HISTORICAL AND GEOMETRICAL ANALYSIS OF MUQARNAS AND PROSPECT OF ITS REFLECTION ON TODAY'S ARCHITECTURE

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

ELENA IMANI

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
BUILDING SIENCE
IN
ARCHITECTURE

JULY 2017

ABSTRACT

HISTORICAL AND GEOMETRICAL ANALYSIS OF MUQARNAS AND PROSPECT OF ITS REFLECTION ON TODAY'S ARCHITECTURE

Imani, Elena

M.Sc. Bilding Science in Architecture.

Supervisor: Prof. Dr. Mualla Erkılıç

July 2017, 221 pages

Muqarnas, one of the key elements in Islamic architecture, is a three-dimensional element that is mostly used in religious buildings for decorating vaults, niches, porches and domes. Muqarnas initially was used as a functional element to provide a gradual and smooth transition between two levels, two sizes or two shapes in buildings, but over time, various *Muqarnas patterns* have developed and acquired ornamental features.

After the developments achieved in digital art and architecture, Muqarnas compositions became the source of inspiration for many designers who try to generate alternative two and three-dimensional patterns in their works. Today in architecture, there is a developing tendency to re-generate Muqarnas compositions through digital technologies in order to benefit from historical patterns of Muqarnas either in facades or interior decorations of modern buildings.

Following a historical research on the types, forms, and elements of Muqarnas in architectural works in Islamic countries this thesis aims, firstly, to analyze and explore

the geometrical characteristics of Muqarnas focusing on two dimensional *motif-based* and *layer-based* properties. The selected modern examples of Muqarnas forms are analyzed, similarly, in geometrical terms, as some cases, in order to find out the similarities /dissimilarities between modern and historical Muqarnas examples.

It is the claim of this thesis that the patterns of historical Muqarnas compositions have their own geometrical characteristics that can be considered as unique guidelines of Muqarnas patterns that are used intentionally or unintentionally in today's architecture. It is claimed, also, in this thesis that apart from some similarities of appearances the geometrical and constructional realization of the historical Muqarnas patterns are completely different from the modern ones due to generative design formation in digital medium.

Keywords: Muqarnas, geometrical analysis, muqarnas patterns, muqarnas guidelines.

ÖZ

MUQARNAS'IN TARİHSEL VE GEOMETRİK ANALİZİ VE BUGÜNÜKÜ MİMARİ GELİŞİMİ ÜZERİNDE YANSIMASI

Imani, Elena

Yüksek Lisans, Yapı Bilimleri, Mimarlık Bölümü

Supervisor: Prof. Dr. Mualla Erkılıç

Temmuz 2017, 221 sayfa

İslam mimarisindeki en önemli unsurlardan biri olan Mukarnas, çoğunlukla dini yapılarda tonoz, niş, revak ve kubbe detay ve dekorasyonu için kullanılan üç boyutlu bir elemandır. Mukarnas yapılarda önceleri iki seviye / boyut /şekil arasındaki kademeli ve sürekli bir geçişi sağlamak için işlevsel bir unsur olarak kullanılmış ancak zamanla çeşitli Mukarnas örüntüleri geliştirilerek daha kompleks detay ve bezemeler oluşturulmuştur.

Dijital sanat ve mimaride elde edilen gelişmelerden sonra Mukarnas kompozisyonları, iki ve üç boyutlu modeller üretmeye çalışan birçok tasarımcı için ilham kaynağı olmuştur. Bugün mimaride modern binaların cephelerinde ve iç süslemelerinde Mukarnas'ın tarihi örneklerinden yararlanılarak digital ortamda üretilmiş çeşitli kompozisyonları görebilmekteyiz.

İslam ülkelerindeki mimari yapılardaki Mukarnas türleri, motifleri ve bileşenleri üzerine tarihsel bir araştırmanın ardından, bu tez, öncelikle iki boyutlu motif ve katman temelli özelliklere odaklanan Mukarnas'ın geometrik özelliklerini analiz etmeyi ve keşfetmeyi amaçlamaktadır. Modern ve tarihi Mukarnas kompozisyonlerı arasındaki benzerlikleri / farklılıkları ortaya çıkarmak üzere bazı örnek yapılar irdelenerek geometrik kurgu ve yönleri ile analiz edilmiştir.

Bu tez, tarihi Mukarnas örüntülerinin bugünün mimarisinde bilinçli ya da istemeden kullanılan Mukarnas kompozisyonlarından farklı olarak düşünülebilecek, kendine has geometrik kugulama/yapma özelliklerine sahip olduklarını savunur. Ayrıca bu tez, tarihi Mukarnas ile biçimsel bazı benzerlikler olmakla beraber, modern dijital ortamda üretilen Mukarnas kompozisyonlarının geometrik kurgularının dijital yöntemle üretilen formasyonlarından kaynaklanan kendine özgü bir sistematiğinin olduğunun da altını çizmektedir

Anahtar kelimeler: Mukarnas, geometrik analiz, Mukarnas kalıpları, Mukarnas kuralları.

To my family

ACKNOWLEDGEMENTS

First and foremost I wish to express an immense depth of gratitude to my advisor, Prof. Dr. Mualla Erkılıç for her continuous guidance and support through this study. I will always be grateful to her for being a great mentor in the process of this thesis. Besides my advisor, I would like to thank the rest of my thesis committee Prof. Dr. Ali Uzay Peker, Assist. Prof. Dr. İpek Gürsel Dino, Assist. Prof. Dr. Pelin Yoncacı and Assist. Prof. Dr. Yasemin Afacan for their comprehensive suggestions and mind opening discussions.

My sincere thanks go to my sister, Semira Imani for her continuous support and patience. She was always there and stood by me through this process. Without her, this study could have never been completed.

I would like to present my gratitude to some of my dear friends, I am indebted to Homa Jabbarion for her precious friendships and generous helps during my presentation. I feel so much luck of having Deniz Sheibaniaghdam, Nesa Masalehdanzadeh, Sahar Shadmand and Negin Foroughi in my life, who have always shared my excitement and bringing their endless motivation and joy through the study process.

Last but not least, I want to thank my lovely parents for their love, support and trust not only in this process but in my life. It has always been precious to be your daughter and I have always felt your endless love.

TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

CHAPTER 1

INTRODUCTION

1.1 Muqarnas in Islamic architecture

Traditional architecture can't be considered as an architectural style on its own but it reveals the ways in which buildings in a specific area are built according to the local needs of that area. On the other hand, traditional architecture is a reflection of human beliefs and cultures therefore, using the elements of traditional architecture in modern architecture is important because it keeps local settlers connected to the past and provides them with a sense of stability and sense of belonging.

Architects try to vitalize their own traditional art as an expression of each countries ancestral origin and symbolize their culture either in exterior or interior decorations of buildings. The role of traditional architecture in today's architectural progress and modern architecture is an undeniable fact. In modern architecture, we can see some architects who try to refer forms of traditional architecture with different expressions. With an emphasis on the effect of tradition on modern architecture, Vellinga and Asquith (2006) state that:

> "what is needed at the beginning of the new millennium is an architectural perspective in which valuable vernacular knowledge is integrated with equally valuable modern knowledge…"

Islamic art and architecture have always been associated with Islamic countries from the advent of Islam and were manifested by Muslims who lived in Islamic countries. Historical and vernacular forms of Islamic architecture are considered as traditional architecture which encompasses a wide range of architectural styles in Islamic countries. Nature and the environment were the factors that influence Islamic architectural development. Islamic architecture mostly used as ornamental forms[1] which display geometrical forms, floral patterns[2] and calligraphic[3] inscriptions to decorate buildings. Geometry is always considered constitutive of Islamic art and architecture. The importance of the relation between geometry and Islamic architecture is described differently in various sources and researches (Ulu & Sener, 2009; Abdullahi & Embi, 2013; Dabbour, 2012). These researchers underline the importance of geometry in Islamic architecture and by referring to the strict idea which lays

[1] Complexity and variation of Islamic ornaments caused the development of a traditional multiple-level design. These ornaments have been formed by incorporating secondary elements with smaller scale into the background areas.
In Islamic ornaments, a basic shape or a background area mostly can be observed, but sometimes deformed so that it is hardly recognizable.

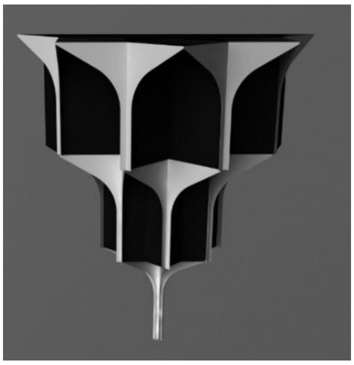
[2] In this multi-level design techniques, there is a primary scrollwork background with curvilinear elements symbolizing leafed and floral forms incorporates secondary floral elements with the similar style. Floral ornaments mostly used in Safavids architecture includes a diversity of stylistic variations. Yazdi bandi is one of the most significant style among the floral techniques that in this style the background of the primary floral scroll includes a thinner parallel secondary scroll with proportionally decreased flower forms. The efflorescence of multiple-level floral design was in the fourteenth century in Timurid architectural style and the most eminent example of this techniques can be found in the fifteenth century.

[3] The first examples of this type of multi-level techniques were belonging to Kufic patterns that essentially they formed inside the rectangular frame so that a smaller secondary calligraphy was merged into the background of primary text and mostly in this technique there is a Strip with the angular Kufi calligraphy on the top of the rising letters such as the alif in cursive scripts. Carved stucco ornament of the Masjid-i Jami in Varamin is on of the prominent examples of the calligraphic ornament that was produced in Persia.

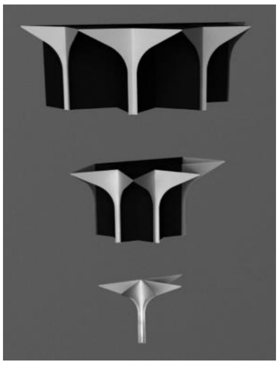
emphasis on the expression, there is no other deity but Allah!. According to this expression using the figures and depicting the deity were forbidden in Islamic art and architectural principles. Therefore, Islamic ornamentation and decorative elements came to include geometric forms instead of living creatures. Master builders or artists in that time started to use single or combined motifs and Geometric Patterns in their architectural decoration such as adorning the surfaces or using complex arrangements. Islamic artists and architects mostly tried to lay emphasis on unity, logic, and order in architectural decoration. Geometrical Islamic decorations have been developed with the help of Islamic mathematicians, astronomers, and other scientists, whose ideas are indirectly reflected in an artistic way. Islamic Geometrical Patterns (IGPs) have been widely used to adorn not only artworks like books, carpets, etc but also, became one of the main features information of Islamic architecture over walls, ceilings, doors, openings, domes, minarets, etc. The importance of geometry in Islamic architecture has been shown by the different designers in their structural and ornamental architectural works.

Among various IGPs that are used to decorate architectural buildings, Muqarnas played a crucial role in architectural decorations due to its strong aesthetic appeal as a decorative motif and flexibility as a structural element. Also, Muqarnas has been used as a transitional method to be applied to various surfaces, facades, etc. Garofalo (2010) and Van and van den (2010), describe Muqarnas and say that the word Muqarnas is the Arabic word for "*stalactite vault*". Muqarnas is a three-dimensional decorative element which consists of both concave and flat parts organized in tiers to create the integrated structure as a whole in order to provide a gradual and smooth transition between two levels, two sizes or two shapes (Figure1, a). The chronology and geographical origins of Muqarnas are unknown. There is a general belief among the scientists and historians that from the late eleventh century onwards Muqarnas had been a common feature of decorative arts in all Muslim lands.

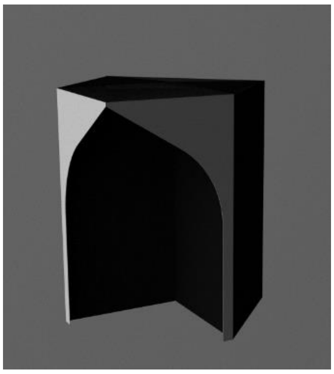
Different kinds of Muqarnas had been used in the decorative traditions of many different regions with different cultures. The construction of Muqarnas with its rich geometry and symmetric design consists of “*tiers*” (Figure 1, b). Tiers themselves consist of elements which can be categorized into cells and intermediate elements[4]. Cells look like small pieces of a vault and provide the body of Muqarnas thus, they are the most important parts of the Muqarnas (Figure 1, c). Cells themselves have been combined by intermediate elements however, cells can be omitted (Figure 1, d).



(a)



(b)



(c)

(d)

Figure 1. A sample of Muqarnas (a), tiers of Muqarnas (b), a cell (c), an intermediate element (d). (Eric Broug, 2015)

(These photos are remoded by author, from a practical introduction to Muqarnas)

[4] intermediate elements are located in the form of either a triangle or two triangles as a curved surface between the roofs of two adjacent cells.

A Muqarnas composition is mostly designed by drawing the plane projection of that Muqarnas pattern. According to Al-Kāshī (1977), the plane projection of a Muqarnas pattern consists of particular geometrical forms repeated in every Muqarnas compositions with various ways of interlacing. These geometrical forms are square, half-square, rhombus, half-rhombus, almond, jug, large biped, and small biped. Further explanation about these geometrical elements will be given in Chapter2.

Muqarnas and its complex composition which derives from its various combinations of three-dimensional motifs in different ways could be an impressive source of inspiration for current artists and architects. However using Muqarnas as a structural element in contemporary buildings is not common but, there are many architects and designers who try to refer Muqarnas and interpret Muqarnas's beauty. They also try to benefit from the patterns of muqarnas in modern period differently in their designs.
Hamekasi, Samavati and Nasri (2011) in accordance with the usage of Muqarnas in contemporary architecture, note that Muqarnas was initially used to fill the corners and they act as a transition element between rectangular and the vaulted section in a dome structure. Over time Muqarnas compositions became wide and variable and in addition to its functional role, it acquired aesthetic qualities and started to be used for covering porches, arches and domes. Muqarnas also creates a mesmerizing space via shadow and lighting along with the symmetric patterns.

Moreover, through the evolution of available technologies and computer aided software, new forms of Muqarnas are employed in modern buildings with new means of expression. Computers have a profound impact on the design of complex Muqarnas compositions and facilitate the process of design, visualization, reconstruction. They can even rapidly modify the Muqarnas forms and virtual models at the very first stages of a designing process. Therefore, such technologies were adapted into the field of architecture in order to make the process of developing the construct of the past.

1.2 Problem definition

There are some limitations related to the efficient and accurate documentation of Muqarnas compositions. Muqarnas in different time periods displays various styles due to the variety of materials and culture it is located in. Hence, most of the documentation of designed Muqarnas compositions were specific to a single style or specific area. According to a survey of available documents about Muqarnas, it seems that there is a very large but scattered analytical literature about Muqarnas which are not organized systematically enough and they mostly are repeated each other. The unknown history behind Muqarnas compositions represents the main obstacle in the documentation process. According to Garofalo (2010):

> "The chronology and geographical origins of the Muqarnas are not yet certain. Some scholars date the Muqarnas to the end of the eighth century in Syria, others to the ninth century in Iran, still others to the eleventh century in North Africa or Baghdad".

The present study, like others, conducted previously is not expected to provide a definitive historical background for the Muqarnas. However, in spite of all its limitations, this study tries to gathering available documents in a more systematic way.

The second problem is related to the restricted analytical study focusing on the designing methods of Muqarnas patterns. Designers or craftsmen who constructed historical Muqarnas compositions didn't keep a proper archive of their drawing methods, and mostly the available ones are informal in the sense that they were conducted due to earlier time's presentation styles.

Following the developments achieved in digital art and architecture, two and three-dimensional IGPs have become the source of inspiration for many designers who try to generate alternative patterns. Today, there is a developing tendency to re-generate IGPs in digital technologies, and Muqarnas could have a direct impact on the formation and development of architectural forms. In these architectural works, Muqarnas is built from orderly geometric elements with a certain degree of complexity that varies from one Muqarnas form to another. Muqarnas compositions are determined by the representations of their plane projections. These two-dimensional regulations govern the formation of Muqarnas structure. However today there are some analytical researches about the basic three-dimensional principles of Muqarnas compositions and how they are generated but, pattern regulations seem to be underestimated by researchers. There is a restricted number of analytical studies of two-dimensional regulations of Muqarnas compositions. These regulations will provide an “analysis” through which Muqarnas patterns should be analyzed and evaluated to enrich the understanding of the two-dimensional regulations.

1.3 Aim of the study

This thesis tries to disclose the meaning of Muqarnas in traditional Islamic architecture. It is intended first, to compile the available documents in order to form a meaningful summary to better understand of the Muqarnas itself then, tries to evaluate the widely scattered literature related to Muqarnas.

This thesis’s main goal is to set a design analysis, based on historical Muqarnas compositions in order to identify the potentials of Muqarnas in the development of architectural forms. This research tries to demonstrate the geometric evolution of the Muqarnas over the years in order to create a descriptive analysis that represent the geometrical and tectonic characteristics of Muqarnas as a potential in the regeneration and formulation of new modern architectural forms. This study aims to answer the questions such as; the differences and similarities between modern forms which are inspired by historical and traditional Muqarnas compositions and the aspects of modern forms that make them perceived as the new generations of Muqarnas.

To discuss these questions, this thesis provides a geometrical analysis and context for the concept of Muqarnas. The forms selected for this study are both the examples of historic and contemporary architecture. Historic examples are based on the plane projection of Muqarnas types which are selected among from the Muqarnas classification presented by Shiro Takahashi in his website (http://www.tamabi.ac.jp). The contemporary forms are selected based on the forms perceived have some similarities with historic and traditional Muqarnas designing methods. Finally, such a study about the relation between historical and contemporary analysis will make us achieved the study goals.

One of the main characteristics of the Muqarnas is its form as a three-dimensional unit that can be presented in a two-dimensional outline. Therefore, this research focuses on two–dimensional plane projection of Muqarnas examples.

1.4 Methodology of the study

This study follows two analytical structures to fulfil research goals as critical analysis and evaluation. The methodology will be conducted on Muqarnas definition and the historical background behind it to summaries the available documents. This will help to explore the origins and traditional development of Muqarnas during Islamic eras in order to evolve the potentials of historical Muqarnas compositions. In the following chapters, architectural approaches of historical and traditional Muqarnas compositions and the latest technological development through CAD system will be discussed.

The analytical methodology will be applied on two groups of Muqarnas compositions in order to compare their pattern characteristics. First group is contained of six historical Muqarnas examples, where they built during 10^{th} _16^{th} century and the other group is the modern compositions from 21^{st} century derived from the forms that seem to have same characteristics with the historical Muqarnas patterns.

1.5 Boundaries of the study

Since this thesis aims to concentrate analytically on the alteration process of the characteristics of Muqarnas patterns, most of the historical examples of Muqarnas patterns and architectural modern buildings or graphical art forms have been analyzed in this thesis. Thus, analysis of mentioned patterns are the boundaries of this study. Today, digital technologies have a great impact on contemporary architecture and designing process. CAD systems open up new opportunities in architecture by developing an architectural design by producing more complex geometric shapes. However, today there is a developing desire to reproduce Islamic forms and patterns with the aid of computer more complex and fast, but this study wants to look at these patterns analytically. The approach of this study doesn't discuss regenerating Muqarnas forms through CAD systems or in the algorithmic ways.

Two-dimensional Muqarnas patterns will be analyzed to disclose the geometrical characteristics of Muqarnas patterns which will be called the guidelines of Muqarnas patterns in this study. These guidelines can be considered as the guidelines of Muqarnas patterns to help designers and architects to be more precise on their designing in the first stages of their conceptual designs. Also, architects can benefit from muqarnas guidelines to achieve a new generation of Muqarnas patterns much easier and faster.

It should be noted that this study has been primarily concerned with two-dimensional Muqarnas patterns instead of three-dimensional compositions. The most important features of Muqarnas is its complex two-dimensional patterns and their mathematical relations therefore, one of the common methods of decoding a Muqarnas composition to its basic elements is projection method which is a Two-dimensional representation. The complexity of the most intricate three-dimensional Muqarnas examples depends on their 2DPPs that are created upon them. This study provided analytical information regarding the geometrical characteristics of Muqarnas patterns, however, more research is required in analyzing of three-dimensional compositions in order to validate and expand on the discoveries of this study. The study can be elaborated in a further analytical survey related to three-dimensional Muqarnas compositions including parametric designs of these patterns with specified materials and techniques, which goes beyond the thesis.

It is better to point out that this study not included the symbolical theory of Muqarnas. However, Muqarnas can be considered as a symbolism geometry holds for Islam but, this study will focus on the geometrical properties of these decorative elements and the impact of them in contemporary designs as a source of inspiration. The findings of this research are restricted to the historical background of Muqarnas and their geometrical features.

To date, there is limited research examining differences as well as geometrical similarities between the historical Muqarnas patterns and modern forms. This thesis tries to find contemporary buildings or graphical art forms that are inspired by Muqarnas in their both exterior and interior designs to explore the potential of Muqarnas as a source of inspiration for contemporary architects or designers. While many researchers have studied new generations of Islamic patterns and Muqarnas as digital works, this study tries to find the examples that are constructed and exist in the 21st century. The modern forms that are selected in this study are selected between the forms those their designers intentionally or unintentionally seem to be inspired by Muqarnas patterns. In some cases, designers clearly acclaim that their design concept was Muqarnas patterns, but the other designers did not mention anything about

Muqarnas and maybe they even did not have any idea about Muqarnas patterns while they were designing the forms.

1.6 Structure of the study

In order to achieve the main goals of this thesis, it is very important to understand Muqarnas composition and the history behind it to find out basic concepts about them. Chapter 2 will explore the origins, the historical evaluation of Muqarnas and different uses of that in Islamic architecture. At this step, the research is attempting to catch, the early examples of Muqarnas, and goes through the main sources in order to discover the culture and traditional concepts of Muqarnas.

In the third chapter, examples of Muqarnas are investigated through two-dimensional or three-dimensional geometrical drawings and photographs that are survived from the past. Further, the structural properties of Muqarnas are going to be analyzed, to introduce the basic elements of Muqarnas and the way that they are combined to construct a Muqarnas composition. Afterward, certain key concepts are determined to use during the evaluation of Muqarnas modeling in architecture through software. For achieving this goal, the Muqarnas transition from craftsman's design to contemporary forms that are generated by computer aided design software is also explained in chapter 3.

In chapter 4, in order to form recognition approaches of Muqarnas, some historical Muqarnas examples have been selected from the classification of Shiro Takahashi. Takahashi classified Muqarnas into three different types: 1) square lattice Muqarnas, 2) pole table Muqarnas, and 3) other style of Muqarnas. For representation purposes, two examples of each style are decomposed into sectors to achieving their two-dimensional composition rules. Analyzing these compositions can reveal valuable information for architects and designers to provide a better understanding of these mathematical patterns.

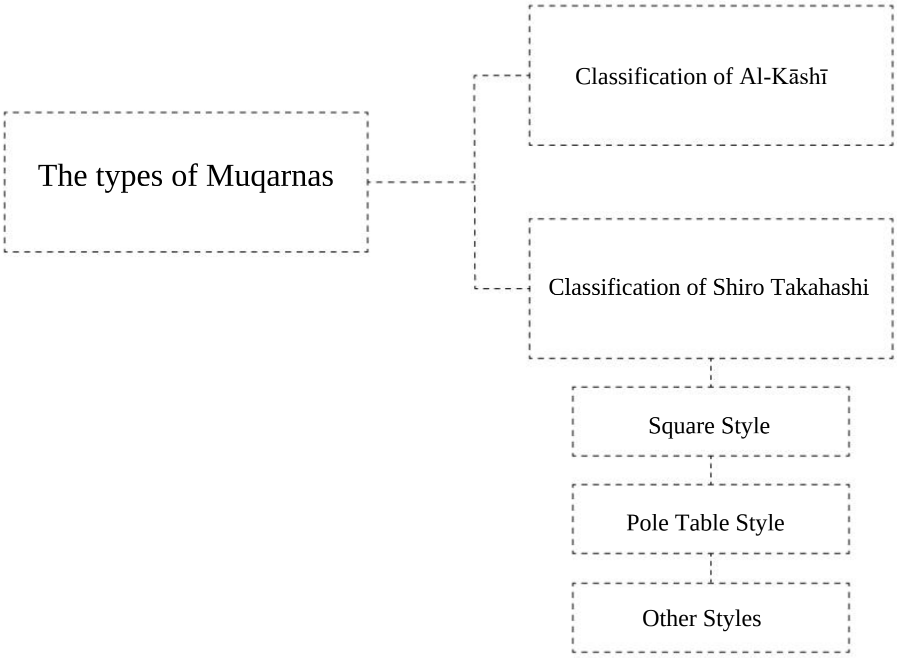


Figure 2. Types of Muqarnas flow-chart

This chart is drawn by author

Finally, the geometrical analysis of the existing examples of contemporary buildings and graphical art forms that are inspired by Muqarnas in their both exterior and interior designs will be formed in order to explore the potential of Muqarnas as a source of inspiration for contemporary architects or designers. The buildings display the influences of Muqarnas not only in Islamic architecture but also, in all over the world.

CHAPTER 2

LITERATURE REVIEW: HISTORICAL BACKGROUND OF MUQARNAS

2.1 Definition of Muqarnas

In order to understand the word ‘Muqarnas’, it is better to take a glance over its etymology in different languages. Necipoglu and Al-asad (1995) state that the word “Muqarnas” is an Arabic word for stalactite vault, derived from “Qurnas[5]”, meaning decorated ceiling with three-dimensional elements (p. 350). Mathematician Dold-Samplonius (1992) refers to Behrens-Abouseif [6]and claimes that the term of Muqarnas derives from the Greek (κorwvιs), Latin (Coronis), French (Corniche), Italian (Cornice), German (Karnies), and it refers to a mason’s technical term in Islamic architecture that does not have any relation with its meaning in any of the Arabic dictionaries (p. 197). Additionally, Samplonius emphasizes the Greek word (κorwvιs) and says that it comes from the word (κorwvh), which can be used as an adjective, meaning ‘curved’ or as a noun, meaning ‘everything’ that is curved or bent (p. 198).

According to the definitions of Muqarnas presented by mathematicians, architects and researchers in various architectural dictionaries, the term Muqarnas has a unique description with different expressions depending on each author’s own words. To sum up, according to (Peterson, 1996, p.206; Esposito, 2003; Necipoglu and Al Asad, 1995, p. 196), Muqarnas is a three-dimensional architectural composition, arranged in tiers

[5] The Arabic word qurnas is defined as something like a nose projecting in a mountain, therefore the relationship between qurnas and Muqarnas will become more obvious if we admit that the qurnas would mean ‘to furnish a structure with projecting overhanging elements’.

[6] Doris Behrens-Abouseif, née Abouseif

and consisting of superimposed layers of tiny niches, often interposed with pendant elements. Muqarnas is mostly used on domes, vaults, niches[7] or squinches[8] under domes in Islamic architecture to provide a widespread method of transition to overwhelm the distinction between vertical, curved and horizontal domes and make a smooth transition from the rectangular plan of the building to the vaulted ceiling. It is also used on a flat surface as a decorative element. Palmer (2008) asserts:

> “Muqarnas is a series of small niche arches that give the effect of a cave ceiling covered with stalactites, yet the ceiling appears to float up above the square room, weightless in appearance ”(p. 7).

In accordance with this definition, Muqarnas or its synonyms, honeycomb work or stalactite work is an original Islamic design element, involving various combinations of three-dimensional shapes, corbeling[9], etc. (Harris, 2006, p. 515).

There is another definition for the term Muqarnas that is presented in (http://www.virtualani.org):

Muqarnas is an Arabic word (Arabic: مقرنص; Persian: مقرنس; Turkish: Mukarnas) that is not only used in Islamic architecture but also was adopted into Armenian architecture, for a form of vaulting, also known as a honeycomb vault. Muqarnas vault is used for decorative purposes and has not any structural function. It is mostly used in domes, entrance portals, niches, capital, and decorative surfaces. Muqarnas can be

[7] Niche is a shallow recess, especially one in a wall to display a statue or other ornament.

[8] A squinch in architecture has a structure like a small arch, that is built across the interior angle of a square room, to carry a superstructure such as a dome, also provide a proper base to admit an octagonal or spherical dome.

[9] In architecture, a corbel (also called bracket or weight-carrying member) is a solid piece of stone, wood or metal that built deeply into the wall to carry and overturn a superlative weight.

defined as a subdivision of a squinch, or cupola[10], or corbel, into a set of many tiny squinches, producing a layering structure. The individual elements are called Alveoles[11]. Sometimes Muqarnas is used as superimposed element, as it has icicle-like[12] structures, it is called a stalactite vault. In order to emphasize the importance of Muqarnas in traditional architecture, Carrillo (2016) claims that:

> “Among the various motifs that Islamic art used to decorate buildings, Muqarnas decoration (a system of projecting niches used for zones of transition and for architectural decoration) is most notable. It came to play a major role in architectural decoration due to its strong aesthetic appeal and flexibility, readily adaptable to various surfaces. This intricate three-dimensional decorative scheme was a device forged and developed in Islamic contexts and thereafter employed in non-Muslim settings” (p. 202).

Hamekasi, Samavati and Nasri (2011) note that the earliest and main motivation behind Muqarnas was to create domes over square buildings because domes were more stable and they could cover a large area in comparison with straight roofs. Therefore, Muqarnas were discovered to be a smooth transition method between two shapes that are different in size, form or their position. They usually fit into domes in more than one step to transform a rectangular part to a vaulted one (p. 132). Muqarnas with its three-dimensional form consists of regular horizontal “layers” of small “unit-surfaces” (Yaghan, 2003, p. 30).

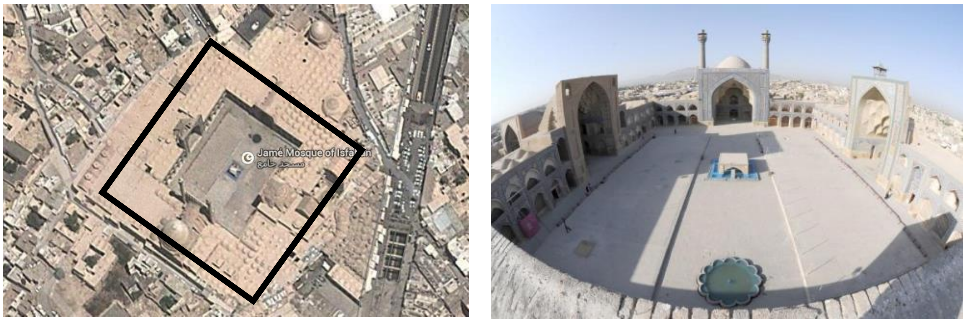
[10] The word cupola derives from Latin word ‘cupula’ it means: "small cup". In architecture the term of cupola, used for a small dome-like structure on top of a building, to provide a lookout or to admit light and air, inside the building.

[11] Alveoles or Alveolus is a general definition term for a concave cavity.

[12] Icicle is a hanging mass of ice formed by the freezing of dripping water.

As a conclusion, according to Moussavi (2009), Muqarnas can be briefly defined as a three-dimensional decorative motifs of Ivans[13] and domes in Islamic traditional architecture, that is constructed of the blocks of stone, brick masonry, stucco or wood which repeat themselves at various rhythmic scales in a self-similar way (p. 326).

[13] An Ivan or Iwan (Persian: ایوان eyvān, Arabic: إيوان Iwan) is a vaulted rectangular space, walled on three sides and usually used as an entrance in Islamic architecture, and three sides of the.



(a) (b)

Figure 3. The examples of Muqarnas that are used in Jameh mosque of Isfahan in Iran. Plan of Jame Mosque of Isfahan (a), the location of Muqarnas (b). (Web 1)

(a) (b)

Figure 4. South Iwan of Jmeh mosque (a), Western Iwan of Jmeh (b). (Web 2)

(a) (b)

Figure 5. Muqarnas on South Iwan (a), Muqarnas on western Iwan (b). (Web 3)

2.2 History of Muqarnas

According to Garofalo (2010), Chronology and geographical origins of Muqarnas are still unknown. Some of the scientists have claimed that Muqarnas firstly has been created in Eighth century in Syria, but others believe that it has been invented in the Ninth century in Iran, however, the third group of scientists believes that its existence maybe comes back to the eleventh century in northern Africa and in Bagdad. However, there is a universal belief that Muqarnas had been a common feature of decorative arts in all Islamic countries of the world in Twelve century and different kinds of it had been used in the decorative traditions of many different regions with different cultures (p. 358).

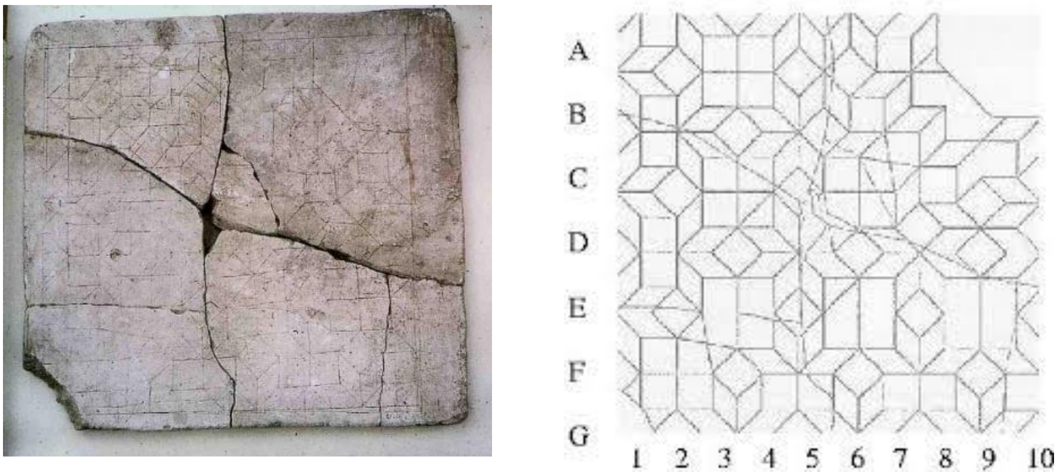
According to the researches of Necipoglu and Al-asad (1995), the oldest example of Muqarnas has emerged around the middle of the 10th century, simultaneously and independently in northeast of Iran and Africa, then it was developed from the 11th century throughout the entire Islamic countries from India to Spain, according to each country different materials and culture (p. 349). The earliest example of a Muqarnas drawing is a 50-cm stucco plate found in 1968 by German archeologist in amid the Takhti Sulayman[14] ruins in Iran that was engraved with a design of one-quarter of a Muqarnas (Necipoglu, 2006, p. 88) (Figure 6, a).

Necipoglu and Al-Asad (1996) note that, this plate has been found in the location of Mughal Palace built by Abaga Khan, one of the rulers of Ilkhanan[15] in 1720s. One can

[14] Takht-e Soleymān (Persian: تخت سلیمان) is an archaeological site in West Azarbaijan (Takab), Iran. It had been destroyed by a fire that occurred in Zoroastrian temple and rebuilt during the Sassanid period and partially during the Ilkhanid period.

[15] The Ilkhanan or Ilkhanate dynasty (Persian: ایلخانان, Ilkhānān) was founded in the 13th century by Hulagu Khan, a grandson of Genghis Khan in Iran to form the southwestern sector of the Mongol Empire.

see squares, triangles, and rhomboids on this plate, which are, respectively, relates to 45, 90, and 135 degrees of Muqarnas units (p. 5) (Figure 6, b).



(a) (b)

Figure 6. Stucco plate found in Takhti Sulayman (Web 4) (a), Harb's plan of plate (Necipoglu and Al-asad. 1996, p.5) (b)

Since the chronological order of the Muqarnas is a controversial subject among the researchers, therefore, there are some other theories about the emergence of Muqarnas in Islamic architecture. For instance, the first examples of Muqarnas in vaults can be seen in Al-Qarawiyyan Mosque[16] (Tabba, 1985, p. 64) (Figure 7). Garofalo (2010) has referred to his study to one of the most correct theories of Muqarnas functions, according to this theory, the first function of Muqarnas had been in the corner tripartite squinch. So, Muqarnas had been used for the first time in reshaping the square based walls to a circular dome. Moreover, he refers to Ecochard and claims that, according to the states of these early structures of corner squinches which date back to 10th

[16] The Al Quaraouiyine Mosque was founded by Fatima Al-Fihri in 859. It is located in Fes, Morocco also contains an associated school or madrasa that is the oldest existing, continually operating educational institution in the world.

century in Iran, can show the future functions of Muqarnas. Also, he mentioned to Hautecoeur and says that cells which were constituting parts of Muqarnas, for the first time had been extracted from the walls in Maghreb[17]. In some areas, ceilings which were made a mixture of elements such as, squinches and other portions of vaults were replaced as the alternative to the squinches (p. 358).

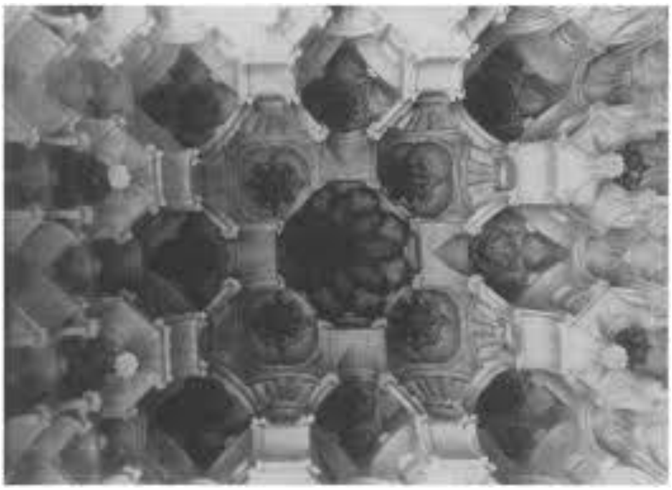


Figure 7. Muqarnas vault of Jami' al-Qarawiyyin in Fez

Source: Tabba, 1985, p. 65

Table 1 shows a classification of buildings with Muqarnas by Carrillo (2016). Carrillo mentioned to some researchers and declared that Muqarnas first has been created in Eastern Islamic countries, such as North – east of Iran, Iraq and Egypt. Some examples of Muqarnas which have been discovered in these areas are as below:

- Sāmānid Ismāʿīl Mausoleum (301-331/914-943) at Bukhara
- The domes of the Saljūḳid Great Mosque of Isfahan (464-480/1072-1088)
- The Shrine of Imām al-Dāwar (477/1085) at Samarra

[17] The Maghreb which was previously called, Barbary Coast or "Barbary States" among Europeans, is usually defined most of the region of Northwest Africa, including the Atlas Mountains and the coastal plains of Morocco, Algeria, Tunisia, and Libya.

- Some Muqarnas elements found in Ḥammām of Abū'l-Suʿud in Fusṭāṭ (Cairo).

Beside this classification, other researchers believe that maybe the Muqarnas has been created, at the same time and independently in North Africa and Near East. These diverse beliefs have caused too many complex questions (p. 203).

Dold-Samplonius (1992) also refers to Tabbaa and states that the first example of a full-fledged Muqarnas has been found in the shrine of Imam al-Dawr, which is located in the north of Samarra in a village of al- Dawr (will be discussed later). But perhaps, this first example of Muqarnas dome is not the first kind or model for all other examples of Muqarnas (p. 200). According to another theory, the earliest example of Muqarnas belongs to Topkapı Scroll, in which the variety and differences in polygons and star polygons have been increased significantly (Necipoglu & Al Asad, 1995, p. 30).

Table 1. Historical classification of buildings with Muqarnas

FIGURES OF BUILDINGS	BUILDINGS with MUQARNAS	PERIOD/DINASTY
 	**Mausoleum of Ismā'īl at Bukhara** **(943-914/331-301)** **943-914 was built in**	**Sāmānids** **(1005-819/395-204)**
 	**Great Mosque of Isfahan South-east and North-east domes** **(1088-1072/480-464)** **1088-1072 was built in**	**Saljūķids or Seljuq** **(1194-1040/590-431)**
 	**Shrine of Imām al-Dāwar at Samarra** **(1085/477)**	**Abu'l-Makārim Sharaf al-Dawla Muslin ibn Quraysh** **(1085-1061/478-453)**
	**Ḥammām of Abū'l-Su'ud, Fusṭāṭ** **Tentatively attributed to the** **Third/Ninth or Fourth/Tenth centuries**	**Abbāsids'** **(1258-750/656-132)**
 	**Minaret's Mosque Badr al-Jamālī Cairo** **(1085/477)**	**Fāṭimids** **(1171-909/567-297)**
 	**Qal'at Banī Ḥammād at Algeria** **(1152-1015/547-405)** **1015 Founded** **1152 partly destroyed in**	**Ḥammādids** **(1152-1015/547-405)** **al-Nāṣir b. 'Alannās** **(1088-1062/481-454)**

(Figures of the buildings are added by author)

In order to investigate the real geographical origin of Muqarnas, we should review recent evidence on this subject and also, we should concentrate only on theories which have been documented in the best way. In this respect, this study classifies the earliest examples of Muqarnas, chronologically, in three groups of Muqarnas in the Persian, Middle Eastern and Arab world, Muqarnas in Al-Andalus and Muqarnas in non-Islamic countries.

2.2.1 Muqarnas in Persian[18], Middle Eastern and Arab world

According to Dadkhah, Safaeipour and Memarian (2012), a historical study of the evolution of Muqarnas shows that the first examples of Muqarnas can be found in various countries of Islamic regions such as Iran, Africa and Iraq from 10th century and Iranian architects could achieve a great progress in constructing these “Muqarnas-like” elements (p. 131).

According to Tabba (1985), the earliest emergence of Muqarnas in Iran is seen the fragments that were found near Nishapur[19] in late 10th century. Then, it developed in the eleventh century with the tripartite squinches of Arab Ata mausoleum at Tim and continued with the various Seljuq[20] domes and ends with Ilkhanid and Timurid [21] Muqarnas domes and portal vaults (p. 61).

[18] Both phrases ‘Iran’ and ‘Persia’ both are used interchangeably to mean the same country. The name ‘Persia’ derives from the word ‘pars’ which is known as ‘Pers’ among Europeans. Iran is a local name and Persia was a kingdom of Iran.

[19]Nishapur or Nishabur (Persian: نیشابور) is a city in the Khorasan Province, in northeastern Iran. It is the capital of the Nishapur County and former capital of Province Khorasan. After Mashhad Nishabur is the largest city in Khorasan province. In 1037 nishabur was selected as the capital of Seljuq dynasty by Tughril.

[20] The Seljuk Empire (also spelled Seljuq) was a Sunni Muslim empire that was founded by Tughril Beg in 1037 and lasted until 1063, also from 1174 until 1194. Seljuk Empire governed a vast area stretching from the Hindu Kush to eastern Anatolia and from Central Asia to the Persian Gulf.

[21] The Timurid dynasty (Persian: تیموریان) or Gurkani (Persian: گورکانیان) was a Sunni Muslim dynasty that was established by Amir Timur and had two significant empires in history, the Timurid Empire (1370-1507) based in Persia and Central Asia and the Mughal Empire (1526-1857) based in the Indian subcontinent.

According to Vermeulen and De Smet (2001), the “Muqarnas-like” element[22] that was found in Nishapur and dates back to the 10th century is formed of nine small niche-shaped panels. Vermeulen and De Smet, refer to the Morris Dimand and state that “the niches are part of a series of multiple or stalactite squinches which supported a dome over a square room”. It seems that the curved surfaces have been joined from their back to other plaster surfaces (Figure 8).

Figure 8. Carved and painted stucco panel, 10th Century, Nishapur, Iran

Source: Vermeulen, U., & De Smet, D, 2001, p. 33

The other example of early Muqarnas like element is found in the squinches of ‘Arab Ata Mausoleum in Tim, Uzbekistan[23] which also dates back to the late 10th century (Figure 9).

[22] Muqarnas arrangements are assembled from the forms of small niches. Muqarnas are made from different materials such as bricks, stone, stucco, plaster, wood. They are also decorated in many ways like painting them. The Muqarnas-like element that was found in Nishapur also was decorated by paint.

[23] In that time Uzbekistan was a part of Iran. Uzbekistan has separated from Greater Iran in 1881. Greater Iran (Persian: ایران بزرگ), is a term that used to refer to the regions of the Caucasus, West Asia, Central Asia, and parts of South Asia which were ruled by the Persian Empires.

Figure 9. Mausoleum of Arab Ata, 997-998, Tim, Uzbekistan

Source: Vermeulen, U., & De Smet, D, 2001, p. 33

Vermeulen and De Smet (2001) also mentioned that the earliest example of entire Muqarnas is related to the dome of Imam al-Dawr mausoleum which is located in Samarra. It was about 12 meters high and built between 1075 and 1090 (p. 27) (Figure 10). As Raby (2004) stated in his study, the minaret of the Great Mosque in Aleppo was built in 1089 and it was also one of the earliest examples of the building that used Muqarnas (p. 291) (Figure 11).

Figure 10. Shrine of Imam al-Dawr, 1075-1090, Iraq

Source: Vermeulen, U., & De Smet, D, 2001, p. 34

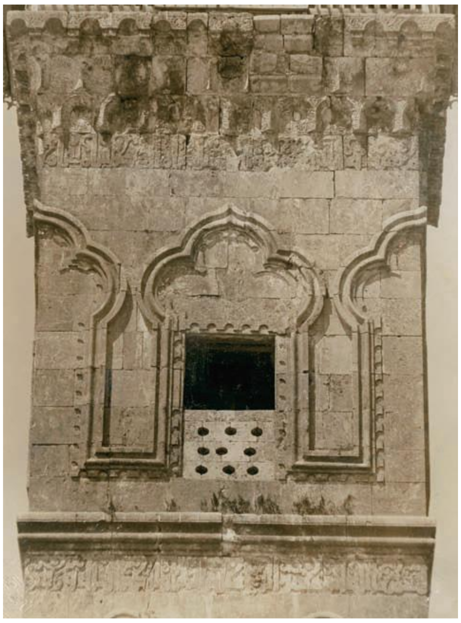


Figure 11. Great Mosque in Aleppo

Source: Raby, 2004, p. 292

According to Bloom (1988), Muqarnas first appeared in Egyptian architecture as decorative elements in the cornice on the minaret of Badr al-Jamali's Mashhad[24] in Cairo, in 478(1085) (Figure 12). The other early examples of Muqarnas in Cairo have belonged to the Aqmar mosque located in North Muizz Street of Ciro in about 1125. Muqarnas has been used as filling for a niche hood (Figure 13) and corner chamfer on the façade of the aqmar mosque[25] (Figure 14).

Figure 12. Badr al-Jamali's mosque (Web 5)

Figure 13. Niche hood of Aqmar mosque (Web 6)

[24] Badr al-Jamali or Juyushi Mosque was built by Badr al-Jamali who was the vizier of the Fatimid Caliphate. The mosque was built to ensure a view of Cairo and completed in 478 H/1085 CE.

[25] The Mosque of Al-Aqmar was built by al-Ma'mun al-Bata'ihi the vizier of Fatimid khalifeh in that time. The mosque was constructed in Cairo in 1125.

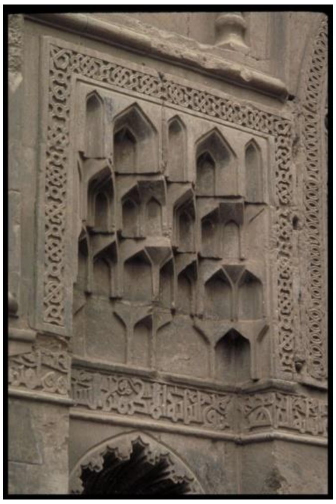
 

Figure 14. Corner chamfer of Aqmar mosque (Web 7)

In contrary with bloom, Vermeulen and De Smet (2001) states that the earliest evidence for the use of Muqarnas in Egypt is the cornice at the top of the minaret of Al-Guyusi which is dated back to 1087 (Figure 15). It is a linear Muqarnas form that was used as a cornice.

Figure 15. Al-Guyusi mosque (Web 8)

From the 12th century, Muqarnas is widespread as decorative elements in all over the Muslim countries. As Deniz (2004) declared, until the 13th century Antique motifs like the acanthus[26] leaves of the capitals[27] or the shell-shaped hemispheres are usually used in the formation of Islamic ornaments in architecture, but after the 13th century even these motifs have been replaced by Muqarnas and its composition (p. 9).

Karabork, Karasaka and Yildiz (2015) assert that Karatay madresseh and Alaadin mosque have the samples of Muqarnas used in their construction and both are located in Konya city of Turkey. Karatay madresseh, was constructed by Emir Celaleddin during Sultan Izzedin Keykavus II period in 1251 (Figure 16). The Alaadin Mosque belongs to the Anatolian Seljuk period[28] (Figure 17) (Karabork, Karasaka& Yildiz, 2015) (p. 135).

Figure 16. Karatay medresseh (Web 9)

Figure 17. Alaadin mosque

Source: Karabork, Karasaka& Yildiz, 2015, p. 134

[26] The Acanthus or Acanthus ornament is one of the most common plant forms in architecture to resemble a foliage ornament or decoration, by carving into stone or wood.

[27] Capital in architecture forms the topmost part of a column and broadening the area of the column's supporting surface.

[28] Between 11th and 13th Centuries when most of the Middle East and Anatolia was ruled by Seljuks, Seljuk architectural style had flourished and developed their own architecture after the 11th century. This style was inspired by Armenians, Byzantines and Persians architecture.

As Vermeulen and De Smet (2001) note, Ernest Herzfeld was one of the first researchers of Muqarnas that classified Muqarnas vaults into two groups: “Mediterranean” type and “Iranian” type. According to his classification, Mediterranean Muqarnas has a vault type structure, that is constructed on a pendentive[29] like the vault which is found in the portal of Mashad al-Dikka in Aleppo (Figure 18). On the other hand, Iranian Muqarnas type is formed above the cornice like what is found in the portal of Bimaristan[30] Nur al-Din Zengi (p. 28) (Figure 19).

Figure 18. Mashhad al-Dikka

1189, Aleppo, Syria

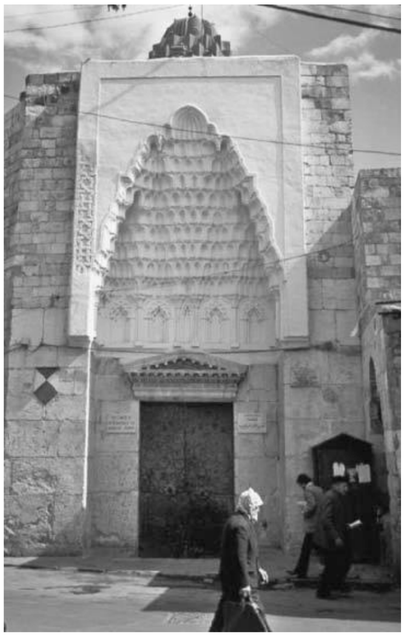


Figure 19. Bimaristan of Nur al-Din

1154, Damascus, Syria

Source of Figure 18: Vermeulen and De Smet, 2001, p. 35

Source of Figure 19: Vermeulen and De Smet, 2001, p. 34

[29] Pendetive in architecture is a curved triangular structure, that has supporting arches to achieving a circular dome over a square room or an elliptical dome over a rectangular room.

[30] Bimaristan is a Persian word (Persian : بيمارستان) meaning "hospital".

Table 2. The first examples classification of historical Muqarnas compositions in Islamic countries derives from the literature

FIGURES OF BUILDINGS	BUILDINGS NAME	LOCATION	DATE PERIODS
	**Muqanas-like element that was found in Nishapur**	**Nishapur**	**10th century**
	**Arab Ata Mausoleum**	**Tim, Uzbekistan**	**998-997**
	**Imam al-Dawr mausoleum**	**Samarra iraq**	**1090-1075**
	**Juyushi Mosque**	**Cairo Egypt**	**1085**
	**Al-Guyusi Mosque**	**Mokattam Egypt**	**1087**
	**Great Mosque**	**Aleppo Syria**	**1089**

(The table is created by author)

Table 3. (Continued) The first examples classification of historical Muqarnas compositions in Islamic countries derives from the literature

FIGURES OF BUILDINGS	BUILDINGS NAME	LOCATION	DATE PERIODS
	Aqmar Mosque	Cairo Egypt	1089
	Bimaristan of Nur al-Din	Damascus, Syria	1154
	Mashhad al-Dikka	Aleppo Syria	1189
	Alaadin mosque	Konya Turkey	1235
	Karatay medresseh	Konya Turkey	1251

(The table is created by author)

2.2.2 Muqarnas in Al-Andalus [31]

So far as we have seen, the researchers attempt to discover the Muqarnas birth, relying on historical facts. As mentioned before, the earliest examples of Muqarnas and Muqarnas like elements came into existence around the eleventh century in Iran and North Africa. From the eleventh century onward, Muqarnas were common all over the Islamic world. According to Saoud (2002), Africa was one of the prominent propagator of Islam in Europe, therefore Islam arrived in Spain through Africa in 726 by Abd-al-Rahman I[32]. After the arrival of Islam in Spain, the architecture of Spain has been influenced by Islamic architecture and Muqarnas as well (p. 5).

Muqarnas in Spain mostly appeared as a construction that was made of stucco. Shafiq (2014) refers to Tabba and mentioned that dome in Muqarnas whether made of wood, stucco, brick, or stone belongs to Islamic architecture (p. 16). Muqarnas vaults were known as prominent features of medieval Islamic architecture from Iran to Spain to catch and reflect the light entering from the windows around the dome (Tabbaa, 1985, p. 61).

Tariq (2014) declared that the Moorish[33] was well known with ribbed dome style [34] that was introduced perfectly in the Great Mosque of Cordoba in the 10th

[31] Al-Andalus (Arabic: الأندلس, Spanish: al-Ándalus; Portuguese: al-Ândalus), also known as Muslim Spain or Islamic Iberia. It was the name of Iberian Peninsula, the part that governed by Muslims in the period between 711 and 1492. And what are today called Spain and Portugal. Al-Andalus was a province of the Umayyad Caliphate.

[32] Abd al-Rahman I, full name: Abd al-Rahman ibn Mu'awiya ibn Hisham ibn Abd al-Malik ibn Marwan (731–788), also, it was known by the Arabs as al-Andalus, was the founder of a Muslim dynasty that ruled a huge part of Iberia for nearly three centuries.

[33] Spain architecture

[34] The Ribbed dome is a kind of vault that is invented in Spain and Iran, the first known examples are found in great mosque of cordoba in the 10th century. Ribbed dome shaped from the rotation of Many arches which are separated with each other and meet at the same point. By rotating a pair of arches,

century (Figure 20). However the famous example of Muqarnas style of dome construction is seen in the interior of Qubbat al-Baadiyyin in Marrakesh, it is combined with Cordoban[35] tradition of ribbed vaults with Muqarnas in the corner (p. 29) (Figure 21).

Figure 20.Mosque of Cordoba (Web 10)

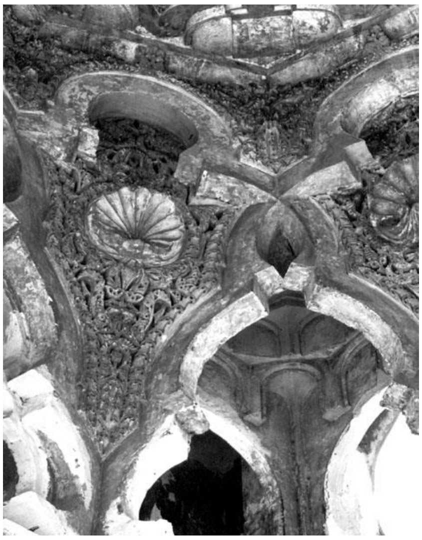


Figure 21. Qubbat al-Baadiyyin in Marrakesh (Web 11)

dome surface has been divided into star-like shapes composed of regular polygon cells. At the center of dome space, an empty regular convex polygon has been formed.

[35] Córdoba (Spanish: [ˈkoɾðoβa]), is a city in Andalusia, Spain, and the capital of the province of Córdoba. It was conquered by Muslims in the 8th century, and then became the capital of the Caliphate of Córdoba.

There is another type of Muqarnas, with pendentives, or stalactites, called muqarbas, that is not taken into account as a Muqarnas composition (Samplonius and Harmsen, 1992, p. 85). Therefore, this study just focused on Muqarnas in Spain.

Tariq (2014) notes that Granada was one of the main cities of Spain that was the host of particular architectural ornamentation including Muqarnas, which was used on arches, in cupolas and as a frieze Art in Spain. The most famous example of Muqarnas dome belonging to Spain is in the Alhambra[36] in Granada (p. 29) (Figure 22). In the Alhambra, Muqarnas are used on different scales. According to the theory that is found on Alhambra (https://prezi.com/), the ceiling of this dome is covered with 5000 carved and ornamented Muqarnas, with a star in the center (Figure 23). Samplonius refers to the analysis of the Muqarnas in the Alhambra made by Goury and says:

> They indicated that only seven different prisms are involved in most constructions (i.e. of the Alhambra), and some five thousand of them were used in the dome of the Hall of the two Sisters[37]. There are only three sections to the prisms: rightangled triangle, rectangle, and isosceles triangle. What makes variety of composition possible is that there are several facets on each prism and that these facets are at different angles from each other and have different curvatures. But at least one of the surfaces is common to more than one unit, so that a large number of different arrangements can be made with a small number of units.

[36] The Alhambra is a palace and fortress that is located in Granada, Spain. It is one of the prominent buildings of not only Spain but Islamic world because of its artificial planning, fascinating gardens and complex decorating. The name Alhambra, is the synonym of “the red,” in Arabic, and it is so called because of the Alhambra’s reddish walls.

[37] The Hall of the Two Sisters was located in the center of chambers in Alhambra where the king and his family lived.

Figure 22. Hall of Two Sisters Alhambra (Web 12)

Besides the ceiling of the dome, also the smaller arches of intimate chambers are covered by mind blowing Muqarnas. The Muqarnas Chamber is used from the western side of the courtyard to a long rectangular space that was used for feasting. It circulated into the courtyard via a roofed Iwan with three large Muqarnas arches that were adorned this space. However, this place has disappeared over time (Figure 24).

Figure 23. Detail of Muqarnas Arch, Alhambra, Granada, Spain (Web 12)

Figure 24. Arch Detail in the Muqarnas Chamber, Granada, Spain (Web 13)

2.2.3 Muqarnas in Non-Islamic countries

According to (Kaprielian, 2005) after the usage of Muqarnas in all over the Muslim world, it spread westwards as well. There are three early examples of using Muqarnas as decorative elements in Christian architecture. The Cappella Palatina in the central royal space of King Roger ll's palace is one of the earliest examples of Sicilian Norman architecture that used Muqarnas in its structure (Figure 25). Mouchroutas palace hall in Constantinople was also built with a Muqarnas ceiling. Finally, during the thirteenth century, numerous stone Muqarnas vaults were formed in Georgian Armenian monastic constructions (p. 182). Another example of Muqarnas as a decorative motif is the Muqarnas in the central niche of the iwan found at the Zisa of Palermo. This place was built in 1164 by Guglielmo I d'Altavilla (r. 1154-66) (Garofalo, 2010,p. 357) (Figure 27).

Figure 25. Palatine chapel (Italian: Cappella Palatina) (web 14)

Figure 26. Detail of Palatine chapel Muqarnas (web 15)

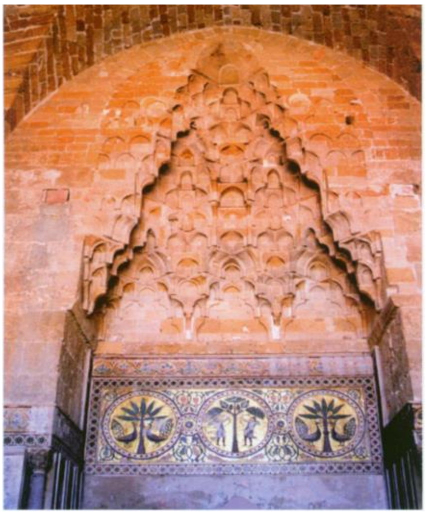


Figure 27. The Muqarnas of the central niche of the iwan in the Zisa, Palermo

Source: Garofalo. 2010, p.358.

2.3 Placement of Muqarnas in historical buildings

As mentioned before, the first function of Muqarnas was to use it as a decorative element on smaller cupolas, niches, arches, and frieze to create domes over square buildings. Then architects tried to use Muqarnas as a structural element for strengthening a part of dome, or constituting a part of a building. Also, (Garofalo, 2010) notes that the curvature form of the Muqarnas played as an acoustical element to diffuse the sound in the prayer hall of the mosque to spread the Quran recitation and reduced the echo. Now we can find Muqarnas on vaults, domes, pendentives, cornices, corbels, capitals to connect two vertical, non-coplanar surfaces (p. 357).

Despite the main purpose of the Muqarnas that is transition between a square base and a dome, Muqarnas also has been used to create a niche on the entrance of the building or used as a decorative element on cornice or beneath a balcony (Yakar, Yilmaz, Armagan and Korkmaz, 2009, p. 203). This chapter will discuss the locations in the building that Muqarnas can be fit in (Figure 28).

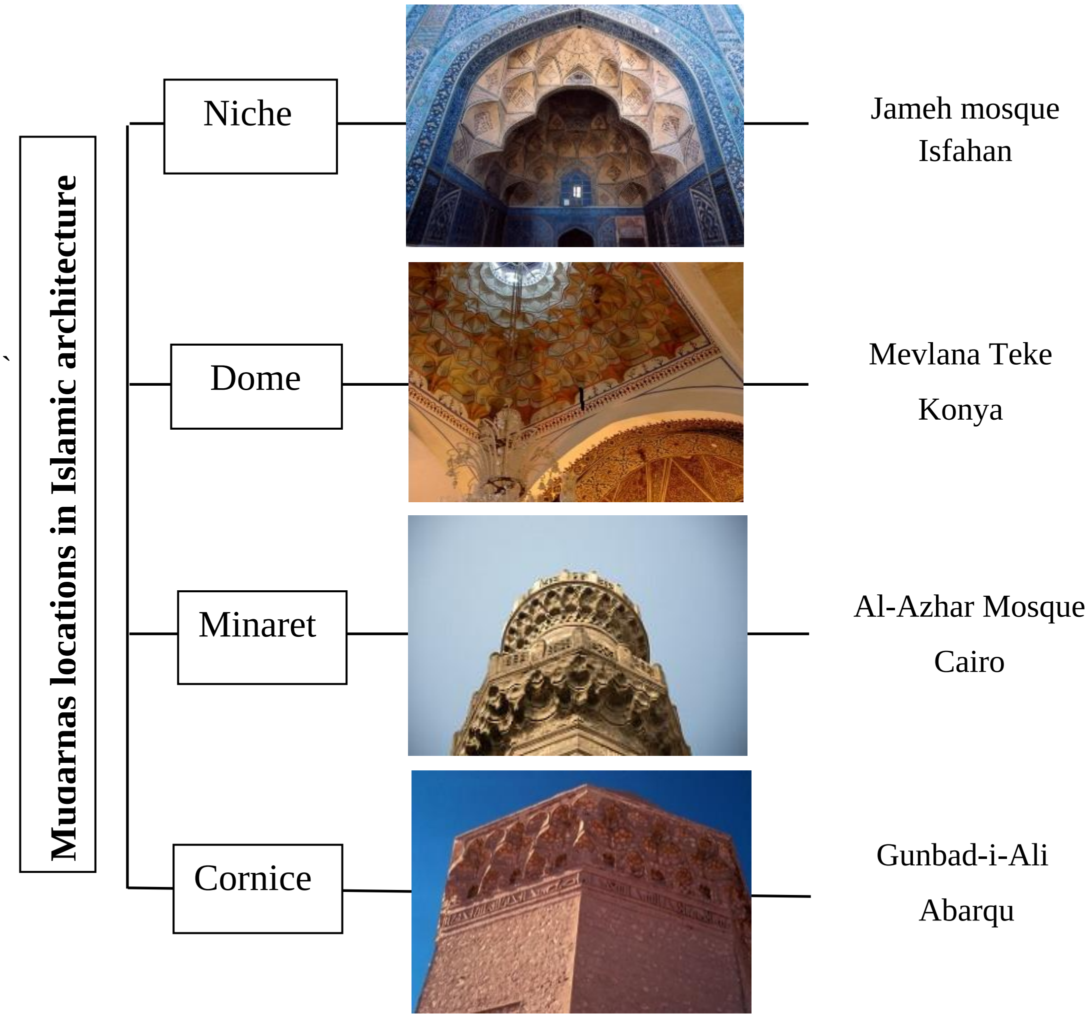


Figure 28. Placement of Muqarnas in Islamic architecture

(This chart is drawn by author)

2.3.1 Muqarnas on Niche

The most well-known use of Muqarnas in Islamic architecture is a kind of decoration element which has been used on the niches or semi vaults in historical buildings especially in mosques and the examples of shallow Muqarnas hoods are seen at the top of niches on the facets of the building (Mcclary, 2014). According to Yakar, Yilmaz, Armagan and Korkmaz (2009), this style of Muqarnas belongs to North Africa and Anatolia[38]. They carved triangular elements from wood or plaster into the downward curve elements on the building with different angles (30° or 60°). This kind of Muqarnas has been applied on the surface facing forward or facing backward.

Using this type of Muqarnas as a smooth transition method on the tripartite squinch was used in the 10th century in Iran for decorating the vaults of niches in the way that they are located symmetrically in relevance to the lateral axis of the building (Garofalo, 2010). The blocks of the Muqarnas merge on to the concave surface as a layer of Muqarnas composition to make a three-dimensional decorative structure (Des & Sakkal, 2016). Sometimes, Muqarnas's purpose is to provide a transition from a vertical surface to more impressive one as a concave semi-vault above the entrance of a building (Dadkhah, Safaeipour and Memarian, 2012). To create three-dimensional decorative motifs of Ivans and domes in Iranian traditional architecture, Muqarnas blocks repeats themselves at various rhythmic scales in a self-similar[39] way (Moussavi, 2009).

The best-known examples of Muqarnas on niche, are located in the entrance niches of the Ivan of the Jame Mosque of Esfahan[40] (Figure 29) and Bimaristan al-Qaymari (Figure 30).

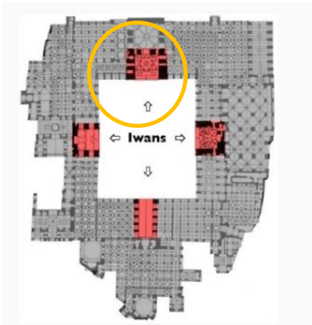
---

[38] Anatolia (Turkish: Anadolu), which is also known as Asian Turkey, Anatolian peninsula or Anatolian plateau, is the westernmost part of Asia, which generates the majority of Turkey. Anatolia is bounded by the Black Sea to the north, the Mediterranean Sea to the south, and the Aegean Sea to the west.

[39] In mathematics, An object is said to be self-similar if the whole form of that object is approximately similar to a part itself and it looks the same on any scale. Self-similarity is a particular property of fractals.

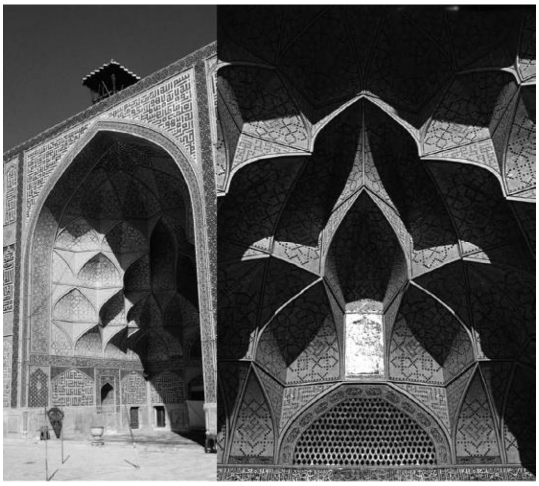
[40] Jāmeh Mosque of Isfahān (Persian: مسجد جامع اصفهان – Masjid-e-Jāmeh Isfahān), is located in Isfahan Province of Iran. It is the largest mosque of Isfahan and one of the prominent traditional buildings of Iran. The construction of the mosque was started around 771 and completed around the 20th century. The site of Jame mosque of Isfahan has been a UNESCO World Heritage Site since 2012.

(a)

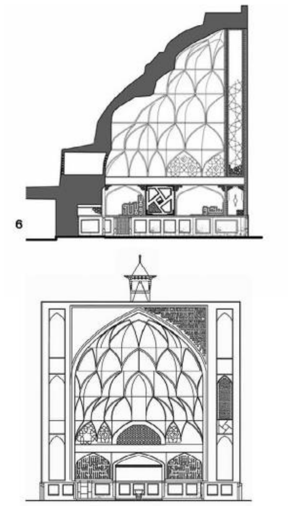


(b)

(c)



(d)



(e)

Source: Dadkhah, Safaeipour and Memarian. 2012, p.133.

Figure 29. Muqarnas on niche, Jameh mosque in Isfahan, Iran

Construction period: from 771 to the end of the 20th century

Muqarnas on niche (Web 16) (a), Plan of Jameh mosque and the location of Muqarnas on niche (Web 17) (b), front view of Muqarnas (Web 17) (c), details of the Muqarnas in west iwan of jameh mosque (d), section of Muqarnas in west iwan of Jameh mosque (e).

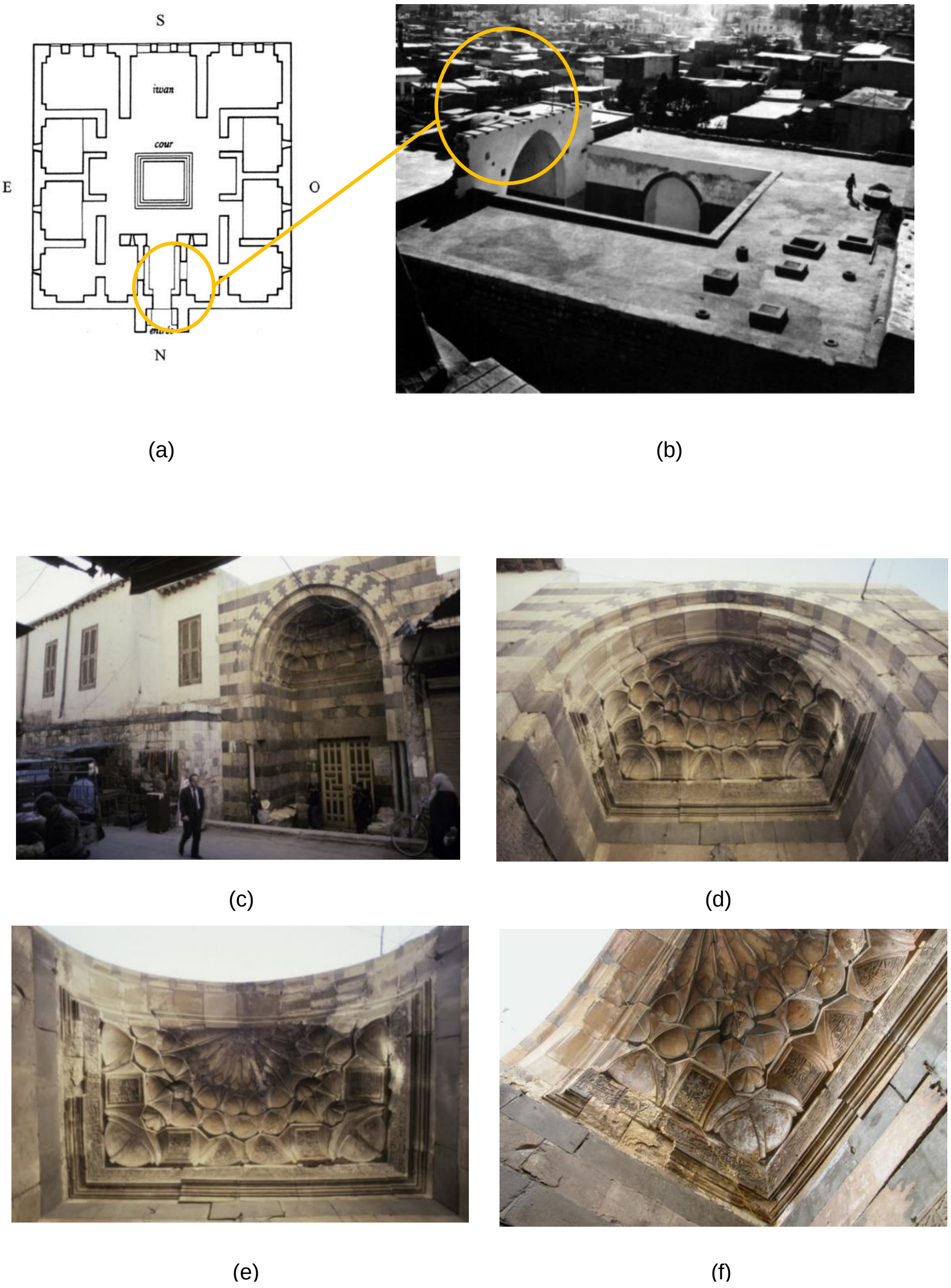


Figure 30. Muqarnas on Niche, Bimaristan al-Qaymari in Damascus, Syria

Construction period: 1248 and 1258

The location of Muqarnas on plan of Bimaristan al-Qaymari (a) (Web 18), Muqarnas on niche in Bimaristan al-Qaymari (b) (Web 18), Muqarnas on niche in the entrance (c), Muqarnas on niche of Bimaristan al-Qaymari (d), underneath view of Muqarnas on niche (e), details of Muqarnas of Bimaristan al-Qaymari from underneath view (f).

2.3.2 Muqarnas on Dome

Muqarnas compositions can also be located in a building, as a vaulting over a hall, and usually are used as part of a double-shell adjustment. Therefore, they are only visible from the inside of a building (Necipoglu, and al-Asad, 1996). Since Muqarnas has been applied to the underneath of domes, they present a downwards-facing shape that could construct a line between the floor and any point on the Muqarnas of dome's surface (Akram, Ismail, and Franco, 2016). For example, in a transition from a circle to a single point, before reaching dome shape, there are many other steps with different shapes. These steps are called layers of Muqarnas. The initial and final shapes of each layer are called layer lines (Hamekasi, Samavati and Nasri, 2011).

Muqarnas can be used as a decorative element, instead of playing a structural role, by hanging on the roof. The shrine of Lady Zumurrud Khatun[41] which is located in Baghdad City is one of the significant architectural landmarks with Muqarnas domes (Figure 32) (Akram, Ismail and Franco, 2016). Also, the dome of Madrasa Al-Nuriyya Al-Kub[42], which is located in Damascus, Syria is another example of buildings that has mind-blowing Muqarnas example (Figure 33).

As Tabbaa declared in his article all of Muqarnas domes have some common features:

- They all consists of small cells that are different from each other.
- They all have obscure structures.

[41] The shrine of Lady Zumurrud Khatun is one of the most significant and mind blowming Muqarnas domes, that is one of the famous landmarks of Seljuk architecture, in Baghdad City. Lady Zumurrud Khatun was married to Caliph Al-Mustadhi Bi-Amrillah. This shrine constructed in around 1202 AD, by an impressive technique and style.

[42] The madrasa of Nur al-Din is located along the Khayattin Souk, inside the walled city of Damascus. It was built in 1167 by Zangid ruler Nur al-Din (1147-1174) as the first royal madrasa complex in Damascus. The complex is composed of a madrasa, a mosque and the founder's mausoleum.

- Layers of stucco, paint, or glazed tiles are usually used at the base of the double shell dome (Tabbaa, 1985).

As Kiani and Amiriparyan (2016) declared, using Muqarnas in domes is an integrated system to cover wide openings that are built on a squared shaped plan in multiple layers with the diverse scale of elements. Therefore, the first layer that is called layer line has the most intensity which is reduced in subsequent vertical layers and is overlapped by different scale degrees to achieve unity in the whole pattern.

According to Samplonius (1992), although, Muqarnas is used as a transitional zone of domes with a square based plan, its main function is ornamental. “The builders, concerned with finding decorative roles for the squinch areas, favored a decoration by the multiplication of parts, a treatment which gave interest to the otherwise blank and shaded inner surface of the squinch”.

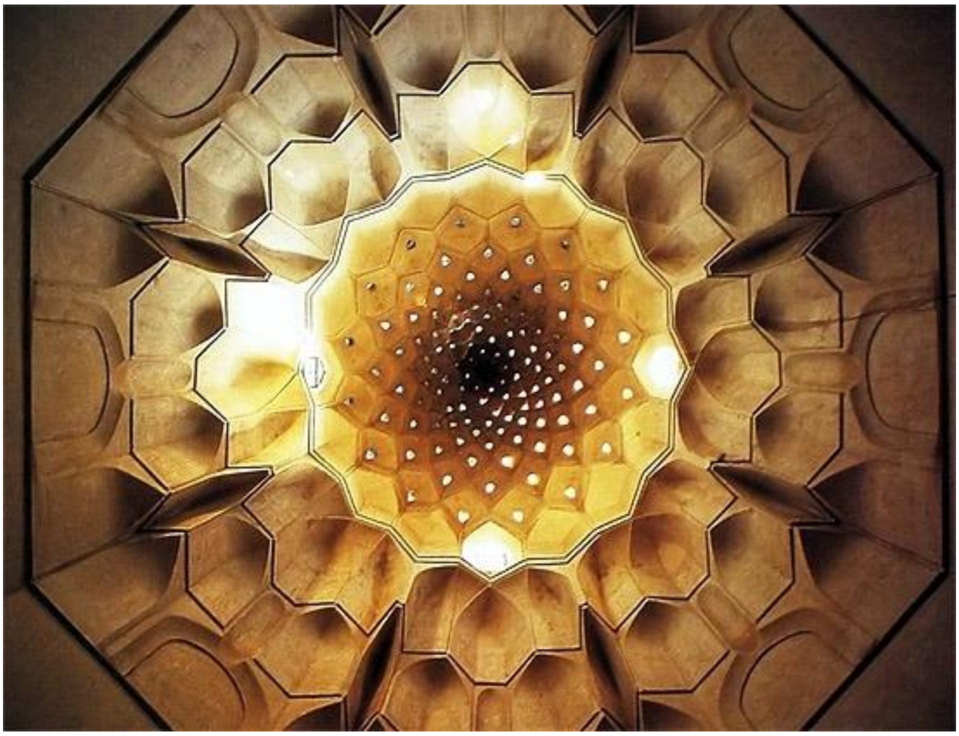
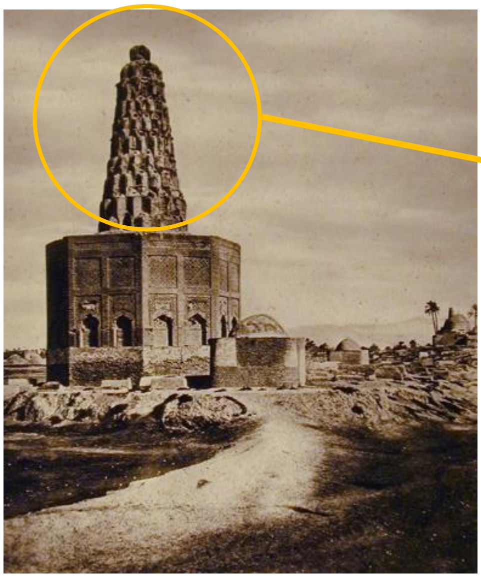
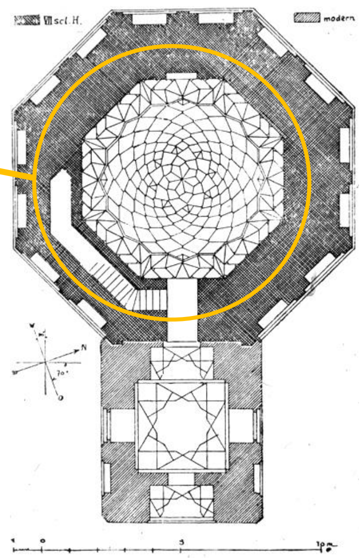


Figure 31. Mausoleum of sitt Zumurrud Khatun (Web 19)

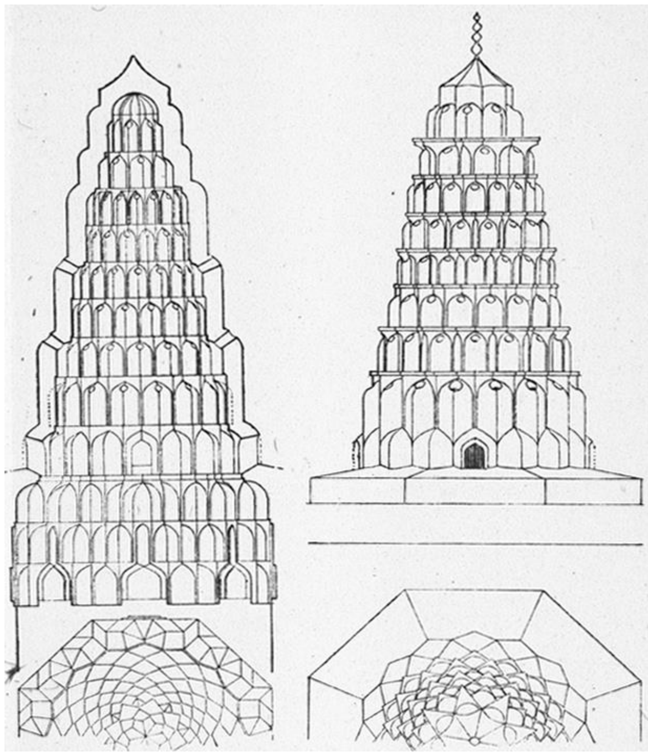


(a)

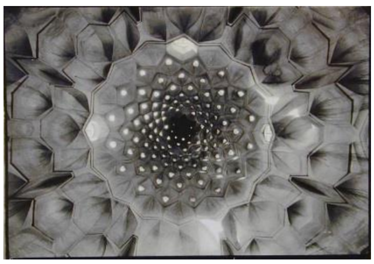


(b)

(c)



(e)

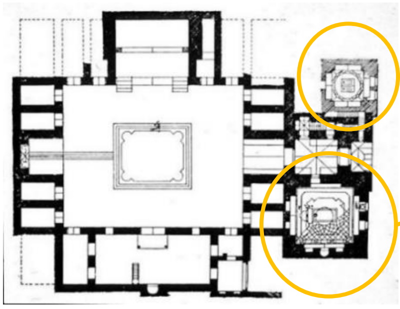


(d)

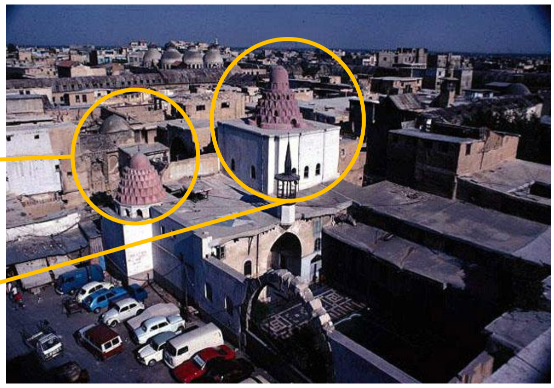
Figure 32. Muqarnas on Dome, Bimaristan al-Qaymari in Baghdad, Iraq

Construction period: 1879-1948

Mugarnas on dome of Bimaristan al-Qaymari (Web 20) (a), The location of Muqarnas on plan of Bimaristan al-Qaymari (web 21) (b), inside vew of Muqarnas on dome (web 22) (c), details of Muqarnas on dome (web 23) (d), Section of Muqarnas on dome (web 23) (e).



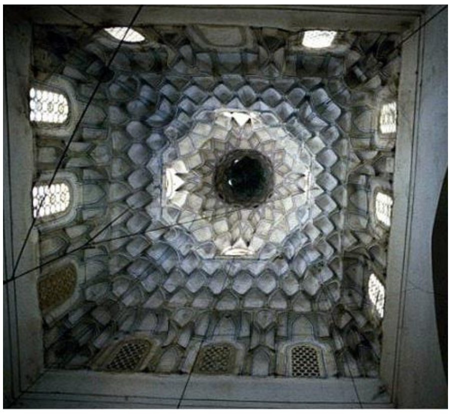
(a)



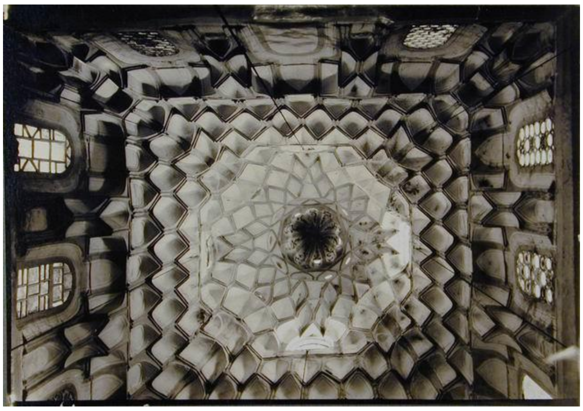
(b)

(c)

(d)



(e)



(f)

Figure 33. Muqarnas on Dome, Madrasa al-Nuriyya al-Kub in Damascus, Syria

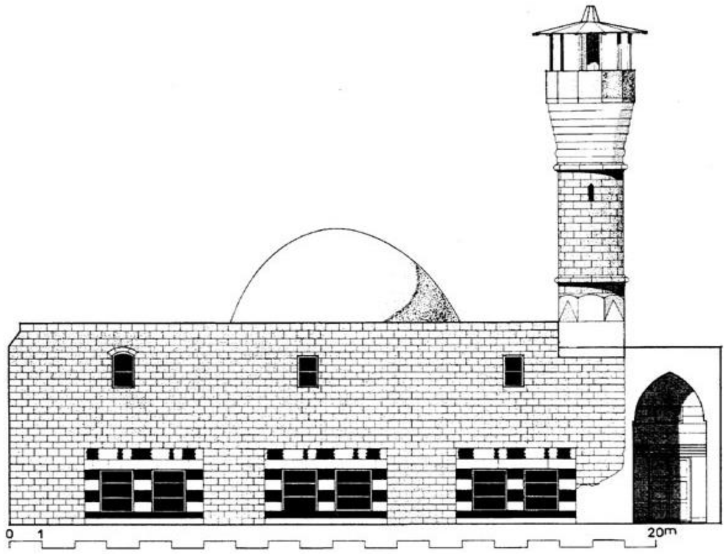
Construction year: 1167

The location of Muqarnas on plan of Madrasa al-Nuriyya al-Kub (Web 24) (a), Muqarnas on dome of Madrasa al-Nuriyya al-Kub (Web 24) (b), The view of Muqarnas on dome of Madrasa al-Nuriyya al-Kub from outside (Web 24) (c), the details of Muqarnas on dome of Madrasa al-Nuriyya al-Kub from outside (Web 24) (d), the view of Muqarnas on dome of Madrasa al-Nuriyya al-Kub from inside (Web 24) (e), the details of Muqarnas on dome of Madrasa al-Nuriyya al-Kub from inside (Web 24) (f).

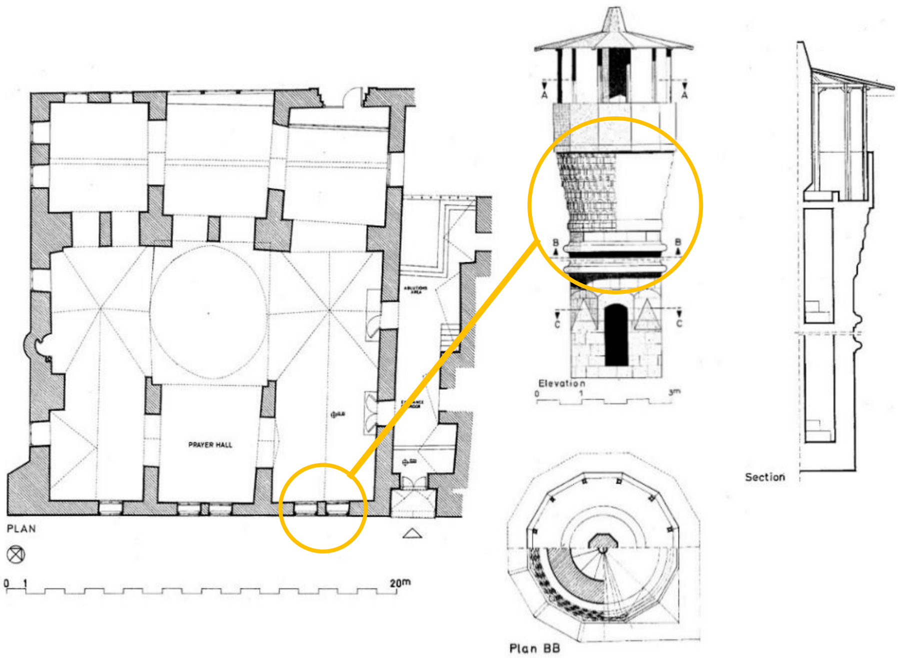
2.3.3 Muqarnas on Minaret

Muqarnas has been used on minarets or the eaves of a building as a smooth transition method between two shapes that are different in size (Hoeven, & Veen, 2010). Also, it can be used on minaret to support a minaret's balcony. (Necipoglu, and Al Asad, 1996). It is located between the shaft and balcony on the minaret of the mosque, a fish scale Muqarnas creates a transition zone (Bloom, 2000). According to Mcclary (2014), Muqarnas usually was used at the top of the shaft with a number of variations and inconsistencies. These irregularities in both, width and alignment of the cells from each layer to the next one, have caused deviations from constant form along the curvature of the shaft, thus the problem of accessibility occurs, and makes accurate measurement difficult. Muqarnas elements on minaret were used in the earlier Byzantine brick buildings and the first example was in the western border regions of the Rūm Saljuq Empire. Later Iranian teams of craftsmen were trained in the native western Anatolian tradition of brick construction that did not have a history of Muqarnas use in that region. The Minaret of Arghun Shah Mosque (Figure 35) and Quwwat al-Islam (Figure 36) are two prominent examples of Muqarnas on the minaret.

Figure 34. The Minaret of Arghun Shah Mosque (web 25)



(a)

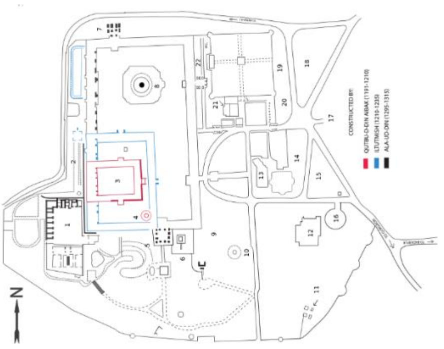


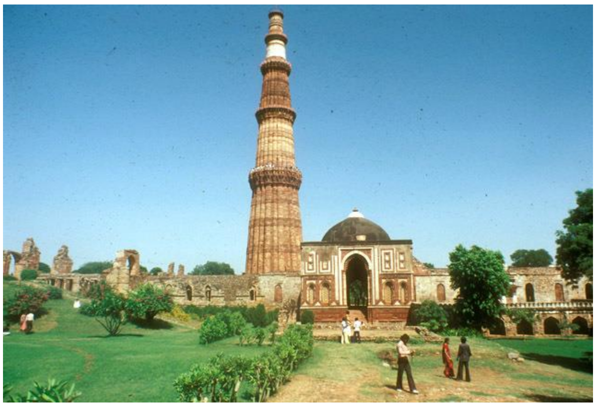
(c) (d)

Figure 35. Muqarnas on minaret example, Arghun Shah Mosque in Tripoli, Lebanon

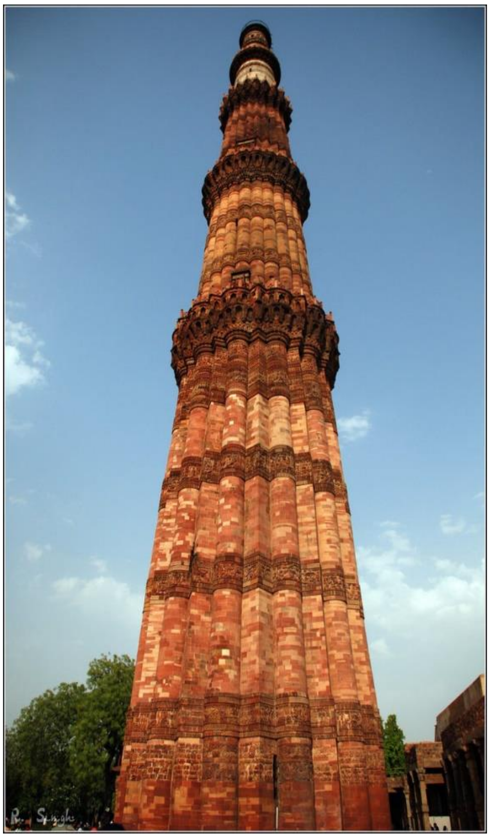
Construction year: 1475

Southeast elevation of Arghun Shah Mosque (web 26)(a), the location of Muqarnas on plan of Arghun Shah Mosque (web 26)(b), Minaret plan, section, elevation of Arghun Shah Mosque (web 26)(c).





(a) (b)



(d)

(c) (e)

Figure 36. Muqarnas on minaret example, Quwwat al-Islam in Delhi, India

Construction year: 1192

The location of Muqarnas on plan of Quwwat al-Islam (Web 27) (a), the view of Quwwat al-Islam from outside (Web 28) (b), the minaret of Quwwat al-Islam (Web 29) (c), the details of Muqarnas on minaret of Quwwat al-Islam (Web 29) (d) and (e)

2.3.4 Muqarnas on Cornice

According to Carrillo (2016), During Turkish Ottoman Dynasty Muqarnas became the more popular decorating element in Islamic architecture. Architects have generalized Muqarnas as a technique for decorating not only squinches but also other architectural elements such as cornices. Kaprielian (2005) asserts that from the eleventh century onwards Muqarnas has played a decorative role in historical buildings beside its transitional method. In order that Muqarnas cornice was used as a part of the ceiling instead of using as an appendant element, it was used as a separating part from the wall and presented a transition between the horizontal and vertical planes of ceiling and wall. Moreover, Muqarnas cornice was used to enlarge the surface area above the building in order to achieve a larger platform for the upper part of a structure. Bekar sultan tomb which is located in Aksaray, Turkey (Figure 38), And Gunbad-i 'Ali which is located in Abarquh, Iran (Figure 39) are historical buildings that have Muqarnas cornice in their upper sections As (Mcclary, 2014) says:

> The functional role of cornice in that building is to increase the size of the roof in order to shed water runoff away from the walls of the tomb and thus reduce erosion of the brick Kufic band of epigraphy below.

Figure 37. Details of Muqarnas on cornice (web 30)

(a)

(b)

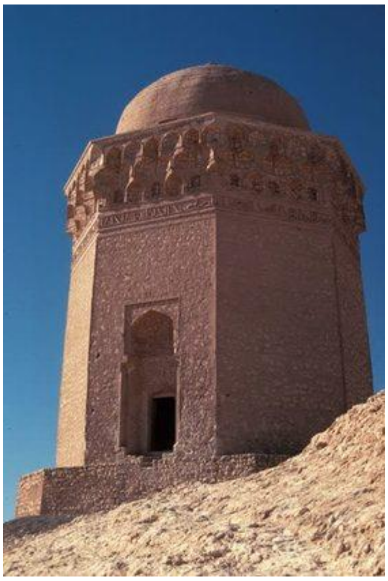
(c)

Figure 38. Muqarnas on cornice example, Bekar sultan tomb in Aksaray, Turkey

Constructed in 13th century

The view of Bekar sultan tomb from outside (web 31) (a), Muqarnas on cornice of sultan tomb (web 31) (b), details of Muqarnas on cornice of sultan tomb (web 32) (c).

(a)



(b)

(c)

Figure 39. Muqarnas on cornice example 2, Gunbad-i 'Ali in Abarquh, Iran

Construction year: 1055

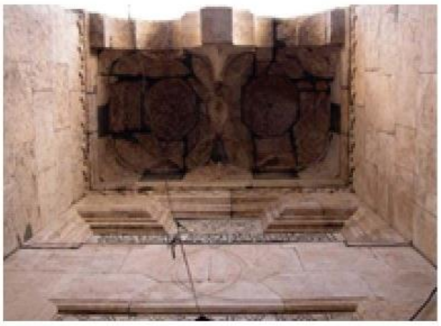
The view of Gunbad-i 'Ali from outside (web 33) (a), Muqarnas on cornice of Gunbad-i 'Ali (web 34) (b), details of Muqarnas on cornice of sultan tomb (web 35) (c).

2.4 Materials of Muqarnas

Garofalo (2010) states that according to the region of different Islamic countries, the materials used in the construction of the Muqarnas can be varied. For example; In Syria, Egypt, and Turkey, Muqarnas mostly is made of stone, the predominant material for building in these areas and great care should be given when applying such material In North Africa, the plaster and wood mostly are used in constructing the Muqarnas. In Iran and Iraq, bricks are the constituent parts of them, which sometimes can be covered with plaster or ceramic. The orthogonal geometry in them make it necessary to use bricks in producing the Muqarnas, but plaster can give greater freedom in selecting the composition because it allows coping with angles that are not exactly 45° or 90°.

In contrary to Garofalo, Mcclary (2014) believes that the materials of Muqarnas compositions were varied depending on their construction time period. He says that Brick has been often used in constructing Muqarnas in Seljuk architecture, but in the architecture of Il-Khanid, plaster is more common. Depending on the materials used in constructing the Muqarnas elements, there can be seen a broad variety of these elements. When the Muqarnas is not produced from prefabricated elements, it is only necessary to fit the two joining elements together, but the elements in other parts of the Muqarnas may have other measures. According to Necipoglu (2005) When a Muqarnas structure has to be inserted into an existing vault, there is need to modify the height of the facets of the elements. These height measures often decrease in the upper tiers with decreasing the curvature of them. So, the part which is above the last tier can be completed in several ways. For example, in some vaults, the original brick is left visible, however in others, the ceiling is covered with plaster and can be ornamented by painting, or the upper part of the vault is filled. Table 3 illustrates material variations of Muqarnas compositions according to their locations.

Table 4. Material variations of Muqarnas compositions according to their locations

Material	Location	Properties of Muqarnas	Figures of Muqarnas
stone	**Syria**	**Madrasa al-'Adiliyya Damascus, 1219 ,Syria**	
	Egypt	**Al-Azhar mosque ,Cairo, Egypt 970**	
	Turkey	**Hunad Hatun mosque Kayseri, 1238 ,Turkey**	

(The table is created by author)

Table 5. (Continued) Material variations of Muqarnas compositions according to their locations

Material	Location	Properties of Muqarnas	Figures of Muqarnas
Plaster or Wood	**North Africa**	**Nasrid Palace of the Alham-bra Granda, 889 ,Spain**	
Brick	**Iran**	**Shaykh 'Abd al-Samad ,Natanz, Iran 1299**	
	Iraq	**Abdullah al-Qa-dir al-Jaylani Mosque ,Baghdad, Iraq 1534**	

(The table is created by author)

2.5 Functiocn of Muqarnas

Muqarnas is used as a multi-functional element for interlocking, load- bearing or niche-shaped vaulting units. Gradually, it became increasingly ornamental and appear as an intricately faceted surface (Omer, 2010). Also, it is often used to vault mihrabs and, on a larger scale, to support and to form domes (Kaptan, 2013).

According to Bloom (2000), when placing a circular dome on a square plan, a triangular shape can remains open, and this can creates a problem. In order to solve this problem, a transitional layer can be applied. This layer can convert the circular base to a square one. Symmetric and aesthetically pleasing three-dimensional forms have been used for such transition. Muqarnas, at first, only was applied to domes, but then it was used on other different architectural elements including smaller cupolas, niches, arches, and also in a decorative frieze.

Irwin (2011) states that some Muqarnas structures are only used for decorative purposes and they have not structural roles in the building. But some of them, besides decorative objectives, can play structural roles in the building. Such as fortifying a part of a building (a dome, or a vault) or constituting a part of a building. Some structures of Muqarnas have been used for acoustic purposes, especially those employed in mosques, where Quran is read and other religious recitals are taken place because the curvature of pieces in the Muqarnas can spread the sound and also can be effective in reducing the echo.

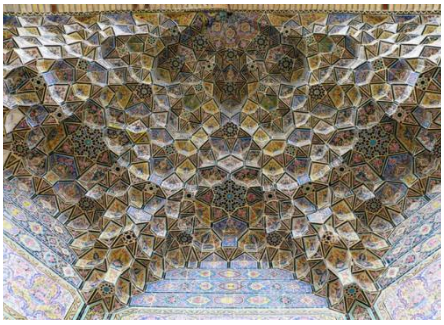
Some examples of Muqarnas variations are shown in figures on next page.

(a)

(b)

(c)

(d)



(e)

(f)

(g)

(h)

Figure 40.The variation of Muqarnas design in Islamic architecture

Madrasa Chahar Bagh, Isfahan, Iran (a), Chehel sutun, Isfahan, Iran (b), Madrasa Chahar Bagh (c), Isfahan, Iran, Shaykh Abd al-Samad (d), Natanz, Iran, Nasir al-Mulk mosque, Shiraz (e) , Iran, Naghsh-i Jahan, Isfahan, Iran (f), Ceiling at KL Tower, Kuala Lumpur, Malaysia (g), Alhambra Palace, Granada, Spain (h).

CHAPTER 3

PROPERTIES OF MUQARNAS

Understanding the mystery of Muqarnas geometry and its complexity needs to have knowledge about the designing thought and designing methods of these geometrical decorative elements. After discussing the historical background of Muqarnas in chapter 2, this chapter tries to reveal the architectural and geometrical properties of Muqarnas and survey about the new methods that are used in today's technologies to make the Muqarnas designing process much faster and easier.

In order to understand the structure of Muqarnas, this chapter tries to study Muqarnas structure in both two-dimensional plane projection which is a substitute for the Muqarnas design and three-dimensional Muqarnas compositions and the way that are constructed as a volume. In this regard, firstly two-dimensional Muqarnas patterns that define a finite set of Muqarnas elements will be discussed. Then the three-dimensional elements of Muqarnas compositions and the way that they are combined to form a Muqarnas structure will be the subject of this chapter.

According to the new developments that are achieved in art and architecture through CAD systems, Muqarnas also is designed and constructed within new digital methods. Therefore, after disclosing the architectural and structural properties of both two-dimensional Muqarnas patterns and three-dimensional Muqarnas composition, this chapter aims to concentrate on the development of Muqarnas patterns designing methods through CAD systems which are used today instead of paper based designing methods. This chapter will discuss these methods that are helped the designers to think more rapidly and may access to more alternatives of their initial ideas in the first stages of their designing process.

3.1 Geometrical and tectonic properties of Muqarnas

As mentioned before, because of Islamic beliefs, traditional master builders started to use singly or combined motifs and geometric patterns instead of living creature figures in their architectural decoration such as adorn the surfaces or using complex arrangements. According to the survey that is presented by The Metropolitan Museum of Art (2000), the first samples of geometric ornamentation already existed in late antiquity in the Byzantine[43] and Sasanian empires[44]. Islamic architects mostly tried to lay emphasis on unity, logic, and order in architectural decoration. Then geometrical Islamic decorations developed by the help of mathematicians, astronomers, and other scientists, whose ideas are indirectly reflected in an artistic way. Compass and Ruler were the first tools to drawing these geometries and circle was the first geometry forms of Islamic patterns.

> "It should be known that geometry enlightens the intellect and sets one's mind right. All its proofs are very clear and orderly. It is hardly possible for enter into geometrical reasoning, because it is well arranged and orderly. Thus, the mind that constantly applies itself to geometry is not likely to fall into error".[45]

[43] The Byzantine Empire or the Eastern Roman Empire was the continuation of the Roman Empire in the East. Its capital city was Constantinople. *Byzantine Empire* can be traced to 330 A.D and fell in 1453 A.D.

[44] The Sasanian Empire (also spelled Sassanian, Sasanid, Sassanid or Neo-Persian Empire), was the last pre-Islamic Persian Empire, established in 224 CE by Ardeshir I and lasted until 651 CE when it was overthrown by the Arab Caliphate.

[45] The famous agewmetrhtoz mhdeiz eisitw, which appears in Elias' commentary on the Categories and was well known to the Arabs. It entered Arabic literature in connection with the introductions to

According to Necipoglu and Alasad (1995), the most interesting fact about Muqarnas is its geometry and the mystery behind it. Most of the scientists and architects, from Jones and Goury in the nineteenth to contemporary researchers, like Notkin and Michel Ecochard, have tried to analyze it. Understanding the mystery of Muqarnas geometry and its complexity because of their mathematical relations is too much complicated so that researchers must have mathematical knowledge to analyzing them.

Dadkhah, Safaeipour and Memarian (2012) state that one of the common methods of decoding a Muqarnas composition to its basic elements is projection method which is a Two-dimensional representation of the complex three-dimensional Muqarnas. This method present us two fixed rules that are valid for all Muqarnas compositions:

1. Using simple and uncompounded geometric elements in every Muqarnas projection.
2. Determining a geometric system for connecting muqarnas elements.

(Dadkhah, Safaeipour and Memarian, 2012) also refer to the definition that is expressed by the fifteenth century mathematician Ghiyāth al-Dīn Jamshīd Masʿūd al-Kāshī [46] and note that the first category consists of two main geometry groups: its basic building blocks that are called cells and the primary elements or intermediate elements that are used between the cells to complete the geometric network (p. 134).

Dold-Samplonius (1992) refers to al-Kāshī and note that a Muqarnas is a roofed (musaqqaf)[47] vault, that consists of cells which themselves consist of elements. Cells

Aristotelian philosophy. Cf al-Farabi, Fi-ma yanbaghi an yuqaddam qabl 'ilm al falsafah, ed. and tr. F. Dieterici: Alfarabi's Philosophische Abhandlungen (Leiden, 1890, 1892), pp. 52, 87.

[46] Ghiyāth al-Dīn Jamshīd Masʿūd al-Kāshī (or al-Kāshānī) (Persian: غیاث الدین جمشید کاشانی Ghiyās-ud-dīn Jamshīd Kāshānī) was a Persian astronomer and mathematician.

[47] Musaqqaf is an Arabic word and it means roofed.

can be divided into a roofed vault with facets that they are the straight planes perpendicular to the horizon and intersect each other's adjacent at a right angle, or half a right angle, or their sum, or another combination of them, in a way that they stand on a plane parallel to the horizon or two surfaces. These two facets with their roof which is the upper part of them are made one cell (bayt). The word bayt is an Arabic word and could be translated with the house (Figure 41). These cells also consist of intermediate elements that are beeing located in the form of either a triangle or two triangles as a curved surface between the roofs of two adjacent cells. These elements consist of facets and a roof themselves (Figure 42) (p. 226).

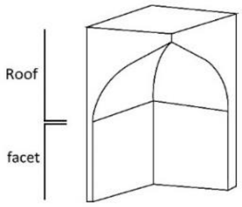


Figure 41. A cell

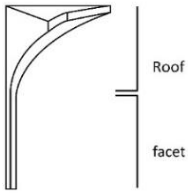


Figure 42. An intermediate element

Figures are redrawn by author

According to Hoeven and Veen (2010) for filling some spaces beside the cells, another kind of building blocks is used. As al-Kāshī mentioned, to describing the other type of building blocks called an intermediate element. The largest facet of the Muqarnas is considered as a module of the Muqarnas, that they are constructed with the same unit of measure to fit all the elements together to connect the roofs of two adjacent elements. As illustrate in Figure 43, there are three ways to connect two cells. With one intermediate element, two intermediate elements and without the connection of an intermediate element. It is not necessary to fill the space between two elements with intermediate elements. (p. 3)

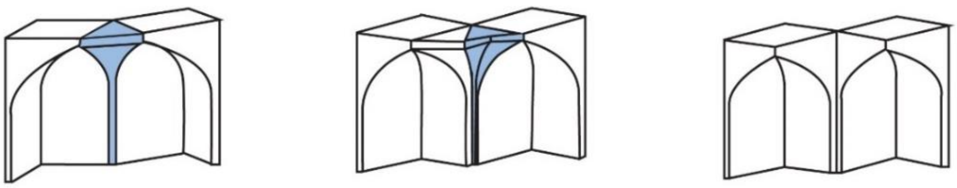


Figure 43. Three ways of joining intermediate element/elements and cells

3.1.1 Two-dimensional Muqarnas structure

Every Muqarnas pattern can be presented as a non-periodic tiling with a lot of variations based on an isometric network (Castera, 2003). These patterns contain the intermediate elements of Muqarnas which have specific shapes with limited variety and number and include specific elements such as Thakht, Pabarik, Tass(Tassé), Shaparak, Shamssé. Figure 44, shows the two-dimensional plane projection of a muqarnas composition with the location of each intermediate elements. According to Navai and Haji ghasemi (2014), Tass, Shaparak, and Takht are the most important intermediate elements of the Muqarnas that are shown in Figure 45.

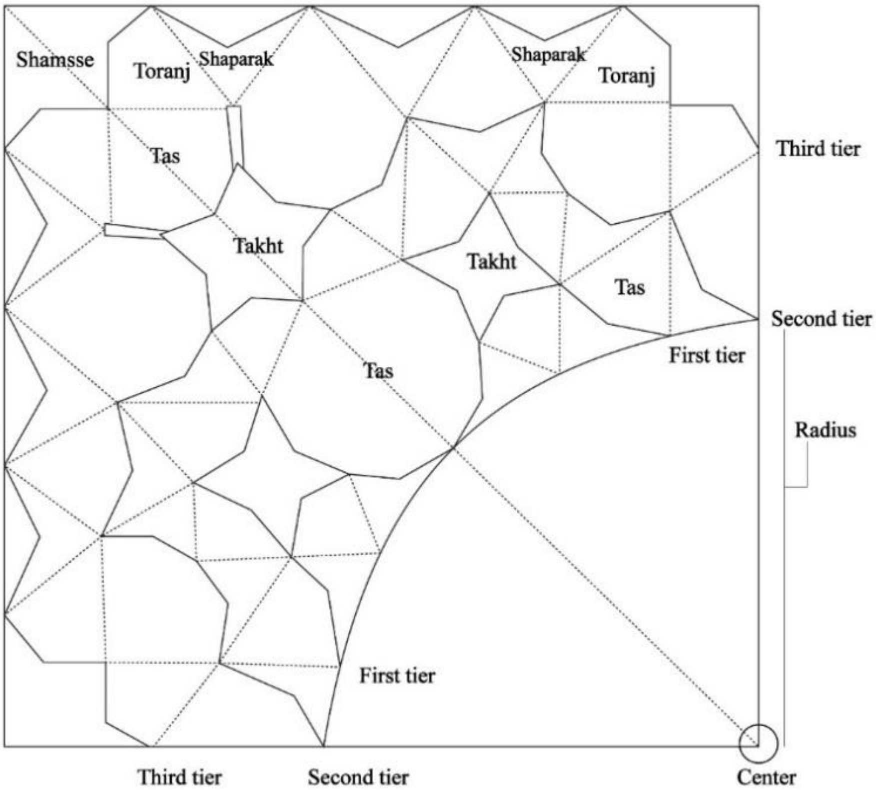


Figure 44. Two-dimensional plane projection of a Muqarnas composition (Web 36)

Figure is redrawn by author

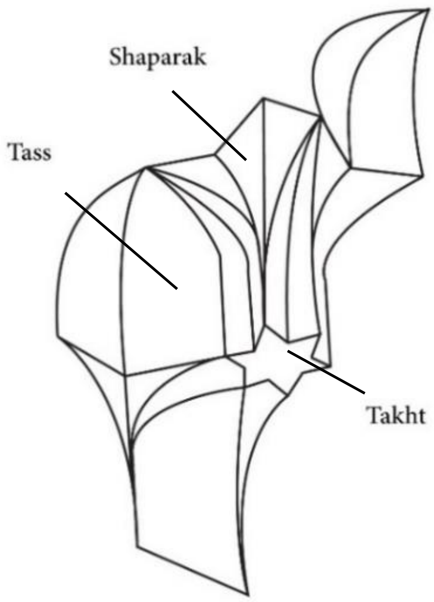


Figure 45. A part of Muqarnas composition

Source: Navai and Haji ghasemi. 2014, p.97.

Figure is redrawn by author

According to the definitions that Dadkhah, Safaeipour & Memarian(2012) and Navai and Haji ghasemi (2014) present about the intermediate elements of the Muqarnas:

Shāparak: Pendetive or shaparak is the most important three-sided element of Muqarnas composition that determines the position of the other elements and connects one point of the lower tier to the two line of upper tier. The angles between the sides of Shaparak are flexible and determine by the needs of composition (Figure 46).

Figure 46. Pendetive or shaparak

Figure is redrawn by author

Takht: takht mostly appears in Muqarnas composition with one of the forms of regular polygon (triangle, square, pentagon, hexagon, etc…), multi sided stars or knotted shape (triangle and other repeated elements in Islamic patterns) (Figure 47). It is the only flat element that is used in Muqarnas composition and the designer defines the location of that in order to be projected in specific spots to facilitate synchronization of the Muqarnas on a hypothetical circle.

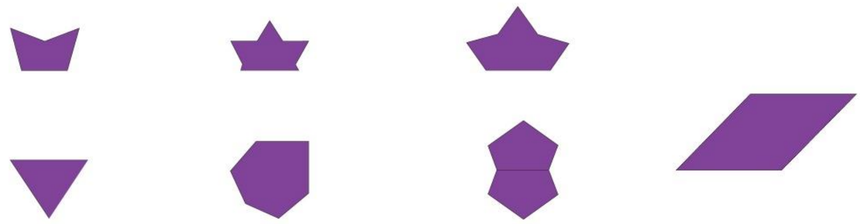


Figure 47. Takht

Figure is redrawn by author

Tāss (Tāssé): Tāss can be found in all tiers but it is mostly seen in the last tier. Tass is located between two Shāparak or two Pābārik and connected two or more line of the lower tier to one point at the uper one. They are curved triangular pieces or niche shape elements, that form the concaved part of Muqarnas composition (Figure 48).

Figure 48. Tāss

Figure is redrawn by author

Squinche: The squinches are located between the pendetives and provide a transition between subsequent tiers. Squinches, are used to provide a transition zone for complex tracks and the complexity of the Muqarnas composition, determines the number of flat elements.

Pabarik: pabarik is located around takht or shamsse. It is a parallelogram in which two sides are equal and the long axis is its axial symmetry.

Shamsséi: Shamsse is always located in the last tier of Muqarnas and has a multi sided star shape or half of it. The number of sides and axis in shamsse must be more than those in Takht (p. 135).

Figure 49, shows the location of elements on three-dimensional Muqarnas composition that is used on mihrab of Hakim mosque in Isfahan, Iran. Figures on the next page illustrate the location of elements in each tier, respectively.

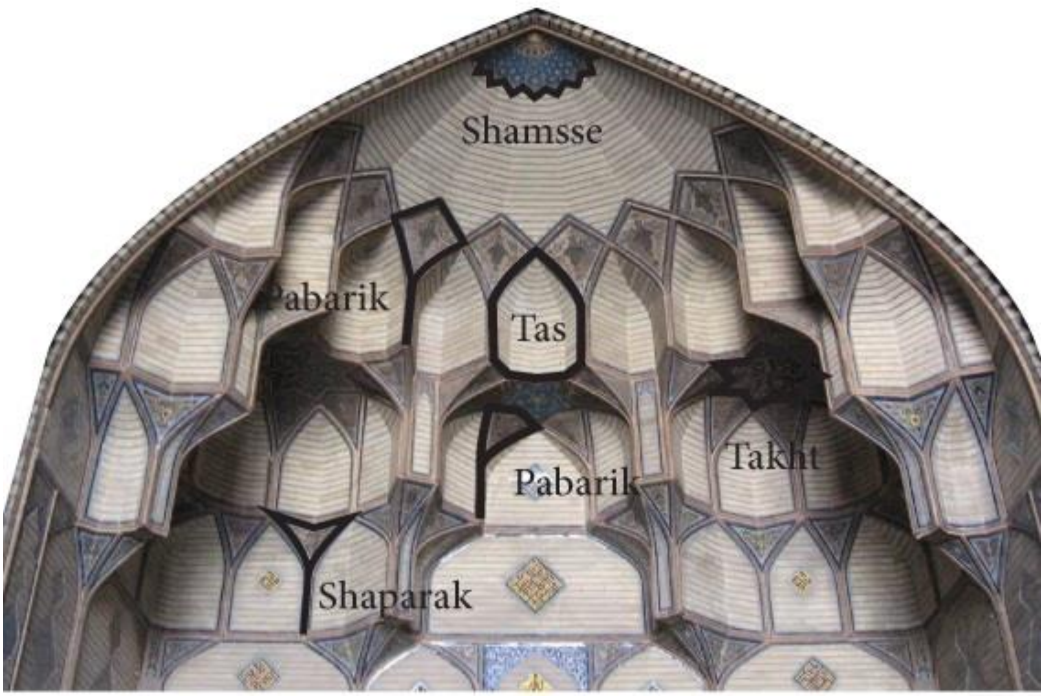


Figure 49. Muqarnas that is used on mihrab of Hakim mosque in Isfahan, Iran

Also, Moussavi (2009) by decomposing a Muqarnas to its constructor modules, presented another definition and asserts that Muqarnas composition must include at least two tiers and these tiers must be at the same height and they consist of changeable but repeatable plan forms which are also called “tracks”. In order to produce a Muqarnas composition, these tracks stacked next to and on top of each other. Each track itself also is composed of four major elements: the pendentive (shaparak), flat surface (takht), squinches (Tas), and extruded vertical planes (Figure 50) (p. 326).

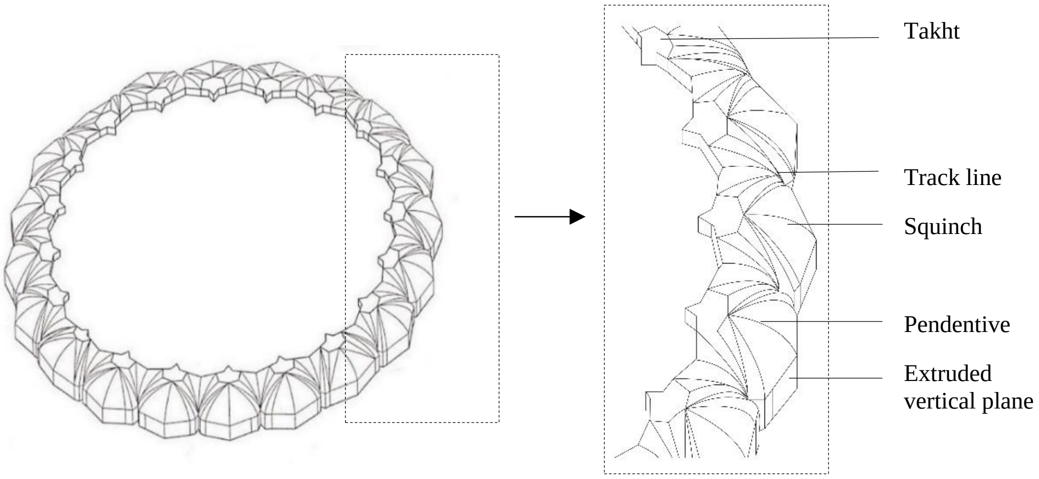
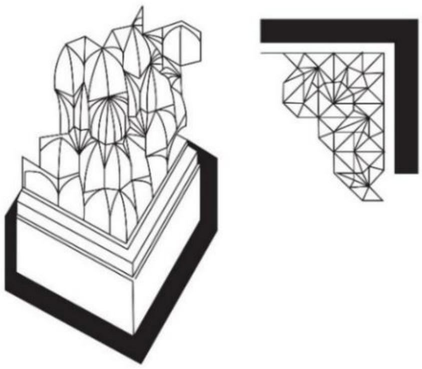


Figure 50. Track elements of Muqarnas

Source: Moussavi. 2009, p.327.

Figures are redrawn by author

When we look at a two-dimensional plane projection of a Muqarnas compositions, it seems that the intermediate elements themselves, are split into the constitutive forms. Samplonius (1992) presented three-dimensional views of the south octagon- vault at the Takht-i Suleiman and its plane projection (Figure 51). As you can see in Figure 52, it is obvious that the three-dimensional Muqarnas design corresponding to the plate can’t consist of only squares and rhomboids but they combined with the split forms of some elements. Like a square can be split into a large biped and a jug, in the same way, a rhombus can be split into a small biped and an almond (p. 203).



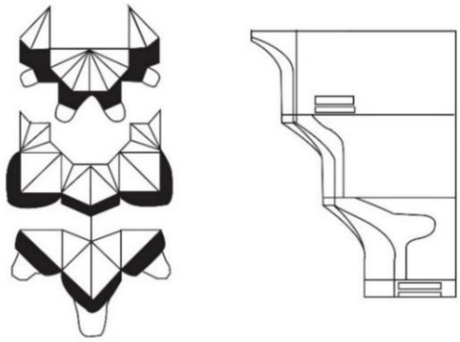


Figure 51. A corner of Takht-i Suleiman Projection

Source: Samplonius. 1992, p.203.

Figure 52. A vault of Takht-i Suleiman

Source: Samplonius. 1992, p.204.

Both figures are redrawn by author

Hoeven and Veen (2010) refer to al-Kāshī and mentioned that the top views of Muqarnas elements consist of a small set of simple geometrical forms such as squares, half-squares, rhombuses, half-rhombuses, almond and, biped. The top views of these elements are based on the square and the rhombus (Figure 53). This thesis firstly discusses the elements that have the square base.

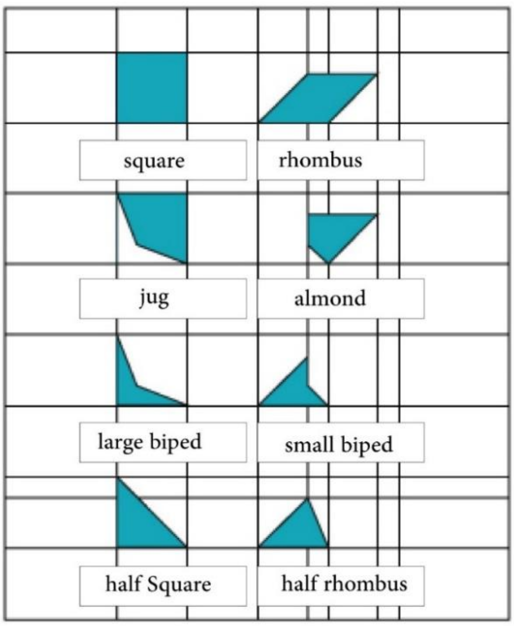


Figure 53. Overview of the basic Muqarnas elements

The figure is redrawn by author

Square: all sides of a square is equal to the module. This element can appear as a cell or as an intermediate element.

Jug: the jug element is a plane projection of a cell and has two longer sides that correspond to the curved sides of the element and have the module as their length, the other smaller sides are equals to the module.

Large Biped: this element mostly combines with a jug and it is used as an intermediate element in Il-Khanid Muqarnas. It is what remains after the subtraction of a jug from the square.
Half square: the last square-based figure is the half square, where the square split over its diagonal. That is mostly appear as a cell or as an intermediate element.

All rhombus based elements can appear as an intermediate element or as a cell. And all of them have at least one angle with 45 degrees.

Almond: almond is appeared as a cell and has two sides with 45-degree angle and one 135 degrees angle and the other two angles are right angles of 90 degrees.

Small biped: It is what remains after the subtraction of an almond from the rhombus. This element appears as an intermediate element and curved sides in this element have 45 degrees angle, the opposite angle is 225 degrees and the other two angles are 45 degrees again.

Half rhombus: it is what the rhombus split over its short diagonal. This element appears as a cell or as an intermediate element (p. 4).

The structure of the different types of Muqarnas are the same but they can have vast variation in the elements and the angles such as Seljuk Muqarnas or the Muqarnas of Topkapı Scroll (Necipoglu and Al Asad, 1995. P. 351).

3.1.2 Three–Dimensional Muqarnas Structure

According to Al-Kāshī, Muqarnas is like a staircase. The elements are the steps of the stair and have to be arranged to form a Muqarnas structure. The structure of the Muqarnas is built from elements arranged in different levels, each level called tier as a translation of Arabic word Tabaqa and consists of adjacent cells. In Figure 54 we observe how the elements join together on one tier. It shows, elements can join directly like A and B, or they join just at the back with an angle between them like C and D. Cells can only join other elements in the same tier at their curved sides. The intermediate elements also mostly join other elements at their curved sides on the same tier, but they can also meet other intermediate elements at the front parts.

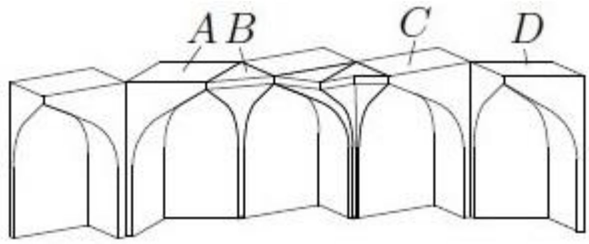


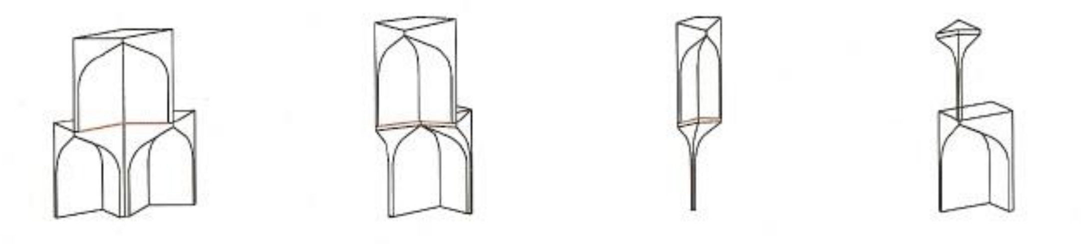
Figure 54. Part of a tier of the muqarnas, Natanz, Iran

The first tier of the Muqarnas that there is not any other tier stand under it is called the base of Muqarnas. The top tier of Muqarnas that there is not any other tier stands on top of that is called center of the Muqarnas. For joining the different tiers of the Muqarnas there are different ways, but in all of them the tiers place on top of each other so that their projections do not overlap:

Curve to curve: as we discussed this way before in the same tier the curved sides of the elements join each other (Figure 55, a).
Front to front: in this way, the front of intermediate elements meets each other on the same tier (Figure 55, b).

Back to curve side: the backside of a cell stands on the curved side of an element in lower tier (Figure 55, c).

Back to front: the backside of a cell stands on the front side of an intermediate element in a lower tier or inverse an intermediate element stands on the front of the cell in a lower tier (Figure 55, d).



(a) On curved side (b) On front (c) On front (d) On front

Figure 55. Combination of Muqarnas elements

3.2 Process of Muqarnas construction

According to Akram, Ismail and Franco (2016), Muqarnas is usually used in undersides of domes, cornices, pendentives, arches, vaults and squinches. As they can show a downward-facing shape, so, one can trace a line between the floor and any point on the Muqarnas surface. One can design Muqarnas geometrically so that a unique visual form can be obtained (p. 135).

Sakkal (1988), declares that Blocks of Muqarnas have been arranged in many compositions in the way that they can enclose partially or completely space, in doing so, there must be a connection between them in the horizontal direction, and also, a connection between on top of each Muqarnas in the vertical direction. In both cases, the first state (horizontal layering called "combination") and the second state (which is called "juxtaposition", the final arrangement is called "cluster" (p. 25).

A short description of the construction process of plaster Muqarnas has been presented by Dadkhah, Safaeipour & Memarian (2012), which can be summarized as below: Firstly, a two-dimensional pattern plan of working sketch with full-scale should be scratched on a plaster or on a wooden slab, according to the Muqarnas vault design. This projection is called Takhmir[48]. Then, tiers rows should be identified by ceramic bars or clay lumps on these plans. And after that, for each, all tiers of the vault, alternative plates of plaster or wood (1-2 inches) should be cast and prepared. Then, the plates must be connected to the vault and wall by Sazoo[49]. All plates are loaded on the dome, above the Muqarnas. A Plumb line[50] is used as correspondence to the full-scale sketch on the floor (Figure 56, a, b, c)

In the next step, Molding and installation of each tier should be done from the base row to the central Shamssé and molds of plaster should be constructed main elements of each tier. These molds sometimes are developed in a way that can be used in the number of elements that architect-artisan (mimar) has intended to produce. Then, an overlaying plaster, tile work (either tile or mirror), or bricks 26 should be constructed with these molds. And after that, a plaster which can be dried quickly is used between the cells, as a liquid grout on the back of the tile work. Finally, these cells should be connected to the vault and also, they should be fitted to a spatial grid of the horizontal and vertical openings, in order to form a whole work (Figure 56, d, e) (p. 136).

[48] Takhmir is a geometrical full-scale sketch of a two-dimensional pattern on plaster or wooden slab on ancient Islamic architecture.

[49] In order to construct a Muqarnas vault, for standing each tier of the vault on two-dimensional plane projection, designers used a suspended column called Sazoo, on the back of Muqarnas, which itself was made of animal fur like horsehair and lined with plaster.

[50] A plumb line (Persian: [شاقول]), is an instrument that has been used in ancient Islamic architeture to ensure that constructions are "plumb", or vertical.

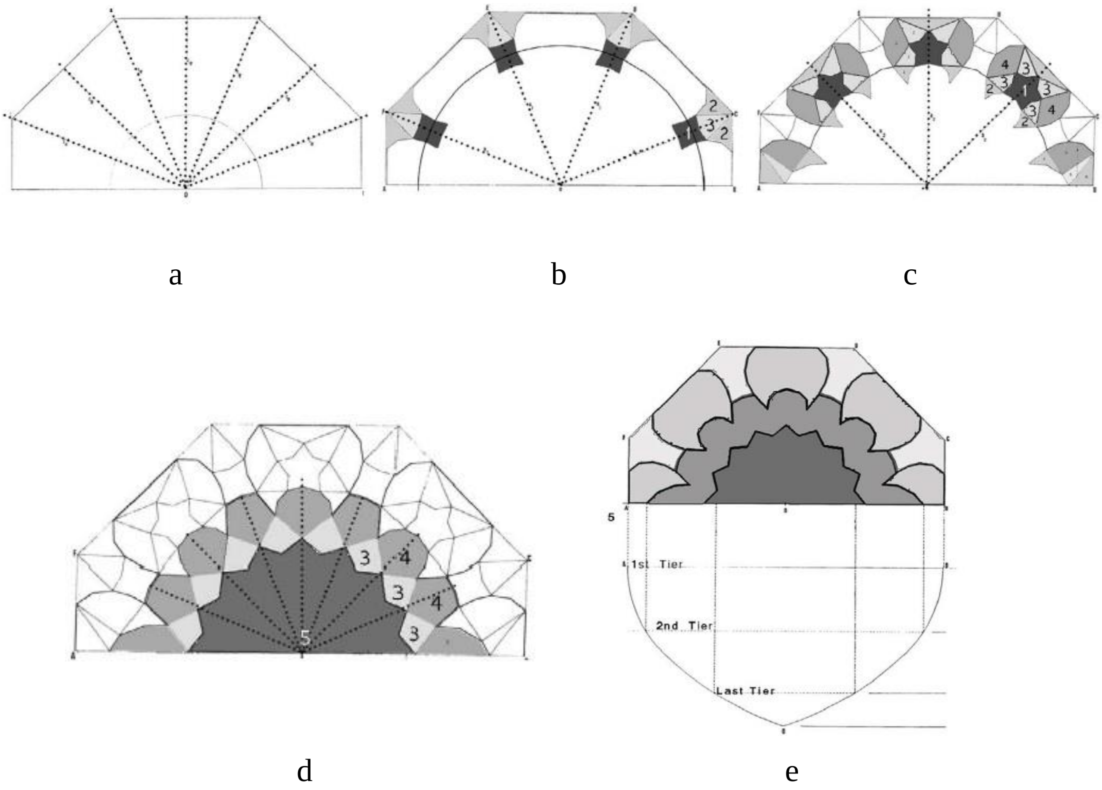


Figure 56. Formation stages of Muqarnas in context of half an octagon

Source: Dadkhah, Safaeipour & Memarian. 2012, p.135.

Yaghan (2001) has stated that Muqarnas consists of orderly horizontal layers with small ‘unit surfaces’, these layers have been accumulated on each other and have been connected by their layer lines in base and top. These layer- lines (top and base) are integrated or have been separated by joints. All horizontal gaps between lines with same height (like stars), have been filled by ‘roof-patches’, which are along with units faces and layers, constructed according to 2DPP and their boundaries have been defined clearly. Muqarnas, depending on their 2DPPs that are created upon them have certain degrees of complexity. Sometimes it needs a great deal of skill and experience to decode Muqarnas drawings into three-dimensional forms (we will discuss later) (Figure 57).

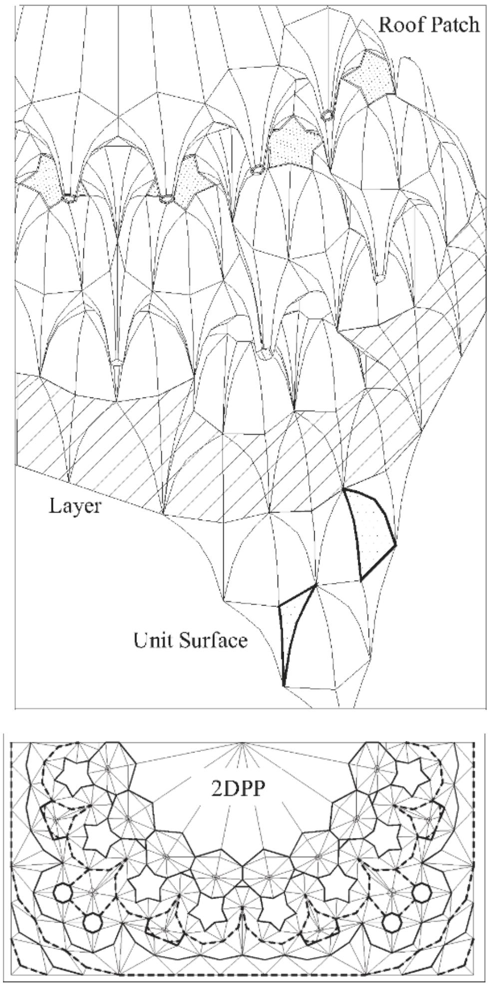


Figure 57. The basic parts of the Muqarnas (Web 37)

For better understanding the construction process of Muqarnas composition another example has been presented in (Figure 58) by Moussavi. The gate of Sheikh Loft Allah mosque is developed by vertical tessellation of a base unit, or a “track “consisted of a series of different units. But these units are repeated and are arranged along a horizontal contour and are in conjunction with a pointed arch. The base unit has been repeated in order to form six different tracks, as shown in Figure 59. (Moussavi, 2009).

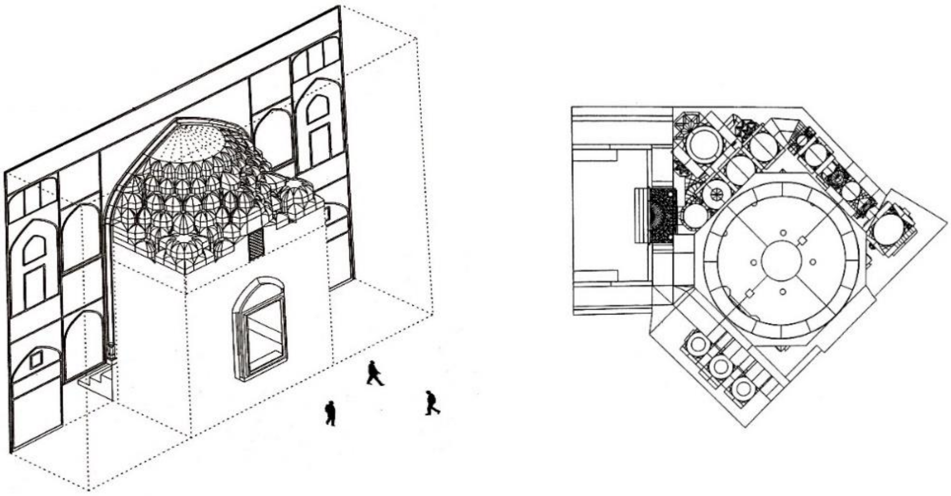


Figure 58. Sheikh Lotf Allah Mosque, Isfahan, Iran

Source: Moussavi. 2009, p.337

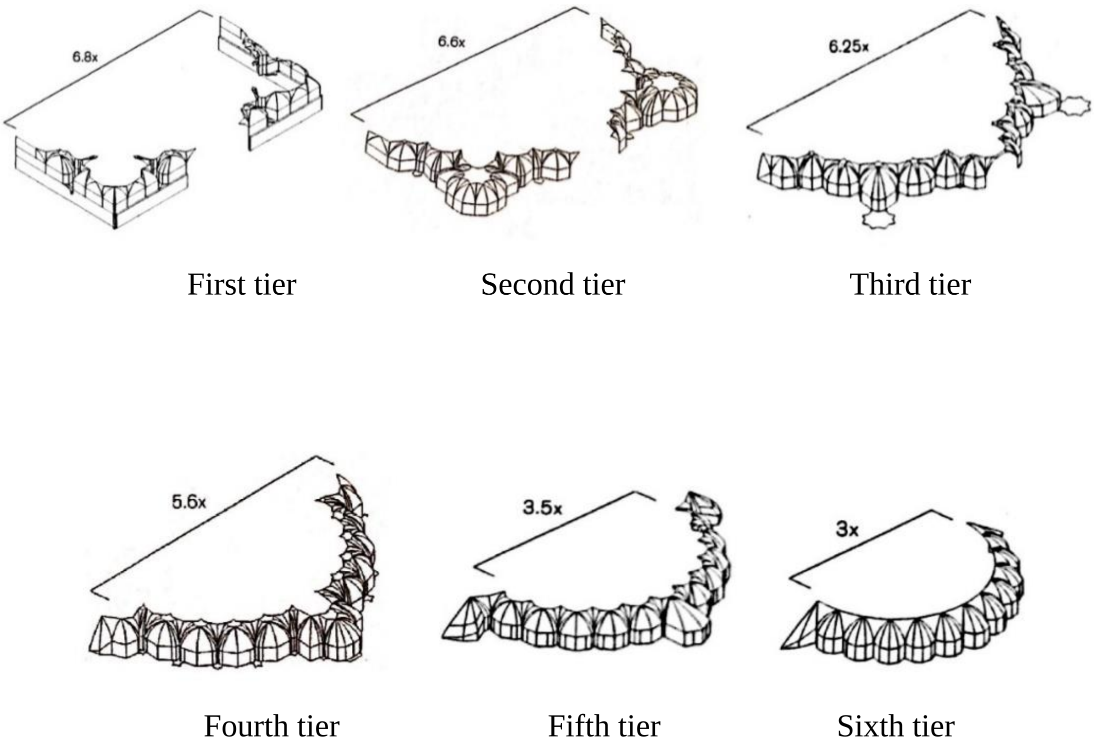


Figure 59. Process for constructing a Muqarnas composition

Source: Moussavi. 2009, p.337.

3.2.1 Decoding the two-dimensional Muqarnas pattern into three-dimensional composition

According to Notkin (1994), in former centuries the available elevations and sections of Muqarnas compositions were not reliable enough. Hence, master builders tried to enlarge the plane projection of Muqarnas patterns proportionally for the perception of their complicated concepts with drawing and assembling all the component units on the paper. Hoeven and Veen (2010) state that the pole table pattern has been appearing and become common all over the western Asia (including Iran) after invasion and fall of the Mongols in the 15th century. This type has not any direct link regarding its architectural structure, there is shown a connection in its own structure. The elements of this type of Muqarnas firstly are produced on the ground, and then they are attached to the architectural structure by ribs. The center (the top of a Muqarnas) of the pole table can be expanded from 4, 5 and 6 segments to 7 and 11. Elements with star shapes also have 7- and 9-pointed stars.

According to Harmsen, Jungblut and Krömker (2007), both kinds of Muqarnas elements such as a cell and intermediate element for connecting the cells, were designed on two-dimensional geometrical projections on the ground in order to convert it into a three-dimensional Muqarnas structure. As Al-Kāshī (1977) mentioned, if you look at the two-dimensional projection of the Muqarnas or the view from underneath, all of them consist of the following shapes: square, rhombus, half-rhombus, almond and its complement to a rhombus, a small biped, jug and its complement to a square, a large biped, and barley-kernels (that is only use on the top tier of a Muqarnas).

Ghazarian and Ousterhout (2001) assert that there is no doubt that the architectural design process in the pre-modern world began with drawing. Figure 60 below illustrates the construction of the central Muqarnas vault that was found onto the south facade of the gavit of Astvatsankal in Armenia. The drawing was represented one-quarter of the Muqarnas vault with the geometry of the plan for the lower three stages of the Vault. As you can see in the picture the lines showing the forms of the blocks geometry.

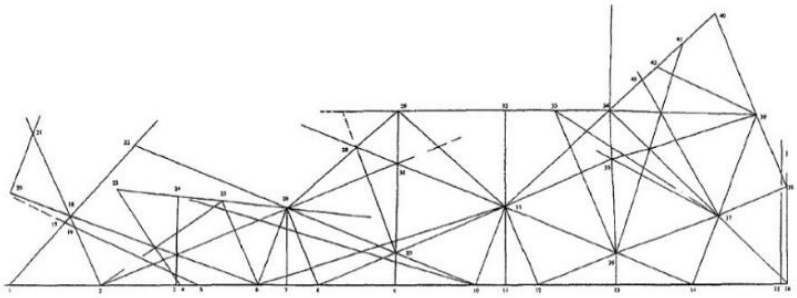
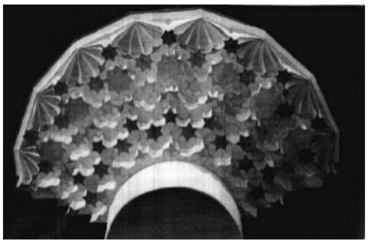


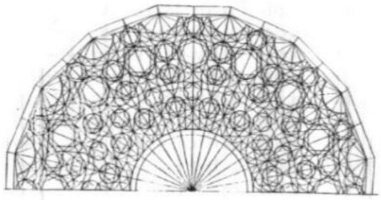
Figure 60. Construction of the Central Muqarnas vault

Source: Ghazarian and Ousterhout. 2001, p.148.

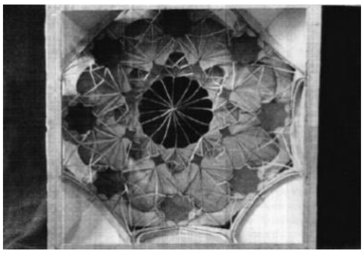
In order to explain the creative designing process of earlier time, Notkin (1994), refers to the system of decoding the two-dimensional Muqarnas pattern of former Bukharan Muqarnas builder, Ustad Shirin Muradov. According to his system of coded design, the two-dimensional presentation of a three-dimensional Muqarnas composition form was shortened according to traditional design conventions. Ustad Shirin applied each Muqarnas plane projection according to the laws of orthogonal projection and using mesh lines and conventional regulations to represents consecutive corbeled tires of modular Muqarnas cells (Figure 61).



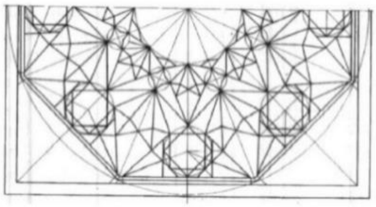
(a)



(b)



(c)



(d)

Figure 61. Ustad Shirin Muradov's sketches of stalactites and their spatial treatment in scale models made of paper. Muqarnas model (a), Muqarnas sketch (b), Muqarnas model (c), Muqarnas sketch (d).

Source: Notkin. 1994, p.148.

According to Hoeven and Veen (2010) a whole collection of Topkapi scrolls containing 114 drawings, was discovered in 1986 in Istanbul, they are painted in sixteenth century and now they are kept in Istanbul's Topkapi Palace Museum Library. Some of them are shown in the Figure 62.

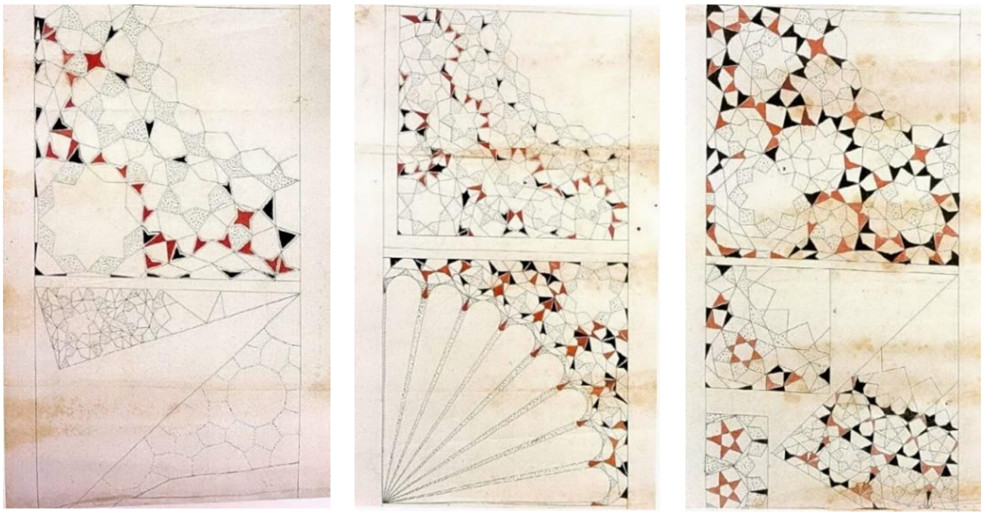


Figure 62. Topkapi scrolls (Web 38)

Yaghan (2000) state that all forms of Muqarnas found to date have been built according to the patterns of 2D plans (2DPPs)[51] , such as Muqarnas found on plaster (Harb, 1978, p. 9–10; Yaghan, 2000, p. 77), Muqarnas drawn on parchments (Necipoglu, 1995, p. 3–27), or form which has been projected on paper. These plans are in regular form and in the most cases, a multi – fold symmetry can be seen with an ordered arrangement of their units (linear, radial, or grid), and also there can be seen as groups of units and focal points (p. 82).

Ulrich Harb was the person who published the details of a Muqarnas design plate that found at the Ilkhanid seasonal palace, Takhti Suleyman and tried to explain the design. Yaghan (2000) refers to Harb and asserts that the plate had a geometric pattern incised on it (Figure 63).

To build a Muqarnas from its two-dimensional plan directly, designer has to handle the combination of elements appear in plans like what Harb did in his article, the lines of the shape recognized by Harb and as he mentioned some figures such as square and rhombus must be split to forming this design (Figure 64). By using Harb's experience yaghan, reconstruct three-dimensional form of the whole vault from the quarter of a vault plan with the help of a computer (Figure 65) (p. 80).

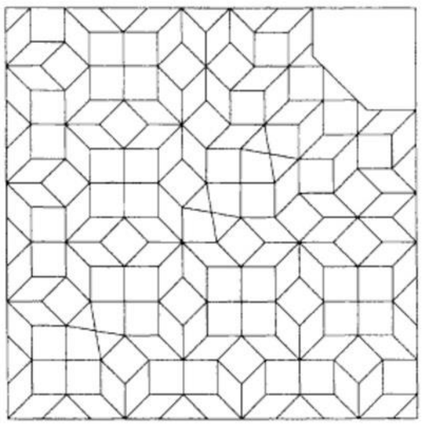


Figure 63. The pattern as found inscribed, representing a quarter vault, drew by Harb

Source: Yaghan. 2000, p.81.

[51] 2DPP means: two-dimensional pattern plan

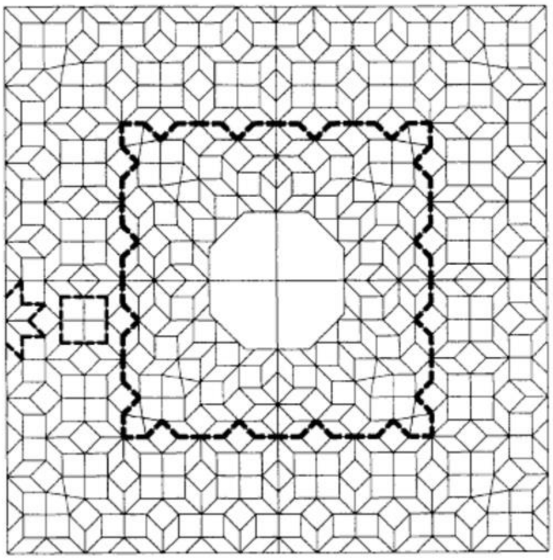


Figure 64. Four quarters, representing the whole vault, drew by Harb

Source: Yaghan. 2000, p.81.

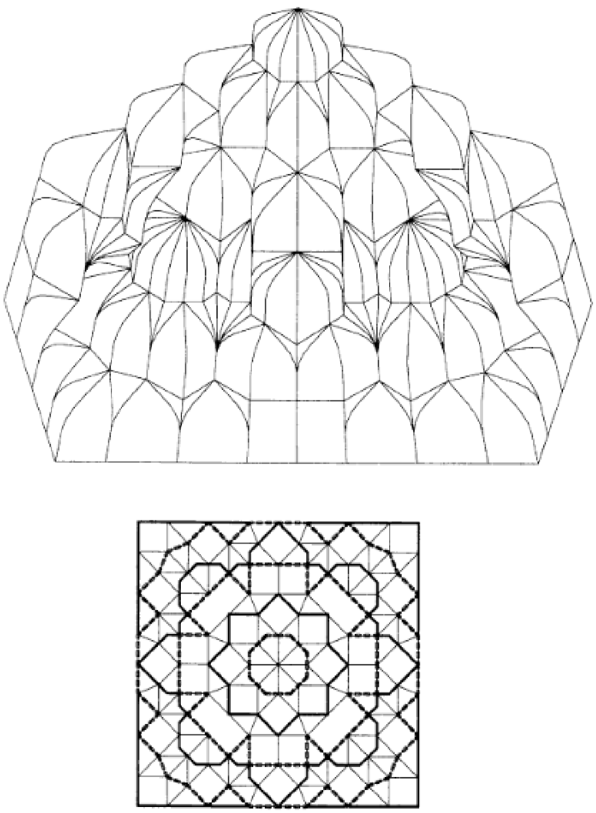
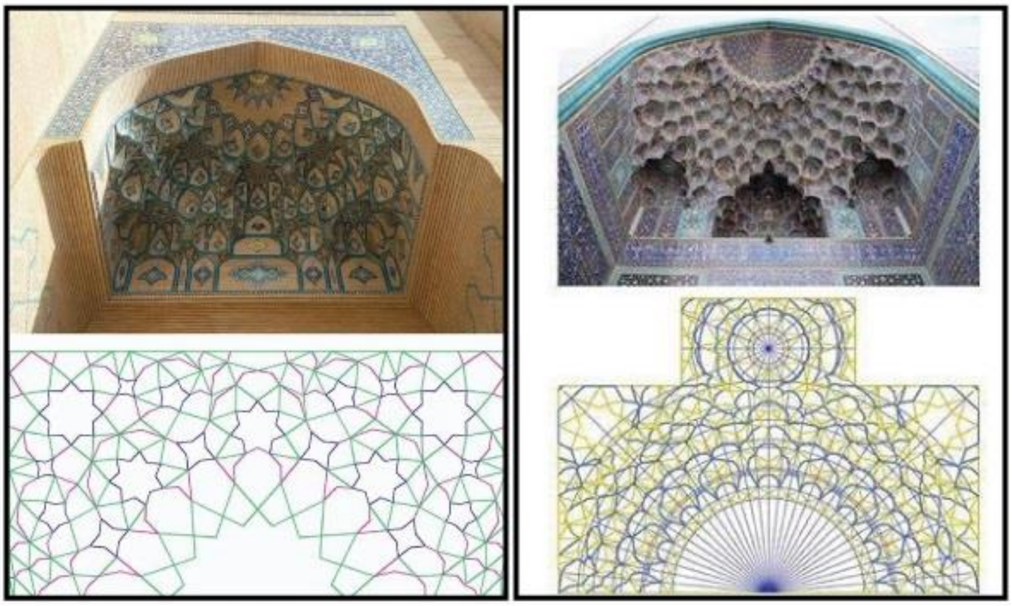


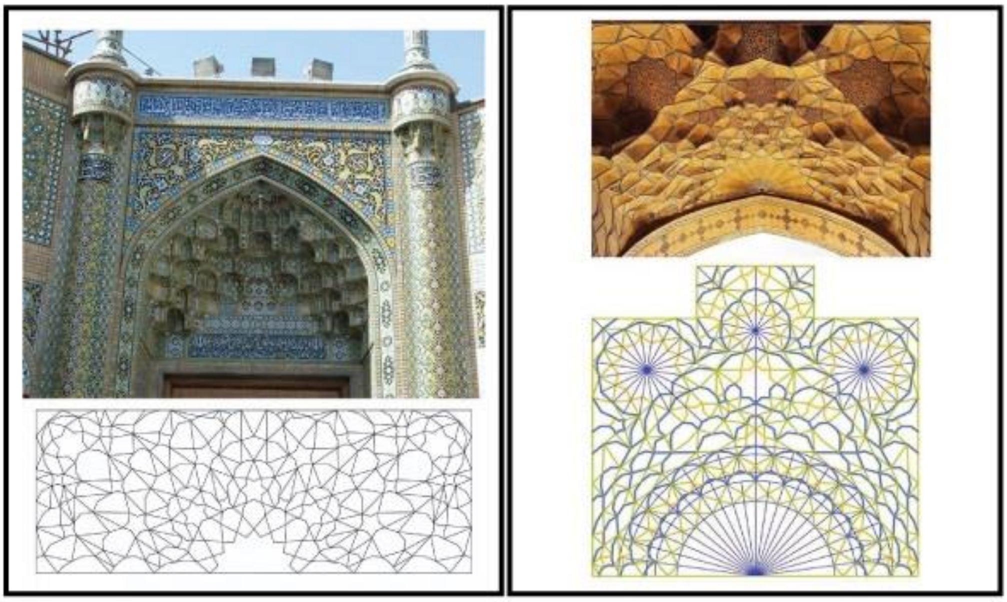
Figure 65. Reconstruction of the Muqarnas of the vault of Takhti-suleyman

Source: Yaghan. 2000, p.80.

Figure 66, is illustrated some of Muhammad golyar's Muqarnas design collection, that are collected by Ali Reza Sarvdalir. (a): A religious academy in Qom, Iran. (b): Entrance Muqarnas, Imam Mosque, Esfahan, Iran. (c): Entrance Muqarnas, Qom, Iran. (d): Jami mosque, Esfahan, Iran.



(a) (b)



(c) (d)

Figure 66. Muhammad Golyar's Muqarnas design collection (Web 39)

3.2.2 Constructional properties of Muqarnas

According to Hamekasi, Samavati and Nasri (2011), for understanding the concept of the Muqarnas it is better to understand their construction methods. There are three ways to constructed Muqarnas: corbeled, superimposed and suspended. As it is shown in Figure, a corbeled Muqarnas cells are made from stone or wooden blocks so that they are carved outwards either before or after installation. This type has a structural role as a part of the building (Figure 67, a). To constructing a superimposed Muqarnas firstly the supporting concave surface is built then Muqarnas elements are applied on surfaces whiteout any gaps between them. Superimposed Muqarnas also acts as a supporting element more than decoration one. Plaster, cement and similar materials are used for this type of construction (Figure 67, b). And the last way is assembling a decorative panel from ceramics on the surface that named suspended Muqarnas. In this way, the panel components are built separately, then attached to the architectural surface with an empty space between them with the help of attachment ribs by horse hair. This type has a decorative role (Figure 67, c).

(a) (b) (c)

Figure 67. Three ways of Muqarnas construction

Corbeled (a), Superimposed (b), Suspended (c)

Source: Hamekasi, Samavati and Nasri. 2011, p.131.

Hamekasi, Samavati and Nasri (2011) refer to Harb and described Muqarnas elements in three different units of measurement 21 (with 42 cm height), 26 (with 52 cm height), and 42 cm (height is undetermined. Only one element of 42 cm (twice 21) has been found. It is obvious that the height of each cell is twice their unit of measurement.

3.3 Digital formation of Muqarnas

There are many variants of designing methods in use today. Most of these methods are help designers with producing complex forms by the aid of computers. Creating a Muqarnas composition is really tough due to its complex shape and geometry. The diversity of its styles also makes it more difficult and complex. This chapter tries to explain reconstruction methods of historical Muqarnas examples and the evaluation of them through CAD system which makes the construction process much easier and faster. As it was mentioned in chapter 2, historical Muqarnas compositions were constructed by using traditional methods which were handed down from one generation's handcrafts to another. But today besides the developments in art and architecture through CAD systems, Muqarnas compositions also, benefit from CAD technologies and new generations of them come to existence. These technologies generate a process that was impossible in conventional methods before.

After we took a glance over the process of Muqarnas construction which is presented in Chapter 3, we use them in this research to create new evolutions in the Muqarnas forms. This chapter tries to develop this process through CAD and discuss how digital technologies engender new theories that make it possible to generate more complex forms of Muqarnas and expand the process of Muqarnas construction.

3.3.1 The evolution of Muqarnas forms through CAD

Digital architecture has been the main topic of most of the architectural researches and theoretical writings in last decades. In the past, there were just paper-based methods in architecture but in the last decades, new method emerged in designing which exploit a computer as a designing tool. The designing process also has been changed from paper-based method to digital method gradually. Sometimes a computer can acts as a designer by suggesting good ideas during a designing process which could be totally different from designer's initial ideas. Computational design generates and materializes a set of design logic which needs a formulation between the result of design and the process that it is driven by that.

The computational design also creates a new method in the complex forms design process and somehow developed them by integration of tools and create fast changes in digital technologies. Currently, computer science has been developed and computational design methods need more information about these technologies. Therefore, designers require computer experts and specialists to split the complex software however, digital designers need the methods associated with computer aided design software and numerical techniques. According to Kipnis (1993), in most of the architectural exegesis, architects and designers make an effort to explain more about digital techniques and methods. Complex forms which are emerging with the help of computational design tools are very popular discourses among the last generation.

In this thesis, the significance of computer aided design software will be underlined to reach last decades architectural designing developments. For this purpose, a brief definition of computer aided design is presented.

According to Pipe (2005), computer aided design is a method of modeling that suggests the use of computer software instead of drawing on the paper to enhance the speed and visual accuracy of designing and create technical illustrations, some other benefits of CAD software are as follows.

- Designer could have final visualization of their products and assemblies of them therefore they could easily speed the designing process.
- In designing with CAD software because of its great accuracy errors are reduced significantly.
- Creating a shape or space with complex geometries and dimensions with various materials in CAD software are easily possible however it could be really tough with drafting.
- CAD software allows to designer re-uses their design data easily and practices more on them.

As Knight, Dokonal and Brown (2005), claimed that the diverse aspects of the objects can be modeled in a design process to give a comprehensive perception of a project to designers and they have a chance to lay out components for sizing, positioning with using both 2D and 3D displays. Almost every commercial designer using CAD systems in their designs. CAD tools simply described the result of the process to facilitate and support design decisions. It can define easily solid models of designer ideas to ensure that assemblies, surfaces, intersections, etc.., are clearly defined.

Nowadays digital logic is able to generate a process that was impossible in conventional methods before. Jabi (2013) declared that to modifying the interactivity of models which have been created by computer-aided tools, design software allows architects to describe relationships between various parameters of their design models. Designers accordingly could access various features of their models and update them immediately. Being an expert in the crafting of different Medias could help designers to be more creative in their fields. According to Stavric and Marina (2011), various possibilities of digital technologies and many topics from different fields influenced architectural design. All the mathematical and geometrical algorithms, logics, forms and structures are now very useful factors for architectures that many computational concepts such as topological space, isomorphic surfaces, motion kinematics and dynamics, key shape animation, parametric design or fractal geometry have been established.

In contemporary architecture, there are many samples of design projects. According to Sola (2012) designing with the aid of generative system methods is an interactive connection between systems and designers that can suggest so many different forms, originated from the initial forms and help a designer to be more creative. Form and shape of the models are considered as a result of generative systems procedure. Designers could access various models of their initial model by using the generative system, in this case, they have a chance to evaluate different feature of their form to selecting a suitable form inside them without time-consuming. However, we could have variation and generation about the forms which have relation with together within the same family of forms.

As mentioned before, digital technologies developed new theories that make it possible to generate more complex forms and topological geometries. According to Sayah (2016), Muqarnas has a complex composition through its various combinations of three-dimensional shapes in different ways. CAD software or algorithmic software are the best designing tools to draw complex shapes and creating modular forms. Also, they increase the efficiency to automate design and explore design ideas. Sayah refers to Gero and presents two main parts of CAD development. “the representation and production of the geometry and topology of designed objects” and “the representation and use of knowledge to support or carry the synthesis of designs”.

Hamekasi, Samavati and Nasri (2011) assert that CAD software also can help architects in reconstructing and visualizing three-dimensional models of Muqarnas from recordings of Islamic architecture that are usually two-dimensional in the forms of maps or sketches. According to Nejad Ebrahimi, Aliabadi and Gharehbaglou (2014), CAD software are building separate discourses about designing in architectural theory. For example, parametric design is a process that enables the expression of parameters. Parameters are describing the dimensions of the objects, and computers are able to automatically modify a design by the values of parameters. Creating Muqarnas as the result of parametric methods, make it possible to produce variety types of it.

3.3.2 Modeling Muqarnas through CAD system

To show how a Muqarnas plan is transformed into a three-dimensional composition, one of the Muqarnas plans of the Topkapi scroll will be used as an example in Necipoglu and Al-Asad (1996) books (Figure 68), as she states:

> "It consists of a quarter vault at the lower right with Muqarnas units filling the remaining part. Forming it into a three-dimensional composition is an interpretative process in which the interpreter needs to supply the missing data. Although the coding of some of the plan units with dots and colors provides some of that information, one still has to make use of the information provided by other sources such as surviving examples of Muqarnas vaults and Al-Kāshī's section on the Muqarnas in his Miftah al-hisab. Even so, no single correct three-dimensional interpretation of the plan exists and what is provided here is only one version of the possible interpretation."

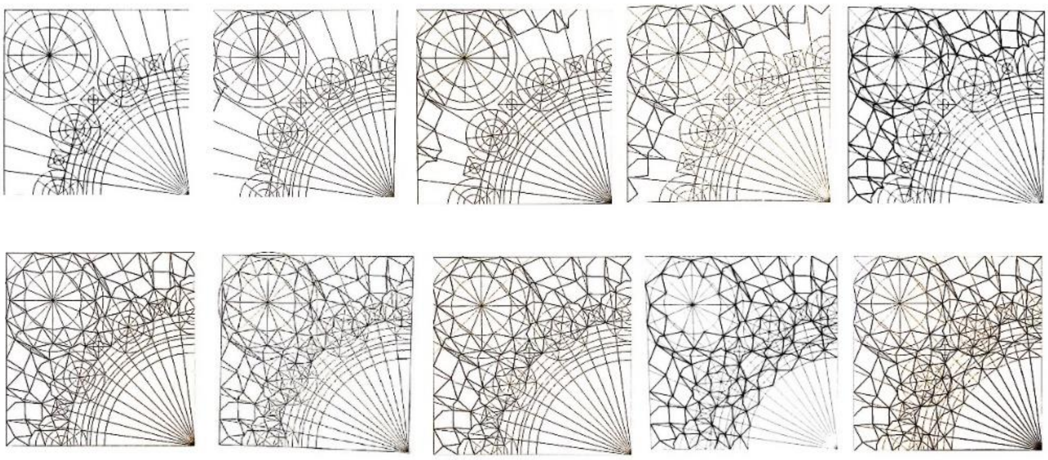


Figure 68. Muqarnas plans of the Topkapi scroll

Source: Necipoglu and Al-Asad. 1996, p.352.

Moreover, according to Necipoglu and Alasad (1996) the transforming process of a row of Muqarnas plan into a three-dimensional composition, is demonstrated in Figure 69. The uppermost tier was selected as a reference tier. And one-unit height was selected for all of the tiers. As she declared:

> It is the most regular tier and consists only of identical units and half units. The height used for the tier equals twice the depth of the units of the uppermost row. Another interpretation based on a different height, or heights, for the tiers would be equally correct.

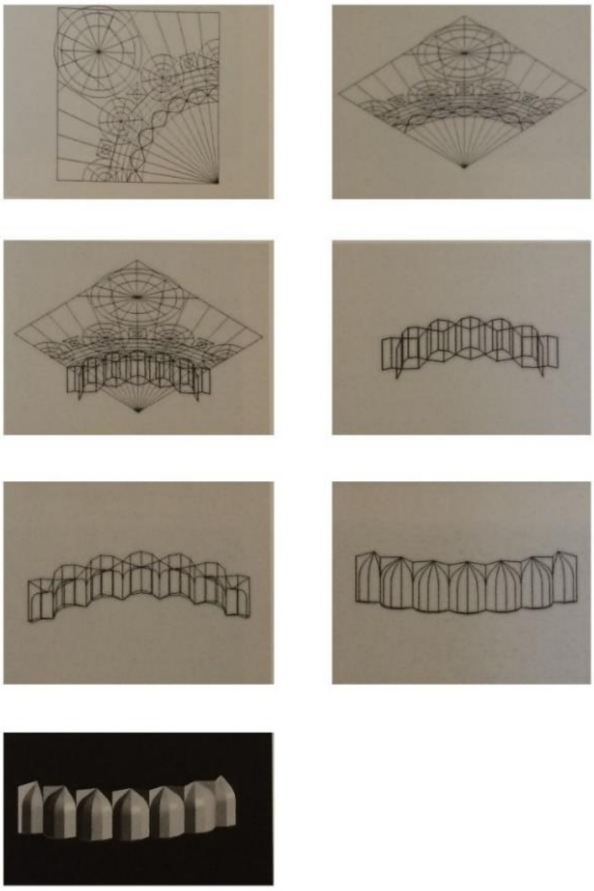


Figure 69. The process of a Muqarnas row transforming into a three-dimensional composition.

Source: Necipoglu and Al-Asad. 1996, p.355.

After projection of plan by specific height for the determined tier, the process is repeated for the other tiers according to the guidelines provided by the Al-Kāshī as well. Placing each tier in their correct position plays a crucial role in transforming process. The heights plane, or roof, of other tiers, should be located at the same level as the bottom plane of the tier situated above it. In the final result, each row maintains its location in plan and is only shifted along the height coordinate (Figure 70).

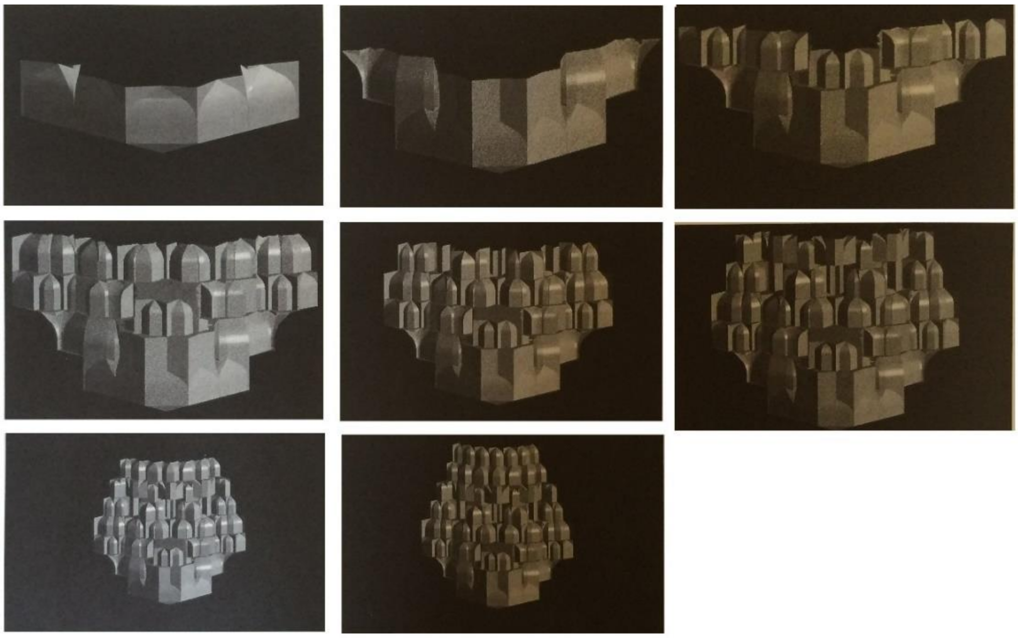
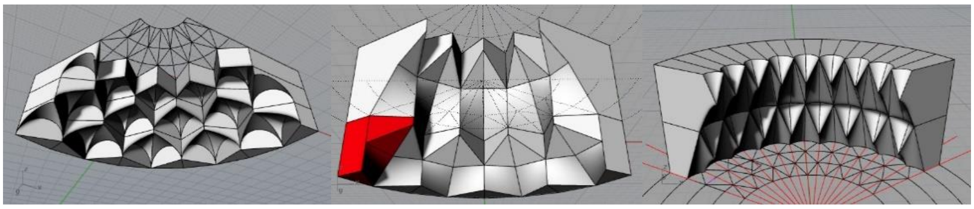


Figure 70. The process of constructing a three-dimensional Muqarnas composition

Source: Necipoglu and Al-Asad. 1996, p.356.

As Sakkal (2001) mentioned in his Ph.D. thesis, Muqarnas blocks are combined with each other in consecutive layers to creating three-dimensional geometries to enclosing the space because of their unique geometry. The Muqarnas geometries can take several variations according to their functions. In one of Design Computing courses, students started to obtain their own idea of generating solid surfaces out of a few components. They tried to understand the relationship between Muqarnas components and using it with transforming a two-dimensional design template to a three-dimensional model. The relationship between the Muqarnas three-dimensional components, and referencing two-dimensional pattern plan underneath are obviously illustrated. (Figure 71, a, b, c)

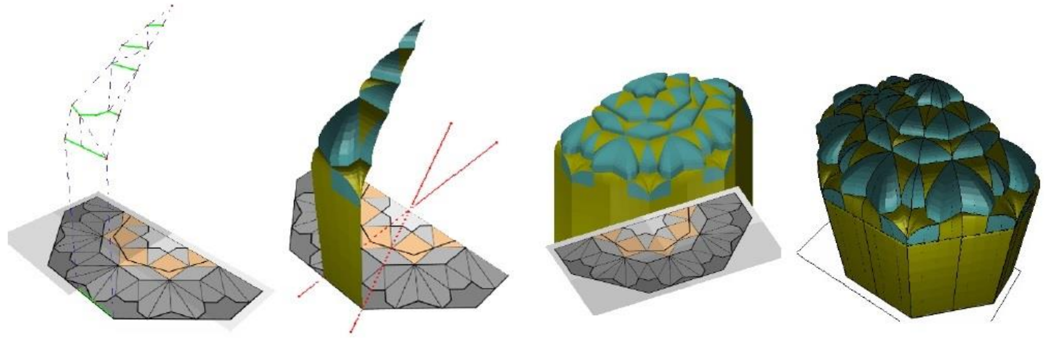


(a) (b) (c)

Figure 71. Muqarnas modeling in Design Computing courses (Web 40)
Modeled by Seda Öznal (a), modeled by Elif Özüçağlıyan (b), modeled by Ekin Arslan (c)

Hamekasi, Samavati and Nasri (2011) explain a modeling method to create new Muqarnas designs beyond the historical forms. In this respect, they define 2DPP as a guiding image and use symmetric or exactly repeated parts of the Muqarnas as a modeler. They specify modeler by drawing lines on the symmetry axes which used as the identity of Muqarnas. Actually, motifs are procedurally repeated and create a Muqarnas. However, motifs consist of layer lines inside them which are useful for making modeling easier. Snapping tools are used to connect layer lines to boundaries and 3D curves as the layer lines. The height of each layer lines was specified with respect to the actual model. There are control points for arranging the layer lines and reshaping the curve using. Control points enable us to place the layers in relatively proper height by approximating the curvature of the vault or dome.

The other step is creating the surface of Muqarnas, in this regard, we need to know the concordance between points on two adjacent layer lines that defines the boundaries for each piece. The concordance is shown in 2DPP with dashed or thinner lines. Layer lines with the connection lines form a Muqarnas which consisted of an arrangement of cells. The cells replaced with patches that incorporate the layer lines and build each layer. The process of Muqarnas modeling between symmetry lines are shown in (Figure 72, a,b). The part that is formed so far is considered as a motif of Muqarnas, by repeating and mirroring this motif as needed, the whole Muqarnas will be obtained (Figure 72, c).



(a) (b) (c)

Figure 72. The process of constructing a Muqarnas vault

Binding the layers (a), a completed motif (b), Figure Completed Muqarnas vault (c)

Source: Hamekasi, Samavati and Nasri. 2011, p.134.

Yaghan (2003) mentioned that for producing Muqarnas forms with the aid of CAD software, they firstly created Muqarnas by using the special software written in Auto LISP[52] for AutoCAD software (Figure 73). Then they transformed the forms into VRML[53] worlds. The final form of each VRML file was re-arranged and edited by designers to achieve smooth and pleasing feature of the forms (Figure 74).

---

[52] AutoLISP is a dialect of the LISP programming language built specifically for use with the full version of AutoCAD and its derivatives, which include AutoCAD Map 3D, AutoCAD Architecture and AutoCAD Mechanical.

[53] VRML (Virtual Reality Modeling Language, pronounced verbal or by its initials, originally—before 1995—known as the Virtual Reality Markup Language) is a standard file format for representing 3-dimensional (3D) interactive vector graphics, designed particularly with the World Wide Web in mind.

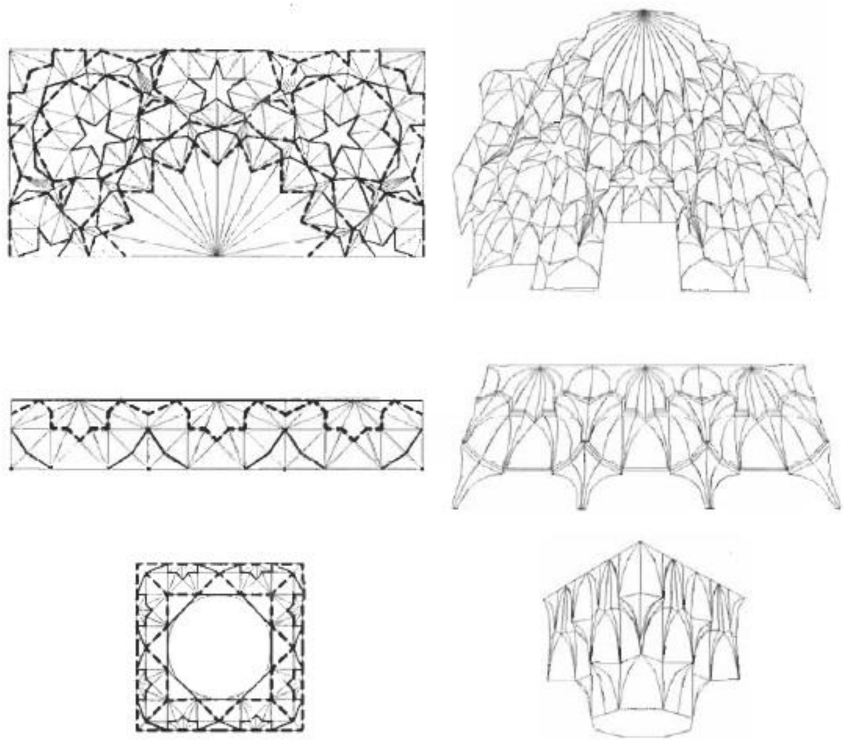


Figure 73. Muqarnas worlds used in the course (from top: portal, cornice, and capital)

Source: Yaghan. 2003, p.33.

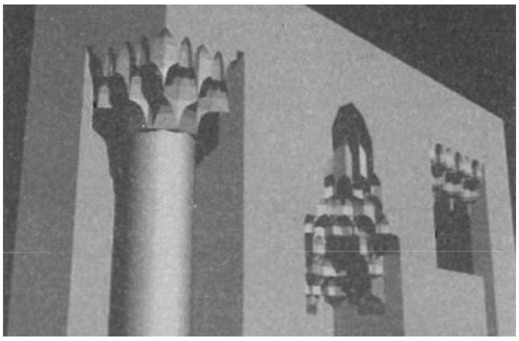


Figure 74. Abstract building containing the three Muqarnas worlds

Source: Yaghan. 2003, p.34.

As Alkandari (2011) presented in his Ph.D. thesis, the study attempted to reduce the complexity of Muqarnas creation by the aid of CAD programs. He started the design process with creating an individual three-dimensional interlacing, Muqarnas element by using Solid Works[54] 3D modeling software. Then by repeating the initial form, he gained multiple repeating units (Figure 75).

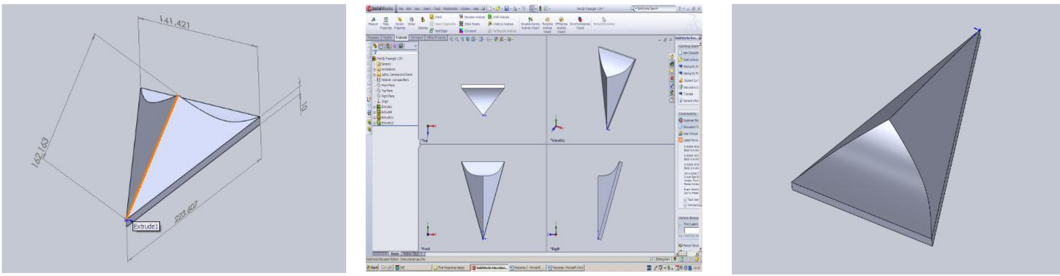


Figure 75. Muqarnas element design process

Source: Alkandari. 2011, p.319.

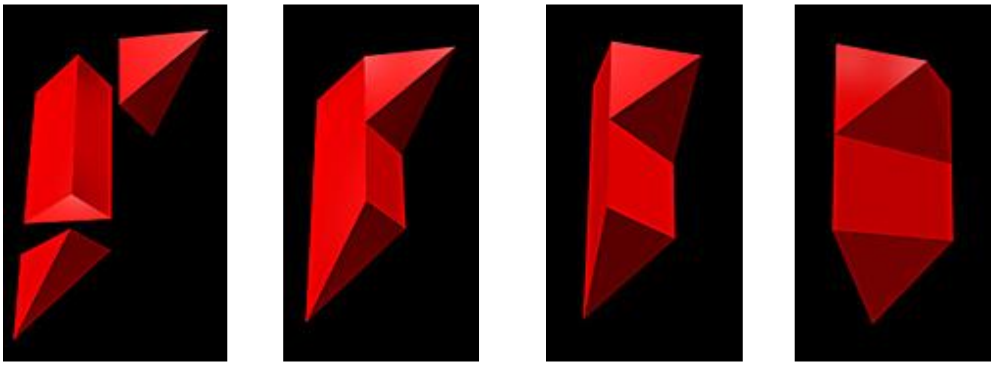
3.3.3 New generation of Muqarnas created through CAD

According to Hamekasi, Samavati and Nasri (2011) Muqarnas were expanded during last decades, with the aid of CAD system. It is possible to automatically generate new Muqarnas models by using various parameters of scale, texture, depth, coloration, and tone with an intellectual distribution of geometry and form. By changing these parameters within the valid range, it is easily extendable to more interesting designs. As Yaghan (2010) says:

> Computers are very useful tools that help producing the actual form for purpose of visualizing and improving design quality. However, the computer-Muqarnas-drafting is still a tedious job.

[54] The SolidWorks (stylized as SOLIDWORKS), is a solid modeling computer-aided design (CAD) and computer-aided engineering (CAE) software program that runs on Microsoft Windows.

Sakkal presented a Muqarnas three-dimensional sample which has produced in Form Writer software that was developed at the University of Washington Architecture School’s computer lab and final results have been retouched in Infini-D[55] software. The initial form of this Muqarnas sample is a triangle that is combined with other triangles to create pyramids and prisms blocks (Figure 76, a). These blocks are combined with each other to create an accurate geometry. The angles of blocks are consecutively increased. As you can see in the figure (Figure 76, b, c, d,) three blocks with 45, 90, and 135 degrees are shown, that they are related to 45 degrees family of Muqarnas blocks.



(a) (b) (c) (d)

Figure 76. Three-dimensional Muqarnas sample which has produced in Form Writer software (Web 41)

With the help of CAD system in this study, we could obtain interior surface, variations by changing parameters that we defined in Form Writer program whiteout changing the initial form. Figure 77 shows the different variation of 45 degrees family blocks with increments of 11degrees 15 seconds of each one.

[55] Infini-D was a 3D computer graphics program, useful for creating print images as well as the web and video media.

Figure 77. Different variation of 45 degrees family blocks (Web 42)

Figure 78 shows a dome-like structure of Muqarnas that are created by 30 degrees family blocks we obtain earlier. As you can see the smaller blocks are fitted at top of the dome. And consecutively the angle of blocks is increased in each layer.

Figure 78. A dome-like structure of Muqarnas (Web 43)

Another parametric Muqarnas design sample was presented by Mete Tüneri with a basic method of modeling. He assumed a simple Muqarnas shape with defining six reference points on it. By using this method and applied it to Rhino[56] and Grasshopper[57] software, designer tried to adding real-time parameters such as a number of rows and row height. With this script and modeling method, creating Muqarnas components with flat surface or curvature are much easier and accurate. Also, this script could easily apply on different initial surfaces without any gaps or overlaps (Figure 79). The system organizes the elements of a surface in the way that connect each other without gaps or overlaps. For creating Concave and convex elements in this project, it uses solid Boolean operations.

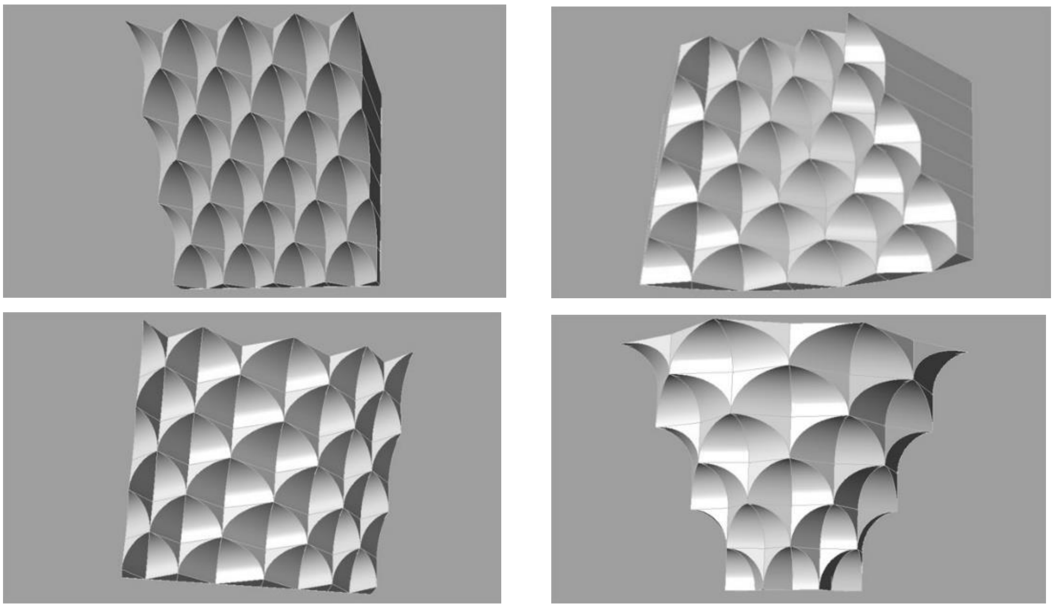


Figure 79. Muqarnas design sample (web 44)

[56] Rhinoceros (typically abbreviated Rhino, or Rhino3D) is a commercial 3D computer graphics and computer-aided design (CAD)application software developed by Robert McNeel & Associates.

[57] Grasshopper is a visual programming language developed by David Rutten at Robert McNeel & Associates, that runs within theRhinoceros 3D computer-aided design (CAD) application.

Owen suggests techniques for designing Muqarnas Forms in his paper. He chose a regular 32-gon and copied each polygon, N-1 times then rotated each one with 360/N degrees around a vertex. With this technique, he reached new generations of traditional Muqarnas dome each time with the following characteristics:

- Once complete this is the plan view of a historical Muqarnas dome.
- Each Muqarnas tile is a rhombus in the plan view.
- All Muqarnas tile edges are the same length in top view and in 3 dimensions, a key property of Muqarnas tiles.
- Each edge is identical, allowing all edges of all Muqarnas tiles to be joined seamlessly to every edge of every other unique Muqarnas tile - including itself.
- There are 16 unique rhombs and Muqarnas tiles for the 32-gon.

He used Domes as the source of Muqarnas geometry and are among their primary original uses, therefore they are a logical place to start exploring more complex forms.

- One method is to make a copy of one of each of the unique Muqarnas tiles
- This group is moved down and out to the periphery of the dome to begin the primary tier.
- It is copied N - 1 times at an angle of 360/N normal to the plan view around the center of the original dome.
- The required Muqarnas tiles to fill the interstices are determined and copied in a similar manner.
- The secondary tier uses a duplication of the Muqarnas tiles used in the primary tier with double the interstitial tiles.
- By repeating this method tertiary and quaternary tiers, etc. may be added.

Figures on next page illustrate the variations of Muqarnas dome compositions that are created by Owen on his paper (Figure 80).

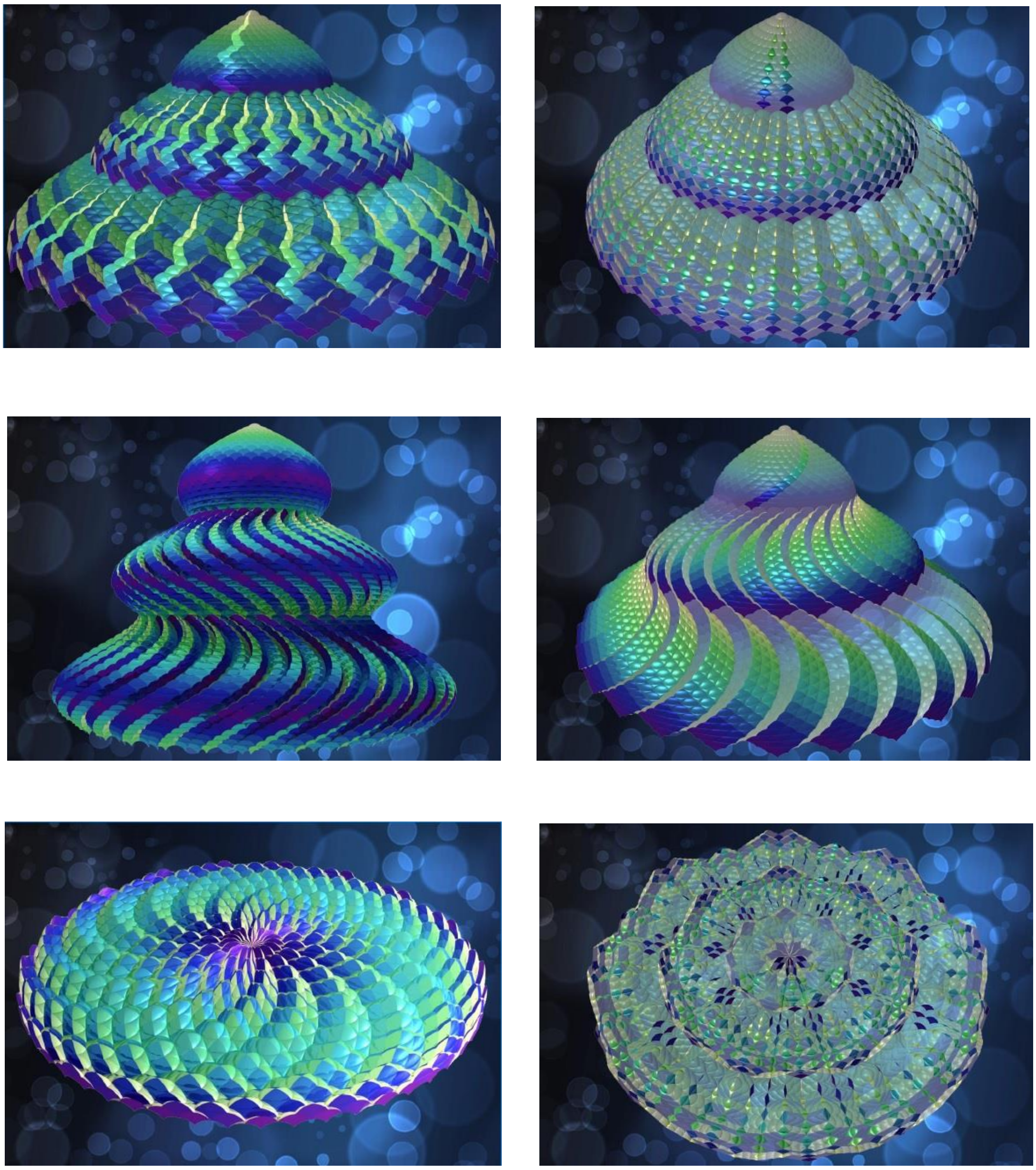


Figure 80. The variations of Muqarnas dome compositions that are created by Owen on his paper (web 45)

This chapter tried to discuss the structure of two-dimensional and three-dimensional Muqarnas compositions and then tried to discuss the generation of Muqarnas patterns through CAD systems which are used today as an innovative method for producing any architectural designing ideas.

The plane projection of two-dimensional historical Muqarnas patterns that we have discussed in Chapter 2 consist of two main geometry groups which are called; elements and intermediate elements to complete the geometric network. The intermediate elements of Muqarnas patterns have specific shapes with limited variety and number and include specific elements. Also when we look at the plane projection of a Muqarnas pattern it is obvious that the elements of pattern can’t consist of only squares and rhomboids but they combined with the split forms of some elements. Like a square can be split into a large biped and a jug, in the same way, a rhombus can be split into a small biped and an almond. These elements and intermediate elements will be the subjects of Chapter 4 in this study. By using two-dimensional plane projection of Muqarnas patterns and their geometrical elements Chapter 4 will try to disclose the geometrical regulations of these patterns.

This chapter tried to define a three-dimensional composition of a Muqarnas and stated that it is like a staircase. The elements are the steps of the stair and have to be arranged to form a Muqarnas structure. The structure of the Muqarnas is built from elements arranged in different levels, each level called tier. The process of three-dimensional Muqarnas is declared that Blocks of Muqarnas have been arranged in many compositions in the way that they can enclose partially or completely space, in doing so, there must be a connection between them in the horizontal direction, and also, a connection between on top of each Muqarnas in the vertical direction. Understanding the structure of Muqarnas as a three-dimensional volume which was mentioned in this chapter could help us to understand the Muqarnas designing process with computer aided systems.

As mentioned in this chapter, new designing methods have developed in architecture that enable architects to make the designing and constructing process much easier and faster by the help of computer. Today almost every designer and architects use CAD systems in their designing process. Besides all of the architectural buildings and decorative forms, Muqarnas designing methods also benefit from CAD technologies.

This chapter tried to discuss the evolution of Muqarnas forms through CAD systems and the new methods of Muqarnas modeling instead of paper-based designing methods that were used in the past. Most of the complex forms which are inspired by Muqarnas compositions will be discussed in Chapter 4. These architectural modern forms are mostly designed with digital technologies. Therefore, this chapter tried to lay emphasis on the significance of computer aided design software to reach last decade's architectural designing developments. And explore the methods that CAD systems helped architects in order to achieve new generation and more complex forms of Muqarnas compositions.

CHAPTER 4

AN OVERVIEW OF THE TRANSFORMATION OF MUQARNAS FROM TRADITIONAL STYLES TO MODERN FORMS

Geometry is the prominent feature of Islamic arts. Designers in the past while had a comprehensive knowledge of geometry they were applying geometrical relations to their designing methods. According to Nejad Ebrahimi, Aliabadi and Gharehbaglou (2014), the dominant guideline of Islamic architect's innovation and complex compositions was using various geometric forms in their decorative designs. In fact, all of the complex geometrical patterns are built by using simple and basic motifs which are repeated according to a specific guideline determined by a designer. All of the motifs are repeated and developed by using interlace together in the same way without any gaps or overlaps in order to create various symmetrical patterns.

In this chapter, analytical structures will be applied step by step on two groups of Muqarnas patterns as a historical group and a modern one which they were built in different eras. The historical group is selected among Muqarnas examples that were built between the 10th _16th centuries in different countries presented by Takahashi, and the modern group is a group of modern buildings or art forms from the 21st century. The modern group is selected between the buildings, graphical art forms and decorative forms that are perceived to convey Muqarnas patterns characteristics. The modern forms also have been selected from different countries with different functions.

The analytical studies of this chapter will be conducted to compare two historical and modern groups in terms of pattern differences in order to identify their relationship.

The analysis of the Muqarnas patterns may vary from one pattern to another. This analysis process depends on the author's cognition of each pattern and will be presented in this section as two following analytical studies;

1. Motif-based analysis of a Muqarnas pattern
2. Layer-based analysis of a Muqarnas pattern

This study tries to disclose the geometrical characteristics of historical Muqarnas patterns by using two analytical ways that are mentioned above. These analytical studies try to decompose two-dimensional Muqarnas patterns in the ways that it is estimated to historical and modern Muqarnas examples are constructed.

The first analytical way that is focused on the motif-based analysis of a Muqarnas pattern is aimed to decompose each Muqarnas pattern to its split forms step by step. The number of steps followed in each pattern will differ from one example to another. The second way which is focused on the layer-based analysis of a Muqarnas pattern tries to decompose each pattern in terms of its layer compositions also, in this case, the number of each step depends on the complexity of each composition. we will discuss comprehensively these ways in next section.

The designing methods of the two-dimensional and three-dimensional historical Muqarnas patterns are unknown. According to the historical background of Muqarnas that is mentioned in Chapter 2, the constructing and designing methods of historical Muqarnas examples date back to the 9th century and onward, therefore, there are not definite and available documents about their designing methods. Researchers always try to estimate their designing ways and methods from the pictures or the examples of the Muqarnas that are survived. Therefore, This thesis doesn't claim that the strategies that are used in this chapter are the definite and certain ways that historical patterns were designed before. This study also like the other researches tries to estimate the two possible ways that historical Muqarnas patterns might be designed. Two analytical strategies which are proposed in this chapter, try to analyze two-dimensional Muqarnas patterns with mathematical and geometrical relations between them.

According to the results derived from the descriptive analysis of these two strategies, this study tries to discover the geometrical characteristics of historical Muqarnas patterns which are called Muqarnas pattern guidelines in this chapter. Muqarnas pattern guidelines might be the possible way that can be used to display the geometrical characteristics which are specific to historical Muqarnas patterns. Finally, this chapter by using these guidelines tries to identify the similarity and differences between historical Muqarnas patterns with modern forms which are intentionally or unintentionally inspired by Muqarnas patterns in their plan, facade or in their three-dimensional compositions.

The modern buildings or art forms that are selected in this chapter, are belong to the 21st century and seem to be the reflection of historical Muqarnas patterns in terms of their geometrical characteristics. The modern forms are selected among those geometrically reminds the author of a Muqarnas pattern. They are first selected according to the mathematical and general Muqarnas geometrical characteristics which are known as types of isometry.

4.1 Interpretation of historical Muqarnas compositions

This chapter tries to disclose the regulation of Muqarnas patterns which was essential and permanent for creating a Muqarnas composition in traditional architecture. As mentioned in chapter 3, all of the Muqarnas patterns were constructed from single or combined motifs repeated in every Muqarnas compositions in various ways of interlacing. This thesis tries to reveal these ways and called them the guideline of Muqarnas patterns.

Alongside with all of the geometrical characteristics which are belong to specific Muqarnas example, Muqarnas also conveys some general and mathematical characteristics which are disclosed by researchers in this field. Muqarnas patterns are related to mathematics and mathematical numbers. In some cases, patterns directly follow mathematical series and numbers. To comprehend this accommodation, it is better to take a look at this mathematical regulations and isometry types which are proved previously as characteristics of Muqarnas patterns. The Golden Ratio and the Fibonaccci Series[58] are some of these mathematical characteristics that Muqarnas patterns convey them in their two-dimensional patterns.

Hejazi and Mehdizadeh (2014) declared that geometrical analysis of many Persian traditional buildings shows that the Golden Ratio, was widely used in Persian architecture and it was the basis of Persian aesthetics. The numerical quantity of Golden ratio can be expressed as $\Phi = \frac{1+\sqrt{5}}{2}$ or, in the ratios of 5:3, 8:5, and a square root of two proportions (Suhaimi, Noraini and Khadizah, 2014) it is also, obvious in Muqarnas geometries proportion as it illustrates in Figure 81.

[58] According to Dabbour (2012), the problem in Leonardo Fibonacci's book that leads him to study about sequence and emersion of Fibonacci Series was:

"How many pairs of rabbits will be produced in a year, beginning with a single pair, if in every month each pair bears a new pair which becomes productive from the second month on?".
The Fibonacci Sequence is the series of numbers: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34,... The next number is found by adding up the two numbers before it. In mathematical terms, the sequence f(n) of Fibonacci numbers is defined:

f(n) = f (n - 1) + f (n - 2) using f (0) = 1 and f (1) = 1

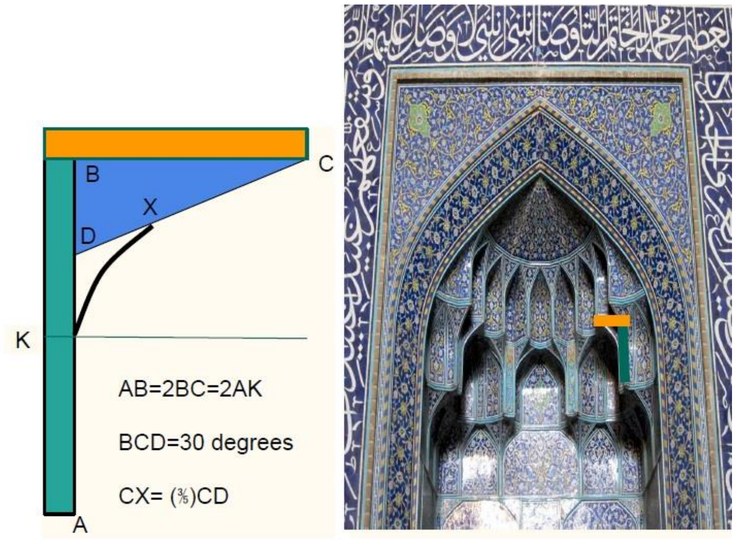


Figure 81. Sheikh Loft Allah Mosque, Isfahan, Iran

Source: By Richard, Hema, and Wayne

As it is obvious from the figure above, the Golden Ratio was used in Muqarnas patterns as one of the bases of Muqarnas aesthetics. The numerical quantity of golden ratio is seen in the relation of Muqarnas elements.

4.1.1 Types of Isometry

The most important and impressive mathematical characteristic of Muqarnas pattern is the symmetric property of its 2DPP. The figure 82, shows the geometric process that produced final Muqarnas form of an eight-fold star in the Alhambra Palace. As shown in Figure 83, the plane projection of Alhambra’s Muqarnas has a symmetrical pattern. The final pattern is created by rotating, reflecting and repeating basic elements. These geometrical relations are called types of isometry.

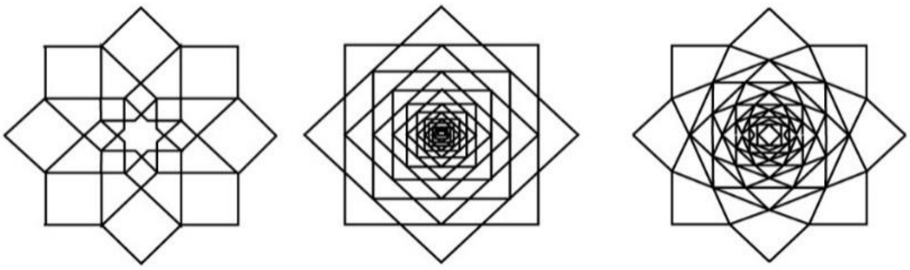


Figure 82. Construction stage of eight pointed patterns based on √2 proportions

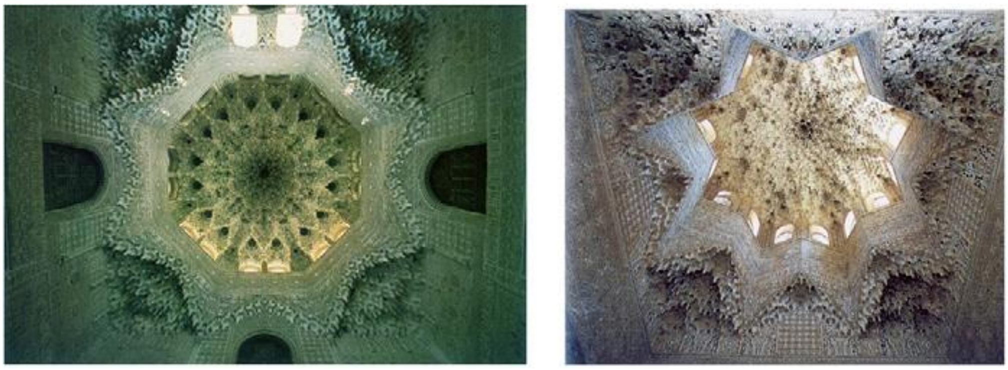


Figure 83. Octagon based on eight pointed patterns in Alhambra

Almost every Muqarnas patterns have the types of isometry. Actually isometric and symmetric characteristics of Muqarnas make it more complex and appealing. According to Morandi (2003), translations, reflections, rotation, glide reflection are four types of isometry.

Translation

Translation is a form of isometry that is a shift in some direction. In this case, both original object and its translation are same in the size, shape and direction. In translation type, the original figure is congruent with the translation form and keep distance and orientation. We recall the notation $T_{\mathbf{v}}$ for translation by V. The inverse of a translation is $T\mathbf{v}^{-1} = \mathrm{T} - \mathrm{V}$ (Figure). If you do a translation, then do another translation, the result is again a translation (Figure 84).

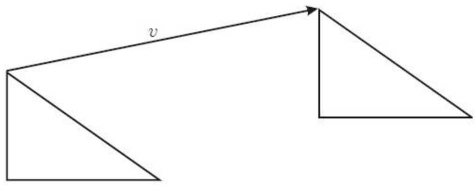


Figure 84. Translation

Reflection

Let's consider a line in $R2$ through the origin parallel to a vector w, the reflection of an image can be considered as a mirror of that image through the considered line. This line can be viewed as the line on which the mirror will locate in order to see the reflection of an image. The result of explanation shows that f is an isometry and it is a linear transformation. The easy way to find an object reflection through a line is a folded graph paper along the line reflection and locate the object (Figure 85).

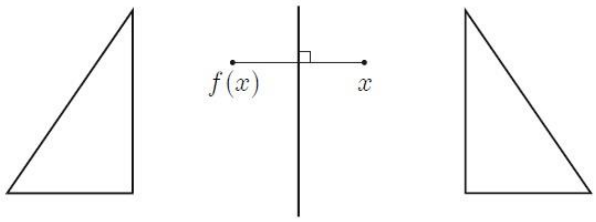


Figure 85. Reflection

Rotation

Rotation type of isometry has two main rules; the first rule is a point which is called the center of rotation. Except for this point, every other point of the image will be moved. And the other rule is an angle. If θ is an angle, then the rotation r by an angle θ will be about the origin. It is obvious that the origin is an isometry and again it is a linear transformation that can be used to describe a rotation about any point (Figure 86).

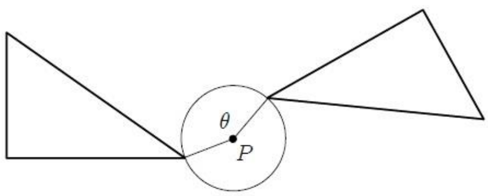


Figure 86. Rotation

Glide Reflection

There is an other type of isometry that comes from combining two types of isometries. By combining reflection and translation types with each other, an-other type of isometry will be achieved; this type of isometry is called glide reflection. On the other hand, if you apply a reflection followed by translation, a glide reflection will be achieved (Figure 87).

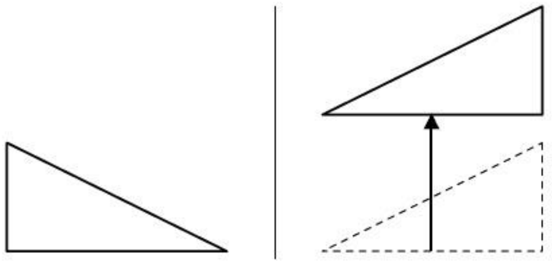


Figure 87. Glide Reflection

In order to discover the guidelines of Muqarnas patterns, it is needed to have a set of historical and traditional Muqarnas patterns to set a design analysis structure and contextualize the concept of them. In this regard, the present section offers an analytical structure to analyze the plane projection of historical Muqarnas compositions which are mentioned in chapter 2 under the title of TWO-DIMENSIONAL MUQARNAS STRUCTURE.

The analytical structures that are presented in this chapter, try to define a designing guideline which is same or at least similar in every single historical Muqarnas compositions. Therefore, this guideline could be determined as a characteristic of Muqarnas. Historical and traditional set of Muqarnas examples will be selected between historic Muqarnas compositions. First of all, it is better to take a look at Muqarnas compositions in traditional architecture.

4.1.2 The types of Muqarnas in historic and traditional architecture

Geometric structures of Muqarnas have not any single pattern or style, but a large variety of designs can be observed, and this can be one of the most interesting aspects of them. One can easily recognize the different geometric forms in the Muqarnas and also can easily identify the way in which they are connected to each other in order to develop a unique geometric design (Akram, Ismail and Franco, 2016). There are two types of Muqarnas composition presented by Al-Kāshī and Shiro Takahashi.

Classification of Al-Kāshī

Jamshid Ghiyath al-Din Al-Kāshī has presented the first mathematical approach to Muqarnas. He was a famous mathematician and astronomer born around 1380 in Kashan, Iran. He is known for his book, *Key of Arithmetic*, which was written in 1427.

Al Hassani (2007) and Amoruso (2016) refer to Al-Kāshī and state that the surface of a Muqarnas has been approximated and the earliest definition of Muqarnas has been presented in this study. Al-Kāshī has identified four types of Muqarnas:

1) Simple Muqarnas (made just from flat surfaces)
2) clay-plastered or Mutayyan Muqarnas (the levels of which are not all the same hight)
3) Curved Muqarnas (characterized by curved surfaces and the ipografia entirely made from triangles and wishbones)
4) Shirazi Muqarnas (made from other polygons, as pentagons, hexagons, octagons and finally multipointed stars)

The first two types only consist of plane surfaces. They both are similar but their tiers have not the same height. Remaining two types also exhibit curved surfaces. The Shirazi Muqarnas is similar to the curved one, but it has many different elements. The top views of all simple, clay plastered and curved Muqarnas, consist of triangles and quadrilaterals, but Shirazi Muqarnas also has other polygons such as pentagons, hexagons, octagons and multi-pointed stars.

Classification of Shiro Takahashi

Another distinct type of Muqarnas has been discovered by Shiro Takahashi (1943) and presented in his online website (http://www.shiro1000.jp/). Takahashi has classified Muqarnas into three different types:

1) Square lattice Muqarnas
2) Pole table Muqarnas
3) Other style of Muqarnas

The most important characteristics of Type 1 (Square lattice Muqarnas) are as follows: it is filled with squares and 45-degree rhombuses, their origin dates back to the 11th century, and they have been spread throughout the entire Islamic world over the thousand years. This type of Muqarnas has four-fold rotational symmetry and is used at domes, iwans and vaults. A beautiful example of this Muqarnas can be found in stalactite decoration of the Alhambra Palace in Granada (14th century). Type 2 (Pole table Muqarnas) is more common in Anatolia, Iran and Central Asia and is used to separate the plan of Muqarnas into segments with the radial cluster of stars and polygons. The central motif of the pole table Type is expanded from 4, 5 and 6 segments to 7 and 11 and the star-shaped motifs also include the 7- and 9-pointed stars. Type 3 (Other style of Muqarnas) which does not belong to the first two types is mostly used in Spain, Morocco and Iran (Figure 88).

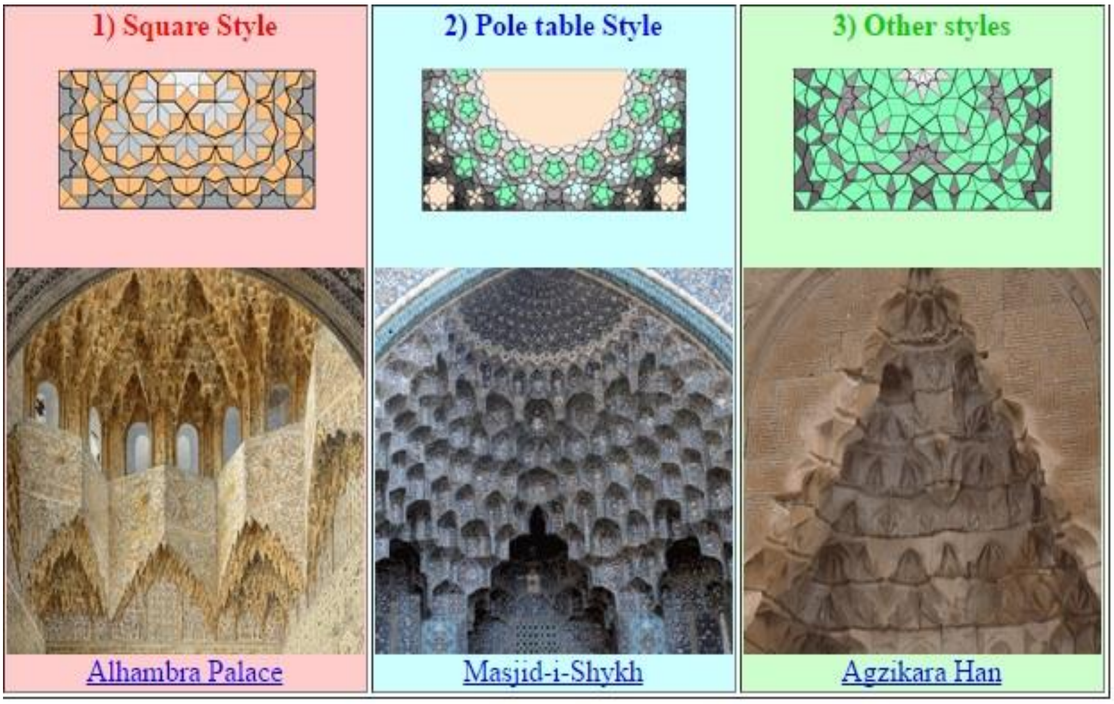


Figure 88. Muqarnas classification according to Takahashi (Web 46)

4.2 Analytical studies of historical Muqarnas patterns

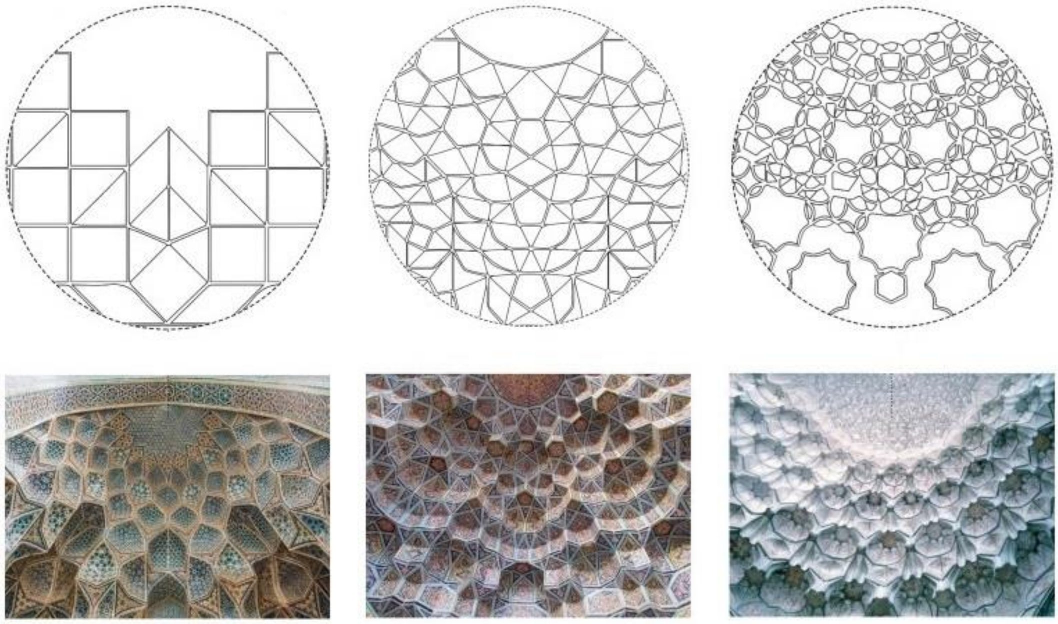
In this chapter, firstly, analytical studies are proposed to illustrate the guidelines of historical and traditional Muqarnas patterns and then these guidelines will be used to analyze the modern forms in the same way with historical examples. In this regard, this section will focus on Shiro Takahashi's classification among the types of Muqarnas in historic and traditional architecture and the six historical examples are selected to be analyzed in this chapter among his three types which are classified as; Square lattice Muqarnas, Pole table Muqarnas and Other style of Muqarnas.

Shiro Takahashi's classifications are diverse and they include around 2000 examples of historical Muqarnas compositions all around the world, without Geographic Separation. Also, all kinds of Muqarnas types in terms of their functions in buildings will be diversely accessible in his classification; such as Muqarnas on niches, domes, minaret and cornice which are mentioned in chapter 2 under the title of PLACEMENT OF MUQARNAS IN HISTORICAL BUILDINGS.

Therefore, why this study focuses especially on Shiro Takahashi's classifications among the other classifications, is its availability of about 2000 examples of historical Muqarnas patterns with diversity of their functions in the buildings. All of the Muqarnas examples of the most historical ones classified by Takahashi are easy to access in www. Shiro 1000.jp.

As mentioned before, Shiro Takahashi classified Muqarnas patterns in three types according to their pattern and geometrical characteristics that are shown in Figure 89;

Square lattice Muqarnas **Pole table Muqarnas** **Other style of Muqarnas**



Shaykh Abd al-Samad Mosque Natanz, Iran

Nasir al-Mulk Mosque Shiraz, Iran

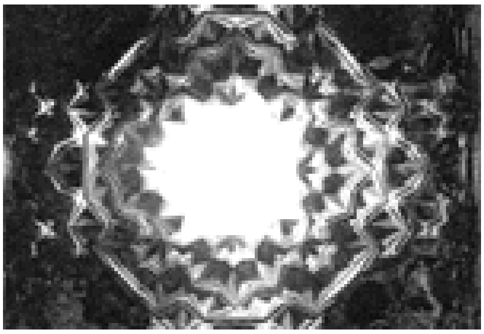
Abd al-Aziz Khan Madrasa Bukhara, Uzbekistan

Figure 89. Three types of Muqarnas patterns (Web 47)

Of the three types he classified, two Muqarnas examples of each type have been chosen to be decomposed by two analytical ways; i.e. their constructional forms and their constructional layers, to achieve their compositional rules. In order to attain a diverse set of selections, Muqarnas compositions made in different countries and different eras will be included in this group. The era is limited with the Muqarnas compositions between 10^{th} _16^{th} centuries because this era contains various types of Muqarnas compositions in terms of their functions and patterns designs. The six historical Muqarnas examples which will be analyzed in this chapter are; Square lattice Muqarnas, Pole table Muqarnas and Other style of Muqarnas as shown in the figures on next page.

Square lattice Muqarnas;

(a)



(b)

Pole table Muqarnas;

(c)

(d)

Other style of Muqarnas;

(e)

(f)

Figure 90. The six historical Muqarnas examples

Tinmal Mosque, Morocco (a), II. Beyazid Mosque, Turkey (b), Murat pasha Mosque, Turkey (c), Zisa Castle, Italy (d), Selimiye Mosque, Turkey (e), Sahabiye Madrasse, Turkey (f).

As mentioned before, the Muqarnas patterns are made from the repetition of motifs, and they are rotationally symmetric around their centers. According to Dadkhah, Safaeipour and Memarian (2012), there are two types of rotation axes in the formation of Muqarnas, radial axis and orbital axis. To identify the location of the axis, a half-circle is centered in the middle of the pattern with the desired radius, and the radial axis is designed by dividing the half-circle into equal parts.

This chapter will present the following strategies to analyze Muqarnas patterns:

1. Motif-based analysis of a Muqarnas pattern
2. Layer-based analysis of a Muqarnas pattern

These two analytical strategies are about Muqarnas 2DPP analysis based on decomposition and try to disclose the regulations of the arrangement of these motifs. The motif-based analysis of a Muqarnas pattern tries to decompose each Muqarnas pattern to its split forms within steps, the number of which can differ from one example to another. The layer-based analysis of a Muqarnas pattern also tries to decompose each pattern to its layer compositions; in this case, the number of each step depends on the complexity of each composition. According to Hamekasi, Samavati and Nasri (2011), the layered structure of Muqarnas is the other important characteristics. The layer lines are determined inside the motifs in its 2DPP map. We can achieve each model's layers by connecting layer lines of each tier of Muqarnas to their boundaries.

However this study has its own analytical ways, but these two analytical strategies are inspired by shape grammar[59] as an analytical tool for analyzing patterns in this study.

---

[59] According to Stiny(1977), The language of two-dimensional shapes defined the characteristics of the painting which is defined by the double (S, M). "S is a specification of a class of shapes and consists of a shape grammar, defining a language of two-dimensional shapes, and a selection rule. M is a specification of material representations for the shapes defined by S and consists of a finite list of painting rules and a canvas shape".

However the analytical ways which are used in this study belong to the author but, they can be considered as strategies which are similar to shape grammar as an analytical tool[60]. In this analytical study, the initial form is called the most basic part of the pattern and the rules are defined as steps of analyze.

In order to explain the pattern analysis based on Motif-based construction of a Muqarnas pattern, the analytical study of the first pattern will be more wide and comprehensive than others. Each step of the first pattern will be clarified by explaining the author's analytical recognitions. Remaining patterns will follow the same analyzing steps.

---

[60] In 1981 Knight suggest a new method for creating new shape grammar by combining existing rules and grammar language based on the existing ones. For achieving a new design style and new grammar language she transformed known existing rules to determine the evolution of the styles to develop new designs, "Therefore, this approach to shape grammar is both analytical and synthetic".

Gehry is one of the architects who applied shape grammar algorithm on his works.by these algorithms he rationalizes the surface forms to recognize requirements of specific construction. Experience music project in Seattle was the first project of Gehry that manipulate surface fabrication strategy by shape grammar language. In the Gehry's Experience music project as respects to the basic rule that was subdivided a square into four smaller squares, firstly subdivision surface algorithm has been recursively re-applied to surface curvature to decompose that for getting smaller, flat square in various sizes. Initial grammar based algorithm that applied on the surface can't produce more subdivisions than necessary in a relatively predictable pattern of the equal sized region. These rules applied in algorithm format for getting efficient results during the design process. Grammar algorithm plays a crucial role to searching the optimal form of subdivision surface instead of splitting the region in the middle simply.

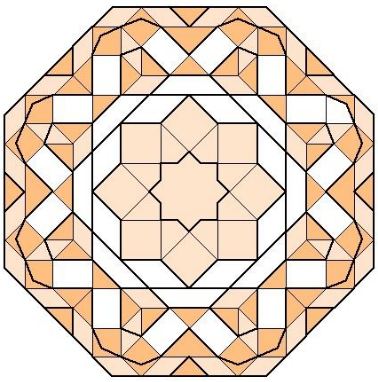
Motif-based analysis of a Muqarnas pattern

Pattern 1 (Square lattice Muqarnas)

Figure 91. Tinmal Mosque

Located in Al Haouz, Morocco, 1153

The analysis of this pattern is evaluated as an octagonal tiled composition (Figure 92). In this section, the pattern is decomposed into its basic parts step by step. In the first step, as it is obvious from the Figure 93, the pattern can be divided into eight sectors that the whole pattern is created by repeating and rotating each of these sectors that are called basic module in this study.



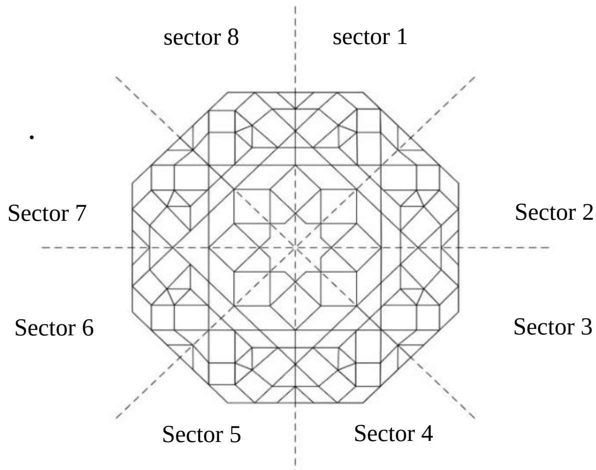


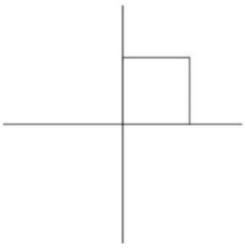
Figure 92. The design of the Tinmal Mosque

Figure 93. Decomposition of the pattern

The pattern is redrawn by author

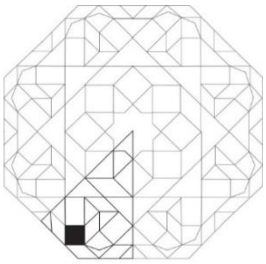
The process of basic elements placement in Tinmal Mosque pattern

As mentioned before, all of the motifs are repeated and developed by being interlaced together, without any gaps or overlaps to create symmetric patterns. As it is obvious from the plane projection of Tinmal Mosque's Muqarnas type, the projection of this pattern is rotationally symmetric around its center. When each basic module of this pattern is decomposed, a square appears as the simplest and the most basic element of Tinmal mosque's Muqarnas pattern (a). The placement of the basic element is shown on both basic module Figure (b) and whole pattern Figure (c). After rotating the square by 45 degrees, Figure (d) will be achieved. According to achieved information about pattern 1, two squares are the simplest parts of this pattern. Figure (e) shows the location of these parts in basic module and Figure (f) shows the whole pattern with the simplest parts.



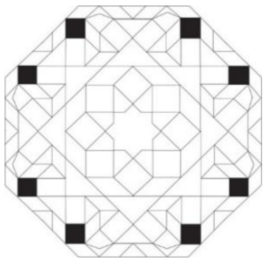
(a)

The most basic element



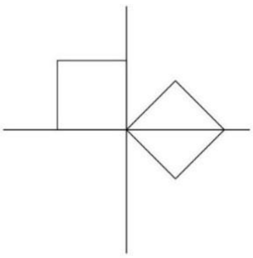
(b)

Basic element in basic module



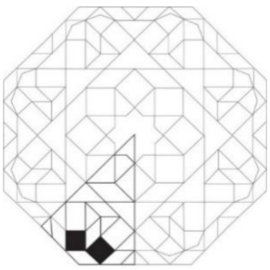
(c)

Basic element in pattern



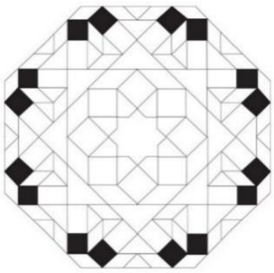
(d)

Basic elements



(e)

Basic elements in basic module



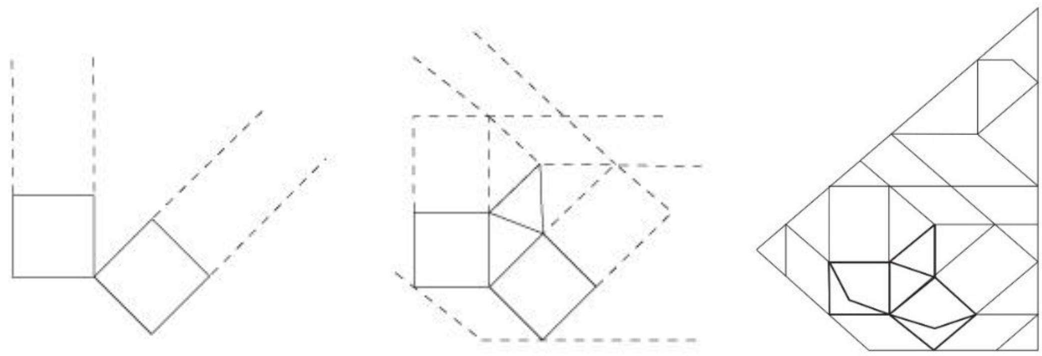
(f)

Basic elements in pattern

Figure 94. Process of locating basic elements in both basic module and whole pattern

The creation process of a basic module of Tinmal Mosque pattern

For generating a basic module of Tinmal Mosque pattern, it seems that the fundamental elements of this pattern are two squares that the whole pattern is not accessible without drawing them initially. These two squares are shown in Figure (g). Other elements are reached by extending the vertical sides of the basic elements shown on Figure (h). By connecting the lines, the basic module will be created as shown in Figure (i).



(g) Vertical sides of basic elements have been extended

(h) The lines are connected and extended

(i) Basic elements are shown in basic module

Figure 95. Process of accessing basic module

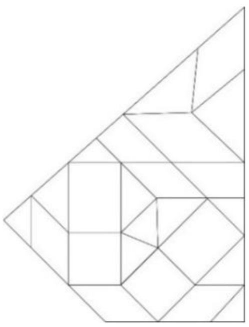


Figure 96. Basic module of this pattern

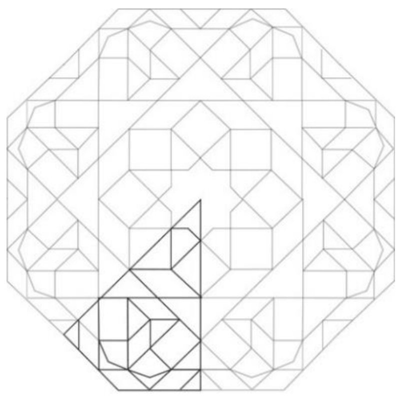


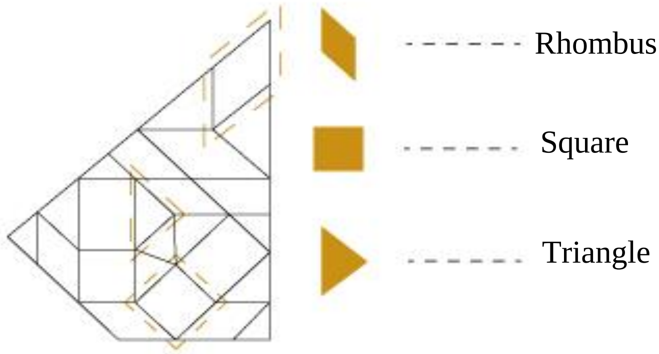
Figure 97. Basic module has shown in whole pattern

Patterns are redrawn by author

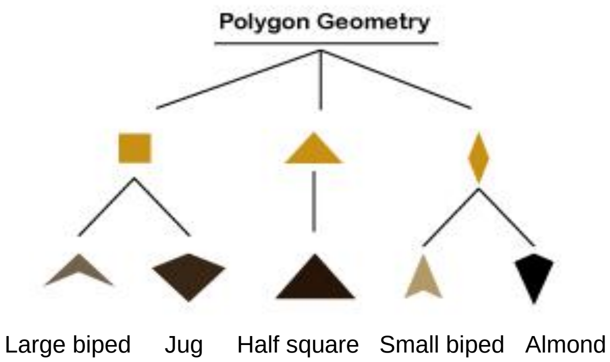
Geometric design process of a Muqarnas form construction

The basic module itself consists of split forms which are mentioned in chapter 3. The split forms of Tinmal mosque pattern are shown in below.

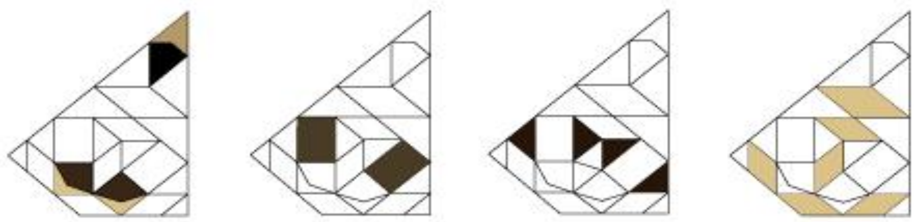
1 Decomposition of the basic module into it's initial forms



2 Initial forms also can be divided into split forms



3 Positioning of split forms in the basic module



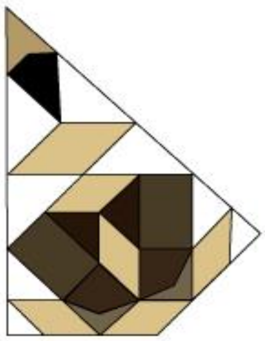
(a) (b) (c) (d)

Patterns are redrawn by author

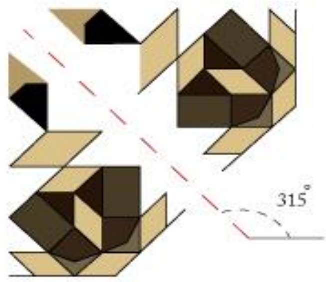
Part to whole relation of Muqarnas

The creation of a Muqarnas pattern by using isometry types that is created from a single basic module.

4

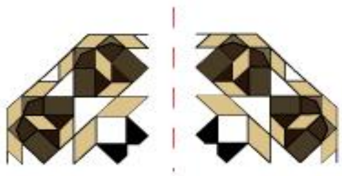


(a). Basic Module

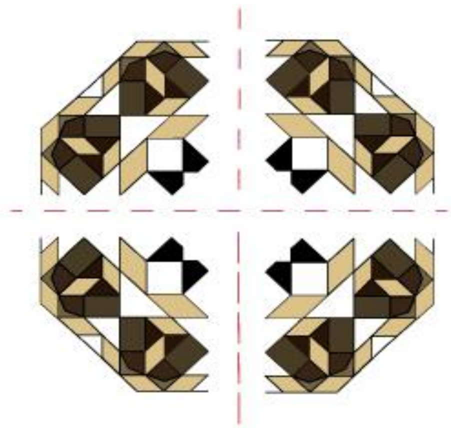


(b). Mirror 315°

5

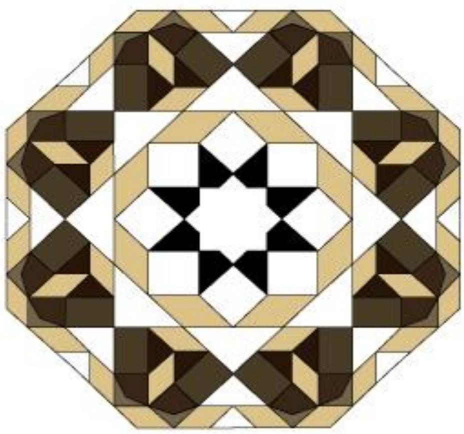


(c). Mirror 90°



(d). Mirror 180°

6

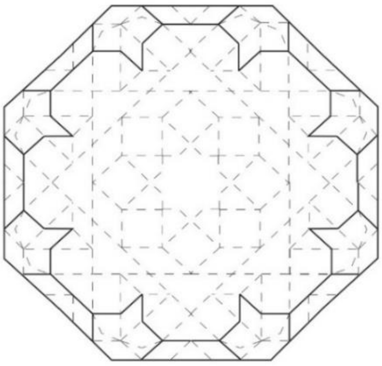


The rotational symmetry of the Tinmal Mosque's Muqarnas pattern

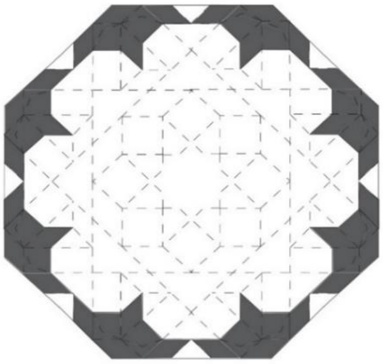
Patterns are redrawn by author

Layer-based analysis of a Muqarnas pattern

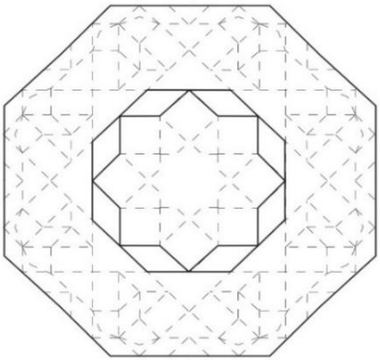
Pattern of Tinmal Mosque is analyzed based on the constructional layers that is estimated to may be the other designing way of historical Muqarnas patterns.



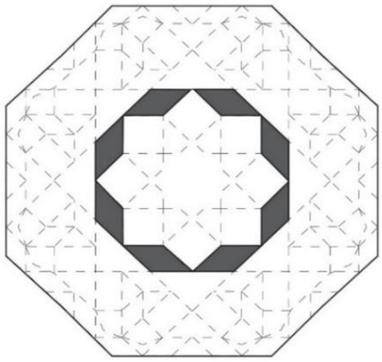
First layer



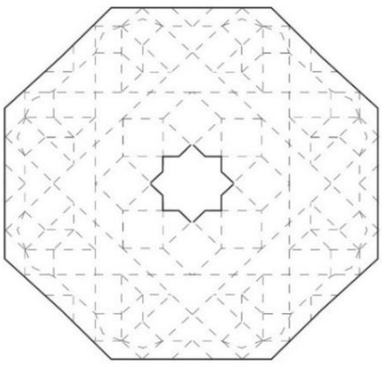
Geometry of first layer



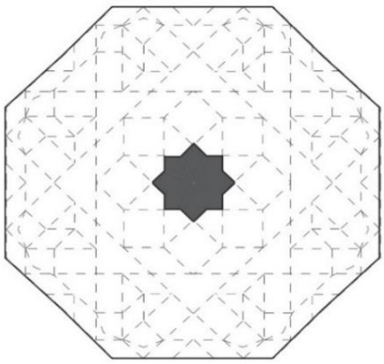
Second layer



Geometry of second layer



Third layer



Geometry of third layer

Patterns are redrawn by author

Motif-based analysis of a Muqarnas pattern

Pattern 2 (Square lattice Muqarnas)

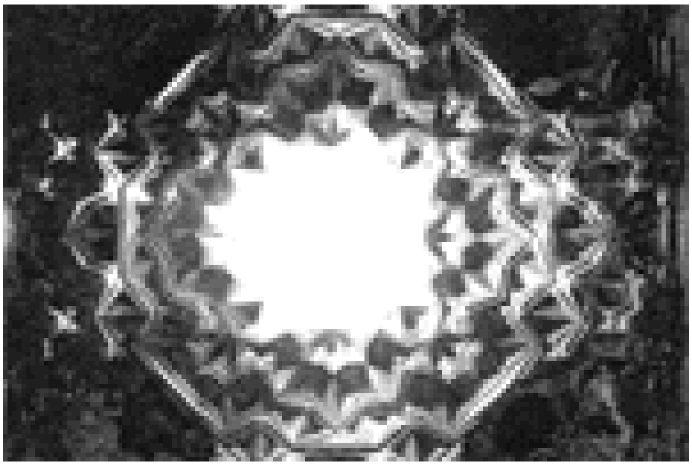
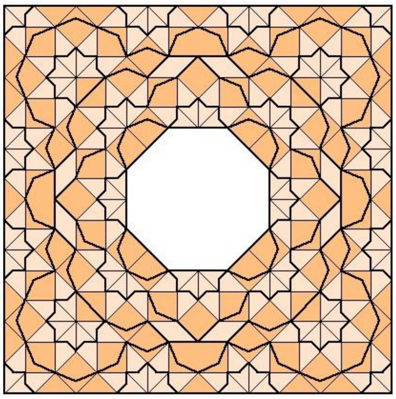


Figure 98. II.Beyazid Mosque

Located in Edirne, Turkey, 1488

The analysis of this pattern is evaluated as a square tiled composition (Figure 99). The pattern is tried to be decomposed to its basic parts step by step. In the first step, as it is obvious from the Figure 100, the pattern can be divided into eight sectors that the whole pattern is created by repeating and rotating each of these sectors that are called basic module in this study.



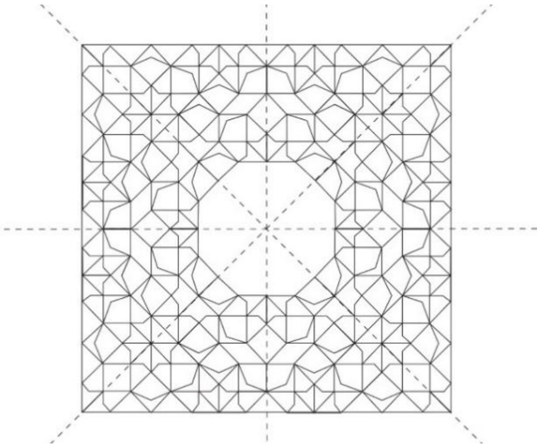
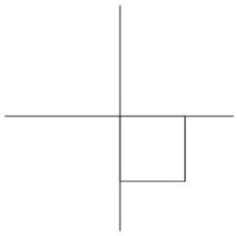


Figure 99. The design of the II. Beyazid Mosque   Figure 100. Decomposition of the pattern

The pattern is redrawn by author

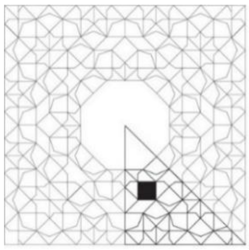
The process of basic elements placement in II. Beyazid Mosque pattern

As it is obvious from the plane projection of II. Beyazid Mosque’s Muqarnas type, the projection of this pattern is rotationally symmetric around its center. When each basic module of this pattern is decomposed, a square appears as the simplest and the most basic element of II. Beyazid Mosque’s Muqarnas pattern (a). The placement of the basic element is shown on both basic module Figure (b) and whole pattern Figure (c). After rotating the square by 45 degrees, Figure (d) will be achieved. According to achieved information about pattern 2, two squares are the simplest parts of this pattern like the first pattern. Figure (e) shows the location of these parts in basic module and Figure (f) shows the whole pattern with the simplest parts.



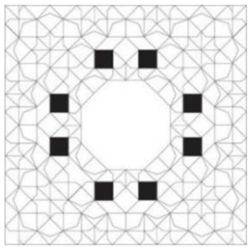
(a)

The most basic element



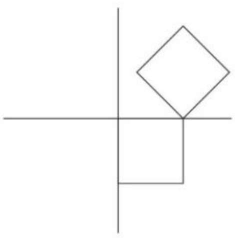
(b)

Basic element in basic module



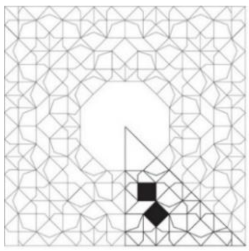
(c)

Basic element in pattern



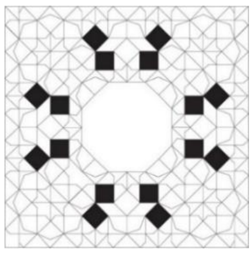
(d)

Basic elements



(e)

Basic elements in basic module



(f)

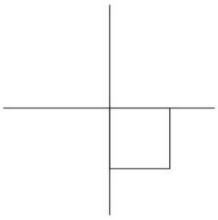
Basic elements in pattern

Figure 101. Process of locating basic elements in both basic module and whole pattern

Patterns are redrawn by author

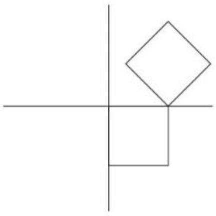
The creation process of a basic module of II. Beyazid Mosque pattern

For generating a basic module of II. Beyazid Mosque pattern, it seems that the fundamental elements of this pattern are two squares that the whole pattern is not accessible without drawing them initially. These two squares are shown in Figure (b). By rotating and mirroring two squares as Figure(c) the Other elements are reached. Finally, by extending the vertical and horizontal sides of the basic elements shown in Figure (d) and connecting the lines, the basic module will be created.



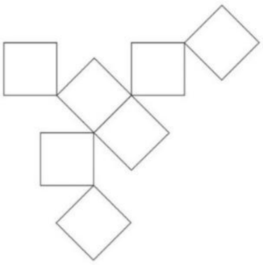
(a)

Basic element is a square



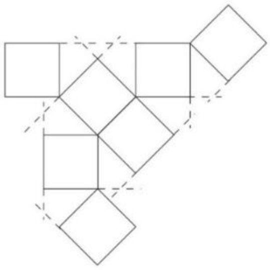
(b)

Rotate 45°



(c)

Rotate -45° and mirror 45°



(d)

Extend the lines of created forms

Figure 102. Process of accessing basic module

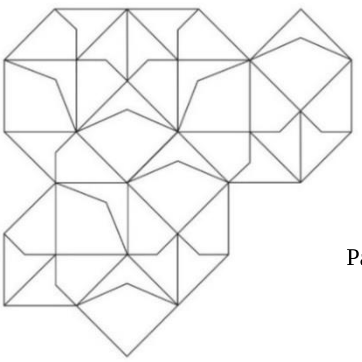


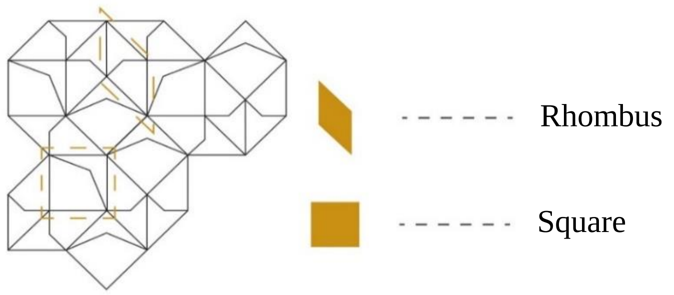
Figure 103. Basic module of this pattern

Patterns are redrawn by author

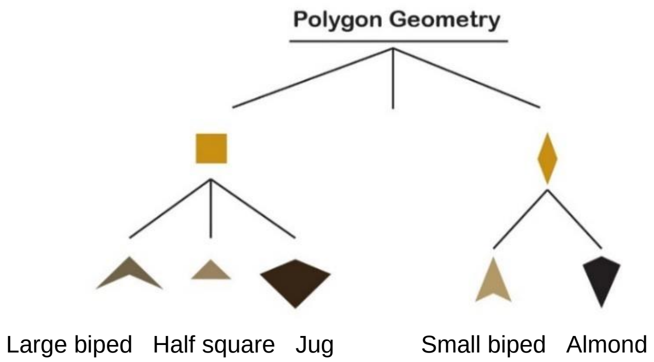
Geometric design process of a Muqarnas form construction

The basic module itself consists of split forms which are mentioned in chapter 3. The split forms of II. Beyazid Mosque pattern are shown below.

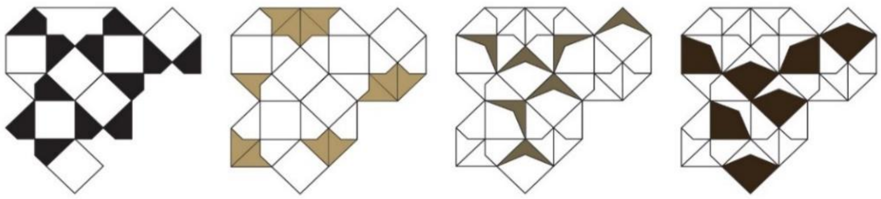
1 Decomposition of the basic module into it’s initial forms



2 Initial forms also can be divided into split forms



3 Positioning of initial elements in the basic module



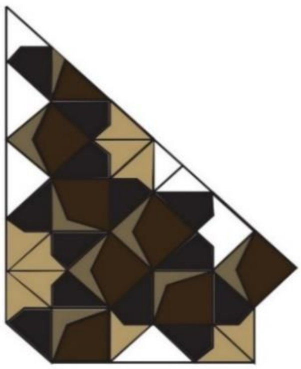
(a) (b) (c) (d)

Patterns are redrawn by autho

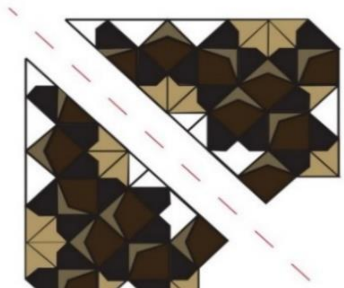
Part to whole relation of Muqarnas

The creation of a Muqarnas pattern by using isometry types that is created from a single basic module.

4

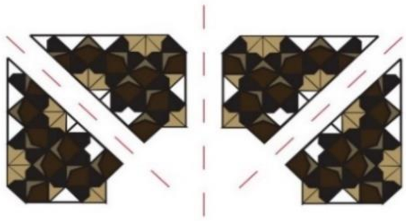


(a). Basic Module

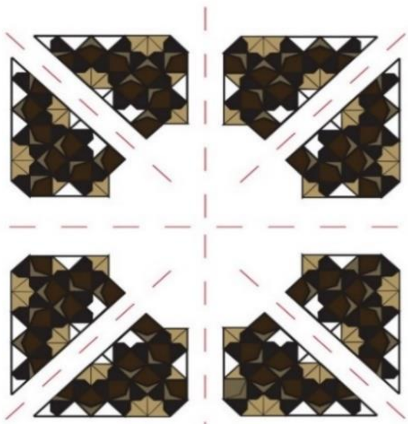


(b). Mirror 315°

5

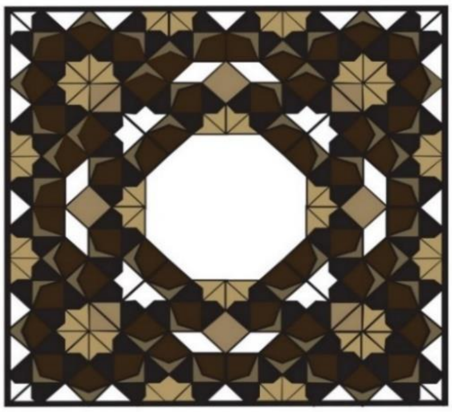


(c). Mirror 90°



(d). Mirror 180°

6

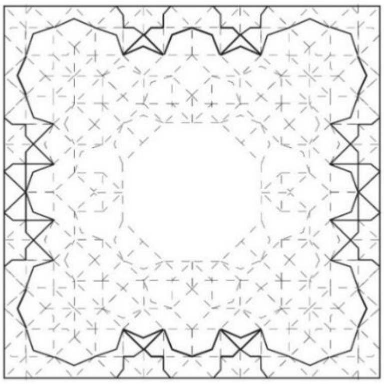


The rotational symmetry of the II.Beyazid Mosque’s Muqarnas pattern

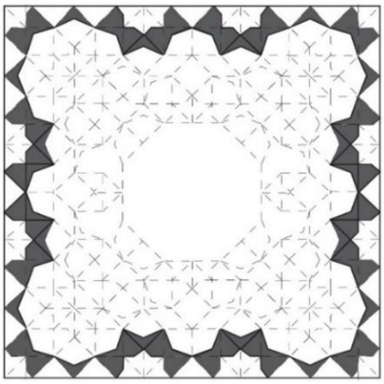
Patterns are redrawn by author

Layer-based analysis of a Muqarnas pattern

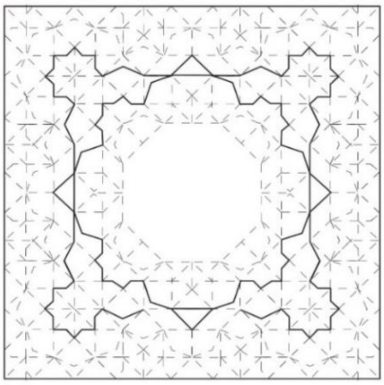
Pattern of II. Beyazid Mosque is analyzed based on the constructional layers that is estimated to may be the other designing way of historical Muqarnas patterns.



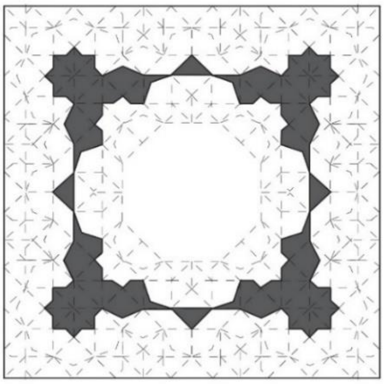
First layer



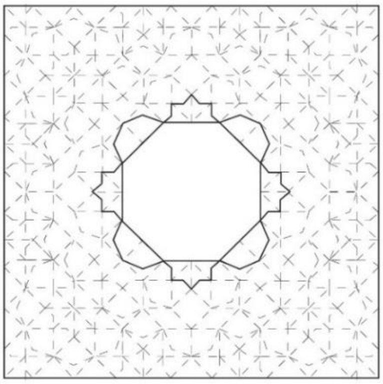
Geometry of first layer



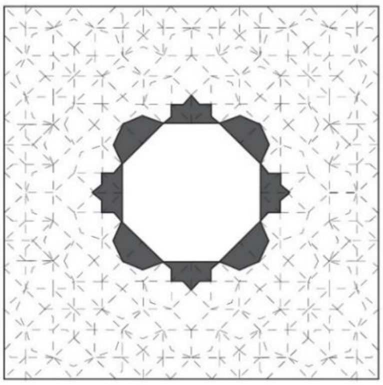
Second layer



Geometry of second layer



Third layer



Geometry of third layer

Patterns are redrawn by author

Motif-based analysis of a Muqarnas pattern

Pattern 3 (Pole table Muqarnas)

Figure 104. Murat pasha Mosque

Located in Istanbul, Turkey, 1570

The analysis of this pattern is evaluated as a rectangular tiled composition (Figure 105). The pattern is tried to be decomposed to its basic parts step by step. In the first step, as it is obvious from the Figure 106, the pattern can be divided into four sectors that the whole pattern is created by repeating and rotating each of these sectors that are called basic module in this study.

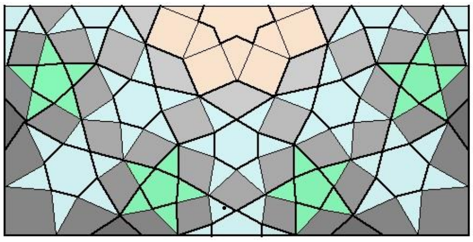


Figure 105. The design of the Murat pasha Mosque

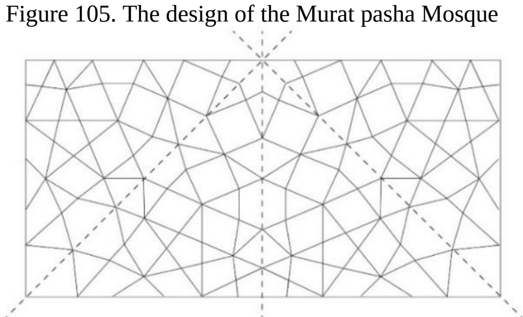
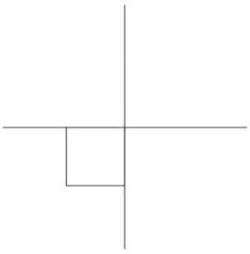
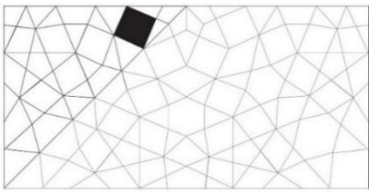


Figure 106. Decomposition of the pattern

The pattern is redrawn by author

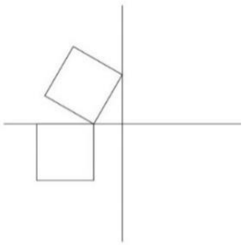
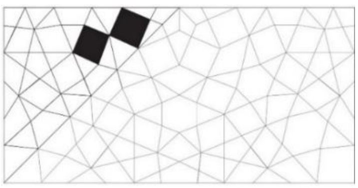
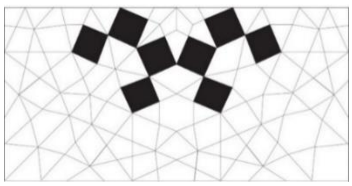
The process of basic elements placement in Murat pasha Mosque pattern

The plane projection of Murat pasha Mosque's Muqarnas type has a symmetric pattern. When each basic module of this pattern is decomposed, a square appears as the simplest and the most basic element of Murat pasha Mosque's Muqarnas pattern (a). The placement of the basic element is shown in both basic module Figure (b) and whole pattern Figure (c). After rotating the square by 45 degrees, Figure (d) will be achieved. According to achieved information about pattern 2, two squares are the simplest parts of this pattern like the first pattern. Figure (e) shows the location of these parts in basic module and Figure (f) shows the whole pattern with the simplest parts.


(a) (b) (c)

The most basic element — Basic element in basic module — Basic element in pattern

(d) (e) (f)

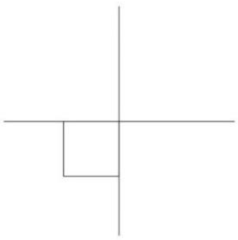
Basic elements — Basic elements in basic module — Basic elements in pattern

Figure 107. Process of locating basic elements in both basic module and whole pattern

Patterns are redrawn by author

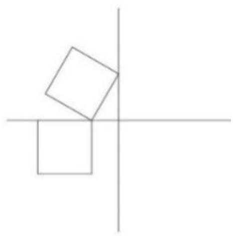
The creation process of a basic module of Murat pasha Mosque pattern

For generating a basic module of Murat pasha Mosque pattern, it seems that the fundamental elements of this pattern are two squares. These two squares are shown in Figure (b). By adding two rhombuses and rotating them as Figure(c) the Other elements are reached. Finally by extending the vertical and horisontal sides of the basic elements shown in Figure (d) the basic module will be created.



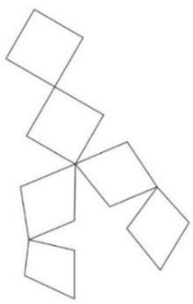
(a)

Basic form is a square



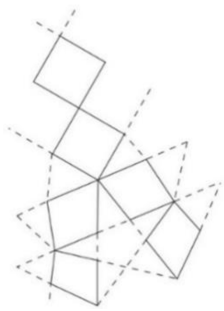
(b)

Rotate 60°



(c)

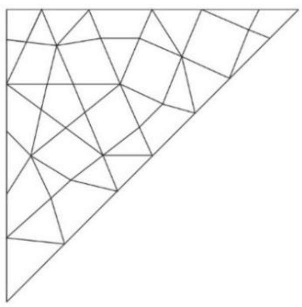
Add two rhombuses and rotate them



(d)

Extend the lines of created forms

Figure 108. Process of accessing basic module



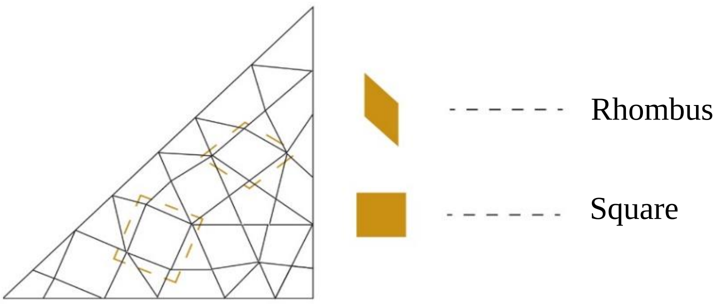
159. Basic module of this pattern

Patterns are redrawn by author

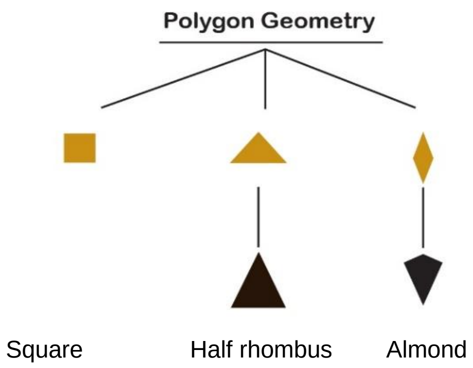
Geometric design process of a Muqarnas form construction

The basic module itself consists of split forms which are mentioned in chapter 3. The split forms of Murat pasha Mosque pattern are shown below.

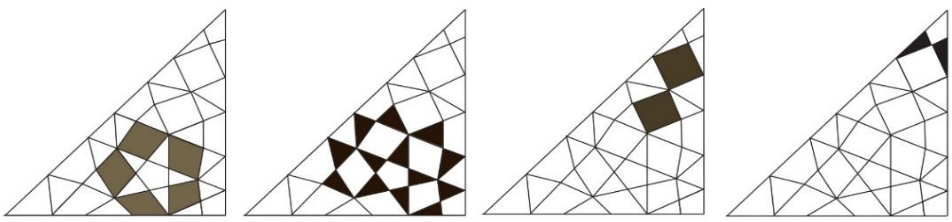
1 Decomposition of the basic module into it's initial shapes



2 Initial forms also can be divided into split forms



3 Positioning of initial forms in basic module



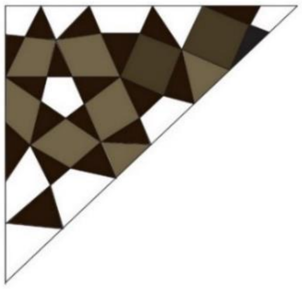
(a) (b) (c) (d)

Patterns are redrawn by author

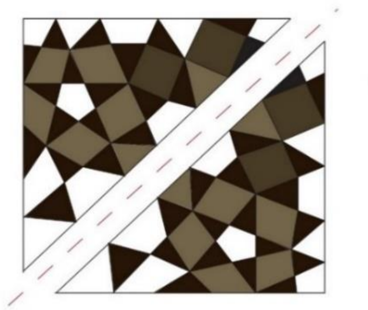
Part to whole relation of Muqarnas

The creation of a Muqarnas pattern by using isometry types that is created from a single basic module.

4

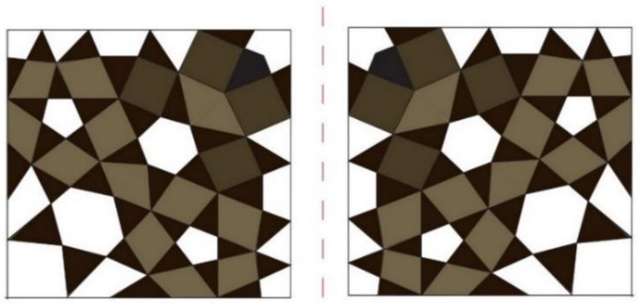


(a). Basic Module



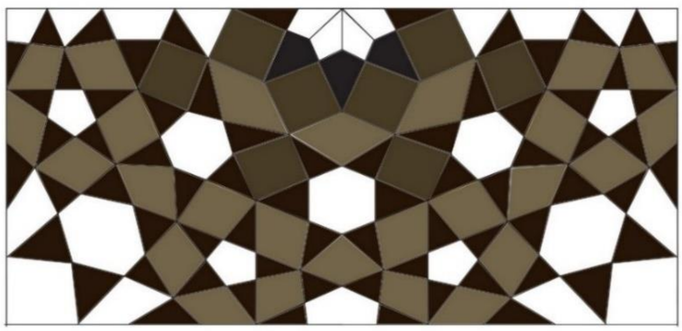
(b). Mirror 45°

5



(c). Mirror 90°

6

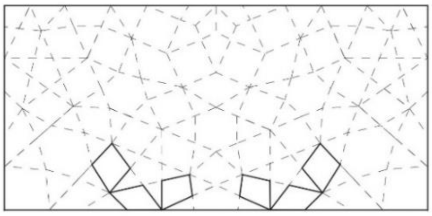


The symmetric pattern of the Murat pasha Mosque Muqarnas pattern

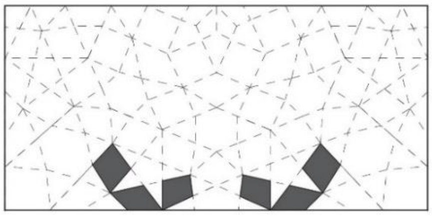
Patterns are redrawn by author

Layer-based analysis of a Muqarnas pattern

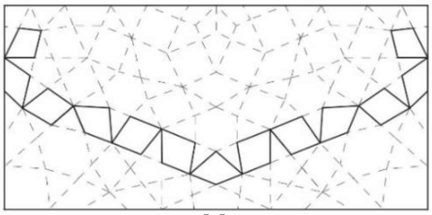
Pattern of Murat pasha Mosque is analyzed based on the constructional layers that is estimated to may be the other designing way of historical Muqarnas patterns.



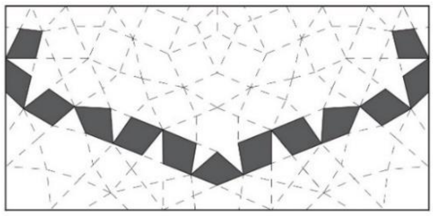
First layer



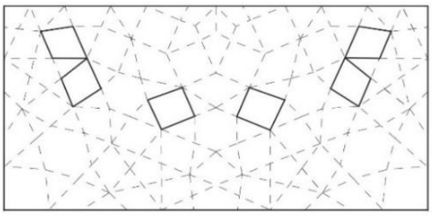
Geometry of first layer



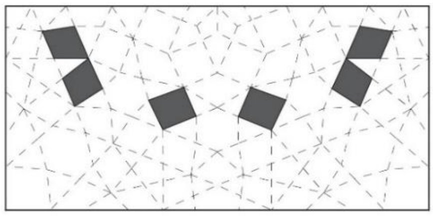
Second layer



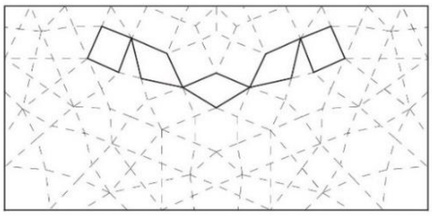
Geometry of second layer



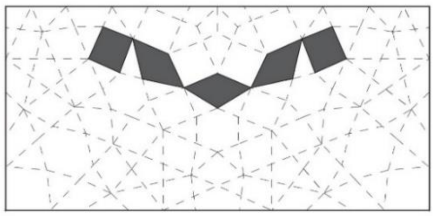
Third layer



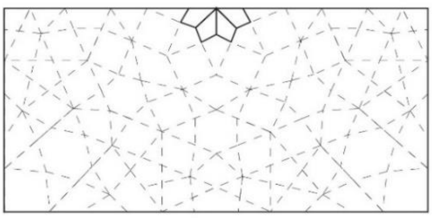
Geometry of third layer



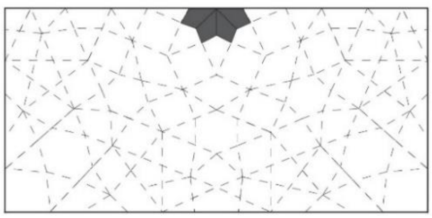
Fourth layer



Geometry of fourth layer



Fifth layer



Geometry of fifth layer

Patterns are redrawn by author

Motif-based analysis of a Muqarnas pattern

Pattern 4 (Pole table Muqarnas)

Figure 109. Zisa Castle

Located in Palermo, Italy, 12th century

The analysis of this pattern is evaluated as a rectangular tiled composition (Figure 110). The pattern is tried to be decomposed to its basic parts step by step. In the first step, as it is obvious from the Figure 111, the pattern can be divided into four sectors that the whole pattern is created by repeating and rotating each of these sectors that are called basic module in this study.

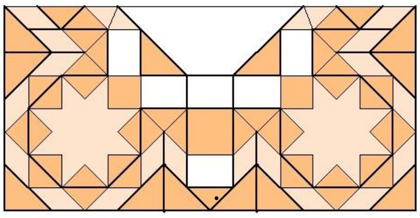


Figure 110. The design of the Zisa Castle

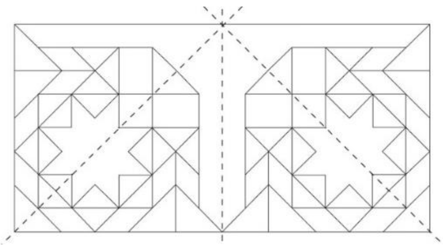
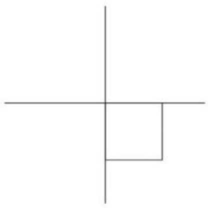


Figure 111. Decomposition of the pattern

The pattern is redrawn by author

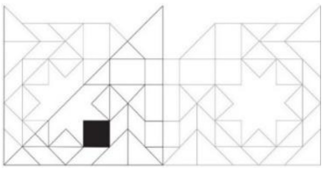
The process of basic elements placement in Zisa Castle pattern

The plane projection of Zisa Castle's Muqarnas type has a symmetric pattern. When each basic module of this pattern is decomposed, a square appears as the simplest and the most basic element of Zisa Castle's Muqarnas pattern (a). The placement of the basic element is shown on both basic module Figure (b) and whole pattern Figure (c). After rotating the square two times by 45 degrees, Figure (d) and Figure (e) will be achieved. According to achieved information about pattern 2, three squares are the simplest parts of this pattern like the first pattern.



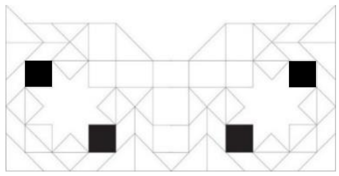
(a)

The most basic element



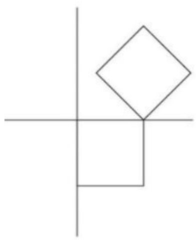
(b)

Basic element in basic module



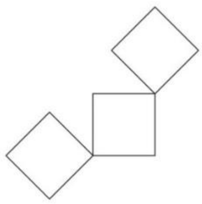
(c)

Basic element in pattern



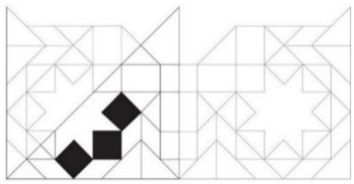
(d)

Basic elements



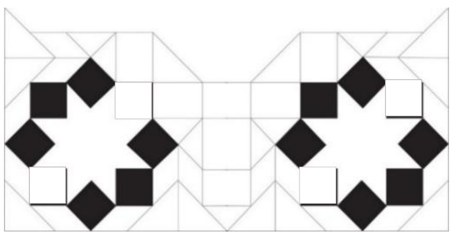
(e)

Basic elements in basic module



(f)

Basic elements in basic module



(g)

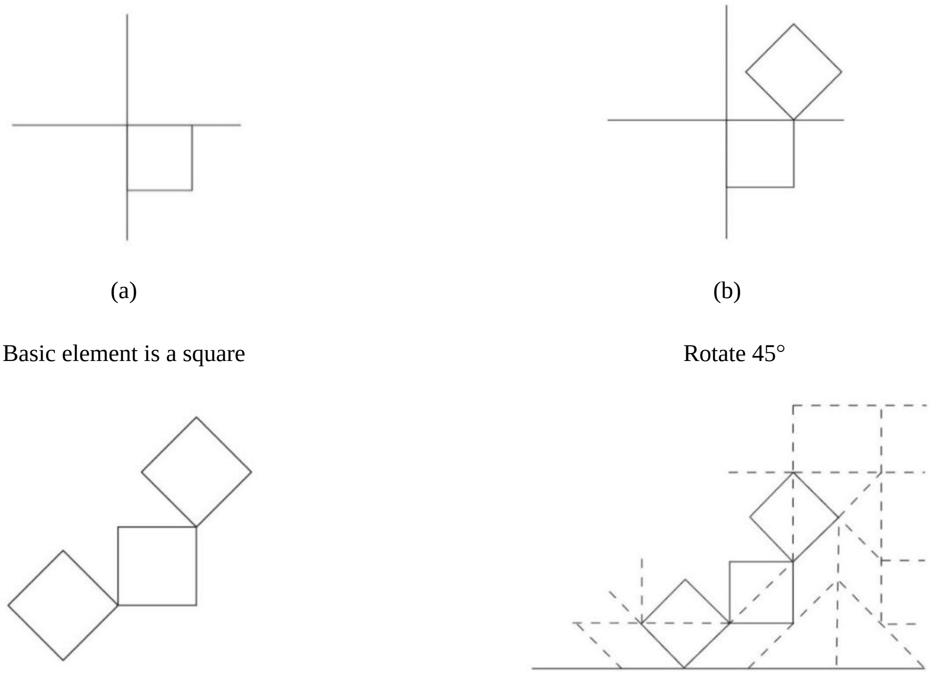
Basic elements in pattern

Figure 112. Process of locating basic elements in both basic module and whole pattern

Patterns are redrawn by author

The creation process of a basic module of Zisa Castle pattern

For generating a basic module of Zisa Castle pattern, it seems that the fundamental elements of this pattern are three squares. These three squares are shown in Figure (c). By extending the vertical and horisontal sides of the basic elements shown in Figure (d) the basic module will be created.



(a)

Basic element is a square

(b)

Rotate 45°

(c)

Rotate -45°

(d)

Extend the lines of created forms

Figure 113. Process of accessing basic module

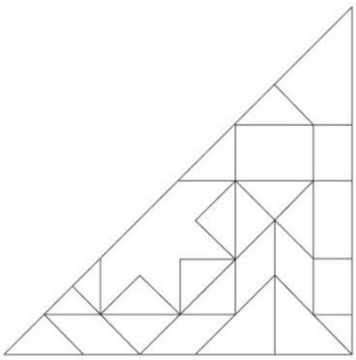


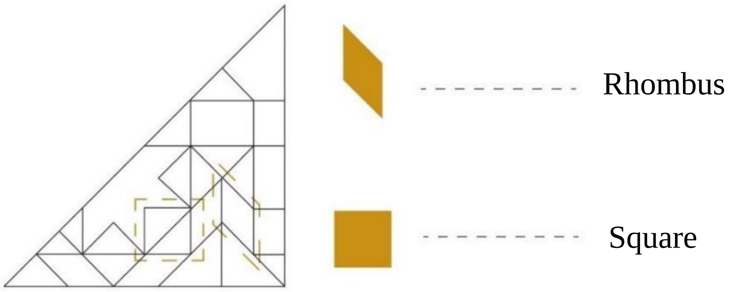
Figure 114. Basic module of this pattern

Patterns are redrawn by author

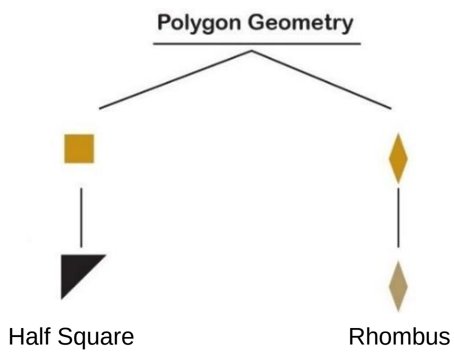
Geometric design process of a Muqarnas form construction

The basic module itself consists of split forms which are mentioned in chapter 3. The split forms of Zisa Castle pattern are shown below

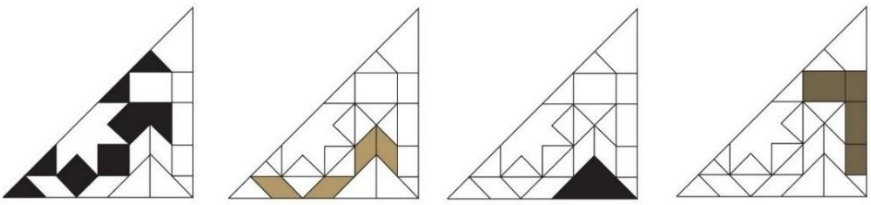
1 **Decomposition of the basic module into it's initial forms**



2 **Initial forms also can be divided into split forms**



3 **Positioning of initial elements in basic module**

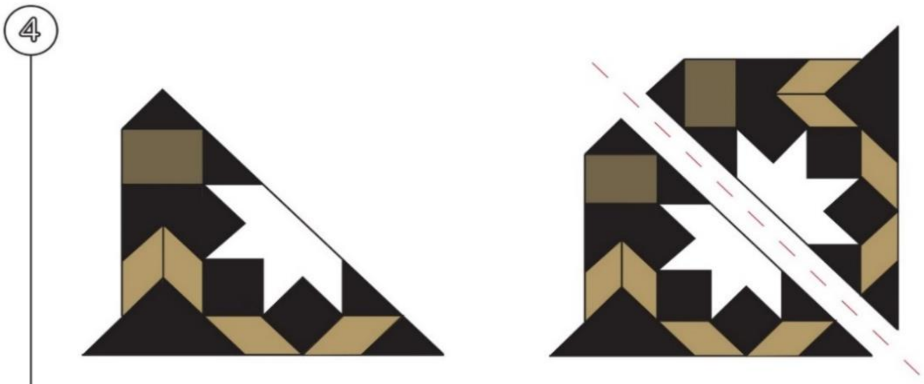


(a) (b) (c) (d)

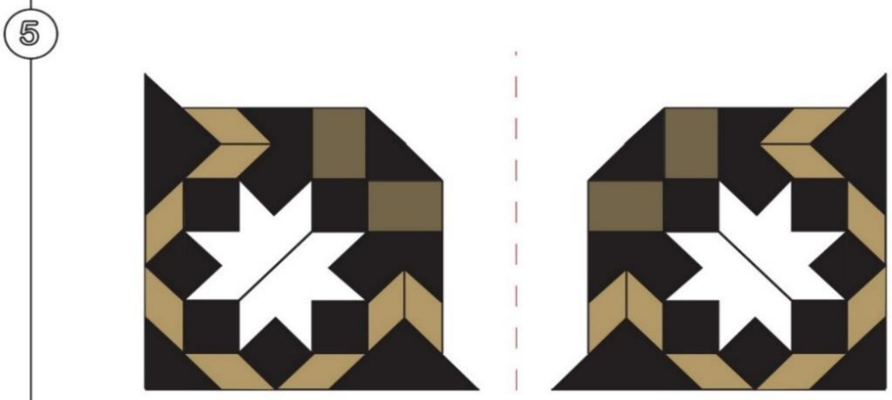
Patterns are redrawn by author

Part to whole relation of Muqarnas

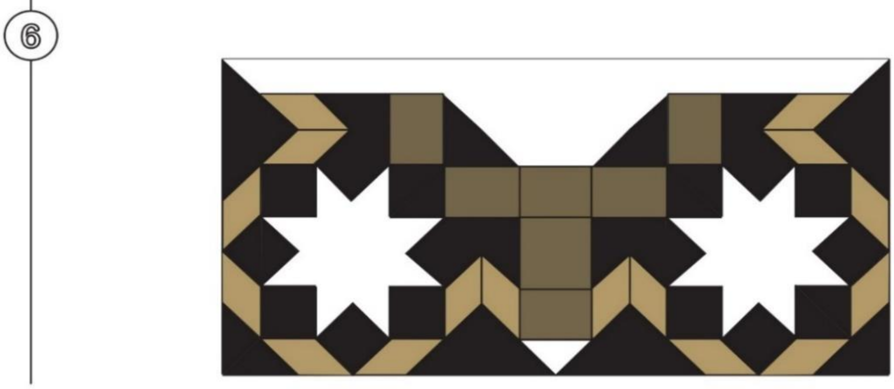
The creation of a Muqarnas pattern by using isometry types that is created from a single basic module.



(a). Basic Module (b). Mirror 315°



(c). Mirror 90°

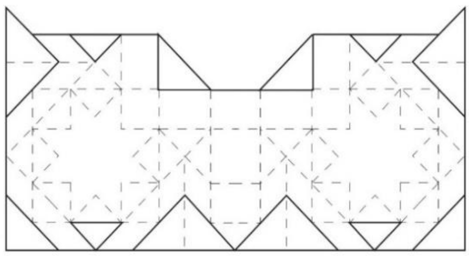


The symmetric pattern of the Zisa Castle Muqarnas pattern

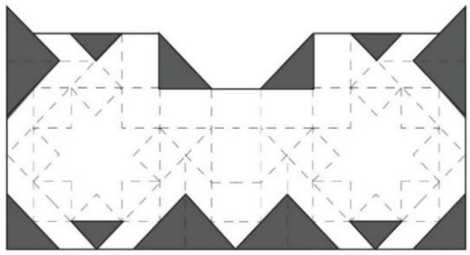
Patterns are redrawn by author

Layer-based analysis of a Muqarnas pattern

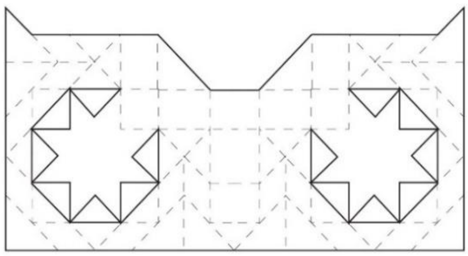
Pattern of Zisa Castle is analyzed based on the constructional layers that is estimated to may be the other designing way of historical Muqarnas patterns.



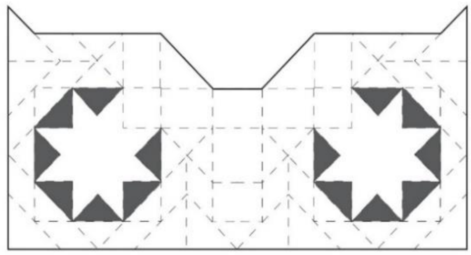
First layer



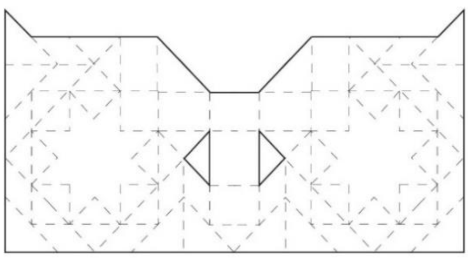
Geometry of first layer



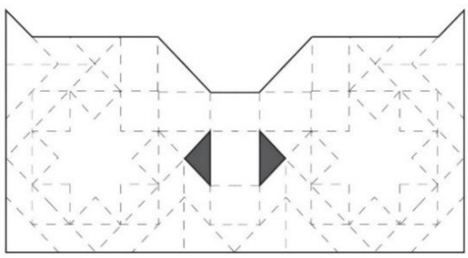
Second layer



Geometry of second layer



Third layer



Geometry of third layer

Patterns are redrawn by author

Motif-based analysisof a Muqarnas pattern

Pattern 5 (Other style of Muqarnas)

Figure 115. Selimiye Mosque

Located in Edirne, Turkey, 1574

The analysis of this pattern is evaluated as a rectangular tiled composition (Figure 116). The pattern is tried to be decomposed to its basic parts step by step. In the first step, as it is obvious from the Figure 117, the pattern can be divided into sixteen sectors that the whole pattern is created by repeating and rotating each of these sectors that are called basic module in this study.

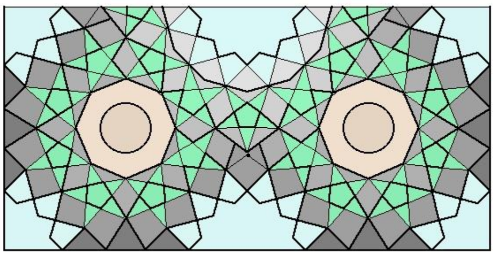


Figure 116. The design of the Selimiye Mosque

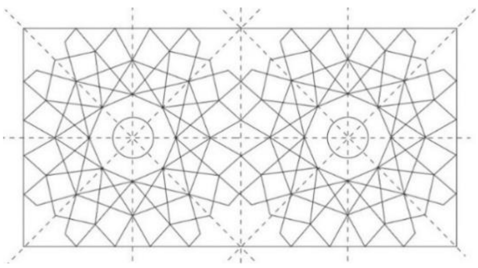


Figure 117. Decomposition of the pattern

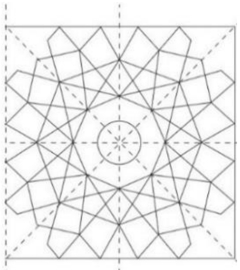
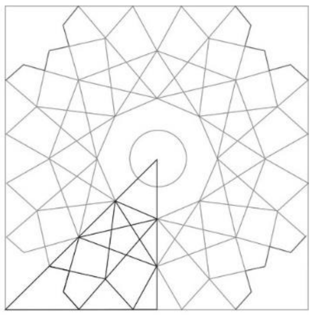


Figure 118. A motif

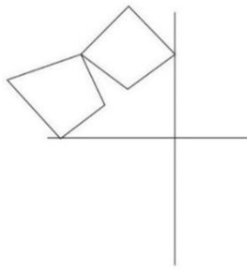
Patterns are redrawn by author

As it is obvious from the basic module of Selimiye Mosque's pattern Figure (a), the plane projection of this pattern is rotationally symmetric around its center. When each basic module of this pattern is decomposed, two rhombuses appear as the simplest and the most basic elements of Selimiye Mosque's Muqarnas pattern (b). The placement of the basic element is shown on both basic module Figure (c) and whole pattern Figure (d).



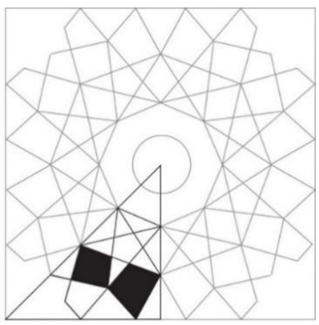
(a)

Basic module of the pattern



(b)

The most basic element



(c)

Basic element in basic module

(d)

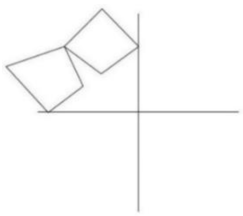
Basic element in pattern

Figure 119. Process of locating basic elements in both basic module and whole pattern

Patterns are redrawn by author

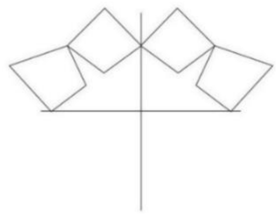
The creation process of a basic module of Selimiye Mosque pattern

For generating a basic module of Selimiye Mosque pattern, it seems that the fundamental elements of this pattern are two rhombuses. These two rhombuses are shown in Figure (a). By mirroring them 90° as shown in Figure (b) and extending the vertical and horisontal sides of the basic elements shown in Figure (c) the pattern will be created. Figure (d) illustrates how a basic module is obtained from the pattern.



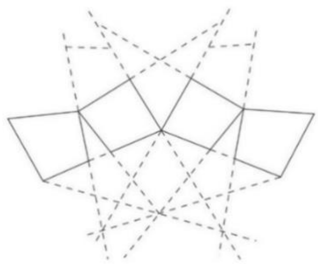
(a)

Basic elements are two rhombuses



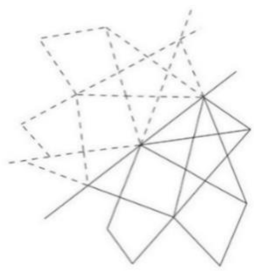
(b)

Mirror 90°



(c)

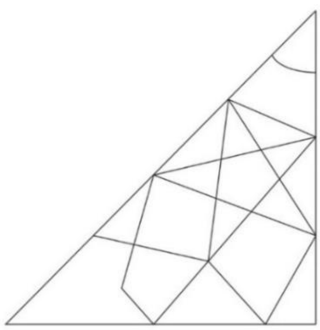
Extend the lines of created forms



(d)

Rotate 130°

Figure 120. Process of accessing basic module



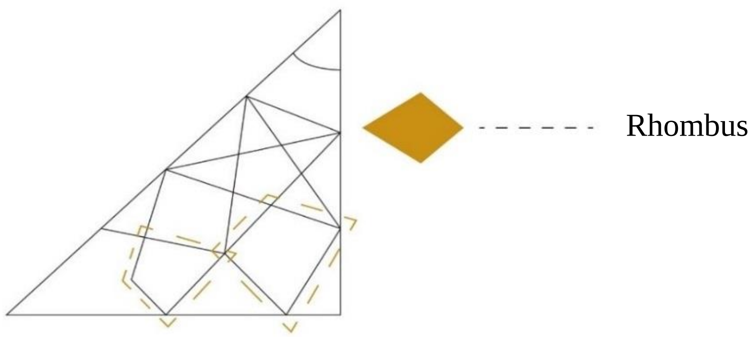
168.Basic module of this pattern

Patterns are redrawn by author

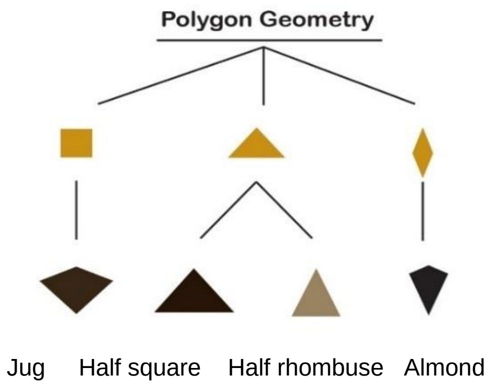
Geometric design process of a Muqarnas form construction

The basic module itself consists of split forms which are mentioned in chapter 3. The split forms of Selimiye Mosque pattern are shown below

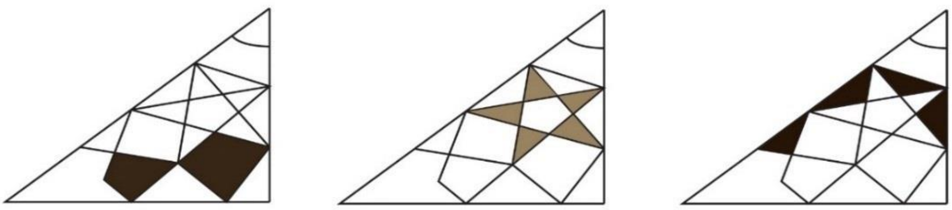
1 Decomposition of the basic module into it's initial forms



2 Initial forms also can be divided into split forms



3 Positioning of initial elements in basic module



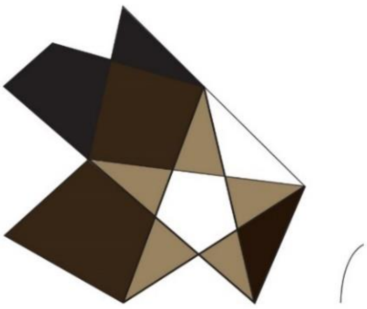
(a) (b) (c)

Patterns are redrawn by author

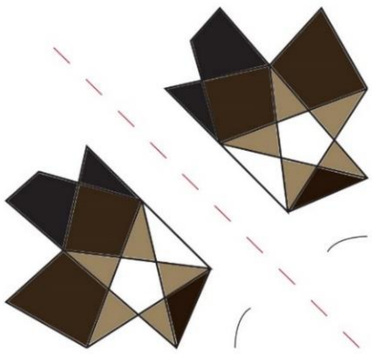
Part to whole relation of Muqarnas

The creation of a Muqarnas pattern by using isometry types that is created from a single basic module.

4

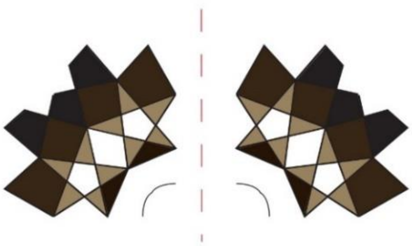


(a). Basic Module

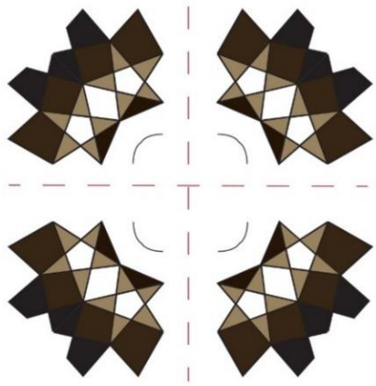


(b). Mirror 315°

5

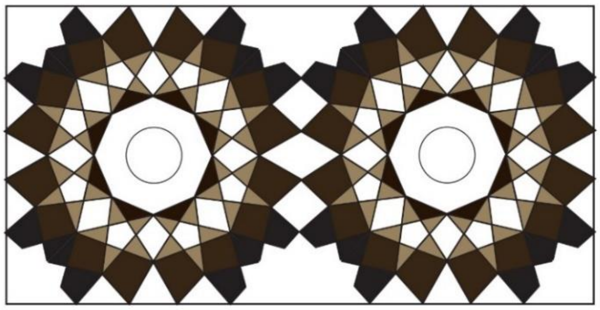


(c). Mirror 90°



(d). Mirror 180°

6

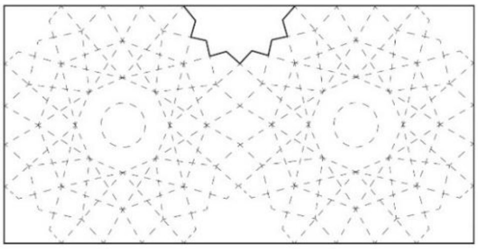


The rotational symmetry of the Selimiye Mosque's Muqarnas pattern

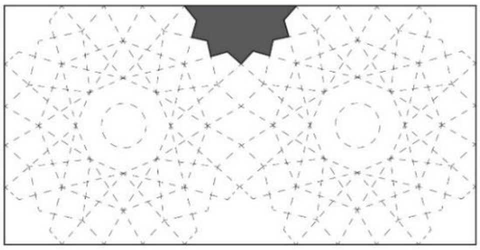
Patterns are redrawn by author

Motif-based analysis of a Muqarnas pattern

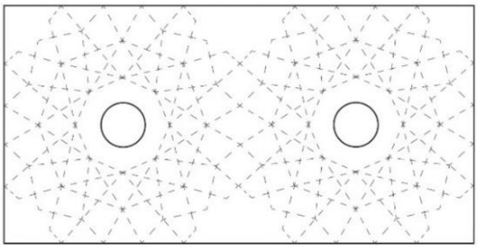
Pattern of Selimiye Mosque is analyzed based on the constructional layers that is estimated to may be the other designing way of historical Muqarnas patterns.



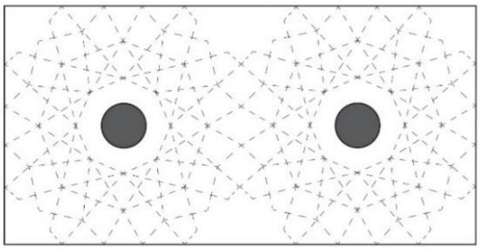
First layer



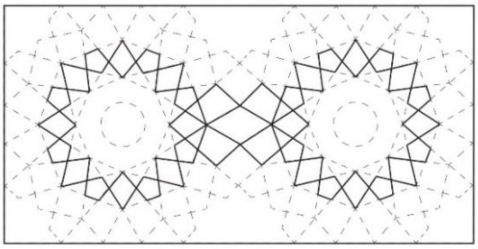
Geometry of first layer



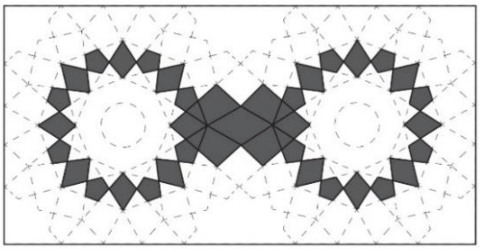
Second layer



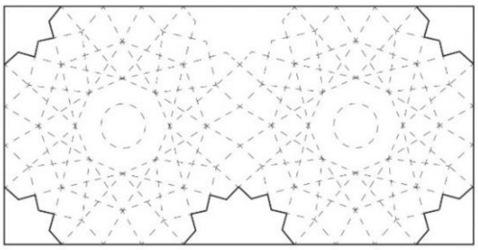
Geometry of second layer



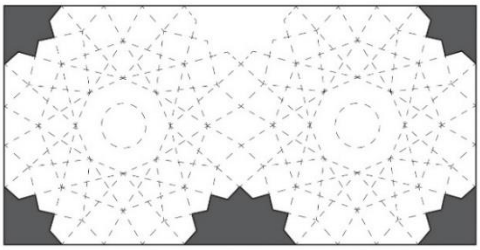
Third layer



Geometry of third layer



Fourth layer



Geometry of fourth layer

Patterns are redrawn by author

Motif-based analysis of a Muqarnas pattern

Pattern 6 (Other style of Muqarnas)

Figure 121. Sahabiye Madrasse

Kayseri, Turkey, 1267

The analysis of this pattern is evaluated as a rectangular tiled composition (Figure 122). The pattern is tried to be decomposed to its basic parts step by step. In the first step, as it is obvious from the Figure 123, the pattern can be divided into four sectors that the whole pattern is created by repeating and rotating each of these sectors that are called basic module in this study.

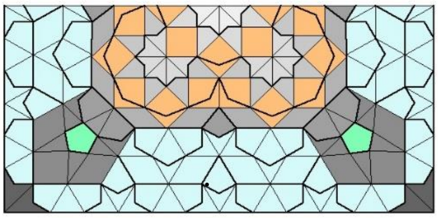


Figure 122. The design of the Sahabiye Madrasse

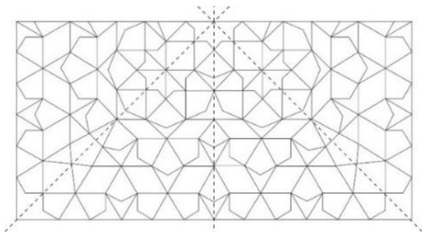
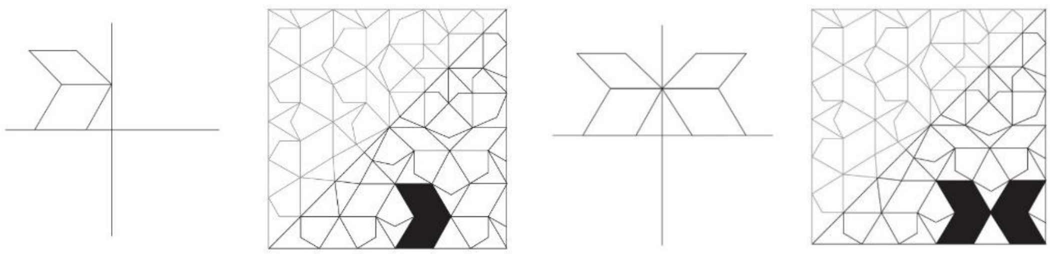


Figure 123. Decomposition of the pattern

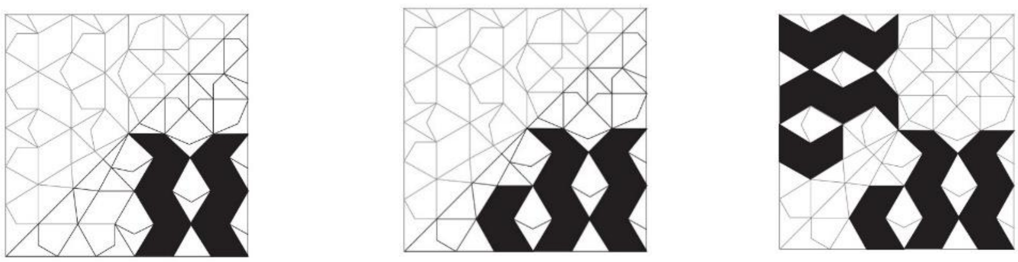
The pattern is redrawn by author

The plane projection of Sahabiye Madrasse's Muqarnas type has a symmetric pattern. When each basic module of this pattern is decomposed, there are some steps to create a basic module. In contrary to the other five examples that have just one or two basic elements, this pattern has a geometric process to create its basic module. This process is shown in figures below step by step.



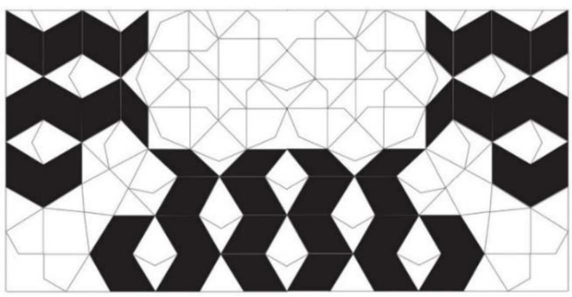
(a) (b) (c) (d)

Basic elements   Basic elements in basic module   Basic elements   Basic elements in basic module



(e) (f) (g)

Figure 124. Process of locating basic elements in basic module

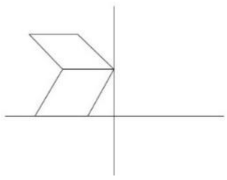


Basic elements in pattern

Patterns are redrawn by author

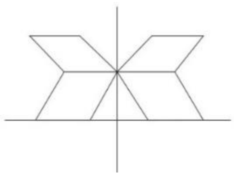
The creation process of a basic module of Sahabiye Madrasse pattern

For generating a basic module of Sahabiye Madrasse's pattern, it seems that the fundamental elements of this pattern are some rhombuses. The process of accessing the rhombuses are shown in Figures below step by step.



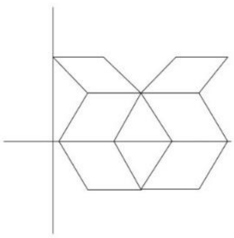
(a)

Two rhombuses



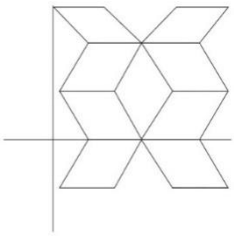
(b)

Mirror 90°



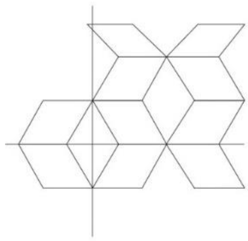
(c)

Mirror 90°



(d)

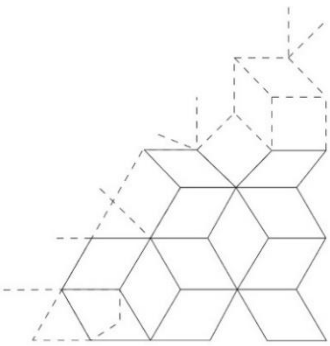
Mirror 180°



(e)

Mirror 90°

Figure 125. Process of accessing basic module



Extend the lines of created forms

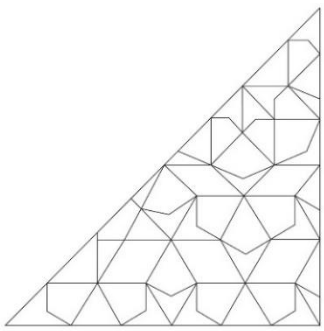


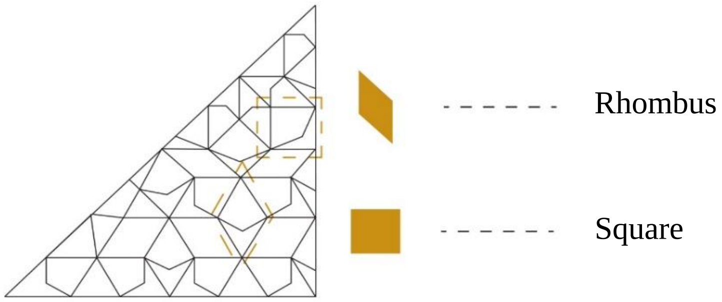
Figure 126. Basic module of this pattern

Patterns are redrawn by author

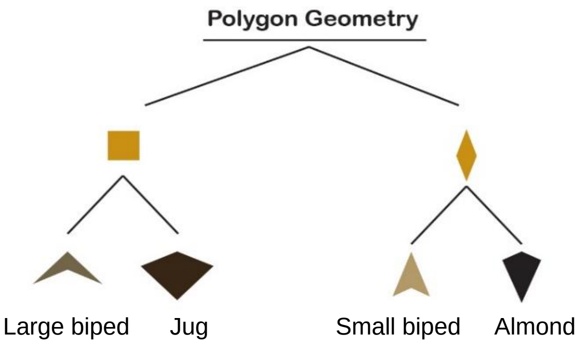
Geometric design process of a Muqarnas form construction

The basic module itself consists of split forms which are mentioned in chapter 3. The split forms of Sahabiye Madrasse pattern are shown below

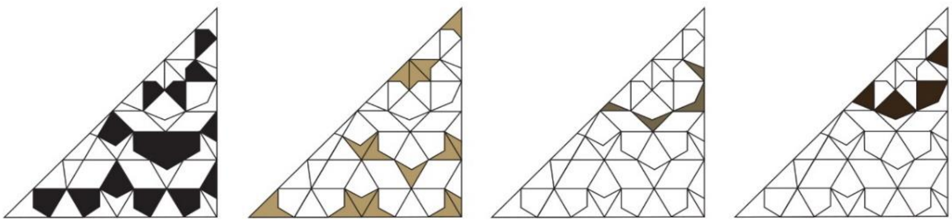
1 Decomposition of the basic module into it’s initial forms



2 Initial forms also can be divided into split forms



3 Positioning of initial elements in basic module

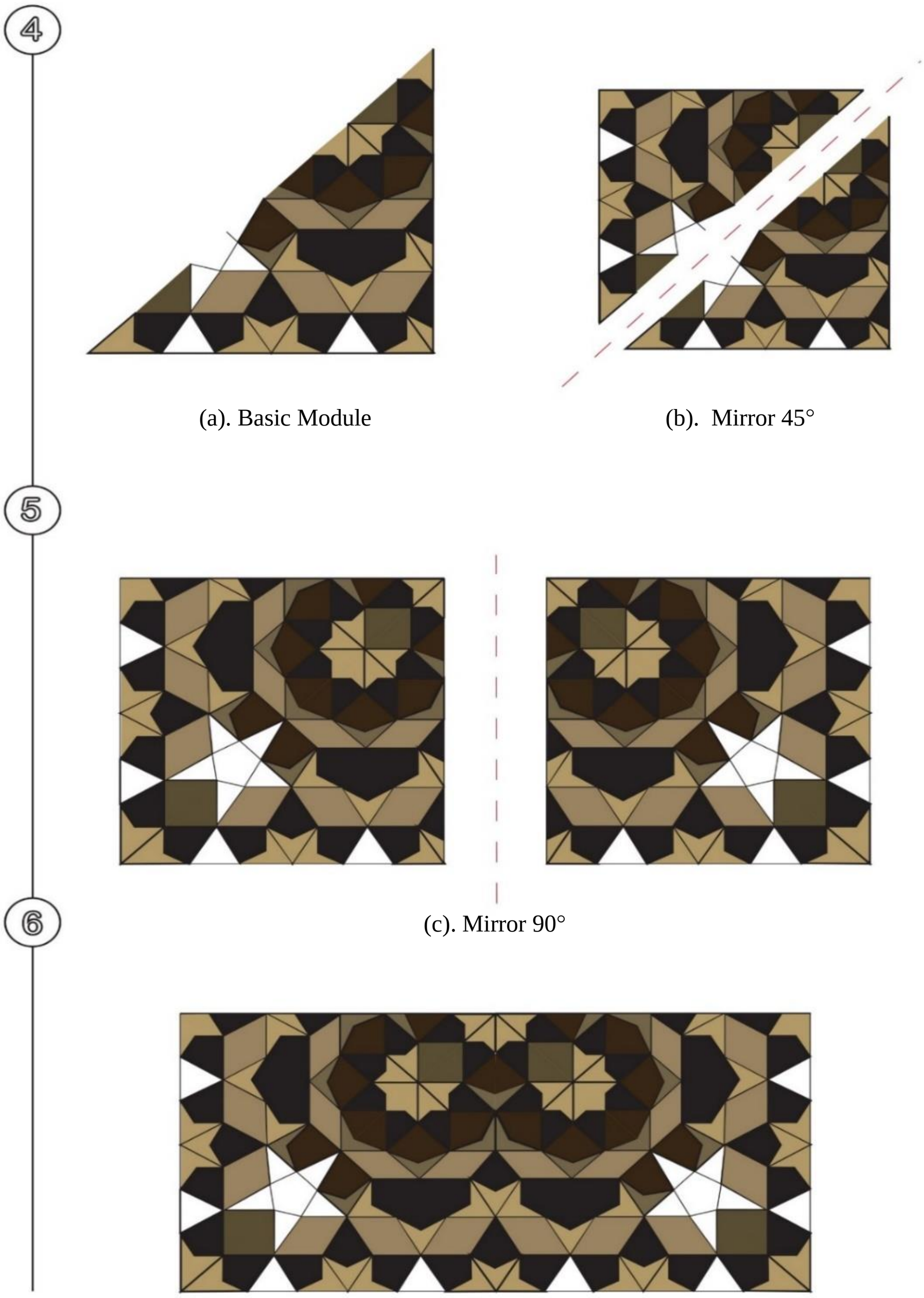


(a) (b) (c) (d)

Patterns are redrawn by author

Part to whole relation of Muqarnas

The creation of a Muqarnas pattern by using isometry types that is created from a single basic module.



(a). Basic Module

(b). Mirror 45°

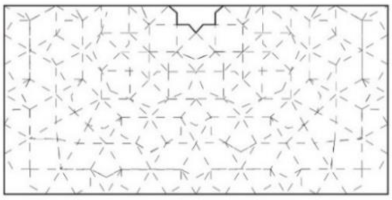
(c). Mirror 90°

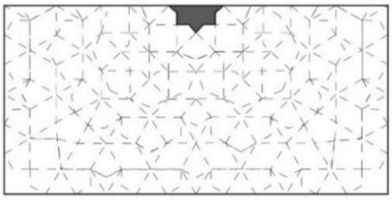
The symmetric pattern of the Sahabiye Madrasse’s Muqarnas pattern

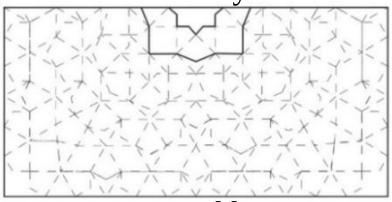
Patterns are redrawn by author

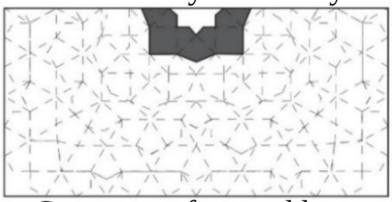
Layer-based analysis of a Muqarnas pattern

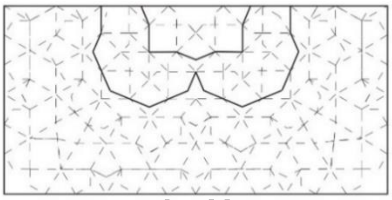
Pattern of Sahabiye Madrasse is analyzed based on the constructional layers that is estimated to may be the other designing way of historical Muqarnas patterns.

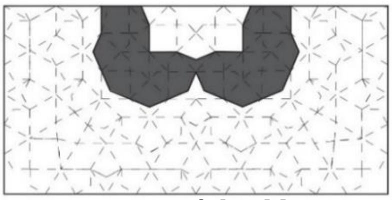

First layer

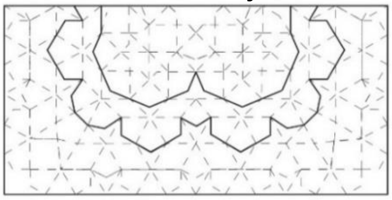

Geometry of first layer

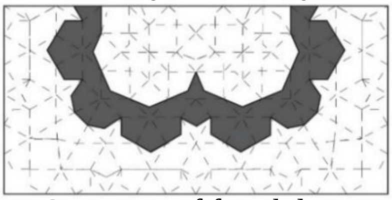

Second layer

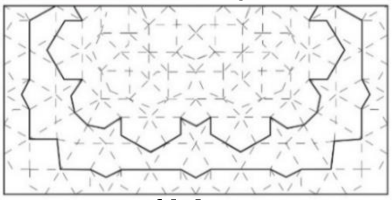

Geometry of second layer

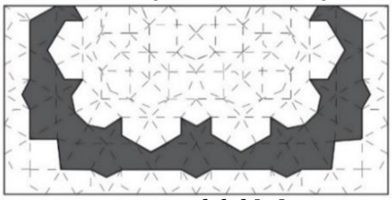

Third layer

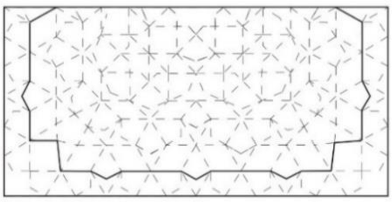

Geometry of third layer

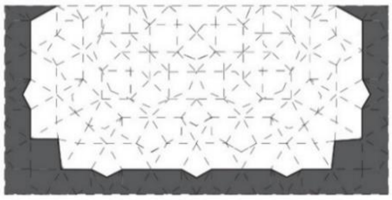

Fourth layer


Geometry of fourth layer


Fifth layer


Geometry of fifth layer


Sixth layer


Geometry of sixth layer

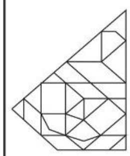
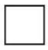
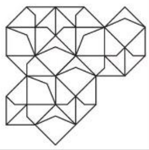
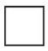
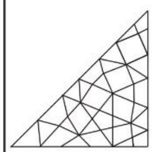
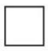
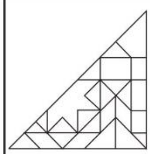
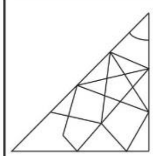
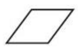
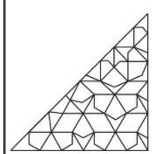
Patterns are redrawn by author

This section tries to generate guidelines for six historical Muqarnas patterns that were analyzed to understand the two-dimensional geometric rules of these Muqarnas patterns.

By evaluating the stated analytical studies of six historical Muqarnas examples, a descriptive and analytical table is obtained. This table is presented on next page as Table 4, and it exhibits six geometrical characteristics of these historical patterns. Six characteristics which are mentioned in Table 4 are; the most basic element, isometry types of basic elements, basic module, types and number of basic module’s elements, isometry types of module, numbers of layers.

All of these six geometrical characteristics that are evaluated in Table 4 are obtained from the analysis results derived from the both motif-based and layer-based analytical studies of six historical Muqarnas examples. However most of historical Muqarnas compositions seem to have the same geometrical characteristics but, this thesis does not claim that these rules were applied on all of the Muqarnas compositions with different functions. There could exist a single Muqarnas composition that is different from all of the evaluated examples and carries different characteristics but still called a Muqarnas composition. There are more than 2000 Muqarnas examples all around the world and it is impossible to access all of them or have a claim about the 100% of Muqarnas patterns. Therefore, this study tries to select Muqarnas examples among very various categories with different functions and different eras to access a wide range of historical Muqarnas patterns.

Table 6. Geometrical characteristics of six Muqarnas patterns

Pattern's buildings	Basic elements	Isometry types of basic sector	Basic module	Number of basic module elements		Isometry types of pattern	Number of layers
pattern 1 Tinmal Mosque		Rotation 45°		------ Large biped ------ Jug ------ Smal biped ------ Almond ------ Half square	5	Mirror 315° Mirror 90° Mirror 180°	3
pattern 2 II.Beyazid Mosque		Mirror 45° Rotation -45° Mirror 45°		------ Large biped ------ Jug ------ Smal biped ------ Almond	5	Mirror 315° Mirror 90° Mirror 180°	3
pattern 3 Murat pasha Mosque		Rotation 60°		------ square ------ Half rhombus ------ Almond ------ Rhombus	4	Mirror 45° Mirror 90°	5
pattern 4 Zisa Castle		Mirror 45° Mirror 45° Mirror 45°		------ Half square ------ Rhombus	2	Mirror 315° Mirror 90°	3
pattern 5 Selimiye Mosque		Mirror 90°		------ Jug ------ Almond ------ Half square Half rhombus	4	Mirror 315° Mirror 90° Mirror 180°	4
pattern 6 Sahabiye Madrasse		Mirror 90° Mirror 90° Mirror 180° Mirror 90°		------ Large Biped ------ Jug ------ Smal biped ------ Almond	4	Mirror 45° Mirror 90°	6

The table is drawn by author

4.2.1 The evaluation of historical Muqarnas patterns based on analytical studies

The table lays emphasis on the fact that in the most complex Muqarnas patterns, the geometry of a finite number of elements is linked to the cohesive logic of the whole. In order to define the geometrical logic of Muqarnas patterns, the characteristics of six Muqarnas patterns in this table are classified as follows;

The most basic element:

The analytical studies which are presented in this section tries to find the most basic elements of each Muqarnas patterns which are estimated to be the pattern's initial idea starting with these basic elements. Every single Muqarnas pattern consists of the most basic element which cannot be omitted but, can be called the fundamental element of each pattern's module. This element defines the whole structure of a pattern. The creation of other elements might also be the results deriving from the extended lines of these basic elements which are illustrated in analytical studies section.

Isometry types of basic elements:

In order to create a basic module of each pattern, basic element (or in some case basic elements) is rotated or reflected through different degrees. The degrees of the basic element's rotation and reflection are illustrated as the isometry types of these basic elements. The isometry types of basic elements could vary in every Muqarnas patterns.

The Basic module:

The basic module is the most important sector of a Muqarnas pattern. Every Muqarnas pattern consists of the rotation and reflection of a basic module. A basic module is the only sector of a Muqarnas pattern which should be repeated to create other sectors of this pattern. A whole Muqarnas pattern is created from the repetition of a sector which is called basic module of that pattern.

Types and number of the basic module's elements:

Every Muqarnas module consists of elements which themselves are created from the basic parts. As mentioned in chapter 2, every basic Muqarnas elements also consist of a small set of simple geometrical forms. The types and numbers of these geometrical forms depend on the basic module's forms and they are presented under the title of Types and number of the basic module's elements.

Isometry types of the module:

The rotation number of each Muqarnas pattern depends on the function of Muqarnas. For example, the pattern of a Muqarnas dome is different from the pattern of a niche. A Muqarnas dome always has a circular 2DPP and a Muqarnas niche has a rectangular 2DPP. Therefore, in order to create a Muqarnas dome's pattern, the basic module should be rotated and created according to the function of Muqarnas in the building. Therefore the number of isometry types of a circular pattern will be more than 2.

Numbers of layers:

The second analysis way in this chapter is the Layer-based analysis of a Muqarnas pattern. This way tries to decompose each pattern in terms of their layer compositions. The numbers of the construction layers of Muqarnas patterns are different and depends on the complexity of each composition. The number of each Muqarnas's layers are shown in this table. The layers of each pattern give really important information for creating a three-dimensional composition of Muqarnas two-dimensional pattern. All of the Muqarnas compositions as a volume consist of different layers with different height. The boundary of each layer indicates the location of elements and intermediate elements of Muqarnas.

This section tries to disclose the arrangement regulations of the motifs of Muqarnas 2DPP. According to the results derived from the analytical studies which were presented in Table 4, and mathematical characteristics of Muqarnas, since the very beginning of Muqarnas emergence, Muqarnas patterns have carried geometrical and mathematical regulations. These regulations are constant in most of the Muqarnas patterns and they just belong to Muqarnas compositions, therefore, these regulations can be considered as the guidelines of Muqarnas patterns. According to the results derived from Table 4;

Guideline 1: In all six historical Muqarnas examples, the most basic elements of these patterns are square and rhombuses.

Guideline 2: All of the basic elements are rotated through the angles of 45°, 60°, 90° and 180°.

Guideline 3: All of the modules are created from minimum 2 and maximum 5 smaller parts such as; square, half-square, rhombus, half-rhombus, almond, jug, large biped, and small biped.

Guideline 4: The basic modules in all of the six historical patterns are rotated through the angles of 45°, 90°, 315° and 180°.

Guideline 5: The number of layers of Muqarnas compositions depends on the complexity of compositions. These six historical Muqarnas examples have minimum 3 and maximum 6 construction layers.

Guideline 6: Golden Ratio was widely used in historical Muqarnas compositions.

Guideline 7: All of the Muqarnas compositions consist of translations, reflections, rotation and glide reflection which are the four types of isometry.

4.3 Interpretation of modern forms which are inspired by Muqarnas based on pattern analysis

Most of the architects, all around the world, have tried to use traditional architectural styles in their contemporary designs (Behnejad, Mottaghirad and Jamali, 2012). James (2012) declared that traditional architecture is a rich source of inspiration for architects. They could easily learn from it to develop their design ideas from the point's view of beauty, harmony and proportion and applied them to their contemporary design for being accepted by clients, and the general public. Michler (2015) refers to Dr. Feist and asserts that it turns out that given the limited technology that people had in the past, of course, they did a good job with the regional traditional design. There have been different types of architectural heritage in different parts of the world, and of course, it had something to do with the local climate. Traditional architects knew how to build for the local climate given the technology at the time.

According to Hejazi and Mehdizadeh (2014), geometry plays a crucial role in the design of traditional Persian architecture. The happy union of art and science was the main factor that Persian architects emphasized them more than strength, stiffness and stability in historical buildings. Today also geometrical patterns or motifs are used in architecture to fill surfaces with regular shapes or polygons without any gaps or overlaps between them (Figure127).

Figure 127. Chahbahar-bagh madrassa, Isfahan, Iran

According to (Omer, 2011), architects trying to revive the concepts of Islamic architecture in their designs with considering of two main rules; first of all, they identify all the Islamic architecture principles and apply them to their construction. Next, they try to match Islamic architectural guidelines with the diverse regions in which they live. Most of the time, a successful architectural solution in a particular ecological setting cannot come up with today's architectural dilemmas. According to Yakar, Yilmaz, Armagan and Korkmaz (2009), Muqarnas compositions can be designed very appropriate in contemporary architecture, for both exterior and interior functions so that they could have an ornamental function in interior design such as designing lamps or display cabinets also.

4.3.1. Analytical studies of modern forms

As mentioned in Chapter 3, advances in digital art and architecture have meant designers can design forms and structure inspired by historical and traditional Muqarnas compositions more easily and rapidly. However historical Muqarnas compositions have potentials to be encoded by algorithms and three-dimensional modeling tools but, innovation in producing historical Muqarnas geometry have been limited to traditional ways. Therefore, today there are many novel designing ways to produce a Muqarnas composition with CAD systems that can be somehow different from the traditional ways of Muqarnas constructing. In addition, the structural complexity of Muqarnas design with CAD systems causes the evolutionary development of forms whose complexity is relative and increases with their various combinations.

However, sometimes the modern forms can be apparently different from historical Muqarnas compositions; so, getting distinction can be really tough but, they usually carry some of the Muqarnas guidelines on them which are mentioned in previous section. These guidelines that were derived from historical Muqarnas compositions, can define the similarity between historical Muqarnas compositions with modern forms. There are some designs which Muqarnas seems to be the main idea behind their design; we represent some of these forms in following examples.

In this section, all of the modern forms selected from the 21st century seem to be the reflection of historical Muqarnas patterns in terms of their geometrical characteristics. These forms are first selected according to the general Muqarnas geometrical characteristics which are mentioned before as types of isometry. In contrary with historical Muqarnas patterns that all have these designing regulations on their plane projections, modern forms can have them on their plan, façade or three-dimensional composition volumes. Therefore, in order to have a diverse set of modern forms, these forms are evaluated in this section not only with their plans but also with other views of them that have carried the Muqarnas characteristics. The modern forms that seem to have isometry types in their designing have been chosen for analyzing in this section.

In total, seven modern forms are selected in this section that their designers intentionally or unintentionally seem to be inspired by Muqarnas patterns. In some cases, designers clearly acclaim that their design concept was Muqarnas patterns, but the other designers did not mention anything about Muqarnas and maybe they even did not have any idea about Muqarnas patterns while they were designing the forms.

The main goal of this section is to determine some modern forms as a new generation of historical and traditional Muqarnas compositions and discuss their similarity and differentiation with historical patterns. This study does not claim that all of the forms are inspired by Muqarnas, but with the general characteristics which are mentioned in the previous section, this thesis estimate that the modern group which are selected to be analyzed may carry some of Muqarnas guidelines. After analyzing these modern forms with the same analyzing ways that were used for analyzing historical Muqarnas examples, the table will be achieved which determines the guidelines of modern forms like historical ones. Then, by comparing the guidelines of these forms with the historical ones, the impact of new designing methods and the transformation of these patterns from past to present will be discussed.

The forms which are selected as modern forms for analyze in this section are illustrated in the next page.

Figure 128. Azadi Tower

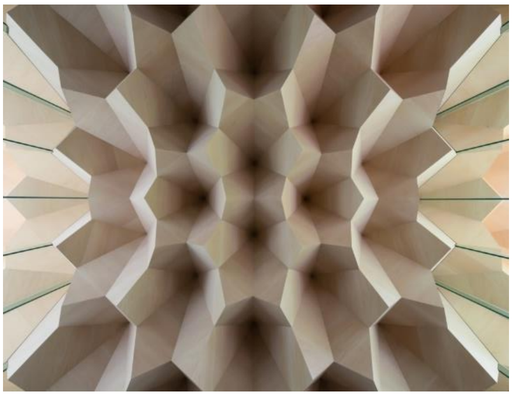


Figure 129. Miyahata Ruins Museum

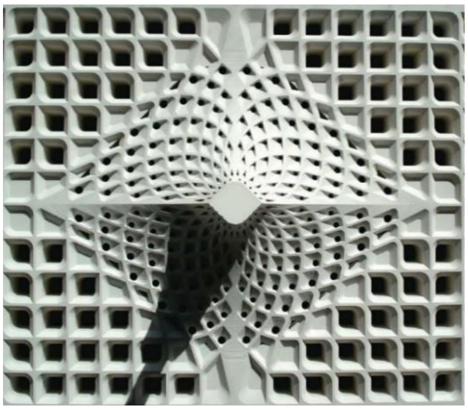


Figure 130. Chhatrapati Shivaji Airport

Figure 131. Museum of Islamic Art, Doha

Figure 132. Muqarnas tower

Figure 133. Hyperbolic Installation

Figure 134. Medical Center

The modern examples are selected among buildings, art forms or decorative elements which they have Muqarnas like composition in their construction. The first four examples that are selected to be analyzed in this chapter are; Azadi Tower, Miyahata Ruins Museum, Chhatrapati Shivaji International Airport and Museum of Islamic Art. These four examples have the most Muqarnas like compositions so that some parts of their structure that are analyzed in this chapter as modern forms clearly remind the visitors of Muqarnas composition. For example, the plane of the Azadi Tower shows an octagonal form with curves in the sides that overload the structure. The octagonal part of this structure is selected to be analyzed because the designer of Azadi Tower directly declared that the main idea of designing this form was Muqarnas. The internal spaces of Miyahata Ruins Museum, Chhatrapati Shivaji International Airport and Museum of Islamic Art, have impressive roof structures which have conveyed the characteristics of Muqarnas in their designs. The geometrical compositions and plane projections of these forms are somehow similar to historical Muqarnas examples. Their two-dimensional patterns have historical Muqarnas example’s geometrical regulations. The other three examples are; Muqarnas Tower, Hyperbolic Installation and Medical International Center. Muqarnas Tower has been selected to be analyzed because, not only Muqarnas Tower’s name directly infers to Muqarnas but, according to the building designers, the façade of this commercial office building has inspired by the traditional “Muqarnas” vaulting technique, however when we look at the facade we couldn’t find any similarity between the form of this facade with historical Muqarnas examples. The other modern example is Hyperbolic Installation. This installation designed from a research on ornamental techniques used within traditional Persian architecture and focus on Muqarnas. However, the main concept behind this form was also Muqarnas but, like Muqarnas Tower this form also doesn’t look alike historical Muqarnas examples. The last example and apparently the most similar examples to the historical Muqarnas patterns is Medical International Center. The windows of this building are decorated with Muqarnas like ornaments. Albeit, the ornaments are very similar to historical Muqarnas examples but, the pattern of them have some differences with each other.

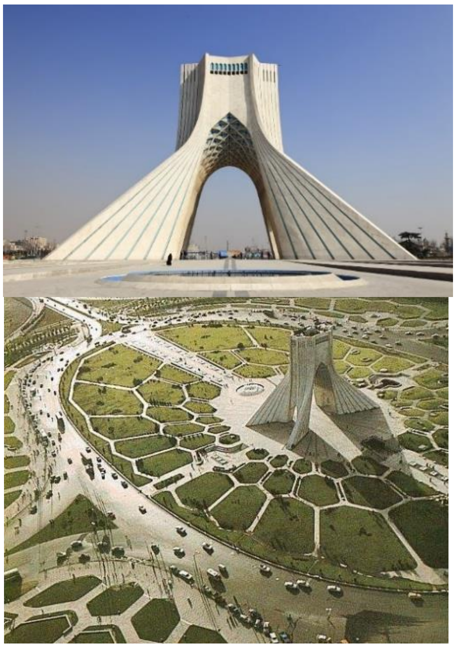
Azadi Tower

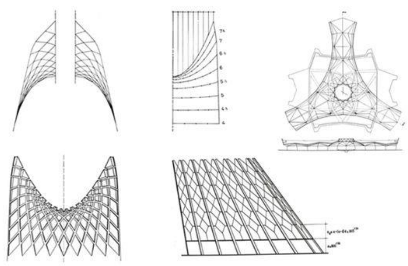
Architects: Hossein Amanat

Location: Tehran, Iran

Project Year: 1971

(a)





(b) (c)

Figure 135. Azadi Tower

Muqarnas vault of Azadi monument, Photo by: Mohammadreza samaee (a), Azadi monument, Tehran (b), details of Azadi Tower (c).

Pattern 1; Azadi Tower

The Azadi Tower which previously known as the Shahyad Tower that is located in Tehran, is one of these building which has been inspired by Muqarnas. Grigor (2003) refers to the architect of Azadi Tower and claims:

> "I find it difficult, sometimes impossible, to explain how ideas come to my mind. I usually sketch freely sometimes hundreds of versions. ... Ctesiphon was what I always liked and came to my mind immediately. The [Muqarnas-work], underlying geometry of domes and vaults and the spaces they define were all in my mind when I started sketching. They were what I had measured and drawn in Ray, as part of my university courses and sketched as [a] hobby most of the weekends. I used to go to sites around Tehran, sketching [domes, walls, mausoleums, and mosques]. I had [also] traveled through Europe and USA . . . seen a lot, including the Titus arch. All these had a share in the soul of this monument."

Azadi monument, is a symbol of victory that was achieved from Iran's Islamic Revolution[61]. Hosein amanat was the architect of Azadi tower, in 1966. It was a monumental gate of Tehran with a wonderful land scape geometry. The landscape of this area has a honeycomb pattern and the structure has a mirrored Muqarnas vault over the museum or the galleries (Behnejad, Mottaghirad and Jamali, 2012) (Figure 136).

[61] The Iranian Revolution refers to events involving the overthrow of the Pahlavi dynasty and its eventual replacement with an Islamic republic.

Motif-based analysis of Azadi Tower

Pattern 1

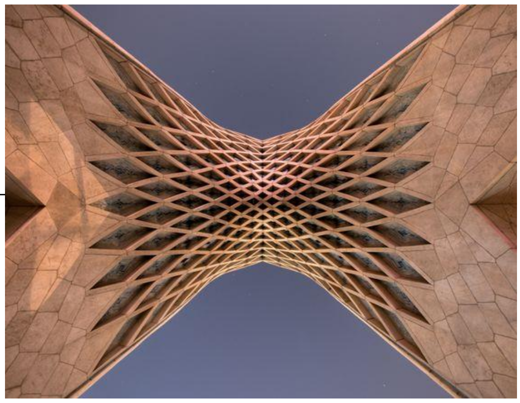


Figure 136. Azadi Tower

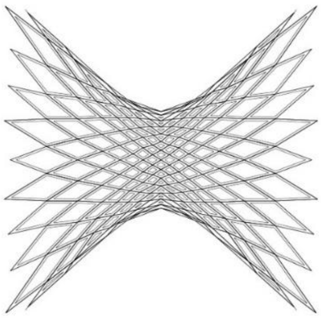


Figure 137. The design of the Azadi Tower

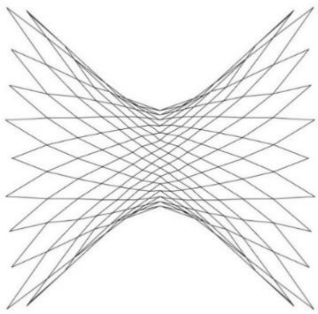
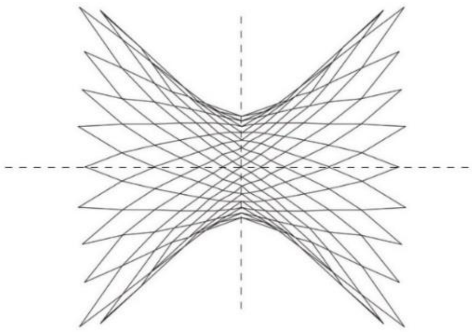


Figure 138. General form of the pattern

According to Hosein Amanat, the plane projection of Azadi tower's pattern is derived directly from Muqarnas pattern; therefore, the designing method of Azadi Tower is somehow similar to the historical Muqarnas patterns. In this case, the analysis of this pattern is evaluated as a rectangular tiled composition (Figure 138). The pattern can be divided into four sectors that are created by repeating and rotating the basic module which is called module pattern.

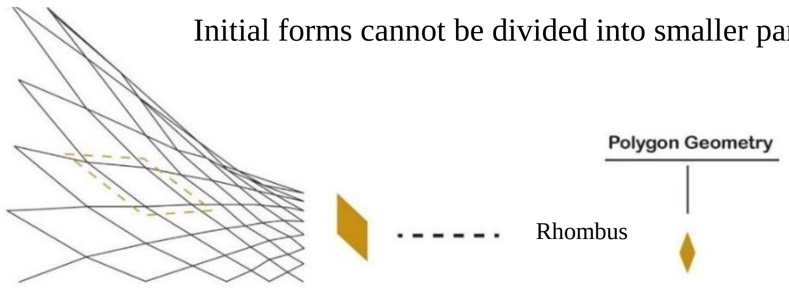


Decomposition of the pattern to its sectors

The Patterns are drawn by author

1 Decomposition of the basic module into it’s initial forms

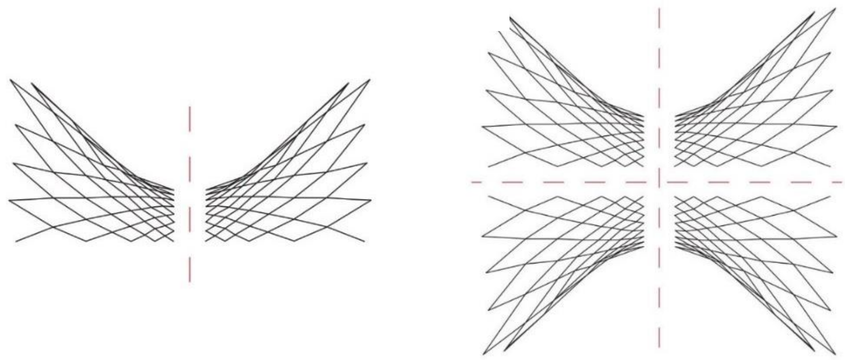
Initial forms cannot be divided into smaller parts



Module pattern of Azadi Tower

Module pattern or the most basic sector of Azadi tower does not have regular shapes. All of the elements of this sector are rhombuses which they all have differentiations in their scales, forms, directions… Creating forms like the basic sector of Azadi tower can be possible only with computer aided design systems.

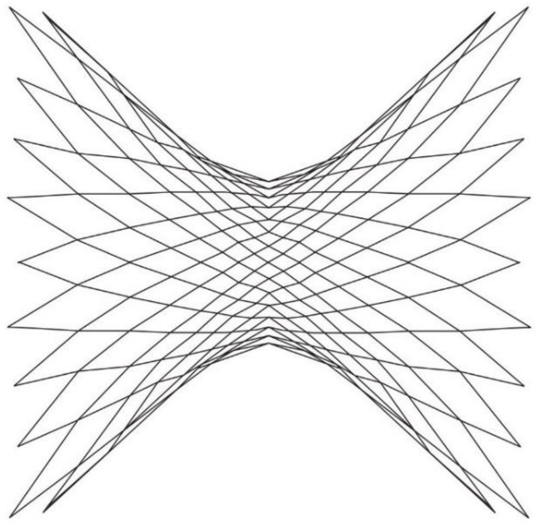
2 The process of accessing basic module



Module is mirrored 90°   Mirror 180°

The Patterns are drawn by author

③ Whole to part relation of Muqarnas

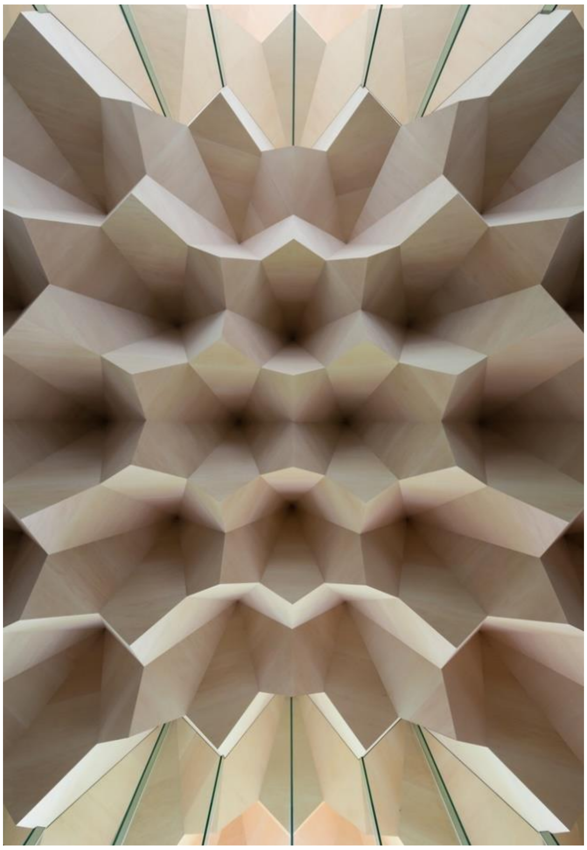


The rotational symmetry of the Azadi tower' pattern

As it is obvious from the pattern of Azadi tower, the plane projection of this pattern is rotationally symmetric around its center exactly like historical Muqarnas patterns. According to the second step in this analyzing process, the whole pattern is created by rotating the basic sector through 90 degrees and 180 degrees respectively.

The rotational symmetry of the Azadi tower pattern shows that, in contrary to historical patterns that all are created from the simplest forms, this pattern was created as a whole initially. Because according to the generative aspects of its basic sector, it is not easy to achieve this pattern like traditional ways from part to whole.

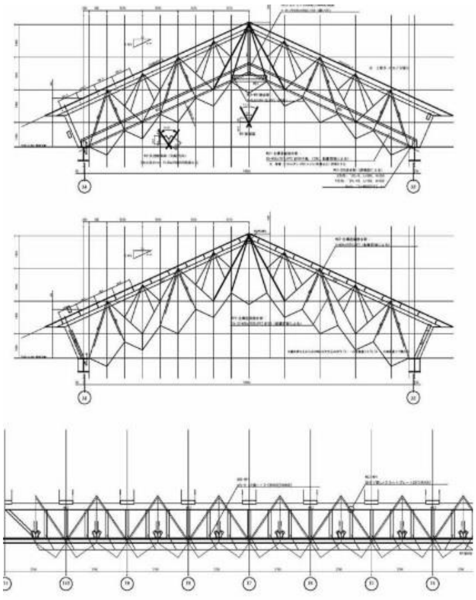
Miyahata Ruins Museum



Architects: Furuichi and Associates

Location : Fukushima, Japan

Project Year : 2015



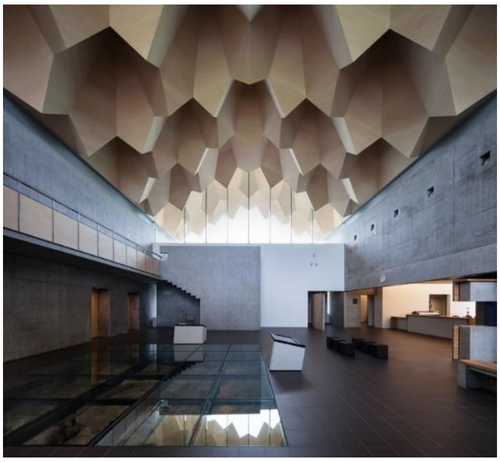


Figure 139. Miyahata Ruins Museum

Source: www.archdaily.com

Pattern 2; Miyahata Ruins Museum

The internal space of Miyahata Ruins Museum has an impressive roof structure which seems to have isometry types on its roof plan design therefor, it has been selected to be analyzed in this section. The design of the roof of Miyahata Ruins Museum is the subject of this study to find the relationship between this design and the historical Muqarnas patterns. According to the rotational symmetry of the Miyahata Ruins Museum, because of the irregular forms of its module pattern which are illustrated as a three-dimensional composition of its roof in Figure 140, the logic behind this pattern’s initial idea has some differences and similarities with historical Muqarnas patterns that will be discussed later.

The pattern analysis of this design will be presented in the following pages to disclose the relation between this form with the historical Muqarnas patterns. The two-dimensional pattern of the roof of Miyahata Ruins Museum will be analyzed as a plane projection of its design.

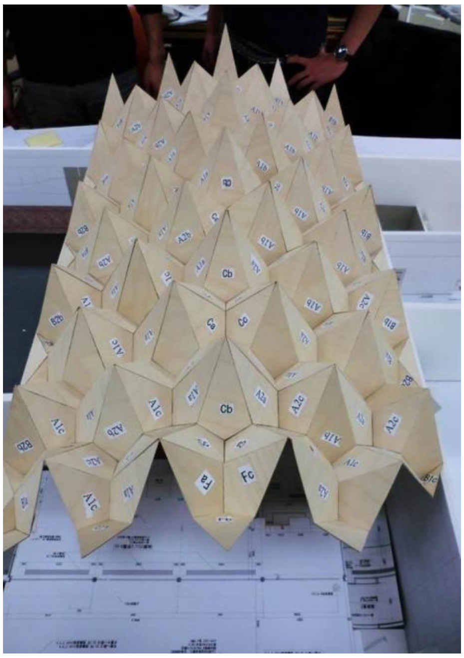


Figure 140. Miyahata Ruins Museum (Web 48)

Motif-based analysis of Miyahata Ruins Museum

Pattern 2

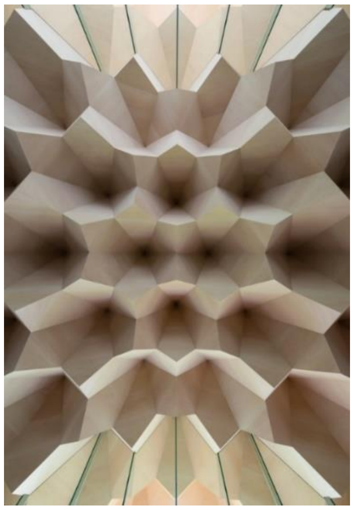
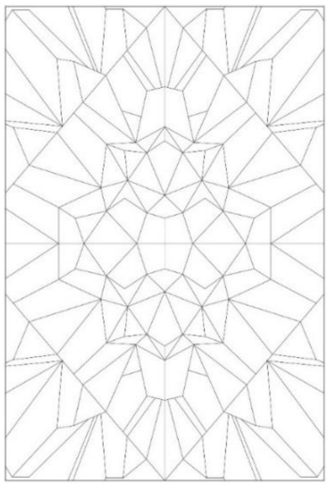


Figure 141. Miyahata Ruins Museum



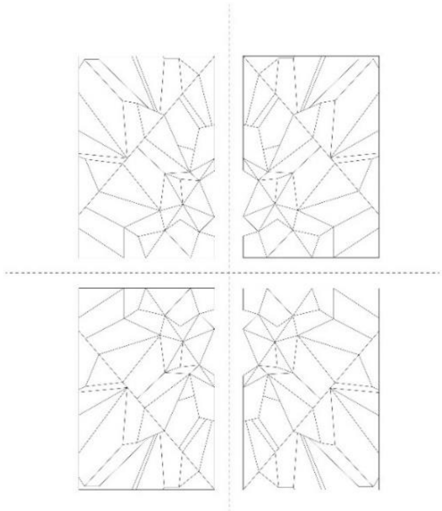


Figure 142. The design of the Muqarnas tower   Figure 143. Decomposition of the pattern to its sectors

The Patterns are drawn by author

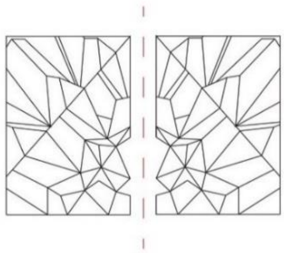
The analysis of Miyahata Ruins Museum pattern is evaluated as a rectangular tiled composition (Figure 142). Like Azadi Tower pattern The pattern can be divided into four sectors that are created by repeating and rotating the basic module which is called module pattern.

1 Decomposition of the basic module into it’s initial forms

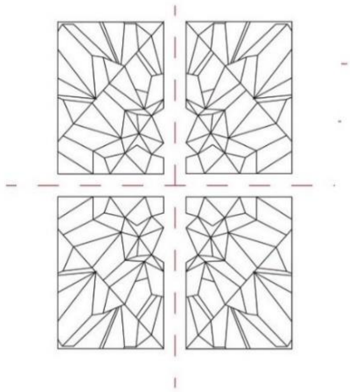
Initial forms also can be divided into small parts

Module pattern of Miyahata Ruins Museum

2 The process of accessing basic module

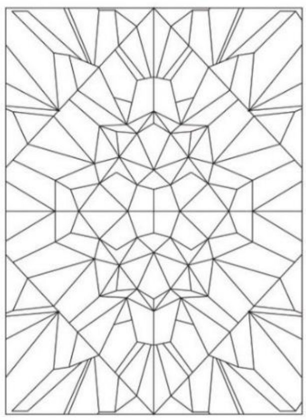


Mirror 90°



Mirror 180°

3



This form also is a kind of pattern which is reached part from the whole. On the other hand, it seems that at first designer created this form as a whole.

The rotational symmetry of the Miyahata Ruins Museum

The Patterns are drawn by author

Chhatrapati Shivaji International Airport– Terminal 2

Architects: Skidmore, Owings&Merrill

Location: India

Project Year: 2014

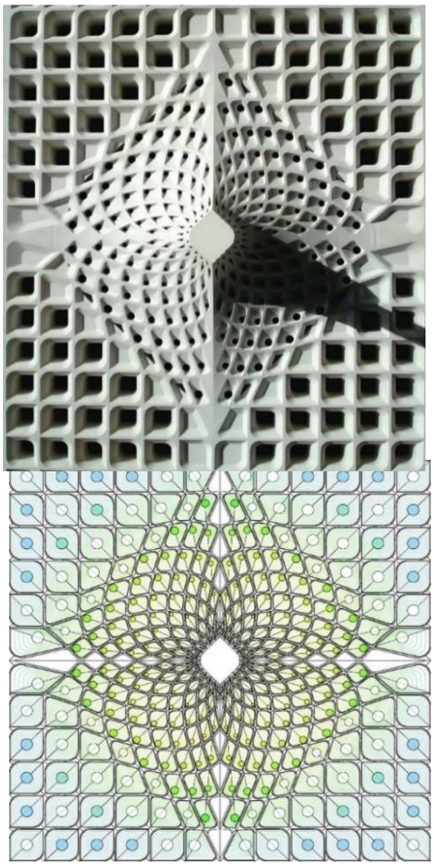


Figure 144. Chhatrapati Shivaji International Airport (Web 49)

Pattern 3; Chhatrapati Shivaji International Airport

The designer of Chhatrapati Shivaji International Airport Terminal 2 declared that the design of this structure is inspired by the form of traditional Indian pavilions and tries to respond to the local setting, history, and culture of the area. However, Muqarnas was not the concept of this design but, the Feature Column model of this structure has a specific design which led this study to assume this design as a generation of Muqarnas. The plan of the column of this structure seems to have isometry types and a symmetric plan.

The pattern analysis of this design will be presented in the following pages to disclose the relation between this form with the historical Muqarnas patterns. The two-dimensional pattern of the plan of Feature Column model of Chhatrapati Shivaji International Airport Terminal 2 will be analyzed as a plane projection of its design. Figure 145.

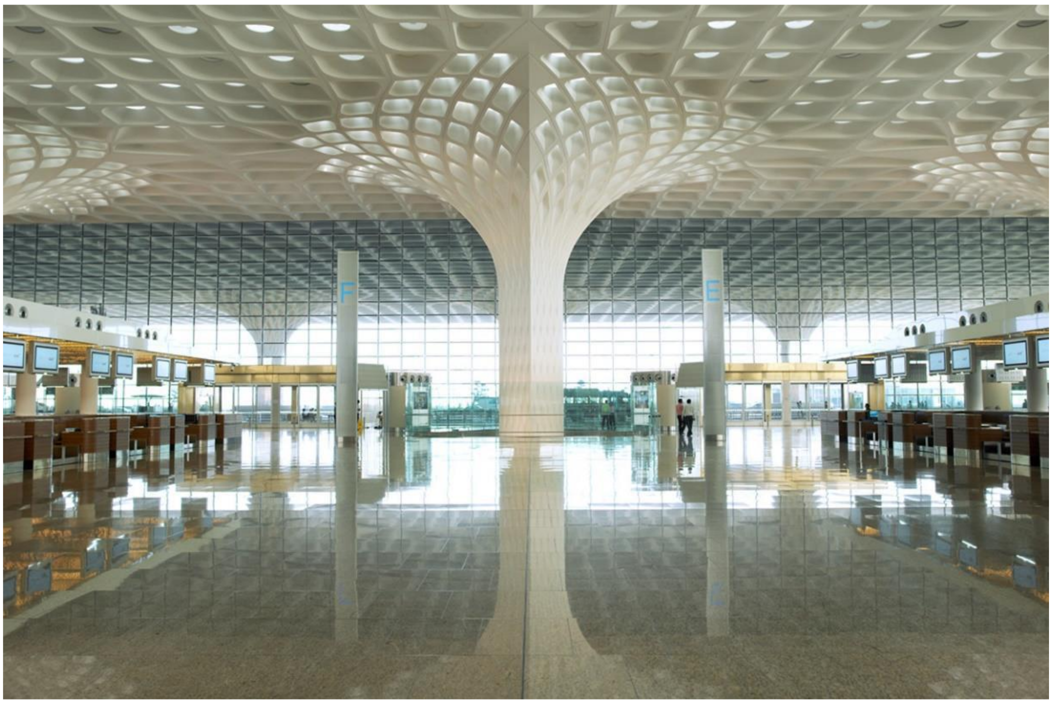


Figure 145. Chhatrapati Shivaji International Airport (Web 50)

Motif-based analysis of Chhatrapati Shivaji International Airport

Pattern 3

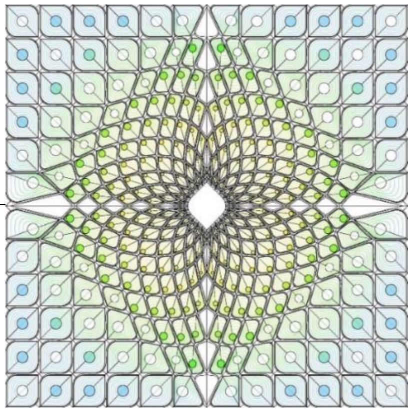


Figure 146. Chhatrapati Shivaji International Airport

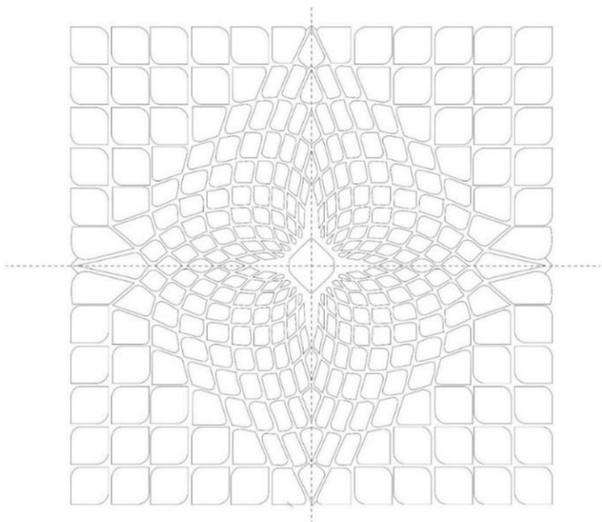


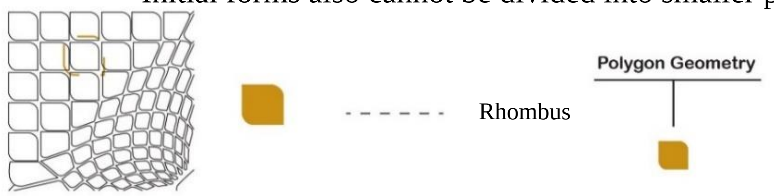
Figure 147. Decomposition of the pattern to its sectors

The Pattern is drawn by author

The analysis of Chhatrapati Shivaji International Airport pattern is exactly like Azadi tower’s pattern and evaluated as a rectangular tiled composition (Figure 147). The pattern can be divided into four sectors that are created by repeating and rotating the basic module which is called module pattern.

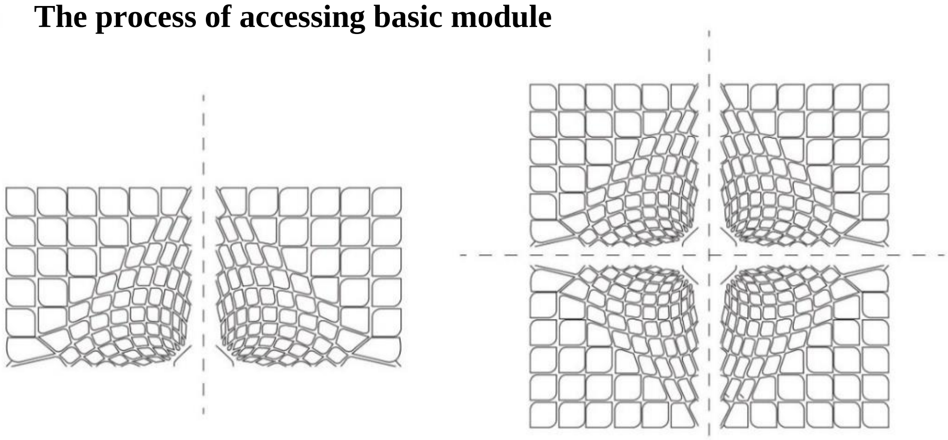
1 **Decomposition of the basic module into it’s initial forms**

Initial forms also cannot be divided into smaller parts



Module pattern of Chhatrapati Shivaji International Airport

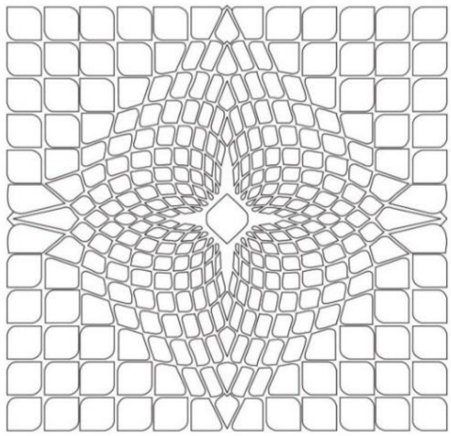
2 **The process of accessing basic module**



Module is mirrored 90°

Mirror 180°

3



The rotational symmetry of the Chhatrapati Shivaji

The Patterns are drawn by author

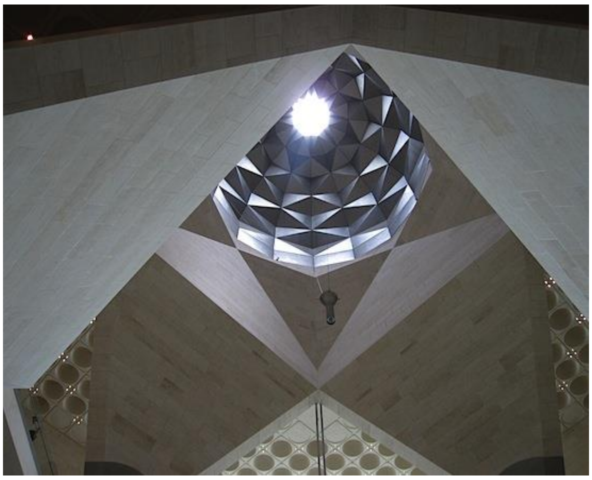
The rotational symmetry of the pattern shows that, exactly like Azadi tower’s pattern, this pattern was created as a whole initially with CAD systems. According to the generative aspects of its basic sector, it is impossible to achieve this pattern like traditional ways from part to whole.

Museum of Islamic Art, Doha

Architects: I. M. Pei

Location : Doha, Qatar

Project Year: 2008



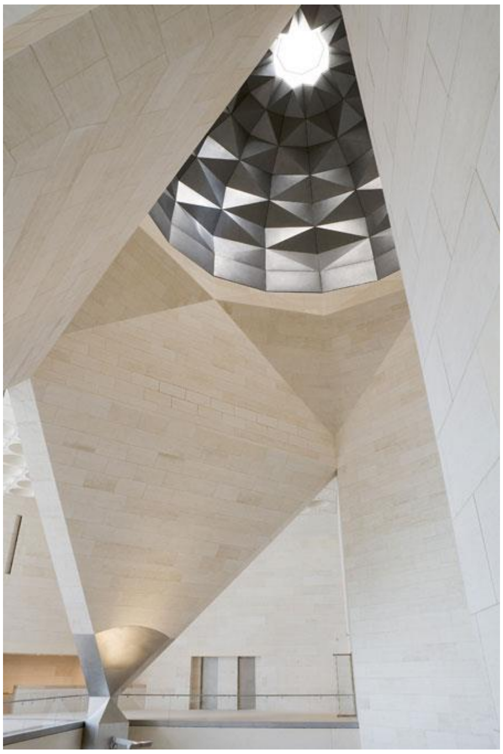


Figure 148. Museum of Islamic Art, Doha (Web 51)

Pattern 4; Museum of Islamic Art, Doha

The design of the Museum of Islamic Art in Doha, is inspired by the Islamic architectural principles of light, geometry and water. The general plan of this building is a square surrounded by an octagon and a circle. The main subject of this section is the designing of the dome above the atrium of this Museum. The dome's has a symmetric plan and the main function of this dome is to reflect sunlight inside the building. As mentioned in chapter 2, one of the functions of Muqarnas dome in traditional architecture was also reflecting and bringing sunlights inside the holy places or the buildings that Muqarnas was used on them.

The dome above the atrium has some geometrical regulations of Muqarnas. However, the general look of this form reminds the viewer of Muqarnas, but this study tries to disclose the similarity and differences between this form as a modern form with historical Muqarnas compositions by the analyzing way in next pages.

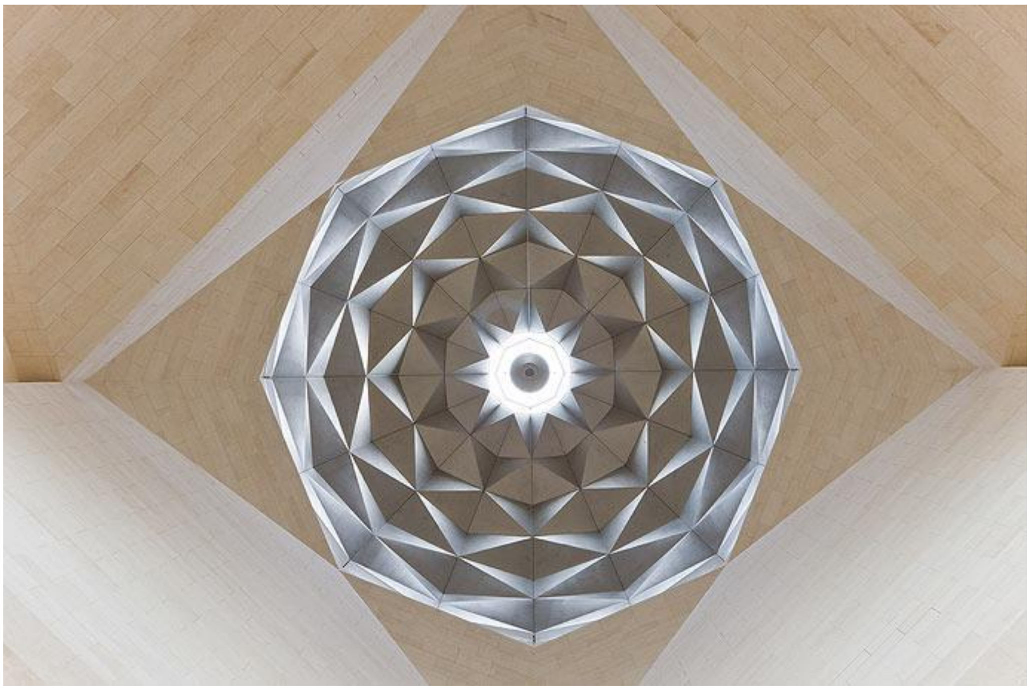


Figure 149. The dome of Museum of Islamic Art (Web 52)

Motif-based analysis of Museum of Islamic Art

Pattern 4

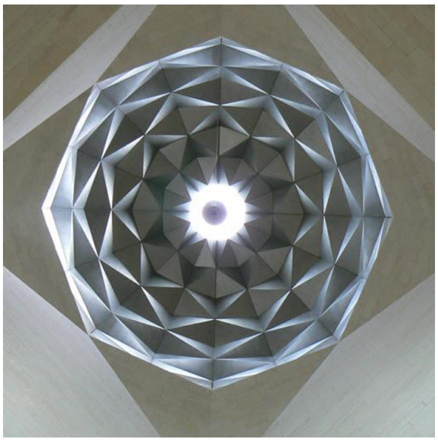


Figure 150. Museum of Islamic Art

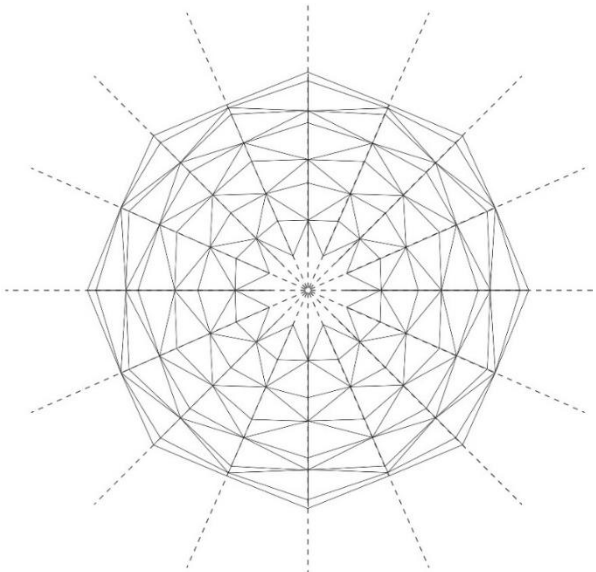
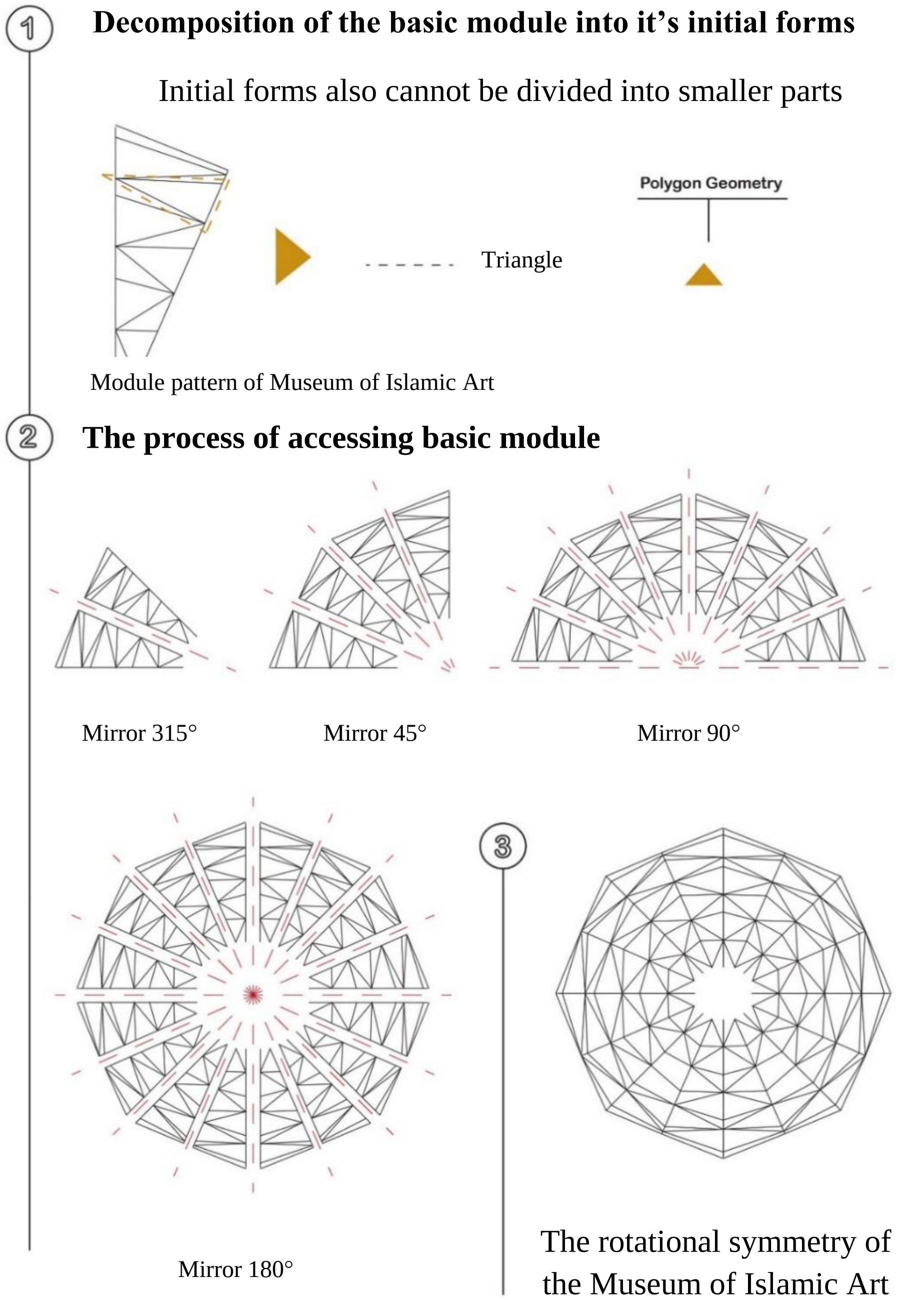


Figure 151. Decomposition of the pattern to its sectors

Pattern is drawn by author

The analysis of Museum of Islamic Art pattern is evaluated as a hexadecane tiled composition (Figure 151). The pattern can be divided into sixteen sectors that are created by repeating and rotating the basic module which is called module pattern.



The Patterns are drawn by author

The pattern of Museum of Islamic Art is the most apparently similar pattern with historical ones. However, it seems that it could be a pattern which reaches the whole from the part, but also, in this case, the module pattern dose not have regular forms. It just consists of triangles with different scales and forms.

The whole pattern of Azadi Tower, Miyahata Ruins Museum, Chhatrapati Shivaji International Airport and Museum of Islamic Art are created by rotating the basic sector through some degrees. The designing method of these modern examples can be considered very similar to the analysis way that we used for the analysis of historical Muqarnas pattern in two styles of Takahashi; square lattice Muqarnas or other styles. As it is obvious from the Figure 152 and Figure 153 which were analyzed in the previous section, these patterns can be considered as new generations of historical Muqarnas patterns which have circular symmetry.

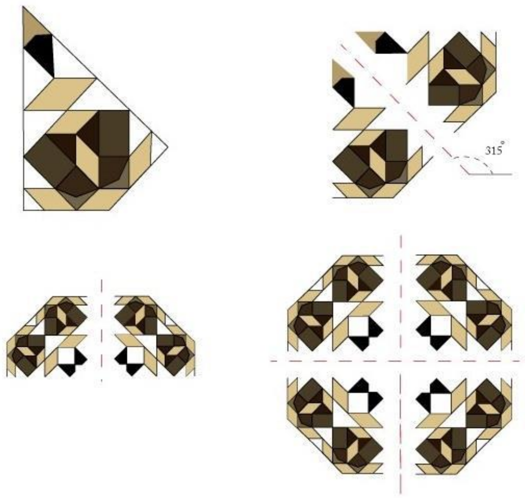


Figure 152. Tinmal Mosque (Square lattice Muqarnas)

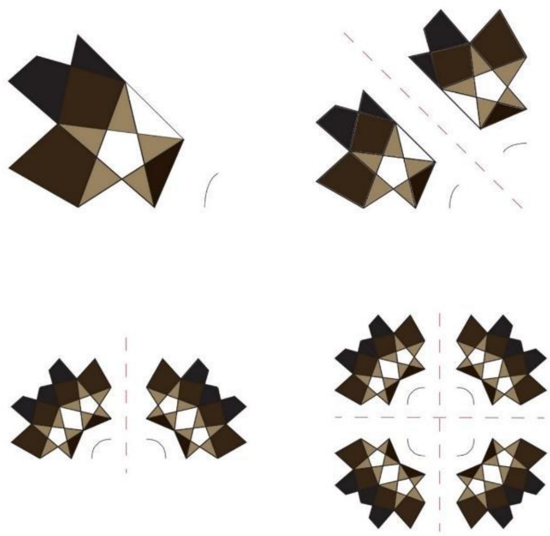
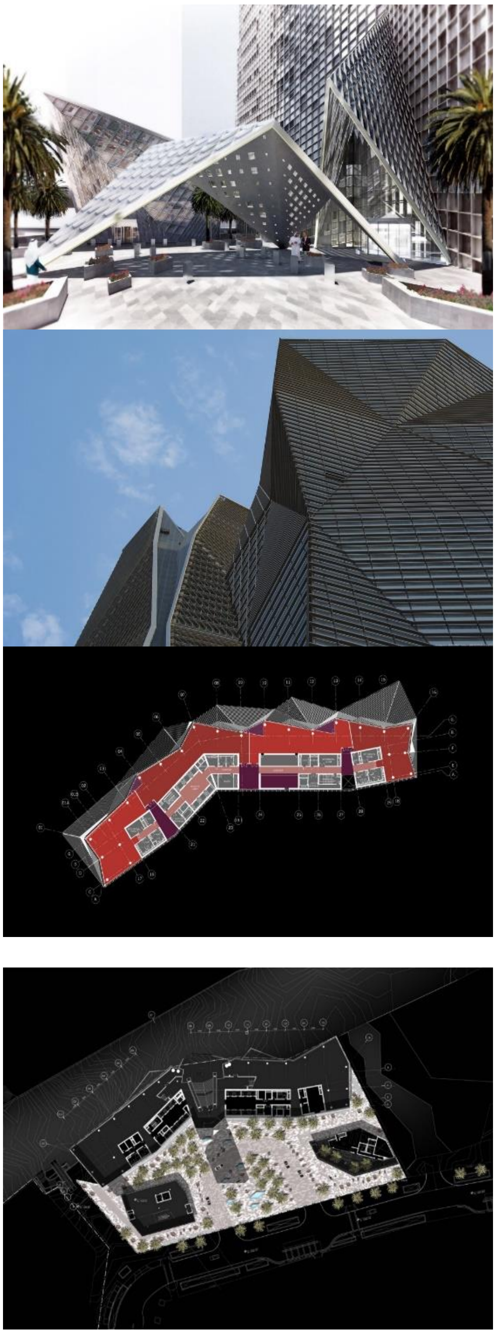


Figure 153. Selimiye Mosque (Other style of Muqarnas)

Muqarnas Tower



Architects: Skidmore, Owings & Merrill

Location: Riyadh Saudi

Project Year: 2013

Figure 154. Muqarnas Tower (Web 53)

Pattern 5; Muqarnas Tower

The Muqarnas Tower is a commercial office building that is constructed for the King Abdullah. The facade of this building is inspired by historical Muqarnas vaulting technique and provide the shading which is needed in the Saudi Arabia's climate.

The facade geometry of Muqarnas tower building conveys the isometry types and gemetrical regulations of historical Muqarnas on it. However, the facade of Muqarnas Tower cannot be considered as a simulation of historical Muqarnas compositions but, it can be considered as a modern generation of them. The pattern analysis of this design will be presented in the following pages to disclose the relation between this form with the historical Muqarnas patterns.

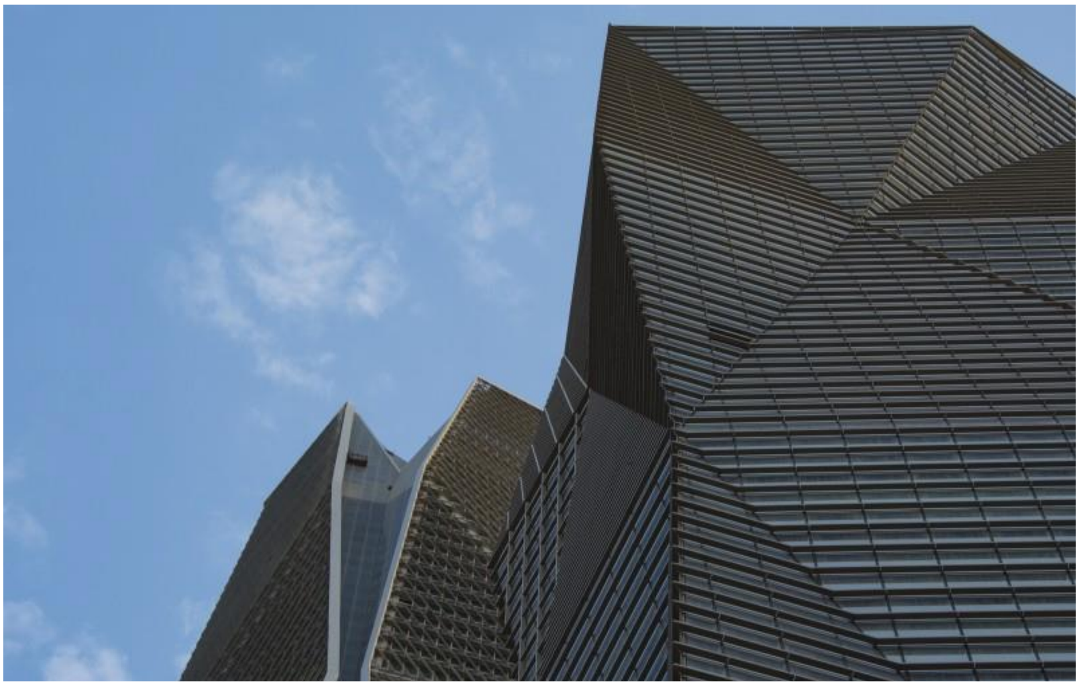


Figure 155. The facade of Muqarnas Tower (Web 54)

Motif-based analysis of Muqarnas Tower

Pattern 5

Figure 156. Muqarnas tower

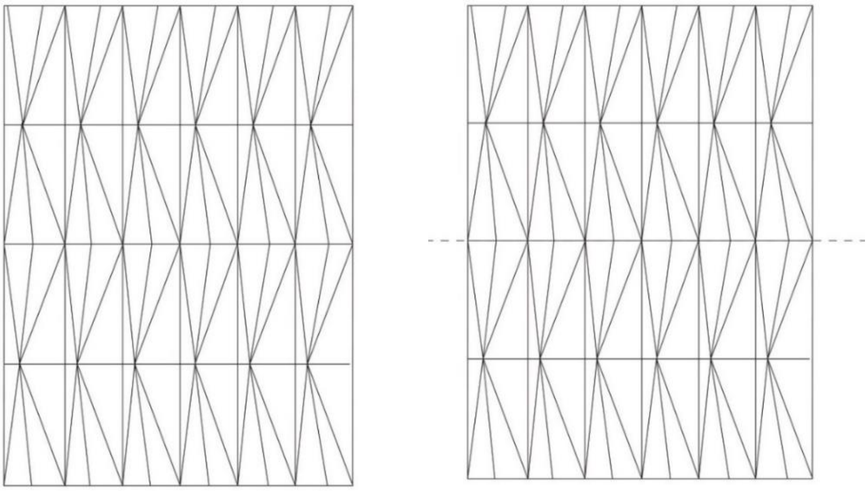


Figure 157. The design of the Muqarnas tower and Decomposition of the pattern to its basic sectors

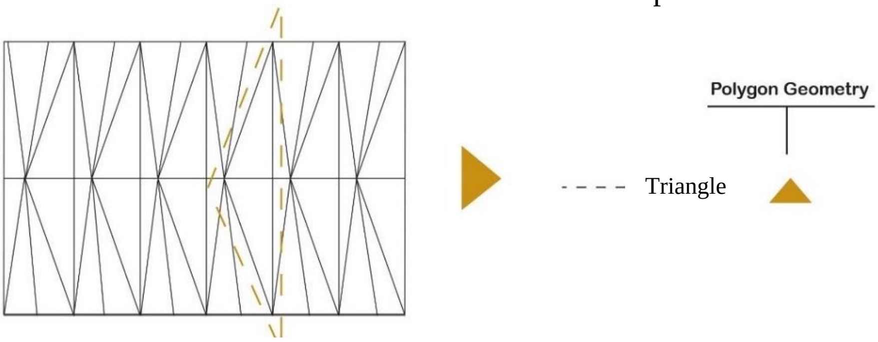
The Patterns are drawn by author

In contrary with historical Muqarnas patterns, the pattern of Muqarnas Tower which has the Muqarnas characteristics on its façade, is not rotationally symmetric around its center. It just consists of repeating and rotating a basic sector in rows.

1

Decomposition of the basic module into it's initial forms

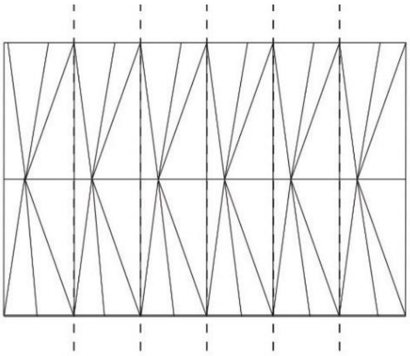
Initial forms also can be divided into smaller parts

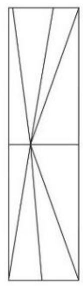


Module pattern of Muqarnas Tower

2

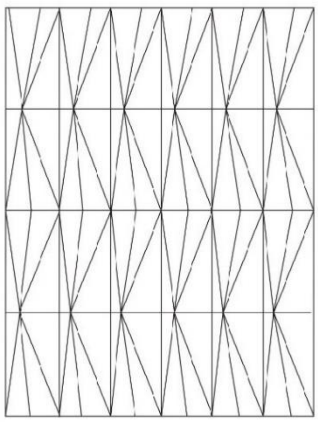
The process of accessing basic module





Module pattern

3



The symmetrical pattern of the Muqarnas Tower totally has different designing logic with historical ones. There is a module pattern which consists of triangles with different scales and forms like other modern patterns, but in this case, it seems that this pattern has the simplest plan among all of the modern patterns that this study has tried to analyze.

The symmetrical pattern of the Muqarnas tower

The Patterns are drawn by author

Hyperbolic Installation Based on Persian Patterns and Muqarnas

Architects: Vincent Pocsik, Zach Schoch, Mike Gross, and Zifan Liu

Location : SCI-Arc, USA

Project Year : 2010

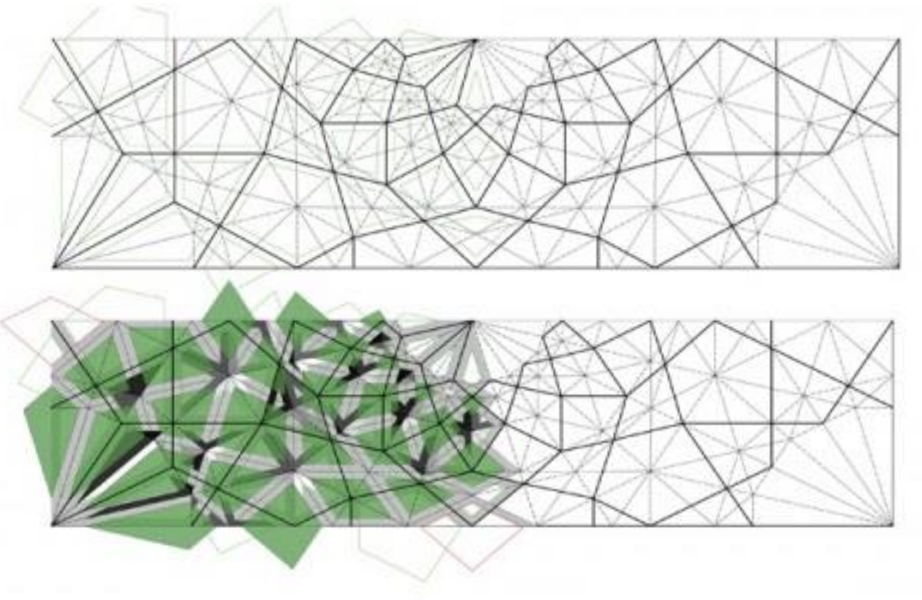


Figure 158. Hyperbolic Installation Based on Persian Patterns and Muqarnas (Web 55)

Pattern 6; Hyperbolic Installation

The Hyperbolic Installation is designed by SCI-Arc students for a purpose of research on ornamental techniques used within traditional Islamic and Persian architecture. According to the designers of this modern form, Muqarnas was the concept of this design. In order to produce this geometrical volume, the designers used Scripting method in parallel to manual two-dimensional geometric patterning which was widely discussed in chapter 3. This form consists of different pieces like the elements of Muqarnas with different scales and shapes. This form also can not consider as a simulation of historical Muqarnas examples but, it is obvious from the plan of the design that this form conveys geometrical regulations of Muqarnas.

The pattern analysis of this design will be presented in the following pages to disclose the relation between this form with the historical Muqarnas patterns.

Figure 159. Hyperbolic Installation (Web 56)

Motif-based analysis of Hyperbolic Installation

Pattern 6

Figure 160. Hyperbolic Installation Based on Persian Patterns and Muqarnas

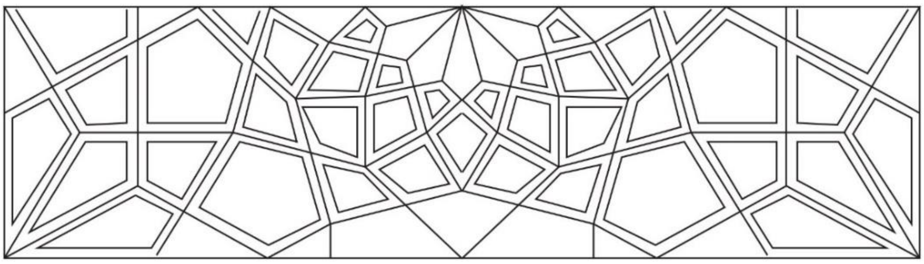


Figure 161. The design of Hyperbolic Installation Based on Persian Patterns and Muqarnas

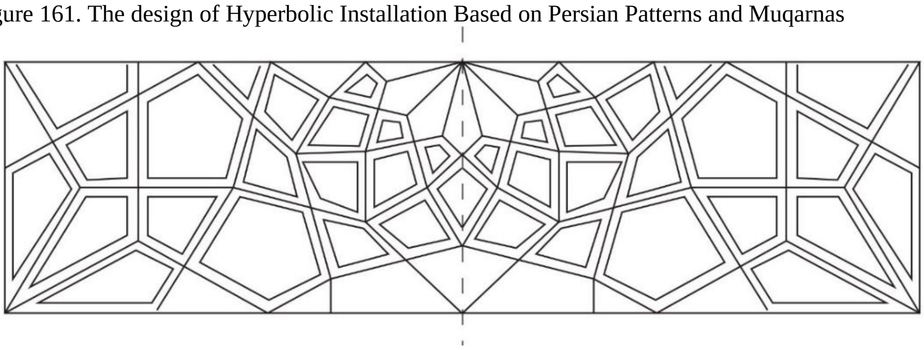


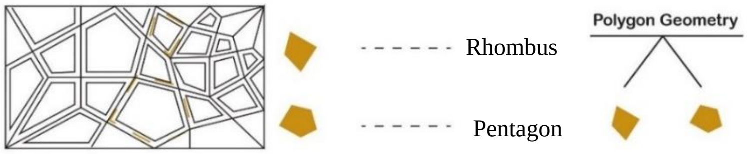
Figure 162. Decomposition of the pattern to its sectors

The Patterns are drawn by author

The analysis of Hyperbolic Installation pattern is evaluated as a rectangular tiled composition (Figure 161). The pattern can be divided into two sectors that are created by rotating the basic module which is called module pattern (Figure 162).

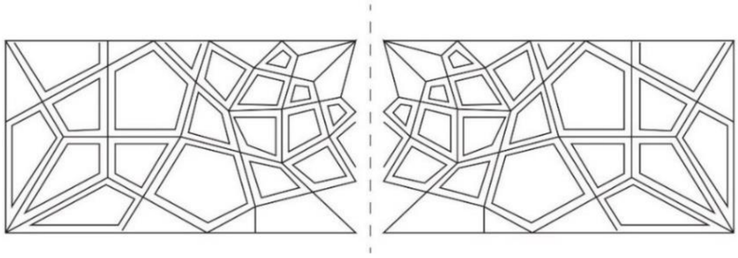
1 Decomposition of the basic module into it’s initial forms

Initial forms also cannot be divided into smaller parts



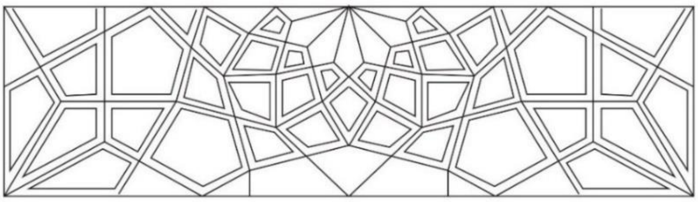
Module pattern of Hyperbolic Installation

2 The process of accessing basic module

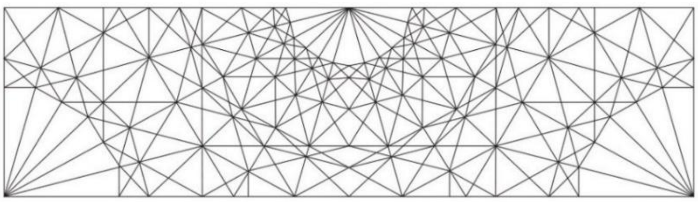


Mirror 90°

3



The symmetrical pattern of the Hyperbolic Installation



The symmetrical pattern of the Hyperbolic Installation with details

The Patterns are drawn by author

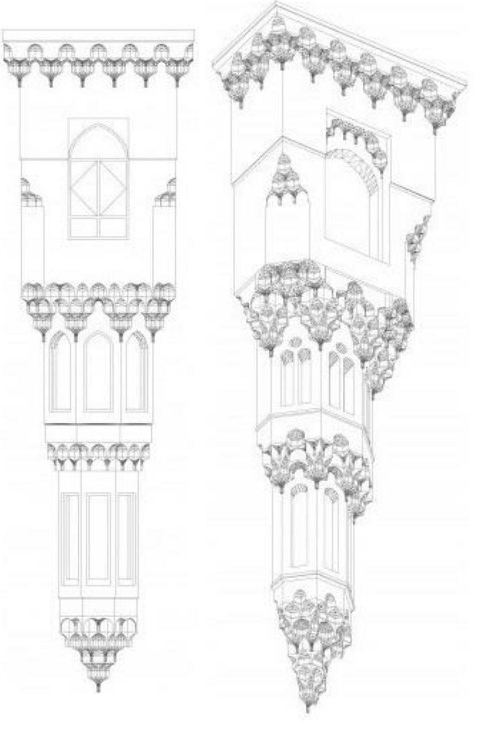
The whole pattern has a generative design and creating it without using CAD systems seems to be really tough but also this case has a rotation guideline of historical patterns. Connecting and interlacing the basic elements are same with historical ones.

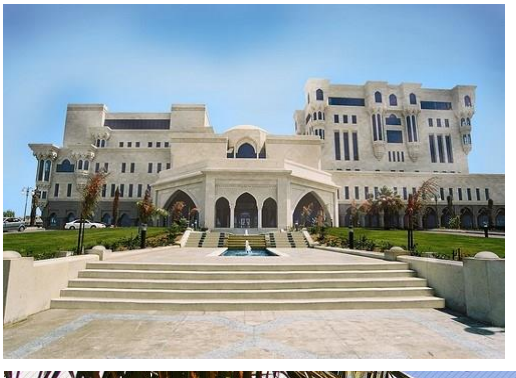
International Medical Center

Location : Jeddah, Saudi Arabia

Architects: Dr. Sami Angawi

Project Year: 2005





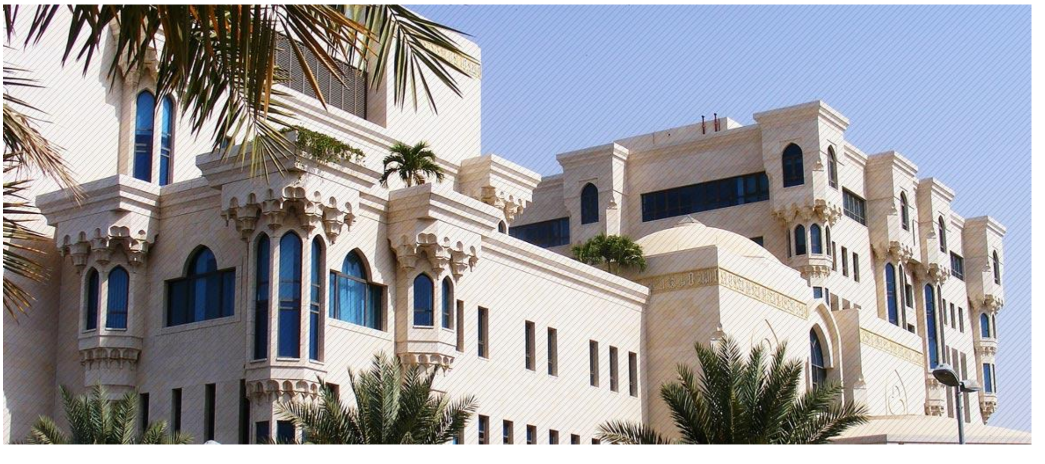


Figure 163. International Medical Center (Web 57)

Pattern 7; International Medical Center

According to the information that is presented in www.bonner-design.com website about the design of International Medical Center:

> "International Medical Center in jaddeh is designed by the Saudi architect Dr. Sami Angawi. It is intended that this hospital demonstrate the appropriateness of using extensive traditional Islamic ornament in contemporary civic architectural projects in the Middle East. Jay Bonner was contracted to design the exterior and interior ornament for this hospital. This building called for the extensive use of traditional ornamental features and a combination of both traditional and state-of-the-art manufacturing technologies. Jay Bonner's work on this project included the extensive use of originally designed muqarnas for both the interior and exterior of the building ..."

This section tries to consider the windows of this buildings as a new generation of Muqarnas and the pattern analysis of this design will be presented in the following pages to disclose the relation between th is form with the historical Muqarnas patterns.

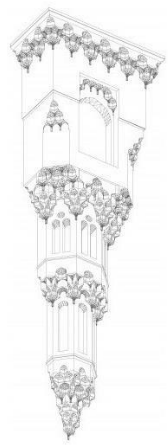


Figure 164. The design for the projecting muqarnas by windows (Web 58)

Motif-based analysis of International Medical Center

Pattern 7

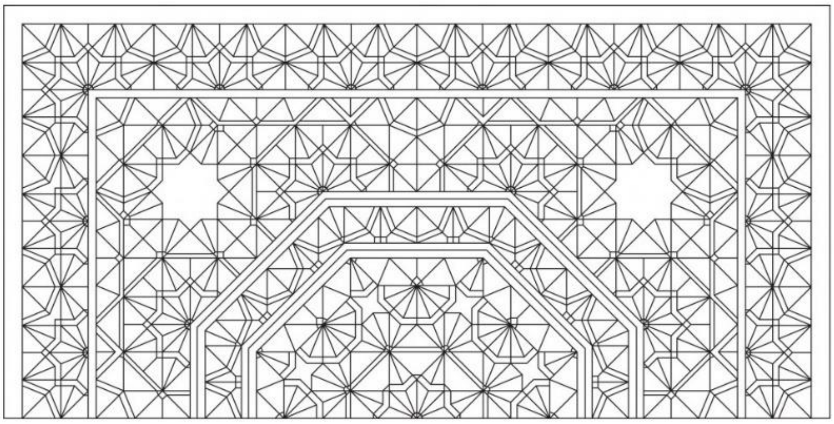


Figure 165. International Medical Center

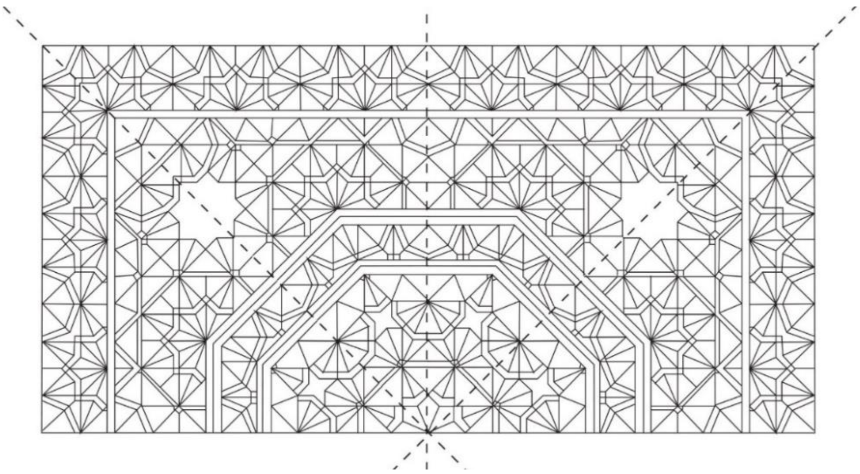


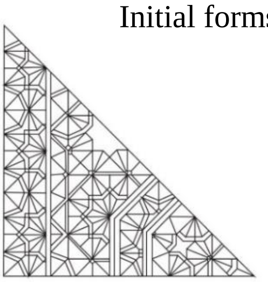
Figure 166. Decomposition of the pattern to its sectors

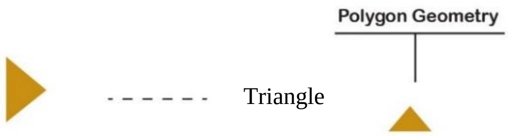
The Patterns are drawn by author

The analysis of International Medical Center pattern is evaluated as a rectangular tiled composition (Figure 165). The pattern can be divided into four sectors that are created by rotating the basic module which is called module pattern. The design guidelines of this pattern seems to be same with historical ones.

1 Decomposition of the basic module into it’s initial forms

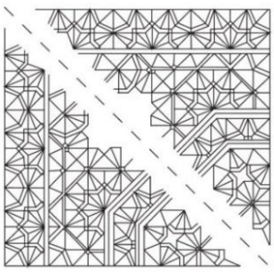
Initial forms also can be divided into small parts

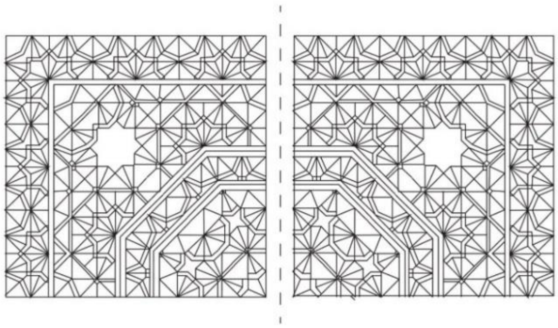




Module pattern of International Medical Center

2 The process of accessing basic module

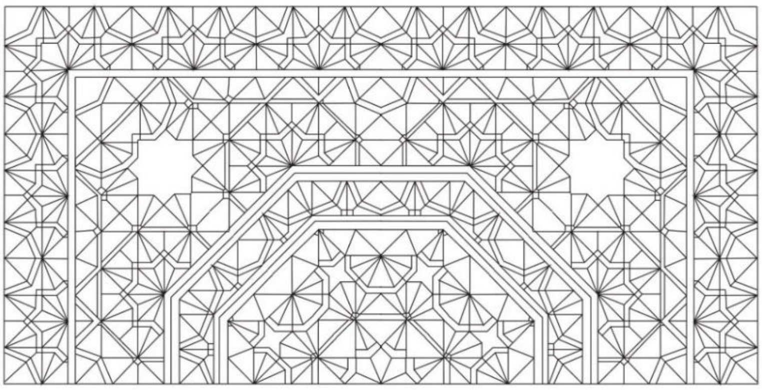




Mirror 315°

Mirror 90°

3



The rotational symmetry of the International Medical Center

The Patterns are drawn by author

The whole pattern has a complex design which creating it without using CAD systems seems to be really tough but also this case has a rotation guideline of historical patterns. Connecting and interlacing the basic elements are same with historical ones.

The whole pattern of Muqarnas Tower, Hyperbolic Installation and International Medical Center are created by rotating the basic sector through degrees. The designing method of these modern examples can be considered similar to the analysis way that we used for the analysis of historical Muqarnas patterns in two styles of Takahashi; Pole table Muqarnas or other styles but in comparision with the first four modern forms, these three examples have different designing methods with historical ones. As it is obvious from the Figure 167 and Figure 168 which were analyzed in the previous section, these patterns can be considered as new generation of historical Muqarnas patterns.

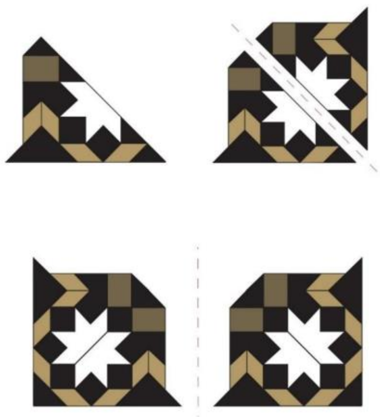
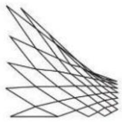
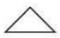
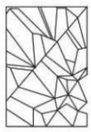
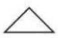
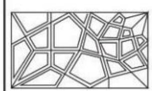
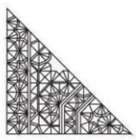


Figure 167. Zisa Castle (Pole table Muqarnas)

Figure 168. Sahabiye Madrasse (Other style of Muqarnas)

For six historical Muqarnas patterns, we defined guidelines which illustrated the characteristics of Muqarnas patterns as an analytical table. The other analytical study presented with the same characteristics for modern forms.

Table 7. Geometrical characteristics of modern patterns

Pattern's buildings	The most basic element	Isometry types of basic elements	Basic module	Types and number of basic module's elements		Isometry types of module	Number of layers
pattern 1 Azadi Tower		—		—	0	Mirror 90 ° Mirror 180 °	7
pattern 2 Miyahata Ruins Museum		—		—	0	Mirror 90 ° Mirror 180 °	3
pattern 3 Chhatrapati Shivaji		—		—	0	Mirror 90 ° Mirror 180 °	8
pattern 4 Museum of Islamic Art		—		—	0	Mirror 315 ° Mirror 45 ° Mirror 90 ° Mirror 180 °	5
pattern 5 Muqarnas Tower		—		 ------ Triangle	1	Rotation 0 °	1
pattern 6 Hyperbolic Installation	 	—		—	0	Mirror 90 °	—
pattern 7 International Medical Center		—		—	0	Mirror 315 ° Mirror 90 °	4

The table is drawn by author

4.3.2 The evaluation of modern forms based on analytical studies

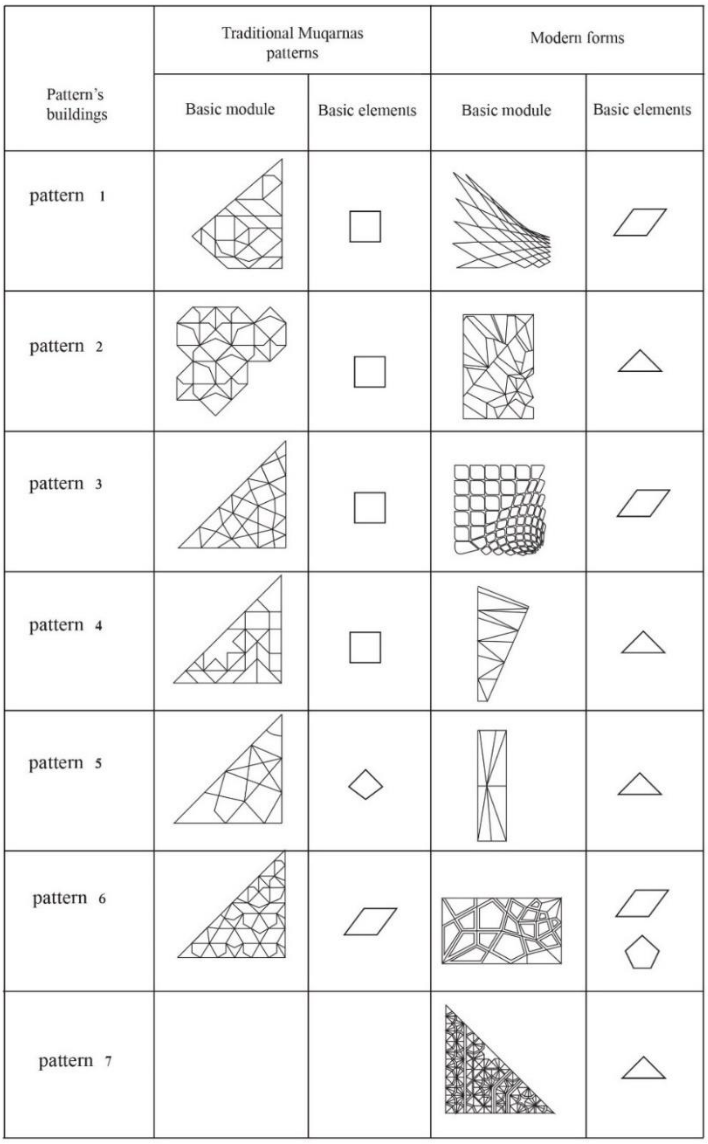
This chapter tries to set design analytical studies, based on historical and traditional Muqarnas compositions and the modern forms in order to study the alteration process of the geometrical characteristics of them over years. The main questions of this study are: the differences and similarities between modern forms and historical Muqarnas compositions and the aspects of modern forms that make them perceived as the new generation of Muqarnas.

According to the descriptive and analytical studies that represented the geometrical and tectonic characteristics of the historical Muqarnas compositions, it seems that historical Muqarnas patterns convey some mathematical characteristics that are the same in all of the Muqarnas patterns. Types of isometry is the prominent and the most general characteristic of these patterns. Translation, Reflection, Rotation and Glide Reflection are four features of isometry types which all the historical Muqarnas compositions have at least two or three of them in their pattern design. Therefore, the modern group in this chapter was firstly selected among those forms which have isometry types in their plan, facade or in their three-dimensional compositions.

According to these characteristics, any patterns or forms that carry those four characteristics could be considered as a Muqarnas pattern. However, that is not enough reason to call a form as a generation of historical Muqarnas patterns. Therefore, to be more precise on the research questions, this study tried to disclose analytically the geometrical and tectonic characteristics of historic Muqarnas patterns. After analyzing the historical Muqarnas patterns the guidelines that can be considered peculiar to Muqarnas pattern are achieved. These guidelines can make the Muqarnas patterns as the potentials in the development of architectural forms.

After generating the guidelines tables for both historical Muqarnas compositions and modern forms in terms of their two-dimensional patterns designs analysis, this thesis tries to find the differences and similarities between them. According to these tables that presented in this chapter before, it seems that all of the historical Muqarnas patterns and modern forms have basic sectors which they consist of basic elements that are illustrated in Table 6.

Table 8. Comparing basic modules and basic elements of historical Muqarnas with modern forms



Pattern's buildings	Traditional Muqarnas patterns		Modern forms	
	Basic module	Basic elements	Basic module	Basic elements
pattern 1				
pattern 2				
pattern 3				
pattern 4				
pattern 5				
pattern 6				
pattern 7				

The basic elements of both historical patterns and modern ones consist of regular geometric forms. Square and rhumboses were mostly used in historical patterns and triangle and rhumboses are mostly used in modern forms. Therefore, using regular shapes as basic elements can be considered as a characteristic of Muqarnas patterns all of which can be traced in the modern forms. However, in some cases, basic elements of modern forms have irregular shapes but, the idea behind them is driven from a regular shape or they were transformed into irregular shapes through CAD systems and generative design procedures.

Figure 169. Basic elements of taraditional patterns

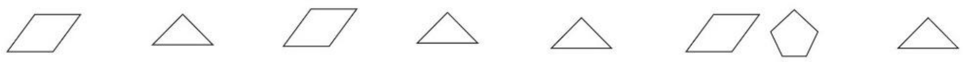
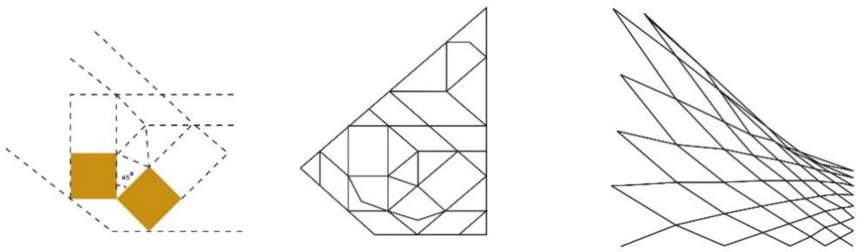


Figure 170. Basic elements of modern forms

As mentioned before, isometry types are the other characteristics of Muqarnas that are the same among historical compositions and modern ones but there is a differentiation between their rotation or reflection degrees. According to Guideline 2 that is mentioned in the previous section, all of the basic elements of historical Muqarnas compositions are rotated through the angles of 45°, 60°, 90° and 180° (Figure 171) but, modern forms don’t have any specific rotation degree in the formation of their basic module.



(a) (b)

Figure 171. Basic module formation process, Tinmal mosque (a), Azadi Tower (b).

All of the modern forms that are analyzed in this section were created through CAD system and it is estimated that most of them had algorithmic design methods. Therefore, according to algorithmic design or parametric design, the designing process is completed via altering the parameters of the entity forms. As Easton (2003) asserts:

> "Most current CAD/CAM/CAE software utilizes a design feature called parametrics, a method of linking dimensions and variables to geometry in such a way that when the values change, the part changes as well. A parameter is a variable to which other variables are related, and these other variables can be obtained by means of parametric equations. In this manner, design modifications and creation of a family of parts can be performed in remarkably quick time compared with the redrawing required by traditional CAD. In the past five years, PTC's success has prompted major CAD players to offer similar functions."

Therefore, modern forms don't have specific rotation or reflection degrees in the formation of their basic modules. Most of the basic elements seem to have parametric design process and the basic elements are generated themselves through altering the process.

According to Guideline 3 and the information that are mentioned in Chapter 3, all the basic elements of the historical Muqarnas patterns were split to constant forms like; square, half-square, rhombus, half-rhombus, almond, jug, large biped, and small biped. The modern forms, in terms of their parametric process and the basic regular forms, have altered smoothly, therefore, they don't keep their regularity during the designing process. Also, Because of the alteration process during the designing procedure, the modern forms don't even have specific numbers of basic module's elements which were minimum 2 and maximum 5 in historical Muqarnas compositions. Thus, the numbers of basic module's elements in modern forms are changing until the entity forms have been obtained.

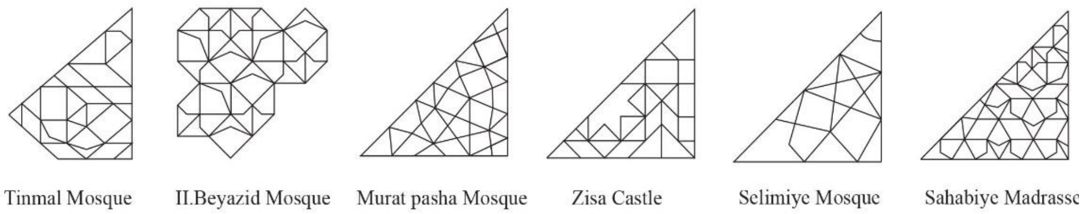


Figure 172. Basic elements of the historical Muqarnas patterns

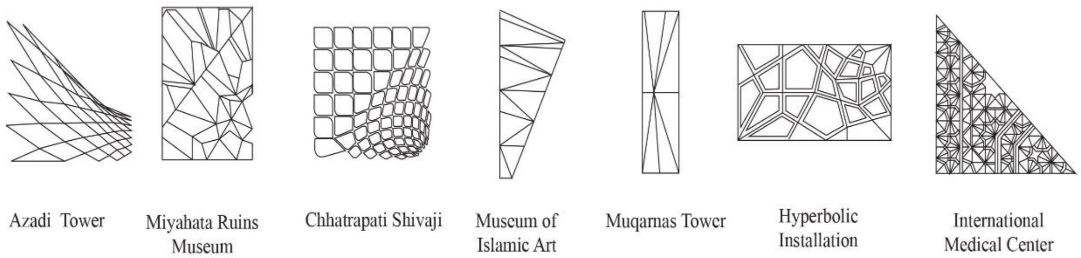
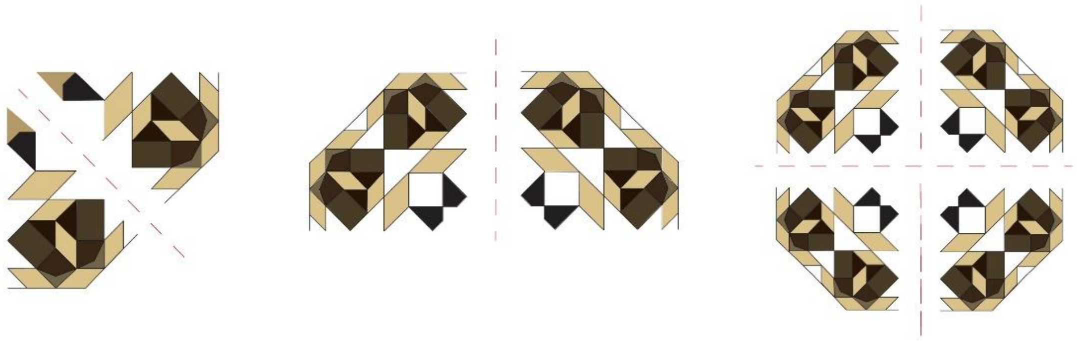


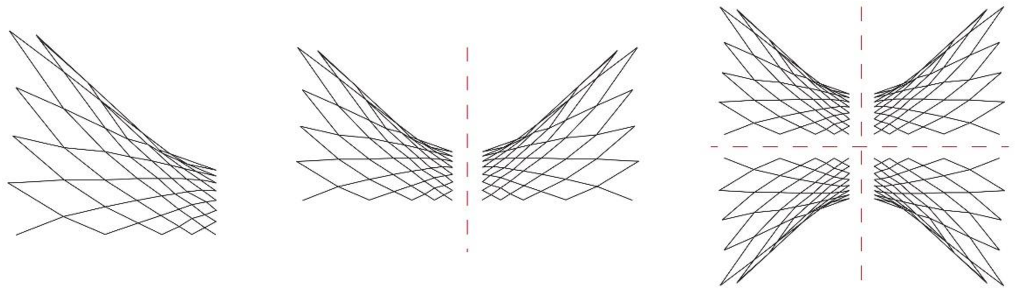
Figure 173. Basic elements of the modern forms

According to the analytical tables, both of the historical patterns and modern forms have basic modules or basic sectors which are mentioned in this section. These basic modules are rotated or reflected according to isometry types regulations and created the whole patterns. Both of historical Muqarnas patterns and modern forms are rotated through the angles of 45°, 90°, 315° and 180° which are mentioned as Guideline 4.



315° 90° 180°

Figure. The rotation process of basic sectors of Tinmal Mosque pattern

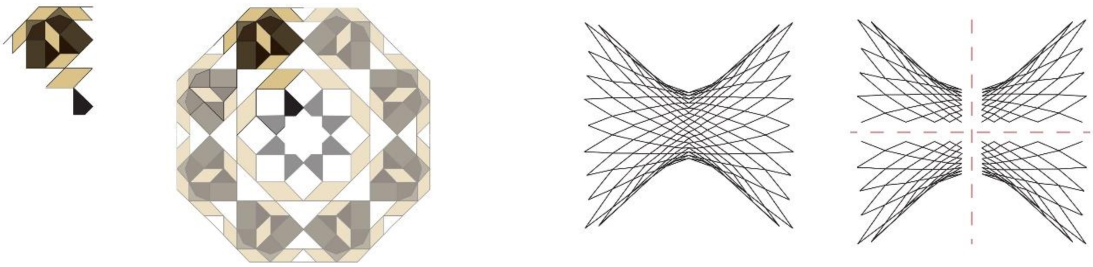


90° 180°

Figure 174. The rotation process of basic sectors of Azadi Tower pattern

According to the results that are derived from Guideline 5, all of the historical Muqarnas patterns can also be analyzed with their construction layers and according to the information that is presented in chapter 3, these layers have limitation. But modern forms rarely have regular layers which can be compared with historical ones. Modern forms are mostly constructed through CAD systems and the process is defined the number of constructing layers without any limitations.

Finally, according to the analyzing studies of this research, designing process of historical Muqarnas patterns seems to be a part to the whole system. It means that the designer or master builder in that time firstly drew the basic module of patterns and then by following the guidelines the final form of the pattern was obtained. But today's designers mostly use CAD systems in their design, therefore, in parametric design or generative design, it seems to achieve the whole pattern firstly. And the process is whole to part.



(a) (b)

Figure 175. Part to whole process, Tinmal Mosque (a), Azadi Tower (b).

Isometry types in plane projection of any Muqarnas pattern in plan, facade or three-dimensional composition can be considered as the aspects of modern forms that make them perceived as new generation of Muqarnas compositions. The six guidelines which are mentioned in this section as the most basic element, isometry types of basic elements, basic module, types and number of basic module's elements, isometry types of module, numbers of layers, define the similarity and differentiation between modern forms with historical Muqarnas patterns.

CHAPTER 5

CONCLUSION

Muqarnas is a three-dimensional decorative element which consists of both concave and flat parts organized in tiers to create the integrated structure as a whole in order to provide a gradual and smooth transition between two levels, two sizes or two shapes. Muqarnas was firstly used as a transitional element in historical religious buildings in Islamic countries but over times it got decorative aspects and even today Muqarnas has been used for decorating architectural buildings. In this thesis, we explored the differences and similarities between modern forms which are inspired by historical Muqarnas compositions and the aspects of modern forms that make them perceived as new generations of Muqarnas.

As technology affected the architecture of the 20^{th} and 21^{st} centuries by CAD systems, technology had an impact in changing, adding and developing Muqarnas forms. Following the developments achieved in digital art and architecture, Muqarnas compositions became the source of inspiration for many designers who try to generate alternative patterns in their works. Today in architecture, there is a developing tendency to re-generate Muqarnas compositions through digital technologies in order to reflect traditional culture symbolically and to be able to benefit from historical patterns of Muqarnas either in facades or interior decorations of modern buildings.

The complexity of geometry of Muqarnas compositions creates many challenges to solve. This study's main goal has been to disclose the geometrical regulations of Muqarnas patterns which were essential and permanent for creating a Muqarnas composition in traditional architecture and define the guidelines which are specific to

Muqarnas patterns. Therefore, by using these guidelines tries to distinguish new forms of Muqarnas patterns of the 21st century also, determines the similarity and differences between historical Muqarnas patterns and the modern forms.

Achieving this goal requires a better understanding of the Muqarnas and its structure. In this regard, Chapter 2 provides a historical background about Muqarnas and chapter 3 gives information about the structures of historical Muqarnas compositions. Based on this knowledge, we provide analytical studies and a set of strategies for both historical patterns and modern forms to overcome the challenges in disclosing Muqarnas regulations and define guidelines that are specific to Muqarnas compositions. The approach presented is based on the two-dimensional plane projection of Muqarnas and aims to concentrate analytically on the alteration process of these geometrical characteristics over years.

For disclosing geometrical characteristics of Muqarnas patterns, two analytical ways are suggested in this study; motif-based analysis of a Muqarnas pattern and layer-based analysis of a Muqarnas pattern. The first way is aimed to decompose each Muqarnas pattern to its split forms and the second way tries to decompose each pattern in terms of its layer compositions. These analytical studies try to decompose two-dimensional Muqarnas patterns in the ways that it is estimated to historical Muqarnas examples were constructed. According to the results derived from the descriptive analysis of these two strategies, this study tries to discover the geometrical characteristics of historical Muqarnas patterns which are called Muqarnas pattern guidelines. Muqarnas pattern guidelines might be the possible way that can be used to display the geometrical characteristics which are specific to historical Muqarnas patterns. Finally, by using these guidelines tries to identify the similarity and differences between historical Muqarnas patterns with modern forms which are intentionally or unintentionally inspired by Muqarnas patterns in their plan, facade or in their three-dimensional compositions.

Based on the findings of this research, there are some similarities between historical Muqarnas examples with modern ones. Isometry types and symmetry act important roles in the formation of Muqarnas patterns for both historical and modern forms. Therefore, this study takes them into consideration in analytical strategies as important characteristics of Muqarnas patterns and select the modern forms which have Isometry types in their patterns. alongside these general geometrical characteristics, there are some similarities which were driven from the analytical studies that are presented in this research. According to the results, both of historical and modern examples of Muqarnas are constructed from regular shapes as basic elements. Also, the basic modules of both types of examples are rotated or reflected through the angles of 45°, 90°, 315° and 180°.

Besides all the similarities between historical Muqarnas examples with modern forms, there are some differentiations between them. According to the analytical studies, because modern forms are mostly created by the aid of digital technologies and parametric design, form alteration and the designing process are completed according to the designing procedure that is defining at the first stages of design. Therefore, in contrary with historical Muqarnas compositions that had specific basic elements with specific rotation degrees, modern forms are completed and transformed during the design process. This process has a specific algorithm that was defined earlier so the basic elements of modern forms have a smooth alteration in their forms and they couldn’t keep their regularity.

According to the analyzing studies of 2DPP, designing process of historical Muqarnas patterns seems to be part to whole process. It means that the designer or master builder in the past firstly, drew the basic module of patterns and then by following the guidelines the final form of the pattern was obtained. But today’s designers mostly use CAD systems in their design, therefore, in parametric design or generative design, it seems to achieve the whole pattern firstly. And the process is whole to part.

Finally, the analytical strategies that are used in this study try to enable the designers and architects to have a better understanding of Muqarnas patterns to use them for generating new generations of Muqarnas patterns. Also, this study underlines the importance of Muqarnas geometrical patterns that are intentionally or unintentionally have been used in today's architecture. This research can motivate future studies that can be focused on new generations of Muqarnas patterns not only as a decorative element but as functional elements in buildings to provide optimizing performance for the buildings by using Muqarnas in the facade, roof, etc.

REFERENCES

Abdullahi, Y., & Embi, M. R. B. (2013). Evolution of Islamic geometric patterns. *Frontiers of Architectural Research*, *2*(2), 243-251.

Adams, W. Y., & Adams, E. W. (2007). *Archaeological typology and practical reality: a dialectical approach to artifact classification and sorting*. Cambridge University Press.

Adam, J. A. (2006). *Mathematics in nature: Modeling patterns in the natural world*. Princeton university press.

Akram, O. K., Ismail, S., & Franco, D. J. The Significant of Islamic Architecture Heritage at Baghdad City, Iraq–Case Studies of Shrine of Lady Zumurrud Khatun and Omar Al-Sahrawardi. *International Journal of Engineering Technology, Management and Applied Sciences, 4*, 133-138.

Al-Hassani. S. (2007). New Discoveries in the Islamic Complex of Mathematics, Architecture and Art. Institute for the History of Arabic Sciences. Aleppo University.

Alkandari, F. A. (2011). *Islamic ceramic ornamentation and process: proposals for a new aesthetic vocabulary in contemporary architectural embellishment within kuwait* (Doctoral dissertation, University of Central Lancashire).

Al-Kâshî, G. A. D. J., al-Demerdash, A. S., & al-Cheikh, M. H. (1977). Miftâh al-Hisâb (Key of Arithmetic). *Arabic edition, with French notes and introduction. Damascus: Damascus Society*.

BEHNEJAD, S. A., MOTTAGHI RAD, A. R. M. I. N., & JAMILI, H. (2012). Traditional Components of Iranian Vernacular Architecture in Contemporary Projects. *The Journal of the International Society for Interdisciplinary Study of Symmetry (ISIS-Symmetry)*, (1-2), 24-29.

Behrens-Abouseif, D. (1989). *Islamic architecture in Cairo: an introduction* . , 171-173. Leiden; New York: E.J. Brill.
Bloom, J. M. (1988). The introduction of the muqarnas into Egypt. *Muqarnas*, *5*, 21-28.

Carrillo, A. (2016). The Sasanian Tradition in ʿAbbāsid Art. *Mirabilia*, (22), 0201-226.

Castera, J. M. (2010). From the Angle of Quasicrystals. In *Proceedings of Bridges 2010: Mathematics, Music, Art, Architecture, Culture* (pp. 215-222). Tessellations Publishing.

Dabbour, L. M. (2012). Geometric proportions: The underlying structure of design process for Islamic geometric patterns. *Frontiers of Architectural Research*, *1*(4), 380-391.

Dadkhah, N., Safaeipour, H., & Memarian, G. (2012). Traditional Complex Modularity in Islamic and Persian Architecture: Interpretations in Muqarnas and Patkâné Crafts, Focusing on their Prefabricated Essence. In *Proceedings of 2012 ACSA FALL CONFERENCE—Offsite: Theory And Practice Of Architectural Production*.

Beyazit, D. (2004). Architectural Decoration of the Artuqids of Mardin During the 12th and 13th Centuries: Between Antique and Islamic Style. *Asiatische Studien Etudes Asiatiques*, 1013-30.

Dold-Samplonius, Y. (1992). Practical Arabic Mathematics: Measuring the Muqamas by al-Eshi.

Emmer, M. (2006). Mathematics and culture V. *west Bloomfield,* MI, U.S.A: Springer

Esposito, J. L. (Ed.). (2004). *The oxford dictionary of Islam*. Oxford University Press.

Easton, N., 2003. Parametric Design Definition. [online] Available at: http://www.designcommunity.com/discussion/25136.html (Accessed 15 June 2017).

Faramarzi, S., & Mohamadianmansoor, S. (2013). Typology and the Formulating Geometric structure of Karbandi in Iran's Architecture. *Journal of Fine Arts*, *48*, 97-109.

Garofalo, V. (2010). A methodology for studying muqarnas: the extant examples in Palermo. *Muqarnas*, *27*, 357-406.

Grigor, T. (2003). Of Metamorphosis Meaning on Iranian Terms. *Third Text*, *17*(3), 207-225.

Ghazarian, A., & Ousterhout, R. (2001). A muqarnas drawing from thirteenth-century Armenia and the use of architectural drawings during the Middle Ages. *Muqarnas*, *18*, 141-154.

Hamekasi, N., Samavati, F. F., & Nasri, A. (2011, August). Interactive modeling of Muqarnas. In *Proceedings of the International Symposium on Computational*

Aesthetics in Graphics, Visualization, and Imaging (pp. 129-136). ACM.

Harmsen, S., Jungblut, D., & Krömker, S. (2007). Seljuk Muqarnas along the Silk Road.

Harris, C. M. (2006). *Dictionary of Architecture and Construction*. McGraw-Hill.

Hejazi, M. (2005). Geometry in nature and Persian architecture. *Building and Environment, 40*(10), 1413-1427.

Hejazi, M., & Saradj, F. M. (2014). *Persian Architectural Heritage: Architecture, Structure and Conservation*. WIT Press.

Irwin, R. (2011). *The Alhambra*. Profile Books.

Jabi, W. (2013). *Parametric design for architecture*. Laurence King Publ..

James, A. (2012). What is Contemporary Traditional Architecture and Why Should We Consider It? Or even teach it? : Buffalo, NY.

Kaprielian, L. (2005). Cultural Re-Invention and the Accumulation of Diversity: The 'Christian' Muqarnas. *Contrapposto,* 179-221.

Kaptan, K. (2013). Early Islamic architecture and structural configurations. *International Journal of Architecture and Urban Development, 3*(2), 5-12.

Karabörk, H., Karasaka, L., & Yaldız, E. (2015). A Case Study: Documentation Method with Close Range Photogrammetry of Muqarnas Which is to be an Ornamentation Type Specific to the Islamic Architecture. *Procedia Earth and Planetary Science*, *15,* 133-140.

Kiani, Z., & Amiriparyan, P. (2016). The Structural and Spatial Analysing of Fractal Geometry in Organizing of Iranian Traditional Architecture. *Procedia-Social and Behavioral Sciences*, *216*, 766-777.

Knight, M., Dokonal, W., Brown, A., & Hannibal, C. (2005). Contemporary digital techniques in the early stages of design. In *Computer Aided Architectural Design Futures 2005* (pp. 165-174). Springer Netherlands.

Michler, A. (2015). *Hyperlocalization of architecture: Contemporary sustainable archetypes*.

Mohamadianmansoor, S., Faramarzi, S., Akbari, M. & Hatamimajd, F. (2012). Karbandi: The ground of applying Dome on different contexts in Iranian architecture. Domes in the world International Scientific Congress, At Florence, Italy

Morandi, P. J. (2003). The Classification of Wallpaper Patterns: From Group Cohomology to Escher's Tesselation s. *New Mexico State University*.

Moussavi, F., & López, D. (2009). *The function of form*. Actar.

Navaie, K., & Haji Ghasemi, K. (2011). Khesht-o Khial; an Interpretation of Iranian Islamic Architecture. *Tehran-Iran: soroush*.

Necipogulu, G., & Roxburgh, D. J. (Eds.). (2000). *Muqarnas: An Annual on the Visual Culture of the Islamic World* (Vol. 17). BRILL.

Necipoğlu, G. (1992). *Geometric Design in Timurid/Turkmen Architectural Practice: Thoughts on Recently Discovered Scroll and Its Late Gothic Parallels*.

Necipoğlu, G., & Al-Asad, M. (1995). *The Topkapı Scroll: Geometry and Ornament in Islamic Architecture: Topkapı Palace Museum Library MS*. Getty Center for the History of Art and the Humanities.

Necipoglu, G. (2005). Muqarnas: An Annual on the Visual Cultures of the Islamic World, Volume 22. Harvard University. Print.

Nejad Ebrahimi, A., Aliabadi, M., & Aghaei, S. (2014). DOMES INTERNAL DECORATIVE ELEMENTS IN PERSIAN ARCHITECTURE CASE STUDY: YAZDI-BANDI. *Alam Cipta, 6,* 113-127

Nejad Ebrahimi, A., Aliabadi, M., & Gharehbaglou, M. (2014). Parametric Design pattern Language and Geometric Patterns in Historical Domes in Persian Architecture. *CIÊNCIA E TÉCNICA VITIVINÍCOLA*, 29, 234-256.

Ebrahimi, A. N., & Aliabadi, M. (2014). The Role of Mathematics and Geometry in Formation of Persian Architecture. *Asian Culture and History*, *7*(1), 220.

Notkin, I. I. (1994). Decoding sixteenth-century muqarnas drawings. *Muqarnas Online*, *12*(1), 148-171.

Omer, S. (2011). Islamic architecture and the prospect of its revival today. *Journal of Architecture, Planning and Construction Management*, *1*(1), 1-17.

Palmer , A L.(2008). *Historical Dictionaries of Literature and the Arts*. Plymouth, Scarecrow Press.

Petersen, A. (1996). *Dictionary of Islamic architecture*. Psychology Press.

Petralla, S. (2012). Arches and ribbed vaults of the Iranian tradition. In *online*

Proceedings of the International Symposium Masons at Work. Architecture and Construction in the Pre-Modern World.

Pipes, A. (Ed.). (2014). *Computer-Aided Architectural Design Futures*. Butterworth-Heinemann.

Raeisi, M., Bemanian, M., & Tehrani, F. (2013). Rethinking the Concept of Karbandi Based on theoretical geometry, practical geometry and building function. *Maremat & Me'mari-e Iran*, *3*. 33-55.

Raby, J. (2004). NUR AL-DIN, THE QASTAL AL-SHU 'AYBIYYA, AND THE "CLASSICAL REVIVAL". *Muqarnas Online*, *21*(1), 289-310.

Sakkal, M. (1988). An introduction to Muqarnas domes Geometry. *Structural Topology 1988 núm 14*, 21-34.

Sakkal. M. (2001). [Computational] Geometry in Islamic Architecture. University of Washington .Department of Architecture.

Saliba, R. (1994). Tripoli, the Old City: Monument Survey-Mosques and Madrasas: A Sourcebook of Maps and Architectural Drawings. Beirut: American University of Beirut, Department of Architecture.

Dold-Samplonius, Y., & Harmsen, S. L. (2005). The muqarnas plate found at Takht-i Sulayman: a new interpretation. *Muqarnas*, *22*, 85-94.

Saoud, R. (2002). A review on Architecture in Muslim Spain and North Africa (756-1500AD). *Fondation for Science Technology and Civilisation*.

Sayah, I. (2016). Creating a parametric muqarnas utilizing algorithmic software. *International Journal of Review in Life Sciences, 6,* 47-53.

Shafiq, J. Architectural Elements in Islamic Ornamentation: New Vision in Contemporary Islamic Art.
Arts and Design Studies, 21, 11-21.

de Solà-Morales, P. (2012). Information, Architecture, Complexity. In *Architecture, Systems Research and Computational Sciences* (pp. 17-24). Springer Basel.

Stavric, M., & Marina, O. (2011). Parametric modeling for advanced architecture. *International journal of applied mathematics and informatics*, *5*(1), 9-16.

Stiny, G. (1977). Ice-ray: a note on the generation of Chinese lattice designs. *Environment and Planning B: Planning and Design*, *4*(1), 89-98.

Abdullah, N., Khadizah, G., & Salleh, S. (2015). Modelling approach in Islamic architectural designs. *Global Journal Al-Thaqafah*, *4*(1), 49-56.

Tabbaa, Y. (1985). The muqarnas dome: its origin and meaning. *Muqarnas, 3,* 61-74.

Takahashi, S. (2004). http://www.tamabi.ac.jp/idd/shiro/muqarnas/. Last Visit: 19th November 2006.

Tariq, R. (2014). Islamic Architecture. VIA University College. Horsens, Denmark

Tepavčević, B., & Stojaković, V. (2012). Shape grammar in contemporary architectural theory and design. *Facta Universitatis-series: Architecture and Civil Engineering*, *10*(2), 169-178.

Ebru, U. (2009). A shape grammar model to generate islamic geometric pattern. In *12th Generative Art International conference, Italy* (pp. 290-297).

Hoeven, S., & Veen, M. (2011). Muqarnas: Mathematics in Islamic Arts. Seminar Mathematics in Islamic Arts, Utrecht University, Faculty of Science, Department of Mathematics.

Vermeulen, U., & De Smet, D. (2001). *Egypt and Syria in the Fatimid, Ayyubid, and Mamluk Eras III: Proceedings of the 6th, 7th and 8th International Colloquium Organized at the Katholieke Universiteit Leuven in May 1997, 1998, and 1999* (Vol. 3). Peeters Publishers.

Yaghan, M. A. (1998). Structural Genuine Muqarnas Dome: Definition, Unit Analysis, and a Computer Generation System. *Journal of King Saud University (Architecture and Planning), 10*, 17-52.

Yaghan, M. A. J. (2000). Decoding the two-dimensional pattern found at Takht-i Sulayman into three-dimensional muqarnas forms. *Iran*, 77-95.

Yaghan, M. A. (2003). Teaching Architectural-Visual-Experience through Virtual Reality Using Vrml: Muqarnas Example. *Journal of King Abdulaziz University, Environmental Design, 1*, 27-42.

Yaghan, M. A. (2010, July). The evolution of architectural forms through computer visualisation: muqarnas example. *Proceedings of Electronic Visualisation and the Arts*, 113–120.

Yakar, M., Yilmaz, H. M., Gulec, S. A., & Korumaz, M. (2009). Advantage of digital close range photogrammetry in drawing of muqarnas in architecture. *Information Technology Journal, 8*, 202-207.

Website references

Web 1:

https://www.khanacademy.org/humanities/ap-art-history/west-and-central-asia/a/the-great-mosque-or-masjid-e-jameh-of-isfahan, (visited: 02.09.2016)

Web 2:

http://orujtravel.com/en/attraction/jameh-mosque/, (visited: 02.09.2016)

Web 3:

http://archnet.org, (visited: 02.09.2016)

Web 4:

http://www.iwr.uni-heidelberg.de/ (Visited: 22.08.2016)

Web 5:

https://www.google.com.tr/search?q=Badr+alJamali+mosque&source=lnms&tbm=isch&sa=X&ved=0ahUKEwjWto61jLnUAhUMOpoKHdUyCCMQ_AUICigB&biw=1366&bih=613#imgrc=YVb3mfSrwBijfM, (visited: 12.04.2017)

Web 6:

http://www.panoramio.com/photo/1425016, (visited: 12.04.2017)

Web 7:

https://tr.pinterest.com/shabbirraj/fatemid-architecture/?lp=true, (visited: 12.04.2017)

Web 8:

http://vegypttours.com/english/EgyptSightseeing.aspx?ID=54&Name=Juyushi%20Mosque, (visited: 12.06.2017)

Web 9:

http://islamicarchit.blogspot.com.tr/ (Visited: 22.04.2017)

Web 10:

https://www.studyblue.com, (visited: 28.08.2016)

Web 11:

http://www.keyword-suggestions.com/, (visited: 28.08.2016)

Web 12:
http://archnet.org, (visited: 28.08.2016)
Web13:
(http://www.flickr.com/photos/amarola/246778736/in/set72157594289870209/),
(visited: 28.08.2016)
Web 14:
https://www.flickr.com, (visited: 28.08.2016)
Web 15:
http://rwrightson.wixsite.com, (visited: 12.06.2017)
Web 16:
http://aparthistory2015.blogspot.com.tr/2015/11/the-great-mosque-masjid-e-jameh-of.html (visited: 02.09.2016)
Web 17:
https://www.khanacademy.org/humanities/art-islam/beginners-guide-islamic-art/a/common-types-of-mosque-architecture (visited: 02.09.2016)
Web 18:
https://www.researchgate.net/publication/28143783_Internement_et_Traitement_de_la_Maladie_Mentale_au_Moyen (visited: 22.09.2016)
Web 19:
www.pinterest.com (visited: 22.09.2016)
Web 20:
https://sabahalnassery.wordpress.com/2016/05/ , (visited: 22.09.2016)
Web 21:
http://www.essentialarchitecture.com/ASIA-WEST/WA-IQ/WA-IQ-004.htm,
(visited: 22.09.2016)
Web 22:
http://archnet.org (Visited: 22.09.2016)
Web 23:
http://www.essentialarchitecture.com/ASIA-WEST/WA-IQ/WA-IQ-004.htm,
(visited: 22.09.2016)

Web 24:
http://archnet.org, (Visited: 22.09.2016)
Web 25:
archnet.com, (Visited: 15.09.2016)
Web 26:
http://archnet.org, (Visited: 23.09.2016)
Web 27:
http://www.polyolbion.org.uk/Hanwell/October15.html (Visited: 23.09.2016)
Web 28:
archnet.com, (visited: 23.09.2016)
Web 29:
http://www.ralf-heiser.info/newgqfwd-qutub-minar-architecture.html, (visited: 23.09.2016)
Web 30:
http://wowturkey.com/forum/viewtopic.php?p=2180429, (Visited: 13.06.2017)
Web 31:
http://www.aksaraykulturturizm.com/tr/foto-galeri/bekar-sultan-turbesi_17-05-2016 (Visited: 23.09.2016)
Web 32:
http://www.aksaraykulturturizm.com/tr/foto-galeri?page=3, (visited: 23.09.2016)
Web 33:
https://hiveminer.com/Tags/abarqu,iran/Interesting (visited: 25.09.2016)
Web 34:
HTTPS://HİVEMİNER.COM/TAGS/ABARQU,İRAN/INTERESTİNG, (visited: 25.09.2016)
Web 35:
archnet.com (visited: 25.09.2016)
Web 36:
http://www.afamnews.ir/ , (visited: 21.11.2016)
Web 37:

www.pinterest.pt (visited: 05.11.2016)
Web 38:
www.kilyos.ee.bilkent.edu.tr/~history/geometry.html (visited: 14.11.2016)
Web 39:
www.pinterest.pt (visited: 16.11.2016)
Web 40:
http://www.designcoding.net/Muqarnas-student-works/ (visited: 22.11.2016)
Web 41:
http://www.sakkal.com/islamic_geometry/muqarnas.html (visited: 22.11.2016)
Web 42:
http://www.sakkal.com/islamic_geometry/muqarnas.html (visited: 22.11.2016)
Web 43:
http://www.sakkal.com/islamic_geometry/muqarnas.html (visited: 22.11.2016)
Web 44:
http://www.designcoding.net/parametric-muqarnas/ (visited: 05.03.2017)
Web 45:
https://www.slideshare.net/danowen777/muqarnas-reconceived-a-brief-survey
(Visited: 25.04.2017)
Web 46:
http://www.shiro1000.jp
Web 47:
https://spatialexperiments.wordpress.com
Web 48:
www. Archdalily.com, (Retrieved: 04.05.2017)
Web 49:
www. Archdalily.com, (Retrieved: 20.10.2016)
Web 50:
www. Archdalily.com, (Retrieved: 04.05.2017)
Web 51:
www. Archdalily.com , (Retrieved: 18.10.2016)

Web 52:
https://livelifeelectric.wordpress.com/tag/iwan-baan/, (Retrieved04.05.2017)
Web 53:
http://www.archdaily.com/, (Retrieved: 15.10.2016)
Web 54:
http://www.skyscrapercenter.com/building/muqarnas-tower/17307, (Retrieved: 04.05.2017)
Web 55:
http://www.evolo.us
Web 56:
http://www.evolo.us/architecture/hyperbolic-installation-based-on-persian-patterns-and-muqarnas/, (Retrieved: 04.05.2017)
Web 57:
www.pinterest.com, (Retrieved: 27.10.2016)
Web 58:
http://bonner-design.com/index.php/projects/international-medical-center-jeddah/, (Retrieved: 04.05.2017)