EXPLORING MATHEMATICAL CREATIVITY IN THE FRACTIONS TOPIC IN A FIFTH GRADE MATHEMATICS CLASS

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ABSTRACT

EXPLORING MATHEMATICAL CREATIVITY
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The purpose of this study was to investigate of 5th grade students' mathematical creativity examples related to fractions topic. Therefore, a 5th grade class in a private school in which the mathematics teacher was also the researcher of the study was studied. Basic qualitative research design was used as a research method with convenience sampling at researcher's convenience. Data were gathered through observation protocol, paper tasks including two quizzes and a questionnaire, and in-class short interviews between February-March 2016. Data were analyzed by using qualitative methods. Appropriateness, fluency, flexibility, and novelty criteria were searched through data in redefinition, problem posing, and problem solving tasks on paper tasks. Although appropriateness and novelty were the two criteria for in-lesson tasks initially, collective fluency and collective flexibility criteria were considered later.

The findings revealed that redefinition task in questionnaire yielded in more responses than other. Problem posing tasks and questionnaire findings showed that students bring their interests, past experiences and daily routines into the context of the problems. In lesson tasks, there were some cases that individual creativity occurred. However, there were some other cases that students interacted with each other such that the idea of a student might be a step for the development of other student's idea. In-class short interviews could not provide much information about students' reasons behind their ideas. Findings have addressed that certain tasks, such as problem posing tasks, can reveal and support students' creativity.

Keywords: Mathematics Education, Creativity, Fractions, Fifth Grade

ÖZ

KESİRLER KONUSUNA İLİŞKİN MATEMATİKSEL YARATICILIĞIN
5.SINIF MATEMATİK DERSİNDE ARAŞTIRILMASI

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Bu çalışmanın temel amacı, 5.sınıf kesirler konusuna ilişkin matematiksel yaratıcılık örneklerini incelemektir. Araştırma tekniği basit nitel araştırma olarak desenlenmiştir ve çalışma grubu kolaylıkla bulunabilen örnekleme yoluyla belirlenmiştir. Araştırmacının 2016 yılında bir özel okulda sınıf matematik öğretmeni olduğu bir 5. sınıftaki öğrenciler çalışma grubunu oluşturmuştur. Veriler, gözlem, yazılı sınavlar ve sınıf içi kısa görüşmeler ile 2016 yılı Şubat ve Mart aylarında toplanmıştır. Veriler nitel yöntemlerle analiz edilmiştir. Yazılı olan yeniden tanımlama, problem kurma ve problem çözme etkinliklerindeki öğrenci yanıtları uygunluk, akıcılık, esneklik ve orijinallik olmak üzere dört kriter bakımından araştırılmıştır. Ders içi etkinlikleri için başta uygunluk ve orijinallik kriterleri ele alınmıştır. Ortak akıcılık ve ortak esneklik kriterleri de sonradan eklenmiştir.

Bulgular yazılı olan yeniden tanımlama etkinliğinin diğerlerinden daha fazla yanıta sahip olduğunu göstermektedir. Soru kağıdında yer alan problem kurma etkinliklerinde problemlerin içeriğinde öğrencilerin ilgi, günlük işleri ve tecrübelerinin yer aldığını göstermiştir. Derste bazı durumlarda bireysel olarak yaratıcılık gözlenmiştir. Fakat, bazı durumlarda öğrenciler iletişim halindeyken bir öğrencinin fikri diğer bir öğrencinin fikrini geliştirmesini sağlamıştır. Sınıf içi kısa görüşmelerde öğrenciler nasıl düşündükleri hakkında yeterli cevaba ulaşılamamıştır. Bulgular problem kurma gibi etkinliklerin öğrencilerin yaratıcılıklarını ortaya çıkarabileceğini ve destekleyebileceğini ortaya çıkarmıştır.

Anahtar Kelimeler: Matematik Eğitimi, Yaratıcılık, Kesirler, Beşinci Sınıf

To the Memory of My Lovely Brother, Çağrı

and Our Childhood

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INTRODUCTION

1.1 Conceptual understanding to make sense of mathematics

Conceptual knowledge is knowledge of wide range of relations which are connected to a specific system in mind (Hiebert & Lefevre, 1986). There are several ways of enhancing conceptual understanding. It can be developed by constructing relationship between knowledge gathered either previously or recently (Hiebert & Lefevre, 1986). Sociomathematical norms, such as explanations that include mathematical claims, also have the potential to foster thinking conceptually (Kazemi, 2002). When teachers put more emphasis on conceptual understanding by giving importance to student expressions, students become better in problem solving and improve their conceptual understanding (Kazemi, 2002). Such expressions could be resulted from imagining, recognizing patterns, detecting connections relevantly, and modeling from new angle which are the essences of conceptual development (Glas, 2002).

The emphasis on conceptual knowledge is also about making sense of mathematics. If students are allowed to build their "own mathematics", then it would help making sense of mathematics in any part of learning (Warrington & Kamii, 2002). Asking questions, letting students attempt to learn, reminding that they are responsible for their learning may help students make sense of mathematics, which also increases their profound comprehension (Reinhart, 2002). What teachers should do in such process is to be careful about the usefulness of the idea of child in other situations (Curcio & Schwartz, 2002).

1.2 From making sense of mathematics to mathematical creativity

While children make sense of mathematics, they may establish new relationships according to their way of understanding (Curcio & Schwartz, 2002). As making sense of mathematics, students generally use words such as "might", "usually", and "probably" in order to explain their ideas. Therefore, each expression becomes an idea to be tested and explored by others (Whitin & Whitin, 2000). Besides, students may come up with their own unique definitions in addition to discover some properties of a concept. Giving their own definitions enables students to bring their personal interests into defined terms (Whitin & Whitin, 2000).

Students are capable of inventing, creating, or finding original solutions. Such abilities could be observable when students are confronted by problems which are interesting for them (Curcio & Schwartz, 2002). Use of imagination to pose and solve problems is not a unique way of being creative. One can also be creative while making sense of mathematics itself (Huckstep & Rowland, 2008). Thus, making sense is both activator and reflector of creativity in mathematics education.

1.3 Creativity in general and in the field of education

Creativity depends on both social and cultural settings (Krummheuer, Leuzinger-Bohleber, Müller-Kirchof, Münz, & Vogel, 2013). Emergence of creative behaviors could depend on experience and environment (Mann, 2009). The basic idea of creativity is to synthesize (Gardner, 2006) and to combine previous experiences in a new form (Vygotsky, 2004). Vygotsky (2004) stated that real substances in the world form the base for being creative because either real substances themselves or understandings built on those real substances enable human beings to be creative. To clarify, he addressed four stages of the relationship between reality and creativity. The first one explains that what a person creates is constructed on the person's experiences (Vygotsky, 2004). The second one stems from "a more complex association, not between the elements of an imaginary structure and reality, but between the final product of imagination and some complex real phenomenon" (Vygotsky, 2004, p. 16). The third one explains the "mutual dependence between imagination and experience", through either "imagination is based on experience" or "experience itself is based on imagination" (Vygotsky, 2004, p. 17). The fourth association explains how “imagination becomes reality” (Vygotsky, 2004, p. 20). In addition, Vygotsky’s study revealed that a child and social side of meaning-making are interdependent to each other; hence, classrooms should show that each child from a different culture and with different experiences is welcomed. In order to achieve this aim, the relation between how meaning-making takes place and the sociocultural situations they take place should be understood according to Vygotsky (Mahn, 2003).

By understanding meaning-making process, students’ mathematical development could be supported as well as creativity. According to Glas (2002), mathematical development occurs onto prior knowledge and is activated by not only the inner effort to organize concepts, but also external requirements by several practice contexts and cultural settings. Therefore, it helps students adopting different viewpoints, which enables representing those differently (Glas, 2002).

When students are given opportunities to experience, they are also given opportunities to be creative in education (Vygotsky, 2004). In mathematics education, students are equipped with several materials in topics such as fractions and geometry. For example, counters, fraction bars, fraction cards, paper folding in fractions and pattern blocks, geoboards, tangrams, and rulers in geometry could be used in mathematics lessons (MoNE, 2009b; Van de Walle, 2007). Thus, students are encouraged to experience with reality (Vygotsky, 2004) and make sense of mathematics with such materials (Glas, 2002); therefore, they can be creative and can express their opinions in a different way depending on their own understanding. They can develop detailed inferences which are unique to themselves regarding these topics (MoNE, 2009b).

It is possible to state that novelty is required for creativity in general (Kaufmann, 2003). In mathematics education, creativity could be associated with “the ability to overcome fixations or rigidity in thinking, to break from mental sets” (Haylock, 1997, p. 69). Such effort could be observed in redefinition tasks (Haylock, 1987, 1997), problem posing tasks (Haylock, 1987, 1997; Silver, 1997) and problem solving tasks in mathematics education (Briggs, 2005; Curcio & Schwartz, 2002; Haylock, 1987, 1997; Silver, 1997).

Schools may be the places that help students to increase their creativity. Teachers are cornerstones of such development. For instance, if they welcome students' different interests, then this goal could be achieved (Maksić & Pavlović, 2011). In addition, creativity could be included in teaching strategies and creativity of students could be analyzed and improved by the teacher. Such methods might provide required atmosphere for a child to be creative and also to enhance it (Lin, 2011).

Apart from student-teacher interactions, there are some other methods to increase creativity of students. For instance, according to Sriraman (2004), complexity attracts creative individuals but it may not take place in the school curriculum. Therefore, elementary mathematics education curriculum, which gives a direction to teachers' practice and affects their expectations from students in accord, is worth to be introduced below.

1.4 Mathematical Creativity in Elementary Mathematics Education Curriculum in Turkey

Ministry of National Education (MoNE) aims to educate students in a way that they will become constructive, creative, and productive people (MoNE, 2009a). Mathematics education helps people in understanding their physical and social environment with richness of knowledge and skills gained in lessons (MoNE, 2009a; MoNE, 2013). Learning mathematics is not only gaining knowledge on some concepts and skills, but also recognizing that mathematics has an important role in real life like solving problems (MoNE, 2013). One of the skills that students are expected to have is thinking creatively with the mathematics curriculum. The curriculum is considered to help thinking creatively and provides appreciation of aesthetics (MoNE, 2009a).

Improving students' creativity is mentioned not only in the main aims of education, but also in the assessment part. Projects and performance assignments improve abilities of students such as creativity, research, communication, and problem solving. Projects are suitable for observing the creativity of students (MoNE, 2009a; MoNE, 2009b). Concept maps and open-ended questions also help students reveal their creativity (MoNE, 2009a). Thus, projects, performance assignments, concept maps, and open-ended questions are seen as methods of inducing students' creativity.

As opposed to traditional education, modern education should encourage students to deal with problems which have been encountered for the first time (Lindqvist, 2003). Ministry of National Education aims to improve students' problem solving abilities. In the revised curriculum, problem refers to non-routine problems. Students could solve those by applying their knowledge and finding a strategy. Students should be provided enough time to think about the problem (MoNE, 2013).

When students solve problems with their methods and become successful, they believe that they could do mathematics. Therefore, they behave more patient and creative in such processes. They learn how to communicate with mathematics and also improve their abilities in thinking (MoNE, 2009a). They are expected to be active participants in learning process by constructing their own knowledge themselves. Therefore, classrooms should welcome different methods of students and let them share their ideas without hesitation. To form such a classroom, open-ended questions and such activities should be more in case students could do mathematics (MoNE, 2013).

Although the revised curriculum does not mention creativity explicitly, it approaches to creativity with giving emphasis on non-routine problem solving, meaning making, and sharing different ideas.

1.5 Statement of the problem

Mathematics education could be seen as a chance for students' creativity development (Švecová, Rumanová, & Pavlovičová, 2014). Problem solving (Briggs, 2005), problem posing (Silver, 1997), and redefinition tasks (Haylock, 1997) in mathematics education could yield in creativity (Silver, 1997). It has been emphasized in the recent mathematics curriculum in Turkey that problem posing is a part of teaching and should be included in teaching process as well as problem solving (MoNE, 2013). However, the formal and intended curriculum sometimes could be apart from each other as the teacher transfers the former by using own understanding, experience, motivation into the latter (Stein, Remillard, & Smith, 2007). Most of the students might not have experienced problem posing yet (Van Harpen & Presmeg, 2011). There are some studies regarding problem posing abilities of students that students had posed problems for the first time during the study in

Turkey (for example Arıkan & Ünal, 2015). Besides, teachers are requesting for more explanations and space about problem posing in curriculum (Kılıç, 2013). Researchers could be informed about students' mathematical thinking and experiences with problem posing tasks (Silver, 1994).

There are many studies conducted in Turkey in order to investigate the relationship between creativity and achievement, perceptions of teachers and pre-service teachers on creativity, experimental studies aiming to improve creative problem solving skills, analysis of student's creative writing, and prospective mathematics teachers' mathematical creativity in problem solving (for example, Batırbek, 2007; Kandemir, 2006; Kıymaz, Sriraman, & Lee, 2011; Olgun, 2012; Yıldırım, 2006). There are some studies conducted abroad about mathematical creativity of adolescents in redefinition, problem posing, and problem solving process tasks on paper separately (for example, Chen, Himsel, Kasof, Greenberger, & Dmitrieva, 2006; Haylock, 1987, 1997; Van Harpen & Presmeg, 2011, 2013). However, students' mathematical creativity in redefinition, problem posing, and problem solving tasks are rather rare.

There is no study in the accessible literature that the effect of experience with reality, either with manipulative or real-life experiences, on mathematical creativity is investigated. However, fractions and geometry topics, where several manipulative can be used in the classroom, may provide a creativity supported environment in school mathematics (MoNE, 2009b; Van de Walle, 2007) to be investigated.

MoNE (2015) says that while teaching new concepts, manipulatives should be utilized such as fraction bars and models which could be obtained from basic daily materials. In order students to comprehend what they did, they should build their knowledge themselves. To serve that purpose, experiences on mathematics should be ordered from basic to complicated and concrete to abstract in elementary school level (MoNE, 2015). Moreover, for the operations regarding fractions, teachers should let students focus on "the meaning of the operations" (Van de Walle, 2007, p.316). "Adequate time or experiences" are key factors for helping students in learning fractions (Van de Walle, 2007, p.293). Thus, as experience is cornerstone for creativity for Vygotsky and meaning making is somehow activate creativity, fractions topic for fifth grade students could be worth to study in terms of creativity.

In this study, middle school students' mathematical creativity will be explored with classroom observation and paper-tasks made up of redefinition, problem posing, and problem solving tasks together with manipulative and real-life examples in the fractions concept.

1.5.1 Research questions

The research questions which are aimed to be answered in this study are:

1. What are the instances of mathematical creativity of 5th grade students in fractions concept in the fractions unit which is designed to elicit mathematical creativity through redefinition, problem posing and problem solving tasks and use of manipulative and real-life experiences?
   1.1 What are the examples of mathematical creativity of 5th graders observed in fractions during the mathematics lesson?
   1.2 What are the examples of mathematical creativity of 5th graders displayed in written tasks of redefinition, problem posing, and problem solving in fractions?
   1.3 How do examples of mathematical creativity of 5th graders in fractions differ in lessons and on paper?

1.6 Significance of the study

Creativity should be included in curriculum from early childhood to higher education level (Sriraman, Yaftian, & Lee, 2011). Because life embodies several problems; every time people face with problems and solve them, they undergo intellectual growth Guilford (1967).

In Turkey, students are expected to use mathematical terminology and language effectively in order to explain their mathematical thoughts properly. With the help of meaning and language of mathematics, students could make sense of relations between people and objects or among objects (MoNE, 2015).

Primary school mathematics education aims at improving students' independent thinking and decision-making in addition to self-regulation abilities. At the end of this process, students are expected to get familiar with and learn about some basic concepts and to obtain essential knowledge in mathematics in order to apply their understanding and knowledge within different mathematical concepts and also in other areas. After they gather such understanding and knowledge, students could make inferences on their own and explain their own mathematical thinking (MoNE, 2009a). In this regard, exploring their mathematical creativity immediately after students graduated from primary schools could increase the occurrence of examples of mathematical creativity.

Students meet with topic of fractions in grade 1 in Turkey (MoNE, 2015). There are many concepts such as “denominator”, “numerator”, “fractional parts”, “unit fraction”, “proper fraction”, “improper fraction”, and “same size unit whole” in this topic (Spangler, 2011, p. 22). However, these concepts are scattered from grade 1 to 6 (MoNE, 2015; MoNE, 2013). For instance, they get familiar with the terms: whole, half, and one fourth in grade 1 and they learn about the operations on fractions throughout grade 4, 5, and 6 (MoNE, 2013; MoNE, 2015). Thus, studying fractions topic in 5th grade might reflect students’ experiences and therefore, it might increase creativity examples of students.

Mathematics lessons aim at learning to value different point of views as well as meaningful learning (MoNE, 2009b). An environment in which students are aware of differences in problem solving styles and are allowed to develop their own methods could feed creativity of students, therefore, creativity supported environment is worth to study (Mann, 2009).

As mathematical creativity is an assurance of improvement in entire mathematics (Sriraman, 2004), exploration of 5th grade students’ mathematical creativity in classroom observation and redefinition, problem posing, and problem solving tasks together is important. Moreover, in order to understand the phenomenon of creativity, interviews are necessary as follow-ups (Levenson, 2011; Mann, 2009; Piffer, 2012). Combining information gathered from classroom observation and students’ answers on paper with interviews could provide worthy information about creativity. Besides, redefinition problems could be included in mathematics course book and welcomed in mathematics lessons if different types of tasks are perceived as useful, depending on the conclusions drawn.

Creativity is attributed to vibrant characteristics of human thinking which could be developed (Nadjafikhah, Yaftian, & Bakhshalizadeh, 2012). Mathematics itself has such a nature that creativity could be visible in a classroom, hence, it could be seen a duty of mathematics educators to work on creativity (Nadjafikhah, Yaftian, & Bakhshalizadeh, 2012). In schools, students might begin to "think as mathematician" while they are expressing their own idea in creativity supported environment. Studies point to describing classrooms yielding in creativity and there is a view that being a creative teacher could feed creative students (Nadjafikhah, Yaftian, & Bakhshalizadeh, 2012). Thus, questions such as how to design a learning environment and how to perform teaching which improve creativity in middle school could be somehow answered with the results of the current study. With the findings of the study, mathematics educators and curriculum designers could be informed to a certain degree about how to design a lesson or behave in a lesson.

1.7 Definition of the terms

According to Vygotsky, there are four stages which explain the relationship between reality and imagination, and therefore creativity. In this study, taking Vygotskian lenses, mathematical creativity is defined operationally as follows:

> *Mathematical creativity*: Any mathematically appropriate product and/or process, which might be resulted from experience with reality, of fluency, flexibility, or novelty observed in classroom and/or any of redefinition, problem posing, and problem solving tasks (see Guilford, 1967; Haylock, 1997; Kaufmann, 2003; Levenson, 2011; Silver, 1997).
>
> *Appropriateness:* An appropriate response is a response related to mathematical content and/or valid under mathematical facts (Haylock, 1997; Nadjafikhah & Yaftian, 2013; Van Harpen & Presmeg, 2013). Posed problems which could be arbitrarily answered such as "Do they know each other?" and responses which are original but not valid under mathematical fact such as "$\sqrt{8}=4$" was eliminated in this study as suggested in other studies (Haylock, 1997; Nadjafikhah & Yaftian, 2013; Van Harpen & Presmeg, 2013), both in classroom observations and tasks including redefinition, problem posing, and problem solving.

Fluency: It is “the number of acceptable responses” (Haylock, 1997, p. 71). In this study, the total number of appropriate responses of a student on redefinition tasks, problem solving tasks, and problem posing tasks was calculated separately.

Flexibility: It is “the number of different kinds of response” (Haylock, 1997, p. 71). In order to determine flexibility, students’ responses on redefinition tasks, problem solving tasks, and problem posing tasks were categorized initially. Then, the number of different categories which involves students’ responses on redefinition, problem posing, and problem solving tasks was counted separately.

Novelty: It is “the statistical infrequency of the responses in relation to the peer group” (Haylock, 1997, p. 71). Novelty is a criterion for creativity in both classroom observation and tasks which include redefinition, problem posing, and problem solving. In classroom, original mathematical inferences or ideas of students was called as novel. In tasks including redefinition, problem posing, and problem solving, the rate of similar responses to the total number of responses was calculated in each task. If a response is given by the number of students less than 10% of the total number of students in the class, then it is novel as done in other study (Van Harpen & Presmeg, 2013).

Redefinition task: Task requiring students for attempting with several responses "by continually redefining the elements of the situation in terms of their mathematical attributes" (Haylock, 1987, p. 71). For instance, requesting students to "State all the things that are the same about the two numbers 16 and 36" could be an example of this task (Haylock, 1997, p. 73).

Problem posing task: According to the information given, students are expected to write about the concerning problems (Haylock, 1987, 1997). The further problem exemplifies problem posing task: "There are 10 girls and 10 boys standing in a line. Make up as many problems as you can that use the information in some way" (Van Harpen & Presmeg, 2011, p.290).

Problem solving task: Task which could be solved by several ways (Haylock, 1997). For example, "given a nine-dot centimetre-square grid draw as many shapes as possible with an area of 2 cm^2" illustrates problem solving task (Haylock, 1997, p. 72).

CHAPTER II

LITERATURE REVIEW

In this chapter, Vygotsky's theory of creativity, definition of creativity, and the studies conducted about creativity are summarized.

2.1 Vygotsky's theory of creativity

Vygotsky is known by his cultural-historical theory. However, he is also known to be the first researcher who combined the ideas on creativity and emotions to understand human behaviors and thinking (Lindqvist, 2003). According to Vygotsky, people are creative regardless of their ages and by means of this attribute, they promote art, science, and technology (Lindqvist, 2003).

For Vygotsky (2004, p. 9), there are two sorts of human behavior: "reproductive" and "combinatorial or creative activity". The first behavior is attached to memory. Human act, as repetitive or reproductive, could be explained by the influences gathered from previous acts stored in the brain. However, human brain could do more. If not, then it would mean that human could not react towards unfamiliar experiences. Therefore, the second behavior is a result of producing new representations or actions. In psychology, such activity addresses a mental skill of combination and it is termed as "imagination or fantasy". Hence, anything in the world made by human was created through imagination of human beings (Vygotsky, 2004).

Creativity is generally linked to great artists, composers, or scientists such as Tolstoy, Edison, and Darwin in daily life but, creativity occurs at any time as human imagine. In fact, creativity could be identified in human beings when they are young children while playing. In their play, children orient their experiences towards play, not by reproducing experiences but by adapting their experiences to play in order to create a new actuality (Vygotsky, 2004). Combining previous experiences in a new manner forms the main idea of creativity (Vygotsky, 2004).

Each step of human development covers unique interpretation and explanation, and therefore has unique attributes of creativity. It is important to improve creativity in school-age children because days to come will be shaped with creative imagination (Vygotsky, 2004). Preparing students for the future could be achieved by exercises of imagination. Yet, imagination is set up onto real substances in the world (Vygotsky, 2004). The relationship between imagination and reality could be examined at four stages (Vygotsky, 2004). At the first stage, creativity, which is caused by imagination, is based on the rich and various past experiences of a person since those experiences give required substances for imagination. Fairy tales could be an example for this step. For example, animals mentioned in fairy tales are from reality but combined with fantastic figures (Vygotsky, 2004). Therefore, students should be provided with more experiences so as to support or enhance their creativity by improving their imagination through gaining more experience with the substances of real life (Vygotsky, 2004).

At the second stage, children bring the final product generated by imagination and complicated real incidents together. This stage could be reached with the help of the first one (Vygotsky, 2004). For instance, drawing a picture of a specific event or place in history based on experiences or others' travel stories is not reproduction but creativity upon that experience. Thus, this stage could happen by social experience and the stories told on experience or traveling itself constitutes reality here (Vygotsky, 2004).

The third stage includes emotional association which could be in two ways. Either emotions could search for accurate appearances, or imagination may affect emotion. An example to first may be labeling blue as cool and red as warm color. Although "blue" and "cool" are not relevant to each other in reality, these words could be called together as individuals feel similar when they hear these words. Such subjective expressions yield in differences in our imagination. An example to the second emotional association could be a child's imagination of a dangerous person in a dark room who is actually not there. Still, based on his imagination a child feels fear which is real at that point (Vygotsky, 2004).

The last stage makes imagination a real thing. Although imagination does not correspond to anything in experience or object, after imagination is materialized, the new object occurs in real world. This stage lasts for a long time, even after substances are gathered from nature and modified throughout years, and they become products of imagination. Lastly, materialized objects become real but still have the potential of changing reality by imagination later on (Vygotsky, 2004).

2.2 Definition of creativity

Although creativity could be easily perceived, it is still hard to give a definition of it (Saracho, 2012). There is no clear definition of mathematical creativity in the literature (Nadjafikhah, Yaftian, & Bakhshalizadeh, 2012; Nadjafikhah & Yaftian, 2013). It is suggested to examine literature on creativity and then, on mathematical creativity, deductively (Nadjafikhah & Yaftian, 2013).

What creativity means is rather unclear; however, it is attached to a positive meaning without hesitation (Huckstep & Rowland, 2008) because happiness might be a common result of creation (Vygotsky, 2004). Vygotsky defines creativity as follows:

> "Any human act that gives rise to something new is referred to as a creative act, regardless of whether what is created is a physical object or some mental or emotional construct that lives within the person who created it and is known only to him" (Vygotsky, 2004, p. 7).

Piffer (2012) proposes that creativity should be redefined according to a person's and product's creativity. Besides, only if it is possible to sum all the creativity scores of products that a person created throughout his/her life, then it may also be possible to talk about his/her creativity. In this regard, studies concerning creativity may be found improper as it is almost impossible to find summation of creativity scores of the products one created. Yet, there are still some approaches attempting to make definitions clearer to some extent.

According to Gardner (2006), synthesizing and creativity are parallel paths to each other. For instance, literacy and discipline are required for both. In addition, multiple representations could be a result of both. Therefore, it is not quite possible to distinguish them sharply. While trying to synthesize generates creativity, synthesizing has a considerable role in creating (Gardner, 2006).

According to Kaufman and Beghetto (2009), the Four C model, which is comprised of Big-C creativity, little-c creativity, mini-c creativity, and pro-c creativity, could explain one's creativity. Big-C creativity refers to creativity of well-known people such as Albert Einstein, Sigmund Freud, and Leo Tolstoy while little-c is about creative acts in everyday life. Mini-c creativity explains unique and meaningful interpretations of individuals, which overlaps with the Vygotskian creative development notion. Last, pro-c clarifies the process until which an individual with little-c creativity reaches Big-C creativity (Kaufman & Beghetto, 2009).

Guilford (1967) states that fluency, flexibility, and elaboration could be attributed to creativity and problem solving, in general. Fluency is derived from information in mind and it is a result of call-back information. Flexibility refers to transforming information in various ways and rearranging information, such as classifying again. When an individual figures out some implications, elaboration occurs (Guilford, 1967).

Apart from these attributes, there is a common idea that novelty is required for creativity. Kaufmann (2003) says, novelty could be explained in two parts: high task novelty and high solution novelty. The former one refers to not having a ready solution accepted earlier and the latter one indicates a requirement for change in or removal of previously accepted opinions. Indeed, Kaufmann (2003, p. 243) distinguishes novelty as "novelty on the stimulus and novelty on the response side". According to responses to tasks, four categories occur: "familiar task-familiar solution", "novel task-familiar solution", "familiar task-novel solution", and "novel task-novel solution" (Kaufmann, 2003, pp. 243-247). This taxonomy suggests that the first category is reflecting usual problem-solving; the second is reflecting intelligence of a person which addresses using previous knowledge in an unfamiliar tasks; the third is reflecting proactive creativity where no external force affects person; and finally the fourth category is reflecting reactive creativity because problem itself may encourage a person to be creative (Kaufmann, 2003).

To conclude, fluency, flexibility, elaboration, and novelty seem to be the most important of the several characteristics of creativity which can be better explained with the Four C model of creativity. However, elaboration could be quite hard to be determined by others.

2.2.1 Definition of mathematical creativity

Huckstep and Rowland (2008) claim that creativity is such a complicated construct that it is important to be cautious and careful while applying to mathematics**.** Mathematics and creativity should be displayed together so as to classify a behavior as mathematically creative (Haylock, 1987).

In early years, creativity for children could be connecting mathematical skills and situations. Later, children may solve problems, find patterns, or produce games in an original way as well (Briggs, 2005). Originality of an idea could be explained by novelty for the person who comes up with it although it is not novel for everyone (Kaufmann, 2003). Therefore, judging a solution, pattern, or game as original is complicated like creativity itself and originality might be judged relatively.

According to Briggs (2005), how much of mathematics learned at schools is used in our daily life is questionable. However, problem-solving is also a component of real life; therefore, it could be seen as creative if several solutions provided (Briggs, 2005). In addition, problem-posing activities might enable students to be more creative in mathematics (Silver, 1997). Hence, problem solving and problem-posing are two methods of addressing creativity in mathematics.

Problem solving and problem posing process could be investigated according to fluency, flexibility, and novelty of ideas (Silver, 1997). However, there is another issue to be considered. For example, 6x5=65 could be categorized as novel but it is clearly not creative, which shows that ordinal mathematical facts are also important (Nadjafikhah & Yaftian, 2013). Hence, appropriateness is another criterion in mathematics to acknowledge creativity (Haylock, 1997). Thus, flexibility, fluency, and novelty could be searched under the light of appropriateness in problem solving and problem-posing processes and products.

Redefinition tasks, which require students to give responses in many ways, are seen as a third way for creativity in mathematics (Haylock, 1987). There is a restriction that students redefine “the elements of the situation in terms of their mathematical attributes” (Haylock, 1987, p. 71). For instance, a task of writing down several times about “What the two numbers 16 and 36 have in common” (Haylock, 1987, p. 72) could be an example of redefinition task.

In general, overcoming fixations and thinking in a flexible and divergent way, which are describing creativity in a way, are ignored in schools to discard customized ideas (Haylock, 1987). At least students should have a chance to show their creativity in their assessments (Haylock, 1987). Students should experience that there is not only one correct solution or one approach of exploration in mathematics (Haylock, 1987). Divergent thinking requires divergent production on a given open-ended task (Haylock, 1997).

Problem-solving, problem-posing, and redefinition are three methods that yield in producing ideas for mathematical creativity (Haylock, 1987, 1997). However, if a student is required to ask a question whose answer is 4, he/she may say “5-1”, “6-2”, “7-3” and could extend it to infinity. However, this does not show any creativity. Therefore, in order to see creativity in mathematics, divergent thinking should be accompanied with flexibility and originality (Haylock, 1997).

To conclude, not only problem solving and problem-posing, but also redefinition may yield in creativity in mathematics and they could be evaluated considering fluency, flexibility, novelty (or originality), and appropriateness.

2.3 Studies about creativity

Here, studies about creativity will be examined under the titles: the general indicator of creativity, explicit and implicit theories of creativity, mathematicians, adolescents, children, and classroom discourse.

2.3.1 The search for the general indicator of creativity

According to Vygotsky (2004), it is not true to match creativity with talented people. Everyone could be creative to some certain degree, which is natural. In fact, imagination of children is more affluent than that of adults since a child disciplines imagination less with faithful attitude even though an adult could imagine more than a child (Vygotsky, 2004).

Creativity has generally been studied in relation to intelligence and achievement (Cropley 1967, 1969; Runco & Albert, 1986). Although people tend to believe that creativity and intelligence tests are correlates of each other, they are actually not (Cropley, 1969). While the threshold theory postulates that there is a relationship between creativity and intelligence up to around IQ of 120, studies investigating threshold theory contradicts with this view (Runco & Albert, 1986). It may be because of differences in measuring creativity and intelligence. Indeed, for average IQ level, there is a positive relationship between IQ and creativity. Although individuals of high IQ do not show high creativity precisely, individuals of low IQ are not likely to show high creativity (Cropley, 1967).

Except from intelligence, achievement is not a predictor of creativity as well. Runco and Albert (1986) state that correlations between creativity, which was measured by divergent thinking test, and achievement, were found significant in high achiever group while insignificant in low achievers. Although some tests include “creativity” in their names, which results in insight that they seem to measure creativity, they measure divergent thinking in real (Piffer, 2012).

Meanwhile, there is also controversy on the criterion for creativity such as divergent thinking. Not only divergent thinking, but also convergent thinking is decisive for creativity (Cropley, 1969; Nadjafikhah & Yaftian, 2013; Piffer, 2012). Therefore, person’s creativity could be determined by none of the divergent thinking tests, IQ tests, and the Consensual Assessment Technique, which is commonly used for assessing creativity by researchers (Kaufman, Baer, & Cole, 2009). In this regard, indirect measurement of creativity is more appropriate than direct measurement (Piffer, 2012).

Whilst creativity is generally attributed to a single concept, it could be multifaceted (Sternberg, 2005). According to Piffer (2012), there are many factors affecting creativity such as IQ level, motivation and persistence, which may affect different people in a different way. To illustrate, analytical thinking could be prerequisite for scientific creativity. On the other hand, there could be some factors such as being open-minded for new experiences and having divergent thinking which are feeding creativity in general. It is still almost impossible to find general factor of creative potential because these factors are quite different from each other. However, some studies which have shown the discovery of general factor of creativity have several problems both conceptually and methodologically (Piffer, 2012).

2.3.2 Explicit and implicit theories of creativity

People's different understandings of creativity could obstruct development of creativity in society. However, people with no information about creativity may help discover what is deficient in scientific studies. There are many studies that attempted to explore creativity theories by using explicit and implicit theories. The former stands for theories developed by psychologists and social scientists while the latter is constructed with person's beliefs (Saracho, 2012).

Educators may believe that creativity could not be associated with all children. If educators think that a child is creative, then they may boost his/her creativity. If not, they may not encourage those children enough. Therefore, studies regarding implicit theories of creativity are important to cope with such biases (Jukić, 2011).

Implicit theories of 27 educational researchers were explored in a study (Maksić & Pavlović, 2011). Through a questionnaire with open-ended questions, participants' answers included few words reflecting their opinion; therefore, the explanations were mostly parallel to explicit theories of creativity. The open-ended questions included in the questionnaire were "What is creativity?","What are the characteristics of creative persons and production", "In what way does creativity manifest itself from childhood to adulthood?" and "To what extent and in what ways can the school support creativity of students?".

Participants commonly mentioned the words: "originality", "novelty", and "difference". The words "uniqueness", "authenticity", "imagination", and “rationality” appeared only once. Besides, some participants preferred to use nouns but not adjectives in the description and they wrote “idea”, “inspiration”, and “personality”. They characterized indicators of creativity by “personal traits”, “creative products”, “behaviors”, “intelligence”, and “knowledge”. The descriptions of creativity in preschool years according to them were “playing, asking questions, giving unusual responses, exploration, drawings, songs, and dramatic expression” (Maksić & Pavlović, 2011, p. 225). It was described for primary school students with “curiosity” but as “specific interests in specific curricular or extra-curricular activities within the school setting” (Maksić & Pavlović, 2011, p. 225).

2.3.3 Mathematical creativity of students

There are a number of studies aiming to explore and increase students’ mathematical creativity in the literature. Problem posing tasks in order to examine mathematical creativity (Van Harpen & Presmeg, 2011, 2013) and taking real-life photos to increase their mathematical creativity in problem posing tasks (Švecová et al., 2014) might constitute one scope of these studies. There are studies providing problem solving tasks to explore mathematical creativity of students as well (such as, Chen, Himsel, Kasof, Greenberger, & Dmitrieva, 2006). Moreover, some of the studies examined research studies conducted about creativity in school mathematics in order to understand mathematical creativity of students, suggesting the use of redefinition tasks in order to discover mathematical creativity (Haylock, 1987, 1997).

In recent years, studies about mathematical creativity of gifted students or high ability students (see Lev & Leikin, 2013; Sarrazy & Novotna, 2013) have been conducted. Moreover, there is a search for whether there is a relationship between mathematical ability and mathematical creativity, or not (Kattou, Kontoyianni, Pitta-Pantazi, & Christou, 2011). There are also studies which tried to explore mathematical creativity in classroom context (such as, Levenson, 2011).

In these studies, mathematical creativity is determined by some criteria which are flexibility, fluency, novelty, overcoming fixations in divergent production, and originality (Chen et al., 2006; Haylock, 1987, 1997; Levenson, 2011; Švecová et al., 2014; Van Harpen & Presmeg, 2011, 2013). Findings of those studies suggest that overcoming fixations could be achieved by problem solving and divergent production tasks (Haylock, 1987, 1997), taking real-life photos help students in posing mathematically creative problems (Švecová et al., 2014), interactions in classroom increase mathematical creativity of students (Levenson, 2011), and mathematical creativity could be dominantly perceived by either context or inclusion of mathematics work of the posed problems depending on the social and cultural settings of the students (Van Harpen & Presmeg, 2011, 2013).

Some of the studies used problem solving questions to search for mathematical creativity. Leikin and Kloss (2011) used a multiple solution test which enables students to give many answers to questions. Participants were 108 tenth grade and 158 eight grade students. Results showed that, tenth graders performed better in the test as they got higher marks in terms of correctness of solution. Moreover, tenth graders were better in fluency. However, flexibility and originality of eight graders and tenth graders changed depending on the task, or no differences occurred (Leikin & Kloss, 2011).

By relating problem posing to creativity of students, Van Harpen and Presmeg (2011) aimed to examine attitudes and abilities of 55 Chinese and 30 U.S. students in problem posing process. Data collection tools were mathematics content test and a mathematical problem posing test. Some students were interviewed as well. For the question "How did you pose these problems?", one group talked about being funny or interesting while the other mentioned the mathematics included. That is, while context was important for U.S. students, involving mathematics work was important for Chinese students. Van Harpen and Presmeg (2013) further compared mathematical creativity of students in relation to their mathematical content knowledge by implementing mathematical content knowledge and mathematical problem posing tests to students in China and U.S. They found that there were no relationships between students' mathematical content knowledge and their abilities in problem posing (Van Harpen & Presmeg, 2013).

Haylock (1987) analyzed several studies about creativity in school mathematics. Two common points about creative abilities, which are "overcoming fixations in mathematical problem-solving and divergent production within mathematical situations" (Haylock, 1987, p. 72) were recognized and were suggested as a base in order to strengthen and honor mathematical creativity in schools (Haylock, 1987, 1997). There could be two types of fixation: "self-restriction" and "adherence to stereotype approaches" (Haylock, 1997, p. 69). Students could not overcome fixations as they were used to use prototype examples in mathematics or as they depended on the algorithms they developed in problem solving tasks (Haylock, 1997).

By combining collective learning and theories of mathematical creativity, Levenson (2011) aimed to explore collective mathematical creativity as both process and product in two 5th grade classes and one 6th grade class, in a public school in middle-income suburb. Students' responses in a lesson were evaluated according to fluency, flexibility, and originality. However, the main point was that a simple idea of a student could lead another student to develop a new one. Therefore, in classroom it could be different to distinguish product or process from creativity. Furthermore, it could be possible to improve individual mathematical creativity by supporting collective mathematical creativity (Levenson, 2011). Therefore, atmosphere in the classroom constitutes an important role in mathematical creativity.

Classroom observation should be supported with interviews in order to get better understanding of the situation in classroom (Levenson, 2011). Interviews could help researchers understand everyday creativity more precisely than standardized tests. Moreover, not only qualitative but also quantitative data could be gathered through interviews (Piffer, 2012).

2.3.4 Classroom environment

In order to support children's creativity, freedom is a must since it is necessary for a child to be creative. Therefore, activities linked to creativity should not be implemented compulsorily. Moreover, they should be emerged with respect to their interests (Vygotsky, 2004).

In order to develop creativity in mathematics, accepting different opinions and not applying mathematical skills based only on memorization should be emphasized. Besides, it is essential that students should be encouraged to go further through unfamiliar ideas and relationships (Mann, 2006). When teachers listen to students' own way of thinking, they could understand students' way of thinking. Moreover, students are given a chance to think about their thinking way again and may revise that idea or put some more on it. Such a process could yield in divergent and multiple ideas in classrooms where students are also evaluating their ideas (Doerr, 2006).

2.4 Fractions

Mathematics education should support that students feel mathematics as a part of real-life and mathematics is worth to deal with it. Students should build their knowledge of mathematics themselves in order to make sense of what they are doing. For that reason, experiences from concrete to abstract order are important issues in primary education level. Concrete materials and manipulatives should be used for satisfying students' different needs and interests. Moreover, it is important to use manipulatives in teaching of new concepts (MoNE, 2015).

Learning mathematics is an active process. Sharing tasks let students observe equal-sized portions of a whole and decrease in number of equal-sized portions while students begin learning fractions, and therefore students could build their knowledge. Even though students build their own knowledge, it depends on the mathematics knowledge of the social group in which students are. Mathematics is dynamic, growing area, and a cultural product. When students share inferences they gained through activities contributes the development of knowledge. While students are interacting with each other, they could think about such cultural aspect of mathematics (MoNE, 2015)

Fractions topic could be harder for some students due to its complexity (Van de Walle, 2007). There is not a direct correspondence between natural numbers and fractions. Students’ previous knowledge about natural numbers could create some problems for students in constructing conceptual understanding of multiplication of fractions (Prediger, 2008; Van de Walle, 2007). Students should begin with their previous knowledge and go further with fractions. They might begin learning fractions with idea of fair share, then; they could get familiar with the definitions or operations (Van de Walle, 2007). Drawing and coloring appropriate part of the shape drawn might show how well students learn about the topic (Peck & Jencks, 1981).

Whole, half, and one fourth concepts introduced to students in grade 1. Part-whole relationship is emphasized and fraction symbols are presented in grade 2. Unit fraction concept is focused in grade 3 and the relationship between numerator and denominator is strengthen. Students are expected to define and use proper and improper fractions in grade 4. Moreover, students add and subtract fractions with same denominator, and solve problems (MoNE, 2015)

Models in fractions serve important role in mathematics education. Besides, if the same activity could be done with two distinct models, then students might find it particular. Circular "pie" pieces, rectangular regions, geoboards, drawings on grids or dot paper, pattern blocks, and paper folding are all could represent area models of fractions. Circular "pie" pieces could be beneficial for learning one whole whereas others could be good representation for distinct wholes (Van De Walle, 2007).

2.5 Other concerns

According to Vygotsky (2004), drawing, by which children could reflect expressions readily, could be an activity for creativity in early childhood. However, school age children’s desire to draw decreases unless they are encouraged to draw at school or home. Children at school age experience new endeavor and adopt oral or literary creation in their new endeavor. Literary creativity of a child could be improved by encouraging him/her about an issue which he/she could easily understand, activates his/her emotions, above all boosts him/her to reflect own inner world (Vygotsky, 2004).

Although socioeconomic disadvantages have an influence on learning mathematics, such obstacles are equalized due to the fact that disadvantaged children become good at group work and problem solving as they keep importance on music, dance, and humor at home which lead to improvement of some talents (Dance & Higginson, 1979). Therefore, low socioeconomic status should not be perceived as having negative effect on children’s creativity.

2.6 Summary of the studies about creativity

Experience has an influence on being creative (Vygotsky, 2004). Moreover, MoNE (2015) says that new concepts should enable students experiencing with concrete materials. For example, models and manipulatives are important for fractions teaching (Van de Walle, 2007).

Research have explored mathematical creativity of students in redefinition, problem posing, and problem solving on paper (Chen et al., 2006; Haylock, 1987, 1997; Van Harpen & Presmeg, 2011, 2013) and in classroom (Levenson, 2011). These studies have shown that creativity could be better observed through these tasks and classroom interactions. Similarly, these and many studies showed that indicators of creativity should be rather fluency, flexibility, and novelty.

There is also a need for studies concerning evaluating both the creativity process and products (Shriki, 2010). Therefore, research studies about creativity in school settings should consider conducting classroom observations as well (de Souza Fleith, 2010). Lack of interviews in creativity research might have hindered what students intend by working on paper and pencil tasks (Mann, 2009). Thus, interviews could help researchers to get worthy information about creativity (Piffer, 2012; Levenson, 2011).

CHAPTER III

METHODOLOGY

The main aim of this study was to investigate 5th grade students' mathematical creativity in the fractions concept. In this chapter, research design and variables, participants and sampling, and data gathering issues were presented. Further, trustworthiness of the study was discussed.

3.1 Design of the study

In order to investigate students' mathematical creativity, a qualitative research design was employed. In qualitative research studies, researchers try to understand participants' perspectives (Fraenkel, Wallen, & Hyun, 2012). Therefore, qualitative study could help the researcher in investigating how students' mathematical creativity examples arose in the 5th grade.

In order to provide insight for mathematical creativity of 5th graders, basic qualitative inquiry was employed in this study (Creswell, 2012). The aim of this inquiry is not to generalize findings to population but deepen the understanding of the central phenomenon. Such an understanding might be resulted from understanding people or environment in detail (Creswell, 2012). For this reason, multiple sources of data were collected to respond to the research questions.

3.1.1 Context of the study

3.1.1.1 School Context

The school in which the study was carried out was selected in researcher's convenience since she was working as a Mathematics teacher there. It was a private school in the south of the Turkey, in a touristic district in the Mediterranean Region.

There were approximately 400 students in total in preschool, primary school and middle school. The preschool is single floor building whereas the primary and middle school has three floors. There are two football courts, three volleyball courts, and a park in the garden. There are many tennis tables in the school. There are science and computer laboratories, cafeteria, and canteen. In the classroom, there are smart boards with internet connection. There were approximately less than 20 students in each class. There were several student clubs such as chess, music, mathematics, and sports. Parents generally had high socioeconomic status and they were encouraging their children for joining these clubs.

There are certain essential issues to initiate creativity in classrooms. First, different point of views should be welcomed in classrooms. Furthermore, applications based on memorization should be avoided. Instead, students should be encouraged to produce different ideas (Mann, 2006; Švecová et al., 2014). Considering these concerns, a 5th grade classroom in this private school in which manipulatives were used were selected at the researcher's convenience in order to understand the real phenomenon.

3.1.1.2 Mathematics Lesson Context

Classes in primary schools in Turkey are taught by primary teacher. When students start the middle school (grades 5 to 8), they meet different teachers for each content area. Therefore, the 5th grade class where the study was conducted was taught mathematics by a mathematics teacher for the first time. The goals of mathematics education stated by MoNE were mentioned to the students in the beginning of the fall semester after meeting with students.

In general, the lessons included discussions about solving problems in real life, estimation practices, and integrating mathematics learned in school to life. "How did you do?" and "Why?" questions were mostly asked by the teacher in the lessons. The aim of the lessons was to improve students' understanding of mathematics. The main characteristics of the classroom culture was listening to others' opinions and appreciating them. When students came up with an idea, they were asking for the way of their thinking. Later, they were explaining their comprehension or own mathematics. Several activities which were in pair, group, or individually were done in lessons. Technology use and concrete examples of mathematics such as paper folding or manipulatives were other concerns of the mathematics lessons. Since there were several topics before fractions concepts, students were used to this atmosphere before the data collection in fractions topic.

In each topic, students were asked about giving daily life examples. They were also requested to talk about what they knew from the last year or what they did different from each other. In the middle of the lesson, either individual-study activity sheets were distributed or group activities took place. Apart from those, since students' enjoyment of mathematics was aimed, several activities were done together as a whole class.

3.1.1.3 Fractions Topic Context

In addition to the classroom atmosphere, the topics covered might be important in displaying mathematical creativity through the usage of manipulative (see MoNE, 2009b; Van de Walle, 2007; Vygotsky, 2004). Fractions topic in the 5th grade is selected for the study because this topic might better welcome the examples of mathematical creativity which students would produce by using manipulative (MoNE, 2009b; Van de Walle, 2007).

It was a challenging task for the teacher that as a researcher she was eager to record a good lesson which yields in creativity in mathematics lesson. Therefore, the MoNE curriculum and several other resources were visited for a thorough preparation including digital resources. MoNE has nine objectives which are given in Table 3.1 for students to achieve.

Table 3. 1

Objectives about Fractions in 5th Grade

Number	Objectives on fractions
1	Order unit fractions.
2	Represent unit fractions on number line.
3	Understand that mixed fraction is sum of a natural number, and convert mixed fraction to improper fraction and vice versa.
4	Compare a natural number and improper fraction.

Table 3. 1 (continued)

Objectives about Fractions in 5th Grade

5	Understand that simplifying and expanding do not change the value of fraction and compose equivalent fractions.
6	Order fractions with same denominator or denominator which is multiple of each other.
7	Calculate proper fraction of the whole and the whole of the proper fraction by using unit fractions.
8	Add and subtract fractions with same denominator or denominator which is multiple of each other.
9	Solve problems requiring addition and subtraction of fractions with same denominator or denominator which is multiple of each other.

MoNE suggested that 6 weeks should be allotted for fractions topic in the 5th grade and study lasted for approximately 6 weeks.

Fractions topic began approximately in the middle of the year. In order to focus on the concept instead of rules, learning process began with sharing tasks. For example, the teacher asked about the share of each person when if 3 people shared 5 toasts. Students' responses (see Figure 3.1) yielded in three different sharing processes (Van de Walle, 2007, p.295).

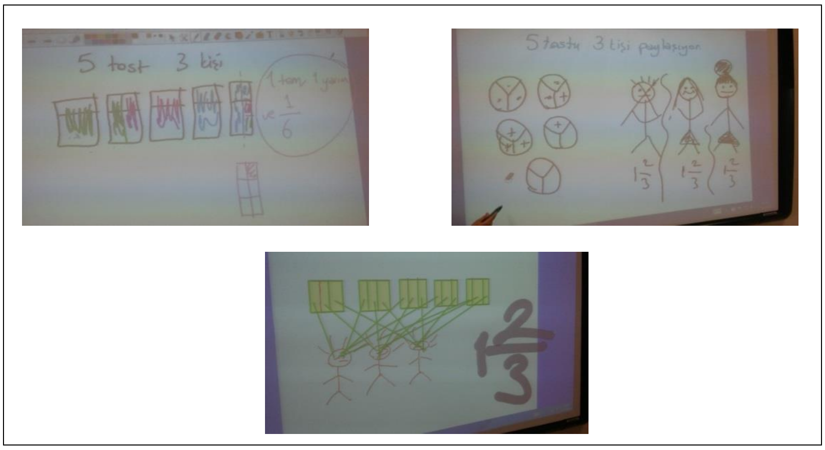


Figure 3. 1 Students' Different Responses to the Case of 3 People Sharing 5 Toasts

Materials in the lessons were paper, fraction strip, rope, fruits, measurement cups, water, and activity sheets. Activities included making fraction strip, paper folding, ordering fraction on a rope, and dividing fruits. First, student made their own fraction strip on the given activity sheet on paper. They colored it and cut into parts. Students used them for ordering unit fractions, that is, the first objective seen in Table 3.1. For the second objective an activity done: Two students represented numbers 0 and 1 and some students were given a paper on which some unit fractions written to locate it correctly. For the third objective in Table 3.1, students played with parts of fruits simulated that they converted mixed fraction to improper fraction and vice versa. Later, Digging Up Improper Fractions activity on Illuminations website (https://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/3-5/DigIt%20AS.pdf) was solved by the students.

For equivalent fractions, they utilized papers and students got familiar with the concept by paper folding. Besides, drawings were made and fraction strips students made before were used in other activities. For the sixth objective in Table 3.1, teacher attached a rope and numbers 0, 1, 2, 3, and 4 on the wall of classroom. Then, students stick several fractions on the wall. If a students had a mistake they thought together to overcome that mistake. Measurement cups were used in addition and subtraction of fractions. Change in fractions was observed by adding or taking water which is a fractional part of the cup. Besides, fraction strips, drawings on number line, and paper were utilized in addition to measurement cups.

In lessons, the course book of the 5th graders was followed mainly. The problems and tests were solved by the students. There were some extra word problems. If applicable, the teacher requested students for giving real-life examples to topics. Moreover, some worksheets were distributed by the teacher. They were including some redefinition tasks such as $\frac{2}{3} = \frac{O}{6} = \frac{10}{O} = \frac{O}{O}$. Similarly, for the homeworks the course book was performed by the students. Sometimes, teacher distributed worksheets and problems.

Students got familiar with the idea that there could be another solution for the redefinition questions throughout the study, although they were unfamiliar with them at the beginning. Generally, they were listening to other answers more carefully in this task as they were wondering if the answer was correct and if their answer was still unique in the classroom. In addition, there was some problem posing tasks in course book (Yaman, Akkaya, & Yeşilyurt, 2013). They were unfamiliar with the problem posing task previously. However, they liked writing their own problems. Besides, problem solving type questions which students were used to in preceding mathematics lessons were also included in the lesson frequently. Therefore, redefinition, problem posing, and problem solving tasks, which might bring about mathematical creativity, were covered beforehand.

3.1.2 Participants of the study

In this study, the researcher was also the teacher and her students of 5^{th} grade class in a private school were participants. The teacher-researcher in this study had one year of a teaching experience at the time of the study. She graduated from an Elementary Mathematics Education Program and started to work in this school one year after her graduation. She had finished her coursework for her Master’s degree at the time of the study.

Purposive sampling is generally employed in qualitative studies because there is a need for interacting with an appropriate group of individuals to satisfy the conditions for conducting research (Fraenkel et al., 2012). Moreover, intentionally selected individuals could help understanding the central phenomenon (Creswell, 2012). Although there were three classes of 5^{th} graders, the teacher-researcher intentionally and purposefully selected the specific classroom in which the study was conducted since the students’ academic achievement was in middle compared to the other two classrooms. Moreover, students seemed to have a desire for explaining different ideas they developed in a specific class. In addition to their desire for using mathematics language, they had a desire for mathematical activities. Students generally liked doing activities in the mathematics lessons. Whenever teacher told them to do an activity together they were up to do and bring any material. Teacher asked them about if they would like to participate her study on fractions before the study began. They volunteered for the study.

There were a total of 18 students (9 males, 9 females) in the class in which the study was conducted. Three students joined the class in the academic year that the study was conducted and they came from public schools. Eight students graduated from one primary teacher’s class in the current school and 7 students attended the other primary teacher's class in the same school. Thus, it might be speculated that the students in the class had very similar mathematical background.

3.2 Data collection tools

In this study, data were collected by observation, open-ended questions, and in-class short interviews. Observation was utilized during the mathematics lessons in the Spring Semester in 2016. The responses of students to the specific tasks were observed in order to find examples of mathematical creativity in the classroom. Two quizzes were implemented during the study in order to get more data to understand the phenomenon of creativity better. A questionnaire was used in order to collect data about mathematical creativity at the end of first part of the unit "Fractions, decimal numbers, and percentage". Moreover, homeworks and notebooks of students and mathematics examinations were reviewed. However, homeworks were not used in this study as data source because they did not produce sufficient data.

3.2.1 Observation

In classroom, two criteria of mathematical creativity, which are appropriateness and novelty, were the focus of the researcher in her observations. Appropriateness is related to whether or not examples of mathematical creativity are valid under mathematical facts (Haylock, 1997; Nadjafikhah & Yaftian, 2013; Van Harpen & Presmeg, 2011). Therefore, the responses were evaluated in terms of their appropriateness in order to label them as mathematically creative act. Novelty, on the other hand, could be determined by the frequency of the responses by a specific student compared to the responses provided by rest of the students (Haylock, 1997). In that sense, if there was an infrequent or original response produced in the classroom, then it was considered as novel, and therefore an example of creativity.

Naturalistic observation, which reflects observing without manipulating or controlling variables or individuals (Fraenkel, Wallen, & Hyun, 2012) approach was adopted in this process. The lessons were video-recorded during the study. Additionally, the researcher observed and took short field notes on what happened in the classroom, including appropriate and novel ideas, strategies, solutions, definitions, and questions posed by students. Another foci of the observation was the use of manipulatives and real-life questions which could be attributed to Vygotsky's theory of creativity. Therefore, in which manipulative use or question solution an example of mathematical creativity appeared was also noted. If there were some cases in which the researcher found an example of mathematical creativity, then those situations were noted as well. Observation protocol is available in Apendix B.

A video recorder was settled into the back wall of the classroom two weeks before fractions topic in order to minimize its effect on students' responses and behaviors in the classroom. Teacher requested them to be in their normal mood as the videos would be watched by only her and no grading would be done.

3.2.2 Paper-and-pencil tasks

There were two types of paper-and-pencil tasks: a questionnaire and two quizzes. At the end of the fractions topic, a questionnaire including three different tasks was implemented within two-lesson-hour, 80 minutes. The three tasks, which were redefinition, problem posing, and problem solving tasks, were combined to encourage students display their mathematical creativity. By examining studies conducted about mathematical creativity (such as, Haylock, 1997; Van Harpen & Presmeg, 2011), 5th grade mathematics textbook (MoNE, 2014), and objectives of 5th grade mathematics curriculum (MoNE, 2013), the items of the questionnaire were written by the researcher. One task was adapted from the National Assessment of Educational Progress (NAEP) and one from a study conducted by Haylock (1997). The third task was written by the researcher in correspondence with the Vygotsky's theory of creativity and other studies about problem posing (Van Harpen & Presmeg, 2011, 2013). The aim of the questionnaire was to examine students' responses in multiple tasks in terms of creativity criteria to find examples of mathematical creativity regarding fractions in the 5th grade mathematics education.

Three beginning-mathematics teachers, one researcher with Master's degree on Science Education, and the supervisor of the researcher reflected their opinions on the questionnaire tasks for whether or not each type of the task, i.e., redefinition, problem posing, and problem solving, was appropriate for the expected action. In addition, they criticized the content of the questions. Therefore, face validity was checked by them.

After revisions made with respect to expert opinions, the questionnaire was piloted twice in the summer of 2015 and was revised taking into consideration two pilot studies. It was piloted with two groups of students in mathematics club of the school at the researcher's convenience; first with 5 students from 7th grade and then, with 10 students from 6th grade of the same school. After the first pilot study, the researcher decided on changing the problem solving task as students gave only one correct answer. The researcher also could not find another way of solution. Therefore, that task could not help researcher in gathering mathematical creativity (see Figure 3.2). Besides, if students misunderstood or could not understand, the researcher noted those situations down in both pilot studies. The tasks were administered in students' classrooms and they were given adequate time that each one of them completed the tasks. Later, they were requested to comment on the questions and necessary changes were made by the researcher.

Items appeared in the first pilot study of the questionnaire about fractions in the order of redefinition, problem posing, and problem solving tasks as given in Figure 3.2.

1. State all the things that are same about two numbers: $\frac{1}{3}$ and $\frac{1}{2}$. (adapted from Haylock, 1997)

2. Select two of the words below (or you may add two and use them) and pose as many as possible problems about fractions.

   bird, ball, tree, book, street, school, French Revolution, African desert, ………, ……

3. I ate 1/2of a loaf of bread. My friend ate 1/3of another loaf of bread. I said that I ate more pizza than you, but my friend said he/she ate more. Under what conditions my friend could be right? (adapted from NAEP, 1992)

Figure 3. 2 Items in the First Pilot Study of the Questionnaire

The items appeared in the second pilot study are given in Figure 3.3.

1. State all the things that are same about two numbers: $\frac{1}{3}$ and $\frac{1}{2}$. (adapted from Haylock, 1997)

2. *a bird, a ball, a tree, a book, a street, a school, the Turkish War of Independence, French Revolution, Fairy chimneys, Nasreddin Hodja,………, ……….*

Use two words above or you may add (at most two) yourself. Pose problems about fractions as much as you can.

3. Joe rode his bicycle from his house to his friend's house. He first rode $\frac{1}{4}$ of the path below and then, he rode $\frac{1}{5}$ of it . Decide where on the path best indicate how far Joe rode to his friend's house and put a sign. (adapted from 2011 NAEP Assessment for 4th Graders)

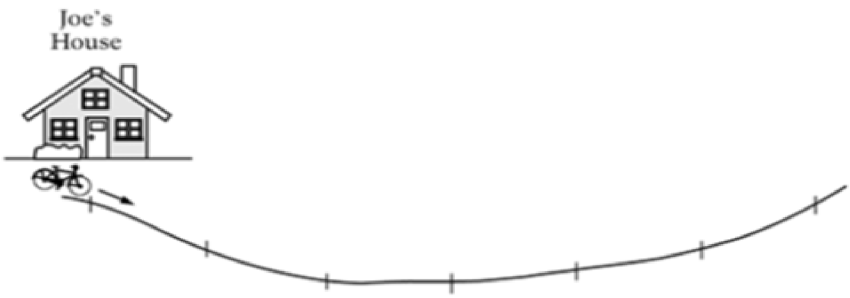


Figure 3. 3 Items in the Second Pilot Study of the Questionnaire

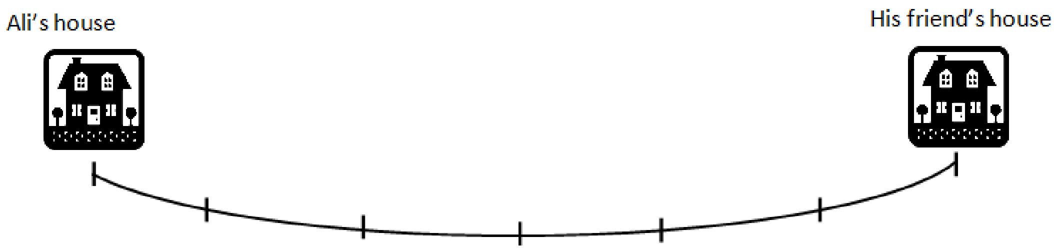
The analysis of pilot studies, participants' written responses, and consideration of their comments resulted in certain changes in the tasks in the questionnaire. Confusing and ambiguous expressions were detected in the analysis and these were removed or rewritten. For the problem posing task, some words were omitted and a new word: "water" was added intentionally by the researcher in case activity done with measurement cups and water evoked something in students' minds and would enable students being creative. Revised version of the questions is given in Figure 3.4.

1. State all the things that are same about two numbers: $\frac{1}{3}$ and $\frac{1}{2}$ (adapted from Haylock, 1997).

2. *a bird, a ball, the Turkish War of Independence, French Revolution, Fairy chimneys, water, Nasreddin Hodja,………, ……….*

First select two words above or you may add (at most two) yourself. Pose problems with those two words about fractions as much as you can. Note that it must be **solvable**.

3.



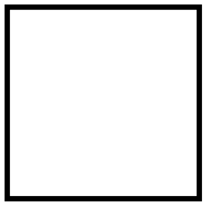
Ali would like to ride a bicycle from his house to a friend's house. He fist rides $\frac{1}{4}$ of the way and then $\frac{1}{5}$ of it. At that moment, **put a sign** about where he is on the picture above. Please explain your solution. (adapted from 2011 NAEP Assessment for 4th Graders)

Figure 3. 4 Final Version of the Questionnaire

During the instruction, the teacher-researcher tried to let students think about different questions in case they could demonstrate or develop their creative thinking by the two quizzes she implemented. Question in Quiz A was taken from Van de Walle (2007) and it was related to unit fractions. Quiz B included two problem posing tasks about addition and subtraction in fraction unit in the mathematics-course book written by MoNE (Yaman, Akkaya, & Yeşilyurt, 2013). Quiz A and B lasted nearly 10 and 40 minutes, respectively. The quizzes were implemented in different lessons and they are given in Table 3.2.

Table 3. 2

Two Quizzes Used in the Study

Quiz A (Van de Walle, 2007)	Ali would like to plant flowers on the $\frac{3}{4}$ of his garden. Please help him by drawing plans which shows how to design it on the square given below. 
Quiz B (MoNE, 2016, p.181)	I. Pose as many problems as possible whose solution is 2+$\frac{3}{4}$. II. Pose as many problems as possible whose solution is 1-$\frac{2}{7}$.

3.2.3 In-Class Short Interviews

The aim of in-class short interviews was to understand how students came up with examples of mathematical creativity. Because the teacher-researcher was concerned about the possible influence of interviews in terms of change in students' behaviors, she decided to conduct in-class short interviews by asking questions to clarify student's responses. These interviews were sometimes conducted in the break.

When there was an example of mathematical creativity in the class, students to whom those creative responses belonged were interviewed immediately after the response in a natural way. If examples of mathematical creativity were noticed in written documents, then the students with example of mathematical creativity were interviewed in the break as if teacher wonders how a student thought about it. These instances were video-recorded.

The main aim of the interview questions was to learn about first, in what ways mathematical creativity occurred and second, if students' experiences with reality had a role in mathematical creativity. If the first question could have been answered, then the second question had been followed by the researcher. Interview protocol is presented in Appendix C.

3.3 Data collection process

Ethics permission was taken in January in 2015 (see Appendix A). Later, the teacher-researcher was presented that permission to school principal and she was permitted by the school principal. Parents and students were informed before the study. After one year the study was began. Detailed information was given in ethics section below.

Data collection process began in February 2016. There were 5 hours of mathematics lessons in a week and the fractions unit took approximately 6 weeks and a total of 29 class hours. All the lessons were video-recorded during the fractions unit. Two quizzes were implemented in February and one in March. They lasted 10 to 40 minutes depending on the content and task. The questionnaire was implemented at the end of the fractions topic in March.

Students were asked to give as many answers as possible for the tasks in the quizzes and the questionnaire before the implementation of these tools, in order to gather data for flexibility and fluency of responses. For each task, one sheet paper was distributed. If there was a need, students were given additional sheet of paper.

3.4 Data analysis

Data of the study were gathered from observations and written documents of 18 students in the class and the transcriptions of course videos were the data of the study. Names of students were coded by numbers such as S1 and S2 for data analysis. All the data including transcriptions and paper-tasks such as questionnaire and quizzes were analyzed separately as in-lesson and on-paper creativity examples.

For the finding instances of in-lesson mathematical creativity, data gathered by observation were reviewed and codes were generated separately for each situation. Codes were checked with co-coder, who is a beginning mathematics teacher with 3 years of a teaching experience, first and then, an expert, who is researcher in mathematics education, gave feedback. By this process, irrelevant data were eliminated and final version of the categories was determined. Considering those categories, excerpts of the lesson was read by the researcher several times and coded.

For the analysis of data obtained from paper-tasks, the same coding process was repeated. After appropriateness criterion was satisfied, fluency which is number of correct response(s) (Haylock, 1997) and flexibility which is "number of different responses" (Haylock, 1997, p.71) criteria were checked in addition to novelty criterion. If any criterion was held, then a response became a mathematical creativity example.

The criteria of mathematical creativity in lesson were appropriateness and novelty while on paper tasks they were appropriateness, fluency, flexibility, and novelty. Analysis of data in lesson began with the first criterion for mathematical creativity, appropriateness, which is about if a response is valid under mathematical facts (Haylock, 1997). Hence, in-lesson and on-paper data were reviewed for disregarding inappropriate responses. If there was an inappropriate response, then it was eliminated.

Figure 3.5 displays responses of two students (S1 and S17) which were not satisfying appropriateness criterion. S1 had a mistake in enumeration and S17 had not colored the correctly partitioned whole as shown in Figure 3.5. Both of them could be considered as novel because there were no other students with similar ideas in the classroom. S1 was the only student representing $\frac{3}{4}$ of a whole by irregular parts. Hence, she might be attributing fraction concept to area concept. On the other hand, S17 represented $\frac{3}{4}$ of a whole by different-sized parts. He first might have thought about area concept as well but then, skipped his drawing. If they could have colored their drawing properly, they could be only two students who associated their previous knowledge with the current one.

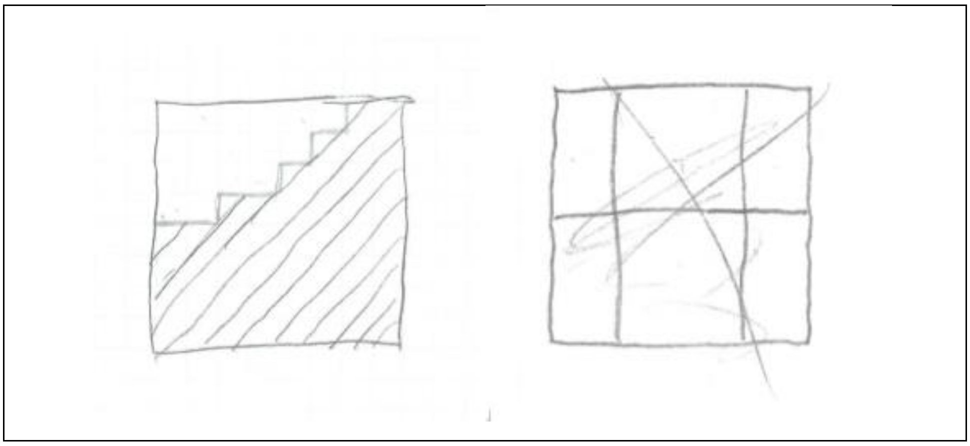


Figure 3. 5 Inappropriate Responses of S1 and S17

Fluency is about counting appropriate responses for each student. Fluency examples were provided with respect to the most number of responses of a student. In other words, let a student gives 6 responses to a task and he/she has 5 appropriate responses, which is the most number of appropriate response in the class, then responses of him/her becomes fluency example.

Flexibility criterion prerequires the determination of categories since it depends on the number of different categories for a task. Again, after eliminating inappropriate responses, then categories were made up of from the total of appropriate responses of students in the class. Categorization enabled to see responses from different windows. For instance, think about that there are four categories for a task, and Ayşe gives 4 appropriate responses in first category while Osman gives 3 appropriate responses in first and second category, then the latter student's responses illustrates flexibility example. It was a quite challenging task but after locating each one of the students’ responses, the number of categories a student's responses located in was counted first. Later, a student with the most number of categories were presented flexibility example. Table 3.3 depicts such process. In this example, Osman provides flexibility example compared to Ayşe.

Novelty, “the statistical infrequency of the responses in relation to the peer group” (Haylock, 1997, p.71), was the focus. If a student gave response(s) which is unique in the class, then, it was labeled as mathematical creativity example. It means that a response or responses should be given by only one student, and therefore there is not same response in the class apart from one student, then it becomes a novelty example.

Table 3. 3

Example for Flexibility Analysis

Students	Number of responses	Number of appropriate responses	Number of different categories responses in
Ayşe	6	4	1
Osman	4	3	2

Additionally, collective mathematical creativity was introduced in the analysis of in-lesson data since there were some cases where one student leaded another student to be creative and such interaction resulted in collective mathematical creativity. In-lesson mathematical creativity instances were exemplified in accord with the following topics respectively: types of fractions, converting improper fractions into mixed numbers, ordering fractions, and problems about fractions.

Students' responses in problem posing tasks and redefinition task in the questionnaire were translated into English first by the researcher and then, her supervisor checked them. Sample student responses for redefinition and problem posing tasks in original version are presented in Appendix D.

3.5 Trustworthiness of the study

Trustworthiness could be mainly based on how accurately a researcher design a study (Merriam, 2009). Internal validity, reliability, and generalizability are three main factors showing a study accounts for it trustworthiness (Merriam, 1998).

Internal validity, which is about how credible research findings are, could be carried out by "triangulation, checking interpretations with individuals interviewed or observed, staying on-site over a period of time, asking peers to comment on emerging findings, and clarifying researcher biases and assumptions." (Merriam, 2009, p. 234). In this study, triangulation was done by "multiple investigators" method (Merriam, 2009, p. 216). That is, not only the researcher, but also two people, who were one mathematics teacher and one instructor in a mathematics education department in a university in Turkey, analyzed data. Later, they worked as a team to reach commonality in findings. There is a postmodernist view that there is not a fixed point to be triangulated yet there are at least 3 sides of facts to understand our world (Merriam, 2009). Hence, it was actually "crystallizing" data as personal analysis was reflecting researcher's own perspective but the change reflects one big perspective which includes the combination of their perspectives.

In qualitative studies, reliability does not refer to seek a "single reality" as in quantitative studies (Merriam, 2009, p. 220). On the contrary, it tries to understand the world by approaching it. As human behavior could not be regarded as the same for all the time, there is no way to expect the same findings if the same study is repeated. Therefore, it is important to present consistent findings over data gathered in qualitative studies (Merriam, 2009). Triangulation done by three people feeds consistency as well. Moreover, the process of data collection, data analysis, and findings was expressed in detail to ensure reliability in this study. At the end, researcher's role and biases which may affect the reliability of findings were presented (in the following section).

Generalizability could be achieved by providing an in-depth description of the context and procedures in the study (Merriam, 2009). Context of the study was described in detail and the findings were represented in detail. Therefore, adequate information was provided for transferability.

3.6 Ethics

Ethical concerns constitute an important part of the data gathering. The study was presented to METU Ethics Committee initially and then to MoNE Ethics Board. After necessary permissions obtained, permission of the school administrator was obtained. Later, Parent Informed Consent, since participants were younger than 18, and Participant Consent Form were given by the researcher. In these forms, the identity of the researcher, the purpose of the study, research procedures, risks, and confidentiality and withdrawal issues were addressed. For further questions, phone number and e-mail address of the researcher was provided at the end.

Confidentiality was ensured in that the identities of participants were not given to other people. Students' names were coded into numbers such as S1 and S2 in the findings. It was assured that data collected would not be harmful for them. Moreover, the researcher informed participants that their works would not be graded.

3.7 Limitations of the study

Generalizability of the results constitutes some limitations for the current study. For instance, the fact that mathematical creativity instances were only searched in fractions topic makes the study not generalizable to other topics. In addition, participants were 5^{th} graders in a private school at researcher's convenience. Thus, it is not possible to think of the study on reflecting any 5^{th} graders in Turkey. Moreover, data were restricted to paper-tasks, observations, and in-class short interviews. Although use of different data sources was employed in order to decrease the limitation of the findings, it is still not possible to say those of data collection tools are sufficient for understanding mathematical creativity examples of 5^{th} graders in fractions topic.

Video-recorder might have an effect on students' participation during the lesson. For this reason, video-recorder was introduced to class 2 weeks before the study in order to minimize effect.

Researcher role and bias directly affect the findings and interpretations of the study and they stand for another limitation for the study. Researcher's role and bias were explained in the next section in order to explain researcher's perspective.

3.8 Researcher's role and bias

I believed in every student and that they could understand mathematics if there are suitable materials for their learning. I encouraged them to go one step further from where they already were in mathematics. I generally talked about not to be afraid of mathematics.

The study began with the thought of being only a researcher. However, I was not only a researcher but also their teacher. It was a challenging part of the study as I was interacting students all the time as a teacher but I was a researcher at the same time. I tried to distinguish both positions in the school in order to make students feel more comfortable with the study. Moreover, there was another reason, not to interrupt the study with a researcher position. I tried to do my best with not only activities but also pursuing the teacher role.

During the lessons, my role was to ask questions and trigger students' ideas on the mathematics. In addition, I valued any idea told and evaluated it for whether it was mathematically valid or not. I tried to increase mathematical communication by letting students express their ideas. I generally chose to facilitate their learning with some activities and questions, and let them do mathematics. However, my facilitator role only changed if there was disrespect to anyone or if students ignored their homework.

Data analysis part was the most important part for me. I watched videos and read paper tasks several times as I did not want to miss any mathematical creativity examples. It was very rare that I labeled a piece of data as irrelevant. Instead, I tried to see from student's perspective and reminded myself that it could be meaningful for a student. However, triangulation with other two people gave me extra perspective and I decided on the mathematical creativity examples after 5 revisions were made.

CHAPTER IV

FINDINGS

In this study, several data collection tools were utilized in order to provide satisfying information to understand the creativity phenomenon better. Findings chapter was mainly divided into two subsections which are the findings of paper-tasks and lesson-tasks. Paper-tasks cover Quiz A, Quiz B, and questionnaire. Lesson-tasks titled as the main topic of the corresponding lesson in which mathematical creativity occurred. Participating students were randomly assigned a number for the purposes of anonymity and they were referred as S and the number throughout the text such as S1 and S2.

The findings of the study were presented under two main sections with respect to criteria set by the researcher so as to find mathematical creativity. There are three criteria for the paper-tasks: fluency, flexibility, and novelty while novelty had been set before as the only criterion for lesson-tasks. However, while conversations in the lessons were transcribed, fluency and flexibility criteria were included as some responses of students showed collective fluency and collective flexibility as mentioned in other study (Levenson, 2011).

Paper tasks were two quizzes and a questionnaire. Fluency and novelty examples were decided on looking at all responses as explained in data analysis section in previous chapter. Yet, flexibility analysis required some effort to categorize responses. Since each task has its unique answers, single categorization was not considered in order to analyze data and responses to each paper-task leaded to different categorization. Therefore, each response was located in the corresponding categories which were decided for that paper-task. Two aspects were taken into consideration for the problem posing task. These could be summarized into two as (i) what data say mathematically and (ii) what data say contextually.

In lesson-tasks, data were collected by video-recorder and the teacher took field notes for some lessons. While analyzing data, lessons were transcribed and essential parts were provided in the second section of this chapter. As the researcher was also the teacher, some explanations about students and the lessons were added to clarify some points in results.

In-class short interviews were mostly embedded in the lesson as the teacher did not want to influence her natural relationship with the students during the study. For the paper-tasks, the teacher asked students about their way of thinking in the next lesson. However, they did not give worthy information as they tended to forget why they did what they did creatively. Still, if there had been any explanation, it was provided under mathematical creativity example.

4.1 Mathematical Creativity on Paper Tasks

Mathematical creativity was analyzed as the products on the paper tasks. The responses were examined firstly by means of appropriateness. Later, mathematical creativity was labeled according to fluency, flexibility, and novelty criteria on the paper. As stated before, fluency is the number of correct responses given by a student. Flexibility is the number of categories in which responses of a student are covered. Some researchers (Van Harpen & Presmeg, 2013) put a limit that if the response is given by only less than 10% of the students, then it is novel. In this research study, the researcher set the same criteria. That is, only if one student gave a specific appropriate response that other students did not give, then it was labeled as novel since less than 10% of 18 students makes 1 student.

4.1.1 Quiz A

Fractions lesson began with unit fractions. Students discussed about equal-sized portions of the whole and the situations in which a given shape was correctly partitioned, or not. In the second lesson, the teacher intentionally asked the question in Figure 4.1 in order to assess students' knowledge.

Ali would like to plant flowers on $\frac{3}{4}$ of his garden. Please help him by drawing plans which show how to design it on the square given below.

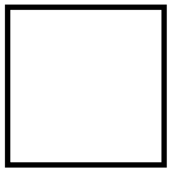


Figure 4. 1 The Question in Quiz A (adapted from Van de Walle, 2007, p.302)

Students asked if they could give more than one answer and if possible, how to do it. Teacher guided students that if they would find more than one answer, then they could draw a square equal to one provided and colored it as well. After a while, each student drew more squares and represented the fraction $\frac{3}{4}$.

One student was absent in the implementation day. Therefore, the answers of 17 students were analyzed. There were a total of 71 responses for this question. The task seemed quite interesting and enjoyable for them probably because it was not full of mathematical operations. Fourteen of the responses were not mathematically correct, and therefore, not appropriate. Therefore, analysis was conducted by 57 appropriate responses.

Categories were determined after excluding colored fractional parts. Lines students had drawn in order to partition a whole was the focus. The five categories that came out of the analysis were shown in Figure 4.2.

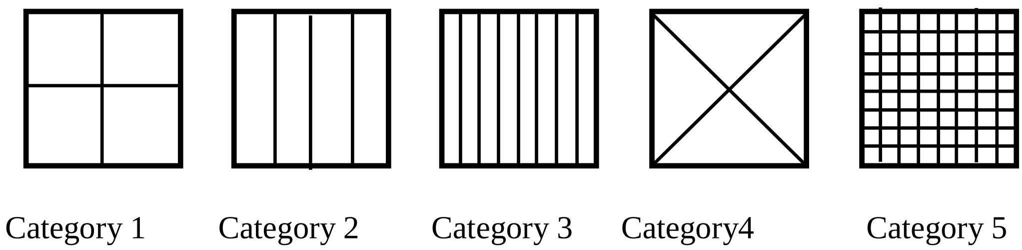


Figure 4. 2 Categories for Quiz A

If a partition can be done by rotation, then they were included in the same category. For instance, the second and the third category were represented with vertical lines in Figure 4.2. However, they were also representing partitioning with horizontal lines.

The number of responses in each category and the categories that each student responded are shown in Table 4.1.

Table 4. 1

Number of Responses in Each Category per Student

Students	Category 1	Category 2	Category 3	Category 4	Category 5	Total Appropriate Response
S1	1	2	0	0	0	3
S2	2	0	0	0	1	3
S3	1	0	0	1	0	2
S4	1	2	0	1	0	4
S5	1	2	0	1	0	4
S6	2	0	0	1	0	3
S7	0	4	0	0	0	4
S8	1	2	0	1	0	4
S9	1	2	0	1	0	4
S10	2	2	0	2	0	6
S11	1	2	0	0	0	3
S12	1	2	0	1	0	4
S13	0	2	0	0	0	2
S14	0	2	2	0	0	4
S15	1	1	0	0	0	2
S16	1	0	0	0	0	1
S17	1	2	0	1	0	4
S18	-	-	-	-	-	-
Total	17	27	2	10	1	57

The Table 4.1 illustrates that among 57 appropriate responses, there are 17, 27, 2, 10, 1 responses in Category 1, 2, 3, 4, 5, respectively. Most of the responses were included in Category 2 while there is only one response in Category 5.

4.1.1.1 Fluency

Fluency was found basically by counting the number of appropriate responses of each student. Students who provided the most number of appropriate responses shows a fluency example. Table 4.2 illustrates the number of appropriate responses of students. To illustrate, there is one student with 1 appropriate response and three students with 2 appropriate responses. However, there is a student giving 6 appropriate responses which was considered as a fluency example.

Table 4. 2

Fluency Analysis for Quiz A

Number of Appropriate Responses	Number of Students
1	1
2	3
3	4
4	8
5	0
6	1
Total	17

Figure 4.3 below presents S10's responses as an example for fluency and therefore, mathematical creativity.

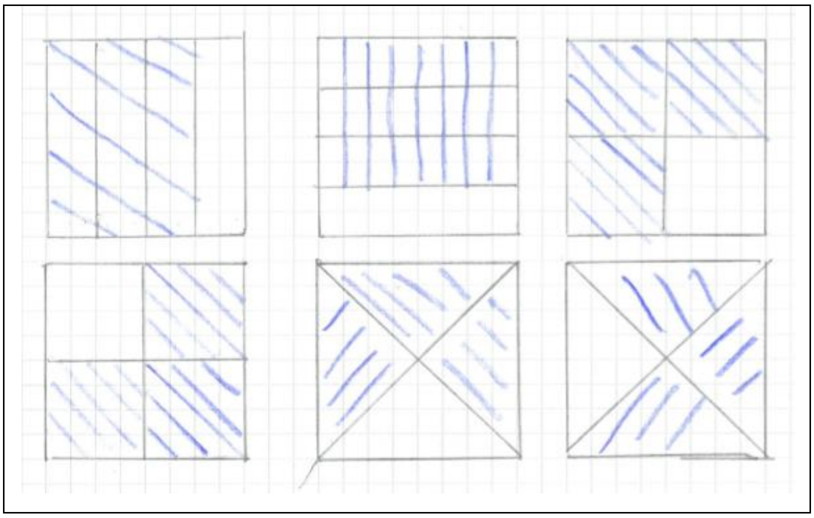


Figure 4. 3 Fluency Example, Response of S10, Quiz A

4.1.1.2 Flexibility

For the flexibility, there is a need for counting how many categories responses of students have produced. Therefore, the number of different categories for each student was counted by using Table 4.1, and Table 4.3 was constructed. As Table 4.1 shows S1 has one response in Category 1 and two responses in Category 2, making two different categories. After repeating the same process for each student, the number of students with responses in one category, two different categories, and three different categories were counted. There were no students with responses in more than three different categories. Indeed, there were 3 students whose responses were categorized into one category and 7 students with responses in two and three categories.

Table 4. 3

Flexibility Analysis for Quiz A

Number of Categories Responses in	Number of Students
1	3
2	7
3	7
Total	17

Table 4.3 states that seven students' responses are in 3 different categories and they stand for flexibility example. As Table 4.1 suggests, students' responses have a tendency to appear in Category 1, Category 2, and Category 4. Moreover, those seven students gave answers in Category 1, Category 2, and Category 4. To illustrate, one student's response was randomly selected to represent flexibility example and provided in Figure 4.4

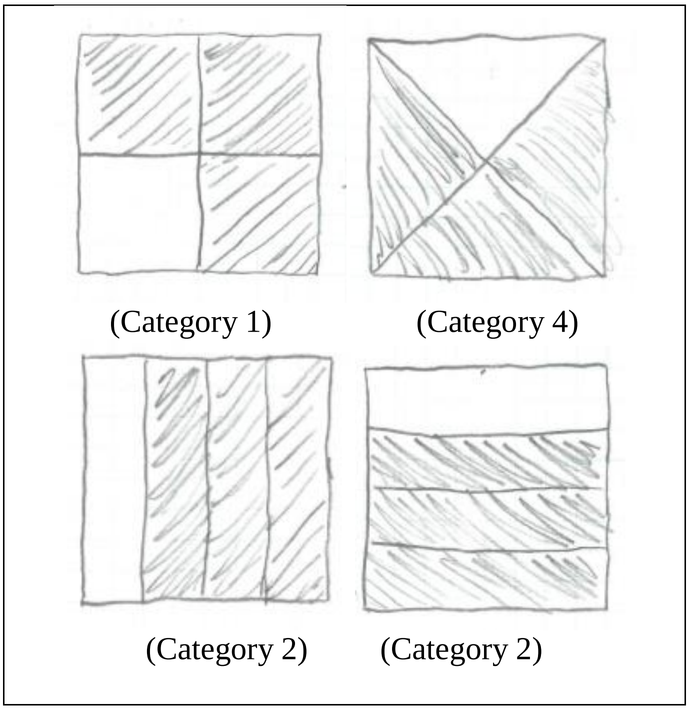


Figure 4. 4 Flexibility Example, Response of S5 (Representing Responses of S4, S8, S9, S10, S12, S17), Quiz A

Table 4.1 shows that students, who presented flexibility example to Quiz A, S4, S8, S9, S12, and S17 had four appropriate responses and S10 had six appropriate responses. However, they all depicted mathematical creativity in terms of flexibility criterion. Thus, this case explains how number of different categories responses in influences flexibility analysis instead of number of appropriate responses.

4.1.1.3 Novelty

In order to mention novelty, there should be only one student's response(s) in a specific category that other students did not produce. Table 4.1 shows that S14 and S2 are the only students with appropriate responses in Category 3 and Category 5, respectively. Therefore, those responses are novel, and so mathematically creative. The related pictures are given in Figure 4.5 and in Figure 4.6.

Figure 4.5 shows the responses of S14 in Category 3. The drawings at top-right and the bottom was included in Category 3. S14 is the only student in the classroom with those responses. Thus, those drawings stand for novelty example.

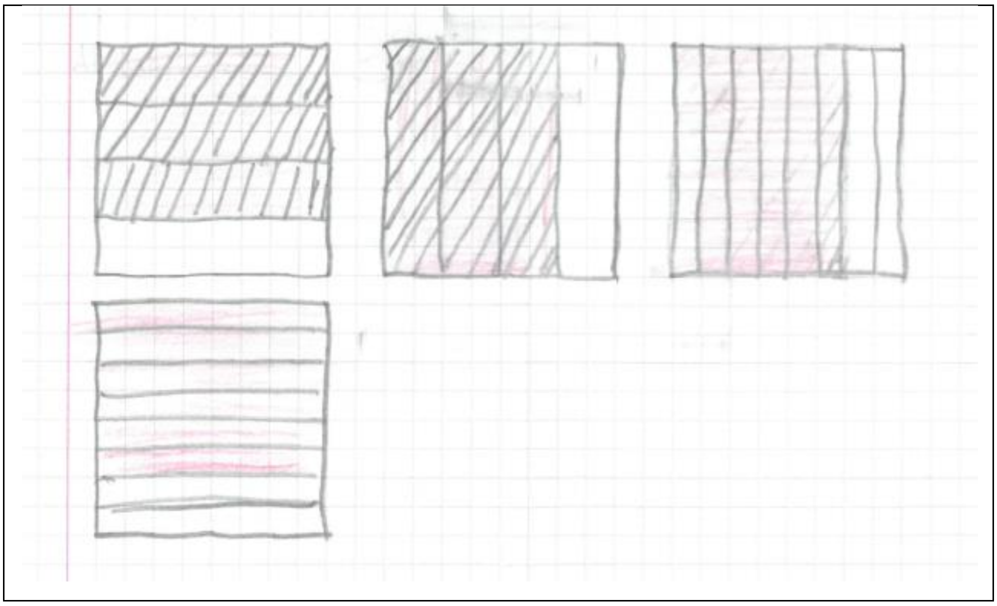


Figure 4. 5 Novelty Example, Response of S14, Quiz A

Figure 4.6 indicates the responses of S2. The drawings at top-right and the bottom was covered by Category 5. Moreover, S2 is the only student in the classroom with those responses, which are novel.

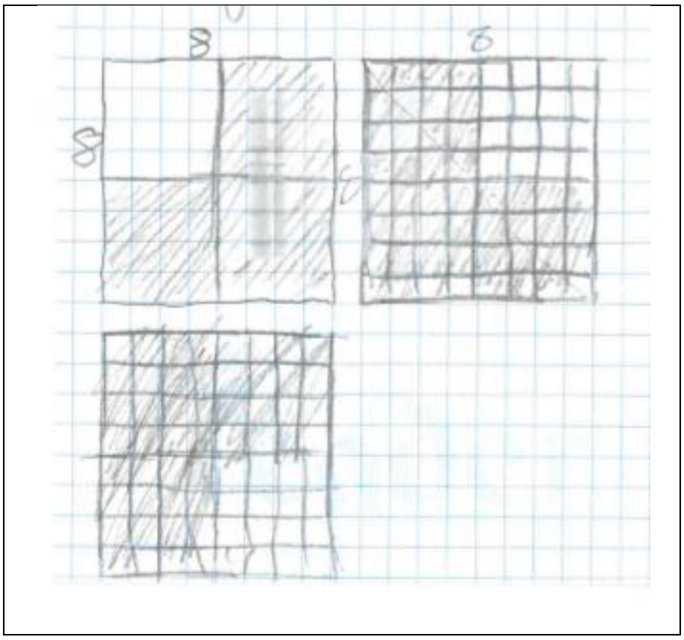


Figure 4. 6 Novelty Example, Response of S2, Quiz A

To sum up, both S14 and S2 had unique responses. They were the only students with the mentioned answers and the idea of those answers in the classroom. Hence, their responses were novel and therefore, mathematically creative in the classroom.

4.1.2 Quiz B

Addition and subtraction in fractions were illustrated with papers, fraction strips, and liquids. While students got familiar with the topic, they solved questions provided at the end of fractions topic in their course book (Yaman, Akkaya, & Yeşilyurt, 2013). Teacher requested students to solve two problem posing questions in the course book in Figure 4.7 separately on a sheet of paper. Students did not deal with problem posing type tasks in mathematics much until this one. In order to prevent interaction among students, which might affect the data gathered, among students, the points that some students were not clear about the task were answered silently.

I. Pose as many problems as possible whose solution is $2+\frac{3}{4}$.

II. Pose as many problems as possible whose solution is $1-\frac{2}{7}$.

Figure 4. 7 Questions (Yaman, Akkaya, & Yeşilyurt, 2013, p.181) in Quiz B

There were 17 students in the classroom during the implementation of the problem posing tasks. Their answers were examined in the order of criteria: appropriateness, fluency, flexibility, and novelty. For the fluency analysis, all appropriate problems were counted. Since categorization is needed for the flexibility and novelty analysis, two subsections, which were mathematical component and contextual component, were constructed first, and then categories were determined. The same process was repeated in the analysis of problem posing task in the questionnaire.

4.1.2.1 Fluency

Fluency is the number of the appropriate responses. For the first question, there were 45 responses of which 22 were appropriate. Table 4.4 shows that 6 of the students could not pose appropriate problems while one of the students, S4, posed 4 appropriate problems. Thus, S4's responses illustrated an example of fluency, which is an indicator of mathematical creativity in Quiz B.

Table 4. 4

Fluency Analysis, Question I in Quiz B

Number of appropriate responses	Number of students
0	6
1	4
2	4
3	2
4	1
Total	17

Four posed problems of S4 for the operation $2+\frac{3}{4}$ are given in Table 4.5.

Table 4. 5

Posed Problems of S4, Question I in Quiz B, Fluency Example

Number	Problems
1	Jale drinks 2 liter water and her friend Esma drinks $\frac{3}{4}$ liter water. How much water do Jale and Esma drink in total?
2	Ali worked 2 hours in the field and his sister Asya worked $\frac{3}{4}$ hours. How many hours did Ali and Asya work in the field?

Table 4. 5 (continued)

d Problems of S4, Question I in Quiz B, Fluency Example

3	Meryem ate 2 of the cookies on the plate. After she played a game, she ate $\frac{3}{4}$ of a cookie. How much cookies did Meryem eat?
4	An architecture drew 2 house models in the first day. The next day, she/he drew $\frac{3}{4}$ of a house model. How much drawing did she/he do in two days?

For the second question, there were 45 responses but only 17 of them were counted as appropriate. Table 4.6 reveals that 8 of the students could not pose any appropriate problems. Once more, S4 posed more number of appropriate problems than the other students with 5 posed problems, which was considered as an indication of fluency.

Table 4. 6

Fluency Analysis, Question II in Quiz B

Number of Appropriate Responses	Number of Students
0	8
1	5
2	2
3	1
4	0
5	1
Total	17

Five problems of S4 for the operation 1-$\frac{2}{7}$ were provided in table 4.7.

Table 4. 7

Posed problems of S4, Question II in Quiz B, Fluency Example

Number	Problems
1	Ziya played football for one hour after work. His best friend Ejder played football for $\frac{2}{7}$ of an hour. How much more time did Ziya play football than Ejder?
2	Melike swam 1 hour in the pool. Her twin Demet swam $\frac{2}{7}$ of an hour in the pool. How much more time did Melike swim in the pool than Demet?
3	Emel prayed at night in one hour. She prayed in the morning in $\frac{2}{7}$ of an hour. How much quickly did Emel pray in the morning than at night?
	A cameraman recorded a video 1 hour in a day. The next day, she/he

Table 4. 7 (continued)

Posed problems of S4, Question II in Quiz B, Fluency Example

4	videotaped $\frac{2}{7}$ of an hour. How much more time did she/he videotape in the first day than the second? (Show your work with fractions)
5	One group of children sat on a sofa. The other group sat $\frac{2}{7}$ of a sofa. How much more space did first group filled than the second group?

Since fluency criterion is about the number of appropriate problems, note that mathematical and contextual categories are involved together in fluency analysis. However, the next two criteria will evaluate problems from those two angles.

4.1.2.2 Flexibility

Responses of 11 students in the first question included at least one appropriate problem. For those problems, 4 categories were formed by considering the mathematics involved in problems. Moreover, the fact that S4 posed four problems for the first question, but mathematical part of the problem was only in one category (operation $2+\frac{3}{4}$) led the researcher to check contextual part as well. It was an indicator that some students focused on context, but some on numbers. In this analysis, the focus was both on mathematical and contextual components of the problem.

Four categories were constructed in the mathematical component. Table 4.8 depicts four categories and corresponding examples.

Table 4. 8

Categories in Mathematical Component, Question I in Quiz B

Number	Categories	Examples
1	No change in terms	$2, \frac{3}{4}$
2	Separating the first term into two	$1, 1, \frac{3}{4}$
3	Changing the first term	$\frac{8}{4}, \frac{3}{4}$ or $1\frac{4}{4}, \frac{3}{4}$
4	Changing both first and second terms	$1\frac{8}{8}+\frac{12}{16}$

Table 4.9 shows the number of categories of responses of students. For example, eight students' responses were in one category and two students' responses were in two categories. Yet, one student (S11) had appropriate problems which were in 3 different categories.

Table 4. 9

Flexibility Analysis (mathematical), Question I in Quiz B

Number of Categories Responses in	Number of Students
1	8
2	2
3	1
Total	11

S11 had shown example of flexibility in her problems which are given in Table 4.10.

Table 4. 10

Posed Problems of S11, Question I in Quiz B, Flexibility Example

Number	Problems
1	Ali ate 2 loafs of bread and Veli ate $\frac{3}{4}$ of a loaf of bread. How many loaf of bread did Ali and Veli eat together? (Category 1)
2	Zülal ate $1\frac{2}{2}$ cake and Sevgi $\frac{3}{4}$ of a cake. How much cake did they eat in total? (Category 3)
3	Add $1\frac{8}{8}$ and $\frac{12}{16}$, and then write fractions in simplest terms. (Category 4)

Contextual part of problems had 3 categories which were (1) home, (2) school, (3) jobs, (4) animals, (5) nature, (6) community, and (7) no context. These are clarified in Table 4.11.

Table 4. 11

Categories in Contextual Component, Question I in Quiz B

Categories	Name	Examples
1	Home	What they ate, bought, did with parents
2	School	Homework, projects, papers, games with teachers and friends
3	Jobs	An architecture, a house painter, a greengrocer
4	Animals	Cows, rabbits, birds
5	Nature	Pine tree, walnut tree, flowers, field
6	Community	Fethiye Municipality
7	No context	

Table 4.12 shows that among 11 students with appropriate responses, six students' responses were in any one of the categories while four students' responses were in any two of the categories in Table 4.11. However, there was a student with responses in three different categories, i.e., Category 1, Category 2, and Category 3. Thus, it was labeled as the flexibility example.

Table 4. 12

Flexibility Analysis (contextual), Question I in Quiz B

Number of Categories Responses in	Number of Students
1	6
2	4
3	1
Total	11

S4's problems appeared in 3 categories, which showed flexibility. S4's problems are listed in Table 4.13.

Table 4. 13

Posed Problems of S4, Question I in Quiz B, Flexibility Example

Number	Problems
1	Jale drinks 2 liter water and her friend Esma drinks $\frac{3}{4}$ liter water. How much water do Jale and Esma drink in total? (Category 1)
2	Ali worked 2 hours in the field and his sister Asya worked $\frac{3}{4}$ hours. How many hours did Ali and Asya work in the field? (Category 5)
3	Meryem ate 2 of the cookies on the plate. After she played a game, she ate $\frac{3}{4}$ of a cookie. How much cookies did Meryem eat? (Category 1)
4	An architecture drew 2 house models in the first day. The next day, she/he drew $\frac{3}{4}$ of a house model. How much drawing did she/he do in two days? (Category 3)

Note that the problems above were written in the previous section, as a fluency example. Thus, S4 illustrated both fluency example and flexibility example in terms of context in Quiz B.

For the second question, 9 students had at least one appropriate problem. There were three categories for mathematical component in a total of 17 problems: (1) no change in terms, (2) separating the second term into two, and (3) changing the first term. Table 4.14 exemplifies the categories.

Table 4. 14

Categories in Mathematical Component, Question II in Quiz B

Number	Categories	Examples
1	No change in terms	$1, \frac{2}{7}$
2	Separating the second term into two	$1, \frac{1}{7}, \frac{1}{7}$
3	Changing the first term	$\frac{27}{27}, \frac{2}{7}$

Table 4.15 shows the number students in each category. While 8 students had responses located in one category, one student's answers were in two categories, showing flexibility example.

Table 4. 15

Flexibility Analysis (mathematical), Question II in Quiz B

Categories	Number of Students
1	8
2	1
Total	9

Table 4.16 indicates posed problems of S12, whose answers were from two categories.

Table 4. 16

Posed Problems of S12, Question II in Quiz B, Flexibility Example

Number	Problems
1	Mrs. Şule used $\frac{1}{7}$ of a packet of rice in the afternoon and $\frac{1}{7}$ of it in the evening. How much rice left? (Category 2)
2	If Ceren eat $\frac{2}{7}$ of an apple, how much of apple left? (Category 1)
3	Şebnem spent $\frac{2}{7}$ of her money. How much of her money left behind? (Category 1)

Contextually, problems were allotted in 7 categories: (1) home, (2) school, (3) jobs, (4) animals, (5) sports, (6) community, and (7) beliefs. Generally, students posed problems about school and home, that is, their daily life. Table 4.17 represents the number of categories that students' responses were found. Six students' responses were in one category while two students' responses were in two categories. However, there was one student with responses in four categories.

Table 4.17

Flexibility Analysis (contextual), Question II in Quiz B

Number of Categories Responses in	Number of Students
1	6
2	2
3	0
4	1
Total	9

Table 4.17 indicates that one student's (S4) responses were in 4 categories in context component. S4's problems are listed in Table 4.18 for being flexibility example as given in the previous section as a fluency example again.

Table 4. 18

Posed Problems of S4, Question II in Quiz B, Flexibility Example

Number	Problems
1	Ziya played football for one hour after work. His best friend Ejder played football for $\frac{2}{7}$ of an hour. How much more time did Ziya play a football than Ejder? (Category 5)
2	Melike swam 1 hour in the pool. Her twin Demet swam $\frac{2}{7}$ of an hour in the pool. How much more time did Melike swim in the pool than Demet? (Category 5)
3	Emel prayed at night in one hour. She prayed in the morning in $\frac{2}{7}$ of an hour. How much quickly did Emel pray in the morning than at night? (Category 7)
4	A cameraman recorded a video 1 hour in a day. The next day, she/he videotaped $\frac{2}{7}$ of an hour. How much more time did she/he videotape in the first day than the second? (Show your work with fractions) (Category 3)
5	One group of children sat on a sofa. The other group sat $\frac{2}{7}$ of a sofa. How much more space did first group filled than the second group? (Category 1)

To conclude, S11 and S4 were showing flexibility examples for the first question and S12 and S4 were showing flexibility examples for the second question in Quiz B according to mathematics and context components, relatively.

4.1.2.3 Novelty

For the first question which required addition of fractions, there were 22 appropriate answers. Contextual and mathematical components of the question were the foci of the analysis. In general, students had a tendency to use 2 and $\frac{3}{4}$. However, some students changed the numbers properly and differed in mathematical component such as dividing whole and fraction part into two ((1+$\frac{1}{4}$) + (1+$\frac{2}{4}$)), expansion ($\frac{8}{4}$+$\frac{3}{4}$), dividing whole into two (1+$\frac{3}{4}$+1), and converting whole into mixed numbers (1$\frac{4}{4}$+$\frac{3}{4}$), (1$\frac{8}{8}$+$\frac{12}{16}$). Taking 4 categories constructed before which were, (1) no change in terms, (2) separating the first term into two, (3) changing the first term, and (4) changing both first and second term, into consideration, students' responses in each category were analyzed. Table 4.19 shows that for only Category 4, there was a response of only one student (S11), which is required for novelty. Therefore, problem posed by S11 reflected novelty example of mathematical creativity.

Table 4. 19

Responses of Students in Each Mathematical Category, Question I in Quiz B

Students	Category 1	Category 2	Category 3	Category 4
S1	*			
S2	**			
S3		*		
S4	****			
S5	**			
S6			*	
S7	*	*		
S8				
S9	*			
S10	*		*	
S11	*		*	*
S12				
S13				
S14	***			
S15				
S16				
S17				
S18	-	-	-	-
Total	16	2	3	1

For the contextual analysis of novelty, seven categories decided before were utilized and the responses of each student were put under corresponding categories in Table 4.20. There was only one appropriate response for each of the categories 4, 6 and 7. Therefore, those responses stand for novelty example of creativity. In Category 4, response of S14; in Category 6, response of S7, and in Category 7, responses of S11 were seen as novel.

Table 4. 20

Responses of Students in Each Contextual Category, Question I in Quiz B

	Categories						
Students	1 Home	2 School	3 Jobs	4 Animals	5 Nature	6 Community	7 No context
S1	*						
S2	**						
S3		*					
S4	**		*		*		
S5		**					
S6			*				
S7	*					*	
S8							
S9					*		
S10	*		*				
S11	**						*
S12							
S13							
S14	**			*			
S15							
S16							
S17							
S18	-	-	-	-	-		
Total	11	3	3	1	2	1	1

As a context, students wrote problems about daily life at school, at home, with family members and students or teachers. However, some students went beyond and thought about animals and community. Here, Fethiye Municipality and rabbit words were seen as original apart from other contexts since there was only one appropriate posed problem in the corresponding category. Moreover, one student, S11, wrote a problem without context and this was considered in Category 7. Original answers were S14 in Category 4, S7 in Category 6, and S11 in Category 7. Note that S11's same problem was seen as novel according to mathematical component of the analysis since she changed both first and second term in the operation. Therefore, her problem was noted as creative in terms of both mathematical and contextual component of the analysis. Novel problems standing for mathematical creativity examples are given in Table 4.21.

Table 4. 21

Novelty Examples by Mathematics and Context, Question I in Quiz B

Foci	Problems
Context	A rabbit ate two carrots while a turtle ate $\frac{3}{4}$ of a carrot. How much carrot did they eat together? (S14)
Context	Fethiye Municipality decided to pave 3 roads with same length with asphalt. They asphalted 2 roads first day since the weather was good. However, since it was rainy, they could asphalt $\frac{3}{4}$ of a road in the following day. If they finished it on the third day, how much road did they pave with asphalt on that day? (S7)
Context & Mathematics	Add $1\frac{8}{8}$ and $\frac{12}{16}$. Then, write them again with lowest terms. (S11)

For the second problem, which was about subtraction, there were 17 appropriate answers. Again, contextual and mathematical components were the foci. Mathematical component had yielded 3 categories: (1) no change in terms, (2) separating the second term into two, and (3) changing the first term. Novelty analysis was also done using those categories. Responses of each student for each category were shown in Table 4.22. In Category 2 and 3, there is only one appropriate problem posed by only one student. Table 4.22 displays that problems posed by S11 and S12 as they were only one in their category. Therefore, they were original for the class.

Table 4. 22

Responses of Students in Each Mathematical Category, Question II in Quiz B

Students	Category 1	Category 2	Category 3
S1			
S2	*		
S3			
S4	*****		
S5			
S6			
S7			
S8			
S9	**		
S10	*		
S11			*
S12	**	*	
S13			
S14	**		
S15			
S16	*		
S17	*		
S18	-	-	-
Total	15	1	1

Responses of S11 and S12 are given in Table 4.23 as novelty examples.

Table 4. 23

Novelty Examples of Students (matheamtical), Question II in Quiz B

Focus	Problems
Mathematics	Teacher Çağla drinks $\frac{27}{27}$ cup of tea while Teacher Elvan drinks $\frac{2}{7}$ cup of tea. How much more tea does Teacher Çağla drink than Teacher Elvan? (S11) Mrs. Şule used $\frac{1}{7}$ of a packet of rice in the afternoon and $\frac{1}{7}$ of it in the evening. How much rice left? (S12)

As a context, seven categories constructed before were utilized for novelty analysis. Table 4.24 displays the responses in each category for each student. In Category 4, 6, and 7, there is only one student's response for each. Hence, they stand for novelty examples of mathematical creativity.

Table 4. 24

Responses of Students in Each Contextual Category, Question II in Quiz B

	Categories						
Students	1 Home	2 School	3 Jobs	4 Animals	5 Sports	6 Community	7 Beliefs
S1							
S2	*						
S3							
S4	*		*		**		*
S5							
S6							
S7							
S8							
S9		*		*			
S10	*						
S11		*					
S12	***						
S13							
S14	*					*	
S15							
S16			*				
S17		*					
S18	-	-	-	-	-	-	-
Total	7	3	2	1	2	1	1

As a context, Table 4.24 shows that original answers correspond to S9 in Category 4, S4 in both Category 5 and 7, and S14 in Category 6. The problems are given in Table 4.25.

Table 4. 25

Novelty Examples of Students (contextual), Question II in Quiz B

Focus	Problems
	A cow milks one liter of milk every day. One day, it gives $\frac{2}{7}$ liter of milk. How much milk deficient does it give on that day than previous day? (S9, Category4)
Context	Ziya played football for one hour after work. His best friend Ejder played football for $\frac{2}{7}$ of an hour. How much more time did Ziya play a football than Ejder? (S4, category5)
	Melike swam 1 hour in the pool. Her twin Demet swam $\frac{2}{7}$ of an hour in

Table 4. 25 (continued)

Novelty Examples of Students (contextual), Question II in Quiz B

the pool. How much more time did Melike swim in the pool than Demet? (S4, Category5)
There are some cakes in charity bazaar. Elif bought a cake in there. Ayşegül bought $\frac{2}{7}$ of a cake. How much more cake did Elif buy than Ayşegül? (S14, Category 6)
Emel prayed at night in one hour. She prayed in the morning in $\frac{2}{7}$ of an hour. How much quickly did Emel pray in the morning than at night? (S4, Category 7)

To sum up, mathematically original answers belonged to S11 for the first question and to both S11 and S12 for the second question. Contextually original answers belonged to S7, S11, and S14 for the first question, while S4 (2 many), S9, S11, S12, and S14 for the second question.

4.1.3 Questionnaire

The questionnaire was implemented at the end of this topic. Three types of mathematics questions; redefinition, problem posing and problem solving tasks, were included according to the related literature as they have the potential to help students reflect their mathematical creativity. In this task, two students were absent. For that reason, answers of 16 students were analyzed according to fluency, flexibility, and novelty as before.

4.1.3.1 Redefinition task

Two of the 18 participating students were absent in the implementation day of this task in Figure 4.8. Therefore, answers of 16 students were analyzed for the redefinition task. Among 130 responses, 23 of them were not appropriate and they were eliminated. A total of 107 appropriate responses were analyzed and 6 categories which are (1) basic fraction concepts, (2) types of fractions (or what it is), (3) what it is not, (4) about mathematical representation, (5) about daily life representation, and (6) operations on fractions were generated.

1. State all the things that are same about two numbers: $\frac{1}{3}$ and $\frac{1}{2}$.

Figure 4. 8 Redefinition Task (adapted from Haylock, 1997) in the Questionnaire

Table 4.26 summarizes appropriate responses of each student for each task. For instance, S1 has one appropriate response in Category 1, two appropriate responses in Category 2, one appropriate response in Category 4, and two appropriate responses in Category 6, making 6 appropriate responses in total. In the following analysis, one student with the most number of responses (fluency), one student with the most number of different categories of responses (flexibility), and one student with a response unique in a category (novelty) were determined by the help of Table 4.26.

Table 4. 26

Number of Responses in Each Category, Redefinition Task in the Questionnaire

Students	Categories						Total Appropriate Response
	1	2	3	4	5	6	
S1	1	2	0	1	0	2	6
S2	0	1	0	0	0	4	5
S3	3	2	1	0	0	2	8
S4	1	1	0	3	5	3	13
S5	0	1	2	0	0	3	6
S6	0	2	0	0	0	4	6
S7	1	1	1	1	0	0	4
S8	0	0	0	0	0	7	7
S9	1	1	0	0	0	0	2
S10	2	3	0	0	0	2	7
S11	0	3	2	4	0	1	10
S12	1	3	0	0	0	2	6
S13	1	2	0	0	0	0	3
S14	1	1	0	3	0	3	8
S15	-	-	-	-	-	-	-
S16	3	0	1	2	0	0	6
S17	-	-	-	-	-	-	-
S18	2	2	2	1	0	3	10
Total	17	25	9	15	5	36	107

4.1.3.1.1 Fluency

Table 4.26 shows that the most number of responses belongs to S4 with 13 appropriate responses. Hence, S4's examples stand for fluency criterion of mathematical creativity. Redefinition task responses of S4 are provided in Table 4.27 as a fluency example.

Table 4. 27

Statements of S4, Redefinition Task in the Questionnaire, Fluency Example

1. They could be both expanded by 3.
2. Their top number is both 1.
3. They are both unit fractions.
4. They are both in one whole.
5. They could be both represented on number line.
6. They could both take part in problems.
7. They are both bigger than $\frac{1}{5}$.
8. They could both represent a slice of a cake.
9. They could both represent some of money.
10. It is possible to fold a paper either $\frac{1}{2}$ or $\frac{1}{3}$ of it.
11. It is possible to write an article on $\frac{1}{2}$ or $\frac{1}{3}$ of a paper.
12. It is possible to divide a rubber into $\frac{1}{2}$ or $\frac{1}{3}$ of it.
13. They are both less than $3\frac{5}{8}$.

4.1.3.1.2 Flexibility

Table 4.26 suggests that S18's responses were in 5 different categories. Thus, S18 exemplifies flexibility criterion for mathematical creativity. S18's 10 appropriate responses were distributed in 5 categories. The responses of S18 are given in Table 4.28.

Table 4. 28

Statements of S18, Redefinition Task in the Questionnaire, Flexibility Example

1. They are both a fraction. (Category 1)
2. The top numbers of both fractions are equal. (Category 1)
3. They could be both expanded by 2. (Category 6)
4. If both of them are expanded by 5, then there is a multiple of 5. (Category 6)
5. They are both unit fractions. (Category 2)
6. They are both proper fractions. (Category 2)
7. They are not both mixed fractions. (Category 3)
8. They are not both improper fractions. (Category 3)
9. Before adding or subtracting them, they must be expanded. (Category 6)
10. They could both represent equal-sized parts of a rectangle. (Category 4)

4.1.3.1.3 Novelty

Example of novelty criterion could be determined here by rareness in a category. In Category 5, there is only one student's (S4) response as can be seen in Table 4.26. Therefore, S4's answers in Category 5 showed novelty example. S4's responses had novelty in this task as only she wrote responses about daily life. She wrote 5 statements in this category and all were unique in the classroom. They are given in Table 4.29.

Table 4. 29

Statements of S4, Redefinition Task in the Questionnaire, Novelty Example

1. They could both represent a slice of a cake.
2. They could both represent some of money.
3. It is possible to fold a paper either $\frac{1}{2}$ or $\frac{1}{3}$ of it.

Table 4. 29 (continued)

Statements of S4, Redefinition Task in the Questionnaire, Novelty Example

4. It is possible to write an article on $\frac{1}{2}$ or $\frac{1}{3}$ of a paper.
5. It is possible to divide a rubber into $\frac{1}{2}$ or $\frac{1}{3}$ of it.

4.1.3.2 Problem posing task

Problem posing task was including some words that could evoke ideas in students' minds. Moreover, there was an option that students could add words themselves in order not to restrict their imagination. By taking into consideration Vygotsky's theory of creativity, the words "a bird", "a ball", and "water" were added as they were convenient for the first stage of creativity and the words "the Turkish War of Independence", "French Revolution", "Fairy chimneys", and "Nasreddin Hodja" were added for second stage of creativity. The task is given in Figure 4.9 below.

> *a bird, a ball, the Turkish War of Independence, French Revolution, Fairy chimneys, water, Nasreddin Hodja,………, ……….*
>
> 2. First, select two words above or you may add (at most two) yourself. Then, pose as much problems as you can with those two words about fractions. Note that it must be **solvable**.

Figure 4. 9 Problem Posing Task in the Questionnaire

For this task, there were 62 posed problems in total. Since only 25 of them were appropriate, analysis was conducted with 25 posed problems. Posed problems were analyzed both contextually and mathematically. It was quite difficult to determine categories because it was an open-ended task. Therefore, each answer was somehow different from each other.

4.1.3.2.1 Fluency

After counting responses of each student, Table 4.30 shows that one student was showing fluency example with 5 appropriate answers.

Table 4. 30

Fluency Analysis, Problem Posing Task in the Questionnaire

Number of appropriate responses	Number of students
0	5
1	4
2	3
3	2
4	1
5	1
Total	16

S14's posed problems are given in Table 4.31.

Table 4. 31

Posed Problems of S14, Problem Posing Task in the Questionnaire, Fluency Example

Number	Problems
1	Ali and Ayşe want to eat a banana. They divided a banana into 4 with a knife. Ali ate $\frac{1}{4}$ and Ayşe ate $\frac{2}{4}$ of it. How much banana did they eat together?
2	In order to eat banana, Selim and Selin divided a banana into 6 with a knife. Selim ate $\frac{2}{6}$ whereas Selin ate $\frac{4}{6}$ of it. How much more banana Selim ate than Selin?
3	Elif and Hamide cut a banana with a knife. Elif ate $\frac{2}{5}$ of it. How much banana left to Hamide?
4	Ahmet cut a banana with a knife and ate $\frac{1}{4}$ of it. How much of the banana left?
5	Ece cut a banana into 8 with a knife. She ate $\frac{3}{8}$ of it. How much banana left?

4.1.3.2.2 Flexibility

Since context was restricted to two words even though they were different two words, the researcher thought to keep them as if they were all the same words in order to check mathematics involved in the problems. Since the context of the problems each student posed was about the two words they had chosen, there could not be problems in different contextual categories for each student. In other words, a student could not pose problems in different categories to be flexible as there is a restriction for using only 2 words. Therefore, contextual categorization for flexibility analysis was skipped.

Although students were expected to achieve nine objectives addressing ordering unit fractions to solving problems requiring addition and subtraction of fractions with same denominator or denominator which is multiple of each other (see also Table 3.2) throughout this research, they had a tendency in writing problems about addition and subtraction of fractions. Only two students posed problems requiring part-whole relationship. Thus, there were four categories constructed: (1) addition of fractions, (2) subtraction of fractions, (3) finding a part of given whole, and (4) finding the whole of a given part. Later, solutions were analyzed relying on how many objectives each solution required. The response covering more categories, then, will be a flexible response. Table 4.32 shows the number of responses in each category.

Table 4.32

Flexibility Analysis (mathematical), Problem Posing Task in the Questionnaire

Categories Number of Responses in	Number of Responses
1	16
2	7
3	2
Total	25

Table 4.32 shows that two students asked problems whose solution required any of 3 categories which were addition, subtraction, finding part of whole, and finding the whole of a given part. Reponses of two students were given in Table 4.33 as a flexibility example of creativity in this task.

Table 4. 33

Responses of S1 and S5, Problem Posing Task in the Questionnaire, Flexibility Examples (mathematical)

Categories involved in	Problem
1,2,3	We are going on a trip to Fairy Chimneys with my classmates. I bring 120 TL to there. I bought my ticket with $\frac{6}{24}$ of my money. I spent $\frac{3}{24}$ of it on eating. I bought gifts to my family with $\frac{5}{24}$ of it. How much money left at the end? (S1)
1,3,4	The wife of Nasreddin Hodja spent $\frac{5}{9}$ of her money for lettuce, $\frac{3}{9}$ of her money for garlic, and $\frac{1}{9}$ of her money for tomatoes in bazaar. If she spent 30 TL for lettuce, how much money did she spend in total? (S5)

4.1.3.2.3 Novelty

According to categories constructed for the flexibility analysis, only one student (S5) posed a problem covering Category 4. Generally, students' responses were in Category 1, 2, and 3. However, S5's problem required finding the whole of a given part. Thus, S5 produced novelty example of problem posing task in terms of mathematics involved. The problem is given in Table 4.34.

Table 4. 34

Responses of S5, Problem Posing Task in the Questionnaire, Novelty Example (mathematical)

Number	Problem
1	The wife of Nasreddin Hodja spent $\frac{5}{9}$ of her money for lettuce, $\frac{3}{9}$ of her money for garlic, and $\frac{1}{9}$ of her money for tomatoes in bazaar. If she spent 30 TL for lettuce, how much money did she spend in total?

In terms of originality in context, the two words selected by each student were the foci. Table 4.35 lists the words students had chosen to pose problems.

Table 4. 35

Two Words Chosen by Each Student, Problem Posing Task in the Questionnaire

Students	Water	School	Ball	Bird	Fairy Chimneys	Nasreddin Hodja	Self-written-words
S1					*		trip
S2							
S3							
S4	*					*	
S5						*	bazaar
S6							
S7	*					*	
S8		*	*				
S9							
S10		*	*				
S11		*				*	
S12				*			zoo
S13	*						me
S14							banana,
S15	-	-	-	-	-	-	knife
S16				*		*	
S17	-	-	-	-	-	-	
S18							
Total	3	3	2	2	1	5	

Only one student, S1, posed problems about Fairy chimneys in the class. Therefore, it was novel. When she was asked about her reason for selecting this word, she replied that she traveled there before. Hence, she thought about her experience in Fairy chimneys and wrote the problems. The problems of S1 are given in Table 4.36.

Table 4. 36

Responses of S1, Problem Posing Task in the Questionnaire, Novelty Examples (contextual)

Number	Problems
1	We went to Fairy Chimneys in Nevşehir with my family for one day. We spent $\frac{2}{12}$ of a day to balloon, $\frac{1}{8}$ of day to eat something, $\frac{1}{6}$ of day to walk around. Rest of the day, we slept on the way turning back. Write down how much of the day we slept with a fraction.
2	We are going on a trip to Fairy Chimneys with my classmates. I bring 120 TL to there. I bought my ticket with $\frac{6}{24}$ of my money. I spent $\frac{3}{24}$ of it on eating. I bought gifts to my family with $\frac{5}{24}$ of it. How much money left at the end?

Among the students who added a word or two words themselves, it was not possible to say that their responses were novel as they were different. Only one student, S14, added two words. Hence, she produced a novelty example as well as she provided a fluency example. When she was asked about her reason to write and use the words banana and knife, , she said "I do not know, it came to my mind while I was thinking about the problems I have to write." The posed problems are provided once more below in Table 4.37 as novelty examples this time.

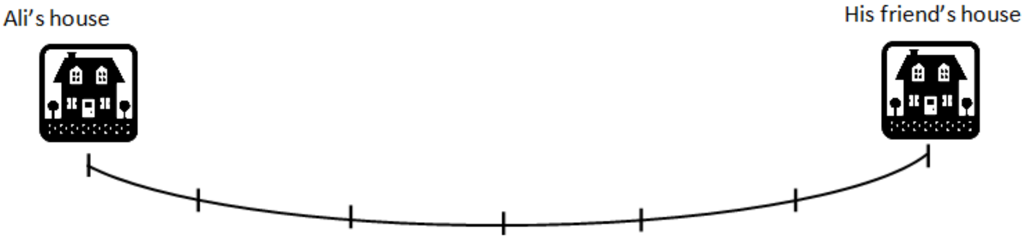
Table 4. 37

Responses of S14, Problem Posing Task in the Questionnaire, Novelty Examples (contextual)

Number	Problems
1	Ali and Ayşe want to eat a banana. They divided a banana into 4 with a knife. Ali ate $\frac{1}{4}$ and Ayşe ate $\frac{2}{4}$ of it. How much banana did they eat together?
2	In order to eat banana, Selim and Selin divided a banana into 6 with a knife. Selim ate $\frac{2}{6}$ whereas Selin ate $\frac{4}{6}$ of it. How much more banana Selim ate than Selin?
3	Elif and Hamide cut a banana with a knife. Elif ate $\frac{2}{5}$ of it. How much banana left to Hamide?
4	Ahmet cut a banana with a knife and ate $\frac{1}{4}$ of it. How much of the banana left?
5	Ece cut a banana into 8 with a knife. She ate $\frac{3}{8}$ of it. How much banana left?

4.1.3.3 Problem solving task

The problem solving task is given in Figure 4.10.



3. Ali would like to ride a bicycle from his house to a friend's house. He fist rides $\frac{1}{4}$ of the way and then $\frac{1}{5}$ of it. At that moment, **put a sign** about where he is on the picture above. Please explain your solution.

Figure 4. 10 Problem Solving Task in Questionnaire (adapted from 2011 NAEP Assessment for 4^{th} Graders)

Considering both mathematical sentences students wrote down to explain their way of thinking and the sign they put on the figure, there were only 3 appropriate responses for this task. Since one student had not explained her answer clearly, it was eliminated as well. Other two solutions by S2 and S18 were appropriate and analysis was conducted with them. Since there is only one appropriate answer of each student, fluency and flexibility were not considered for this problem. Moreover, as the two responses were different from each other, they both were labeled as original for the group. Hence, the responses of S2 and S18 constituted novelty examples. Figure 4.11 and Figure 4.12 display the responses of S2 and S18 respectively.

S12 and S18 were asked about the way they found. S12 said that "I added $\frac{1}{4}$ and $\frac{1}{5}$ what made $\frac{9}{20}$. It was less than a half. Therefore, I found the half way on the picture first, then, I put a sign on the left side of middle." S18 told "I thought every piece as 10 units, making 60 units in total. I added $\frac{1}{4}$ and $\frac{1}{5}$ by expanding denominators of the fractions to 60. I got $\frac{27}{60}$. On the picture, first two pieces made 20 and I tried to find 7 units out of 10 units on the third piece."

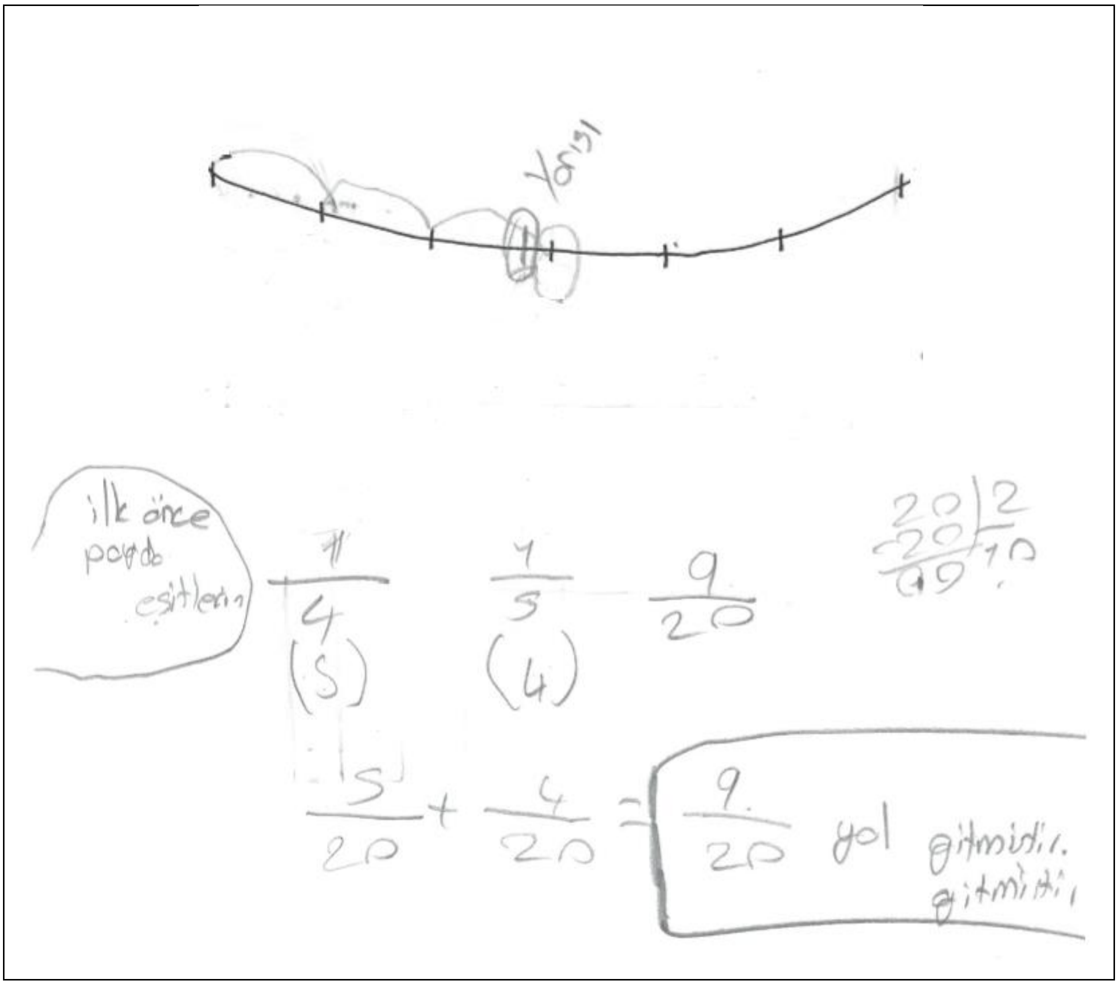


Figure 4. 11 Response of S2, Problem Solving Task in the Questionnaire, Novelty Example

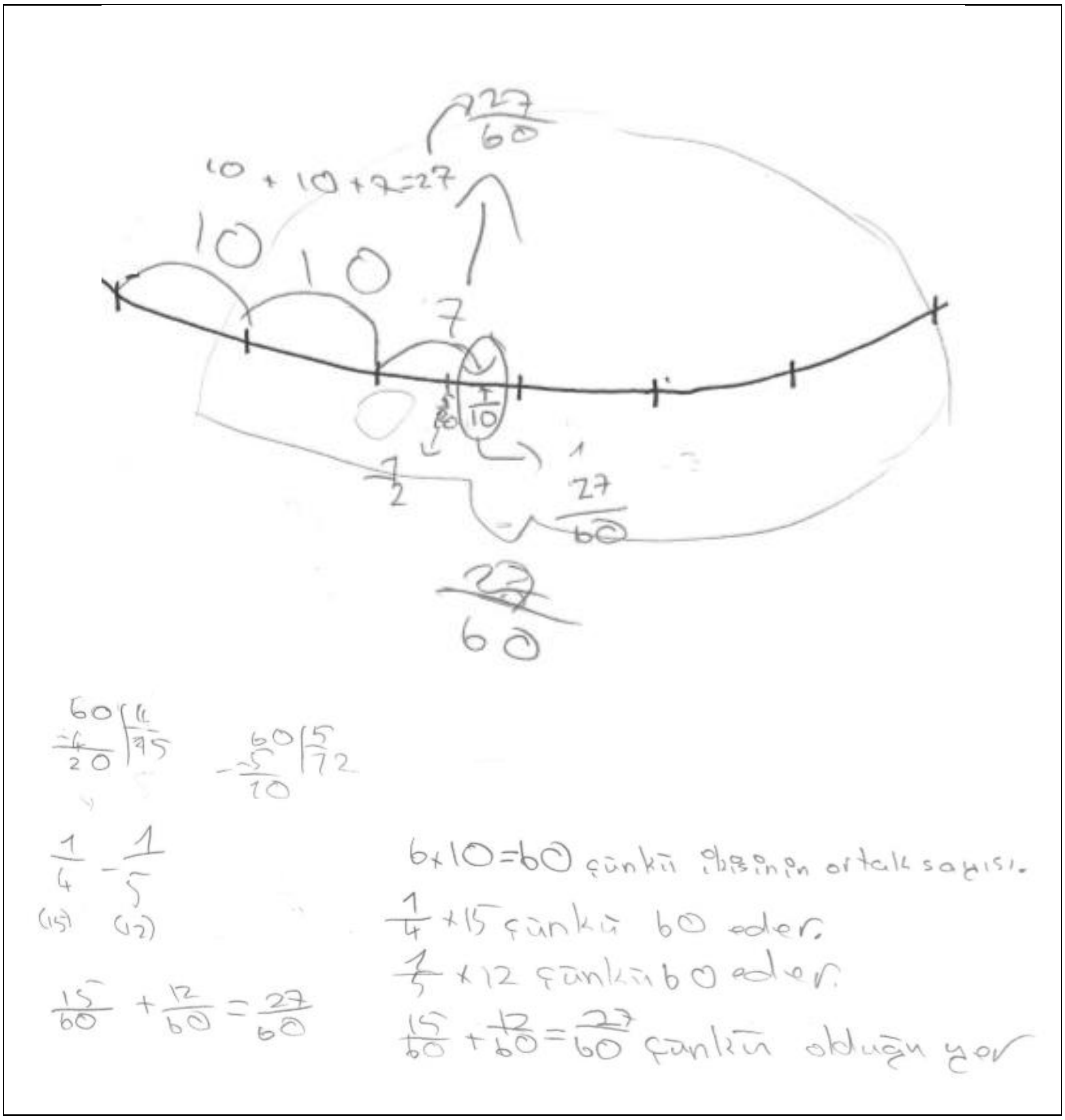


Figure 4. 12 Response of S18, Problem Solving Task in the Questionnaire, Novelty Example

4.2 Mathematical Creativity in the Lesson

Mathematical creativity was investigated during the lesson according to novelty, fluency, and flexibility criteria similar to the paper-tasks. The difference between paper tasks and lesson-tasks is that the idea development process was missed during paper-tasks but researcher could have an opportunity for observing that process in lesson-tasks. Therefore, mathematical creativity might be searched both in process and product. It was noticed that since students were in an interaction in the lesson, one student’s mathematical creativity could benefit from others’ ideas.

In this section, mathematical creativity examples are given by providing excerpts from the lesson. Analysis of short interviews was also included here. Collective mathematical creativity is introduced in this part. In addition to analysis of novelty, fluency, and flexibility, how students interact with a question in the lesson together was analyzed through collective fluency and collective flexibility (Levenson, 2011).

4.2.1 Types of fractions (collective and individual mathematical creativity)

Types of fractions were the main topic in the lesson. After giving the mathematical definitions of proper fractions, improper fractions, and mixed numbers, students were requested to give examples about them. Firstly, proper fractions were exemplified. Later, improper fractions were the focus. The students gave different examples but they were not labeled as novel since they were almost similar to each other. Yet, mixed numbers yielded a difference as in the episode provided below.

> Teacher: Let us write examples of mixed numbers now. What could those be?
>
> S10: $1\frac{1}{2}$.
>
> Teacher: Ok. Another?
>
> S17: $1\frac{1}{4}$.
>
> Teacher: Yes. What else S16?
>
> S16: $1\frac{3}{8}$.
>
> Teacher: Then, it seems that it has to begin with 1 whole…
>
> Students: Noooo…
>
> Teacher: What else?
>
> S2: $9\frac{9}{9}$, that is, 10 whole.
>
> Teacher: Is it correct (to students, while writing $9\frac{9}{9}$ on the board)?
>
> Students: Yesss.
>
> Teacher: How did you come up with this idea?
>
> S2: I don't know…
>
> Teacher: Ok. (In order not to interrupt lesson more, passing on another student) Any other?

S9: $12\frac{2}{4}$.

Teacher: Well, go on S6.

S1: $8\frac{5}{45}$.

Teacher: Good, I think these are enough.

Examples of S10, S17, and S16 were different but it was possible to categorize these responses in the same category. In other words, these responses were similar even though they showed different quantities. However, teacher let students construct unlike examples by warning that the whole part of the examples was all 1. This resulted in new responses. S2 stated that it could be even 10 whole. Then, other mixed numbers were written on the board. This episode could be considered as an example for collective fluency as each student gave different answers. Moreover, students were eager to give other examples although the teacher was satisfied with those responses. This episode was continued with the one below.

Students: Teacher, please listen to our example as well!

Teacher: Ok, last ones.

S10: $1\frac{800}{1000}$.

S18: $12\frac{10}{11}$.

Teacher: Why?

S18: Each number is getting smaller one by one.

Teacher: You said $12\,\frac{10}{11}$ in order to make numbers smaller one by one?

S18: Yes.

Teacher: Did you think it as if it was a pattern (knowing that student is interested in patterns a lot)?

S18: Yes, exactly.

This episode was an example for not only collective flexibility but also (individual) novelty. Previous excerpt might have yielded in one category but this one added two more categories as thousands are introduced to mixed numbers. In addition, patterns were another category here. In brief, mixed numbers with (1) numbers less than one hundred, (2) numbers less than one thousand, and (3) numbers as patterns were the three different categories. Since novelty is relative to the group, third category, mixed numbers with numbers as patterns was decided as novel (original). S18 gave an example of novelty in the lesson as $12\frac{10}{11}$. Other attempts were random numbers but this was intentionally chosen. Teacher knew that he was generally asking questions about patterns in free times in the form of intelligence questions either to the teacher or to whole class. The lesson continues with the conversation given below.

Teacher: Is it correct (looking at whole class)?

Students: Yes (after some discussions).

S15: Can it be millions?

Teacher: Of course, it could be.

S15: 90 000 000$\frac{1}{1000}$.

Teacher: Great! And you again, S18 (he was raising his hand willingly for moments)?

S18: 400 $\frac{4}{40}$ because 4 is my favorite number. I multiply it with 100 firstly, then with 10. One more, $12\frac{4}{8}$. 4 is my best and so I want it to see at the end.

Teacher: Lastly, you say S10.

S10: $92\frac{1}{3}$.

Teacher: Ok, everybody. Great job, please note these examples…

This part of the lesson enabled researcher to think of the forth category: Mixed numbers with numbers less than millions. Collective flexibility could be explained in terms of those four categories at the end, which were mixed numbers with (1) two-digit-numbers, (2) three-digit-numbers, (3) numbers as patterns, and (4) between four-digit-numbers and eight-digit-numbers. Such abundance of answers (12 many) also reflected collective fluency. For S18, he was still thinking deeply about patterns and relating the two topics. His examples were selected as novelty examples. Table 4.38 summarizes the examples given for mixed numbers in the lesson.

Table 4. 38

Mathematical Creativity Examples, Students' Examples on Mixed Numbers

Students	Examples	Category Number
S10, S17, S16	$1\frac{1}{2}, 1\frac{1}{4}, 1\frac{3}{8}$	1
S2,S9,S1	$9\frac{9}{9}, 12\,\frac{2}{4}, 8\frac{5}{45}$	1
S10	$1\,\frac{800}{1000}$	2
S18	$12\,\frac{10}{11}, 400\,\frac{4}{40}, 12\frac{4}{8}$	3
S15	$90\,000\,000\,\frac{1}{1000}$	4
S6	$92\,\frac{1}{3}$	1
Total	Collective Fluency	Collective Flexibility

This episode might exemplifies collective fluency, collective flexibility, and novelty (individual). Individual creativity examples observed by novelty criteria in the lesson were as follows. When S18 put his interest into example, novelty was observed as well. This could be explained by the first category of Vygotsky's theory of creativity as he emphasized the role of previous experiences individuals have in revealing creativity (Vygotsky, 2004). Thus, it could be said that collective and individual mathematical creativity were observed in this part of the lesson.

4.2.2 Converting improper fractions to mixed numbers (individual mathematical creativity)

In the second week of the study, the goal was to convert improper fractions to mixed numbers and vice versa. For this aim, the activity sheet "Digging up improper fractions" was distributed to students. While the teacher was walking around and checking solutions of students, S6 came near by the teacher and asked whether her idea was correct or not for the question shown in Figure 4.13.

$$\frac{5}{2} = \boxed{} = \frac{}{}$$

Figure 4. 13 A Question Asked by a Student on Activity Sheet: Digging Up Improper Fractions

$$\frac{7}{2} = \boxed{} = \frac{}{}$$

Figure 4. 14 A Question Done on the Board on Activity Sheet: Digging Up Improper Fractions

The teacher appreciated her idea and asked her to solve the next question (see Figure 4.14) with her method on the board. Figure 4.15 summarizes the process of S6's creativity.

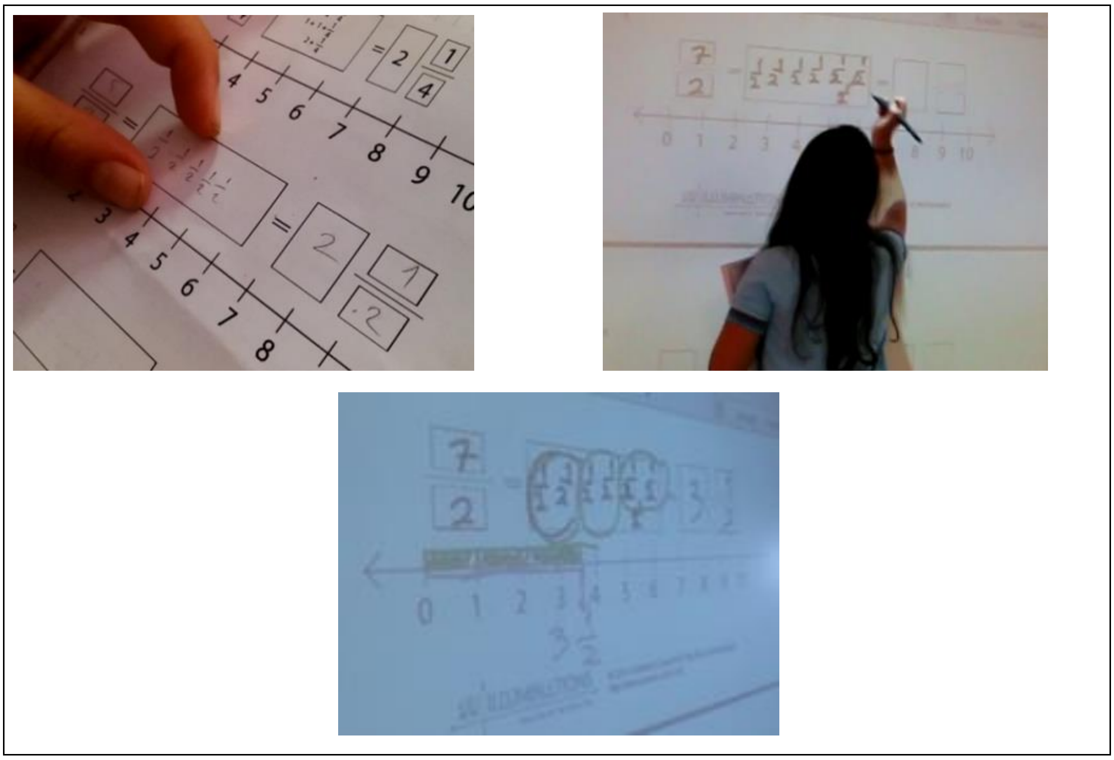


Figure 4. 15 S6's Creativity Process

S6 found the unit of the improper fraction, $\frac{1}{2}$ of $\frac{7}{2}$, and then, wrote it 7 times (as the numerator was 7). Later, she circled two units as two of $\frac{1}{2}$s composed a whole. Figure 4.15 represents that repeating this process where she got 3 wholes and a half.

Some of the students divided numerator by denominator from their previous knowledge. When the teacher asked about how they knew such method, one of them acknowledged that either their previous teacher or their family had taught. Still, the teacher utilized drawings and used conceptual ideas instead of rules while changing improper fractions to mixed numbers. Apart from division or drawing methods with which the students were familiar, S6's method was different and mathematically appropriate. Hence, this attempt could be categorized as novel and therefore, mathematically creative. When the teacher asked how she came up with that idea, she told that "I don't know, it just came to my mind." Even though this novelty example was emerged in lesson, it was not collective but individual mathematical creativity.

4.2.3 Ordering fractions (collective and individual mathematical creativity)

The topic of the lesson was ordering fractions in the fourth week of the study. An activity sheet involving restricted-response type items, which was the redefinition task, was distributed to students. It was observed that students were interested in one specific item, __ > 5 >__, more than others. Several responses were gathered for this item such as $\frac{6}{1}$ and $\frac{4}{1}$, $\frac{30}{5}$ and $\frac{15}{5}$, $\frac{11}{2}$ and $\frac{3}{3}$, and $\frac{90}{8}$ and $\frac{20}{5}$, respectively for the first and second blanks. These responses might be classified as collective fluency because this question produced several responses compared to the other questions.

While students were stating their answers, S11 was working on something on her activity sheet. After a while, she came up with the following response: $\frac{26}{5}$ and $\frac{7}{6}$. It seemed that $\frac{26}{5}$ was an original answer to the teacher-researcher during the class and the following conversation occurred between the teacher and S11.

Teacher: How did you think of $\frac{26}{5}$, S11?

S11: I wrote 5 to the bottom number. If I wrote 25 to the top number, then it [intending $\frac{25}{5}$] equaled to 5. I added 1[to the top number, 25]. Therefore, it [$\frac{26}{5}$] became bigger [than 5].

This episode clarifies S11's thinking. This novelty example could be an individual creativity example.

4.2.4 Ordering problems (individual mathematical creativity)

At the beginning of the lesson, the topic of ordering fractions was introduced to students in the fifth week of the study to remind the topic covered in the previous week. Teacher drew a number line and represented fractions on the number line. Later, she reminded for case of natural numbers that the number on the right gets bigger. Moreover, closeness to a half was mentioned as a strategy for one ordering example. The problem in Figure 4.16 in the mathematics course book (Yaman, Akkaya, & Yeşilyurt, 2013) was written down on the board and discussed with the students.

Fatih, Metin, and İlhan are planning to run together at the weekend. Fatih runs $\frac{4}{10}$ km. Metin runs $\frac{9}{10}$ km, and İlhan runs $\frac{7}{10}$ km. Who runs the most? Who runs the least?

Figure 4. 16 Problem Solving Task from the Course Book (Yaman, Akkaya, & Yeşilyurt, 2013, p.168)

Conversation among the teacher, S14, and S18 is given below:

S14: Each fraction shows that a whole is partitioned into ten parts. The bottom number is same for all. Since 9 is the biggest top number among these [fractions], he [Metin] runs the most.

Teacher: Do you memorize a rule for that?

S14: No, I thought that 9 parts of a whole is bigger [than 4 and 7 parts].

Teacher: Ok, good. Any other idea?

S18: Teacher, I thought in a different way.

Teacher: How?

S18: I thought about closeness to 1. For Fatih 6 [tenths] are while for Metin 1 tenth is needed. Metin is nearest to one whole and Fatih is furthest. Therefore, Fatih runs the least and Metin runs the most.

Teacher: Well done, thank you.

After this episode, the teacher explained S18's method one more time to class. It was an interesting way of solution for the other students. Since novelty is relative to the group, S18's answer was unique in the class and therefore it was novel. Here, S18's answer stands for individual mathematical creativity example depending on the novelty criterion of creativity.

4.2.5 Fraction problems (individual mathematical creativity)

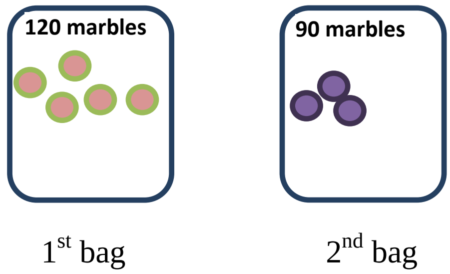
The teacher began a lesson with a warm-up question given in Figure 4.17 at the end of fourth week of the study.

Semih ate $\frac{1}{3}$ of a pizza and Mustafa ate $\frac{1}{2}$ of it. Semih said to Mustafa "I ate more than you did." Under what conditions this could be right?

Figure 4. 17 Warm-up Question (adapted from NAEP, 1992; Van de Walle, 2007) in the Lesson

When the teacher asked this question, students gave answers such as, "If one of the pizza is bigger than the other" and "If Semih's pizza is bigger" because they were familiar with this type of question from the quiz they had before. Then, the teacher continued, "For that reason, we should be careful about whole because it can change the result" while drawing circles and coloring them.

After this warm-up, the main topic, fractional parts of the whole, was introduced. Teacher informed students that they would need to find a part of whole as they did before on shapes by drawing. However, this time they could arrange numbers as well in addition to visual representation. Then, she wrote the problem in Figure 4.18 from course book (Yaman, Akkaya, & Yeşilyurt, 2013, p.170) on the board:



The picture shows that there are 120 marbles in the first bag and 90 marbles in the second bag. Some marbles will be taken out from each bag. Which fractions of marbles in each bag do result in same amount of marbles taken?

Figure 4. 18 Fractional Parts of the Whole Question (Yaman, Akkaya, & Yeşilyurt, 2013, p.170) from the Course Book

The teacher walked around to check students' answers one by one without disturbing any students. Not many students could solve the problem. Among four appropriate responses, one of them was novel. Given answers were: $\frac{1}{12}$ and $\frac{1}{9}$, $\frac{1}{4}$ and $\frac{1}{3}$, $\frac{1}{2}$ and $\frac{2}{3}$, and $\frac{15}{30}$ and $\frac{20}{30}$. The novelty example stands for the last response given by S1, $\frac{15}{30}$ and $\frac{20}{30}$. She stated that "I thought about bigger numbers. $\frac{15}{30}$ of first bag and $\frac{20}{30}$ of second bag made 60 marbles each." This answer could be categorized as individual mathematical creativity.

4.3 Summary of the findings

For paper tasks, it could be stated that redefinition task in questionnaire yielded more responses than other tasks. Besides, students had a tendency in replying problem solving tasks with one answer. Only few students responded problem solving task in questionnaire with more than one answer.

Problem posing tasks in paper tasks showed that students bring their interests, favorites, and daily routines into the contexts of the problems. Students generally posed problems regarding contexts: home, school, and jobs. There were some expressions observed different compared to others which are Fethiye Municipality, charity bazaar, rabbit and cows, and praying. In addition, some students were reminded about their past experiences such as S1 on questionnaire (their travel to Fairy chimneys) and S18 in types of fractions lesson (as patterns was the topic of previous semester in 5th grade). Lastly, one student added two words: banana and knife in order to pose problems. Real-life examples, manipulatives, and experiences somehow might be the predictor of mathematical creativity taking these cases into consideration.

Apart from context, mathematical part of the problems constitutes importance for analysis of the problems. Some students play with numbers and changed given numbers into mathematically equal different numbers, or divided numbers into pieces and then inserted into the contexts of the problems.

In lesson, mathematical creativity examples were found from the beginning of the lesson until the end of topic including sub-topics of types of fractions, converting improper fractions to mixed numbers, ordering fractions, ordering problems, and fraction problems. Unlike paper-tasks, collective mathematical creativity was welcomed in lesson tasks. Therefore, examples gathered from the lesson were sometimes considered as collective creativity and sometimes as individual creativity. If there was no conversation among the students, then the creativity example found was labeled as individual creativity. However, if there was a conversation, then it was difficult to determine whether it was an example for collective fluency, flexibility, novelty, or all of them together because ideas of one student might be a step for the development of other student's idea. Students' ideas sometimes collaborated or sometimes differentiated in lesson where mathematical creativity was nurtured.

When a mathematical creativity example was found, a student was asked about how he/she came up with the idea. Yet, students generally said that "I was thinking on it, it just came to my mind, I do not know." Only some students explained the reason behind their response such as their trip on Fairy chimneys, the fact that his favorite number is 4, and that he was thinking about patterns. Therefore, in-class short interview data was not fruitful in terms of students' real-life experiences so as to be creative as Vygotsky had said before. If researcher could have collected data through in-depth interviews, valuable information might have been drawn.

Creativity on paper and an independent response in lesson was searched as a product. However, interviews on paper tasks and conversation in lesson were enabled researcher to check creativity as process. Examples of fluency, flexibility, and novelty were provided under corresponding task. Besides, collective fluency and collective flexibility examples were explored and provided in lesson. Collective creativity is special to classroom environment as interaction was prevented on paper tasks in order not to affect data gathered. Flexibility might be harder criterion to determine at first sight compared to others as there is a flexibility example in Quiz A that among S4, S8, S9, S10, S12, S17 presented flexibility example, S10 gave 6 responses and the others gave 4 responses. They still did same thing for flexibility criterion. Since fluency and novelty depends on the number of responses provided by other students, it reflects the cultural feature of creativity such that it depends on the relative group.

There were students who showed fluency example but not any novelty example, and vice versa. Some of the students showed only flexibility example. Some students provided both flexibility and novelty examples in a task. Thus, the findings here showed that there were many ways in which students were able to show their creativity.

CHAPTER V

DISCUSSION, IMPLICATIONS, AND RECOMMENDATIONS

The main aim of the study was to explore mathematical creativity examples of 5th graders on fractions topic. The findings of the study will be evaluated in accordance with research questions.

5.1 Mathematical Creativity Examples

The abundance of mathematical creativity examples wase provided in previous chapter based on the appropriateness, fluency, flexibility, and novelty criteria as suggested in other studies (Haylock, 1997; Nadjafikhah & Yaftian, 2013; Silver, 1997). Moreover, redefinition task, problem solving task, and problem posing task in mathematics were fruitful paper-tasks in terms of data presented as mathematical creativity examples as other studies put emphasis on those tasks (such as Chen et al., 2006; Haylock, 1987, 1997; Van Harpen & Presmeg, 2011, 2013). Besides, mathematical creativity in lesson provided worthy information as in another study (Levenson, 2011) in the process was elaborated and the interaction among students was visible by the teacher.

According to Vygotsky (2004), anyone could be creative up to a certain point and therefore, creativity must not be attributed to talented people. Besides, he explains the creativity of individuals via their experiences with reality. In the current study, there were some cases that experience worked for creativity. Manipulatives could be a part of in-class experience and real-life could be perceived as out-of-class experience for students. As Vygotsky points to experience factor on creativity; manipulatives and real-life could be base for giving creativity examples in mathematics lessons as Figure 5.1 suggests.

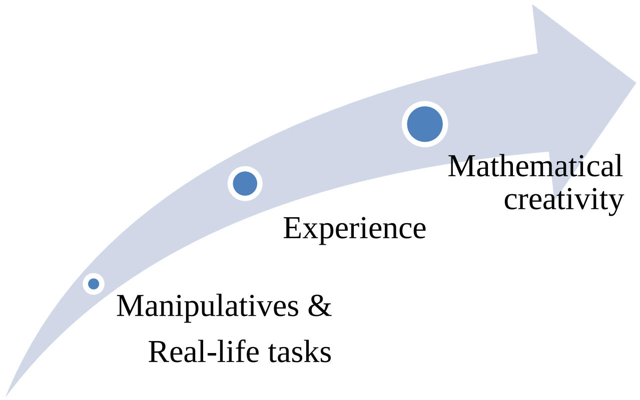


Figure 5. 1 Proposal for the Occurence of Mathematical Creativity Examples

In paper-tasks, each student was given opportunity to reflect any idea they had because more than one response was requested. When comparing number of responses to each paper-task, students gave responses to redefinition task at most, problem posing task later, and problem solving task the least. Students gave responses to redefinition task in paper-and-pencil test the most. It could be due to the nature of the task that it was open to any idea. The question was about writing down anything about two unit fractions. Therefore, students could use those two numbers in any context and operation depending on their knowledge. The findings might suggest that problem solving tasks are not as effective as problem posing and redefinition tasks in reaching students' creativity.

In both problem posing tasks in Quiz B and in the second task in questionnaire, the mathematics that students could do was restricted since the task in Quiz B provided the operation they had to use and the task in the questionnaire required problem about fractions. Although students posed problems about various topics in mathematics in other studies due to no content restriction (Van Harpen & Presmeg, 2011, 2013), it was not possible to categorize problems similar to those studies in the current study due to the restriction of the topic to fractions. Therefore, categories in fractions topic under related objectives were done in more detailed way.

The words students used in problem posing tasks showed that students generally tended to use words related to their experiences in or out of class. One student explained that she was reminded about her last trip to a Fairy chimney when she saw that word, thus, she posed problems about that context. Apart from this, there were two words, which were banana and knife, added by one student to pose problems. It could be because of the activity about converting improper fractions to mixed numbers in the lesson. In that activity, every student worked in pairs and brought some fruits to the classroom. They told which fraction they would like to work with to the teacher, and then, teacher cut fruits into equal-sized portions. Later, students held cut-fruits in their hands as a whole as if they had not been cut. Then, they converted fractions. Therefore, the student might have added those two words because of this activity even though she did not tell the reason explicitly. If so, Vygotsky's theory of creativity (2004) could help us to better understand mathematical creativity since this case exemplifies the role of past experience on creativity.

In problem solving task, there were very rare appropriate responses probably because it was a challenging task for 5^{th} graders. Two appropriate answers for the problem posing task in this study were quite different and original. Since they satisfied novelty criterion, they were mathematical creativity examples.

Students were interacted each other during the lesson. Therefore, process and product of creativity was involved together as Levenson (2011) stated. One's product could start process of the other one and one's original response could be resulted from others' responses as fluency or flexibility examples. Thus, individual and collective mathematical creativity were closely intertwined in the lesson.

Mathematical creativity examples differed on paper compared to the ones in lesson because there were fewer examples illustrated in lesson than on paper tasks. It could be due to several reasons. First, it could be the case that students could perform several responses on paper whereas it might not be possible to give opportunity to each student to express their opinion, which puts a limit on their verbal responses. The time that students have to think about the task in the lesson to respond is limited and the context of the classroom might not satisfy all students' needs to produce responses. For example, regular noise in the classroom might not help some of the students in producing creative responses. Students might focus on the task better on paper than on board.

There were some examples that students showed individual mathematical creativity in terms of novelty criterion in both lesson and on paper. Those situations might help authors to learn about individual creativity process. However, it is not possible to talk about creativity or creative potential of students since its more complex issue and it depends on several factors.

S4 was mostly mentioned in the fluency, flexibility, and novelty examples. It might be possible that there is a student who could give more creativity examples in one criterion in a task. However, it would not be possible to conclude that one student is more creative than others because some students were more interested in questions at a specific lesson and they feel more comfortable than others. Moreover, there are some students who gave creativity examples both on paper and in lesson, only on paper, only in lesson, or none.

From wider perspective, 16 students out of 18 students who participated the study somehow showed creativity examples. It is good for that each student could give mathematical creativity examples up to some point. It could be due to the fact that mathematical creativity is searched in several tasks: in-lesson, on-paper (redefinition, problem posing, and problem solving tasks) with respect to several criteria (appropriateness, fluency, flexibility, novelty, collective fluency, collective flexibility), and in-class short interviews. No matter how successful in mathematics, students could be given an opportunity for reflecting the creativity examples.

To conclude, this study provides worthy information in that it takes into consideration four criteria in the accessible literature in order to find mathematical creativity in several tasks on paper and also in lesson together. Further, short in-class interviews were conducted. Even though those interviews were not so much helpful for understanding situation, it might lead other studies to consider all cases in a study.

5.2 Implications

Teachers might focus more on materials, activities, and real-life examples in order to help students experience with reality in mathematics since Vygotsky attributes experience with reality and creativity (Vygotsky, 2004). Students should be given enough time to think about concept and response in lessons.

This study could help teachers in that they could be aware of mathematical creativity and design a lesson considering their students' needs and interests. They could present questions which students could think deeper and give many responses. Moreover, teachers could emphasize on redefinition and problem posing tasks in addition to problem solving tasks which is very common. Teachers can consider several correct answers for assessment of students' learning instead of one common way of solution in order to increase students' awareness of originality. Moreover, teacher educators could increase prospective teachers' awareness of those concerns. Awareness of teachers and prospective teachers

Curriculum developers could take the existence of mathematical creativity into consideration and they could locate redefinition tasks into course books as they recently embedded problem posing tasks in materials in Turkey. Besides, redefinition, problem posing, and problem solving tasks might be included in mathematics lessons and course books.

5.3 Recommendations for Further Research

This study provides an understanding of the type of activities that could be prepared in the future studies about mathematical creativity. Specifically, redefinition, problem posing, and problem solving tasks could be studied and deep information could be gathered.

Examining mathematical creativity examples on fractions topic in other grade levels or of different participants (such as public school students) in 5th grade could help researchers in understanding the nature of mathematical creativity in different classroom cultures. Students can give very different responses in other types of schools, cities and countries. The categories found in the analysis in this study can be extended and exact novelty examples can be determined at the end.

Studying creativity in other topics in mathematics is important in order to understand mathematical creativity better. Thus, there is a need for future studies focusing on other topics in mathematics, and also studying with higher grade students so as to let students choose any topic they want for abundance of responses and categories.

Since in-class short interviews were held in this study, mathematical creativity of 5th graders on fractions with in-depth interviews could provide accurate information about the phenomenon. In addition, mathematical creativity on fractions in other grade levels could contribute mathematical creativity literature. Studying mathematical creativity of 5th graders on fractions in different schools, or cultures could be worth for it since cultural aspects and beliefs and attitudes of students may be related to creativity phenomenon (Massarwe, Verner, & Bshouty, 2011)

Mathematical creativity of 5th graders on other topics and mathematical creativity on other topics might be further steps for approaching creativity. Besides, studying creativity in other subjects might provide worthy information about creativity in education area.

REFERENCES

Arikan, E. E. & Unal, H. (2015). An investigation of eighth grade students' problem posing skills (Turkey sample). International Journal of Research in Education and Science (IJRES), 1(1), 23-30.

Batırbek, M. (2007). *The analysis of children's writing: A study on the creative potential and rhetorical structure of written texts* (Unpublished master's thesis). Middle East Technical University, Ankara, Turkey.

Briggs, M. (2005). Creative mathematics. In A. Wilson (Ed.), *Creativity in primary education* (pp. 102-111). Exeter, UK :Learning Matters Ltd.

Chen, C., Himsel, A., Kasof, J., Greenberger, E., & Dmitrieva, J. (2006). Boundless creativity: Evidence for the domain generality of individual differences in creativity. *Journal of Creative Behavior, 40*(3), 179-199.

Creswell, J. W. (2012). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research* (4^{th} ed.). Boston: Pearson Education, Inc.

Cropley, A. J. (1967). Creativity: A new kind of intellect? *The Australian Journal of Education, 11*, 120-125.

Cropley, A. J. (1969). Creativity, intelligence, and intellectual style. *Australian Journal of Education, 13,* 3-7.

Curcio, F. R. & Schwartz, S. L. (2002). There are no algorithms for teaching algorithms. In A. R. Teppo (Ed.), *Reflecting on NCTM's principles and standards in elementary and middle school mathematics* (pp. 122-126). Reston, VA: The National Council of Teachers of Mathematics, Inc.

Dance, R. A. & Higginson, W. C. (1979). *Mathematics creativity and the socioeconomically disadvantaged.* Distributed by ERIC Clearinghouse, [Washington, D.C.]

de Souza Fleith, D. (2010). Teacher and student perceptions of creativity in the classroom environment. *Roeper Review, 22*(3), 148-153. doi:10.1080/02783190009554022

Doerr, H. M. (2006). Examining the tasks of teaching when using students' mathematical thinking. *Educational Studies in Mathematics, 62*(1), 3-24.

Fraenkel, J. R., Wallen, N. E., & Hyun, H. H. (2012). *How to design and evaluate research in education* (8th ed.). New York, NY: McGraw-Hill.

Gardner, H. (2006). Five minds for the future. *The creating mind* (pp. 77-101). Boston, Massachusetts: Harvard Business School Press.

Glas, E. (2002). Klein's model of mathematical creativity. *Science & Education, 11*, 95-104.

Guilford, J. P. (1967). Creativity: Yesterday, today, and tomorrow. *The Journal of Creative Behavior, 1*(1), 3-14.

Haylock, D. (1997). Recognising mathematical creativity in schoolchildren. *ZDM, 29*(3), 68-74.

Haylock, D. W. (1987). A framework for assessing mathematical creativity in schoolchildren. *Educational Studies in Mathematics, 18*(1), 59-74.

Hiebert, J. & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1-27). Hillsdale, London: Lawrence Erlbaum Associates, Inc.

Huckstep, P. & Rowland, T. (2008). Being creative with the truth? Self-expression and originality in pupils' mathematics. *Research in Mathematics Education, 3*(1), 183-196. doi: 10.1080/14794800008520092

Jukić, T. (2011). Implicit theories of creativity in early education. *Croatian Journal of Education, 13*(2), 38-65.

Kandemir, M. A. (2006). *OFMA matematik eğitimi öğretmen adaylarının yaratıcılık eğitimi hakkındaki görüşleri ve yaratıcı problem çözme becerilerinin incelenmesi* (Unpublished master's thesis). Balıkesir University, Balıkesir, Turkey.

Kattou, M., Kontoyianni, K., Pitta-Pantazi, D., & Christou, C. (2011). Does mathematical creativity differentiate mathematical ability? In M. Pytlak, T. Rowland, & E. Swoboda (Eds.), Proceedings of the 7th Congress of European Research in Mathematics Education (pp. 1056–1065). Rzeszów, Poland: University of Rzeszów.

Kaufman, J. C., Baer, J., & Cole, J. C. (2009). Expertise, domains, and the Consensual Assessment Technique. *Journal of Creative Behavior* (43), 223-233.

Kaufman, J. C. & Beghetto, R. A. (2009). Beyond big and little: The four C model of creativity. *Review of General Psychology, 13*(1), 1-12.

Kaufmann, G. (2003). What to measure? A new look at the concept of creativity. *Scandinavian Journal of Educational Research, 47*(3), 235-251. doi:10.1080/0031383032000079227

Kazemi, E. (2002). Discourse that promotes conceptual understanding. In A. R. Teppo (Ed.), *Reflecting on NCTM's principles and standards in elementary and middle school mathematics* (pp. 49-54). Reston, VA: The National Council of Teachers of Mathematics, Inc.

Kılıç, Ç. (2013). Turkish Primary School Teachers' Opinions about Problem Posing Applications: Students, the Mathematics Curriculum and Mathematics Textbooks. Australian Journal of Teacher Education, 38(5).

Kıymaz, Y., Sriraman, B, & Lee, K. H. (2011). Prospective secondary mathematics teachers' mathematical creativity in problem solving: A Turkish study. In B. Sriraman & K. H. Lee (Eds.), *The elements of creativity and giftedness in mathematics* (pp. 173-191). Rotterdam, the Netherlands: Sense Publishers.

Krummheuer, G., Leuzinger-Bohleber, M., Müller-Kirchof, M., Münz, M., & Vogel, R. (2013). Explaining the mathematical creativity of a young boy: an interdisciplinary venture between mathematics education and psychoanalysis. *Educ Stud Math, 84*, 183-199. doi: 10.1007/s10649-013-9505-3

Leikin, R., & Kloss, Y. (2011). Mathematical creativity of 8th and 10th grade students. In M. Pytlak, T. Rowland, & E. Swoboda (Eds.), Proceedings of the 7th Congress of European Research in Mathematics Education (pp. 1084-1094). Rzeszów, Poland: University of Rzeszów.

Lev, M., & Leikin, R. (2013). The connection between mathematical creativity and high ability in mathematics. In B. Ubuz, Ç. Haser, & M. A. Mariotti (Eds.), Proceedings of the 8th Congress of European Research in Mathematics Education (pp. 1204-1213). Ankara, Turkey: Middle East Technical University.

Levenson, E. (2011). Exploring collective mathematical creativity in elementary school. *Journal of Creative Behavior, 45*(3), 215- 234.

Lin, Y. -S. (2011). Fostering creativity through education- A conceptual framework of creative pedagogy. *Creative Education, 2*(3), 149-155. doi:10.4236/ce.2011.23021

Lindqvist, G. (2003). Vygotsky's theory of creativity. *Creativity Research Journal, 15*(2-3), 245-251. doi:10.1080/10400419.2003.9651416

Mahn, H. (2003). Periods in child development: Vygotskys's perspevtive. In A. Kozulin, B. Gindis, V. S. Ageyev, & S. M. Miller (Eds.), *Vygotsky's educational theory in cultural context* (pp. 119-137). Cambridge, United Kingdom: The Press Syndicate of the University of Cambridge.

Maksić, S. & Pavlović, J. (2011). Educational researchers' personal explicit theories on creativity and its development: a qualitative study. *High Ability Studies, 22*(2), 219-231.

Mann, E. L. (2006). Creativity: The essence of mathematics. *Journal for the Education of the Gifted*, *30*(2), 236-260.

Mann, E. L. (2009). The search for mathematical creativity: Identifying creative potential in middle school students. *Creativity Research Journal, 21*(4), 338-348. doi: 10.1080/10400410903297402

Massarwe, K., Verner, I., & Bshouty, D. (2011). Fostering creativity through geometrical and cultural inquiry into ornaments. In B. Sriraman & K. H. Lee (Eds.), *The elements of creativity and giftedness in mathematics* (pp. 217-230). Rotterdam, the Netherlands: Sense Publishers.

Merriam, S. B. (1998). *Qualitative research and case study applications in education (Rev. ed.).* San Francisco, CA: Jossey-Bass.

Merriam, S. B. (2009). Qualitative research: A guide to design and implementation. San Francisco, CA: Jossey-Bass

Milli Eğitim Bakanlığı [Ministry of Education] (2009a). İlköğretim matematik dersi 1-5. sınıflar öğretim programı [Elementary mathematics curriculum for 1-5 graders]. Ankara, Turkey.

Milli Eğitim Bakanlığı [Ministry of Education] (2009b). İlköğretim matematik dersi 6-8. sınıflar öğretim programı [Elementary mathematics curriculum for 6-8 graders]. Ankara, Turkey.

Milli Eğitim Bakanlığı [Ministry of Education] (2013). İlköğretim matematik dersi 5-8. sınıflar öğretim programı [Elementary mathematics curriculum for 5-8 graders]. Ankara, Turkey.

Milli Eğitim Bakanlığı [Ministry of Education] (2015). İlkokul matematik dersi 1, 2, 3 ve 4. sınıflar öğretim programı [Elementary mathematics curriculum for 1,2,3 and 4 graders]. Ankara, Turkey.

National Center for Education Statistics (NAEP) (2014). *Assessment for 4th Graders 2011*. Retrieved from https://nces.ed.gov/NationsReportCard/nqt/Search/SearchOptions. Question ID: 2011-4M12 #12 M149301.

Nadjafikhah, M., & Yaftian, N. (2013). The frontage of creativity and mathematical creativity. *Procedia – Social and Behavioral Sciences, 90*, 344-350. doi:10.1016/j.sbspro.2013.07.101

Nadjafikhah, M., Yaftian, N., & Bakhshalizadeh, S. (2012). Mathematical creativity: some definitions and characteristics. *Procedia – Social and Behavioral Sciences, 31,* 285-291.

Olgun, E. (2012). *A non-formal learnıng program for the contribution of creative problem solving skills: A case study* (Unpublished master's thesis). Bilkent University, Ankara, Turkey.

Peck, D. M. & Jencks, S. M. (1981). Conceptual issues in the teaching and learning of fractions. *Journal for Research in Mathematics Education,, 12*(5), 339-348.

Piffer, D. (2012). Can creativity be measured? An attempt to clarify the notion of creativity and general directions for future research. *Thinking Skills and Creativity, 7,* 258-264.

Prediger, S. (2008). Discontinuities for mental models - a source for difficulties with the multiplication of fractions.

Reinhart, S. C. (2002). Never say anything a kid can say. In A. R. Teppo (Ed.), *Reflecting on NCTM's principles and standards in elementary and middle school mathematics* (pp. 59-64). Reston, VA: The National Council of Teachers of Mathematics, Inc.

Runco, M. A. & Albert, R. S. (1986). The threshold theory regarding creativity and intelligence: An empirical test with gifted and nongifted children. *The Creative Child and Adult Quarterly, 11*(4), 212-218.

Saracho, O. (2012). Creativity theories and related teachers' beliefs. *Early Child Development and Care, 182*(1), 35-44. doi:10.1080/03004430.2010.535899

Sarrazy, B., & Novotná, J. (2013). Mathematical creativity and highly able students: What can teachers do? In B. Ubuz, Ç. Haser, & M. A. Mariotti (Eds.), Proceedings of the 8th Congress of European Research in Mathematics Education (pp. 1245-1253). Ankara, Turkey: Middle East Technical University.

Shriki, A. (2010). Working like real mathematicians: developing prospective teachers' awareness of mathematical creativity through generating new concepts. *Educ Stud Math, 73,* 159-179. doi:10.1007/s10649-009-9212-2

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14*(1), 19-28.

Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM, 29*(3), 75-80.

Sriraman, B. (2004). The characteristics of mathematical creativity. *The Mathematics Educator, 14*(1), 19-34.

Sriraman, B., Yaftian, N., & Lee, K. H. (2011). Mathematical creativity and mathematics education: A derivative of existing research. In B. Sriraman & K. H. Lee (Eds.), *The elements of creativity and giftedness in mathematics* (pp. 119-130). Rotterdam, the Netherlands: Sense Publishers.

Stein, M. K., Remillard, J., & Smith, M. S. (2007). How curriculum influences student learning. In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 319-370). Charlotte, NC: Information Age Publishing.

Sternberg, R. J. (2005) Creativity or creativities? *International Journal of Human-Computer Studies, 63,* 370-382.

Švecová, V., Rumanová, L., & Pavlovičová, G. (2014). Support of pupils' creative thinking in mathematical education. *Procedia – Social and Behavioral Sciences, 116,* 1715-1719.

Van de Walle, J. A. (2007). *Elementary and middle school mathematics: Teaching developmentally* (6th ed.). Boston: Pearson Education, Inc.

Spangler, D. B. (2011). *Strategies for teaching fractions using error analysis for intervention and assessment.* United States of America: Corwin.

Van Harpen, X. & Presmeg, N. (2011, July 10-15). Insights into students' mathematical problem posing processes. In B. Ubuz, *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education, Turkey, 4*, 289-296.

Van Harpen, X. Y. & Presmeg, N. C. (2013). An investigation of relationships between students' mathematical problem-posing abilities and their mathematical content knowledge. *Educ Stud Math, 83*, 117-132. doi: 10.1007/s10649-012-9456-0

Vygotsky, L. S. (2004). Imagination and creativity in childhood. *Journal of Russian and East European Psychology, 42*(1), 7-97.

Yaman, H., Akkaya, R., & Yeşilyurt, Ü. (2013). Ortaokul matematik 5. sınıf ders kitabı [Middle school mathematics course book for grade 5]. In Ş. Zöhra (Ed.), *4. Ünite: Kesirler* [*Unit 4: Fractions*] (pp. 153-181). Ankara, Özgün Matbaacılık Sanayii ve Ticaret Anonim Şirketi.

Yıldırım, B. (2006). *Öğretmenlerin yaratıcılığa bakış açısı ve anasınıfı çocuklarının yaratıcılık düzeylerinin, öğretmenin yaratıcılık düzeyine gore incelenmesi* (Unpublished master's thesis). Hacettepe Univeristy, Ankara, Turkey.

Warrington, M. A. & Kamii, C. (2002). Multiplication with fractions: A Piagetian, Constructivist approach. In A. R. Teppo (Ed.), *Reflecting on NCTM's principles and standards in elementary and middle school mathematics* (pp. 65-69). Reston, VA: The National Council of Teachers of Mathematics, Inc.

Whitin, P. & Whitin, D. J. (2000). Making personal connections to geometric ideas. In J. Creed & R. D. Disroe (Eds.), *Math is language too: Talking and writing in the mathematics classroom* (pp. 70-92). Urbana, Illinois: the National Council of Teachers of English.

APPENDICES

APPENDIX A: PERMISSION OBTAINED FROM METU APPLIED ETHICS RESEARCH CENTER

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

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13.01.2015

Gönderilen : Doç.Dr. Çiğdem Haser
İlköğretim Bölümü

Gönderen : Prof. Dr. Canan Sümer
IAK Başkan Vekili

İlgi : Etik Onayı

Danışmanlığını yapmış olduğunuz İlköğretim Bölümü öğrencisi Çağla Adıgüzel'ın "İlköğretim Çağındaki Öğrencilerin Kesirler Konusuna İlişkin Matematiksel Yaratıcılık Örneklerinin İncelenmesi" isimli araştırması "İnsan Araştırmaları Komitesi" tarafından uygun görülerek gerekli onay verilmiştir.

Bilgilerinize saygılarımla sunarım.

Etik Komite Onayı

Uygundur

13/01/2015

APPENDIX B: OBSERVATION PROTOCOL

Aim of the observation: The aim of the observation is to explore mathematical creativity examples of 5^{th} grade students regarding fractions topic in lesson.

Corresponding questions:

1. What is the atmosphere of the lesson?
2. Are there any materials used? for what purposes?
3. Are real life examples being used?
4. Do students participate lessons and how is the attitude of teacher towards it?
5. What are the mathematical creativity examples of students?
    - Which material was being used?
    - After which real life example, mathematical creativity example was explored?
    - Except from the stiuations in which materials were used and real life examples were utilized, do mathematical creativity examples occur?

APPENDIX C: IN-CLASS SHORT INTERVIEW PROTOCOL

Date: Time: Place: Participant:

Questions

1. How did you come up with this idea/ example/ problem/ solution/ strategy?
2. Do you think that an activity we did in lesson before let you produce this idea/ example/ problem/ solution/ strategy?
3. Do you think that real life examples might affect you producing idea/ example/ problem/ solution/ strategy?
4. Are there any other factors affecting you producing such idea/ example/ problem/ solution/ strategy?
5. Do you have anthing to add?

APPENDIX D: STUDENTS' REPONSES ON REDEFINITION AND PROBLEM POSING TASKS IN ORIGINAL VERSION

1) Jale 2L su içmiştir. Arkadaşı Esma ise $\frac{3}{4}$ L su içmiştir. $2+\frac{3}{4}$

Jale ve Esma kaç L su içmiştir?

2) Ali tarlada 2 saat çalışmıştır. Kardeşi Asya ise $\frac{3}{4}$ saat çalışmıştır. Ali ve kardeşi Asya kaç saat çalışmıştır?

3) Meryem tabaktaki kurabiyelerden 2 tanesini yemiştir. Oyun oynadıktan sonra ise kurabiyenin $\frac{3}{4}$'ünü yemiştir. Meryem ne kadar kurabiye yemiştir?

4) Bir mimar 1. gün 2 adet ev modeli çizmiştir. 2. gün ise 1 çizimin $\frac{3}{4}$'ünü çizmiştir. Mimar iki günde ne kadar çizim yapmıştır?

$2\frac{3}{4}$

1- Semih ve Serkan pasta yapmaya karar vermişlerdir. Mutfağı hazırladıklarından sonra Pastalarını yapmışlardır. Yaptıkları pastalar aynı boydadır. Semih pastasının hepsini, Serkan ise $\frac{3}{4}$'ü yemiştir. Sonra Semih aynı boyda bir pasta daha yemiştir.

Buna göre Semih ve Serkan toplamda ne kadar pasta yemişlerdir?

2- Fethiye Belediyesi 3 aynı uzunlukta olan yolu asfaltlamaya karar vermiştir.

1. Gün hava güzel olduğu için 2 yolu asfaltlamışlardır. Ama 2. gün yağmur dolayısıyla yolun sadece $\frac{3}{4}$'ü asfaltlayabilmişler. 3 gün ise bitirmişlerdir. Buna göre 1. ve 2. gün ne kadar yol asfaltlamışlardır?

- İkiside 3 ile genişletilebilir.
- İkisininde payları eşittir.
- İkiside birim kesirdir.
- İkiside bir tamın içindedir.
- İkisinide sayı doğrusunda gösterebiliriz.
- İkiside problem çözerken problemlerde bulunabilir.
- İkiside $\frac{1}{5}$'dan büyüktür.
- İkiside bir pasta dilimi olabilir.
- veya bir miktar paranın $\frac{1}{2}$'i yada $\frac{1}{3}$'i olabilir.
- Bir kağıdı $\frac{1}{3}$'e veya $\frac{1}{2}$'e katlaya katlayabiliriz.
- Bir makaleyi $\frac{1}{2}$ sayfaya belkide $\frac{1}{3}$'i sayfaya yazabiliriz.
- Bir silgiyi $\frac{1}{2}$'e yada $\frac{1}{3}$'e bölebiliriz.
- İki kesirde $3\frac{5}{8}$'ten küçüktür.

- Ali ile Ayşe muz yiyecektir. Birlikte muz bıçak ile 4'e bölmüştür. Ali muzun $\frac{1}{4}$'ini Ayşe $\frac{2}{4}$'sini yemiştir. ikisi toplam ne kadar muz yemiştir?
- Selim ve Selin muz yiyecektir. Muz'u bıçak ile 6'ya bölmüştür. Selim muzun $\frac{2}{6}$'sini Selin muzun $\frac{4}{6}$'ünü yemiştir. Selim selin'den ne kadar fazla muz yemiştir?
- Bir manavcı muzların $\frac{2}{4}$'sini satmıştır. Manavcı muzu bıçak ile bölmüştür. Geriye ne kadar muz kalmıştır?
- Elif ile Hamide muzu bıçak ile bölmüştür. Elif muzun $\frac{2}{5}$'sini yemiştir. Hamide'ye muzun kaçta kaçı kalmıştır?
- Ahmet bir muzu bıçak ile bölmüştür ve muzun $\frac{1}{4}$'ini yemiştir. Geriye muzun kaçta kaçı kalmıştır?
- Ece bir muzu bıçak ile 8'e bölüyor. Ece muzun $\frac{3}{8}$'ünü yemiştir geriye muzun kaçta kaçı kalmıştır?

1- Okulumuzda düzenlenen Peri Bacalarına geziye Okuldaki kız öğrencilerinden $\frac{1}{3}$'i, erkek öğrencilerinden ise $\frac{3}{12}$'ü geliyor. Buna göre bu geziye toplam kaç öğrencinin geldiğini kesir olarak gösteriniz

($\frac{7}{12}$)

2- Ailemle tatilde Nevşehir/Peri Bacalarını 1 günlüğüne gezmeye gittik. Günün $\frac{2}{12}$'sini balonla gezmeye harcadık. $\frac{1}{8}$'iyle yemek yedik. $\frac{1}{16}$'i ile de hem gezip hem de yemek yedik. Günün geriye kalanını ise yolda ve uyuyarak geçirdik. Buna göre yolda ve uyuyarak geçirdiğimiz vakti kesir olarak yazınız.

($\frac{13}{24}$)

3- Sınıfça gittiğimiz Peri bacalarına toplam 120 TL götürdüm Paramın $\frac{6}{24}$'i ile yol paramı ödedim $\frac{3}{24}$'ünü yemeğe harcadım. $\frac{5}{24}$'i ile de Aileme hediyeler aldım. Buna göre yanıma aldığım paranın geriye kaç TL kaldığını bulunuz. (50 TL)

APPENDIX E: TURKISH SUMMARY

KESİRLER KONUSUNA İLİŞKİN MATEMATİKSEL YARATICILIĞIN 5.SINIF MATEMATİK DERSİNDE ARAŞTIRILMASI

GİRİŞ

Kavramsal öğrenme daha önceden veya yeni edinilmiş bilgiler arasında ilişki kurarak gelişir (Hiebert & Lefevre, 1986). Günümüzde matematik öğretmenleri hesap yapmaktan öte öğrencilerin matematiği anlamasını istemekte ve öğrencilere matematiği keşfetme şansı tanımaktadır (Kazemi, 2002). Kavramsal öğrenme içinde matematiği anlamlandırmayı da barındırmaktadır. Eğer öğrencilere kendi bilgilerini inşa etmeleri ve matematik yapmaları için izin verilirse, matematiği anlamlandırmalarına imkân sağlanmış olur (Warrington & Kamii, 2002).

Öğrenciler matematiği anlamlandırırken kendi anlama biçimini yansıtan yeni ilişkiler kurabilir (Curcio & Schwartz, 2002). Bununla beraber, öğrenciler zaman zaman kendilerine özgü tanımlar yapıp ve bir kavrama ait özellikleri bulabilirler (Whitin & Whitin, 2000). Öğrenciler keşfetme ve yaratıcı fikirler üretme yeteneğine sahiptir.

Bu tür yetenekler matematik eğitiminde yeniden tanımlama, problem kurma ve problem çözme etkinliklerinde ortaya çıkabilir (Bkz. Briggs, 2005; Curcio & Schwartz, 2002; Haylock, 1987, 1997; Silver, 1997; Van Harpen & Presmeg, 2011, 2013). Ayrıca, uygunluk, akıcılık, esneklik ve orijinallik gibi kriterler göz önünde bulundurularak bu etkinlikler incelenebilir (Bkz. Haylock, 1997; Silver, 1997).

Vygotsky (2004)’ e göre, öğrencilere tecrübe etmeleri için olanak verilirse yaratıcı olmaları için de olanak sağlanmış olur. Örneğin, sayı pulları, kesir çubukları, kesir kartları, geometri tahtası ve tangram seti gibi birçok somut materyal, matematik eğitiminde öğrenciler derste kullansın diye tasarlanmıştır (MEB, 2009b; Van de Walle, 2007). Yani, öğrenciler somut deneyim kazanmaları (Vygotsky, 2004) ve yaptıkları şeyleri anlamlandırmaları (Glas, 2002) için teşvik edilmektedir. Böylece, öğrenciler yaratıcı olabilir ve kendi anlayış şekillerine bağlı olarak farklı fikirlerini açıklayabilirler.

Matematik eğitiminde yaratıcılığı geliştirmek için, farklı fikirleri kabul etmek ve ezbere dayalı işlem yapmaktan kaçınmak önemsenmelidir. Öğrenciler farklı fikir ve ilişkiler hakkında düşünmek için teşvik edilmelidir (Mann, 2006). Öğretmenler, öğrencilerin düşünce yolunu dinleyerek onları anlayabilirler. Böylece, düşünmeleri için öğrencilere tekrar fırsat vermiş olurlar. Öğrenciler de fikirlerini değiştirebilir veya üstüne bir şeyler koyabilirler (Doerr, 2006).

Türk Eğitim Sisteminde, kesirler konusu 6 yıla yayılmıştır (MEB, 2009b, 2013, 2015) Birinci sınıfta “tam, yarım, çeyrek” gibi kavramlar öğretilir. Parça-bütün ilişkisi ve kesir sembolleri ise ikinci sınıfta ele alınır. Üçüncü sınıfta birim kesir işlenir ve pay ile payda arasındaki ilişkiyi güçlendirmek hedeflenir. Öğrencilerden, basit ve bileşik kesirleri dördüncü sınıfta tanımlamaları beklenmektedir. Ayrıca, paydası aynı olan kesirlerle toplama ve çıkarma işlemi yapmayı ve problem çözmeyi de dördüncü sınıfta öğrenmeleri beklenmektedir (MEB, 2015)

Kesirler konusunun yıllara yayılmış olması, öğrencilerin tecrübe kazanmaları açısından önemli olduğu kadar kullanılan modeller de önemlidir. Eğer bir etkinlik iki farklı modelle yapılırsa öğrenciler o dersi özel bulabilirler. Yuvarlak kesir takımı, şerit kesir takımı, geometri tahtası, noktalı kağıt üzerine çizim yapma, örüntü blokları ve kağıt katlama kesirlerin alan modelleridir. İlki bir bütünü öğrenmek için faydalıyken diğerleri farklı bütünleri öğrenmede yararlıdır (Van de Walle, 2007).

Sonuç olarak, kesirler konusu ilgili tecrübe ve kullanılabilecek somut materyaller bakımından yaratıcılığı tecrübeye bağlı olarak araştırmak için uygun bir konudur.

Çalışmanın Amacı

Bu çalışmanın temel amacı, 5.sınıf kesirler konusuna ilişkin matematiksel yaratıcılık örneklerini incelemektir. Bu amaçla kesirler konusu işlenirken sınıf gözlemi yapılacak ve öğrencilere yazılı sınavlar uygulanacak, gerekli görüldüğü takdirde bazı öğrencilerle görüşme yapılacaktır. Görüşmeler esnasında derste materyal ve günlük hayat problemleri kullanmanın matematiksel yaratıcılığa etkisi araştırılaraktır.

Çalışmada aşağıdaki araştırma sorularına cevap aranacaktır:

2. Beşinci sınıf öğrencilerinin kesirler konusunda, matematiksel yaratıcılığı ortaya çıkarmak için yeniden tanımlama, problem kurma ve problem çözme etkinlikleri ile somut materyal ve gerçek hayat tecrübeleri kullanarak tasarlanan, sundukları matematiksel yaratıcılık örnekleri nelerdir?
   2.1 Beşinci sınıf öğrencilerinin kesirler konusunda sınıfta gözlemlenen matematiksel yaratıcılık örnekleri nelerdir?
   2.2 Beşinci sınıf öğrencilerinin kesirler konusunda yeniden tanımlama, problem kurma ve problem çözme etkinliklerinde sundukları matematiksel yaratıcılık örnekleri nelerdir?
   2.3 Beşinci sınıf öğrencilerinin kesirler konusunda sundukları matematiksel yaratıcılık örnekleri sınıf ortamında ve kağıt üzerinde nasıl değişmektedir?

Bu sorulara yanıt ararken öğrencilerin matematiksel yaratıcılık örneği sunmalarında tecrübelerinin etkisi göz önünde bulundurulacaktır (Vygotsky, 2004).

Çalışmanın Önemi

Matematik dersleri anlamlı öğrenmenin yanı sıra farklı bakış açıklarına değer vermeyi öğretmeyi amaçlamaktadır (MEB, 2009). Bu nedenle, öğrencilerin değişik bakış açılarının farkında olduğu bir sınıf da matematiksel yaratıcılığı destekleyebilir ki böyle bir ortam çalışmaya değerdir (Mann, 2009).

Türkiye'de ilköğretim matematik eğitimi (1-4) öğrencilerin bağımsız düşünme, karar verme ve öz-düzenleme becerilerini geliştirmeyi amaçlamaktadır. Bu sürecin sonunda, öğrencilerin temel matematik kavramlarına alışması ve bazı temel bilgileri edinmesi beklenmektedir. Böylece kendi bilgi ve becerilerini başka matematiksel kavramlarda veya disiplinler arası uygulayabilirler. Yani, ilköğretim matematik eğitimi sonunda öğrenciler bazı çıkarımlarda bulunabilir ve kendi matematiksel düşüncelerini açıklayabilirler (MEB, 2009). Bu nedenle, öğrencilerin matematiksel yaratıcılıklarını ilköğretim (1-4) seviyesini tamamlayınca yani beşinci sınıfta araştırmak, onların matematiksel yaratıcılıklarını anlamamıza yardımcı olabilir.

Beşinci sınıf öğrencilerinin matematiksel yaratıcılık örneklerini hem sınıf ortamında gözlem yaparak hem de yazılı sınavları değerlendirerek keşfetmek önemlidir. Çünkü matematiksel yaratıcılık matematik alanında gelişmelere bir nebze de olsa imkan verir (Sriraman, 2004). Gözlem protokolü ve yazılı sınavlardan elde edilen bilgileri sentezlemek matematiksel yaratıcılık hakkında önemli bilgiler sunabilir (Levenson, 2011; Mann, 2009; Piffer, 2012).

Tanımlar

Matematiksel yaratıcılık: Gerçeklik ile deneyim kazanmaktan kaynaklanabilecek olan akıcılık, esneklik ve orijinallik gibi kriterlere bağlı olarak yeniden tanımlama, problem kurma ve problem çözme etkinlikleri ile kağıt üzerinde veya ders içinde ortaya çıkan matematiksel olarak uygun ürün veya süreçtir (Bkz. Guilford, 1967; Haylock, 1997; Kaufmann, 2003; Levenson, 2011; Silver, 1997).

Uygunluk: Verilen yanıtın matematiksel içerik ve/veya kurallar bakımından geçerli olmasıdır (Bkz. Haylock, 1997; Nadjafikhah & Yaftian, 2013; Van Harpen & Presmeg, 2013). Örneğin, "$\sqrt{8} = 4$" yanıtı orijinal bir yanıt olabilir fakat matematiksel olarak uygun olmadığı için diğer çalışmalarda olduğu gibi bu çalışmada da elenmiştir (Nadjafikhah & Yaftian, 2013).

Akıcılık: Uygun olarak verilmiş olan yanıt sayısıdır (Haylock, 1997, s.71). Bu çalışmada, her bir etkinlikte her bir öğrenci için uygun cevap sayısı hesaplanarak en akıcı olan/lar seçilecektir.

Esneklik: Farklı yanıtların sayısıdır (Haylock, 1997, s.71). Bunun için uygun olan yanıtlar belirlendikten sonra yanıtlar kategorilendirilmiş ve her bir etkinlikte her bir öğrenci için vermiş olduğu yanıtların ait olduğu kategori sayısı hesaplanmıştır. Sonunda, en çok esnek olan/lar seçilmiştir.

Orijinallik: Yanıtların istatistiksel olarak grup içindeki nadirliğidir (Haylock, 1997, s.71). Orijinallik için, daha önce yapılan bir çalışmada (Van Harpen & Presmeg, 2013), öğrenci tarafından verilen bir yanıtın sınıftaki öğrenci sayısının %10'undan daha az verilmesi sınırı getirilmiştir. Bu çalışmada da aynı sınırlama kullanılmıştır.

Yeniden tanımlama etkinliği: Matematiksel özellikleri bakımından bir durumu birden çok kez tekrar tanımlama etkinliğidir (Haylock, 1987). "16 ve 36 sayıları için aynı olan şeyleri belirtiniz" sorusu bu tür etkinliklere örnektir (Haylock, 1997, s.73)

Problem kurma etkinliği: Verilen bilgiler doğrultusunda, öğrencilerin soru kurması istenir (Haylock, 1987, 1997).

Problem çözme etkinliği: Birden çok yol ile çözülebilen problemleri kapsamaktadır (Haylock, 1987, 1997).

YÖNTEM

Araştırma Deseni

Beşinci sınıf öğrencilerinin kesirler konusuna ilişkin matematiksel yaratıcılık örneklerini kağıt üzerinde ve derslerde inceleyip ayrıntılı bir bilgiye sahip olmak için basit nitel araştırma yöntemi (Creswell, 2012) kullanılmıştır.

Araştırmanın Bağlamı

Bu çalışma fiziksel şartları bakımından iyi durumda olan bir özel okulda gerçekleştirilmiştir. Sınıf içi kuralları bakımından da matematiksel yaratıcılık desteklenmiştir. Öğrencilere verdikleri yanıtların ardından "Nasıl düşündün, Niye" gibi sorular yöneltilmiştir. Öğrenciler başkalarının fikirlerini dinleyerek takdir ediyorlardı. Kesirler konusundan önce birçok konu beraber işlendiği için bu ortama öğrenciler alışıktı. Ayrıca, hemen hemen her konuda öğrencilerden günlük hayat örnekleri vermeleri istenmiştir. Geçmiş yıllardan gelen bilgileri de zaman zaman sorulmuştur.

Tablo 1'de görüldüğü üzere, beşinci sınıf kesirler konusunda toplam dokuz kazanım öğretilmeyi planlanmaktadır.

Tablo 1

Kesirlerle İlgili Kazanımlar

Sayı	Kazanım
1	Birim kesirleri sıralar.
2	Birim kesirleri sayı doğrusunda gösterir.
3	Tam sayılı kesrin, bir doğal sayı ile bir basit kesrin toplamı olduğunu anlar ve tam sayılı kesri bileşik kesre, bileşik kesri tam sayılı kesre dönüştürür.
4	Bir doğal sayı ile bileşik kesri karşılaştırır.
5	Sadeleştirme ve genişletmenin kesrin değerini değiştirmeyeceğini anlar ve bir kesre denk olan kesirler oluşturur.
6	*Pay ve paydaları eşit veya birbirinin katı olan kesirleri sıralar.*
7	Bir çokluğun istenen basit kesir kadarını ve basit kesir kadarı verilen bir çokluğun tamamını birim kesirlerden yararlanarak hesaplar.
8	Paydaları eşit veya birinin paydası diğerinin paydasının katı olan iki kesrin toplama ve çıkarma işlemini yapar.
9	Paydaları eşit veya birinin paydası diğerinin paydasının katı olan kesirlerle toplama ve çıkarma işlemleri gerektiren problemleri çözer.

Bu kazanımlar ışığında sınıfta yapılan etkinlikler şöyle sıralanabilir: kâğıttan kesir çubuğu oluşturma, kağıt katlama (denk kesirler için), kağıt üzerinde yazılı olan kesirleri ip üzerinde sıralama, meyveleri eş parçalara ayırıp birleştirme (bileşik kesri tam sayılı kesre, tam sayılı kesri bileşik kesre dönüştürmek için), çizim yapma, ölçme kapları ile su aktarma (kesirlerle toplama ve çıkarma işlemi yapmak için).

Derste ve ödev olarak genellikle Milli Eğitim Bakanlığı tarafından dağıtılan ders kitabından (Yaman, Akkaya, & Yeşilyurt, 2013) yararlanılmıştır. Ek olarak bazı derslerde çalışma yaprağı dağıtılmıştır. Çalışma yaprakları bazen yeniden tanımlama soruları içermekteydi. Ayrıca ders kitabında problem çözme soruları olduğu gibi problem kurma soruları da bulunmaktaydı. Yani, veri toplanmadan önce öğrenciler veri toplama araçlarına (yeniden tanımlama, problem kurma ve problem çözme) az da olsa alışıklardı.

Katılımcılar

Araştırmanın çalışma grubu kolaylıkla bulunabilen örnekleme yoluyla belirlenmiştir (Fraenkel, Wallen, & Hyun, 2012). 2016 yılında öğretmen-araştırmacı ve özel okulda okuttuğu 5. sınıflardan birindeki öğrenciler çalışma grubunu oluşturmuştur.

Veri Toplama Araçları

Veriler gözlem, yazılı sınavlar ve sınıf-içi kısa görüşmeler ile toplanmıştır. Veriler nitel yöntemlerle analiz edilmiştir.

Veri Toplama Süreci

2016 yılı Şubat ayı ile Mart ayı arasında toplam yaklaşık olarak 6 hafta boyunca veri toplanmıştır. Kesirler konusu boyunca dersler görüntülü olarak kaydedilmiştir. Böylelikle, öğrenciler sınıf ortamında gözlenmiştir. Ayrıca, bu süreçte iki quiz (Quiz A ve Quiz B) ile kesirler konusunun sonunda yazılı sınav yapılmıştır. Dersler ve yazılı sınavlar değerlendirildikten sonra matematiksel yaratıcılık örneği veren öğrencilerle görüşme yapılmıştır.

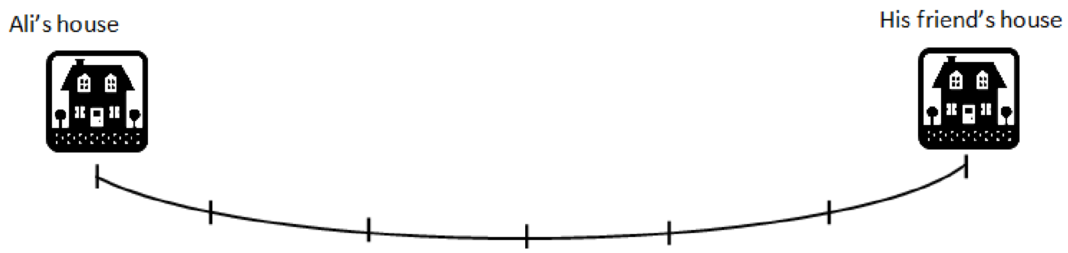
Yazılı sınavların iki kez pilot çalışması yapılmıştır. Açık olmayan noktalar netleştirilmiş ve problem çözme sorusu amacına ulaşmadığı için değiştirilmiştir. Gerekli değişiklikler yapıldıktan sonra oluşturulan yazılı sınavın son hali Şekil 1'de verilmiştir. Bu sınav iki ders saati sürmüştür.

1. Bu iki sayı ile ilgili aynı olan her şeyi yazınız: $\frac{1}{3}$ ve $\frac{1}{2}$ (Haylock, 1997).

2. *kuş, top, Kurtuluş Savaşı, Fransız İhtilali, Peri bacaları, su, Nasrettin Hoca, ,………, ……….*

Öncelikle yukarıda bulunan kelimelerden ikisini seçin veya kendiniz (en fazla iki kelime) ekleyebilirsiniz. Bu iki kelime ile kesirlerle ilgili kurabildiğiniz kadar çok problem kurunuz. Not: Sorular **çözülebilir** olmalı.

3.



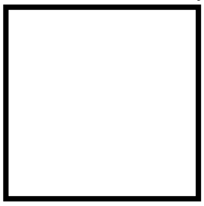
Ali kendi evinden arkadaşının evine doğru bisiklet sürmek istiyor. Ali gideceği yolun önce $\frac{1}{4}$ 'ini,daha sonra $\frac{1}{5}$'ini gidiyor. Yukarıda verilen şekil üzerinde Ali'nin nerede olduğunu **işaretleyiniz**. Çözümünüzü açıklayınız (2011 NAEP Assessment for 4th Graders.)

Şekil 1 Yazılı Sınavın Son Hali

Kesirler konusu işlenirken ders esnasında uygulanan iki quiz Tablo 2' de verilmiştir. Quiz A on dakikada Quiz B ise 40 dakikada uygulanmıştır.

Tablo 2

Uygulanan İki Quiz

Quiz A (Van de Walle, 2007)	Ali bahçesinin $\frac{3}{4}$ ‘üne çiçek dikmek istiyor. Aşağıdaki kare üzerinde çizimler yaparak lütfen ona yardımcı olunuz. 
Quiz B (MEB, 2016, s.181)	III. Sonucu 2+ $\frac{3}{4}$ olan problemler kurunuz (Kurabildiğiniz kadar çok). IV. Sonucu 1- $\frac{2}{7}$ olan problemler kurunuz (Kurabildiğiniz kadar çok).

Veri Analizi

Kağıt üzerindeki yeniden tanımlama, problem kurma ve problem çözme etkinliklerindeki öğrenci yanıtları uygunluk, akıcılık, esneklik ve orijinallik olmak üzere dört kriter bakımından araştırılmıştır. Ders içi etkinlikleri için başta uygunluk ve orijinallik kriterleri ele alınmıştır. Ama ortaklaşa akıcılık ve ortaklaşa esneklik kriterleri de sonradan eklenmiştir.

Bu analizde öncelikle uygunluk kriteri sağlanmıştır. Daha sonra akıcılık kriteri için her bir soruda her bir öğrencinin cevap sayısı belirlenmiş ve en çok uygun cevabı veren öğrencinin yanıtları akıcılık örneği olarak gösterilmiştir.

Esneklik kriteri için ise, uygun öğrenci cevaplarından ortaya çıkan kategoriler ele alınmıştır. Daha sonra her bir öğrencinin cevaplarının yer aldığı kategorilerin sayısına bakılarak en çok kategori sayına sahip olan öğrenci cevapları esneklik örneği olarak verilmiştir.

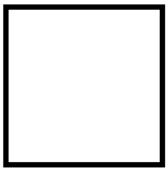
Son olarak, orijinallik kriteri için bir öğrencinin verdiği cevabı sınıf içinde başka bir öğrenci de verdi mi diye araştırılmıştır. Bunun için de, esneklik analizinde oluşturulan kategorilerden yararlanılmış eğer bir kategoride yalnız bir öğrencinin cevabı bulunuyorsa orijinal olarak belirtilmiştir.

Sınıf içindeki etkinliklerde aynı süreç tekrarlanmıştır. Bunun için video kayıtları incelenerek ders içindeki konuşmalar pasaj olarak yazılmıştır. Bu pasajlara bakarak bazı durumlarda ortaklaşa akıcılık ve ortaklaşa esneklik gibi kriterler de analize dahil edilerek, matematiksel yaratıcılık örnekleri sunulmuştur. Aşağıda Quiz A için elde edilen bulgular açıklanmaktadır.

Quiz A

Kesirler konusuna birim kesirler ile giriş yapılmıştır. Öğrenciler bir bütünün eşit büyüklükteki parçaları üzerine düşünmüş ve bazı şekillerin doğru olarak parçalara ayrılıp ayrılmadığını tartışmışlardır. İkinci derste öğretmen öğrencilerin bilgisini ölçmek amacıyla Şekil 2'de verilen soruyu uygulamıştır.

Ali bahçesinin $\frac{3}{4}$ 'üne çiçek dikmek istiyor. Aşağıdaki kare üzerinde çizimler yaparak lütfen ona yardımcı olunuz.

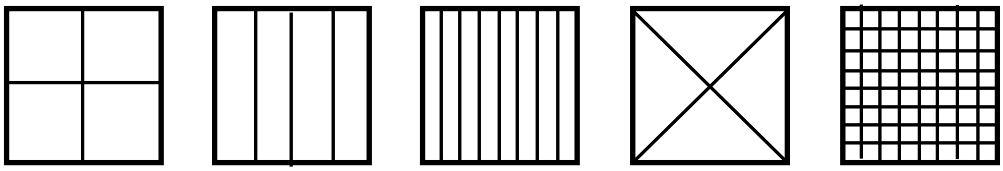


Şekil 2 Quiz A'nın içerdiği soru (Van de Walle, 2007, s.302)

Öğrenciler birden çok cevap verip veremeyeceklerini ve bunu nasıl yapabileceklerini sormuştur. Bu sırada öğretmen birden fazla cevabı olanların sorudaki kareye eş başka kareler çizerek ilgili bölümü taramalarını istemiştir. Bir süre sonra, her öğrenci yeni kareler çizerek $\frac{3}{4}$ kesrini göstermişlerdir.

Uygulama gününde bir öğrenci eksiktir. Bu sebeple, diğer 17 öğrencinin verdikleri yanıtlar analiz edilmiştir. Bu soruya toplamda 71 yanıt verilmiştir. Fakat bu yanıtların 14 tanesi uygunluk kriterinin sağlamadığı için elenmiştir. Kalan 57 yanıt üzerinden diğer kriterler değerlendirilmiştir.

Kategoriler belirlenirken taralı kısımlar göz ardı edilmiş ve öğrencilerin bütünü parçalamak için kullandığı çizgiler dikkate alınmıştır. Elde edilen 5 kategori Şekil 3'de gösterilmektedir.



Kategori 1 Kategori 2 Kategori 3 Kategori 4 Kategori 5

Şekil 3 Quiz A için Elde Edilen Kategoriler

Eğer parçalama işlemi döndürme işlemi ile elde edilebiliyorsa bu tür cevaplar aynı kategoriye dahil edilmiştir. Örneğin, Şekil 3'de verilen Kategori 2 ve Kategori 3 dikey çizgilerle belirtilmiş olsa da aynı zamanda aynı parçalama işleminin yatay çizgilerle elde edilmiş halini de yansıtmaktadır. Her bir kategoride yer alan yanıtların sayısı ve öğrenci yanıtlarını içeren kategoriler ayrı ayrı Tablo 3'de gösterilmektedir.

Tablo 3

Her Bir Kategoride Yer Alan Yanıtların Sayısı

Öğrenciler	Kategori 1	Kategori 2	Kategori 3	Kategori 4	Kategori 5	Toplam uygun yanıt
S1	1	2	0	0	0	3
S2	2	0	0	0	1	3
S3	1	0	0	1	0	2
S4	1	2	0	1	0	4
S5	1	2	0	1	0	4
S6	2	0	0	1	0	3
S7	0	4	0	0	0	4
S8	1	2	0	1	0	4
S9	1	2	0	1	0	4
S10	2	2	0	2	0	6
S11	1	2	0	0	0	3
S12	1	2	0	1	0	4
S13	0	2	0	0	0	2
S14	0	2	2	0	0	4
S15	1	1	0	0	0	2
S16	1	0	0	0	0	1
S17	1	2	0	1	0	4
S18	-	-	-	-	-	-
Total	17	27	2	10	1	57

Tablo 3’e göre, verilen 57 uygun yanıttan 17, 27, 2, 10, 1 yanıtları sırasıyla Kategori 1, 2, 3, 4, 5 içinde toplanmıştır. Yani, yanıtların büyük çoğunluğu ikinci kategoride yer alırken Kategori 5’de yalnızca bir uygun yanıt saptanmıştır.

Akıcılık

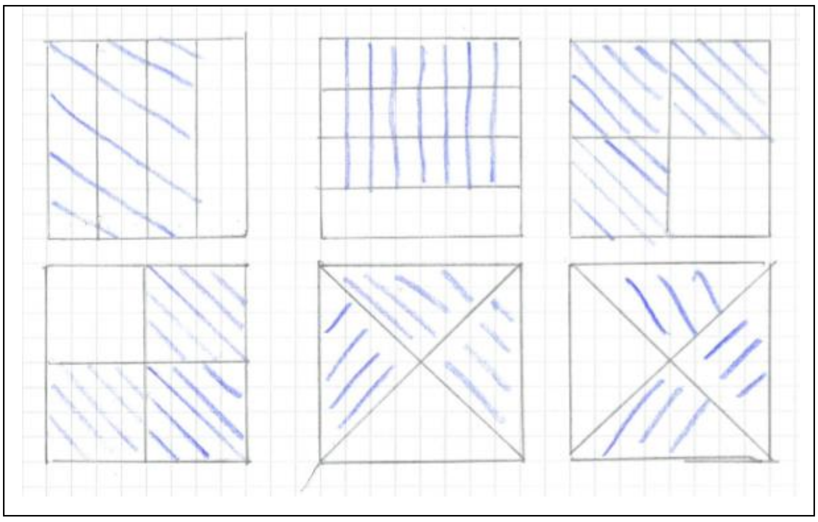
Akıcılık basitçe öğrencilerin uygun yanıt sayısı hesaplanarak bulunmuştur. En çok uygun cevabı veren öğrencinin yanıtları akıcılık örneği olarak belirlenmiştir. Tablo 4 öğrencilerin uygun yanıt sayısını göstermektedir. Örnek olarak, bir öğrenci 1 uygun yanıta sahipken üç öğrenci 2 uygun yanıt vermiştir. 6 uygun yanıtı olan öğrencinin yanıtları akıcılık örneği olarak seçilmiştir.

Tablo 4

Quiz A için Akıcılık Analizi

Uygun yanıt sayısı	Öğrenci sayısı
1	1
2	3
3	4
4	8
5	0
6	1
Toplam	17

Şekil 4’te S10’nun yanıtları akıcılık örneği, yani matematiksel yaratıcılık örneği, olarak sunulmuştur.



Şekil 4 Akıcılık Örneği, S10’un Yanıtları

Esneklik

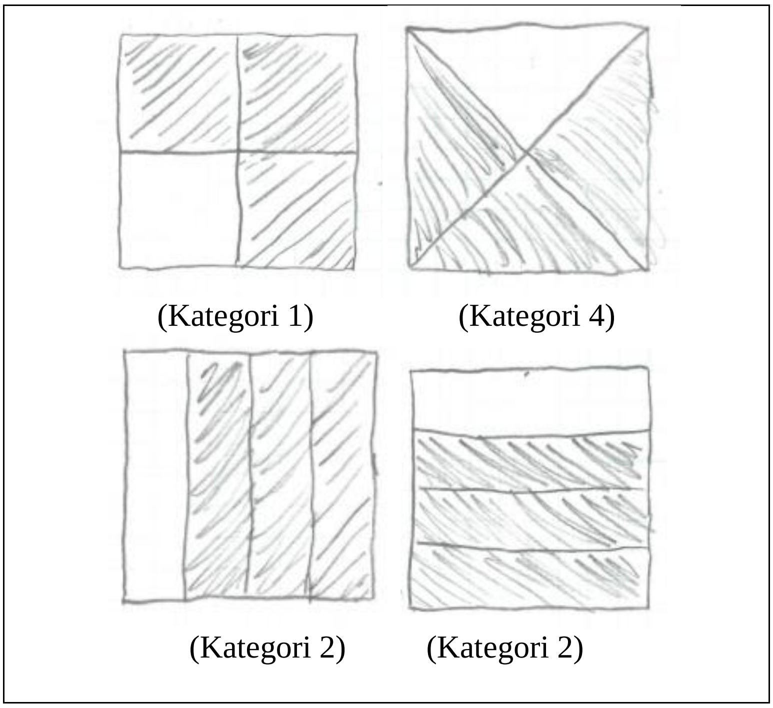
Esneklik için her bir öğrencinin verdiği yanıtların kaç farklı kategoride toplanmış olduğunu saymak gerekmektedir. Tablo 3'den yararlanarak, bu sayı her bir öğrenci için hesaplanmış ve Tablo 5 oluşturulmuştur. Tablo 3'e bakarak, S1'in Kategori 1'de bir, Kategori 2'de ise iki uygun yanıtının olduğunu söyleyebiliriz. Bu da S1'in yanıtları iki farklı kategoride yer alıyor demektir. Bu sayma işlemi her bir öğrenci için tamamlandıktan sonra, kaç öğrencinin yanıtının bir kategoride veya iki, üç farklı kategoride yer aldığı hesaplanmıştır. Yanıtları üç farklı kategoriden daha fazla sayıda farklı kategoride yer alan öğrenci yoktur. Üç öğrencinin yanıtları bir kategoride, yedi öğrencinin yanıtları iki kategoride, kalan yedi öğrencinin yanıtları da üç kategoride toplanmaktadır.

Tablo 5

Quiz A için Esneklik Analizi

Yanıtların bulunduğu kategori sayısı	Öğrenci sayısı
1	3
2	7
3	7
Toplam	17

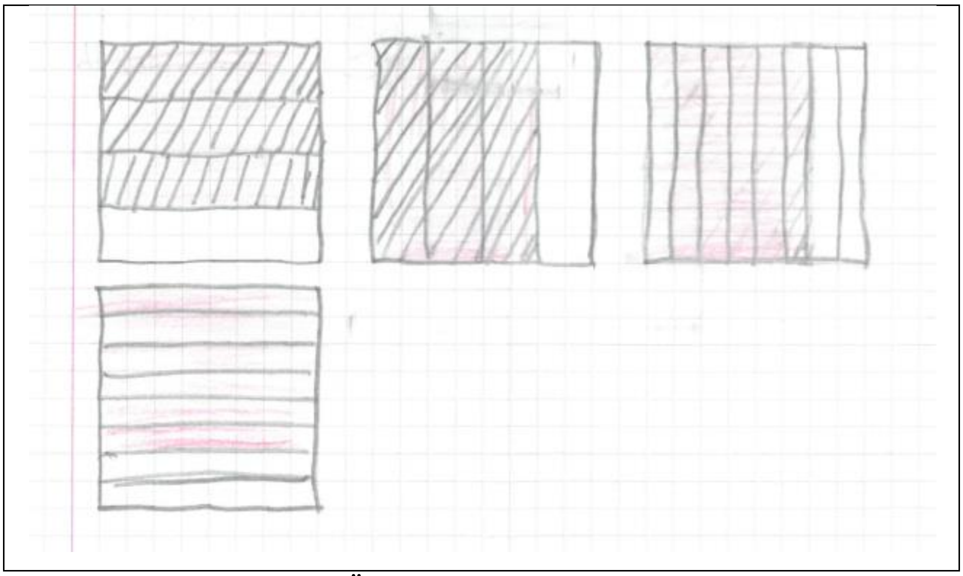
Tablo 5'in gösterdiği üzere, yedi öğrencinin cevabı üç farklı kategoride yer almaktadır ve bu öğrencilerin yanıtları esneklik örneği teşkil etmektedir. Yedi öğrenciyi (S4, S8, S9, S12, S10, and S17) temsilen bir öğrencinin (S5) yanıtları rastgele olarak seçilmiştir. Şekil 5, bu soru için esneklik örneği olarak hazırlanmıştır.



Şekil 5 Esneklik Örneği, S5'in yanıtları (S4, S8, S9, S10, S12, S17'nin yanıtlarını da temsil etmektedir)

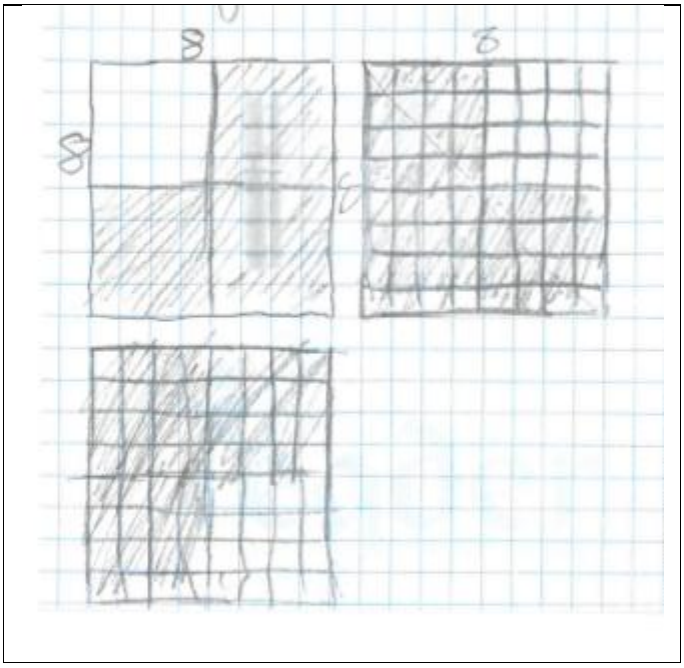
Orijinallik

Orijinallikten bahsetmek için bir kategoride yalnızca bir öğrenciye ait yanıtların olması gerekmektedir ki bu da verilen yanıtın yalnızca bir öğrenci tarafından düşünüldüğünün göstergesidir. Tablo 3'e göre, S14 ve S2'nin yanıtları sırasıyla Kategori 3 ve Kategori 5'te tektir. Bu nedenle, S14 ve S2'nin bu kategorilerde yer alan yanıtları ayrı ayrı orijinallik örneği olarak Şekil 6 ve Şekil 7'de gösterilmektedir.



Şekil 6 Orijinallik Örneği, S14'ün yanıtları

Şekil 6'da verilen yanıtlardan sağ üst ve altta yer alan çizim Kategori 3'de yer almaktadır ve S14'ün bu çizimleri matematiksel yaratıcılığın orijinallik kriterine örnektir.



Şekil 7 Orijinallik Örneği, S2'nin Yanıtları

Şekil 7'de sağ üst ile altta yer alan çizimler Kategori 5'te yer alan tek yanıttır. Bu sebeple, S2'nin çizdiği bu tasarımlar da Quiz A için orijinallik örneğidir.

Sonuç olarak, S14 ve S2'nin vermiş olduğu özgün yanıtlar orijinallik örneğidir.

BULGULAR VE TARTIŞMA

Bulgular soru kâğıdındaki yeniden tanımlama etkinliğinin diğerlerinden daha fazla yanıta sahip olduğunu göstermektedir. Quiz B ve soru kâğıdında yer alan problem kurma etkinliklerinde ise problemlerin içeriğinde öğrencilerin ilgi ve günlük işlerinin yer aldığını göstermiştir. Bazı öğrenciler de tecrübelerinden bahsetmişlerdir. Bu da Vygotsky'nin yaratıcılık teorisini (Vygotsky, 2004) doğrular niteliktedir.

Derste bazı durumlarda bireysel olarak yaratıcılık gözlenmiştir. Fakat bazı durumlarda öğrenciler iletişim halindeyken, bir öğrencinin fikri diğer bir öğrencinin fikrini geliştirmesini sağlamıştır. Sınıf-içi kısa görüşmelerde yeterli cevaba ulaşılamamıştır; öğrenciler nasıl düşündüklerini genellikle "Aklıma geldi" şeklinde yanıtlamışlardır.

Çalışmada beşinci sınıf öğrencilerinin matematiksel yaratıcılık örnekleri vermede farklılık gösterdikleri fark edilmiştir. Örneğin, bazı öğrenciler akıcılık kriterinde, bazıları esneklik kriterinde, bazı öğrenciler orijinallik kriterinde, bazı öğrenciler birkaçında ve bazı öğrenciler ise sadece ders içi etkinliklerinde yaratıcılık örneği vermiştir.

Kâğıt üzerinde ve ders içinde elde edilen bulgular arasında bazı farklılıklar vardır. Kâğıt üzerinde daha çok örnek bulunmuştur. Bunun durumun birçok nedeni olabilir. Ders içinde sınırlı sayıda söz hakkı alan öğrenciler kâğıt üzerinde istedikleri kadar yanıt verebilir. Ayrıca, öğrencilerin düşünmek için daha fazla zamanı vardır. Belki bazı öğrencilerin vereceği örnekleri ders içindeki sesler etkiliyor ve kâğıt üzerinde daha çok odaklanma şansı buluyorlardır. Bunun dışında, ders içinde gözlemlenen ortaklaşa akıcılık ve ortaklaşa esneklik durumları sadece sınıf içine özgüdür. Çünkü kağıt üzerinde öğrencilerin birbirinden etkilenme durumu toplanan verilerin sağlıklı sonuçlar vermesi için baştan engellenmiştir.

Katılımcı olan 18 öğrenciden 16'sı herhangi bir durumda matematiksel yaratıcılık örneği sergilemiştir. Bu da, öğrencilerin matematiksel başarısının çok da etkisi olmadan her birinin yaratıcılık örneği verebileceğini gösterebilir.

Çalışmanın bulguları ilkokul müfredatlarının yenilenmesi aşamasında ve kitap yazarları tarafından kitapların yenilenmesi çalışmalarında değerlendirilebilir. Aynı zamanda ortaokul öğretmenleri de ortaokula yeni başlayan öğrencilerin yaratıcılık potansiyelleri hakkında bulgular ışığında fikir sahibi olup öğretimlerinde yaratıcılığı destekleyecek etkinliklere yer verebilirler.

APPENDIX F: TEZ FOTOKOPİSİ İZİN FORMU

<u>ENSTİTÜ</u>

Fen Bilimleri Enstitüsü □

Sosyal Bilimler Enstitüsü ■

Uygulamalı Matematik Enstitüsü □

Enformatik Enstitüsü □

Deniz Bilimleri Enstitüsü □

<u>YAZARIN</u>

Soyadı : ADIGÜZEL
Adı : Çağla
Bölümü : İlköğretim Fen ve Matematik Alanları Eğitimi

<u>TEZİN ADI</u> (İngilizce) : Exploring Mathematical Creativity in the Fractions Topic in a Fifth Grade Mathematics Class

<u>TEZİN TÜRÜ</u> : Yüksek Lisans ■ Doktora □

1. Tezimin tamamından kaynak gösterilmek şartıyla fotokopi alınabilir. □
2. Tezimin içindekiler sayfası, özet, indeks sayfalarından ve/veya bir bölümünden kaynak gösterilmek şartıyla fotokopi alınabilir. □
3. Tezimden bir (1) yıl süreyle fotokopi alınamaz. ■

<u>TEZİN KÜTÜPHANEYE TESLİM TARİHİ</u>: