasyon çalışmaları

Alanyazında argümantasyon ve ispat arasındaki ilişki araştırmacılar arasında hala tartışma konusudur. İspat genel anlamda tümdengelime dayanmakta iken argümantasyonda her türlü muhakemenin (tümdengelim, tümevarım, abdüksiyon vb.) görülmesi araştırmacıları ikiye bölmüştür. Kimi araştırmacılar argümantasyon ile ispat arasında bir süreklilik (continuity) olduğunu (Boero, 2007; Douek, 1999; Garuti, Boero, Lemut, & Mariotti, 1996; Mariotti, Bartolini Bussi, Boero, Ferri, & Garuti, 1997; Raman, 2002) savunurken, kimileri bu kavramların ikiye ayrıldığını (Balacheff, 1991; Douek, 1999; Mariotti, 2006; Pedemonte, 2007) savunmaktadır. Örneğin Balacheff (1991) argümantasyon da odağın sınıftaki akranları ikna etmek olduğunu, ispatta ise odağın söylenen ifadenin doğruluğu olduğunu, dolayısıyla bu iki kavram arasında sosyal açıdan farklılık olduğunu savunmaktadır. Diğer yandan birçok araştırmacı argümantasyon ve ispat arasında yapısal süreklilik (structural continuity) sağlanabileceğini savunmaktadırlar (Boero, 2007; Garuti, Boero, Lemut, & Mariotti, 1996; Mariotti, Bartolini Bussi, Boero, Ferri, & Garuti, 1997; Raman, 2002). Örneğin, Boero (2010) eğer argümantasyon ve ispat arasındaki çıkarımlar aynı mantık yapısında (tümevarım, tümdengelim) olurlarsa yapısal sürekliliğin sağlanacağını savunmuştur. Bu demektir ki, argümantasyon süreci boyunca yapılan muhakeme ile tartışma sonunda ortaya ürün olarak sunulan ispat, muhakeme çeşidi açısından birbiriyle uyumlu ise argümantasyon ve ispat arasındaki yapısal süreklilik sağlanmıştır.

2.5 Teknoloji ve argümantasyon üzerine yapılan çalışmalar

Alanyazında dinamik geometri programlarının geometri çalışmalarında kullanılmasının birçok faydasından bahsedilmiştir. Bunlar arasında dinamik geometri programlarının kullanıcıya kesin çizimleri yapabilme fırsatını vermesi ve böylelikle kullanıcının aradaki ilişkileri daha net görmesini sağlaması (Vincent, 2002), soyut geometri kavramlarını anlamlandırmaya yardımcı olması (Hollebrands, Laborde, & Sträβer, 2008), kullanıcıların keşfetmelerine, varsayımlarda bulunmalarına olanak sağlaması, geometri başarısını desteklemesi (Hollebrands, 2007; Laborde, Kynigos, Hollebrands, & Sträβer, 2006; NCTM, 2000), motivasyonu arttırması (Lampert, 1993; Ruthven, Hennessy, & Deaney, 2005) ve kullanıcılar arasındaki etkileşimi desteklemesi (Vincent, Chick & McCrae, 2005) olarak sıralanabilir.

Alanyazında, teknoloji ortamında gerçekleştirilen argümantasyon çalışmalarına da rastlanmaktadır (Hewit, 2010; Hollebrands, Conner & Smith, 2010; Inglis, Mejia-Ramos & Simpson, 2007; Prusak, Hershkowitz & Schwarz, 2012). Örneğin, Prusak, Hershkowitz ve Schwarz (2012) iki öğretmen adayına fikir ayrılığı yaşayacakları bir problem durumu verip onlara durumu çözmeleri için teknolojik destek sağlayarak ürettikleri argümanları incelemişlerdir. Daha sonra Toulmin modelini adapte ederek akran tartışması analizi için bir model geliştirmişlerdir. Bir diğer teknoloji destekli çalışmada ise Cabri dinamik geometri yazılımı ispat sürecinin değerlendirilmesinde kullanılmıştır (Baccaglini-Frank & Mariotti, 2009). Araştırmacılar, katılımcıların sürükleme aracını kullanma sebeplerine odaklanmış böylelikle sürükleme şemalarını belirlemişlerdir. Ayrıca bu araştırmacılar katılımcıların dinamik şekiller ile ileri sürdükleri varsayımların kağıt kalem ortamında ileri sürdükleri varsayımlardan daha gelişmiş olduğu sonucuna varmışlardır (Baccaglini-Frank & Mariotti, 2009). Bir diğer çalışmada ise Mariotti (2006) katılımcılara teknoloji ortamında açık uçlu problem çözdürmüş ve geliştirdikleri varsayımları incelemiştir. Çalışma sonucunda Mariotti (2006) dinamik geometri yazılımının muhakeme ve ispat süreçlerine olumu katkıda bulunduğu sonucuna varmıştır.

3. YÖNTEM

3.1. Araştırmanın Deseni

Ortaokul matematik öğretmen adaylarının teknoloji ortamında veya kağıt-kalem ile geometri soruları çözerken kullandıkları argümantasyon yapılarının incelendiği bu araştırmada nitel araştırma yöntemilerinden durum çalışması deseni kullanılmıştır. Alanyazına bakıldığında Yin'in (2003) dört durum çalışmasından bahsettiği görülmektedir. Bunlar bütüncül tek durum deseni, bütüncül çoklu durum deseni, iç içe geçmiş tek durum deseni ve iç içe geçmiş çoklu durum desenidir. Bu çalışmada durum çalışması türlerinden *bütüncül çoklu durum deseni* kullanılmıştır.

3.2 Pilot Çalışma

Uygulamanın nasıl yapılacağını netleştirmek için öncelikle 2013 yılı bahar döneminde ortaokul matematik öğretmenliği programında öğrenim gören dokuz gönüllü öğretmen adayı ile pilot çalışma yapılmıştır. Bu çalışmanın amacı çözülecek geometri problemlerinin argümantasyon yöntemine uygunluğunu kontrol etmek, geometri problemlerinin çözülebileceği gerekli süreleri belirlemek, her bir uygulamada kaç geometri problemin çözülebileceğine karar vermek, uygulamada kullanılan materyallerin açık ve anlaşılır olup olmadığını incelemek ve her bir grupta olması gereken katılımcı sayısını belirlemektir.

Katılımcılar elverişli örnekleme yöntemiyle seçilmiş ve GeoGebra bilen beş kişi GeoGebra grubuna, diğer dört kişi Kağıt-Kalem grubuna atanmıştır. Katılımcılar ikili üçlü çalışma gruplarına bölünmüştür. GeoGebra grubunda her bir çalışma grubu bir çalışma kağıdı ve bir bilgisayar ile soruları çözmüşlerdir. Kağıt-Kalem grubunda ise yine her bir çalışma grubu bir çalışma kağıdı ve çizim araçları (cetvei, pergel ve gönye) ile soruları çözmüşlerdir. Pilot çalışmada katılımcılar üç uygulamada üçgenler, dörtgenler ve çemberler konularından seçilerek hazırlanmış 10 geometri problemi çözmüşlerdir. Her bir uygulama sonunda seçilen bir çalışma grubu ile görüşme yapılmıştır, diğer katılımcılardan ise yansıtıcı düşünme yazısı yazmaları istenmiştir. Uygulamalar ve görüşmeler kamera ve ses kayıt cihazları ile kaydedilmiştir.

Araştırmacı pilot çalışma sonucunda ana uygulamanın nasıl yapılacağı konusunda çıkarımlarda bulunmuştur. Öncelikle bir uygulamada katılımcılara iki geometri problemi çözdürülmesi uygun görülmüştür çünkü bir uygulamada daha fazla problem çözdürülmesi katılımcıları yormuş ve argüman üretme isteklerini azaltmıştır. Uygulamadaki geometri problemlerinin argümantasyon yöntemine uygunluğu, tartışmaya ne kadar açık olduğu, ne kadar farklı çözüm yolunun olduğu incelenerek iki üçgen ve iki çember geometri problemi ana uygulama için seçilmiştir. Küçük çalışma gruplarının kaçar kişiden oluşması gerektiğini belirlemek için ikili ve üçlü grupların tartışmaları incelenmiştir. Üç kişiden oluşan grupta katılımcılardan bazılarının tartışmaya dahil olmadığı, dinleyici konumuna geçtiği gözlemlenmiştir. Bu nedenle küçük çalışma gruplarının ikişer kişi ile oluşturulmasına karar verilmiştir. Katılımcıların tartışmalarını ve etkileşimini desteklemek için her bir çalışma grubuna bir bilgisayar, bir çalışma kağıdı, birer tane çizim aracı verilmesine karar verilmiştir. Aksi takdirde bireysel çalıştıkları gözlemlenmiştir. Uygulamadan bir gün sonra yapılan görüşmelerde katılımcıların bazı detayları unuttuğu farkedilmiştir. Bu nedenle, katılımcılara görüşme öncesinde sınıftaki tartışma video kaydına hızlıca göz gezdirme fırsatı verilmesine karar verilmiştir.

3.3 Ana Uygulama

Bu bölümde ana uygulamada yer alan katılımcılar, veri toplama araçları ve veri analizi hakkında bilgi verilecektir.

3.3.1 Katılımcılar

Araştırmanın katılımcıları amaca yönelik olarak Ankara'daki bir devlet üniversitesinin eğitim fakültesinin, Ortaokul Matematik Öğretmenliği Programı'nda eğitim gören son sınıf öğrencileri arasından seçilen 16 öğretmen adayıdır. Patton'a (2002) göre zengin bilgi vereceği düşünülen adaylar seçildiğinden dolayı amaçlı örnekleme yöntemi, derinlemesine bilgi sağlamak açısından güçlü bir yöntemdir. Bu çalışmada zengin veri toplamak önemli olduğu için çalışmanın katılımcıları amaçlı örnekleme yöntemi kullanılarak belirlenmiştir. Uygulama katılımcıların ders saatleri dışında ayarlanacak saatlerde uygulanacağı ve haftalık ders programlarına göre boş zaman ayarlanacağı için yalnızca bir devlet üniversitesinden katılımcı seçilmiştir. Böylelikle en azından katılımcıların ders programlarının birbirine yakın olması öngörülmüştür. Uygulama GeoGebra grubu ve Kağıt-Kalem grubu olmak üzere iki grup ile gerçekleştirilmiştir. GeoGebra grubuna 'Dinamik geometri uygulamaları ile geometri keşfetme' dersini almış, GeoGebra programını kullanmayı bilen sekiz öğretmen adayı seçilmiştir. Böylelikle GeoGebra grubundaki katılımcılara programı kullanmayı öğretmeye gerek kalmamıştır. Kağıt-Kalem grubuna ise GeoGebra programını bilmeyen gönüllü adaylar arasından sekiz öğretmen adayı seçilmiştir.

3.3.2 Veri Toplama

Ana uygulama 2013-2014 eğitim öğretim yılının sonbahar döneminde yapılmıştır. Araştırmanın amacına uygun olarak zengin veri sağlamak için 'çoklu veri toplama araçları' kullanılmıştır (Creswell, 2007). Uygulamaların ve görüşmelerin video ve ses kayıtları, yansıtıcı düşünme yazısına ek olarak katılımcıların not aldıkları çalışma kağıtları ve her türlü çizimleri veri toplama kaynağı olarak kullanılmıştır. Veri toplama araçları ve süreci ilgili detaylı bilgi takip eden bölümlerde verilmiştir.

3.3.2.1 Veri Toplama Araçları

3.3.2.2 Geometri problemleri

Bu çalışmada dört geometri problemi kullanılmıştır. Bu problemlerin gayret gerektiren, birden fazla çözüm yolu olan, hem GeoGebra ile hem kağıt kalemle çözülebilen, argüman üretilebilecek problemler olmasına çalışılmıştır. Geometri problemlerinden birisi Ceylan'ın (2012) yüksek lisans tezinde uyguladığı bir üçgen problemidir. Bir diğeri Iranzo-Domènech'in (2009) doktora tezinde kullandığı bir üçgen problemidir. Üçüncü ve dördüncü geometri problemleri ise Posamentier ve Salkind'in (1988) 'Challenging problems in Geometry' isimli kitabından seçilmiş çember problemleridir.

3.3.2.3 Görüşmeler

Yarı-yapılandırılmış görüşme soruları her bir uygulama sonrasında araştırmacı tarafından uygulamanın videosu izlenerek hazırlanmıştır. Uygulama süresince katılımcıların argüman öğelerinin hepsini her bir argümanda belirtmedikleri görülmüştür. Belirtilmeyen öğelerin (örneğin gerekçe) içeriğini sorgulamak görüşmeler yapılmıştır. Araştırmacı GeoGebra ve Kağıt-Kalem grubundan birer çalışma grubu seçmiş, onların kendi aralarındaki ikili tartışmasını ve sınıf tartışmasını izleyerek görüşme soruları hazırlamıştır. Bu aşamada katılımcıların argümanları belirlenerek argümanlarındaki eksik öğeler (iddia, veri, gerekçe) not alınmış ve katılımcıların bu eksik öğeler hakkındaki düşüncelerini açığa çıkaracak sorular hazırlanmıştır. Ayrıca görüşme yapılan katılımcılara uygulamada bahsi geçen iddialardan verilip bu iddiaları savunmaları, eğer savunmuyorlarsa çürütmeleri istenmiştir.

3.3.2.4 Yazılı Kaynaklar

Katılımcıların geometri problem çözümleri ve mülâkatlar sırasında kullandıkları her türlü yazılı kağıt veri toplama aracı olarak kullanılmıştır.

Yansıtıcı düşünme yazısı soruları, sınıf tartışmasında yer alan argümanlar ile ilgili görüşme soruları arasından seçilerek düzenlenmiştir. Bu sorular bütün katılımcılara gönderilmiştir. Dolayısıyla görüşme için seçilen çalışma gruplarının ikili çalışmalarına özgü sorular dışındaki sorular düzenlenerek hazırlanmıştır. Araştırmacı, katılımcılara görüşme sorularını gönderirken üzerinde kendi çözümleri olan çalışma kağıtlarının fotokopilerini de göndermiştir. Katılımcılar yansıtıcı düşünme yazılarını e-posta ile araştırmacıya göndermişlerdir.

3.3.3 Veri Toplama Süreci

Araştırmacı her bir grup ile (GeoGebra ve Kağıt-Kalem grubu) ikişer uygulama gerçekleştirmiştir. İlk uygulama yaklaşık üç saat sürmüş ve üçgenlerle ilgili iki geometri problemi çözülmüştür. Katılımcılar ikişerli çalışma gruplarına ayrılmış, dolayısıyla her bir grupta iki kişiden oluşan dört çalışma grubu olacak şekilde organize edilmiştir. Bu çalışma gruplarının her birine bir kamera ve bir ses kayıt cihazı ayarlanmış, sınıf tartışmasını kaydetmek üzere de tahtayı gören bir kamera ayarlanmıştır. İlk önce birinci geometri probleminin çalışma kağıdı çalışma gruplarına birer tane verilmiş ve çalışma gruplarının soruyu çözmesi için yeteri kadar süre verilmiştir. Katılımcılar ikişerli olarak yeteri kadar tartıştıktan sonra sınıf tartışmasına geçilmiştir. Sınıf tartışmasında önce gönüllü grupların sonra diğer grupların çözümleri sınıfça araştırmacının yönlendirmeleriyle argümantasyon yöntemlerine uygun olarak gerçekleştirilmiştir. Araştırmacı argümantasyonu yönetirken farklı çözüm yollarını ve öğrencilerin gerekçelerini sorgulayan sorular sormuş, katılımcıların sürekli aktif olmalarını sağlamıştır. Tartışma bittikten sonra aynı şekilde ikinci geometri sorusu da çözülmüştür. Uygulama yaklaşık üç saat sürmüştür. Uygulama sonrası araştırmacı aynı gün seçtiği bir çalışma grubunun bütün video görüntülerini ve sınıf tartışmalarının görüntülerini izleyerek görüşme soruları hazırlamıştır. Ertesi gün seçilen çalışma grubu ile görüşme yapılarak bu görüşme yine video ile kaydedilmiştir. Görüşme öncesi katılımcıların sınıftaki uygulama videosunu hızlıca gözden geçirmeleri ve yapılanları hatırlamaları sağlanmıştır.

Üçgen sorularının çözüldüğü ilk uygulamada gerçekleştirilen aşamalar aynı şekilde ikinci uygulamada, iki çember sorusu için de gerçekleştirilmiştir. Araştırmacı uygulamalar bittikten sonra yansıtıcı düşünme yazısı sorularını hazırlamış ve bütün katılımcılara online olarak göndermiştir. Katılımcılar cevaplarını e-posta yoluyla en kısa sürede araştırmacıya göndermişlerdir.

3.3.4 Verilerin analizi

Bu çalışmada da veri analizi, veri toplama süreciyle eş zamanlı olarak gelişmiştir. Bunun için Creswell'in (2009) ileri sürdüğü data analiz süreçleri takip edilmiştir. Öncelikle ses ve görüntü kayıtları alınan bütün uygulamalar ve görüşmeler yazıya aktarılmış ve diğer veri kaynaklarıyla birlikte bilgisayar dosyaları olarak organize edilmiştir. Daha sonra mülâkatların yazıya aktarılan kayıtları, çalışma kağıtları, katılımcıların yazılı kayıtları tekrar tekrar okunup incelenerek büyük boyutlardaki veriyi daha anlamlı hale getirebilmek adına kodlamalar yapılmıştır. Alanyazındaki çalışmaların veri analizleri incelenerek veriler için uygun analizler belirlenmiştir. Bu analizler yapılırken gerektiğinde kodlayıcı tutarlılığı (intercoder reliability) uygulanmış ve verilerin daha sağlıklı analiz edilmesi için çalışılmıştır. Daha sonra elde edilen bulgular yorumlanarak anlamlandırılmış genel bir çerçeve sunulmuştur.

4. BULGULAR

Bu araştırmanın bulguları ortaokul matematik öğretmen adaylarının argümantasyon yapıları, lokal argüman tipleri ve lokal argümantasyon çeşitleri olmak üzere üç bölümde açıklanmıştır.

4.1 Ortaokul matematik öğretmen adaylarının argümantasyon yapıları

Bu analiz için Knipping'in (2008) kendi çalışmasında kullandığı global argüman yapıları analizi adapte edilerek kullanılmıştır. Knipping (2008) ve Reid ve Knipping (2010) dört çeşit global argümantasyon yapısının varlığından söz etmişlerdir: Kaynak yapı (Source-structure), Rezervuar yapı (Reservoir-structure), Spiral yapı (Spiral-structure) ve Toplanma yapıdır (Gathering-structure). Bu çalışmada Knipping'in (2008) geliştirdiği çizim şekli kullanılarak global argümantasyon şemaları her bir geometri problem çözümü için çizilmiştir. Daha sonra bu çizimler bahsi geçen global argümantasyon yapıları ile karşılaştırılmış, Knipping'in (2008) Toplanma-yapısı hariç diğer yapı çeşitlerine rastlanmış, bunun yanında bazı global argümantasyon yapılarının bu sınıflandırmaya uymadığı, farklı şekil oluşturduğu görülmüştür. Bunlar Çizgi/Hat-yapı (Line-structure) ve Bağımsız argümanlar-yapısıdır (Independent arguments-structure). Bu argümanların geometri problemleri içinde dağılımı ve kaç kez gözlemlendiği aşağıdaki tabloda gösterilmiştir.

Tablo 4.1 Çalışma gruplarında gözlenen global argümantasyon yapıları

	GeoGebra Grubu	Kağıt-Kalem Grubu
Geometri Problemi 1	1 Rezervuar-Yapı 1 Spiral-Yapı	1 Rezervuar-Yapı 1 Çizgi/Hat-Yapı 8 Bağımsız Argümanlar-Yapısı
Geometri Problemi 2	3 Spiral-Yapı 2 Bağımsız Argümanlar-Yapısı	2 Spiral-Yapı 2 Bağımsız Argümanlar-Yapısı 1 Kaynak-Yapı
Geometri Problemi 3	1 Rezervuar-Yapı 1 Çizgi/Hat-Yapı 4 Bağımsız Argümanlar-Yapısı	1 Rezervuar-Yapı 1 Çizgi/Hat-Yapı 6 Bağımsız Argümanlar-Yapısı 1 Kaynak-Yapı
Geometri Problemi 4	2 Spiral-Yapı 5 Bağımsız Argümanlar-Yapısı 1 Kaynak-Yapı 1 Rezervuar-Yapı	3 Spiral-Yapı 5 Bağımsız Argümanlar-Yapısı

Tabloda görüldüğü gibi Knipping'in (2008) sınıflandırmasından kaynak-yapı tüm uygulamada üç kez, rezervuar-yapı beş kez, spiral-yapı on bir kez gözlemlenmiştir. Bunlardan iki grupta da en çok gözlemlenen spiral-yapı olmuştur. Bu çalışmada ortaya çıkan çizgi/hat-yapı üç kez, bağımsız argümanlar-yapısı ise otuz iki kez gözlemlenmiştir.

Global argümantasyon yapıları geometri problemi bazında incelendiğinde GeoGebra grubu ile Kağıt-Kalem grubu arasında benzerlik ve farklılıkların olduğu görülmektedir. Birinci geometri probleminde iki grupta da birer rezervuar-yapı bulunmaktadır. Bunun yanında GeoGebra grubunda bir spiral-yapı var iken Kağıt-Kalem grubunda bir çizgi/hat-yapı ve sekiz bağımsız argümanlar-yapısı bulunmaktadır. İkinci geometri problemi çözümünde iki grupta da spiral-yapı ve bağımsız argümanlar yapısı bulunmakta iken Kağıt-Kalem grubunda fazladan bir kaynak-yapı argümantasyon görülmüştür. Üçüncü geometri probleminin çözümü incelendiğinde iki grupta yine benzer global argümantasyonlar (rezervuar-yapı, çizgi/hat-yapı ve bağımsız argümanlar-yapısı) gözlemlenmiştir. Yine Kağıt-Kalem grubunda fazladan bir kaynak-yapı argümantasyon gözlemlenmiştir. Son geometri probleminin çözümünde GeoGebra grubunda global argümantasyon yapısı açısından çeşitlilik görülürken Kağıt-Kalem grubunda sadece spiral-yapı ve bağımsız argümanlar-yapısı gözlemlenmiştir.

4.2 Ortaokul matematik öğretmen adaylarının lokal argüman tipleri

Bu analizde global argümantasyon yapılarının içindeki lokal argümanlarda (iddia, veri, gerekçe içeren argümanlar) Toulmin'in (1958) argüman elemanlarının (iddia, veri, gerekçe) sınıfta söyleniş sırası yani argüman elemanlarının akış sırası incelenmiştir. Bunun için araştırmacı verileri tekrar tekrar okuyarak herbir argüman içinde hangi elemanın daha önce söylendiğini değerlendirmiş her bir elemana numara vermiştir (iddia ilk önce söylenmişse iddiaya '1', sonra gerekçe söylenmişse gerekçeye '2', en son verilerden bahsedilmişse verilere '3' numarası verilmiştir). Analiz sonucunda dokuz adet lokal argüman tipi ortaya çıkmıştır. Bu argüman tipleri isimlendirilirken argüman elemanlarının İngilizce isimlerinin baş harfleri, söyleniş sırasına göre yazılmıştır. Örneğin veri için 'data' kelimesinin baş harfi olan 'D', iddia için 'claim' kelimesinin baş harfi olan 'C' ve gerekçe için 'warrant' kelimesinin baş harfi olan 'W' kullanılmıştır. Yani önce gerekçe, sonra veri, daha sonra iddia elemanlarının söylendiği bir argüman 'WDC' lokal argüman tipi olarak kodlanmıştır. Eğer bir argüman elemanı ifade edilmemişse onun yeri boş bırakılmıştır (Ör:CD). Tüm argümanlar kodlandıktan sonra belirlenen lokal argüman tipilerinin her bir grupta kaçar tane olduğu incelenmiş, aşağıdaki tablo oluşturulmuştur.

Tablo 4.2 GeoGebra ve Kağıt-Kalem gruplarındaki argümanların lokal argüman tiplerine göre dağılımı

	GeoGebra Grubu	**Kağıt-Kalem Grubu**
1. DCW	**11**	**10**
2. DWC	**29**	**38**
3. CDW	1	4
4. WDC	2	2
5. CD	1	2
6. DC	2	3
7. CW	**10**	**8**
8. WC	4	3
9. C	3	3

Tablodaki verilere göre ortaokul öğretmen adaylarının en çok kullandıkları lokal argüman tiplerinin sırasıyla DWC, DCW and CW dir. Lokal argüman tipleri grup bazında karşılaştırıldığında iki grupta da benzer şekilde dağıldıkları sonucuna varılmıştır. Araştırmacı lokal argüman tiplerinin geometri problemleri arasındaki dağılımını da incelemiştir.

Tablo 4.3 Lokal argüman sayılarının her bir geometri sorusuna göre dağılımı

	GeoGebra				**Kağıt-Kalem**			
	GP 1	**GP 2**	**GP 3**	**GP 4**	**GP 1**	**GP 2**	**GP 3**	**GP 4**
1. DCW	2	1	3	**5**	3	2	2	3
2. DWC	**3**	**10**	**7**	**9**	**8**	**11**	**8**	**11**
3. CDW	0	1	0	0	1	0	1	2
4. WDC	1	1	0	0	2	0	0	0
5. CD	0	0	0	1	0	0	2	0
6. DC	1	0	0	1	2	0	0	1
7. CW	**5**	1	**4**	0	**4**	1	**3**	0
8. WC	3	0	1	0	2	1	0	0
9. C	1	0	0	2	1	0	0	1

Lokal argüman tiplerinin iki grupta da herbir geometri problemine göre dağılımı incelendiğinde, ortaokul matematik öğretmen adaylarının en çok DWC lokal argüman tipini kullandıkları görülmektedir. Bu sonuçlara göre bir grupta GeoGebra dinamik geometri programı kullanılmasına rağmen öğretmen adaylarının argümanlarının lokal argüman tiplerine göre dağılımının benzer olduğu ve iki grupta da en çok DWC lokal argüman tipini kullandığı sonucuna varılmıştır.

Lokal argüman tipleri geometri problemlerinin konularına (üçgenler, çemberler) göre de karşılaştırılmış (Bkz. Tablo 4.4)

Tablo 4.4 Matematik içeriğine göre lokal argümanların argüman tiplerine göre dağılımı

	GeoGebra Grubu		Kağıt-Kalem grubu	
	Üçgenler	**Çemberler**	**Üçgenler**	**Çemberler**
1. *DCW*	**3**	**8**	**5**	**5**
2. *DWC*	**13**	**16**	**19**	**19**
3. *CDW*	1	0	1	3
4. *WDC*	2	0	2	0
5. *CD*	0	1	0	2
6. *DC*	1	1	2	1
7. *CW*	**6**	**4**	**5**	**3**
8. *WC*	3	1	3	0
9. *C*	1	2	2	1

Lokal argüman tiplerinin matematik içeriğine göre dağılımı incelendiğinde GeoGebra ve Kağıt-Kalem gruplarındaki öğretmen adaylarının benzer şekilde argüman ürettikleri görülmüştür. En sık gözlenen lokal argüman tipi üçgen problemlerinde de çember problemlerinde de DWC dir. Daha sonra en sık kullanılan lokal argüman tipinin kimi geometri problemlerinde DCW, kimilerinde CW olduğu görülmektedir. Bunun yanında bir grupta GeoGebra kullanılmasının, lokal argüman tipi dağılımında bir fark yaratmadığı görülmektedir.

4.3 Ortaokul matematik öğretmen adaylarının lokal argümantasyon nitelikleri

Ortaokul matematik öğretmen adaylarının lokal argümantasyonları Knipping'in (2008) geliştirdiği sınıflandırmaya göre incelenmiştir. Bu sınıflandırmada lokal argümanların gerekçe (warrant) kısımları yani sunulan muhakeme incelenmiştir. Bu sınıflandırmada gerekçeler öncelikle kavramsal argümantasyon ve görsel argümantasyon olarak ikiye ayrılmakta; görsel argümantasyon ise ampirik-görsel ve kavramsal-görsel argümantasyon olarak ikiye ayrılmaktadır. Lokal argümantasyonlar öncelikle her bir geometri problemi için ayrı ayrı incelenmiştir. Aşağıdaki tabloda birinci geometri problemi çözümünde oluşturulan lokal argümanların gerekçeleri sınıflandırılmıştır. Örneğin ‘2’ sayısı ikinci lokal argümanın gerekçesidir.

Tablo 4.5 Birinci geometri problem çözümünde oluşturulan argümanların lokal argümantasyon çeşitlerine göre dağılımı

Lokal Argümantasyon		**Görsel Argümantasyon**		**Kavramsal Argümantasyon**	**Gerekçe elemanı olmayan arg.**	**Yeni Durum**
		Ampirik-Görsel	**Kavramsal-Görsel**			
Argüman numaraları	**GeoGebra Grubu**	2, 5, 7,9, 10, 11,12, 13, 14, 16	6	8, 15	1, 3	4
	Kağıt-Kalem Grubu	9, 14, 18, 20, 23, 24	----	5, 6, 7, 8, 11, 12, 13, 15, 16, 17, 19, 21, 22	1, 2, 3, 10, 25	4

Tablo 4.5’te GeoGebra grubundaki katılımcıların çoğunlukla *ampirik-görsel* gerekçeler sundukları görülmektedir. Diğer yandan Kağıt-Kalem grubundaki katılımcıların daha çok *kavramsal argümantasyona* uygun gerekçeler kullandıkları görülmektedir. Bunun yanında iki grupta da gerekçesi olmayan argümanlar bulunmaktadır. Ayrıca iki grupta da 4 numaralı argümanların gerekçeleri bu sınıflandırmaya girmediği için tabloda yeni durum sütununda yer almışlardır. Ortaya çıkan bu yeni durum “*soruda verilenleri gerekçe gösterme*” olarak isimlendirilmiştir.

Tablo 4.6’da ikinci geometri problemi çözümünde elde edilen argümanların gerekçelerinin Knipping’in (2008) sınıflandırmasına göre dağılımı görülmektedir.

Tablo 4.6 İkinci geometri problem çözümünde oluşturulan argümanların lokal argümantasyon çeşitlerine göre dağılımı

Lokal Argümantasyon		**Görsel Argümantasyon**		**Kavramsal Argümantasyon**
		Ampirik-Görsel	**Kavramsal-Görsel**	
Argüman numaraları	**GeoGebra Grubu**	19, 20, 22, 23	17, 24, 28, 29, 30	18, 21, 25, 26, 27
	Kağıt-Kalem Grubu	27, 30, 33, 39	32, 35, 36	26, 28, 29, 31, 34, 37, 38, 40

Tablo 4.6'da görüldüğü gibi GeoGebra grubundaki katılımcılar daha çok görsel argümantasyon kullanmışlardır. Kağıt-Kalem grubundaki katılımcılar ise ikinci geometri probleminin çözümünde hem görsel hem kavramsal argümantasyon kullanmışlardır.

Üçüncü geometri probleminin çözümünde oluşturulan argümanların gerekçeleri Tablo 4.7'deki gibi dağılmaktadır.

Tablo 4.7 Üçüncü geometri problem çözümünde oluşturulan argümanların lokal argümantasyon çeşitlerine göre dağılımı

Lokal Argümantasyon		Görsel Argümantasyon		Kavramsal Argümantasyon		
		Ampirik-Görsel	**Kavramsal-Görsel**		**Gerekçe elemanı olmayan arg.**	**Yeni Durum**
Argüman numaraları	**GeoGebra Grubu**	32, 33, 34, 35, 36, 41, 42, 45	37, 38, 43, 44	39	---	31, 40
	Kağıt-Kalem Grubu	42, 44, 48, 52, 45, 47, 53, 54	41, 43, 50, 46	51, 56	49, 55	---

Tablo 4.7'de görüldüğü gibi iki gruptaki katılımcılar da üçüncü geometri problemini çözerken benzer çeşitte gerekçeler sunmuşlardır. Daha detaylı söylemek gerekirse katılımcılar gerekçe sunarken daha çok görsel argümantasyona başvurmuşlardır. Görsel argümantasyonlardan ise daha çok ampirik-görsel argümantasyonu kullanmışlardır. Kağıt-Kalem grubunda gerekçesi olmayan iki argüman bulunmakta iken, GeoGebra grubunda bu sınıflandırmaya uymayan, *GeoGebra ölçümlerini gerekçe sunma* olarak tanımladığımız (31. Argümanın gerekçesi) ve *GeoGebra eylemlerini gerekçe sunma* (Ör: sürükleme) olarak tanımladığımız (40. argümanın gerekçesi) yeni durumları ile karşılaşılmıştır.

Son olarak dördüncü geometri probleminin çözümünde oluşturulan argümanların gerekçeleri Tablo 4.8'deki gibi dağılmaktadır.

Tablo 4.8 Dördüncü geometri problem çözümünde oluşturulan argümanların lokal argümantasyon çeşitlerine göre dağılımı

Lokal Argümantasyon		**Görsel Argümantasyon**		**Kavramsal Argümantasyon**	
		Ampirik-Görsel	**Kavramsal-Görsel**		**Gerekçe elemanı olmayan argümanlar**
Argüman numaraları	**GeoGebra Grubu**	51, 52, 53, 54, 59, 62	55, 56, 63	47, 48, 57, 58, 60,	46, 49, 50, 61
	Kağıt-Kalem Grubu	57	64, 65, 69, 70, 71, 73, 74	59, 60, 62, 63, 66, 67, 68, 72	58, 61

Tablo 4.8 incelendiğinde GeoGebra grubundaki katılımcıların daha çok görsel argümantasyon kullandıkları, görsel argümantasyon çeşitlerinden ise daha çok ampirik-görsel gerekçe sundukları görülmektedir. Kağıt-Kalem grubunda ise görsel ve kavramsal argümantasyon kullanımının eşit sayıda olduğu görülmektedir. Bu katılımcıların kullandığı görsel argümantasyon çeşidinin ise çoğunlukla kavramsal-görsel olduğu görülmektedir. Son olarak iki grupta da gerekçe elemanı olmayan argümanların olduğu görülmektedir.

Araştırmacı geometri problemlerinin çözümlerini ayrı ayrı inceledikten sonra bu problemlerin konularına göre (Üçgenler, Çemberler) de inceleme yapmıştır. Bunun için birinci ve ikinci geometri problemleri üçgen konusunda oldukları için birlikte ele alınırken, çember konusunda hazırlanan üçüncü ve dördüncü geometri problemleri birlikte ele alınmıştır.

Üçgen problemlerinde GeoGebra grubundaki katılımcıların daha çok *ampirik-görsel argümantasyon* kullandığı sonucuna varılmıştır. Diğer yandan Kağıt-Kalem grubundaki katılımcıların ise daha çok *kavramsal argümantasyonu* tercih ettikleri sonucuna varılmıştır. GeoGebra programı insanları kendi çizimleri üzerinde deneme yanılma yolu ile düşünmeye teşvik etmektedir. Bu nedenle

GeoGebra grubundaki katılımcıların spesifik çizimler üzerinden yorum yapmayı tercih etmeleri beklenen bir sonuçtur. Benzer şekilde Kağıt-Kalem grubundaki katılımcıların kavramsal bilgileri kullanmaya yönelmesi de beklenen bir durumdur. Çünkü katılımcılar kendi çizmlerini kavramsal bilgileriyle desteklemeye ve çıkarım yapmaya çalışmışlardır.

Çember problemlerinde bu durumun GeoGebra grubu için aynı olduğu görülmektedir. Dolayısıyla matematiksel içeriğin GeoGebra grubundaki katılımcılarda farklı lokal argümantasyon tercih etmeye neden olmadığı görülmüştür. Kağıt-Kalem grubunda ise durum farklılık göstermiştir. Katılımcılar çember problemlerinde daha çok görsel argümantasyon kullanmışlardır. Ampirik-görsel ve kavramsal-görsel argümantasyonlardaki argüman sayıları incelendiğinde Kağıt-Kalem grubunda neredeyse eşit sayılarda oldukları görülmektedir. Ayrıca Kağıt-Kalem grubundaki katılımcıların problemlerdeki dinamik soruları GeoGebra programı kullanmadan zihinlerinde doğru bir şekilde hayal edebilmeleri, teorem ve matematiksel kurallar yerine görsel çizimler üzerinden yorum yaparak gerekçe sunmaları, beklenmeyen bir sonuç olarak karşımıza çıkmaktadır.

5. TARTIŞMA

5.1 Argümantasyon yapısı ile ilgili sonuçlar

Matematik öğretmen adaylarının argümantasyon süreçleri her bir geometri problemi için GeoGebra ve Kağıt-Kalem grubu karşılaştırılarak incelenmiştir. Buna göre her bir geometri problemi için grupların argümantasyon yapıları arasında benzerlikler ve farklılıklar bulunmaktadır. Bütün geometri problemleri incelendiğinde genel olarak aşağıdaki sonuçlar elde edilmiştir.

Bu çalışmada global argümantasyon yapılarını incelemek için Knipping (2008)'in sınıflandırması geliştirilerek kullanılmıştır. Knipping'in sınıflandırmasında yer alan yapılardan üç tanesi bu çalışmada gözlemlenmiş (Kaynak-yapı, Rezervuar-yapı ve Spiral-yapı), bunun dışında bu sınıflandırmaya uymayan iki yeni yapı ortaya çıkmıştır. Bunlar Çizgi/Hat yapı ve Bağımsız-

Argümanlar yapısı olarak isimlendirilmiştir. Bu yeni yapılar geometri alanında argümantasyon çalışmalarına katkıda bulunması açısından önemlidir.

Bir diğer önemli sonuç tercih edilen global argüman yapıları ile ilgilidir. Kaynak-yapı, rezervuar-yapı ve spiral-yapı daha karmaşık global argümantasyon yapılarıdır ve bu çalışmada çoğunlukla öğretmenin sorgulatması ve teşviki ile gözlemlenmiştir. Bu yapılar arasında en çok rastlanan ise spiral-yapı olmuştur. Bunun en önemli nedeni araştırmacının sürekli farklı çözüm yolları bulmayı teşvik etmesi ve argümantasyona sevk eden davranışları olabilir. Bu davranışlar öğrenci iletişimini destekleme, gerekçe sunmayı teşvik etme, cevapları sorgulama (Kosko, Rougee, & Herbst, 2014; Vincent, Chick, & McCrae, 2005; Wood, 2003), bütün grupları takip etme, yargılayıcı dönütlerden kaçınma (Cross, 2009) olarak sıralanabilir. Bu bulgu argümantasyon yönteminin uygulanmasında öğretmen faktörünün önemini bir kez daha ortaya çıkarmaktadır (Conner, 2007; Forman ve diğerleri, 1998; Heinze & Reiss, 2007; Hunter, 2007; Yackel & Cobb, 1996).

Önemli bulgulardan bir diğeri katılımcıların argüman yapılarının sayıları karşılaştırıldığında daha çok bağımsız argümanlar-yapısı gibi basit yapıları kullanmalarıdır. Genel olarak bütün geometri problemlerinde iki grupta da (GeoGebra ve Kağıt-Kalem) bağımsız argümanlar-yapısının görülmesi katılımcıların matematiksel muhakeme yapma konusunda zayıf olduklarının göstergesi olabilir. Çünkü katılımcılar argümanları arasındaki ilişkileri kuramamakta ve argüman yapıları basit kalmaktadır. Alanyazında matematiksel muhakeme "matematiksel kavramlar ve ilişkiler üzerinde amaçlı olarak yapılan çıkarım" (Conner ve arkadaşları, 2014b, s. 183) olarak tanımlanmıştır. Matematik öğretmen adaylarının öğrencilerine matematik anlatabilmeleri için mutlaka matematiksel muhakeme becerilerini geliştirmeleri gerekmektedir. Bu çalışmadaki öğretmen adaylarının daha çok bağımsız argümanlar-yapısını kullanmaya eğilimli olmalarının ve dolayısıyla matematiksel muhakemede biraz zayıf olmalarının nedeni eğitim yaşamları boyunca argümantasyon veya tartışma yöntemlerinin uygulandığı derslere tanıdık olmamaları olabilir. Türkiye'deki eğitim sistemi ve uygulanan müfredat incelendiğinde, öğrencileri çoktan seçmeli ulusal sınavlara hazırlamayı amaç edindikleri görülmektedir. Bu sınavlarda kısa sürede doğru cevabı bulmak önemli olduğundan öğrencilerin benzer alıştırma sorularından sürekli çözdüğü, kuralları ezberlediği bir çalışma yöntemine alışık olduğu söylenebilir. Bunun sonucunda öğrenciler ispat, tartışma, argümantasyon gibi yüksek seviyede muhakeme gerektirecek çalışmalara katılmamış ve matematiksel muhakeme becerilerini geliştirmemiş olarak yetişmektedirler. Bu becerilerin geliştirilmesi matematiği kavramsal öğrenme açısından önemli olduğu kadar geleceğin öğrencilerini bu becerilere sahip şekilde yetiştirmek adına önemlidir. Bu nedenle ortaokul matematik öğretim programına öğretmenlerin argümantasyon uygulamaları yapacağı ve matematiksel muhakemelerini geliştirecekleri dersler eklenmesi yerinde olacaktır.

5.2 Lokal argüman tipleri ile ilgili sonuçlar

Ortaokul matematik öğretmen adaylarının geliştirdikleri lokal argümanlar (iddia, veri ve gerekçe içeren argümanlar), argüman elemanlarının argümantasyon sürecinde söylenme sırasına göre analiz edilmiştir. Böylelikle argümanların nasıl bir muhakeme sonucu ortaya çıktığı konusunda ipuçları elde edilmiştir. Bulgular incelendiğinde ortaokul matematik öğretmen adaylarının teknoloji ve kağıt-kalem ortamlarında en sık kullandıkları üç lokal argüman tipi sırasıyla Veri-Gerekçe-İddia (DWC), Veri-İddia-Gerekçe (DCW) ve İddia-Gerekçe (CW) olarak gözlemlenmiştir. Bu aslında beklenen bir sonuçtu çünkü katılımcılar ilk önce ellerindeki verilerden bahsetmiş, daha sonra bir gerekçe sunmuş ve bir iddiada bulunmuşlardır (DWC). Bunun yanında bazen iddia ile gerekçenin yerinin değiştiği (DCW) bazen de verilerden bahsedilmediği durumlarla karşılaşılmıştır. CW lokal argüman tipinde verilerden bahsedilmemesinin nedeni, o verilerin artık sınıfça bilinen paylaşılan bilgi (taken-as-shared) olması olabilir (Simon & Blume, 1996; Yackel & Cobb, 1996; Yackel, Ramussen, & King, 2000). Katılımcıların bazı argümanlarında veriyi ve/veya gerekçeyi paylaşılan bilgi (taken-as-shared) olması nedeniyle sunmadığı gözlemlenmiştir (CD, DC, WC and C). Bu nedenle çğretmenlerin bu argüman elemanlarını sunmayan öğrencileri değerlendirirken bu bilgilerin sınıfça paylaşılan bilgi olup olmadığına dikkat etmesi önerilmektedir.

Çünkü öğrencinin bahsetmemesi o bilgiyi bilmediği anlamına gelmeyebilir. Bazen sınıfça kabul edilmesi sebebiyle söylemeye gerek duymamış olabilirler. Sonuç olarak argümantasyon yöntemini uygulayan öğretmenlerin öğrencilerin gerçekten bilip bilmediğini anlamak için sürekli 'Neden?' sorusuyla sorgulama yapması önerilmektedir (Vincent, 2002; Wood, 2003).

Lokal argümanlar GeoGebra ve Kağıt-Kalem grubu arasında karşılaştırıldığında iki grubunda benzer şekilde lokal argüman sundukları gözlemlenmiştir. Benzer şekilde lokal argümanlar her bir geometri problemi için ayrı ayrı incelendiğinde yine en sık kullanılan lokal argüman tiplerinin DWC, DCW ve CW tipleri olduğu görülmüştür. Lokal argüman tipleri ayrıca üçgen soruları ve çember soruları arasında da karşılaştırılmış, yine iki grubun benzer şekilde lokal argüman tiplerini kullandığı gözlenmiştir. Dolayısıyla teknoloji kullanımının, geometri problemi çeşidinin veya geometri konusunun argüman elemanı söyleniş sırası açısından önemli bir değişikliğe neden olmadığı sonucuna varılabilir. Argüman elemanı sunma şeklinin, alışkın olunan muhakeme şekliyle alakalı olduğu düşünülmektedir. Daha açık söylemek gerekirse, Türkiye'deki öğrenciler eğitim hayatları boyunca argümantasyon gibi soruşturma tabanlı öğrenme yöntemlerine çok alışık olmadıkları için argümanlarını benzer şekilde ifade ederek sunmaktadırlar. Farklı öğrenme ortamlarında bulunmaları (teknoloji, Kağıt-Kalem) onların argüman tiplerinde değişikliğe neden olmamaktadır.

5.3 Lokal argümantasyon çeşitleri ile ilgili sonuçlar

Ortaokul matematik öğretmen adaylarının local argümantasyon çeşitleri, argümanlarında sundukları gerekçelere (warrant) odaklanılarak Knipping'in (2008) sunduğu sınıflandırmaya göre incelenmiştir. Genel olarak GeoGebra grubundaki katılımcıların ampirik-görsel argümantasyon, Kağıt-Kalem grubundaki katılımcıların ise kavramsal argümantasyon kullanmaya meyilli oldukları gözlemlenmiştir. GeoGebra grubunda daha çok ampirik-görsel argümantasyon kullanılmasının nedenlerinden birisi GeoGebra'nın sürükleme yapma olanağı olabilir. Kullanıcılar sürükleme yaparak belirli şekiller üzerinde konuşmakta, o şekiller üzerindeki ölçümler/değerler üzerinden yorum yapmaktadırlar. Sonuç olarak bu yorumları yeterli bir gerekçe olarak kabul etmektedirler. Bu nedenle çıkarımlarını çoğunlukla teorik olarak desteklememektedirler (Chazan, 1993). Bir diğer neden teknoloji ortamında teorik gerekçe sunmaya alışkın olmamaları olabilir (Chazan, 1993; Harel & Sowder, 1998; Hollebrands, Conner, & Smith, 2010). Diğer yandan Kağıt-Kalem grubunda katılımcılar gerekçelerini teorik olarak desteklemek zorunda kaldıkları için teorem, aksiyom, kural ve benzeri ilişkileri gerekçe olarak sunmuşlardır ve kavramsal argümantasyon kullanmışlardır. Alanyazında sürükleme, ölçme gibi ampirik delillerin kullanımının önemli olduğu (Arzarello ve diğerleri, 2002; Chazan, 1993; De Villiers, 2003; Healy & Hoyles, 2001) fakat gerekçelendirme açısından yeterli olmadığı savunulmaktadır (Hoyles & Healy, 1999). Araştırmacılar ampirik delillerin teorik olarak desteklenemesinin gerekli olduğunu savunmaktadırlar (Arzarello ve diğerleri, 2002).

Bu çalışmada Knipping'in (2008) sınıflandırmasına tam olarak uymayan birkaç çeşit gerekçe de sunulmuştur. Bu gerekçeler 'soruda verilenleri gerekçe sunma', 'GeoGebra ölçümlerini gerekçe sunma' ve 'GeoGebra hareketlerini gerekçe sunma' olarak sınıflandırılmıştır. İsimlerinden de anlaşılacağı gibi katılımcılar bazı durumlarda soruda verilenleri, bazı durumlarda ise GeoGebra ile yaptıkları ölçümleri ve sürüklemeleri delil olarak göstermişlerdir. Bu gerekçeler herhangi bir muhakeme veya teorik bilgi ile desteklenmediği için iddialarını savunmaları açısından yeteri kadar güçlü değildir (Chazan, 1993). Sayıları çok az olan bu tip gerekçeler, ortaokul matematik öğretmen adaylarının genel anlamda kullandığı bir lokal argümantasyon çeşidi olarak kabul edilmemektedir. Diğer yandan, bu çalışmanın bulgularının ileride yapılacak çalışmalara kaynak olabileceği düşünülerek bu tip gerekçelerin de mevcudiyetinin dikkate alınması tavsiye edilmektedir.

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EDUCATION

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WORK EXPERIENCE

Year	Place	Enrollment
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FOREIGN LANGUAGES

Advanced English

PUBLICATIONS

Erkek, Ö., & Işıksal-Bostan, M. (2015). The Role of Spatial Anxiety, Geometry Self-Efficacy and Gender in Predicting Geometry Achievement. *Elementary Education Online, 14*(1), 164-180.

Erkek, Ö., Işıksal-Bostan, M., & Çakıroğlu, E. (2017). A Study on Prospective Teachers' Spatial Visualization Ability and Spatial Anxiety. *Kastamonu Eğitim Fakültesi Dergisi, 25*(1), 33-50.

CONFERENCE PARTICIPATION

Erkek, Ö., & Isiksal-Bostan, M. (2015). Does the use of Geogebra advantageous in the process of argumentation? *Paper presented at 9th Congress of European Research in Mathematics Education*, Prague, Czech Republic.

Erkek, O., & Isiksal, M. (2014). Uzamsal görselleştirme testinin Türkçe'ye uyarlanması. Paper presented at the *13. Matematik Sempozyumu: Bilimin Çeliği Matematik*, Karabük, Turkey.

Erkek, O., & Isiksal-Bostan, M. (2013). Scale Development: Looking into Geometry Achievement of 8 Graders from a Different Perspective Paper presented at *Main Conference of European Conference on Educational Research*, İstanbul, Turkey.

Erkek, O., & Isiksal, M. (2013). 8. Sınıf öğrencilerinin iki ve üç boyutlu geometri sorularını çözerken yaptıkları hatalar üzerine bir çalışma. Paper presented at the *12. Matematik Sempozyumu: Toplumda Matematik*, Ankara, Turkey.

Erkek, O., & Isiksal, M. (2012). Exploring the relationships among 8th grade students' geometry achievement, geometry self-efficacy and spatial anxiety. Paper presented at *Emerging Researchers' Conference of European Conference on Educational Research*, Cadiz, Spain.

Erkek, O., Isiksal, M. & Cakiroglu, E. (2011). The relationship between Preservice Teachers' Spatial Anxiety and Geometry Self-Efficacy in terms of Gender and Undergraduate Program. Paper presented at *Emerging Researchers' Conference of European Conference on Educational Research*, Berlin, Germany.

Dursun, O., Isiksal, M. & Cakiroglu, E. (2010). Investigating the relationship between preservice teachers' spatial ability and spatial anxiety. Paper presented at *Emerging Researchers' Conference of European Conference on Educational Research*, Helsinki, Finland.

Dursun, O., Isiksal, M. & Cakiroglu, E. (2010). İlköğretim Öğretmen adaylarının uzamsal yeteneklerinin cinsiyet ve öğretmenlik programlarına göre incelenmesi (Investigating the spatial abilities of elementary school preservice teachers in terms of gender and undergraduate programs). Paper presented at a conference *IX. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi (IX National Science and Mathematics Education Conference)*, İzmir, TURKEY.

PROJECTS

- *Adaptive Curriculum Project (At SEBIT as Educational Designer) in 2008*
- *TTNET Vitamin Project (At SEBIT as Educational Designer) in 2008*
- *ODTÜ-BAP-07.03.2010.108*

  İlköğretim öğretmen adaylarının uzamsal görselleştirme yetenekleri, uzamsal kaygıları ve geometriye yönelik öz-yeterlikleri arasındaki ilişkinin incelenmesi. As a Researcher. Coordinator: Assist. Prof. Dr. Mine Işıksal University Supported Scientific Research, METU-BAP Coordination University: METU.
- *ODTÜ-BAP-05.06.2011.003*

  8. sınıf öğrencilerinin uzamsal kaygıları, geometri başarıları ve geometriye yönelik öz-yeterlikleri arasındaki ilişkinin incelenmesi. As a Researcher. Coordinator: Assoc. Prof. Dr. Mine Işıksal. University Supported Scientific Research, METU-BAP Coordination University: METU.
- *ODTÜ-BAP-05.06.2014.003*

  (Doctoral Dissertation) Dinamik geometri yazılımı ile İlköğretim Matematik Öğretmen Adaylarının Argümantasyon Süreçlerinin İncelenmesi. As a Researcher. Coordinator: Assoc. Prof. Dr. Mine Işıksal. University Supported Scientific Research, METU-BAP Coordination University: METU.

J: TEZ FOTOKOPİSİ İZİN FORMU

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Uygulamalı Matematik Enstitüsü □

Enformatik Enstitüsü □

Deniz Bilimleri Enstitüsü □

YAZARIN

Soyadı : Erkek
Adı : Özlem
Bölümü : İlköğretim

TEZİN ADI (İngilizce) : An analysis of prospective middle school mathematics teachers' argumentation structures in technology and paper-pencil environments.

TEZİN TÜRÜ : Yüksek Lisans □ Doktora ■

1. Tezimin tamamından kaynak gösterilmek şartıyla fotokopi alınabilir. □

2. Tezimin içindekiler sayfası, özet, indeks sayfalarından ve/veya bir bölümünden kaynak gösterilmek şartıyla fotokopi alınabilir. □

3. Tezimden bir bir (1) yıl süreyle fotokopi alınamaz. ■

TEZİN KÜTÜPHANEYE TESLİM TARİHİ: