ONE-WAY ANOVA FOR TIME SERIES DATA WITH NON-NORMAL INNOVATIONS: AN APPLICATION TO UNEMPLOYMENT RATE DATA

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

ÖZGECAN YILDIRIM

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
STATISTICS

AUGUST 2017

Approval of the thesis:

ONE-WAY ANOVA FOR TIME SERIES DATA WITH NON-NORMAL INNOVATIONS: AN APPLICATION TO UNEMPLOYMENT RATE DATA

submitted by **ÖZGECAN YILDIRIM** in partial fulfillment of the requirements for the degree of **Master of Science in Statistics Department, Middle East Technical University** by,

Prof. Dr. Gülbin Dural Ünver
Dean, Graduate School of **Natural and Applied Sciences** ______________

Prof. Dr. Ayşen Dener Akkaya
Head of Department, **Statistics** ______________

Assoc. Prof. Dr. Ceylan Yozgatlıgil
Supervisor, **Statistics Department, METU** ______________

Prof. Dr. Birdal Şenoğlu
Co-supervisor, **Statistics Dept., Ankara Uni.** ______________

Examining Committee Members:

Assoc. Prof. Dr. Özlem İlk Dağ
Statistics Department, METU ______________

Assoc. Prof. Dr. Ceylan Yozgatlıgil
Statistics Department, METU ______________

Prof. Dr. Mehmet Yılmaz
Statistics Department, Ankara University ______________

Assoc. Prof. Dr. Berna Burçak Başbuğ Erkan
Statistics Department, METU ______________

Assoc. Prof. Dr. Seher Nur Sülkü
Econometrics Department, Gazi University ______________

Date: ______________

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: ÖZGECAN YILDIRIM

Signature :

ABSTRACT

ONE-WAY ANOVA FOR TIME SERIES DATA WITH NON-NORMAL INNOVATIONS: AN APPLICATION TO UNEMPLOYMENT RATE DATA

Yıldırım, Özgecan
M.S., Department of Statistics
Supervisor : Assoc. Prof. Dr. Ceylan Yozgatlıgil
Co-Supervisor : Prof. Dr. Birdal Şenoğlu

August 2017, 89 pages

ANOVA is a well-known approach when examining the equality of three or more than three groups' means. However, like every parametric test, some assumptions are to be satisfied so that the appropriate and reliable conclusions are obtained. The major emphasis of this thesis is the non-validated model assumptions of the one-way ANOVA, where the independency and normality assumptions are considered as non-validated. Indeed, in real life applications, it is not realistic to validate all of those assumptions. That's why, in the literature there exists a number of studies related to the non-validated assumptions. For this thesis, a test statistic for one-way ANOVA is introduced when the underlying distribution of the error terms is Student's t and the each group, which are compared for the equality of their means, follows AR(1) process. In addition to one-way ANOVA test statistic, a test statistic for the linear contrasts is introduced as well. A comprahansive simulation study is done to investigate the performances of the corresponding test statistics. Finally, a real life data related to the unemployment rate are analysed in order to illustrate the application of the subjects stated under the scope of this thesis.

Keywords: One-Way ANOVA, Linear Contrasts, AR(1) Model, Student's t Distribution

ÖZ

NORMAL DAĞILIMA SAHİP OLMAYAN HATA TERİMLİ ZAMAN SERİLERİ İÇİN BİR YÖNLÜ VARYANS ANALİZİ: İŞSİZLİK ORANI VERİSİNE UYGULAMA

Yıldırım, Özgecan
Yüksek Lisans, İstatistik Bölümü
Tez Yöneticisi : Doç. Dr. Ceylan Yozgatlıgil
Ortak Tez Yöneticisi : Prof. Dr. Birdal Şenoğlu

Ağustos 2017 , 89 sayfa

Varyans analizi, sayısı üç veya üçten fazla olan grupların ortalamalarının eşitliğinin araştırılmasında yaygın olarak kullanılan bir yaklaşımdır. Fakat, bütün parametrik testlerde olduğu gibi, burada da uygun ve güvenilir sonuçlara ulaşabilmek için bazı varsayımların sağlanması gerekmektedir. Bu tezin temel vurgusu model varsayımları sağlanmayan bir yönlü varyans analizidir, burada normallik ve bağımsızlık varsayımlarının sağlanmadığı durumlar üzerine yoğunlaşılmıştır. Aslında, gerçek hayat uygulamalarında bu varsayımların hepsinin birden sağlanması gerçekçi değildir. Bu nedenledir ki, literaturde sağlanmayan varsayımlar üzerinde yapılan bir çok çalışma bulunmaktadır. Bu çalışmada, hata terimlerinin dağılımının Student's t ve deneme ortalamaları karşılaştırılan grupların AR(1) yapıda olduğu durumlar için test istatistiği tanıtılmıştır. Bir yönlü varyans analizi test istatistiğine ek olarak, lineer bağıntılar için de bir test istatistiği tanıtılmıştır. Söz konusu test istatistiklerinin performanslarını inceleyen geniş kapsamlı bir simulasyon çalışması yapılmıştır. Son olarak da, bu tez kapsamında belirtilen konuların uygulamasına örnek teşkil etmesi için işsizlik oranı veri seti analiz edilmiştir.

Anahtar Kelimeler: Bir Yönlü Varyans Analizi, Lineer Bağıntılar, AR(1) Model, Stu-

dent's t Dağılımı

To my unique family

ACKNOWLEDGMENTS

Actually there are lots of people in my life to deserve thanks because of their encouregement, support and love throughout this thesis study. But the biggest thanks are for my supervisor Associate Professor Ceylan Yozgatlıgil and co-supervisor Professor Birdal Şenoğlu. It was really great honor for me to have a chance to work with them and benefit their massive knowledge related not only scientific context but also real life. They had very important role in expanding my horizon of the statistics. Actually, I feel very lucky to be a student of them. They always supported and motivated me even in my worst moments. They were always helpful, smiling and very understanding for every step of this thesis. I also would to thank them for their endless support, constant patience and valuable guidance from the beginning to the end of this study.

I also would like to express my great thanks to my eximining commitie members, Assoc. Prof. Dr. Özlem İlk Dağ, Prof. Dr. Mehmet YILMAZ, Assoc. Prof. Dr. Berna Burçak Başbuğ Erkan, and Assoc. Prof. Dr. Seher Nur Sülkü for their time and great contributions with their precious suggestions about this study.

I would also thank my colleagues from METU and Ankara University. My special thanks go to Elif Akça because of her endless support and really motivated speeches. I know that she will always be with me no matter the distances between us.

Lastly, my special gratitude are for my family members, Güler YILDIRIM, Mehmet YILDIRIM, Esra YILDIRIM and Mertcan YILDIRIM. They make me feel loved and special.

TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

LIST OF ABBREVIATIONS

a.k.a	also known as
ANOVA	Analysis of Variance
AR (1)	The First Order Autoregressive Process
df	Degrees of Freedom
FANOVA	Functional One-Way Analysis of Variance
FDA	Functional Data Analysis
LS	Least Squares
LSE	Least Squares Estimation
ML	Maximum Likelihood
MLE	Maximum Likelihood Estimation
MSE	Mean Squared Error
MS_{Error}	Mean Squares of Error
MS_{Total}	Mean Squares of Total
$MS_{Treatment}$	Mean Squares of Treatment
RE	Relative Efficiency
SS_{Error}	Sum of Squares of Error
SS_{Total}	Sum of Squares of Total
$SS_{Treatment}$	Sum of Squares of Treatment
VaR	Value at Risk
WN	White Noise

CHAPTER 1

INTRODUCTION

There has been a number of studies done on the analysis of variance (ANOVA) framework, which is used for comparing the equality of the means of the groups more than two. Even the mean of the group itself does not play a big role, but within the comparision cases it can carry significant information related to the data belongs to. Indeed, comparison along with the means can sometimes act very important role for the data analysis process. For these reasons, tests on the means have wide range of usage area on the medicine, engineering, educational analysis etc. Another reason of this kind of widespread usage area is sourced by the simple implementation of it. However, like every parametric test, analysis of variance test also requires some assumptions to be validated in order to get valid results. The starting point of this study actually those assumptions. Once you aim to use this test you should be sure that the error terms of the ANOVA model should be distributed independently and identically normal. The compared groups are to be also independent from each other and the variances should also be homogeneous. Actually the literature encapsulates lots of study on unfulfilled assumptions of the variance analysis. Sometimes studies concentrated on one non validated assumption, and sometimes more than one non validated assumptions, i.e., for example, it is tried to come up with a solution of the question that what to do when faced with dependency and heterogeneity simultaneously. There are many studies in the literature focusing on the problems stated above. Some of the important of ones are presented below in order to have an idea on the gaps in the literature and have an idea on the ways of handling of possible problems.

When the case is comparing means, literature suggests to start with the simple tests on the means, which are actually the one sample or two sample tests on the means, because the problem investigated in this study is also a part of these tests. For this reason, those studies found a place in our revison of the literature step. One of those studies was done by Zwiers and Storch (1995). In their study, it was tried to propose a new test statistic for serially correlated samples in order to test the mean. They made a revision of the existing tests on this issues, used AR(1) model with gaussian error terms, and finally recommended new test statistics to overcome the correlation problem. Some of the one sample tests are examined in order to specify effect of the dependency structure by Gastwirth and Rubin (1971). The results demonstrate that the corresponding tests are seriously affected even there exists a slight dependence. Likewise, Albers (1978) also studied on the dependency problem while testing the mean.

The earlier studies related to the variance analysis are as follows:

"We most certainly want to dispel notions that correlation in data does not affect inferences drawn from F test of ANOVA" (Scariano and Davenport, 1987). This expression actually stress the importance of the independency assumption on the one way ANOVA. Adke (1986) stated that one-way ANOVA procedure is invalid when the dependency occurs between observations. This study focuses on the gaussian stationary first order sequences. Baldessari (1987) adressed the properties of ANOVA under dependency. This paper presents the sufficient condition on the covariance matrix when the data is normaly distributed. Another study which pays attention to the depencence across the distributions in one way ANOVA was done by Iorio et al. (2004). In this paper, a new probability model which represents this dependency was proposed. Correlated error terms issue was studied by considering the two way analysis of variance framework as well (Anderson et al., 1981).

One of the studies dealing with dependency is done by Pavur (1988). In his article, according to certain correlations within and between groups, it was shown that one-way ANOVA design model can be rewritten with a difference sourced by a design matrix multiplied by a constant. This study also provides ways to determine the even small correlations for multiple comparison procedures.

Considering the observations as a form of time series is another way to handle dependency problem. One of the studies take this in care was done by MacNeill and Umphrey (1987). Their discussion was mainly on one way classification when the error terms are formed AR(1) process.

Keselman et al. (2003) illustrated the effect of using trimmed means on one way analysis, factorial completely randomized and correlated designs to distort the effect of non-normality and non-constant variance. Clinch and Keselman (1982) compared the type I errors and powers of three test statistics, one of which is ANOVA F test statistisc, under some violated assumption cases. Nonnormality, heterogeneous variances and unequal sample sizes were the concerns of this study. The rank transformation approach was studied by Olejnick and James (1985). By examining the numerous simulation studies, which were created by changing ratio of population variances, sample sizes, distributional forms and difference in population means, type I error and the power of the test statistic were evaluated. Another way of handling dependency problem was practiced by Shumway (1970). Unlike the time domain studies, this study worked on frequency domain.

One of the recent studies on the problem of dependency for ANOVA belongs to Lund et al. (2016). They proposed a new test statistic for autocorrelated data to use one way ANOVA. In other words, their focus was to apply one way variance analysis when the data is autocorrelated. Their approach slightly different than the previous studies. Their proposed test statistic is not based on the raw data, actually based on the one step ahead prediction errors. Moreover, they showed that their test statistic is distributed as classical F with customary degrees of freedoms. This study contains comprehensive simulation study including type I errors and power comparisons of the test statistics provided by previous studies. In another similar study, the aim is to test the equality of the seasonal means for time series data (Liu et al., 2016). With the wide range of simulation studies, the new test statistic was provided by their work. This time the null hypothesis assess the equality of the seasonal means. The main objective is actually similar with these two studies. This similarity is sourced by the fact that the test statistic is based on the one step ahead prediction errors.

A number of study takes a part in literature which are based on functional data analysis (FDA). By using FDA, experimental design analysis can be easily done. The reason why a new method is required is that independence within compared group assumption is not validated when dealing with time dependent or correlated data. That's why if the observations are obtained in time, which simply implies that the independence assumption is not met, with the help of FDA method this problem tried to be overcomed. Horváth and Rice (2015) worked on this issue in their study. In their work, the one way analysis of variance was studied by using one way functional ANOVA (FANOVA). In their article, the new method was derived to test whether the mean curves of the multiple functional population are same or not. Their method was supported with simulation studies. Another study done by Górecki and Smaga (2015) in which some new test statistics were proposed for functional data in one way ANOVA and comparison were made between new and existing test in one way ANOVA for functional data.

Literature covers many studies on the effect of non-normality over the test statistic for the analysis of variance as well. Şenoğlu and Tiku (2001) showed the effect of non-normality on the test statistic while working with two way classification model. Their study is based on the method of modified likelihood (MML) and generalized logistic distribution and weibull distribution are used for the distribution of the errors of specified model. Moreover, type I error and power properties of their defined test statistics were presented. Yılmaz (2004) also studied on experimental design framework for her master's thesis. In their work, generalized secant hyperbolic distribution was used for the distribution of the error terms for the one way and two way classification models. Their study was also based on MML methodology and type I error and power of the proposed test statistic were studied. The skew normal distribution situation case was studied by Çelik et al. (2015). Their concentration was one way anova model with the error terms distributed skew normal. The earlier studies were done by Pearson (1931) and Geary (1947). In their work, it was tried to be found out the effect of nonnormality of the error terms of the ANOVA model on the distribution of the test statistics, i.e. their study focused on the effect of nonnormality on the significance levels. The effect of nonnormality was also considered by David and Johnson (1951), Srivastava (1959) and Tiku (1971). Their studies paid particular attention to the type 2 errors. Indeed, it was desired to find out how the power function is affected by the non-normality stiuations.

Up to now, either the studies adress the non normality or the dependency are tried to be presented. However, the number of studies combines these two cases are limited. That is, there exists a few studies done on behalf of one way analysis when nonnormality and correlation structures are experienced at the same time as far as we know. The first one is studied by Pavur and Lewis (1982). Lognormal, uniform, double exponential and cauchy distributions were the concerns of this study. By using these non-normal distributions with the correlation structure, performance of the corrected F statistic was examined, where the corrected F statistic is nothing but the usual F statistic multiplied by a constant. The second study was done by Şenoğlu and Bayrak (2016). Their aim was to make one way classifion by using linear contrasts when the distribution of error terms is gamma and the observations in each group follow AR(1) model. In their paper, dependency was stressed by the observations from time series and the non-normality was assured by gamma error terms. That is, it addresses the dependency and non-normality problems simultaneously. The proposed test statistic of this work was evaluated with type I error and power studies of the proposed test statistic.

As far as our revision of literature, we could not encounter with the case of treatment observation as time series and the distribution of the error term is student's t. For this reason, the aim of this study is to introduce the test statistic when the model itself is AR(1) and the distribution of the error terms is student's t simultaneusly. That is, the focus is to present a test statistic for finding out the equality of the treatment means of the groups, which are modelled by time series AR(1), with error terms from student's t distribution. The reason of choosing first order autoregressive model is that it provides easiness in calculation because of its simple form. Student's t distribution is chosen for this study because of its wide range use in a number of areas especially, when it is important to modelling the extreme cases such as finance, weather, network etc.

The importance of the heavy-tailed distributions especially arises with financial field. Because of the fact that the extreme cases are more likely to occur when using heavy tailed model compared to normal model, heavy-tailed distributions have become act very important role in financial areas in order to capture extreme situations easily.

In their paper, Platen and Sidorowicz (2007) documented that log-return distribution of indices performs well under student's t distribution. Similar conclusion was reached by Champet et. al (2013). They stated that stock log returns provides heavy tails.

Another study was presented by Lin and Shen (2006). The main concern was to evaluate the performance of student's t distribution. The purpose is to determine whether it provides accurate value at risk (VaR) for risk management. The findings supports that student's t distribution can provide accurate VaR estimates. Moreover, with the motivation of the conclusions of the previous studies, it can be stated that financial asset returns are heavy-tailed.

Another area of usage of heavy-tailed distribution is the weather. Heavy-tailed aspects of the daily weather conditions were studied by Sardeshmuckh et al. (2015). A revision related to the heavy tails of climate or weather variable was presented by Katz (2002).

Anderson and Meerschaert (1998) disscussed the implementation of heavy-tailed models to hydrology. Another area of use of heavy-tailed distributions was studied by Hernández-Campos et al. (2004). Practical techniques for use of heavy-tailed distributions were presented in network field.

Taking the range of use of the student's t distribution into account, which is one of the heavy-tailed distribution, the distributional focus of this study is based on student's t distribution.

The outline of this study is listed below:

In chapter 2, historical background of the methodologies used in this study are presented in a detailed way. That is, one-way ANOVA under experimental desing framework, linear contrasts method and autoregressive models are discussed. The assumptions of the related fields are clearly stated. In addition to the assumptions, hypothesis of the ANOVA and linear contrasts are pointed out. Classical normal theory test statistics are appointed seperately for ANOVA and linear contrasts. Moreover, estimation methods used in this study while estimating the model and distribution parameters are also provided.

In chapter 3, the objective model of this study is included. The related model assumptions are yielded exoterically. Furthermore, hypotheses about what we are exploring are also given for ANOVA and linear contrasts one by one. The test statistics for one-way ANOVA and linear contrasts are introduced as well.

In chapter 4, related simulation studies are presented and the results of these studies are evaluated. The performances of the estimation methods are disscused. Type I errors of the introduced test statistics are interpretted. In addition to this, the power of the test statistics introduced by this study for one-way ANOVA and linear contrasts are compared with the corresponding normal theory test statistics. The results are evaluated in detail.

In chapter 5, the illustrative real life example is supplied in order to make a practice with the introduced study. This chapter actually serves as a guide of the usage of the introduced procedures stated throughout the entire study. The description of the concerned data is given. Moreover, the results are handled by using the introduced procedures and they are comprehensively interpreted.

In chapter 6, the aim and the importance of this study is stressed. This chapter operates as a summary of the whole study.

CHAPTER 2

BASIC CONCEPTS

In this chapter, experimental design and the time series frameworks are studied in detail. Actually, the related concepts of the experimental design and time series used for the futher parts of this study are presented. The one way analysis of variance and linear contrasts concepts under the experimental design framework is investigated. Likewise, the first order autoregressive processes are examined under the time series concept.

2.1 Experimental Design

One-way analysis of variance (one-way ANOVA) model is presented with the required assumptions of the test statistic used under the experimental design methodology and the related hypothesis test is also supplied. The linear contrasts concept is introduced with the hypothesis, assumptions and test statistic as well.

2.1.1 One-Way Analysis of Variance

Analysis of variance concept is used for finding out the potential difference on the means of treatments more than two. This is done by separating the total variance into the components. One-way ANOVA is the specific and the simplest form of the ANOVA in which only one factor effect is investaged. Under this concept, treatment units are assumed to be approximately homogeneous within a treatment. The homogeniety actually means that treatment units are to be as similar as possible. That's why one-way ANOVA often called as completely randomized design (Şenoğlu and Acıtaş, 2004).

The model considered under one-way ANOVA is as follows:

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij}, \qquad i = 1, 2, 3, ..., k; \quad j = 1, 2, ..., n_i \tag{2.1}$$

where y_{ij} is the jth unit belongs to ith treatment and each treatment with n_i unit , μ is the grand mean, τ_i is the ith treatment effect and ε_{ij} is the random error of jth observation of the ith treatment. Here μ is a fixed parameter and it can find a place in the formula of μ_i, where $\mu_i = \mu + \tau_i$. In this formulation, μ_i represents the true mean of the ith treatment.

The equation (2.1) is a fixed effect model because $\sum_{i=1}^{k} \tau_i = 0$ which implies that $\mu = (1/k) \sum_{i=1}^{k} \mu_i$.

Therefore, the model (2.1) can be written as:

$$y_{ij} = \mu_i + \varepsilon_{ij}, \qquad i = 1, 2, 3, ..., k; \quad j = 1, 2, ..., n_i. \tag{2.2}$$

If the model (2.1) is the concern, the related hypothesis can be stated as follows:

$$\begin{aligned} H_0 &: \tau_1 = \tau_2 = ... = \tau_k = 0 \qquad (2.3) \\ H_1 &: \tau_i \neq 0 \quad \text{for at least one i.} \end{aligned}$$

The hypothesis actually aims to test the equality of treatment effect to 0. Treatment means can also be used for the same target. That is, the hypothesis can be written based on treatment means as in stated below:

$$\begin{aligned} H_0 &: \mu_1 = \mu_2 = ... = \mu_k \qquad (2.4) \\ H_1 &: \mu_i \neq \mu_j \quad \text{for at least one pair of i and j.} \end{aligned}$$

The model assumptions are crucial to get valid conclusion for one-way ANOVA. That's why, these assumptions are listed below:

1. ε_{ij}'s are to be distributed normal with 0 mean and σ^2 variance.

2. Variances of the error terms are to be homogenues.

3. Error terms are to be independent from each other (ε_{ij}~NID(0,σ^2), independently and identically).

For the sake of example, the data structure is given as follows.

	Treatments			
Observations	1	2	$\cdots$	*k*
1	y_{11}	y_{21}	$\cdots$	y_{k1}
2	y_{12}	y_{22}	$\cdots$	y_{k2}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	y_{1n}	y_{2n}	$\cdots$	y_{kn}

The least square estimators of one-way ANOVA model (2.1) can be found in a way that stated below.

Let S represents the sum of squared error (Assume that the size of each treatment equals to *n*). That is; S be

$$S = \sum_{i=1}^{k} \sum_{j=1}^{n} \varepsilon_{ij}^2 = \sum_{i=1}^{k} \sum_{j=1}^{n} (y_{ij} - \mu - \tau_i)^2. \tag{2.5}$$

Then, in order to obtain the unknown parameters, the partial derivatives with respect unknown parameters can be found in a way that stated below, by minimizing the S,

$$\begin{aligned}
\frac{\partial S}{\partial \mu} &= (-2) \sum_{i=1}^{k} \sum_{j=1}^{n} (y_{ij} - \mu - \tau_i) = 0 \\
\frac{\partial S}{\partial \tau_i} &= (-2) \sum_{j=1}^{n} (y_{ij} - \mu - \tau_i) = 0.
\end{aligned} \tag{2.6}$$

From the equation (2.6), least square (LS) estimators of μ and τ_i can be found as respectively,

$$\tilde{\mu} = \bar{y}_{..} \tag{2.7}$$

$$\tilde{\tau}_i = \bar{y}_{i.} - \bar{y}_{..}, \quad i = 1, 2, ..., k. \tag{2.8}$$

Similar to that LS estimator of μ_i in equation (2.2) can be easily obtained as:

$$\tilde{\mu}_i = \bar{y}_{i.}, \qquad i = 1, 2, 3, ..., k. \tag{2.9}$$

Therefore, the LS estimator of σ^2 can be gathered after corrected for bias in a way that supplied below:

$$\tilde{\sigma}^2 = \frac{\sum_{i=1}^{k} \sum_{j=1}^{n} (y_{ij} - \bar{y}_{i.})^2}{N - k} \tag{2.10}$$

where $\bar{y}_{i.} = \frac{\sum_{j=1}^{n} y_{ij}}{n}$, $\quad \bar{y}_{..} = \frac{\sum_{i=1}^{k} \sum_{j=1}^{n} y_{ij}}{N}$ and *N*=*nk*.

Parameters of ANOVA model can also be estimated by using maximum likelihood (ML) method. The related estimators are actually same with the LS estimators which are stated in equations (2.7) and (2.8). Likewise, the ML estimator of σ^2 is the same with LS estimator after corrected for bias as well (See equation (2.10)).

The related test statistic for testing the null hypothsesis given in (2.3) and/or (2.4) can be reached by partitioning the total sum of square into sum of square treatment and sum of square error. That is,

$$\sum_{i=1}^{k} \sum_{j=1}^{n} (y_{ij} - \bar{y}_{..})^2 = \sum_{i=1}^{k} \sum_{j=1}^{n} (\bar{y}_{i.} - \bar{y}_{..})^2 + \sum_{i=1}^{k} \sum_{j=1}^{n} (y_{ij} - \bar{y}_{i.})^2. \tag{2.11}$$

This equation (2.11) represents the following,

$$SS_{Total} = SS_{Treatment} + SS_{Error}. \tag{2.12}$$

If the $SS_{Treatment}$ and SS_{Error} is divided by the relative degrees of freedom, the following equation (2.13) is handled. These are mean square treatment and mean square error. Therefore,

$$\begin{aligned} F_{Treatment} &= \frac{SS_{Treatment}/(k-1)}{SS_{Error}/(N-k)} \\ &= \frac{MS_{Treatment}}{MS_{Error}}. \end{aligned} \tag{2.13}$$

Therefore the test statistic can be rewritten and it follows F distribution with (*k-1*) and (*N-k*) degrees of freedoms:

$$F_{Treatment} = \frac{n\sum_{i=1}^{k}(\bar{y}_{i.} - \bar{y}_{..})^2/(k-1)}{\sum_{i=1}^{k}\sum_{j=1}^{n}(y_{ij} - \bar{y}_{i.})^2/(N-k)} \sim F_{k-1,N-k}. \tag{2.14}$$

The equation (2.14) can be proved as follows.
Consider the following model,

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij}, \qquad i = 1, 2, 3, ..., k; \quad j = 1, 2, ..., n.$$

The distribution of the y_{ij} can be written as $y_{ij} \sim N(\mu_i, \sigma^2)$ because of the distribution of the error terms $\varepsilon_{ij} \sim N(0, \sigma^2)$. Moreover, under the null hypothesis in (2.4), $y_{ij} \sim N(\mu, \sigma^2)$. Therefore it can be stated that:

$$\begin{aligned} \frac{y_{ij} - \mu}{\sigma} &\sim N(0,1), \\ \Rightarrow \left(\frac{y_{ij} - \mu}{\sigma}\right)^2 &\sim \chi^2_{(1)}, \\ \Rightarrow \sum_{i=1}^{k}\sum_{j=1}^{n}\left(\frac{y_{ij} - \mu}{\sigma}\right)^2 &\sim \chi^2_{(N)}. \end{aligned}$$

Write the LS estimator of μ instead of μ, therefore the degrees of freedom of the χ^2 distribution decreases by the amount of one.

$$\begin{aligned} \sum_{i=1}^{k}\sum_{j=1}^{n}\left(\frac{y_{ij} - \bar{y}_{..}}{\sigma}\right)^2 &\sim \chi^2_{(N-1)}, \\ \frac{SS_{Total}}{\sigma^2} &\sim \chi^2_{(N-1)}, \end{aligned} \tag{2.15}$$

where $SS_{Total} = \sum_{i=1}^{k}\sum_{j=1}^{n}(y_{ij} - \bar{y}_{..})^2$.

Likewise, the $SS_{Treatment}$ and the SS_{Error} can also be written as respectively,

$$\begin{aligned} \sum_{i=1}^{k}\sum_{j=1}^{n}\left(\frac{\bar{y}_{i} - \bar{y}_{..}}{\sigma}\right)^2 &\sim \chi^2_{(k-1)}, \\ \frac{SS_{Treatment}}{\sigma^2} &\sim \chi^2_{(k-1)}, \end{aligned} \tag{2.16}$$

where $SS_{Treatment} = \sum_{i=1}^{k} \sum_{j=1}^{n} (\bar{y}_i - \bar{y}_{..})^2$,

$$\begin{aligned} \sum_{i=1}^{k} \sum_{j=1}^{n} \left(\frac{y_{ij} - \bar{y}_{i.}}{\sigma} \right)^2 &\sim \chi^2_{(N-k)}, \\ \frac{SS_{Error}}{\sigma^2} &\sim \chi^2_{(N-k)}, \end{aligned} \tag{2.17}$$

where $SS_{Error} = \sum_{i=1}^{k} \sum_{j=1}^{n} (y_{ij} - \bar{y}_{i.})^2$.

Moreover, recall the equation (2.12),

$$SS_{Total} = SS_{Treatment} + SS_{Error}.$$

In addition to this, the summation of degrees of freedom of $SS_{Treatment}$ and SS_{Error} equals to the degrees of freedom of SS_{Total}, and represented by,

$$N - 1 = (k - 1) + (N - k).$$

Furthermore, according to the Cohchran Theorem, the statements in the equations (2.16) and (2.17) are independent from each other and the ratio of two independent χ^2 distrubuted random variables divided by their own degrees of freedom creates F distribution. Therefore,

$$\begin{aligned} F_{Treatment} &= \frac{(\frac{SS_{Treatment}}{\sigma^2})/(k-1)}{(\frac{SS_{Error}}{\sigma^2})/(N-k)}, \\ &= \frac{SS_{Treatment}/(k-1)}{SS_{Error}/(N-k)}, \\ &= \frac{MS_{Treatment}}{MS_{Error}} \sim F_{(k-1),(N-k)}. \end{aligned}$$

With the help of the related information presented above, one-way ANOVA table can be obtained in a way that stated below:

Table 2.1: One-Way ANOVA Table

Source of Variation	df	SS	MS	F
Treament	*k-1*	$SS_{Treatment}$	$MS_{Treatment}$	$F_{Treatment}$
Error	*N-k*	SS_{Error}	MS_{Error}	
Total	*N-1*	SS_{Total}		

2.1.2 Linear Contrasts

Linear contrasts method is one of the frequently used comparison method in the literature especially when the specific relations on the treatment means is tried to be investigated. That is, when the null hypothesis (2.4) which claims the equality of the treatment means is rejected, some pairwise or multiple comparisons are done in order to specify the source of inequality of the means. One of the well known method among these is the linear contrasts method. Actually, the comparison of pairwise treatment means are done by using the linear combinations of the treatment means. These contrasts are to be constructed before designing experiment. Moreover, if there exists *k* treatment means, then only *k*-1 orthogonal linear contrasts can be builted. The limitations on the linear contrasts framework are supplied below:

Let's consider the two contrasts are defined as $C_1 = \sum_{i=1}^{k} c_{1i}\mu_i$ and $C_2 = \sum_{i=1}^{k} c_{2i}\mu_i$. Therefore, the conditions

$$\sum_{i=1}^{k} c_{1i} = 0 \quad \text{and} \quad \sum_{i=1}^{k} c_{2i} = 0 \tag{2.18}$$

are to be satisfied. Moreover, in order to hold the orthogonality of the linear contrasts,

$$\sum_{i=1}^{k} c_{1i}c_{2i} = 0. \tag{2.19}$$

Let's consider the following null hypothesis as an example,

$$\begin{aligned} C_1 &: \quad \mu_3 - \mu_1 = 0 \\ C_2 &: \quad \mu_1 - 2\mu_2 + \mu_3 = 0 \end{aligned}$$

$\sum_{i=1}^{k} c_{1i} = (-1+0+1) = 0$, $\sum_{i=1}^{k} c_{2i} = (1-2+1) = 0$ and $\sum_{i=1}^{k} c_{1i}c_{2i} = (-1+0+1) = 0.$

Two conditions stated in equations (2.18) and (2.19) are satisfied for linear contrasts. Therefore, it can be stated that C_1 and C_2 are named as orthogonal linear contrasts.

Here the main goal is to test the null hypothesis:

$$H_0 : \sum_{i=1}^{k} c_i\mu_i = m. \tag{2.20}$$

Constant "*m*" is often taken as 0, and this hypothesis is tested with the test statistic stated below:

$$t_c = \frac{\sum_{i=1}^{k} c_i\hat{\mu}_i - m}{\sqrt{V(\sum_{i=1}^{k} c_i\hat{\mu}_i)}} \sim N(0,1). \tag{2.21}$$

When V(μ_i) is not known, which is the function of σ^2, mean squared error (MSE) is used instead of σ^2. Then,

$$t_c = \frac{\sum_{i=1}^{k} c_i\hat{\mu}_i - m}{\sqrt{\hat{V}(\sum_{i=1}^{k} c_i\hat{\mu}_i)}} \sim t_{N-k}. \tag{2.22}$$

The null hypothesis given in (2.20) can be tested by using F statistic rather than the t statistic as,

$$F_c = \frac{SS_c/1}{MSE} = \frac{MS_c}{MSE}, \quad \text{where} \quad SS_c = \frac{n(\sum_{i=1}^{k} c_i\bar{y}_{i.})^2}{\sum_{i=1}^{k} c_i^2}. \tag{2.23}$$

Therefore, the test statistic (2.23) for the null hypothesis (2.20) is compared with the tabulated F with degrees of freedom *k-1*, df_{error}, which is $F_{k-1,df_{error}}$.

2.2 Time Series Analysis

Time series anaysis pays particular attention to the data whose units are collected in time order. In other words, the measurements for a variable is obtained in a sequence of time, for example time sequence can be day, month, year or etc. In this section, related time series process studied in this study is presented. Moreover, the model and the related assumptions are introduced and some estimation methods are supplied.

2.2.1 The First Order Autoregressive Process

This study focuses on the first order autoregressive process, that is also known as AR(1) model. The related model is presented below:

$$y_t = a + \phi y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim WN(0, \sigma^2); \quad t = 1, 2, ..., n. \tag{2.24}$$

The model (2.24) concentrated on the relation between time point *t* and previous time point *t*-1. This relation is represented by the coefficient ϕ. In the model (2.24), y_t symbolizes the data point for time *t*, likewise y_{t-1} symbolizes the data point for *t*-1. Moreover, ε_t represents the error terms of the conducted model. For this model, if ϕ equals 1 and *a* equals 0 model (2.24) becomes a random walk model. Furthermore the model (2.24) can be stated in a way that supplied below (under the stationarity):

$$y_t = (1 - \phi)\mu + \phi y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim WN(0, \sigma^2); \quad t = 1, 2, ..., n. \tag{2.25}$$

The difference between model (2.24) and (2.25) is sourced by parameters *a* and μ because *a* does not represent the process mean however μ does. That is, *a* and μ can be expressed as follows:

$$a = (1 - \phi)\mu \quad \text{and} \quad \mu = \frac{a}{1 - \phi}. \tag{2.26}$$

Because it is one of the important assumptions for the error terms the properties of the white noise process are to be known. That's why, the conditions on a process, say ε_t, to be white noise are listed below:

$$\begin{aligned} i. &\quad E(\varepsilon_t) = 0 \\ ii. &\quad V(\varepsilon_t) = \sigma^2 \\ iii. &\quad Cov(\varepsilon_t, \varepsilon_{t+h}) = 0 \quad \text{for all} \quad h \neq 0. \end{aligned}$$

Properties of Stationarity

Stationarity is a crucial concept for time series analysis because of the fact that almost all statistical inferences are done under the stationarity condition. Actually, it often appears as an assumption (Wei, 2006). The logic behind the stationarity is that the characteristic of the process does not diversify in time. If the related process does not meet the stationarity assumption, then suitable technique should be applied to validate stationarity before the statistical inference step. There exists two types of stationarity definition, which are weak and strong stationarity. Applicational problem of the strong stationarity makes it useless. That's why, the term stationarity implies the weak stationarity.

The conditions of weak stationarity are listed as below:

$$\begin{aligned} i. &\quad E(y_t) = \mu \quad \text{for all t,} \\ ii. &\quad V(y_t) = \sigma^2 < \infty \quad \text{for all t,} \\ iii. &\quad Cov(y_t, y_{t-h}) = \gamma_h \quad \text{for all t,} \\ iv. &\quad Corr(y_t, y_{t-h}) = \rho_h \quad \text{for all t.} \end{aligned} \tag{2.27}$$

Parameter Estimation in Time Series Model

Parameter estimation procedure for time series models is done by conditional maximum likelihood or unconditional maximum likelihood procedures which are actually the approximations. Moreover, least square estimation methodology is also used as well as Yule's Walker estimation procedure (Wei, 2006). These estimation techniques are not provided under the context of this study. The related estimation techniques used for this study is supplied under the Section 3.1.

Process Mean, Autocovariance Function (ACF) and Partial Autocovariance Function (PACF)

Consider the model (2.25).

The process mean of the y_t series can be found as follows:

$$\begin{aligned} E(y_t) &= E((1-\phi)\mu) + E(\phi y_{t-1}) + E(\varepsilon_t), \\ &= (1-\phi)\mu + \phi E(y_{t-1}), \\ &= E(y_{t-1}) \quad \text{because of stationarity,} \\ &= (1-\phi)\mu + \phi E(y_t), \\ &= \mu. \end{aligned} \tag{2.28}$$

The autocovariance function (γ_h) can be obtained as,

$$
\begin{aligned}
\gamma_{(h)} &= Cov(y_t, y_{t-h}), \\
&= E\left((y_t - \mu)(y_{t-h} - \mu)\right), \\
&= E\left((\phi(y_{t-1} - \mu) + \varepsilon_t)(y_{t-h} - \mu)\right), \\
&= E(\phi(y_{t-1} - \mu)(y_{t-h} - \mu) + E((y_{t-h} - \mu)\varepsilon_t), \\
&= \phi\gamma_{(h-1)}; \quad \text{for } h \geq 1. \qquad (2.29)
\end{aligned}
$$

Therefore, the variance of y_t can be written in terms of γ_h when *h=0*. That is,

$$
\begin{aligned}
V(y_t - \mu) &= V(\phi(y_{t-1} - \mu) + \varepsilon_t), \\
V(y_t - \mu) &= \phi^2 V(y_{t-1} - \mu) + V(\varepsilon_t), \\
\gamma_{(0)} &= \phi^2\gamma_{(0)} + \sigma^2, \\
&= \frac{\sigma^2}{1 - \phi^2}. \qquad (2.30)
\end{aligned}
$$

Here the $\gamma_{(0)}$ represents the variance of the stationary y_t series.

The autocorrelation function can be obtained as follows:

$$\rho_h = \phi\rho_{h-1}; \quad \text{for } h \geq 1.$$

CHAPTER 3

PARAMETER ESTIMATION AND HYPOTHESIS TESTING IN ONE-WAY ANOVA FOR TIME SERIES DATA WITH NON-NORMAL INNOVATIONS

In this chapter, the objective model is given with the related assumptions. Moreover, parameter estimation part is included. Therefore, the test statistics for one-way ANOVA and linear contrasts are introduced.

Consider the model,

$$y_{i,t} = \phi y_{i,t-1} + (1-\phi)\mu_i + \varepsilon_{i,t}, \quad i = 1, 2, ..., c; \quad \text{and} \quad t = 1, 2, ..., n, \tag{3.1}$$

where *c* is the number of treatment and each treatment has *n* observations. y_{it} is the t^{th} observation for i^{th} treatment. Moreover, $y_{i,t-1}$ is the $t\text{-}1^{\text{th}}$ unit of the treatment *i*. ε_{it} represents the error term of the t^{th} unit of the treatment *i* as well. As it can be seen from the model (3.1), each treatment follows the first order autoregressive process and the autoregressive coefficient is common for all treatment and symbolized by ϕ. However, the treatment means are different from each other, and indicated with μ_i, which insinuates that each treatment mean is distinguished from each others'. By using the model (3.1), our main target is to identify the differences in the mean of treatment with one-way variance analysis.

Assumptions of our model are listed below:

$i.$ $\varepsilon_{i,t}$ are independently and identically distributed as Student's t$(\nu, 0, \sigma^2)$,

$ii.$ $y_{i,t}{}'s$ follow AR(1) process,

iii Variances of the error terms are homogeneous.

There exits differences between our assumptions and the classical one-way ANOVA assumptions. The first one is the distribution of the error terms of the model. For the classical assumption, error terms are to be distributed as normal, however, in our assumption error terms are distributed as Student's t. The second difference within the assumptions is associated with the independency assumption. In classical assumption, error terms are to be distributed independently and identically, which alludes the independency of the treatment units $(y_{i,j})$. However, although in our case error terms are distributed independently and identically, the treatment units $(y_{i,t})$ are not independent, on the contrary, they follow AR(1) process.

3.1 Parameter Estimation of the Model Parameters

In this part, the model and the distribution parameters are estimated by using two well known parameter estimation methodologies. The first one is maximum likelihood estimation (MLE) method and the least squares estimation (LSE) is the second estimation method used for this study. For both, the shape parameter of the student's t distribution is assumed to be known.

3.1.1 Maximum Likelihood Estimation

In our model (3.1), error terms are independently and identically distributed as Student's t. Consider the model (3.1),

$$y_{i,t} = \phi y_{i,t-1} + (1-\phi)\mu_i + \varepsilon_{i,t}, \quad i = 1, 2, ..., c; \quad t = 1, 2, ..., n.$$

Hence, the error terms can be rewritten as stated below:

$$\varepsilon_{i,t} = y_{i,t} - \phi y_{i,t-1} - (1-\phi)\mu_i. \tag{3.2}$$

By the guideness of the information related with the distribution of the error terms, the probability density function can be obtained in a way that is stated below:

$$\varepsilon_{i,t} \sim \text{t}\ (\nu,0,\sigma^2)$$

with probability density function stated as,

$$f(\varepsilon_{i,t}) = \frac{1}{\sigma}\frac{\Gamma\frac{\nu+1}{2}}{\Gamma\frac{\nu}{2}\sqrt{\pi\nu}}\left(1+\frac{(\varepsilon_{i,t}/\sigma)^2}{\nu}\right)^{-\frac{\nu+1}{2}}, \nu > 0; \quad \sigma > 0; \quad -\infty < \varepsilon < \infty.$$

Therefore, the log-likelihood function symbolized by lnL, which is $lnL(\nu, \mu, \phi, \sigma; y_{i,t}, y_{i,t-1})$, can be obtained in a way that is stated below:

$$\begin{aligned} lnL = \quad & = ncln(\Gamma(\nu+1)/2) - ncln(\sigma) - ncln(\nu\pi)/2 - \\ & ncln(\Gamma\nu/2) - \frac{\nu+1}{2}\sum_{i=1}^{c}\sum_{t=1}^{n} ln\left(1+\frac{\varepsilon_{i,t}/\sigma)^2}{\nu}\right) \end{aligned} \tag{3.3}$$

by substituting (3.2) into the equation (3.3),

$$\begin{aligned} lnL = \quad & lnL(\nu, \mu, \phi, \sigma; y_{i,t}, y_{i,t-1}) = ncln(\Gamma(\nu+1)/2) - ncln(\sigma) - ncln(\nu\pi)/2 \\ & -ncln(\Gamma\nu/2) - \frac{\nu+1}{2}\sum_{i=1}^{c}\sum_{t=1}^{n}\delta_{i,t}(\nu, \mu, \phi, \sigma) \end{aligned} \tag{3.4}$$

where $\delta_{i,t}(\nu, \mu, \phi, \sigma) = ln\left(1+\frac{((y_{i,t}-\phi y_{i,t-1}-(1-\phi)\mu_i)/\sigma)^2}{\nu}\right)$.

Therefore, in order to obtain the estimators which maximize the lnL, the partial derivatives with respect to each unknown parameters are equated to 0. Then,

$$\frac{\partial lnL}{\partial \mu_i} = -\frac{\nu+1}{2}\sum_{t=1}^{n}\frac{-2\nu^{-1}\sigma^{-2}(y_t - \phi y_{t-1} - (1-\phi)\mu_i)(1-\phi)}{1+(y_t-\phi y_{t-1}-(1-\phi)\mu_i)^2\nu^{-1}\sigma^{-2}} = 0,$$

$$\frac{\partial lnL}{\partial \phi} = -\frac{\nu+1}{2}\sum_{i=1}^{c}\sum_{t=1}^{n}\frac{2\nu^{-1}\sigma^{-2}(y_{i,t} - \phi y_{i,t-1} - (1-\phi)\mu_i)(\mu_i - y_{i,t-1})}{1+(y_{i,t}-\phi y_{i,t-1}-(1-\phi)\mu_i)^2\nu^{-1}\sigma^{-2}} = 0,$$

$$\frac{\partial lnL}{\partial \sigma^2} = -\frac{nc\sigma^{-2}}{2} - \frac{\nu+1}{2}\sum_{i=1}^{c}\sum_{t=1}^{n}\frac{-(y_{i,t} - \phi y_{i,t-1} - (1-\phi)\mu_i)^2\nu^{-1}\sigma^{-4}}{1+(y_{i,t}-\phi y_{i,t-1}-(1-\phi)\mu_i)\nu^{-1}\sigma^{-2}} = 0.$$

Since there are no closed form solutions for the parameters stated above, the maximum likelihood estimators of the unknown parameters are tried to be obtained by using numerical methods. To do so, with the help of iterative methods the loglikelihood function (3.3) is maximized with the optimal values for the unknown parameters.

3.1.2 Least Squares Estimation

In this part, the same model and distribution parameters are tried to be estimated this time by using the least squares (LS) methodology, which does not require distributional assumption. Consider the model (3.1),

$$y_{i,t} = \phi y_{i,t-1} + (1-\phi)\mu_i + \varepsilon_{i,t} \quad \Rightarrow \quad \varepsilon_{i,t} = y_{i,t} - \phi y_{i,t-1} - (1-\phi)\mu_i.$$

Then, let define S in a form that is stated below:

$$S = \sum_{i=1}^{c}\sum_{t=1}^{n}\left(y_{i,t} - \phi y_{i,t-1} - (1-\phi)\mu_i\right)^2. \tag{3.5}$$

Therefore, partial derivatives with respect to the unknown parameters are equated to 0 so that the LS estimators can be obtained by,

$$\begin{aligned}\frac{\partial S}{\partial \mu_i} &= -2\sum_{t=1}^{n}\left(y_{i,t} - \phi y_{i,t-1} - (1-\phi)\mu_i\right)(1-\phi) = 0,\\ \frac{\partial S}{\partial \phi} &= 2\sum_{i=1}^{c}\sum_{t=1}^{n}\left(y_{i,t} - \phi y_{i,t-1} - (1-\phi)\mu_i\right)(\mu_i - y_{i,t-1}) = 0.\end{aligned}$$

Hence, by solving the equations stated above, the LS estimators are obtained as,

$$\tilde{\mu}_i = \sum_{t=1}^{n}\frac{y_{i,t} - \tilde{\phi} y_{i,t-1}}{(1-\tilde{\phi})n} \quad \text{and} \quad \tilde{\phi} = -\frac{\sum_{i=1}^{c}\sum_{t=1}^{n}(y_{i,t} - \tilde{\mu}_i)(\tilde{\mu}_i - y_{i,t-1})}{\sum_{i=1}^{c}\sum_{t=1}^{n}(y_{i,t-1} - \tilde{\mu}_i)^2}. \tag{3.6}$$

The LS estimator of σ^2 is to be adjusted for bias, because $V(\varepsilon_{i,t}) = \sigma^2\frac{\nu}{\nu-2}$ after corrected for bias, we derive it as,

$$\tilde{\sigma}^2 = \frac{\sum_{i=1}^{c}\sum_{t=1}^{n}\left(y_{i,t} - \tilde{\phi} y_{i,t-1} - (1-\tilde{\phi})\tilde{\mu}_i\right)^2}{N-c-1}\frac{\nu-2}{\nu}. \tag{3.7}$$

As it can be clearly seen from the equations (3.6) and (3.7), obtaning the LS estimators are to be done with solving equations simultaneously. To do so, numerical methods are used, that's why the estimators are the approximations.

3.2 Hypothesis Testing in One-Way ANOVA

In this subsection, considering the model (3.1), the equality of the means of the treatments are tested with the hypothesis test written below:

$$\begin{aligned} H_0 &: \mu_1 = \mu_2 = ... = \mu_c \\ H_1 &: \mu_i \neq \mu_j \quad \text{for at least one pair of i and j.} \end{aligned} \tag{3.8}$$

The corresponding test statistic is reported as,

$$F = \frac{\sum_{i=1}^{c} \left(\frac{\hat{\mu_i} - \hat{\mu}}{\sqrt{\hat{V}(\hat{\mu}_i)}} \right)^2 / (c-1)}{\sum_{i=1}^{c} \sum_{t=1}^{n} \left(\frac{y_{i,t} - \hat{\mu_i}}{\sqrt{\hat{V}(y_{i,t})}} \right)^2 / (nc-c)} \sim F_{c-1, nc-c}. \tag{3.9}$$

where $\hat{\mu_i}$ is the estimator of the i^{th} treatment mean, $\hat{\mu}$ is the estimator of the grand mean, which is actually the average of the treatment means and $y_{i,t}$ corresponds to the t^{th} unit of the i^{th} treatment. The variance term on the numerator of the test statistic is the estimated variance of the estimated i^{th} treatment mean and it can be estimated by several ways. In this study, variance of the treatment means are tried to be estimated by using 3 different ways. For each way, corresponing test statistics are supplied in following sections. However, the variance term on the denumerator part of the test statistic is found in a way that is stated below:

Consider the model (3.1), and the variance of the $y_{i,t}$ can be found as:

$$\begin{aligned} V(y_{i,t}) &= V(\phi y_{i,t-1} + (1-\phi)\mu_i + \varepsilon_{i,t}), \\ &= V(y_{i,t-1}) \quad \text{because of the stationarity of the } y_i's, \end{aligned}$$

Here, $(1-\phi)\mu_i$ is constant and the response at time t-1 ($y_{i,t-1}$) and the error term in time t ($\varepsilon_{i,t}$) are independent. Therefore, the variance of $y_{i,t}$ can be found as:

$$\begin{aligned} V(y_{i,t}) &= V(\phi y_{i,t-1}) + V((1-\phi)\mu_i) + V(\varepsilon_{i,t}), \\ &= \phi^2 V(y_{i,t}) + V(\varepsilon_{i,t}), \\ &= V(\varepsilon_{i,t})/(1-\phi^2) \quad \text{where} \quad V(\varepsilon_{i,t}) = \sigma^2 \frac{\nu}{\nu-2} \\ &= \frac{\sigma^2}{1-\phi^2} \frac{\nu}{\nu-2}. \end{aligned}$$

Since the parameters are unknown, in order to estimate the variance of the $y_{i,t}$, the estimators are to be placed in the formula as:

$$\hat{V}(y_{i,t}) = \frac{\hat{\sigma}^2}{1-\hat{\phi}^2} \frac{\nu}{\nu-2}. \tag{3.10}$$

It should be noted that, the parameter ν is assumed to be known.

3.2.1 The Variance of Treatment Means Obtained by Simulation

The variance of the treatment means symbolized by V($\hat{\mu}_i$) can be found by simulation. That is, a number of simulations are done and that specified size of treatment means are obtained. Moreover, by using the tretment means handled by simulation the estimated variance can be gathered by using simple sample variance formula.

Consider that the number of simulation runs is *J*, that is *J* treatment means are at hand. Then, one can easily estimate the related variance by using the formula provided below:

$$\hat{V}(\hat{\mu}_i) = \frac{\sum_{j=1}^{J} \left(\hat{\mu}_{i,j} - \bar{\hat{\mu}}_i\right)^2}{J-1}. \tag{3.11}$$

3.2.2 The Variance of Treatment Means Obtained by Fisher Information Matrix

Another way to estimate the variance of the treatment means,V(μ_i), is to use related diagonal of the inverse of the Fisher Information Matrix (a.k.a. information matrix).

Recall that the loglikelihood function, lnL, given in 3.3,

$$\begin{aligned} lnL = \quad & ncln(\Gamma(\nu+1)/2) - ncln(\sigma) - ncln(\nu\pi)/2 - ncln(\Gamma\nu/2) - \\ & \frac{\nu+1}{2} \sum_{i=1}^{c} \sum_{t=1}^{n} ln\left(1 + \frac{(\varepsilon_{i,t}/\sigma)^2}{\nu}\right). \end{aligned}$$

General form of Fisher information matrix:

$$I = \begin{bmatrix} -E\left(\frac{\partial^2 lnL}{\partial \mu^2}\right) & -E\left(\frac{\partial^2 lnL}{\partial \mu \partial \phi}\right) & -E\left(\frac{\partial^2 lnL}{\partial \mu \partial \sigma}\right) \\ -E\left(\frac{\partial^2 lnL}{\partial \phi \partial \mu}\right) & -E\left(\frac{\partial^2 lnL}{\partial \phi^2}\right) & -E\left(\frac{\partial^2 lnL}{\partial \phi \partial \sigma}\right) \\ -E\left(\frac{\partial^2 lnL}{\partial \sigma \partial \mu}\right) & -E\left(\frac{\partial^2 lnL}{\partial \sigma \partial \phi}\right) & -E\left(\frac{\partial^2 lnL}{\partial \sigma^2}\right) \end{bmatrix}.$$

The first diagonal element of the Fisher Information Matrix is

$$I_{11} = -E\left(\frac{\partial^2 lnL}{\partial \mu^2}\right) = -E\left(\sum_{t=1}^{n} \frac{(\nu+1)(1-\phi)^2}{\nu\sigma^2}\left(\frac{1}{1+\frac{(\varepsilon_{i,t}/\sigma)^2}{\nu}} - \frac{2}{\nu\sigma^2}\frac{\varepsilon_t^2}{\left(1+\frac{(\varepsilon_{i,t}/\sigma)^2}{\nu}\right)^2}\right)\right).$$

This term can be written as follows:

$$I_{11} = \frac{\nu+1}{\nu}\frac{n}{\sigma^2}(1-\phi)^2\left(T^{*} - \frac{2}{\nu\sigma^2}T^{**}\right), \tag{3.12}$$

since there is no closed form of those expectation terms, with the help of the numerical approaches those can be easily obtained in calculations.

The second diagonal element of the Fisher Information Matrix is

$$\begin{aligned} I_{22} &= -E\left(\frac{\partial^2 lnL}{\partial \phi^2}\right) = \frac{\nu+1}{2}\sum_{i=1}^{c}\sum_{t=1}^{n}\frac{2}{\nu\sigma^2}E(\mu_i - y_{i,t-1})^2\left(T^{*} - \frac{2}{\nu\sigma^2}T^{**}\right), \\ I_{22} &= -E\left(\frac{\partial^2 lnL}{\partial \phi^2}\right) = \frac{\nu+1}{\nu-2}\frac{nc}{1-\phi^2}\left(T^{*} - \frac{2}{\nu\sigma^2}T^{**}\right). \end{aligned}$$

The third diagonal element of the Fisher Information Matrix is

$$I_{33} = \; = \; -E\left(\frac{\partial^2 lnL}{\partial \sigma^2}\right) = \frac{nc}{\sigma^2}\left(\frac{\nu+1}{\nu\sigma^2} - 1\right)\left(3T^{***} - \frac{2}{\nu\sigma^2}T^{****}\right)$$

where

$$
\begin{aligned}
T^{*} &= E\left(\frac{1}{1+\frac{(\varepsilon_{i,t}/\sigma)^2}{\nu}}\right), \\
T^{**} &= E\left(\frac{\varepsilon_{i,t}^2}{\left(1+\frac{(\varepsilon_{i,t}/\sigma)^2}{\nu}\right)^2}\right), \\
T^{***} &= E\left(\frac{\varepsilon_{i,t}^2}{1+\frac{(\varepsilon_{i,t}/\sigma)^2}{\nu}}\right), \\
T^{****} &= E\left(\frac{\varepsilon_{i,t}^4}{\left(1+\frac{(\varepsilon_{i,t}/\sigma)^2}{\nu}\right)^2}\right).
\end{aligned}
$$

The off-diagonals can be obtained as,

$$
I_{12} = I_{21} = -E\left(\frac{\partial^2 lnL}{\partial\mu\partial\phi}\right) = \frac{\nu+1}{2}\sum_{t=1}^{n} E(A+B) = \frac{\nu+1}{2}\sum_{t=1}^{n} E(A)+E(B),
$$

where

$$
\begin{aligned}
A &= \frac{2\nu^{-1}\sigma^{-2}\left(y_{i,t}+(1-2\phi)y_{i,t-1}-2(1-2\phi)\mu_i\right)}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})}, \\
B &= \frac{4\nu^{-2}\sigma^{-4}(1-\phi)(\mu_i-y_{i,t-1})\varepsilon_{i,t}^2}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})^2}.
\end{aligned}
$$

Rearrange the equation *A*,

$$
A = \frac{y_{i,t-1}-y_{i,t}+2\varepsilon_{i,t}}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})}2\nu^{-1}\sigma^{-2}. \tag{3.13}
$$

Under the random shock representation the part *A* can be rewritten as,

$$
A = \frac{\sum_{j=1}^{\infty}\phi^{j-1}(1-\phi)\varepsilon_{i,t-j}+\varepsilon_{i,t}}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})}2\nu^{-1}\sigma^{-2}
$$

with expectation

$$
\begin{aligned}
E(A) = {} & 2\nu^{-1}\sigma^{-2}\sum_{j=1}^{\infty}\phi^{j-1}(1-\phi)E\left(\frac{\varepsilon_{i,t-j}}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})}\right)+ \\
& E\left(\frac{\varepsilon_{i,t}}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})}\right).
\end{aligned}
$$

The expectation stated above can be written as follows because the first part includes two independent variables which are $\varepsilon_{i,t}$ and $\varepsilon_{i,t-j}$. Thus, the expected *A* is,

$$
E(A) = \sum_{j=1}^{\infty}\phi^{j-1}(1-\phi)\frac{E(\varepsilon_{i,t-j})}{E(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})} + E\left(\frac{\varepsilon_{i,t}}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})}\right). \tag{3.14}
$$

Since $E(\varepsilon_{t-j}) = 0$, the first part of the E(A) equals to 0. The second part of the E(A) is also 0 by numerical solution.

The part *B* can be decomposed as follows:

$$B = \frac{4\nu^{-2}\sigma^{-4}(1-\phi)\mu_i\varepsilon_{i,t}^2}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})^2} - \frac{4\nu^{-2}\sigma^{-4}(1-\phi)y_{i,t-1}\varepsilon_{i,t}^2}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})^2},$$

with expectation

$$E(B) = 4\nu^{-2}\sigma^{-4}(1-\phi)\left(\mu_i E\left(\frac{\varepsilon_{i,t}^2}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})^2}\right) - E\left(\frac{y_{i,t-1}\varepsilon_{i,t}^2}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})^2}\right)\right).$$

Since $y_{i,t-1}$ and $\varepsilon_{i,t}$ are independent from each other, which points out the independency of the $y_{i,t-1}$ and $\varepsilon_{i,t}^2$, so E(B) can be written as,

$$\begin{aligned} E(B) &= 4\nu^{-2}\sigma^{-4}(1-\phi)\mu_i E\left(\frac{\varepsilon_{i,t}^2}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})^2}\right) - \\ &\quad 4\nu^{-2}\sigma^{-4}(1-\phi)E(y_{i,t-1})E\left(\frac{\varepsilon_{i,t}^2}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})^2}\right). \end{aligned}$$

Moreover, since $E(y_{i,t-1}) = \mu_i$,

$$\begin{aligned} E(B) &= 4\nu^{-2}\sigma^{-4}(1-\phi)\mu_i E\left(\frac{\varepsilon_{i,t}^2}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})^2}\right) - \\ &\quad 4\nu^{-2}\sigma^{-4}(1-\phi)\mu_i E\left(\frac{\varepsilon_{i,t}^2}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})^2}\right) = 0. \end{aligned}$$

Therefore, since both E(A) and E(B) are equal to 0 then $I_{12} = 0$, which means that μ_i and ϕ are independent from each other.

The other off-diagonal element of the Fisher Information Matrix is,

$$I_{13} = I_{31} = -E\left(\frac{\partial^2 lnL}{\partial\mu\partial\sigma}\right) = \frac{\nu+1}{2}\sum_{t=1}^{n} E(C-D) = \frac{\nu+1}{2}\sum_{t=1}^{n} E(C) - E(D)$$

where

$$\begin{aligned} C &= \frac{4\nu^{-1}\sigma^{-3}(1-\phi)\varepsilon_{i,t}}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})} \\ D &= \frac{4\sigma^{-5}\nu^{-2}(1-\phi)\varepsilon_{i,t}^3}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})^2} \end{aligned}$$

with expectations of C and D,

$$\begin{aligned}
E(C) &= 4\nu^{-1}\sigma^{-3}(1-\phi)E\left(\frac{\varepsilon_{i,t}}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})}\right)\\
E(D) &= 4\nu^{-2}\sigma^{-5}(1-\phi)E\left(\frac{\varepsilon_{i,t}^3}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})^2}\right).
\end{aligned}$$

Both E(C) and E(D) are 0 by numerical solutions, therefore I_{13} equals to 0, which indicates the independency of μ_i and σ.

The last off-diagonals can also be shown similarly as,

$$I_{23} = I_{32} = -E\left(\frac{\partial^2 lnL}{\partial\sigma\partial\phi}\right) = \frac{\nu+1}{2}\sum_{i=1}^{c}\sum_{t=1}^{n}E(F+G) = \frac{\nu+1}{2}\sum_{i=1}^{c}\sum_{t=1}^{n}E(F)+E(G)$$

where

$$\begin{aligned}
F &= -4\sigma^{-3}\nu^{-1}(\mu_i - y_{i,t-1})\frac{\varepsilon_{i,t}}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})}\\
G &= 4\sigma^{-5}\nu^{-2}(\mu_i - y_{i,t-1})\frac{\varepsilon_{i,t}^3}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})^2}.
\end{aligned}$$

Recall that because of the stationarity, $E(y_{i,t-1}) = \mu_i$ and $y_{i,t-1}$ and $\varepsilon_{i,t}$ are independent,by using these facts rearrange F, G, E(F) and E(G) respectively;

$$\begin{aligned}
F &= -4\sigma^{-3}\nu^{-1}\left(\mu_i\frac{\varepsilon_{i,t}}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})} - y_{i,t-1}\frac{\varepsilon_{i,t}}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})}\right),\\
G &= 4\sigma^{-5}\nu^{-2}\left(\mu_i\frac{\varepsilon_{i,t}^3}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})^2} - y_{i,t-1}\frac{\varepsilon_{i,t}^3}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})^2}\right),
\end{aligned}$$

with expectations

$$\begin{aligned}
E(F) &= -4\sigma^{-3}\nu^{-1}\left(\mu_i E\left(\frac{\varepsilon_{i,t}}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})}\right) - E\left(y_{i,t-1}\right)E\left(\frac{\varepsilon_{i,t}}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})}\right)\right),\\
E(G) &= 4\sigma^{-5}\nu^{-2}\left(\mu_i E\left(\frac{\varepsilon_{i,t}^3}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})^2}\right) - E\left(y_{i,t-1}\right)E\left(\frac{\varepsilon_{i,t}^3}{(1+\varepsilon_{i,t}^2\nu^{-1}\sigma^{-2})^2}\right)\right).
\end{aligned}$$

From the equations above, it can be obviously seen that E(F) and E(G) are both equal to 0. Therefore, I_{23} is 0, which points out the independency of the ϕ and σ.

Since all the off diagonals are equal to 0, the inverse of the Fisher Information Matrix is simply found as below:

$$I^{-1} = \begin{bmatrix} \frac{1}{I_{11}} & 0 & 0 \\ 0 & \frac{1}{I_{22}} & 0 \\ 0 & 0 & \frac{1}{I_{33}} \end{bmatrix}.$$

Therefore, the variance of the treatment mean in the numerator of the equation (3.9) can be estimated by 1 over the first diagonal element of the Fisher Information Matrix named as (3.12). Then, the variance obtained by Fisher Information Matrix is given as,

$$\hat{V}(\hat{\mu}_i) = \frac{1}{\frac{\nu+1}{\nu}\frac{n}{\hat{\sigma}^2}\left(1-\hat{\phi}\right)^2\left(T^* - \frac{2}{\nu\hat{\sigma}^2}T^{**}\right)}. \tag{3.15}$$

3.2.3 The Variance of Treatment Means Obtained by Observed Information Matrix

The asymtotic covariance matrix of the estimators can be obtained by hessian matrix as well. That is, the asymptotic variance of the $\hat{\mu}_i$ can be obtained from the related diagonal element of the inverse of the negative hessian matrix evaluated at $\hat{\mu}_i$.

The hessian matrix can be shown as follows:

$$H = \begin{bmatrix} \frac{\partial^2 lnL}{\partial^2\mu} & \frac{\partial^2 lnL}{\partial\mu\partial\phi} & \frac{\partial^2 lnL}{\partial\mu\partial\sigma} \\ \frac{\partial^2 lnL}{\partial\phi\partial\mu} & \frac{\partial^2 lnL}{\partial\phi^2} & \frac{\partial^2 lnL}{\partial\phi\partial\sigma} \\ \frac{\partial^2 lnL}{\partial\sigma\partial\mu} & \frac{\partial^2 lnL}{\partial\sigma\partial\phi} & \frac{\partial^2 lnL}{\partial\sigma^2} \end{bmatrix}.$$

Therefore, in order to test the null hypothesis stated with (3.8), the test statistic (3.9) is used, in which the $\hat{V}(\hat{\mu}_i)$ is obtained from the related diagonal of the inverse of the negative hessian matrix.

3.3 Hypothesis Testing in Linear Contrasts

In this section, the model given by (3.1) is considered and the linear combinations of the treatment means are tested by using the contrast vectors. The related hypothesis is given by,

$$H_0 : \sum_{i=1}^{c} l_i \mu_i = 0 \tag{3.16}$$
$$H_1 : \sum_{i=1}^{c} l_i \mu_i \neq 0$$

This null hypothesis (3.16) is tested with the test statistic given below:

$$t = \frac{\sum_{i=1}^{c} l_i \hat{\mu}_i}{\sqrt{\sum_{i=1}^{c} \hat{V}(l_i \hat{\mu}_i)}}. \tag{3.17}$$

Test statistic (3.17) asymptotically follows standard normal distribution (Şenoğlu and Bayrak, 2016).

Indeed, the estimated variance of the treatment means can also be obtained by three different ways for the test statistic of linear contrasts (3.17). The first one is using simulated variances (3.11), using the related diagonal element of the inverse Fisher Information Matrix (3.15) is the second way of estimating the variance of the treatment mean. The last way is using the observed information matrix, that is, using the related diagonal of the inverse of the negative hessian matrix (3.2.3).

CHAPTER 4

SIMULATION STUDY

In this chapter, various simulation scenarios are created for different models and parameters of distribution values so that the efficiency of functions and the conducted tests can be easily captured. In the first part of the simulation studies, model and distributional parameters are tried to be estimated by using the maximum likelihood estimation (MLE) method and least squares estimation (LSE) method. This part studies the simulated means, simulated mean square errors (MSE) and simulated relative efficiencies (RE). The comprehansive simulation results related with the simulated means, simulated MSE and RE are observed for different sample sizes, degrees of freedom and AR(1) coefficients. The second part of this chapter encapsulates the results of the empirical type 1 errors of the conducted test statistic by using simulated variances, variance found from Fisher information matrix and variance found from observed information matrix for one-way ANOVA. Moreover, the related power tables are also provided under this chapter. The third part of this chapter supplies the results of the empirical type 1 errors of the conducted test statistic by using simulated variances, variance found from Fisher information matrix and variance found from observed information matrix for the linear contrasts. The related power tables are given as well. The calculations of this chapter are done in R Statistical Software environment (R Core Team, 2016).

4.1 Comparison of the Estimators of the Model Parameters

In this subsection, simulated means of the maximum likelihood (ML) estimators and least squares (LS) estimators are obtained. L-BFGS-B method , which is one of the quasi Newton method, is used for obtaining ML estimators (Nash, J. C. and Varadhan, R., 2011). LS estimators are found by applying Newton algorithm (Hasselman, B., 2016). That is, since there is no closed form expression of the estimators for each estimation method, the approximate estimation results are presented on the tables stated under this part of the chapter. Moreover, here for example, $\tilde{\mu}$ and $\hat{\mu}$ represent the estimator of treatment mean by using the method of LS and ML respectively.

In here, for the sake of easiness only three treatments are used and the treatment means are set to be 0. For each run, specified size of independent errors are generated with the specified degrees of freedom with 0 mean. The scale parameter of the errors is used as 1 for all simulation scenarios (σ=1). By using these random errors, three time series each belongs to a treatment are generated.

In order to represent the wide range of correlation structure, AR(1) coefficient is chosen to be ϕ=-0.8, -0.4, -0.2, 0.0, 0.2, 0.4, 0.8 from negative to positive. The sample sizes are chosen to be n=50, 100, 200 and 500 so that the effect of sample size on the estimation can be easily judged. For simplicity, length of each treatment is set equally, this is n_i=*n*. Moreover, degrees of freedom effect on the estimations are tried to be kept under control by using diversified value for ν,where ν is chosen as 3, 6, 12, 24. For each scenario, 1,000 Monte Carlo runs are simulated. Moreover for each case, mean square error is obtained in order to judge the performances of the estimation methods accurately. Furthermore, the comparability of the MLE and LSE methods is evaluated by the relative efficiency which is found by the ratio of the variance of ML estimators and variance of LS estimators multiplied by 100, which is for example $RE = 100 * \frac{V(\hat{\mu}_{MLE})}{V(\tilde{\mu}_{LSE})}$. The results are disscused in detailed at the end of this part. The related tables are presented below:

Table 4.1: Simulated Means, MSE's and RE's of the Estimators; ν=3 , ϕ=-0.8

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	0.000	-0.795	0.958	0.001	-0.796	0.968
100	-0.002	-0.797	0.968	-0.001	-0.798	0.985
200	-0.001	-0.798	0.992	-0.002	-0.799	0.991
500	0.001	-0.800	0.988	-0.001	-0.800	0.997

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.020	0.003	0.061	0.010	0.002	0.007	48	64	11
100	0.009	0.001	0.036	0.005	0.001	0.003	55	60	9
200	0.005	0.001	0.097	0.002	0.000	0.002	43	59	2
500	0.002	0.000	0.014	0.001	0.000	0.001	53	56	5

Table 4.2: Simulated Means, MSEs and REs of the Estimators; ν=3 , ϕ=-0.4

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	0.004	-0.409	0.965	0.007	-0.405	0.967
100	-0.005	-0.400	0.969	-0.004	-0.400	0.979
200	-0.004	-0.403	0.979	-0.001	-0.402	0.989
500	0.000	-0.400	0.986	0.000	-0.400	0.997

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.031	0.006	0.060	0.016	0.004	0.008	51	68	12
100	0.014	0.003	0.036	0.008	0.002	0.004	54	65	9
200	0.008	0.001	0.029	0.004	0.001	0.002	54	59	6
500	0.003	0.001	0.011	0.002	0.000	0.001	48	53	6

Table 4.3: Simulated Means, MSEs and REs of the Estimators; ν=3 , ϕ=-0.2

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	0.005	-0.218	0.954	0.003	-0.213	0.962
100	0.000	-0.209	0.965	-0.002	-0.204	0.982
200	-0.005	-0.202	0.974	-0.002	-0.201	0.989
500	-0.002	-0.202	0.988	0.000	-0.201	0.996

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.041	0.006	0.051	0.022	0.005	0.008	53	89	14
100	0.019	0.003	0.029	0.010	0.002	0.004	51	61	12
200	0.011	0.001	0.023	0.005	0.001	0.002	51	61	8
500	0.004	0.001	0.016	0.002	0.000	0.001	54	56	4

Table 4.4: Simulated Means, MSEs and REs of the Estimators; ν=3 , ϕ=0.0

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	0.007	-0.023	0.969	0.005	-0.013	0.967
100	0.002	-0.011	0.972	0.001	-0.005	0.984
200	-0.005	-0.006	0.986	-0.003	-0.003	0.990
500	0.001	-0.001	0.986	0.001	0.000	0.997

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.062	0.007	0.060	0.031	0.004	0.007	50	62	10
100	0.031	0.004	0.039	0.016	0.002	0.003	52	61	8
200	0.016	0.002	0.033	0.008	0.001	0.002	48	61	5
500	0.006	0.001	0.013	0.003	0.000	0.001	48	54	5

Table 4.5: Simulated Means, MSEs and REs of the Estimators; ν=3 , ϕ=0.2

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	-0.013	0.175	0.965	-0.007	0.185	0.965
100	-0.002	0.190	0.974	-0.003	0.196	0.982
200	0.001	0.192	0.978	-0.001	0.195	0.993
500	0.005	0.197	0.995	0.001	0.199	0.998

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.102	0.007	0.064	0.050	0.005	0.008	49	68	11
100	0.047	0.003	0.051	0.025	0.002	0.004	53	60	7
200	0.024	0.002	0.019	0.012	0.001	0.002	50	61	9
500	0.010	0.001	0.021	0.005	0.000	0.001	49	55	3

Table 4.6: Simulated Means, MSEs and REs of the Estimators; ν=3 , ϕ=0.4

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	-0.001	0.364	0.964	-0.002	0.380	0.963
100	-0.007	0.385	0.975	-0.003	0.392	0.979
200	-0.007	0.391	0.981	-0.002	0.395	0.991
500	0.000	0.397	0.984	0.001	0.398	0.996

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.191	0.007	0.054	0.091	0.004	0.008	48	65	12
100	0.081	0.003	0.043	0.042	0.002	0.004	52	58	7
200	0.041	0.001	0.025	0.022	0.001	0.002	53	58	7
500	0.016	0.001	0.010	0.009	0.000	0.001	56	52	7

Table 4.7: Simulated Means, MSEs and RE of the Estimators; ν=3 , ϕ=0.8

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	0.101	0.804	0.975	-0.058	0.769	0.968
100	-0.079	0.782	0.973	-0.012	0.787	0.983
200	0.002	0.789	0.979	-0.020	0.794	0.990
500	-0.005	0.796	0.986	-0.003	0.798	0.996

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	83.034	0.012	0.078	0.793	0.003	0.008	1	18	9
100	6.819	0.002	0.033	0.374	0.001	0.003	5	37	9
200	0.358	0.001	0.025	0.179	0.000	0.002	50	59	6
500	0.150	0.000	0.014	0.081	0.000	0.001	54	50	5

Table 4.8: Simulated Means, MSEs and REs of the Estimators; ν=6 , ϕ=-0.8

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	0.003	-0.794	0.996	0.002	-0.795	0.974
100	-0.001	-0.796	1.000	-0.001	-0.796	0.987
200	-0.001	-0.798	0.999	-0.001	-0.798	0.993
500	-0.001	-0.800	0.998	-0.001	-0.800	0.996

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.010	0.003	0.008	0.008	0.002	0.006	85	86	63
100	0.005	0.001	0.004	0.004	0.001	0.003	85	89	58
200	0.002	0.001	0.002	0.002	0.001	0.001	85	87	65
500	0.001	0.000	0.001	0.001	0.000	0.001	89	82	64

Table 4.9: Simulated Means, MSEs and REs of the Estimators; ν=6 , ϕ=-0.4

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	-0.001	-0.404	0.994	0.001	-0.402	0.971
100	0.002	-0.401	0.999	0.002	-0.399	0.988
200	0.001	-0.401	0.997	0.000	-0.400	0.992
500	-0.001	-0.402	1.000	-0.001	-0.402	0.997

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.015	0.006	0.008	0.013	0.005	0.006	87	89	66
100	0.008	0.003	0.005	0.007	0.003	0.003	85	92	54
200	0.004	0.001	0.002	0.003	0.001	0.001	85	86	70
500	0.002	0.001	0.001	0.001	0.000	0.000	83	91	62

Table 4.10: Simulated Means, MSEs and REs of the Estimators; ν=6 , ϕ=-0.2

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	-0.007	-0.208	0.996	-0.007	-0.206	0.971
100	-0.005	-0.205	0.998	-0.005	-0.204	0.989
200	0.002	-0.202	0.999	0.002	-0.202	0.992
500	0.001	-0.200	1.000	0.000	-0.200	0.998

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.023	0.007	0.008	0.019	0.006	0.006	85	90	62
100	0.011	0.003	0.004	0.009	0.003	0.002	86	86	62
200	0.005	0.002	0.002	0.004	0.001	0.001	84	85	62
500	0.002	0.001	0.001	0.002	0.001	0.001	87	85	55

Table 4.11: Simulated Means, MSEs and REs of the Estimators; ν=6 , ϕ=0.0

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	0.010	-0.020	0.991	0.010	-0.018	0.971
100	-0.004	-0.010	0.998	-0.003	-0.009	0.985
200	0.002	-0.003	0.998	0.001	-0.003	0.992
500	-0.001	-0.002	1.000	0.000	-0.002	0.997

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.031	0.007	0.008	0.027	0.006	0.006	86	91	66
100	0.015	0.003	0.004	0.013	0.003	0.003	85	88	55
200	0.007	0.002	0.002	0.006	0.001	0.001	86	87	51
500	0.003	0.001	0.001	0.003	0.001	0.001	85	86	56

Table 4.12: Simulated Means, MSEs and REs of the Estimators; ν=6 , ϕ=0.2

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	-0.001	0.177	0.991	-0.002	0.180	0.968
100	0.003	0.185	1.000	0.006	0.187	0.987
200	-0.001	0.191	0.998	-0.001	0.193	0.993
500	-0.002	0.199	0.999	-0.002	0.199	0.997

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.049	0.007	0.008	0.042	0.006	0.006	87	88	62
100	0.023	0.004	0.005	0.019	0.003	0.003	86	87	53
200	0.011	0.002	0.002	0.010	0.001	0.001	87	91	60
500	0.004	0.001	0.001	0.004	0.001	0.000	86	85	63

Table 4.13: Simulated Means, MSEs and REs of the Estimators; ν=6 , ϕ=0.4

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	-0.005	0.364	0.992	-0.006	0.368	0.968
100	0.003	0.382	1.000	0.003	0.383	0.987
200	0.006	0.391	0.998	0.006	0.392	0.993
500	-0.001	0.396	1.001	-0.002	0.396	0.998

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.086	0.008	0.008	0.075	0.007	0.006	87	88	67
100	0.043	0.003	0.004	0.037	0.003	0.003	86	87	60
200	0.020	0.002	0.002	0.017	0.001	0.001	86	87	65
500	0.008	0.001	0.001	0.007	0.000	0.000	87	85	59

Table 4.14: Simulated Means, MSEs and REs of the Estimators; ν=6 , ϕ=0.8

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	0.463	0.806	1.006	0.010	0.754	0.968
100	-0.047	0.778	0.993	0.017	0.777	0.981
200	0.006	0.788	0.998	0.003	0.789	0.991
500	-0.007	0.796	1.001	-0.004	0.797	0.998

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	69.772	0.013	0.008	0.647	0.005	0.006	1	24	63
100	5.916	0.003	0.004	0.326	0.002	0.003	6	56	69
200	0.206	0.001	0.002	0.173	0.001	0.001	84	84	58
500	0.076	0.000	0.001	0.066	0.000	0.001	88	83	60

Table 4.15: Simulated Means, MSEs and REs of the Estimators; ν=12 , ϕ=-0.8

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	0.002	-0.795	0.998	0.002	-0.795	0.977
100	0.000	-0.796	0.997	0.000	-0.796	0.987
200	0.001	-0.798	0.998	0.000	-0.798	0.993
500	0.001	-0.798	0.999	0.001	-0.798	0.997

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.008	0.003	0.004	0.007	0.002	0.004	96	96	87
100	0.004	0.001	0.002	0.004	0.001	0.002	98	95	91
200	0.002	0.001	0.001	0.002	0.001	0.001	97	97	90
500	0.001	0.000	0.000	0.001	0.000	0.000	98	95	92

Table 4.16: Simulated Means, MSEs and REs of the Estimators; ν=12 , ϕ=-0.4

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	-0.006	-0.409	0.993	-0.007	-0.408	0.972
100	-0.004	-0.405	0.995	-0.003	-0.405	0.984
200	0.000	-0.400	0.997	0.000	-0.400	0.992
500	0.000	-0.402	1.000	0.000	-0.401	0.998

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.012	0.005	0.004	0.012	0.005	0.005	99	99	85
100	0.006	0.003	0.002	0.005	0.003	0.002	99	95	91
200	0.003	0.002	0.001	0.003	0.001	0.001	95	96	88
500	0.001	0.001	0.000	0.001	0.001	0.000	96	96	90

Table 4.17: Simulated Means, MSEs and REs of the Estimators; ν=12 , ϕ=-0.2

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	0.001	-0.214	0.996	0.001	-0.213	0.975
100	-0.002	-0.204	0.998	-0.002	-0.204	0.988
200	0.000	-0.203	0.998	0.000	-0.203	0.993
500	0.000	-0.202	0.999	0.000	-0.202	0.997

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.016	0.006	0.005	0.016	0.006	0.005	96	98	86
100	0.008	0.003	0.002	0.008	0.003	0.002	94	97	88
200	0.004	0.002	0.001	0.004	0.002	0.001	96	98	90
500	0.002	0.001	0.000	0.002	0.001	0.000	96	96	91

Table 4.18: Simulated Means, MSEs and REs of the Estimators; ν=12 , ϕ=0.0

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	0.002	-0.017	0.995	0.001	-0.017	0.972
100	0.002	-0.009	0.996	0.001	-0.008	0.985
200	-0.001	-0.008	0.999	-0.002	-0.007	0.994
500	-0.001	-0.003	0.999	-0.002	-0.003	0.997

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.024	0.007	0.004	0.024	0.006	0.004	97	97	84
100	0.012	0.004	0.002	0.012	0.004	0.002	97	98	93
200	0.006	0.002	0.001	0.006	0.002	0.001	95	96	92
500	0.002	0.001	0.000	0.002	0.001	0.000	98	95	90

Table 4.19: Simulated Means, MSEs and REs of the Estimators; ν=12 , ϕ=0.2

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	-0.010	0.171	0.994	-0.009	0.172	0.972
100	0.000	0.187	0.997	-0.001	0.188	0.986
200	0.001	0.194	0.999	0.001	0.195	0.994
500	-0.005	0.197	1.000	-0.004	0.197	0.998

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.040	0.007	0.005	0.039	0.007	0.005	97	98	88
100	0.018	0.004	0.002	0.018	0.003	0.002	99	96	88
200	0.009	0.002	0.001	0.009	0.002	0.001	99	95	93
500	0.004	0.001	0.000	0.003	0.001	0.000	97	97	93

Table 4.20: Simulated Means, MSEs and REs of estimators; ν=12 , ϕ=0.4

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	-0.002	0.363	0.991	-0.003	0.364	0.969
100	0.005	0.381	0.996	0.005	0.382	0.986
200	0.006	0.391	0.998	0.007	0.391	0.993
500	0.002	0.396	0.999	0.002	0.396	0.997

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.068	0.008	0.005	0.066	0.008	0.005	98	99	83
100	0.033	0.003	0.002	0.031	0.003	0.002	95	97	90
200	0.017	0.001	0.001	0.016	0.001	0.001	95	97	88
500	0.007	0.001	0.000	0.007	0.001	0.000	95	96	91

Table 4.21: Simulated Means, MSEs and REs of the Estimators; ν=12 , ϕ=0.8

n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
	Mean					
50	0.022	0.806	1.008	-0.001	0.751	0.973
100	0.068	0.782	0.997	0.035	0.777	0.984
200	-0.001	0.789	0.999	0.008	0.789	0.993
500	0.000	0.795	0.999	0.002	0.795	0.997

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	52.634	0.012	0.005	0.549	0.006	0.005	1	26	78
100	6.202	0.003	0.002	0.296	0.002	0.002	5	53	86
200	0.147	0.001	0.001	0.139	0.001	0.001	94	96	91
500	0.059	0.000	0.000	0.056	0.000	0.000	96	96	90

Table 4.22: Simulated Means, MSEs and REs of the Estimators; ν=24 , ϕ=-0.8

n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
	Mean					
50	0.002	-0.792	0.994	0.002	-0.792	0.974
100	0.003	-0.797	0.998	0.003	-0.797	0.988
200	-0.001	-0.796	0.999	-0.001	-0.796	0.994
500	0.000	-0.799	0.999	0.000	-0.799	0.997

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.006	0.003	0.004	0.006	0.003	0.004	98	99	94
100	0.003	0.001	0.002	0.003	0.001	0.002	99	98	96
200	0.002	0.001	0.001	0.002	0.001	0.001	99	99	98
500	0.001	0.000	0.000	0.001	0.000	0.000	99	99	97

Table 4.23: Simulated Means, MSEs and REs of the Estimators; ν=24 , ϕ=-0.4

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	-0.005	-0.411	0.997	-0.005	-0.411	0.977
100	-0.003	-0.403	0.997	-0.003	-0.403	0.987
200	0.000	-0.402	0.999	0.000	-0.402	0.994
500	0.000	-0.401	1.000	0.000	-0.401	0.998

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.011	0.006	0.004	0.011	0.006	0.004	99	100	94
100	0.006	0.003	0.002	0.006	0.003	0.002	99	99	97
200	0.003	0.001	0.001	0.003	0.001	0.001	99	99	97
500	0.001	0.001	0.000	0.001	0.001	0.000	98	99	97

Table 4.24: Simulated Means, MSEs and REs of the Estimators; ν=24 , ϕ=-0.2

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	0.003	-0.211	0.995	0.003	-0.211	0.974
100	0.000	-0.208	0.998	0.000	-0.208	0.988
200	0.001	-0.204	0.997	0.001	-0.204	0.992
500	0.001	-0.203	1.000	0.001	-0.203	0.998

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.016	0.006	0.004	0.016	0.006	0.005	99	99	92
100	0.008	0.003	0.002	0.008	0.003	0.002	99	100	97
200	0.004	0.002	0.001	0.004	0.002	0.001	100	98	97
500	0.001	0.001	0.000	0.001	0.001	0.000	100	99	98

Table 4.25: Simulated Means, MSEs and REs of the Estimators; ν=24 , ϕ=0.0

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	0.004	-0.021	0.995	0.003	-0.020	0.974
100	0.000	-0.009	0.997	0.000	-0.009	0.986
200	0.002	-0.006	0.997	0.001	-0.006	0.992
500	0.000	-0.003	0.999	0.000	-0.003	0.997

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.023	0.007	0.004	0.022	0.007	0.005	99	99	95
100	0.011	0.003	0.002	0.011	0.003	0.002	99	99	95
200	0.006	0.002	0.001	0.005	0.002	0.001	99	100	97
500	0.002	0.001	0.000	0.002	0.001	0.000	99	99	97

Table 4.26: Simulated Means, MSEs and REs of the Estimators; ν=24 , ϕ=0.2

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	0.007	0.170	0.993	0.008	0.170	0.972
100	-0.003	0.186	0.996	-0.003	0.186	0.986
200	-0.002	0.192	0.999	-0.002	0.192	0.994
500	-0.001	0.197	0.999	-0.001	0.197	0.996

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.033	0.007	0.004	0.032	0.007	0.005	98	98	94
100	0.017	0.003	0.002	0.017	0.003	0.002	99	100	95
200	0.009	0.002	0.001	0.009	0.002	0.001	100	99	98
500	0.003	0.001	0.000	0.003	0.001	0.000	99	100	97

Table 4.27: Simulated Means, MSEs and REs of the Estimators; ν=24 , ϕ=0.4

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	0.010	0.366	0.996	0.011	0.366	0.974
100	-0.004	0.382	0.997	-0.003	0.382	0.987
200	0.002	0.391	0.998	0.001	0.391	0.993
500	-0.002	0.397	1.000	-0.001	0.397	0.998

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	0.058	0.007	0.004	0.058	0.007	0.005	100	99	95
100	0.030	0.003	0.002	0.030	0.003	0.002	99	99	95
200	0.014	0.001	0.001	0.014	0.001	0.001	99	98	96
500	0.006	0.001	0.000	0.006	0.001	0.000	99	100	97

Table 4.28: Simulated Means, MSEs and REs of the Estimators; ν=24 , ϕ=0.8

	Mean					
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$
50	-0.234	0.809	1.010	0.008	0.749	0.974
100	0.014	0.781	0.999	0.005	0.775	0.986
200	-0.002	0.789	0.998	-0.001	0.789	0.992
500	0.004	0.795	1.000	0.005	0.795	0.998

	MSE						RE		
n	$\tilde{\mu}_{LSE}$	$\tilde{\phi}_{LSE}$	$\tilde{\sigma}_{LSE}$	$\hat{\mu}_{MLE}$	$\hat{\phi}_{MLE}$	$\hat{\sigma}_{MLE}$	μ	ϕ	σ
50	49.491	0.013	0.005	0.550	0.006	0.005	1	29	86
100	3.979	0.003	0.002	0.258	0.002	0.002	6	54	91
200	0.144	0.001	0.001	0.143	0.001	0.001	99	99	95
500	0.056	0.000	0.000	0.056	0.000	0.000	100	99	99

When the tables stated in this chapter are examined, both methods give acceptable results on estimation of the parameters. For each method, even for small sample sizes, the estimates of the treatment means are near to 0, as it should be. Moreover, as it is expected, increase in sample size results in better estimation values. Estimation of the scale parameter also converges to its real value 1. These two method performs also well on the estimation of AR(1) coefficient. For each value of the coefficient, no matter negative or positive, estimated value converges to the real ones. Here also increase in sample size provides better approximations in estimating autoregressive coefficients. However, when the AR(1) coefficient is near to 1, LS estimators of the treatment means are seen to be biased. In other words, when the correlation between treatment unit with the previous unit is positively large, then the estimation done by least squares method will be biased, that is estimations diverge from their real values. Actually this problem only occurs for the small sample size cases since when the length of the treatments is getting larger, LS estimators perform well. Therefore, because of the small sample performances of the LS estimators with high positive autoregressive coefficient, ML estimators are preferable in this study.

Mean square error values are presented because being unbiased is not enough to characterize the estimators. Mean square error should be one of the criteria to evaluate the performance of an estimator just because it not only measure the biasness but also measure the variance. Those two quantities and the MSE itself need to be as small as possible. In this point of view, MSE values are compared and it can be said that MSE of the ML estimators, for each case no matter the degrees of freedom of the error term and autoregressive coefficient, are smaller than the MSE of the LS estimators. Moreover, like the small sample performances of the LS estimators on estimating the unknown parameters, MSE values of the LS estimators are also unacceptable when the AR (1) coefficient is near to 1. This is one of the other reason why the maximum likelihood estimation method is used for the further analysis of this study. Besides these, one can clearly see that when the error terms are produced from student's t distribution with large degrees of freedom, that is, when the error terms behaves the characteristics of normal distribution, MSE values of the LS estimators approach to the MSE values of ML estimators.

Finally, the relative efficiencies of the estimators one coming from LS and the other coming from ML is the concern, it can be obviously seen that relative efficiency of ML estimators are better than LSs'. It is expected that the relative efficiency of ML estimators is getting smaller or in other words the relative efficiency of the LS estimators is getting larger when the size of samples are boosted. For some scenarios, the relative efficiency of LS estimators declines when the sample size is increased. This is actually another result of interest. In the further studies, it can be analysed in a detailed way.

4.2 Simulated Type I Error and Power of the One-Way ANOVA Test Statistic

This part of the study is related to the one-way ANOVA conclusions. Type I errors of the introduced test statistic for ANOVA are presented by three ways. That are one for using simulated variances, another one for information matrix and the last one is for observed information matrix. The next part is related to the power of the introduced test statistic. The last part contains power comparison among the other test statistics. The first one is actually normal classical test statistic (2.14), named as F_0. The other one is based on LS estimators of the model (3.1) in the test statistic of normal classical F statistic, named as F_{LSE}. The reason of this kind of comparison is that it is aimed to observe that how the power is affected if the non-validated assumptions are ignored. That is, once you disregard the independency and normality assumption while working on one-way ANOVA, how the power of the test statistic is affected is tried to be observed.

4.2.1 Simulated Type I Error of One-Way ANOVA Test Statistic

In this section, it is aimed to determine the underlying distribution of the introduced test statistic for the one-way ANOVA. It is actually desired to be F distribution with the corresponding degrees of freedoms. Here, the desired distribution that the test statistic to follow is F with c-1 and nc-c degrees of freedoms, where n is the size of each treatment (size of treatments are taken to be equal for the sake of easiness) and c is the number of treatment that is chosen to be 3. Besides this, type I error can help to test the performance of the test statistic by looking at its probability of rejection of null hypothesis when in fact it is true. That is, wrong decision making process of the test statistic is evaluated with type I error. Reaching the type I error of this section is done by comparing the calculated test statistic with the tabulated test statistic. If the test statistic is greater than the tabulated value then the null hypothesis can be rejected. Therefore, the proportion of those rejection out of total trials (runs) is nothing but the type I error. It is expected to be close to 0.05, because significance level is chosen to be 0.05 ($\alpha = 0.05$). As it is declared in Section 4.2, type I error of the related test statictic is found by using three different ways. Furhermore, in order to observe the size of treatment and student's t degrees of freedom effect, *n* is chosen to be 50, 100, 200, 500 and 1,000 and ν is chosen to be 6, 9, 12 and 24.

4.2.1.1 Simulated Type I Error of the Test Statistic Based on Simulated Variances

The variance of the treatment means can be obtained by simulation with 10,000 Monte Carlo runs. The variance of these means can be easily calculated, the corresponding variance values are presented in Table 4.29. This part comprises the test statistic that uses the simulated variances for the variance of the treatment means in the test statistic (3.9). The null hypothesis emphasises the equality of treatment means. The algorithm is as follows:

Step 1: Generating Random Numbers

1.1 Generate specified size of independent errors terms from student's t distribution.

1.2. Generate treatment observations ($y_{i,t}$) by using equation (3.1).

Step 2: Parameter Estimation and Hypothesis Testing

2.1 Estimate related parameters of the model and distribution.

2.2 Calculate test statistic from the equation (3.9) with the variance coming from simulation and compare it with critical tabulated values.

Step 3: Type I Error

3.1 Calculate type I error by proportinating the number of rejected test statistic to 10,000.

Table 4.29: Simulated Variance of The Treatment Mean: $\hat{V}(\hat{\mu})$

n	ν=6						
	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.00835	0.013260	0.01840	0.02665	0.040850	0.07487	0.65074
100	0.00397	0.00665	0.00909	0.01278	0.02140	0.03635	0.32173
200	0.00199	0.00331	0.00447	0.00654	0.01013	0.01807	0.16183
500	0.00080	0.00131	0.00179	0.00261	0.00398	0.00719	0.06611
1000	0.00040	0.00066	0.00089	0.00128	0.00197	0.00360	0.03207
	ν=9						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.00755	0.01254	0.01697	0.02410	0.03762	0.06839	0.59717
100	0.00372	0.00619	0.00853	0.01193	0.01888	0.03408	0.29950
200	0.00189	0.00311	0.00423	0.00612	0.00925	0.01662	0.14652
500	0.00074	0.00123	0.00165	0.00237	0.00372	0.00670	0.05865
1000	0.00037	0.00060	0.00084	0.00121	0.00188	0.00330	0.03090
	ν=12						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.00728	0.01207	0.01606	0.02361	0.03610	0.06484	0.57339
100	0.00364	0.00595	0.00814	0.01166	0.01829	0.03198	0.29424
200	0.00180	0.00298	0.00397	0.00576	0.00913	0.01634	0.14195
500	0.00072	0.00121	0.00157	0.00230	0.00364	0.00640	0.05806
1000	0.00037	0.00060	0.00082	0.00115	0.00181	0.00320	0.02913
	ν=24						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.00693	0.01084	0.01542	0.02174	0.03418	0.06037	0.53533
100	0.00341	0.00553	0.00748	0.01083	0.01691	0.03055	0.27260
200	0.00168	0.00282	0.00375	0.00532	0.00858	0.01473	0.13759
500	0.00067	0.00111	0.00152	0.00218	0.00332	0.00597	0.05430
1000	0.00034	0.00055	0.00076	0.00107	0.00167	0.00305	0.02703

Table 4.30: Simulated Type I Error of the One-Way ANOVA Test Statistic Based on Simulated Variances

	ν=6						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.049	0.045	0.046	0.046	0.045	0.049	0.049
100	0.052	0.050	0.045	0.047	0.046	0.046	0.048
200	0.049	0.048	0.050	0.047	0.049	0.048	0.048
500	0.052	0.049	0.050	0.052	0.048	0.047	0.047
1000	0.050	0.054	0.049	0.050	0.052	0.049	0.047
	ν=9						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.052	0.045	0.048	0.049	0.046	0.046	0.047
100	0.051	0.050	0.048	0.050	0.046	0.046	0.050
200	0.048	0.046	0.046	0.046	0.050	0.051	0.053
500	0.052	0.047	0.052	0.054	0.048	0.048	0.051
1000	0.052	0.052	0.053	0.048	0.050	0.050	0.047
	ν=12						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.049	0.045	0.046	0.045	0.047	0.045	0.049
100	0.053	0.047	0.050	0.049	0.048	0.047	0.047
200	0.048	0.046	0.052	0.048	0.046	0.048	0.051
500	0.050	0.046	0.054	0.050	0.046	0.048	0.048
1000	0.048	0.048	0.047	0.047	0.050	0.049	0.048
	ν=24						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.051	0.045	0.045	0.045	0.048	0.046	0.047
100	0.048	0.050	0.049	0.046	0.046	0.045	0.047
200	0.050	0.048	0.048	0.051	0.047	0.047	0.050
500	0.048	0.053	0.050	0.052	0.054	0.052	0.050
1000	0.049	0.048	0.046	0.049	0.052	0.046	0.050

The results from Table 4.30 point out that for each degrees of freedom and/or AR(1) coefficient situation test statistic based on simulated variances performs well under F distribution. That is, by looking at Table 4.30, it can be stated that the underlying distribution of the test statistic is F because in all situations type I error is acceptably close to 0.05.

4.2.1.2 Simulated Type I Error of the Test Statistic Based on Fisher Information Matrix

For this case, the variance of the treatment means are found by using the related diagonal of the inverse of the information matrix. Procedure of calculating type I error with variances achieved from information matrix leads to fluctation, that's why average of the 10 type I errors are used. The algorithm is as follows:

Step 1: Generating Random Numbers

1.1 Generate specified size of independent errors terms from student's t distribution.

1.2. Generate treatment observations ($y_{i,t}$) by using equation (3.1).

Step 2: Parameter Estimation and Hypothesis Testing

2.1 Estimate related parameters of the model and distribution

2.2 Calculate test statistic from the equation (3.9) with variance coming from Fisher Information Matrix

2.3 Compare the test statistic with critical tabulated values. Conclude whether null hypothesis is rejected or not.

Step 3: Type I Error

3.1 Do the steps from 1 to 2 1,000 times, calculate type I error by proportinating the number of rejected test statistic to 1,000.

3.2 Do step 3.1 10 times, and take a simple average of these 10 type 1 errors as a final type I error to tabulate.

Table 4.31: Simulated Type I Error of the One-Way ANOVA Test Statistic Based on Fisher Information Matrix

	ν=6						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.058	0.053	0.062	0.056	0.091	0.061	0.161
100	0.050	0.050	0.061	0.050	0.064	0.058	0.126
200	0.055	0.052	0.053	0.050	0.064	0.053	0.073
500	0.047	0.054	0.050	0.053	0.048	0.0.55	0.064
1000	0.050	0.050	0.050	0.056	0.054	0.056	0.063
	ν=9						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.047	0.054	0.048	0.054	0.066	0.059	0.163
100	0.052	0.052	0.061	0.062	0.070	0.071	0.099
200	0.051	0.048	0.057	0.052	0.055	0.050	0.084
500	0.057	0.049	0.048	0.051	0.051	0.056	0.054
1000	0.050	0.047	0.047	0.053	0.055	0.054	0.050
	ν=12						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.049	0.057	0.039	0.067	0.072	0.059	0.211
100	0.052	0.057	0.048	0.062	0.051	0.056	0.129
200	0.055	0.057	0.048	0.062	0.051	0.056	0.129
500	0.051	0.049	0.053	0.052	0.050	0.050	0.050
1000	0.053	0.055	0.046	0.049	0.059	0.049	0.053
	ν=24						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.059	0.055	0.049	0.061	0.065	0.079	0.178
100	0.052	0.050	0.054	0.059	0.049	0.070	0.091
200	0.049	0.051	0.048	0.053	0.048	0.056	0.104
500	0.050	0.048	0.055	0.047	0.049	0.049	0.069
1000	0.054	0.049	0.051	0.050	0.050	0.050	0.051

The findings mark that except for positively high autoregressive coefficient cases, type I error approaches to its theoretical value. However, this problem, which arises with high AR(1) coefficient, can be eliminated by enhancing the size of treatments. Moreover, in general Table 4.31 points out that the introduced test statistic follows F distribution with the desired degrees of freedoms.

4.2.1.3 Simulated Type I Error of the Test Statistic Based on Observed Information Matrix

In this part, the test statictic based on observed information matrix is evaluted by interpreting its type I error. The algorithm is same with the 4.2.1.2 with a small difference sourced by the fact that the variance of treatment means are achieved here by using the related diagonal element of negative inverse of hessian matrix instead of information matrix. The type I error of test statistic with variance coming from observed information matrix for one-way ANOVA is supplied below. This table is also constructed by averaging the 10 type I errors in order to remove fluctuations occured in simulations. The algorithm is as follows:

Step 1: Generating Random Numbers

1.1 Generate specified size of independent errors terms from student's t distribution.

1.2. Generate treatment observations ($y_{i,t}$) by using equation (3.1).

Step 2: Parameter Estimation and Hypothesis Testing

2.1 Estimate related parameters of the model and distribution

2.2 Calculate test statistic from the equation (3.9) with variance coming from Observed Information Matrix

2.3 Compare the test statistic with critical tabulated values. Conclude whether null hypothesis is rejected or not.

Step 3: Type I Error

3.1 Do the steps from 1 to 2 1,000 times, calculate type I error by proportinating the number of rejected test statistic to 1,000.

3.2 Do step 3.1 10 times , and take a simple average of these 10 type 1 errors as a final type I error to tabulate.

Table 4.32: Simulated Type I Error of the One-Way ANOVA Test Statistic Based on Observed Information Matrix

	ν=6						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.058	0.051	0.062	0.058	0.092	0.057	0.160
100	0.050	0.054	0.059	0.053	0.065	0.057	0.130
200	0.057	0.050	0.052	0.050	0.065	0.054	0.073
500	0.046	0.054	0.052	0.053	0.050	0.054	0.063
1000	0.052	0.052	0.050	0.057	0.053	0.055	0.064
	ν=9						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.049	0.055	0.046	0.050	0.065	0.057	0.159
100	0.053	0.049	0.063	0.062	0.073	0.071	0.101
200	0.052	0.049	0.060	0.051	0.055	0.048	0.081
500	0.058	0.049	0.047	0.051	0.050	0.057	0.055
1000	0.051	0.047	0.048	0.054	0.055	0.054	0.049
	ν=12						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.048	0.060	0.040	0.076	0.078	0.061	0.222
100	0.054	0.059	0.050	0.063	0.054	0.058	0.125
200	0.056	0.052	0.045	0.053	0.055	0.052	0.108
500	0.053	0.048	0.054	0.050	0.050	0.049	0.049
1000	0.053	0.055	0.046	0.050	0.060	0.049	0.054
	ν=24						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.059	0.058	0.052	0.068	0.068	0.078	0.191
100	0.055	0.052	0.057	0.059	0.050	0.073	0.097
200	0.049	0.049	0.049	0.057	0.049	0.057	0.105
500	0.052	0.049	0.056	0.047	0.048	0.048	0.071
1000	0.054	0.050	0.051	0.049	0.049	0.049	0.050

The same final conclusion can be done here with the 4.2.1.2. There exists only a problem with the small size of treatment when AR(1) coefficient is close to 1. However, this can be eliminated by increasing the size of treatment. Therefore, Table 4.32 indicates an acceptable type I errors.

4.2.2 Simulated Power of the One-Way ANOVA Test Statistic

This part of the study encapsulates the power of the introduced test statsistic with the variance from Fisher Information Matrix for one-way ANOVA. Actually, there is no computational difference in using observed or Fisher information matrices, we prefer to use the test statistic based on Fisher information matrix. The test statistic based on simulated variances is not preferred because in each time additional simulation is needed to observe simulated variances of the treatment means. That's why, power tables are constructed by using the test statistic based on Fisher information matrix. In order to obtain the power of the test statistic, the simulation scenerio is formed in an opposite way of the null hypothesis so that the probability of rejection of the null hypothesis is obtained under the wrong null hypothesis. This is done by using constant "d", by increasing d, which causes the increase in the difference between treatment means, the power of the test statistic is achieved. The first treatment mean is hold constant as 0, second treatment mean is increased by $2d$ and the third treatment mean is increased by $3d$ in each scenario. The simulation number is taken to be 1,000 and the treatment size is chosen to be 100 in each scenario.

Table 4.33: Simulated Power of the F_{MLE} ; *n*=100

	$\nu = 6$	$\nu = 9$	$\nu = 12$	$\nu = 24$
d				
		$\phi = -0.8$		
0.00	0.058	0.069	0.065	0.050
0.04	0.210	0.281	0.298	0.272
0.08	0.686	0.782	0.786	0.769
0.12	0.976	0.979	0.984	0.987
		$\phi = -0.4$		
0.00	0.053	0.049	0.070	0.068
0.05	0.178	0.239	0.295	0.290
0.10	0.644	0.729	0.780	0.773
0.15	0.958	0.967	0.978	0.988
		$\phi = -0.2$		
0.00	0.051	0.042	0.071	0.069
0.06	0.181	0.232	0.317	0.314
0.12	0.660	0.737	0.810	0.805
0.18	0.966	0.970	0.989	0.988
		$\phi = 0.0$		
0.00	0.047	0.033	0.071	0.074
0.07	0.167	0.200	0.310	0.309
0.14	0.619	0.681	0.792	0.789
0.21	0.952	0.964	0.982	0.988
		$\phi = 0.2$		
0.00	0.041	0.046	0.077	0.055
0.08	0.134	0.153	0.280	0.283
0.16	0.524	0.571	0.711	0.731
0.24	0.897	0.917	0.956	0.970
		$\phi = 0.4$		
0.00	0.063	0.059	0.060	0.055
0.11	0.171	0.260	0.376	0.173
0.22	0.644	0.617	0.744	0.600
0.33	0.928	0.942	0.974	0.949
		$\phi = 0.8$		
0.00	0.120	0.192	0.178	0.114
0.40	0.328	0.333	0.242	0.574
0.80	0.604	0.792	0.877	0.965
1.20	0.969	0.988	1.000	0.999

Actually, when d equals to 0, the power table turns into the type I error table. According to Table 4.33, as the difference between the treatment means are increased, independent from the AR(1) coefficient, the power of the introduced test statistic for here called as F_{MLE} is increasing.

4.2.3 Simulated Powers of the Other Test Statistics for One-Way ANOVA

The main aim of this part of the study is to observe that how the power of the test statistics are affected if the non-validated assumptions are ignored. The first test statistic, called as F_0, ignores both correlation and non-normality. The second test statistic, F_{LSE} is the same test statistic with F_0 but it is based on LS estimators of the model (3.1). These two test statistics are,

$$F_0 = \frac{n\sum_{i=1}^{c}(\bar{y}_{i.} - \bar{y}_{..})^2/(c-1)}{\sum_{i=1}^{c}\sum_{j=1}^{n}(y_{ij} - \bar{y}_{i.})^2/(nc-c)} \quad \text{and} \tag{4.1}$$

$$F_{LSE} = \frac{n\sum_{i=1}^{c}(\tilde{\mu}_i - \tilde{\mu})^2/(c-1)}{\sum_{i=1}^{c}\sum_{j=1}^{n}(y_{ij} - \tilde{\mu}_i)^2/(nc-c)}. \tag{4.2}$$

Firstly, we assume both of these test statistics follow F distribution with the degrees of freedom, *c*-1 and *nc*-*c* respectively. By examining the type I errors of these test statistics, we try to ensure that these test statistics follow F distribution. To do so, ν is randomly chosen as 9, simulation number is 1,000 and the size of the treatment is chosen as 100.

Table 4.34: Simulated Type I Errors of F_0 and F_{LSE}

F	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
F_0	0.000	0.002	0.009	0.048	0.140	0.272	0.722
F_{LSE}	0.000	0.002	0.010	0.046	0.143	0.270	0.740

According to results of Table 4.34, test statictics F_0 and F_{LSE} do not follow F distribution with the *c*-1 and *nc*-*c* degrees of freedoms, except for ϕ equals to 0. The case of when ϕ is 0 is nothing but the case of that there exists no correlation. In other words, when the autoregressive coefficient is 0, only non-normality problem is the concern, and therefore, non-normality does not have a vital effect on the type I error when the size of sample is large enough, actually because of central limit theorem

(Pavur and Lewis, 1982). That's why the power comparision is done for only ϕ:-0.10, 0.00 and 0.10.

Table 4.35: Simulated Power of the F_0, F_{LSE} and F_{MLE}; *n*=100

	$\nu=6$			$\nu=9$			$\nu=12$			$\nu=24$		
d	F_0	F_{LSE}	F_{MLE}	F_0	F_{LSE}	F_{MLE}	F_0	F_{LSE}	F_{MLE}	F_0	F_{LSE}	F_{MLE}
$\phi=-0.10$												
0.00	0.028	0.026	0.050	0.026	0.025	0.040	0.022	0.022	0.071	0.020	0.022	0.070
0.06	0.098	0.098	0.153	0.127	0.125	0.194	0.142	0.141	0.285	0.144	0.140	0.277
0.12	0.428	0.433	0.585	0.516	0.515	0.631	0.566	0.563	0.731	0.596	0.596	0.735
0.18	0.836	0.834	0.926	0.893	0.890	0.942	0.914	0.916	0.967	0.947	0.947	0.975
$\phi=0.0$												
0.00	0.047	0.047	0.047	0.048	0.046	0.033	0.047	0.047	0.071	0.051	0.051	0.074
0.07	0.163	0.162	0.167	0.220	0.221	0.200	0.235	0.238	0.310	0.246	0.247	0.309
0.14	0.591	0.592	0.619	0.682	0.682	0.681	0.704	0.704	0.792	0.728	0.728	0.789
0.21	0.938	0.938	0.952	0.959	0.959	0.964	0.963	0.963	0.982	0.979	0.979	0.988
$\phi=0.10$												
0.00	0.087	0.089	0.043	0.087	0.088	0.030	0.097	0.097	0.073	0.087	0.088	0.080
0.08	0.251	0.249	0.169	0.304	0.304	0.198	0.339	0.341	0.326	0.344	0.348	0.331
0.16	0.713	0.712	0.640	0.790	0.789	0.704	0.807	0.807	0.815	0.824	0.824	0.822
0.24	0.973	0.972	0.961	0.977	0.978	0.967	0.988	0.988	0.989	0.992	0.992	0.990

According to Table 4.35, when there exists no autocorrelation, the introduced test statistic(F_{MLE}) performs well in terms of its power. Even the AR(1) coefficient is positive but too small as 0.10, the F_0 and F_{LSE} does not follow F, that's why the powers for ϕ=0.10 can be misleading.

4.3 Simulated Type I Error and Power of the Linear Contrasts Test Statistic

In this part of the study, type I errors of the introduced test statistic for linear contrasts are presented, which are found by three ways. At the end of this part, the power calculation results are tabulated so that the comparison between the test statistic introduced by this study for linear contrasts with the corresponding classical theory test statistic can be done.

4.3.1 Simulated Type I Error of the Linear Contrasts Test Statistic

The significance level is used as 0.05 and acceptance rejection criteria is decided by the 0.025th and 0.975th quantile of the standard normal distribution. For each run the test statistic is compared with the theoretical quantiles, if it is bigger than the 0.975th quantile or smaller than the 0.025th quantile, then it is in the rejection region. Actually, the empirical type I error is the sample proportion of these rejections out of total number of simulation (out of total number of runs). If the conducted test statistic is acceptable, then the empirical type 1 error is expected to be close to the theoretical one, which is 0.05 in our case.

To calculate the empirical type I errors, treatment means should be adjusted in a way that the null hypothesis is hold because of the fact that type I error is the probability that rejecting the null hypothesis, when in fact it is true. In our case, the null hypothesis claims that the average of the first and third treatment mean is equal to the second treatment mean. Simulation scenarios are constructed in the view of this fact. In order to hold the null hypothesis, and also for the sake of easiness, three treatments are generated with each mean of 0. In this part degrees of freedom of student's t distribution are set to be ν=6, 9, 12 and 24 in order to see the performance of the test statistic under the different degrees of freedom range. This part of the simulation studies are divided into three parts as it is mentioned above.

4.3.1.1 Simulated Type I Error of the Test Statistic Based on Simulated Variances

By using the simulated variances, the empirical type I errors can be found, which are stated in Table 4.36. In each simulation run, the introduced test statistic is calculated with the simulated variances, and it is compared to the tabulated values in order to decide whether it is in the rejection region or not. Actually, the values stated in the Table 4.36 is the proportion of the number of rejected test statistics out of total test statistics (out of 10,000). The corresponding tables are as follow:

Table 4.36: Simulated Type I Error of the Linear Contrasts Test Statistic Based on Simulated Variances

	ν=6						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.051	0.050	0.051	0.055	0.049	0.049	0.052
100	0.051	0.050	0.056	0.050	0.049	0.051	0.052
200	0.053	0.048	0.052	0.050	0.049	0.048	0.051
500	0.048	0.054	0.053	0.050	0.048	0.051	0.050
1000	0.051	0.049	0.050	0.049	0.048	0.053	0.049
	ν=9						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.050	0.051	0.053	0.050	0.047	0.050	0.050
100	0.052	0.055	0.050	0.050	0.052	0.052	0.049
200	0.051	0.049	0.051	0.049	0.049	0.049	0.053
500	0.052	0.052	0.050	0.052	0.049	0.051	0.049
1000	0.050	0.051	0.050	0.045	0.052	0.046	0.051
	ν=12						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.050	0.051	0.052	0.048	0.051	0.049	0.045
100	0.048	0.052	0.050	0.050	0.049	0.051	0.047
200	0.049	0.048	0.051	0.049	0.050	0.053	0.052
500	0.049	0.049	0.052	0.050	0.048	0.046	0.049
1000	0.051	0.053	0.048	0.050	0.052	0.048	0.050
	ν=24						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.049	0.052	0.050	0.050	0.051	0.048	0.052
100	0.051	0.048	0.050	0.050	0.051	0.050	0.051
200	0.051	0.049	0.050	0.050	0.049	0.046	0.050
500	0.048	0.049	0.052	0.049	0.051	0.051	0.050
1000	0.051	0.049	0.050	0.049	0.052	0.050	0.050

As it can be clearly seen in Table 4.36, approximation of the distribution of the introduced test statistic to the normal distribution can be accepted because the type I error in each simulation scenario is close to theoretical value of the type I error, which is 0.05.

4.3.1.2 Simulated Type I Error of the Test Statistic Based on Fisher Information Matrix

The variance of treatment means can be found from the Fisher Information Matrix. In each simulation run, treatment mean, autoregressive coefficient and the scale parameter of the error terms are found. Besides this, for each run of the simulations, the variances of the treatment means are obtained by using the related diagonal element of the inverse Fisher Information Matrix. Actually these variances are the estimated variances of the treatment means since in order to calculate the variance of the treatment means, the necessary parameters are to be estimated. The null hypothesis is same with the hypothesis stated in section 4.3.1. While contructing the table related to this part, we faced with some fluctuation on the type I error values, that is, for the same criterias the resulting type I error is differentiated. The reason of this kind of situation may sometimes sourced by the nature of the simulation. Therefore, we recommend averaging the 10 type I error values to get rid of this fluctuations.

Table 4.37: Simulated Type I Error of the Linear Contrasts Test Statistic Based on Fisher Information Matrix

	ν=6						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.054	0.058	0.060	0.065	0.068	0.076	0.120
100	0.049	0.053	0.057	0.052	0.055	0.060	0.085
200	0.050	0.054	0.051	0.051	0.056	0.052	0.065
500	0.050	0.050	0.053	0.049	0.048	0.051	0.053
1000	0.049	0.051	0.051	0.052	0.053	0.053	0.053
	ν=9						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.058	0.061	0.061	0.064	0.065	0.072	0.128
100	0.054	0.055	0.058	0.059	0.058	0.063	0.086
200	0.047	0.047	0.055	0.053	0.059	0.057	0.069
500	0.052	0.050	0.050	0.053	0.052	0.054	0.054
1000	0.051	0.051	0.049	0.051	0.052	0.056	0.055
	ν=12						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.058	0.057	0.065	0.060	0.065	0.070	0.127
100	0.054	0.053	0.056	0.054	0.060	0.059	0.083
200	0.052	0.048	0.051	0.050	0.055	0.055	0.065
500	0.049	0.049	0.053	0.051	0.051	0.051	0.052
1000	0.052	0.052	0.044	0.050	0.053	0.052	0.052
	ν=24						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.060	0.057	0.056	0.064	0.065	0.075	0.126
100	0.051	0.052	0.057	0.054	0.058	0.060	0.084
200	0.051	0.050	0.050	0.053	0.053	0.057	0.070
500	0.051	0.047	0.049	0.048	0.051	0.052	0.061
1000	0.051	0.048	0.053	0.051	0.052	0.056	0.055

When the table of empirical type I error with variances based on information matrix is investigated, the normal approximation of the test statistic is acceptable. For each autoregressive coefficient; increase in treatment size results in better approximation. In some cases, a problem arises with large AR(1) coefficient, actually when it is near to 1, which can be solved by increasing size of sample. What the type I error should be is near to its theoretical value (0.05), and moreover the values on the table are acceptable on this point of view. Therefore, it can be concluded that the test statistic with variance obtained from Fisher Information Matrix follows standard normal distribution.

4.3.1.3 Simulated Type I Error of the Test Statistic Based on Observed Information Matrix

The variances of the treatment means can be found by using the observed information matrix in test statistic for linear contrasts. The related diagonal element of the inverse of the negative hessian matrix is used in place of variance of treatment means. The logic behind the algorithm of this part is actually same with in section 4.3.1.2 but there exists small difference which is sourced by the test statistic. In here the test statistic based on the variances found by hessian matrix rather than information matrix. The fluctuations are also the concern of this part. The same remedial measure is applied for this type I errors as well. That is, the values stated in Table 4.38 is in fact the average of 10 type I errors. The corresponding table of type I error is supplied below.

Table 4.38: Simulated Type I Error of the Linear Contrasts Test Statistic Based on Observed Information Matrix

	ν=6						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.057	0.059	0.062	0.065	0.068	0.076	0.120
100	0.050	0.053	0.057	0.054	0.055	0.061	0.087
200	0.051	0.055	0.052	0.051	0.057	0.053	0.064
500	0.050	0.051	0.053	0.049	0.048	0.051	0.053
1000	0.049	0.052	0.052	0.051	0.054	0.054	0.053
	ν=9						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.061	0.061	0.063	0.065	0.066	0.073	0.129
100	0.055	0.055	0.059	0.059	0.058	0.064	0.086
200	0.048	0.047	0.055	0.053	0.060	0.056	0.068
500	0.052	0.050	0.049	0.054	0.052	0.054	0.054
1000	0.052	0.051	0.050	0.050	0.052	0.056	0.055
	ν=12						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.059	0.058	0.066	0.060	0.066	0.072	0.129
100	0.055	0.054	0.056	0.056	0.059	0.059	0.084
200	0.052	0.049	0.051	0.049	0.056	0.055	0.066
500	0.049	0.050	0.052	0.052	0.051	0.051	0.053
1000	0.052	0.052	0.044	0.05	0.053	0.052	0.052
	ν=24						
n	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
50	0.062	0.059	0.057	0.066	0.067	0.077	0.127
100	0.052	0.054	0.058	0.055	0.059	0.060	0.085
200	0.051	0.050	0.051	0.053	0.053	0.057	0.070
500	0.051	0.0.47	0.053	0.051	0.052	0.052	0.062
1000	0.051	0.047	0.053	0.051	0.052	0.056	0.055

When Table 4.38 is studied, it can be clearly stated that the test statistic based on observed information matrix follows the desired distribution which is standard normal. The reason of this kind of inference is that the related type I errors can be considered as near to the theoretical value of it, which is set to be 0.05, for this work. Size of sample effect can also be noticed, as increase in sample size makes the type I errors to approach 0.05. On the other hand, when the autoregressive coefficient is close to 1, some deviations are observed in type I errors for each degrees of freedom cases. But, this can be eliminated by augmented length of treatment.

4.3.2 Simulated Power of the Linear Contrasts Test Statistic

In this subsection, the empirical power of the introduced test statistic with variance based on Fisher information matrix, here called as t_{MLE} is studied. 1,000 Monte Carlo simulations are run in order to form the empirical power table. In each run, test statistics are compared with the tabulated values in order to reach a conclusion. Here, the treatment means are created in an opposite way of the null hypothesis. In this study, the null hypothesis claims that the treatment mean of the second treatment is the average of the first's and third's. The aim of examining the power of a test statistic is to see the performance of the test statistic under the condition that the null hypothesis is false. Power of a test statistic is actually the probability of rejecting the null hypothesis when it is false in fact. These simulations are aimed to make the null hypothesis is wrong. This is done with the constant "*d*". By adding *d* to first and third treatments and subtracting 2*d* from the second treatment to make the null hypothesis wrong. Actually, constant *d* reduces the power to type I error when it equals to 0. According to different *d* values, the rejecting probabilities are tabulated. Moreover, Table 4.39 is created under different values of the student's t degrees of freedom and AR(1) coefficient in order to judge the performance of the powers under different situations. In this part, the sample size effect on the power is not considered that's why the simulations are carried out with only sample of size $n = 100$. In each calculation, rejecting probabilities are calculated with the standard normal tabulated values corresponding the 0.025th and 0.975th quantiles.

Table 4.39: Simulated Power of the t_{MLE} ; *n*=100

	$\nu = 6$	$\nu = 9$	$\nu = 12$	$\nu = 24$
d				
		$\phi = -0.8$		
0.00	0.069	0.051	0.055	0.050
0.04	0.369	0.380	0.371	0.427
0.08	0.875	0.898	0.909	0.928
0.12	0.995	0.996	0.996	1.000
		$\phi = -0.4$		
0.00	0.071	0.052	0.056	0.050
0.05	0.352	0.363	0.364	0.409
0.10	0.856	0.885	0.891	0.912
0.15	0.993	0.995	0.996	1.000
		$\phi = -0.2$		
0.00	0.070	0.053	0.053	0.050
0.06	0.368	0.383	0.382	0.436
0.12	0.874	0.900	0.909	0.928
0.18	0.994	0.996	0.996	1.000
		$\phi = 0.0$		
0.00	0.072	0.054	0.055	0.053
0.07	0.352	0.366	0.370	0.413
0.14	0.856	0.886	0.892	0.913
0.21	0.992	0.995	0.996	1.000
		$\phi = 0.2$		
0.00	0.076	0.057	0.055	0.053
0.08	0.316	0.321	0.337	0.356
0.16	0.793	0.837	0.831	0.861
0.24	0.987	0.991	0.993	0.999
		$\phi = 0.4$		
0.00	0.074	0.061	0.056	0.054
0.09	0.249	0.251	0.264	0.279
0.18	0.645	0.696	0.681	0.743
0.27	0.941	0.961	0.962	0.974
		$\phi = 0.8$		
0.00	0.096	0.080	0.078	0.078
0.24	0.256	0.257	0.263	0.275
0.48	0.595	0.632	0.629	0.688
0.72	0.898	0.923	0.924	0.941

Table 4.39 points out that as the difference in the means increase the power of the test statistic is increasing.

4.3.3 Simulated Power of the Other Test Statistics for Linear Contrasts

This part of the simulation studies chapter is created for observing the effect of ignoring non-validated assumptions on the power of the test statistic for linear contrasts concept. To do so, t_0 and t_{LSE} statistics, whose formula is given below, are compared with the introduced test statistic, named as t_{MLE}, which uses the variance coming from Fisher Information Matrix. The test statistic t_0 actually estimates the treatment mean from the normal classical formula and estimates σ^2 from the model (3.1) by LS methodology. The test statistic t_{LSE} uses the LS estimators of (3.1) for both treatment mean and also σ^2. Moreover, in order to get reasonable result from the power comparisions, we should ensure that the t_0 and t_{LSE} statistics follow the F distribution with *c*-1 and *nc*-*c* degrees of freedoms for this reason the type I error of these two test statistics are observed with *n* is 100 and ν is 9 scenario. The power comparisions are done for the test statistics stated below,

$$t_0 = \frac{\sum_{i=1}^{c} l_i \bar{y}_{i\cdot}}{\sqrt{\sum_{i=1}^{c} l_i^2 \frac{\tilde{\sigma}^2}{n}}} \text{ and } t_{LSE} = \frac{\sum_{i=1}^{c} l_i \tilde{\mu}_i}{\sqrt{\sum_{i=1}^{c} l_i^2 \frac{\tilde{\sigma}^2}{n}}}.$$

Table 4.40: Simulated Type I Errors of t_0 and t_{LSE}

t	ϕ=-0.8	ϕ=-0.4	ϕ=-0.2	ϕ=0.0	ϕ=0.2	ϕ=0.4	ϕ=0.8
t_0	0.003	0.012	0.037	0.081	0.162	0.298	0.742
t_{LSE}	0.004	0.012	0.039	0.081	0.161	0.300	0.744

While constructing the power comparision table, ϕ is chosen to be -0.05, 0.00 and 0.05.

Table 4.41: Simulated Power of the t_0, t_{LSE} and t_{MLE}; *n*=100

	$\nu=6$			$\nu=9$			$\nu=12$			$\nu=24$		
d	t_0	t_{LSE}	t_{MLE}	t_0	t_{LSE}	t_{MLE}	t_0	t_{LSE}	t_{MLE}	t_0	t_{LSE}	t_{MLE}
							$\phi=-0.05$					
0.00	0.072	0.072	0.060	0.061	0.057	0.050	0.049	0.048	0.059	0.044	0.044	0.055
0.07	0.428	0.424	0.428	0.384	0.384	0.401	0.398	0.396	0.429	0.371	0.373	0.435
0.14	0.916	0.916	0.933	0.910	0.910	0.916	0.940	0.940	0.954	0.933	0.933	0.954
0.21	0.998	0.998	0.998	0.998	0.998	1.000	0.999	0.999	0.999	0.999	0.999	0.999
							$\phi=0.00$					
0.00	0.115	0.114	0.048	0.090	0.088	0.059	0.064	0.065	0.054	0.067	0.067	0.066
0.07	0.442	0.442	0.371	0.417	0.418	0.367	0.400	0.402	0.360	0.426	0.428	0.414
0.14	0.877	0.877	0.841	0.892	0.892	0.873	0.908	0.908	0.902	0.907	0.907	0.903
0.21	0.998	0.998	1.000	0.996	0.996	0.998	0.994	0.994	0.995	0.998	0.998	0.998
							$\phi=0.05$					
0.00	0.155	0.158	0.062	0.121	0.122	0.056	0.108	0.107	0.061	0.088	0.090	0.059
0.07	0.428	0.429	0.293	0.431	0.429	0.314	0.417	0.418	0.313	0.389	0.385	0.319
0.14	0.864	0.865	0.781	0.863	0.864	0.808	0.881	0.880	0.831	0.882	0.883	0.831
0.21	0.992	0.992	0.987	0.997	0.997	0.996	0.994	0.994	0.992	0.998	0.998	0.994

The output of Table 4.41 can be interpretted as when the t_0 and t_{LSE} follow F distribution actually when making comparison is logical, the test statistic introduced by this study performs well in terms of its power. That is, when type I error of the statistics t_0 and t_{LSE} is not around the 0.05, comparing those test statistics is not logical that's why in power of these two test statistics are observed higher than t_{MLE}. Another point here is that while interpretting the outputs, it is to be recalled that the σ^2 is estimated by using the model (3.1).

CHAPTER 5

APPLICATION

In this chapter of the thesis, we apply the introduced process of the one-way ANOVA methodology to the real life data set so that the procedure discussed for this study is illustrated. In the first part of the chapter the data set is described, and the analysis and the final conclusions are done in the next part of this chapter.

5.1 Data Description

The data set is obtained from International Monetary Fund Data Bank (IMF, International Financial Statistics, 2017). We use three different countries, which are Bulgaria, El Salvador and Pakistan. The yearly unemployment rates of these countries are the main interest. The data are taken for the time interval between 1990 and 2016, which means each country has 27 observations. Even the size of the treatments are small to conduct a time series analysis, this is not considered as a problem because the main emphasize of the this part is to illustrate the application of the methodologies introduced throughout the study.

5.2 Statistical Analysis

In this section, there countries are examined under time series framework. The acf and pacf plots, normality test, stationarity test, homogenity of variances, independency test and the quantile-quantile plot of the residuals of the conducted models are provided. Here, the data of each country are performed by the natural logarithm transformation twice and all the analysis are done with these transformed data.

5.2.1 Order Selection of the Unemployment Rate Series

By investigating the autocorrelation and partial autocorrelation plots, the order of the time series each belongs to an unemployment series of countries can be determined.

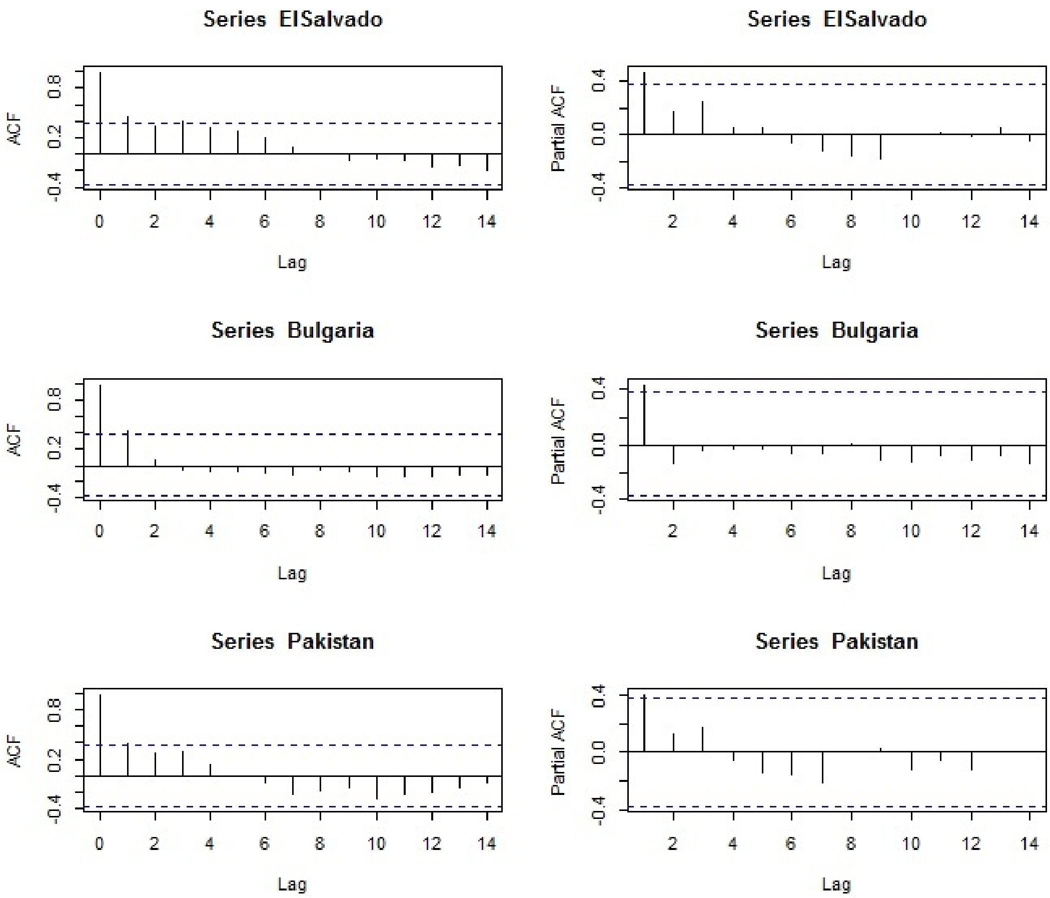

Figure 5.1: ACF and PACF Plots of Countries

The ACF and PACF plots suggest that each series follow AR(1) process because of the fact that there is only one spike at lag 1 when the PACF of each series examined.

5.2.2 Stationarity Test

In order to get valid and reliable results, the series analysed are to be stationary. For this reason, stationarity tests are to be conducted for each series that we are interested. To do so, Phillips-Perron test whose null hypothesis claims that there exists a unit root aganist the alternative hypothesis of stationarity is used. The results of this test are given as follows:

- Phillips-Perron Unit Root Test for the series belongs to El Salvador

  Dickey-Fuller = -4.3302, Truncation lag parameter = 2, p-value =**0.01143**

- Phillips-Perron Unit Root Test for the series belongs to Bulgaria

  Dickey-Fuller = -5.2556, Truncation lag parameter = 2, p-value = **0.01**

- Phillips-Perron Unit Root Test for the series belongs to Pakistan

  Dickey-Fuller = -4.3706, Truncation lag parameter = 2, p-value =**0.01001**

The results of Philips-Perron tests point out the stationarity of the each series since the p-values of each test is less than the significance level of $\alpha = 0.05$.

5.2.3 Determining the Proper Distribution Fit

Normality Test

In this section normality of the error terms are examined. By using the Shapiro-Wilk test for normality, whose null hypothesis claims the normality, it is tried to reach a conclusion about whether the distribution of the error terms is normal or not. The series for El Salvador, Bulgaria and Pakistan are fitted to AR(1) model and the corresponding p-values belongs to Shapiro-Wilk tests are presented below respectively;

- Shapiro-Wilk normality test for the error terms of the series belongs to El Salvador

  W = 0.91987, p-value = **0.03919**

- Shapiro-Wilk normality test for the error terms of the series belongs to Bulgaria

  W = 0.89915, p-value = **0.01279**

- Shapiro-Wilk normality test for the error terms of the series belongs to Pakistan

  W = 0.81663, p-value = **0.0002694**

According to the results of Shapiro-Wilk tests, the error terms of models, each belongs to a country, are not distributed normally because of the fact that their p-values are smaller than the significance level of 0.05, that's why the null hypothesis of each test is rejected.

Examining the Q-Q Plots

In this section, it is tried to determine the degrees of freedom of the student's distribution for each error terms of the models. To do so, the quantile-quantile plots are used.

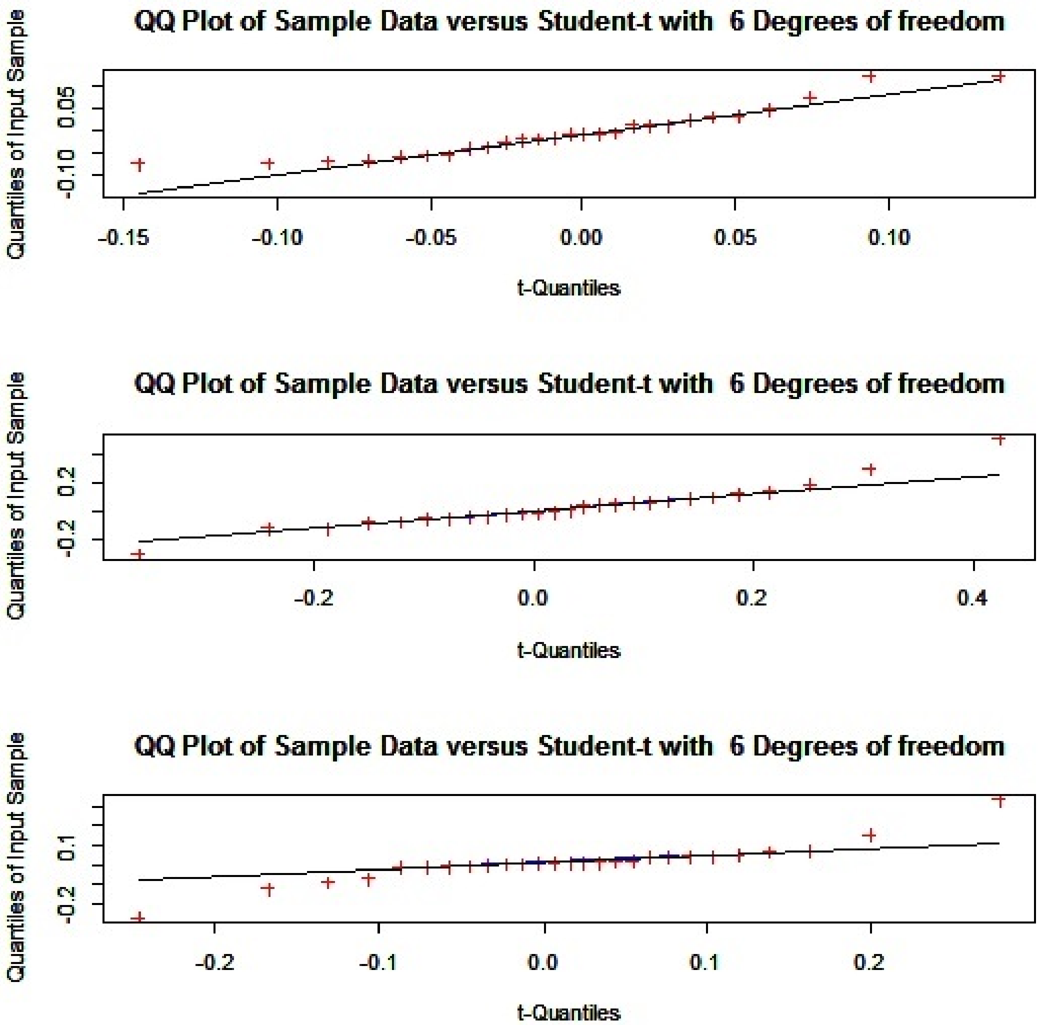

Figure 5.2: Q-Q Plots of Error Terms for respcetively El Salvador, Bulgaria and Pakistan

Figure 5.2.3 points out that the error terms of the models perfom well under the Student's t distribution with 6 degrees of freedom.

5.2.4 Parameter Estimation and Hypothesis Testing in One-Way ANOVA

According to the results from the Sections 5.2.1, 5.2.2 and 5.2.3, it can be said that each of the series follow stationary AR(1) process and the error terms of each conducted AR(1) models do not follow normal distribution instead the error terms of the models behave well under student's t distribution with 6 degrees of freedom. That's why, in this section we apply the parameter estimation process stated in Section 3. Moreover by using the estimated paramters the representative model can be gathered in a way that is stated below

$$\hat{y}_{i,t} = 0.469\hat{y}_{i,t-1} + (1 - 0.469)\hat{\mu}_i \tag{5.1}$$

where $\hat{\mu}_i$'s are the treatment means and $\hat{y}_{i,t}$ is the ln(ln(unemployment rate)).

In the following parts, the one-way ANOVA assumptions are checked.

Normality Assumption

The error terms follow student's t with $\nu = 6$ and this is provided by Section 5.2.3.

Variance Homogenity Assumption

One of the most important assumption of the ANOVA model is that the variance of the error terms of each group are to be homogeneous. In order to test the homogenity of the variaces of the error terms of the model (5.1), Levene's test is used because the normality assumption is not validated. The result of this test is as follows:

$$\begin{aligned} H_0 &: \sigma_1^2 = \sigma_2^2 = \sigma_3^2 \\ H_1 &: \sigma_i^2 \neq \sigma_j^2 \quad \text{for at least one pair of i and j.} \end{aligned}$$

Levene's Test for Homogeneity of Variance (center = median)

	df	F value	Pr(>F)
Series	2	2.2616	**0.111**

The p-value of the Levene's test suggests that the null hypothesis claiming the equality of the variances is failed to be rejected at 0.05 significance level. Therefore, it can be concluded that the variances of the error terms for the series are equal.

Independency

The independency of the error terms of the conducted models are tested by Box-Pierce test. The null hypothesis of the Box-Pierce test claims the independency of the error terms. The related test results are supplied respectively for El Salvador, Bulgaria and Pakistan.

- Box-Pierce test for the model for El Salvador: p-value = **0.2618**
- Box-Pierce test for the model for Bulgaria: p-value = **0.3178**
- Box-Pierce test for the model for Pakistan: p-value = **0.02381**

The results of the Box-Pierce tests support the independencies of the error terms for each model at 0.01 significance level.

Since all the model assumptions of the ANOVA are satisfied, the introduced test statistic can be applied. Therefore, the null hypothesis stated below is tested with the introduced test statistics provided in this thesis as,

$$\begin{aligned} H_0 &: \mu_1 = \mu_2 = \mu_3 \\ H_1 &: \mu_i \neq \mu_j \quad \text{for at least one pair of i and j.} \end{aligned}$$

By using the test statistic (3.9), with the $\hat{V}(\hat{\mu}_i)$ is obtained by Fisher Information

Matrix,

$$F = \frac{\sum_{i=1}^{c} \left(\frac{\hat{\mu}_i - \hat{\mu}}{\sqrt{\hat{V}(\hat{\mu}_i)}} \right)^2 / (c-1)}{\sum_{i=1}^{c} \sum_{t=1}^{n} \left(\frac{y_{i,t} - \hat{\mu}_i}{\sqrt{\hat{V}(y_{i,t})}} \right)^2 / (nc-c)} \sim F_{c-1,nc-c}.$$

By using the estimated values of the unknown parameters with degrees of freedom as 6, the calculated F statistic is found as, F=32.69422. Since calculated F statistic is greater than the F tabulated value, which is $F_{3-1,27*3-3} = F_{2,78} = 3.113792$, the null hypothesis can be rejected which suggests that the mean of these three log-log uneployment rates are not equal to each other.

Since the null hypothesis which claims the equality of the log-log unemployment rates of the countries is rejected, now the source of this inequality is tried to be discussed by using linear contrasts methodology. In order to have an idea of the means of the related unemployment rates, the following plot is obtained. Here, the red lie belongs to ln(ln(unemployment rate)) of El Salvador, the blue one is for Bulgaria and the other one is for Pakistan. By looking at Figure 5.3, the linear contrasts are determined.

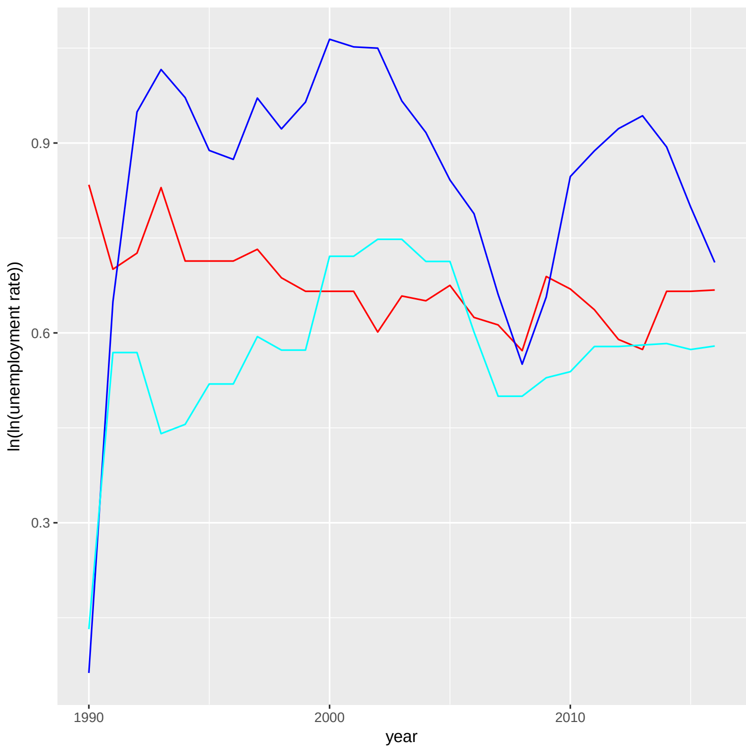

Figure 5.3: Time Series Plot of ln(ln(Unemployment Rate)) of Countries

The related null hypothesis are as follows,

$$H_0 : \mu_1 = \mu_3, \tag{5.2}$$

$$H_0 : \mu_1 - 2\mu_2 + \mu_3 = 0. \tag{5.3}$$

where the μ_1, μ_2 and μ_3 represent the mean of log-log unemployment rate of EL Salvador, Bulgaria and Pakistan respectively.

The first test statistic is,

$$t_1 = \frac{\sum_{i=1}^{c} l_{1i}\hat{\mu_i}}{\sqrt{\sum_{i=1}^{c} \hat{V}(l_{1i}\hat{\mu_i})}} = -1.894165 \tag{5.4}$$

where l_1=(1, 0, -1), for testing null hypothesis stated under (5.2). This resut points out that mean of log-log unemployment rate of El Salvador and Pakistan are equal for the specifed time interval since the absolute related value of the calculated test statistic is not greater than 1.96, then it is failed to be rejected at 0.05 significance level.

Therefore, we try to investigate the relation between those two countries with Bulgaria, and specify the null hypothesis as the average mean of log-log unemployment rates of El Salvador and Pakistan equals the mean of the log-log unemployment rate of Bulgaria. Actually, since mean of log-log unemployment rates of El Salvador and Pakistan are equal to each other, the average of them also gives the value of mean of log-log unemployment rate of El Salvador or Pakistan. That is, the null hypotesis actually test the equality of the mean of log-log unemployment rate of the Bulgaria with El Salvador or Pakistan. The related test statistic is,

$$t_2 = \frac{\sum_{i=1}^{c} l_{2i}\hat{\mu_i}}{\sqrt{\sum_{i=1}^{c} \hat{V}(l_{2i}\hat{\mu_i})}} = -9.810371 \tag{5.5}$$

where l_2=(1,-2,1), for 5.3.

The corresponding result marks the rejecting the null hypothesis because the absolute of the calculated test statistic is greater than the tabulated one, which is 1.96. This is also implying that the mean of the log-log unemployment rate of the Bulgaria is not equal to the mean of the log-log unemployment rate of El Salvador or Pakistan.

Moreover, it can be clearly stated that the difference in the means of the unemployment rates is sourced by the inequality of the mean of the log-log unemployment rate of Bulgaria with the other two countries' means of log-log unemployment rate.

Here we do not aim to interpret the result under the scope of economy, our main target is to illustrate the process of one-way ANOVA with non-validated assumptions case by using the introduced procedures stated througout the study.

CHAPTER 6

CONCLUSION

In this thesis study, the main motivation is one-way ANOVA test under the first order autoregressive model. The objective is actually to introduce a test statistic for one-way ANOVA when the normality and the independency assumptions are failed to validate. The reason why the normality assumption is not met is sourced by the fact that error terms are assumed to be distributed as Student's t in this thesis. Moreover, because of each treatment is assumed to be followed AR(1) process, which causes the autocorrelated observations of the treatments, the independency assumption is not met as well. Under the same assumptions, a test statistic for linear contrasts concept is also introduced.

The first chapter of this thesis includes an introduction and revision of literature related to the studied topics of this thesis. The previous studies found a place in the literature is discussed and our point of view is presented. This part also includes a content of the study.

The second part of the study encapsulates the methodologies related to thesis topic. The one-way analysis of variance and the linear contrasts concepts are given in detailed with their assumptions, parameter estimation techniques, hypothesis tests and the test statistics. Moreover, the related time series model which is AR(1) process and its model and assumptions are given. In addition to this, parameter estimation procedure of the AR(1) model is given by name because the related technique of estimation is given in the next chapter.

The third part is actually the main part of this study because of the fact that it contains the objective model of the study. Related model assumptions, hypothesis tests and the affilated test statistics are welcomed in this part. Furthermore, derivations of the model and distribution parameter estimation procedure are also included. Here, the test statistics for one-way ANOVA and linear contrasts are introduced, each procedure has three type of test statistics with respect to way of obtaining the variance of the treatment means. Actually three way are used for estimating the variance of the treatment means. The first way is the simplest one because it uses the basic simulated variances in place of variance of treatment means. Using the related diagonal of the inverse of the Fisher Information Matrix is the second way of finding the variance of treatment means. The last way of obtaining variance of the treatment means is using the related diagonal of the negative of inverse hessian matrix, actually this is called as observed information matrix.

The next chapter contains the simulation studies. This part is divided into three sections. The estimation of the parameters creates the first section of this chapter. In this part, in order to judge the performances of LS and ML estimators, MSEs and REs of the parameters are obtained. The results from the related tables show that for small sample sizes, performances of the LS estimations under the AR(1) coefficient near to 1 are not satisfying. The related conditions affect the MSE values of the LS estimators, that is when the correlation is positively large and size of treatments are small, LS estimations are not acceptable. Therefore, because of the small sample performances of LS estimators under large AR(1) coefficient, the ML estimators are preferred for the upcoming studies for this thesis. The values of REs are expected to approximate 100 as size of treatment is increased just because the performances of the LS estimators become closer to the ML ones'. By investigating the tables in this point of view, except for some scenarios the REs of LS estimators become competitive to the ML estimators as the size of treatment is increasing.

The second part of the simulation studies is formed by the related context of one-way ANOVA. The type I error tables, which are one for simulated variances, second one for variance coming from Fisher Information Matrix and the last one is for variance coming from observed information matrix, are stated. The results of Table 4.30 shows that the test statistic based on simulated variances perfoms well with the desired degrees of freedom of F distribution. However, while contrusting the type I error tables by using Fisher and observed information matrices, we faced with a fluctuation. Therefore, this problem is tried to be solved by using the remedial measure of averaging the 10 type I errors. Furthermore, Tables 4.31 and 4.32 point out that except for the small sample size under large AR(1) coefficient cases, performances of the test statistics using these two variances are satisfying. In this part of the chapter, power of the introduced test statistic, which uses the variance coming from Fisher Information Matrix, is studied. The results mark that as the difference in the treatment means is increased, the power of the test statistic approaches to 1 as it is expected. Moreover, the test statistic based on LS estimators and normal classical test statistics are compared with respect to their power. The outputs of this comparison can be interpretted as, when these three test statistics are comparable, the power of the test statistic introduced for this study is promising.

The third part of the simulation studies chapter encapsulates the related contents of linear contrasts. In the first section of this part, the type I error tables are studied. Table 4.36, which uses the simulated variances of treatment means, shows that the test statistic behaves well under the standard normal distribution. The fluctuation problem also occurs when dealing with Fisher and observed information matrices, and the same remedial measure is applied here as well. Therefore, according Tables 4.37 and 4.38, except for the cases in which the treatment sizes are small and the AR(1) coefficent is near to 1, these test statistics are seen to follow standard normal distribution. The power of the test statistic, which uses the variance coming from the Fisher Information Matrix, is given. When Table 4.39 is examined, it can be obviously seen that as the constant "d" raising, the power of the test statistic converges to 1. That is, the intoduced test statistic can detect almost all the wrong null hypothesis to reject. In addition to this, power comparisions are done with normal test statistic and LS based test statistic for linear contrasts. According to the related power table, the introduced test statistic performs well.

The application chapter includes the real life application of the introduced hypothesis testing procedure. To do so, the unemployment rate data for the countries El Salvador, Bulgaria and Pakistan for the time interval 1990 and 2016. Related model assumptions and the analysis are given in detail.

To conclude, this thesis study is motivated by the non-validated model assumptions of the one-way ANOVA. The non-normality and the dependency are the main objectives. The case of that error terms are distributed independently and identically Student's t and the treatments follow AR(1) process is studied. Three test statistics are introduced depending on the way of finding the variances of the treatment means, which are using simulated variances, using Fisher Information Matrix and using the observed information matrix. By using the same ways, three test statistics are introduced for linear contrasts concepts as well. Related type I errors and powers of the test statistics are investigated as well as the power comparisons of the corresponding classical test statistics. At the end, it can be observed that the performances of the introduced test statistics are promising. In the future works, this study can be generalized to AR(p) process and also other distribution families can be studied. Also, other type desings like two way anova will be considered as well as assuming different models for each treatment cases can be studied. The related R codes of the study can be supplied if they are requested.

REFERENCES

[1] Adke, S.R. (1986). One way ANOVA for dependent observations. *Communications in Statistics - Theory and Methods*, 15(5), 1515-1528.

[2] Albers, W. (1978). Testing the mean of a normal population under dependence. *The Annals of Statistics*, 6(6), 1337-1344.

[3] Anderson, H. A., Bjørn, E. J. and Schou, G. (1981). Two-Way analysis of variance with correlated errors. *International Statistical Review*, 49(2), 153-167.

[4] Anderson, P. L. and Meerschaert, M. M. (1998). Modeling river flows with heavy tails. *Water Resources Research*, 34(9), 2271-2280.

[5] Baldessari, B. A. (1987). Experimental desings with dependent observations: New results and comments on previous results. *Communications in Statistics - Theory and Methods*, 16(3), 785-803.

[6] Champagnat, N., Deaconu, M., Lejay, A., Navet, N., and Boukherouaa, S. (2013). *An Empirical Anaysis of Heavy-Tails Behaviour of Financial Data: The Case for Power Laws*. Retriewied from: https://hal.inria.fr/hal-00851429/document

[7] Clinch, J. J. and Keselman, H. J. (1982). Parametric alternatives to the analysis of variance. *Journal of Educational Statistics*, 7(3), 207-214.

[8] Çelik N., Şenoğlu B. and Arslan, O., (2015), Estimation and testing in one-way ANOVA models using skew normal distributions, *Revista Colombiana De Estadistica*, 38, 75-91.

[9] David, F. N. and Johnson, N. L. (1951). he Effect of non-normality on the power function of the F-Test in the analysis of variance. *Biometrica*, 38(1/2), 43-57.

[10] Gastwirth, J. L. and Rubin, H. (1971). Effect of dependence on the level of some one-sample tests. *Journal of the American Statistical Association*, 66(336), 816-820.

[11] Geary, R. C. (1947). Testing for normality. *Biometrica*, 34(3/4), 209-242.

[12] Górecki, T. and Smaga, Ł. (2015). A Comparision of tests for the one-way ANOVA problem for functional Data. *Computational Statistics*, 30(4), 987-1010.

[13] Hasselman, B. (2016). nleqslv: Solve Systems of Nonlinear Equations. R package version 3.0.3. https://CRAN.R-project.org/package=nleqslv

[14] Hernández-Camposa, F., Marronb, J. S., Samorodnitskyd, G. and Smitha, F. D. (2004) Variable heavy tails in internet traffic. *Performance Evaluation*, 58, 261–284.

[15] Horváth, L. and Rice, G. (2015). Testing equality of the means when the observations are from functional time series. *Journal of Time Series Analysis*, 36(1), 84-108.

[16] IMF Data, International Financial Statistics. (2017). Retrieved from http://data.imf.org

[17] Iorio, M. D., Müller, P. and MacEachern, S. N. (2004). An ANOVA model for dependent random measures. *Journal of the American Statistical Association*, 99(465), 205-215.

[18] Katz, R.W., 2002. Do weather or climate variables and their impacts have heavy-tailed distributions? Preprints, AMS 16th Conference on Probability and Statistics in the Atmospheric Sciences, Orlando, FL, pp. J84–J87.

[19] Keselman, H. J., Wilcox, R.R. and Lix, L.M. (2003). A Generally robust approach to hypothesis testing in independent and correlated groups. *Psychophysiology*, 40 (4), 586-96.

[20] Lin, C. H. and Shen, S. S. (2006) Can the student-t distribution provide accurate value at risk?. *The Journal of Risk Finance*, 7(3), 292-300.

[21] Liu, G., Shao, Q., Lund, R. and Woody, J. (2016). Testing for seasonal means in time series data. *Environmetrics*, 27(4), 198-211.

[22] Lund, R. , Liu, G. and Shao, Q. (2016). A new approach to ANOVA methods for autocorrelated data. *American Statistical Association*, 70(1), 55-62.

[23] Nash, J. C. and Varadhan, R. (2011). Unifying Optimization Algorithms to Aid Software System Users: optimx for R. Journal of Statistical Software, 43(9), 1-14. URL http://www.jstatsoft.org/v43/i09/.

[24] Olejnik, S. F. and Algina, J. (1985). Type I error rate and power of rank transform ANOVA when the populations are non-normal and have equal variance. *Florida Journal of Educational Research*, 27(1), 61-81.

[25] Pavur, R. J. and Lewis, T. O. (1982). Test procedure for the analysis of experimental designs with correlated nonnormal data. *Communications in Statistics - Theory and Methods*, 11(20), 2315-2334.

[26] Pavur, R. (1988). Type I error rates for multiple comparision procedures with dependent data. *The American Statistician*, 42(3), 171-173.

[27] Pearson, E. S. (1931). The anaysis of variance in cases of non-normal variation. *Biometrica*, 23(1/2), 114-133.

[28] Platen, E. and Sidorowicz, R. (2007). Empirical evidence on Student's-t log-returns of diversified world stock indices. Quantitative Finance Research Centre, (194).

[29] R Core Team (2016). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/.

[30] Sardeshmukh, P. D., Compo, G. P. and Penland, C. (2015). Need for caution in interpreting extreme weather statistics. *Journal of Climate*, 28, 9166-9187.

[31] Scariano, S. M. and Davenport, J. M. (1987). The effects of violations of independence sssumptions in the one-way ANOVA. *The American Statistician*, 41(2), 123-129.

[32] Shumway, R. H. (1970). Applied regression and analysis of variance for stationary time series. *Journal of the American Statistical Association*, 65(332), 1527-1546.

[33] Srivastasa, A. B. L. (1959). Effect of non-normality on the power of the analysis of variance test. *Biometrica*, 46(1/2), 114-122.

[34] Sutradhar, B. C., MacNeill, I. B., and Sahrmann, H. F. (1987), "Time Series Valued Experimental Designs: One-Way Analysis of Variance With Autocorrelated Errors," *Time Series and Econometric Modelling*, eds. I. B. MacNeill and G. J. Umphrey, 113–129, New York, Springer-Verlag.

[35] Şenoğlu, B., and Acıtaş, Ş. (2014). *İstatistiksel Deney Tasarımı Sabit Etkili Modeller* (3. Baskı). Ankara: Nobel.

[36] Şenoglu, B. and Bayrak, Ö. T. (2016). Linear contrasts in one-way classification AR(1) model with gamma innovations. *Hacettepe Journal of Mathematics and Statistics*, 45(6), 1743-1754.

[37] Şenoglu, B. and Tiku, M. (2001). Analysis of variance in experimental design with nonnormal error distributions. *Communication Statistical Theory Methods* 30, 1335–1352.

[38] Tiku, M. L. (1971). Power function of the F-Test under non-normal stiuations. *Journal of the American Statistical Association*, 66(336), 913-916.

[39] Wei, W. S. W. (2006). *Time Series Analysis: Univariate and Multivariate Methods*, Pearson Education, Inc.

[40] Yılmaz, Y. E. (2004). Experimental design with short-tailed and long-tailed symmetric error distributions. METU Ms. Thesis.

[41] Zwiers, F. W. and Vonstorch, H. (1995). Taking serial correlation into account in tests of the mean. *Journal of Climate*, 8, 336-351.