MULTIOBJECTIVE MISSILE RESCHEDULING PROBLEM

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ABSTRACT

MULTIOBJECTIVE MISSILE RESCHEDULING PROBLEM

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In this thesis, we address dynamic missile allocation problem for a naval task group (TG). We consider rescheduling of surface to air missiles (SAMs) in response to disturbances during the engagement process where a set of SAMs have already been scheduled to a set of attacking anti-ship missiles (ASMs). To produce an updated schedule, we propose mathematical models that consider efficiency of air defense and stability of the schedule.

In the first part of thesis, we present a new biobjective mathematical model that maximizes the probability of no-leaker and minimizes total deviation from the existing schedule. We analyze the computational complexity of the problem and develop exact and heuristic solution procedures. In the second part of thesis, we develop a semi-autonomous decision making framework to update the engagement allocation plan due to rapid decision making requirement of a dynamic air defense scenario. The approach is a based on an artificial neural network (ANN) method that includes an adaptive learning algorithm to structure prior articulated preferences of decision maker (DM). Assuming that the DM's preferences are consistent with a quasi-concave utility function, ANN chooses one of the non-dominated solutions in each rescheduling time point and updates the existing schedule. In the third part of thesis, we consider a different stability criterion that minimizes total number of tracking changeover for SAM systems. We formulate the biobjective model and generate solutions by new exact and heuristic methods.

Keywords: air defense, missile allocation problem, naval task group, rescheduling.

ÖZ

ÇOK AMAÇLI GÜDÜMLÜ MERMİ YENİDEN ÇİZELGELEME PROBLEMİ

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Bu tezde, bir deniz görev grubu (TG) için satıhtan havaya güdümlü mermilerin (SAM) dinamik tahsis problemi çalışılmıştır. Deniz görev grubuna saldıran gemisavar füzelerine (ASM) karşı başlangıçta oluşturulan güdümlü mermi tahsis planının angajmanlar başladıktan sonra ortaya çıkan etmenlerle bozulması sonucu yeniden çizelgeleme durumu ele alınmıştır. Güncellenmiş bir tahsis planı oluşturmak için hava savunmasının etkinliğini ve tahsis planına tutarlılığını dikkate alan iki amaçlı modeller önerilmiştir.

Tezin ilk kısmında, hava tehditlerinin tamamını imha etme olasılığını ençoklayan ve ilk tahsis planı ile yeni tahsis planı arasındaki değişim miktarını enazlayan iki amaçlı model sunulmuştur. Problemin hesaplama karmaşıklıkları analiz edilmiş, kesin ve sezgisel çözüm metotları geliştirilmiştir. Tezin ikinci kısmında, dinamik hava savunma senaryosuna ait hızlı karar verme ihtiyacı nedeniyle angajman tahsis planını güncelleyen yarı otonom bir karar verme sistemi geliştirilmiştir. Geliştirilen yöntem, daha önceden karar vericiden alınmış tercih bilgisini kullanan adaptif öğrenme algoritmasını da içeren yapay sinir ağı temeline dayanmaktadır. Karar vericinin tercihlerinin bir kuvazi konkav değer fonksiyonu ile uyumlu olduğunu varsayarak, yapay sinir ağı her bir yeniden çizelgeleme zamanında etkin çözümü seçerek mevcut planı güncellemektedir. Tezin üçüncü kısmında, satıhtan havaya güdümlü mermi sistemlerinin hedefleme değişiklik sayısını enazlayan farklı bir kararlılık kriteri düşünülmüştür. İki amaçlı model geliştirilmiş, çözümler yeni kesin ve sezgisel metotlarla bulunmuştur.

Anahtar Kelimeler: hava savunma, güdümlü mermi tahsis problemi, deniz görev grubu, yeniden çizelgeleme.

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CHAPTER 1

INTRODUCTION

Air defense flourished in the field of military operations has received great attention along with the advancements in aviation and weapon technology. The appearance of air defense concepts dates to time that man took to the air. As early air defenses are relied on the massive and uncoordinated fire systems, today's current air defense weapon systems possess destructive power, great ranges and high speeds.

There has been considerable interest in naval air defense for many years since controlling seas with effective air defense systems is a critical power for nations. The famous Turkish Admiral Barbaros Hayreddin indicates the importance of naval forces as "Whoever rules the waves rules the world". The prominence of air defense in navies is introduced with the incident destroyed German battleship, Ostfriesland, by United States air forces. The sinking a target battleship by air attacks demonstrated the vulnerability of ships to air attacks (Bolkcom and Pike (1996)). The advances in air bombing technologies during World War II revealed the necessity of enhancement in the defensive capabilities of navies. A major change in air defense history is the development of surface to air missiles (SAMs) with the integration of radar units that detect and track targets. Nations strive to increase the effectiveness of air defense systems with new improvements. Researches on missiles have grown rapidly such as in the development of naval air defense systems that are capable of reaching a target over 200 miles away with high speeds.

The fundamental air threat for Navies are anti-ship missiles (ASMs) that can be subsonic, supersonic or low altitude missiles. The vulnerability of navies by ASMs first appeared with the sinking of Israeli destroyer Eilat by the Sytx ASMs of Egyptian Navy in 1967. Despite ships having powerful defense systems, several successful ASMs attacks have been recorded in history. For instance, in 1987, the USS Stark was attacked by two Exocet ASMs during the Iran–Iraq War and was nearly sunk.

For decades, there has been significant progress in design and technology of ASMs. Modern ASMs are extremely fast, accurate and can be launched at great ranges, from the air, surface or sub-surface. Also, the proliferation of ballistic missiles has become a big problem for naval air defense. Competing technological improvements and formidable offensive capabilities of ASMs led navies to increase the capability of countering every potential threat and develop new tactics continuously.

In a typical naval mission, ships are dispatched to a region and defend all units while remaining in operational area over long period of time. While conducting mission, combatant and auxiliary ships are grouped together to achieve the mission called as task group (TG). Modern and well equipped ships in terms of air defense are essential for success in a naval warfare. Equipping all platforms with expensive air defense systems is not the most cost efficient solution. Moreover, there may be saturating attacks to a ship that onboard defense systems cannot withstand. Coordination of air defense units in a TG in terms of surveillance, identification and allocation of SAMs are crucial in addition to capability and number of air defense systems. Full coordination between ships composing TG enables all defense systems operate as one.

The engagement process, in an air defense operation, begins after an allocation plan is generated. Optimum allocation of SAMs with full coordination in TG is important to utilize full potential in air defense operations since TG may counter a number of and different types of ASMs in a dynamic operational environment. Cooperative air defense ensures collecting data from multiple sensors and combining them to achieve best decision for allocation. The gathering information taken from several units supports decision making process of allocating SAMs against ASMs. A set of SAM rounds are scheduled against ASMs after an air attack is detected. The missile allocation plan is developed according to the initial state of the defensive and attacking units. Future states of the units cannot be known at the time schedule is generated. During the engagement process, states may change quickly. This creates a dynamic environment to be handled and make the initial schedule infeasible or inefficient.

An effective TG in terms of air defense has the capability coordinating the defense quickly to develop a new efficient engagement schedule due to new states with available air defense systems. In real life, initial schedules are rarely carried out as planned and the changes on the engagement schedules are realized by commander's intuitive decisions. TG as a coherent unit should react rapidly and accurately to the dynamic nature of warfare. As reported by Lagrone (2015), a launch failure in Raytheon SM-2 air defense system is occurred onboard USS Sullivans during a training exercise on July 18, 2015. If similar incident becomes during a real life operation, it requires change on the initial schedule since optimal initial schedule is generated as if broken SAM system is available during the engagement process.

Real time change of tactical information within engagement process requires adaption capability for a TG to a new environment. The aim of this study is to develop efficient air defense plans for a TG by dynamic allocation of SAM rounds against ASMs in response to unforeseen events during the engagement process. Our approach provides rescheduling of SAM rounds where a set of them have already been scheduled to a set of attacking ASMs. Hence, defensive units are coordinated with the status of every missile engagement and a new allocation plan is generated according to the continuous change on units within the engagement process.

1.1 Contribution of the Dissertation

The problem that we consider is a specific Weapon Target Allocation (WTA) problem where SAMs and ASMs correspond to weapons and targets respectively. WTA can be considered as a class of resource allocation problem where main objective is the maximization of the total expected damage given to the targets with available number of weapons.

Research to date on WTA problems consider that allocation plan can be implemented as initially planned. The main focus in those problems is on how to establish an initial plan. A number of studies consider building allocation strategy with respect to time stages. But also in those studies the subset of weapons to be fired in the time stages are determined at the beginning of the engagement process. There is no study in the literature that considers generating solutions in the event of change during the engagement process. No tools that update initial engagement schedule have been developed despite military operations are unavoidably subject to unexpected changes. By keeping the initial schedule, the possible changes during the engagement process are ignored.

Naval air defense is a cooperative endeavor of humans and weapon systems. Control and protection of air space from air attacks are realized through a sequential process. The improvement on air defense efficiency depends on the success conducting the activities begin with detection of the air threats and end with their neutralization. Although the capability of each unit improves control and protection of air space, the better implementation of these activities with dynamic allocation provides higher efficiency of air defense.

In this study, we address a missile rescheduling problem for air defense of a TG. To the best of our knowledge, our study is the first attempt that deals with disturbances for the disruption management of air defense operations. With new states of air defense environment, the rescheduling of SAM rounds in response to changes provides a better schedule in terms of efficiency of air defense. But it provides a new schedule that deviates from the initial schedule and changes shoot order of SAM systems. Two measures namely efficiency of the system and the stability of the schedule are the main concerns of rescheduling decision. We consider these measures as the objectives of the problem. While increasing the efficiency of air defense, we take into account the difference between the new schedule and the initial schedule as a second objective. Therefore, formulation of the rescheduling problem is based on two objectives such as efficiency of air defense and stability of the schedule. For stability objective, we consider two main issues. First one is to have a new schedule that increases efficiency of air defense without much deviation from the existing schedule. Second one is to consider the shoot order of SAM systems and change on target tracking in SAM systems while rescheduling SAM rounds and increasing the efficiency of air defense.

The motivation behind this study is to develop an autonomous decision aid that contributes air defense operation of TG that consists of a number of operations that must be performed under time and resource constraints. To cope with complex air threats for a task group, it is required that air defense systems to be efficiently managed. The proposed approach assists the command and control and the decision making process since current operational systems generally provide little support for decision making. The proposed solution procedures are fast enough to provide a timely engagement solution before the next engagement and overcome the inherent complexity of naval command control process and underlying resource allocation problem.

The foremost use of the proposed approach is to embed it as an element of autonomous decision making unit inside the Threat Evaluation and Weapon Assignment (TEWASA) systems working for all the ships in TG. Such an autonomous system could be used in the training, the analysis, and the test areas of the navies. The approach can contribute navies to evaluate their air defense capabilities in performing tasks with changed conditions in every potential theater air defense operations. The efficiency of the air defense systems can be analyzed with real-time status of engagements and real-time changes on units.

1.2 Organization of the Dissertation

The organization of the thesis is as follows.

In Chapter 2, we review the studies relevant to our research. Literature review consists of two parts. Firstly, we give literature on WTA problems. We classify the problems and describe the main features. Secondly, we concentrate on rescheduling problems. Literature review on rescheduling problems includes analyzing the approaches for efficiency and stability objectives.

We present biobjective missile rescheduling problem (BMRP) in Chapter 3. The problem environment and the elements of naval air defense operations are explained. The basic assumptions are defined and formulation of BMRP is given. We show theoretical results about the computational complexity of BMRP. We explain the solution approaches and present the procedures. Lastly, we give the computational results on varying size problems.

In Chapter 4, we propose a dynamic update scheme for engagement allocations in the presence of disturbances. The approach is based on choosing one of the non-dominated solutions in each rescheduling time point from the results of BMRP model. We suggest an artificial neural network approach that includes an adaptive learning algorithm to structure prior articulated preferences of the DM. In addition, we assume that DM utility is consistent with non-decreasing quasi-concave function to eliminate some of the efficient solutions uninteresting to the DM. The solution procedure generates a non-dominated solution most preferred by DM in each rescheduling time point and update the existing schedule.

In Chapter 5, we formulate a different stability criterion that considers shoot sequence of SAM systems. We call this problem as biobjective missile rescheduling problem with sequence-dependent stability measure (BMRP-S). First, an exact solution procedure that solves BMRP-S is developed. In the solution approach, feasible schedules are generated by solving a mathematical model with probability of no-leaker objective. Non-dominated solutions are obtained by revising shoot order of SAM systems in each feasible schedule. To meet the solution time requirement, we next propose a heuristic approach that reallocates SAM rounds in the existing schedule.

Chapter 6 compares BMRP and BMRP-S models. We find all objective function values in each model to show the effect of different stability measures on the performance metrics and outcome of the engagement process.

In Chapter 7, we present our concluding remarks and further research directions.

CHAPTER 2

LITERATURE REVIEW

This chapter consists of literature review related to our research. First, we review the literature on MAP in detail. We classify the models, determine the main approaches and analyze the important features of models. The survey on rescheduling literature includes different research areas such as machine rescheduling, vehicle rescheduling and airline rescheduling. We identify the main concerns on those problems in order to use them in our models.

2.1 Weapon Target Allocation (WTA)

WTA is an optimization problem that attracts researchers over fifty years. WTA problem maximizes total expected damage given to targets or minimizes the expected survival value of targets while satisfying the number of weapons limit. The first known analytical approach is developed by Flood (1957) as a target assignment problem in a nonlinear integer programming formulation. The minimization of the expected value of survival is formulated similar to personnel assignment problem. Manne (1958) suggest a simplification to this nonlinear model to solve it with Lagrange multiplier methods.

Lloyd and Witsenhausen (1986) prove that WTA problem is NP-Hard in the simplest form. The mathematical formulation of WTA problem that is proved as NP-Hard is as follows:

$$min \sum_{j=1}^{|N|} v_j \prod_{i=1}^{|M|} \left(1-p_{ij}\right)^{X_{ij}} \qquad (2.1)$$

subject to

$$\sum_{i=1}^{|T|} X_{ij} = 1 \qquad (2.2)$$

$j = 1......, |N|$ and $X_{ij} = 0,1$.

where number of targets and weapons denoted by $|N|$ and $|M|$ respectively. V_i is the value of target i, p_{ij} is the probability that weapon i destroys target j and X_{ij} equals 1 if weapon i is assigned to target j. Objective function (2.1) minimizes total expected value of surviving targets. The product part of objective function calculates the survival probability of target j. Constraint set (2.2) ensures that each weapon can be allocated only one target.

Missile Allocation Problem (MAP) is a specific version of WTA problem and can be stated as given an existing missiles and a set of targets, what is the optimal allocation of missiles to targets? MAP has many characteristics and inclusion of these characteristics with different assumptions reveals various models in the literature. One of the comprehensive literature survey on WTA is Matlin (1970)'s study. He reviews WTA problems and presents a classification on WTA from the attacker's perspective. He classifies the literature into three major categories such as allocation models, game models and special feature models. Another survey on MAP is Ecker and Burr (1972) study that extensively review the target coverage and missile allocation models. They focus on much more on defensive asset based problems.

We consider the main approaches and important features of models in order to classify the literature. As a first level of classification, we categorize the literature into two groups. The first group is the static version of WTA. In the static version of WTA models, all weapons are allocated simultaneously and the damage assessment is made after the last engagement is accomplished. On the other hand, in the dynamic version of the WTA, the planning decision is based on the outcomes of time stages throughout the engagement time horizon. Thus, in the dynamic version, time stages are defined to perform assessments on the previous stage and to make allocation for the present stage.

2.1.1 Static WTA Models

The research and literature on static WTA mainly covers the optimization of missile allocation using analytical approaches. Numerous articles are proposed in literature due to different features of parameters and scenarios. We classify the static version of WTA models into three categories such as allocation models, game theoretical models and simulation models.

2.1.1.1 Allocation Models

Allocation models build on allocation strategy without knowing the opposite side's course of action. There are two main concerns that are treated as objective functions in allocation models. Those are survivability of the units and the cost of utilized missiles. The survivability objective functions may be the minimization of expected leakage value, maximization of probability for surviving targets, maximization of probability of no-leaker, expected damage value given to enemy forces or maximization of the expected number of unsuccessful threats. The examples of objectives that consider cost are the minimization of the total number of interceptors, minimization of cost or number of missiles utilized or minimization of the total assignment cost. In those models, different aspects with different assumptions inspire the subject of many researches in literature. For instance, some researchers analyze layered defense systems. Different coordination levels such as full and partial coordination capabilities or autonomous systems are covered in the literature. The order of different shooting policies are investigated in order to maximize the value of target killed and to determine the order of shooting. Due to complexity of the problems and nonlinearity in their formulations, various different solution procedures are suggested.

Shumate and Howard (1974) introduce proportional defense model as a defense strategy. They address the problem that defense balances its interceptors to defend targets with different values. The approach ensures that offense have to pay a price to damage the assets and determines which units will be defended. To optimize allocation of interceptors for defense, they suggest a dynamic programming approach. Another study on WTA problem is introduced by Burr et al. (1985). They propose the prim-read defense models for both single target and multi-target case. The problem is formulated in order to minimize the total number of interceptors used in defensive units against the unknown number of sequentially incoming attacking units. They define an upper bound on the maximum expected damage per attacking weapon and assume that defense does not explicitly know the attack size. They formulate multi target version of the problem and solve the model with greedy algorithm.

Soland (1987) considers sequential engagements and arriving simultaneously attacking reentry vehicles. He analyzes the number of remaining interceptors after each wave. He implements stochastic dynamic programming to calculate the expected fraction of target destroyed. For the defense of target i, when the attack size is a_i, the distribution of d_i defenders as possible as is the optimal defense strategy. The theorem is called as quasi-uniform defense. The theorem basically states that if $a_i > 0$ then defense allocates

$$\left\lceil \frac{d_i}{a_i} \right\rceil \text{ interceptors to each of } r_i = \left\lceil d_i - \left\lfloor \frac{d_i}{a_i} \right\rfloor a_i \right\rceil \text{ attackers}$$

and

$$k_i = \left\lfloor \frac{d_i}{a_i} \right\rfloor \text{ interceptors to each of } a_i - r_i \text{ attackers}$$

Layered defense systems have been frequently considered in the WTA literature. Mizrahi (1981) propose an approach to calculate the attrition of targets. He considers a number of sequential attacks to penetrate layers. He examines different cases with respect to number of missiles to be fired and missile allocation situations. The targets are assumed to be identical and the survivability is calculated due to large number of attacking missiles. Nunn et al. (1982) analyze layered defense system with Markov chain formulation. They assume that each layer has its own success probability against attacks and show that the distribution of survivors at each stage is binomial. The transition matrix of penetration in each layer is defined and number of surviving units are approximated according to layer, attacker parameters and given probabilities. Orlin (1987) suggests a missile allocation model for attacking side against a layered regional defense that has perfect defensive weapons. He transforms the formulation into min-cost network flow problem. The objective is the difference between maximization of defensive target destroyed and the cost of utilized offensive weapons. The problem is solved with a specific attrition algorithm and hybrid algorithm. Menq et al. (2007) propose a multi layered defense for ballistic defense system. They use Markov decision model to formulate the problem. They build the model to decide how many interceptors will be allocated to each layer. They define number of incoming objects as states. The transition probabilities are constructed due to kill probabilities.

Some researchers focus on firing policies and order of shooting strategies for WTA problems. For instance, Friedman (1977) suggests a model to determine the order of shooting to enemy units in order to maximize the survival probability of the single unit. He assumes that many attacking targets shoots at a particular single defense unit and time between shoots are exponentially distributed. The engagement process continues until one of the sides is destroyed. He calculates the winning probabilities with algebraic procedure. Manor and Kress (1997) model greedy shooting strategy problem with incomplete damage information in Markovian process. They inspire from the multi-armed bandit problem. The fire allocation problem is developed as a special case of the finite horizon multi-armed bandit without discount factor. They define each target by an arm and the shot against target corresponds to the pull of arm. Glazebrook and Washburn (2004) review shoot-look-shoot (SLS) policies in WTA problems. They investigate shooting strategies to maximize the expected number of targets killed. They formulate the problem as a stochastic dynamic programming and implement Markov decision process for finite and infinite time horizon. The state of the firing process is defined as (S,T) where S and T are the sets of remaining shots and live targets, respectively. The largest amount of target value that can be killed with all remaining shots $V(S,T)$, the shortest horizon to make a specific number of shots $H(S,T)$ and the maximum expected number that can be killed with s shots, t targets, and remaining n salvos $F_n(s,t)$ is calculated with perfect and imperfect information. Glazebrook et al. (2007) examine the policies for shooting problems in order to maximize the value of target killed. They inspire from the multi-bandit problem to shoot which target, how many times and in which order. They also consider the disengagement case due to return value of shootings. They use stochastic dynamic programming to evaluate the different shooting policies. Kim and Cha (2010) suggest a model for fire scheduling of available weapons to targets with time-dependent kill probabilities. They investigate the problem from the attacker perspective. In their formulation, the fire sequence of targets is determined with the consideration of the destruction of targets decreases as time passes. The decreasing rate of the destruction probability of the attack against each target is defined. They call set of firing operations against a target as jobs. They optimize the beginning time of jobs due to this model construction.

A few researches propose artificial neural network (ANN) method to model WTA formulations. Wacholder (1989) presents ANN approach for many weapons to many targets scenario with known attack size. The total expected leakage value of surviving targets in defense is minimized subject to maximum available number of interceptors. The solution approach is based on a combination of Hopfield and Tank's neural network method and Lagrange multipliers differential method. Bertsekas et al. (2000) suggest using ANN to approximate cost-to-go function with a Markovian decision model in order to find the optimum allocation of defensive units if the attacking units come in discrete attack waves. The objective is to maximize expected surviving assets at the end of the engagement process. They formulate the problem as a stochastic shortest path problem and use neuro-dynamic programming with policy iteration methods to solve the problem. The defined state has two components. The first component is $i = \{A_1, ..., A_n, I, M\}$ where A_t is the number of surviving assets of type t, n is the number of asset types, I is the number of interceptors and M is the number of missiles. The second component of the state is defined as current attack vector $a = \{a_1, ..., a_A\}$.

The formulation of WTA is converted to network flow formulations in some studies in the literature. Ahuja et al. (2007) formulate WTA problem by using network flow formulations and suggest lower bounding solution methods. They propose exact and heuristic algorithms for WTA problems. The objective function is transformed to separable convex objective functions. A construction heuristic that solves a sequence of minimum cost flow problems is developed to determine the lower bound on optimal solutions. They use a specific branch and bound method and solve the moderate size test problems exactly in a few seconds. Kwon et al. (2007) formulate the weapon target assignment problem in order to minimize the total assignment cost with the limited number of available rounds. They reformulate the nonlinear integer programming model. By changing parameters and the decision variable, the problem is transformed into an integer programming model. They use LP relaxation and generate convex hull to solve the problem.

The study on naval air defense problems in literature is scant. Kohlberg and Greer (1996) propose tactical missile defense to minimize cost of number of missiles utilized. The problem is formulated for different cases according to constraints, cost and coverage of targets. The approach minimizes cost or maximizes effectiveness of a tactical ballistic missile defense (TBMD) system. Lagrange multipliers method is incorporated to solve the problem. Nguyen et al. (1997) develop an analytical model to optimize allocation of defensive resources against ASMs. The perfect coordination between groups of ships is considered. The objective is formulated as the maximization of the expected number of unsuccessful ASMs. They propose a quasi-uniform model of cohesion to allocate missiles when the attack size is known. Washburn (2005) introduces a method to allocate anti-ballistic missiles (ABMs) to inter-continental ballistic missiles (ICBMs). In the problem, attacking units have decoys to deceive the defense units. He assumes that if an ABM engages to an ICBM, it definitely destroys the ICBM. With different value function of defense units, optimization is performed with the maximization of probability for surviving targets.

Karasakal (2008) models missile allocation problem of a TG in full coordination to maximize effectiveness of air defense. The SLS engagement policy is assumed in the formulation of the air defense problem. He presents a linearization process and suggests two integer programming models to solve the nonlinear integer programming problem. Karasakal et al. (2011) propose a missile allocation model for air defense of TG. The model is based on SLS engagement policy in a coordinated way of defensive units. The formulation provides an allocation and scheduling plan of SAMs to ASMs over the non-overlapping time slots. The maximization of probability of no-leaker for the whole task group is considered. They develop construction and improvement heuristics to solve the problem.

The only study with multiobjective optimization in WTA literature is Brown et al.'s (2011) study. They develop an operational planning model to optimize assignment of tomahawk cruise missiles. They describe different objectives to cover the maximization of the utilization ability of tasks and effective installation of task parts. They use value function, hierarchical approach, Pareto optimization and heuristic approaches to optimize the multiobjective model.

2.1.1.2 Game Theoretical Models

Game theoretical approaches have been frequently used for WTA formulations, since WTA is the concern of two sides having opposite desires. Game models of WTA consider allocation problem in both defense and attack side and draw conclusions about each side with given strategy. Several extensions are proposed in the literature by considering the case in which the offense or defense knows the size of units, positions and types of weapons.

Danskin (1967) introduce the theory of max-min in WTA problems. The max-min problem is to find x and y vectors with $x \in E^n$ and $y \in E^n$ in the following formulation:

$$\max_{x} \ \min_{y} \ \sum_{i=1}^{n} f_i\left(x_i, y_i\right)$$

subject to

$$i = 1......, n, \ x_i \in X \text{ and}$$

where $f_i\left(x_i, y_i\right)$ represent the remaining value of target i if the target is defended by x_i defensive units and attacked by y_i offensive units. The offense wishes to minimize the total remaining value of targets and allocate its units with respect to this minimization. The defense with the knowledge of allocation of offensive units maximizes the total remaining value of targets.

Randolph and Swinson (1969) address discrete max-min problem in an attack and defense situation rather than continuous version of max-min problems. Soland (1973) formulates the min-max discrete missile allocation problem by minimization of damage for defense by assuming offense has optimal attack strategy. He considers that attacking side knows the predetermined defense levels and defense side knows the number of missiles in which attack units hold. The problem of choosing antiballistic missile (ABM) is examined with a certain limit on budget. Haaland and Wigner (1977) propose an allocation approach for defense of ABM. They construct a min-max model and solve it with Lagrange multipliers method. They minimize the maximum damage that the attack can cause. The approach provides optimum allocation in distribution of defending missiles with the size of attack.

Bracken and Brooks (1985) formulate optimal attack and defense of intercontinental ballistic missiles (ICBMs). Both sides are allowed to allocate their missiles wherever they desire. The proportion of surviving defensive missiles with respect to the number of attacking units is analyzed in a game theoretical approach together with preferential strategies. Soland (1987a) formulates missile allocation problem of defensive units with respect to the cost objective. A game model with three phases is considered. Firstly, defense allocates their missiles with the objective of minimum cost, then attacking units maximize the total expected damage with observation of defense planning, finally defense minimizes the total value of target destroyed with respect to known attacking strategy.

Bracken et al. (1987) introduce the preferential defense game model. In the model, attack allocates missiles with the knowledge of the optimum choice of defense preferential strategy. The robustness of preallocated missiles for defense is examined with the assumptions of known attack size. O'meara and Soland (1990) present algorithms for optimal attack and defense strategies. They assume the defense allocates interceptors to maximize expected total value of surviving targets with the knowledge of attack size. Offense attempts to minimize expected total survival value of defensive units. The optimal value of min-max problem with the given scenario is evaluated for defensing of many targets.

Brown et al. (2005) formulate the missile allocation and defense platform location problem with the objective of minimization of maximum total expected damage for defensive platforms. The mathematical model of the problem is solved optimally by proving the total unimodularity of the model. They form different scenarios with the knowledge of position and plans for opposite units. They consider that defense attempts to optimize defensive prepositioning while assuming attack observes the preparations and optimize allocations of its units.

2.1.1.3 Simulation Models

Simulation has been applied on WTA problems for analyzing the behavior of elements in the systems. Simulation represents real life processes more realistically with assumptions and scenarios. Studies on air defense problems in literature are presented below.

Hoyt (1985) presents a Monte-Carlo simulation model to determine whether a defense system can neutralize a number of enemy missiles within a specified time. The approach is developed to assist decision makers to evaluate the probability of success of ballistic missile defense (BMD) system. The variation on the number of interceptors is analyzed and the effectiveness of air defense is evaluated during a given time period.

Beare (1987) proposes optimization model together with a simulation to defend a number of assets against air raids. The model includes choosing the most effective allocation to defend a given set of assets against a range of air threats. A Monte Carlo simulation is integrated to optimize allocation of air defense weapons.

Martin et al. (1995) propose the “Simulation, Evaluation, Analysis, and Research on Air Defense Systems” (SEAROADS) model to analyze air defense capability of a frigate using a Monte Carlo simulation. The model evaluates an engagement between a given ship configuration and an air threat with different settings and analyzes the performance of air defense by comparison of different strategies.

2.1.2 Dynamic WTA Models

Dynamic WTA is a multi-stage problem in which the result of each stage is assessed and used for the future stages. Most of the studies on WTA focus on static version of the problem. WTA through multi-stage process is barely studied in the literature. There are a few researches that build the allocation strategy with respect to time periods. Cai et al. (2006) survey the literature of dynamic WTA problems. They present the shortages of current research and define the characteristic of problems.

Wacholder (1989) presents the mathematical formulation of dynamic WTA problem. The formulation minimizes the expected leakage value of targets killed over time. He states that the closed mathematical solution of the dynamic WTA appears very difficult, thus he concentrates on static version of the model to solve the problem.

The first known generic dynamic WTA problem is introduced by Hosein and Athans (1990). They investigate the WTA problem with time stages. They present the dynamic version of the problem with two stages together with analytical results and asymptotic results as the numbers of weapons and targets go to infinity. In the formulation, for each stage a number of weapons are chosen and allocated to targets in order to minimize the value of surviving enemy targets.

$$min\ F_1 = \sum_{\vec{\omega} \in \{0,1\}^N} \Pr\left[\vec{\mu} = \vec{\omega}\right] F_2^*\left(\vec{\mu}, \vec{\omega}\right)$$

where $\mu_i = 1$ if target i survives in stage 1 and $F_2^*\left(\vec{\mu}, \vec{\omega}\right)$ denotes the optimal cost of problem with initial target state $\vec{\mu}$ and initial weapon state $\vec{\omega}$. The weapon state of the system in stage 2 is defined as the set of available weapons after stage 1. Thus, $w_j = 1$ if weapon j is not used in stage 1 and $w_j = 1 - \sum_{i=1} X_{ij}$. The decision variable is defined as $x_{ij} = 1$ if weapon j is assigned to target i in stage 1. They analyze the optimal strategy for different cases as changing the number of stages, number of weapons and number of targets. They conclude that the dynamic version of the problem is computationally complex and difficult to solve even if the kill probabilities are fixed for each stage and number of weapons used in each stage is same.

Khosla (2001) considers target based dynamic WTA for defensive platforms. The model minimizes the surviving targets subject to resource availability over a given period of time. He assumes that only one weapon is assigned to a target over all time stages. He incorporates the genetic algorithm and simulated annealing algorithm to solve the problem. Jinjun et al. (2006) propose dynamic WTA optimization model to minimize the expected loss of warships. The proposed model is based on Hosein and Athans (1990) study. The status of targets and number of available weapons are updated in each stage. The objective is the minimization of the total threat of the targets. They use simulation to evaluate the performance of the model since the dynamic nature of the problem complicates the solution. Jie et al. (2009) introduce asset based dynamic WTA. The objective function value of surviving assets with the remaining weapons, assets and targets is calculated for each stage. They specify the assignment pairs by permutation of all available engagements and use heuristic approaches to solve the problem.

2.2 Rescheduling Problems

Rescheduling is the process of updating the original schedule because of changes in the problem environment. The real-time events that disrupt the initial schedule are called as disturbances in rescheduling problems. Rescheduling updates an existing schedule in response to disturbances or other changes. The disturbances may make performing the initial schedule impossible or rescheduling may become essential to increase the system performance. Rescheduling studies have been considered in many research areas. We focus on manufacturing and transportation rescheduling problems to analyze the main concerns of those problems related to our research.

2.2.1 Rescheduling in Manufacturing Problems

Rescheduling has been frequently studied in manufacturing systems in literature. Quelhadj and Petrovic (2009) present a classification for real time events that disrupt the initial schedule such as resource related (e.g. unavailability of materials, machine breakdown) and job related (e.g. rush jobs, cancellation). Vieria et al. (2003) review the rescheduling literature and present a framework about rescheduling strategies, policies and methods. They describe two common strategies in rescheduling environment. These are dynamic scheduling and predictive-reactive scheduling. In dynamic scheduling, initially no production schedule is generated. Instead, jobs are assigned with respect to dispatching rules when necessary. Predictive-reactive scheduling method is the most common rescheduling strategy in literature (Mehta and Uzsoy (1998); Ouelhadj and Petrovic (2009)). In this approach, rescheduling is utilized in response to real time events. Wu and Li (1995) describe the iterative process of predictive-reactive scheduling method. In general, response to a new event is determined according to the impact of it. Three types of policies are typically examined in the literature for rescheduling strategy: periodic, event driven and hybrid (Sabuncuoglu and Bayiz (2000); Vieira et al. (2003)). Most of the work in rescheduling problems use event-driven policy that reschedules the system after new event happens such as machine failure (Vieria et al. (2003)). Two common rescheduling methods are used for responding to real time events such as complete rescheduling, creating a new schedule and schedule repair, updating the initial schedule with local adjustments (Sabuncuoglu and Bayiz (2000); Cowling and Johannson (2002)).

In machine rescheduling problem there are mainly two concerns when adapting the schedule to new environment. The first one is keeping the system performance high which is called as efficiency measure. Efficiency of the system is specified with regard to the problem characteristics and the preferences of the decision makers. In production problems, mostly flow time measures are considered as efficiency criteria. Total flow time, tardiness or lateness values are the examples of objectives for rescheduling problems (Azizoglu and Alagoz (2005); Sabuncuoglu and Karabuk (1999)). While retaining the schedule efficiency high, difference between the new and the initial schedule is the second concern which is named as stability measure. The impact of schedule change is measured with stability criteria. Number of jobs processed on different machines with respect to initial schedule, positional disruption of jobs, sequence changes, time deviations are considered as schedule disruption measures (Wu et al. (1993); Azizoglu and Alagoz (2005)). For instance, Hoogeveen et al. (2012) consider three disruption measures. They define $P_j(\alpha)$ and $P_j(\pi)$ as the position of job j in the original schedule α and in the new schedule π respectively. The stability measures are as follows:

$$D_j(\alpha,\pi) = \left|P_j(\pi) - P_j(\alpha)\right|$$

where $D_j(\alpha,\pi)$ is the absolute positional disruption that represents the absolute difference between its position in α and π.

$$P_j(\alpha,\pi) = P_j(\pi) - P_j(\alpha)$$

where $P_j(\alpha,\pi)$ is the difference between its position in π and its position in α.

$$\Delta_j(\alpha,\pi) = \left|C_j(\pi) - C_j(\alpha)\right|$$

where $C_j(\alpha)$ and $C_j(\pi)$ is the completion time of job j in the original schedule α and in the new schedule π respectively. $\Delta_j(\alpha,\pi)$ is the absolute completion time disruption that represents the absolute difference between its completion time in α and its completion time π.

In general, these two measures; efficiency and stability criteria are conflicting with each other in machine rescheduling problems. Early studies in the literature concentrate on revising the existing schedule without considering the difference between initial and new schedule (Hall and Potts (2004)). Wu et al. (1993) present one machine rescheduling formulation with efficiency and stability criteria. They measure the difference between new and initial schedule by the starting time deviations and sequence differences. The efficiency measure is specified as the makespan of the system. They suggest local search procedures to solve the biobjective rescheduling problem.

Abumaizar and Svestka (1997) develop rescheduling algorithm for affected operations in a job shop to find a new schedule. They define starting time deviation and sequence deviation as stability criteria to measure the difference between new and initial schedule. The total completion time is considered as efficiency criteria. Azizoglu and Alagoz (2003) formulate rescheduling problem that considers total flow time as efficiency objective and number of jobs processed between initial and new schedule as stability objective. Their model provides rescheduling of jobs on identical parallel machines with the machine eligibility constraints.

Hall and Potts (2004) consider inserting new jobs to initial schedule when a disruption occurs in manufacturing facilities. They evaluate the disruption value with different measures such as the maximum sequence disruption, the total sequence disruption, the maximum time disruption and total time disruption of the jobs. Yuan and Zhao (2013) propose a biobjective rescheduling model for a single machine that process jobs due to release dates. The set of original jobs and new jobs have been scheduled to minimize makespan and minimize total sequence disruption of jobs. Liu and Ro (2014) propose a rescheduling model on a single machine when an unexpected disruption happens. They measure the disruption of the initial schedule as the maximum time deviation instead of total time deviation. They consider makespan and maximum lateness values for the efficiency of the system.

2.2.2 Rescheduling in Transportation Problems

Transportation systems are inevitably subject to unexpected disturbances such as technical failures, extraordinary passengers, accidents, track delays and unplanned stops. Thus, many rescheduling approaches have been developed in the literature in recent years for transportation systems such as railway, airline and road based services (see Visentini et al. (2013); Alwadood et al. (2012); Kroon et al. (2014) for reviews). In general, rescheduling is considered as a schedule recovery method in transportation problems.

Visentini et al. (2013) address that total delay cost associated with flights, aircrafts and passengers are commonly examined as an objective function in airline rescheduling problems literature. For instance, Bratu and Barnhart (2006) develop models to find the optimal trade-off between airline operating costs and passenger delay costs. Akturk et al. (2014) propose aircraft rescheduling model that minimizes summation of tardiness, swap, additional fuel and carbon emission cost by incorporating cruise speed control and swapping aircrafts.

In road and railway rescheduling problems, the main concern is based on the minimization of the total delay of the network or total operating cost with delay. Li et al. (2004) introduce vehicle rescheduling problem that considers disruptions with regard to vehicle breakdowns. The model minimizes the operating and delay costs. Törnquist and Persson (2007) suggest a railway traffic rescheduling model that considers total network delay and total cost of delay as objectives. Pacciarelli et al. (2014) develop a decision support system based on partitioning of networks, local scheduler, dispatching rules and coordination strategy by minimization of delay in traffic management of railway networks. Spliet et al. (2014) suggest a vehicle rescheduling formulation that considers incorporation of the deviation cost when a route deviates from a location onwards. They consider the objective as the minimization of total traveling cost and cost of deviating from the master schedule.

Stability is often used in production management as a rescheduling performance measure. However stability, in transportation problems, has not been a major consideration because of complex nature of the disruption management (Visentini et al. (2013)). Studies in transportation problems have not integrated the impact of schedule changes or deviation from the initial schedule as a separate objective functions. In the transportation context, stability is considered within the efficiency objective that is based on the ability to return to normal operation after a disturbance occurs (D'Ariano (2008)). Stability is implicitly taken into account in the objectives while minimizing the cost of disturbances.

CHAPTER 3

BIOBJECTIVE MISSILE RESCHEDULING PROBLEM

3.1 Problem Definition

Consider a TG consists of several ships that are dispatched to a region in order to control sea. The ships equipped with a number of SAM systems that can be either a self-defense or an area-defense system. Assume that the sensors of air defense systems detect an air attack by a number of ASMs. To provide a response to ASM attacks, a sequence of operations called as detect-to-engage sequence is followed. TG uses a variety of search radars to detect ASMs. The detection process starts with producing information from every sensor of the ships. The information is constituted for each target ASM. The processing includes identification of type, speed and range of the ASMs. Also, sensors of air defense systems determine the target ship of each ASM. The detection process is performed by a central unit called Naval Tactical Data System (NTDS). NTDS collects data from each of the sensors of the ships in TG and produces the air picture by collecting, analyzing, and correlating the data and share information via real-time and in full coordination. By the communication data link, the picture is supplied to the ships and each ship in the link is capable of using the processed target data. Command and control, C^2 system coordinates TG to ensure maximum efficiency and probability of success.

Engagement process of a SAM to an ASM starts with tracking of the target. To predict the ASM's future position and missile intercept point, ASMs are illuminated and tracked. By using ASM course and speed information, the prediction on interception point is solved by fire control systems. A fire control computer processes all data and provides a solution to the fire control problem. Once the firing solution is solved, SAM round is ready to launch. Each engagement takes a constant setup time. The setup time includes target illumination radar track time, fire control solution time and launch delay time.

The maximum distance of interception depends on the maximum effective range of SAM systems. Each SAM system has specific maximum and minimum effective ranges. A self-defense SAM system can only defend the ship it is stationed and an area defense SAM system can defend other ships within their effective ranges. Therefore, a ship can be defended by both onboard SAM systems and area defense SAM systems.

Figure 3.1 shows a picture of a TG with four ships while an air attack with five attacking ASMs takes place. Ship 1 is a helicopter carrier with no defense system. Ship 3 has SAM 2 area defense system and SAM 3 self-defense system. The effective range of SAM 2 area defense system is depicted in dashed line. SAM 2 can engage all of the ASMs. SAM 1, SAM 2 and SAM 4 are self-defense systems. The circles around the ships indicate the effective range of the self defense systems. Since ship 1 has no defense systems, it can only be protected by SAM 2 area defense system. The lines between ASMs and ships show the target ships of ASMs. ASM 1 and ASM 2 attack to ship 2, ASM 3 attacks to ship 3, ASM 4 attacks to ship 1 and ASM 5 attacks to ship 4. Note that we do not take into account the air defense close-in weapon systems such as Phalanx, Korkut as they are the last line of defense against any leaker ASM that has not been neutralized by the previous SAM engagements scheduled centrally.

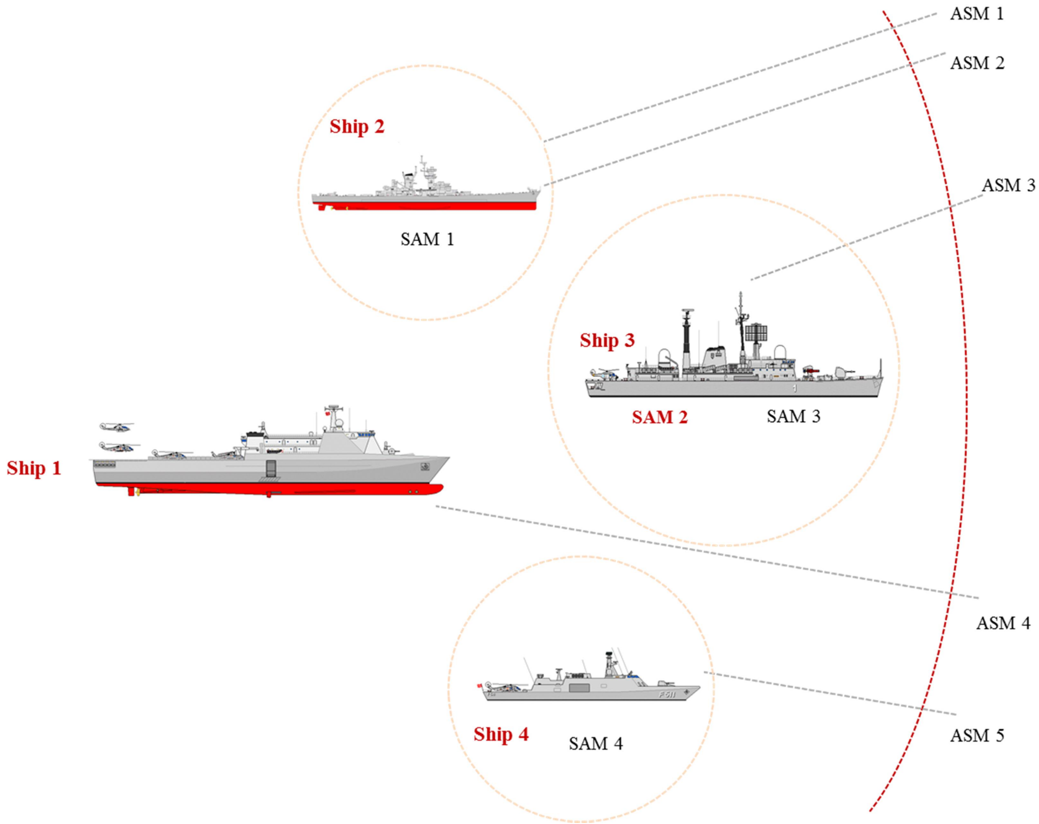

Figure 3.1 Depiction of a TG and attacking ASMs.

Air defense operation process starts right after an air attack is observed. TEWASA system embedded inside the central command and control system of TG plans the allocation of all SAMs onboard ships in TG. An optimized missile allocation schedule is created and orders are given to ship via NTDS or engagement link in order to carry out the engagement schedule. Thus, a set of SAMs are scheduled against ASMs after the air attack is evaluated. The initial schedule includes a firing schedule against ASMs for each SAM system. Thus, SAM systems have shoot order plan to carry out the engagements scheduled.

An example of the shoot order of SAM systems for the initial schedule is depicted in Figure 3.2. The target ASMs and time of shoots are given for each SAM system's schedule. For instance, SAM system 1 first launches one round against ASM 1 and two rounds against ASM 2. Since SAM system 2 is an area defense system, its shoot schedule includes to be engaged against ASM 4, ASM 1 and ASM 5.

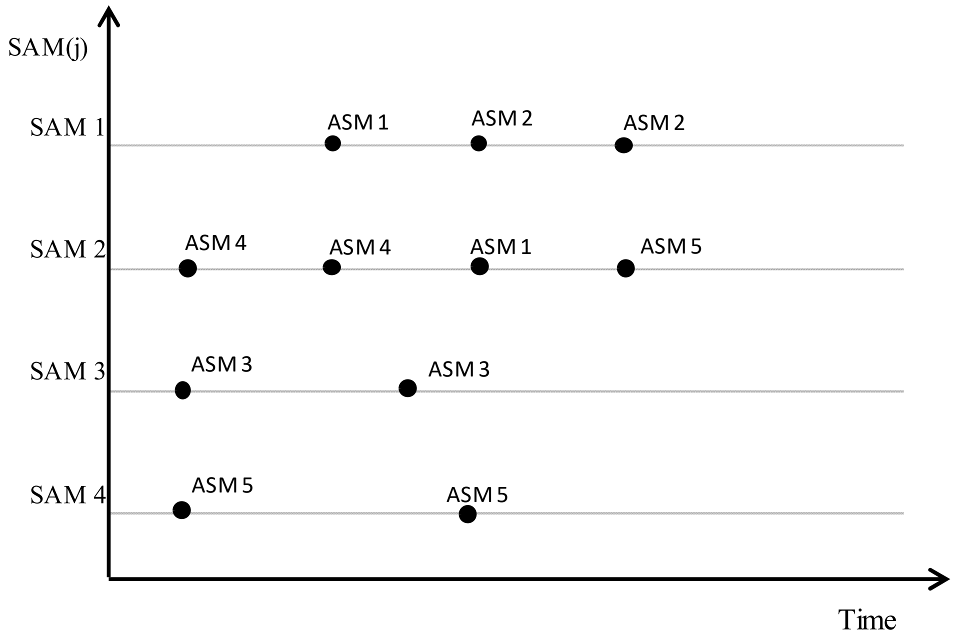

Figure 3.2 Sample engagement schedule of SAM systems.

According to the initial schedule, the engagement process starts. Each SAM system fires the rounds against the target ASMs according to the starting time in the initial schedule. We consider three disturbances during the engagement process.

1. Destroying the target ASM in early stages*:* If a SAM destroys the target ASM, the subsequent allocated SAMs in the original schedule will not be used against the already destroyed ASM. The remaining SAM rounds that are initially allocated for destroyed ASMs can be scheduled for other ASMs that are still threats for TG. If the original schedule is kept, other ships of TG may not utilize the potential benefit because of not using those remaining available SAM rounds.

2. System breakdown: SAM systems including ship sensors have lots of electronic and mechanical parts that guide the missile from its launcher to its

target. The malfunction of one or more of those components may cause a SAM system to be unavailable. For instance, if the illumination radar of a SAM system is out of order or launchers are unable to perform firing the missile, SAMs of that system become incapable of neutralizing the targets. The effectiveness of air defense strategy will decrease with these unavailable SAMs since the prior scheduling is performed by not taking into account malfunctions of SAMs.

3. A new incoming ASM after the engagement process started: Another disturbance during the engagement process is the occurrence of a new target ASM that is not considered in the original engagement schedule. In such a case, change on the allocation plan is required. If the original schedule is kept, the new threat will destroy its intended target.

We name these three disturbances as “destroyed ASM”, “breakdown of a SAM system”, and “new target ASM” throughout the thesis. Before formulation of the problem, we present the main assumptions as follows:

- We possess the optimized initial engagement allocation plan at the beginning of the engagement process.
- A disturbance is occurred after the initial schedule is started and before the completion time of the engagement process.
- At a time point, at most one disturbance occurs.
- If an ASM is destroyed, initially allocated and not fired SAM rounds in the initial schedule become available for other ASMs that are still not destroyed.
- If an ASM is not destroyed by all of the allocated SAM rounds, it destroys its target ship.
- If a ship is destroyed, on board SAM systems become certainly unavailable.
- If a SAM system breaks down within the engagement process, it is unavailable until the completion time of the engagement process.

- If a new incoming ASM is detected, then type, distance, speed and target of new incoming ASMs are identified.
- After ASMs are detected, they are assumed to be classified according to their flight profiles and single shot kill probability matrix between SAM systems against ASMs are generated. Full coordination between ships in terms of allocation of SAM rounds is considered.
- Missile allocation policy is based on SLS tactic.
- An engagement between a SAM round and an ASM may be ongoing at the rescheduling time point. This means that before the rescheduling time point, a SAM round is fired and there is still time for interception to occur. There will be no allocation to these ASMs up to the completion time of the engagement according to SLS policy.

3.2 Problem Formulation

Suppose that there are n incoming ASMs indexed by $i \in N = \{1,............n\}$ and there are m SAM systems indexed by $j \in M = \{1,............m\}$. Let V denote the set of valid combinations of the ASM and the SAM systems, i.e. $(i, j) \in V$ if SAM system j can engage ASM i. The time that ASM i reach its target is t_i and the maximum of these time values determines the engagement time horizon. Hence, $H = \max_{i \in N} \lceil t_i \rceil$ where $t_i = \frac{pf_i}{Va_i}$. pf_i and Va_i are the present distance of ASM and velocity of ASM respectively. We assume that each engagement takes a constant setup time, Δ_c.

The maximum and minimum ranges of a SAM system are denoted by $ra_j^{\max}$ and $ra_j^{\min}$ respectively. Vs_j is the velocity of SAM system j. The present distance, pf_i and the constant velocity of ASMs, Va_i are known by TG. Earliest beginning time of the first engagement is q_{ij} and r_{ij} is the latest ending time of the last engagement.

SAM systems can intercept with ASMs within a specific engageability interval $\left[q_{ij}, r_{ij}\right]$. If $pf_i \le ra_j^{\max}$, SAM system can engage ASM at the beginning of engagement process and $q_{ij} = 0$. Otherwise, q_{ij} is calculated according to occurrence of interception at the maximum effective range of SAM system:

$$q_{ij} = \frac{\left(pf_i\text{-}ra_j^{\max}\right) - Va_i.\left(\frac{ra_j^{\max}}{Vs_j} + \Delta_c\right)}{Va_i}$$

The latest ending time of the last engagement between SAM and ASM pairs, r_{ij} is calculated according to the occurrence of interception at the minimum range of SAM system:

$$r_{ij} = \frac{pf_i\text{-}ra_j^{\min}}{Va_i}$$

The time horizon is divided into equal non-overlapping time slots and unit duration of each time slot is δ. Each engagement can start at the beginning of these time slots, indexed by $k \in K = \{1, ...H\}$. Figure 3.3 shows the time slots, time horizon and starting time of slots. The τ_k represents the beginning of time slot k.

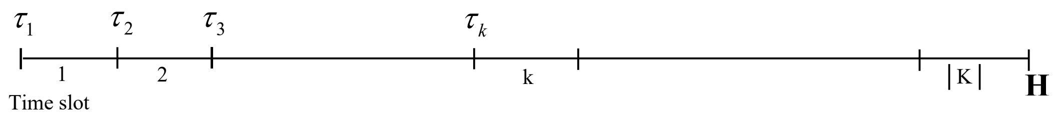

Figure 3.3 Engagements time horizon.

We assume that at most one disturbance can occur in each time slots. This is a reasonable assumption in the problem definition since we take the unit duration of time slots in computational experiments as small as possible such as one second. Moreover, even in the case of more than one disturbance in a time slot, the formulation of the problem enables to find solutions.

The sum of constant setup time and variable flight time gives engagement duration and is denoted by Δ_{ijk}. The engagement duration, is calculated as follows:

$$\Delta_{ijk} = \frac{pf_i - (\tau_k + \Delta_c).Va_i}{Va_i + Vs_j} + \Delta_c$$

Flight time depends on the velocities of ASM-SAM pairs and starting time slot of the engagement. SAMs scheduled against each ASM has to be performed in non-overlapping time slots due to SLS firing policy. SLS policy ensures saving SAM rounds for possible future attacks. SLS policy can be defined as shooting at a target, look to see if it is killed and then shooting again if necessary. Thus, SAM systems do not shoot until the completion time of the previous shoot. We refer to Glazebrook and Washburn (2004) for a comprehensive review and extension on SLS policy. An example given by Glazebrook and Washburn (2004) for SLS policy is as follows. If a single shot kill probability of destroying a target is 0.9, shooting twice at the target provides 0.99 destroying probability with the expense of two shots. With SLS policy, destroy probability is still 0.99 if the time window is enough for looking the result of the engagement, but the average expenditure of shots is only 1.1. Thus, cost objective is ensured by using SLS policy.

The maximum number of engagement between valid ASM and SAM pairs, μ_{ij} is calculated by dividing the engageability interval by the minimum duration of a single engagement in accordance with SLS firing policy. Since the engagement between ASM and SAM pairs can only be achieved within $[q_{ij}, r_{ij}]$ engageability interval, set S_{ij} is defined as the time slots for which SAM *j* can be scheduled to engage ASM *i*.

$$S_{ij} = \left\{ k \in K : (i,j) \in V \text{ and } \left[\tau_k, \tau_k + \Delta_{ijk}\right] \subseteq \left[q_{ij}, r_{ij}\right] \right\}$$

Instead of multiple allocations of SAMs at the same time, SLS tactic provides no overlap of the engagements against each ASM. To ensure that engagements are scheduled due to SLS tactic, the specific set, J_{ik} is defined for each ASM i and time slot k as follows:

$$J_{ik} = \left\{ (j,\rho) : (i,j) \in V, \rho \in S_{ij} \text{ and } \left[\tau_k, \tau_k + \Delta\right] \subseteq \left[\tau_\rho, \tau_\rho + \Delta_{ij\rho}\right] \right\}$$

It states that, if an engagement between ASM i and SAM j starts at time slot ρ that is prior to time slot k and finishes after the end time of slot k, then (j,ρ) pairs are in the set J_{ik}. In short, the set includes all (j,ρ) pairs that blocks time slot k of ASM i

The single shot kill probability (sskp) of SAM j against ASM i when the engagement begins at the beginning of slot k is denoted by p_{ijk}. The maximum number of engagement between ASM and SAM pairs at rescheduling time point, u_{ij}^{RT} are determined according to SLS firing policy. Each SAM system has a number of available rounds, d_j at the beginning of engagement process and f_j numbers of rounds are fired until the rescheduling time point. The parameter $x_{ijk} = 1$ if SAM j is scheduled to start the engagement process against ASM i at the beginning of time slot k in the initial schedule and $x_{ijk} = 0$ otherwise. The decision variable $Y_{ijk} = 1$ if SAM j is scheduled to start the engagement process against ASM i at the beginning of time slot k and $Y_{ijk} = 0$ otherwise.

When a disturbance happens, the time slot is set as rescheduling time point. The set of current ASMs, available SAM systems and engagement time horizon is updated at rescheduling time point. Assume that there are n^d destroyed ASMs indexed by $i^d \in D = \left\{1,........n^d\right\}$ and n^n new incoming ASMs indexed by $i^n \in I = \left\{1,........n^n\right\}$. The broken SAM systems are indexed by . $j^b \in B = \left\{1,........m^b\right\}$. Finished time slots at rescheduling time point are indexed by $k^f \in K = \left\{1,........t^f\right\}$. Set $A = \left(N \cup I / D\right)$ includes current ASMs and set $S = \left(M / B\right)$ includes available SAM systems at rescheduling time point. The remaining time slots at rescheduling time point is in set $T = \left(K / F\right)$. The mathematical formulation of the biobjective missile rescheduling problem (BMRP) is as follows:

(BMRP)

$$min \ \ Z_{ND} = \sum_{i \in A} \sum_{j \in S} \sum_{k \in T} \left| Y_{ijk} - x_{ijk} \right| \tag{3.1}$$

$$max \ \ Z_{PNL} = \prod_{i \in A} \left(1 - \prod_{k \in T \ j \in S} \left(1 - p_{ijk}\right)^{Y_{ijk}} \right) \tag{3.2}$$

subject to

$$\sum_{\substack{k \in T \\ i \in A}} Y_{ijk} \le d_j - f_j \qquad \forall j \in S \tag{3.3}$$

$$\sum_{(j,p) \in J_{ik}} Y_{ijp} \le 1 \qquad \forall i \in A, k \in T \tag{3.4}$$

$$\sum_{k \in S_{ij}} Y_{ijk} \le \mu_{ij}^{RT} \qquad \forall \left(i, j\right) \in V \tag{3.5}$$

$$Y_{ijk} \in \left\{0,1\right\} \qquad \forall \left(i, j\right) \in V, k \in T \tag{3.6}$$

Objective function (3.1) is the minimization of schedule disruption value that is the total number of changed allocations for all SAM systems. It minimizes the total difference of scheduled engagements between initial and new schedule. Objective function (3.2) maximizes the probability of no-leaker for whole task group. Constraint set (3.3) enforces limit on the number of SAM rounds to schedule. Constraint set (3.4) prevents allocation of SAMs to ASMs until the previous engagement finishes. Within the engagement interval, constraint set (3.5) restricts the number of rounds of each SAM that can be scheduled to each valid ASM. Constraint set (3.6) ensures that Y_{ijk} can only have binary values.

BMRP considers survival probability of ships as the efficiency measure. The inner part of objective function (3.1) $\prod_{k\in T}\prod_{j\in S}\left(1-p_{ijk}\right)^{Y_{ijk}}$ is the probability that an ASM cannot be destroyed over all allocated SAM rounds against that ASM. The no-leaker probability of each ASM is calculated by $1-\prod_{k\in T}\prod_{j\in S}\left(1-p_{ijk}\right)^{Y_{ijk}}$. The multiplication of no-leaker probability of ASMs gives the no-leaker probability of TG.

BMRP considers the total number of changed SAM round allocations with respect to initial schedule as a stability objective. The disruption on the schedule occurs according to difference between new and initial schedule in the allocation of SAM round j against an ASM i at time slot k. Thus, an allocation that exits in the new schedule and does not exist in the initial schedule brings one disruption since $Y_{ijk}=1$ and $x_{ijk}=0$. If an allocation exits in the initial schedule and discarded in the new schedule, it also brings one disruption since $Y_{ijk}=0$ and $x_{ijk}=1$.

In solution procedure, the initial schedule is generated with respect to the maximization of probability of no-leaker value of TG while satisfying number of rounds available for each SAM system and ensuring SLS firing policy. The x_{ijk} parameters are obtained from the results of the initial schedule.

3.3 Multiobjective Optimization

In this section, we present the basic definitions about multiobjective optimization problems.

A multiobjective optimization problem involves more than one objective to be optimized and these objectives generally conflict with each other.

A multiobjective problem with p objectives can be defined as follows:

$$min\ \left(Z_1(x), Z_2(x)........., Z_n(x)\right)$$

subject to

$$x \in X$$

where $x \in R^n$ is a feasible solution, $Z_i(x)$ is the i^{th} objective function value of solution x and X is the set of all feasible solutions.

For each solution x in the decision variable space X, there is a point in the objective space Z. A solution x is said to dominate $x^{'}$ if and only if $Z_i(x) \geq Z_i(x^{'})$ for all i and $Z_i(x) > Z_i(x^{'})$ for at least one objective i. If there exists no solution that dominates $x \in X$ then x is said to be non-dominated.

The set of non-dominated solutions in decision space X is called as Pareto optimal set and the set of non-dominated solutions in objective space Z is called as Pareto Front.

We refer to Steuer (1986) study for a comprehensive review of the multiple criteria optimization theory.

3.4 Computational Complexity

Lloyd and Witsenhausen (1986) prove that WTA problem is NP-Hard. Our problem differs from WTA problem in terms of both objectives and constraints. In this section, we show theoretical results about the computational complexity of BMRP.

Theorem 3.1. BMRP is NP-Hard.

Proof: In mathematical formulation of the BMRP, first we consider as if only efficiency objective (3.2) exists. Assume that there is only one ASM threat, and multiple SAM systems $j \in M = \{1,...,m\}$. The objective function is the maximization of $1 - \prod_{k \in T} \prod_{j \in S} \left(1 - p_{jk}\right)^{Y_{jk}}$ and it can be converted to the minimization of $\prod_{k \in T} \prod_{j \in S} \left(1 - p_{jk}\right)^{Y_{jk}}$ The non-linearity of objective function can be linearized by taking the logarithm since $\ln(a) \le \ln(b)$ if and only if $a \le b \quad \forall a,b \in R$.

By taking the logarithm of the equation, objective function becomes $min \sum_j \sum_k \ln\left(1 - p_{jk}\right) Y_{jk}$. Let we denote $a_{jk} = -\ln\left(1 - p_{jk}\right)$. To keep the objective as maximization, we transform the objective function as $Max \sum_j \sum_k a_{jk} Y_{jk}$. The decision variable $Y_{jk} = 1$, if SAM j is scheduled to start the engagement process against the ASM at the beginning of time slot k and otherwise $Y_{jk} = 0$.

Assume that the single shot kill probability does not depend on time slots. Hence, $a_j = -\ln\left(1 - p_j\right)$ and decision variable $Y_{jk} = 1$ if SAM round is chosen to allocate against the ASM until the completion of the engagement time horizon. The problem becomes choosing SAM rounds to allocate against the ASM until the completion of the engagement time horizon. The time horizon, H can be considered as capacity and according to velocities and shoot sequence of SAM systems, each SAM systems have an engagement duration size, t_j. To ensure the available round limit and maximum number of engagement limit, we can simplify the problem by considering each SAM systems have only one available round $d_j = 1$.

The problem is to choose a set of SAMs to allocate against ASM until it reaches the target ship. The resulting restricted MAP is exactly 0/1 knapsack problem where

$Y_k = 1$ if SAM round is chosen to allocate against the ASM. SAM rounds correspond to items and input is a collection of SAM systems that each SAM $j \in M = \{1,...,m\}$ has reward $a_j > 0$, size $t_j > 0$, and a knapsack capacity $H > 0$. If a SAM round j is scheduled against ASM, it brings profit a_j and takes t_j time slots from the time horizon H. However, size of SAM, t_j, depends on the sequence of its allocation. As time passes, the value of t_j decreases since ASM is close to its target ship. Dean et al. (2008) investigate the 0/1 stochastic knapsack problem when the size of an item is determined while trying to place it in the knapsack. They place items in the knapsack sequentially and the size of an item is identified with respect to sequence of placing. They consider deterministic item profits and prove that the problem is NP-Hard since the problem reduces to the classical knapsack problem in the deterministic item size case.

The producing one of the extreme point of BMRP is equivalent to the selecting items to place to a knapsack with item size depending on the sequence of placing it. The known strongly NP-hard problem, 0/1 stochastic knapsack problem, is actually just a special case of the BMRP formulation with one objective. Since resulting restricted BMRP model includes solution of stochastic 0/1 knapsack problem, BMRP is at least as hard as stochastic 0/1 knapsack problem and generating other non-dominated solutions bring additional computational complexity. Thus, by restriction, BMRP is strongly NP-Hard. □

Theorem 3.2. BMRP can be solved optimally in polynomial time in case of one ASM threat, multiple SAM systems, and when all the engagement durations are less than the unit duration of time slots.

Proof: In Theorem 1, we transform the formulation of BMRP in case of one ASM threat, multiple SAM systems. The objective function is converted to $Max \sum_j \sum_k a_{jk} X_{jk}$

Consider that when a SAM system j scheduled at time slot k, it only blocks itself. In other words $\Delta_{jk} \leq \delta \quad \forall j,k$. Thus, for each time slot, the scheduled SAM in time slot k does not affect other time slots and constraint set (3.4) can be formulated as $\sum_{j \in S} Y_{jk} \leq 1 \quad \forall k \in T$.

Since there is only one ASM, constraint sets (3.3) and (3.5) are $\sum_{k \in T} Y_{jk} \leq d_j - f_j \quad \forall j$ and $\sum_{k \in T} Y_{jk} \leq u_j^{RT} \quad \forall j$ respectively. One of them is redundant and smallest of $d_j - f_j$ and μ_j^{RT} is taken. Instead of constraint sets (3.3) and (3.5), the constraint $\sum_{k \in T} Y_k \leq s_j$ is formulated where $s_j = min(d_j - f_j, u_j^{RT})$. The resulting formulation of the problem is as follows:

$$max \sum_{j \in S} \sum_{k \in T} a_{jk} Y_{jk} \tag{3.7}$$

subject to

$$\sum_{k \in T} Y_{jk} \leq s_j \qquad \forall j \in S \tag{3.8}$$

$$\sum_{j \in S} Y_{jk} \leq 1 \qquad \forall k \in T \tag{3.9}$$

$$Y_{jk} \in \{0,1\} \qquad \forall j \in S, k \in T \tag{3.10}$$

The coefficient matrix of the above resulting model is totally unimodular. Hence, it can be solved in polynomial time. From a different point of view, the transformed BMRP formulation with one objective is exactly the generalized assignment problem with all jobs having unit size. $Y_{jk} = 1$ indicates that job k is assigned to machine j. The maximum number of jobs in the machine j has to be less than s_j and job k can be assigned to at most one machine. Since the generalized assignment problem with unit size jobs is solvable in polynomial time (Krumke and Thielen (2013)), we can solve BMRP with one objective in polynomial time in case of one ASM threat, multiple SAM systems, and when unit duration of time slots is greater than all engagement durations. We show that generating the extreme point with maximum efficiency is solvable in polynomial time in case of one ASM threat, multiple SAM systems and when unit duration of time slots is greater than all engagement durations. BMRP includes several non-dominated solutions. All non-dominated solutions of BMRP can be found by generating solutions from maximum Z_{ND} to minimum Z_{ND}. By choosing set of Y_{jk} with specific Z_{ND}, assignment of job k to machine j leaves problem polynomially solvable. As a result, BMRP is solvable in polynomial time in case of one ASM threat, multiple SAM systems, and when unit duration of time slots is greater than all engagement durations. □

Theorem 3.3. BMRP can be solved in polynomial time in case of one SAM system, one ASM threat and constant available rounds. Computational complexity of the BMRP is $O\left(k^s e\right)$ where e is the maximum number of Z_{ND}, k is the number of time slots, s is the minimum of available rounds and maximum number of engagements within the engageability interval.

Proof: Consider the problem in case of one incoming ASM and one SAM system. Also we take into account only the efficiency objective. The problem becomes the decision of in which time slots engagements are scheduled. The decision variable $Y_k = 1$ if the SAM system is scheduled to start the engagement process against the ASM at the beginning of time slot k and $Y_k = 0$ otherwise. Also p_k is the single shot kill probability of SAM system if engagement starts at the beginning of time slot k.

By linearization, the objective function of the MAP formulation becomes. $max \sum_{k \in T} a_k Y_k$ where $a_k = -\ln\left(1 - p_k\right)$ and the round constraint is $\sum_k Y_k \le min\left(d - f, u\right) = s$. The transformed formulation is as follows:

$$max \sum_{k \in T} a_k Y_k \qquad (3.11)$$

subject to

$$\sum_{k \in T} Y_k \leq s \qquad (3.12)$$

$$\sum_{p \in J_k} Y_p \leq 1 \qquad \forall k \qquad (3.13)$$

There are k blocking constraints in constraint set (3.13) and they enforce the SLS policy. The schedule of the engagements should not overlap and J_k set includes the time slots that block the k^{th} time slot according to the duration of the engagements. Since set J_k is the subset of set K for each time slot k, each constraint set (3.13) with at least two variables can be reformulated by writing all binary combinations of variables (we do not need to write constraint with one variable $Y_k \leq 1$ since it is trivial). Thus, if a blocking constraint includes η variables, for $\eta \geq 2$, in the new formulation there will be $\binom{\eta}{2}$ number of constraints for this blocking constraint. Thus, each blocking constraint including more than one variable constitutes edge constraints for a graph G, $Y(\mathrm{u}) + Y(\mathrm{v}) \leq 1 \;\; \forall (u,v) \in E$. The number of vertices, $\left|V(G)\right|$, is equal to the number of decision variables, so $\left|V(G)\right| = k$.

We convert BMRP formulation with one objective to the maximum weighted independent set problem in polynomial time. However, there is an additional round constraint (3.12), $\sum_{k \in T} Y_k \leq s$. If s is a constant then we can solve the problem in $O\left(\left|V(\mathrm{G})\right|^s\right)$ by checking all subsets with no more than s vertices, so it becomes polynomial-time solvable because the number of s-subsets of $V(\mathrm{G})$ is bounded above by $\left|V(\mathrm{G})\right|^s$ (Courcelle et al. (2000)).

In conclusion, if a BMRP problem has only one ASM and SAM system and available round of SAM system is a predefined constant value, the complexity of BMRP formulation with one objective is $O\left(k^s\right)$ and solvable in polynomial time. Besides, for all possible number of disruption values, we can find results of BMRP in polynomial time and the computational complexity is $O\left(k^s e\right)$. □

Corollary 3.1. In case of one SAM system, one ASM threat, if the number of available rounds of SAM system to be scheduled against ASM depends on the unit duration of time slot then BMRP is NP-hard.

Proof: If s depends on, $|V(\mathrm{G})|$, the problem of deciding whether a graph G has an s-clique, where s depends on $|V(\mathrm{G})|$ is NP-Hard (Dabrowski et al. (2011)). Thus, if number of available round, d, or the maximum number of engagement between ASM and SAM, μ, depend on unit duration of time slots, δ, in other words, k, (since $k = \frac{H}{\delta}$ where H is time horizon), BMRP is NP-Hard. □

3.5 Solution Procedures

In this section, we develop solution methods to solve the problem. To generate exact Pareto front, we first use augmented ε-constraint method. Since objective functions are nonlinear, we describe linearization process of objective functions. We give implementation of augmented ε-constraint method to linearized formulation of BMRP. Secondly, we present the procedures and steps of the two newly proposed heuristic algorithms.

3.5.1 Augmented ε-Constraint Method

BMRP model includes two non-linear objective functions. To solve the problem, we linearize both objectives and use augmented ε-constraint method to generate all non-dominated solutions. For the linearization of stability objective, Z_{ND} we define two new binary deviation variables, $\beta_{ijk}^{+}, \beta_{ijk}^{-}$. The linearized objective function and the new constraint are as follows:

$$min \ Z_{LND} = \sum_{i \in I} \sum_{j \in S} \sum_{k \in T} \beta_{ijk}^{+} + \beta_{ijk}^{-}$$

subject to

$$Y_{ijk} - x_{ijk} = \beta_{ijk}^{+} - \beta_{ijk}^{-} \qquad \forall i \in I, j \in S, k \in T$$

The nonlinearity term of the efficiency objective function, Z_{PNL}, is linearized by defining piecewise linear functions. By taking the logarithm of the equation and defining a new constraint set, the problem is converted to mixed integer programming problem (Karasakal (2004)).

The probability of no-leaker for each ASM can be denoted by h_i. $h_i = \left(1 - \prod_{\substack{k \in T \\ j \in S, (i,j) \in V}} \left(1 - p_{ijk}\right)^{Y_{ijk}} \right)$. Thus $Z_{PNL} = \prod_{i \in I} h_i$. The objective function becomes $Z_{PNL} = \sum_{i \in I} ln\left(h_i\right)$ by taking the logarithm of the equation.

The new set of constraints can be defined as $1 - \prod_{k \in T} \prod_{j \in S} \left(1 - p_{ijk}\right)^{Y_{ijk}} \geq h_i$. The logarithm of both sides in the constraint is taken and the equation is simplified by defining new parameter and variable.

$\sum_{k \in T} \sum_{j \in S} a_{ijk} Y_{ijk} \geq b_i$ where $a_{ijk} = -ln\left(1 - p_{ijk}\right)$ and $b_i = -ln\left(1 - h_i\right)$. With new objective function and new introduced constraint, the model is as follows:

$$Max \;\; Z_{PNL} = \sum_{i \in I} ln\left(h_i\right)$$

subject to

$$\sum_{k \in T} \sum_{j \in S} a_{ijk} Y_{ijk} \geq -ln\left(1 - h_i\right)$$

Since objective function and right hand side of constraint include h_i, the ratio of those enable to define linear piecewise functions, $c_i = \dfrac{ln\left(h_i\right)}{-ln\left(1-h_i\right)}$

c_i is a concave function and by using piecewise linear functions, linear approximation can be performed. To generate piecewise linear functions, b_i is partitioned into l parts with Z_l values and bounding constraints are added for each part of b_i values.

We refer to Kwon et al. (1999), Karasakal (2004) and Winston (2004) for further information about the linearization process of efficiency objective.

The linearized formulation of BMRP is as follows:

(L-BMRP)

$$min \;\; Z_{LND} = \sum_{i \in I} \sum_{j \in S} \sum_{k \in T} \beta^{+}_{ijk} + \beta^{-}_{ijk} \qquad (3.14)$$

$$max \;\; Z_{LPNL} = \sum_{p=1}^{p=l} \sum_{i \in I} c_p . b_{ip} \qquad (3.15)$$

subject to

$$\sum_{k \in T} \sum_{j \in S} a_{ijk} Y_{ijk} \geq \sum_{p=1}^{p=l} b_{ip} \qquad \forall i \in A \qquad (3.16)$$

$$0 \leq b_{ip} \leq Z_1 \qquad \forall i \in A,\; p = 1 \qquad (3.17)$$

$$0 \leq b_{ip} \leq Z_p \text{-} Z_{p-1} \qquad \forall i \in A, p = 2, ..., l \qquad (3.18)$$

$$Y_{ijk} - x_{ijk} = \beta^{+}_{ijk} - \beta^{-}_{ijk} \qquad \forall i \in A, j \in S, k \in T \qquad (3.19)$$

$$\beta^{+}_{ijk}, \beta^{-}_{ijk} \in \{0,1\} \qquad \forall i \in A, j \in S, k \in T \qquad (3.20)$$

and

(3.3), (3.4), (3.5), (3.6)

To solve L-BMRP, efficiency objective function is chosen to be optimized while stability objective is formulated as a constraint. Augmented ε-L-BMRP formulation is as follows:

$$Max\ Z_{LPNL} - \mu.Z_{LND}$$

subject to

$$Z_{LND} \leq \theta - \varepsilon$$

(3.3)-(3.6) and (3.16)-(3.20)

The multiplication of Z_{LND} with small number μ avoids generation of inefficient solutions. ε is a small value that avoids generating weakly non-dominated solutions. The parameter θ is updated due to Z_{LND} value of non-dominated point in each iteration and non-dominated points are produced. The solution procedure of augmented ε-constraint method is as follows:

Solution Procedure:

Step 1: Initiate engagement process with initial schedule.
Step 2: Observe the disturbance within the engagement process.
Step 3: Set $\theta = \infty$
Step 4: Solve augmented ε-L-BMRP.
Step 5: If a feasible solution is found, add this solution to the non-dominated solution set. Otherwise, STOP.

Step 6: Set θ to Z_{LND} value of last generated efficient solution, $\theta = Z_{LND}$ and go to Step 4.

3.5.2 Heuristic Approach

Missile allocation problems must be solved within a few seconds in order to use the models in real-life. We already prove that computational complexity of the BMRP is NP-hard. Also, our experiments show that in several seconds we cannot solve small size problems such as problem with 4 ASMs and 4 SAM systems. Hence, augmented ε-constraint method do not enable us to use the model results during the engagement process. To generate non-dominated solutions and meet the solution time requirement, we develop two heuristic procedures. First one is New and Replace Heuristic (NRH). NRH allocates the available SAM rounds that are available and not included in the initial schedule. Second one is Change and Exchange Heuristic (CEH). CEH revises the initial schedule with switching the target of SAM systems.

3.5.2.1 New and Replace Heuristic (NRH)

In this section, we present a heuristic algorithm that concentrates new allocation and replacement of SAM rounds in the existing schedule. The first objective of BMRP $Z_{ND} = \sum_{i \in A} \sum_{j \in S} \sum_{k \in T} \left| Y_{ijk} - x_{ijk} \right|$ is the schedule disruption value that calculates the total number of disruption in the initial schedule. At the beginning of the engagement process, each SAM system has available rounds, d_j and the optimum initial schedule determines the number of SAM rounds to be fired. If a SAM destroys the target ASM, the subsequent allocated SAMs in the initial schedule are not fired to the destroyed ASM. The remaining SAM rounds that are included in the initial plan can be scheduled for other ASMs that are still threats for TG. On the other hand, at rescheduling time point some of the SAM rounds are already fired and missed their targets. Thus, for each SAM system, we have a new available number of SAM rounds at rescheduling time point.

NRH algorithm consists of two parts. The first part is allocation of a new on hand SAM round j, to an ASM i at time slot k. This new allocation brings one disruption since $Y_{ijk} = 1$ and $x_{ijk} = 0$. The second part includes discarding an initially allocated SAM round j from a target ASM i and replacing a different SAM round j' to the ASM i engagement plan. Thus both SAM system j and j' is disrupted in this case. Since $\left|Y_{ijk} - x_{ijk}\right| = |0-1|$ and $\left|Y_{ij'k} - x_{ij'k}\right| = |1-0|$, two disruptions occur.

We inspired from the Cauchy's mean theorem (Cauchy (1821)) in developing the heuristic procedure. In this theorem, the product of positive numbers of constant sum attains its maximum value when they are equal. Since the probability of no-leaker of TG is the product of no-leaker probability of each ASM, h_i, we concentrate on the ASM that has minimum probability of no-leaker. Thus, the probability of no-leaker of ASMs and single shot kill probability of SAM systems are two main concerns while allocating a SAM round. We first try to allocate SAM rounds against ASM that has minimum probability of no-leaker value and we increase no-leaker probability value of ASM which has minimum.

At rescheduling time point, we have a non-dominated solution with no disruption, $Z_{ND} = 0$ which is the extreme point of the efficient frontier. We start generating solutions from minimum Z_{ND} to maximum Z_{ND} in the objective space. For each possible number of disruptions on the schedule, we try to achieve the maximum value of Z_{PNL}. We start from zero disruption, Z_{ND}, increase Z_{ND} by one in each iteration $Z_{ND} = \{1, 2, ..., max\ |Z_{ND}|\}$. Thus, for each integer Z_{ND} value, we generate solutions up to the maximum number of possible disruptions. We also consider the combination of new allocation and replacement values for each specific Z_{ND} in scheduling of SAM rounds. For instance, for $Z_{ND} = 5$, the possible combinations of number of new allocation and replacement values are $\{5,0\},\{3,1\},\{1,2\}$ respectively since new allocation brings one and replacement brings two disruptions. For each combination, we generate a solution with $Z_{ND} = 5$. We choose the solution with maximum probability of no-leaker among those. After all solutions are generated for different Z_{ND}, we determine the non-dominated solutions. The steps of the NRH algorithm are as follows:

Steps of the NRH Algorithm

Step 0. Calculate available on hand SAM rounds, on_j, that can be allocated in addition to original schedule for each SAM system at rescheduling time point, *RT*.

- Calculate number of allocated SAM rounds, a_j at the beginning of engagement process, $a_j = \sum_{i \in A} \sum_{k \in T} x_{ijk}$.
- If an ASM is destroyed, calculate the subsequent allocated SAMs, sa_j, that are allocated to the destroyed ASM initially and will not be launched against the destroyed ASM.

  If ASM i is destroyed then $sa_j = \sum_{i \in D} \sum_{k \geq RT} x_{ijk}$. Otherwise $sa_j = 0$.
- $on_j = d_j - a_j + sa_j$ where d_j is the available round of SAM systems at the beginning of the engagement process.

Set $Z_{ND} = 0$ and calculate Z_{PNL} according to the initial schedule. The resulting solution is the first non-dominated point with minimum disruption value.

New Allocation Part

Step 1. Set $Z_{ND} = Z_{ND} + 1$. Use the initial schedule. Determine number of the maximum replacement value, max^r, number of maximum new allocation value,

max^n with respect to specified number of disruption Z_{ND} and initialize number of total new replacement value, tn^r.

- $max^r = \left\lfloor |Z_{ND}| / 2 \right\rfloor$, $max^n = |Z_{ND}|$, $tn^r = 0$.

Step 2. Calculate the total number of new allocation value, $tn^n = |Z_{ND}| - 2tn^r$. If $tn^n = 0$, go to step 15. Otherwise, set the number of current new allocation value, $cn^n = 0$.

Step 3. Create set A that includes available ASMs for allocation. Calculate the probability of no-leaker, h_i, for each ASM in set A.

Step 4. If $A = \{ \}$ then tn^n number of SAM rounds cannot be allocated against ASMs, stop. Otherwise select the ASM i from set A with lowest h_i, $i = \underset{i \in A}{argmin}(h_i)$.

Step 5. Create set S that consists of SAM systems. Set S includes SAM systems that have valid combination against selected ASM i. Also, the SAM systems in set S must have available SAM rounds, $S_i = \{ j | (i, j) \in V \text{ and } on_j > 0 \}$. Sort the SAM systems in the set due to single shot kill probability against ASM i.

Step 6. If $S = \{ \}$, there is no available SAM to allocate against ASM i, discard ASM i from set A, $A = A \setminus \{i\}$ and go to step 4. Otherwise, choose the available SAM system j from set S with maximum single shot kill probability to allocate against the selected target ASM i.

Step 7. Determine the available time slot for selected ASM i to allocate SAM round j. Start from the rescheduling time point, *RT*, and search for available time slot to allocate SAM round j, set $p = RT$.

Step 8. If $p = H$, no available time slots exist to schedule SAM round j, discard SAM round j from set S, $S = S \setminus \{j\}$ and go to step 6.

Step 9. Check availability of SAM round j to schedule at time slot p, if $p \notin S_{ij}$ then $p = p + 1$ and go to step 8.

Step 10. If p is between starting time slot of an already allocated SAM round and finishing time slot of an already allocated SAM round, p is not available to allocate new SAM round, $p = p + 1$, go to step 8.

Step 11. If p is before the starting time slot of an already allocated SAM round and $p + \left\lceil \frac{\Delta_{ijp}}{\delta} \right\rceil$ is after the starting time slot of an already allocated SAM round, p is not available to allocate new SAM round, $p = p + 1$, go to step 8.

Step 12. If p is before the starting time slot of an already allocated SAM round and $p + \left\lceil \frac{\Delta_{ijp}}{\delta} \right\rceil$ is before the starting time slot of an already allocated SAM round or p is after the finishing time slot of an already allocated SAM, then p is to be determined as the starting time slot of new SAM round to be scheduled. Allocate SAM round to ASM at the beginning of time slot p. Update no-leaker probability of ASM i and update on hand available rounds of SAM system j, $on_j = on_j - 1$.

Step 13. Set $cn^n = cn^n + 1$, if $cn^n = tn^n$ then tn^n number of new allocation is performed. Otherwise go to step 3.

Replacement Part

Step 14. If $tn^r = 0$, keep the result as a possible non-dominated solution and stop. Otherwise, set the number of current replacement value, $cn^r = 0$.

Step 15. Create set E that includes current threat ASMs.

Step 16. If $E = \{ \ \}$, then tn^r number of SAM cannot be replaced against ASMs, stop. Otherwise select the ASM with lowest h_i, $i = \underset{i \in E}{argmin}\left(h_i\right)$.

Step 17. Create set M that includes the already allocated SAM rounds to the selected ASM. Determine the SAM $j^{'}$ from set M with minimum sskp that is to be removed from the allocation plan of selected ASM. If $M = \{ \ \}$, then it could not be achieved for ASM i that a SAM round replaced with another one with higher sskp. Discard ASM i from set E, $E = E \setminus \{i\}$ and go to step 16.

Step 18. Create set R where it consists of SAM systems. Set R only includes SAM systems that have valid combination against selected ASM i. Also, the SAM systems in set R must have available SAM rounds, and single shot kill probability of SAM rounds j in set R against ASM i are greater than the single shot kill probability of SAM round $j^{'}$ $R_i = \left\{ j \middle| (i,j) \in V, on_j > 0 \text{ and } sspk(i,j) > sspk(i,j^{'}) \right\}$. Thus, replacement of a SAM round j from set R instead of SAM round $j^{'}$ provides a better probability of no-leaker value for ASM i.

Step 19. Choose the available SAM system j with maximum single shot kill probability to allocate against the selected target ASM, if $R = \{ \ \}$ then there is not available SAM round that has higher sskp, so discard SAM j' from the set, $M = M \setminus \{j'\}$ and go to step 17.

Step 20. Determine the available time slots for selected ASM to allocate SAM round j. Start from the rescheduling time point and search for available time slot to allocate SAM j, set $r = RT$.

Step 21. If $r = H$, no available time slots exist to schedule SAM round j, discard SAM round j from the set R, $R = R \setminus \{j\}$ and go to step 19.

Step 22. Check availability of SAM round j to schedule at time slot r, if $r \notin S_{ij}$ then $r = r + 1$ and go to step 21, otherwise go to step 23.

Step 23. If r is between starting time slot of an already allocated SAM round and finishing time slot of an already allocated SAM round, r is not available to allocate new SAM round, $r = r + 1$, go to step 21.

Step 24. If r is before the starting time slot of an already allocated SAM round and $r + \left\lceil \frac{\Delta_{ijp}}{\delta} \right\rceil$ is after than the starting time slot of an already allocated SAM round, r is not available to allocate new SAM round, $r = r + 1$, go to step 21.

Step 25. If r is before the starting time slot of an already allocated SAM round and $r + \left\lceil \frac{\Delta_{ijp}}{\delta} \right\rceil$ is before than the starting time slot of an already allocated SAM round or r is after the finishing time slot of an already allocated SAM, then r is to be determined starting time slot of new SAM round to be scheduled. Remove SAM round j' and allocate SAM round j to the allocation plan of ASM i at the beginning of time slot r. Update $on_j = on_j - 1$ and $on_{j'} = on_{j'} + 1$.

Step 26. Set $cn^r = cn^r + 1$, if $cn^r = tn^r$ then tn^r number of new allocation is performed, otherwise go to step 15.

Step 27. If $tn^r < max^r$ then $tn^r = tn^r + 1$ and go to step 2, otherwise keep the result as a possible non-dominated solution.

Step 28. If $Z_{ND} < Z_{ND}^{max}$ where Z_{ND}^{max} is the maximum number of disruption, go to step 1, otherwise stop.

3.5.2.2 Change and Exchange Heuristic (CEH)

New and Replace Heuristic considers allocating available and not used SAM rounds against the target ASMs that has the minimum probability of no-leaker value. However, at rescheduling time point there may not be any available SAM rounds to allocate against ASMs. For instance, if at the beginning of an engagement process, number of allocated SAM rounds, a_j is equal to the available number of SAM rounds, d_j and if there is not any subsequent allocated SAM round reserved for the destroyed ASM initially, then new allocation or replacement of SAM rounds against ASMs is not possible. Thus, NRH cannot produce any non-dominated solution since the number of available SAM round, on_j, is zero. But, the probability of no-leaker value of ASMs may differ from each other at rescheduling time point and we may change the target ASM of some SAM systems. This can balance the probability of no-leaker value of ASMs and increase the efficiency of the system. In other words, discarding a SAM round from the schedule of an ASM having greater probability of no-leaker value and assigning this SAM against an ASM that has a smaller probability of no-leaker can provide a better solution for the problem. In addition, exchanging two different SAM rounds between two ASMs may also increase the efficiency of the system. Hence, the heuristic algorithm is based on two parts; changing the target ASM of a SAM round, exchanging two different SAM rounds between allocation plans of two ASMs. As in Cauchy's mean theorem (Cauchy

(1821)), we examine whether it is possible to change or exchange allocated SAM rounds between the ASM that has the minimum probability of no-leaker and the ASM that has the maximum probability of no-leaker value. We change the target ASM of SAM systems and it enables us increasing the probability of no-leaker of ASM and also increasing the probability of no-leaker of TG. The steps of the CEH algorithm are as follows:

Steps of the CEH Algorithm

Change Part

Step 1. Use the initial schedule. Create set A that consists of current ASMs. Calculate probability of no-leaker value of TG, Z_{PNL}.

Step 2. If $A = \{ \}$, stop. Otherwise, calculate the probability of no-leaker value of ASMs in set A, and select ASM i with the minimum probability of no-leaker value, h_i where $i = \underset{i \in A}{argmin}(h_i)$.

Step 3. Create set $A^{'}$ that includes ASMs that have a greater probability of no-leaker value than h_i. Set $A^{'}$ is created in order to select the ASM with the maximum probability of no-leaker value. So, the SAM rounds of ASMs in set $A^{'}$ can be allocated to the schedule of ASM i that has minimum probability of no-leaker value.

Step 4. If $A^{'} = \{ \}$, go to step 2. Otherwise, calculate probability of no-leaker value of ASMs in set $A^{'}$, and select the ASM $i^{'}$ with maximum probability of no-leaker value, $h_i^{'}$ where $i^{'} = \underset{i \in A^{'}}{argmin}(h_i)$

Step 5. Create set $R^{'}$ that includes allocated SAM rounds for ASM $i^{'}$ after the rescheduling time point.

- Discard the SAM rounds from set $R^{'}$ that cannot engage ASM i. Thus, we ensure that $R^{'}$ contains only SAM rounds that can engage ASM i and a new SAM round can be allocated to the schedule of ASM i from the engagement list of ASM $i^{'}$.

Step 6. If $R^{'} = \{\ \}$, discard ASM $i^{'}$ from set $A^{'}$, $A^{'} = A^{'} \setminus \{i^{'}\}$ and go to step 4. Otherwise select the SAM round, $j^{'}$, from the engagement list of ASM $i^{'}$ with minimum single shot kill probability. We choose the SAM round to be discarded with the minimum sskp from the engagement list of ASM $i^{'}$ not to ruin the efficiency of the ASM $i^{'}$. Thus, we ensure slow but controllable improvements on efficiency of the system, $j^{'} = \underset{j \in R^{'}}{argmin}\left(sskp\left(i^{'}, j^{'}\right)\right)$.

Step 7. Determine the SAM round $j^{'}$ that can engage ASM i according to available time slots. Start from the rescheduling time point and search for available time slot to allocate SAM $j^{'}$, set $p = RT$.

Step 8. If $p=T$, no available time slots exist to schedule SAM round $j^{'}$, discard SAM round $j^{'}$ from set $R^{'}$, $R^{'} = R^{'} \setminus \{j^{'}\}$ and go to step 6.

Step 9. Check the availability of SAM round $j^{'}$ to schedule at time slot p. If $p \notin S_{ij}$ then $p = p+1$ and go to step 8, otherwise go to step 10.

Step 10. If p is between starting time slot of an already allocated SAM round and finishing time slot of an already allocated SAM round, p is not available to allocate a new SAM round, $p = p+1$, go to step 8.

Step 11. If p is before the starting time slot of an already allocated SAM round and $p+\left\lceil \frac{\Delta_{ijp}}{\delta} \right\rceil$ is later than the starting time slot of an already allocated SAM round, p is not available to allocate a new SAM round, $p = p+1$ go to step 8.

Step 12. If p is before the starting time slot of an already allocated SAM round and $p+\left\lceil \frac{\Delta_{ijp}}{\delta} \right\rceil$ is before than the starting time slot of an already allocated SAM round or p is after the finishing time slot of an already allocated SAM, then p is to be determined as the starting time slot of a new SAM round $j^{'}$ to be scheduled. Remove SAM round $j^{'}$ from the schedule of ASM $i^{'}$. Allocate SAM round $j^{'}$ to ASM i at the beginning of time slot p. Update probability of no-leaker of ASM i and $i^{'}$.

Step 13. Calculate the new probability of no-leaker value of TG, Z_{n-PNL}. If $Z_{n-PNL} > Z_{PNL}$, calculate the number of disruptions, Z_{ND}, with respect to the initial schedule. Keep the solution as a possible non-dominated solution and update the engagement allocation plan, and go to step 14. Otherwise, keep the current engagement allocation plan since changes on the engagement allocation plan generates a worse solution. Discard SAM round $j^{'}$ from set $R^{'}$, $R^{'} = R^{'} \setminus \{j^{'}\}$ go to step 6.

Exchange Part

Step 14. Create set E that includes current ASMs to select the ASM with minimum probability of no-leaker value. Calculate probability of no-leaker value of TG, Z_{PNL}.

Step 15. If $E = \{\ \}$, go to step 1. Otherwise, calculate the probability of no-leaker value of ASMs in set *E*, and select the ASM with minimum probability of no-leaker value, $i = \underset{i \in E}{argmin}\left(h_i\right)$

Step 16. Create set M that includes the allocated SAM rounds for ASM i after the rescheduling time point in the initial schedule.

Step 17. If $M = \{\ \}$, there is no available SAM round to be removed from the engagement schedule of ASM i. Thus, exchange of SAM rounds is not possible. Then, discard ASM i from set E, $E = E \setminus \{i\}$ and go to step 15. Otherwise select the SAM round j from the engagement list of ASM i with minimum single shot kill probability. In this step, we determine the SAM round to be removed from the engagement schedule of ASM i in order to replace a better SAM round in terms of sskp, $j = \underset{j \in M}{argmin}\left(sskp\left(i, j\right)\right)$.

Step 18. Create set $E^{'}$ that includes ASMs that have greater probability of no-leaker value than h_i. That is the SAM rounds of ASMs in set $E^{'}$ can be allocated to the engagement plan of ASM i.

Step 19. If $E^{'} = \{\ \}$, go to step 17. Otherwise, calculate the probability of no-leaker value of ASMs in set $E^{'}$ and select the ASM with maximum probability of no-leaker value, $i^{'} = \underset{i \in E^{'}}{argmin}\left(h_{i^{'}}\right)$.

Step 20. Create set $M^{'}$ that includes allocated SAM rounds for ASM $i^{'}$ after the rescheduling time point.

- Discard the SAM rounds from set $M^{'}$ that cannot engage ASM i. Thus, we ensure that $M^{'}$ contains only SAM rounds that can engage ASM i and a new SAM round can be replaced to the schedule of ASM i from the engagement list of ASM $i^{'}$

- Discard the SAM rounds from set $M^{'}$ that the single shot kill probability between SAM round in set $M^{'}$ and ASM i is less than the single shot kill probability between SAM round j and ASM i. So, we ensure that removing SAM round j and replacing a SAM round $j^{'}$ to the schedule of ASM i always make an improvement to the probability of no-leaker of ASM i which has the minimum h_i.

Step 21. If $M^{'}=\{\ \}$, go to step 19. Otherwise select the SAM round $j^{'}$ from the engagement list of ASM $i^{'}$ with the minimum sskp. We choose the SAM round to be removed with the minimum sskp from the engagement list of ASM $i^{'}$ not to ruin the efficiency of ASM $i^{'}$. Thus, we ensure slow but controllable improvements on the efficiency of the system, $j^{'} = \underset{j^{'} \in M^{'}}{argmin}\left(sskp\left(i^{'}, j^{'}\right)\right)$.

Step 22. Determine the SAM round that can engage to ASM i according to available time slots. Start from the rescheduling time point and search for an available time slot to allocate SAM $j^{'}$, set $p = RT$.

Step 23. If $p = H$, no available time slot exists to schedule SAM round $j^{'}$, discard SAM round $j^{'}$ from set $M^{'}$, $M^{'} = M^{'} \setminus \{j^{'}\}$ and go to step 21.

Step 24. Check availability of SAM round $j^{'}$ to schedule at time slot p, if $p \notin S_{ij^{'}}$ then $p = p+1$ and go to step 23, otherwise go to step 25.

Step 25. If p is between starting time slot of an already allocated SAM round and finishing time slot of an already allocated SAM round, p is not available to allocate a new SAM round, $p = p+1$, go to step 23.

Step 26. If p is before the starting time slot of an already allocated SAM round and $p+\left\lceil \frac{\Delta_{ijp}}{\delta} \right\rceil$ is after the starting time slot of an already allocated SAM round, p is not available to allocate new SAM round, $p = p+1$, go to step 23.

Step 27. If p is before the starting time slot of an already allocated SAM round and $p+\left\lceil \frac{\Delta_{ijp}}{\delta} \right\rceil$ is before the starting time slot of an already allocated SAM round or p is after the finishing time slot of an already allocated SAM, then p is to be determined as the starting time slot of new SAM round j' to be scheduled.

Step 28. Determine the SAM round j can engage to ASM i' according to available time slots. Start from the rescheduling time point and search for available time slot to allocate SAM j to engagement plan of ASM i', set $r = RT$.

Step 29. If $r = H$, no available time slots exist to schedule SAM round j. Thus, SAM round j cannot be added to the schedule of ASM i'. Discard ASM i' from set E', $E' = E' \setminus \{i'\}$ and go to step 19.

Step 30. Check availability of SAM round j' to schedule at time slot r, if $r \notin S_{ij'}$ then $r = r+1$ and go to step 29, otherwise go to step 31.

Step 31. If r is between starting time slot of an already allocated SAM round and finishing time slot of an already allocated SAM round, r is not available to allocate new SAM round, $r = r+1$, go to step 29.

Step 32. If r is before the starting time slot of an already allocated SAM round and $r+\left\lceil \frac{\Delta_{ijp}}{\delta} \right\rceil$ is after than the starting time slot of an already allocated SAM round, r is not available to allocate a new SAM round, $r = r+1$, go to step 29.

Step 33. If r is before the starting time slot of an already allocated SAM round and $r+\left\lceil \frac{\Delta_{ijp}}{\delta} \right\rceil$ is before the starting time slot of an already allocated SAM round or r is after the finishing time slot of an already allocated SAM, then r is to be determined as the starting time slot of a new SAM round j to be scheduled.

Step 34. Remove SAM round j from the schedule of ASM i and remove SAM round $j^{'}$ from the schedule of ASM $i^{'}$. Allocate SAM round $j^{'}$ to ASM i at the beginning of time slot p. Allocate SAM round j to ASM $i^{'}$ at the beginning of time slot r. Update probability of no-leaker of ASM i and $i^{'}$. Calculate new probability of no-leaker value of TG, Z_{n-PNL}. If $Z_{n-PNL} > Z_{PNL}$, keep the solution as a possible non-dominated solution and go to step 14. Otherwise, keep the former engagement allocation plan because changes on the engagement allocation plan generate a worse solution. Discard SAM round $j^{'}$ from set $M^{'}$, $M^{'} = M^{'} \setminus \left\{ j^{'} \right\}$ and go to step 21.

NRH and CEH algorithms produce feasible solutions that are possible non-dominated solutions. CEH algorithm works on feasible solutions generated by NRH algorithm. Hence, CEH algorithm starts after then NRH algorithm. A solution for specified number of new allocation and replacement is produced by NRH algorithm. CEH algorithm uses this solution and generates other feasible solutions by changing and exchanging the place of allocated SAM rounds within the existing solution. If any schedule generated by CEH algorithm is a dominated solution, it keeps the former solution and tries to generate a new one.

3.6 Computational Results

In this section, we present computational results in order to show the effectiveness of the rescheduling model for destroyed ASM, breakdown of a SAM system and new ASM target cases. We generate sample problems using the properties of real weapon systems in open literature. We define seven different SAM systems and seven different ASM systems. The feature of the SAM systems and ASMs are given in Appendix A. The single shot kill probability matrix for SAM systems and ASMs are created by using open sources. We define each problem set by the number of ASM and SAM systems. We randomly generate the sample problems by using different random number streams for the type of ASM, the type of SAM, the initial detection range of ASM, the target ship of ASM and the available rounds of SAMs. We run each problem instance using five different seeds set. First of all, we find the optimal initial schedule with respect to the probability of no-leaker of TG. We perform rescheduling for initial plan in the event of disturbances.

We solve problems for all three cases by augmented ε-constraint method using IBM ILOG CPLEX version 12.6 in Java platform. The objective function values are calculated for each case. We analyze the results according to the increment on the objective function values. In order to evaluate the improvement on efficiency and the disruption on stability of the schedule, we define different metrics and analyze the results. We define the metrics below:

Maximum improvement on efficiency (MIE) metric is the difference between maximum probability of no-leaker and the minimum probability of no-leaker value among the non-dominated solutions. We also calculate the percentage improvement on efficiency with maximum percentage improvement on efficiency (MPIE) metric. It shows percentage improvement on Z_{PNL} objective function.

Assume there are n non-dominated solutions indexed by s and included in set NS.

$$\alpha = \underset{s \in NS=\{1,..,n\}}{argmax}\left(Z_{PNL}^{s}\right),\ Z_{PNL}^{max} = Z_{PNL}^{\alpha} \text{ and } \beta = \underset{s \in NS=\{1,..,n\}}{argmin}\left(Z_{PNL}^{s}\right),\ Z_{PNL}^{min} = Z_{PNL}^{\beta}$$

$$MIE = Z_{PNL}^{max} \text{-} Z_{PNL}^{min} \text{ and } MPIE = \frac{Z_{PNL}^{max} - Z_{PNL}^{min}}{Z_{PNL}^{min}}$$

Maximum number of disruption on schedule (MNDS) metric indicates difference between the maximum Z_{ND} and minimum Z_{ND} values among the non-dominated solutions.

$$MNDS = Z_{ND}^{\beta} - Z_{ND}^{\alpha}$$

The relationship between Z_{PNL} and Z_{ND} is measured by average percentage improvement on efficiency with one disruption (APIE). It shows percentage improvement of efficiency on average with only one disruption of engagement schedule. The measure is calculated by

$$APIE = \frac{MPIE}{MNDS}$$

For a sample problem, the values of these metrics are presented in Figure 3.4. There are five non-dominated solutions in the Pareto front. If we keep the initial schedule the probability of no-leaker is 0.45. The efficiency of the system can be at most 0.84 with eight disruptions on the schedule. With one change on schedule, 10.82% efficiency increase is ensured on the average. If the initial schedule is kept on, to be able to maintain the stability, the total probability of no-leaker decreases. If we change initial schedule, we acquire a higher total probability of no-leaker value.

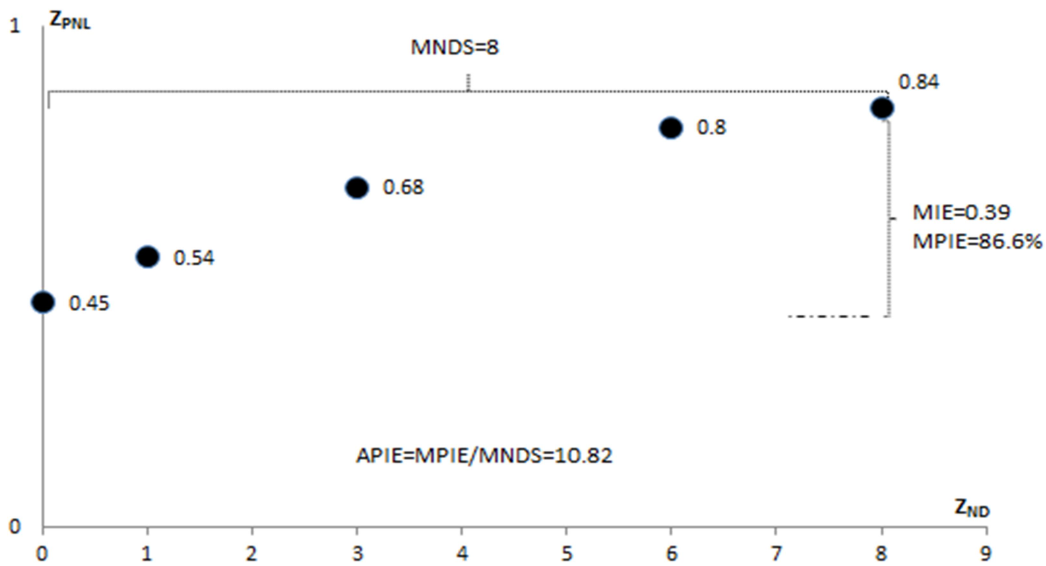

Figure 3.4 Improvement on metrics for analyzing the rescheduling approach.

Secondly, we test our heuristic algorithm in all problem sets. The solution of the mathematical model and the heuristic algorithm is compared for all cases. For large size problems, we cannot solve the BMRP model with augmented ε-constraint method since the problem is NP-hard. To compare the performance of the heuristic algorithm, three performance metrics are used. These metrics are the Hyper Volume Ratio (HVR), Inverted Generational Distance (IGD) and Percentage of Found Solutions (FS). HVR measures the ratio of the region enclosed by the non-dominated set of heuristic results and the region enclosed by the solution of augmented ε-constraint method Pareto front.

$$HVR = \frac{vol\left(\bigcup_{i \in heuristic} v_i\right)}{vol\left(\bigcup_{i \in \varepsilon-cons} v_i\right)}$$

where v_i is the objective space dominated by solution i with respect to a reference point.

The Inverted Generational Distance (Bosman and Thiernes (2003)) is the average Euclidean distance between non-dominated solutions of heuristics and their closest non-dominated front member of augmented ε-constraint method.

$$\text{IGD} = \frac{1}{|A|}\sum_{i \in A}\left(\min_{j \in PF}\left\|Z^i - Z^j\right\|_2\right)$$

where A denotes the nondominated set generated by heuristics and PF denotes the Pareto front set and $\left\|Z^i - Z^j\right\|_2$ represents the Euclidean distance between nondominated solutions of heuristics, Z^i and their closest nondominated front member of mathematical model, Z^j.

The third metric is the percentage of found solutions (FS) represents the solution generated by heuristics. We also present total number of non-dominated solutions generated by the heuristic and the augmented ε-constraint method.

3.6.1 Computational Results for Destroyed ASM Case

In this case, we observe outcome of an engagement between a SAM round and ASM with respect to initial schedule. We start from the first engagement and generate a random number from the uniform distribution. We assume that the engagement between SAM and ASM pair ends up with destroyed ASM if the random number value is less than the single shot kill probability of SAM against ASM. We set the ASM as destroyed and set the rescheduling time point, *RT*, as the starting time of the following time slot. Available rounds, remaining time slots and upper bound on the number of engagements are updated with respect to the rescheduling time point. For a sample problem, Figure 3.5 shows the picture of engagement process between SAM and ASM pairs until the end of time slot 4. The shapes with dashed border, thin border and bold border indicate the place of units at time slot k=1, k=3 and k=4 respectively. At the beginning of the engagement process, SAM 3 is fired against

ASM 3 and SAM 5 is fired against ASM5. At the beginning of time slot 3, SAM 3 and ASM 3 pairs complete the engagement process. At time slot 4, the engagement process between SAM 5 and ASM 5 accomplishes. Also, SAM 1 is fired against ASM 1 at time slot 4. ASM 3 is destroyed by SAM 3 at time slot 3 and ASM 5 is destroyed by SAM 5 at time slot 4.

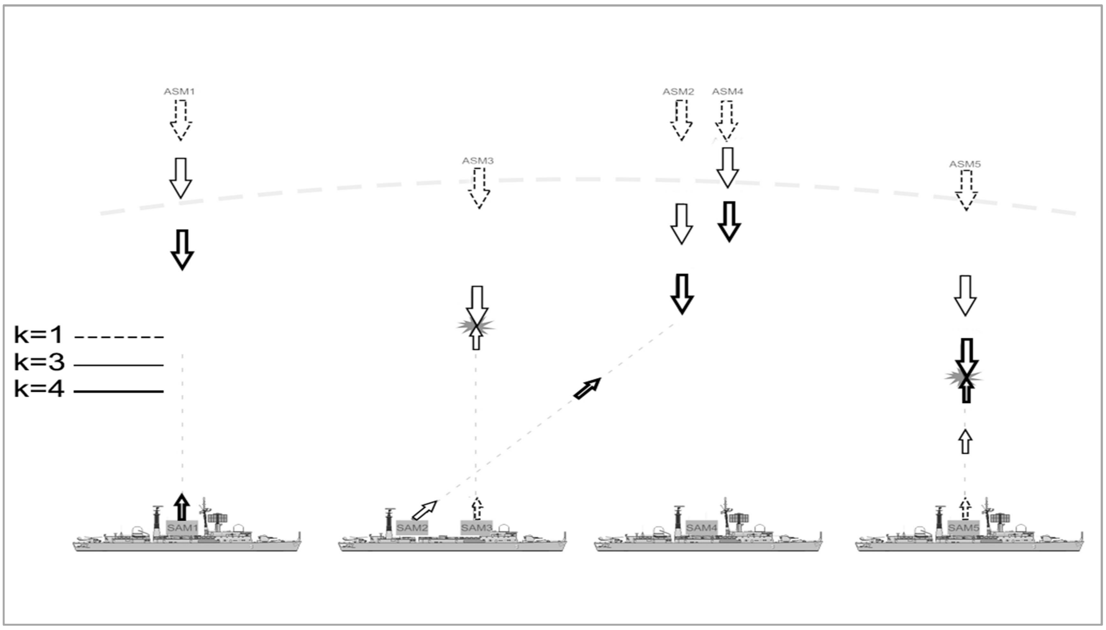

Figure 3.5 An illustration of air defense operation for the sample problem.

We present the computational results in Table 3.1. Average MIE values change between 0.091 and 0.312. The highest average MIE is attained when number of ASM is 6 and number of SAM system is 3. The lowest average MIE value, 0.091 and the lowest minimum MIE value, 0.019, are in the problem set with number of ASM is 3 and SAM system is 6. We get higher average MIE results as the number of ASM increases and the number of SAM system decreases. We have the highest maximum MIE value, 0.501 in a problem consists of 6 ASM and 3 SAM systems.

The similar results of MIE values are attained on the values of average MPIE metric. The efficiency of the TG improves 118.13% in the problems set with 6 ASM and 3 SAM systems. The lowest average MPIE value, 15.53%, is in problem set includes 3

ASM and 6 SAM systems. The highest maximum MPIE value, 244.39% is attained in problem set with 5 ASM and 4 SAM systems. Thus, we can increase the efficiency of the TG more than three times in this problem. We get the lowest minimum MPIE value, 2.17%, in problem set 3 ASM and 6 SAM systems. The improvement decreases when the number of SAM system increase or number of ASM decreases. Therefore, the effect of rescheduling approach is more notable with limited number of SAMs and with many threats.

MNDS value partially depends on number of non-dominated solutions. If number of non-dominated solutions increases, MNDS values also increase since the first extreme solution in the Pareto front has $Z_{ND} = 0$. The rescheduling time point, allocated SAM rounds for destroyed ASM and the available time slots according to the initial schedule affect the number of non-dominated solutions and also MNDS values. According to the results, we can say that there is not one to one relation between MNDS values and the number of SAM systems or the number of ASMs. However, we may say that MNDS values increases when problem size gets larger. The highest average MNDS value, 10 is in the problem set with 6 ASMs and 6 SAM systems. The lowest average MNDS value, 3.4 is in the problem set with 3 ASMs and 4 SAM systems.

APIE values depend on both MIE and MNDS values. We have the highest average APIE value, 28.15%, in problem set with 6 ASM and 3 SAM systems. The lowest average APIE value is attained as 3.42% in problem set with 3 ASM and 5 SAM systems. The highest maximum APIE value, 71.2%, is acquired in a problem consists of 5 ASM and 3 SAM systems. Thus, only one change on the initial schedule can increase 71.2% of the initial efficiency of the system. The lowest minimum APIE value, 0.54%, is in the problem having 3 ASM and 6 SAM systems. This result shows that in some problems we may need to change schedule more times in order to increase the survival probability. On the other hand, for some problems only one change in the initial engagement plan can be sufficient for efficiency of TG.

Table 3.1 Minimum, average, maximum values of metrics for destroyed ASM.

ASM			SAM			
			3	4	5	6
3	MIE	min	0.045	0.070	0.046	0.019
		ave	0.140	0.123	0.108	0.091
		max	0.194	0.212	0.174	0.225
	MPIE (%)	min	15.76	7.76	7.53	2.17
		ave	24.62	23.17	18.25	15.53
		max	33.39	66.89	24.78	51.20
	MNDS	min	1	2	5	3
		ave	3.8	3.4	5.6	4.2
		max	6	5	8	6
	APIE (%)	min	4.71	2.59	1.51	0.54
		ave	8.75	6.46	3.42	4.59
		max	19.52	13.38	4.96	17.07
4	MIE	min	0.092	0.129	0.062	0.047
		ave	0.202	0.197	0.120	0.130
		max	0.473	0.320	0.181	0.204
	MPIE (%)	min	10.42	16.67	13.42	6.56
		ave	92.30	64.65	28.73	20.64
		max	215.90	222.27	61.41	39.83
	MNDS	min	4	5	3	2
		ave	6	5.8	4.2	4
		max	10	7	5	6
	APIE (%)	min	1.04	3.33	2.76	2.19
		ave	20.31	10.98	7.87	4.97
		max	51.33	37.04	20.47	9.96
5	MIE	min	0.141	0.040	0.049	0.068
		ave	0.225	0.206	0.204	0.164
		max	0.316	0.380	0.372	0.347
	MPIE (%)	min	59.58	2.76	22.41	8.22
		ave	99.26	67.92	53.28	32.40
		max	213.60	244.39	82.99	76.06
	MNDS	min	3	2	3	4
		ave	5.6	4.6	6.8	5.4
		max	9	7	10	7
	APIE (%)	min	10.37	1.38	6.25	1.64
		ave	17.73	12.12	7.83	5.98
		max	71.20	40.73	10.37	10.87
6	MIE	min	0.230	0.179	0.152	0.074
		ave	0.312	0.202	0.206	0.182
		max	0.501	0.222	0.268	0.254
	MPIE (%)	min	70.94	58.11	33.31	34.39
		ave	118.13	78.73	76.88	46.27
		max	168.20	98.09	109.20	66.01
	MNDS	min	2	4	5	4
		ave	5.4	6.6	8	10
		max	8	9	13	14
	APIE (%)	min	8.87	6.83	6.66	3.44
		ave	28.15	12.89	10.03	5.64
		max	59.50	19.62	15.60	12.79

We present performance of heuristic approaches in Table 3.2. The last row of the each ASM and SAM combinations in Table 3.2 includes the number of non-dominated solutions generated by heuristics and augmented ε-constraint method respectively.

Table 3.2 Performance of heuristic approach for destroyed ASM.

		SAM			
ASM	Performance Metrics	3	4	5	6
3	HVR	0.9987	1	0.9995	0.9999
	IGD	0.0009	0	0.0007	0.0003
	FS (%)	95.83	100.00	96.77	95.65
	No. of Solutions*	23/24	22/22	30/31	22/23
4	HVR	1	0.9908	0.9998	0.9995
	IGD	0	0.0021	0.0009	0.0005
	FS (%)	100.00	97.06	92.00	95.83
	No. of Solutions	30/30	33/34	23/25	23/24
5	HVR	0.9947	0.9998	0.9963	0.9997
	IGD	0.0025	0.0001	0.0007	0.0002
	FS (%)	95.83	96.15	96.88	93.55
	No. of Solutions	23/24	25/26	31/32	29/31
6	HVR	0.9959	0.9921	0.9974	0.9937
	IGD	0.0005	0.0187	0.0025	0.0037
	FS (%)	93.10	88.89	95.00	91.49
	No. of Solutions	27/29	32/36	38/40	43/47

*No.of solutions metric includes the number of non-dominated solutions generated by heuristics and augmented ε-constraint method

The heuristic approaches generate all of the non-dominated solutions in problem sets with number of ASM and SAM systems are 3 and 4, 4 and 3 respectively. So, HVR values are 1 and IGD values are 0 in these problems. In problem sets with ASM and SAM combinations 3-3, 3-5, 3-6, 4-4, 4-6, 5-3, 5-4 and 5-5 only one non-dominated solution cannot be attained. Average HVR values are almost 1 and IGD values are nearly zero in these problems. Thus, the dominated solution approximates the non-dominated solution in these problems. In problem sets with 6 ASM, 4 SAM and 6 ASM and 6 SAM, four solutions cannot be generated by heuristics. We analyze the results of each problem in these problem sets and in only one problem two solutions cannot be attained by heuristics. In conclusion, almost all of the non-dominated solutions can be generated in all problem sets and even if a non-dominated solution cannot be found, a solution near to the non-dominated solution is attained by heuristics.

We compare the elapsed times of ε-constraint method and heuristic approaches. The results of elapsed times are depicted in Table 3.3. The elapsed times of problems depend on number of non-dominated solutions. Also, the problem characteristics such as valid engagement between SAM systems and ASMs, number of SAM rounds in each SAM systems, rescheduling time point affect the complexity of problem. So, the problem becomes computationally complex in some problems even though it is a small size problem in terms of the number of ASMs and the number of SAM systems. The results shows that in all problem sets, augmented ε-constraint method run times are significantly larger than those of heuristic approaches. Heuristic approaches find non-dominated solutions at most 0.4 seconds.

Table 3.3 Elapsed times (sec) for destroyed ASM.

	SAM							
ASM	3		4		5		6	
	ε-cons	heuristic	ε-cons	heuristic	ε-cons	heuristic	ε-cons	heuristic
3	2.70	0.16	3.00	0.11	4.43	0.28	4.30	0.28
4	4.31	0.18	4.63	0.22	5.24	0.34	5.61	0.20
5	5.46	0.11	6.49	0.12	13.10	0.40	10.23*	0.31
6	6.96	0.14	10.00	0.16	13.96**	0.14	13.61**	0.17

*One problem cannot be solved within 3600 seconds.
**Two problems cannot be solved within 3600 seconds.

The results show that the run times of augmented ε-constraint method increase when problems size increase. Besides, in problem set with 5 ASMs and 6 SAM systems, one problem cannot be solved by augmented ε-constraint method in two hours. Also, two problems cannot be solved within two hours in problem sets with 6 ASMs, 5 SAM systems and 6 ASMs and 5 SAM systems.

3.6.2 Computational Results for Breakdown of a SAM System Case

In the case of breakdown of a SAM system, we assume one of the SAM systems becomes unavailable to shoot after the engagement process starts. We randomly determine the time of breakdown during the engagement process and randomly choose the broken SAM system. We set the SAM as broken and set the rescheduling time point, *RT*, as the starting time of the following time slot. The unavailable SAM system is discarded from the engagement allocation plan of ASMs and probability of no-leaker values of each ASM is calculated. Available rounds, remaining time slots and upper bound on the number of engagements are updated with respect to the rescheduling time point.

We solve small size problems with augmented ε-constraint method and the results are given in Table 3.4. Average MIE value is 0.194 in problem set with 3 ASM and 3 SAM systems. Average MIE values decrease to 0.122 when the number of SAM system is 6 and the number of ASM is 3. The highest average MIE is achieved in problem set with 6 ASM and 3 SAM systems. Also, the highest maximum MIE value, 0.657, is attained in the problem including 6 ASM and 3 SAM systems. So, the probability of no-leaker value of TG increases 0.657 in this problem. The lowest minimum MIE value, 0.03, is in the problem with 3 ASM and 4 SAM systems.

Average MPIE values are highest in problem set with 6 ASM and 3 SAM systems. On the average, the probability of no-leaker value increases 122.27% by updating the whole schedule. The lowest average MIE value, 24.77%, is in problem set with 3 ASM and 6 SAM systems. The lowest minimum MPIE value, 6.26%, is in problem set with 3 ASM and 5 SAM systems. We get the highest maximum MPIE value, 199.35%, in the problem consists of 5 ASM and 3 SAM systems. Thus, we acquire about three times increment on percentage improvement in this problem. Therefore, if the number of SAM systems in TG gets larger, the effect of breakdown of a SAM system gets smaller. The necessity of rescheduling approach is more apparent while TG has small number of SAM systems.

The maximum MNDS values are between 4 and 12 and the minimum average MNDS values are between 1 and 6 among the problem sets. The lowest average MNDS value, 2.67, is in problem set having 3 ASM and 4 SAM systems. The highest average MNDS value, 8, is acquired in problem set with 5 ASM and 6 SAM systems. MNDS values vary with regard to the problem characteristics. The rescheduling time point, the feature of broken SAM system and the available time slots according to the initial schedule affect the number of non-dominated solutions and also MNDS values.

The values of APIE depend on both MIE and MNDS values. The highest average APIE value, 32.62% is in problem set with 5 ASM and 4 SAM systems. Thus, instead of changing the whole schedule only one change in the initial schedule can increase the survival probability 32.62% in this problem set.

The lowest average APIE value is attained as 4.01% in problem set with 3 ASM and 6 SAM systems. The highest maximum APIE value, 60% is in a problem consists of 5 ASM and 4 SAM systems. The lowest maximum, 0.65% is in a problem having 4 ASM and 5 SAM systems.

The results indicate that in some problems, the efficiency of TG can be increased significantly by slight changes on the initial schedule. So, only one change in the initial engagement plan can be sufficient for efficiency of TG. In some problems, change on initial engagement plan may not provide satisfactory probability of no-leaker value up to a certain degree. So, the initial plan may be changed entirely. On the other hand, disruption of schedule, in other words, number of difference on allocated SAM rounds between new and initial schedule may be crucial for air defense of TG.

Table 3.4 Minimum, average, maximum values for breakdown of a SAM system.

ASM			SAM			
			3	4	5	6
3	MIE	min	0.081	0.030	0.046	0.045
		ave	0.194	0.181	0.176	0.122
		max	0.409	0.336	0.335	0.196
	MPIE (%)	min	51.82	17.60	6.26	15.48
		ave	65.64	44.50	30.90	24.77
		max	96.00	96.00	67.07	31.85
	MNDS	min	2	1	3	6
		ave	5.25	2.67	6.67	6.33
		max	9	4	10	7
	APIE (%)	min	6.64	4.97	1.94	2.21
		ave	19.19	18.19	4.54	4.01
		max	48.00	32.00	9.58	5.31
4	MIE	min	0.123	0.085	0.043	0.110
		ave	0.196	0.187	0.175	0.137
		max	0.236	0.313	0.328	0.172
	MPIE (%)	min	51.12	13.35	7.83	18.06
		ave	82.13	45.38	44.57	36.33
		max	120.07	108.43	109.31	50.92
	MNDS	min	5	4	1	4
		ave	6.00	6	7	4.67
		max	7	7	12	6
	APIE (%)	min	7.30	1.91	0.65	4.52
		ave	14.62	10.35	8.53	7.67
		max	24.01	27.11	15.62	10.01
5	MIE	min	0.113	0.140	0.118	0.117
		ave	0.206	0.190	0.184	0.163
		max	0.299	0.243	0.298	0.250
	MPIE (%)	min	32.33	28.40	28.82	43.57
		ave	115.84	73.63	64.54	58.12
		max	199.35	132.50	94.64	77.73
	MNDS	min	4	1	1	4
		ave	5.5	3.67	6.76	8
		max	7	6	13	12
	APIE (%)	min	4.62	4.73	5.72	4.42
		ave	27.23	32.62	16.02	8.34
		max	49.84	60.00	28.82	10.89
6	MIE	min	0.017	0.136	0.158	0.067
		ave	0.243	0.192	0.197	0.163
		max	0.657	0.255	0.232	0.286
	MPIE (%)	min	72.80	31.52	16.55	50.05
		ave	122.27	92.54	65.50	61.86
		max	174.91	127.08	118.62	68.44
	MNDS	min	3	6	2	4
		ave	4.67	7	5.67	7.33
		max	7	8	10	10
	APIE (%)	min	17.01	3.94	6.13	5.00
		ave	28.34	13.98	12.71	10.11
		max	43.73	19.83	23.72	16.77

The performance of heuristic approach is presented in Table 3.5. The algorithms generate all of the non-dominated solutions in problem sets with ASM-SAM combinations of 3-3, 3-4, 4-3, 4-4, 4-6, 5-3, 6-3 and 6-5. So, HVR values are 1 and IGD values are 0 in these problems.

In problem sets 3-5, 3-6, 4-5, 5-4, 5-5 and 6-4 only one non-dominated solution cannot be attained. The HVR values are almost 1 and IGD values are nearly zero in these problems. Thus, the dominated solution approximates the non-dominated solution in these problems. In problem sets with 5 ASM and 6 SAM systems, 6 ASM and 6 SAM systems, two non-dominated solutions cannot be generated by heuristics. The heuristic approaches generate nearly all the non-dominated solutions in all problem sets. Hence, the performance of the heuristics are also quite well in breakdown of a SAM system case.

Table 3.5 Performance of heuristic approach for breakdown of a SAM system.

		SAM			
ASM	Performance Metric	3	4	5	6
3	HVR	1	1	0.9999	0.9998
	IGD	0	0	0.0001	0.0002
	FS (%)	100.00	100.00	95.00	95.24
	No. of Solutions	20/20	19/19	19/20	20/21
4	HVR	1	1	0.998	1
	IGD	0	0	0.0008	0
	FS (%)	100.00	100.00	96.67	100.00
	No. of Solutions	19/19	16/16	29/30	17/17
5	HVR	1	0.9976	0.9988	0.9862
	IGD	0	0.0002	0.0003	0.0014
	FS (%	100.00	91.67	93.75	92.00
	No. of Solutions	21/21	21/22	15/16	23/25
6	HVR	1	0.9968	1	0.9968
	IGD	0	0.0001	0	0.0008
	FS (%)	100.00	95.45	100.00	90.91
	No. of Solutions	13/13	21/22	19/19	20/22

The result of a problem with 5 ASM and 5 SAM systems is depicted in Figure 3.6. Only one non-dominated solution cannot be generated by heuristics which is the extreme point of the Pareto front.

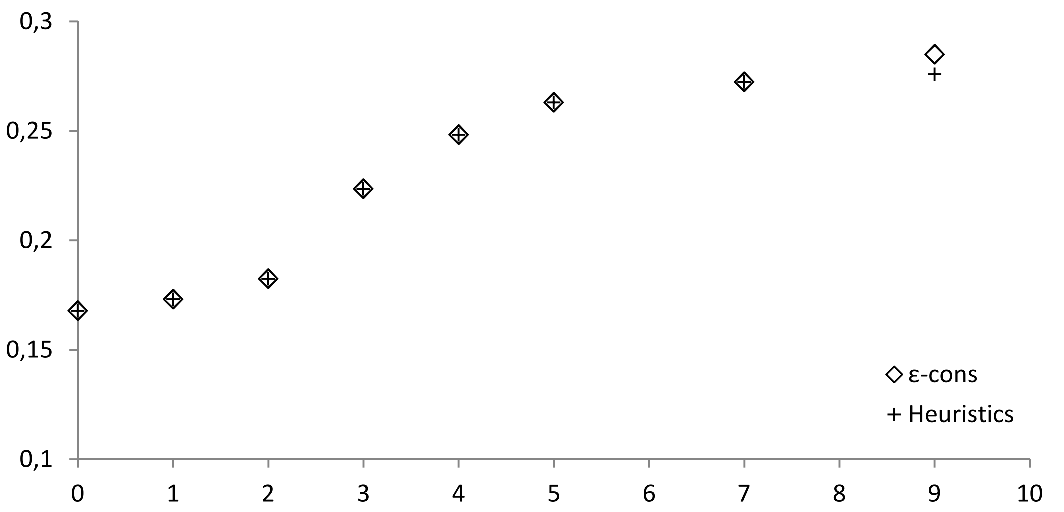

Figure 3.6 Non-dominated solutions of a problem with 5 ASM and 5 SAM systems.

We present the elapsed times of augmented ε-constraint method and heuristic algorithm in Table 3.6. In all problem sets, augmented ε-constraint method run times are greater than those of heuristic approaches. We cannot solve a problem within two hours in problem sets with 5 ASM and 5 SAM systems. Also two problems in problem set with 6 ASM and 6 SAM systems cannot be solved by augmented ε-constraint method in two hours. Since several problems cannot be solved within the time limit with augmented ε-constraint method, the non-dominated solutions of large size problems may not be found by augmented ε-constraint method. Heuristic approaches find non-dominated solutions at most 0.21 seconds.

Table 3.6 Elapsed times (sec) for breakdown of a SAM system.

ASM	SAM 3		SAM 4		SAM 5		SAM 6	
	ε-cons	heuristic	ε-cons	heuristic	ε-cons	heuristic	ε-cons	heuristic
3	2.05	0.08	4.64	0.07	6.12	0.21	7.57	0.25
4	5.25	0.09	4.69	0.08	7.27	0.19	11.25	0.21
5	8.62	0.10	6.29	0.10	9.24	0.13	11.45*	0.20
6	7.16	0.12	12.16	0.10	10.21	0.21	16.44**	0.14

*One problem cannot be solved within 3600 seconds.
**Two problems cannot be solved within 3600 seconds.

3.6.3 Computational Results for New ASM Target Case

We assume that sensors of TG detect an unexpected incoming ASM after the engagement process is started and the initial allocation plan is in operation. If the initial schedule is kept, new ASM destroys its target ship since it is not considered at the beginning of the engagement process. To solve the model, we first update the current threats. We randomly determine the target ship of new incoming ASM, distance of the ASM, and velocity of ASM. We find the valid combinations, the available time slots of engagements and the maximum number of engagements between the new ASM and SAM systems. We randomly determine the time of arrival from Poisson distribution. We set the rescheduling time point, *RT*. Available rounds, remaining time slots and upper bound on the number of engagements are updated with respect to the rescheduling time point.

We use the same problems of first and second case and solve with augmented ε-constraint method. In this case, without rescheduling the efficiency of TG is zero since new threat has zero probability of no-leaker value. Thus, MPIE and APIE cannot be calculated. To find the MPIE and APIE performance metrics, we assume that one of the available SAM round is allocated against the new ASM. The existing schedule is set with this new allocated SAM round and the metrics relevant to efficiency objective is attained according to this assumption. The results are given in Table 3.7.

The highest average MIE value, 0.236, is in problem set with 6 ASM and 3 SAM systems. It decreases to 0.179 when number of ASM and SAM system is 6. The lowest average MIE value, 0.142, is attained in problem set with 3 ASM and 6 SAM systems. The problem with 6 ASM and 5 SAM systems has the highest maximum MIE value, 0.600 and the lowest minimum MIE value, 0.012.

The highest average MPIE value, 150.48%, in the problem set with 6 ASM and 3 SAM systems. Thus, if the whole schedule is updated, the efficiency of the system can be improved nearly two and half on the average. The lowest average MPIE value, 48%, is occurred in the problem set with 3 ASM and 6 SAM systems.

The minimum and maximum average MNDS values are 3.33 and 9.33 in problem sets with 4 ASM-5 SAM systems and 6 ASM-6 SAM systems respectively. In problem set with 3 ASM and 3 SAM systems, the lowest MNDS value, 1, is acquired. So, only one non-dominated solution is generated in this problem. The problem with 5 ASM and 6 SAM systems has the highest MNDS value, 13.

The average APIE values are between the range of 8.62% and 35.65%. The minimum and maximum APIE values are between the range of 1.73%-16.20% and 13.29-63.0% respectively. In some problems, one unit change on schedule improves the efficiency of the system in the event of a new incoming threat and it may be sufficient for survival of TG.

As a result, we conclude that our rescheduling approach can increase the survival probability of TG by making accurate change on the engagement allocation plan. Preference of the DM about efficiency and stability of schedule determines the updating decision.

Table 3.7 Minimum, average, maximum values of metrics for new ASM target.

ASM			SAM 3	SAM 4	SAM 5	SAM 6
3	MIE	min	0.172	0.046	0.107	0.052
		ave	0.220	0.193	0.146	0.142
		max	0.271	0.330	0.203	0.280
	MPIE (%)	min	52.85	43.08	43.04	17.27
		ave	83.80	63.75	51.34	48.00
		max	98.12	99.17	64.00	80.00
	MNDS	min	1	4	4	2
		ave	5.2	5.67	5	8.5
		max	10	7	7	12
	APIE (%)	min	9.81	7.18	6.15	1.73
		ave	24.94	11.20	11.30	12.58
		max	52.85	14.17	16.00	40.00
4	MIE	min	0.090	0.047	0.136	0.069
		ave	0.221	0.204	0.169	0.157
		max	0.391	0.286	0.237	0.260
	MPIE (%)	min	56.80	58.28	41.56	42.50
		ave	127.51	89.96	63.85	51.19
		max	200.14	144.12	75.00	75.00
	MNDS	min	6	6	3	3
		ave	6.67	8.67	3.33	7.8
		max	8	10	4	11
	APIE (%)	min	9.47	6.75	10.39	4.22
		ave	19.51	10.29	20.13	8.62
		max	33.36	14.41	25.00	15.00
5	MIE	min	0.139	0.057	0.027	0.089
		ave	0.226	0.203	0.163	0.156
		max	0.296	0.282	0.413	0.248
	MPIE (%)	min	80.99	43.82	34.57	54.47
		ave	146.58	111.69	110.45	77.48
		max	189.03	200.62	209.76	98.39
	MNDS	min	5	4	5	5
		ave	6.5	6.5	7.6	8.67
		max	7	10	11	13
	APIE (%)	min	16.20	4.38	4.32	6.81
		ave	22.10	20.60	16.33	10.10
		max	27.00	33.44	33.88	15.92
6	MIE	min	0.166	0.145	0.012	0.044
		ave	0.236	0.203	0.185	0.179
		max	0.286	0.261	0.600	0.302
	MPIE (%)	min	122.75	74.78	55.74	53.98
		ave	150.48	131.89	108.47	96.28
		max	177.39	189.00	145.82	159.52
	MNDS	min	5	3	7	8
		ave	6.33	6	8.8	9.33
		max	8	9	11	12
	APIE (%)	min	15.34	8.31	6.97	6.75
		ave	25.35	35.65	12.42	9.82
		max	35.48	63.00	17.64	13.29

We also test the performance of the heuristic approach in new ASM target case and generate non-dominated solutions. Comparison of the heuristic approach and the augmented ε-constraint method and results of performance metrics are depicted in Table 3.8. The heuristics find all of the non-dominated solutions in problem sets with ASM-SAM system combinations of 3-4, 3-5, 4-5, 6-4 and 6-6. Although only one non-dominated solution cannot be generated in the problem set with 3 ASM and 3 SAM systems, the worst average value, 0.0024, is occurred in this problem. Also, the minimum HVR value is also in this problem set.

Table 3.8 Performance of heuristic approach for new ASM target.

		SAM			
ASM	Performance Metric	3	4	5	6
3	HVR	0.9919	1	1	0.9998
	IGD	0.0024	0	0	0.0002
	FS (%)	96.97	100.00	100.00	97.22
	No. of Solutions	32/33	20/20	18/18	35/36
4	HVR	0.9933	1	1	0.9995
	IGD	0.0018	0	0	0.0005
	FS (%)	90.00	100.00	100.00	97.62
	No. of Solutions	18/20	26/27	20/20	41/42
5	HVR	0.997	0.9997	0.9997	0.9979
	IGD	0.0003	0.0001	0.0001	0.0004
	FS (%)	95.00	97.44	97.44	96.43
	No. of Solutions	19/20	27/29	38/39	27/28
6	HVR	0.9981	1	0.9929	1
	IGD	0.0001	0	0.0017	0
	FS (%)	93.75	100.00	91.43	100.00
	No. of Solutions	15/16	13/13	32/35	28/28

In problem set with 6 ASM and 5 SAM systems, 32 of 35 non-dominated solutions are found by heuristics. The heuristic algorithms yield the minimum performance in this problem set. The average HVR values of all problem sets are greater than 0.99. Also, The HVR values average IGD values are nearly zero in all problem sets. Thus, found solutions are diverse enough despite a few non-dominated solutions cannot be generated in some problem sets. As a result, the performances of the heuristics are also quite well in new threat case.

The elapsed times of augmented ε-constraint method and heuristic algorithm are given in Table 3.9. Heuristic approach finds non-dominated solutions at most 0.52 second. When the problem size gets larger, the run times of the augmented ε-constraint method increase. Moreover, we cannot solve a problem within two hours in problem sets with 5 ASM, 5 SAM systems and 5 ASM and 6 SAM systems. Also two problems in problem set with 6 ASM-5 SAM systems and 6 ASM-6 SAM systems cannot be solved by augmented ε-constraint method in two hours.

Table 3.9 Elapsed times (sec) for new ASM target.

	SAM							
ASM	3		4		5		6	
	ε-cons	heuristic	ε-cons	heuristic	ε-cons	heuristic	ε-cons	heuristic
3	6.51	0.13	6.36	0.25	7.40	0.16	9.53	0.13
4	4.92	0.07	9.82	0.13	8.64	0.11	11.34	0.23
5	9.15	0.09	13.93	0.15	14.01*	0.15	11.46*	0.17
6	10.19	0.09	5.41	0.12	11.52**	0.15	50.16**	0.52

*One problem cannot be solved within 3600 seconds.

**Two problems cannot be solved within 3600 seconds.

In all disturbance cases, we define each problem set by the number of ASM and SAM systems varied in the range of between three and six. In a different air defense scenario, attacking side may want to suppress air defense of TG using saturating tactics. The number of ASMs attacking to TG may be greater than number of SAM systems in a naval air defense operation. The computational results show that with limited number of SAM systems and many threats, the effect of responding to the disturbances and rescheduling of SAM rounds increase the effectiveness of the air defense. Hence, when there is a large number of attacking ASMs, it is obvious that the dynamic tactical planning and rescheduling of SAM rounds during the engagement process is more crucial to destroy ASMs and survive ships.

The solution procedure of BMRP includes generating whole Pareto front with respect to efficiency and stability objectives. After the disturbance, our aim is to maximize efficiency of air defense while minimizing deviation from the initial schedule. In an air defense operation, keeping the initial schedule of SAM systems as much as possible is essential since the initial preparations such as slewing sensors and weapons, tracking targets, solution of fire control problem and starting time of these sequential operations are determined due to initial schedule. The updated schedule with a particular degree of deviation from the initial schedule is more realistic than updating the whole schedule by ignoring the deviation. The result of complete rescheduling may usually be inapplicable in real life. For instance, with complete rescheduling, new beginning time of shoots in a SAM system schedule against ASMs can be earlier than the one at the initial schedule. It may cause the SAM system to not be able to track on target ASMs due to the restrictions imposed by operations such as receiving information from sensors, guidance and implementing the track management. Hence, while desiring to increase the effectiveness of air defense by rescheduling, an inexecutable new schedule can be attained with complete rescheduling.

CHAPTER 4

A DYNAMIC APPROACH TO BMRP IN A SEMI-AUTONOMOUS DECISION MAKING FRAMEWORK

Decision making in multiobjective optimization problems requires preference information from the DM. The preference based methods for multiobjective optimization problems can be classified as a priori, interactive and a posteriori with respect to time of DM preferences are included in the model (Mavrotas (2009)). In a priori methods, DM expresses his/her preferences before the solution process. In interactive approaches, preferences are articulated progressively during the solution process and after several iterations the most preferred one is found. In a posteriori methods all non-dominated solutions of the problem are generated then the preferences are considered in order to find the most preferred one.

BMRP involves two objectives to be optimized and these objectives conflict with each other. A number of non-dominated solutions are generated in a rescheduling time point and numerous disturbances may occur during the engagement process. To update the engagement allocation plan in each rescheduling time point, we must develop a semi-autonomous decision making framework by choosing one of the non-dominated solutions from the results of BMRP. In real life, initial schedule is changed within the engagement process and there is not enough time to interact with DM as the schedule has to be updated in a few seconds. We should extract preference information before the engagement process. Hence, we construct a decision model with an artificial neural network method by asking DM to assign utility values to priory generated non-dominated solutions. In each rescheduling time point, the structured ANN acts like a DM for choosing the most preferred solution. To update the schedule in response to a disturbance, non-dominated solutions are presented to ANN. However, finding all non-dominated solutions of BMRP is hard, time consuming and not practical. To discard not preferable non-dominated solutions while generating, we assume that DM utility is consistent with non-decreasing quasi-concave function. The cone domination principle of the non-decreasing quasi-concave function is incorporated into the solution procedure. The dominated cones in objective space are generated according to the preference of ANN and we reduce the feasible objective region iteratively. It ensures discarding Pareto optimal solutions that are no interest to the DM and reducing the computational times.

4.1 Artificial Neural Network

Initial schedule of SAM systems is changed within the engagement process and there is no enough time to interact with DM during the engagement process just because the problem has to be solved in a few seconds. Also a posterior approach is not applicable since interaction with DM is still required during the engagement process. However, it is essential to decide an efficient solution most preferred by DM. Thus, we have to extract preference information before the engagement process. The prior articulated preferences of DM can be used to construct a decision model that represents the utility of DM properly. In order to find a decision model from prior decision examples of DM, one of the most popular approach is preference disaggregation. Preference disaggregation is based on inferring a decision model from a set of evaluated examples by the DM (Jacquet-Lagrezea and Siskos (2001)). The preference disaggregation approaches aim at learning the cognitive behavior of the DM. Doumpos and Zopounidis (2013) review the literature on the implementation of statistical learning methods for disaggregation of preference information. They also discuss the connection of machine learning methods with preference disaggregation.

ANNs are one of the most popular approaches in machine learning and they can model highly complex problems with unknown underlying structure. An ANN consists of a set of nodes that are connected with links. The weight of the links indicates the strength of the connection. If the nodes are hierarchically structured into layers and directed arcs connect lower layers to higher layers, the neural network is called as feedforward artificial neural network (FFANN). FFANN are very successful in representing complex patterns (Sun et al. (2000)). Multilayer perceptron stands for a neural network with one or more hidden layers.

A neural network propagates in forward and backward phase. In the forward phase, the input value is propagated through the network in proportion to the weights until it reaches the output node. In the backward phase, the actual output of the network and the desired output are compared and an error value is produced. The error value is propagated through the network in the backward direction. The direction of search in weight space is calculated and with this information weight values are updated in each iteration. The back propagation algorithm proposed by Rumelhart et al. (1986) is one of the important method for weight update in ANN.

The objective and weight relationship of ANN provides a representation of the DM's preferences. A number of studies in literature show structuring DM preferences with ANN in multiobjective problems. Wang and Malakooti (1992) propose a feedforward neural network model to capture the DM’s preferences by an adaptive learning algorithm. Sun et al. (1996) propose an interactive approach with incorporating adaptive neural network to solve continuous solution space problems. They either assign a utility value to the solutions or make pairwise comparisons to calculate the principal eigenvector of the reciprocal comparison matrix. Sun et al. (2000) combine the augmented Tchebycheff approach and ANN strategy. They train the network with response of DM and use all the response of DM as an output. The proposed approach is based on the reduction of weights in Tchebycheff method with respect to result of the ANN. Chen and Lin (2003) propose a feedforward ANN approach to solve continuous solution space problem. They ask to indicate pairwise comparison in terms of approximate ratios or intervals.

To incorporate the DM's preferences into the model, we model an adaptive learning algorithm instead of back propagation algorithm in an ANN structure. Because, the back propagation algorithm uses steepest descent method and constant learning parameter to update the weights. It may require too many epochs to learn the desired behavior of the data (Wang and Malakooti (1992)). Also, it may converge into local minima. We define a multilayer neural network structure with four hidden nodes. Two input nodes are used since objective function values Z_{PNL} and Z_{ND} are taken as input of ANN. The utility value of each solution is taken as output of the ANN. The utility values are calculated from a non-decreasing quasi-concave utility function, $\mathrm{f}(z) = \max \sum_{i=1}^{p=2} w_i \left(z_i - z_i^{IP}\right)^2$ where z_i is the i^{th} objective function value and z_i^{IP} is the ideal point of i^{th} objective function. Firstly, the weight of objective functions are set as $w_1 = 0.5$ and $w_2 = 0.5$. The network is trained in forward phase by normalizing objective functions and utility values. We compute the rescaled utility values and normalize each of the objective function with respect to ideal and nadir points as follows.

$$z_i^{'} = \frac{z_i - z_i^{nad}}{z_i^{ideal} - z_i^{nad}}$$

We use sigmoid function in activation of nodes. To find the direction in updating weights of network, Polak-Ribiere conjugate gradient direction method is implemented since it has a superlinear convergence rate and it provides faster convergence then steepest descent (Sun et al. (2000)). To specify the line, the golden section search algorithm is used. It evaluates the points starting at a distance of delta and doubling in distance each step along the search direction. When the minimum interval is found, it reduces the size of the interval and determines the best point within the interval. Additional information for the adaptive learning algorithm can be found in Wang and Malakooti (1992) and Sun et al. (2000).

The ANN with multilayer structure includes input nodes, hidden nodes and output nodes which are indexed by i, j and k respectively. Let the input value of hidden node j is $v_j(n)$ and input value of output node is $v_k(n)$. $y_j(n)$ denotes the signal produced as an output of hidden node j and $y_k(n)$ is the output value of output node k. Assume there are n number of input nodes and N number of hidden nodes. The steps of the adaptive learning algorithm are as follows:

Steps of the adaptive learning algorithm

Step 0 (Initialization):

Set w^0 randomly and $t = 0$.

Step 1 Train the network in forward phase:

In forward phase, the produced input signal for hidden node j is $v_j^t = \sum_{i=0}^{n} w_{ji}^t . \mathrm{y}_i^t$. The output signal value of hidden node j is $y_j^t = \varphi_j(\mathrm{v}_j^t)$ where $\varphi(.)$ is the sigmoid function utilized for the activation of all nodes. Thus, the output value is $y_j^t = \frac{1}{\left(1+e^{-v_j(\mathrm{t})}\right)}$. The input signal value of output node k is: $v_k^t = \sum_{i=0}^{n} w_{ji}^t \, \mathrm{y}_i^t + \sum_{j=0}^{N} w_{kj}^t \, \mathrm{y}_j^t$ and actual output value is $y_k^t = \varphi_k(\mathrm{v}_k^t)$.

Step 2 Error calculations:

Performance of the error function is as follows:

$$E\left[w^t\right] = 0.5\sum_{p=1}^{P}\left(z^p - y^p\right)^2$$

where P is the number of pattern and z^p is the desired output. By taking the derivative of the total error and according to the chain rule,

$$\frac{\partial E}{\partial w_{ji}} = \frac{\partial E}{\partial e_j} \cdot \frac{\partial e_j}{\partial y_j} \cdot \frac{\partial y_j}{\partial v_j} \cdot \frac{\partial v_j}{\partial w_{ji}}$$

Step 3 Calculation of weight correction

Step 3.1 Calculation of g^t from Chain rule:

- For w_{kj} ; $g^t = -\sum_{p=1}^{P} \left(z^p - y_k^p\right).\left(1 - y_k^p\right).y_k^p.v_j^p$
- For w_{ki} ; $g^t = -\sum_{p=1}^{P} \left(z^p - y_k^p\right).\left(1 - y_k^p\right).y_k^p.v_i^p$
- For w_{ji} ; $g^t = -\sum_{p=1}^{P} \left(z^p - y_k^p\right).\left(1 - y_k^p\right).y_k^p.y_j^p.\left(1 - y_j^p\right).v_i^p.w_{jk}.y_i$

Step 3.2 Finding the Polak-Ribiere conjugate direction. If $t = 0$, $d(t) = -g(t)$, otherwise;

$$\beta^{t-1} = \frac{\left[g^t - g^{t-1}\right].\left(g^t\right)^T}{\left[g^{t-1}.\left(g^{t-1}\right)^T\right]} \text{ and } d^t = -\left(g^t\right)^T + \beta^{t-1}.d^{t-1}$$

Step 3.3 Calculation of interval L for line search

- If $E\left[w^t\right] < E\left[w^t + \gamma d^t\right]$, then $L = \gamma$ and go to step 3.4,
- If $E\left[w^t + \gamma d^t\right] < E\left[w^t + 2\gamma d^t\right]$, then $L = 2\gamma$ and go to step 3.4, otherwise
- $w^t = w^t + 2\gamma d^t$ and repeat step 3.3.

Step 3.4 Golden section line search

- $w^{t'} = w^t + 0.382Ld^t$ and $w^{t''} = w^t + 0.618Ld^t$

- If $E\left[w^{t'}\right] < E\left[w^{t''}\right]$ then $w^{t''} = w^{t'}$ and $w^{t'} = w^{t} + 0.236Ld^{t}$ otherwise $w^{t} = w^{t'}$, $w^{t'} = w^{t''}$ and $w^{t''} = w^{t} + 0.382Ld^{t}$

Step 3.5 Find the weights within the interval

- If $\left|E\left[w^{t'}\right] - E\left[w^{t''}\right]\right| < \rho$ then $w^{t} = \left(w^{t'} + w^{t''}\right)/2$, go to step 4 otherwise $L = 0.618L$ and go to step 3.4

Step 4 Check for termination

- If $E\left[w^{t+1}\right] \le \varepsilon$, then $w^{*} = w^{t+1}$,

otherwise go to step 5

Step 5 Restarting

- If $t > N(n+2)+n+1$, then set $t = 0$; $w^{0} = w^{t+1}$ and go to step 1,

otherwise $t+1 = t$ and go to step 1.

We construct the topology of the ANN by training the non-dominated solutions. A set of non-dominated solutions that are assumed to be generated in a past air attack and in a rescheduling time point are used to train the network. The scaled objective function values and the corresponding scaled utility values of fourteen non-dominated solutions are presented in Table 4.1. The last column of the Table 4.1 indicates the result of ANN. The results show that ANN completely represents the utility function. It ranks all the solutions correctly and generate values almost same as the desired output values. The approach ensures that ANN learns the DM preferences.

Table 4.1 Input and output values of ANN.

No	efficiency (input 1)	stability (input 2)	utility (desired output)	ANN results
1	0.2	0	0.6792	0.67997
2	0.28	0.08	0.7393	0.73997
3	0.29	0.15	0.7361	0.73418
4	0.3	0.23	0.7263	0.72407
5	0.31	0.31	0.7146	0.71419
6	0.32	0.38	0.6948	0.69627
7	0.33	0.46	0.6684	0.67103
8	0.33	0.54	0.6326	0.63422
9	0.34	0.62	0.5895	0.58784
10	0.37	0.69	0.5644	0.56773
11	0.38	0.77	0.5101	0.50852
12	0.39	0.85	0.4547	0.45144
13	0.39	0.92	0.3897	0.38637
14	0.41	1	0.3248	0.32966

4.2 Quasi-Concave Utility Function

In multiobjective problems, it is very hard to know explicitly the functional form and parameters of the DM utility. But, the existence of an underlying utility function of some form can be assumed. A non-decreasing quasi-concave value function has considerably used in order to represent human behavior (Silberberg (1978); Crouch (1979)). The property of a diminishing marginal rate of substitution is peculiar to quasi-concave functions. It is obvious that efficiency and stability objectives have the property of diminishing marginal rate of substitution. Because, at the minimum efficiency value, DM may sacrifice more units of stability and at the higher efficiency values DM do not allow same increment on the disruption of the schedule while maintaining the same level of utility. Korhonen et al. (1984) introduce a solution approach for discrete multiobjective problems while DM has an implicit non-decreasing quasi-concave utility function. We briefly review their theorem and present definitions about quasi-concave functions as follows:

Definition 1. A function $f(x)$ is said to be non-decreasing if $f(x) \geq f(y)$ for all $x > y$ where $x, y \in X$.

Definition 2. A real valued function $f(x)$ defined over a convex set X in R is called quasi-concave if $f(\lambda x + (1-\lambda) y) \geq \min\{f(x), f(y)\}$ for all $x, y \in X$ and for all $\lambda \in [0,1]$.

Theorem 4.1. (Korhonen et al. (1984))

Assume $f(x)$ is a real-valued, quasi-concave and non-decreasing function defined in p dimensional Euclidean space R^p. Consider distinct points $x_i \in R^p, i = 1, ...m$ and $y \in R^p$, and assume that $f(x_k) < f(x_i)$, $i \neq k$. Then if $\varepsilon \geq 0$ in the following linear programming problem;

$$Max\ \varepsilon$$

subject to

$$\sum_{i=1}^{m} \mu_i . (x_k - x_i) - \varepsilon \geq y - x_k$$

$$\mu_i \geq 0 \quad \forall i = 1....m$$

it follows that $f(x_k) \geq f(y)$.

The theorem states that the pairwise preference decision such as x_i is preferred to x_k generates a convex cone. The points inferior to this cone are called as cone dominated solutions. If any point, y falls in the cone or is dominated by the cone, then it is at most as preferred as x_k and less preferred than x_i. The characteristic of non-decreasing quasi-concave utility function provide determining the cone of inferior solutions. The feasible region of the solution space is reduced with respect to DM responses by excluding the cone dominated points. Lokman et al. (2014) propose an interactive method for quasi-concave and non-decreasing functions. They obtain the region that is not cone dominated by adding some inequalities to the problem and exclude cone dominated solutions in each iteration by using these inequalities.

To generate non-dominated solutions, one of the method that we use is augmented ε-constraint method. In each iteration, we define a constraint to BMRP model according to results of pairwise comparison. The non-dominated solutions belongs to cone dominated regions are excluded from solution space and the non-dominated solutions that are not preferred are discarded while generating a new non-dominated solution. Figure 4.1 shows the inequality and cone dominated region for a pairwise comparison result. In the objective space x axis represents Z_{ND} objective function and y axis represents Z_{PNL} objective function. If y^s is preferred to y^m, the new constraint, (C1) ensures not generating cone dominated solutions, namely, less preferred solutions.

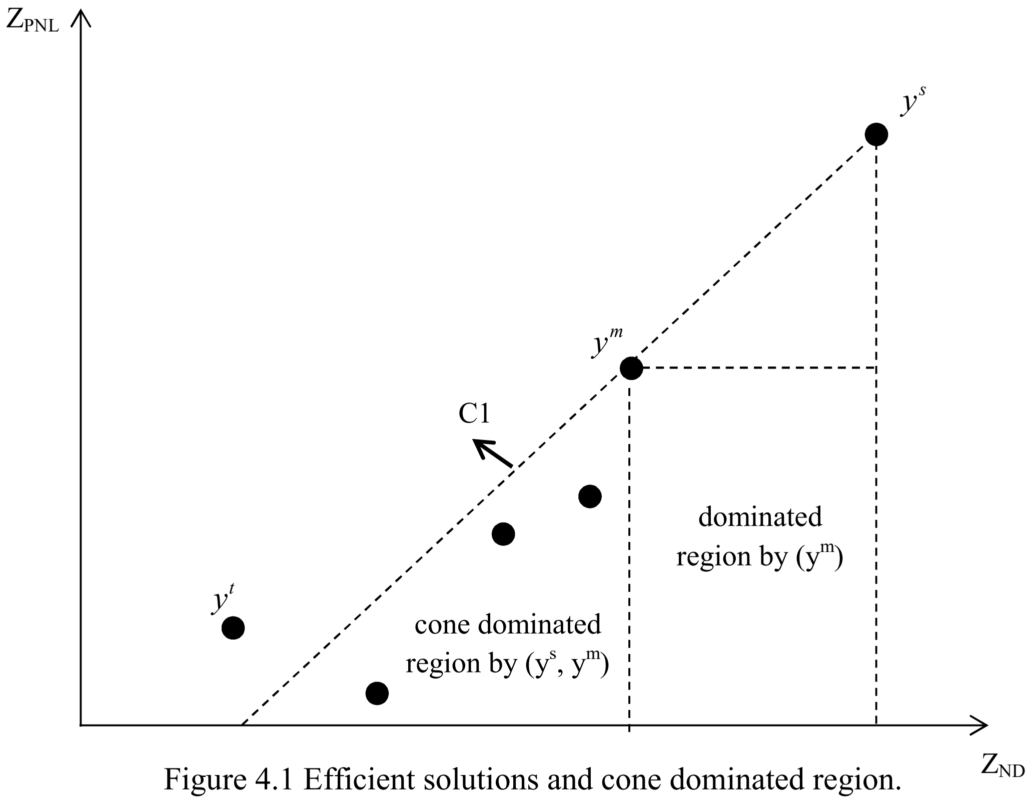

Figure 4.1 Efficient solutions and cone dominated region.

To discard cone dominated solutions, one constraint is added to the augmented *ε-L-BMRP* model. The formulation of the constraint is as follows:

$$y^s_{PNL}.(y^m_{ND} - y^t_{ND}) + y^s_{ND}.(y^t_{PNL} - y^m_{PNL}) \geq y^t_{PNL}.y^m_{ND} - y^m_{PNL}.y^t_{ND} + \varepsilon \quad \text{(C1)}$$

4.3 Solution Procedure

We develop two solution procedures augmented ε-constraint method and heuristic approach to generate results of BMRP model in each rescheduling time point. The dynamic allocation of SAM rounds in each rescheduling time point is constituted by solution of BMRP formulation.

Figure 4.2 depicts the representation of update scheme for engagement allocation plan. Three disturbances occur during the engagement process. In each rescheduling time point, several non-dominated solutions are generated and one of them is chosen among the non-dominated solutions.

The schedule that is generated at the beginning of the engagement process according to the probability of no-leaker of TG is called as initial schedule in BMRP. After the first disturbance happens, the initial schedule is updated and the new schedule is called existing schedule. Thus the initial schedule is the schedule used before the first disturbance occurred and the new schedule is the schedule created by rescheduling.

In first rescheduling time point, the number of disruption values for each non-dominated solutions of BMRP are calculated with respect to initial schedule. In the event of the following disturbances, the existing schedules are updated and the number of disruption values for each non-dominated solutions of BMRP are generated with respect to the existing schedules.

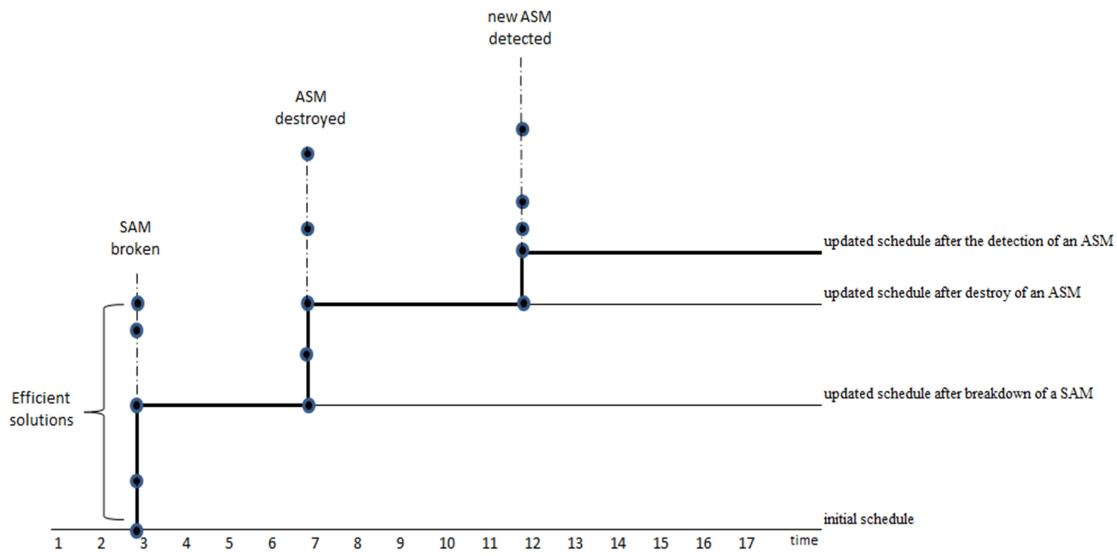

Figure 4.2 The representation of updated schedule in time horizon.

4.3.1 Solution Procedure with Augmented ε-Constraint Method

The solution method with augmented ε-constraint method consists of three main parts. The first part is the solution of BMRP with augmented-ε-constraint method. In this part, non-dominated solutions are generated. The pairwise comparison of two solutions with ANN is the second part of approach. The last part is the use of pairwise comparison result of ANN for quasi-concave utility function in order to reduce the solution space and eliminate non-preferred solution in the next iteration.

The steps of generating non-dominated solutions and choosing one of them in a rescheduling time point is presented as follows:

Steps of the solution procedure with augmented ε-constraint method

Step 0 (Initialization)

Set the iteration counter, t=0 and set $\theta = \infty$.

Solve augmented ε-L-BMRP model, if $Z_{ND} = 0$, Pareto Front has only one solution, Stop. Otherwise, set the point with maximum disrupted allocation as initial incumbent solution, $z^c = y^0$.

Step 1 (Solve augmented *ε-L-BMRP*)

Set t=t+1.

Solve augmented ε-L-BMRP model. If infeasible Stop, otherwise set the generated point as challenger solution $z^{inc} = y^{t+1}$. Set $\theta = Z_{ND}$ and go to step 2.

Step 2 (Comparison)

Ask neural network to compare z^c and z^{inc}.

If $z^{inc} \succ z^c$, go to step 3.

If $z^{inc} \prec z^c$, $z^{inc} = z^c$ and go to step 1.

Step 3 (Solve augmented *ε-L-BMRP* with new constraint)

Set t=t+1.

Solve augmented ε-L-BMRP model with constraint C1. If infeasible Stop, otherwise set $z^c = y^t$ and $\theta = Z_{ND}$. Go to step 2.

The solution procedure is based on generating solutions from maximum Z_{ND} to minimum Z_{ND} in the objective space. The algorithm starts with generating the first non-dominated solution and specifying the initial point, as current incumbent point, z^{inc}. Step 0 produces one of the extreme points of the efficient frontier that has maximum Z_{ND}. In Step 1, in order to present ANN to compare two solutions, we solve augmented *ε-L-BMRP* model. If the problem is infeasible, then we have only one non-dominated solution. Otherwise the new non-dominated solution y^{t+1} is generated that has at least one Z_{ND} and y^{t+1} is set as current challenger point, z^c. Since there are two non-dominated solutions, we ask ANN to compare y^0 and y^1. If

ANN decides to prefer the solution with lower Z_{ND}, we do not need cone domination constraints since all of the solutions with higher Z_{ND} have already been generated. On the other hand, if ANN prefers the solution that has higher Z_{ND} value, then a new constraint is incorporated to the model to exclude cone dominated solutions. Step 2 is the decision part of the algorithm. If the challenger point y^1 is preferred, then new incumbent point is y^1. We again solve augmented *ε-L-BMRP* model and generate new challenger point y^2. If y^0 is preferred rather than challenger point y^1, then y^0 is still incumbent point and the ($y^0 \succ y^1$) characterize a cone dominated region and the points dominated by this cone is discarded by including new constraint. The solution procedure generates and determines the most preferred non-dominated solution. The command and control unit orders the updated engagement allocation plan to the SAM systems and new schedule is performed. In each rescheduling time point, the existing engagement schedule is updated according to the most preferred solution.

4.3.2 Solution Procedure with Heuristic Approach

In Chapter 3, we propose NRH and CEH heuristic algorithms. We suggest using those algorithms to generate non-dominated solutions of the problem. Since heuristic approaches performance is quite well and generate nearly all the non-dominated solutions due to preliminary runs, we implement those to solve large size problems.

In heuristic approach, we start from zero number of disruption, Z_{ND} and increase Z_{ND} by one in each iteration. We generate solutions up to the maximum number of possible disruption for each integer Z_{ND} value. We choose the solution with maximum probability of no-leaker among these ones. After all solutions are generated for different Z_{ND}, we determine the non-dominated solutions. We find numerous solutions and obtain non-dominated ones among them in heuristic approach. Thus, in contrast with ε-constraint method, the non-dominated solutions are not generated step by step. This indicates that cone dominated solutions cannot be excluded in each iteration. Instead, ANN as a DM provides us not to choose these cone dominated solution by utility function representation. The steps of the procedure are as follows:

Steps of the solution procedure with heuristic approaches

Step 0 (Generate solutions)

Generate non-dominated solutions with NRH and CEH algorithms.

If Pareto Front has only one solution, then $Z_{ND} = 0$, stop. Otherwise,

Set *n*=number of non-dominated solutions and go to step 2.

Step 1 (Comparison)

Ask neural network to find most preferred solution. $z^* = \underset{n}{argmax}\{U(z_n)\}$.

where $U(z_n)$ is the utility value of solution n.

4.4 Computational Experiments

In this section, the computational results are presented in order to show the effectiveness of the rescheduling approach. The procedure for generating sample problems, the performance measures and discussion on the results are presented.

4.4.1 Generation of the Problems

We generate sample problems by using the properties of real weapon systems in open literature. We define seven different SAM systems and seven different ASM systems. Each problem set is specified with the number of ASM and SAM systems. By using different random number streams, 16 different problem sets are created with the number of ASM and SAM systems ranging from 3 to 6. Each problem set includes five different problems that are randomly generated. The maximum numbers of engagements that can be done against ASMs are calculated according to effective range of SAM systems, velocity and distance of ASMs. The numbers of available rounds on SAM systems are generated from a discrete uniform distribution in the interval [1-9]. The unit duration of time slots are set as 1 second. We create a sample single shot kill probability matrix for ASM and SAM systems from a uniform distribution in the interval [0.05, 0.80]. The setup time of each engagement is taken as 9 seconds. Time taken by each feasible engagement is calculated by the summation of constant setup time and variable flight time.

The type of disturbance is determined by going on upon the time horizon. In destroyed ASM case, we start from the first engagement and generate a random number from the uniform distribution. We assume that the engagement between SAM and ASM pair ends up with destroyed ASM if the random number value is less than the single shot kill probability of SAM against ASM. In breakdown of a SAM system case, we suppose that SAM systems may become unavailable to shoot after the engagement process starts. The breakdown probability of each SAM systems is chosen randomly between 0.05 and 0.20 (Bolkcom and Pike (1996)). If a SAM system is unavailable to shoot, we specify the time of the breakdown from Erlang distribution (Li et al. (1998)). The SAM system is set as broken and the starting time of the following time slot is set as rescheduling time point, *RT*. Likewise, we assume that sensors may detect an unexpected incoming ASM after the engagement process is started and the initial allocation plan is in operation. The time of arrival is randomly determined from Poisson distribution. Also the target ship of new incoming ASM, distance of the ASM, and velocity of ASM is randomly chosen. We assume that in each time slot only one disruption can occur. In all cases, current threats, available rounds, remaining time slots and upper bound on the number of engagements between ASM and SAM systems are updated with respect to the rescheduling time point. If a disturbance happens, the solution procedure is applied to update the existing schedule.

The problems are solved using version 12.6 of IBM ILOG CPLEX in Java platform on a personal computer with Intel i5-7200 CPU, 2.5 GHz and 8 GB of RAM.

4.4.2 Performance Measures

In each rescheduling time point, the objective function values of all non-dominated solutions are calculated. The results are analyzed according to the increment on the survival probability and schedule disruption. In order to evaluate the improvement on efficiency of TG and disruption on the schedule, we define four different measures. The increase on Z_{PNL} and Z_{ND} objectives in each rescheduling time point are found according to the chosen most preferred non-dominated solution. The defined performance measures are as follows.

Average improvement on efficiency (AIE) metric is the average value of difference between Z_{PNL} value of chosen non-dominated point and Z_{PNL} value of existing engagement allocation plan for all rescheduling time point. We also calculate the percentage improvement on efficiency with average percentage improvement on efficiency (APIE) metric. It shows average percentage improvement on Z_{PNL} objective function. Assume there are $|D|$ disturbances indexed by d. The probability of no-leaker value of chosen non-dominated solution in disturbance d is $Z_{PNL}^{ch,d}$. The probability of no-leaker value of keeping the existing schedule in disturbance d is $Z_{PNL}^{kp,d}$. The improvement on efficiency in a rescheduling time point is calculated as $Z_{PNL}^{ch,d}$-$Z_{TC}^{kp,d}$. Then,

$$AIE = \frac{\sum_{d=1}^{|D|} Z_{PNL}^{ch,d}\text{-}Z_{PNL}^{kp,d}}{|D|} \quad \text{and} \quad APIE = \frac{\sum_{d=1}^{|D|} \frac{Z_{PNL}^{kp,d}}{Z_{PNL}^{ch,d}\text{-}Z_{PNL}^{kp,d}}}{|D|}$$

Average number of disruption on schedule (ANDS) metric indicates average difference between the Z_{ND} value of chosen non-dominated solution and minimum Z_{ND} value. The number of disruption value of chosen non-dominated solution in disturbance d is $Z_{ND}^{ch,d}$ and keeping existing schedule has no disruption value. The increase on number of disruptions in a rescheduling time point is calculated as $Z_{ND}^{ch,d}$. Then,

$$ANDS = \frac{\sum_{d=1}^{|D|} Z_{ND}^{ch,d}}{|D|}$$

To evaluate the relationship between efficiency and stability, we calculate average percentage improvement on efficiency with one number of disruption, (APIED) metric. The value of APIED is the ratio of percentage improvement on efficiency and number of disruption on schedule. It shows percentage improvement of efficiency on the average with only one change of engagement schedule.

$$APIED = \frac{\sum_{d=1}^{|D|} \frac{Z_{PNL}^{ch,d} - Z_{PNL}^{kp,d}}{Z_{ND}^{ch,d}}}{|D|}$$

We also define outcome metrics that evaluates the results at the end of engagement time horizon. The defined metrics are average number of ships survived (NSS), average percentage of survived ships (PSS) and average percentage of destroyed ASM (PDA). The values of those metrics by keeping the initial schedule and updating engagement allocation plan with BMRP model in each rescheduling time point are calculated.

4.4.3 Computational Results with Augmented ε-constraint Method

Table 4.2 shows the performance metrics of the problem sets. AIE values change between 0.037 and 0.150. The highest average AIE, 0.15, is attained when the number of ASM is 4 and the number of SAM system is 3. The lowest average AIE, 0.037, is in the problem set with number of ASM is 6 and SAM system is 3. We get the highest average APIE, 30.22%, with 6 ASM and 5 SAM systems and lowest average APIE, 10.76%, with 3 ASM and 6 SAM systems. The highest maximum APIE value, 64.45%, is attained in problem set with 4 ASM and 4 SAM systems. The AIE and APIE result show that we may greatly increase the survival probability of TG. On the other hand, if there are no alternative solutions in Pareto front and we cannot generate non-dominated solutions, we cannot update the existing schedule. For instance, in problems sets with 6 ASM, 3 SAM systems and 6 ASM, 4 SAM systems we get lowest minimum APIE value 0.0%. We can also conclude that the improvement on efficiency does not depend on the number of SAM systems or number of ASMs. Because, type and time of disturbance affect the increment on efficiency of the TG. If a problem set has a new threat case, efficiency of TG may increase since updating engagement plan increases with destroying new incoming ASM.

ANDS metric indicates average number of disruption on the existing schedule. The average ANDS values change between 0.84 and 1.75. ANDS values depend on the DM utility function and the chosen non-dominated point. Also, in a rescheduling point ANDS may be a higher value or may be zero if there is no option to update the schedule. The average of those values in all rescheduling time point determines the ANDS metric. The highest maximum ANDS value, 2.57, is attained in problems set with 6 ASM and 6 SAM systems. We get the lowest minimum ANDS value, 0.25, in problem set with 3 ASM and 6 SAM systems.

The relation between efficiency and stability objectives can be evaluated with APIED metric. The problem with 4 ASM and 3 SAM systems has the highest maximum APIED, 19.51%. This shows that on the average only one allocation change in the schedule can highly improve the efficiency of TG. In problem set with 6 ASM, 3 SAM systems and 6 ASM, 4 SAM systems, APIED value is 0 in one problem. Average APIED values are between 1.68% and 8.86%.

Table 4.2 The performance metric results of problem sets.

ASM			SAM			
			3	4	5	6
3	AIE	min	0.030	0.009	0.010	0.006
		ave	0.095	0.108	0.101	0.092
		max	0.153	0.266	0.243	0.221
	APIE (%)	min	3.47	0.94	1.09	0.69
		ave	17.54	13.11	12.74	10.76
		max	26.89	28.65	33.37	24.93
	ANDS	min	1.00	0.50	0.40	0.25
		ave	1.13	0.96	1.02	0.84
		max	1.40	1.33	1.80	1.25
	APIED (%)	min	1.72	0.45	1.02	0.60
		ave	3.94	4.88	3.79	3.79
		max	7.14	13.66	8.45	8.45
4	AIE	min	0.077	0.051	0.009	0.034
		ave	0.150	0.100	0.114	0.085
		max	0.230	0.208	0.216	0.166
	APIE (%)	min	12.74	6.88	1.03	4.50
		ave	27.92	25.88	19.65	12.12
		max	40.01	64.45	40.12	22.99
	ANDS	min	1.00	0.80	0.33	0.80
		ave	1.28	1.39	1.04	0.95
		max	2.00	1.67	1.67	1.14
	APIED (%)	min	3.22	2.47	0.45	1.86
		ave	8.86	3.92	5.78	4.87
		max	19.51	8.50	12.56	8.85
5	AIE	min	0.047	0.006	0.005	0.049
		ave	0.100	0.085	0.099	0.110
		max	0.186	0.183	0.212	0.173
	APIE (%)	min	17.93	9.08	0.60	6.91
		ave	30.21	25.10	19.55	17.23
		max	58.80	44.61	33.33	28.68
	ANDS	min	0.80	0.60	0.29	1.00
		ave	1.16	1.31	1.22	1.50
		max	1.60	1.67	2.00	1.83
	APIED (%)	min	1.89	0.32	0.54	3.07
		ave	4.98	3.66	3.83	5.30
		max	10.66	8.60	7.30	7.23
6	AIE	min	0.000	0.000	0.073	0.029
		ave	0.037	0.051	0.109	0.098
		max	0.073	0.080	0.134	0.243
	APIE (%)	min	0.00	0.00	20.94	7.85
		ave	16.31	17.44	30.22	25.36
		max	36.33	25.01	40.56	55.69
	ANDS	min	0.26	0.33	1.17	0.86
		ave	0.95	1.10	1.75	1.61
		max	1.33	1.57	2.13	2.57
	APIED (%)	min	0.00	0.00	2.81	1.67
		ave	1.68	2.41	5.83	3.98
		max	3.95	3.66	10.92	8.00

The results of outcome metrics are given in Table 4.3. For each number of ASM and number of SAM system combination, the outcomes of engagement time horizon in terms of destroyed ASMs and survived ships are calculated. In our computational experiments, we assume that a ship can contain at most two SAM systems. Thus, high number of SAM system indicates high number of total ship at the beginning of engagement process. Also it affects the number of ships survived. For instance, in problem set with 3 ASMs, average number of ships survived increases from 1.4 to 3.4 when number of SAM systems increase 3 to 6. If we evaluate the rescheduling approach, the difference between updating the schedule and no rescheduling is more apparent in some problem sets. For instance, the rescheduling approach increases number of ships survived from 0.2 to 1.4 in problem set with 5 ASMs and 3 SAM systems. On the other hand, in problem set with 3 ASMs and 3 SAM systems, average number of ships survived value with updating the schedule is nearly same as without rescheduling. The average number of ships survived value is doubled from 1.5 to 3 by using rescheduling model in problem set with 6 ASMs and 6 SAM systems.

For PSS metrics, if rescheduling is not used only 30% of ships are survived in problem set with 4 ASM and 3 SAM systems. By BMRP model, 80% of ships are survived. We get the highest average PSS, 95.00%, in problem set with 3 ASMs and 6 SAM systems by BMRP. The lowest average PSS, 50.00%, is attained in problem set with 6 ASMs and 3 SAM systems. Another metric to assess the performance of rescheduling model is percentage of destroyed ASM (PDA). In all problem sets, PDA increases by rescheduling approach. We get the highest PDA, 96%, in problem set with 4 ASM and 5 SAM systems. In problem set 4 ASM and 3 SAM systems, we get the highest difference for PDA between rescheduling model and keeping the existing schedule. The value increases from 63% to 92% in this problem set. In conclusion, computational runs show that our model ensures utilizing the capability of air defense systems better for destroying ASMs and surviving ships by increasing the probability of no-leaker of TG.

Table 4.3 The outcome metric results of problem sets.

ASM			SAM			
			3	4	5	6
3	NSS	e-cons*	1.4	2.2	2.8	3.4
		no res**	1.3	1.8	2.2	2.8
	PSS (%)	e-cons	70.00	80.00	93.33	95.00
		no res	70.00	66.67	73.33	76.67
	PDA (%)	e-cons	85.00	90.00	95.00	95.00
		no res	80.00	80.00	78.33	80.00
4	NSS	e-cons	1.6	2.2	2.8	3.75
		no res	0.6	1.6	2.2	3.25
	PSS (%)	e-cons	80.00	83.33	93.33	93.75
		no res	30.00	56.67	73.33	81.25
	PDA (%)	e-cons	92.00	92.00	96.00	95.00
		no res	63.00	76.00	79.00	85.00
5	NSS	e-cons	1.4	2	2.6	3.25
		no res	0.2	1.8	2.2	2.5
	PSS (%)	e-cons	70.00	73.33	86.67	85.42
		no res	30.00	66.67	73.33	66.67
	PDA (%)	e-cons	86.67	86.67	93.33	91.67
		no res	62.67	72.67	79.33	74.17
6	NSS	e-cons	1	2	2.6	3
		no res	0.8	1.2	2	1.5
	PSS (%)	e-cons	50.00	76.67	86.67	79.17
		no res	40.00	43.33	66.67	39.58
	PDA (%)	e-cons	62.86	88.10	94.29	88.69
		no res	56.19	68.10	82.86	65.48

*outcome metric values with rescheduling by augmented ε-constraint method during the engagement process

**outcome metric values without rescheduling during the engagement process

We present the elapsed times of solution methods in Table 4.4. The elapsed times of problems depend on number of non-dominated solutions and number of rescheduling time point. Also, the problem characteristics such as valid engagement between SAM systems and ASMs, number of SAM rounds in each SAM systems may affect the complexity of problem. So, the problem may become computationally complex in some problems even though it is a small size problem in terms of number of ASMs and number of SAM systems. The results show that the run times increase when problem sizes increase. Besides in problem set with 4 ASMs and 6 SAM systems, with 5 ASMs and 6 SAM systems and with 6 ASMs and 6 SAM systems, one problem cannot be solved in two hours.

Table 4.4 The average elapsed times (sec) of problem sets

ASM	**SAM**			
	3	4	5	6
3	4.52	6.00	7.69	14.33
4	7.93	11.76	14.56	28.01*
5	10.40	15.79	21.12	26.60*
6	12.74	17.62	35.26	38.36*

*One problem cannot be solved within 3600 seconds.

To evaluate the effect of DM's behavior on outcome of the solutions, we extend computational results by changing the weight of objectives in the utility function since the importance given to efficiency and stability may change among DMs when updating the existing schedule. In previous results, we consider that both objectives have equal importance in the underlying quasi-concave utility function of DM, $\mathrm{f}(z) = \max \sum_{i=1}^{p=2} w_i^2 . \left(z_i - z_i^{IP}\right)^2$. To compare the results for different weight vectors, we use problem set with 6 ASMs and 6 SAM systems. We first randomly determine weight values with $w_{ND} < w_{PNL}$. This implies a behavior of more willing to change the existing schedule. This consideration involves taking the risk of deviation from the existing schedule and targeting for the sake of its potential benefit. Hence, training ANN with giving greater importance to efficiency than stability in utility function creates a DM who is risk seeking. On the other hand, the DM who is more conservative to update the schedule is represented by randomly generating weight values with $w_{PNL} < w_{ND}$. Thus the DM created with greater weight on stability while training the ANN is called as risk averse person. We structure the ANNs by calculating desired output values with these weight functions and construct different ANNs as DMs.

The average values of performance metrics are presented in Table 4.5. AIE value is 0.092 if we use equal weights. It increases to 0.143 if DM is more willing to change the schedule and decreases to 0.043 if stability of the schedule is more desired. Same results are attained in the average percentage improvement on efficiency metric values. The result is 38.80% when efficiency is more considered, 27.81% with equal weights and 12.02% when stability is more desired. The average increase on the stability objective values are compared with the average number of disrupted schedule metric. With DM who is risk averse, on average only 0.17 number of disruption is occurred. The ANDS values are 1.98 with equal weights and 2.52 when DM is more willing to update the schedule. We get the maximum average percentage of improvement on efficiency with one number of disruptions as 3.59% with equal weights.

Table 4.5 The results of average performance metric values.

	$w_1 = w_2$	$w_{ND} < w_{PNL}$	$w_{ND} > w_{PNL}$
AIE	0.092	0.143	0.043
APIE (%)	27.81	38.80	12.02
ANDS	1.98	2.52	0.17
APIED (%)	3.59	3.38	3.04

We present the outcome metric results of engagement process in Table 4.6. If the schedule is not changed on the average only 2 ships and 48.33% of ships are survived and 70.48% of ASMs are destroyed. The number of ships survived increases to 2.5 when stability is more important to DM, 2.8 with equal weights and 3.2 when efficiency is more important to DM. The percentage of survived ships increases to 88.33% if the weight vector is chosen as $w_1 < w_2$. The percentage of killed ASM equals to 70.48%, if we do not update the schedule. If we update the schedule with weight function $w_1 > w_2$, the percentage of destroyed ASM increases to 84.52%, with equal weights increases to 88.10% and increases to 94.29% with weight function $w_1 < w_2$.

Table 4.6 The outcome metric values for different weight vector.

	no rescheduling	$w_1 > w_2$	$w_1 = w_2$	$w_1 < w_2$
NSS	2	2.5	2.8	3.2
PSS (%)	48.33	66.67	76.67	88.33
PKA (%)	70.48	84.52	88.10	94.29

The results show that total effectiveness of rescheduling approach of TG increases with respect to efficiency if prior articulated preferential information is based on giving more sacrifice from stability. If the number of change in SAMs allocation from the existing schedule is more desired, the stability of the engagement allocation plan is more controlled and less increment on efficiency is achieved.

4.4.4 Computational Results with Heuristic Approach

We use heuristic algorithms to generate non-dominated solutions for large size problems. The results are given in Table 4.7. The highest average AIE value, 0.107 is in problem set with 7 ASMs and 7 SAM systems. AIE values decrease when number of SAM system is 7 and number of ASMs increases from 7 to 10. The lowest average AIE, 0.045, is in the problem set with number of ASM is 10 and SAM system is 7. On the contrary, we get the highest average APIE, 43.38%, with 10 ASMs and 7 SAM systems. APIE values change between 5.41% and 70.44%. The results show that we may greatly increase the survival probability of TG in large size problems by using our model. The highest maximum APIE value, 70.44%, is attained in problem set with 10 ASMs and 7 SAM systems. We get the lowest minimum APIE value, 5.41%, in problem set with 8 ASM and 10 SAM systems. The results show that the improvement on efficiency does not depend on the number of SAM system or number of ASM as well as small size problems.

Table 4.7 The performance metrics with heuristic approach.

ASM			SAM			
			7	8	9	10
7	AIE	min	0.017	0.031	0.051	0.051
		ave	0.107	0.081	0.090	0.090
		max	0.200	0.127	0.154	0.154
	APIE (%)	min	11.05	5.48	8.31	8.31
		ave	26.95	26.34	16.01	16.01
		max	40.32	47.50	30.07	30.07
	ANDS	min	1.25	1.10	1.36	1.36
		ave	1.55	1.46	1.60	1.60
		max	1.67	2.00	2.00	2.00
	APIED (%)	min	0.84	1.63	2.23	2.23
		ave	6.49	3.83	3.74	3.74
		max	13.01	5.91	5.89	5.89
8	AIE	min	0.024	0.029	0.052	0.025
		ave	0.081	0.073	0.089	0.059
		max	0.232	0.206	0.159	0.111
	APIE (%)	min	8.97	5.76	9.51	5.41
		ave	31.11	17.13	18.93	13.84
		max	61.37	28.75	28.09	25.05
	ANDS	min	1.33	1.18	1.33	0.90
		ave	1.62	1.51	1.49	1.24
		max	2.44	2.14	1.91	1.75
	APIED (%)	min	1.46	1.47	2.80	1.17
		ave	3.98	3.03	4.96	2.81
		max	11.01	7.18	9.20	4.80
9	AIE	min	0.007	0.034	0.028	0.046
		ave	0.064	0.093	0.066	0.073
		max	0.164	0.206	0.120	0.117
	APIE (%)	min	13.90	10.16	10.43	7.05
		ave	33.78	32.13	21.73	17.43
		max	49.44	53.93	38.19	32.82
	ANDS	min	0.91	1.55	1.30	1.27
		ave	1.42	1.83	1.51	1.47
		max	2.00	2.22	1.78	1.90
	APIED (%)	min	0.44	1.59	1.60	2.54
		ave	3.67	5.13	3.68	4.09
		max	6.90	8.31	6.35	6.13
10	AIE	min	0.009	0.005	0.012	0.028
		ave	0.045	0.066	0.061	0.073
		max	0.123	0.132	0.116	0.104
	APIE (%)	min	26.01	21.39	17.24	13.28
		ave	43.38	27.43	23.36	21.59
		max	70.44	32.38	29.59	38.36
	ANDS	min	1.25	1.30	1.30	1.08
		ave	1.56	1.53	1.58	1.52
		max	2.11	1.91	2.33	1.86
	APIED (%)	min	0.57	0.33	0.77	1.67
		ave	2.30	3.36	3.04	3.64
		max	5.86	7.30	4.44	5.49

ANDS values change between 0.90 and 2.44. The average ANDS values are nearly around 1.5 in all problem sets. The highest maximum ANDS value, 2.44, is attained in problems set with 8 ASM and 7 SAM systems. We get the lowest minimum ANDS value, 0.90, in problem set with 8 ASM and 10 SAM systems. The problem with 7 ASM and 7 SAM systems has the highest maximum APIED, 13.01%. Average APIED values are between 2.30% and 6.49%.

The results of overall metrics for large size problems are given in Table 4.8. The highest difference between rescheduling and keeping the existing schedule is in problem set with 9 ASMs and 8 SAM systems. The number of ships survived value increases from 2.2 to 4.2 and percentage of survived ships value increases from 46% to 88% in this problem set. On the other hand, in problem set with 7 ASMs and 10 SAM systems, average numbers of ships survived values are nearly same for updating the schedule and no rescheduling. If we don't implement our model, only 48% of ships are survived in problem set with 9 ASM and 7 SAM systems. But, we can survive 81% of ships in this problem set by using the rescheduling model. We get the highest average percentage of survived ships, 96.67%, in problem set with 7 ASMs and 9 SAM systems for our model. The lowest average percentage of survived ships, 76.00%, is attained in problem set with 10 ASMs and 7 SAM systems.

The metric of percentage of killed ASM helps to evaluate the effectiveness of rescheduling approach. In all problem sets, percentage of killed ASM increases with the rescheduling approach. We get the highest percentage of killed ASM, 97.5%, in problem set with 7 ASM and 9 SAM systems. In problem set 9 ASM and 8 SAM systems, we get the highest difference for percentage of killed ASM between rescheduling model and keeping initial schedule. The value increases from 68.89% to 91.78% in this problem set.

We present the elapsed times of heuristic approach in Table 4.9. We cannot solve large size problems exactly with augmented ε-constraint method. However, we solve large size problems with heuristic approach less than one second. So, we can use our rescheduling model for large size problem sets in terms of the number of ASMs and number of SAM systems.

Table 4.8 The results of overall metrics for large size problems.

ASM			SAM			
			7	8	9	10
7	NSS	heur	3.4	4.4	5.2	5.2
		no res	2.8	3.8	4.2	5
	PSS (%)	heur	81.00	92.00	96.67	96.65
		no res	66.00	79.00	76.67	83.62
	PDA (%)	heur	88.93	89.64	97.50	97.50
		no res	67.86	86.79	79.29	87.14
8	NSS	heur	3.6	4.2	5	5.4
		no res	2.2	3.6	4	4.6
	PSS (%)	heur	87.00	88.00	92.67	90.48
		no res	53.00	74.00	73.33	76.95
	PDA (%)	heur	93.33	92.78	95.56	93.06
		no res	73.06	84.17	84.17	83.61
9	NSS	heur	3.4	4.2	4.8	5.6
		no res	2	2.2	3.4	5.2
	PSS (%)	heur	81.00	88.00	88.67	93.81
		no res	48.00	46.00	61.33	86.48
	PDA (%)	heur	89.56	91.78	93.78	96.00
		no res	69.11	68.89	77.56	91.78
10	NSS	heur	3.2	3.8	4.6	5.6
		no res	1.4	3	3.4	4
	PSS (%)	heur	76.00	79.00	85.33	93.81
		no res	34.00	62.00	63.33	65.81
	PDA (%)	heur	80.73	88.73	90.55	96.18
		no res	63.82	75.64	70.18	77.09

*outcome metric values with rescheduling by heuristic method during the engagement process
**outcome metric values without rescheduling by heuristic method during the engagement process

Table 4.9 Elapsed times (sec) of large size problems by heuristic method.

	SAM			
ASM	7	8	9	10
7	0.302	0.475	0.500	0.527
8	0.322	0.586	0.561	0.726
9	0.272	0.537	0.592	0.657
10	0.277	0.336	0.587	0.637

CHAPTER 5

BIOBJECTIVE MISSILE RESCHEDULING PROBLEM WITH SEQUENCE DEPENDENT STABILITY MEASURE

BMRP concentrates on generating new schedule that increases efficiency of air defense without much deviation from the existing schedule. However, after a disturbance happens, updating the original schedule as much as possible during the engagement process may be preferable. DM may need to reschedule SAM rounds with excessively disrupting the original schedule. To handle the disturbances, rescheduling with small deviation from the initial schedule may result a complicated schedule than large deviation from the initial schedule. This can be easily illustrated by the following example. Assume that a disturbance happens and the initial schedule is updated only with a few changes on SAM allocations. If new schedule includes that SAM systems consecutively change their targets during the engagement process, targeting of SAM systems become time consuming and ineffective. Instead of measuring the deviation from the initial schedule, a schedule that makes SAM systems operations more effective can be more preferable according to DMs. The disturbance may be an opportunity to get a stable schedule in terms of shoot order of SAM systems without considering the deviation from the initial schedule. Thus, defining accurate measure for stability is very important since schedule change may bring benefit to air defense.

The launch process of a SAM round that has semi-active radar includes tracking of the target illumination radar, the solution of radar control problem and launch delay (Karasakal et al. (2011)). Each target must be illuminated in order to launch the SAM round. The fire control radar tracks the target after acquisition of target from search radar and target is illuminated. If a SAM round misses the target, next allocated SAM round is fired against the ASM according to firing policy. If consecutive ASMs are identical in the shoot schedule of SAM systems, the tracking does not change from one to another ASM. While tracking and illuminating a new target, there is a risk of not being able to acquire the new target. Targeting risk and resolution of fire control problem of a SAM system can be avoided by shooting same ASM consecutively. Thus, sequencing decision should not simply be disregarded since change on target tracking may cause multiple radar, illuminator and launcher positioning. By defining a new stability measure that considers change of target tracking for SAM systems, new engagement plan yields improvement on targeting of SAM systems.

In the first part of our study, the decision on scheduling is made without considering shoot order of SAM systems against ASMs. BMRP minimizes total deviation from the initial schedule. The efficiency objective of BMRP is the minimization of probability of no-leaker value of TG. The model considers the total number of changed SAM allocations with respect to initial schedule as a stability objective. The formulation of stability in BMRP is $Min\ Z_{ND} = \sum_{i \in A} \sum_{j \in S} \sum_{k \in T} \left| Y_{ijk} - x_{ijk} \right|$ where the decision variable $Y_{ijk} = 1$ if SAM j is allocated to start engagement process against ASM i at the beginning of time slot k and parameter $x_{ijk} = 1$ if in the initial schedule SAM j is allocated to start engagement process against ASM i at the beginning of time slot k. The number of disruptions is measured by four different types. The allocation of a new SAM round in addition to initial schedule brings one disruption since $Y_{ijk} = 1$ and $x_{ijk} = 0$. Discarding an initially allocated SAM round j from a target ASM i and replacing a different on hand SAM round $j^{'}$ to the ASM i engagement plan yields two disruptions since $\left| Y_{ijk} - x_{ijk} \right| = \left| 0 - 1 \right|$ and $\left| Y_{ij^{'}k} - x_{ij^{'}k} \right| = \left| 1 - 0 \right|$ Also, discarding an initially allocated SAM round j from a target

ASM i and replacing it to ASM i' engagement plan yields two disruptions since $\left|Y_{ijk} - x_{ijk}\right| = |0-1|$ and $\left|Y_{i'jk} - x_{i'jk}\right| = |1-0|$. If a SAM round j in the engagement plan of ASM i and a SAM round j' in the engagement plan of ASM i' are exchanged, four disruptions are occurred in total.

In this part of our research, we develop a new stability measure that considers shoot order of SAM systems and change on target tracking for SAM systems. We define "tracking changeover" as the process of converting target of a SAM system from one ASM to another. The "total number of tracking changeover" for TG, Z_{TC}, is calculated by summation of total number of tracking changes over all SAM systems. In the next section, we evaluate two stability measures, the total number of tracking changeover (Z_{TC}) and the total number of disruption (Z_{ND}) on an illustrative example problem.

5.1 An Example

We generate a specific example problem in order to compare total number of tracking changeover (Z_{TC}) and total number of disruption (Z_{ND}) measures. Consider a TG with four ships and six SAM systems and counters an air attack containing six ASMs. The types of ASMs are given in Table 5.1 and illustration of air defense operation is presented in Figure 5.1.

Table 5.1 Feature of ASMs.

	ASM1	ASM2	ASM3	ASM4	ASM5
Velocity(m/sec)	289	350	306	289	306
Target	Ship 2	Ship 1	Ship 1	Ship 3	Ship 2

Within the engagement interval, upper bounds on the number of engagements of ASM and SAM system pairs are calculated according to velocities, minimum and maximum effective range of SAM systems and velocities and initial distance of ASMs. SAM systems 2, 4 and 5 are area-defense systems. The number of SAM rounds for each SAM system, *d*, is shown in Figure 5.1.

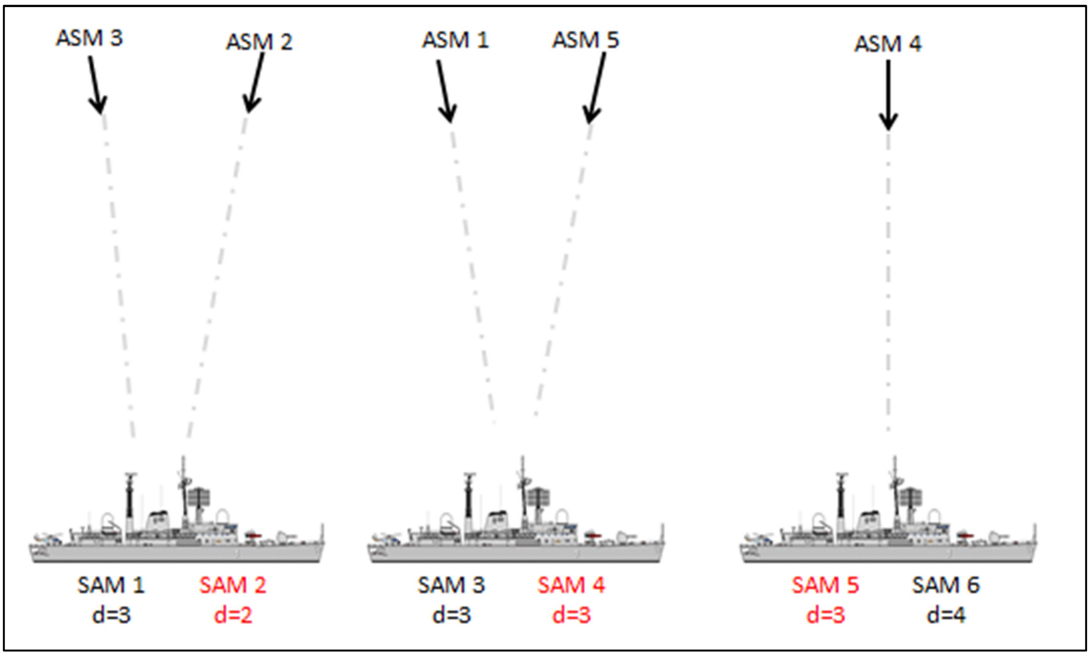

Figure 5.1 A TG and attacking ASMs.

We assume a disturbance occurs at time slot 9 as breakdown of SAM system 6. The initial engagement schedule after time slot 9 is presented in Figure 5.2. The *x* axis represents times slots and *y* axis includes target ASMs. For each ASMs, the scheduled SAM systems are indicated in the rectangle shapes.

According to SLS policy, until a SAM round intercepts with an ASM, there is no scheduled SAM for this ASM. Thus, there is no overlap between rectangles. The starting and finishing points of rectangles show the starting and finishing time slots of engagements. The size of rectangles represent the duration of engagements. Since SAM 6 is broken down, it cannot shoot to ASM 2 and ASM 5 that was initially allocated.

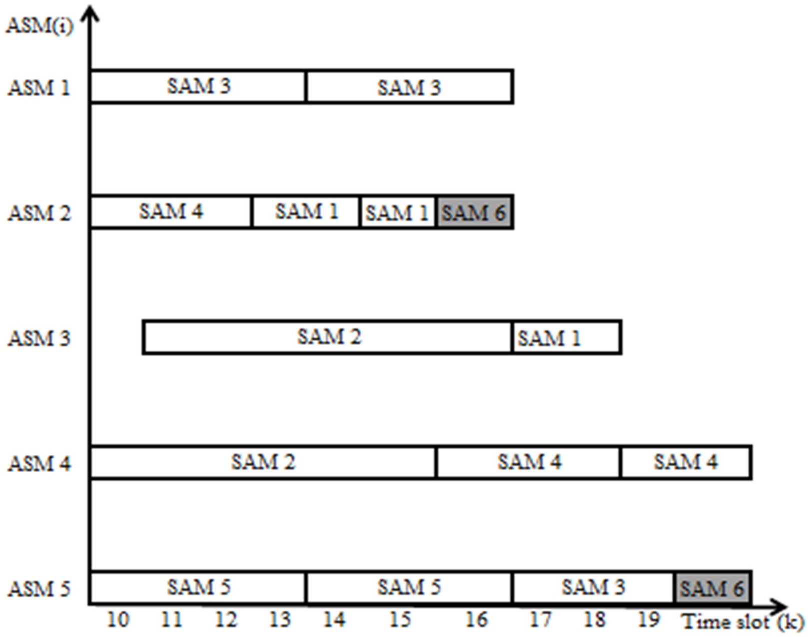

Figure 5.2 The initial engagement schedule after breakdown of a SAM system.

The corresponding shoot order of SAM systems for initial schedule is presented in Figure 5.3. Figure 5.3 shows the time of shoots and target ASMs for each SAM systems. For each SAM system schedule, the target ASMs and time of shoots are given.

The solution keeping the initial schedule is a non-dominated solution with respect to BMRP model since it has minimum deviation from the initial schedule. The objective function values are $Z_{PNL} = 0.41$ and $Z_{ND} = 0$. The total number of tracking changeover value is 4. All SAM systems except SAM system 5 have one tracking changeover. For instance, in SAM system 1, the first two are fired to ASM 2. Then, the SAM system changes its target and shoots to ASM 3. Thus, SAM system 1 has one tracking change. SAM system 5 has no tracking changeover since its only target is ASM 5.

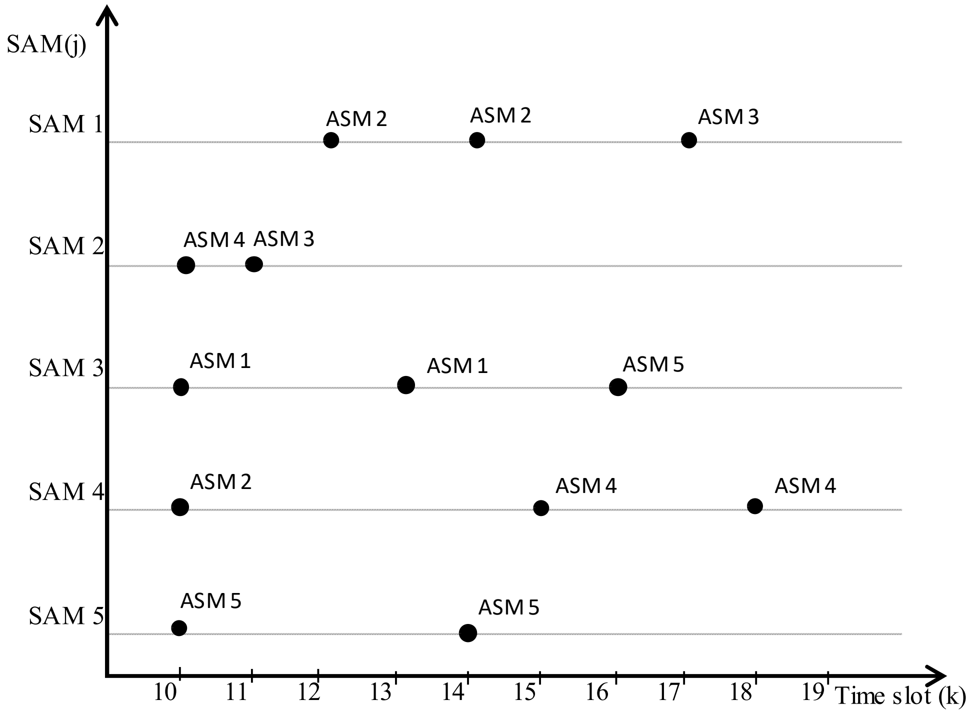

Figure 5.3 The initial schedule of SAM systems after breakdown of SAM system 6.

We generate non-dominated solution of BMRP. In addition to keeping the initial schedule, we obtain a new solution. The objective function values for non-dominated solutions of BMRP are given in Table 5.2. We also calculate total number of tracking changeover for both non-dominated solution and present in Table 5.2.

Table 5.2 The non-dominated solutions of BMRP.

Z_{PNL}	Z_{ND}	Z_{TC}
0.41	0	4
0.68	4	6

The total number of tracking changeover is 4 in the initial schedule and 6 in the updated schedule. Figure 5.4 shows engagement schedules of updated schedule with $Z_{ND} = 4$ and $Z_{TC} = 6$. SAM 4 and SAM 5 shoot order is updated with exchange of

SAM rounds in the engagement schedule of ASM 4 and ASM 5. SAM system 4's tracking changeover increases from two to three and SAM system 5's tracking changeover increases from zero to one. Thus, $Z_{TC} = 4$ changes to $Z_{TC} = 6$ from initial schedule to the updated schedule. Also four disruptions are occurred since two SAM rounds are exchanged $Z_{ND} = 4$.

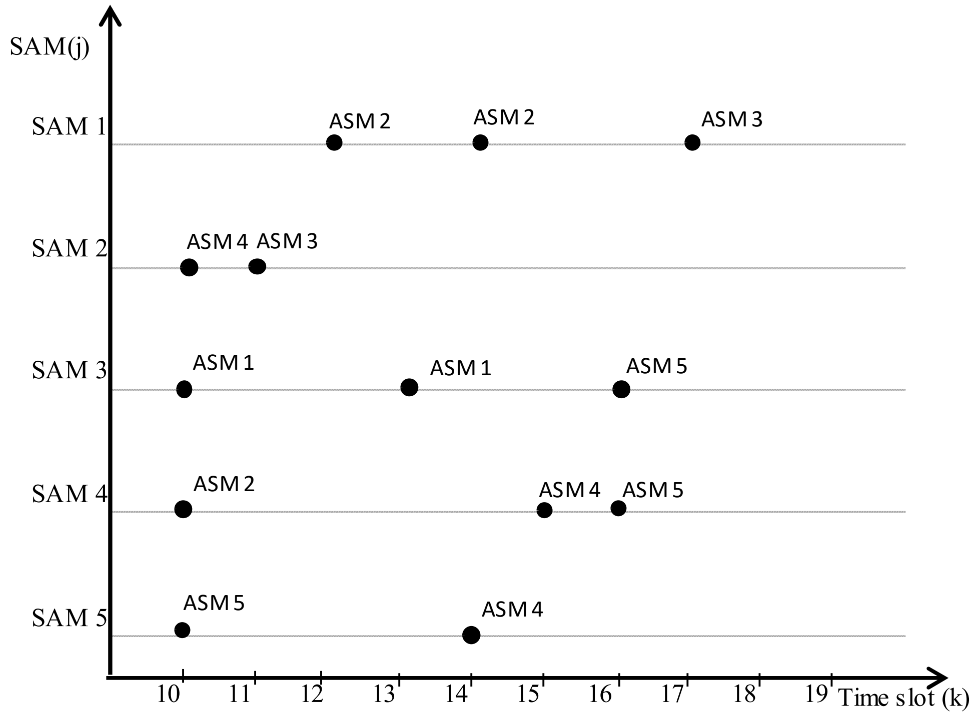

Figure 5.4 Non-dominated solution with $Z_{PNL} = 0.68$, $Z_{ND} = 4$ and $Z_{TC} = 6$.

To find a solution with minimum total number of tracking changeover, we generate a feasible updated schedule of the sample problem. We present the updated schedule in Figure 5.5. All SAM systems have only one target ASM. Hence, total number of tracking changeover is 0. The schedule has $Z_{TC} = 0$, $Z_{PNL} = 0.65$. On the other hand, there are eight disruptions in the initial schedule. Thus, according to BMRP, the solution is dominated by solution with $Z_{PNL} = 0.68$, $Z_{ND} = 4$. If we develop a new mathematical model consists of two objectives with probability of no-leaker value, Z_{PNL} and total number of tracking change, Z_{TC}, the updated schedule will be a non-dominated solution of the model to be proposed since it has minimum total tracking changeover.

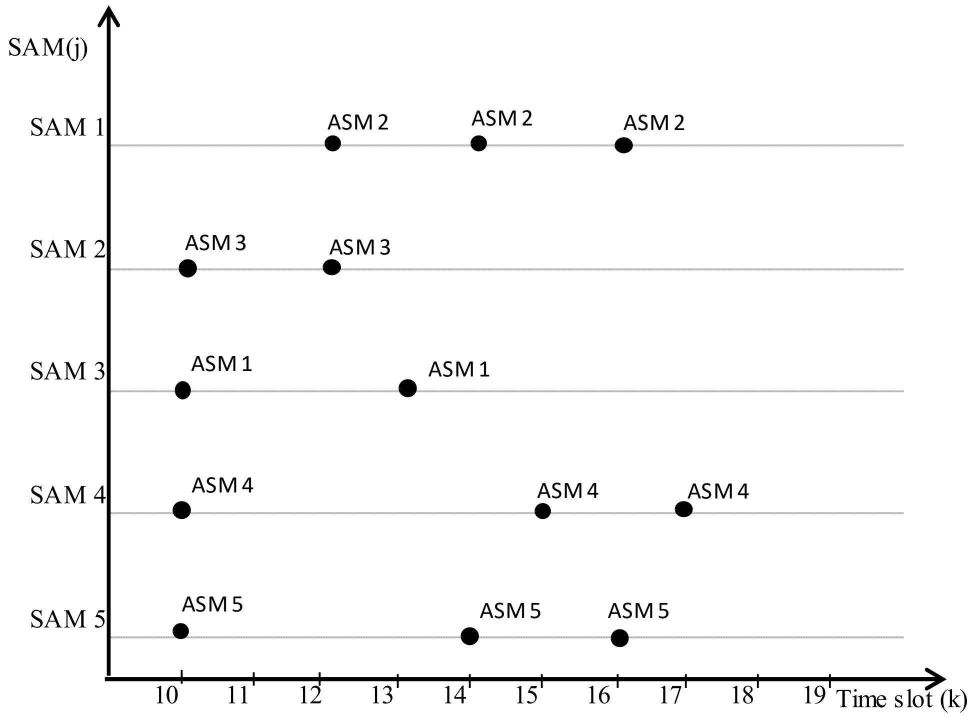

Figure 5.5 Feasible solution with $Z_{PNL} = 0.65$, $Z_{ND} = 8$ and $Z_{TC} = 0$.

The overall results are presented in Table 5.3. The last two columns of the table show either the solution is a dominated (D) or a non-dominated (ND) one according to two models.

Table 5.3 The solutions of the sample problem

Z_{PNL}	Z_{ND}	Z_{TC}	BMRP	New Model
0.41	0	4	ND*	D**
0.65	8	0	D	ND
0.68	4	6	ND	ND

* Non-dominated solution
** Dominated solution

The results indicate that BMRP model cannot obtain solution that has 0.65 probability of no-leaker value without tracking changeovers for SAM systems.

BMRP and new model generate the non-dominated solution with 0.68 probability of no-leaker value that is barely greater than 0.65. Moreover, the solution with keeping the initial schedule is a dominated solution according to new model. It has 4 total number of tracking changeover and 0.41 probability of no-leaker value and dominated by the solution with $Z_{TC} = 0$ and $Z_{PNL} = 0.65$ in the new model.

Since BMRP considers only deviation from the initial schedule and does not concentrate on tracking changeover, solutions that may be preferable under certain circumstances cannot be captured by BMRP. In the schedule with $Z_{TC} = 0$ and $Z_{PNL} = 0.65$, none of the SAM systems shoots to different ASMs and the efficiency objective is very close to highest value. But, it is identified as a dominated solution due to BMRP since the amount of deviation from the initial schedule is high.

We consider total number of tracking changeover for all SAM systems as a stability measure. The point that is of concern for this study is the need to integrate sequencing decisions with rescheduling of SAMs for a TG. With the need to get an updated schedule having the ability to reduce the impacts of a disruption and increase the effectiveness of air defense, we suggest a new biobjective model. The resultant schedule of new model is a stable one among schedules due to minimum total tracking changeovers. Besides, we propose a solution approach to generate efficient and stable schedules with regard to shoot order of SAMs.

5.2 Biobjective Missile Rescheduling Problem with Sequence-Dependent Stability Measure (BMRP-S)

In the first part of our study, our aim is to generate stable schedules that consider the shoot sequence of SAM systems. We develop a mathematical model that maximizes the probability of no-leaker value of TG and maximizes number of shoots to same ASMs consecutively for each SAM systems, or equally minimizes total number of tracking changeover for SAM systems. The model considers the order of ASMs in

SAM systems schedule. It finds the sequence of shoots and maximizes shooting to same ASMs consecutively for all SAM systems.

We present the indices, parameters and decision variables along with the mathematical formulation of BMRP-S as follows:

Indices, sets and parameters

i	index for inital incoming ASMs $(i = 1, ..., n)$
N	set of initial incoming ASMs
D	set of destroyed ASMs
I	set of new incoming ASMs
j	index for SAM systmes on warships $(j = 1, ..., m)$
M	set of SAM systems
B	set of broken SAM systems
k	index for time slots $(k = 1, ..., t)$
K	set of time slots
F	set of finished time slots
RT	rescheduling time point
A	set of current ASMs at RT, $\left(A = (N \cup I) / D\right)$
S	set of current SAM systems at RT, $(S = M / D)$
T	set of time slots after at RT, $(T = K / F)$
$(i, j) \in V$	valid combinations of ASMs and SAMs
q_{ij}	earliest begining time of first engagement between ASM i and SAM system j
r_{ij}	latest ending time of last engagement between ASM i and SAM system j
Δ_{ijk}	engagement duration of (i, j) pair if engagement starts in time slot k
S_{ij}	set of time slots that SAM j can be scheduled to ASM i, $\left\{k \in K : (i, j) \in V \text{ and } \left[\tau_k, \tau_k + \Delta_{ijk}\right] \subseteq \left[q_{ij}, r_{ij}\right]\right\}$
J_{ik}	set of combinations of (j, ρ) that time slot k for ASM i will be blocked, $\left\{(j, \rho) : (i, j) \in V, \rho \in S_{ij} \text{ and } \left[\tau_k, \tau_k + \Delta\right] \subseteq \left[\tau_\rho, \tau_\rho + \Delta_{ij\rho}\right]\right\}$
p_{ijk}	single shot kill probability of SAM j for ASM i if engagement starts in time slot k
d_j	avaliable rounds on SAM j
f_j	fired rounds of SAM up to the RT

μ_{ij}^{RT}	upper bound on number of engagements of SAM j against ASM i at RT
$tc_{ii^{'}jkk^{'}}$	a parameter equals to -1 if $i=i^{'}$ or eqauls to 1 if $i \neq i^{'}$
M	a large number

Decision variables

$$Y_{ijk} = \begin{cases} 1, \text{ if SAM } j \text{ is scheduled to start engegament process against ASM } i \\ \quad \text{at the begining of time slot } k \\ 0, \text{ otherwise} \end{cases}$$

$$\beta_{ii^{'}jkk^{'}} = \begin{cases} 1, \text{ if ASM } i \text{ is allocated at time slot } k, \text{ ASM } i^{'} \text{ is allocated at time slot } k^{'} \\ \quad \text{in the SAM } j \text{ shoot schedule and ASM } i \text{ is followed by ASM } i^{'}. \\ 0, \text{ otherwise} \end{cases}$$

Q_{ijk} = the order value of ASM i in the SAM j shoot schedule if ASM i is allocated at time slot k

(BMRP-S)

$$min \quad \sum_{i^{'}}\sum_{i}\sum_{j}\sum_{k}\sum_{k^{'}} tc_{ii^{'}jkk^{'}}.\beta_{ii^{'}jkk^{'}} \tag{5.1}$$

$$max \quad \prod_{i\in A}\left(1-\prod_{\substack{k\in T\\ j\in S,(i,j)\in V}}\left(1-p_{ijk}\right)^{Y_{ijk}}\right) \tag{5.2}$$

subject to

$$\sum_{\substack{k\in T\\ i\in A}} Y_{ijk} \leq d_j - f_j \qquad \forall j \in S \tag{5.3}$$

$$\sum_{(j,p)\in J_{ik}} Y_{ijp} \leq 1 \qquad \forall i \in A, k \in T \tag{5.4}$$

$$\sum_{k\in S_{ij}} Y_{ijk} \leq \mu_{ij}^{RT} \qquad \forall (i,j) \in V \tag{5.5}$$

$$\sum_{i\in A} Y_{ijk} \leq 1 \qquad \forall j \in S,\ k \in T \tag{5.6}$$

$$Q_{ijk} \le MY_{ijk} \qquad \forall i \in A, j \in S, k \in S_{ij} \qquad (5.7)$$

$$Q_{ijk} - \sum_{i'} \sum_{p \le k} Y_{i'jp} \le M\left(1 - Y_{ijk}\right) \qquad \forall i \in A, j \in S, k \in S_{ij} \qquad (5.8)$$

$$\sum_{i'} \sum_{p \le k} Y_{i'jp} - Q_{ijk} \le M\left(1 - Y_{ijk}\right) \qquad \forall i \in A, j \in S, k \in S_{ij} \qquad (5.9)$$

$$\left(Q_{ijk} - Q_{i'jk'}\right) - 1 \le M\left(1 - \beta_{ii'jkk'}\right) \qquad \forall i, i' \in A, j \in S, k, k' \in S_{ij} \qquad (5.10)$$

$$1 - \left(Q_{ijk} - Q_{i'jk'}\right) \le M\left(1 - \beta_{ii'jkk'}\right) \qquad \forall i, i' \in A, j \in S, k, k' \in S_{ij} \qquad (5.11)$$

$$Y_{ijk} \in \{0,1\} \qquad \forall (i,j) \in V, k \in S_{ij} \qquad (5.12)$$

$$\beta_{ii'jkk'} \in \{0,1\} \qquad \forall i, i' \in A, j \in S, k, k' \in S_{ij} \qquad (5.13)$$

$$Q_{ijk} \in N^0 \qquad \forall i \in A, j \in S, k \in S_{ij} \qquad (5.14)$$

The objective function (5.1) minimizes total number of tracking changeover in SAM systems. The efficiency objective (5.2) is the maximization of probability of no-leaker for TG. The limit on total number of SAM rounds is enforced by constraint set (5.3). Constraint set (5.4) provides allocation of SAMs due to SLS tactic. Constraint set (5.5) limits the maximum number of SAM rounds that can be scheduled against each ASM. Constraint set (5.6) avoids allocation of SAM rounds to same time slots for each SAM system. Constraint Set (5.7) to (5.9) calculates the order value of allocated ASM i in time slot k. If ASM i is allocated in time slot k of SAM system j shoot schedule, then $Y_{ijk} = 1$. To satisfy Constraint set (5.8) and (5.9), total number of allocated ASMs until time slot k, $\sum_{i'} \sum_{p \le k} Y_{i'jp}$, is equal to the order value of allocated ASM i in SAM j shoot schedule, $Q_{ijk} = \sum_{i'} \sum_{p \le k} Y_{i'jp}$. Constraint set (5.10) and (5.11) ensure that if ASM i is not followed by ASM i' in shoot schedule of SAM system j $\left(Q_{ijk} - Q_{i'jk'}\right) \ne 1$, then $\beta_{ii'jkk'} = 0$. If ASM i is followed by ASM i' in the shoot schedule of SAM system j, $\left(Q_{ijk} - Q_{i'jk'}\right) = 1$, then objective function (1) forces $\beta_{ii'jkk'}$ to be 1 with constraint set (5.10) and (5.11).

5.3 Exact Solution Approach

We formulate a new biobjective missile rescheduling problem to find the shoot sequence of SAM systems. The number of variables and constraints increase with the number of time slots, SAM systems and ASMs. Results obtained in our experimental study show that the computation time to solve the problem is extremely long even for small size instances. For example, problems with 3 ASM and 3 SAM systems cannot be solved within an hour. Thus, we propose a new exact solution procedure in order to solve BMRP-S. The solution procedure has three stages:

The first stage is the generation of feasible solutions in the range of maximum Z_{PNL} and minimum Z_{PNL} by defining upper bound on probability of no-leaker objective in each step. First, the solution with maximum Z_{PNL} is generated by solving below mathematical model. We call the formulation of problem as MAP-z and it is given as follows:

(MAP-z)

$$max\ \ Z_{PNL} = \prod_{i \in A} \left(1 - \prod_{\substack{k \in T \\ j \in S, (i,j) \in V}} \left(1 - p_{ijk}\right)^{Y_{ijk}} \right) \tag{5.15}$$

subject to

$$\sum_{\substack{k \in T \\ i \in A}} Y_{ijk} \le d_j - f_j \qquad \forall j \in S \tag{5.16}$$

$$\sum_{(j,\rho) \in J_{ik}} Y_{ij\rho} \le 1 \qquad \forall i \in A, k \in T \tag{5.17}$$

$$\sum_{k \in S_{ij}} Y_{ijk} \le \mu_{ij}^{RT} \qquad \forall (i,j) \in V \tag{5.18}$$

$$\sum_{i \in A} Y_{ijk} \le 1 \qquad \forall j \in S,\ k \in T \tag{5.19}$$

$$Y_{ijk} \in \{0,1\} \qquad \forall (i,j), k \in S_{ij} \tag{5.20}$$

To obtain solutions with different probability of no-leaker value, we restrict Z_{PNL} by adding the following constraint to the model.

$$Z_{PNL} \le Z_{PNL}^{cur} - \varepsilon \qquad (5.21)$$

In constraint (5.21) Z_{PNL}^{cur} value is an upper bound on probability of no-leaker objective. It limits objective function value to have a solution with different Z_{PNL}. In the first step, Z_{PNL}^{cur} is set as 1 which is the maximum value that probability of no-leaker value can take. Thus, first solution has the maximum probability of no-leaker value. Then, in each step, Z_{PNL}^{cur} is updated by equating to the probability of no-leaker value that is previously found. By updating the right hand side of constraint (5.21) in each step, we obtain new solution that has less probability of no-leaker value than previously found solution. The iteration continues until the generation of solution with minimum probability of no-leaker value. The shoot order of SAM systems is not considered in this stage.

In the second stage, we concentrate on total tracking change value of solutions. An algorithm, namely, SAM Round Swap (SRS) algorithm is developed to find the minimum Z_{TC} value for each solution having different probability of no-leaker value. SRS algorithm includes two exchange cases.

The first one is exchanging allocated SAM rounds for each ASM. We choose a pair of SAM rounds in an engagement schedule of ASM and try to exchange their order. We search for whether there exist available time slots to allocate both SAM rounds. If exchange is available, we keep the new ASM engagement schedule as a part of possible solution. All combinations of exchanged SAM rounds are produced for each ASM.

The second case is exchanging two SAM rounds between engagement schedules of two ASMs. We keep the same probability of no-leaker value to find the minimum Z_{TC} in each feasible solution. If two different SAM systems have same single shot kill probability value against two ASMs, then those different SAM rounds can be exchanged between engagement schedule of ASMs and the probability of no-leaker value of whole TG does not change. For each pair of ASMs, we check whether there exists same singe shot kill probability value for different SAM systems. If exchange of allocated SAM rounds between ASMs is possible, we update current schedule and find all possible engagement schedules of new updated solution. ASM engagement schedules with different shoot order of SAM rounds constitute a number of different allocation plans of TG. We find all combination of ASM schedules to evaluate Z_{TC} values. SRS algorithm finds the solution with minimum Z_{TC} by updating shoot orders of SAM systems without changing the probability of no-leaker of TG.

The last stage includes discarding infeasible solutions that are previously produced. In exchange part, ASM engagement schedules are created by checking available time slots within their own schedule. However, combination of different ASM schedules may create an engagement schedule with a SAM system shooting twice at a time slot. Thus, if there exists such an engagement schedule, it must be discarded from the feasible solution set. We check starting time slots of SAM rounds in each SAM system and determine the feasible ones among all solutions. Finally, Z_{TC} values are calculated for all solutions in the feasible set and the solution with minimum Z_{TC} value is chosen as a possible candidate for non-dominated solution set. The computational complexity of SRS algorithm is $O(r^{2n})$ where r is the number of allocated SAM rounds for an ASM and n is the number of ASMs.

5.3.1 Exact Solution Algorithm

We present the notation of the algorithm as follows:

Sets

R_i	set of allocated SAM rounds for ASM *i*
Q_i	set of all possible allocated SAM sequences for ASM *i*
E	set of engagement allocation plan of naval task group
NS	set of non-dominated solutions for BMRPs

Parameters and variables

Z_{PNL}	probability of no-leaker value for naval task group
Z_{TC}	total number of tracking change value for all SAM systems
Z_{PNL}^{cur}	current probability of noleaker value
Z_{TC}^{cur}	current total number of tracking change
cs	current solution

Steps of the Algorithm

Stage 1:

Initialization

Step 0. Set $Z_{PNL}^{cur} = 1$ and $Z_{TC}^{cur} = \infty$

Solution

Step 1. Solve MAP-z model. If infeasible, stop. Otherwise, set solution ω as *cs* and set Z_{PNL}^{cur} as probability of no-leaker value of solution ω. *cs*=ω and $Z_{PNL}^{cur} = Z_{PNL}^{\omega}$. Calculate Z_{TC}. If $Z_{TC} = 0$, stop.

Stage 2:

Exchange allocated SAM rounds for ASM i

Step 2. For current schedule, *cs*, change the order of allocated SAM rounds in the set, R_i, for each ASM i. Find all possible SAM sequences and add to set Q_i for each ASM *i*.

Step 3. Select one of the allocation plans from set Q_i for each ASM *i* and find all combinations of engagement allocation plan of TG and add to set *E*.

Step 4. Determine whether the solution in set *E* includes that a SAM system shoot more than once at same time slot. Discard these infeasible solutions from set *E*.

Exchange allocated SAM rounds between two ASMs

Step 5. For each pair of ASMs, check whether there exists same singe shot kill probability value for different SAM systems. If two different SAM systems have same single shot kill probability value against two ASMs, then those different SAM rounds can be exchanged between schedule of ASMs and the probability of no-leaker value of TG does not change. If exchange of allocated SAM rounds between ASMs is possible, update current schedule, *cs*. Go to step 2, with new engagement allocation plan of ASMs.

Stage 3:

Find non-dominated solutions

Step 6. Calculate total number of tracking changes, Z_{TC} for each solution in set *E*.

Step 7. Select the solution, λ, with minimum Z_{TC} objective value from set *E*.

Step 8. If $Z_{TC}^{\lambda} < Z_{TC}^{cur}$, add the solution λ to *NS* and set $Z_{TC}^{cur} = Z_{TC}^{\lambda}$. Add constraint $Z_{PNL} \leq Z_{PNL}^{cur} - \varepsilon$ (5.21) to MAP-z. Go to step 1.

Theorem 5.1 Proposed solution procedure generates all exact non-dominated solutions of a biobjective missile rescheduling problem with sequence-dependent stability measure.

Proof: Note that solution of MAP-z find a feasible solution of BMRP-S. Updating the upper bound on probability of no-leaker value in each iteration of MAP-z find feasible solutions of BMRP-S with different Z_{PNL}. Thus, all solutions with different probability of no-leaker value are generated by MAP-z. Assume that set *P* includes all solutions produced by MAP-z. The exchange of SAM rounds within the engagement schedule of an ASM and the interchanges of SAM rounds that has same single shot kill probability against both ASMs does not change Z_{PNL} but minimizes Z_{TC}. SRS algorithm minimizes Z_{TC} by creating all possible engagement schedules for each solution that MAP-z produces. Z_{TC} values of all solutions in set P are minimized by SRS algorithm. Let α be a non-dominated solution. In one iteration MAP-z finds a solution, μ, that the probability of no-leaker value equals to the probability of no-leaker value of α, $Z^{\mu}_{PNL} = Z^{\alpha}_{PNL}$. Let $Z^{\mu}_{TC} > Z^{\alpha}_{TC}$. Since SRS algorithm minimizes Z_{TC} value from solution μ without changing the Z_{PNL}, the solution λ with minimum total tracking changeover and with Z^{α}_{PNL} probability of no-leaker value is generated by SRS algorithm. $Z^{\lambda}_{TC} = Z^{\alpha}_{TC}$, $Z^{\lambda}_{PNL} = Z^{\alpha}_{PNL}$. Hence λ and α are same non-dominated solutions. If $Z^{\lambda}_{TC} < Z^{\alpha}_{TC}$ then α is not a non-dominated solution which contradicts with definition of α. This completes the proof that the solution procedure generates all non-dominated solutions of BMRP-S. □

5.3.2 An Example Problem

We next describe the solution procedure for BMRP-S on an example problem. We generate non-dominated set of BMRP-S and evaluate the total number of disruptions. Figure 5.6 depicts TG with SAMs and attacking ASMs. TG consists of four ships and all ships are equipped with SAM systems. There are four incoming ASMs and five SAM systems. SAM 2 and SAM 4 are area-defense system and can defend other ships within their effective ranges. The target ship of each ASM is known by defensive units. The ranges, velocities and target ships of ASMs are given in Figure 5.6. The maximum number of engagement between valid ASM and SAM pairs is calculated in accordance with SLS firing policy. We present the defense and the attack information in Tables 5.4 and 5.5 respectively. The engagement time horizon is 275 seconds. The optimal initial schedule is generated and each SAM system has an order of shoot plan to carry out the engagement schedule.

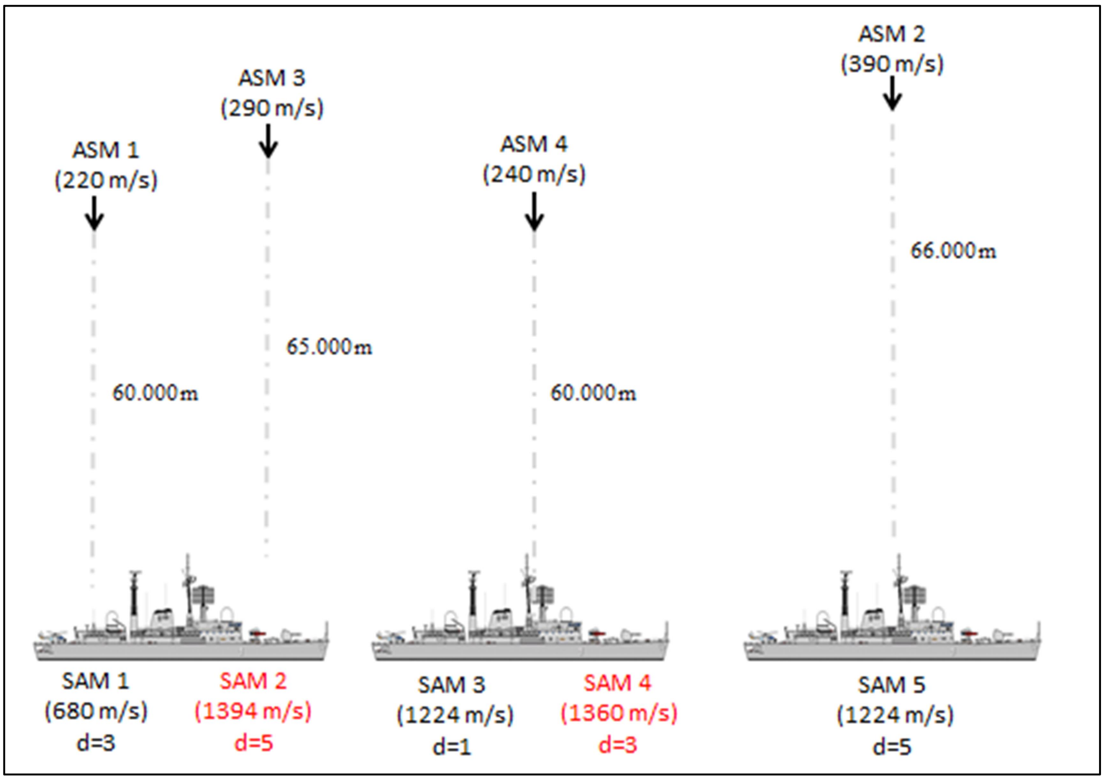

Figure 5.6 An illustration of defensive and attacking units for sample problem.

Table 5.4 Attack information.

ASM	Target Ship	Distance (m)	Speed (m/sec)
1	1	60000	220
2	3	65000	290
3	1	60000	240
4	2	66000	390

Table 5.5 Defense information.

SAM System	Hosting Ship	Minimum Range (m)	Maximum Range (m)	Speed (m/sec)	Round	Type
SAM 1	1	1500	12000	680	3	Self
SAM 2	1	3000	100000	1394	5	Area
SAM 3	2	1500	18000	1224	1	Self
SAM 4	2	5000	38000	680	3	Area
SAM 5	3	1500	18000	1224	5	Self

We assume that at the beginning of time slot 116, surveillance units detect an unexpected incoming ASM. The feature of ASM 6 is given in Table 5.6.

Table 5.6 New ASM features.

i	ASM6
Velocity (m/sec)	290
Target	Ship 3
Distance (m)	60000

Since it is not considered at the beginning of the engagement process, SAM rounds should be scheduled against ASM 6. The initial schedule after rescheduling time point, t=116, is presented in Figure 5.7. Total number of tracking changeover is five, $Z_{TC} = 5$.

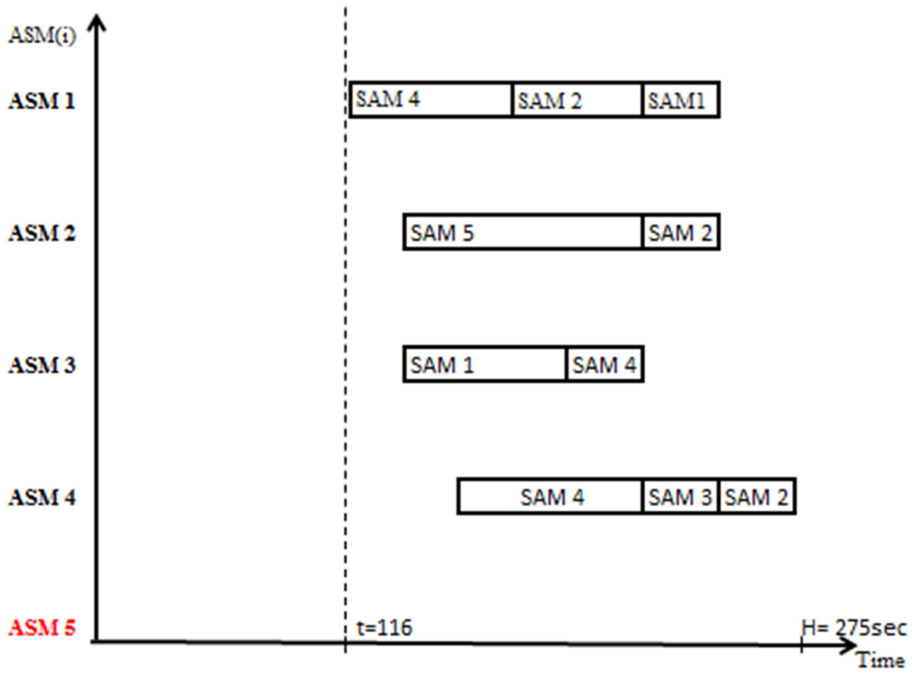

Figure 5.7 Initial schedule after rescheduling time point.

Firstly, we solve MAP-z model without bound on probability of no-leaker value. We find the solution with maximum probability of no-leaker value presented in Figure 5.8.

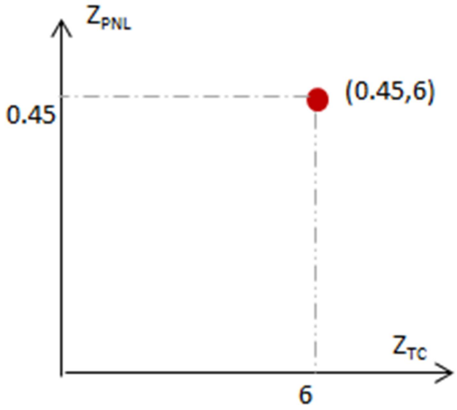

Figure 5.8 Solution with maximum probability of no-leaker.

Secondly, we use SRS algorithm to find the solution having minimum Z_{TC} for 0.45 probability of no-leaker value. All possible ASM engagement plans are created and we choose the feasible solution with minimum Z_{TC}. The solution produced by SRS algorithm is shown in Figure 5.9. SRS algorithm decreases number of total tracking changeover form 6 to 5.

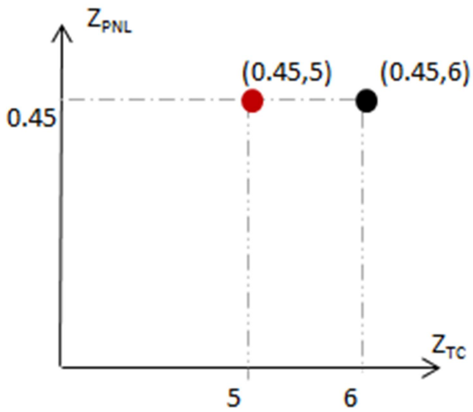

Figure 5.9 The solution produced by SRS algorithm.

Next, we solve MAP-z by limiting probability of no-leaker value with constraint $Z_{PNL} \leq 0.45 - \varepsilon$ where ε is a small number. We attain the solution with $Z_{PNL} = 0.36$ and $Z_{TC} = 7$ as depicted in Figure 5.10. SRS algorithm is used for the solution with 0.36 probability of no-leaker value and Z_{TC} is minimized and decreases from 7 to 5.

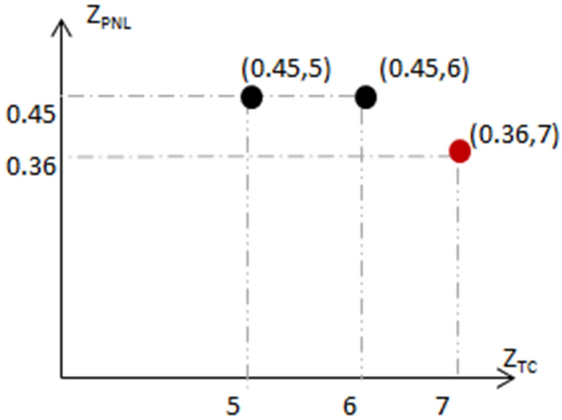

Figure 5.10 Solution with $Z_{PNL} = 0.36$ and $Z_{TC} = 7$.

Figure 5.11 shows the solution that SRS algorithm is produced. Since the solution is dominated by solution having $Z_{TC} = 5$ and $Z_{PNL} = 0.45$, it is not added to the non-dominated set. The next iteration continues by solving MAP model with the constraint that Z_{PNL} is less than 0.36.

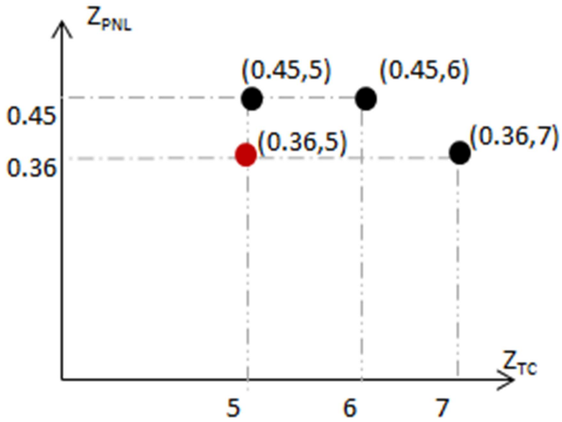

Figure 5.11 Solution with $Z_{PNL} = 0.36$ and $Z_{TC} = 5$.

We generate all non-dominated solutions of sample problem that are depicted by rectangles in Figure 5.12. The solution with $Z_{PNL} = 0.456$ and $Z_{TC} = 6$ is generated in the first iteration. SRS algorithm revises the shoot order of SAM systems and the solution having $Z_{TC} = 5$ with the probability of no-leaker value equals 0.456 is obtained. The MAP-z model is solved with a bound probability of no-leaker less than 0.456. New feasible solution with $Z_{PNL} = 0.363$ is generated. The algorithm continues generating solutions until reaching minimum probability of no-leaker value. Non-dominated solutions are determined from the set of feasible solutions. We also calculate Z_{ND} values of each feasible solution. The Z_{ND} values are given in the brackets. The non-dominated solutions of BMRP are shown by triangle shapes in Figure 5.12. The dominated solutions with respect to both models are depicted by diamond shapes.

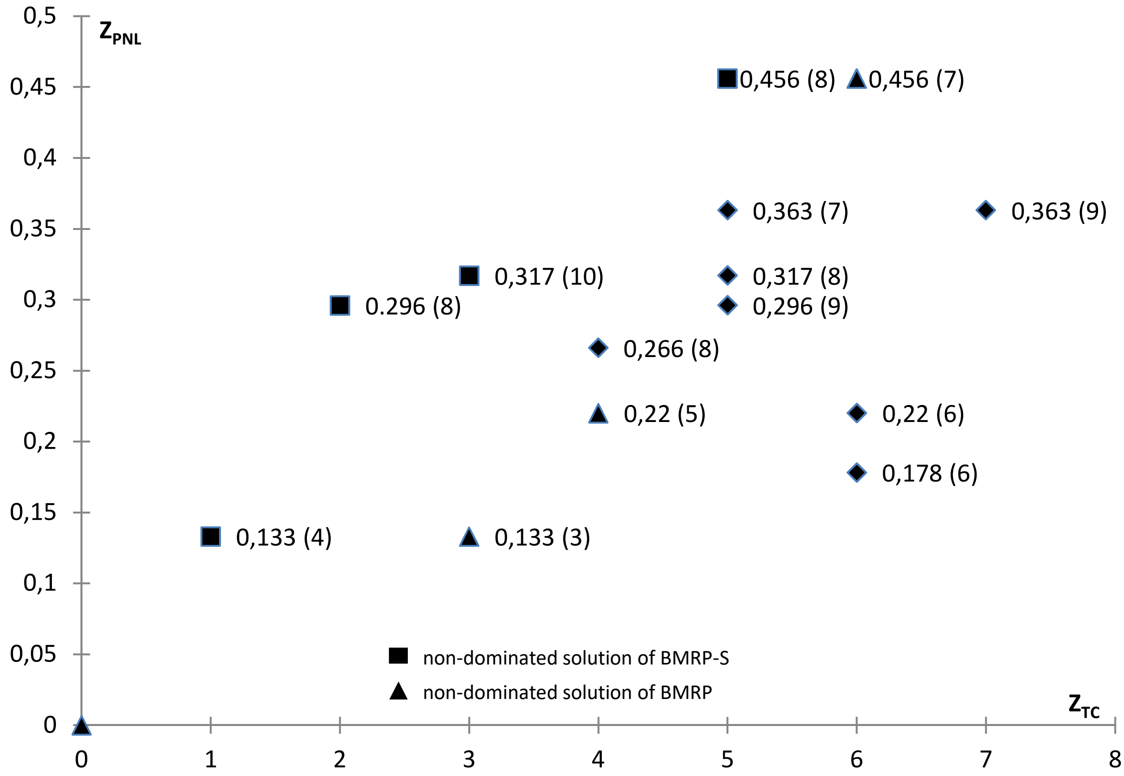

Figure 5.12 Non-dominated and dominated solutions of example problem.

There are four non-dominated solutions of BMRP-S and four non-dominated solutions of BMRP. The non-dominated solutions of both models are given in Table 5.7.

Table 5.7 Non-dominated solutions of BMRP and BMRP-S.

Z_{PNL}	Z_{TC}	Z_{ND}	BMRP-S	BMRP
0.456	5	8	ND*	D**
0.456	6	7	D	ND
0.317	3	10	ND	D
0.296	2	8	ND	D
0.220	4	5	D	ND
0.133	1	4	ND	D
0.133	3	3	D	ND
0	5	0	D	ND

* Non-dominated solution
** Dominated solution

An interesting result is the first two solutions have same probability of no-leaker value, 0.456. The solution with $Z_{ND} = 7$ and $Z_{TC} = 6$ is a dominated solution with respect to BMRP-S and non-dominated solution with respect to BMRP. On the contrary, the solution with $Z_{ND} = 8$ and $Z_{TC} = 5$ is dominated and non-dominated according to BMRP and BMRP-S, respectively. The same result is observed in the solutions with 0.133 probability of no-leaker value. This shows that revising shoot order of SAM systems may increase the deviation from the initial schedule, but it may yield a preferable solution according to tracking changeover consideration.

5.4 Heuristic Approach

The solution procedure cannot solve BMRP-S in polynomial time. The computational times of SRS algorithm and MAP-z exponentially increase as number of ASMs and SAM system increase. For instance, a sample problem with 6 ASM and 6 SAM systems cannot be solved in an hour. Also, the solution of a very small size problem with 3 ASM and 3 SAM systems takes more than a minute. The schedule cannot be updated with those solution times within the engagement process.

However, a solution procedure must enable us to update the existing schedule in response to disturbances. We have to generate solutions in a few seconds to use the model in real-life. Thus, we develop a heuristic procedure that generate non-dominated solutions and meets the solution time requirement.

5.4.1 Reallocation Heuristic Algorithm

In this section, we present a heuristic algorithm, namely Reallocation Heuristic Algorithm (RHA) that attempts to generate non-dominated solutions. RHA is based on reallocating SAM rounds in the existing schedule. RHA takes into consideration that the maximization of the probability of no-leaker value of TG, Z_{PNL} and the minimization of the total number of tracking changeover for SAM systems, Z_{TC}. We define five operations within the RHA that reallocate SAM rounds in the initial schedule. In each operation, the heuristic algorithm concentrates on increasing Z_{PNL} or decreasing Z_{TC}. The parts of heuristic algorithm are as follows:

New Allocation: The first operation is new allocation of a SAM round in addition to initial schedule. At the beginning of the engagement process each SAM system has available round, d_j. Some of them are initially allocated against ASMs, some of them are used up to the rescheduling time point. Thus, at rescheduling time point there may be SAM rounds in each SAM system that are available for the initial schedule.

In new allocation part, an on hand SAM round is allocated to the engagement plan of ASM. We choose the target ASM with minimum probability of no-leaker value. SAM system with maximum single shot kill probability against chosen ASM is determined to allocate. The summary of the new allocation part is as follows:

Algorithm of New Allocation

Step 1. Calculate the available SAM rounds that can be allocated in addition to the initial schedule.

Step 2. Find the ASM i that has the minimum probability of no-leaker value. If there is no available ASM to allocate a SAM round, stop.

Step 3. Find the SAM system j to allocate against ASM i in addition to the initial schedule. If there is no available SAM system go to step 2 and find another ASM. Otherwise, choose SAM system j that has maximum single shot kill probability against ASM i is according to following criteria.

- SAM system has to get at least one available on hand round.
- There must be valid combination between SAM system j and ASM i.

Step 4. Search for available time slot to allocate SAM j to ASM i. If there is no available time slot to allocate, go to step 3 and find another SAM system to allocate against ASM i. Otherwise allocate SAM round j against ASM i and update the available number of SAM rounds. Keep the updated schedule and add to the solution set. Initiate change and exchange operations with updated schedule. If all possible new allocations are achieved, stop. Otherwise go to step 1.

Total number of SAM rounds in the initial schedule increases with new allocation of an on hand SAM round. The allocation of on hand SAM round j, to an ASM i at time slot k exactly increases Z_{PNL} value. However, after allocation Z_{TC} value either increases or remains constant. It depends on time slot of allocation, previous and next target ASMs of the SAM system. Figure 5.13 shows the new allocation operation of an on hand SAM round into the initial schedule. The rectangles demonstrate initially allocated SAM rounds to the target ASMs. The values above the rectangles shows the single shot kill probabilities of SAM systems against ASMs. PNL denotes the probability of no-leaker value of each ASM. ASM 3 with smallest probability of no-leaker and SAM system 1 that has maximum single shot kill probability are chosen for new allocation.

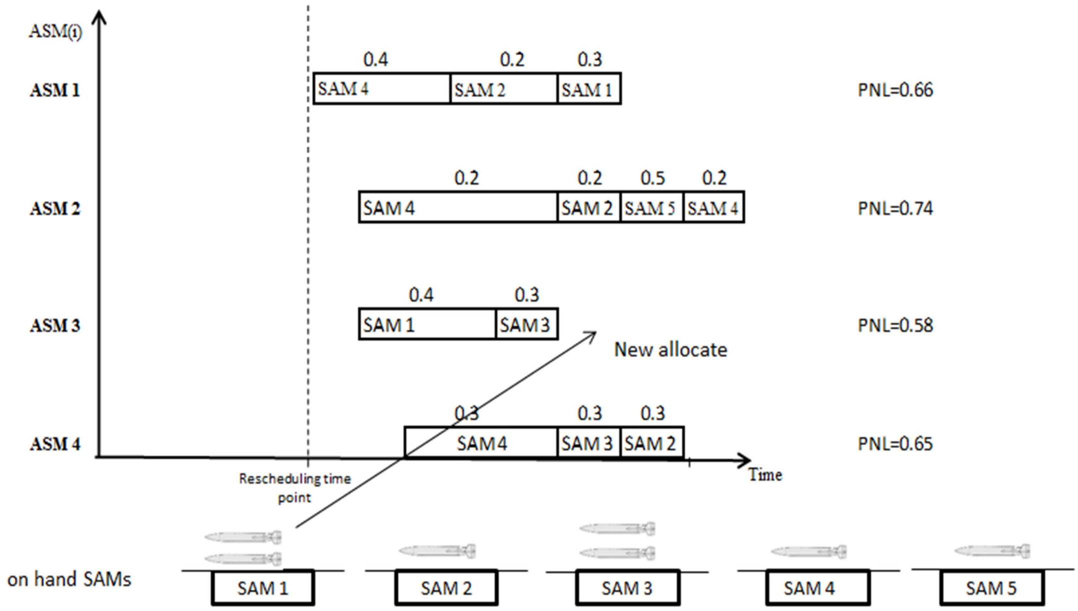

Figure 5.13 New allocation operation.

Replacement: The second operation is called as replacement. In replacement part, we also use on hand SAM rounds that are available to allocate engagement plan of an ASM. This operation includes discarding an initially allocated SAM round from a target ASM and replacing a different on hand SAM round to the engagement plan of that ASM. In replacement operation, total number of SAM rounds does not change.

The summary of the replacement part is as follows:

Algorithm of Replacement

Step 1. Calculate the available SAM rounds that can be allocated in addition to initial schedule.

Step 2. Find the ASM i that has the minimum probability of no-leaker value. If there is no available ASM to allocate a SAM round, stop.

Step 3. Determine the SAM round j' to be removed from the schedule of ASM i. If there is no SAM round to be removed, go to step 2 and find another ASM. Otherwise, choose the SAM round j' whose single shot kill probability against ASM i is minimum.

Step 4. Find the SAM round j to allocate against ASM i instead of SAM round j'. If there is no available SAM round go to step 2 and find another ASM. Otherwise, choose SAM system j that has maximum single shot kill probability against ASM i according to following criteria.

- SAM system j has to get at least one available round.
- There must be valid combination between chosen SAM system j and ASM i.
- Sskp between SAM system j and ASM i has to be greater than sskp between SAM system j' and ASM i.

Step 5. Search for available time slot to allocate SAM j to ASM i. If there is no available time slot to allocate, go to step 4 and find another SAM system to allocate against ASM i. Otherwise remove SAM round j' from the allocation plan of ASM i and allocate SAM round j against ASM i. Update the number of available SAM rounds. Keep the updated schedule and add to the solution set. Initiate change and exchange operations with the new updated schedule. If maximum number of replacement is achieved, go to the discard part. Otherwise go to the step 1.

We choose the target ASM with minimum probability of no-leaker value. SAM round with larger sskp is replaced instead of initially allocated SAM round. Thus, replacement of an initially allocated SAM round j with a different SAM round j' in the schedule of ASM i increases Z_{PNL}. Z_{TC} value either increases, decreases or remains constant. It depends on time slot of allocation, chosen SAM round to be removed, chosen SAM round to be allocated, the previous and the next target ASMs of SAM systems. Figure 5.14 shows the replacement operation in an engagement schedule. SAM 3 is removed and SAM 1 is replaced in the engagement schedule of ASM 3 which has the minimum probability of no-leaker value.

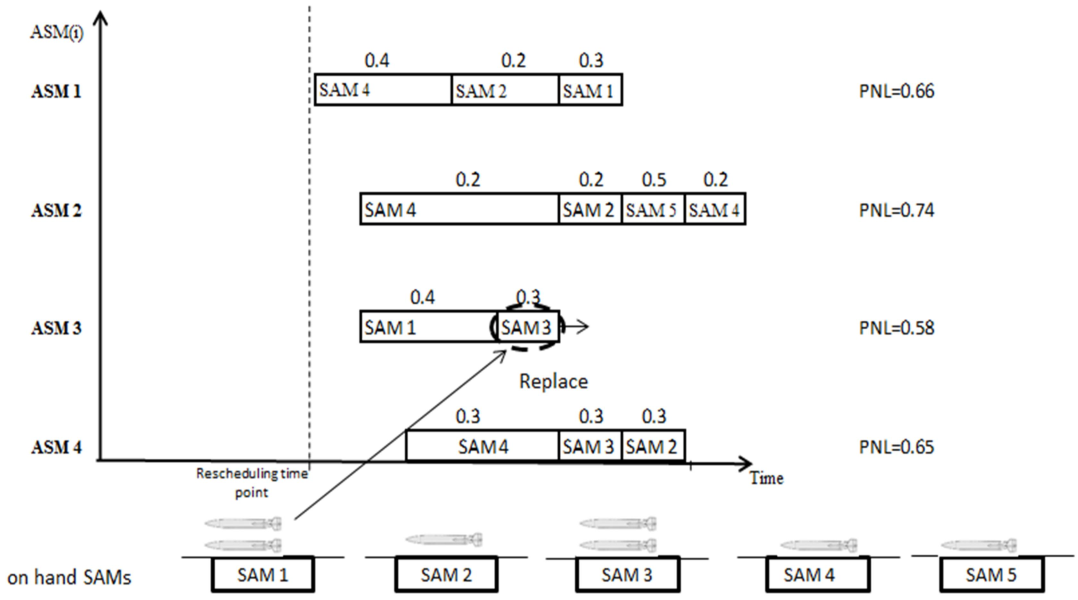

Figure 5.14 Replacement operation.

Removal: The removal part is based on discarding a SAM round from the initial schedule. In removal operation, total number of SAM rounds in the initial schedule decreases. Firstly, ASM with maximum probability of no-leaker value is chosen. To decrease Z_{TC} value, SAM round with the maximum number of tracking changeover is determined from the chosen ASM engagement plan. The chosen SAM round is removed and the total number of tracking changeover is calculated. Removing of a SAM round j from ASM i at time slot k exactly decreases Z_{PNL} and Z_{TC} value.

After removing a SAM round, the result is added to the solution set. The summary of the removal part is given as follows:

Algorithm of Removal

Step 1. Find ASM i that has the maximum probability of no-leaker value. If there is no available ASM to discard a SAM round, stop.

Step 2. Determine the SAM round j' to be removed from the schedule of ASM i. If there is no SAM round to be removed, go to step 1 and find another ASM. Otherwise, choose the SAM round j' whose tracking changeover is maximum.

Step 3. Remove SAM round j' from the allocation plan of ASM i. Keep the updated schedule and add to the solution set. Initiate change and exchange operations with new updated schedule. If minimum Z_{TC} is achieved, stop. Otherwise go to the step 1.

Figure 5.15 shows the removal operation in an engagement schedule. ASM 2 has the maximum probability of no-leaker value. SAM 4 has the maximum tracking changeover among the allocated SAM rounds in the ASM 2 engagement schedule. It is discarded and the result is added to solution set according to decrease on total number of tracking changeover.

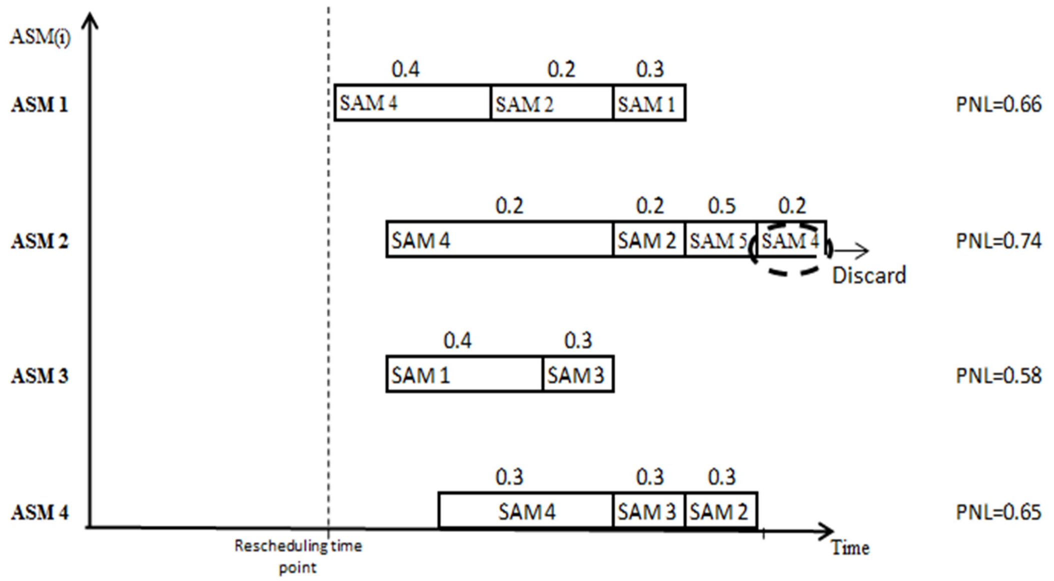

Figure 5.15 Removal operation.

Change: The change part considers changing the target ASM of a SAM round. In this operation, a SAM round is chosen from one of the ASM engagement plan and it is allocated into another ASM engagement plan. We choose two ASMs with the maximum and the minimum probability of no-leaker values. SAM round with the maximum tracking changeover is chosen in order to change its target from ASM with the maximum probability of no-leaker to ASM with the minimum probability of no-leaker. The change operation may increase or decrease Z_{PNL} and Z_{TC} values. If change of SAM target decreases Z_{PNL} and increases Z_{TC}, it yields a dominated solution. Thus, after target ASM of SAM round is changed, we check whether the solution is dominated. If change on the engagement plan increases the probability of no-leaker value or decreases total number of tracking changeover, we do not update the schedule and keep the previous schedule.

The summary of the change operation is as follows:

Algorithm of Change

Step 1. Find ASM i that has the minimum probability of no-leaker value. If there is no available ASM to allocate a SAM round from other ASMs, stop.

Step 2. Find the ASM $i^{'}$ that has the maximum probability of no-leaker value. If there is no available ASM $i^{'}$ go to step 1 and choose another ASM to allocate a new SAM round to its initial engagement plan. Otherwise ASM $i^{'}$ is chosen according to the following criterion.

- The probability of no-leaker value of chosen ASM $i^{'}$ has to be greater than the probability of no-leaker value of ASM i.

Step 3. Determine SAM round $j^{'}$ from the initial engagement plan of ASM $i^{'}$ to allocate against ASM i. If there is no available SAM round go to step 2 and find another ASM. Otherwise, choose SAM round $j^{'}$ according to the following criteria.

- There must be valid combination between chosen SAM system $j^{'}$ and ASM i.
- Total tracking changeover value of SAM system $j^{'}$ is the maximum among the SAM rounds in the schedule of ASM $i^{'}$.

Step 4. Search for available time slot to allocate SAM $j^{'}$ to ASM *i*. If there is no available time slot to allocate, go to step 3 and find another SAM system to allocate against ASM *i*. Otherwise allocate SAM round $j^{'}$ against ASM *i*. Calculate new Z_{TC} and Z_{PNL}. If new solution is a dominated solution, do not update the initial schedule. Otherwise, add solution to the solution set. If the maximum number of change operation is achieved, stop. Otherwise go to step 1.

Figure 5.16 shows the change operation in an engagement schedule. ASM 2 and ASM 3 have the maximum and the minimum probability of no-leaker value respectively. SAM 4 has the maximum tracking changeover among the allocated SAM rounds in the ASM 2 engagement schedule. It is removed from ASM 2 and allocated to ASM 3 engagement plan. If the result is not a dominated solution, it is added to the solution set.

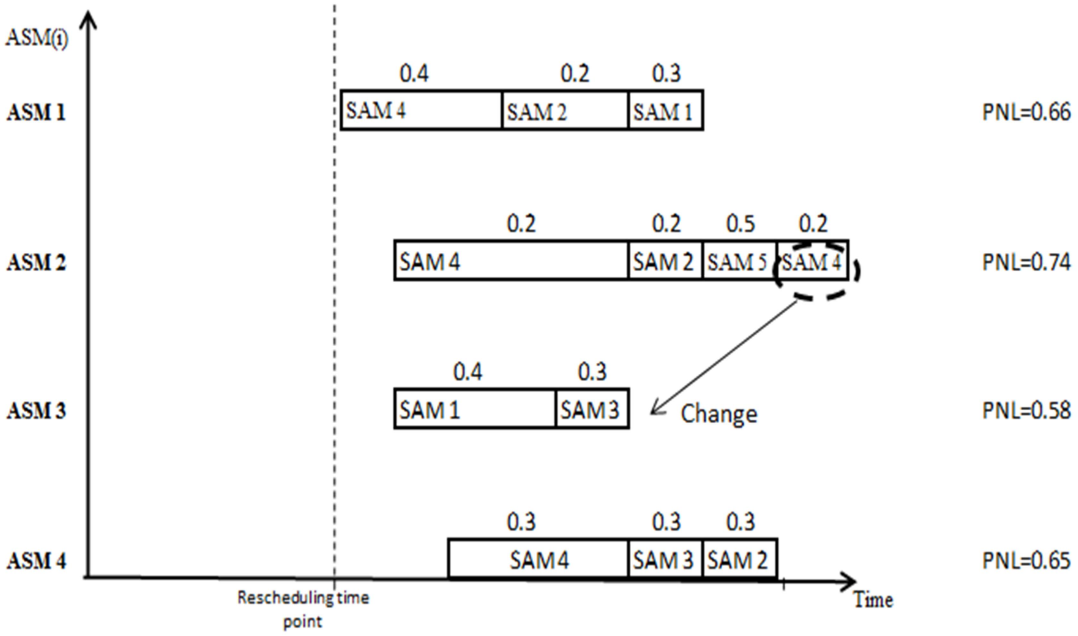

Figure 5.16 Change operation.

Exchange: Exchange operation swaps two different SAM rounds between allocation plans of two ASMs. The target ASMs of two SAM systems are exchanged in this part. The tracking changeover values of SAM systems are considered while choosing SAM rounds from engagement schedule of ASMs. We aim to decrease Z_{TC} or increase Z_{PNL}. Exchange operation may result in decrease and increase on Z_{TC} and Z_{PNL} values. Since the result of exchange operation may be a dominated solution, we check whether the solution is dominated as in the change part.

Algorithm of Exchange

Step 1. Find ASM i that has the minimum probability of no-leaker value. If there is no available ASM to allocate a SAM round from other ASMs, stop.

Step 2. Determine SAM round j from the initial engagement plan of ASM i to remove and to allocate against another ASM.

- Select SAM j that has the maximum total tracking changeover among SAM rounds in the initial engagement plan of ASM i .

Step 3. Find ASM $i^{'}$ that has the maximum probability of no-leaker value.

- The probability of no-leaker value of chosen ASM $i^{'}$ has to be greater than the probability of no-leaker value of ASM i. If there is no available ASM $i^{'}$, go to step 1 and choose another ASM to exchange SAM rounds from their initial engagement plans.

Step 4. Determine SAM round $j^{'}$ that has the maximum total tracking changeover. If there is no available SAM round go to step 3 and find another ASM. Otherwise choose SAM round $j^{'}$ according to the following criteria from the initial engagement plan of ASM $i^{'}$ to exchange with SAM round j.

- There must be a valid combination between SAM system $j^{'}$ and chosen ASM i.
- SAM system $j^{'}$ has to be different than SAM system j.

Step 5. Search for available time slot to allocate SAM $j^{'}$ to ASM i and SAM j to ASM $i^{'}$ If exchange is not possible, go to step 4 and find another SAM system to allocate against ASM i. Otherwise allocate SAM $j^{'}$ to ASM i and SAM j to ASM $i^{'}$. Calculate new Z_{TC} and Z_{PNL}. If the new solution is a dominated solution, do not update the initial schedule. Otherwise, add solution to the solution set. If the maximum number of exchange operation is achieved, stop. Otherwise go to the step 1.

Figure 5.17 demonstrates the exchange operation in an engagement schedule. ASM 2 and ASM 3 have the maximum and the minimum probability of no-leaker value respectively. SAM 4 has the maximum tracking changeover among the allocated SAM rounds in ASM 2 engagement schedule. SAM 3 has the maximum tracking changeover among the allocated SAM rounds in ASM 3 engagement schedule. The SAM rounds are exchanged between engagement plans of ASMs. The result is added to the solution set if it is not a dominated solution.

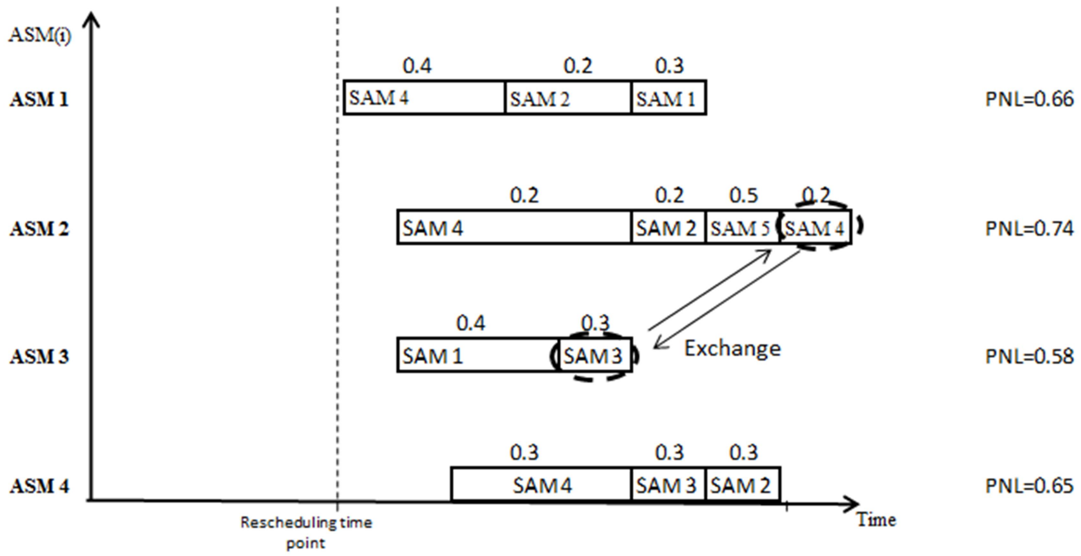

Figure 5.17 Exchange operation.

The flow chart of the heuristic algorithm is presented in Figure 5.18. Heuristic algorithm starts with a new allocation part. The solutions are generated by allocating on hand SAM rounds up to the maximum possible number of allocation. Secondly, replace part of the heuristic procedure is done. On hand SAM rounds up to the maximum possible number of replacement are used to generate solutions. In removal part, SAM round is discarded from the initial schedule.

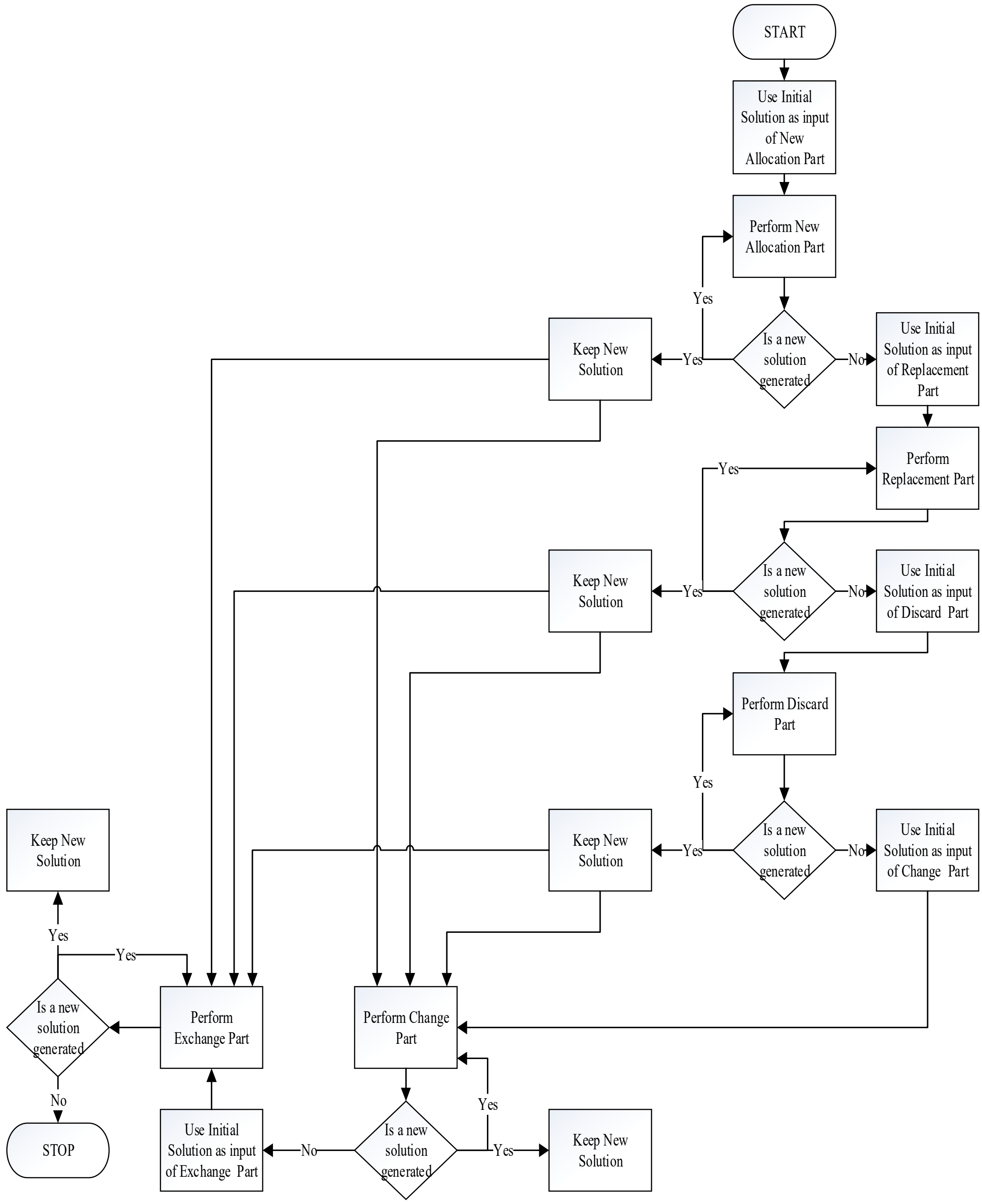

Figure 5.18 Flow chart of the heuristic algorithm.

The operation continues until minimum total number of tracking changeover is found. New allocation, replacement and removal operations generate solutions by adding new SAM rounds to the initial schedule or discarding allocated SAM rounds from the initial schedule. The result of new allocation, replacement and removal operations generates a different allocation plan from the initial allocations in terms of SAM rounds. On the other hand, change and exchange operations generate solutions by changing the place of allocated SAM rounds within the initial schedule. Thus, we perform change and exchange operations for each solution produced by new allocation, replacement and removal operations. All solutions generated by each part of heuristic procedure are combined and finally non-dominated solutions are determined in the solution set.

5.5 Computational Results

In this section, the results of computational experiments are presented to show the effectiveness of the proposed rescheduling model. For each disturbance case, improvement on efficiency of TG and increase on total tracking changeover are calculated. Performance of the heuristic approach is evaluated by applying it to each disturbance case. Lastly, we consider that during the engagement process any disturbance may occur. The schedule is updated in each rescheduling time point according to preference of DM. Comparison of results with rescheduling and keeping the initial schedule during the engagement process are presented.

We define seven different SAM systems and seven different ASMs by using real weapon systems in open literature. The SAMs and ASMs are randomly chosen among these systems in each problem set. The target ship of ASMs and available rounds of SAM systems are randomly selected.

First of all, we consider each disturbance separately. The optimal initial schedule is created with respect to the probability of no-leaker of TG. For destroyed ASM case, the outcome of an engagement between a SAM round and an ASM with respect to the initial schedule is observed. Observation starts with termination of the first engagement. We generate a random number from uniform distribution and assume that the engagement between SAM and ASM pair ends up with destroyed ASM if the random number value is less than the single shot kill probability of SAM against ASM. We set the ASM as destroyed and set the rescheduling time point as the finishing time of engagement. In the case of breakdown of a SAM system, it is assumed that one of the SAM system becomes unavailable to shoot after the engagement process starts. The broken SAM system and time of the breakdown are randomly determined. We set the SAM as broken and set the rescheduling time point as the starting time of the following time slot. The unavailable SAM system is discarded from the engagement allocation plan of ASMs. In the new ASM target case, we assume that sensors of TG detect an unexpected incoming ASM after the engagement process is started and the initial allocation plan is in effect. We randomly determine the time of arrival from Poisson distribution and set the rescheduling time point. We find the valid combination, the available time slots of engagements and the maximum number of engagements between the new ASM and SAM systems. The target ship, distance and the velocity of the new ASM are randomly determined. We solve BMRP-S for each disturbance case.

The problem sets are created with the combination of three and six ASMs and SAM systems. We solve all problem sets for all three cases by exact method using IBM ILOG CPLEX version 12.6 in Java platform on a personal computer with Intel i5-7200 CPU, 2.5 GHz and 8 GB of RAM. We run each problem instance by using five different seed set. The metrics defined in Chapter 1, maximum improvement on efficiency, MIE and maximum percentage improvement on efficiency, MPIE are used to evaluate change on Z_{PNL} objective. To evaluate stability objective Z_{TC}, maximum tracking changeover, MTC metric is defined. MTC indicates difference between the maximum Z_{TC} and minimum Z_{TC} values among the non-dominated solutions. Assume there are n non-dominated solutions indexed by s and included in set NS. Then, $MTC = Z_{TC}^{max} - Z_{TC}^{min}$ where

$$Z_{TC}^{max} = Z_{TC}^{\alpha},\ \alpha = \underset{s \in NS = \{1,..,n\}}{argmax} \left(Z_{TC}^{s} \right) \text{ and } Z_{TC}^{min} = Z_{TC}^{\beta} \quad \beta = \underset{s \in NS = \{1,..,n\}}{argmin} \left(Z_{TC}^{s} \right)$$

The relationship between Z_{PNL} and Z_{TC} is measured by average percentage improvement on efficiency with one tracking changeover, *APIEC*. The measure is calculated by $APIEC = \frac{MPIE}{MTC}$. It shows percentage improvement of efficiency on average with only one tracking changeover.

The computational results for destroyed ASM case are given in Table 5.8. Average MIE values are between 0.107 and 0.244. The highest average MIE and APIE is provided when the number of ASM is 6 and the number of SAM system is 3. The lowest average MIE and APIE is in the problem set when number of ASM is 3 and SAM system is 6.

The average MIE and APIE results increase if the number of ASM increases or the number of SAM system decreases. The highest maximum MIE value, 0.704 and lowest minimum MIE value, 0.016 are attained in the problem set with 6 ASMs-6 SAM systems and 3 ASMs-6 SAM systems respectively.

The highest average MTC value, 9 is in problem set with 6 ASMs and 6 SAM systems. On the other hand, the lowest average MTC value, 2.4 is in problem set with 3 ASM and 3 SAM systems. It shows that average MTC increases when the number of ASMs and SAM systems increase.

APIEC values depend on both MIE and MTC values. We have the highest average APIEC value, 18.61%, in problem set with 6 ASM and 3 SAM systems. The highest maximum APIEC value, 42.52% is attained in problem set with 6 ASM and 6 SAM systems. It can be stated that in this problem one tracking changeover provides 42.52% improvement on efficiency.

Table 5.8 Minimum, average, maximum values of metrics for destroyed ASM.

			SAM	
ASM			**3**	**6**
3	MIE	min	0.088	0.016
		ave	0.135	0.107
		max	0.193	0.193
	MPIE (%)	min	10.27	2.07
		ave	21.66	15.13
		max	41.76	28.54
	MTC	min	1	3
		ave	2.4	4
		max	4	6
	APIEC (%)	min	2.57	0.52
		ave	10.19	4.19
		max	15.48	8.05
6	MIE	min	0.068	0.371
		ave	0.244	0.148
		max	0.545	0.704
	MPIE (%)	min	7.37	152.50
		ave	89.27	72.44
		max	153.19	340.16
	MTC	min	4	8
		ave	6.4	9
		max	11	10
	APIEC (%)	min	0.92	15.93
		ave	18.61	12.57
		max	38.30	42.52

The computational results for breakdown of a SAM system case are given in Table 5.9. Average MIE values increase from 0.197 to 0.326 when number of ASM increases from 3 to 6 in problem sets with 3 SAM systems. Also average MPIE increases from 36.78% to 141.78% in those problems sets.

The highest average MIE, MPIE and APIEC values are attained in problem sets with 6 ASMs and 3 SAM systems. The lowest average MIE, MPIE and APIEC values are attained in problem sets with 3 ASMs and 6 SAM systems.

Table 5.9 Minimum, average, maximum values of metrics for breakdown of a SAM system.

			SAM	
ASM			**3**	**6**
3	MIE	min	0.061	0.061
		ave	0.197	0.167
		max	0.539	0.261
	MPIE (%)	min	0.83	6.80
		ave	36.78	28.26
		max	76.79	45.92
	MTC	min	1	1
		ave	2.6	3.2
		max	4	6
	APIEC (%)	min	0.21	1.13
		ave	17.79	13.98
		max	34.50	32.40
6	MIE	min	0.180	0.000
		ave	0.326	0.225
		max	0.589	0.399
	MPIE (%)	min	47.66	31.40
		ave	141.82	85.64
		max	196.45	192.35
	MTC	min	3	0
		ave	5.6	8.3
		max	10	10
	APIEC (%)	min	15.89	3.32
		ave	26.96	9.97
		max	45.34	21.37

The average MTC values increase when the number of SAM systems and ASMs increase. The highest average and maximum MTC values are 8.3 and 10 respectively in problem set with 6 ASMs and 6 SAM systems. The highest maximum MPIE value, 199.45%, is attained in the problem consists of 6 ASM and 3 SAM systems. Thus, about three times increment on percentage improvement is achieved in this problem.

The computational results for new target ASM case are presented in Table 5.10. The highest average MIE value, 0.288, is in problem set with 6 ASM and 3 SAM systems. It decreases to 0.154 when number of ASM and SAM system is 6. The highest average MPIE value, 108.11%, in the problem set with 6 ASM and 3 SAM systems. Thus, if the whole schedule is updated, the efficiency of the system is doubled in this problem set.

Table 5.10 Minimum, average, maximum values of metrics for new ASM target.

			SAM	
ASM			**3**	**6**
3	MIE	min	0.060	0.027
		ave	0.295	0.157
		max	0.685	0.348
	MPIE (%)	min	10.43	3.95
		ave	55.15	46.47
		max	115.56	159.83
	MTC	min	1	1
		ave	3.6	4.6
		max	6	12
	APIEC (%)	min	1.74	1.68
		ave	21.68	15.26
		max	44.72	53.28
6	MIE	min	0.133	0.062
		ave	0.288	0.154
		max	0.695	0.266
	MPIE (%)	min	50.43	3.67
		ave	108.11	44.91
		max	153.57	87.46
	MTC	min	3	6
		ave	5.75	8.6
		max	12	13
	APIEC (%)	min	4.20	1.38
		ave	27.73	11.97
		max	44.90	22.02

The lowest average MPIE value, 48%, is in the problem set with 3 ASM and 6 SAM systems. The lowest and highest average MTC values are 3.6 and 8.6 in problem sets with 3 ASM-3 SAM systems and 6 ASM-6 SAM systems respectively. The results indicate that in all disturbances the effect of rescheduling approach is more notable with limited number of SAMs and with many threats. The value of improvement on efficiency decreases when the number of SAM system increases or number of ASM decreases. MTC values increase when number of SAM systems or number of ASM increase. In some problems, the efficiency of TG can be sufficiently improved by slight increase on tracking changeover on the initial schedule.

In each disturbance case, we solve problems with the heuristic approach. The solution of the exact method and the heuristic algorithm is compared for all cases. To compare the performance of the algorithms, we use same metrics in Chapter 3. These metrics are the Hyper Volume Ratio (HVR), Inverted Generational Distance (IGD) and Number of Found Solutions (NFS). We present performance of the heuristic approaches in Tables 5.11, Table 5.12 and Table 5.13 for destroyed ASM, breakdown of a SAM system and new target case respectively. The last row of each ASM and SAM combinations in the tables includes the number of non-dominated solutions generated by heuristic and exact method respectively.

The heuristic approach generates nearly all non-dominated solutions in all cases. In problem sets with ASM-SAM combinations of 3-3 and 3-6 for destroyed ASM and breakdown of a SAM system cases only two solutions cannot be generated. In problem set with 6 ASMs and 6 SAM systems in new ASM target case, 22 non-dominated solutions are generated out of 29.

The average HVR values are greater than 0.98 in all problem sets and all disturbance cases except problem set with 6 ASM and 6 SAM systems in new ASM target case. The lowest minimum HVR value, 0.9312 and lowest average HVR value, 0.9593 are attained in problem set with 6 ASM and 6 SAM systems in new ASM target case. Since HVR values are close to 1 in all problem sets, the dominated solution approximates the non-dominated solutions. Also, IGD values are nearly zero in all problem sets. Thus, found solutions are diverse enough despite a few non-dominated solutions cannot be generated in some problem sets. As a result, the performances of the heuristics are quite well in all cases and in all problem sets.

Table 5.11 Performance of heuristic for destroyed ASM case.

			SAM	
ASM	Performance Metrics		**3**	**6**
3	HVR	min	0.9896	0.9794
		ave	0.9968	0.9946
		max	1.0000	1.0000
	IGD	min	0.0000	0.0000
		ave	0.0025	0.0039
		max	0.0075	0.0133
	No. of Solutions		13/15	14/18
6	HVR	min	0.9352	0.8176
		ave	0.9862	0.9635
		max	1.0000	1.0000
	IGD	min	0.0000	0.0000
		ave	0.0007	0.0066
		max	0.0024	0.0328
	No. of Solutions		20/22	23/28

Table 5.12 Performance of heuristic for breakdown of a SAM system case.

			SAM	
ASM	Performance Metrics		**3**	**6**
3	HVR	min	0.9770	0.9690
		ave	0.9918	0.9921
		max	1.0000	1.0000
	IGD	min	0.0000	0.0000
		ave	0.0023	0.0056
		max	0.0100	0.0205
	No. of Solutions		13/15	15/17
6	HVR	min	0.9827	0.9605
		ave	0.9943	0.9861
		max	1.0000	1.0000
	IGD	min	0.0000	0.0000
		ave	0.0005	0.0032
		max	0.0016	0.0100
	No. of Solutions		20/22	23/25

Table 5.13 Performance of heuristic for new ASM target case.

			SAM	
ASM	Performance Metrics		**3**	**6**
3	HVR	min	0.9716	0.9526
		ave	0.9884	0.9844
		max	1.0000	1.0000
	IGD	min	0.0000	0.0000
		ave	0.0333	0.0118
		max	0.1500	0.0342
	No. of Solutions		16/19	22/27
6	HVR	min	0.9843	0.9312
		ave	0.9969	0.9593
		max	1.0000	1.0000
	IGD	min	0.0000	0.0000
		ave	0.0003	0.0061
		max	0.0015	0.0100
	No. of Solutions		22/27	22/29

Elapsed times of the exact method and the heuristic algorithm are given in Table 5.14. Elapsed times of problem sets depend on number of non-dominated solutions. Also, the problem characteristics such as the valid engagement between SAM systems and ASMs, the number of SAM rounds in each SAM systems, the rescheduling time point affect the complexity of the problems. When the problem size gets larger, the run times of the exact and the heuristic method increase. The results shows that in all problem sets, exact method run times are greater than those of heuristic approaches. We cannot solve a problem in two hours with 6 ASM and 6 SAM systems for destroyed ASM and breakdown of a SAM system case. Two problems cannot be solved within two hours with 6 ASM and 6 SAM systems for new ASM target case. The heuristic approach finds non-dominated solutions less than one second in problem set with 3 ASMs and 3 SAM systems for all cases. The highest average elapsed time of heuristic, 1.89 is in new ASM target case with 6 ASMs and 6 SAM systems. Elapsed times of the exact method are quite large even problem sizes are small in terms of the number of ASMs and SAM systems. Results show that exact method cannot be used in real life since problems cannot be solved within the time limit.

Table 5.14 Elapsed times (sec) of exact and heuristic solution procedures.

		SAM					
		3			6		
ASM		Destroyed ASM	Broken SAM	New ASM	Destroyed ASM	Broken SAM	New ASM
3	Exact	10.65	19.08	56.32	23.15	44.45	89.85
	Heuristic	0.35	0.24	0.67	1.32	1.54	1.67
6	Exact	63.24	68.59	91.08	185.01*	170.23*	246.48**
	Heuristic	1.48	1.72	1.22	1.71	1.81	1.89

*One problem cannot be solved within 3600 sec.
**Two problems cannot be solved within 3600 sec

Results given above are produced by considering only occurrence of one disturbance. In addition to this, we take into account that any disturbance may occur during the engagement process. We start from the beginning of the engagement time horizon and observe the disturbances. Thus, after a disturbance happens, the engagement allocation plan is rescheduled according to the DM's preferences. The same ANN method as discussed in Chapter 4 is used to structure the DM's preferences. The objective functions Z_{PNL} and Z_{TC} are taken as input of ANN. The utility value of each solution is taken as output of the ANN. We calculate the utility values from function $f(z) = \max \sum_{i=1}^{p=2} \left(z_i - z_i^{IP}\right)^2$ where z_i is the i^{th} objective function value and z_i^{IP} is the ideal point of i^{th} objective function. We iterate the ANN structure in forward phase and construct the topology of the ANN. In each rescheduling time point, we generate non-dominated solutions by heuristic approach. The best solution is decided by ANN in each rescheduling time point. The existing schedule is updated and rescheduling continues until the end of time horizon when a disruption occurs. At the engagement time horizon, number of ships survived (NSS), percentage of survived ships (PSS) and percentage of destroyed ASM (PDA) values are calculated as in Chapter 4. To compare the rescheduling results with keeping the initial schedule results, we calculate the metrics without updating the schedule. Table 5.15 presents the average results for each number of ASM and number of SAM system combination. The number of ASMs and SAM systems are chosen between 7 and 10. We run each problem set with five different seed.

Table 5.15 The average results of rescheduling and not rescheduling.

			SAM			
ASM			**7**	**8**	**9**	**10**
7	NSS	reschedule	3.6	3.8	4.8	5.2
		not reschedule	2.2	2.4	4	4.2
	PSS (%)	reschedule	80.00	80.00	88.00	84.76
		not reschedule	48.33	51.00	73.33	71.24
	PDA (%)	reschedule	84.29	83.93	89.64	87.14
		not reschedule	65.71	65.00	79.64	71.43
8	NSS	reschedule	3.8	3	4.2	5
		not reschedule	1.2	2.2	3.6	4
	PSS (%)	reschedule	91.00	63.00	76.67	87.62
		not reschedule	28.00	44.00	66.00	66.29
	PDA (%)	reschedule	91.11	75.28	85.83	88.61
		not reschedule	60.56	62.78	74.17	76.94
9	NSS	reschedule	3	2.4	3.8	4.8
		not reschedule	1.2	2	2.8	3
	PSS (%)	reschedule	70.00	50.00	70.67	79.05
		not reschedule	29.00	41.00	51.33	47.52
	PDA (%)	reschedule	82.67	71.11	79.56	81.56
		not reschedule	55.78	60.44	64.67	57.33
10	NSS	reschedule	3.2	2.4	2.8	4
		not reschedule	1.6	1.4	2.4	3
	PSS (%)	reschedule	70.00	51.00	52.67	61.71
		not reschedule	36.00	30.00	44.00	51.24
	PDA (%)	reschedule	75.27	63.27	66.36	66.91
		not reschedule	62.18	54.36	56.36	60.36

In computational experiments, it is assumed that a ship can contain at most two SAM systems. Thus, high number of SAM system indicates higher number of total ship at the beginning of engagement process. In all problem sets, average number of survived ships increase with rescheduling model. The highest average NSS, 5.2 is attained is in problem set with 7 ASMs and 10 SAM systems. The average percentage of survived ships increase more than two times in problem sets with 8 ASMs-7 SAM systems and 9 ASMs-7 SAM systems. If the schedule is not updated, only 28% of ships are survived in problem set with 8 ASMs and 7 SAM systems. Percentage of survived ships increases to 91% in this problem set by using the rescheduling model.

The lowest average percentage of survived ships, 50.00%, is attained in problem set with 9 ASMs and 8 SAM systems. When the number of SAM systems decrease, the effect of rescheduling is more apparent. For instance, rescheduling increases PSS value from 48.33% to 80.00% and from 71.24% to 84.76% in problem set with 7 ASMs-7 SAM systems and 7 ASMs-10 SAM systems respectively. Also percentage of destroyed ASM increases by rescheduling approach. The highest average PDA, 89.64% is in problem set with 7 ASMs and 9 SAM systems. In problem set 8 ASMs and 7 SAM systems, we get the highest difference for PDA between rescheduling model and keeping the initial schedule. The value increases from 60.56% to 91.11% in this problem set.

CHAPTER 6

COMPARISON OF MODELS

In this chapter, we compare the results of BMRP and BMRP-S models by solving same problem sets with both models. Also, we maximize the probability of no-leaker value of TG without considering stability measure in each rescheduling time point. All SAM allocations are rescheduled only with respect to efficiency of TG. We call this problem as complete rescheduling problem (CRP). Solution of CRP is the extreme point of BMRP and BMRP-S with maximum Z_{PNL}. While solving problems with BMRP or BMRP-S, we additionally calculate the stability objective of the other model. We find all objective function values in each model to see how concentrating on different stability measure affects the performance metrics and outcome of the engagement process. The different size problems are solved with exact methods of each model instead of heuristic approaches to have precise comparison. In each rescheduling time point, the increase on Z_{PNL}, Z_{ND} and Z_{TC} values are found and average of those values are calculated.

We generate problem sets with the combination of three and six ASMs and SAM systems. Each problem sets include five problem instances that are randomly created by using different seed sets. We assume that during the engagement process any disturbance may occur. The engagement process is started with respect to initial schedule and disturbances are observed during the engagement process. After a disturbance happens, the engagement allocation plan is rescheduled according to DM preferences. The same ANN method as discussed in Chapter 4 is used to structure DM preferences. For BMRP model, Z_{PNL} and Z_{ND} objective functions are taken as input of ANN. On the other hand, Z_{PNL} and Z_{TC} objective functions are taken as input of ANN in BMRP-S model. The existing schedule is updated and rescheduling continues until the end of time horizon when a disruption occurs.

Table 6.1 shows the increment on objective functions of each problem set. The average value of difference between Z_{PNL} value of chosen non-dominated point and Z_{PNL} value of initial engagement allocation plan for all rescheduling time point is presented in the first column of each problem set. The second column of each problem set indicates average difference between the Z_{ND} value of chosen non-dominated solution and initial schedule's Z_{ND} value. Similarly, last column is the difference between Z_{TC} value of chosen non-dominated solution and Z_{TC} value of the initial schedule. If in a rescheduling time point, keeping the initial schedule is the only solution we discard those solutions while taking average of the objective functions.

The results show that average increase on Z_{PNL} value is the highest with complete rescheduling. In all problem sets BMRP-S model has higher average increase on Z_{PNL} than BMRP model. The average increase on Z_{ND} value is smallest with BMRP model in all problem sets since BMRP keeps disruption on the initial schedule at a certain level. Similarly, BMRP-S model has the smallest average increase on Z_{TC} value in all problem sets. In addition to this, the average increase on Z_{TC} values are between 2.3 and 2.8 among all problem sets in BMRP-S. On the other hand, the range of Z_{ND} values is between 3.2 and 3.6 in BMRP model. Thus, the values of Z_{TC} in BMRP-S are smaller than Z_{ND} values of BMRP model. The main reason behind this result is that increasing Z_{PNL} value by rescheduling exactly increases Z_{ND} but may decrease Z_{TC} since at rescheduling time point the initial schedule has $Z_{ND} = 0$ and $Z_{TC} \geq 0$.

The average increase on Z_{PNL} value with complete rescheduling is greater than 0.2 in all problem sets. For instance, in problem set with 5 ASM and 4 SAM systems, average increase on Z_{PNL} equals to 0.273 by complete rescheduling. Those values are 0.211 and 0.240 in BMRP and BMRP-S models respectively. But, complete rescheduling produces 0.273 increase on probability of no-leaker with 7.1 disruption on initial schedule and 6.1 tracking changeover on the average. So, complete rescheduling may not be realistic and possible during the engagement process. The average value of increment on Z_{PNL} is 0.240 in BMRP-S model with 2.5 number of tracking changeover and 5.5 number of disruption on the initial schedule. For BMRP model, the value of increment on Z_{PNL} is 0.211 in with 3.9 number of disruption on the initial schedule and 5.1 number of tracking changeover.

Table 6.1 Average increase on objective function values in three models.

		SAM					
ASM		**4**			**5**		
		Z_{PNL}	Z_{ND}	Z_{TC}	Z_{PNL}	Z_{ND}	Z_{TC}
4	CRP	0.224	6.7	5.2	0.201	6.5	5.6
	BMRP	0.152	3.3	4.9	0.141	3.2	5.2
	BMRP-S	0.178	5.2	2.3	0.149	5.0	2.8
5	CRP	0.273	7.1	6.1	0.216	7.1	6.4
	BMRP	0.211	3.9	5.1	0.143	3.6	5.8
	BMRP-S	0.240	5.5	2.5	0.171	5.9	2.8

The lowest average increase on Z_{PNL} is attained in problems set with 4 ASM and 5 SAM systems for all models. On the contrary, the highest average increase on Z_{PNL} is attained in problems set with 5 ASM and 4 SAM systems for all models. The results indicate the value of improvement on efficiency decreases when the number of SAM system increases or number of ASM decreases.

We also calculate the outcome metrics of the engagement process for both three models. The same outcome metrics of Chapter 4 such as the number of ships survived (NSS), percentage of survived ships (PSS) and percentage of destroyed ASM (PDA) are used to compare the model results. To find the difference between rescheduling model results and keeping the initial schedule results, we calculate the metric values without updating the schedule.

Table 6.2 shows the average results of outcome metrics. NSS, PSS and PDA values increase with rescheduling in all problem sets. The NSS value increases from 1.6 to 2.7 by complete rescheduling in problem set with 4 ASMs and 5 SAM systems. BMRP-S and BMRP survives 2.3 and 2.0 ships on the average in this problem set. The highest increments on NSS with rescheduling models are achieved in problem set with 5 ASMs and 4 SAM systems. Adversely, the gap between complete rescheduling and no rescheduling for all metrics is the lowest in the problem set with 4 ASMs and 5 SAM systems.

The benefit of rescheduling is more notable with limited number of SAM systems and higher number of ASMs. In all problem sets, results of BMRP and BMRP-S are nearly same but BMRP-S model attain slight better results than BMRP model in all metrics. For instance, on the average BMRP-S survives 81.1% of ships and BMRP survives 78.3% of ships in problem set with 5 ASMs and 4 SAM systems.

PDA results are higher than 90% in all problem sets with CRP and BMRP-S models. The lowest values of all metrics with no rescheduling are attained in problem set with 5 ASMs and 4 SAM systems. In this problem set, PDA increase from 64.6% to about 90.4% with BMRP-S model. In problem set with 4 ASMs and 5 SAM systems, the values of metrics for BMRP and BMRP-S are close to those of CRP. The PSS and PDA values are higher than 90.00% with CRP, BMRP and BMRP-S.

Table 6.2 The average results of outcome metrics for rescheduling models and no rescheduling.

ASM		SAM					
		4			5		
		NSS	PSS (%)	PDA (%)	NSS	PSS (%)	PDA (%)
4	CRP	2.7	88.2	93.2	3.2	94.2	96.7
	BMRP	2.0	79.1	89.0	2.8	90.5	93.3
	BMRP-S	2.3	81.6	91.1	2.9	91.1	95.0
	No Res	1.6	58.2	78.5	2.4	74.6	82.0
5	CRP	2.5	87.7	92.8	3.0	91.2	94.4
	BMRP	2.0	78.3	86.6	2.7	88.5	93.3
	BMRP-S	2.2	81.1	90.4	2.8	89.8	91.2
	No Res	1.3	56.6	64.6	2.2	73.33	79.33

The computational experiments show that BMRP-S model produces relatively better results than BMRP model in terms of efficiency of TG. This means that at rescheduling time points, chosen non-dominated point of BMRP-S has frequently higher Z_{PNL} value than chosen non-dominated point of BMRP. We examine the solutions of the problem sets and present some conclusions on the results as follows.

BMRP and BMRP-S include efficiency objective function as Z_{PNL}. BMRP considers the total number of changed SAM allocations with respect to initial schedule as a stability objective. On the other hand, the total number of tracking changeover is considered as a stability measure for BMRP-S. Figure 6.1 and 6.2 show two solutions denoted by μ and λ in the objective spaces of BMRP and BMRP-S respectively.

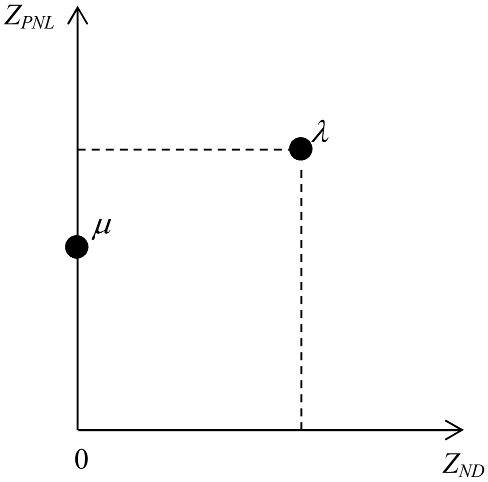

Figure 6.1 Case 1: Objective space of BMRP.

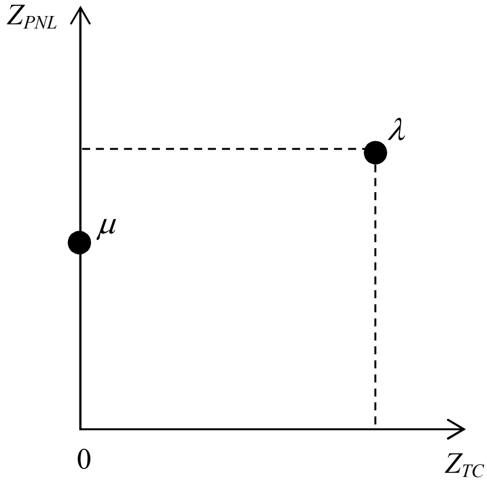

Figure 6.2 Case 1: Objective space of BMRP-S.

At rescheduling time point, the solution keeping the initial schedule, μ is the extreme point of Pareto front for BMRP model. Since there is no deviation from the initial schedule, $Z_{ND}^{\mu} = 0$. The solution λ is the adjacent efficient solution of μ for BMRP.

The initial schedule has also no tracking changeover for all SAM systems, $Z_{TC}^{\mu} = 0$. The solution λ is generated by rescheduling of SAM rounds. The total number of tracking changeover increases by updating the schedule. So, $Z_{TC}^{\lambda} > 0$ and $Z_{ND}^{\lambda} > 0$.

The solution μ is not dominated by solution λ for BMRP and BMRP-S. If the selection is done between λ and μ in BMRP-S model or in BMRP model, one of them may be preferred by DM. Figure 6.3 and Figure 6.4 show a similar case except tracking changeover value of the initial schedule.

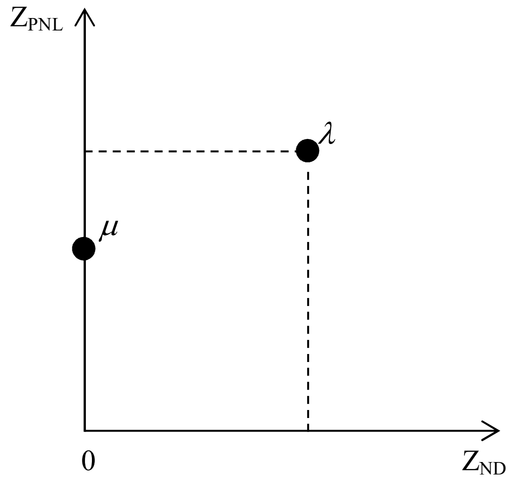

Figure 6.3 Case 2: Objective space of BMRP.

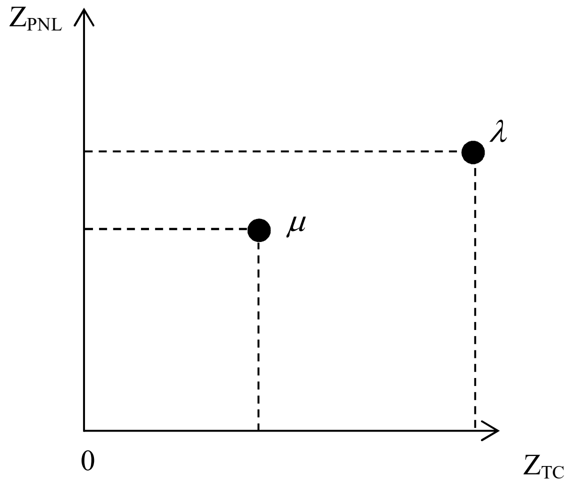

Figure 6.4 Case 2: Objective space of BMRP-S.

The initial schedule has more than one number of tracking changeover, $Z_{TC}^{\mu} > 0$ at the rescheduling time point and the total number of tracking changeover increases by updating the initial schedule. The generated solution λ has $Z_{TC}^{\lambda} > 0$ and $Z_{ND}^{\lambda} > 0$. Similar to the first case, both solution μ and solution λ can be chosen by DM with BMRP and BMRP-S models. Above results show that deviation from the initial schedule increases the total number of tracking changeover. The updated schedule does not dominate the initial schedule in both BMRP and BMRP-S models. In generating non-dominated solutions, if the total number of tracking changeover increases with increasing number of disruption, BMRP and BMRP-S are in same

Figure 6.5 and 6.6 show that generating a solution with rescheduling yields a different case. The generated solution λ is the adjacent efficient solution of μ for BMRP and $Z_{PNL}^{\lambda} > Z_{PNL}^{\mu}$ and $Z_{ND}^{\lambda} > Z_{ND}^{\mu}$. By rescheduling, change on the initial schedule produces a better solution in terms of probability of no-leaker value although it causes deviation. Figures 6.5 and 6.6 show the solution μ and solution λ in objective spaces of BMRP and BMRP-S, respectively. The initial schedule of SAM systems at rescheduling time point includes more than one target ASM,

$Z_{TC}^{\mu} \geq 0$ and increase on the probability of no-leaker value of TG decreases the total number of tracking changeovers.

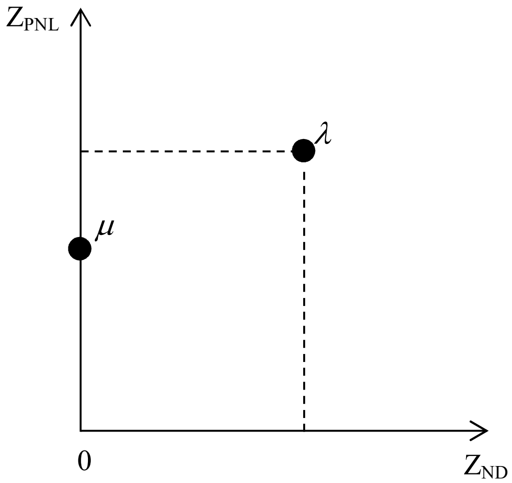

Figure 6.5 Case 3: Objective space of BMRP.

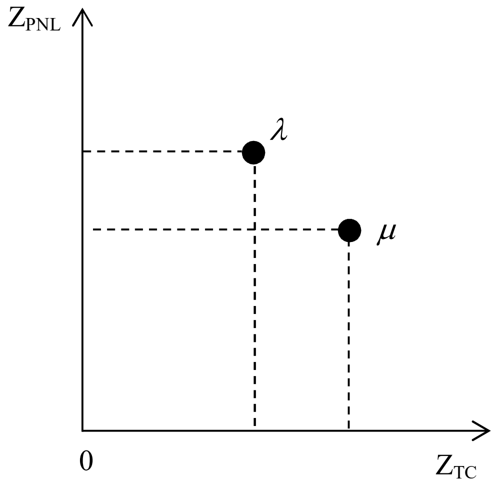

Figure 6.6 Case 3: Objective space of BMRP-S.

Rescheduling with BMRP-S yields a better solution in terms of both objectives. Thus, $Z_{PNL}^{\lambda} > Z_{PNL}^{\mu}$ and $Z_{TC}^{\lambda} < Z_{TC}^{\mu}$ and μ is dominated by λ for BMRP-S. The utility value of solution λ is always greater than solution μ and it is exactly preferred by DM if a choice decision is made between them in BMRP-S. On the other hand, in BMRP, μ which has lower probability of no-leaker value may be chosen by DM if the utility value of μ is greater than the utility value of λ.

In case of the increasing probability of no-leaker value of TG by changing the initial schedule always yields more disrupted initial schedule but may result more stable schedule in terms of tracking changeover. Also, with more disruption in the initial schedule, the solution λ may be dominated by another solution according to BMRP-S. We present uttermost of this case in Figures 6.7 and 6.8. Assume that α is the extreme point of the Pareto front of BMRP with maximum efficiency. Then $i = \underset{i=1,\ldots,n \in NS}{argmax}\left\{Z_{PNL}^{i}\right\}$ and $Z_{PNL}^{\alpha} = Z_{PNL}^{i}$. Consider the maximum deviation from the initial schedule to have maximum Z_{PNL} decreases total number of tracking changeover. If the solution α has $Z_{TC}^{\alpha} = 0$, then it is the only non-dominated solution of BMRP-S. The best solution according to BMRP-S, α has the maximum probability of no-leaker value as presented in Figure 6.8.

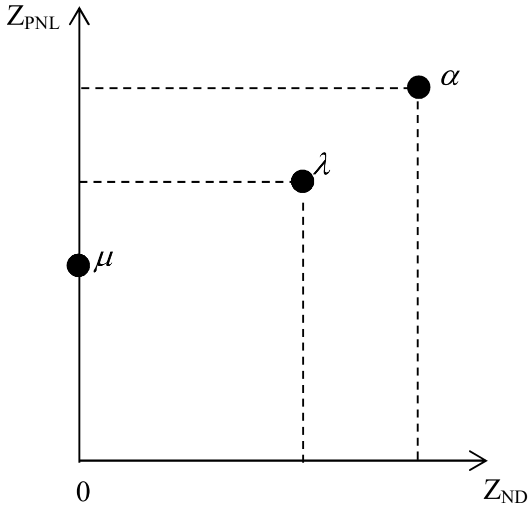

Figure 6.7 Case 4: Objective space of BMRP.

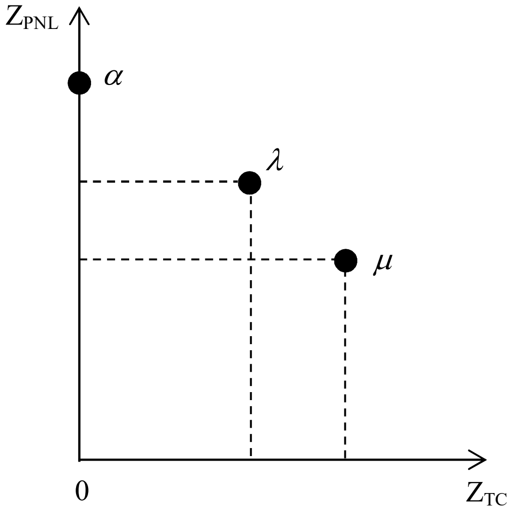

Figure 6.8 Case 4: Objective space of BMRP-S.

On the other hand, the initial schedule in the rescheduling time point may be the solution with maximum total tracking changeovers. In this case, DM may want to reduce total number of tracking changeover and may sacrifice from the efficiency of TG even the result deviates from the initial schedule. Figures 6.9 and 6.10 show the solution ω that is generated by rescheduling to decrease tracking changeover that also decreases efficiency of TG. The solution ω is dominated by solution μ according to BMRP since $Z_{PNL}^{\mu} > Z_{PNL}^{\omega}$ and $Z_{ND}^{\omega} > Z_{ND}^{\mu}$. Thus, ω cannot be chosen in BMRP model. However, for BMRP-S it may be preferable according to DM. To have more stable schedule in terms of tracking changeovers, probability of no-leaker value of TG decreases. This yields deviation from the initial schedule. In this case, BMRP have more chance than BMRP-S while choosing better solutions in terms of efficiency objective.

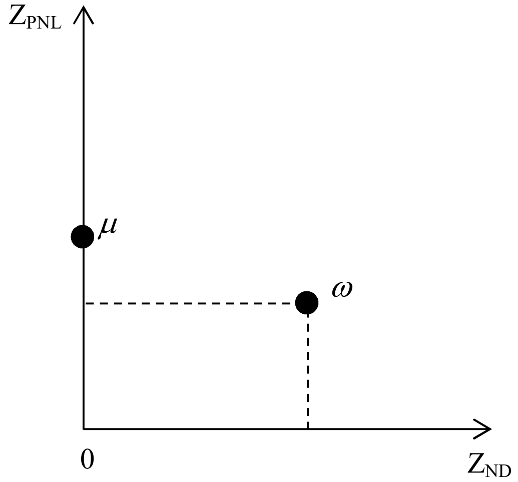

Figure 6.9 Case 5: Objective space of BMRP.

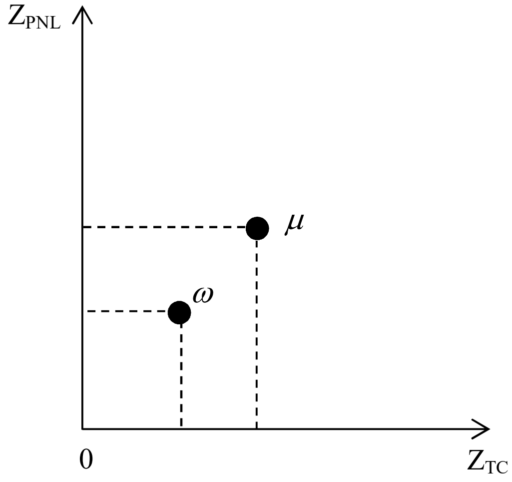

Figure 6.10 Case 5: Objective space of BMRP-S.

We consider generating only one solution by changing the initial schedule up to this point. The comparison is done between these two solutions to understand the difference between BMRP and BMRP-S models. However, at the rescheduling point there exist several non-dominated solutions of both models. Since DM may want to increase efficiency of TG with rescheduling, DM always chooses solutions with higher Z_{PNL} value. Dispersions of the non-dominated solutions in BMRP and BMRP-S are the main indicative factor at Z_{PNL} value of chosen non-dominated points of two models. In some instances, with slight deviation from the initial schedule higher Z_{PNL} value can be attained. This solution may be less preferred by BMRP-S since it causes too many tracking changeover. The solution chosen by BMRP-S may have lower Z_{PNL} value than those of BMRP. But, BMRP model has no option to increase Z_{PNL} value with decreasing Z_{ND} since the initial schedule is always a non-dominated solution of BMRP model. BMRP-S model may yield a decrease in Z_{TC} with increasing efficiency of TG. This increases the chance of choosing better solutions in terms of efficiency objective by BMRP-S model with respect to BMRP model.

CHAPTER 7

CONCLUSION AND FURTHER RESEARCH

In this dissertation, we study dynamic missile allocation problem for air defense of a TG. We develop solution procedures that provide an efficient air defense plan by rescheduling SAM rounds against ASMs in response to disturbances during the engagement process. The considered disturbances that disrupt the existing schedule during the engagement process are destroyed ASM, breakdown of a SAM system and new target ASM. We build the allocation strategy according to the time stages to update the schedule.

Rescheduling of SAMs in the presence of disturbances substantially increases air defense effectiveness but rescheduling causes deviation from the initial schedule and changes shoot order of SAM systems within the engagement process. Immediate change of targets for SAM systems may lead to not able to intercept and destroy ASMs. Hence, two main measures, efficiency of system and stability of schedule, are incorporated to the formulation of the problems.

First of all, we propose a biobjective missile rescheduling model (BMRP) that minimizes the total number of difference in allocations between the new and the initial schedule and maximizes the probability of the no-leaker value of TG. We generate all non-dominated solutions since DM may be interested in selecting optimal solution by screening all non-dominated solutions.

We show BMRP is an NP-Hard problem. We also investigate computational complexity of special cases. In order to solve BMRP, we use augmented ε-constraint method and evaluate the non-dominated solutions of the problem. In all cases, the effect of rescheduling approach is more notable with limited number of SAMs and with many threats. The improvement on efficiency increases if number of SAM system decreases and number of ASM increases.

We see that small size problems such as having five ASMs and five SAM systems cannot be solved in the time limit with augmented ε-constraint method. To utilize the model in real life cases, we develop two heuristic algorithms namely NRH and CEH algorithms. We test our heuristic approach in all disturbance cases. The heuristic approach yields highly successful results. Computation time of the heuristic approach is less than half a second in all problem sets and in all cases. Therefore one can conclude that for large size problems, heuristic algorithm is a very attractive alternative.

In the second part of our research, we consider a real life engagement process and all three disturbance cases are integrated. We develop a decision aid framework that uses results of BMRP and update the existing schedule by choosing one of the non-dominated solution in the occurrence of disturbances. We go on upon the time horizon to observe an occurrence of disturbance. We start from the beginning of the engagement time horizon and detect the type of disruption. The rescheduling decision is time-sensitive and the amount of information to be processed before is large. Thus, we suggest an artificial neural network approach that includes an adaptive learning algorithm to structure prior articulated preferences of DM and choose one of the non-dominated solutions in each rescheduling time point. It is assumed that DM has a non-decreasing quasi-concave utility function since efficiency and stability objectives have the property of diminishing marginal rate of substitution. While generating a non-dominated solution, the solutions belong to dominated cones are eliminated in each iteration.

The solution procedure is based on the results of BMRP model in each rescheduling time point. We solve the BMRP model either with the augmented ε-constraint method or the heuristic approach. We solve varying size problems and evaluate the performance of our model. Results of the computational tests have revealed that our dynamic update scheme for missile allocation problem can greatly increase total effectiveness of TG. Our model yields an increased number of ASMs and survived ships.

We next introduce a new biobjective missile rescheduling problem with sequence dependent stability measure (BMRP-S). The point that is of concern for this study is the need to integrate sequencing decisions with missile allocation. We consider decision of sequence of ASMs in SAM system shoot schedule. By defining a new stability measure that considers change of target tracking for SAM systems, the new engagement plan yields improvement on targeting of SAM systems. The formulation of BMRP-S maximizes the probability of no-leaker value of TG and maximizes number of shoots to same ASMs consecutively for each SAM systems, or equally minimizes the total number of tracking changeovers for SAM systems. A drawback of our formulation is the fact that the number of variables and constraints grow with the number of time slots, SAM systems and ASMs.

Results obtained in our experimental study show that the computation time to solve the problem is extremely large even for small size instances. Hence, we propose a new exact solution procedure in order to solve BMRP-S. We generate non-dominated solutions by combining a mathematical model with the probability of no-leaker objective and an algorithm that obtains the minimum total number of tracking changeover by revising shoot order of SAM systems. To get a solution procedure that enables us to update the existing schedule in a few seconds, we develop a new heuristic procedure for BMRP-S. We solve varying size problems using both the exact and the heuristic procedures, evaluate the results of BMRP-S and analyze the performance of heuristic procedure.

In the last part of our study, we compare the results of BMRP and BMRP-S models by solving same problem sets with both models. Additionally, we only take into account the efficiency objective and we completely reschedule all SAM round allocations as any disturbance occurs. We find all objective function values in each model to see how concentrating on different stability measure affects the performance metrics and outcome of the engagement process.

To the best of our knowledge, our rescheduling approach is the first attempt for weapon target allocation problems and multiobjective rescheduling strategy throughout the engagement process is the first reported study in the literature. Missile rescheduling models can be used as a threat evaluation and weapon assignment system (TEWASA) module in the decision making process during the engagements.

This research can be extended in several directions. Allocation of SAM rounds against ASMs is a kind of resource allocation problem. The approach presented in this research can be used for the real-time constrained resource management problems in a variety of application areas. The models and heuristics may also be useful in the context of rescheduling problems that include time critical applications require fast response.

We consider SLS firing policy in formulation of the problems. The incorporating different firing policies affects the changes on shoot sequence and deviation from the schedule. For instance, other firing policies such as shoot-look-shoot-shoot (SLSS) or modified shoot-look-shoot (m-SLS) can be analyzed with rescheduling approach.

To structure prior articulated preferences of DM, we use ANN method. One can develop a different preference elicitation approach for ANN. DM may not give an exact utility value to the efficient solutions or taking precise information may be impracticable from DM. Asking DM to make pairwise comparisons and use this information in ANN should be more applicable.

A noteworthy extension of our study might be developing initial robust engagement allocation plans. Robust scheduling can minimize the effects of disturbances on the air defense effectiveness. A typical solution to generate a robust schedule for missile allocation problem might be to consider both efficiency and stability measures due to several disturbances. A robustness measure that represents the efficiency of the realized schedule and deviation from the initial schedule can be defined and the robustness of the schedule can be measured by different combinations of disturbances. Also, the worst case performance of the robust schedules with different firing policies can be evaluated in missile rescheduling problems.

Another interesting further research might be integrating the probability of occurrence of disturbances to the definition of the problem with a stochastic formulation. With the probability of disturbances, a multi-stage stochastic formulations can be developed for dynamic allocation of SAM rounds. The states of SAM systems and ASMs may be defined and updated with the outcome of the engagements and occurrence of disturbances in each stage.

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APPENDIX A

PROPERTIES OF WEAPON SYSTEMS

The features of SAM systems and ASMs used in computational studies are given in Table A.1 and A.2 respectively.

Table A.1 Features of SAM systems

SAM System	Minimum Range (m)	Maximum Range (m)	Speed (m/sec)	Type
SeaSparrow	1500	16000	850	Self-Defense
ESSM	1500	18000	1224	Self-Defense
Aster-15	1500	30000	986	Self-Defense
Barak	1500	12000	680	Self-Defense
SM-1	5000	38000	680	Area Air Defense
SM-2	5000	170000	850	Area Air Defense
Aster-30	3000	100000	1394	Area Air Defense

Table A.2 Features of ASMs

ASM	Speed (m/sec)
Harpoon	289
Exocet	306
Polyphem	221
Gabriel	238
Penguin	238
SS-N26	1190
Maveric	850

The steps of the sample problem generation are as follows:

1. Determine the number of ASMs, number of SAMs, the unit duration of time slot and the setup time of an engagement.

2. Choose SAM systems randomly from seven SAM systems and ensure that at least one area defense system is selected.

3. Choose ASMs randomly from seven ASMs.

4. Determine present distance of ASMs randomly ranging from 5 to 40 km.

5. Determine the target ship of ASMs randomly.

6. Generate number of SAM rounds (no more than 9 missiles for each SAM systems).

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