STRATEGIC ENVIRONMENTAL QUALITY INVESTMENTS IN MULTI-TIER SUPPLY CHAINS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

PINAR YALÇIN

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
OPERATIONAL RESEARCH

MAY 2018

Approval of the thesis:

STRATEGIC ENVIRONMENTAL QUALITY INVESTMENTS IN MULTI-TIER SUPPLY CHAINS

submitted by **PINAR YALÇIN** in partial fulfillment of the requirements for the degree of **Master of Science in Operational Research Department, Middle East Technical University** by,

Prof. Dr. Halil Kalıpçılar
Dean, Graduate School of **Natural and Applied Sciences** ________________

Assoc. Prof. Dr. Cem İyigün
Head of Department, **Operational Research** ________________

Assist. Prof. Dr. Özgen Karaer
Supervisor, **Industrial Engineering Department, METU** ________________

Assist. Prof. Dr. Tim Kraft
Co-supervisor, **Darden School of Business, University of Virginia, USA** ________________

Examining Committee Members:

Assoc. Prof. Dr. İsmail S. Bakal
Industrial Engineering Department, METU ________________

Assist. Prof. Dr. Özgen Karaer
Industrial Engineering Department, METU ________________

Assist. Prof. Dr. Tim Kraft
Darden School of Business, University of Virginia, USA ________________

Assoc. Prof. Dr. Pelin Bayındır
Industrial Engineering Department, METU ________________

Assist. Prof. Dr. Ayşe Selin Kocaman
Industrial Engineering Department, Bilkent University ________________

Date: ________________

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: PINAR YALÇIN

Signature :

ABSTRACT

STRATEGIC ENVIRONMENTAL QUALITY INVESTMENTS IN MULTI-TIER SUPPLY CHAINS

YALÇIN, PINAR
M.S., Department of Operational Research
Supervisor : Assist. Prof. Dr. Özgen Karaer
Co-Supervisor : Assist. Prof. Dr. Tim Kraft

May 2018, 91 pages

We study a buyer's strategies to incentivize environmental quality investment in a three-tier supply chain. Environmental quality of the end product depends on the environmental performances of both the tier 1 and tier 2 suppliers. A higher environmental quality produces an increased demand for the end product; and hence for the whole supply chain. In this setting, we compare the effectiveness of a delegation vs. a full control model from the buyer's cost-sharing perspective. Our analysis considers how the buyer's and the suppliers' decisions are impacted by the market opportunity for improved quality and the division of the supply chain margin between the three parties.

Keywords: Environmental quality investment, Game theory, Multi-tier supply chain

ÖZ

ÇOK BASAMAKLI TEDARİK ZİNCİRİNDE STRATEJİK ÇEVRESEL KALİTE YATIRIMLARI

YALÇIN, PINAR
Yüksek Lisans, Yöneylem Araştırması Bölümü
Tez Yöneticisi : Dr. Öğr. Üyesi Özgen Karaer
Ortak Tez Yöneticisi : Dr. Öğr. Üyesi Tim Kraft

Mayıs 2018 , 91 sayfa

Çalışmamız üç basamaklı bir tedarik zincirinde alıcı firmanın çevresel kalite yatırımlarını teşvik etmek için kullanabileceği stratejileri incelemektedir. Bitmiş ürün kalitesi hem birinci hem de ikinci basamaktaki tedarikçi firmaların kalite performansından etkilenmektedir. Yüksek kalite, bitmiş ürünün ve dolayısıyla tüm tedarik zincirinin talebinin artmasını sağlamaktadır. Bu şartlar altında, alıcı firmanın maliyet paylaşma perspektifinden delegasyon ve tam kontrol modellerinin etkinliği incelenmektedir. Çalışmamız, alıcı ve tedarikçi firmaların kararlarının, pazardaki yüksek kalite fırsatından ve tedarik zinciri paydaşları arasındaki kar marjı dağılımından ne şekilde etkilendiğini incelemektedir.

Anahtar Kelimeler: Çevresel kalite yatırımları, Oyun teorisi, Çok basamaklı tedarik zinciri

To those who come up with a pearl out of restlessness...

ACKNOWLEDGMENTS

First and foremost, I would like to express my sincere gratitude to my supervisor Assist. Prof. Dr. Özgen Karaer for her continuous support, guidance, encouragement and trust. I have learned a lot from her and it has been an honor to be mentored by her.

I am also very grateful to my co-supervisor Assist. Prof. Dr. Tim Kraft for his valuable insights and important contributions during this study.

I would like to convey my thanks to my jury members Assoc. Prof. Dr. İsmail S. Bakal, Assoc. Prof. Dr. Pelin Bayındır and Assist. Prof. Dr. Ayşe Selin Kocaman for allocating their time to my thesis.

I am greatly thankful to my dear husband Emre Yalçın. During this study, he has always been there for me. Without his support and patience, it would not be possible to complete this study. I am grateful for having him and his everlasting love.

I want to share my deepest gratitude with my family, my father Aziz Şimşek, my mother Besime Şimşek and my brother Can Şimşek. I owe them my never ending eagerness to improve myself, my curiosity, and my ambition. I am thankful for their trust in me.

I also want to express my gratitude to Middle East Technical University and especially to Department of Industrial Engineering. Since my bachelor years, I have always felt privileged to be a part of that unique culture.

Finally, I would like to thank Emine Gündoğdu for being a great friend and helping me whenever I needed.

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CHAPTER 1

INTRODUCTION

Consumers are increasingly more sensitive to the environmental performance of products. According to a study conducted by Cone Communications, 91% of global consumers expect companies to operate responsibly to address social and environmental issues (Cone Communications/Ebiquity Global CSR Study, 2015). Moreover, in the same study, it is found that 84% of consumers globally prefer responsible products whenever possible. This increasing consumer awareness has led the manufacturers to strive for environmentally-friendly, responsibly-produced products.

As companies continue to develop their products to meet consumer demands, many corporations are working to improve the environmental performance of their suppliers. One way to do this is through industry consortium. For example, the Responsible Business Alliance (RBA), members including Amazon, IBM, Apple, Samsung, Dell, Sony, Microsoft and Philips, are working to create an industry-wide standard on social, environmental and ethical issues in the electronics industry. Individual companies are also implementing supply chain improvement and compliance programs to improve environmental and social responsibility standards of their suppliers. For example, Unilever aims improving their suppliers who are expected to adopt Unilever's Responsible Sourcing Policy and provide compliance across their extended supply chain (Unilever Responsible Sourcing Policy, 2017).

Many brand owner electronics companies design their products but outsource their manufacturing. Tier-one suppliers, who manufacture some components or assemble the units, also procure some of the components from tier-two suppliers. In the end, the overall environmental performance of the product becomes contingent on the performance of all the supply chain partners. To improve the environmental and social responsibility performance of their products, brand owners adopt different approaches. For example, in their Supply Chain Social and Environmental Responsibility (SER) Progress Report, Dell discusses how it takes a comprehensive approach to monitor SER performance in its supply chain (Supply Chain Social and Environmental Responsibility Progress Report, 2017). This includes auditing both first-tier and sub-tier supplier facilities. Furthermore, all product lines across multiple tiers of the supply chain are tracked. As another example, Sony requires primary suppliers to adhere to the Sony Supply Chain Code of Conduct (Sony Supply Chain Code of Conduct, 2018). However, Sony monitors only its immediate suppliers while it expects first-tier suppliers to ensure secondary and further suppliers to comply with the Code.

In this thesis, we focus on a multi-tier supply chain and characterize a buyer's strategies in managing the environmental quality of her products. Environmental quality is affected by both the tier 1 and tier 2 suppliers' environmental performance. When the environmental quality of the product is higher, the demand for the product increases and the whole supply chain benefits from the demand increase. Increasing their quality, however, requires the suppliers to incur a lump sum investment cost and increase their unit cost. The buyer can utilize cost-sharing in order to motivate investment by the upstream partners. We analyze the effectiveness of two different strategies from the buyer's cost-sharing perspective: (i) a full control model in which the buyer incentivizes both the tier 1 and the tier 2 supplier herself; and (ii) a delegation model in which the buyer only subsidizes the tier 1 supplier and designates the tier 1 supplier to incentivize the tier 2 supplier. We consider the following research questions: (1) What is a buyer's optimal cost-sharing strategy to develop the capabilities of his suppliers in a multi-tier supply chain under each model? (2) When does a buyer prefer a full-control model over a delegation model? And does the buyer's preference produce the higher environmental quality in the market?

According to our analysis, under a full control model, we observe that the buyer's cost-sharing decision is mainly shaped by the relative market awareness of quality and the unit cost impact of quality for the suppliers. The buyer uses cost-sharing as long as the relative market awareness of quality is higher than at least one of the suppliers' unit cost impact of quality. However, we find that if the buyer cost-shares at a high rate with the high-margin supplier, then the other supplier may not invest in quality and instead choose to free-ride on the high-margin supplier's investment. In order to avoid that situation, the buyer prefers to lower her cost-sharing rate with the high-margin supplier although she is tempted to be more generous.

When a full control model and a delegation model are compared, we find that the buyer generally prefers a full control model when she and the tier 2 supplier capture high portions of the supply chain margin and as long as the relative market awareness of quality is high enough. Conversely, a delegation model is mostly preferred when the buyer is less powerful and the suppliers earn comparable margins. When we compare the overall quality performance of the two models, we see that although a full control model produces a higher overall quality most of the time, the buyer may prefer a delegation model due to its economic concerns in some cases.

The rest of the thesis is organized as follows; in the next chapter, we review the related literature. In Chapter 3, we define the problem and its settings, state our assumptions and formulate our model. In Chapter 4, we provide our analytical results. In Chapter 5, we conduct numerical experiments and compare the models. Finally, in Chapter 6, we conclude our study.

CHAPTER 2

LITERATURE REVIEW

In this study, we compare two different business models for effective environmental quality investment in a multi-tier supply chain. In this respect, our work is closely related with three streams of literature: research on multi-tier supply chains, research on upstream investment incentives in supply chains and research on environmental and social responsibility in supply chains.

Multi-tier Supply Chains

Supply chain literature is typically focused on two-tier settings. Though, not as extensive a literature, there is also significant amount of work focusing on supply chains of three or more tiers. Below we highlight a few papers that are closest in setting and research questions to this thesis.

Kayis et al. (2013) study the component procurement decisions in a three-tier supply chain containing a manufacturer and two serial suppliers. They examine control and delegation strategies conducted by the manufacturer. In the study, supplier cost information is private, and the manufacturer uses his distribution assumptions around their cost structures. In order to maximize his profit, the manufacturer considers two basic contracts: a price-only contract and a simple quantity discount contract. Under these two different contracts, the optimal procurement strategy, i.e, either control or delegation, is investigated. The primary goal of this study is to find the effective contracts for the manufacturer's management strategy under asymmetric cost information. Therefore, although comparing delegation and control models seems similar to our study, it serves to the purpose of effective contract management, which is far from our environmental quality improvement goal.

Another study in the literature of contract management under cost asymmetry is conducted by Guo et al. (2010). They analyze a three-tier supply chain to understand the effect of cost asymmetry on supply chain performance. Three outsourcing models with different material, information and cash flows are compared: turnkey (similar to our delegation model), turnkey with integration, and in-house consignment (similar to our control strategy). In this paper, one-period and two-period contracts are analyzed under these three outsourcing models. The outcomes show that the preferred model mostly depends on the prior cost information and the contract duration. The models considered in the paper seem similar to ours; however, here the focus is on the cost information asymmetry. In our study, however, we work in a complete information environment.

Dong et al. (2014) study contracting on quality in a multi-level supply chain that contains a brand owner, a manufacturer and a component supplier. In the study, quality is defined as the conformance to the desired performance and it is affected by all tiers of the supply chain. Both the brand owner and the manufacturer inspect the incoming products and they penalize the upstream partner if a defective item is detected. In this setting, the authors examine four types of quality incentive mechanisms. They consider the brand owner's contracting choice with only the manufacturer or both the manufacturer and the supplier. Moreover, they look for the short-term and long-term contracting effects. The quality is defined as conformance quality. Hence, quality is considered as a parameter rather than a variable. Even though the considered contracting choices seem similar to our models, the main concern is satisfying the required conformance quality level, not improving a performance quality level as in our study. Moreover, this study differs from ours since the model contains a penalty mechanism.

Ang et al. (2017) study the optimal sourcing decisions of a three tier supply chain in order to find effective disruption management strategies. In the study, disruption is assumed to occur only in tier 2 level. There are two types of tier 2 suppliers; reliable ones that provide the requested quantity vs. unreliable suppliers that may fail to deliver the requested quantity (i.e., disruption) but are less costly. A disruption results in a deficiency affecting the tier 1 and ultimately the manufacturer. However, the manufacturer cannot enforce which tier 2 supplier is to be chosen. The authors show that the choice of the tier 1 and the tier 2 suppliers affect the shape, and subsequently the risk, of the supply network. In this study, the authors focus on the disruption risk related to the second level supplier while we focus on environmental performance of both the tier 1 and tier 2 suppliers.

Upstream Investment Incentives in Supply Chains

In the literature, there are a number of studies that focus on upstream investment decisions and related interactions between partners in a supply chain. Since our work investigates the buyer-supplier(s) cost sharing agreements and incentives, our discussion below focuses on this subset of works in the literature.

Li & Wan (2017) study the interaction between supplier competition and supplier cost improvement efforts in order to enhance supply base performance. They model their supply chain as composed of a buyer facing two ex ante identical suppliers. The buyer needs to source a product with a unit demand from the suppliers. Each supplier can invest in cost reduction, and the realized production cost is random and stochastically decreasing in the cost-reduction effort. The authors work on two scenarios that differ based on the observability of the suppliers' effort. If the supplier activities are internal, then they are considered unobservable; whereas if the efforts are external (e.g., training, technology investments, etc.), they are considered observable. This paper is related to our study since it considers supply chain performance improvement; however, it focuses on cost reduction while we pay attention to total quality improvement. Moreover, the authors work on a dyadic supply chain rather than a multi-tier one.

Dong et al. (2015) compare an external-based and an inspection-based quality management approach and characterize the most effective one. They analyze two different cases: a dyadic supply chain consisting of a manufacturing brand owner and his direct component supplier, and a multi-tier supply chain containing a brand owner, a contract manufacturer and a component supplier. In this study, the definition of quality and the model used are nearly the same as the study of Dong et al. (2014) discussed above. Furthermore, the multi-tier setting with inspection-based approach is similar to our delegation case. However, the authors' perspective is different since they compare two different cases and choose between the effective way of supplying a predetermined quality. On the other hand, in our study, we focus on environmental performance improvement driven by market incentives.

Lee & Li (2012) study the supplier quality management strategy of a buyer for sourced quality in a two-tier supply chain. The main concern of the study is developing an integrated framework in order to improve sourced quality. The authors analyze strategic relations between three instruments: cooperation, incentives and inspection. Afterwards, they compare four strategies that combines these instruments in different ways. In the paper, quality is conformance-based and contractible. Moreover, the model contains not only buyer investment but also supplier punishment if a defective item is detected. The paper shares with this thesis a buyer's quality improvement perspective, but it differs in main assumptions regarding the quality definition and the supply chain structure.

Chen et al. (2015) examine a cooperative quality investment (CQI) strategy in outsourcing. They work on a two-tier supply chain composed of one contract manufacturer (CM) and two competing original equipment manufacturers (OEMs), both of whom outsource their production from the CM. The end product demand is affected by not only the price and the quality of the product but also the competitor's price and quality. Therefore, all three parties are motivated to improve the CM's quality to increase their demand. The authors analyze three different decision structures: the CM has full control of the quality decision, the OEMs have full control of the quality decision with either cooperation or no cooperation. The OEMs share some portion of the CM's quality improvement costs, similar to our study. Despite some similarities, the paper differs from this thesis in a few aspects. First of all, the main focus of the study is the effective implementation of a CQI strategy. Secondly, the study is conducted in a competitive environment and competition between the OEMs has an important effect on both the model and the results. Furthermore, the quality level is determined at the beginning of the game. Although quality is improved, the level of improvement is determined at the very beginning. In our study, the quality improvement level is an outcome of the actions taken by the supply chain members.

Another study about quality investment in a supply chain is conducted by Zhu et al. (2007). They analyze a supply chain that comprises a brand owner and his manufacturing supplier. In this setting, both parties are motivated to improve their quality since they both incur additional costs if a nonconformity is detected. The authors analyze the the quality investment when the buyer invests in the supplier's quality improvement. Moreover, they investigate the interaction between the quality improvement decision and operational decisions such as ordering and production sizes. Among other things, this study differs from our work in terms of the type of quality studied; i.e., conformance quality. Its main focus is on decreasing the number of nonconforming units while taking into account the order and production sizes. This thesis, however, focuses on increasing environmental quality which can be categorized as performance quality. Moreover, we are interested in developing strategies for the buyer in a three-tier supply chain.

Aust (2015) works on a three-echelon supply chain composed of one supplier, one manufacturer and one retailer. He characterizes the variables that influence the consumer demand: price, product quality and retail service. After modeling the system, Aust (2015) compares five different game scenarios in terms of different power distributions and relationships. The model elements such as quality definition, customer demand function and profit functions of the parties are very similar to ours. But, it differs from our model since it includes retail service as a demand variable. Although this study is a good example of game theoretic approach in a three-tier supply chain, it only analyzes the supply chain as it is. On the other hand, in our study we do not just analyze the game theoretic behaviours of the supply chain participants, but search for the effective methods to encourage upstream investment for environmental quality improvement.

Environmental and Social Responsibility in Supply Chains

Research on environmental investment and responsible sourcing issues is rapidly expanding in recent years. Since this thesis is mainly motivated by the rising environmental awareness in supply chains, this literature is closely related to our work.

Zhang et al. (2017) focus on sourcing relationships that arise in a three-tier mineral supply network where the raw materials cause social responsibility violations. The supply chain consists of upstream suppliers who provide either certified minerals (i.e., non-conflicting minerals) or non-certified minerals; and, downstream suppliers who are considered as certified if they source from responsible upstream suppliers, and non-certified if not. Manufacturers face penalties such as reputational damage or legislative punishments when they do not trace their sources. In the study, the authors examine two main questions: 1) is it possible to reach a supply chain that utilizes 100% certified products if the manufacturers who buy non-certified products are increasingly penalized, 2) which manufacturers as a subset should be penalized to achieve a higher rate of certified products? Additionally, the researchers analyze the effect on certified product amount if the manufacturers share their audit results with each other. In the paper, the authors analyze the equilibrium sourcing decisions of a supply chain that contains multiple players at each tier while in our model the supply chain has only one participant at each level and the downstream partner utilizes cost-sharing with her upstream supplier(s) in order to increase the overall quality.

Guo et al. (2015) study the supplier selection problem of a buyer in an environment that contains suppliers that may not uphold certain standards and hence cause violations. The authors analyze the trade-off between expensive responsible suppliers and cheap but potentially risky suppliers. The responsibility level of the suppliers are known by the buyer. The authors analyze four possible sourcing strategies and present their optimality conditions. They examine how the optimal sourcing strategy is affected by three factors: responsibility violation cost, customers' willingness-to-punish for violations, and customers' willingness-to-pay for responsibility. Moreover, they characterize the effective actions taken by regulators, customers or NGOs to motivate the buyer for responsible sourcing. The main concern of this study is finding the optimal supplier selection strategy of the buyer in a dyadic setting. On the other hand, we analyze the effective strategy to encourage the current supplier(s) to invest in environmental quality, hence to increase supply chain's overall environmental quality level.

Plambeck & Taylor (2014) explore the effective mechanisms buyers can utilize to motivate their suppliers to adhere to the required labor and environmental standards. They declare that obvious approaches (increasing auditing, publicizing negative audit reports, etc.) may produce counter-intuitive outcomes such as a decrease in the supplier's compliance effort. The authors explain this unexpected outcome by the suppliers' increasing effort for hiding their nonconformity. They also analyze some alternative methods such as squeezing the supplier's margin, increasing wages for workers or commitment to lower the level of auditing in order to increase the supplier's compliance effort. Moreover, the authors recommend NGOs to penalize the suppliers for a supplier-related violation or to conduct audits together with the buyer in order to improve the suppliers' compliance. This paper has a similar approach to ours since it seeks to develop strategies for the buyer to improve his upstream performance. Nevertheless, in this paper the suppliers compliance is triggered by audits while in our work the buyer motivates his suppliers through monetary incentives.

Huang et al. (2017) study social responsibility management in a three-tier supply chain that contains a buying manufacturer, a tier 1 supplier with no violation risk and a tier 2 supplier with responsibility violation risk. The responsibility level of the tier 2 supplier is assumed to be endogenous and motivated by the efforts of the tier 1 and tier 2 suppliers in the model. The authors examine two possible manufacturer strategies to be conducted together or separately: full control and delegation strategies. In their model, a social responsibility violation at the tier 2 supplier is modeled as a probability distribution function. Moreover, there is a portion of the end customers that are considered as socially conscious stop buying from the manufacturer if a responsibility violation occurs. Although the approach of the paper is very similar to our work in terms of comparing delegation vs. full control strategies from the perspective of the downstream partner, the model is built on different dynamics. In our model, we focus on the environmental performance of a product that both suppliers contribute to whereas here the authors study the responsibility violation risk that can only arise in the tier 2 supplier. Furthermore, the effort level in the paper is independent from violation risk reduction while we relate our incentive rate with quality improvement.

One of the closest studies to our work is Karaer et al. (2017). In this study, the authors investigate effective mechanisms to encourage a supplier to invest in environmental quality from the buyer's perspective. They focus on identifying the most effective mechanisms a buyer can use, and/or a nonprofit can facilitate to encourage investment by the upstream partner. The authors compare a unit premium, cost sharing and competition scenarios under the assumption of complete information. They then develop strategies for a nonprofit about how to promote the highest investment in the market. We share their approach in modeling the environmental quality and its effects in the markets and borrow the main dynamics under their cost-sharing scenario. Our sole focus, however, is to characterize effective strategies for the downstream partner in a three-tier supply chain in terms of interacting with either one or all of his upstream partners.

CHAPTER 3

MODEL DESCRIPTION AND ANALYSIS

3.1 MODEL DETAILS

In this chapter, we introduce the details of our model. We study a three-tier supply chain composed of a buyer, a supplier and a component supplier that produces for the first-tier supplier. The buyer sells the end product directly to the customers. The environmental performance of the end product is affected by the environmental quality level of the upstream performance in the supply chain. Due to the environmental awareness in the market, a product of higher environmental quality attracts more customers and hence reaches a higher demand. To seize this opportunity, the buyer, completely reliant on her supply chain partners, encourages her suppliers to invest in environmental quality. Although increase in demand increases the revenue for all parties in the chain, the suppliers face both variable and fixed lump-sum costs to invest in environmental quality. To incentivize her suppliers, the buyer offers to share the cost of the quality investment.

In our setting, the buyer sells the end product at a price p while buying them from supplier 1 at a wholesale price of w_1. Supplier 1 buys the components produced by supplier 2 at a wholesale price of w_2. Suppliers have unit manufacturing costs of m_1 and m_2, respectively. According to given notation, the buyer's margin is $\hat{p} = p - w_1$, supplier 1's margin is $\hat{w}_1 = w_1 - w_2 - m_1$ and supplier 2's margin is $\hat{w}_2 = w_2 - m_2$. We assume that all margins are exogenous and fixed; similar to Huang et al. (2017). In this regard, we take the perspective of a supply chain in which there are already existing wholesale price agreements in place between the buyer, supplier 1, and supplier 2.

The environmental quality level of the end product is influenced by the quality levels of the components used in the end product; i.e., the quality levels set by both suppliers. We define the quality level of supplier 1 and supplier 2 as q_1 and q_2, respectively. Since we assume both suppliers have similar contribution to the end product quality, we define the overall quality level of the end product as $q = q_1 + q_2$.

The consumer demand for the end product is given by

$$D = K - ap + d(q_1 + q_2) = \theta + d(q) \tag{3.1}$$

where K is intrinsic market potential, a is price awareness of consumers, d is environmental quality awareness of consumers (we define $\theta = K - ap$ for easy reference). We model the demand as linear in both price and quality. We assume that the consumer demand increases when the quality increases but the price does not change.

In order to improve quality, both supplier 1 and supplier 2 face additional quality-driven unit costs, i.e., cq_1 and cq_2, respectively. Moreover, supplier 1 and supplier 2 need to incur investment costs of yq_1^2 and yq_2^2, respectively. The investment cost function demonstrates diminishing returns on the suppliers' efforts to improve environmental quality. In this setting, the buyer can only indirectly influence the suppliers' quality decisions by sharing their investment costs to build quality. The buyer has two primary choices of interaction by cost sharing: full control or delegation.

As displayed in Figure 3.1, under the full control model, the buyer offers to share both supplier 1's and supplier 2's investment costs at a rate of γ_1 and γ_2, respectively. Here, the buyer works directly with both suppliers. The sequence of events can be summarized as follows:

1. Buyer offers to share costs with supplier 1 and supplier 2 at rates γ_1 and γ_2,

2. After observing both γ_1 and γ_2, supplier 1 and supplier 2 make their quality investment decisions q_1 and q_2 simultaneously.

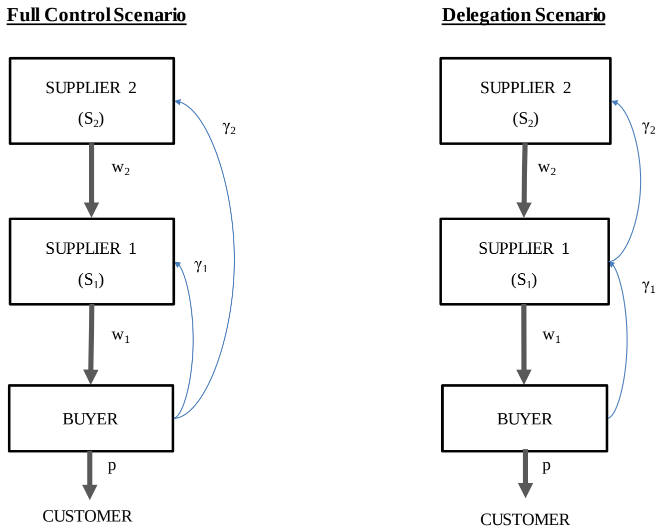

Figure 3.1: Full Control vs. Delegation Models

The profit functions of the buyer and the suppliers are given below:

$$\pi_B^F = (\theta + dq_1 + dq_2)\hat{p} - \gamma_1 y q_1^2 - \gamma_2 y q_2^2 \quad (3.2)$$

$$\pi_{S_1}^F = (\theta + dq_1 + dq_2)(\hat{w}_1 - cq_1) - (1 - \gamma_1) y q_1^2 \quad (3.3)$$

$$\pi_{S_2}^F = (\theta + dq_1 + dq_2)(\hat{w}_2 - cq_2) - (1 - \gamma_2) y q_2^2 \quad (3.4)$$

Under the delegation model, the buyer shares only supplier 1's investment cost at a rate of γ_1 while supplier 1 is delegated to provide incentive to supplier 2 at a rate γ_2 of his choosing. The sequence of events for the delegation model is as follows:

1. Buyer offers to share costs with supplier 1 at a rate γ_1,
2. After observing γ_1, supplier 1 makes his quality decision q_1, and offers to share costs of supplier 2 at rate γ_2,
3. After observing γ_1, γ_2 and q_1, supplier 2 makes his quality decision q_2.

Under this scenario, the profit functions are provided below:

$$\pi_B^D = (\theta + dq_1 + dq_2)\hat{p} - \gamma_1 y q_1^2 \quad (3.5)$$

$$\pi_{S_1}^D = (\theta + dq_1 + dq_2)(\hat{w}_1 - cq_1) - (1 - \gamma_1) y q_1^2 - \gamma_2 y q_2^2 \quad (3.6)$$

$$\pi_{S_2}^D = (\theta + dq_1 + dq_2)(\hat{w}_2 - cq_2) - (1 - \gamma_2) y q_2^2 \quad (3.7)$$

Under the delegation model, since the buyer provides incentive only to supplier 1, we only include the term $\gamma_1 y q_1^2$ in the buyer's profit function. In supplier 1's profit function, we include $\gamma_2 y q_2^2$ as a cost because he is the party responsible for incentivizing supplier 2. Note that the profit function of supplier 2 is the same as in the full control model.

The notation used in our model is summarized in Table 3.1 below:

Table 3.1: Notation

Decision Variables	
q_i	Quality level of supplier i, $q_i > 0$ ($q = q_1 + q_2$)
γ_i	Portion of the supplier i's investment cost that his partner is willing to share, $0 \leq \gamma_i \leq 1$
Parameters	
K	Intrinsic market potential
a	Consumers' price awareness effect on demand, $a > 0$
p	Buyer's retail price, $p > 0$
d	Consumers' quality awareness effect on demand, $d > 0$
w_i	Current unit wholesale price to the supplier i, $w_i > 0$
m_i	Supplier i's unit manufacturing cost, $m_i \geq 0$
c	Quality-driven unit cost increase to suppliers, $c > 0$
y	Suppliers' investment cost factor to build quality q_i, $y > 0$
Consolidated Terms	
θ	$\theta = K - ap$
$\hat{p}$	$p - w_1$
$\hat{w}_i$	$\hat{w}_1 = w_1 - w_2 - m_1,\ \hat{w}_2 = w_2 - m_2$

3.2 FULL CONTROL MODEL

In this section, we analyze the full control model. We use backward induction to obtain the subgame perfect equilibrium. The proofs of all analytical results are available in Appendices.

Before examining the multi-tier supply chain, we characterize the behaviour of a simple two-tier supply chain as a benchmark in Lemma 1 below:

Lemma 1. *For a supply chain composed of a buyer and a tier 1 supplier,*

(i) If $\frac{d}{\theta} > \frac{c}{\hat{w}}$ *and* $\frac{(cd+2y)(d\hat{w}-c\theta)}{cd} \leq 2d\hat{p}$*, then* $q^* = \frac{d\hat{w}-c\theta}{2cd} > 0$*, and* $\gamma^* = 1$*,*

(ii) If $\frac{d}{\theta} > \frac{c}{\hat{w}}$ *and* $(d\hat{w} - c\theta) \leq 2d\hat{p} < \frac{(cd+y)(d\hat{w}-c\theta)}{cd}$*, then* $q^* = \frac{2d\hat{p}+d\hat{w}-c\theta}{4(cd+y)} > 0$ *and* $1 > \gamma^* = \frac{(cd+y)(2d\hat{p}-d\hat{w}+c\theta)}{y(2d\hat{p}+d\hat{w}-c\theta)} \geq 0$*,*

(iii) If $\frac{d}{\theta} > \frac{c}{\hat{w}}$ *and* $2d\hat{p} < (d\hat{w} - c\theta)$*, then* $q^* = \frac{d\hat{w}-c\theta}{2(cd+y)} \geq 0$ *and* $\gamma^* = 0$*,*

(iv) If $\frac{d}{\theta} \leq \frac{c}{\hat{w}}$*, then* $q^* = 0$ *and* $\gamma^* \in [0, 1]$*.*

Lemma 1 demonstrates the possible investment outcomes in a two-tier supply chain. We observe that when the relative market awareness of quality (i.e., $\frac{d}{\theta}$) is higher than the supplier's unit cost impact of quality (i.e., $\frac{c}{\hat{w}}$), then the supplier invests in quality even if the buyer does not cost-share with him. The buyer will cost-share only if the supplier has a market opportunity (i.e., $\frac{d}{\theta} > \frac{c}{\hat{w}}$) and $(d\hat{w} - c\theta) \leq 2d\hat{p}$ holds. The latter condition implies that when the supplier captures a high portion of the supply chain margin, the buyer is not inclined to support the supplier.

Next, we continue with our three-tier supply chain problem. We start the analysis from the second stage (i.e., the supplier investment step) before analyzing the buyer's investment decision.

3.2.1 Suppliers' Investment Decisions

For the second stage, the best responses of supplier 1 and supplier 2 are provided in Proposition 1 below:

Proposition 1. *Under the full control model, for a given (γ_1, γ_2), the supplier investment stage has the following quality equilibria:*

$$(q_1^*(\gamma_1,\gamma_2), q_2^*(\gamma_1,\gamma_2)) = \begin{cases} \Big(\frac{2y(1-\gamma_2)(d\hat{w}_1-c\theta)+cd(2d\hat{w}_1-d\hat{w}_2-c\theta)}{3c^2d^2+4y^2(1-\gamma_1)(1-\gamma_2)+4cdy(2-\gamma_1-\gamma_2)}, \\ \frac{2y(1-\gamma_1)(d\hat{w}_2-c\theta)+cd(2d\hat{w}_2-d\hat{w}_1-c\theta)}{3c^2d^2+4y^2(1-\gamma_1)(1-\gamma_2)+4cdy(2-\gamma_1-\gamma_2)}\Big) \\ \qquad \textit{if } \frac{2cd^2+2dy(1-\gamma_2)}{d^2\hat{w}_2+cd\theta+2\theta y(1-\gamma_2)} > \frac{c}{\hat{w}_1} \ (Con\ 1a) \\ \qquad \textit{and } \frac{2cd^2+2dy(1-\gamma_1)}{d^2\hat{w}_1+cd\theta+2\theta y(1-\gamma_1)} > \frac{c}{\hat{w}_2} \ (Con\ 1b), \\ \Big(\frac{d\hat{w}_1-c\theta}{2cd+2y(1-\gamma_1)}, 0\Big) \\ \qquad \textit{if } \frac{d}{\theta} > \frac{c}{\hat{w}_1} \ (Con\ 2a) \quad \textit{and} \quad \frac{2cd^2+2dy(1-\gamma_1)}{d^2\hat{w}_1+cd\theta+2\theta y(1-\gamma_1)} \le \frac{c}{\hat{w}_2} \ (Con\ 2b), \\ \Big(0, \frac{d\hat{w}_2-c\theta}{2cd+2y(1-\gamma_2)}\Big) \\ \qquad \textit{if } \frac{d}{\theta} > \frac{c}{\hat{w}_2} \ (Con\ 3a) \quad \textit{and} \quad \frac{2cd^2+2dy(1-\gamma_2)}{d^2\hat{w}_2+cd\theta+2\theta y(1-\gamma_2)} \le \frac{c}{\hat{w}_1} \ (Con\ 3b), \\ (0,0) \\ \qquad \textit{if } \frac{d}{\theta} \le \frac{c}{\hat{w}_1} \ (Con\ 4a) \quad \textit{and} \quad \frac{d}{\theta} \le \frac{c}{\hat{w}_2} \ (Con\ 4b). \end{cases}$$

In Proposition 1, we see that there are four possible investment cases that may arise in the supplier stage. We will refer to the case where both suppliers invest as Case 1, where only supplier 1(2) invests as Case 2(3), and where neither invests as Case 4. There are several conditions that are linked with each equilibrium cases. In order to understand the relationship and transitions between the cases we analyze the relationship between some of these conditions in Lemma 2:

Lemma 2. *(i) If $\frac{d}{\theta} = \frac{c}{\hat{w}_i}$, then $\frac{2cd^2+2dy(1-\gamma_i)}{d^2\hat{w}_i+cd\theta+2\theta y(1-\gamma_i)} = \frac{d}{\theta} = \frac{c}{\hat{w}_i}$ for any $\gamma_i \in [0,1]$,*

(ii) If $\frac{d}{\theta} > \frac{c}{\hat{w}_i}$, then $\frac{d}{\theta} > \frac{2cd^2+2dy(1-\gamma_i)}{d^2\hat{w}_i+cd\theta+2\theta y(1-\gamma_i)} > \frac{c}{\hat{w}_i}$ for any $\gamma_i \in [0,1]$,

(iii) If $\frac{d}{\theta} < \frac{c}{\hat{w}_i}$, then $\frac{c}{\hat{w}_i} > \frac{2cd^2+2dy(1-\gamma_i)}{d^2\hat{w}_i+cd\theta+2\theta y(1-\gamma_i)} > \frac{d}{\theta}$ for any $\gamma_i \in [0,1]$,

where $i \in \{1,2\}$.

From Lemma 2, we see that if $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$, then $\frac{d}{\theta} > \frac{2cd^2+2dy(1-\gamma_1)}{d^2\hat{w}_1+cd\theta+2\theta y(1-\gamma_1)} > \frac{c}{\hat{w}_1}$. Under this condition, when $Con\ 2b$ holds then $\frac{c}{\hat{w}_1} < \frac{c}{\hat{w}_2}$ $(\hat{w}_1 > \hat{w}_2)$ must hold. Similarly, when $\frac{d}{\theta} > \frac{c}{\hat{w}_2}$, then $Con\ 3b$ implies $\frac{c}{\hat{w}_2} < \frac{c}{\hat{w}_1}$ $(\hat{w}_2 > \hat{w}_1)$. With this relationship, we find that Case 2 (Case 3) may arise when $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$ $(\frac{d}{\theta} > \frac{c}{\hat{w}_2})$, under the condition that $\frac{c}{\hat{w}_1} < \frac{c}{\hat{w}_2}$ $(\frac{c}{\hat{w}_1} > \frac{c}{\hat{w}_2})$. Note that in a simple two-tier chain, the condition $\frac{d}{\theta} > \frac{c}{\hat{w}}$ is sufficient to guarantee a supplier to invest. However, we cannot claim the same for a three-tier supply chain. When the margin of one of the suppliers is larger than the other's, the low-margin supplier may avoid investing in quality even though he would prefer to invest if he was by himself. Therefore, we conclude that suppliers' margins and their relative magnitude are critical in driving investment decisions.

Next, we characterize the supplier stage best response $q_1^*(\gamma_1, \gamma_2)$ and $q_2^*(\gamma_1, \gamma_2)$ decisions with respect to γ_1 and γ_2.

Lemma 3. *(i) When $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$ and $\frac{d}{\theta} > \frac{c}{\hat{w}_2}$, the supplier investment stage has the following equilibria:*

I. Case 1: $q_1^*(\gamma_1, \gamma_2) = \frac{2y(1-\gamma_2)(d\hat{w}_1 - c\theta) + cd(2d\hat{w}_1 - d\hat{w}_2 - c\theta)}{3c^2d^2 + 4y^2(1-\gamma_1)(1-\gamma_2) + 4cdy(2-\gamma_1-\gamma_2)}$ and $q_2^*(\gamma_1, \gamma_2) = \frac{2y(1-\gamma_1)(d\hat{w}_2 - c\theta) + cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta)}{3c^2d^2 + 4y^2(1-\gamma_1)(1-\gamma_2) + 4cdy(2-\gamma_1-\gamma_2)}$ if $0 \le \gamma_1 < \bar{\gamma}_1$ and $0 \le \gamma_2 < \bar{\gamma}_2$,

II. Case 2: $q_1^*(\gamma_1, \gamma_2) = \frac{d\hat{w}_1 - c\theta}{2cd + 2y(1-\gamma_1)}$ and $q_2^*(\gamma_1, \gamma_2) = 0$ if $\bar{\gamma}_1 \le \gamma_1 \le 1$,

III. Case 3: $q_1^*(\gamma_1, \gamma_2) = 0$ and $q_2^*(\gamma_1, \gamma_2) = \frac{d\hat{w}_2 - c\theta}{2cd + 2y(1-\gamma_2)}$ if $\bar{\gamma}_2 \le \gamma_2 \le 1$, where $\bar{\gamma}_1 = 1 + \frac{cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta)}{2y(d\hat{w}_2 - c\theta)}$ and $\bar{\gamma}_2 = 1 + \frac{cd(2d\hat{w}_1 - d\hat{w}_2 - c\theta)}{2y(d\hat{w}_1 - c\theta)}$.

(ii) When $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$ and $\frac{d}{\theta} \le \frac{c}{\hat{w}_2}$, Case 2 occurs and the supplier investment equilibrium is $q_1^*(\gamma_1, \gamma_2) = \frac{d\hat{w}_1 - c\theta}{2cd + 2y(1-\gamma_1)}$, $q_2^*(\gamma_1, \gamma_2) = 0$, $\forall \gamma_1, \gamma_2 \in [0, 1]$.

(iii) When $\frac{d}{\theta} \le \frac{c}{\hat{w}_1}$, and $\frac{d}{\theta} > \frac{c}{\hat{w}_2}$, Case 3 occurs and the supplier investment equilibrium is $q_1^*(\gamma_1, \gamma_2) = 0$, $q_2^*(\gamma_1, \gamma_2) = \frac{d\hat{w}_2 - c\theta}{2cd + 2y(1-\gamma_2)}$, $\forall \gamma_1, \gamma_2 \in [0, 1]$.

(iv) When $\frac{d}{\theta} \le \frac{c}{\hat{w}_1}$ and $\frac{d}{\theta} \le \frac{c}{\hat{w}_2}$, Case 4 occurs and the supplier investment equilibrium is $q_1^*(\gamma_1, \gamma_2) = 0, q_2^*(\gamma_1, \gamma_2) = 0$, $\forall \gamma_1, \gamma_2 \in [0, 1]$.

With Lemma 3, we find the effect of the buyer's cost-sharing rates (γ_1 and γ_2) on the suppliers' quality investment decisions. We find that when $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$ and $\frac{d}{\theta} > \frac{c}{\hat{w}_2}$, we may observe three different investment scenarios in the supplier investment stage. While both suppliers invest in Case 1, we observe asymmetric investment in Case 2 and Case 3. Here, we may find a transition between cases with respect to γ_i when either $\bar{\gamma}_1 \in [0, 1]$ or $\bar{\gamma}_2 \in [0, 1]$. That means, when the unit cost impact of quality, i.e. $\frac{c}{\hat{w}_i}$, where $i \in \{1, 2\}$, is lower than the relative market awareness of quality, i.e. $\frac{d}{\theta}$, for both suppliers, then the buyer may cause the lower margin supplier to avoid investment by generously cost-sharing (i.e., over $\bar{\gamma}_i$) with the high-margin supplier. Moreover, these points, i.e. $\bar{\gamma}_1$ and $\bar{\gamma}_2$, become the transition points from Case 1 to Case 2 and from Case 1 to Case 3, respectively.

When $\frac{d}{\theta} > \frac{c}{\hat{w}_i}$ and $\frac{d}{\theta} \leq \frac{c}{\hat{w}_j}$, only the high-margin supplier, who is S_i in this case will invest, where $i, j \in \{1, 2\}$. Under these conditions, the buyer cannot influence the weak supplier to invest, which means case transition does not happen. Therefore, either Case 2 or Case 3 happens. Lastly, when $\frac{d}{\theta} \leq \frac{c}{\hat{w}_1}$ and $\frac{d}{\theta} \leq \frac{c}{\hat{w}_2}$, there is no quality investment in the chain, independent of the buyer's cost-sharing rate. Thus, we conclude that case transition may occur only when $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$ and $\frac{d}{\theta} > \frac{c}{\hat{w}_2}$.

We provide the following graphs to show the case transition with respect to γ_i where $i \in \{1, 2\}$. Other parameters are taken as fixed. We choose the following parameter set in order to ensure $\frac{d}{\theta} > \frac{c}{\hat{w}_i}$ where $i \in \{1, 2\}$: $d = \theta = 0.4, c = 0.3, \hat{w}_1 = 0.4, \hat{w}_2 = 0.33$, and we get $\bar{\gamma}_1 = 0.84$ and $\bar{\gamma}_2 = 1.20$.

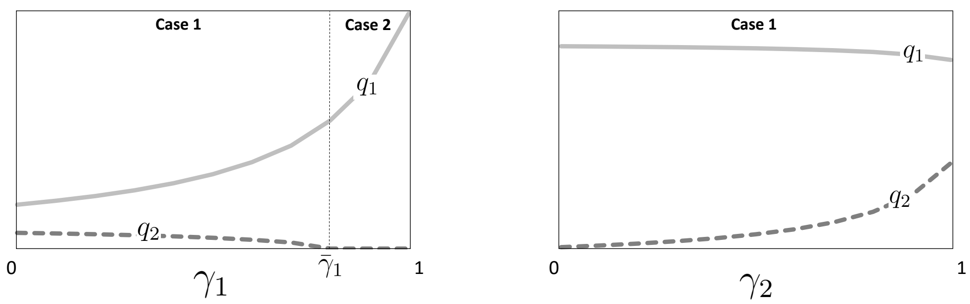

Figure 3.2: q_1 and q_2 behaviour wrt γ_1 and γ_2; $\frac{d}{\theta} = 1$, $\frac{c}{\hat{w}_1} = 0.75$, and $\frac{c}{\hat{w}_2} = 0.91$. On the left, where $\gamma_2 = 0.5$, equilibrium case changes when γ_1 exceeds $\bar{\gamma}_1$; i.e., $0 \leq \gamma_1 < \bar{\gamma}_1$, on the right, where $\gamma_1 = 0.5$, no transition since $0 < \gamma_2 \leq \bar{\gamma}_2$

In Figure 3.2, we present an example for Lemma 3(i) I and II. We see the case transition from Case 1 to Case 2 when γ_1 exceeds $\bar{\gamma}_1 = 0.84$. We observe that q_1 increases more rapidly after the transition and q_2 becomes zero.

Above, we identified the thresholds $\bar{\gamma}_1$ and $\bar{\gamma}_2$ that cause the investment stages to shift between Case 1, Case 2 and Case 3. In order to understand the transition between these cases in detail, we analyze the thresholds $\bar{\gamma}_1$ and $\bar{\gamma}_2$ in the next Lemma, under the conditions $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$ and $\frac{d}{\theta} > \frac{c}{\hat{w}_2}$ since only then case transitions are observed.

Lemma 4. *With* $\bar{\gamma}_1 = 1 + \frac{cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta)}{2y(d\hat{w}_2 - c\theta)}$, $\bar{\gamma}_2 = 1 + \frac{cd(2d\hat{w}_1 - d\hat{w}_2 - c\theta)}{2y(d\hat{w}_1 - c\theta)}$ *defined as in Lemma 3, we have the following equilibrium cases when* $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$ *and* $\frac{d}{\theta} > \frac{c}{\hat{w}_2}$*:*

(i) When $\hat{w}_1 < \frac{2d\hat{w}_2 - c\theta}{d}$ *and* $\hat{w}_2 < \frac{2d\hat{w}_1 - c\theta}{d}$*, then* $\bar{\gamma}_1 > 1$ *and* $\bar{\gamma}_2 > 1$.

In this scenario, we observe Case 1 that is described in Lemma 3(i) I; i.e., $q_1^*(\gamma_1, \gamma_2) = \frac{2y(1-\gamma_2)(d\hat{w}_1 - c\theta) + cd(2d\hat{w}_1 - d\hat{w}_2 - c\theta)}{3c^2d^2 + 4y^2(1-\gamma_1)(1-\gamma_2) + 4cdy(2-\gamma_1-\gamma_2)} > 0$ *and* $q_2^*(\gamma_1, \gamma_2) = \frac{2y(1-\gamma_1)(d\hat{w}_2 - c\theta) + cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta)}{3c^2d^2 + 4y^2(1-\gamma_1)(1-\gamma_2) + 4cdy(2-\gamma_1-\gamma_2)} > 0, \ \forall \gamma_1, \gamma_2 \in [0, 1]$.

(ii) When $\frac{2d\hat{w}_2 - c\theta}{d} \le \hat{w}_1 < \frac{2d\hat{w}_2(y+cd) - c\theta(2y+cd)}{cd^2}$, $0 < \bar{\gamma}_1 \le 1$ *and* $\bar{\gamma}_2 > 1$.

In this scenario, for $0 \le \gamma_1 < \bar{\gamma}_1$ *and* $\forall \gamma_2 \in [0, 1]$*, we observe Case 1; i.e.,* $q_1^*(\gamma_1, \gamma_2) = \frac{2y(1-\gamma_2)(d\hat{w}_1 - c\theta) + cd(2d\hat{w}_1 - d\hat{w}_2 - c\theta)}{3c^2d^2 + 4y^2(1-\gamma_1)(1-\gamma_2) + 4cdy(2-\gamma_1-\gamma_2)}$ *and* $q_2^*(\gamma_1, \gamma_2) = \frac{2y(1-\gamma_1)(d\hat{w}_2 - c\theta) + cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta)}{3c^2d^2 + 4y^2(1-\gamma_1)(1-\gamma_2) + 4cdy(2-\gamma_1-\gamma_2)}$.

For $\bar{\gamma}_1 \le \gamma_1 \le 1$ *and* $\forall \gamma_2 \in [0, 1]$*, we observe Case 2; i.e.,* $q_1^*(\gamma_1, \gamma_2) = \frac{d\hat{w}_1 - c\theta}{2cd + 2y(1-\gamma_1)}$ *and* $q_2^*(\gamma_1, \gamma_2) = 0$.

(iii) When $\hat{w}_1 \ge \frac{2d\hat{w}_2(y+cd) - c\theta(2y+cd)}{cd^2}$*, then* $\bar{\gamma}_1 \le 0$ *and* $\bar{\gamma}_2 > 1$.

In this scenario, we encounter Case 2; i.e., $q_1^*(\gamma_1, \gamma_2) = \frac{d\hat{w}_1 - c\theta}{2cd + 2y(1-\gamma_1)}$ *and* $q_2^*(\gamma_1, \gamma_2) = 0, \ \forall \gamma_1, \gamma_2 \in [0, 1]$.

(iv) When $\frac{2d\hat{w}_1 - c\theta}{d} \le \hat{w}_2 < \frac{2d\hat{w}_2(y+cd) - c\theta(2y+cd)}{cd^2}$, $\bar{\gamma}_1 > 1$ *and* $0 < \bar{\gamma}_2 \le 1$.

In this scenario, $\forall \gamma_1 \in [0, 1]$ *and for* $0 \le \gamma_2 < \bar{\gamma}_2$*, we observe Case 1; i.e.,* $q_1^*(\gamma_1, \gamma_2) = \frac{2y(1-\gamma_2)(d\hat{w}_1 - c\theta) + cd(2d\hat{w}_1 - d\hat{w}_2 - c\theta)}{3c^2d^2 + 4y^2(1-\gamma_1)(1-\gamma_2) + 4cdy(2-\gamma_1-\gamma_2)}$ *and* $q_2^*(\gamma_1, \gamma_2) = \frac{2y(1-\gamma_1)(d\hat{w}_2 - c\theta) + cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta)}{3c^2d^2 + 4y^2(1-\gamma_1)(1-\gamma_2) + 4cdy(2-\gamma_1-\gamma_2)}$.

For $\bar{\gamma}_2 \le \gamma_2 \le 1$ *and* $\forall \gamma_1 \in [0, 1]$*, we observe Case 3; i.e.,* $q_1^*(\gamma_1, \gamma_2) = 0$ *and* $q_2^*(\gamma_1, \gamma_2) = \frac{d\hat{w}_2 - c\theta}{2cd + 2y(1-\gamma_2)}$,

(v) When $\hat{w}_2 \ge \frac{2d\hat{w}_2(y+cd) - c\theta(2y+cd)}{cd^2}$*, then* $\bar{\gamma}_1 > 1$ *and* $\bar{\gamma}_2 \le 0$.

In this scenario, we encounter Case 3; i.e., $q_1^*(\gamma_1, \gamma_2) = 0$ *and* $q_2^*(\gamma_1, \gamma_2) = \frac{d\hat{w}_2 - c\theta}{2cd + 2y(1-\gamma_2)}, \ \forall \gamma_1, \gamma_2 \in [0, 1]$.

The values of $\bar{\gamma}_1$ and $\bar{\gamma}_2$ can be negative or positive depending on the magnitude of the difference between $\hat{w}_1$ and $\hat{w}_2$.

Now, when we look more closely into the three cases in Lemma 3-(i), we see that when $\bar{\gamma}_1 > 1$ and $\bar{\gamma}_2 > 1$, both suppliers invest even if the buyer does not cost-share. In our analysis in Lemma 4, we also find that if $\bar{\gamma}_i$ is in $[0, 1]$ than we must have $\bar{\gamma}_j > 1$. Thus, when $\bar{\gamma}_i \in [0, 1]$, both suppliers invest when $\gamma_i < \bar{\gamma}_i$. But after the threshold

$\bar{\gamma}_i$ (i.e., for $\gamma_i > \bar{\gamma}_i$), only supplier i will continue while supplier j stops investing. This situation shows that the low-margin supplier (here supplier j) becomes a "free-rider" when the buyer generously cost-shares with the high-margin supplier. Recall in Lemma 1 that as long as $\frac{d}{\theta} > \frac{c}{\hat{w}}$, the single supplier invests even if the buyer does not cost-share. However, in a three-tier supply chain, we observe that the lower margin supplier may avoid investing when the other supplier is stronger.

Next, we continue with the first stage of the full control scenario: the buyer's cost-sharing decision.

3.2.2 Buyer's Cost-sharing Decision

By backward induction, we use the suppliers' best response investment decisions to characterize the buyer's optimal cost-sharing strategy. The buyer's profit function is recalculated by inserting the q_1 and q_2 expressions found in Lemma 3.

Proposition 2. *Under the full control model, the buyer's equilibrium cost-sharing decision and the subsequent suppliers' investments are as follows:*

(i) When $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$ *and* $\frac{d}{\theta} > \frac{c}{\hat{w}_2}$*,*

I. When $\hat{w}_1 < \frac{2d\hat{w}_2 - c\theta}{d}$ *and* $\hat{w}_2 < \frac{2d\hat{w}_1 - c\theta}{d}$*, Case 1 emerges and both suppliers invest in quality independent of the buyer's cost-sharing decision.*

II. When $\frac{2d\hat{w}_2 - c\theta}{d} \leq \hat{w}_1 < \frac{2y(d\hat{w}_2 - c\theta) + cd(2d\hat{w}_2 - c\theta)}{cd^2}$*, either both suppliers invest in quality (Case 1) or only supplier 1 invests (Case 2) depending on the buyer's cost-sharing rate according to the conditions described in Lemma 4(ii).*

III. When $\hat{w}_1 \geq \frac{2y(d\hat{w}_2 - c\theta) + cd(2d\hat{w}_2 - c\theta)}{cd^2}$*, Case 2 emerges and the subgame perfect equilibrium is*

$$q_1^* = \begin{cases} \frac{d\hat{w}_1 - c\theta}{2cd + 2y} & \text{if } \frac{(cd+y)(2d\hat{p} - d\hat{w}_1 + c\theta)}{y(2d\hat{p} + d\hat{w}_1 - c\theta)} < 0; \\ \frac{2d\hat{p} + d\hat{w}_1 - c\theta}{4(cd+y)} & \text{if } 0 \leq \frac{(cd+y)(2d\hat{p} - d\hat{w}_1 + c\theta)}{y(2d\hat{p} + d\hat{w}_1 - c\theta)} < 1; \\ \frac{d\hat{w}_1 - c\theta}{2cd} & \text{if } \frac{(cd+y)(2d\hat{p} - d\hat{w}_1 + c\theta)}{y(2d\hat{p} + d\hat{w}_1 - c\theta)} \geq 1, \end{cases}$$

$q_2^* = 0$, $\gamma_1^* = min\left(max\left(\frac{(cd+y)(2d\hat{p} - d\hat{w}_1 + c\theta)}{y(2d\hat{p} + d\hat{w}_1 - c\theta)}, 0\right), 1\right)$*, and* $\gamma_2^* \in [0, 1]$.

IV. *When* $\frac{2d\hat{w}_1-c\theta}{d} \leq \hat{w}_2 < \frac{2y(d\hat{w}_1-c\theta)+cd(2d\hat{w}_1-c\theta)}{cd^2}$, *either both suppliers invest in quality (Case 1) or only supplier 2 invests (Case 3) depending on the buyer's cost-sharing rate according to the conditions described in Lemma 4(iv).*

V. *When* $\hat{w}_2 \geq \frac{2y(d\hat{w}_1-c\theta)+cd(2d\hat{w}_1-c\theta)}{cd^2}$, *Case 3 emerges and the subgame perfect equilibrium is*

$$q_1^* = 0, \quad q_2^* = \begin{cases} \frac{d\hat{w}_2-c\theta}{2cd+2y} & \text{if } \frac{(cd+y)(2d\hat{p}-d\hat{w}_2+c\theta)}{y(2d\hat{p}+d\hat{w}_2-c\theta)} < 0; \\ \frac{2d\hat{p}+d\hat{w}_2-c\theta}{4(cd+y)} & \text{if } 0 \leq \frac{(cd+y)(2d\hat{p}-d\hat{w}_2+c\theta)}{y(2d\hat{p}+d\hat{w}_2-c\theta)} < 1;, \\ \frac{d\hat{w}_2-c\theta}{2cd} & \text{if } \frac{(cd+y)(2d\hat{p}-d\hat{w}_2+c\theta)}{y(2d\hat{p}+d\hat{w}_2-c\theta)} \geq 1, \end{cases}$$

$\gamma_1^* \in [0,1]$, *and* $\gamma_2^* = min\left(max\left(\frac{(cd+y)(2d\hat{p}-d\hat{w}_2+c\theta)}{y(2d\hat{p}+d\hat{w}_2-c\theta)}, 0\right), 1\right)$.

(ii) *When* $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$ *and* $\frac{d}{\theta} \leq \frac{c}{\hat{w}_2}$, *Case 2 emerges and the subgame perfect equilibrium is*

$$q_1^* = \begin{cases} \frac{d\hat{w}_1-c\theta}{2cd+2y} & \text{if } \frac{(cd+y)(2d\hat{p}-d\hat{w}_1+c\theta)}{y(2d\hat{p}+d\hat{w}_1-c\theta)} < 0; \\ \frac{2d\hat{p}+d\hat{w}_1-c\theta}{4(cd+y)} & \text{if } 0 \leq \frac{(cd+y)(2d\hat{p}-d\hat{w}_1+c\theta)}{y(2d\hat{p}+d\hat{w}_1-c\theta)} < 1; \\ \frac{d\hat{w}_1-c\theta}{2cd} & \text{if } \frac{(cd+y)(2d\hat{p}-d\hat{w}_1+c\theta)}{y(2d\hat{p}+d\hat{w}_1-c\theta)} \geq 1, \end{cases}$$

$q_2^* = 0$ *with* $\gamma_1^* = min\left(max\left(\frac{(cd+y)(2d\hat{p}-d\hat{w}_1+c\theta)}{y(2d\hat{p}+d\hat{w}_1-c\theta)}, 0\right), 1\right)$, *and* $\gamma_2^* \in [0,1]$.

(iii) *When* $\frac{d}{\theta} \leq \frac{c}{\hat{w}_1}$ *and* $\frac{d}{\theta} > \frac{c}{\hat{w}_2}$, *Case 3 emerges and the subgame perfect equilibrium is*

$$q_1^* = 0, \quad q_2^* = \begin{cases} \frac{d\hat{w}_2-c\theta}{2cd+2y} & \text{if } \frac{(cd+y)(2d\hat{p}-d\hat{w}_2+c\theta)}{y(2d\hat{p}+d\hat{w}_2-c\theta)} < 0; \\ \frac{2d\hat{p}+d\hat{w}_2-c\theta}{4(cd+y)} & \text{if } 0 \leq \frac{(cd+y)(2d\hat{p}-d\hat{w}_2+c\theta)}{y(2d\hat{p}+d\hat{w}_2-c\theta)} < 1; \\ \frac{d\hat{w}_2-c\theta}{2cd} & \text{if } \frac{(cd+y)(2d\hat{p}-d\hat{w}_2+c\theta)}{y(2d\hat{p}+d\hat{w}_2-c\theta)} \geq 1, \end{cases}$$

with $\gamma_1^* \in [0,1]$, *and* $\gamma_2^* = min\left(max\left(\frac{(cd+y)(2d\hat{p}-d\hat{w}_2+c\theta)}{y(2d\hat{p}+d\hat{w}_2-c\theta)}, 0\right), 1\right)$,

(iv) *When* $\frac{d}{\theta} \leq \frac{c}{\hat{w}_1}$ *and* $\frac{d}{\theta} \leq \frac{c}{\hat{w}_2}$, *Case 4 emerges and the subgame perfect equilibrium is* $q_1^* = 0$, $q_2^* = 0$ *with* $\gamma_1^* \in [0,1]$ *and* $\gamma_2^* \in [0,1]$.

In Proposition 2, we obtain the equilibrium γ_1^* and γ_2^* decisions and the respective q_1^* and q_2^* values for equilibrium in Case 2, Case 3, and Case 4 described in Lemma 3.

In item (ii), since supplier 2's unit cost impact of quality (i.e., $\frac{c}{\hat{w}_2}$) is too high to improve her quality, she does not invest in quality under any circumstances; therefore, $q_2^*(\gamma_1, \gamma_2) = 0$, $\forall \gamma_1, \gamma_2 \in [0,1]$. In this case, we find that the buyer's cost-sharing offer is contingent on the expression $\frac{(cd+y)(2d\hat{p}-d\hat{w}_1+c\theta)}{y(2d\hat{p}+d\hat{w}_1-c\theta)}$. This expression is decreasing in $\hat{w}_1$. Thus, when $\hat{w}_1$ is high enough, the buyer prefers not to share costs, and thus $q_1^* = \frac{d\hat{w}_1-c\theta}{2cd+2y}$. As $\hat{w}_1$ decreases however, the buyer's cost-sharing offer increases. We observe a symmetric behaviour in part (iii) for Case 3.

In item (iv), since the relative market awareness of quality (i.e., $\frac{d}{\theta}$) is less than both suppliers' unit cost impact of quality (i.e., $\frac{c}{\hat{w}_1}$ and $\frac{c}{\hat{w}_2}$), neither supplier 1 nor supplier 2 are motivated to invest in quality. In such a situation, the buyer cannot influence the suppliers by cost-sharing. Therefore, $q_1^* = q_2^* = 0$ for all $\gamma_1^* \in [0,1]$ and $\gamma_2^* \in [0,1]$.

In item (i) we can only structure the pure equilibrium Case 2 and Case 3. By Lemma 4, we know that equilibrium may transition between Case 1 and Case 2 (or Case 1 and Case 3) depending on the buyer's cost-sharing offer. Thus, the buyer's profit function becomes a piecewise function in her cost-sharing rates. Therefore, we check its continuity below:

Lemma 5. *π_B^F is continuous in $\gamma_i \in [0,1]$, where $i \in \{1,2\}$.*

With Lemma 5, we show that π_B^F is continuous even when the function is piecewise.

In Proposition 2, we reached the optimality conditions and equilibrium for Case 2, Case 3, and Case 4. But Case 1 poses as a more complicated problem. We introduce Lemma 6 below in order to discuss the buyer's profit function under Case 1:

Lemma 6. *π_B^F is neither jointly concave nor jointly convex in (γ_1, γ_2) in the region $0 \leq \gamma_1 < \bar{\gamma}_1$ and $0 \leq \gamma_2 < \bar{\gamma}_2$, when $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$ and $\frac{d}{\theta} > \frac{c}{\hat{w}_2}$, where $\bar{\gamma}_1 = 1 + \frac{cd(2d\hat{w}_2-d\hat{w}_1-c\theta)}{2y(d\hat{w}_2-c\theta)}$ and $\bar{\gamma}_2 = 1 + \frac{cd(2d\hat{w}_1-d\hat{w}_2-c\theta)}{2y(d\hat{w}_1-c\theta)}$.*

In order to analyze the equilibrium behavior in more detail, we will conduct numerical analyses in Chapter 4.

3.3 DELEGATION MODEL

Next, we will analyze the delegation scenario. As in the full control subsection, we use backward induction to obtain the subgame perfect equilibrium. We start the analysis from the third stage; i.e. the second supplier investment stage. We then continue with the first supplier investment stage. Finally, we study the buyer's investment decision.

3.3.1 Second Supplier's Investment Decision

For the third stage, the best response of supplier 2 is provided in the Proposition 3, below:

Proposition 3. *Under the delegation model, for a given* $(q_1, \gamma_1, \gamma_2)$*, the best response of the second supplier is the following:*

$$q_2^*(q_1, \gamma_1, \gamma_2) = \left(\frac{d\hat{w}_2 - cdq_1 - c\theta}{2cd + 2y(1 - \gamma_2)} \right)^+ \tag{3.8}$$

Here, supplier 2 decides her quality level according to the other supply chain partners' decisions on quality and incentive levels. Her investment q_2 decreases with q_1 while it increases with γ_2. On the other hand, when q_1 is increased possibly because of a high margin of the supplier 1 or a high incentive level provided to supplier 1 by the buyer, supplier 2 becomes less willing to improve her quality level. Note that supplier 2 will not invest if q_1 is too high, no matter how high of a cost-sharing offer γ_1 he receives.

3.3.2 First Supplier's Investment Decision

In the second stage, we analyze the supplier 1's best response, which depends on the buyer's cost-sharing rate. At that step, supplier 1 must decide on both his quality investment level and the cost-sharing rate with supplier 2, foreseeing his best response quality decision $q_2^*(q_1, \gamma_1, \gamma_2)$. Hence, supplier 1 has two decision variables to determine.

Lemma 7. $\pi_{S_1}^D$ *is neither jointly concave nor jointly convex in* (q_1, γ_2) *in the region* $q_1 \in [0, 1]$ *and* $\gamma_2 \in [0, 1]$.

Consequently, we cannot characterize supplier 1's best response in closed-form. Therefore, we continue our equilibrium analysis by conducting numerical experiments in the next section.

CHAPTER 4

NUMERICAL STUDY

In this chapter, we present numerical experiments to further characterize the equilibrium behaviour of the buyer and the suppliers, and its change with respect to the various model parameters. We first study the full control model, then we continue with the delegation model. Finally, we further work on a comparison section.

We study the effect of each individual parameter on equilibrium decisions, starting with d, consumers' quality awareness effect on demand. Later, we focus on the change in the base market potantial, $\theta = K - ap$, where $K = 1$ and $p = 1$. Here, we keep $K = 1$ since it is the intrinsic market potential. Consequently, θ is directly dependent on a; i.e., consumers' price awareness effect on demand. We base our discussions on θ instead of a in order to be consistent with our analytical derivations and results. Moreover, in order to standardize the total market incentives for investment, we set the total of the supply chain partners' margins to 1; i.e., $\hat{w}_1 + \hat{w}_2 + \hat{p} = 1$. Afterwards, we discuss the effect of c; i.e., the quality-driven unit cost increase to suppliers. Finally, we analyze the effect of y; i.e., the suppliers' investment cost factor to build quality q_i. Without loss of generality, we set $m_1 = m_2 = 0$; besides, we only study the cases where $\hat{w}_1 \geq \hat{w}_2$ for the full control model.

In our setting, the margin difference between the buyer and the suppliers can be as important as the parameter in focus. In order to understand the effect of being dominant or weak in the chain, we analyze different supply chain configurations in terms of margin, and hence bargaining power. Accordingly, we will examine the scenarios summarized in Table 4.1:

Table 4.1: Numerical Study Margin Split Scenarios

Dominant Buyer Scenarios	$\hat{p}$	$\hat{w}_1$	$\hat{w}_2$
1	0.37	0.33	0.3
2	0.46	0.28	0.26
3	0.4	0.34	0.26
Dominant supplier 1 - Weak supplier 2 Scenarios	$\hat{p}$	$\hat{w}_1$	$\hat{w}_2$
4	0.33	0.37	0.3
5	0.28	0.46	0.26
6	0.34	0.4	0.26
Dominant supplier 1 - Weak Buyer Scenarios	$\hat{p}$	$\hat{w}_1$	$\hat{w}_2$
7	0.3	0.37	0.33
8	0.26	0.46	0.28
9	0.26	0.4	0.34

As seen in the table, there are three categories investigated. The first one contains buyer-dominant scenarios where supplier 1 is always stronger than supplier 2. The second category contains three scenarios where supplier 1 is dominant and the buyer is stronger than supplier 2. And finally, the third category contains dominant supplier 1 and weak buyer scenarios. In these nine scenarios, we interchange three different margin split sets between the supply chain partners, so that we can make a healthy comparison between the scenarios. Next, we first analyze the full control model.

4.1 FULL CONTROL MODEL NUMERICAL ANALYSIS

In our analysis, we evaluate the equilibrium decisions by changing one parameter at a time and taking others as given as shown in the table below:

Table 4.2: Numerical Study Base Parameters

K	d	θ	y	c	$\hat{w}_1$	$\hat{w}_2$	$\hat{p}$
1.0	0.5	0.5	0.5	0.25	0.33	0.3	0.37

For the base case, the equilibrium stands as Case 1; i.e., the buyer cost-shares with both suppliers and both q_1^* and q_2^* are positive. The equilibrium values are summarized below:

Table 4.3: Numerical Study Base Case Equilibria

γ_1^*	γ_2^*	π_F^B	q_1^*	q_2^*	q^*	$\pi_F^{S_1}$	$\pi_F^{S_2}$
0.92	1.00	0.3072	0.2582	0.2209	0.4791	0.1685	0.1657

Now, we analyze how the equilibrium is influenced by the change in the quality awareness effect on demand, d.

4.1.1 Consumers' quality awareness effect on demand (d)

Recall that consumers' quality awareness effect on demand is one of the key elements in the demand function and as it increases, the demand at a given quality level increases. In our analysis, since we study numerical examples where $\hat{w}_1 > \hat{w}_2$, we will observe three of the cases we described in the analytical section. These are Case 1, where both suppliers invest in quality, i.e., $q_1^* > 0$, $q_2^* > 0$; Case 2, where only the stronger supplier invests in quality, i.e., $q_1^* > 0$, $q_2^* = 0$; or Case 4, where neither supplier invests in quality, i.e., $q_1^* = 0$, $q_2^* = 0$ ($\gamma_1^* \in [0, 1]$ and $\gamma_2^* \in [0, 1]$ in all cases).

In this study, we analyze the equilibrium behavior for different d values between 0 and 1 by 0.01 increments for all nine scenarios. Across all scenarios, for lower values of d, we observe Case 4 up to a threshold, that we refer as $\bar{d}_1$. There is no cost-sharing and no quality investment in the market. The market potential of environmental quality is just not strong enough to motivate any of the supply chain partners. Beyond the threshold, we observe Case 2 where the buyer starts fully cost-sharing with the high-margin supplier, i.e., supplier 1. As d increases further beyond the second threshold, referred as $\bar{d}_2$, the buyer also cost-shares with supplier 2, and we observe Case 1 as the equilibrium. This behavior is a general pattern observed in the nine scenarios we analyzed. The transition points; $\bar{d}_1$ and $\bar{d}_2$ are provided in Table 4.4.

According to our analysis, $\bar{d}_1$, which is the threshold for the case transition from Case 4 to Case 2, directly depends on $\hat{w}_1$. As $\hat{w}_1$ increases, supplier 1 becomes stronger and we observe case transitions at lower d values, that means $\bar{d}_1$ decreases. As seen from Table 4.4, the $\bar{d}_1$ values are higher in the dominant buyer scenarios since $\hat{w}_1$ values are lower than in the other categories. Moreover, since $\hat{w}_1$ values in the second category and in the third category are the same, the $\bar{d}_1$ values are also the same. From

Table 4.4: Case Transition Thresholds ($\bar{d}_1$ and $\bar{d}_2$) in Each Scenario

Scenarios	$\hat{p}$	$\hat{w}_1$	$\hat{w}_2$	$\bar{d}_1$	$\bar{d}_2$
1	0.37	0.33	0.3	0.38	0.46
2	0.46	0.28	0.26	0.45	0.53
3	0.4	0.34	0.26	0.37	0.58
4	0.33	0.37	0.3	0.34	0.47
5	0.28	0.46	0.26	0.28	0.56
6	0.34	0.4	0.26	0.32	0.57
7	0.3	0.37	0.33	0.34	0.41
8	0.26	0.46	0.28	0.28	0.51
9	0.26	0.4	0.34	0.32	0.4

our analytical results in Chapter 4 (see Proposition 1 and Lemma 3), in order for a case transition from Case 4 to Case 2 occur, $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$ must hold. When we check our $\bar{d}_1$ values, we see that $\bar{d}_1$ is in fact equal to $\lceil \frac{c\theta}{\hat{w}_1} \rceil$. The rounding is because we enumerate the d values in $[0, 1]$ interval by 0.01 increments. Consequently, we can call $\bar{d}_1$ as the break-even point for transition to Case 2.

When we look at $\bar{d}_2$, which is the threshold for transition from Case 2 to Case 1, we see $\hat{w}_2$ is the primary factor that determines $\bar{d}_2$. As $\hat{w}_2$ increases, since supplier 2 becomes stronger, $\bar{d}_2$ decreases. However, $\hat{w}_2$ is not the only factor that affects $\bar{d}_2$. If supplier 2 was the sole supplier, then the threshold for her investment would be $\bar{d}_2 = \lceil \frac{c\theta}{\hat{w}_2} \rceil$ in a similar fashion to $\bar{d}_1$. However, since now there are two suppliers, the lower margin supplier, supplier 2, behaves as a free-rider. As also discussed in Chapter 4, supplier 2 behaves as a free-rider when the buyer subsidizes supplier 1 over a threshold $\bar{\gamma}_1 = 1 + \frac{cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta)}{2y(d\hat{w}_2 - c\theta)}$, which depends on $\hat{w}_1$ as well as $\hat{w}_2$. Thus, for transition from Case 2 to Case 1, $\bar{\gamma}_1$ plays a critical role. Case 2 occurs when $\bar{\gamma}_1 \leq \gamma_1^* \leq 1$ and $0 \leq \gamma_2^* \leq \bar{\gamma}_2$. Whenever $0 \leq \gamma_1^* \leq \bar{\gamma}_1$ is satisfied, the equilibrium transitions to Case 1. Therefore, $\bar{d}_2$ is higher than the break-even point; i.e., $\bar{d}_2 = \lceil \frac{c\theta}{\hat{w}_2} \rceil$, and it is influenced by more than one market condition[1]

Next, we analyze the equilibrium cost-sharing strategies. Across all nine scenarios, we observe that the buyer may sustain full cost-sharing with supplier 1 or may gradually decrease her share throughout the Case 2 region. In the region where both suppliers invest; i.e., the Case 1 region, the buyer is inclined to keep her support of

[1] For example, between scenarios 1 and 4 with $\hat{w}_2 = 0.3$ in both, we have $\hat{w}_1^1 < \hat{w}_1^4$ and $\bar{d}_2^1 < \bar{d}_2^4$.

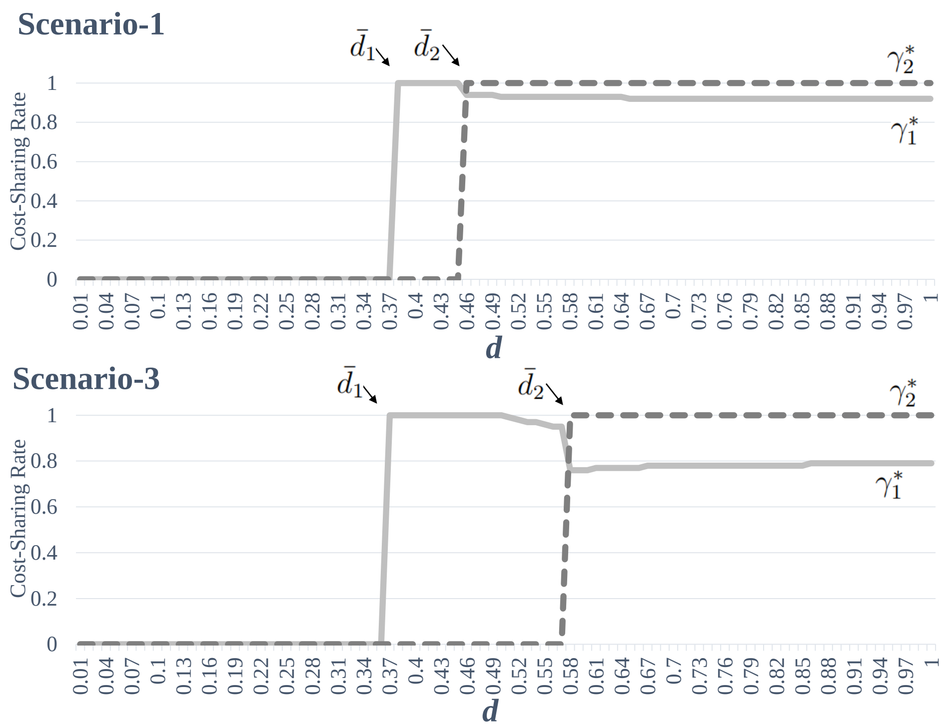

Figure 4.1: Equilibrium Cost-Sharing Decisions under Scenario-1 ($\bar{d}_1^1 = 0.38$, $\bar{d}_2^1 = 0.46$) and Scenario-3 ($\bar{d}_1^3 = 0.37$, $\bar{d}_2^3 = 0.58$)

supplier 2 whereas she may first increase then decrease her subsidy on supplier 1.

We first focus on Case 2 observed between $\bar{d}_1$ and $\bar{d}_2$. When $d \geq \bar{d}_1$, supplier 1 is willing to invest in quality even if the buyer does not cost-share with him. Here, the buyer immediately starts fully subsidizing supplier 1 at $\bar{d}_1$ to maximize improvement in q_1 and therefore her profit. If supplier 1's margin is low, the buyer fully subsidizes supplier 1; i.e., $\gamma_1 = 1$, throughout the whole Case 2 region. This situation is seen for scenarios 1 and 2, where the buyer is the strongest partner in the supply chain and supplier 1's margin is not as high as in the other scenarios. For the other seven scenarios, the buyer does not fully subsidize supplier 1 all the way; hence, gradually she steps back and reduces γ_1^* below 1 as d increases. Figure 4.1 provides examples for either pattern through scenarios 1 and 3.

When supplier 1 is strong enough; i.e., in scenarios 3 to 9, the buyer avoids fully investing in supplier 1 throughout Case 2 region because of the diminishing marginal return of q_1 improvement in her profit. For example, in Scenario-3, as d turns to 0.51, the buyer lowers γ_1^* slightly. Beyond $d = 0.51$, she cannot justify the cost of fully subsidizing an already high-margin supplier 1. In order to visualize the buyer's trade-off on cost-sharing with supplier 1, we provide the marginal change in the buyer's profit as γ_1 increases for $d = 0.51$ and $d = 0.57$ in Figure 4.2. The buyer's marginal gain turns to negative for $\gamma_1 > 0.99$ at $d = 0.51$, and that is the reason $\gamma_1^* = 0.99$ for $d = 0.51$. Similarly, at $d = 0.57$, $\gamma_1^* = 0.95$ since the buyer's marginal gain turns to negative for $\gamma_1 > 0.95$. Hence, when supplier 1 is strong enough, as d increases, the buyer's support decreases due to diminishing returns on her investment.

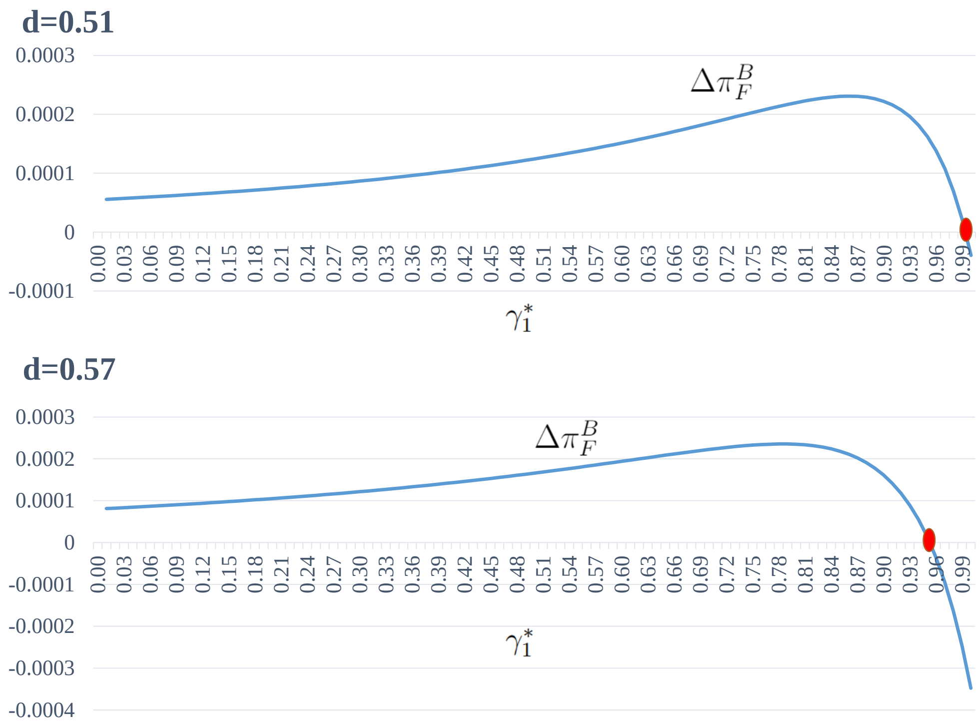

Figure 4.2: Buyer's Marginal Gain from γ_1 Increasing in Scenario-3

Now, we consider the quality improvement as d increases between $\bar{d}_1$ and $\bar{d}_2$. When the equilibrium turns to Case 2, q^* improves dramatically at first and then continues to improve despite a slight decrease in the improvement rate. In Case 2, since supplier 2 is not investing in quality yet, $q_2^* = 0$; therefore, $q^* = q_1^*$. When $\gamma_1^* = 1$, the increase in q^* is steep. In all scenarios except 1 and 2, as d increases beyond $\bar{d}_1$, γ_1^* decreases gradually in the Case 2 region. As γ_1^* decreases, q^* continues to increase, but at a decreasing rate. The buyer and supplier 1 also increase their profits in this range. The evolution of the equilibrium quality in scenarios 1 and 3 are presented in Figure 4.3 as examples.

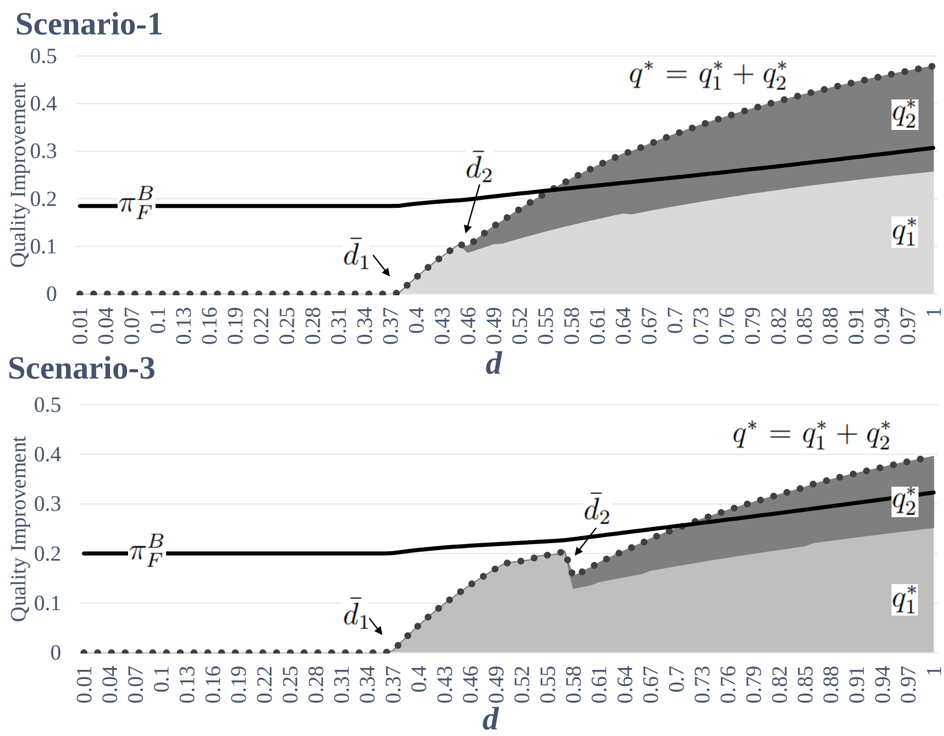

Figure 4.3: Equilibrium Quality Investment Decisions under Scenario-1 ($\bar{d}_1^1 = 0.38$, $\bar{d}_2^1 = 0.46$) and Scenario-3 ($\bar{d}_1^3 = 0.37$, $\bar{d}_2^3 = 0.58$)

With the transition from Case 2 to Case 1, the buyer arrives at a point where she will be more profitable if she supports both suppliers instead of the stronger one. In fact, supplier 2 becomes willing to invest in quality improvement at a lower d than $\bar{d}_2$ as mentioned before. However, since the buyer prefers supporting supplier 1 at a higher rate than $\bar{\gamma}_1$, supplier 2 remains a free-rider [2]. When it is in the buyer's best interest[3], she starts fully subsidizing supplier 2; i.e., $\gamma_2^* = 1$, and dramatically decreases γ_1^*; i.e., under $\bar{\gamma}_1$, in all scenarios except one[4]. In the transition, the decrease in γ_1^* directly depends on the difference between the suppliers' margins; i.e., $\hat{w}_1 - \hat{w}_2$. We observe that the greatest drop in γ_1^* occurs in Scenario-5 where the $\hat{w}_1 - \hat{w}_2$ difference is the largest. Besides, there is no drop in γ_1^* in Scenario-2 since the $\hat{w}_1 - \hat{w}_2$ difference is very small.

[2] In Scenario-1, at $d = 0.43$, if the buyer cost-shares with both suppliers with $\gamma_1 = 0.76$ and $\gamma_2 = 1$, her profit becomes $\pi_F^B = 0.1904$. However, in the optimal case, she fully cost-shares only with supplier 1; i.e., $\gamma_1 = 1$ and $\gamma_2 = 0$; hence, her profit is $\pi_F^B = 0.1944$.

[3] In Scenario-1, at $d = 0.46$, the optimal case; i.e., $\gamma_1 = 0.94$ and $\gamma_2 = 1$, produces $\pi_F^B = 0.1984$. On the other hand, if the buyer goes on subsidizing supplier 1 and do not cost-share with supplier 2; i.e., $\gamma_1 = 1$ and $\gamma_2 = 0$, her profit becomes $\pi_F^B = 0.1980$.

[4] In Scenario-2, the buyer fully subsidizes both supplier 1 and supplier 2 since both $\hat{w}_1$ and $\hat{w}_2$ are very low compared to $\hat{p}$.

When we analyze the effect of the decrease in γ_1^* on q^* during the transition to Case 1, we see that the total quality $q^* = q_1^* + q_2^*$ decreases at the transition point in most of the cases. Here, supplier 1's quality drops abruptly due to the sudden decrease in γ_1^* despite the increase in γ_2^*. However, the increase in q_2^* cannot make up for this sudden drop and total quality decreases. Only in Scenario-2, does q^* not drop but instead improve during the transition because γ_1^* stays equal to 1. The general observation of q^* decreasing during the transition to Case 1 points out a very important result: Increasing market awareness can in fact decrease the total quality improvement in the market due to the buyer's profit maximizing strategies. To illustrate our discussion about the γ_1^* decrease and it's effect on q^*, we provide the buyer's cost-sharing decision and quality improvements in Scenario-5 in Figure 4.4.

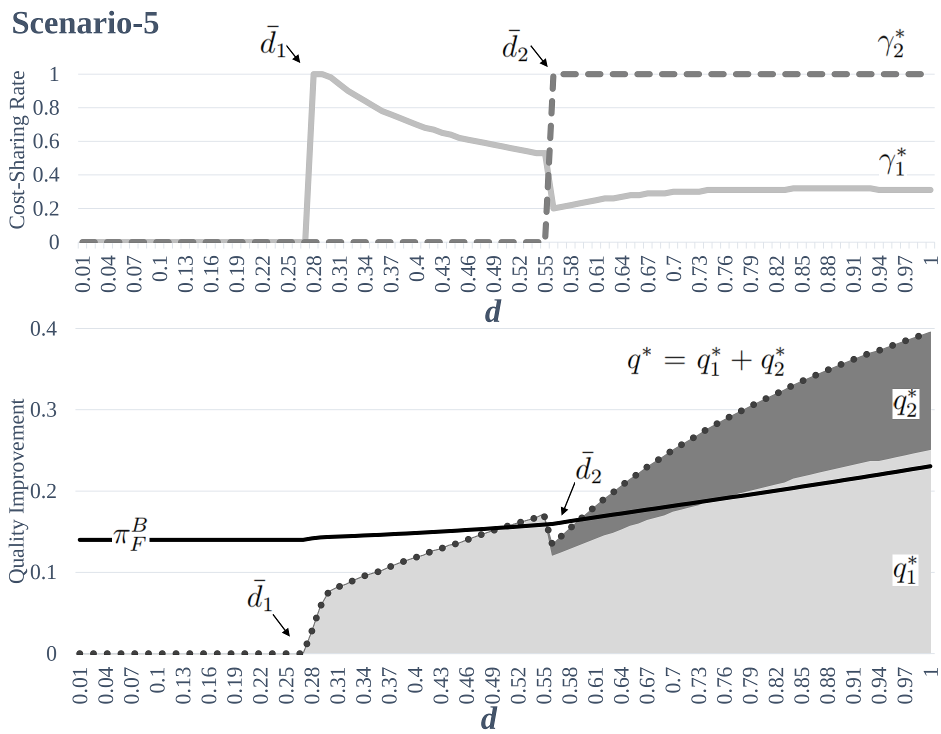

Figure 4.4: Equilibrium Cost-Sharing and Quality Investment Decisions under Scenario-5 ($\bar{d}_1^5 = 0.28$, $\bar{d}_2^5 = 0.56$)

In the range of Case 1, the buyer can afford to increase her support on supplier 1 and still keep supplier 2 invested as d increases. Only in scenarios 1 and 2, we do not see this trend. In the former, she does not prefer to increase her support for an equally strong supplier 1, in the latter she does not have to sacrifice from her support for supplier 1 in transition at all. When we look into the buyer's support of supplier

2, we see that in most of the scenarios, the buyer fully subsidizes supplier 2 for the whole Case 1 region. In scenarios 7 and 9, however, since the buyer is weak, she decreases her support of the relatively stronger supplier 2 as d increases. The change in the buyer's cost-sharing decisions with respect to d for Scenario-9 is provided in Figure 4.5 below.

After the first drop in total quality with the transition to Case 1, $q^* = q_1^* + q_2^*$ increases as d increases further. As the market awareness of quality increases, both suppliers increase their quality investment. In scenarios 7 and 9, even though γ_2^* drops for high values of d, since the quality awareness is high in the market, supplier 2's investment keeps increasing. Ultimately, q^* improves as d increases in the Case 1 region for all scenarios. The change in total quality for Scenario-9 is presented in Figure 4.5.

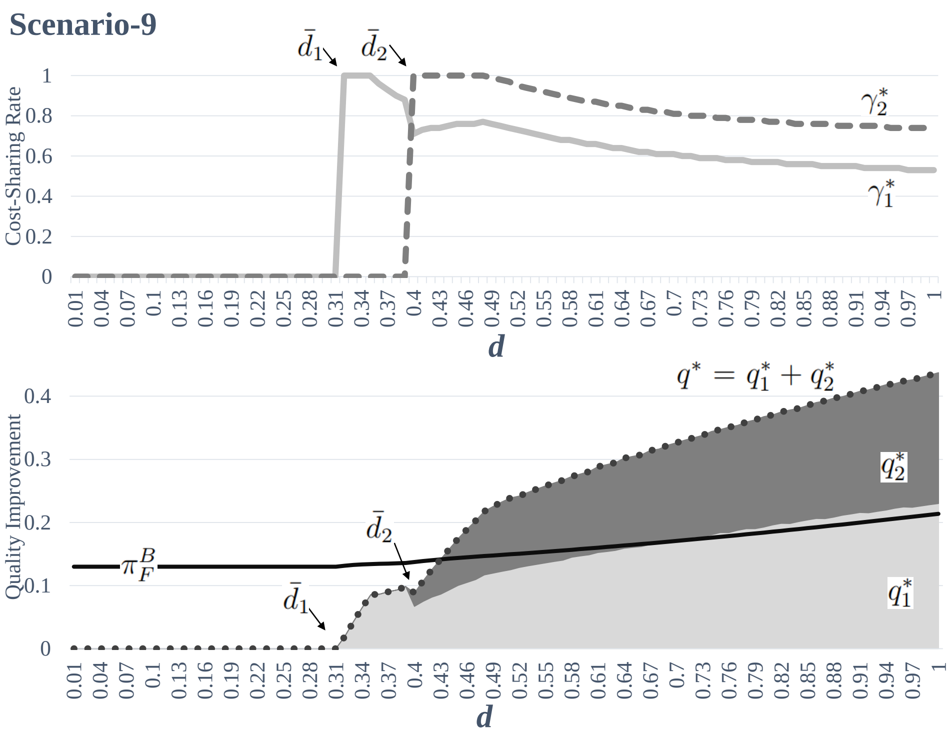

Figure 4.5: Equilibrium Cost-Sharing and Quality Investment Decisions under Scenario-9 ($\bar{d}_1^9 = 0.32$, $\bar{d}_2^9 = 0.4$)

According to our analysis, we see that as the consumers' quality awareness effect on demand, d, gets higher, in general, all supply chain partners become more inclined to invest in quality. For the lower values of d, the market potential is not attractive enough for investment, so we observe Case 4 as the equilibrium. For higher values of d, the stronger supplier, supplier 1, starts investing in quality with full support from the buyer, at least at first, and the equilibrium turns to Case 2. Depending on the margin splits, the buyer either continues to fully subsidize supplier 1 or decrease γ_1^* as d increases during Case 2. For higher values of d, supplier 2 becomes willing to invest in q_2 but due to supplier 1's high investment rates, supplier 2 stays as a free-rider for a while. Whenever d reaches a point where the buyer becomes more profitable by supporting both suppliers, she rearranges γ_1^* and γ_2^* to induce supplier 2 to invest in quality. As the quality awareness in the market increases further, the buyer keeps rearranging the cost-sharing rates but in general she keeps fully subsidizing supplier 2 as long as she can afford to.

When the buyer cost-shares only with supplier 1, the total quality improves due to both supplier 1's and the buyer's investments. When the buyer begins to support both suppliers, the total quality may decrease at first since the buyer dramatically decreases her cost-sharing rate with supplier 1. That shows increasing market awareness can in fact decrease the total quality investment in the market due to the free-rider phenomenon observed among the suppliers.

4.1.2 Market potential (θ)

In this section, we analyze the effect of the base market potential, $\theta = K - ap$, where $K = 1$ and $p = 1$ in equilibrium. We base our discussions on θ instead of a; i.e., consumers' price awareness effect on demand, in order to be consistent with our analytical derivations and results. Market potential, θ, represents the starting market potential of the end-product, before any investment.

We analyze the equilibrium behaviour for different θ values between 0 and 1 by 0.01 increments[5] for the nine scenarios introduced at the beginning of the chapter. In our numerical experiments (with $\hat{w}_1 > \hat{w}_2$), we observe three equilibrium cases. Case 1 is the equilibrium for lower θ values; Case 2 is the equilibrium for medium values of θ; and Case 4 is the equilibrium for higher values of θ. To simplify our discussion, we provide Table 4.5 showing the case transition thresholds, $\bar{\theta}_1$ and $\bar{\theta}_2$, for each scenario.

[5] Here, $a = 1 - \theta$ and it changes from 1 to 0 as θ increases from 0 to 1 (since $\theta = K - ap$, where $K = 1$ and $p = 1$ on equilibrium).

Table 4.5: Case Transition Thresholds ($\bar{\theta}_1$ and $\bar{\theta}_2$) in Each Scenario

Scenarios	$\hat{p}$	$\hat{w}_1$	$\hat{w}_2$	$\bar{\theta}_1$	$\bar{\theta}_2$
1	0.37	0.33	0.3	0.55	0.66
2	0.46	0.28	0.26	0.49	0.56
3	0.4	0.34	0.26	0.45	0.68
4	0.33	0.37	0.3	0.54	0.74
5	0.28	0.46	0.26	0.46	0.92
6	0.34	0.4	0.26	0.46	0.8
7	0.3	0.37	0.33	0.61	0.74
8	0.26	0.46	0.28	0.5	0.92
9	0.26	0.4	0.34	0.64	0.8

We see that both the cost-sharing decisions of the buyer and the investment decisions of the suppliers have similar patterns to the case of d, but in reverse. The reason for this reverse pattern is that the equilibrium decisions are mainly driven by the trade-off between $\frac{d}{\theta}$ and $\frac{c}{\hat{w}_i}$, where d and θ have inverse effects on the trade-off. Mirroring the case of $\bar{d}_1$ and $\bar{d}_2$, respectively, $\bar{\theta}_2$ and $\bar{\theta}_1$ are driven by the margin values of the parties in the chain, the diminishing returns nature of the investment cost function, and the free-rider issue in the investment stage.

The buyer's cost-sharing strategy follows a reverse pattern when we compare its change with respect to d as θ increases. In the Case 1 equilibrium region, the buyer increases her support on both suppliers to sustain their investment levels against an increased θ. At an increased θ, there exists the risk of supplier 2 turning to a free-rider as the low margin supplier. In that respect, γ_2^* is always higher than γ_1^*. As θ reaches high enough values to seriously mitigate investment, the buyer fully subsidizes supplier 2 and reduces supplier 1's support in order to keep both suppliers in the game. Despite the buyer's support efforts in the Case 1 region, both q_1^* and q_2^*, and thus q^* decrease as θ increases in this zone[6].

As θ increases further, the cost of supporting both suppliers becomes too heavy for the buyer. Therefore, she changes her strategy to support only supplier 1 and stop subsidizing supplier 2 at $\bar{\theta}_2$. A dramatic increase in γ_1^* is observed with the transition to Case 2. The increase in γ_1^* directly depends on the margin difference between $\hat{w}_1 - \hat{w}_2$. As the difference increases, the jump observed in γ_1^* during the transition

[6] As seen from Proposition 1, both $q_1^*(\gamma_1, \gamma_2)$ and $q_2^*(\gamma_1, \gamma_2)$ are decreasing in θ on equilibrium.

also increases. Hence, converse to d, largest increase in γ_1^* is observed in Scenario-5 while the smallest change is zero for Scenario-2. Meanwhile, total quality increases abruptly in the transition to Case 2; i.e., at $\bar{\theta}_1$, due to the sudden increase in γ_1^*.

For $\theta \geq \bar{\theta}_2$, the buyer stops subsidizing supplier 1 since the $\frac{d}{\theta} \geq \frac{c}{\hat{w}_1}$ condition no longer holds, and supplier 1 has no intention to invest anymore. Hence, total quality investment becomes 0. As an example, the evolution of the cost-sharing decisions and the quality investment decisions for Scenario-4 is provided below in Figure 4.6.

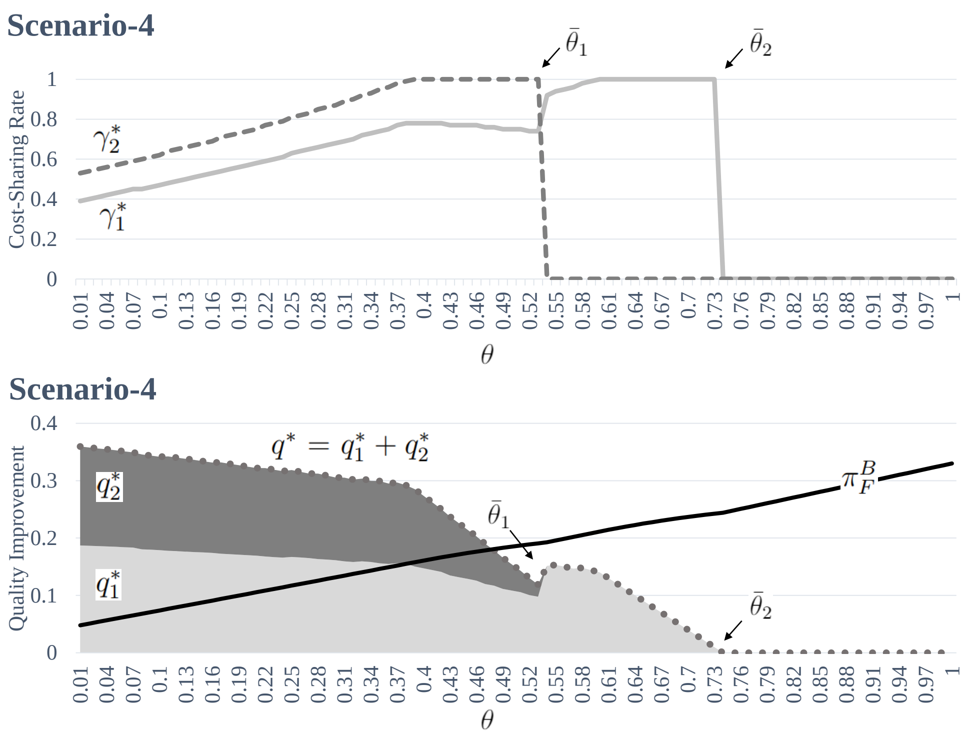

Figure 4.6: Equilibrium Cost-Sharing and Quality Investment Decisions under Scenario-4: $\bar{\theta}_1^4 = 0.54$, $\bar{\theta}_2^6 = 0.74$

According to our analysis, we see that as the base market potential, θ, increases, all supply chain partners become less willing to invest in quality. Since the market potential is one of the key elements of the demand function, as it increases, all supply chain partners' profits increase. However, as the base market potential increases, quality awareness becomes relatively less effective, and investing in quality becomes less attractive for the suppliers. The base market potential creates an inertia against the quality investment decisions; its size represents the current customers that will now be served with an increased cost due to quality. In this respect, it works against the market potential, customers (and margins), to be acquired with investment in quality.

4.1.3 Quality-driven unit cost (c)

In this section, we analyze the effect of the quality-driven unit cost, c, on the equilibrium decisions. As c increases, the suppliers' unit cost increases with an increase in quality. This situation negatively affects both the buyer's and the suppliers' profit. Hence, as c increases, we expect the supply chain partners to become less willing to invest in quality since it becomes a costly and less attractive investment.

We analyze the equilibrium decisions for different c values between 0 and 1 by 0.01 increments for nine scenarios. We observe that c has a very similar effect on the equilibrium to that of θ. Hence, we observe the equilibrium transitions from Case 1 to Case 2 and then to Case 4 as c increases. In Table 4.6, we provide the thresholds, $\bar{c}_1$ and $\bar{c}_2$, for case transitions in each scenario.

Table 4.6: Case Transition Thresholds ($\bar{c}_1$ and $\bar{c}_2$) in Each Scenario

Scenarios	$\hat{p}$	$\hat{w}_1$	$\hat{w}_2$	$\bar{c}_1$	$\bar{c}_2$
1	0.37	0.33	0.3	0.28	0.33
2	0.46	0.28	0.26	0.25	0.28
3	0.4	0.34	0.26	0.23	0.34
4	0.33	0.37	0.3	0.27	0.37
5	0.28	0.46	0.26	0.23	0.46
6	0.34	0.4	0.26	0.23	0.4
7	0.3	0.37	0.33	0.3	0.37
8	0.26	0.46	0.28	0.25	0.46
9	0.26	0.4	0.34	0.32	0.4

We see that both the cost-sharing decisions of the buyer and the investment decisions of the suppliers have a similar pattern to the case of θ since both have a similar effect on the condition $\frac{d}{\theta} \geq \frac{c}{\hat{w}_i}$, which determines the equilibrium. Again, similar to the case of θ, the equilibrium decisions are mainly driven by the margin levels of the supply chain partners, the diminishing returns nature of the investment cost function, and the free-rider issue in the investment stage.

According to our analysis, as c increases, the equilibrium decisions change in the same pattern as they do with respect to θ. The buyer first starts increasing the cost-sharing rates of both suppliers in the Case 1 region; after a point, she fully subsidizes supplier 2 but starts decreasing her cost-sharing rate with supplier 1. The buyer practices this strategy in order to prevent supplier 2 from becoming the free-rider. However, as c increases over $\bar{c}_1$, the equilibrium switches to Case 2 where only supplier 1 stands in the game. In the Case 2 region, the buyer can increase her support of supplier 1 and starts to fully subsidize him. But when c gets beyond $\bar{c}_2$, the market incentives become unrewarding for supplier 1 and there is no investment in the market. Total quality decreases with c throughout the zones of Case 1 and Case 2 (except a small jump in transition to Case 2), and it stays at zero in the Case 4 region. As an example, the change in the cost-sharing decisions and the quality investment decisions for Scenario-3 is provided below in Figure 4.7.

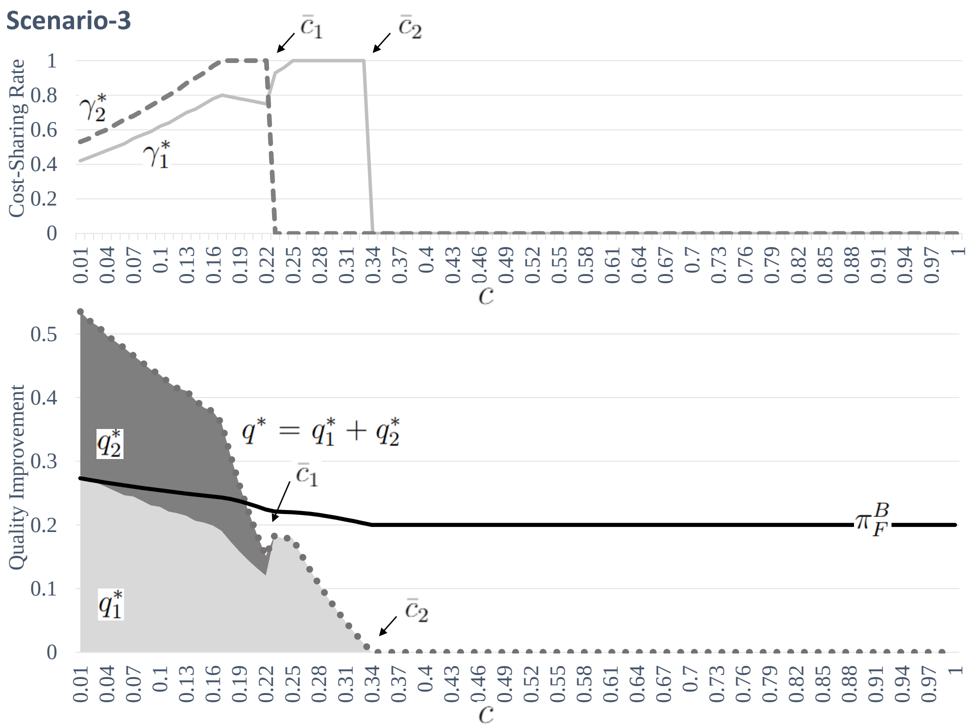

Figure 4.7: Equilibrium Cost-Sharing and Quality Investment Decisions under Scenario-3: $\bar{c}_1^4 = 0.23$, $\bar{c}_2^4 = 0.34$

4.1.4 Suppliers' investment cost factor to build quality (y)

In this part, we focus on the suppliers' investment cost factor to build quality; i.e., y. As y increases, the willingness to invest in quality decreases. Hence, we expect a drop in the equilibrium quality as y increases.

We analyze the equilibria for different y values between 0.1 and 5 by 0.1 increments for nine scenarios. We observe two different equilibrium patterns: In most of the scenarios, the equilibrium is Case 2 at first, and it transitions to Case 1 as y increases. In some of the scenarios, the equilibrium is Case 1 for all y values. In order to structure our discussion, we provide the equilibrium cases and applicable transition thresholds; i.e., $\bar{y}$, for each scenario below in Table 4.7.

Table 4.7: Equilibrium Cases and Thresholds ($\bar{y}$) in Each Scenario

Scenarios	$\hat{p}$	$\hat{w}_1$	$\hat{w}_2$	Equilibrium	$\bar{y}$
1	0.37	0.33	0.3	Case 1	-
2	0.46	0.28	0.26	Case 2 -> Case 1	2.2
3	0.4	0.34	0.26	Case 2 -> Case 1	2.1
4	0.33	0.37	0.3	Case 2 -> Case 1	0.3
5	0.28	0.46	0.26	Case 2 -> Case 1	1.9
6	0.34	0.4	0.26	Case 2 -> Case 1	2.0
7	0.3	0.37	0.33	Case 1	-
8	0.26	0.46	0.28	Case 2 -> Case 1	0.6
9	0.26	0.4	0.34	Case 1	-

We observe that different equilibrium patterns are mainly shaped by the market power of the partners, especially by the low-margin supplier, supplier 2. When supplier 2 is not powerful enough, he has a tendency to behave as a free-rider. Then, the buyer prefers supporting only supplier 1 for lower y values; hence, the equilibrium is Case 2 at first, and as y increases, it transitions to Case 1. The buyer changes her cost-sharing strategy from Case 2 to Case 1 because sharing the convex investment cost of a single supplier becomes more costly compared to subsidizing two suppliers with moderate investments. In that case, the buyer decreases γ_1 and fully subsidizes supplier 2 at first, but as y increases, she adjusts the cost-sharing rates according to the margin split. When supplier 2 is powerful against supplier 1, she does not behave as a free-rider at all. In that case, the buyer cost-shares with both suppliers; i.e., the equilibrium is Case 1 for all y values, and the buyer keeps cost-sharing with both suppliers although she decreases her support as y increases.

When we focus on the scenarios where a transition is observed, we see that as $\hat{w}_2$ increases, $\bar{y}$ decreases. As supplier 2 becomes stronger, the buyer becomes more inclined to motivate supplier 2 to invest and avoid the high cost of subsidizing supplier 1.

When we analyze the quality investment decisions, we see that q_1^* is always in a decreasing trend as y increases. That decreasing trend is observed because both supplier 1 and the buyer becomes less willing to invest in q_1. On the other hand, q_2^* has an increasing trend at first in all of the scenarios due to the buyer's high cost-sharing rate. However, as y increases, the buyer may drop supplier 2's subsidy rate like she did for supplier 1; in that case, q_2^* decreases. Eventually, total quality is in a decreasing trend as expected because of the increased cost factor. The evolution of equilibrium for Scenario-4 is presented below in Figure 4.8.

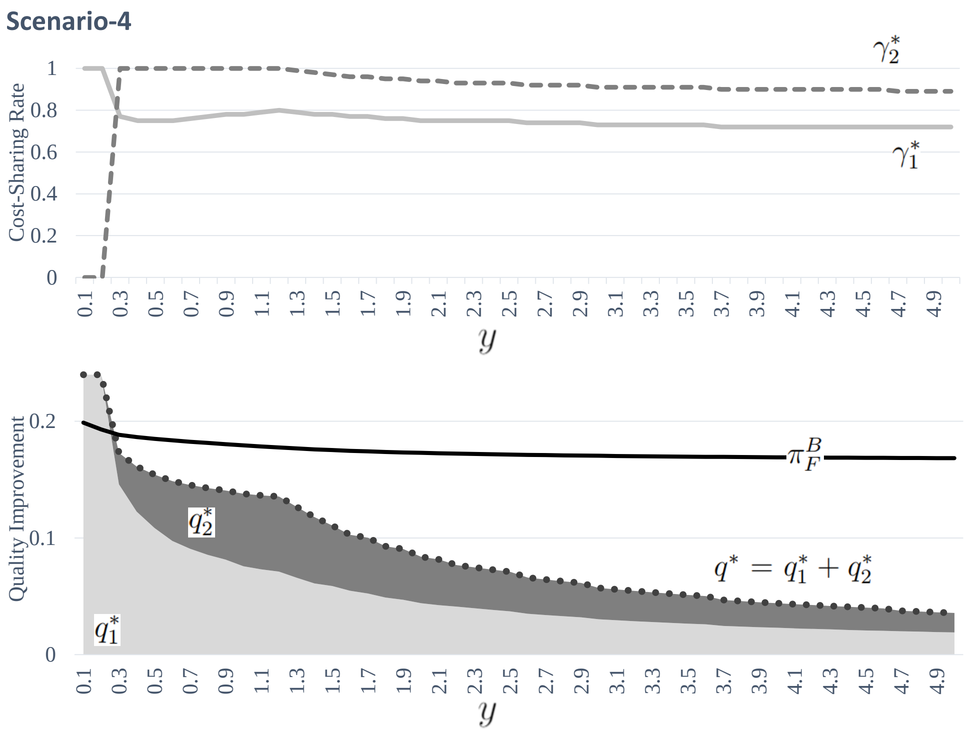

Figure 4.8: Equilibrium Cost-Sharing and Quality Investment Decisions under Scenario-4: $\bar{y}^4 = 0.3$

According to our analysis, we see that as the suppliers' investment cost factor to build quality, y, increases, the supply chain partners become less willing to invest in quality since the investment cost factor has a negative effect on the profit of all partners. Even when there is an imbalance in terms of margin between suppliers, the buyer may prefer to cost-share with both to avoid the high cost of subsidizing only the strong supplier.

4.2 DELEGATION MODEL NUMERICAL ANALYSIS

In this subsection, we analyze the same margin split scenarios we studied in Section 4.1. However, since here the suppliers do not have symmetric positions as they do under full control, we may observe different outcomes when $\hat{w}_2 > \hat{w}_1$. Therefore, in total, we analyze 18 scenarios that include 9 scenarios above and the 9 other scenarios that are symmetric to them in terms of $\hat{w}_1$ and $\hat{w}_2$. In order to simplify our discussion, we will call the initial 9 scenarios as Set-1 and the other group as Set-2.

4.2.1 Consumers' quality awareness effect on demand (d)

As also observed under the full control model, an increase in quality awareness (d) stimulates the suppliers' willingness to invest in quality. We expect a similar equilibrium behavior under the delegation model as well. As the market awareness of environmental quality increases, the chain should move from zero-investment towards investment by both suppliers.

Next, we focus on the equilibrium case transitions and the cost-sharing decisions as d increases. For Set-1, we observe two different equilibrium patterns. In both of the scenarios, the equilibrium starts with Case 4 at low d values and then turns to Case 2 at the break-even point of $d = \frac{c\theta}{\hat{w}_1}$. The first pattern is Case 4, Case 2 and Case 1 equilibrium sequence as d increases. This pattern is observed for three scenarios where $\hat{w}_1 - \hat{w}_2$ is the highest: scenarios 5, 6 and 8. Here, supplier 1 is the strongest partner in the supply chain; and therefore, when d increases to a higher level, supplier 1 starts investing in his own quality. With the transition to Case 2, the buyer starts fully supporting supplier 1 at the beginning but then she pulls back as d increases. As d increases and γ_1^* decreases, although supplier 2 becomes willing to invest under high subsidy rates, supplier 1 keeps investing in his own quality instead of trying to incentivize supplier 2. As d increases further, supplier 1 becomes inclined to change his cost-sharing strategy to also support supplier 2; i.e., Case 1. However, such a transition would cause q_1^* to drop and the buyer prefers to delay this transition by increasing γ_1^*. Eventually, the equilibrium switches to Case 1, the buyer keeps supporting supplier 1 in a decreasing trend as d increases. Meanwhile, supplier 1 fully subsidizes supplier 2 throughout Case 1 since supplier 2's margin is low. An example of Scenario-8 is provided below in Figure 4.9.

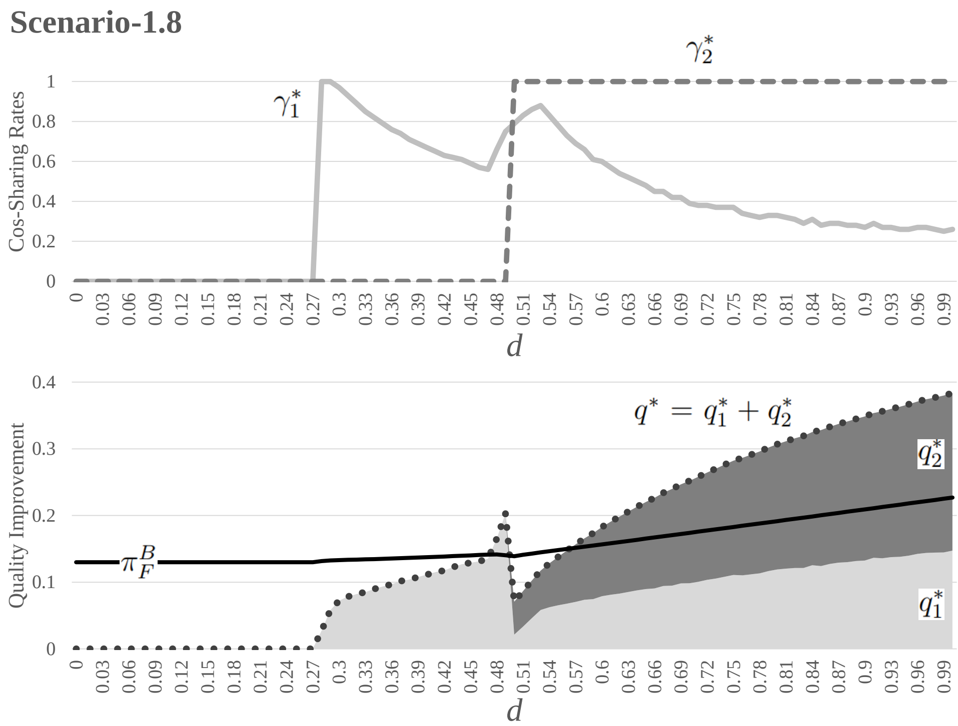

Figure 4.9: Equilibrium Cost-Sharing and Quality Investment Decisions under Set-1 Scenario-8

We observe the second pattern as Case 4, Case 2, Case 3 and Case 1 in scenarios 1, 2, 3, 4, 7 and 9, where the suppliers have comparable margins. Since $\hat{w}_1 > \hat{w}_2$ for these scenarios, supplier 1 is the first one to consider investment among the two suppliers. However, as d increases, supplier 1 prefers to shift the responsibility of investment over to supplier 2 and to support him with cost-sharing. This way, supplier 1 manages to avoid the reduction in his unit margin that would occur if he invested. As the market quality awareness, d, increases further, the equilibrium switches to Case 1. In all of these cases, we observe that the buyer highly subsidizes supplier 1 at the Case 2 region. When d is high enough, the equilibrium switches to Case 1. In that region, both the buyer and supplier 1 start to fully subsidize their upstream partner at first. As d increases further, the partners arrange cost sharing decisions according to the margin distribution between partners, in general in a decreasing trend. An example of Scenario-3 is provided below in Figure 4.10.

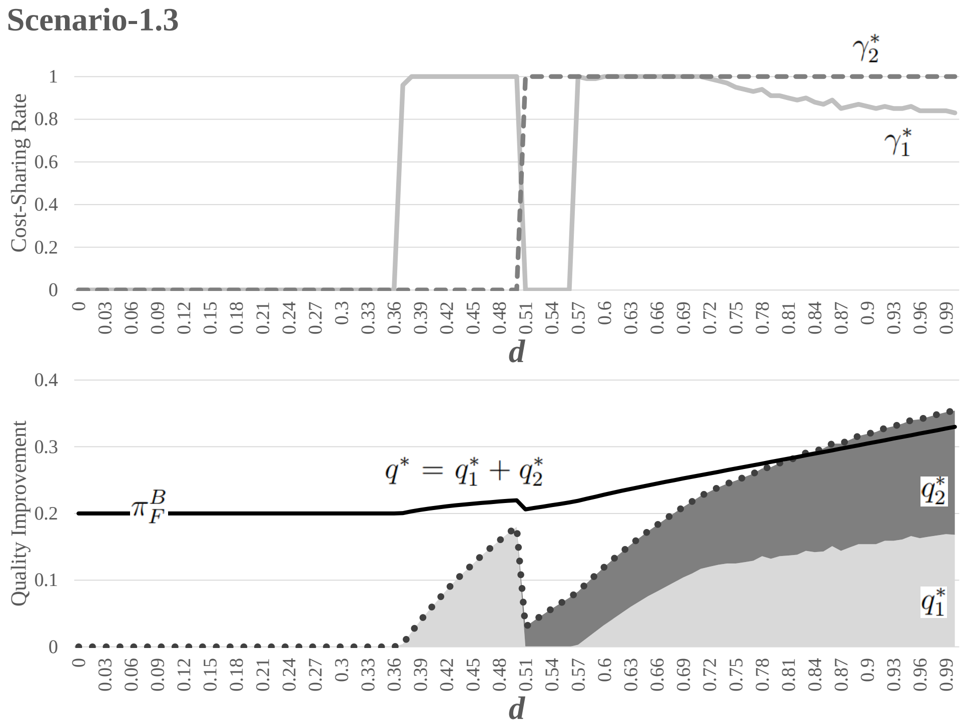

Figure 4.10: Equilibrium Cost-Sharing and Quality Investment Decisions under Set-1 Scenario-3

Because of the "gaming" done by supplier 1, the total quality investment does not necessarily increase with the market awareness d. When supplier 1's margin is significantly higher than supplier 2's, during the transition from Case 2 to Case 1, q^* first dramatically increases and then sharply decreases. Second, when the suppliers' margins are comparable, at the transition from Case 2 to Case 3, q^* drops suddenly but then continues increasing.

Throughout Set-1, the case transitions are related to the suppliers' market power. As the market awareness of quality increases, the high-margin supplier 1 starts investing by himself and then the chain transitions to both suppliers investing for a higher d. However, when supplier 1 does not have a significant margin advantage over supplier 2, he prefers to delegate the investment to supplier 2 at medium d values in order to avoid a margin reduction himself. As the margins get closer, the tendency of supplier 1 to push the investment up the chain increases. Thus, we observe that the high-margin supplier is not necessarily the one that does the investment under delegation, in contradiction to what happens under full control.

For Set-2, we observe the same equilibrium pattern for all 9 scenarios: Case 4, Case 3 and Case 1. Since $\hat{w}_2 > \hat{w}_1$, supplier 2 is already the designated supplier for investment. When d reaches the break-even point of $d = \frac{c\theta}{\hat{w}_2}$, supplier 2 starts to invest. At that point, supplier 1 starts to fully support supplier 2. Depending on the margin difference; i.e., $\hat{w}_1 - \hat{w}_2$, supplier 1 keeps fully subsidizing supplier 2 but eventually decreases γ_2 as d increases. Similar to the full control model, the low margin supplier 1, starts investing in his own quality at a higher d value than where he would invest if he were the sole supplier in the chain. When the equilibrium turns to Case 1, the buyer highly subsidizes supplier 1 as long as his own margin permits. Similar to the Set-1 scenarios, the case transitions are related to the suppliers' margins in the Set-2 group. As $\hat{w}_2$ increases, the equilibrium turns to Case 3 at lower d values; and as $\hat{w}_1$ increases, the equilibrium turns to Case 1 at more moderate d values (an example of Scenario-2.2 is provided in Figure 4.11).

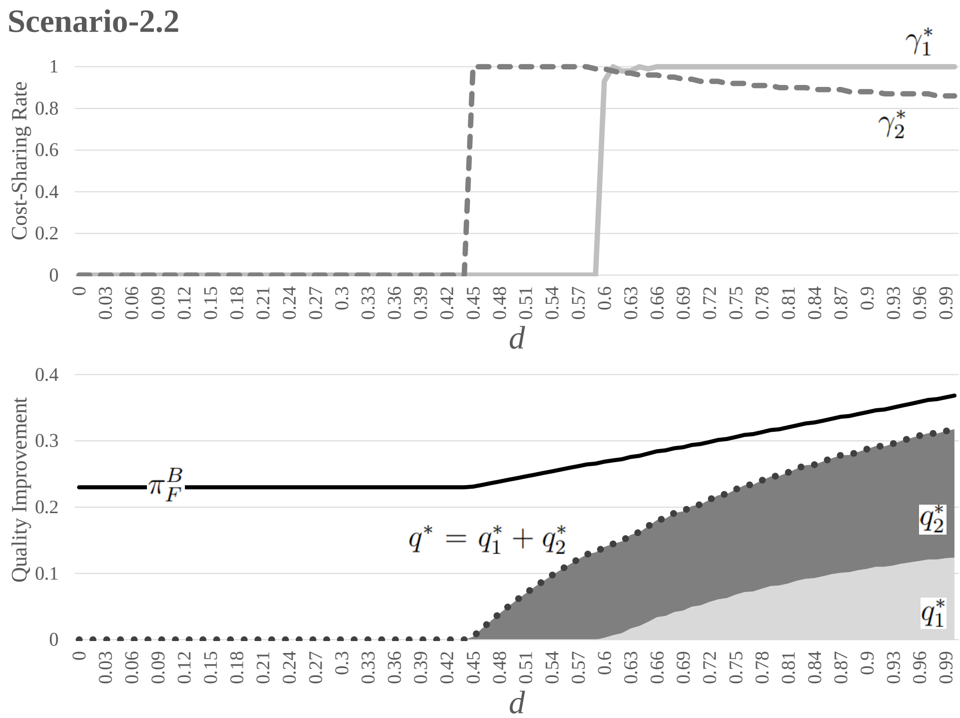

Figure 4.11: Equilibrium Cost-Sharing and Quality Investment Decisions under Set-2 Scenario-2

In Set-2, we always see a monotonic increase in q^* with respect to d. In the Case 3 region only an increase in q_2 is observed and then both q_1 and q_2 increase in the Case 1 area. Therefore, the total quality, q^*, gradually increases as d increases.

4.2.2 Market potential (θ)

As we also observe under the full control, the effect of the market potential on the equilibrium has a similar effect to that of d, but in the reverse direction. The equilibrium decisions are mainly driven by the trade-off between $\frac{d}{\theta}$ and $\frac{c}{\hat{w}_i}$, where d and θ have opposite effects on the trade-off. As the base market potential, θ, increases, the motivation to invest in quality decreases throughout the supply chain.

In Set-1, for scenarios 5, 6 and 8 we observe Case 1, Case 2 and eventually Case 4 in equilibrium. In rest of the scenarios, we observe the equilibrium transitions from Case 1 to Case 3 to Case 2 and then to Case 4 respectively. Under all scenarios, as θ increases, willingness-to-invest decreases, and hence triggers the above Case transitions. As an example, the change in the cost-sharing decisions and the quality investment decisions for Scenario-1 is provided in Figure 4.12.

In the Case 1 region, the buyer increases her support of supplier 1, while supplier 1 increases his support of supplier 2 to compensate for the diminishing willingness to invest in quality as θ increases. Supplier 1 starts to fully support supplier 2; i.e., $\gamma_2^* = 1$, in the Case 1 region as θ increases. With that move, the buyer drops her cost-sharing offer; i.e., γ_1, since with supplier 1's full subsidy supplier 2's improvement on q_2 sustains the total quality at a sufficiently high level. As θ increases further in the Case 1 region, the buyer has to increase back her cost-sharing with supplier 1. However, as θ increases, supplier 1 prefers not to invest at all and pushes this responsibility to his upstream partner. Hence, the equilibrium shifts to Case 3. As θ increases further, since supplier 2 is the low-margin supplier among the two, he stops investing at a point where supplier 1 still has incentives and thus has to take action and invest again. Thus, the equilibrium shifts to Case 2. Eventually, θ reaches a point where neither supplier is motivated to invest and the equilibrium shifts to Case 4.

When we analyze the Set-2 scenarios, the equilibrium sequence we observe is Case 1, Case 3 and Case 4. The reasoning behind these transitions mirror the behavior

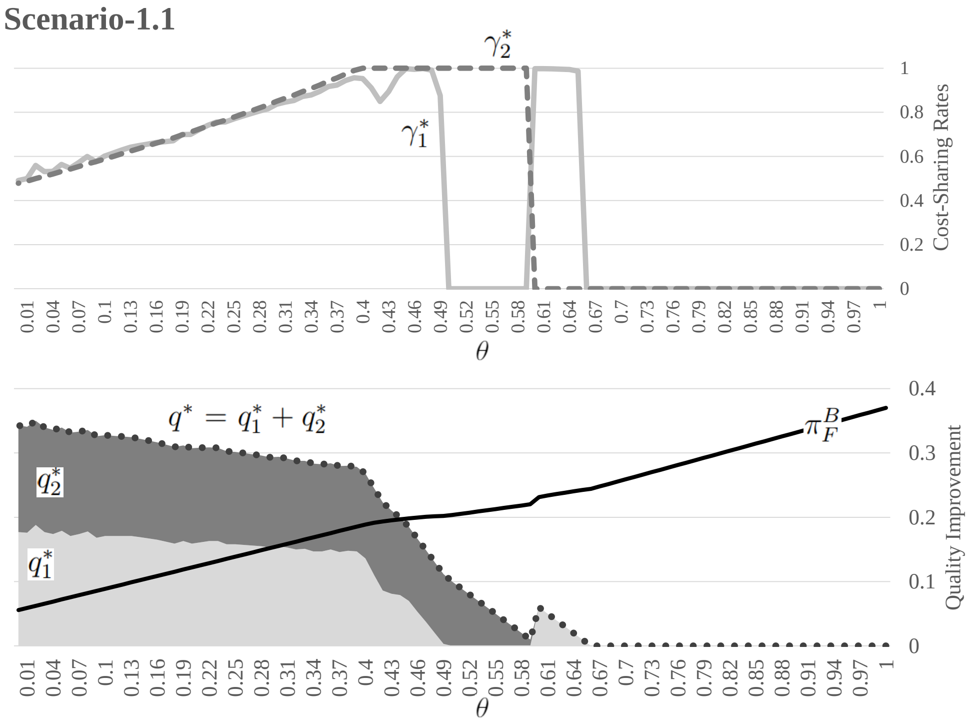

Figure 4.12: Equilibrium Cost-Sharing and Quality Investment Decisions under Set-1 Scenario-1

we observed with respect to d in the reverse direction. Hence we skip the detailed discussion for brevity.

4.2.3 Quality-driven unit cost (c)

Similar to the full control model, the effect of the quality-driven unit cost, c, on the equilibrium is similar to the effect of the market potential, θ. Both affect the investment trade-off for a supplier; i.e., $\frac{d}{\theta}$ vs. $\frac{c}{\hat{w}_i}$, in the same direction.

As c increases, the equilibrium investment levels tend to decrease in the same pattern as they do with respect to θ. In Set-1, we observe two different patterns similar to the ones encountered in the case of the market potential θ. As expected, both q_1 and q_2 have decreasing trends in general. Similarly, in Set-2, with the increase in c, we observe the same behaviour that is seen with a change in θ for all of the scenarios. An example of the change in the cost-sharing decisions and the quality investment decisions for Scenario-1 is provided in Figure 4.13.

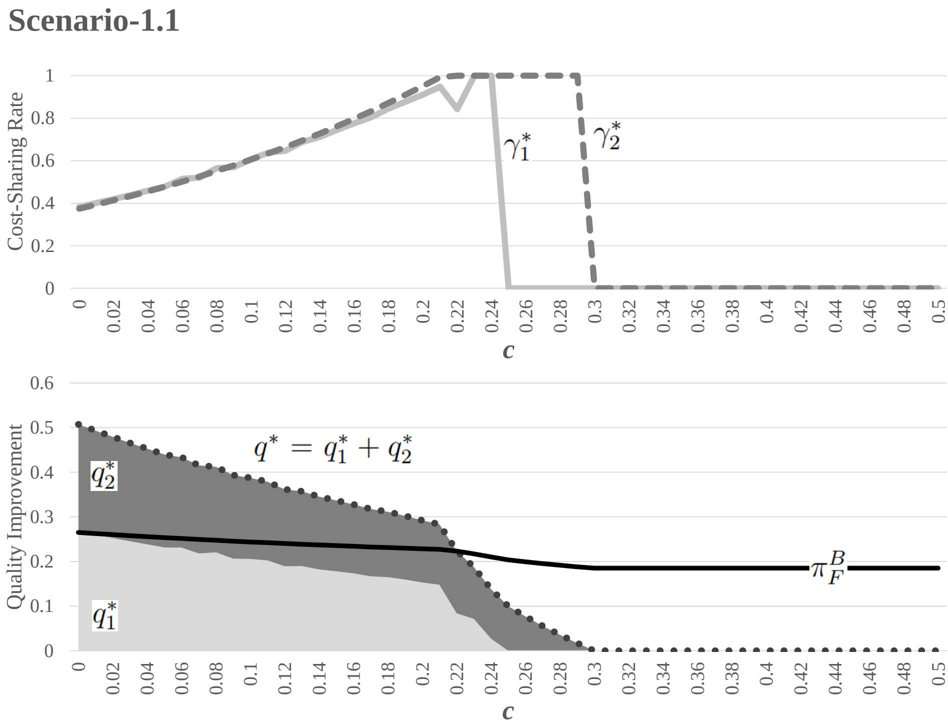

Figure 4.13: Equilibrium Cost-Sharing and Quality Investment Decisions under Set-1 Scenario-1

4.2.4 Suppliers' investment cost factor to build quality (y)

As also observed under the full control model, with an increase in the cost of investment y, the willingness to invest in quality decreases throughout the supply chain. Moreover, the increase in y also affects the severity of diminishing returns with the quality investment, which may entice both suppliers to invest and avoid a significant cost burden on one party. Hence, we expect both suppliers to invest but in a decreasing trend as y increases in general.

We analyze the equilibria for different y values between 0 and 5 by increments of 0.1 for eighteen scenarios. We observe three different equilibrium patterns that are mainly shaped by the market power difference between the suppliers for Set-1 and one common pattern for Set-2.

When we focus on the Set-1 scenarios, we observe that when supplier 1 is substantially strong; i.e., in scenarios 5, 6 and 8, the equilibrium is Case 2 at low y values and it turns to Case 1 when y increases. As supplier 1's margin becomes comparable to supplier 2's as in scenario 3, the equilibrium becomes Case 2, Case 3 and Case 1 as y increases; and for the rest of the scenarios, the equilibrium pattern is Case 3 to Case 1 as y increases. In each scenario, we observe that neither supplier is fond of investing by himself as y increases; hence, the equilibrium turns to Case 1 eventually for high values of y.

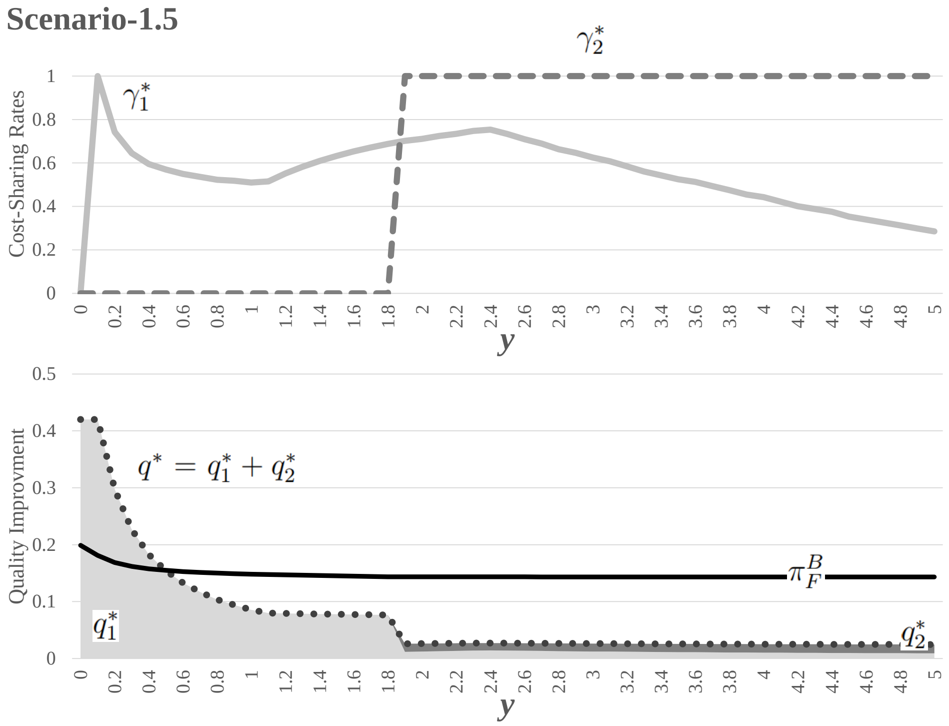

Figure 4.14: Equilibrium Cost-Sharing and Quality Investment Decisions under Set-1 Scenario-5

In the Case 2 region, the buyer cost-shares with supplier 1 in order to cause an improvement in q_1 but generally in a decreasing trend because of the high cost of investment. As y increases, supplier 1's willingness to invest decreases and the buyer has to increase her support on supplier 1. As y increases further, faced with a high y, supplier 1 prefers a split lump-sum investment to avoid the steep cost to achieve a high quality level only by himself. Therefore, he encourages supplier 2 to invest in q_2 by fully subsidizing him as well. With the case transition to Case 1, q_1 instantaneously drops while some improvement is observed in q_2. After supplier 1 changes his strategy, the buyer slightly increases her cost-sharing rate for a while in order to motivate supplier 1 to improve q_1. However, with an increase in y, the buyer decreases her subsidy rate and q_1 slightly decreases. We observe that supplier 1's cost-sharing offer is driven by supplier 2's bargaining power in the chain; i.e., if supplier 2 is the weakest in the supply chain, supplier 1 fully subsidizes him. Otherwise, his subsidy rate eventually decreases as y increases. Total quality is in a decreasing trend in general. An example of the cost-sharing and quality improvement decisions for Scenario-5 are provided in Figure 4.14.

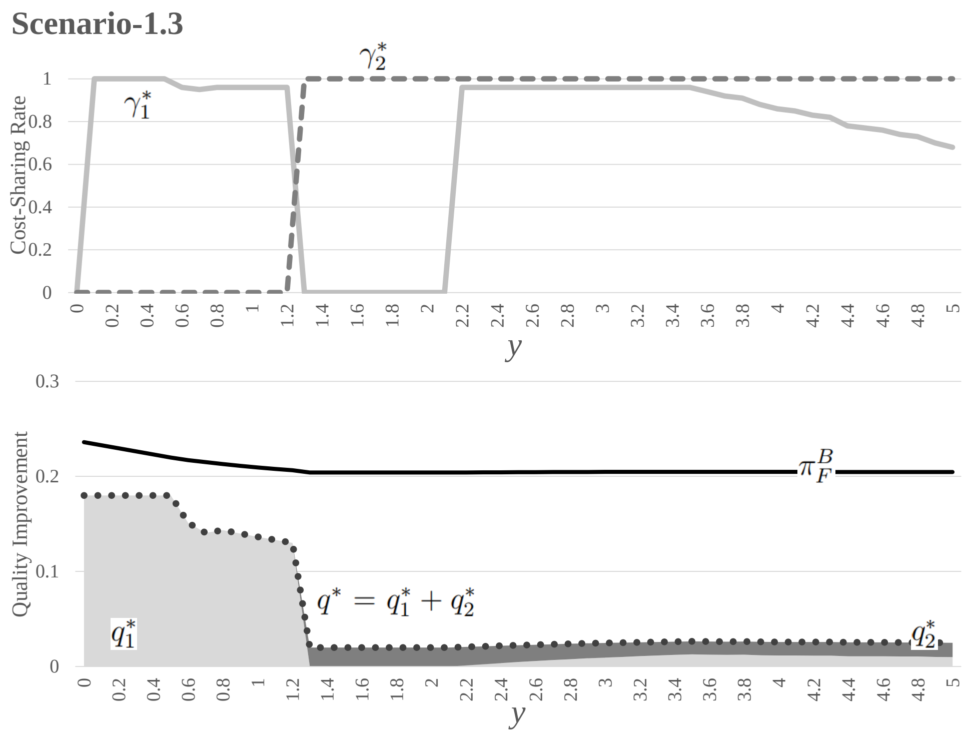

Figure 4.15: Equilibrium Cost-Sharing and Quality Investment Decisions under Set-1 Scenario-3

In Scenario-1.3, the supplier margin difference is the fourth highest among the Set-1 scenarios after the first category explained above. Therefore, we consider this scenario as an intermediary case. Here, the equilibrium is Case 2 at low y values since an improvement in q_1 is more rewarding when supplier 1 is stronger than supplier 2. With that motivation, the buyer starts highly subsidizing supplier 1 but in a decreasing trend. As y increases, supplier 1 wants to unload the total burden of investment, and we observe the transition to Case 3. Here, supplier 1 prefers to contribute to supplier 2's investment in q_2 with a full-subsidy offer. As y increases further, since the cost of subsidizing upstream increases, the motivation to invest in quality is too weak for both suppliers, the buyer participates in cost-sharing again. Here, the buyer is willing to fully subsidize supplier 1 in order to maximize her profit. However, with a high support of supplier 1 and consequently high q_1, supplier 2 is inclined to behave as a free-rider. Therefore, the buyer prefers cost-sharing at a rate[7] that enables supplier 1 to subsidize supplier 2 where both parties invest; i.e., the equilibrium is Case 1. As y increases, the buyer decreases her cost-sharing rate while supplier 1 continues to fully subsidize supplier 2. In this scenario, the total quality improvement generally has a decreasing trend in line with the cost-sharing decisions as shown in Figure 4.15.

In the third equilibrium pattern, we observe the same behaviour explained above for Scenario-1.3 with the exception of the Case 2 part. Therefore, we do not go into detail once more.

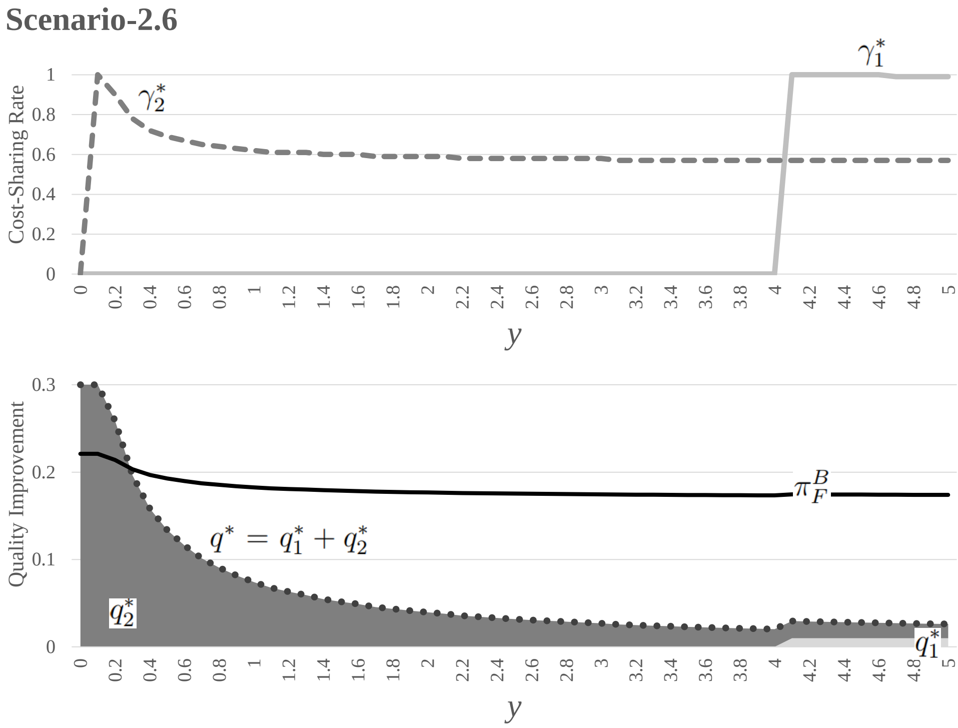

Figure 4.16: Equilibrium Cost-Sharing and Quality Investment Decisions under Set-2 Scenario-6

In Set-2, for all of the scenarios, we observe the same pattern: Case 3 to Case 1. Since supplier 2 is the one with the high-margin, he is the designated supplier for investment. In this situation, supplier 1 offers cost-sharing to supplier 2 at a rate that is driven by their relative leverage (i.e., margins) in the chain. When supplier 1 has a comparable margin, his subsidy rate increases to 1. As the difference $\hat{w}_2 - \hat{w}_1$ in-

[7] That is less than $\bar{\gamma}_1 = 1 + \frac{cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta)}{2y(d\hat{w}_2 - c\theta)}$ which is introduced in Full Control section.

creases, supplier 1 drops his subsidy at a faster rate as y increases. In the Case 1 region, the buyer's cost-sharing offer is contingent on supplier 1's margin; i.e., $\hat{w}_1$. When supplier 1 is the weakest partner in the supply chain, the buyer fully subsidizes supplier 1. In general, the total quality improvement has a decreasing trend except the slight increase observed during the transition to Case 1. An example of the cost-sharing and quality improvement decisions for Scenario-6 are provided in Figure 4.16.

A general observation for y is that as y increases q^* drops substantially but never hits zero; we also observed this in the full control model. This is because, y does not affect the market incentives of the supplier(s); i.e., $\frac{d}{\theta}$ versus $\frac{c}{\hat{w}_i}$, but only the investment level since y is in the denominator in our theoretical findings for q^*.

4.3 COMPARISON

In this section, we focus on the buyer's preferences between the full control and the delegation models. We also determine when her preferences coincide (or not coincide) with the environmentally-preferable model; i.e., the one that produces the higher quality.

In our analyses, we study 10,044 cases. We restrict our parameters to values between 0 and 1, and set the total supply chain margin equal to 1.00 with $p = 1.00$, and $m = 0$. We use the following parameter set: $K = 1.00$, $d \in \{0.3, 0.5, 0.7, 0.9\}$, $c \in \{0.2, 0.25, 0.3\}$, $y \in \{0.3, 0.5, 0.7\}$, $\theta = 0.5$ with $\hat{w}_1 \in [0.02 : 0.02 : 0.70]$, $\hat{w}_2 \in [0.02 : 0.02 : 0.70]$, and $\hat{p} \in [0.28 : 0.02 : 0.44]$.

In our experiments, some cases produce identical outcomes in terms of the buyer's decision and overall quality investment in both models. For the observations that result in equilibrium in Case 2 under both models, we observe the same outcome. Here, we get identical outcomes since only supplier 1 invests in quality, and the buyer is the party to support him. Similarly, the observations that result in equilibrium in Case 4 in both models has the same outcome. In this case, neither supplier invests in quality, which produces the same equilibrium in both models. Therefore, we exclude such observations[8] from our comparison in order to obtain a clearer understanding about the buyer's preference.

For the remaining observations, we see that 63% (i.e., 3573 observations) of the time, the full control model is preferred over the delegation model in terms of the buyer's economic concerns. However, when we look at the total quality produced in both models, we see that in 93% (i.e., 5252 observations) of the cases, the full control model produces higher quality than the delegation model. Hence, we conclude that the high-quality alternative is not necessarily preferred by the buyer.

Table 4.8: Equilibrium Cases Under Each Model

F / D	Case 1	Case 2	Case 3	Case 4	Total
Case 1	1672	34	311	-	2017
Case 2	21	3535	49	-	3605
Case 3	2	-	3601	9	3612
Case 4	-	-	-	810	810
Total	1695	3569	3961	819	10044

We analyze the full control and delegation models in terms of produced equilibrium cases in Table 4.8. Here, the first column of the table shows the equilibrium cases under the full control model and the first row represents the equilibrium cases under the delegation model. We see that the equilibrium cases overlap in both models most of the time. However, there are some differences. For example, under the full control model we observe 2017 Case 1 equilibrium observations. 1672 of these observations produce the Case 1 equilibrium under the delegation model too; however, 34 of the cases get equilibrium at Case 2. When we go into detail, we see that in these cases supplier 2 is very weak compared to supplier 1; therefore, supplier 1 does not prefer cost-sharing with him. Moreover, we observe 311 cases get equilibrium at Case 1 under the full control model, and get equilibrium at Case 3 under the delegation model. In those cases supplier 2 is more powerful than supplier 1 on average. Hence, under the delegation model, supplier 1 prefers not to invest and instead shifts the investment responsibility to supplier 2, and hence producing Case 3 equilibrium.

In Table 4.9, we present the equilibrium cases with respect to the preferred model. In the table, F represents the observations when the full control model is preferred while D represents when the delegation model is preferred observations, and I repre-

[8] In total, 4368 items are excluded.

Table 4.9: Equilibrium Cases wrt Preferred Model

Case Model	Nb of Cases	%	$\hat{w}_1$	$\hat{w}_2$	$\hat{p}$	γ_1^*	γ_2^*	q_1^*	q_2^*	q^*
Case 1	1853	18%	0.3321	0.3222	0.3457	0.7847	0.8051	0.1414	0.1675	0.3089
F	698	38%	0.3316	0.3115	0.3569	0.7208	0.7959	0.1548	0.1249	0.2796
D	1155	62%	0.3324	0.3287	0.3390	0.8233	0.8106	0.1334	0.1933	0.3267
Case 2	3605	36%	0.5148	0.1286	0.3567	0.6036	0.0000	0.2364	0.0000	0.2364
F	70	2%	0.3566	0.2723	0.3711	0.9591	0.0000	0.1983	0.0000	0.1983
I	3535	98%	0.5179	0.1257	0.3564	0.5966	0.0000	0.2372	0.0000	0.2372
Case 3	3776	38%	0.1350	0.5090	0.3560	0.0000	0.5848	0.0000	0.2283	0.2283
F	2805	74%	0.1002	0.5384	0.3614	0.0000	0.5449	0.0000	0.2597	0.2597
D	948	25%	0.2386	0.4211	0.3403	0.0000	0.7069	0.0000	0.1358	0.1358
I	23	1%	0.1113	0.5409	0.3478	0.0000	0.4126	0.0000	0.2098	0.2098
Case 4	810	8%	0.3146	0.3128	0.3726	0.0000	0.0000	0.0000	0.0000	0.0000
I	810	100%	0.3146	0.3128	0.3726	0.0000	0.0000	0.0000	0.0000	0.0000
Total	10044	100%	0.3222	0.3222	0.3557	0.3614	0.3684	0.1110	0.1167	0.2277

Case Model	$\hat{w}_1$	$\hat{w}_2$	$\hat{p}$	d	c	y	θ	$\frac{d}{\theta}$	$\frac{c}{\hat{w}_1}$	$\frac{c}{\hat{w}_2}$
Case 1	0.3321	0.3222	0.3457	0.7604	0.2326	0.5281	0.5000	1.5209	0.7345	0.7594
F	0.3316	0.3115	0.3569	0.7785	0.2384	0.4920	0.5000	1.5570	0.7787	0.8322
D	0.3324	0.3287	0.3390	0.7495	0.2290	0.5499	0.5000	1.4990	0.7079	0.7154
Case 2	0.5148	0.1286	0.3567	0.5902	0.2507	0.4942	0.5000	1.1804	0.5025	3.3617
F	0.3566	0.2723	0.3711	0.6086	0.2707	0.4257	0.5000	1.2171	0.7644	1.0141
I	0.5179	0.1257	0.3564	0.5898	0.2503	0.4956	0.5000	1.1797	0.4973	3.4082
Case 3	0.1350	0.5090	0.3560	0.5943	0.2516	0.4917	0.5000	1.1887	3.2567	0.5122
F	0.1002	0.5384	0.3614	0.6192	0.2489	0.4981	0.5000	1.2384	3.9561	0.4722
D	0.2386	0.4211	0.3403	0.5192	0.2595	0.4724	0.5000	1.0384	1.1262	0.6297
I	0.1113	0.5409	0.3478	0.6565	0.2500	0.5087	0.5000	1.3130	5.7794	0.5546
Case 4	0.3146	0.3128	0.3726	0.3030	0.2794	0.5000	0.5000	0.6059	1.0203	1.0328
I	0.3146	0.3128	0.3726	0.3030	0.2794	0.5000	0.5000	0.6059	1.0203	1.0328
Total	0.3222	0.3222	0.3557	0.6000	0.2500	0.5000	0.5000	1.2000	1.6225	1.6225

sents the indifferent/identical observations. We review each case separately in order to understand the buyer's motivation to select a model. In Case 1, when both suppliers invest in quality, we see that the delegation model is preferred in 62% of the cases (i.e., 1155 observations). The delegation model is preferred when the margin of the supply chain partners are closer compared to those that favor the full control model. Hence, when the suppliers' leverage is close to the buyer's, she prefers a delegation model since she can get both suppliers to invest but share the costs of only one. Moreover, we observe the delegation model produces a higher q_2 on average, and the full control model produces lower γ_1 and γ_2 values despite the higher buyer margin. In Case 2, when only supplier 1 invests in quality, we see that mostly (i.e., 3535 of the observations which is 98% of all Case 2), the equilibrium is identical for both models since in each model the buyer cost-shares with supplier 1. As mentioned before, supplier 1 captures a large portion of the market share on average for the Case 2 equilibrium. In the few observations when full control model is preferred, the equilibrium is either Case 1 or Case 3 under the delegation model. In fact, for these cases, the buyer is the highest-margin party in the chain, followed by supplier 1. Under delegation, supplier 1 wants to avoid investment or share the load with supplier 2 but the full control model pushes the higher margin supplier 1 to invest, so the buyer is better off.

In Case 3, when only Supplier 2 invests in quality, we see that in majority of the cases (i.e., 2805 of the observations which is 74% of all Case 3), the preferred model is full control. Here, supplier 2 holds most of the market power and when his margin is considerably higher than supplier 1, the buyer prefers full control in order to effectively incentivize supplier 2 herself. We observe that delegation is preferred when the average margin difference of the suppliers is lower. Hence, we deduce that the more powerful supplier 2 gets in the market, the more the buyer becomes inclined to prefer the full control model. Finally, whenever quality awareness in the market is low, neither supplier invests (i.e., Case 4) since there is no demand opportunity. Thus, both models produce the same equilibrium in this case.

In general, we observe that the average quality awareness in the market, d, is higher in cases that favor full control. Consequently, we may state that the buyer is more willing to subsidize both suppliers herself when the market opportunity is higher.

Table 4.10: Preferred Model wrt Equilibrium Cases

Model Case	Nb of Cases	%	$\hat{w}_1$	$\hat{w}_2$	$\hat{p}$	γ_1^*	γ_2^*	q_1^*	q_2^*	q^*
F	3573	36%	0.1504	0.4888	0.3607	0.1596	0.5833	0.0341	0.2282	0.2624
Case 1	698	20%	0.3316	0.3115	0.3569	0.2471	0.7959	0.1548	0.1249	0.2796
Case 2	70	2%	0.3566	0.2723	0.3711	0.9591	0.0000	0.1983	0.0000	0.1983
Case 3	2805	79%	0.1002	0.5384	0.3614	0.0000	0.5449	0.0000	0.2597	0.2597
D	2103	21%	0.2901	0.3703	0.3396	0.4521	0.7638	0.0733	0.1674	0.2406
Case 1	1155	55%	0.3324	0.3287	0.3390	0.8233	0.8106	0.1334	0.1933	0.3267
Case 3	948	45%	0.2386	0.4211	0.3403	0.0000	0.7069	0.0000	0.1358	0.1358
I	4368	43%	0.4781	0.1626	0.3593	0.4828	0.0022	0.1920	0.0011	0.1931
Case 2	3535	81%	0.5179	0.1257	0.3564	0.5966	0.0000	0.2372	0.0000	0.2372
Case 3	23	1%	0.1113	0.5409	0.3478	0.0000	0.4126	0.0000	0.2098	0.2098
Case 4	810	19%	0.3146	0.3128	0.3726	0.0000	0.0000	0.0000	0.0000	0.0000
Total	10044	100%	0.3222	0.3222	0.3557	0.3614	0.3684	0.1110	0.1167	0.2277

In Table 4.10, we analyze the buyer's preferred model with respect to the equilibrium cases. We see that in the full control preferred cases, supplier 2 holds the highest margin in the chain on the average. In fact, the average $\hat{w}_2$ is 0.49, while $\hat{w}_1$ is 0.15 and $\hat{p}$ is 0.36. Moreover, 79% of the cases (i.e., 2805 observations) produce equilibrium in Case 3 and the average $\hat{w}_2$ is 0.54 for these cases. We realize that in the full-control-preferred cases, the average $\hat{p}$ is higher than in delegation. Therefore, we conclude that when the buyer and supplier 2 capture most of the supply chain margin, the buyer is inclined towards the full control model. In this situation, the buyer herself steps in and cost-shares with a motivated supplier 2 instead of relying on supplier 1 to support his upstream partner.

In cases that favor delegation, we observe that the average margins of the supply chain partners are closer to each other; i.e., the average values are $\hat{w}_1 = 0.29$, $\hat{w}_2 = 0.37$, and $\hat{p} = 0.34$. We see that 55% of the cases (i.e., 1155 observations) produce equilibrium at Case 1. Here, supplier 1 captures a higher portion of the supply chain margin; and, the buyer margin is lower compared to the full-control-preferred cases. Hence, we conclude that when supplier 1's margin is higher than or close to the buyer's margin, the buyer prefers to avoid supporting both suppliers and to instead push the responsibility of supporting supplier 2 to supplier 1.

Table 4.11: Margin Leader and Preferred Model Summary

Leader	**Nb of Cases**	**%**	**F**		**D**		**I**	
BM	2052	20%	573	28%	686	33%	793	39%
S1M	3960	39%	289	7%	331	8%	3340	84%
S2M	3960	39%	2698	68%	1039	26%	223	6%
EQ	72	1%	13	18%	47	65%	12	17%
Total	10044	100%	3573	36%	2103	21%	4368	43%

In Table 4.11, we present the most powerful partner in the market. In the table, BM represents the cases where the buyer is the margin leader, $S1M$ ($S2M$) represents the cases supplier 1 (supplier 2) is the margin leader and EQ represents the cases both suppliers have equal margin which is higher than buyer margin. In the cases where the buyer is the most powerful partner, the full control model is preferred when supplier 1 is distinctively less powerful than supplier 2; on the other hand, the delegation model is preferred when the suppliers' margins are comparable. When supplier 1 is the margin leader, in most of the observations we see that both models (i.e., 84% of the cases) produce identical outcomes. Conversely, when supplier 2 is the most powerful partner in the chain, 68% of the cases (i.e., 2698 observations) favor the full control model as the buyer prefers supporting a powerful tier 2 supplier herself.

In order to understand the buyer's preference when she is the least powerful partner in the chain, we also check all comparable observations. When she is the weakest party, we see that the buyer mostly prefers the delegation model, since her margin is less than (at least one of) the suppliers. Consequently, she prefers the delegation model as the suppliers already have power to invest in their quality. [9]

Table 4.12: Full Control Preferred Cases vs. Produced Quality

Pref. Model	Nb of Cases	$\hat{w}_1$	$\hat{w}_2$	$\hat{p}$	π^{B*}	γ_1^*	γ_2^*	q_1^*	q_2^*	q^*
F	3573	0.1504	0.4888	0.3607	0.2329	0.1596	0.5833	0.0341	0.2282	0.2624
F-Q	3539	0.1474	0.4916	0.3609	0.2329	0.1593	0.5793	0.0323	0.2302	0.2625
D-Q	34	0.4606	0.1988	0.3406	0.2377	0.1897	1.0000	0.2195	0.0266	0.2461
*D-Q**					*0.2304*	*0.7068*	*0.0000*	*0.3927*	*0.0000*	*0.3927*

Table 4.13: Delegation Preferred Cases vs. Produced Quality

Pref. Model	Nb of Cases	$\hat{w}_1$	$\hat{w}_2$	$\hat{p}$	π^{B*}	γ_1^*	γ_2^*	q_1^*	q_2^*	q^*
D	2103	0.2901	0.3703	0.3396	0.2245	0.4521	0.7638	0.0733	0.1674	0.2406
F-Q	1713	0.2869	0.3705	0.3427	0.2301	0.5012	0.7304	0.0768	0.1745	0.2513
*F-Q**					*0.2237*	*0.5703*	*0.8035*	*0.1103*	*0.1818*	*0.2921*
D-Q	207	0.3539	0.3488	0.2973	0.2088	0.4462	0.8451	0.1088	0.1982	0.3069
I	183	0.2485	0.3931	0.3584	0.1895	0.0000	0.9852	0.0000	0.0659	0.0659

In Table 4.12 and Table 4.13, we present the observations comparing the preferred model and the produced quality. In the tables, *F-Q* represents the cases which produces higher quality under the full control model while *D-Q* represents the cases that produces higher quality under the delegation model. Moreover, *D-Q** (*F-Q**) represents the average figures that would arise if the delegation (full control) model were used instead of the full control (delegation) model in the cases that it dominated. As seen from Table 4.12, 34 observations produce higher quality under the delegation model although the buyer prefers a full control model. In these cases, if the delegation model was preferred, supplier 2 would become a free-rider and the buyer would highly cost-share with supplier 1. In that situation, subsidizing supplier 1 under delegation becomes more costly than subsidizing both suppliers under full control since in the latter, the suppliers' quality improvements are moderate resulting in a smaller

[9] The buyer is the weakest partner in 518 observations, in 448 of which delegation model is preferred.

burden for the buyer. Hence, the buyer prefers the full control model even if that choice produces a lower overall quality than that of in delegation.

We see in Table 4.13 that for delegation-preferred cases, most of the time (i.e., 1713 observations that is 81% of delegation preferred cases) the full control model produces a higher overall quality. In these observations, we see that the buyer is never the margin leader, either supplier 1 or supplier 2 has a margin advantage over the buyer. Therefore, the buyer prefers to assign supplier 1 to undertake the responsibility of cost-sharing with the tier 2 supplier.

In summary, we find that a downstream buyer may utilize both full control and delegation models in order to improve the environmental quality of his upstream suppliers. We see that the full control model is more favorable than the delegation model in general. Especially when the buyer and supplier 2 hold most of the supply chain margin, full control is more preferable since the buyer is willing to support supplier 2 by cost-sharing herself instead of relying on supplier 1. Moreover, the buyer is more willing to subsidize both suppliers herself when the market opportunity is higher. On the other hand, delegation is more preferable when the supply chain partners' powers are closer and/or the buyer is the weakest party in the chain. In that situation, since both suppliers have the power and motivation to improve quality, the buyer interfaces with only supplier 1 and pushes the responsibility of supporting supplier 2 to supplier 1. Consequently, we find that the buyer may prefer the less environmentally-sound model due to her economic concerns.

CHAPTER 5

CONCLUSION

We study a multi-tier supply chain in order to characterize the strategies of a brand owner company in managing the environmental quality of its products. When the environmental quality of a product is higher, the demand for the product increases and all partners benefit from the demand increase. However, the suppliers incur the cost of quality improvement. The buyer can only encourage the suppliers by sharing their environmental quality improvement costs. There are two alternative models that the buyer can implement: (i) full control, and (ii) delegation. In the former, she interfaces with each tier herself, whereas in the latter she only cost-shares with tier 1 and delegates the responsibility of tier 2 over to tier 1.

In our study, in order to emphasize the nature of a multi-tier supply chain, we first characterize the equilibrium of a two-tier supply chain. We then define the best responses of the partners in a three-tier supply chain under the full control model and try to figure out partners' behavior under delegation model. Moreover, we conduct numerical study in order to further understand the equilibrium under each model with respect to changing market parameters. Finally, we compare the two models and summarize our findings about the buyer's behavior in terms of market dynamics.

According to our analysis, in a dyadic supply chain the supplier's investment decision directly depends on the relation between the relative market awareness of quality and the unit cost of quality investment. Whenever the relative market awareness of quality exceeds the unit cost of quality investment, the supplier invests in quality without the buyer's support. However, in a multi-tier supply chain, the suppliers' relative market powers also play a critical role in their investment decisions. In fact, for the high-margin supplier the investment decision is driven by the same dynamics as in a two-tier supply chain. On the other hand, the low-margin supplier is tempted to avoid investment and free-ride on the high-margin supplier's investment when the high-margin supplier is highly supported by the buyer. In order to avoid that situation, the buyer sometimes lowers her cost-sharing rate with the high-margin supplier although she is willing to be more generous. Hence, we conclude that in a multi-tier supply chain a buyer may face inefficiency due to a free-rider phenomenon.

Under a full control model, we observe both suppliers invest in quality when their market powers are comparable and the relative market awareness of quality is high enough. When a supplier is distinctively more powerful than the other, then only the high-margin supplier invests in quality and the buyer only cost-shares with the high-margin supplier. On the other hand, when both suppliers invest in quality, the buyer shares costs with both suppliers while sharing a higher rate of the low-margin supplier's costs. Here, the buyer highly subsidizes the low-margin supplier since his investment cost is moderate. Moreover, the high-margin supplier already is motivated to invest in his quality. It is important to state that the low-margin supplier can be influenced by the buyer's cost-sharing effort only if his unit cost of quality improvement is below the relative market awareness. Otherwise, he does not invest in quality no matter what the buyer offers.

In our numerical study, we analyze the effect of market characteristics on the equilibrium behavior. We observe that in both models, quality awareness effect on demand (i.e., d) affects the suppliers' quality investments positively while market potential (i.e., θ), quality-driven unit cost (i.e., c), and lump-sum investment cost (i.e., y) have discouraging effects on the suppliers' quality investments. Even if the market awareness of quality has a positive influence on quality investment in general, it may cause a decrease in the total quality investment due to the free-rider phenomenon observed among the suppliers. Another point to mention is that under the delegation model, when the tier 1 supplier is the high-margin supplier and the difference between two suppliers is not distinctive, supplier 1 may prefer pushing the quality improvement responsibility to the tier 2 supplier. Hence, we see the high-margin supplier is not necessarily the one that undertakes the investment under delegation, in contradiction to what happens under full control.

When the two cost-sharing models are compared, we find that the buyer mostly prefers a full control model when she and the tier 2 supplier holds most of the market power. On the other hand, when the buyer's power is lower and the supply chain partners have comparable margins, the buyer prefers delegation model, since the suppliers are already motivated to invest. Also, it is seen that although a full control model produces a higher overall quality in general, the buyer may choose adopting a delegation model from an economic point of view. Conversely, in a few select cases, the buyer may prefer full control over delegation even if delegation produces a higher overall quality.

In our research, we model a full information environment. We assume all parties in the supply chain have the information of the other's cost and price levels. In reality, a buyer may not be able to determine the cost information of her upstream partners, especially in a multi-tier supply chain. Therefore, information asymmetry can be studied for further investigation. Alternatively, the end product price can be modeled as variable according to the produced overall quality level. Moreover, we assume the overall quality is the summation of tier 1 and tier 2 suppliers' quality. In future research, each supplier's contribution to the overall quality can be weighted according to their influence on the quality of the end product.

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APPENDIX A

PROOF OF LEMMA 1

In a two-tier supply chain the profit functions of the buyer and the supplier will be $\pi_B = (\theta + dq)\hat{p} - \gamma y q^2$ and $\pi_S = (\theta + dq)(\hat{w} - cq) - (1-\gamma) y q^2$.

We will use backward induction to solve this problem. In the second stage of the problem, we find the supplier's best response q as follows: $\frac{d\pi_S}{dq} = -c\theta - 2cdq + d\hat{w} - 2qy(1-\gamma)$ and $\frac{d^2\pi_S}{dq^2} = -2cd - 2y(1-\gamma) < 0$. Therefore, $q^*(\gamma) = \frac{(d\hat{w}-c\theta)^+}{2(cd+y(1-\gamma))}$.

In the first stage, we use the best response of the supplier $q^*(\gamma)$ from above and we find the following buyer's profit function:

$$\pi_B = \begin{cases} \hat{p}\theta + \frac{d\hat{p}(d\hat{w}-c\theta)}{2(cd+y(1-\gamma))} - \frac{y\gamma(d\hat{w}-c\theta)^2}{4(cd+y(1-\gamma))^2} & \text{if } \frac{d}{\theta} > \frac{c}{\hat{w}}; \\ \hat{p}\theta, & \text{otherwise.} \end{cases}$$

When we focus on the case where $q^*(\gamma) > 0$ (i.e., $\frac{d}{\theta} > \frac{c}{\hat{w}}$), we have the following first order derivation of buyer profit function:

$$\frac{d\pi_B}{d\gamma} = \frac{y(d\hat{w}-c\theta)}{4(cd+y(1-\gamma)^3)}[c^2 d\theta + cy\theta(1+\gamma) + cd^2(2\hat{p}-\hat{w}) + 2dy\hat{p}(1-\gamma) - dy\hat{w} - dy\gamma\hat{w}]$$

The expression on left-hand side is nonnegative since $\frac{d}{\theta} > \frac{c}{\hat{w}}$ and $0 \leq \gamma \leq 1$. When we rearrange the right-hand side, we have the following expression:

$$c^2 d\theta + cy\theta + 2cd^2\hat{p} + 2d\hat{p}y - cd^2\hat{w} - d\hat{w}y + \gamma y[c\theta - d\hat{w} - 2d\hat{p}]$$

Here, $\gamma y[c\theta - d\hat{w} - 2d\hat{w}]$ part is negative since $c\theta < d\hat{w}$. Hence, FOC is decreasing in γ. As a result, the FOC of the buyer's profit function is either positive-positive, positive-negative or negative-negative as in the feasible range $[0, 1]$ of γ. Thus, the buyer's profit function is unimodal in γ. In that situation, the γ value that makes the

FOC function zero is $\gamma_0 = \frac{(cd+y)(2d\hat{p}-d\hat{w}+c\theta)}{y(2d\hat{p}+d\hat{w}-c\theta)}$. If this value turns out to be nonpositive, then the buyer's profit is decreasing in γ; $\forall\gamma \in [0,1]$, and γ_0^* = 0. If this value turns out to be greater than 1, then the buyer's profit is increasing in γ; $\forall\gamma \in [0,1]$, and $\gamma^* = 1$. Note that $\gamma_0 \geq 0$ if and only if $2d\hat{p} - d\hat{w} + c \geq 0$; and $\gamma_0 \leq 1$ if and only if $2cd^2\hat{p} - (cd + 2y)(d\hat{w} - c\theta) \leq 0$. Based on these conditions, we have parts (i), (ii), and (iii) stated in the Lemma. We calculate the equilibrium q^* from the best-response function derived above. When $\frac{d}{\theta} \leq \frac{c}{\hat{w}}$, $q^*(\gamma) = 0$, $\forall\gamma \in [0,1]$. Hence, we have $q^* = 0$ and $\gamma^* \in [0,1]$ since the buyer's profit remains $\theta\hat{p}$, $\forall\gamma \in [0,1]$. ■

APPENDIX B

PROOF OF PROPOSITION 1

Both suppliers maximize their profits. Therefore, to understand the behaviour of the profit functions we check the first and the second order derivatives of the suppliers' profit functions.

$$
\begin{aligned}
\frac{d\pi_{S_1}^F}{dq_1} &= -c\theta - 2cdq_1 - cdq_2 + d\hat{w}_1 - 2yq_1 + 2yq_1\gamma_1 \\
\frac{d^2\pi_{S_1}^F}{dq_1^2} &= -2cd - 2y(1-\gamma_1) < 0 \\
\frac{d\pi_{S_2}^F}{dq_2} &= -c\theta - cdq_1 - 2cdq_2 + d\hat{w}_2 - 2yq_2 + 2yq_2\gamma_2 \\
\frac{d^2\pi_{S_2}^F}{dq_2^2} &= -2cd - 2y(1-\gamma_2) < 0
\end{aligned}
$$

Here, since the second order conditions of suppliers' profit functions are negative, the first order conditions are sufficient to find the optimal quality values. Thus, we find the best responses of supplier 1 and supplier 2 as follows:

$$
(q_1^*(\gamma_1,\gamma_2,q_2), q_2^*(\gamma_1,\gamma_2,q_1)) = \left(\left(\frac{d\hat{w}_1 - cdq_2 - c\theta}{2cd + 2y(1-\gamma_1)}\right)^+, \left(\frac{d\hat{w}_2 - cdq_1 - c\theta}{2cd + 2y(1-\gamma_2)}\right)^+\right) \tag{B.1}
$$

We realize that the suppliers have two alternative course of actions: either investing in quality or not investing. Therefore, when we analyze the combination of two suppliers actions, we figure out four possible cases that may occur:

(i) Case 1: If we assume $q_1^*(\gamma_1,\gamma_2,q_2) > 0$ and $q_2^*(\gamma_1,\gamma_2,q_1) > 0$, we can solve for q_1 and q_2 from the best response expressions in Equation (B.1). Then, we get the following expressions:

$$
q_1 = \frac{2y(1-\gamma_2)(d\hat{w}_1 - c\theta) + cd(2d\hat{w}_1 - d\hat{w}_2 - c\theta)}{3c^2d^2 + 4y^2(1-\gamma_1)(1-\gamma_2) + 4cdy(2-\gamma_1-\gamma_2)}
$$

$$q_2 = \frac{2y(1-\gamma_1)(d\hat{w}_2 - c\theta) + cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta)}{3c^2d^2 + 4y^2(1-\gamma_1)(1-\gamma_2) + 4cdy(2-\gamma_1-\gamma_2)}$$

Here, we see that the denominator of these equations are the same and positive (since $\gamma_{1,2} \in [0,1]$). Therefore, we focus on the numerators to guarantee positivity and find the following expressions:

$$2y(1-\gamma_2)(d\hat{w}_1 - c\theta) + cd(2d\hat{w}_1 - d\hat{w}_2 - c\theta) > 0$$

$$2y(1-\gamma_1)(d\hat{w}_2 - c\theta) + cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta) > 0$$

This will be satisfied if and only if $\frac{2cd^2+2dy(1-\gamma_2)}{d^2\hat{w}_2+cd\theta+2\theta y(1-\gamma_2)} > \frac{c}{\hat{w}_1}$ $(Con1a)$ and $\frac{2cd^2+2dy(1-\gamma_1)}{d^2\hat{w}_1+cd\theta+2\theta y(1-\gamma_1)} > \frac{c}{\hat{w}_2}$ $(Con1b)$.

(ii) Case 2: If we assume $q_1^*(\gamma_1, \gamma_2, q_2) > 0$ and $q_2^*(\gamma_1, \gamma_2, q_1) = 0$, then we get the following expressions:

$$q_1 = \frac{d\hat{w}_1 - c\theta}{2cd + 2y(1-\gamma_1)} > 0$$

$$q_2 = \frac{d\hat{w}_2 - cdq_1 - c\theta}{2cd + 2y(1-\gamma_2)} \leq 0$$

This will be satisfied if and only if $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$ $(Con2a)$ and $\frac{2cd^2+2dy(1-\gamma_1)}{d^2\hat{w}_1+cd\theta+2\theta y(1-\gamma_1)} \leq \frac{c}{\hat{w}_2}$ $(Con2b)$.

(iii) Case 3: If we assume $q_1^*(\gamma_1, \gamma_2, q_2) = 0$ and $q_2^*(\gamma_1, \gamma_2, q_1) > 0$, then we get the following expressions:

$$q_1 = \frac{d\hat{w}_1 - cdq_2 - c\theta}{2cd + 2y(1-\gamma_1)} \leq 0$$

$$q_2 = \frac{d\hat{w}_2 - c\theta}{2cd + 2y(1-\gamma_2)} > 0$$

This will be satisfied if and only if $\frac{d}{\theta} > \frac{c}{\hat{w}_2}$ $(Con3a)$ and $\frac{2cd^2+2dy(1-\gamma_2)}{d^2\hat{w}_2+cd\theta+2\theta y(1-\gamma_2)} \leq \frac{c}{\hat{w}_1}$ $(Con3b)$.

(iv) Case 4: If we assume $q_1^*(\gamma_1, \gamma_2, q_2) = 0$ and $q_2^*(\gamma_1, \gamma_2, q_1) = 0$, then we get the following expressions:

$$q_1 = \frac{d\hat{w}_1 - c\theta}{2cd + 2y(1-\gamma_1)} \leq 0$$

$$q_2 = \frac{d\hat{w}_2 - c\theta}{2cd + 2y(1-\gamma_2)} \leq 0$$

This will be satisfied if and only if $\frac{d}{\theta} \leq \frac{c}{\hat{w}_1}$ $(Con4a)$ and $\frac{d}{\theta} \leq \frac{c}{\hat{w}_2}$ $(Con4b)$. ■

APPENDIX C

PROOF OF LEMMA 2

(i) When we plug-in $d = \frac{c\theta}{\hat{w}_i}$ ($i \in \{1,2\}$) to $\frac{2cd^2+2dy(1-\gamma_i)}{d^2\hat{w}_i+cd\theta+2\theta y(1-\gamma_i)}$, we find the expression $\frac{2c^3\theta^2+2c\theta\hat{w}_i y(1-\gamma_i)}{2c^2\theta^2\hat{w}_i+2\theta\hat{w}_i y(1-\gamma_i)}$, which simplifies into $\frac{c}{\hat{w}_i}$. Therefore, $\frac{d}{\theta} = \frac{c}{\hat{w}_i} = \frac{2cd^2+2dy(1-\gamma_i)}{d^2\hat{w}_i+cd\theta+2\theta y(1-\gamma_i)}$.

(ii) When $\frac{d}{\theta} > \frac{c}{\hat{w}_i}$, by plugging $c\theta$ into the denominator of $d\hat{w}_i$ in $\frac{2cd^2+2dy(1-\gamma_i)}{d^2\hat{w}_i+cd\theta+2\theta y(1-\gamma_i)}$, we get the relationship $\frac{2cd^2+2dy(1-\gamma_i)}{dc\theta+cd\theta+2\theta y(1-\gamma_i)} > \frac{2cd^2+2dy(1-\gamma_i)}{d^2\hat{w}_i+cd\theta+2\theta y(1-\gamma_i)}$ since $d\hat{w}_i > c\theta$. By rearranging the left-hand-side of the inequality, we reach $\frac{2cd^2+2dy(1-\gamma_i)}{dc\theta+cd\theta+2\theta y(1-\gamma_i)} = \frac{d(2cd+2y(1-\gamma_i))}{\theta(2cd+2y(1-\gamma_i))} = \frac{d}{\theta} > \frac{2cd^2+2dy(1-\gamma_i)}{d^2\hat{w}_i+cd\theta+2\theta y(1-\gamma_i)}$.

Similarly, since $d > \frac{c\theta}{\hat{w}_i}$, we can write $\frac{2cd^2+2dy(1-\gamma_i)}{d^2\hat{w}_i+cd\theta+2\theta y(1-\gamma_i)} > \frac{cd^2+\frac{c\theta}{\hat{w}_i}(cd+2y(1-\gamma_i)}{d^2\hat{w}_i+cd\theta+2\theta y(1-\gamma_i)}$. By rearranging, $\frac{cd^2\hat{w}_i+c\theta(cd+2y(1-\gamma_1)}{d^2\hat{w}_i^2+\theta\hat{w}_i(cd+2y(1-\gamma_i))} = \frac{c(d^2\hat{w}_i+\theta(cd+2y(1-\gamma_i)}{\hat{w}_i(d^2\hat{w}_i+\theta(cd+2y(1-\gamma_i))} = \frac{c}{\hat{w}_i}$. Thus, we can conclude $\frac{2cd^2+2dy(1-\gamma_i)}{d^2\hat{w}_i+cd\theta+2\theta y(1-\gamma_i)} > \frac{c}{\hat{w}_i}$.

(iii) When $\frac{d}{\theta} > \frac{c}{\hat{w}_i}$, by the relationship $\frac{2cd^2+2dy(1-\gamma_i)}{d^2\hat{w}_i+cd\theta+2\theta y(1-\gamma_i)} > \frac{2cd^2+2dy(1-\gamma_i)}{dc\theta+cd\theta+2\theta y(1-\gamma_i)}$, we reach $\frac{2cd^2+2dy(1-\gamma_i)}{d^2\hat{w}_i+cd\theta+2\theta y(1-\gamma_i)} > \frac{2cd^2+2dy(1-\gamma_i)}{dc\theta+cd\theta+2\theta y(1-\gamma_i)} = \frac{d(2cd+2y(1-\gamma_i))}{\theta(2cd+2y(1-\gamma_i))} = \frac{d}{\theta}$.

Similarly, $\frac{cd^2+\frac{c\theta}{\hat{w}_i}(cd+2y(1-\gamma_i)}{d^2\hat{w}_i+cd\theta+2\theta y(1-\gamma_i)} > \frac{2cd^2+2dy(1-\gamma_i)}{d^2\hat{w}_i+cd\theta+2\theta y(1-\gamma_i)}$ since $\frac{c\theta}{\hat{w}_i} > d$. By rearranging, $\frac{cd^2\hat{w}_i+c\theta(cd+2y(1-\gamma_i)}{d^2\hat{w}_i^2+\theta\hat{w}_i(cd+2y(1-\gamma_i))} = \frac{c(d^2\hat{w}_i+\theta(cd+2y(1-\gamma_i)}{\hat{w}_i(d^2\hat{w}_i+\theta(cd+2y(1-\gamma_i))} = \frac{c}{\hat{w}_i} > \frac{2cd^2+2dy(1-\gamma_i)}{d^2\hat{w}_i+cd\theta+2\theta y(1-\gamma_i)}$. ■

APPENDIX D

PROOF OF LEMMA 3

From Lemma 2, we saw the relations between $\frac{d}{\theta}$ vs $\frac{c}{\hat{w}_1}$ and $\frac{d}{\theta}$ vs $\frac{c}{\hat{w}_2}$ play a critical role in determining the equilibrium case. Therefore, we try to categorize our analysis in terms of possible relation pairs between these three expressions.

(i) When possible cases under $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$ analyzed, only Case 1, Case 2 or Case 3 can be observed. The requirements for these cases are analyzed below:

I. From Proposition 1(i), two conditions must hold in order to have Case 1, i.e., $q_1^* > 0$ and $q_2^* > 0$:

a) The first condition is $\frac{2cd^2+2dy(1-\gamma_1)}{d^2\hat{w}_1+cd\theta+2\theta y(1-\gamma_1)} > \frac{c}{\hat{w}_2}$. By reordering, we get $\frac{2cd^2\hat{w}_2+2d\hat{w}_2 y(1-\gamma_1)-cd^2\hat{w}_1-c^2d\theta-2cy\theta(1-\gamma_1)}{d^2\hat{w}_1\hat{w}_2+cd\hat{w}_2\theta+2\hat{w}_2 y\theta(1-\gamma_1)} > 0$. Since the denominator is positive, the numerator determines the sign of the expression. When $2cd^2\hat{w}_2 + 2d\hat{w}_2 y(1-\gamma_1) - cd^2\hat{w}_1 - c^2d\theta - 2cy\theta(1-\gamma_1) > 0$ condition holds, then the expression becomes positive. By reordering, we have $cd(d\hat{w}_2 - d\hat{w}_1) + cd(d\hat{w}_2 - c\theta) + 2y(1-\gamma_1)(d\hat{w}_2 - c\theta) > 0$. This expression can be positive or negative depending on the difference between $\hat{w}_1$ and $\hat{w}_2$ under the initial conditions. Therefore, in order to guarantee the first condition to hold, we find the following requirement: $\gamma_1 < 1 + \frac{cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta)}{2y(d\hat{w}_2 - c\theta)}$.

b) The second condition is $\frac{2cd^2+2dy(1-\gamma_2)}{d^2\hat{w}_2+cd\theta+2\theta y(1-\gamma_2)} > \frac{c}{\hat{w}_1}$. This condition brings the second requirement as $\gamma_2 < 1 + \frac{cd(2d\hat{w}_1 - d\hat{w}_2 - c\theta)}{2y(d\hat{w}_1 - c\theta)}$ (the derivation is similar to the derivation of the first condition).

Here, we call the expressions above as $\bar{\gamma}_1 = 1 + \frac{cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta)}{2y(d\hat{w}_2 - c\theta)}$ and $\bar{\gamma}_2 = 1 + \frac{cd(2d\hat{w}_1 - d\hat{w}_2 - c\theta)}{2y(d\hat{w}_1 - c\theta)}$. Since $\gamma_1,\ \gamma_2 \in [0, 1]$, we simplify the conditions we gathered above for Case 1 as $0 \leq \gamma_1 < \bar{\gamma}_1$ and $0 \leq \gamma_2 < \bar{\gamma}_2$.

II. From Proposition 1(ii) we need the following conditions to have Case 2, where $q_1^* > 0$ and $q_2^* = 0$: $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$ and $\frac{2cd^2+2dy(1-\gamma_1)}{d^2\hat{w}_1+cd\theta+2\theta y(1-\gamma_1)} \leq \frac{c}{\hat{w}_2}$. The first condition $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$ trivially holds by assumption in part (i). The second condition is reordered as $\frac{2cd^2\hat{w}_2+2d\hat{w}_2 y(1-\gamma_1)-cd^2\hat{w}_1-c^2d\theta-2cy\theta(1-\gamma_1)}{d^2\hat{w}_1\hat{w}_2+cd\hat{w}_2\theta+2\hat{w}_2 y\theta(1-\gamma_1)} \leq 0$. In a similar manner to that in part (i-I-a), the numerator can be rearranged into $\gamma_1 \geq 1+\frac{cd(2d\hat{w}_2-d\hat{w}_1-c\theta)}{2y(d\hat{w}_2-c\theta)}$. By recalling $\bar{\gamma}_1 = 1+\frac{cd(2d\hat{w}_2-d\hat{w}_1-c\theta)}{2y(d\hat{w}_2-c\theta)}$, the condition for Case 2 is simplified as $\bar{\gamma}_1 \leq \gamma_1 \leq 1$.

III. Similarly, from Proposition 1(iii) we need the following conditions to have Case 3, where $q_1^* = 0$ and $q_2^* > 0$: $\frac{2cd^2+2dy(1-\gamma_1)}{d^2\hat{w}_1+cd\theta+2\theta y(1-\gamma_1)} \geq \frac{c}{\hat{w}_2}$ and $\frac{2cd^2+2dy(1-\gamma_2)}{d^2\hat{w}_2+cd\theta+2\theta y(1-\gamma_2)} < \frac{c}{\hat{w}_1}$, and we reach the following condition similar to part II: $\gamma_2 \leq 1+\frac{cd(2d\hat{w}_1-d\hat{w}_2-c\theta)}{2y(d\hat{w}_1-c\theta)}$. Hence, we reach the simplified condition for Case 3 as $\bar{\gamma}_2 \leq \gamma_2 \leq 1$.

(ii) When $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$ and $\frac{d}{\theta} \leq \frac{c}{\hat{w}_2}$, we may have only Case 2 for all $\gamma_1,\ \gamma_2 \in [0,1]$. From Lemma 2(ii), $\frac{d}{\theta} \geq \frac{2cd^2+2dy(1-\gamma_1)}{d^2\hat{w}_1+cd\theta+2\theta y(1-\gamma_1)} \geq \frac{c}{\hat{w}_1}$. Since $\frac{d}{\theta} \leq \frac{c}{\hat{w}_2}$, we have $\frac{c}{\hat{w}_2} \geq \frac{2cd^2+2dy(1-\gamma_1)}{d^2\hat{w}_1+cd\theta+2\theta y(1-\gamma_1)}$, which is $Con2b$ introduced in Proposition 1(ii). The first condition ($\frac{d}{\theta} > \frac{c}{\hat{w}_1}$) is already the second requirement, i.e., $Con2a$, for Case 2. Therefore, we conclude under these conditions, only Case 2 may occur. Thus $q_1^*(\gamma_1,\gamma_2) = \frac{d\hat{w}_1-c\theta}{2cd+2y(1-\gamma_1)},\ \ q_2^*(\gamma_1,\gamma_2) = 0,\ \forall\gamma_1,\ \gamma_2 \in [0,1]$.

(iii) When $\frac{d}{\theta} \leq \frac{c}{\hat{w}_1}$ and $\frac{d}{\theta} > \frac{c}{\hat{w}_2}$, only Case 3 may occur. Thus, $q_1^*(\gamma_1,\gamma_2) = 0,\ \ q_2^*(\gamma_1,\gamma_2) = \frac{d\hat{w}_2-c\theta}{2cd+2y(1-\gamma_2)},\ \ \forall\gamma_1,\ \gamma_2 \in [0,1]$. The derivation is similar to part (ii).

(iv) When $\frac{d}{\theta} \leq \frac{c}{\hat{w}_1}$ and $\frac{d}{\theta} \leq \frac{c}{\hat{w}_2}$, this means only Case 4 may happen. Thus, $q_1^* = 0$ and $q_2^* = 0,\ \forall\gamma_1, \gamma_2 \in [0,1]$. ■

APPENDIX E

PROOF OF LEMMA 4

We defined the thresholds as $\bar{\gamma}_1 = 1 + \frac{cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta)}{2y(d\hat{w}_2 - c\theta)}$ and $\bar{\gamma}_2 = 1 + \frac{cd(2d\hat{w}_1 - d\hat{w}_2 - c\theta)}{2y(d\hat{w}_1 - c\theta)}$. Since we set $\gamma_1 \in [0,1]$ and $\gamma_2 \in [0,1]$, we want to figure out the conditions that characterize $\bar{\gamma}_1$ and $\bar{\gamma}_2$ with respect to the feasible spaces for γ_i, $i = 1, 2$.

(i) For $\bar{\gamma}_1 = 1 + \frac{cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta)}{2y(d\hat{w}_2 - c\theta)} > 1$, the condition $\frac{cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta)}{2y(d\hat{w}_2 - c\theta)} > 0$ must hold. Since cd, y, and $d\hat{w}_2 - c\theta$ parts are positive, the condition will be satisfied when $2d\hat{w}_2 - d\hat{w}_1 - c\theta > 0$, that can be reordered as $\hat{w}_1 < \frac{2d\hat{w}_2 - c\theta}{d}$. Similarly, for $\bar{\gamma}_2 > 1$, the condition $2d\hat{w}_1 - d\hat{w}_2 - c\theta > 0$ must hold and that means $\hat{w}_2 < \frac{2d\hat{w}_1 - c\theta}{d}$. Under these conditions, the situation described in Lemma 3 (i)-I emerges, and therefore, Case 1 occurs $\forall \gamma_1, \gamma_2 \in [0,1]$.

(ii) For $0 < \bar{\gamma}_1 = 1 + \frac{cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta)}{2y(d\hat{w}_2 - c\theta)} \leq 1$, we analyze two conditions separately. First, $1 + \frac{cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta)}{2y(d\hat{w}_2 - c\theta)} > 0$ condition requires $cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta) + 2y(d\hat{w}_2 - c\theta) > 0$, which can be reordered as $\hat{w}_1 < \frac{2y(d\hat{w}_2 - c\theta) + cd(2d\hat{w}_2 - c\theta)}{cd^2}$. Second, $1 + \frac{cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta)}{2y(d\hat{w}_2 - c\theta)} \leq 1$ condition requires $2d\hat{w}_2 - d\hat{w}_1 - c\theta \leq 0$, which simplifies to $\hat{w}_1 \geq \frac{2d\hat{w}_2 - c\theta}{d}$.

As we mentioned before, $\bar{\gamma}_i$ is a transition point as long as $\bar{\gamma}_i \in [0,1]$. Here, since we have $\bar{\gamma}_1 \in [0,1]$ we will specify two cases: for $0 \leq \gamma_1 < \bar{\gamma}_1$ and $0 \leq \gamma_2 \leq 1$ we observe Case 1 and for $\bar{\gamma}_1 \leq \gamma_1 \leq 1$ and $0 \leq \gamma_2 \leq 1$ we observe Case 2.

Here, we also check $\bar{\gamma}_2$. $\bar{\gamma}_2 \leq 1$ condition requires $2d\hat{w}_1 - d\hat{w}_2 - c\theta \leq 0$, which implies $\hat{w}_2 \leq \frac{2d\hat{w}_1 - c\theta}{d}$. Remember we reached $\hat{w}_2 \leq \frac{d\hat{w}_1 + c\theta}{2d}$ condition from $\bar{\gamma}_1 \leq 1$ requirement. When we combine two conditions on $\hat{w}_2$, we have $\frac{2d\hat{w}_1 - c\theta}{d} < \hat{w}_2 < \frac{d\hat{w}_1 + c\theta}{2d}$. But we see that requires $3d(d\hat{w}_1 - c\theta) < 0$ which contradicts with our initial assumption $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$. Therefore, we conclude that, when $\bar{\gamma}_1 \leq 1$, then $\bar{\gamma}_2 > 1$. Thus, when $\bar{\gamma}_1 \leq 1$, we will only observe a transition between Case 1 and Case 2.

(iii) For $\bar{\gamma}_1 \leq 0$, the condition $1 + \frac{cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta)}{2y(d\hat{w}_2 - c\theta)} \leq 0$ condition must hold. Then the numerator part $cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta) + 2y(d\hat{w}_2 - c\theta)$ must be nonpositive since the denominator is already positive. When the numerator is reordered, we have the following requirement: $\hat{w}_1 \geq \frac{2y(d\hat{w}_2 - c\theta) + cd(2d\hat{w}_2 - c\theta)}{cd^2}$.

By analyzing the condition $cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta) + 2y(d\hat{w}_2 - c\theta) \leq 0$, we get $(cd + 2y)(d\hat{w}_2 - c\theta) + cd^2(\hat{w}_2 - \hat{w}_1) \leq 0$. As we already know $cd + 2y$ and $d\hat{w}_2 - c\theta$ are positive, $\hat{w}_2 - \hat{w}_1$ part must be negative and that means $\hat{w}_1 > \hat{w}_2$. Then the expression $cd(2d\hat{w}_1 - d\hat{w}_2 - c\theta)$ will always be positive, which results in $\bar{\gamma}_2$ to be greater than 1. Thus, when $\hat{w}_1 \geq \frac{2y(d\hat{w}_2 - c\theta) + cd(2d\hat{w}_2 - c\theta)}{cd^2}$, then $\bar{\gamma}_1 \leq 0$ and $\bar{\gamma}_2 > 1$. In that situation, we only observe Case 2, $\forall \gamma_1,\ \gamma_2 \in [0, 1]$.

(iv) For $0 < \bar{\gamma}_2 = 1 + \frac{cd(2d\hat{w}_1 - d\hat{w}_2 - c\theta)}{2y(d\hat{w}_1 - c\theta)} \leq 1$, $1 + \frac{cd(2d\hat{w}_1 - d\hat{w}_2 - c\theta)}{2y(d\hat{w}_1 - c\theta)} > 0$ condition requires $\hat{w}_2 < \frac{2y(d\hat{w}_1 - c\theta) + cd(2d\hat{w}_1 - c\theta)}{cd^2}$. And $\gamma_2 = 1 + \frac{cd(2d\hat{w}_1 - d\hat{w}_2 - c\theta)}{2y(d\hat{w}_1 - c\theta)} \leq 1$ condition requires $2d\hat{w}_1 - d\hat{w}_2 - c\theta \leq 0$ that is rewritten as $\hat{w}_2 \geq \frac{2d\hat{w}_1 - c\theta}{d}$. Under these conditions, the conditions for $\bar{\gamma}_1 \leq 1$ situation contradicts with $\bar{\gamma}_2$ conditions as explained for the converse case in part (ii). Therefore, when $\bar{\gamma}_2 \leq 1$, then $\bar{\gamma}_1 > 1$.

Consequently, we have $\bar{\gamma}_2 \in [0, 1]$ and that will cause two cases to happen: for $0 \leq \gamma_1 \leq 1$ and $0 \leq \gamma_2 < \bar{\gamma}_2$ we observe Case 1 and for $0 \leq \gamma_1 \leq 1$ and $\bar{\gamma}_1 \leq \gamma_1 \leq 1$ we observe Case 3.

(v) For $\bar{\gamma}_2 \leq 0$, $\hat{w}_2 \leq \frac{2y(d\hat{w}_1 - c\theta) + cd(2d\hat{w}_1 - c\theta)}{cd^2}$ condition must hold. The derivation is similar to part (iii). Again similarly, when $\bar{\gamma}_2 \leq 0$, then $\bar{\gamma}_1 > 1$. Therefore, when $\hat{w}_2 \leq \frac{2y(d\hat{w}_1 - c\theta) + cd(2d\hat{w}_1 - c\theta)}{cd^2}$, we will observe only Case 3 $\forall \gamma_1, \gamma_2 \in [0, 1]$.

■

APPENDIX F

PROOF OF PROPOSITION 2

(i) When $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$ and $\frac{d}{\theta} > \frac{c}{\hat{w}_2}$, we structure the following cases as described in Lemma 4:

I. When $\hat{w}_1 < \frac{2d\hat{w}_2 - c\theta}{d}$ and $\hat{w}_2 < \frac{2d\hat{w}_1 - c\theta}{d}$, we cannot characterize the buyer's best response. Thus, we extend our analysis by conducting a numerical study in the next chapter.

II. When $\frac{2d\hat{w}_2 - c\theta}{d} \leq \hat{w}_1 < \frac{2y(d\hat{w}_2 - c\theta) + cd(2d\hat{w}_2 - c\theta)}{cd^2}$, we cannot fully characterize the buyer's best response again. Hence, we analyze that situation in numerical study chapter.

III. When $\hat{w}_1 \geq \frac{2y(d\hat{w}_2 - c\theta) + cd(2d\hat{w}_2 - c\theta)}{cd^2}$, for $\bar{\gamma}_1 \in [0,1]$ and $\bar{\gamma}_2 \in [0,1]$, Case 2 occurs. In this situation, the buyer profit function becomes:

$$\pi_F^B = \theta\hat{p} \, + \, d\hat{p}\left(\frac{d\hat{w}_1 - c\theta}{2cd + 2y(1-\gamma_1)}\right) - \gamma_1 y \left(\frac{d\hat{w}_1 - c\theta}{2cd + 2y(1-\gamma_1)}\right)^2$$

In order to understand the behavior of this function, we analyze the first and second order derivatives:

$$\frac{d\pi_F^B}{d\gamma_1} = \frac{y(d\hat{w}_1 - c\theta)(2d\hat{p}(cd + y(1-\gamma_1)) - (d\hat{w}_1 - c\theta)(y(1+\gamma_1) + cd))}{4(cd + y(1-\gamma_1))^3}$$

$$\frac{d^2\pi_F^B}{d\gamma_1^2} = \frac{y^2(d\hat{w}_1 - c\theta)(2d\hat{p}(cd + y(1-\gamma_1)) - (d\hat{w}_1 - c\theta)(y(1+\gamma_1) + cd))}{2(cd + y(1-\gamma_1))^4}$$

As we can see above, both $\frac{d\pi_F^B}{d\gamma_1}$ and $\frac{d^2\pi_F^B}{d\gamma_1^2}$ depend on γ_1, which means we cannot guarantee concavity of the buyer profit function. Since $d\hat{w}_1 - c\theta \geq 0$, $y > 0$, and $cd + y(1-\gamma_1) > 0$, the sign of $\frac{d\pi_F^B}{d\gamma_1}$ depends on the sign of the following expression:

$$2d\hat{p}(cd + y(1-\gamma_1)) - (d\hat{w}_1 - c\theta)(y(1+\gamma_1) + cd)$$

Here, we see this expression is decreasing in γ_1. Therefore, the expression is either positive-positive, positive-negative or negative-negative. That means π_B^F is a unimodal function of γ_1. Then, we can find the optimal γ_1 as follows:

a) When $\frac{d\pi_F^B}{d\gamma_1}$ is nonpositive for $\gamma_1 = 0$, the buyer profit function will be decreasing and concave; therefore, the maximizing γ_1 value will be 0 in that case. Here, by plugging $\gamma_1^* = 0$ into $q_1^*(\gamma_1, \gamma_2)$ introduced in Lemma 4(iii), we reach the optimal quality level as $q_1^* = \frac{d\hat{w}_1 - c\theta}{2cd+2y}$.

b) When $\frac{d\pi_F^B}{d\gamma_1}$ is positive at $\gamma_1 = 0$ and negative at $\gamma_1 = 1$, the buyer profit function will be increasing at the beginning and then become decreasing after the point of $\gamma_1^0 = \frac{(cd+y)(2d\hat{p} - d\hat{w}_1 + c\theta)}{y(2d\hat{p} + d\hat{w}_1 - c\theta)}$, which makes $\frac{d\pi_F^B}{d\gamma_1} = 0$. In that situation γ_1^0 is the buyer's optimal cost-sharing rate; i.e., γ_1^*. By plugging γ_1^* into quality expression, we have $q_1^* = \frac{2d\hat{p} + d\hat{w}_1 - c\theta}{4(cd+y)}$.

c) When $\frac{d\pi_F^B}{d\gamma_1}$ is positive for $\gamma_1 = 1$, the buyer profit function will be increasing and convex. In that case, $\gamma_1^* = 1$, will be the maximizer in $[0, 1]$. Here, we reach the optimal quality level as $q_1^* = \frac{d\hat{w}_1 - c\theta}{2cd}$.

As it is seen above, optimal γ_1 value changes according to the profit buyer function's behavior. Therefore, we want to synthesize partial optimal γ_1 values as follows: $min\left(max\left(\frac{(cd+y)(2d\hat{p} - d\hat{w}_1 + c\theta)}{y(2d\hat{p} + d\hat{w}_1 - c\theta)}, 0\right), 1\right)$.

Now, let us explain that expression in detail. Consider part a). In order $\frac{d\pi_F^B}{d\gamma_1} \leq 0$ to hold, we need $2d\hat{p}(cd + y(1 - \gamma_1)) - (d\hat{w}_1 - c\theta)(y(1 + \gamma_1) + cd) \leq 0$ for $\gamma_1^* = 0$. That requires $(cd + y)(2d\hat{p} - d\hat{w}_1 + c\theta) \leq 0$ to hold. In that situation, γ_1^0 expression becomes nonpositive. So, we may denote $\gamma_1^* = max\left(\frac{(cd+y)(2d\hat{p} - d\hat{w}_1 + c\theta)}{y(2d\hat{p} + d\hat{w}_1 - c\theta)}, 0\right)$ for that part. When we consider part c), we know that when $\frac{d\pi_F^B}{d\gamma_1}$ is positive for $\gamma_1 = 0$, it will be positive for $\gamma_1 = 1$, too. Hence, we will have $(cd + y)(2d\hat{p} - d\hat{w}_1 + c\theta) > 0$, resulting in $\gamma_1^0 > 0$. Since we define $\gamma_1^* \in [0, 1]$, we will have $min\left(\frac{(cd+y)(2d\hat{p} - d\hat{w}_1 + c\theta)}{y(2d\hat{p} + d\hat{w}_1 - c\theta)}, 1\right)$. Consequently, when we combine these two condition we have $\gamma_1^* = min\left(max\left(\frac{(cd+y)(2d\hat{p} - d\hat{w}_1 + c\theta)}{y(2d\hat{p} + d\hat{w}_1 - c\theta)}, 0\right), 1\right)$.

Finally, $\gamma_2^* \in [0, 1]$, as $q_2^* = 0$ and π_B^F is independent of q_2 and γ_2 in that case.

IV. When $\frac{2d\hat{w}_1 - c\theta}{d} \leq \hat{w}_2 < \frac{2y(d\hat{w}_1 - c\theta) + cd(2d\hat{w}_1 - c\theta)}{cd^2}$, we cannot fully characterize the buyer's best response again. Hence, we analyze that situation in numerical study chapter.

V. When $\hat{w}_2 \geq \frac{2y(d\hat{w}_1 - c\theta) + cd(2d\hat{w}_1 - c\theta)}{cd^2}$, for $\gamma_1 \in [0, 1]$ and $\gamma_2 \in [0, 1]$, Case 3 occurs and the buyer profit function becomes:

$$\pi_F^B = \theta\hat{p} \; + \; d\hat{p}\left(\frac{d\hat{w}_2 - c\theta}{2cd + 2y(1 - \gamma_2)}\right) - \gamma_2 y\left(\frac{d\hat{w}_2 - c\theta}{2cd + 2y(1 - \gamma_2)}\right)^2$$

The rest follows similarly to the proof of part III.

(ii) By Lemma 3 (ii), when $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$ and $\frac{d}{\theta} \leq \frac{c}{\hat{w}_2}$, Case 2 is the equilibrium; i.e., $q_1^*(\gamma_1, \gamma_2) > 0$ and $q_2^*(\gamma_1, \gamma_2) = 0$. The buyer's subgame perfect equilibrium becomes same as in part (i)-III.

(iii) By Lemma 3 (iii), when $\frac{d}{\theta} \leq \frac{c}{\hat{w}_1}$ and $\frac{d}{\theta} > \frac{c}{\hat{w}_2}$, $q_1^*(\gamma_1, \gamma_2) = 0$ and $q_2^*(\gamma_1, \gamma_2) > 0$, and the subgame perfect equilibrium becomes same as the part (i)-V.

(iv) Under $\frac{d}{\theta} \leq \frac{c}{\hat{w}_1}$ and $\frac{d}{\theta} \leq \frac{c}{\hat{w}_2}$ conditions, $q_1^*(\gamma_1, \gamma_2) = 0$, $q_2^*(\gamma_1, \gamma_2) = 0$ from Lemma 3 (iv). Therefore, the buyer profit function becomes $\pi_F^B = \theta\hat{p}$. In this case, buyer profit becomes independent of γ_1 and γ_2. Hence, $\gamma_1^* \in [0, 1]$ and $\gamma_2^* \in [0, 1]$. ■

APPENDIX G

PROOF OF LEMMA 5

Since the buyer profit function is a piecewise function in Proposition 2 items (i)-ii. and (i)-iv., we check the continuity at the transition points. For this reason, we analyze the left and right limits of π_B^F where $\gamma_i = \bar{\gamma}_i$ and $i \in \{1, 2\}$:

$$\lim^{+}_{\gamma_i \to \bar{\gamma}_i^-} \pi_F^B = \lim^{+}_{\gamma_i \to \bar{\gamma}_i^+} \pi_F^B = \frac{(d^2 \hat{w}_j (cd(2\hat{p} + \hat{w}_i - 2\hat{w}_j) - 2\hat{w}_j y) + cd(4\hat{w}_j y - cd\theta(\hat{w}_i - 3\hat{w}_j)) - c^2\theta^2(cd + 2y))}{2c^2 d^2}$$

Here, we see that even though the respective buyer profit function on the left hand side of the transition point is different than the buyer profit function on the right hand side, the limit values are the same at $\bar{\gamma}_i$. Hence, we conclude that the buyer profit function is continuous at the transition points γ_i. Note that the buyer's profit function is already continuous in each piece; i.e. under Case 1, 2, 3 or 4. Thus, the buyer's profit function is continuous in $\gamma_i \in [0, 1]$, $i \in \{1, 2\}$. ■

APPENDIX H

PROOF OF LEMMA 6

When $\frac{d}{\theta} > \frac{c}{\hat{w}_1}$ and $\frac{d}{\theta} > \frac{c}{\hat{w}_2}$, for Case 1 ($q_1^* > 0$ and $q_2^* > 0$) the buyer profit function becomes:

$$\pi_F^B = \theta\hat{p} \;-\; y\gamma_1 \left(\frac{2y(1-\gamma_2)(d\hat{w}_1 - c\theta) + cd(2d\hat{w}_1 - d\hat{w}_2 - c\theta)}{3c^2d^2 + 4y^2(1-\gamma_1)(1-\gamma_2) + 4cdy(2-\gamma_1-\gamma_2)} \right)^2$$

$$-\; y\gamma_2 \left(\frac{2y(1-\gamma_1)(d\hat{w}_2 - c\theta) + cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta)}{3c^2d^2 + 4y^2(1-\gamma_1)(1-\gamma_2) + 4cdy(2-\gamma_1-\gamma_2)} \right)^2$$

$$+\; d\hat{p} \left(\frac{2y(1-\gamma_2)(d\hat{w}_1 - c\theta) + cd(2d\hat{w}_1 - d\hat{w}_2 - c\theta)}{3c^2d^2 + 4y^2(1-\gamma_1)(1-\gamma_2) + 4cdy(2-\gamma_1-\gamma_2)} \right)$$

$$+\; d\hat{p} \left(\frac{2y(1-\gamma_1)(d\hat{w}_2 - c\theta) + cd(2d\hat{w}_2 - d\hat{w}_1 - c\theta)}{3c^2d^2 + 4y^2(1-\gamma_1)(1-\gamma_2) + 4cdy(2-\gamma_1-\gamma_2)} \right)$$

In order to understand the behavior of this function, we analyzed the first and the second derivatives of the function. Moreover, we checked the Hessian matrix for the joint concavity test:

$$H = \begin{bmatrix} \frac{d^2\pi_F^B}{d\gamma_1^2} & \frac{d^2\pi_F^B}{d\gamma_1\gamma_2} \\ \frac{d^2\pi_F^B}{d\gamma_1\gamma_2} & \frac{d^2\pi_F^B}{d\gamma_2^2} \end{bmatrix}$$

A function is jointly concave under the following conditions:
$1)\ \frac{d^2\pi_F^B}{d\gamma_1^2} < 0, \quad 2)\ \frac{d^2\pi_F^B}{d\gamma_2^2} < 0, \quad 3)\ \frac{d^2\pi_F^B}{d\gamma_1\gamma_2} < 0, \quad 4)\ \frac{d^2\pi_F^B}{d\gamma_1^2} * \frac{d^2\pi_F^B}{d\gamma_2^2} - (\frac{d^2\pi_F^B}{d\gamma_1\gamma_2})^2 > 0.$

On the other hand, a function is jointly convex under the following conditions:
$1)\ \frac{d^2\pi_F^B}{d\gamma_1^2} > 0, \quad 2)\ \frac{d^2\pi_F^B}{d\gamma_2^2} > 0, \quad 3)\ \frac{d^2\pi_F^B}{d\gamma_1\gamma_2} > 0, \quad 4)\ \frac{d^2\pi_F^B}{d\gamma_1^2} * \frac{d^2\pi_F^B}{d\gamma_2^2} - (\frac{d^2\pi_F^B}{d\gamma_1\gamma_2})^2 < 0.$

According to these rules, we determine the behaviour of the buyer profit function for a set of parameters in the following table:

Table H.1: Concavity Test for Buyer Profit Function

Parameters	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6
d	0.4	0.4	0.4	1	1	1
$\hat{w}_1$	0.4	0.4	0.4	0.38	0.38	0.35
$\hat{w}_2$	0.4	0.4	0.35	0.35	0.35	0.35
γ_1	0.5	0.8	0.95	0.7	0.7	0.6
γ_2	0.9	0.9	0.6	0.3	0.9	0.8
$\frac{d^2\pi_F^B}{d\gamma_1^2}$	0.0044	-0.0212	-0.2474	-0.1837	-0.1069	-0.0177
$\frac{d^2\pi_F^B}{d\gamma_2^2}$	-0.2143	-0.1751	0.0047	0.0060	-0.3269	-0.1800
$\frac{d^2\pi_F^B}{d\gamma_1\gamma_2}$	0.0072	0.0341	-0.0065	0.0122	0.1025	0.0364
$\frac{d^2\pi_F^B}{d\gamma_1^2} * \frac{d^2\pi_F^B}{d\gamma_2^2} - (\frac{d^2\pi_F^B}{d\gamma_1\gamma_2})^2$	-0.0010	0.0026	-0.0012	-0.0013	0.0244	0.0019
Joint Concavity	NO	NO	NO	NO	NO	NO
Joint Convexity	NO	NO	NO	NO	NO	NO

According to Table H.1, the buyer profit function is neither jointly concave nor jointly convex. ■

APPENDIX I

PROOF OF PROPOSITION 3

In the second supplier's investment stage, we have the following formulation:

$$\begin{aligned}
\frac{d\pi_{S_2}^F}{dq_2} &= -c\theta - cdq_1 - 2cdq_2 + d\hat{w}_2 - 2yq_2 + 2\gamma_2 yq_2 \\
\frac{d^2\pi_{S_2}^F}{dq_2^2} &= -2cd - 2y(1-\gamma_2) < 0
\end{aligned}$$

Here, since the second order condition of the second supplier's profit function is negative, the first order condition provides the optimal quality value. Thus, we find the best response of supplier 2 as $q_2^*(q_1, \gamma_1, \gamma_2) = \left(\frac{d\hat{w}_2 - cdq_1 - c\theta}{2cd + 2y(1-\gamma_2)}\right)^+$. ■

APPENDIX J

PROOF OF LEMMA 7

We first put the best response of supplier 2; i.e., $q_2^*(q_1, \gamma_1, \gamma_2) = \left(\frac{d\hat{w}_2 - cdq_1 - c\theta}{2cd + 2y(1-\gamma_2)}\right)^+$ (assuming that it is positive), into the profit function of supplier 1 and come up with the following profit function for supplier 1:

$$\pi_D^{S_1} = \hat{w}_1\theta - cq_1\theta + dq_1\hat{w}_1 - cdq_1^2 - q_1^2 y + q_1^2 y\gamma_1 + \frac{d\hat{w}_1(d\hat{w}_2 - c(dq_1+\theta))}{2(cd+y-y\gamma_2)} - \frac{y\gamma_2(d\hat{w}_2 - c(dq_1+\theta))^2}{4(cd+y-y\gamma_2)^2} + \frac{cdq1(-dw2+c(dq_1+\theta))}{2(cd+y-y\gamma_2)}$$

In order to understand the behavior of this function, we check the Hessian matrix for the joint concavity test:

$$H = \begin{bmatrix} \frac{d^2\pi_D^{S_1}}{dq_1^2} & \frac{d^2\pi_D^{S_1}}{dq_1\gamma_2} \\ \frac{d^2\pi_D^{S_1}}{dq_1\gamma_2} & \frac{d^2\pi_{S_1}^{S_1}}{d\gamma_2^2} \end{bmatrix}$$

We try to figure out concavity or convexity characteristics of the supplier 1's profit function for a set of parameters in Table J.1.

As seen from the table, supplier 1 profit function is neither jointly concave nor jointly convex. ■

Table J.1: Concavity Test for Supplier 1 Profit Function

Parameters	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6
d	0.4	0.4	0.4	1	1	1
$\hat{w}_1$	0.25	0.3	0.4	0.35	0.45	0.3
$\hat{w}_2$	0.35	0.35	0.35	0.3	0.4	0.4
γ_1	1	0.5	0.4	0.1	0.5	1
γ_2	0.8	0.8	0.7	0.1	1	1
$\frac{d^2\pi_D^{S_1}}{dq_1^2}$	-0.2000	-0.7000	-0.7880	-1.3139	-0.9235	-0.5000
$\frac{d^2\pi_D^{S_1}}{d\gamma_2^2}$	-0.0000	-0.0000	-0.0000	0.0197	-0.4642	-2.0825
$\frac{d^2\pi_D^{S_1}}{dq_1\gamma_2}$	-0.0028	-0.0043	-0.0034	-0.0276	0.4207	0.6661
$\frac{d^2\pi_D^{S_1}}{dq_1^2} * \frac{d^2\pi_D^B}{d\gamma_2^2} - (\frac{d^2\pi_D^B}{dq_1\gamma_2})^2$	-0.0000	-0.0000	-0.0000	-0.0266	0.2517	0.5976
Joint Concavity	NO	NO	NO	NO	NO	NO
Joint Convexity	NO	NO	NO	NO	NO	NO

APPENDIX K

COMPARISON SECTION PARAMETER TABLES

Table K.1: Preferred Model wrt Equilibrium Cases - Parameters

Case Model	$\hat{w}_1$	$\hat{w}_2$	$\hat{p}$	d	c	y	θ	$\frac{d}{\theta}$	$\frac{c}{\hat{w}_1}$	$\frac{c}{\hat{w}_2}$
F	0.1504	0.4888	0.3607	0.6501	0.2473	0.4955	0.5000	1.3003	3.2729	0.5531
Case 1	0.3316	0.3115	0.3569	0.7785	0.2384	0.4920	0.5000	1.5570	0.7787	0.8322
Case 2	0.3566	0.2723	0.3711	0.6086	0.2707	0.4257	0.5000	1.2171	0.7644	1.0141
Case 3	0.1002	0.5384	0.3614	0.6192	0.2489	0.4981	0.5000	1.2384	3.9561	0.4722
D	0.2901	0.3703	0.3396	0.6457	0.2428	0.5149	0.5000	1.2914	0.8964	0.6768
Case 1	0.3324	0.3287	0.339	0.7495	0.2290	0.5499	0.5000	1.4990	0.7079	0.7154
Case 3	0.2386	0.4211	0.3403	0.5192	0.2595	0.4724	0.5000	1.0384	1.1262	0.6297
I	0.4781	0.1626	0.3593	0.537	0.2557	0.4965	0.5000	1.0740	0.6221	2.9526
Case 2	0.5179	0.1257	0.3564	0.5898	0.2503	0.4956	0.5000	1.1797	0.4973	3.4082
Case 3	0.1113	0.5409	0.3478	0.6565	0.2500	0.5087	0.5000	1.3130	5.7794	0.5546
Case 4	0.3146	0.3128	0.3726	0.303	0.2794	0.5000	0.5000	0.6059	1.0203	1.0328
Total	0.3222	0.3222	0.3557	0.6000	0.2500	0.5000	0.5000	1.2000	1.6225	1.6225

Table K.2: Margin Leader and Preferred Model Details

Leader Model	Nb of Cases	%	$\hat{w}_1$	$\hat{w}_2$	$\hat{p}$	γ_1^*	γ_2^*	q_1^*	q_2^*	q^*
BM	2052	20%	0.2958	0.2958	0.4084	0.5108	0.5179	0.0973	0.1034	0.2007
F	573	28%	0.2443	0.3376	0.4181	0.5012	0.8008	0.0909	0.1818	0.2727
D	686	33%	0.2866	0.3199	0.3936	0.5786	0.8715	0.0903	0.1573	0.2476
I	793	39%	0.3410	0.2447	0.4143	0.4592	0.0077	0.1080	0.0001	0.1081
S1M	3960	39%	0.5107	0.1467	0.3425	0.5208	0.1466	0.2169	0.0209	0.2377
F	289	7%	0.4296	0.2517	0.3187	0.4365	0.9066	0.1998	0.0575	0.2573
D	331	8%	0.4012	0.2891	0.3097	0.5782	0.9626	0.1456	0.1994	0.3449
I	3340	84%	0.5286	0.1235	0.3479	0.5224	0.0000	0.2254	0.0000	0.2254
S2M	3960	39%	0.1467	0.5107	0.3425	0.1183	0.5057	0.0123	0.2189	0.2312
F	2698	68%	0.0996	0.5471	0.3533	0.0534	0.5004	0.0041	0.2571	0.2612
D	1039	26%	0.2543	0.4304	0.3153	0.3124	0.6246	0.0360	0.1621	0.1982
I	223	6%	0.2153	0.4454	0.3393	0.0000	0.0151	0.0000	0.0212	0.0212
EQ	72	1%	0.3500	0.3500	0.3000	0.7089	0.7490	0.1030	0.1478	0.2508
F	13	18%	0.3462	0.3462	0.3077	1.0000	1.0000	0.0737	0.0737	0.1475
D	47	65%	0.3511	0.3511	0.2979	0.8094	0.8709	0.1374	0.2060	0.3434
I	12	17%	0.3500	0.3500	0.3000	0.0000	0.0000	0.0000	0.0000	0.0000
Total	10044	100%	0.3222	0.3222	0.3557	0.3614	0.3684	0.1110	0.1167	0.2277

Table K.3: Margin Leader and Preferred Model Details - Parameters

Leader Model	$\hat{w}_1$	$\hat{w}_2$	$\hat{p}$	d	c	y	θ	$\frac{d}{\theta}$	$\frac{c}{\hat{w}_1}$	$\frac{c}{\hat{w}_2}$
BM	0.2958	0.2958	0.4084	0.6000	0.2500	0.5000	0.5000	1.2000	0.9195	0.9195
F	0.2443	0.3376	0.4181	0.7164	0.2477	0.4812	0.5000	1.4328	1.1310	0.7734
D	0.2866	0.3199	0.3936	0.6767	0.2405	0.5192	0.5000	1.3534	0.8688	0.7592
I	0.3410	0.2447	0.4143	0.4496	0.2599	0.4970	0.5000	0.8991	0.8106	1.1638
S1M	0.5107	0.1467	0.3425	0.6000	0.2500	0.5000	0.5000	1.2000	0.5028	3.1231
F	0.4296	0.2517	0.3187	0.7401	0.2469	0.4785	0.5000	1.4803	0.5854	0.9948
D	0.4012	0.2891	0.3097	0.7562	0.2263	0.5580	0.5000	1.5124	0.5694	0.7901
I	0.5286	0.1235	0.3479	0.5724	0.2526	0.4961	0.5000	1.1448	0.4890	3.5384
S2M	0.1467	0.5107	0.3425	0.6000	0.2500	0.5000	0.5000	1.2000	3.1231	0.5028
F	0.0996	0.5471	0.3533	0.6271	0.2472	0.5008	0.5000	1.2543	4.0277	0.4580
D	0.2543	0.4304	0.3153	0.5874	0.2498	0.4979	0.5000	1.1748	1.0286	0.5860
I	0.2153	0.4454	0.3393	0.3305	0.2854	0.5000	0.5000	0.6610	1.9373	0.6564
EQ	0.3500	0.3500	0.3000	0.6000	0.2500	0.5000	0.5000	1.2000	0.7149	0.7149
F	0.3462	0.3462	0.3077	0.5000	0.2654	0.4077	0.5000	1.0000	0.7667	0.7667
D	0.3511	0.3511	0.2979	0.7043	0.2394	0.5255	0.5000	1.4085	0.6823	0.6823
I	0.3500	0.3500	0.3000	0.3000	0.2750	0.5000	0.5000	0.6000	0.7864	0.7864
Total	0.3222	0.3222	0.3557	0.6000	0.2500	0.5000	0.5000	1.2000	1.6225	1.6225

Table K.4: Preferred Cases vs. Produced Quality - Parameters

Model	$\hat{w}_1$	$\hat{w}_2$	$\hat{p}$	d	c	y	θ	$\frac{d}{\theta}$	$\frac{c}{\hat{w}_1}$	$\frac{c}{\hat{w}_2}$
F	0.1504	0.4888	0.3607	0.6501	0.2473	0.4955	0.5000	1.3003	3.2729	0.5531
F-Q	0.1474	0.4916	0.3609	0.6485	0.2475	0.4949	0.5000	1.2969	3.2995	0.5475
D-Q	0.4606	0.1988	0.3406	0.8235	0.225	0.5588	0.5000	1.6471	0.4967	1.1331
D	0.2901	0.3703	0.3396	0.6457	0.2428	0.5149	0.5000	1.2914	0.8964	0.6768
F-Q	0.2869	0.3705	0.3427	0.6689	0.2419	0.5174	0.5000	1.3379	0.8966	0.6766
D-Q	0.3539	0.3488	0.2973	0.6845	0.2355	0.5618	0.5000	1.3691	0.6937	0.6813
I	0.2485	0.3931	0.3584	0.3842	0.2598	0.4388	0.5000	0.7683	1.1241	0.6736