MEASURING LONGEVITY RISK ON SECOND PILLAR PENSION SYSTEM: TURKEY CASE

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ABSTRACT

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Pension systems, that have had the brightest period almost all over the world after World War II, began to have some difficulties with accessing maturity times of these systems. Financial crisis and doubts about the sustainability have caused countries to revise their pension systems. As a result, many countries have applied multipillar pension system which was developed by the World Bank in 1994. Second pillar pension system (SPPS) is the second step of the multi pillar pension system. Today, longevity risk is one of the series problems that faced by pension systems because of the increasing life expectancies especially in the last few decades. Longevity risk refers to the uncertainty surrounding future developments in mortality and life expectancy. The aim of this study, to analyze all aspects of SPPS and longevity risk on this system, and also to evaluate the longevity risk on pensions in Turkey. For these purposes, firstly, pension systems and longevity risk are explained in detail. And then, SPPS pensions are calculated in 2057 when insured persons were assumed to retire. For pension calculations, gender specific life tables are generated with Lee-Carter Method. Then, Monte Carlo Simulation is applied in order to measure the longevity risk on pensions. Finally, sensivity analysis are done to see the time effect on SPPS pensions.

Keywords : Lee-Carter method, longevity risk, second pillar pension system.

ÖZ

İKİNCİ SÜTUN EMEKLİLİK SİSTEMLERİ ÜZERİNDEKİ UZUN ÖMÜRLÜLÜK RİSKİNİN ÖLÇÜLMESİ: TÜRKİYE UYGULAMASI

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II. Dünya Savaşı'ndan sonra refah dönemini yaşayan emeklilik sistemlerinin olgunlaşması bazı promlemleri de beraberinde getirmiştir. Finansal krizler ve sistemin sürdürülmesine dair endişeler söz konusu sistemlerin yeniden gözden geçirilmesine sebep olmuştur. Sonuç olarak, Dünya Bankası tarafından geliştirilen çok sütunlu emeklilik sistemi birçok ülke tarafından uygulanmaya başlanmıştır. İkinci sütun emeklilik sistemi, çok sütunlu emeklilik sisteminin ikinci basamağıdır. Bugün, uzayan yaşam beklentisiyle birlikte artan uzun ömürlülük riski emeklilik sistemlerinin en önemli problemlerinden birisidir. Uzun ömürlülük riski, gelecekteki ölüm ve yaşam beklentisinin belirsizliğinden kaynaklanmaktadır. Bu çalışmanın amacı, ikinci sütun emeklilik sistemini ve bu sistem üzerindeki uzun yaşam riskini tüm yönleriyle analiz etmek ve Türkiye'deki emekli aylıkları üzerindeki uzun ömürlülük riskini belirlemektir. Bu doğrultuda, emeklilik sistemleri ve bu sistemler üzerindeki uzun yaşam riski anlatılmış ve ayrıca sigortalı insanların emekli olacağı varsayılan 2057 yılına ait ikinci sütun emekli aylıkları hesaplanmıştır. Hesaplamalar için Lee-Carter yöntemi ile cinsiyete özel hayat tabloları üretilmiştir. Emekli aylıkları üzerindeki uzun yaşam riskini ölçebilmek için ise Monte Carlo simülasyonuna başvurulmuştur. Son olarak, ikinci sütun emekli aylıkları üzerindeki zaman etkisini görebilmek adına duyarlılık analizleri yapılmıştır.

Anahtar Kelimeler: Lee-Carter yöntemi, uzun ömürlülük riski, ikinci sütun emeklilik sistemleri

To My Family

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LIST OF ABBREVIATIONS

ACF	Autocorrelations Function
APC	Age-Period-Cohort
ARIMA	Autoregressive Integrated Moving Average
DB	Defined Benefit
DC	Defined Contribution
EGR	Economic Growth Rate
GDP	Gross Domestic Product
GLM	Generalized Linear Modelling
G7	Group of Seven
ISSA	International Social Security Association
OECD	Organization for Economic Co-operation and Development
PACF	Particular Autocorrelations Function
PAYG	Pay as You Go
P-Splines	Penalised Spline Regression
SSI	Social Security Institution
SPPS	Second Pillar Pension System
SPSS	Statistical Package for the Social Science
SVD	Singular Value Decomposition
TL	Turkish Lira
TRHA	Türkiye Kadın-Erkek Hayat Anüite
WB	World Bank

CHAPTER 1

INTRODUCTION

The social security concept is related to the human need for eliminating or mitigating the results of various risks. Even though this need is as the same age as the human history, social security systems in the modern sense were emerged with the industrialization process. Today, however social security systems are composed of social insurance, social assistance and social services, social insurance is the most common and most effective implementation of it. In additon, invalidity, old-age and survivors insurances, which constitute the social insurance, are named as a pension system. After the Second World War (between 1950-1975) pension systems had lived their brightest period. In this period, the expansion of the coverage of pension systems and the rise in pensions have caused to increase the number of the retired persons and liabilities. With all these, changing demographic, social and labour environment and the maturation of pension systems have caused them to enter a difficult fiscal turn especially in the second half of 1970's.

The financial crisis and concerns over the sustainability have caused to re-examine the pension systems in many countries especially from the beginning of 1990's. As a result, countries have made some parametric reforms to increase incomes and/or to decrease expenditures and some radical structural reforms such as transition to a new pension system.

The framework of the structural reforms in the pension systems constitutes the multi-pillar pension modal proposal that set by the World Bank (WB) with a report called "Averting the Old Age Crisis" in 1994. In the report, the first pillar pension system is financed by pay as you go (PAYG) method and benefits are determined with defined benefit plans. This system is managed by the state and participation is mandatory. In addition, the second pillar pension system (SPPS) is financed by funded method and the benefits are determined with defined contribution (DC) plans. This system is managed by private insurance companies and participation is mandatory. It can be designed as occupational pension plans or individual savings accounts. Moreover, the third pillar pension system is also financed by funded method and benefits are also determined with DC plans. This system is also managed by private insurance companies, but the participation is voluntary.

On the other hand, the Turkish pension system has also begun to suffer financial problems at the beginning of 1990's. As a result, the first comprehensive parametric reform was implemented in 1999 and the second in 2008. Both of them are designed to increase revenues and decrease expenditures. Furthermore, the implementations such as private pension system and auto-enrollment of private pension system can be given as examples of the structural reforms.

It can be said that, the existing Turkish Pension System has a multi-pillar pension structure. It mainly consist of mandatory public pension system as a first pillar. Additionally, the voluntary private pension system has been implemented since 2003 and this system constitutes the third pillar. On the other hand, workers who are employed under item (a) and (c) of paragraph one of article 4 Law number 5510 and who are under the age of 45 were started automatically include in private pension system since January 1, 2017 [7]. So, auto-enrollment private pension system has started for many workers in Turkey. This implementation could be thought as a first step to Turkish wide-ranging second pillar. So, in this study, this new system is called SPPS in Turkey. Moreover, there are some occupational pension systems, which are specific to certain occupational groups, could be thought as narrow-scoped second pillar.

Over the last decades, the expected life times have been increasing rapidly in the most of the countries. The life expectancy at birth had been increased 23.8 years from 1950 to 2015 in the world [99]. Moreover, even if it would be in a slower pace, it is expected to rise over time without an upper limit. Of course, longer life expectancy is an excellent new for humanity. But, it is very important to know how long people will live in order to calculate pensions and annuities. Therefore, if the necessary precautions were not taken, the financement of the pension systems would be affect from these life gains negatively. Unfortunately, lots of pension funds around the world, do not fully take into account of future improvements in life expectancy and mortality. This may lead to incorrect and inadequate calculation of liabilities and consequently huge financial loses for insurance companies or retired persons would be inevitable. Therefore, it is so obvious that, prediction of future expected life times is a very important issue for all aspects of pension system.

On the other hand, predicting future mortality and life expectancy depend on a lot of unknown parameters, in other words, they are uncertain. This uncertainity produces a longevity risk. The longevity risk can be defined as the uncertainity surrounding future development in mortality and life expectancy. This is the risk of populations living longer then expected. So, mortality rates and life expectancies are the main inputs to measure the longevity risk.

However modelling mortality is a very long history, forecasting mortality and life expectancy is a more recent work. There are three different approaches to predict age specific mortality and life expectancy and these are expectation, explanation and extrapolation. Extrapolative models are the most commonly used methods which are employed by actuaries in order to calculate pension liabilities. Recently, two- and three-factor underlying models are the more popular ones within the extrapolative approaches. The Lee-Carter model is a strong two-factor extrapolative model projecting future mortality. Since its introduction in 1992, the Lee-Carter model, its variants and extentions have been extensively used in actuarial and demographic applications. They have been applied to the populations in United States, in Group of seven (G7) countries as well as in many countries. On the other hand, Generalized Linear Modelling (GLM) and Penalised Spline Regression (P-Splines) are the other two-factor extrapolative models. Moreover, the Lee-Carter Age-Period-Cohort (APC) model is an encouraging three-factor extrapolative model even if it has not yet comprehensively evaluated.

1.1 Literature Survey

In the literature, there are many studies about forecasting mortality and life expectancy in all three approaches: expectation, extrapolation and explanation. However, actuaries and demographers use predominantly extrapolative ones. So, there are lots of studies about the extrapolation method since it is the basis of a lot of mortality and life expectancy forecasting method. Generally, time series methods are used in extrapolation. In time series methods, zero-factor underlying models are rarely used. Besides, one-factor underlying models are seen inadequte so they are used in the case of lack of necessary data. On the other hand, two- and three-factor underlying models are commonly used in recent years. One of the two-factor underlying model is called GLM. Renshaw (1991) [82] had seen that a lot of models, that are used by actuaries for graduation purposes are specific examples of GLMs. Therefore, in the studies done by Renshaw et al. (1996) [84] and Sithole et al. (2000) [95] GLMs are used to forecast mortality without using additional models. In these studies, Poisson GLM is used in order to model and forecast forces of mortality. The other one of the one-factor underlying model is called as P-Splines model. In the study Eilers and Marx (1996) [35], this model was suggested as the extension of GLM models. Moreover, in the study Currie et al. (2004) [29], it is shown how the P-Splines model can be extended in order to smooth and forecast of two-dimensional life tables. On the other hand, the most commonly used two-factor underlying method is called as the Lee-Carter model. It was introduced by the study Lee and Carter (1992) [62]. In this study the Lee-Carter model is fitted to the matrix of United States mortality rates from 1933 to 1987 using the Singular Value Decomposition (SVD) method. This method accounts almost all the variance over time in age-specific mortality rates as a group. In the model, the intensity of mortality is modeled as a random walk with drift and forecasted. Besides, in the study, forecasts of age-specific rates are derived from the forecasts of the mortality index and other life table variables are derived and presented. As a result, the method performs very well on within-sample forecasts. Moreover, in the length of base period from 30 to 90 years, the forecasts are insensitive to reductions. But, for the base periods of 10 to 20 years, some instabilities appear.

The original approach of Lee-Carter model have a lot of variants, extentions and generalizations up to now. And they had been applied by many other countries. The original Lee-Carter model is applied to the populations in group of seven (G7) countries in the study Tuljapurkar et al. (2000) [97], in Australia with the studies Booth and Tickle (2003) [13] and Booth and Tickle (2004) [14], in Sweden with the study Lundstrom and Et. Qvist (2004) [66], in Italia with the study Haberman and Russolillo (2005) [43], in Turkey with the study Yıldırım and Sucu (2013) [109]. In some studies an extention of the Lee-Carter model is used for the countries with limited data. For instance, in the study Lee and Rofman (1994) [63] it is applied to Chile and in the study Li et al. (2004) [65] it is applied to China and South Korea. Moreover, in the study Wilmoth (1993) [104], the parameters of the Lee-Carter model are estimated with Weighted Least Squares instead of SVD. This approach is used successfully in Japan with the study Wilmoth (1993) [104], in Austria with the study Carter and Prskawetz (2001) [19] and in Nordic countries with the study Koissi et al. (2006) [57]. In addition, Wilmoth (1993) [104] and Alho (2000) [2] are proposed to use Maximum Likelihood Estimation in order to find the parameters in the original Lee-Carter method. This formulation of the Lee-Carter method is usually called as Poisson log-bilinear model and it is used in Belgium with the study Brouhns et al. (2002) [17], in United Kingdom and Wales with the study Renshaw and Haberman (2003) [83] and in the Nordic countries with the study Koissi et al. (2006) [57]. Besides, in the study Booth et al. (2002), [12] expanded SVD method is used to the Australian mortality rates. Furthermore, in the study Koissi and Shapiro (2006) [55] a fuzzy formulation of the Lee-Carter method is proposed and in the thesis Yıldırım (2010) [108], Turkish mortality is modelled both the Lee-Carter and Fuzzy Lee-Carter method. Additionaly, in the original Lee-Carter method, the age-specific pattern of mortality change is kept fixed over the years however it is so unrealistic. Therefore, in the study Koissi and Shapiro (2008) [56] a modification of Lee-Carter model, that accommodates variations in age-specific parameters, is proposed. With this study, predicting mortality rates requires forecasting both mortality index and age-specific parameters.

In the study Renshaw and Habermann (2006) [84], a three-factor underlying model is proposed and it is called Lee-Carter APC method. In the study, the Lee-Carter model is extended to a APC basis and it is applied to the 1961-2000 United Kingdom population mortality rates for each gender. The model successfully captured APC effects in the mortality data. But it needs further examination.

On the other hand, in the literature, there are a lot of studies about measuring the longevity risk. For example, in the study of Selçuk-Kestel and Ben [89], longevity risk for Turkish pension system is compared and evaluated with the existing mortality tables commonly used in Turkey and modified version of Türkiye Kadın-Erkek Hayat Anüite (TRHA) 2010 (that is most recent one among the most common mortality tables) with Lee-Carter method. So, the longevity risk is measured with static and dynamic expected lifetimes. A hypothetical pension fund portfolio is used to show the impact of longevity risk in Turkish pension system and the longevity risk is quantified by using the discounted present value of future liabilities methodology on the hypothetical pension fund portfolio. In the study, there are two important conclusions. First of all, male longevity risk is higher than females in Turkey. And second of all, male longevity risk is persistent over years. In addition, in the study of Antolin (2007) [4], effects of longevity risk on employer-provided defined benefit (DB) private pension plans liabilities are examined. The results of estimating Lee-Carter model for mortality and life expectancy for several OECD countries such as England, France, Spain, Netherlands, Sweden and United States are provided. Furthermore, the uncertainty surrounding future mortality and life expectancy is outcome by means of Monte Carlo Simulations of the Lee-Carter model. In order to measure the impact of longevity risk on employer-provided DB pension plans, estimations of the range of increase in the net present value of annuity payments for a theoretical DB pension fund are presented.

In the study, it is concluded that, the gap in the net present value of annuity payments is inversely related with the age of pensioner both in the case of taking into account mortality and life expectancy improvement or not. However, when using mortality tables that account for mortality and life expectancy improvements, the gap is increased. So, in other words in the study it is concluded that, the exposure of the pension fund to improvements in life expectancy is larger the younger the individual is today. Besides, in the study Bisetti and Favero (2014) [9], the impact of longevity risk on Italian pension system is evaluated by the predictions that based on Lee-Carter model and the projections of pension payments for different cohorts of retired persons. They measure the longevity risk by the difference between the upper bound of the total old-age pension expenditure and its mean estimate. They concluded that, the impact of longevity risk is sizeably reduced by the indexation of retirement age to life expectancy at retiment but it is not fully eliminated.

1.2 The Aim of the Study

This study is inspired from the studies done by Habermann and Russolillo (2005) [43] and Antolin (2007) [4]. However, this is the first study in literature that calculates the longevity risk on auto-enrollment private pension system or in other words SPPS in Turkey.

The aim of this thesis is to cognize all aspects of second pillar pension system (SPPS), define the longevity risk on pension systems in detail and measure the longevity risk on SPPS in Turkey.

To achieve this aim, we firstly generate gender specific future life tables in 5-year age groups for Turkey. We use the Lee-Carter model, that is the most commonly used mortality forecasting model in recent years, to construct these life tables. So, we produce the gender specific future life expectancies for Turkey. After that, we generate 10,000 Monte Carlo Simulations on Lee-Carter model error terms to see the volatility of these future life expectancies. Next, we calculate SPPS pensions for each gender with a baseline scenario to see the impact of longevity risk surrounding future mortality and life expectancy. Finally, we make a sensivity analysis in order to see the time effect on SPPS pensions.

We expect the outcomes of the study can be utilized by all parties in the pension systems (regulator, insureds and insurance companies) so that the pensions and liabilities are calculated taking into account the future life expectancies at retirement age.

The rest of the study is organized as follows. In Chapter 2, the pension systems are explained in detail, SPPS model is presented theoretically with regard to WB definition and Turkish pension system is introduced in all its parts. In Chapter 3, the life tables and their constructions are described and mortality modelling and forecasting methods are investigated. In Chapter 4, the longevity risk on SPPS in Turkey is calculated and sensivity analysis are done. Finally in Chapter 5 presents the results.

CHAPTER 2

GENERAL OVERWIEV OF PENSION SYSTEMS

Social security is the product of seeking assurance against events that are dangerous for the economic or physiological life that an individual may encounter. The concept of social security is related to the idea of eliminating or mitigating the consequences of various events identified as “risk” [42].

Although the need for social security is at the same age as the history of humanity, the emergence as an idea (beyond the intellectual extent) of social security has taken some time. Along with the industrialization process, the desire to develop the industry has caused new risks to arise by increasing the severity of occupational, physiological and socio-economic risks faced by employees. Until then, the existing social protection schemes became inadequate against the emerging risks, so the necessity of the state to undertake the task was arised. As a result of all these developments, modern sense of social security was born and social security systems were created by the states [6].

Modern social security systems consist of social insurance, social benefits and social services. But, the weak use of social benefits and social services in practice and the widespread and effective use of social insurance, has resulted in the same use of social security systems as social insurance [54].

Social insurance consist of; **invalidity insurance**, designed to enable individuals who are unable to work because they have lost completely or in a certain way of their working labor, **old-age insurance**, designed to prevent or reduce the loss of income in future years and **survivors insurance**, designed to maintain the lives of those who are liable to look after the death of the insured person. These insurances constitute pension systems and they are considered as pension insurance as a whole. Today, pension systems are seen as the backbone of social security systems in almost every country in the world.

2.1 Historical Development of Pension Systems

It is possible to examine the historical development of pension systems into two main periods [77].

The **first period** included the country-based phase of the pension systems and was started by Bismarck in 1889 with the emergence of old - age and invalidity insurances. At the same time, these insurances are regarded as the beginning of the pension systems in history. The Bismarck model, which began to be implemented in Germany, set an example for other European countries to respond to the growing demand for workers with the Industrial Revolution [11].

The pension systems, that have been spreading at the global level by the Industrial Revolution, have been influenced by economic, social, political and regional conditions over time. The emergence of the economic and social crises have been the main reasons that shaping up the pension systems. In this point of view, it is possible to separate the pension systems, that applied at the global level in the first period, into three main categories.

Firstly, social and political crises in the Industrial Revolution have played an important role in the development of **Bismarckian pension model** [39].

The following years, the First World War, Great Economic Depression in 1929 and the Second World War were occured respectively. These developments have caused many people to die in Europe, a significant portion of the rest of the people have lost their labor force and all of these led to an increase in social and economic problems. After all of these, a report named as "Social Insurance and Related Services" was prepared by William Henry Beveridge in England in 1942. The Report forms the basis of the full coverage and compulsory social insurance system. The **Beveridge pension model** has given a different perspective to pension systems [39, 93].

On the other hand, another pension model adopted in the first period is the **national aid fund**. In this model, funds managed by the state are offered to the insured as a collective payment in retirement [37].

The **second period** includes the reforming phase of the pension systems established in the first period. It began with the Chilean reform in 1981 and still continues. A report which named as "Averting the Old Age Crisis" published by WB in 1994 [81] has been very influential in determining the main criteria of the pension system in this period. With this report multi-pillar pension system concept have got a place on the global level.

2.2 Purpose of Pension Systems

Pension systems have two main purposes. These are the **prevention of poverty** and the **distribution of income in a balanced manner** between working life and retirement life [46]. Pension systems fulfill their main purposes with functions such as saving, insurance and redistribution of income [81].

Together with the saving function, the individual income is provided to distribute in insured's own life. In this way, insureds, who earn more income during the working age period than the old age period, are prevented from spending all their income that they have earned during their working life. In other words, it is tried to ensure that the insureds continue the standard of life in the old age period as close as possible with the working age period.

Together with the insurance function, insureds are guaranteed against risks such as invalidity, old-age and death. Thus, it is tried to prevent insureds from falling into poverty in part or whole against these risks.

Finally, with the function of redistribution of income in the society, it is tried to provide social justice in society by partially prevent of the insureds, who struggled with poverty in the working age period, to encounter with the same problems in old age period. Income redistribution can be implemented in two different ways as redistribution between generations and redistribution within the same generation.

2.3 Classification of Pension Systems

Classification of pension systems can be done according to many different measures. However, the main determinants are the financing of the system and the benefits of the system.

2.3.1 Classification of pension systems according to financial structures

In a typical pension system, financing resources are provided from employee and employer premiums and state contributions [59]. There are two basic methods of financing pension systems. These are, PAYG method that aims to establish short term equilibrium and funded method that aims to establish long term equilibrium [40].

2.3.1.1 Pay as you go method

In PAYG financing method, expenses incurred in a certain period are financed by the revenues obtained in the same period. More clearly, in this financing method, the liabilities to be paid in the current period to passive insureds are collected from the active insureds existing in the system in the same current period [1].

This method allows to suplement to the insured in the low income group up to a minimum income level with the premiums collected from the insured in the high income group. Thus, it ensures that all individuals in the community are protected against social and economic risks. With this feature, PAYG method ensures distribution of revenues both within the generations and the same generation [18].

In PAYG method, the use of the revenues obtained in a certain period immediately

[1] In terms of long-term (invalidity, old-age and survivors insurances) and short-term (work accident and occupational disease, disease and maternity insurances) insurances branches, the pensioners and/or beneficiaries are called passive insureds. On the other hand, the insured persons that actively pay premiums to the related social security institution in the current period are called active insureds.

to meet the expenses in the same period greatly impedes the accumulation of funds. Therefore, in PAYG pension systems, keeping income and expenses in balance is very important to ensure the sustainability of the system.

2.3.1.2 Funded method

Funded method has a completely different operating structure than the PAYG method. In this method, revenues from active insureds are accumulated by collective or individual funds and they are operated with various investment instruments. The benefits are paid from the fund which is collected and assessed throughout the working life on behalf of the insured/insureds. Thus, funded pension systems provide a balanced distribution of income to one's own life. In this method, redistribution of revenues within the generations is not possible.

In funded pension systems, a fixed premium or contribution rate to be paid by insureds is determined by actuarial calculations which based on mathematical and statistical calculations. Thus, it is aimed to keep the system's incomes and expenses constantly in balance.

In funded method, funds can be collected in two different ways as individually and collectively. In the individual funded method, premiums paid on behalf of the insureds are collected on an individual basis and this fund is the source of future retirement incomes and benefits [32]. In this method, premiums which are paid by insureds and benefits to be obtained in retirement are calculated taking into account the age, civil status and other personal characteristics of the insureds. This leads to individual differences in the same pension system [106]. On the other hand, in the collective fund method, premiums paid on behalf of all insureds are accumulated on a common basis and this fund is the source of future retirement incomes and benefits [32]. In this method, contrary to the individual funded method, premium accounts are made irrespective of individual differences between insured persons [98].

2.3.2 Classification of pension systems according to benefit structures

In pension systems, benefits to be obtained by passive insureds are determined by different methods. Among these methods, we can examine the most obvious ones under two different headings as the method based on defined benefit (DB) and the method based on defined contribution (DC).

2.3.2.1 Defined benefit method

In DB pension systems, pensions are calculated according to the insureds' salaries and / or number of premium payment days. They are not closely associated with to the premiums received. In this method, it is aimed to provide a ”defined” benefit during the retirement period.

The benefits in DB pension systems guarantee the pension payment regardless of how long the retired person would live. And the responsibility of any inadequacy of revenues is entirely undertaken by the employer or the state [45]. Moreover, DB method is generally applied in PAYG pension systems.

The benefit can be defined in different ways in DB pension systems. For example; universal flat rate system entitles same pension to the retired persons over a certain age regardless of their salary and number of premium payment days. In additon, employment-related flat rate system entitles same pension to the retired persons who have worked for the same employer. Moreover, means-tested system entitles higher pensions to the low income group insureds, on the contrary earnings-related systems entitles higher pensions to the insureds that pay more and longer [81].

2.3.2.2 Defined contribution method

In DC pension systems, pensions are calculated according to the premiums and the investment returns of these premiums. In these systems, the amount of benefit to be entitled in retirement is not certain and is directly linked with the premiums paid on behalf of the insureds on the contrary of DB pension systems.

DC pension systems do not give the employer or the state the responsibility to provide a certain level of pension. So, the insureds undertake relatively high risk regarding to the DB pension systems. Insureds undertake economic risks such as investment risk, inflation risk and interest rate risk, as well as the demographic risks such as long life risk (because pensions are generally calculated according to the life expectancy in DC pension systems). For this reason, even if the insureds contribute to the pension system as a percentage of their salaries during the working period, pensions may be disproportionate to the final salary. Thus, in DC pension systems, pensioners are exposed to a higher poverty risk in retirement [45, 30]. Moreover, DC method is generally applied in funded pension systems.

2.4 Basic Parameters of Pension Systems

The parameters that determine the outlines of pension systems are classified under 3 main headings. These are **eligibility conditions**, **benefit parameters** and **contribution rates** [88].

Eligibility conditions represent the minimum conditions that insureds must meet in order to be able to retire. These conditions are;

i. **Retirement age** represents the normal age at which insureds can retire.

ii. **Insurance period** represents the term that begins with the beginning of insured status and ends with the termination of the insurance.

iii. **Number of the premium payment days** represents the number of days that insureds contribute premium to the pension system.

On the other hand, **benefit parameters** represent the variables used in pension calculations. Benefit parameters are also determine the basic expenditures of pension systems. These are;

i. **Wage base** represents the wage on which pension benefits are calculated. It is generally the average wage that earned during a specified period of time.

ii. **Replacement rate** represents the percentage of wage base that earned per year of service.

iii. **Valorization of past earnings** represents the parameter that is used to revalue past salaries to the retirement date.

iv. **Indexation** represents the parameter that determines how pensions are increased after retirement.

Finally, **contribution rates** represent the rates that determine the amount of premiums paid by workers, employers and / or the state with regard to insured's insurable earning. Moreover, insurable earning refers the earning level that could be premium collected. A range is usually determined by setting upper and lower limits to the insurable earning. Within this range, premiums are collected according to the premium rates that determined by insurable earnings of the insureds. These premiums constitute the main income of the pension systems.

2.5 Main Factors Affecting Pension Systems

Demographic structures and **labor market conditions** are the most significant factors affecting pension systems. They greatly affect organizational structure, main objective and performance of the pension systems.

2.5.1 Demograpfic factors

Fertility rates, mortality (death) rates and migration are key determinants of demographic factors. Demographic trends give an idea of the number of potential insureds and retired persons in the pension system by determining the age distribution of the population over the coming years. Therefore, demographic factors play an important role in determining the long - term dynamics of pension systems and directly affect the sustainability of these systems.

Among the most important demographic indicators that will provide insight into the future of the pension systems are; **life expectancy at birth and 65 years old**. In general, life expectancy means how long a human being will live on average. It gives important clues to the future active / passive insured ratio [2] in pension systems. Moreover, it provides to estimate the retirement period.

During the twentieth century, life expectany increased dramatically all over the world. This increase can be attributed too many factors. Developments in the health technology, increments in access to health care, socio-economic development and improvements the quality of life are the most important factors that caused mortality rates to decrease. This significant decrease in mortality rates around the world has had a positive impact on life expectancy. Hovewer, it has burdened a significant longevity risk on retirement systems. So today, pension systems in many countries face difficulties in fulfilling their actuarial liabilities.

Figure 2.1 shows the life expectancy at birth in the countries allocated to the groups by income levels for each five years between 1950 and 2015.

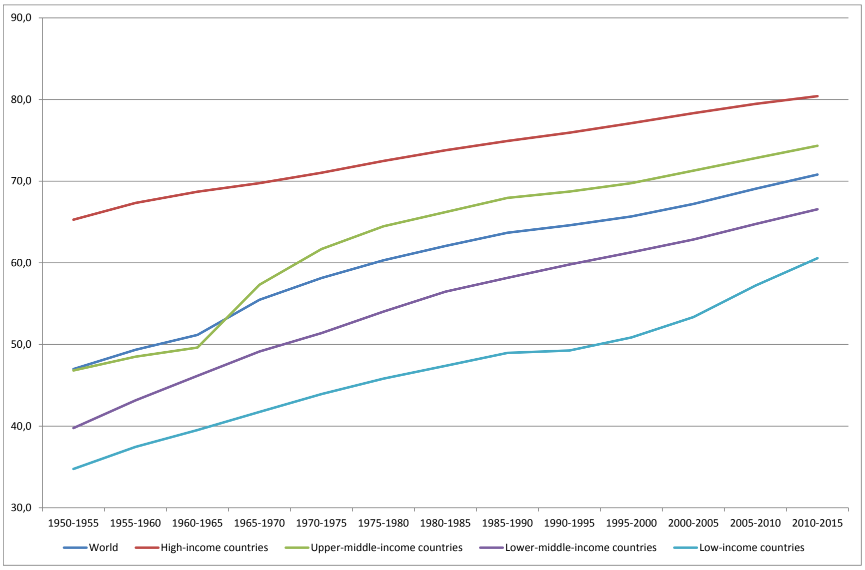

Figure 2.1: Life expectancy at birth for each five years between 1950 and 2015 [99]

We can see from Figure 2.1 that, the life expectancy at birth which is 47 years in the world between 1950 and 1955, increased gradually and reached 70.8 years between 2010 and 2015. This means that the life expectancy at birth has increased by 51 percent over the last 50 years.

It is also clearly observed that, the life expectancy at birth has increased in direct proportion to the income level. The life expectancy at birth is 65.3 years at high income

[2] Active / passive insured ratio means the ratio of workers to pensioners. It is calculated by dividing the number of persons actively contributing to the system by the number of pensioners. It shows how many workers finance one pensioner [92].

countries while 46.8 years at upper middle income countries, 39.8 years at lower middle income countries and 34.8 years at low income countries between 1950 and 1955. The life expectancy at birth has increased dramatically in all countries and it has also remained higher in countries with higher income levels. It is 80.4 years at high income countries while 74.3 years at upper middle income countries, 66.6 years at lower middle income countries and 60.6 years at low income countries between 2010 and 2015.

Nowadays, life expectancy at 65 years old is important in terms of the length of time in retirement since many countries is accepted the 65 years old as the retirement age. Figure 2.2 shows the life expectancy at 65 years old in the countries allocated to the groups by income levels for each five years between 1950 and 2015 which illustrates that the life expectancy at 65 years old which is 11.3 years in the world between 1950 and 1955, increased gradually and reached 16.6 years between 2010 and 2015. This means that the life expectancy at 65 years old has increased by 46 percent over the last 50 years.

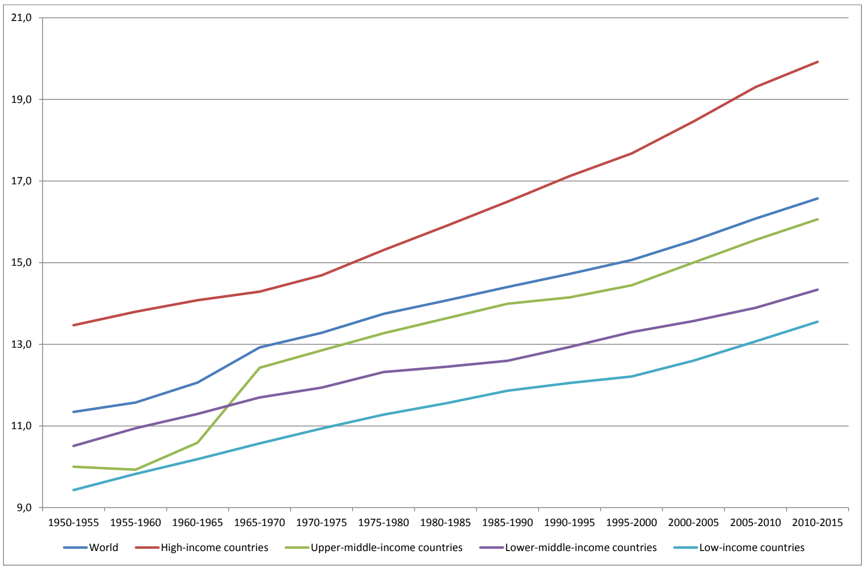

Figure 2.2: Life expectancy at 65 years old for each five years between 1950 and 2015 [99]

Moreover, it is also clearly noticed that the life expectancy at 65 years old has increased in direct proportion to the income level. The life expectancy at 65 years old is 13.5 years at high income countries while 10 years at upper middle income countries, 10.5 years at lower middle income countries and 9.4 years at low income countries between 1950 and 1955. The life expectancy at 65 years old has increased dramatically in all countries and it has also remained higher in countries with higher income levels as the same as the life expectancy at birth. It is 19.9 years at high income countries while 16.1 years at upper middle income countries, 14.3 years at lower middle income countries and 13.6 years at low income countries between 2010 and 2015.

In summary, as observed in both figures, life expectancy has increased significantly in all countries, regardless of income level. And, this is one of the most important reasons for the financial crisis in pension systems.

2.5.2 Labour market conditions

Pension systems are inherently in direct interaction with labour market. Because, the amount of the population participating in the labour force constitutes a significant part of the potential participants of these systems. Moreover, workers who are in the registered employment in the labour force are the active insureds in these systems.

2.6 Financial Crisis in Pension Systems

Social security systems, in particular the pension systems, have the most brilliant period after Second World War. In this period, social security understanding has developed. The inclusion of family as a protection unit instead of individual and the expansion of covered social risks have increased the coverage of pension systems. On the other hand, pensions have also been raised in order to provide a more sufficient social security guarantee [48].Together with all these developments, increasement in the passive insureds and the liabilities due to the maturing pension systems have caused financial difficulties. So, pension systems have entered in a difficult fiscal turn especially in the period of second half of 1970's, which is called the welfare state crisis.

Pension systems are being unsustainable because of basically the same reasons. International Social Security Association (ISSA) collects the causes of the pension system's financial crisis under seven headings in the report that is called as "Demografic Changes and Social Security: Challenges and Opportunities" [49]. In this report, aging of the population in other words longevity risk is seen as a fundamental problem for pension systems. In addition, changes in family structure, transformations in the labor market, urbanization, inconsistency in the life cycle, migration and changes in social structure are seen as other factors affecting the pension systems.

The problems that are mentioned in the report issued by the ISSA are not at the same level of difficulty for all countries [49]. Developed and developing countries are facing with different problems at different levels in their pension systems. Developed countries are mainly faced with problems due to demographic factors. On the other hand, developing countries are faced with problems that arise mainly due to structural distortions in their economies and labour markets. However, the increase of pension expenditures should be emphasized in all developed and developing countries.

Table 2.1 shows public and mandatory private expenditures on old-age and survivors cash benefits in percentage of GDP in Organization for Economic Co-operation and Development (OECD) countries since 1980. The proportion of old-age and survivors

Table 2.1: Public and Mandatory Private Expenditures on Old-Age and Survivors Cash Benefits, in Percentage of Gross Domestic Product (GDP) [75]

Countries	1980	1985	1990	1995	2000	2005	2010	2013
Australia	3.6	3.4	3.1	6.0	7.2	5.2	5.3	5.7
Austria	10.4	11.3	11.3	12.2	12.0	12.0	13.1	13.4
Belgium	8.8	9.2	9.0	9.2	8.7	8.8	9.7	10.2
Canada	3.1	3.8	4.2	4.6	4.2	4.0	4.3	4.5
Chile	-	-	8.3	7.6	8.3	4.9	4.7	4.4
CzechRepublic	-	-	5.6	5.9	7.1	6.9	8.6	9.0
Denmark	5.7	5.5	6.1	7.2	6.3	6.5	9.2	10.4
Estonia	-	-	-	-	6.0	5.3	7.6	6.4
Finland	5.4	7.2	7.2	8.6	7.4	8.1	9.8	11.1
France	9.2	10.3	10.6	11.9	11.6	12.2	13.2	13.9
Germany	10.4	10.3	9.5	10.3	10.8	11.1	10.6	10.1
Greece	5.2	8.3	9.5	9.2	10.4	11.4	13.3	-
Hungary	-	-	-	-	7.5	8.4	9.6	10.3
Iceland	-	-	3.5	4.1	4.4	4.7	5.0	5.9
Israel	-	-	-	4.5	4.6	4.8	4.8	4.8
Italy	9.3	11.7	12.3	14.2	14.4	14.6	15.8	16.7
Japan	3.9	4.8	5.0	6.3	7.8	8.9	10.6	10.9
Korea	-	-	0.9	1.3	1.9	1.8	2.4	2.8
Latvia	-	-	-	0.0	8.7	5.5	9.3	7.5
Luxembourg	8.6	8.1	7.7	8.4	7.1	7.3	7.6	8.5
Mexico	-	0.2	0.5	0.7	0.8	1.2	1.8	-
Netherlands	6.0	5.8	6.3	5.4	4.7	4.7	5.0	5.4
NewZealand	7.0	7.4	7.2	5.6	4.9	4.2	4.6	4.8
Norway	4.5	4.6	5.5	5.4	4.7	4.8	5.2	5.8
Poland	-	-	5.0	9.2	10.5	11.3	11.1	-
Portugal	3.7	4.0	4.8	7.1	7.8	10.0	12.0	14.0
SlovakRepublic	-	-	-	6.2	6.3	6.1	6.8	7.2
Slovenia	-	-	-	-	10.3	9.7	11.0	11.8
Spain	6.1	7.3	7.7	8.8	8.4	7.9	9.8	11.4
Sweden	6.7	7.2	7.3	7.9	6.9	7.2	7.3	7.7
Switzerland	7.1	7.5	7.4	9.3	10.0	10.6	10.8	11.2
Turkey	1.2	1.3	2.4	2.7	4.0	6.0	7.7	8.1
UnitedKingdom	5.3	5.3	4.6	5.2	5.5	5.8	6.7	6.9
UnitedStates	6.0	6.0	5.8	6.0	5.6	5.7	6.6	6.9
OECD – Total	5.8	6.4	6.5	7.2	7.1	7.2	8.2	8.6

cash benefits to GDP is an important indicator that provides information on the trends of countries' expenditures on public and mandatory private pension systems since a significant part of the expenditures in any pension system is derived from old-age and survivors benefits.

As seen in Table 2.1, the share of public and mandatory private expenditures in GDP are increased compared to 1990's share in all other countries, excluding only 3 (Chile, Netherlands, New Zealand) out of 34. Moreover, it is observed that the share of the total old-age and survivors cash benefits of the public and mandatory private sector in the total GDP in all OECD countries, which was 5.8 percent in 1980, increased by 2.8 percentage points to 8.6 percent in 2013. On the other hand, when we look at the data in 2013, it is seen that expenditures in Austria, Belgium, Denmark, Finland, France, Germany, Hungary, Italy, Japan, Portugal, Slovenia, Spain and Sweden have exceeded 10 percent of GDP. In Turkey, which has a very young population structure compared to developed countries, the share of expenditures in GDP in the 1980s, which was 1.2 percent, increased considerably over the years, and reached 8.1 percent by the year 2013. This is an important proof that pension systems in many countries are financially unsustainable.

2.7 Reforms in Pension Systems and Transition to Multipillar Pension Systems

The financial crisis in the pension systems around the world, especially since the early 1990s, and concerns over the sustainability of these systems have caused many countries to re-examine their pension systems. As a result, countries have also made some structural reforms, including more radical changes such as the new financing methods for the system, as well as some parametric reforms to increase income and / or decrease expenditure.

2.7.1 Parametric reforms in pension systems

Parametric reforms are not intended to change the structure of pension systems. These reforms change the basic parameters within the existing structure. Parametric reforms can be analyzed under two main headings. First is one-sided reforms that increase incomes or decrease expenditures, and the second is two-sided reforms that both increase incomes and reduce expenditures.

Reforms to increase incomes of pension systems are made with the changes in contribution rates that are collected over the insurable earnings. But many countries refrain from raising them. Because, increasing the contribution rates collected from the workers decreases the monthly net salaries, furthermore increasing contribution rates collected from the employers causes unregistered employment. For these reasons, countries generally prefer to expand the insurable earning gap by increasing the upper and lower limits of it rather than increasing the contribution rates in order to increase incomes.

On the other hand, reforms to decrease the expenditures are made with the changes in benefit parameters. Changes in this way earnings are measured to calculate benefits and changes in the valorization rates of past earnings are the ways of making these reforms. Significant decreases in the system expenditures can be achieved with these changes. In addition, they generally do not get much reaction in the society because of the interpretation of these changes requires technical knowledge. This situation causes such changes to be favoured by many countries.

Moreover, reforms that aimed both to increase the incomes and to decrease the expenditures are made by changing the eligibility conditions for a pension. Linking pensions to higher life expectancy, promoting the late retirement, changing the number of premium payment days and insurance period are the ways of making these reforms. Although these reforms are subject to serious criticism by the society, they are often preferred by countries because they have a dual effect on the financial well-being of their pension systems.

Table 2.2 shows parametric reforms in OECD countries since 1990. As seen in the Table, reforms about “eligibility conditions” are mostly prefered by OECD countries. In addition, among the reform options of eligibility conditions, increasing retirement age has been the most preferred one both for males and females. The table also draws attention to the fact that at least one parametric reform has been carried out in all OECD countries since 1990. Moreover, countries have often tried to overcome the financial crisis in their pension systems with multiple parametric reforms.

Table 2.2: Parametric Reforms in OECD Countries, 1990-September 2015 [69, 70, 71, 72, 73]

Countries	**Eligibility Conditions**				**Benefit Parameters**		**Cont. Rates and Insurable Earnings**
	Ret. Age (M)	**Ret. Age (F)**	**Late Ret. Incentives**	**Others**	**Benefit Cal.**	**Indexation**	
Australia	*	*	*			*	*
Austria	*	*	*		*	*	
Belgium	*	*	*	*			
Canada	*	*	*		*		*
Chile							
CzechRep.	*	*		*		*	*
Denmark	*	*	*				
Estonia	*	*					*
Finland	*	*	*	*	*	*	*
France	*	*	*	*	*	*	*
Germany	*	*	*				*
Greece	*	*	*	*	*	*	*
Hungary	*	*	*		*	*	
Iceland	*	*					*
Israel							*
Italy	*	*	*	*		*	*
Japan	*	*		*	*	*	
Korea	*	*	*				
Luxembourg	*	*	*	*			*
Mexico							
Netherlands	*	*	*				
NewZealand	*	*					*
Norway				*	*	*	
Poland	*	*	*		*	*	
Portugal	*	*	*	*	*	*	*
SlovakRep.	*	*			*	*	
Slovenia	*	*		*		*	
Spain	*	*	*	*	*		
Sweden	*	*	*		*		*
Switzerland		*	*				*
Turkey	*	*			*	*	
Un.Kingdom	*	*	*				
Un.States	*	*					

Ret.=retirement, M=male, F=female, Cal.=calculations, Cont.=contribution

2.7.2 Structural reforms in pension systems and transition to multi-pillar pension system

In addition to the parametric reforms, more comprehensive and radical structural reform proposals have been put forward to change the current structure of pension systems which can not adapt to the changing demographic and socio-economic conditions in the long run, under the guidance of international organizations such as the WB, European Commission and OECD [60]. Multi-pillar pension model proposal constitutes the framework of these structural reforms that led to emergence of the second-period pension systems.

Foundations of the multi-pillar pension system laid by WB with the report "Averting the Old Age Crisis" in 1994 [81]. According to the report, solely implementation of DB public pension system which is financed by PAYG method was insufficient to meet the basic objectives of the pension systems (preventation of poverty and distribution of income in a balanced manner between working life and retirement life) in almost all developed and developing countries until 1990s. However, in the Report PAYG pension system is considered to be important in terms of ensuring the redistribution of income both within generations and within the same generation. Therefore, it is not suggested to abolish the PAYG pension model completely. In addition, it is emphasized that the responsibility of the main objectives of the pension systems should be divided among the different pension pillars. Moreover, a healthier and manageable system should be constructed by defining different roles for the state in these pillars. In summary, multi-pillar pension system that consists of an ideal combination of different financing methods and different benefit structures and the state shares responsibility with the private sector is suggested by WB. On the other hand, it is emphasized by WB that the proposed structure is not an ideal structure for all countries and that countries should establish their own pension systems by taking into account social, economic and cultural needs and circumstances of their own society [100] .

According to the Report, single-pillar systems should be replaced by two- or three-pillar pension systems. The **first pillar pension system** should provide social justice by providing "minimum income guarantee" to **all** individuals and should be planned to cover the **entire** population. In addition, this pillar should mainly be under state responsibility and/or guarantee and should be financed by premiums or taxes. The **SPPS** should be based on mandatory participation. Insured, employer and even the state should participate in the financing of the system. This pillar should ensure continuity of income, that obtained during working life, in old age. Moreover this pillar, in which retirement benefits are closely associated with income, should be managed by the private sector on the basis of the insurance rules together with the participation of the social partners in the management. On the other hand, completely voluntary and private sector based structure is being constructed in the **third pillar pension system**. In this pillar, it is aimed to provide a higher and differentiated pension guarantee and premiums can be paid by the insureds and / or the employers. Individual priority is the basic philosophy of this pillar.

This three pillar pension model has been revised by WB with the Report which is titled as "Old Age Income Support in the 21st Century: International Perspective on

Pension Systems and Reform" in 2005 [46]. In the new pension model developed by WB, the zeroth column and the fourth column are added to the existing three pillar pension system. So, multipillar pension systems have become a five pillar structure. In the **zeroth pillar** premiums are not paid by insureds and/or employers. Financing is made with the taxes. Main purpose of this pillar is to prevent poverty. This pillar seeks to reduce the impact of poverty in old age especially for the long-term unregistered workers. The non-financial **fourth pillar** includes access to informal support like family support, other formal social programs like housing and/or health care and other individual financial and non-financial assets like reverse mortgages where available and home ownership [47]. Below, Table 2.3 summarizes the main framework of the 5-pillar pension system, which is clearly defined by World Bank with its borders and target group.

Table 2.3: Multipillar Pension Taxonomy [46]

	Target group			**Main criteria**	
Pillar	**Lifetime poor**	**Informal sector**	**Formal sector**	**Features**	**Participation**
0	**X**	X	x	Basic or social pension, at least social assistance, publicly managed	Universal
1			**X**	DB, publicly managed	Mandated
2			**X**	Fully funded DB or fully funded DC, privately managed, occupational or personal pension plans	Mandated
3	x	**X**	**X**	Partially or fully funded DB or funded DC, privately managed, occupational or personal pension plans	Voluntary
4	**X**	**X**	X	Access to informal support, other formal social programs and other individual financial and nonfinancial assets, publicly managed	Voluntary

The size and thickness of x show the importance of each pillar for each target group in the following increasing order of importance: x, X, **X**.

As seen in the Table, the main objective of multipillar pension system is to determine main target groups in the society and to put into practice the special pension systems in different pillars. According to WB, multipillar pension systems are able to meet the needs of key target groups identified within the community and they can also provide better protection against economic, demographic and political risks. Therefore, restricting the pillars in the direction of the determined purpose and target groups will enable the retirement systems to be designed more effectively and efficiently.

In addition, as shown in the Table, each pillar has been structured according to different funding methods and different benefit structures in order to ensure the financial sustainability of the pension systems. Thus, it is aimed that the advantages of all methods are utilized, while the effects of disadvantages are limited. On the other hand, rather than full public responsibility, the risk is shared among public, employer and private pension companies.

2.8 Second Pillar Pension System

2.8.1 Structure of second pillar pension system

SPPS is a DC funded system which is managed by private insurance companies. Additionally, participation in the system is compulsory.

The system can be designed as occupational pension schemes or individual savings accounts. Individual savings accounts are insurance systems that require individuals to save money in order to finance their retirement, in other words, they are based on individual savings. On the other hand, occupational pension schemes are employer-based pension systems created at the company or industry level. Countries prefer the method of maximizing their pensions, provided that they do not cause any deterioration in the labor market [81].

There are two basic elements that distinguish SPPS from other saving systems. Firstly, on the contrary of other privately managed systems participation is compulsory. And secondly, investments are prepared as direct retirement savings plans, unlike general savings instruments such as bank accounts, mutual funds, life insurance policies [78].

2.8.2 Aim and basic philosophy of second pillar pension system

The primary objective of SPPS is to diversify pension insurance, that insures individuals against various risks they may face during their retirement. Thus, it creates more sustainable pension systems. The secondary purpose of the system is to fulfill the saving function of pension systems. Thus, it ensures that the income, that individual's earn through the working life, is spread in a balanced manner to the expected life span [86].

Since SPPS is regulated on the basis of DC, the amount of premiums contributed and the benefits to be deserved at retirement are actuarially related. Besides, since the system is financed according to the funded method, premiums are invested, so that premiums take a share of economic developments relative to the success of investments. Moreover, with this system, saving rates are increased, so financial markets and economic growth are positively affected. On the other hand, with the private management of the system, it is intended to keep the system as far away from the economic and political damage that the public pension systems are exposed to [81]. Thus, the transfer a part of the pension system to the private sector ensures that a part of the state responsibility in the pension system is shared with the private sector. In addition, while the risk in DB public pension systems remains on the state, the risk remains on the insureds or insurers in DC pension systems. Therefore, it is also possible to share the total risk of pension systems between state, insureds and insurance companies with SPPS.

2.8.3 Implementation of second pillar pension system

Implementation of SPPS usually occurs in the following way [88] :

i. Primarily, premiums collected (from insured, employer and / or state) on a regular basis every month on behalf of the insured are transferred to the pension funds to be selected from the standard packages by insureds in individual savings accounts, and employers in occupational pension systems of their own volition. As it is clearly intended, pension funds prepared by insurance companies are not usually the only one kind. They offer various alternatives that include different investment instruments at different rates within the legal limits.

ii. Then the collected premiums are deducted by insurance companies at pre-determined rates.

iii. After the deduction of the administrative fees from the collected premiums, the remaining amount is started to invest by the fund managers in the determined pension fund.

iv. The benefits to be earned when insured persons are retired are determined by paid premiums and investment incomes.

v. At retirement, the insured typically purchases an annuity from insurance company who guarantees the retiree a pension benefit throughout his or her lifetime. The benefit may be indexed or not. In addition, the benefits entitled in retirement can be taken as scheduled withdrawals or retirement bonuses in one instead of annuities.

vi. Pensions are indexed to certain parameters (usually inflation and/or wage increase) if they are given as annuities. In case, the insured lives longer than expected life expectancy, pensions are continued to be given. So, the longevity risk of this situation remains on the pension companies. On the other hand, in the case of a scheduled withdrawals, the total benefit is divided into the life expectancy and the pension of the current year is determined. After that year's pensions are deducted from the total accumulation, the remainder continues to be invested. This process is repeated the following year. And the resulting pension rising or falling depending on realized invesment returns.

2.8.4 Advantages and disadvantages of second pillar pension system

Second pillar pension system has many advantages. First of all, as the financing of SPPS according to the funded method has removed the transfer of premiums from generation to generation, this pillar of the pension systems becomes independent of the population aging that is a very important problem of developed countries. Additionally, in comparison with the public pension systems in SPPS, the benefits to be earned in retirement are determined according to different principles, and the responsibilities undertaken by the private sector instead of the state provides diversification within the pension system. So, distribution of the risk over different methods is provided in this way. Taking all of these into consideration, the most important advantage of SPPS is that it reduces the actuarial obligations on public pension systems and makes pension systems more financially sustainable as a whole. It is also an advantage of the system to save insureds at higher rates in the case of extra premium collection for the SPPS, and to support capital accumulation and hence financial market development. On the other hand, the fact that the administration is taken over by the private sector keeps this pillar of pension system out of populist politics.

Second pillar pension system has many disadvantages as well as many advantages. The main drawback of the system is mainly due to the financing of existing pension systems according to the PAYG method in many countries around the world. As is known, contrary to the PAYG method, in funded retirement systems, individuals finance the retirement benefits with the premiums that they have been paid in their working life. Thus, the construction of SPPS over the existing PAYG pension system makes problem of financing the pensions of existing retired persons and so causes transition cost. States that do not want to bear the transition cost have to collect premiums for SPPS. Another disadvantage of the system is in the management issue. The management of funds by the private sector brings many management problems such as high administrative expenses, inefficient management of funds, inadequacy of capacity and administrative success differences among pension companies. On the other hand, insureds who decide on which fund to invest their premiums are also faced with the risk of inadequate or incorrect information. For this reason, the effective control of the system, complete preparation of regulatory legislation and government participation in the system as regulator and supervisor play an important role in the success of the system. With all these, the system links the pension income mainly to the success of investments. In particular, during periods of economic crisis, insureds face a negative return risk due to the negative macroeconomic outlook.

As a result, it is seen that the features that compose SPPS are the source of both advantages and disadvantages. And, domination of advantages or disadvantages of the system are largely dependent on the demographic and socio-economic structures of the countries.

2.9 Turkish Pension System

It is known that the Ottoman Empire entered the industrialization process too late compared to Europe. This situation has defered the Turks to achieve a modern pension system. In Turkey, a mandatory pension system was firstly established in 1921 with the establishment of Ereğli Coal Basin Workers Union. Later, in the first years of the Turkish Republic, a large number of pension and charity funds were set up, although they were too narrow-scoped.

The Labor Law that was introduced in 1936 was the first law that establish the social insurance institutions and put basic principles about social insurances in Turkey. However, the system foreseen in the Law could not be constituted until 1945 because of the Second World War [93].

After the Second World War, pension and charity funds were merged with the "Workers' Insurance Institution Law number 4792" in 1945. In addition, with the acceptance of "Old-age Insurance Law number 5417" in 1950 and "Disability, Old-age and Survivor's Insurance Law number 6900" in 1957 public pension system has been started to establish and the scope of the pension system has also been expanded. Moreover, "Social Security" concept was firstly mentioned in constitutional terminology in 1961 Constitution. Below, Table 2.4 shows the primary laws about social security branches in Turkey.

Table 2.4: Primary Laws on Social Security Branches in Turkey [93]

Enactment Year	**Law Name**
1945	The Work Accidents, Occupational Diaseases and Maternity Insurance Law
1945	The Workers' Insurance Institution Law
1950	The Old Age Insurance Law
1951	The Sickness and Maternity Insurance Law
1957	The Disability, Old Age and Survivor's Insurance Law

Furthermore, by Retirement Fund Law eleven retirement funds were revoked in 1949. Therefore, an integrated social security structure is constituted for public servants.The Social Insurances Law dated 1964 unified the arrangements in various laws relating the insurance branches of those who work under the status of a worker. The Social Insurance Institution for Tradesmen and Craftsmen and Other Self-Employed People (Bağ-Kur) was founded in 1971. Besides, in order to provide social security for people who are working in agricultural sector, important changes were made with the Social Security Law for Agricultural Workers and the Social Security Law for Self-Employed in Agriculture in 1983. As a consequence, the social security benefits for insureds were regulated in 5 different laws [93]. Table 2.5 shows these 5 different Law names and their enactment dates.

Table 2.5: Laws Regulating the Social Security Benefits of Insurance Holders Before 2006 [93]

Enactment Year	**Law Name**
1949	The Retirement Fund Law
1964	The Social Insurances Law
1971	The Social Insurance Institution Law for Tradesmen and Craftsmen and Other Self-Employed People
1951	The Social Security Law for Agricultural Workers
1957	The Social Security Law for Self Employed in Agriculture

In time, these different laws have led to inequality between the insureds. So, in order to unify norms and create a sustainable social security system in Turkey, the Social Security Institution (SSI) was establish in 2006 by SSI Law number 5502. With this Law, Social Insurance Institution, Retirement Fund and Bağ-Kur were united under a single roof.

On the other hand, from the beginning of the 1990s, the pension system in Turkey has begun to suffer financial problems. However there are many reasons for these financial problems, the main reasons are early or young retirement, high replacement rates and high pensions compared to paid premiums and working days, retrospectively premium amnesties, low premium payout trends by insureds and insufficient institutional capacity of SSIs [51]. Moreover, this system does not include the entire population and can not adequately overcome the poverty in old-age. On the other hand, the aging tendency of the population is one of the main important factors affecting the financial sustainability of the Turkish pension system. Figure 2.3 shows the life expectancy at birth in Turkey for each five years between 1950 and 2015.

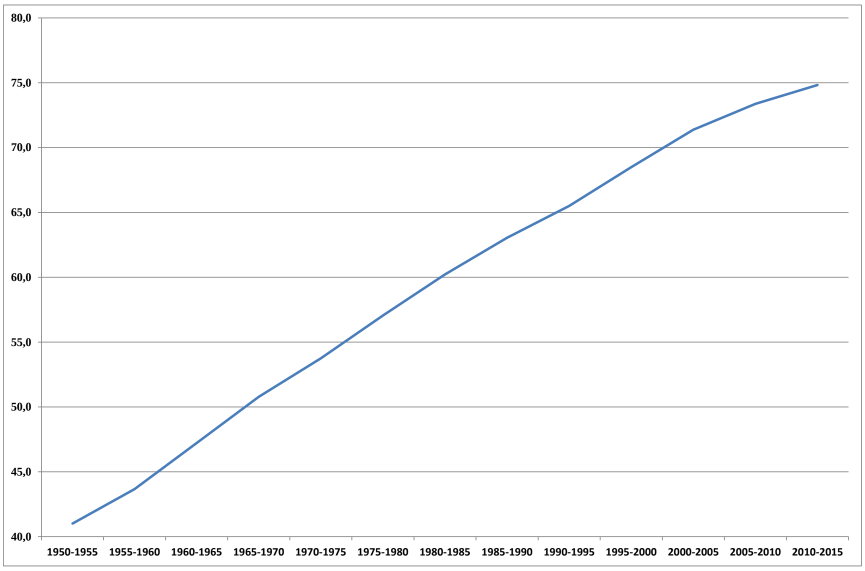

Figure 2.3: Life expectancy at birth for each five years between 1950 and 2015 in Turkey [99]

It can be seen from the Figure that, the life expectancy at birth which is 41.0 years in Turkey between 1950 and 1955, increased gradually and reached 74.8 years between 2010 and 2015. This means that the life expectancy at birth has increased by 82.5 percent over the last 50 years.

When it is compared to Figure 2.1 and Figure 2.3, it can be seen that, the life expectancy at birth in Turkey is 6 years less than the world average in between 1950-1955. However, life expectancy at birth in Turkey is 4 years ahead of the world average in between 2010-2015. Therefore, it is clearly observed that, the life expectancy at birth in Turkey has increased much more rapidly than the world average. Moreover, it is also seen that the life expectancy at birth in Turkey is almost as the same level as with the lower middle income countries in between 1950 and 1955, while it is as the same level as with the upper middle income countries in between 2010 and 2015. This is a clear indication of the positive effect of the developing economy on the expected lifetimes in Turkey.

Figure 2.4 shows the life expectancy at 65 years old in Turkey for each five years between 1950 and 2015.

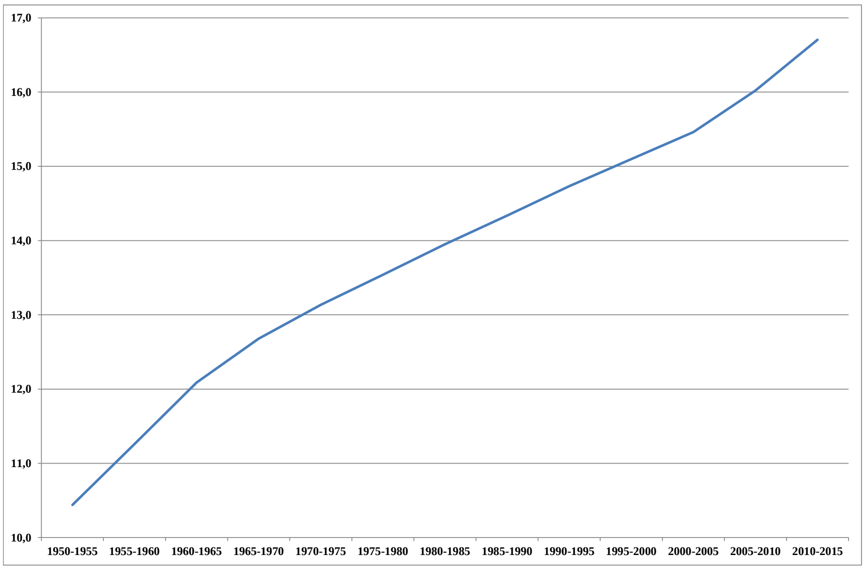

Figure 2.4: Life expectancy at 65 years old for each five years between 1950 and 2015 in Turkey [99]

It can be seen from the Figure that, the life expectancy at 65 years old which is 10.4 years in Turkey between 1950 and 1955, increased gradually and reached 16.7 years between 2010 and 2015. This means that the life expectancy at 65 years old has increased by 60 percent over the last 50 years.

Comparisons on Figure 2.2 and Figure 2.4 show that the life expectancy at 65 years old in Turkey is 0.9 years less than the world average in between 1950-1955. However, life expectancy at 65 years old in Turkey is at almost the same level as the world average in between 2010-2015. Moreover, it is also seen that the life expectancy at 65 years old in Turkey is almost as the same level as with the lower middle income countries in between 1950 and 1955, while it is as the same level as with the upper middle income countries in between 2010 and 2015. This is again a clear indication of the positive effect of the developing economy on the expected lifetimes in Turkey. On the other hand, when we look at Figure 2.2, it can be seen that, the life expectancy at

65 years old in high income countries is 19.9 years in between 2010-2015. This means that, the expected life expectancy at 65 years old in Turkey most probably will increase further in the coming years.

As a result, in order to solve the financial problems and bring the system to a sustainable state, the first comprehensive parametric reform with parametric changes was implemented in 1999 and the second in 2008. Both of the reforms were designed to generate revenues and reduce expenses. With these reforms, basic parameters of the system such as retirement age and number of premium payment days have been increased, insurable earnings have been extended and replacement rates and valorization of past earnings coefficient have been reduced.

The reform in 1999 was made by "Unemployment Insurance Law number 4447". With this Law, the retirement age has been increased to 58 in women and 60 in men [3]. Replacement rates are determined as 2 percent for every 360 days of the first 10 years and 1.5 percent for every 360 days thereafter for Social Insurance Institution and Bağkur insureds. The lower limit of replacement rate was reduced from 70 percent of the minimum salary to 35 percent of the average earnings. In addition, the difference between the upper and lower limits of the insurable earning was determined as three times.

The reform in 2008 was made by "Social Insurance and Universal Health Insurance Law number 5510". The current parameters of the public pension system had been determined by this Law. The details of 2008 reforms are given under the Section 2.9.1. For this reason it is not mentioned separately in this section.

Nowadays, despite all of these reforms, it can be seen that the financing deficit of the pension system in Turkey is not significantly improved. The most important reason for this situation is that; the majority of the reforms make no effect on the existing insureds. Table 2.6 shows the financial situation of SSI in Turkey.

As seen in Table 2.6, the ratio of pension payments to GDP have increased significantly especially in recent years. This is an important indicator of the increasing expense of SSI. Besides, in the third column of the Table, it is seen that the ratio of deficit of SSI to GDP has decreased considerably within the years. While this is encouraging, the fourth column of the Table shows that the ratio of deficit except state contribution to GDP has remained at almost the same level over the years. This clearly shows that the improvement of the SSI deficit is due to the state contribution, which started to be applied in 1 October, 2008 by Law No. 5510.

The existing Turkish Pension System mainly consists of mandatory public pension. In addition, the voluntary individual pension system has been in operation since 2003, although it has not yet reached the expected level. Moreover, the workers, who are employed under item (a) and (c) of paragraph one of article 4 of Law number 5510 and who are under age of 45, were started automatically include in private pension system

[3] Previously in Turkey for a long period of time insureds qualify for a pension with a certain number of premium payment days and insurance period without regard to age conditions. Women were qualified for pensions at the earliest at the age of 38, and men at the earliest at the age of 43. Even those insureds who are less than 18 years old before the date of April 1, 1981, were able to retire before that age limit [33]

Table 2.6: Financial Situation of Social Security Institution [90], [91]

Year	Pension Payments/GDP	SSI Deficit/GDP	Deficit except State Contribution/GDP
2003	-0,2	-3,0	-3,0
2004	-0,2	-2,8	-2,8
2005	-0,2	-2,9	-2,9
2006	-0,2	-2,4	-2,4
2007	-0,2	-3,0	-3,0
2008	-0,2	-2,7	-2,9
2009	-0,2	-3,0	-4,2
2010	-0,3	-2,4	-3,8
2011	-0,6	-1,3	-2,9
2012	-0,6	-1,2	-2,9
2013	-0,6	-1,3	-3,0
2014	-0,7	-1,1	-2,9
2015	-1,3	-0,6	-2,5

since the beginnig of 2017. This new implementation is called as auto-enrollment private pension system. On the other hand, it is not possible to talk about an occupational pension system that covers the whole of society or occupational groups, while there are some occupational pension systems specific to certain occupational groups.

2.9.1 First pillar pension system in Turkey

The first pillar of the existing Turkish pension system consists of a public pension system in which management and supervision are made by the state. The system based on mandatory participation, the benefits are determined on the basis of DB and the expenses are financed according to the PAYG method.

The structure and functioning of the existing public pension system in Turkey is determined by the "Social Insurance and Universal Health Insurance Law number 5510". In Article 4 of this Law, the persons who are obliged to be insured in the public pension system are explained in detail. Besides, the insureds are generally grouped under three different headings. First of these are the ones who are employed by one or more employer through a service contract (specified under item (a) of paragraph one of Article 4 (4-1/a)), second of these are the ones who are individuals working on his/her own name and account without being bound by a service contract and the village and quarter headmen (specified under item (b) of paragraph one of Article 4 (4-1/b)) and third of these are the ones who work for public administration (specified under item (c) of paragraph one of Article 4 (4-1/c)). In addition, those who do not work in a manner to require being subject to mandatory insurance in this Law or working as an insurance holder but less than 30 days a month or not working full time can make an optional insurance in the first pillar pension system.

Premiums are determined by insureds' insurable earnings. According to the Article

82 of Law number 5510, lower limit of the daily insurable earning in calculation of premiums to be collected and benefits to be granted by is one thirtieth of the minimum wage and the upper limit is 6.5 times the lower limit of daily earning. Moreover, according to the Article 81 of Law number 5510, the rate of invalidity, old-age and survivors insurance premiums is 20 percent of the insurable earning of the insurance holder. Insurance holder's share is 9 percent of it, on the other hand employer's share is 11 percent of it. In addition to these, the State contributes to the SSI, at a rate of one fourth of the invalidity, old-age and survivors insurances premium collected by the Institution per month.

In Turkish first pillar pension system, the benefits of the insured in old-age are old-age pension (per month) and lump sum (as a single payment). Among the necessary conditions for the insured to benefit from old-age insurance are; age requirement and number of premium payment days. According to the Article 28 of Law number 5510, old-age pension are provided to the individual who is over 58 if the individual is female or over 60 if the individual is male and minimum 9000 number of premium payment days of invalidity, old-age and survivors insurance are required. However, the number of premium payment days condition are applied as 7200 premium payment days for the insurance holders under item (a) of paragraph one of Article 4. The present age condition is gradually increasing in the following years, and since 1 January 2048, it is equalized at 65 years for both men and women. The age condition shall be applied as;

i. 59 for females and 61 for males between 1/1/2036 and 31/12/2037,

ii. 60 for females and 62 for males between 1/1/2038 and 31/12/2039,

iii. 61 for females and 63 for males between 1/1/2040 and 31/12/2041,

iv. 62 for females and 64 for males between 1/1/2042 and 31/12/2043,

v. 63 for females and 65 for males between 1/1/2044 and 31/12/2045,

vi. 64 for females and 65 for males between 1/1/2046 and 31/12/2047,

vii. 65 for both females and males as of 1/1/2048.

On the other hand, there are early retirement and partial pension opportunities under different circumstances.

According to the Article 29 of Law number 5510, the old-age pensions of insurance holders are found by multiplying the average monthly earning with the replacement rate. Average monthly earning is thirty times the average daily earning, calculated by the sum of insurance holder's insurable earning, found by valorization with the valorization coefficient realized every year, for the years passed from the year of the earning up to the date of requesting pension, divided by the total paid premium days excluding the nominal service period and actual service period increment. Replacement rate are applied as 2 percent for each 360 days of total number of premium payment days of the insurance holder, passed subject to invalidity, old-age and survivors insurances. Periods less than 360 days are considered proportionally in this calculation.

However, the replacement rate can not be over 90 percent. On the other hand, pensions are determined by increasing at the change rate in the general consumer prices index of the final base year announced by Turkish Statistical Institute based on previous six-month period, effective from the date of January and July payment dates of each year.

2.9.2 Third pillar pension system in Turkey

As a part of the social security reform in Turkey, individual pension system (Turkish synonym is bireysel emeklilik sistemi (BES)), which is based on voluntary participation, was accepted for the first time in 2001 with the "Individual Pension Savings and Investment System Law number 4632" but the system was actually put into practice in 2003. In this system, the benefits are determined on the basis of DC and the system is financed according to the funded method. It is obvious that the individual pension system applied in Turkey with all these features is included in the definition of the third pillar pension system. Therefore, the first step to a multi-pillar pension system was taken in Turkey with the implementation of this system.

Individual pension system, which has been established as a complement to the public pension system that has existed for many years in Turkey, has been established in order to encourage individuals to invest their retirement savings, to increase welfare levels of insureds by providing additional income during retirement period, to increase employment by providing long term resources to the economy and to contribute to economic development .

In Turkish individual pension system, insurance holders are entitled to retire on conditions that they stay in the system at least for 10 years and complete the age of 56. Retired persons could demand the accumulation of their private pension accounts to be paid in the framework of a program as a lump sum or as salary.

Individuals, who want to participate in the individual pension system force pension contracts with private companies that are licensed in the field of pension. These companies are subject to the supervision of the Undersecretariat of Treasury. Private pension system is basically based on 3 main method. First is the payment of contributions to individual pension accounts. Second is directing the contributions in order to invest in the selected funds in the legal framework. And the third is the payment of the accumulated contributions and investment incomes of these contributions to the insurance holders. In addition, according to the "Regulation on the Individual Pension System" numbered 28462, some amounts can be deducted off the contributions and the accumulated fund amount. These deductions that are determined in the related regulation are entrance fee, administrative expenses fee and fund management fee.

Individual pension contracts; can be arranged as group contracts or individual contracts. These contracts can be employer-contributed, employer-employee contributed, or only employee contributed. In addition, the implementation of state contribution to individual pension system has been begun since January 1, 2013. With this implementation, 25 percent of the contributions paid to the individual pension account on behalf of the participant, except for the contributions paid by the employer, are paid into the participant's account as state contribution. The state contributions are invested with the investment instruments determined by the Undersecretariat of Treasury. Depending on the time spent in the system since the beginning of the implementation, the state contribution and some or all of the invesment amount of this contribution are paid to the insurance holders. These are the entitled state contribution rates according to spent time in the system;

i. %15 from 3 years to 6 years,

ii. %35 from 6 years to 10 years,

iii. %60 for 10 years or more,

iv. %100 in case of retirement, death or invalidity

It can be understood from the ratios, the incentives were arranged to keep insurance holders on the system as long as possible.

On the other hand, significant changes were made in the implementation of the private pension system by "Amendment to the Private Pension Savings and Investment System Law number 6740" published in the Official Gazette dated 25 August 2016. With this Law, workers who are employed under item (a) and (c) of paragraph one of article 4 of Law number 5510 and who have not completed the age of 45 are automatically included in a retirement plan by employer's pension contract. So, auto-enrollment private pension system has started in Turkey.

In the new system, premiums are collected from the employee and are determined at the rate of 3 percent of the employees' insurable earnings. However, it is left to the authority of the Council of Ministers to increase this ratio up to 2 times, to decrease this ratio up to 1 percent or to limit the premiums to a fix amount. Employee contributions are deducted from the salaries of the employees by the employer and transferred to the insurance companies. If the employers do not transfer the premiums to the companies on time or transfers them late, the employers are held responsible for the financial loss of the employees' savings. In addition, the employees may request from the employers to contribute a higher amount of premium than the amount specified in the pension contract for auto-enrollment private pension system. Employees have the right to withdraw from the pension contract within 2 months from the date on which he / she is included in the pension plan. In case of withdrawal, the premiums and invesment incomes are paid to the employer within 10 working days. Employess who do not use the right to withdraw can request a break in the payment of premiums in cases determined by the Undersecretariat of Treasury.

When the employee's job is changed, if the new workplace has a retirement plan under this Law, the employee's saving and the number of premium payment days earned are transferred to the pension contract at the new workplace. If there is no retirement plan in the new workplace;

i. If the employee requests, he / she can continue to pay the contributions within the scope of the contract prepared in the previous workplace,

ii. If the employee do not request to continue, the pension contract is terminated.

With all that, the state contributions (25 percent of the contributions paid to the individual pension account on behalf of the employee) are provided in auto-enrollment private pension system as in the private pension system. In addition, if the employee does not use the right to withdraw, 1000 Turkish Lira (TL) is provided as an additional state contribution for once only. When the employee retires, the employee, who has chosen to take his / her savings within the scope of annual income insurance contract for at least 10 years, is entitled to an additional state contribution payment of 5 percent of his / her savings. In addition, there is no other deductions from the funds other than the fund management fee for the pension companies.

On the other hand, the transition to the auto-enrollment private pension system was determined gradually by a regulation. According to the Regulation, the gradual transition starts on Januvary 1, 2017 for employed persons who works for an employer with an employee number of 1000 or more and it is to be completed on 1 January, 2019 with the inclusion of different working groups at different times.

Turkish private pension system that has implemented for nearly 13 years, is still in its infancy. One of the important indicators of this situation is that, average age of contracts in force as of 2016 year-end is observed to be 3.4 [34]. Moreover, the ratio of the total value of pension funds to the GDP is a very important indicator for pension funds.

Figure 2.5 shows the total assets in funded and private pension arrangements as a percentage of GDP in OECD countries at which only 7 OECD countries (Australia, Switzerland, United States, Iceland, Canada, Netherlands, Denmark) reached total asset/GDP ratios above 100 percent in 2016. Besides, only 16 out of 35 countries' asset/GDP ratios have risen above 20 percent such that this rate is considered to be the minimum level to have a "mature pension fund market" by OECD. In the figure, it can be seen that, the weighted average of total asset/GDP ratio is 125.7 percent and the simple average is 49.5 percent while Turkey has 5.5 percent. So, we can say that, Turkey is far behind compared to OECD countries. This is an important indicator of weak effectiveness of pension funds and it can be said that pension fund market is rather far away from "mature pension fund market criteria" in Turkey.

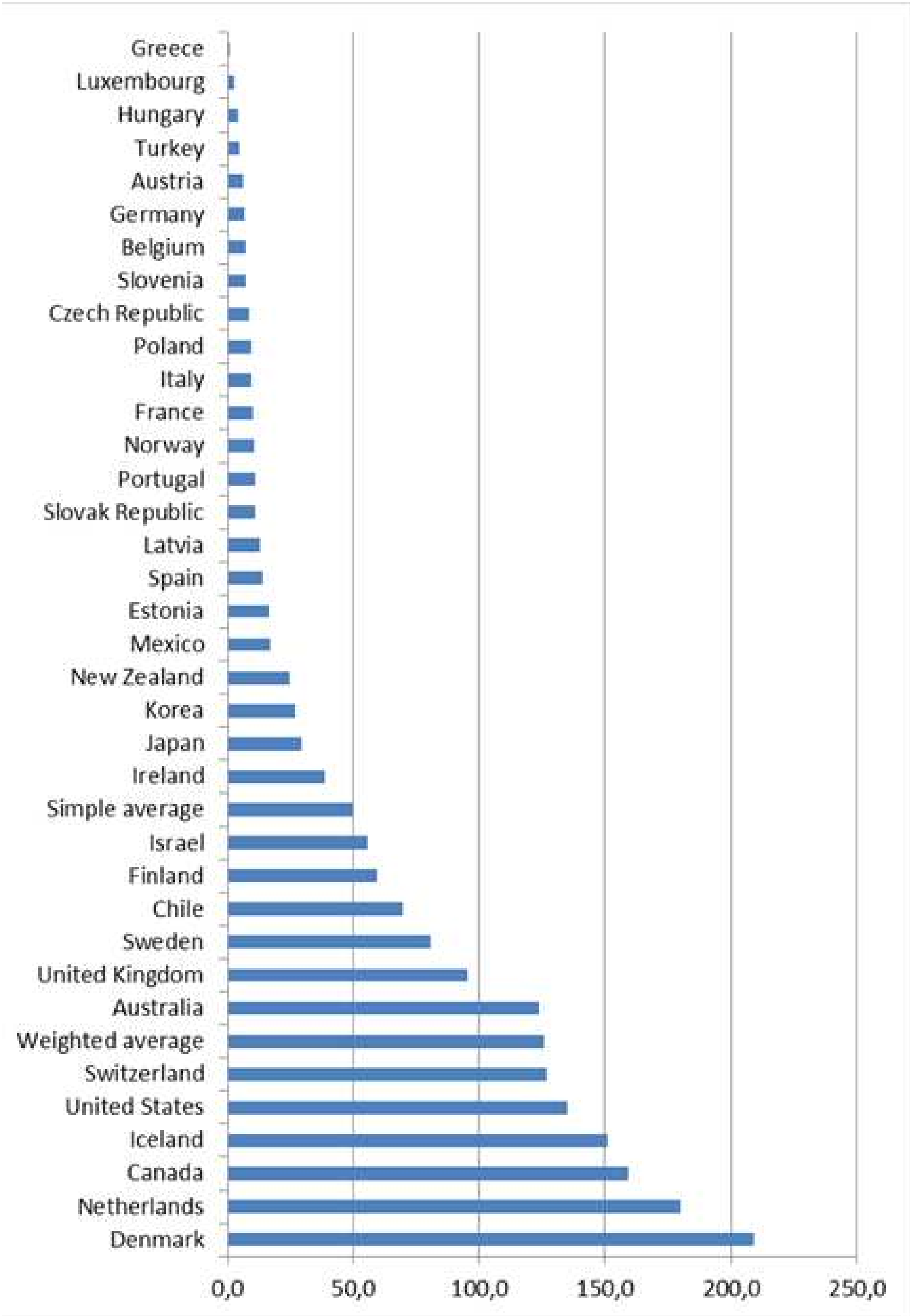

Figure 2.5: Total Assets in Funded and Private Pension Arrangements in 2016 [74]

2.9.3 Implementations of second pillar pension system in Turkey

Many insurance systems that could be included in the definition of SPPS were established in the historical development of Turkish pension system. But the limited coverage and insufficient benefits caused to make no progress in it's comprehensive implementation.

Savings and Charity Funds of Workers Union (Amelebirliği Biriktirme ve Yardımlaşma Sandığı), Primary School Teachers' Health and Social Aid Fund (İlkokul Öğretmenleri Sağlık ve Sosyal Yardım Sandığı), Army Aid Institution (Ordu Yardımlaşma Kurumu "OYAK"), Police Maintenance and Assistance Fund (Polis Bakım ve Yardımlaşma Sandığı "POLSAN"), Bank and Insurance Funds (Banka ve Sigorta Sandıkları) are the examples of SPPS implementations that so far established for relatively wide population and still continue of their operations in Turkey. On the other hand, Public Servant Assistance Institution (Memur Yardımlaşma Kurumu "MEYAK") ended its operations due to failure in fund management even though it covers wide population, too. Moreover, there are SPPS implementations covering smaller population such as The Central Bank of Turkey Staff Pension Fund Foundation (TC Merkez Bankası Mensupları Sosyal Yardım Sandığı Vakfı) and Ereğli Iron and Steel Incorporation Members Assistance Fund Foundatin (Ereğli Demir ve Çelik Fabrikaları T.A.Ş. Mensupları Yardımlaşma Sandığı Vakfı) [3].

CHAPTER 3

METHODOLGY: MEASURING THE LONGEVITY RISK ON A PENSION SYSTEM

As mentioned in the previous chapter, expected life time has been increasing rapidly in most of the countries around the world, over the last decades. The length of time that people are expected to live at birth has increased from 47 years to 70.8 years during the last sixyfive years [1]. Moreover, the life expectancy is expected to rise over time without an upper limit even if it would be in a slower pace [27].

Of course, gains of life expectancy are great news for humanity. But, it is very important to know how long people will live for many products. In the private sector, the most obvious examples are pensions and annuities [85]. So, if the necessary precautions were not taken, pension systems were affected from these life gains negatively in the aspect of retirement finances. Unfortunately, a lot of pension funds around the world, do not fully take into consideration of future improvements in life expectancy and mortality [4]. This may lead to miscalculation of liabilities and miscalculation of the liabilities may cause catastrophic financial loses for insurance companies or retired persons. So, it is so clear that, prediction of future life expectancy is very important issue for all sides in pension systems [89].

On the other hand, predicting mortality and life expectancy depends on a lot of unknown parameters. So, they are uncertain. From this point of view it can be said that, longevity risk is occurred from the risk that future mortality and life expectancy outcomes turn out different than expected [4]. In other words, it is the risk of populations living longer than expected [50] . The risk arises from the simple uncertainty about future mortality rates [85].

Therefore, mortality rates and life expectancies are the basic inputs in measuring the longevity risk. Especially, the life expectancy at retirement age is the vital component in calculating pensions.

[1] See Figure 2.1

3.1 Life (Mortality) Tables

Life tables (also called mortality tables) are probably the most largely used method of analysis in demographic calculations. They describe the mortality experience of a certain group of people. They show the probabilities of a member of a particular population living to or dying at a precise age, in a terse way. Moreover, population life tables are the mortality experience of the total population of a country. On the other hand, there are also seperate life tables for men and women and every country has its own life table [80], [41], [44].

The beginning of the life tables in a modern sense are based on many years ago. John Graunt's "Bills of Mortality" which had been published in 1662 and Edmund Halley's famous table for the city of Breslau which had been published in 1693 are the first examples of modern life tables. During the next hundred years, several life tables were constructed such as the French tables of Deparcieux published in 1746, of Buffon published in 1749, of Mourgue published in the 1790's and of Duvillard published in the 1790's, the Northhampton table of Richard Price published in 1783, and in the United States Wigglesworth's table published in 1793. The first official English life table was published in 1843 and several countries have established series of life tables dating back almost two centuries in Europe (Sweden in 1755, Netherlands in 1816, France in 1817, Norway in 1821, Germany in 1871 and Switzerland in 1876) [21]. On the other hand, Turkish Statistical Institute has produced the first Turkish life tables at national and provincial level in 2014 [96].

Life tables can be assorted as cohort or current life tables according to the reference year of the table, complete or abridged life tables according to the length of age intervals and single or multiple decrement life tables according to the number of characteristics considered [36].

i. **Cohort Life Table** is also called as generation life table or longitudinal life table. It shows the mortality experience of a real cohort from birth until no lives remain in the group. The main advantage of this sort of life table is its conceptual simplicity. On the other hand, it's major disadvantage is the requirement of long period of waiting. The life span of a cohort can be anywhere near 100 years or more. This reason makes this table limited in use and suitable to study with the groups that have shorter life span such as plants or insects [28, 31, 36]

ii. **Current Life Table** is also entitled as cross sectional life table, period life table or specified life table. This table uses data for a single cross section of time to represent an entire generation. It's main advantage is providing measures localized in time like change in expectation of life at birth from one year to the next. On the other hand, conceptual complexity is the main disadvantage of it [36].

iii. **Complete Life Table** is also called as unabridged life table. It contains data for every single years of age from birth to the last applicable age [105, 36].

iv. **Abridged Life Table** contains data by five or ten years intervals [36].

v. **Single Decrement Life Table** takes in consideration only one cause of death and only one characteristic at a time. It is concerned with general experience of a cohort by age [36].

vi. **Multiple Decrement Life Table** describes the seperate and combined effects of more than one characteristic. In contrast to the single decrement life table, it may consider more than one cause of death or more than one characteristic at a time [36].

3.1.1 Construction of the complete life table

Construction of a general life table is based on census and mortality statistics figures of local populations under the hypothesis of a closed demographic system [87]. A typical life table generally has 7 different columns.

Table 3.1: Typical Life Table Columns

1	**2**	**3**	**4**	**5**	**6**	**7**
x	l_x	d_x	q_x	L_x	T_x	e_x^0

Here, the symbol x represents the exact age. The basic definitions, notations and connections between the general life table functions as follows [8, 36];

The Survivorship Function (l_x), represents the number of persons that are alive at age x. For this function, l_0 called with a special name as "radix". The radix of life tables arbitrarily is set to 100,000.

Number of Deaths in the interval (x,x+1) (d_x), represents the number of deaths between exact ages x and x+1 for persons that alive at age x. That is;

$$d_x = l_x - l_{x+1}.$$

Mortality Rate at age x (q_x) is the probability the person at exact age will die within one year following of that age. It represents age-specific risk of death. That is;

$$q_x = \frac{(l_x - l_{x+1})}{l_x} = \frac{d_x}{l_x} = 1 - p_x.$$

where (p_x) represents age-specific risk of life. It is the probability the person at exact age x will survive up to his next birthday $x + 1$. That is;

$$p_x = \frac{l_{x+1}}{l_x}.$$

Person-Years lived by the cohort from age x to x+1 (L_x), represents the number of persons that alive at age x at any time in the stationary population. In other words, L_x

is the sum of the years lived by the l_{x+1} persons who survive in the interval x to x+1, and the d_x persons who die during that interval. L_x is always be the same in each year under a stationary condition. L_x becomes the mid-year population when the death at an exact age x are assumed to be uniformly distributed. That is;

$$L_x = l_x + 0.5(l_x - l_{x+1}),$$
$$L_x = 0.5(l_x + l_{x+1}) \quad \text{for} \quad x \geq 2. \tag{3.1}$$

This means that, it is assumed that a person dying between the age x and $x+1$ lives 0.5 years on average. So, L_x can be defined as;

$$L_x = l_x - 0.5d_x.$$

Person-Years lived after exact age x (T_x), represents the number of persons that alive at age x or older at any time in the stationary population. In other words, T_x is the sum of the numbers in the L_x column from age x to the last row (maximum age) in the table. That is;

$$T_x = \sum_{y=x}^{w} L_y = L_x + L_{x+1} + L_{x+2} + ... + L_w.$$

Here w is the attainable highest age.

So, L_x can also be written such that;

$$L_x = T_x - T_{x+1}.$$

Life Expectation at age x (e_x^0), represents the average number of additional years a person at exact age x is expected to live under the dominant mortality condition. That is;

$$e_x^0 = \frac{T_x}{L_x} = e_x + 0.5.$$

Here (e_x), represents the curtate expectation of life that implies the average number of complete years of life lived. However, the cohort l_0 after age x by each age of l_x persons attaining that age. It is given as;

$$e_x = \frac{L_{x+1} + L_{x+2} + ... + L_w}{l_x}.$$

Moreover, there are some specified assumptions that are used in construction of a life table. For example, the death must be the only factor causing the number of cohort at different ages to decrease in the construction of it. The cohort originates from some standard number at birth such as 10,000, 100,000 or 1,000,000 that is called the **radix** of life table. Moreover, persons die according to pre-determined mortality schedule at each age that is unchanged and fixed [5, 36].

3.1.2 Abridging the complete life table

The life tables that are based on single years of age are too large. So, demographers who do not need to calculate mortality with such precision generally use broader age groups. Ordinarily, five year age groups are used such as 0-4, 5-9, 10-14, etc. Life tables that are using broader age groups are called **abridged life tables** [44].

An abridged version of complete life tables can be easily calculated. The abridgement of the complete life table is simplified by an important property of l_x, T_x and e_x functions which describe exact age x. Here, x is the beginning of the age interval x to $x+n$ where n denotes the lenght of age interval. For instance, the life expectancy at age 20, e_{20} has the same value regardless of the age interval. In other words, e_{20} has the same value whether the age interval is 20-21 or 20-25. And the same applies for l_x and T_x. So, the values l_x, T_x and e_x can be extracted at 5-year intervals from the complete life table [5].

In contrast, q_x, d_x and L_x functions describe the age interval x to $x+n$. So, in abridged life tables the notations of these functions are a little bit different according to life tables. When, the width of an age group is denoted in years by the symbolthe life table quantities q_x, d_x and L_x are written ${}_nq_x$, ${}_nd_x$ and ${}_nL_x$ respectively in the abridged life table. On the other hand, for example, ${}_5q_{20}$ is the probability of dying between ages 20 and 25 and it will clearly be somewhat larger than the probability of dying between ages 20 and 21, q_{20}. Therefore, ${}_nq_x$, ${}_nd_x$ and ${}_nL_x$ must be recalculated in the abridged life table [5, 44]. They are defined as follows [44]:

i. $({}_nd_x)$, represents the number of deaths between exact ages x and $x+n$ years for persons that alive at age x.

ii. $({}_nq_x)$, represents the probability that the person at exact age x will die with n year following of that age.

iii. $({}_nL_x)$, represents the number of person-years lived between exact ages x and $x+n$.

And the calculations are as follows [5]:

$$ {}_nd_x = l_x - l_{x+n}, $$

$$ {}_nq_x = \frac{l_x - l_{x+n}}{l_x} = \frac{{}_nd_x}{l_x} = 1 -_n p_x. $$

Here, $({}_np_x)$ represents the probability that the person at exact age x will live with n year following of that age. That is,

$$ {}_np_x = \frac{l_{x+1}}{l_x}, $$

$$ {}_nL_x = l_x + \frac{1}{n}(l_x - l_{x+n}) = T_x - T_{x+n}. $$

Moreover, there are two difficulties with the calculation of ${}_nL_x$ in abridged life tables. The one is the assumption that deaths are distributed evenly over each age group is so unrealistic for the youngest age group. (In complete life table, it is assumed that deaths are distributed evenly over each year of life (see Equation 3.1).) Indeed, in this age group (0-4 years) most of the deaths occur to children whose ages are under 1 year. So, generally the youngest age group is seperated into two parts: under 1 year, and 1-4 years. The following formula is used for these two age groups;

$$ {}_nL_x = (n)(l_n - a_nd_x). $$

Here, the fraction a is often used as 0.85 for 0-1 years group and 0.60 for 1-4 years group [44, 103].

The other one is the ambiguousness about the age of the oldest person survives. Therefore, the width of the oldest age group (open-age interval) is often unknown. Some assumption must be made to calculate ${}_nL_x$ for the oldest age group. There are three possible ways to handle this problem. The first one is to make an assumption about the oldest age. When the oldest age is denoted by w, then (l_w) is equal to 0. But, there is a problem about this assumption. Deaths are not most likely to be evenly distributed over the age range between the lowest age in the oldest age group and age w. The second way is to make an assumption about the average number of years that a person that reaches the start of the oldest age group has left to live. For instance, it is suppossed that the oldest age group consist of persons aged 95 years and over. Then it is made an assumption about e_{95} and calculated ${}_nL_{95}$ using the following formula;

$$ {}_nL_{95} = l_{95}e_{95}. $$

The third way uses the fact that, logically, ${}_nq_x$ must be equal to 1 for the oldest age group. This means that, it is assumed that deaths are evenly distributed across the oldest age group [44].

3.2 Methods of Mortality Modelling and Forecasting

Modelling mortality is a very long history. After Gompertz published his law of mortality in 1825, a lot of models have been proposed. On the other hand, forecasting mortality and life expectancy is a more recent work. In this area sophisticated methods have been developed and applied in only few decades. But, it has risen in importance because the life expectancy has increased with unexpected rapidity and exceeded previously forecasted limits [76, 100].

Mortality forecasting generally involves the specification of an underlying model of data and a model for forecasting. Age, period (or time) and cohort are commonly employed to sort out the underlying model as zero-, one-, two- or three-factor models. Zero-factor models are basically an aggregate measure or an age specific rate where each age is treated independently. In this models there is no underlying model. One-factor models treat mortality rates (period or cohort) as a function of age. Two-factor models take usually age and period into consideration. Such models are employed by most recent models of mortality forecasting. In this model, age and cohort may be modelled as an alternative. On the other hand, three-factor models express mortality rates as a function of age, period and cohort [15].

There are three general approaches in mortality forecasting; **expectation**, **explanation** and **extrapolation**. In practice, the difference between these approaches is not always obvious. But generally; in the first approach (**expectation**), mortality forecasting depends on an expert opinion. Expert assumes a baseline scenario generally with alternative low and high scenarios. A lot of official statistical agencies had prioritized to the expectation approach, but several are now starting to use extrapolative ones. The main advantage of this approach is the conjunction of demographic, epidemiological and other relevant knowledge. On the other hand, its subjectivity and potential for bias are the disadvantages of it. Expectations are usually not a good method for mortality forecasting, either in the individual or population level. Individual expectations are related only the very short-term future and they have limited applicability. On the other hand, the conservativeness of expert-opinion-based targets of population expectations has caused inaccuracy. In other words, in population level expectation, the validity of forecasts and their uncertainty has been open to question and has yet to be evaluated [101, 15].

In **explanatory approaches**, casual forecasting methods involving econometric techniques that based on variables such as environmental or economic factors are used. This approach needs valuable medical knowledge and information on behavioral and environmental change. The main advantage of this method is the consideration of limiting factors and feedback mechanisms. But in practice, the relationship between risk factors and mortality is imperfectly understood. So this situation makes their use in mortality forecasting less than reliable and they have rarely used. Besides, this approach is not convinient for overall mortality forecasting and demands decomposition by cause of death. Now, causal modelling is limited with the few causes of death for which the determinants are well understood. Even if the determinants are measurable and well understood, causal models can not be extensively used because of a lack of sufficient data on the determinants. Moreover, even if well-defined explanatory models become available for most or all causes of death, it could still not be possible to forecast overall mortality simply as the sum of the independent forecasts. Because, in this case comorbidities and dependencies between causes would need to be taken into consideration [15].

On the other hand, in **extrapolative approaches**, some projections that are based on historical trends in mortality are used. Extrapolative models are the most common method that is used by actuaries and demographers [22, 23, 24, 25, 26]. In extrapolative approach, it is assumed that future trends will be a continuation of the past. This assumption is usually reasonable in mortality forecasting because of historical regularities. However, some exceptions may be occur such as temporary increases in mortality due to deaths from an illness, some disasters or some economic reasons. This discussion takes extrapolative methods from single linear extrapolation to methods based on two-factor models [15].

Moreover, the extrapolative models are based on time series methods for one or more observed indicators. The type of time series model which is employed can be extremely simple, for example constant future levels of the indicators. On the other hand, we can find complicated models of the ARIMA-type, for instance a random walk with drift for life expectancy at birth [52].

Most recent developments have focused on two- and three-factor underlying models with the extrapolative approach. Forecasts based on zero-factor underlying models are generally uncommon. These models provide no information about changes in the age pattern. And independency of age-specific rates may produce irregular and unreasonable age patterns. Furthermore, one-factor models can be used more widely in the case of lack of necessary data. These models are also inadequate. However they have the advantage of smoothness across age, they have serious problems for forecasting. For example, parameter uninterpretability and over-parametrization necessitating multivariate time series models in order to avoid unreasonable trends and forecast age patterns. Two-factor models can be estimated using main components whereby matrix decomposition identifies independent mortality components or age patterns and their importance over time. The estimated age parameters are assumed to be fixed in forecasting and time series methods are used in order to extrapolate the time-varying parameter. Two-factor models has a major advantage. The methods which are using principal components techniques have proved to be successful. The **Lee- Carter model** is the most common used two-factor model for forecasting mortality. This model involves a single time-varying parameter, produces reasonable forecasts in general. It permits a changing age pattern of mortality. Moreover, recent related developments include across-age smoothness, improve estimation and offer convenient enhancements in forecast accuracy. **GLM**, which is including dynamic parametrizations, have been less successful because of the nonlinearities in time that may lead to unreasonable forecast trends. **P-Splines Model** have been successful in applications to date. On the other hand, three-factor models have the advantage of incorporating cohort effects. But these models usually suffer from data unaccsessibility and difficulties in forecasting cohort trends, in practice. The **Lee-Carter APC Method** looks encouraging but it has not yet comprehensively assessed [15, 16].

3.2.1 Lee-Carter Model

The Lee-Carter model is a strong approach to project mortality rates. It has a major advance in mortality projection [27]. In this model, mortality projections describe the logarithm of a time series of age specific mortality rates m(x,t) as the sum of an age-specific and time independent component alpha(x) and another component of a production of a time-varying parameter kappa(t) and age specific component beta(x). Here, kappa(t) reflects the general level of mortality. Besides, beta(x) refers to how slowly or rapidly mortality at each age varies when the general level of mortality changes [43].

Lee-Carter model has been extensively used in actuarial and demographic applications since its introduction in 1992. Moreover, the original approach of Lee-Carter method has had several variants, extentions or generalizations up to now and they have been used in a lot of studies. It has been used as the main method of stochastic forecasts of the United States social security system and other aspects of the United States federal budget (see Congressional Budget Office of the United States (1998) [20]). United Nations Population Division has made long-term projections for all countries with a variant of the Lee-Carter model until 2300 [10].

Lee-Carter model is one of the most commonly used model in order to project future mortality. It has a lot of advantages. First of all it's simplicity. It's parameters are easily interpretable and besides, a simple random walk with drift forecast has commonly been suitable for the single extrapolated parameter. In addition, it has fewer parameters than other stochastic mortality models. The parsimonious model structure constrains the behavior of future mortality rates and causes a stable age pattern of mortality in the projections. This effectively avoids the mortality crosovers which is a non-monotonicity in adulthood and senescent mortality over age and various anti-intuitive behaviours that may be found in some other stochastic methods. Moreover, the model involves minimal subjective judgment and it produces stochastic forecasts with probabilistic prediction intervals. On the other hand, the model has some disadvantages, too. First disadvantage of the model is comes from being an extrapolative estimation method. In other words, the future mortality structure is predicted based on the past mortality behaviour. So that, the existence of possible structural changes will be ignored, if the observed pattern of historical structure and development does not resemble the future. Secondly, it assumes that the ratio of the rates of mortality change at different ages remains invariant over time (in other words, b_x is assumed to be constant over time), however evidence of substantial age-time interaction has been found. Additionaly, Lee-Carter forecast rates are lack of across-age smoothness and become increasingly rough over time. And this may be problematic in practical applications. Besides, the rigorous model structure may generate overly narrow confidence intervals that may result in underestimation of the risk of more extreme outcomes. This defeats the original aim of going on to a stochastic framework. Lastly, the situations such as changes in socio-economic conditions, changes in lifestyles or the emergence of new diases are not included in the model [15, 38, 64, 61, 108].

In the study Lee and Carter (1992), a new demographic model of mortality that represents mortality level by a single index is developed. Then, demographic model is fitted to U.S. data so the performance of it was evaluated historically. After that, the index of mortality and generated associated life-table values at five-year intervals is forecasted with using standard time series methods. In the study, a log-bilinear model with the variables x (age) and t (time) is suggested to be able to estimate the force of mortality (age specific mortality rates) $m(x, t)$ at age x and time t. And the model is written as follows;

$$\ln(m(x,t)) = \alpha(x) + \beta(x)\kappa(t) + \varepsilon(x,t). \tag{3.2}$$

Here, $\alpha(x)$ and $\beta(x)$ are appropriately chosen sets of age-specific constants, $\kappa(t)$ is time-varying index and $\varepsilon(x,t)$ is the error index that reflects particular age-specific historical influences that are not captured by the model. To be more specific, $\kappa(t)$ is an index of the level of mortality, $\alpha(x)$ indicates the general age shape of the age specific mortality rates and $\beta(x)$ indicates which rates decline rapidly and which rates decline slowly in response to changes in $\kappa(t)$ [61]. Actually in theory, $\beta(x)$ could be negative for some ages but in practice especially over the long periods this situation does not seem. Besides, $\varepsilon(x,t)$ is assumed normally distributed with mean 0 and variance σ^2.

On the other hand, for a unique solution of Equation 3.2, $\beta(x)$ and $\kappa(t)$ are normalized such as;

$$\sum \beta(x) = 1 \quad \text{and} \quad \sum \kappa(t) = 0. \tag{3.3}$$

So, under the estimation in Equation 3.3, $\alpha(x)$ are the average over time of the $In(m(x,t))$;

$$\alpha(x) = \frac{1}{t_n - t_1} \sum_{t_1}^{t_n} \ln(m(x,t)).$$

The model can not be fitted by simple methods such as the ordinary least squares since the lack of any regressors; on the right side of Equation 3.2 there are only the estimated parameters and unknown index $\kappa(t)$. Therefore, in the study, SVD that provides to break a matrix into simpler meaningful partitions is used in order to obtain the parameters $\beta(x)$ and $\kappa(t)$ (The SVD method is applied to the matrix of $In(m(x,t))$–$\alpha(x)$). With this way, the fitted mortality rates can not give actual numbers of deaths. Moreover, $\kappa(t)$ have been estimated in order to minimize errors in logarithms of mortality rates. It does not give the mortality rates themselves. So, $\kappa(t)$ is reestimated. Again, $\alpha(x)$ and $\beta(x)$ is found with the same way as above. But this time $\kappa(t)$ have been found by an iterative search and have differed from the direct SVD estimates. Here for each year $\kappa(t)$ is found for the given actual population age distribution, the implied number of deaths would be equal to the actual number of deaths. This estimation have been written as follows:

$$D(t) = \sum [N(x,t)e^{\alpha(x)+\kappa(t)\beta(x)}].$$

Here $D(t)$ is the observed number of deaths at year t, $N(x,t)$ is the age distribution for a given population at year t and $\kappa(t)$ is the reestimated parameter.

*The **SVD** method is introduced by G. Golub and W. Kahan in 1965. It is presented as a method which decomposes the singular values, pseudoinverses and the ranks of* *matrixs [108]. It is a factorization of a complex or real matrix. It takes a rectangular matrix which defined as $m \times n$ matrix A. Here, n rows represents the genes and p columns represents the experimental conditions [68].*

Theorem 3.1 (*Singular Value Decomposition of a Matrix [58]*)**.** *Let A be a $m \times n$ real matrice. In this case, there are $m \times m$ and $n \times n$ orthogonal matrices such that;*

$$A_{m\times n} = U_{m\times m} S_{m\times n} V^T_{n\times n}.$$

Here, S is a diagonal $m \times n$ matrix and $s_{11} \geq s_{12} \geq ...s_{pp} \geq 0$ and $p = min(m, n)$.

*The columns of U are called the **left singular vectors**, the columns of V are called **right singular vectors** and the diagonal values of S are called the **singular values** [58].*

Calculating the SVD is to find the eigenvalues and eigenvectors of AA^T and A^TA. The eigenvectors of A^TA make up the columns of V, on the other hand the eigenvectors of AA^T make up the columns of U. In addition to this, the singular values in S are square roots of eigenvalues from AA^T and A^TA. The singular values are always real numbers. And if the matrix A is a real matrix, then U and V are real matrixs, too [68].

In Lee-Carter method, in order to forecast mortality rates, estimating only the reestimated $\kappa(t)$ values is sufficient (since $\alpha(x)$ and $\beta(x)$ values are assumed invariant over time). The reestimated $\kappa(t)$ is modelled and forecasted with using Box-Jenkins time series methods. In almost all applications and in the study of Lee and Carter (1992), the random walk with drift has been found to be suitable (but in the orijinal study, it is emphasized that other ARIMA models could be preferred for different data sets);

$$\kappa(t) = \kappa(t-1) + d + e(t).$$

Here, d is the drift parameter and $e(t)$ is an error term. Forecast mortality rates are found with using estimated $\alpha(x)$ and $\beta(x)$ and forecasted $\kappa(t)$ [102, 15].

3.2.2 Generalized Linear Modelling

In the study Renshaw (1991) [82], it is observed that many of the models used by actuaries for graduation purposes are specific examples of GLMs. The GLM provides the ideal setting for actuarial graduation techniques both those offer a more unified approach to graduation and offer a more comprehensive set of modelling methods. On the other hand, Renshaw et. al. (1996) [84] and Sithole et al. (2000) [95] forecasted mortality within the GLM structure without using additional models. They used Poisson GLM to model and forecast forces of the mortality. The force of mortality at age x and in year t ($\mu(x, t)$) was modeled by a log link function such as;

$$\ln \mu(x,t) = \beta_0 + \sum_{j=1}^{s} \beta_j L_j(x') + \sum_{i=1}^{r} \alpha_i(t'^i) + \sum_{i=1}^{k}\sum_{j=1}^{l} \gamma_{ij} L_j(x') t'^i. \qquad (3.4)$$

where α_i, β_j and γ_{ij} are unknown parameters, $L_j(x')$ represent Legendre polynomials of degree j, and x' and t' are the transformed ages and calendar years that respectively map x and t onto the interval $[-1, 1]$ [1].

Moreover, the GLM model which was used Renshaw et. al. (1996) and Sithole et al. (2000) has become an inspiration for many studies. Booth and Tickle (2008) further discuss these studies.

3.2.3 Penalized Splines Regression

P-splines model was suggested as the extension of GLM models by Eilers and Marx (1996) [35]. On the other hand, Currie et al. (2004) [29] showed how the P-Splines method can be extended to the smoothing and forecasting of two-dimensional mortality tables. The P-Splines approach to mortality projection can be summarised as follows. A rich enough set of basis splines in two dimension is chosen and this basis fit to data with using a penalised likelihood, choosing the level of the penalty in order to enforce reasonable smoothness. The fitting is carried out over the whole region of the (x, c)-plane which is covering the region of the data and the region of the projection. Because of the operation of the penalty, a well-behaved projection is guaranteed in the latter region. The fitted surface of values can be regarded as the 'mean sheet' of the regression. 'Standart deviation sheet' is obtained from the variance matrix of the estimated parameters. This 'standart deviation sheet' incorporates all of the information about parameter uncertainty since it is based on the variance matrix of the parameter estimates [24].

3.2.4 Lee-Carter Age-Period-Cohort Method

In the study Renshaw and Haberman (2006) [84], the Lee-Carter model were extended to a APC basis. The proposed APC methodology was applied to the 1961-2000 United Kingdom population mortality rates for each gender.

The Lee-Carter APC model is a bilinear model in the variables x (age), t (period) and c (cohort). The model is as follows;

$$\ln \mu(x,t,c) = \alpha(x) + \beta_1(x)\kappa(t) + \beta_2(x)I(c) + z(x,t,c).$$

where $\mu(x,t,c)$ is the force of mortality at age x in year t for generation c and $z(x,t,c)$ is the random error term. The $\alpha(x)$ coeffificients represent the average level of the $log\mu(x,t,c)$ surface over time. Moreover, the $\beta_1(x)$ and $\beta_2(x)$ coefficients describe the pattern of deviations as $\kappa(t)$ and $I(c)$ vary, respectively. On the other hand, the $\kappa(t)$

represents the change in overall mortality over time and $I(c)$ represents the change in mortality between generations [84, 26].

The model does not define unique choices for the parameters. Because the parameters $\beta_1(x)$ and $\kappa(t)$ along with the $\beta_2(x)$ and $I(c)$ appear with their products $\beta_1(x)\kappa(t)$ and $\beta_2(x)I(c)$. Moreover, there is a relationship between x, t and c such as $c = t - x$. So, the constraints have to be applied to the fitted parameters because of producement unique solutions. The following constraints are suggested;

$$\sum_x \beta_1(x) = 1, \sum_x \beta_2(x) = 1 \quad \text{and either} \quad I(t_1 - t_k) = 0 \quad (\text{or} \quad \kappa(t_1) = 0).$$

The model can be fitted using standard likelihood methods. Projected mortality rates produced by fitting time-series models to the fitted $\kappa(t)$ and $I(c)$ parameters. And finally, ARIMA process is used to project this time-series forward [84, 26].

The model successfully captured APC effects in United Kingdom mortality. It represents an important development over age-period and age-cohort models. But, out-of-sample testing by CMIB (2007) indicated that some of the theoretical features of the model needed further examination [15, 26].

CHAPTER 4

IMPLEMENTATION: MESURING LONGEVITY RISK IN TURKEY

As mentioned in Chapter 2, the developments such as rise in welfare, increased access to health services and development of health technologies resulted in a rapid increase in the expected life times in Turkey likewise almost every part of the world. The life expectancy at birth has increased 33.8 years from 1950 to 2015 in Turkey (Figure 2.3). Therefore, this increase has led to an incremental longevity risk on Turkish pension system. Today, making pension calculations without any projections about the future can cause important fiscal loses.

From this point of view, in this section it is aimed to examine the impacts of longevity risk on Turkish SPPS. For this purpose, firstly Lee-Carter method is used to find expected life times at retirement age and secondly Monte Carlo simulations are used to measure the longevity risk.

Haberman and Russolillo (2005) studied mortality forecasting with Lee-Carter Method for the Italian population. In the paper, they investigate the feasibility of using the Lee-Carter methodology to construct mortality forecasts for the Italian population. And then, they fit the model to the matrix of Italian death rates for each gender from 1950 to 2000. They forecast a time-varying index of mortality in an ARIMA framework and after that they use it to generate projected life tables. For the purpose of comparison, they introduce an alternative approach to forecast life expectancies on a period basis. As a result they conclude that, Lee-Carter modelling of the underlying mortality rates is a superior method in theoretical terms. In this study, exactly the same way of Lee-Carter model fitting to the Italian population is applied to data that obtained from the life tables which were prepared for both sexes in each year from 1932 to 2015 for Turkey in the study Yildirim (2014) [107]. Besides, Antolin (2007) studied longevity risk on private pensions for several OECD countries. He uses Monte-Carlo simulations of the Lee-Carter model of mortality to measure uncertainty surrounding mortality and longevity outcomes. He finds a non-negligible longevity risk with future changes in mortality and life expectancy. In this study, after the calculation of future mortality rates with Lee-Carter model, Antolin's (2007) study is taken as an example. Monte-Carlo simulation is also applied to measure uncertainty surrounding future mortality and life expectancy prospects for Turkey.

4.1 Data and Notation

The general population censuses in Turkey are made by Turkish Statistical Institute. Population censuses had been carried out almost every five years from 1927 to 2007 and they were started to be done every year after 2007. Although the population censuses in Turkey are almost the same age as the history of the Turkish Republic, the present data does not have enough detail to analyze the population in past years. Besides, since different methods have been used in the population censuses which were made before and after the year 2007, there are inconsistencies in the data set. So, it is not convenient to use the actual past population data to measure longevity risk in Turkey.

For this reason, $l_{x,t}$ values for the Turkish population are taken from Yildirim (2014) for each sex and each year from 1932 to 2015. In this study, the Lee-Carter model is fitted to the matrix of Turkish mortality rates from 1932 to 2015 for males and from 1933 to 2015 for females due to the lack of female data in 1932. After that, the forecasts of these single parameters are used to generate predictions for both of the age and period distribution of mortality for next years. In particular, the study is focused on the life expectancy at 65 years old, that is assumed to be the retirement age, to measure the longevity risk on pensions. Microsoft Excel and Statistical Package for the Social Science (SPSS) are employed to run these analysis.

The survivorship function, $l_{x,t}$'s for five-year age groups under 105 years old are used for analysis. Firstly, number of deaths, $d_{x,t}$ are calculated from the $l_{x,t}$ and then total number of person-years lived, $L_{x,t}$ are calculated from the $d_{x,t}$ for each gender by the years and five-year age groups. After that, number of deaths are denoted by a 5×1 matrix, where the first number refers to the age interval, and the second number refers to the time interval. The age interval is denoted by; $x = x_1, x = x_2, ..., x = x_k$ and it is grouped in classes as $[0, 1-4, 5-9, 10-14, ..., 100-104, 105+]$. And the time interval is denoted by $t = t_1, t_1 + 1, ..., t_1 + h - 1 = t_n$ where $h = t_n - t_1 + 1$; $t_1 = 1932$ for males, $t_1 = 1933$ for females and $t_n = 2015$. From these data, the force of the mortality rates are calculated with the formula that; $m_{x,t} = \dfrac{d_{x,t}}{L_{x,t}}$.

4.2 Lee-Carter Model Fitting

This part of the study includes the Lee-Carter model fitting to the Turkish population data. Because there are no regressors, the Lee-Carter model can not be fitted by ordinary regression methods. Therefore, a close approximation which is suggested by Lee and Carter (1992) is used in order to find a least square solution to Equation 3.2 as follows;

$$\ln(m_{x,t}) = \alpha(x) + \beta(x)\kappa(t) + \varepsilon(x,t).$$

Here, the error terms $\varepsilon(x,t)$ are assumed to be homoschedastic. As in the study of Lee-Carter (1992), the sum of $\beta(x)$ is equal to 1, and the sum of $\kappa(t)$ is equal to 0.

Under these assumptions, $\alpha(x)$ must be the average values over time of the $\ln(m_{x,t})$;

$$\alpha(x) = \frac{1}{h}\sum_{t=t_1}^{t_n} \ln(m_{x,t}) = \ln[\prod_{t=t_1}^{t_n} m_{x,t}^{\frac{1}{h}}]$$

Figure 4.1 shows the calculated values in five-year age groups of $\alpha(x)$ for each gender.

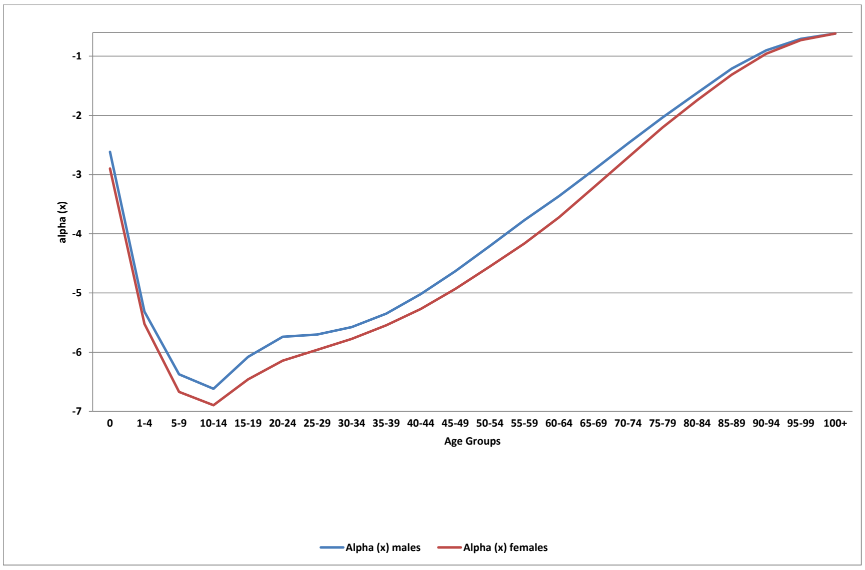

Figure 4.1: $\alpha(x)$ values for males and females in five-year age groups

Here, $\alpha(x)$ represents the general age shape of the age specific mortality rates. It can be said that higher $\alpha(x)$ values reflects higher mortality rates. Besides, it can be seen from Figure 4.1, $\alpha(x)$ male values begin with approximately -2.92 for the newborns because of higher potential death risk until age 1 but then they decrease until the age interval 15-19. With the age interval 15-19, $\alpha(x)$ values begin to increare again and the upward trend continues as the age increases. Consequently, they reach approximately -1.01 for ages older than 100. So, it can be said that, calculated $\alpha(x)$ values are suitable for real life conditions.

Again, it can be seen from the figure that, $\alpha(x)$ female values begin with approximately -3.20 value for newborns and then they decrease until the age interval 15-19. With the age interval 15-19, $\alpha(x)$ values begin to increase again and they reach approximately -1.03 at the oldest age. The point to be highlighted here is that, $\alpha(x)$ values for males are bigger than $\alpha(x)$ values for females at each age groups. This means that, males have higher mortality rates than females at each age interval.

Moreover, $\kappa(t)$ must be equal to the sum over age of $(\ln(m_{x,t}) - \alpha(x))$. And each $\beta(x)$ is found seperately for each age group by regressing $(\ln(m_{x,t}) - \alpha(x))$ on $\kappa(t)$. In other words, $\beta(x)$ is estimated from $(\ln(m_{x,t}) - \alpha(x)) = \beta(x)\kappa(t)^{(1)} + \varepsilon(x,t)$ using the least squares estimation. Here, $\kappa(t)^{(1)}$ refers to the $\kappa(t)$ which is estimated in the previous step. More precisely, $\beta(x)$ is chosen in order to minimize;

$$\sum_{x,t} (\ln(m_{x,t}) - \alpha(x) - \beta(x)\kappa(t)^{(1)})^2 \Longrightarrow \beta(x) = \frac{\sum_{t=t_1}^{t_n} \kappa(t)^{(1)}(\ln(m_{x,t}) - \alpha(x))}{\sum_{t=t_1}^{t_n} (\kappa(t)^{(1)})^2}.$$

Figure 4.2 shows the calculated values of $\kappa(t)$ in each year for each gender.

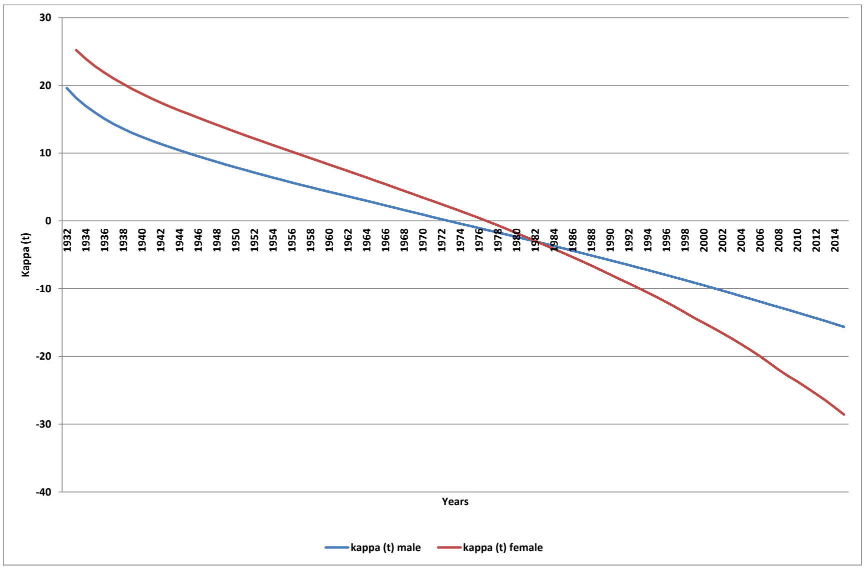

Figure 4.2: $\kappa(t)$ values for males and females for each year between 1932-2015

Here, $\kappa(t)$ is an index which describes the variation in the level of mortality at year t. More precisely, it represents the changes the general level of mortality. It can be seen from Figure 4.2, $\kappa(t)$ values decreases throughout the years for each gender. So, we can clearly say that, the level of mortality decreases from 1932 to 2015 for each gender. Again, it can be seen from the figure that male $\kappa(t)$ values begin with approximately 19.61 and it declines until reaching -15.64 in 2015. Besides, female $\kappa(t)$ values begin with approximately 25.21 and it declines until reaching -25.58 in 2015.The point to be highlighted here is that, $\kappa(t)$ values for females decreases with bigger acceleration than $\kappa(t)$ male values. It says us, female mortality rates decreases faster than male mortality rates.

Additionally, Figure 4.3 shows the calculated values in five-year age groups of $\beta(x)$ for each gender.

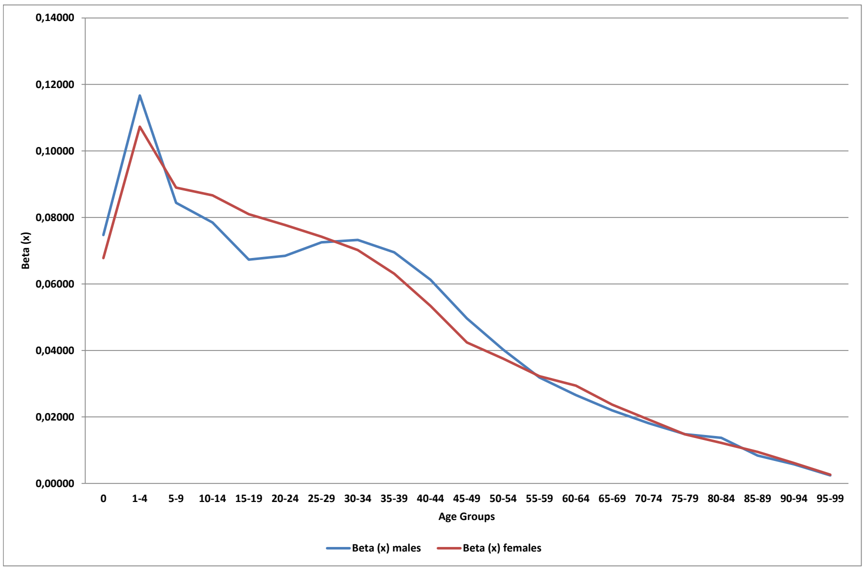

Figure 4.3: $\beta(x)$ values for males and females in five-year age groups

Here, $\beta(x)$ values describe the tendency of mortality at age x to change when $\kappa(t)$, in other words the general level of mortality changes. More precisely, when $\beta(x)$ value is large for some x, then the mortality rate at age x varies substantially when the $\kappa(t)$ changes such as with $x = 0$ for infant mortality. On the contrary, when $\beta(x)$ value is small for some x, then the mortality rate at age x varies little when the $\kappa(t)$ changes such as with mortality at older ages. At Figure 4.3, it can be seen that, for each gender $\beta(x)$ values are very large at the age $x = 0$ in an other saying for the infant mortality. Besides, for each gender, as age increases, $\beta(x)$ values decreases and for the oldest age it is very close to 0. Another point to be highlighted here is that, the shapes of $\beta(x)$ lie for males and females are quite similar.

4.2.1 Reestimating $\kappa(t)$

First stage estimation of $\kappa(t)$ is based on minimizing errors in the logarithms of mortality rates instead of the mortality rates themselves. In this way, the fitted mortality rates generally are not lead to the actual number of deaths when applied to given population age distributions. In other words, important discrepancies can occur between the actual and predicted deaths. The discrepancies can be removed by keeping $\alpha(x)$ and $\beta(x)$ as estimated in the first stage but calculating $\kappa(t)$ in another way. For this aim, a new set of $\kappa(t)$ are calculated with the methodology from Section 3 of Lee and Carter (1992). For each year, a new estimate of $\kappa(t)$ is found by an iterative search such that the implied number of deaths is equal the actual number of deaths. More precisely, the estimated $\kappa(t)$'s are adjusted so that the actual total observed deaths $\sum_{t=t_1}^{t_k} d_{x,t}$ equal to the total expected deaths $\sum_{t=t_1}^{t_k} L_{x,t}e^{\alpha(x)+\beta(x)\kappa(t)}$ for each year t.

The iterative search proceeds as follows [43]:

1. In each period, the total expected deaths $\sum_{t=t_1}^{t_k} L_{x,t}e^{\alpha(x)+\beta(x)\kappa(t)^{(1)}}$ are compared to the actual total observed deaths $\sum_{t=t_1}^{t_k} d_{x,t}$
2. This comparison outcomes one of three possible states:
   i. If $\sum_{t=t_1}^{t_k} L_{x,t}e^{\alpha(x)+\beta(x)\kappa(t)^{(1)}} > \sum_{t=t_1}^{t_k} d_{x,t}$, the expected deaths are need to be decreased, the estimated $\kappa(t)$ would adjusted so that the new estimate of $\kappa(t)$, let's say $\kappa(t)^{(2)}$, will be equal to; $\kappa(t)^{(1)}(1-d)$, if $\kappa(t)^{(1)} > 0$; $\kappa(t)^{(1)}(1+d)$, if $\kappa(t)^{(1)} < 0$, where $\kappa(t)^{(1)}$ is the first estimate of $\kappa(t)$.
   ii. If $\sum_{t=t_1}^{t_k} L_{x,t}e^{\alpha(x)+\beta(x)\kappa(t)^{(1)}} = \sum_{t=t_1}^{t_k} d_{x,t}$, the iterations are stopped here.
   iii. If $\sum_{t=t_1}^{t_k} L_{x,t}e^{\alpha(x)+\beta(x)\kappa(t)^{(1)}} < \sum_{t=t_1}^{t_k} d_{x,t}$ the expected deaths are need to be increased, the estimated $\kappa(t)$ is adjusted so that the new estimate of $\kappa(t)$, let's say $\kappa(t)^{(2)}$, will be equal to; $\kappa(t)^{(1)}(1+d)$, if $\kappa(t)^{(1)} > 0$; $\kappa(t)^{(1)}(1-d)$, if $\kappa(t)^{(1)} < 0$, where $\kappa(t)^{(1)}$ is the first estimate of $\kappa(t)$.
3. Turn to Step 1.

This iterative process is run with using Microsoft Excel and the new estimates of $\kappa(t)$ is found.

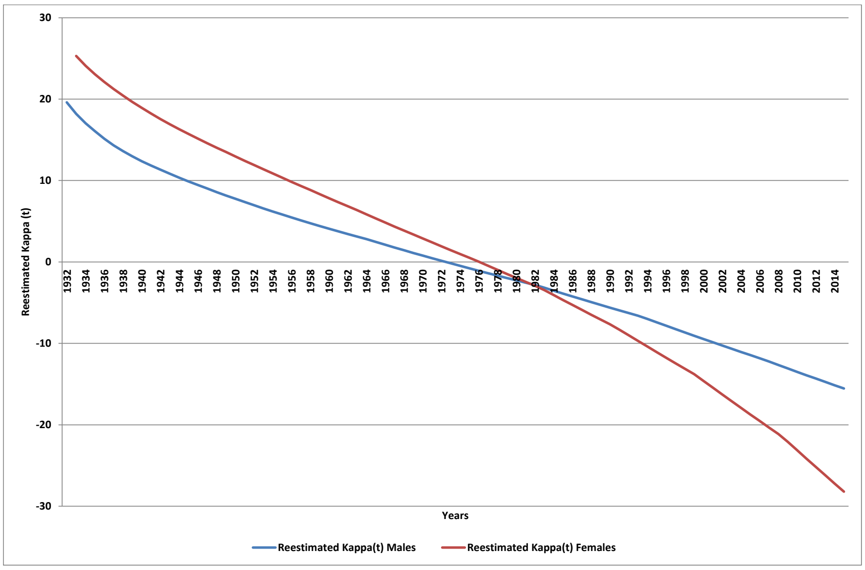

Figure 4.4: Reestimated $\kappa(t)$ values for males and females for each year between 1932-2015

Figure 4.4 shows the calculated values of reestimated $\kappa(t)$ in each year for each gender. It can be seen from Figure 4.4 that male reestimated $\kappa(t)$ values begin with approximately 19.61 and it declines until reaching -15.53 in 2015. Besides, female reestimated $\kappa(t)$ values begin with approximately 25.29 and it declines until reaching -28.20 in 2015. When we compare the $\kappa(t)$ and reestimated $\kappa(t)$ values from Figure 4.2 and Figure 4.4, we can say that, the shapes of $\kappa(t)$ and reestimated $\kappa(t)$ lie for each gender are quite similar. But the point to be highlighted here is that, reestimated $\kappa(t)$ values for females changes much more than reestimated $\kappa(t)$ male values in comparison to $\kappa(t)$ values.

4.2.2 ARIMA methodology

In this section, standard Box and Jenkins methodology is used to obtain future forecasts of mortality rates. It is known that, the reestimated time-dependent parameter, $\kappa(t)$, can be modelled as a stochastic process. Here, $\kappa(t)$ is projected with ARIMA methodology until the year 2057. The choice of this time is explained in Section 4.4.1 in detail.

Furthermore, SPSS 22 is used to implement ARIMA methodology to the reestimated $\kappa(t)$ values for each gender seperately. ARIMA (2,3,0) and ARIMA(1,2,0) is approved for males and females respectively [1].

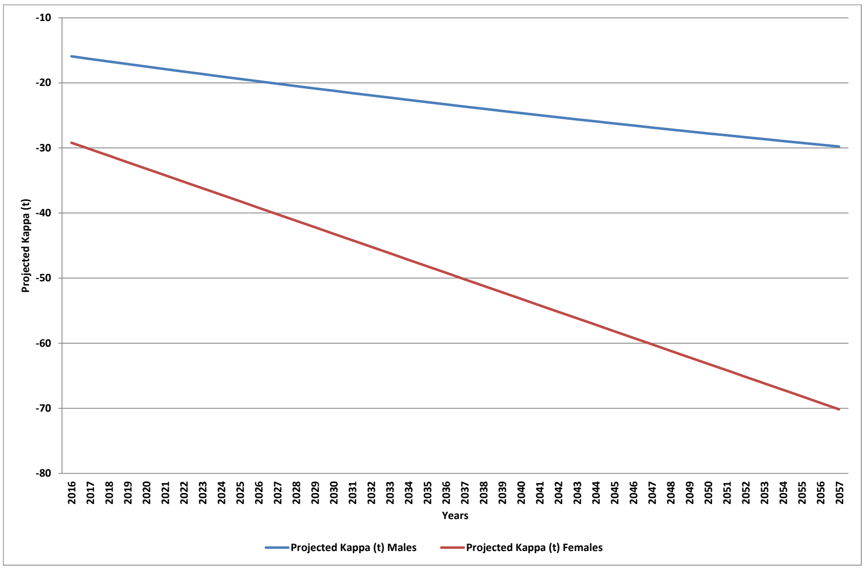

Figure 4.5: Projected $\kappa(t)$ values for males and females for each year between 2016-2057

[1] It can be seen some details about ARIMA methodology in Appendix A and Appendix B for females and males respectively.

It can be seen from Figure 4.5 that, projected $\kappa(t)$ values continue to decrease until the year 2057 for each gender. It begins with approximately -15.93 and reaches approximately -29.78 for males along the years from 2016 to 2057. And for females it starts approximately -29.20 and decreases to approximately -70.16 throughout the years.

Besides, this figure says that general level of mortality will decrease throughout the years for each gender. But if we look at it seperately for males and females, we recognize that projected $\kappa(t)$ values for females decreases with bigger acceleration than males. This situation is thought to be a result of bigger acceleration of reestimated $\kappa(t)$ values for females in accordance with males between the years 1932-2015.

4.2.3 Projecting lifetables

In this section, projected lifetables from 2016 to 2057 are constructed with using projected $\kappa(t)$ values. So, it is aimed to find forecasts of life expectancies at age 65 for each gender at 2057. Because the available data are in five-year age groups, projected lifetable values will be five-year intervals, too.

Firstly the following formula is used to calculate the projected lifetable values;

$$m(x, 2015+s) = m_{x,2015} exp\{\beta(x)(\kappa(2015+s) - \kappa(2015))\} \quad (s = 1, ..., 42);$$

Using this formula, the central death rates for all ages $x = x_1, x = x_2, ..., x = x_k$ that are classes as $[0, 1-4, 5-9, 10-14, ..., 100-104, 105+]$ and all times $t = 2016, 2017, ..., 2057$ are calculated for each gender.

Projected life tables could build and life expectancies at 65 years old could be calculated with these projected force of mortality rates [53]. Mortality rate at age x, $q(x)$, is computed from $m(x)$ for each gender and each year between 2016-2057 according to the formula;

$$q(x) \cong \frac{w(x)m(x)}{1 + (1 - f(x))w(x)m(x)}.$$

Here, $f(x)$ represents the average number of years lived within the age interval $[x, x+1)$ for people who are dying in that interval. It is assumed that $f(x) = 1/2$ for all age groups except x_2 and x_3. For x_2, in other words for ages 0 to 1, it is assumed that $f(x) = 0.85$ and for x_3, in other words for ages 1 to 5, it is assumed that $f(x) = 0.60$ for each gender [103]. Morever, $w(x)$ represents the distance of the age interval, such as $w(x = x_1) = 1, w(x = x_2) = 4, w(x = x_3) = 5, w(x = x_4) = 5$ ... and so on.

After that, probability of surviving from age x to $x + 1$, $p(x)$ is computed from $q(x)$ for each gender and each year between 2016-2057. Then, from calculated $p(x)$ and an arbitrary l_0, the survivorship function $l(x)$'s are constructed. In this study, it is assumed that l_0 is equal to 100,000. The following formula calculates the $l(x)$'s;

$$l(x + w_x) = l(x)p(x).$$

And, number of deaths in the interval $(x, x + 1)$ for persons who are alive at age x, $d(x)$ are;

$$d(x) = l(x) - l(x + w_x) = l(x)q(x).$$

Furthermore, person-years lived by the cohort from age x to $x + 1$, $L(x)$, and person-years lived by the cohort after exact age x, $T(x)$ are calculated respectively as;

$$L(x) = w(x)(l(x) - (1 - f(x))d(x),$$
$$T(x) = \sum_{x=x_1}^{x_k} L(x).$$

And finally, life expectancy at 65 years old will be found by;

$$e(x) = \frac{T(x)}{l(x)}.$$

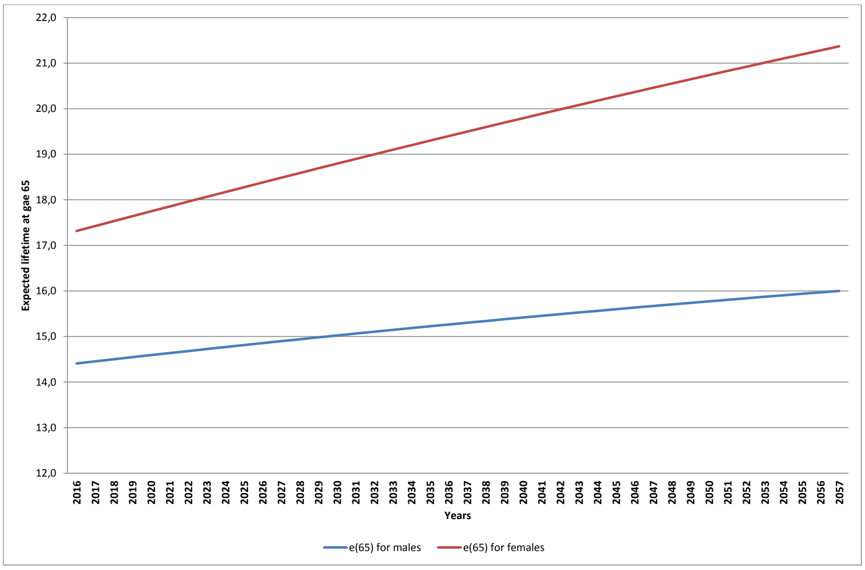

Figure 4.6: $e(65)$ values for males and females for each year between 2016-2057

It can be seen from Figure 4.6 that, expected lifetime at 65 years old is 14.4 years for males and 17.3 years for females at 2016. Throughout the years between 2016-2057, it increases regularly for each gender. In fact, at 2057 it reaches 16.0 and 21.4 by increasing 1.6 years and 4.1 years for males and females respectively. The life expectancy at 65 years old in females is increasing with a greater acceleration in parallel with the increase in projected $\kappa(t)$ along the years.

4.3 Uncertainty Surrounding Life Expectancy Prospects

In order to evaluate the uncertainty surrounding future life expectancy prospects based on the projections done previously, Monte Carlo simulations are applied to the Lee-Carter model.

Monte Carlo simulation is a numerical experimentation method to get the statistics of the output variables of a system computational model from the given the statistics of the input variables. The values of the input random variables are sampled with regard to their distributions, and the output variables are calculated using the computational model, in each experiment. In this manner, a number of experiments are executed and the results are used to calculate the statistics of the output variables. If a statistical estimate is to be used upon repeated experiments, the values should be sampled with regard to the probability distribution of the random variable. In addition to this, the values should appear to be random. In recent years, many computer programme has a random number generator. Common method of generating random numbers starts with a seed value and then compute successive values by using a recursive formula [67].

In this study, 10,000 Monte Carlo simulations are applied to the error terms that obtained from the errors of fitting the Lee-Carter model to the historical data. We already know that, $\varepsilon(x,t)$ has the normal distribution with 0 mean. First of all, the variances of $\varepsilon(x,t)$ are found in each year and age interval for males and females seperately. Then, new error terms, $\varepsilon^{'}(x,t)$ are produced with Monte Carlo simulations in each age interval for each gender [2]. New error terms are used for projection years. Consequently, an interval for expected lifetimes at 65 years old from the Monte Carlo Simulation results are found for each gender between the projection years.

Figure 4.7 shows the calculated values of minimum and maximum expected life times at 65 years old, $e(65)$, for males and females between the years 2016-2057.

It can be seen from Figure 4.7 that, maximum life expectancy for males begins with 14.8 years and at the end of projection it reachs to 16.4 years with an increase of 10.8 percent. Besides, minimum life expectancy for males is 14.0 years at 2016 and with an incresase of 11.3 percent it reachs to 15.6 years until 2057. On the other side, maximum life expectancy for females begins with 17.8 years and at the end of projection it reachs to 21.7 years with an increase of 22.2 percent. Moreover, minimum life expectancy for females is 16,8 years at 2016 and with an incresase of 24.6 percent it reachs to 21.0 years until 2057. It is obvious that, for each scenario females life expectancy at 65 years old is longer than males. In other words, females live more

[2] As a result, 220,000 new error terms are found for 22 different age intervals, $[0, 1-5, 5-9, ..., 95-99, 100+]$ from 10,000 Monte Carlo simulations.

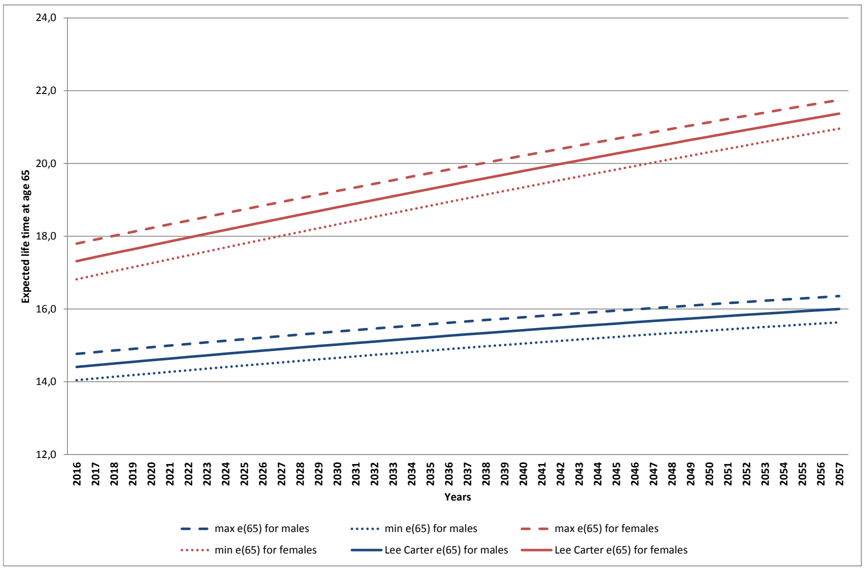

Figure 4.7: Estimated $e(65)$ values for males and females for each year between 2016-2057

than males for projection years in each scenario. If we look at the Figure in detail, in maximum life expectancy, females live 3 more years than males at 2016, besides difference between them increases steadily and reaches 5.3 years at 2057. On the other hand, in minimum life expectancy, females live 2.8 more years than males at 2016 and again the difference between them reaches 5.3 years at 2057. As a consequence, we can say from the results of Monte Carlo simulations, females have much more longevity risk than males.

4.4 Measuring the Impact of Longevity Risk on Second Pillar Pension System in Turkey

4.4.1 Construction of baseline scenario

In this section, baseline scenario is constructed in order to measure the impact of longevity risk on SPPS in Turkey. We already know that, as of 1 January 2017, participation in the private pension system in Turkey has become automatic. And this can be taken as SPPS to be implemented since January 1, 2017 in Turkey. Basic assumptions for the calculation of pensions as follows;

i. All insured persons are assumed to be employed under item a of paragraph one of Article 4 of Law Number 5510. So, all related calculations are made in compliance with this Article of this Law.

ii. The pensions are calculated for the insured males and females that begin to work at the age of 25 and retire at the age of 65. More precisely, whole working life is assumed to be 40 years for all male and female insureds.

iii. Time value of money is assumed to be equal for each year throughout the projection years. It means, all the calculations are made over the real values of money.

iv. Real economic growth rate (EGR) is 3.2 percent for 2016 (Turkish Statistical Institute). And, it is assumed to be 5 percent between the years 2017-2023, then to decrease linearly to 1.5 percent till 2050 and then to be fixed 1.5 percent between the years 2050-2057.

v. Premiums for SPPS are assumed to be 3 percent of monthly insurable earnings along the years between 2017-2057.

vi. Real investment gain is assumed to be 2 percent along the years between 2017-2057.

4.4.2 Monthly pension calculations with regard to the baseline scenario

First of all, intervals for daily insurable earnings are taken from SSI Statistical Yearbook, 2016. It begins with the minimum daily insurable earning, ends with maximum daily insurable earning and there are 20 different daily insurable earning categories (EC) between them. Finally, average daily insurable earnings (ADIE) for males and females are added to these earning categories. Thus, calculations are made for 24 different insurable earning categories. ADIE's are calculated by taking arithmetical mean from these daily insurable earning intervals. EC's are shown in Table 4.1 [3]

Table 4.1: Daily Insurable Earning Categories [94]

Categories	Min	Max	Av.	Categories	Min	Max	Av.
$\mathbf{EC_1}$	-	54.9	54.9	$\mathbf{EC_{13}}$	219.0	234.0	226.5
$\mathbf{EC_2}$	54.9	69.0	62.0	$\mathbf{EC_{14}}$	234.0	249.0	241.5
$\mathbf{EC_3}$	69.0	84.0	76.5	$\mathbf{EC_{15}}$	249.0	264.0	256.5
$\mathbf{EC_4}$	84.0	99.0	91.5	$\mathbf{EC_{16}}$	264.0	279.0	271.5
$\mathbf{EC_5}$	99.0	114.0	106.5	$\mathbf{EC_{17}}$	279.0	294.0	286.5
$\mathbf{EC_6}$	114.0	129.0	121.5	$\mathbf{EC_{18}}$	294.0	309.0	301.5
$\mathbf{EC_7}$	129.0	144.0	136.5	$\mathbf{EC_{19}}$	309.0	324.0	316.5
$\mathbf{EC_8}$	144.0	159.0	151.5	$\mathbf{EC_{20}}$	324.0	339.0	331.5
$\mathbf{EC_9}$	159.0	174.0	166.5	$\mathbf{EC_{21}}$	339.0	356.8	356.8
$\mathbf{EC_{10}}$	174.0	189.0	181.5	$\mathbf{EC_{22}}$	356.8	-	253.5
$\mathbf{EC_{11}}$	189.0	204.0	196.5	$\mathbf{EC_{23}}$	85.24		
$\mathbf{EC_{12}}$	204.0	219.0	211.5	$\mathbf{EC_{24}}$	79.69		

[3] EC_1 represents minimum ADIE and EC_{22} represents maximum ADIE. Besides, EC_{23} represents ADIE for males and EC_{24} represents ADIE for females.

It is taken from SSI Statistical Yearbook, 2016 that an insured male or female works averagely 26 days a month. So, number of insured days (NID) in a month is excepted as 26. Thus, for males and females average monthly insurable earnings (AMIE) are calculated in each earning intervals as follows;

$$\text{AMIE}_{2016,EC_a} = \text{ADIE}_{2016,EC_a} \times \text{NID};$$

$for \quad a = 1, ..., 24.$

And then, average yearly insurable earnings (AYIE) for males and females are calculated for 2016;

$$\text{AYIE}_{2016,EC_a} = \text{AMIE}_{2016,EC_a} \times 12;$$

$for \quad a = 1, ..., 24.$

After that, AYIE's are increase by 30 percent over the previous year's EGR throughout the years 2016-2057. So, AYIE are calculated for each year till 2057 as follows;

$$\text{AYIE}_{t,EC_a} = \text{AYIE}_{t-1,EC_a} \times (1 + (\text{EGR}_{t-1} \times 0,3));$$

$for \quad t = 2017, ..., 2057 \quad and \quad a = 1, ..., 24.$

As mentioned above, it is assumed that SPPS is begun to implement in Turkey as of 1 January, 2017. Thus, premiums (P) have started to be collected as 3 percent of the AYIE from the beginnig of 2017 until the end of 2057 such as;

$$\text{P}_{t,EC_a} = \text{AYIE}_{t,EC_a} * 0.03;$$

$for \quad t = 2017, ..., 2057 \quad and \quad a = 1, ..., 24.$

It is also assumed that, the real investment gain will be 2 percent for all projection years. Besides, In Act 4632 "Individual Pension Saving and Investment System Law", administrative cost is limited with only fund operation expenses and it is evaluated maximum 0.85 percent of premiums. So, in order to calculate the total amount of funds (TAF) for each EC the following formula is used;

$$\text{TAF}_{2017,EC_a} = \text{P}_{2017,EC_a} \times [1 - (\frac{0.85}{100})]$$

$$\text{TAF}_{t,EC_a} = [\text{P}_{t-1,EC_a} \times 1.02 + \text{P}_{t,EC_a}] \times [1 - (\frac{0.85}{100})];$$

$for \quad t = 2018, ..., 2057 \quad and \quad a = 1, ..., 24.$

TAF's at 2057 are shown in Table 4.2 [4].

[4] Prices are given in TL.

Table 4.2: Total Amount of Funds at Retirement with Regard to Earning Categories

Categories	**TAF**	**Categories**	**TAF**
$\mathbf{EC_1}$	34,167.6	$\mathbf{EC_{13}}$	140,967.8
$\mathbf{EC_2}$	38,558,4	$\mathbf{EC_{14}}$	150,303.2
$\mathbf{EC_3}$	47,613.7	$\mathbf{EC_{15}}$	159,638.6
$\mathbf{EC_4}$	56,949.1	$\mathbf{EC_{16}}$	168,974.0
$\mathbf{EC_5}$	66,284.5	$\mathbf{EC_{17}}$	178,309.4
$\mathbf{EC_6}$	75,619.9	$\mathbf{EC_{18}}$	187,644.9
$\mathbf{EC_7}$	84,955.3	$\mathbf{EC_{19}}$	196,980.3
$\mathbf{EC_8}$	94,290.8	$\mathbf{EC_{20}}$	206,315.7
$\mathbf{EC_9}$	103,626.2	$\mathbf{EC_{21}}$	216,534.8
$\mathbf{EC_{10}}$	112,961.6	$\mathbf{EC_{22}}$	222,089.4
$\mathbf{EC_{11}}$	122,297.0	$\mathbf{EC_{23}}$	53,050.0
$\mathbf{EC_{12}}$	131,632.4	$\mathbf{EC_{24}}$	49,595.9

As the amount of premiums increases, TAF, that accumulate until retirement, increase proportionally. Such that, TAF at retirement in the maximum income level (EC_{22}) which is 6.5 times the minimum income level (EC_1) is again 6.5 times in comparison with the EC_1's TAF. Actually, this is the natural result of a DC pension system. Moreover, in EC_{23} which represents the average earning for males, TAF at retirement is 53,050.0 TL while in EC_{24} which represents the average earning for females, the TAF at retirement is 49,595.9 TL. This means that, males accumulate approximately 7 percent more funds at retirement than females. This is due to the fact that, the females average earnings are lower than males.

On the other hand, pensions are calculated from the total amount of funds for each EC with regard to the expected lifetimes at 65 years old. Based on the earlier results found for e(65) to be for male and female 16 and 21.4, respectively, pension payments for each gender are calculated.

In Table 4.3, male and female monthly pensions can be seen for each earning categories. It is clear that, for every same earning categories, males have much more pensions than females. This is due to the fact that, the life expectancy of females at 65 years old is higher than males. More precisely, although both males and females have been paid the same amount of premiums along the same period, females would have lower pensions in DC pension systems because of the longer life expectancy. Besides, it can be seen from the Table that, males with average insurable earning have 276.3 TL for each month as pension while females have 193.4 TL. This is the result of two main reasons. Firstly, because of the higher average insurable earnings, males accumulate more fund along the same time than females. Secondly as mentioned above, females live longer than males at retirement period so they deserve less pension in DC SPPSs. Thus, it can be said that, females are more disadvantaged compared to males in SPPS where financing is done according to the funded method and retirement benefits are determined on the basis of the DC method. It is important to note that, living longer is the main reason for this disadvantageousness. Moreover, when we examine the Table more closely, it can be seen that the difference between male and female pensions

Table 4.3: Monthly Pensions with Regard to Earning Categories (TL)

Categories	Male Pensions	Female Pensions	Difference
EC_1	178.0	133.2	44.7
EC_2	200.8	150.4	50.5
EC_3	248.0	185.7	62.3
EC_4	296.6	222.1	74.5
EC_5	345.2	258.5	86.7
EC_6	393.8	294.9	99.0
EC_7	442.5	331.3	111.2
EC_8	491.1	367.7	123.4
EC_9	539.7	404.1	135.6
EC_{10}	588.3	440.5	147.8
EC_{11}	637.0	476.9	160.0
EC_{12}	685.6	513.3	172.3
EC_{13}	734.2	549.7	184.5
EC_{14}	782.8	586.1	196.7
EC_{15}	831.4	622.5	208.9
EC_{16}	880.1	658.9	221.1
EC_{17}	928.7	695.3	233.3
EC_{18}	977.3	731.7	245.6
EC_{19}	1,025.9	768.2	257.8
EC_{20}	1,074.5	804.6	270.0
EC_{21}	1,127.8	844.4	283.4
EC_{22}	1,156.7	866.1	290.6
EC_{23}	276.3		82.9
EC_{24}	193.4		82.9

increases as income level rises. Such that the difference in minimum insurable earning category is 44.7 TL while the difference in maximum insurable earning category reaches 290.6 TL. In general, for each earning category males have 33.6 percent more pension. On the other hand, when we look at the average insurable earning categories, we can see that males have 82.9 TL more pension than females for each month. This means that on average males have 42.9 percent more pension than females in SPPS. Besides, in the minimum insurable earning category male pension is 178.0 TL while female pension is 133.2 TL. On the other hand, in the maximum insurable earning category male pension is 1,156.7 TL while female pension is 866.1 TL. As well as the total amount of funds, pensions in EC_{22} which is 6.5 times the minimum income level EC_1 is again 6.5 times in comparison with the EC_1's pensions. This exactly means that, more premium gives more pension. So it can be said that, lower-income people at the working life would have lower income at retirement in SPPS .This conflicts with one of the aims of retirement systems that is preventing poverty in old age.

4.4.3 Monthly pension comparisons with Lee-Carter method and Monte Carlo simulations

After pension calculations with respect to Lee-Carter method results, Monte Carlo simulation results are used to measure the longevity risk on SPPS pensions. As known, in Section 4.3, the uncertainty surrounding future life expectancy prospects is evaluated by 10,000 Monte Carlo simulations. One of the simulation results is the maximum e(65) which is 16.35 for males and 21.75 for females at 2057. The minimum is 15.63 for males and 20.96 for females at 2057. Here, pensions of SPPS is calculated with respect to only the maximum and the minimum life expectancies for each gender because, the other 99,998 simulation results would be in this pension interval. So, it is aimed to understand the effect of any change in life expectancy on SPPS pensions. Monthly pensions in the conditions of Lee- Carter, maximum and minimum e(65)s are shown in Table 4.4 with regard to earning categories.

Table 4.4: Monthly Pension Comparisons in Lee-Carter, Maximum and Minimum Life Expectancies at 65 Years Old with Regard to Earning Categories (TL)

Cat	Lee Carter			Max LE			Min LE		
	M	**F**	**Dif**	**M**	**F**	**Dif**	**M**	**F**	**Dif**
$\mathbf{EC_1}$	178.0	133.2	44.7	174.1	130.9	43.2	182.1	135.9	46.3
$\mathbf{EC_2}$	200.8	150.4	50.5	196.5	147.8	48.7	205.5	153.3	52.2
$\mathbf{EC_3}$	248.0	185.7	62.3	242.6	182.5	60.2	253.8	189.3	64.5
$\mathbf{EC_4}$	296.6	222.1	74.5	290.2	218.2	72.0	303.6	226.4	77.1
$\mathbf{EC_5}$	345.2	258.5	86.7	337.8	254.0	83.7	353.3	263.6	89.8
$\mathbf{EC_6}$	393.8	294.9	99.0	385.3	289.8	95.5	403.1	300.7	102.4
$\mathbf{EC_7}$	442.5	331.3	111.2	432.9	325.6	107.3	452.9	337.8	115.0
$\mathbf{EC_8}$	491.1	367.7	123.4	480.5	361.3	119.1	502.6	374.9	127.7
$\mathbf{EC_9}$	539.7	404.1	135.6	528.0	397.1	130.9	552.4	412.0	140.3
$\mathbf{EC_{10}}$	588.3	440.5	147.8	575.6	432.9	142.7	602.1	449.2	153.0
$\mathbf{EC_{11}}$	637.0	476.9	160.0	623.2	468.7	154.5	651.9	486.3	165.6
$\mathbf{EC_{12}}$	685.6	513.3	172.3	670.8	504.4	166.3	701.7	523.4	178.3
$\mathbf{EC_{13}}$	734.2	549.7	184.5	718.3	540.2	178.1	751.4	560.5	190.9
$\mathbf{EC_{14}}$	782.8	586.1	196.7	765.9	576.0	189.9	801.2	597.6	203.5
$\mathbf{EC_{15}}$	831.4	622.5	208.9	813.5	611.8	201.7	851.0	634.8	216.2
$\mathbf{EC_{16}}$	880.1	658.9	221.1	861.0	647.5	213.5	900.7	671.9	228.8
$\mathbf{EC_{17}}$	928.7	695.3	233.3	908.6	683.3	225.3	950.5	709.0	241.5
$\mathbf{EC_{18}}$	977.3	731.7	245.6	956.2	719.1	237.1	1,000.2	746.1	254.1
$\mathbf{EC_{19}}$	1,025.9	768.2	257.8	1,003.8	754.9	248.9	1,050.0	783.2	266.8
$\mathbf{EC_{20}}$	1,074.5	804.6	270.0	1,051.3	790.6	260.7	1,099.8	820.4	279.4
$\mathbf{EC_{21}}$	1,127.8	844.4	283.4	1,103.4	829.8	273.6	1,154.2	861.0	293.2
$\mathbf{EC_{22}}$	1,156.7	866.1	290.6	1,131.7	851.1	280.6	1,183.8	883.1	300.8
$\mathbf{EC_{23}}$	276.3		82.9	270.3		80.3	282.8		85.6
$\mathbf{EC_{24}}$	193.4		82.9	190.1		80.3	197.2		85.6

Cat=category, M=male, F=female, Dif=difference.

When we compare monthly pensions in Lee-Carter method with Monte Carlo simulation results, we can clearly see that, when life expectancy increases pensions decrease for each earning category as seen in Table 4.4. Additionally, considering the maximum life expectancy the difference between male and female pensions are less than the difference in minimum life expectancy. It comes from the higher difference between life expectancies at 65 years old for males and females in the maximum life expectancy results.

Moreover, when we compare the pensions again, it is noticed that Lee-Carter male pensions are 2.16 percent higher for each earning category than maximum life expectancy condition. On the other hand, they are 2.35 percent less for each earning category than minimum life expectancy condition. This is due to the fact that the age difference between males is 0.35 year in the maximum life expectancy condition and 0.37 year in the minimum life expectancy condition when compared with the Lee-Carter method. The same situation applies to females as seen in the Table. Such as, Lee-Carter female pensions are 1.73 percent higher for each earning category than maximum life expectancy condition. Besides, when we again compare Lee-Carter life expectancy with minimum life expectancy, it is noticed that Lee-Carter female pensions are 1.97 percent less for each earning category than minimum life expectancy condition. This is due to the fact that the age difference between males is 0.38 in the maximum life expectancy condition and 0.41 in the minimum life expectancy condition when compared with the Lee-Carter method. Thus, it can be said that when the volatility of longevity risk is up, the volatility of pensions is up together. And of course, the converse is also true. In other words, when the volatility of longevity risk is down, the volatility of pensions is down together.

4.5 Sensivity Analysis

The independent parameter values and assumptions of any model are subject to any changes. In general, sensitivity analysis is the investigation of these potential changes and their impacts on conclusions [79]. In other words, it is a technique that used to see how different values of an independent variable effect a particular dependent variable under a given set of assumptions in a model. With the sensivity analysis it could be determined how changes in one variable impact the outcome. So, in sensivity analysis only one variable is changed but all other assumptions are kept fixed.

Here, only retirement year would be changed and all other baseline scenario assumptions would kept fixed for pension calculations with Lee-Carter method results. Thus, it is aimed to see the impact of the retirement year (time) on pensions in SPPS. Besides, in order to avoid the calculation crowd, sensivity analysis are done only with regard to average insurable earning categories for males and females seperately.

As it is already known, entry age is chosen as 25 and retirement age is chosen 65 in the baseline scenario. So, the retirement year is 2057. Here, from the assumption that entry age can range between 25 to 40, retirement year is changed with every year until 2042, i.e. 2057, 2056, 2055, 2054, 2053,...,2042. Thus, pensions that deserved by the insureds who have the same total fund and retire in different year along the last 15 years are compared. Below, Table 4.5 shows this comparison.

Table 4.5: Pensions in Different Years with Regard to Average Insurable Earning Categories

Years	M Pensions (TL)	F Pensions (TL)	M Dif	F Dif
2057	276.3	193.4	-	-
2056	276.8	194.2	0.2	0.4
2055	277.4	195.0	0.4	0.8
2054	278.0	195.9	0.6	1.3
2053	278.5	196.7	0.8	1.7
2052	279.1	197.6	1.0	2.1
2051	279.7	198.4	1.2	2.6
2050	280.3	199.3	1.4	3.0
2049	280.9	200.2	1.7	3.5
2048	281.5	201.1	1.9	4.0
2047	282.1	202.0	2.1	4.4
2046	282.8	203.0	2.4	5.0
2045	283.4	203.9	2.6	5.4
2044	284.1	204.9	2.8	5.9
2043	284.7	205.8	3.0	6.4
2042	285.4	206.8	3.3	6.9

M=male, F=female, Dif=difference. In the columns M Dif and F Dif rates are given as percent according to 2057.

In Table 4.5, the first two columns illustrate pensions for males and females who retire in each year between 2042 and 2057 by paying premiums on average insurable earnings during their working lives are shown. Besides, in the last two columns, there are the ratios of the awarded pensions between 2042 and 2056 to the awarded pension in 2057 for each gender.

It can be seen that, while male pensions 285.4 TL in 2042, it decreases 3.3 percent along the years and reachs to 276.3 TL in 2057. On the other hand, while female pensions 206.8 TL in 2042, it decreases 6.9 percent along the years and reaches to 193.4 TL in 2057. Here the point to note is that, female pension difference ratios are greater than males. This is due to the fact that the life expectancy at 65 years old for females has increased more than males. Moreover it is obvious that, as the retirement year progresses the amount of pensions decreases with an increasing acceleration for each gender. As a result, pensions with a later retirement year would experienced a larger impact from longevity risk because uncertain developments in life expectancy and mortality would affected pension funds for a longer time.

CHAPTER 5

CONCLUSION

Especially in the last few decades, the life expectancy has significantly increased for each gender almost all over the world. So today, calculating pensions without any projections about the future mortality and life expectancy can cause catastrophic loses in pension systems. From this point of view, this study aims to examine the impacts of longevity risk on Turkish SPPS. To achieve this aim, the Lee-Carter model, which is most commonly used two-factor underlying model, is preferred in order to calculate the future mortality and life expectancy and Monte Carlo simulations is applied to measure uncertainity surrounding Turkish mortality and life expectancy forecasts.

In this study, the Lee-Carter model is fitted to the matrix of Turkish mortality rates. Firstly, the Lee-Carter model parameters are calculated and these are concluded that; males have bigger mortality rates than females at each age interval, female mortality rates decrease faster than male mortality rates and the mortality rate at age 0 varies much when the time-varying parameter changes, on the contrary it varies little when the time-varying parameter changes at the oldest age. Secondly, time-varying parameter values are reestimated with an iterative process and it is seen that, reestimated time-varying parameter values for females change much more than males in comparison to time-varying parameter values. Thirdly, time-varying parameter values are forecasted with ARIMA methodology and forecasted values say us that, general level of mortality will decrease throughout the years for each gender. Especially for females it decreases with a bigger acceleration for males. And finally, forecasted lifetime expectations at 65 years old are found and it is shown that, the female life expectancy at 65 years old is increasing with a bigger acceleration in parallel with the increase in projected time-varying parameter values along the years.

However, the aim of this study is not producing a set of projections. It is the evaluation the uncertainity surrounding future life expectancy forecasts. For this aim, 10,000 Monte Carlo simulations are applied to the error terms which were obtained from the errors of fitting Lee-Carter model to the historical data. As a result after a series of calculations with using Lee-Carter model again, an interval for life expectancies at 65 years old from the Monte Carlo simulation results are found for each gender between the projection years. And, it is concluded that, for each scenario females life expectancy at 65 years old is longer than males. In other words, females live more than males for projection years in each scenario. As a consequence, we can say from the Monte Carlo simulation results, females have much more longevity risk than males.

Next, in order to measure the impact of longevity risk a baseline scenario is constructed. Within the baseline scenario, firstly the total amount of SPPS funds are calculated for 24 different insurable earning categories. After the calculations, it is found that, as the amount of premiums increases, the total amount of funds, that accumulate until retirement, increase proportionally. Besides, it is seen that, at retirement males accumulate more funds than females. This is the direct result of lower average earnings in females. Secondly, pensions are calculated for each earning category with regard to the expected lifetimes at 65 years old. As a result, it is concluded that, males have much more pensions for every same earning category. In other words, even if both males and females have the same total amount of funds, females would have lower pensions in DC pension systems because of the longer life expectancy. Thus, it can be said that, females have more disadvantages in DC and funded pension systems. It is important to note that, living longer is the main reason for this disadvantageousness. In addition, when the earning categories are compared, it is concluded that more premiums give more pensions. So, it is obvious that, lower income workers would have lower pensions at retirement.

After pension calculations with Lee-Carter method results, Monte Carlo simulation results are used to measure the longevity risk on SPPS pensions. For this aim only the minimum and the maximum life expectancy results for each gender are employed in order to calculate pensions. Monthly pensions that are calculated with respect to Lee-Carter model are compared with monthly pensions that are calculated with respect to the Monte Carlo simulation results, and it is clearly seen that, when life expectancy increases, pensions decrease for each earning category. Moreover, maximum and minimum life expectancy pensions are compared with the Lee-Carter pensions and it is concluded that, when the volatility of longevity risk is up, the volatility of pensions is up together, and vice versa.

On the other hand, in sensivity analysis, it is aimed to see the impact of the "retirement year" in other words "time" on pensions. Consequently, it is seen that, pensions with a later retirement year experience more longevity risk since the uncertain developments in life expectancy and mortality effect pension funds for a longer time.

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APPENDIX A

SPSS outputs details about ARIMA methodology

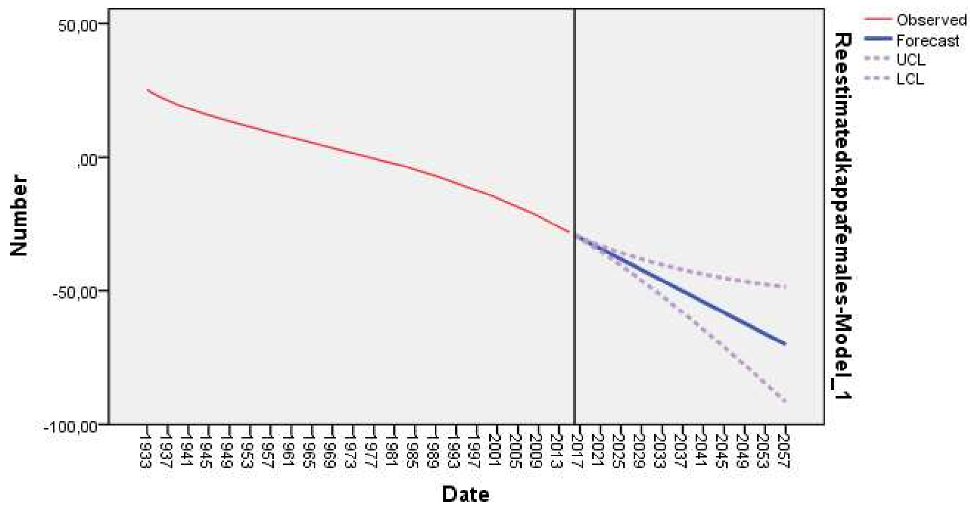

Figure A.1: SPSS outputs details about ARIMA methodology for females

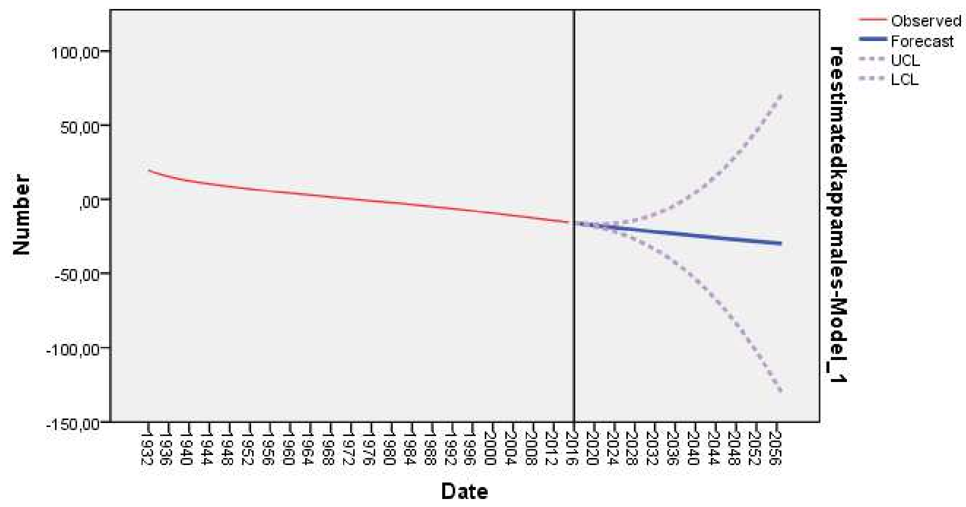

Figure A.2: SPSS outputs details about ARIMA methodology for males

In Figure A.1, it can be seen a red line until the year 2057. It represents the reestimated

$\kappa(t)$ values for females that are calculated in Subsection 4.2.1 with Lee-Carter Method. On the other hand, it can be seen a blue line after the year 2017 until the year 2057 that represents the projected $\kappa(t)$ values for females with ARIMA (1,2,0). Besides, there are two different dotted blue lines over and below the blue line. The over one represents the upper confidence limit (UCL) for the projected $\kappa(t)$ values and the below one represents the lower confidence limit (LCL) for the projected $\kappa(t)$ values for females.

On the other hand, in Figure A.2, it can be seen a red line until the year 2057. It represents the reestimated $\kappa(t)$ values for males that are calculated in Subsection 4.2.1 with Lee-Carter Method. On the other hand, it can be seen a blue line after the year 2017 until the year 2057 that represents the projected $\kappa(t)$ values for males with ARIMA (2,3,0). Besides, there are two different dotted blue lines over and below the blue line. The over one represents the upper confidence limit (UCL) for the projected $\kappa(t)$ values and the below one represents the lower confidence limit (LCL) for the projected $\kappa(t)$ values for males.

APPENDIX B

SPPS pension calculation example

Here is an example of SPPS pension calculation for EC_1 (minimum earning category);

$$\textbf{AMIE}_{2016,EC_1} = \text{ADIE}_{2016,EC_1} \times \text{NID} \Longrightarrow \text{AMIE}_{2016,EC_1} = 54.9 \times 26 = 1,427.4;$$

$$\textbf{AYIE}_{2016,EC_1} = \text{AMIE}_{2016,EC_1} \times 12 \Longrightarrow \text{AYIE}_{2016,EC_1} = 1,427.4 \times 12 = 17,128.8;$$

$$\textbf{AYIE}_{t,EC_1} = \text{AYIE}_{(t-1),EC_1} \times (1 + (\text{EGR}_{(t-1)} \times 0.3)) \quad \text{for} \quad \text{t} = 2017, ..., 2057 \Longrightarrow$$

$$\text{AYIE}_{2017,EC_1} = 17,128.8 \times (1 + (3.2 \times 0.3) = 17,293.2;$$

$$\text{AYIE}_{2057,EC_1} = 25,371.8 \times (1.5 \times 0.3)) = 25,486.0;$$

$$\textbf{P}_{t,EC_1} = \text{AYIE}_{t,EC_1} * 0.03 \quad \text{for} \quad \text{t} = 2017, ..., 2057 \Longrightarrow \text{P}_{2017,EC_1} = 17,293.2 \times 0.03 = 518.8;$$

On the other hand;

$$\text{P}_{2056,EC_1} = 25,486.0 \times 0.03 = 761.2;$$

$$\text{P}_{2057,EC_1} = 25,371.8 \times 0.03 = 764.6;$$

$$\textbf{TAF}_{2017,EC_1} = \text{P}_{2017,EC_1} \times [1 - (\frac{0.85}{100})] \Longrightarrow \text{TAF}_{2017,EC_1} = 518.8 \times [1 - (\frac{0.85}{100})] = 514.4;$$

$$\textbf{TAF}_{2057,EC_1} = ([\text{P}_{2056,EC_1} \times 1.02 + \text{P}_{2057,EC_1}] \times [1 - (\frac{0.85}{100})]) \times (1 + 0.02); \Longrightarrow$$

$$\text{TAF}_{2057,EC_1} = ([761.2 \times 1.02 + 764.6] \times [1 - (\frac{0.85}{100})]) \times 1.02 = 34,167.6;$$

As a result, **male monthly pension** for EC_1 is;

$\text{TAF}_{2057,EC_1} \div 16 \div 12 = 178.0$ for 16 years expected lifetimes at 65 years old.

And **female monthly pension** for EC_1 is;

$\text{TAF}_{2057,EC_1} \div 21.4 \div 12 = 133.2$ for 21.4 years expected lifetimes at 65 years old.