THE EFFECTS OF USING ALGEBRA TILES ON SIXTH GRADE STUDENTS’ ALGEBRA ACHIEVEMENT, ALGEBRAIC THINKING AND VIEWS ABOUT USING ALGEBRA TILES

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF SOCIAL SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

BÜŞRA ÇAYLAN

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
THE DEPARTMENT OF ELEMENTARY SCIENCE AND
MATHEMATICS EDUCATION

JUNE 2018

Approval of the Graduate School of Social Sciences

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ABSTRACT

THE EFFECTS OF USING ALGEBRA TILES ON SIXTH GRADE STUDENTS' ALGEBRA ACHIEVEMENT, ALGEBRAIC THINKING AND VIEWS ABOUT USING ALGEBRA TILES

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June 2018, 185 pages

The purpose of the study was to investigate the effects of using algebra tiles on sixth grade students' algebra achievement, algebraic thinking and views about using algebra tiles. The study was conducted in one public middle school in Hendek, Sakarya in the spring semester of 2017-2018 academic year and employed a pretest-posttest control group design with 40 sixth grade students. Two classes taught by the same mathematics teacher were randomly assigned as experimental group (EG) and control group (CG). EG students interacted with algebra tiles and CG students received regular instruction without any manipulatives during the seven hours of algebra instruction. Prior Algebra Knowledge Test (PAKT) and Algebra Achievement Test (AAT) were administered to EG and CG students as pretest and posttest respectively. EG students' views were gathered by Views about Algebra Tiles Questionnaire (VATQ). PAKT, AAT and VATQ were developed by the researcher.

EG and CG did not differ in PAKT. The Independent Samples T-test showed that there was no statistically significant mean difference between EG and CG in AAT. When EG and CG students' responses to each question in the AAT were examined in detail, it was concluded that EG students performed better than CG students in AAT and more EG students responded to the questions correctly than CG students. The findings addressed that algebra tiles might have limited but positive effect on sixth grade students' algebraic thinking. EG students expressed that algebra tiles helped them learn meaningfully and understand better the concepts, and made lessons enjoyable.

Keywords: Algebra Tiles, Algebra Achievement, Algebraic Thinking, Students' Views, Middle School Students

ÖZ

CEBİR KAROSU KULLANIMININ ALTINCI SINIF ÖĞRENCİLERİNİN CEBİR BAŞARISI, CEBİRSEL DÜŞÜNMELERİ VE CEBİR KAROSU KULLANIMINA İLİŞKİN GÖRÜŞLERİ ÜZERİNDEKİ ETKİLERİ

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Bu çalışmanın amacı cebir karosu kullanımının altıncı sınıf öğrencilerinin cebir başarısı ve cebirsel düşünmeleri üzerindeki etkilerini incelemektir. Bu çalışma aynı zamanda öğrencilerin cebir karosu kullanımına ilişkin görüşlerini incelemeyi de amaçlamıştır. Çalışma, 2017-2018 eğitim-öğretim yılının bahar döneminde Sakarya'nın Hendek ilçesinde bir devlet ortaokulunda 40 altıncı sınıf öğrencisinin katıldığı öntest-sontest kontrol gruplu deneysel desen ile gerçekleştirilmiştir. Aynı matematik öğretmeni tarafından okutulan iki sınıf deney (DG) ve kontrol (KG) grubu olarak rastgele atanmıştır. DG öğrencileri derslerde cebir karolarını somut materyal olarak kullanırken, KG öğrencileri herhangi bir somut materyal kullanmamış, öğretmen olağan dersini yapmıştır. Veri toplama araçları olarak, araştırmacının geliştirdiği Cebir Ön Bilgi Testi öntest olarak ve Cebir Başarı Testi sontest olarak her iki gruba da

uygulanmıştır. Ayrıca, Cebir Karosu Kullanımına İlişkin Öğrenci Görüş Formu ile DG öğrencilerinin görüşleri alınmıştır.

DG ve KG öğrencilerinin cebir ön bilgileri arasında bir fark bulunmamıştır. Bağımsız örneklem T-testinin sonuçları DG ile KG öğrencilerinin sontest puanları arasında istatistiksel olarak anlamlı bir fark bulunmadığını göstermiştir. Ayrıca, deney ve kontrol grubu öğrencilerinin Cebir Başarı Testi'ndeki yanıtları nitel veri analizi ile detaylı bir şekilde incelenmiş ve öğrencilerin cebirsel düşünmeleri hakkında bilgi toplanmıştır. Buna göre, deney grubu öğrencilerinin soruların çoğunda kontrol grubu öğrencilerinden daha iyi performans gösterdiği ve daha fazla soruya doğru yanıt verdiği görülmüştür. Bu durum cebir karolarının altıncı sınıf öğrencilerinin cebirsel düşünmesinde sınırlı ama pozitif bir etkiye sahip olduğunu göstermektedir. Ayrıca, DG öğrencileri derslerde cebir karoları kullanımının konuyu daha iyi öğrenmelerine ve anlamalarına yardımcı olduğunu ve dersleri eğlenceli hale getirdiğini belirtmişlerdir.

Anahtar kelimeler: Cebir Karoları, Cebir Başarısı, Cebirsel Düşünme, Öğrenci Görüşleri, Orta Okul Öğrencileri

To My Parents

ACKNOWLEDGMENTS

I would like to express my deepest gratitude to my supervisor Assoc. Prof. Dr. Çiğdem HASER for her invaluable guidance, suggestions and feedbacks throughout this study. She was just a phone call or email away whenever I need help and I learned many things from her.

I would like to thank to dear committee members Assist. Prof. Dr. Zerrin TOKER and Assist. Prof. Dr. Işıl İŞLER BAYKAL for their valuable comments and contributions for my study.

I am forever thankful to my family. Thanks for believing in me in my undergraduate and graduate studies and opportunities you provided for me. Without them, I could never have reached this current level of success. This thesis is heartily dedicated to them, representing my appreciation.

I am also grateful to my friends and colleagues for their encouragements and moral support.

I want to express my sincere thanks to the teacher and students for their participation in the study.

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LIST OF ABBREVIATIONS

AAT: Algebra Achievement Test

CG: Control Group

EG: Experimental Group

METU: Middle East Technical University

MoNE: Ministry of National Education

NCTM: National Council of Teachers of Mathematics

PAKT: Prior Algebra Knowledge Test

VATQ: Views about Algebra Tiles Questionnaire

CHAPTER 1

INTRODUCTION

1.1 Introduction

One of the important areas in school mathematics is algebra. Initial ideas in algebra could be considered as the focus of early algebra which is "compass algebraic reasoning and algebra-related instruction among young learners-from approximately 6 to 12 years of age" (Carraher & Schliemann, 2007, p. 670). Algebra teaching and learning should start in early elementary school where students should be given opportunities to have experiences with algebra to prepare them for algebra in middle and high school (NCTM, 2000).

In algebra, it is necessary to think not only a few numbers, but sets of numbers. For this reason, algebra seems more abstract than arithmetic (Palabıyık & Akkuş, 2011). Since algebra seems less concrete for students, they find it difficult in school mathematics and they encounter serious obstacles in the mathematics learning process (NCTM, 2000). The difficulty comes from working with variables and their notations (Kieran & Chalouh, 1993).

Edge and Kant (1992, as cited in Thornton, 1995) stated that words represent something that is touched or experienced, and therefore, learning a language is easy. Therefore, when you see a word like banana or computer, you can visualize it. On the other hand, they stated that learning mathematics is difficult because it is generally taught with no recognizable meaning. You cannot visualize anything when you see 2x or x^2 if you do not know the meanings of the symbols. In this case, as Edge and Kant addressed, learning mathematics might be described as learning reading without knowing the meanings of the words. Hence, it can be said that "conceptualizing variables and manipulating them are key features of algebra learning" (Akkuş, 2004, p. 7). Manipulatives could be the tools to make the algebra learning process meaningful and effective for students by providing a concrete base for learning.

In the literature, there are several definitions of manipulatives. Moyer (2001) defines manipulatives as “materials designed to represent explicitly and concretely mathematical ideas that are abstract” (p. 176). According to Hynes (1986), manipulatives are “concrete models that incorporate mathematical concepts, appeal to several senses and can be touched and moved around by students” (p. 11). Manipulative use enables students to transit from concrete thinking to abstract thinking (Fennema, 1973). Manipulatives enhance learning by providing students with characteristics they can see, hear and touch, increase motivation of students and lessens the rather less interesting characteristics of mathematics for students. They are specifically useful when students are introduced new mathematical concepts (Kober, 1991).

Using manipulatives enables students to understand mathematics concepts even when they have abstract nature (Larbi & Okyere, 2014). Using manipulatives also results in increase in students’ mathematics achievement (Sowell, 1989). In addition, Turkish Middle Grades Mathematics Curriculum gives importance to the use of manipulatives in the learning process. In the curriculum, it is stated that using manipulatives improves psychomotor skills of the students and helps them make abstraction and create meaning from concrete experiences. Furthermore, it is emphasized that with the help of manipulatives, students can express mathematical thoughts and their communication skills improve (MoNE, 2013).

According to NCTM (2000), “many students profit from hands-on collaborative learning that manipulatives afford” (p. 20). Collaborative learning enables students to come across ideas and questions of their group mates, check for their own understanding, and comprehend the concepts deeply (Mercier & Higgins, 2013). Using manipulatives to solve tasks in groups enhances learning in cooperative learning groups because using manipulatives motivate and entertain students (Mulryan, 1994).

Research studies have shown that students who use manipulatives in mathematics lessons have higher algebraic abilities such as, representing algebraic expressions and interpreting them, making connections between concepts while solving equations and communicating algebraic concepts, than those who do not. In addition, using manipulatives help middle school students establish meaningful connections in algebraic thinking (Chappell & Strutchens, 2001). Therefore, it can be said that manipulatives should be used in algebra learning process especially when students are about to move towards abstract concepts. One of the manipulatives that can be used in this process is algebra tiles.

Algebra tiles are used to visualize operations with mathematical expressions including variables and numbers (Karakırık & Aydın, 2011). They enable students to figure out mathematical problems algebraically. With the help of algebra tiles, students can visualize polynomial operations, solve equations (Heddens & Speer, 2001 as cited in Saraswati, Putri & Somakim, 2016), have a better understanding of the concepts (Thornton, 1995), and learn the concepts meaningfully (Larbi & Okyere, 2016). They are able to reach the formal solution of linear equation with one variable easily with the help of algebra tiles (Saraswati, Putri & Somakim, 2016).

Previous studies have presented the positive effect of use of manipulatives on students' learning of algebra. However, studies showing the effect of specific manipulatives for specific concepts at earlier levels are rare. Manipulatives are helpful especially when the topics are learned for the first time in order to scaffold students' learning of abstract concepts (Akkaya, 2006 as cited in Çağdaşer, 2008). Therefore, it is important to explore the effect of manipulatives, and algebra tiles in specific, when students meet the key concepts of algebra for the first time.

1.2 Purpose of the Study and Research Questions

The purpose of the study is to investigate the effects of using algebra tiles on sixth grade students' algebra achievement, algebraic thinking and views about using algebra tiles. The effect of using algebra tiles on students' algebra achievement was investigated by pretest-posttest control group design. The effects on the algebraic thinking were investigated through deeply analyzing students' responses in the posttest. This study also aims to investigate students' views about using algebra tiles by a questionnaire. In order to investigate the effects of using algebra tiles on sixth grade students' algebra achievement, algebraic thinking and to explore their views about using algebra tiles, the following research questions and hypotheses were formulated:

1. Is there a statistically significant mean difference between posttest scores of algebra achievement test for 6^{th} grade students who use algebra tiles and those who do not use algebra tiles?

   H_0: There is no significant difference between posttest scores of algebra achievement test for 6^{th} grade students who use algebra tiles and those who do not use algebra tiles.

   H_1: There is a significant difference between posttest scores of algebra achievement test for 6^{th} grade students who use algebra tiles and those who do not use algebra tiles.

2. How does students' algebraic thinking differ in the algebra achievement test for those who use algebra tiles and who do not use algebra tiles?
3. What are the 6^{th} grade students' views about use of algebra tiles in mathematics lessons?

1.3 Significance of the Study

There are a number of studies in Turkey which address the potential benefits of the use of manipulatives while teaching algebra such as algebra tiles, pattern blocks, balance, colored papers, seesaw, matchstick, and computer-assisted visual materials in mathematics lessons (see Akyüz & Hangül, 2013; Işık & Çağdaşer, 2009; Koğ & Başer, 2012; Palabıyık & Akkuş, 2011; Yıldız, 2012). However, there are not sufficient studies which specifically examine the effectiveness of use of algebra tiles. The present study focuses on using algebra tiles as manipulatives in algebra teaching process. Moreover, in this study, algebra tiles were not used as a part of the lesson for practice or exercise. Instead, they were used from beginning to end of algebra teaching and learning process.

Algebra tiles provide geometric interpretation of symbol manipulation and combine algebraic and geometric concepts. Thus, with the help of algebra tiles, students can see algebraic concepts from a geometric perspective and realize that these mathematical concepts are related to each other (Leitze & Kitt, 2000). In addition, algebra tiles are a visual and hands-on way to explore new concepts at the introductory level for the students. They enable students to state the rules of algebra from their own experiences (Okpube, 2016). Moreover, since algebra tiles can be easily made cutting the cardboards (Karakırık & Aydın, 2011), teachers can create algebra tiles by themselves when the resources are inadequate. Creating algebra tiles is inexpensive and they can be easily replicated. From this point of view, it is beneficial to investigate effect of using algebra tiles which can be easily produced and used by teachers.

The studies about algebra tiles abroad were conducted with middle school students to teach solving linear equations with one variable (Magruder, 2012; Saraswati et al., 2016) or with high school students to teach factoring by using algebra tiles (Sharp, 1995; Thornton, 1995). In addition, there are some studies about using algebra tiles in polynomial multiplication (Goins, 2001; Johnson, 1993). However, there are not sufficient studies in the accessible literature related to using algebra tiles of the students who encounter algebra for the first time. Thus, it is believed that this study may contribute to the literature by providing knowledge on students' algebra achievement when they use algebra tiles, which is a common and easy-to-access manipulative.

In the 2013 version of Turkish Middle Grades Mathematics Curriculum, the curricular context in which this study took place, objectives related to algebra learning area take part in 6^{th} grade level for the first time and students are introduced algebraic expressions, variable, term, constant term, and coefficient concepts (MoNE, 2013). If the students do not learn basic algebraic concepts at this grade level conceptually and symbolically, they may not understand the other algebraic concepts in coming years. Research showed that when the students successfully completed algebra course which they took in middle schools, they got higher performance on mathematics tests and they understood advanced mathematics much easier (Wang & Goldschmidt, 2003). Therefore, exploring the effects of manipulative usage on 6^{th} grade students' algebra achievement might provide information on the strength of this achievement for future mathematics achievement.

Besides, in the light of the findings of this study, it can be determined whether using algebra tiles in mathematics lessons is effective in terms of students' algebraic thinking or not and how Turkish students react to use of algebra tiles in algebra learning process. Therefore, outcomes of this study might provide information to middle school teachers, teacher educators and program makers.

1.4 Definitions of Terms

Manipulatives: "Concrete models that incorporate mathematical concepts, appeal to several senses and can be touched and moved around by students" (Hynes, 1986, p. 11). In this study, manipulatives refer to algebra tiles used for 6^{th} grade algebra topics. In this study, the terms *algebra tiles* and *manipulatives* are used interchangeably.

Algebra Tiles: Algebra tiles are manipulatives that are used to visualize operations with mathematical expressions including variables and numbers (Karakırık & Aydın, 2011).

Algebra Achievement: Sixth grade students' achievement scores on algebra achievement test which was prepared by the researcher and which includes questions addressing students' algebraic thinking based on the 6th grade objectives.

Algebraic Thinking: "The use of any of a variety of representations that handle quantitative situations in a relational way" (Kieran, 1996, p. 4).

Group Work: Tasks that are completed by small learning groups of 2 or 4 students who work together in the experimental group classroom.

1.5 My Motivation to Conduct the Study

During my teaching practice in the last year of the Elementary Mathematics Education Program, I had a chance to use manipulatives in a real classroom environment. The objective which would be accomplished by the 6th grade students was multiplying an algebraic expression with a natural number. For this reason, I decided to use algebra tiles and I implemented a task by using them in the class myself. During the class, students had an experience and explored the rationale behind the rule with the help of algebra tiles themselves. Algebra tiles gained their interest and enabled them to concentrate on the topic. Afterward, I thought whether students' achievement could be improved by using algebra tiles and whether they could understand algebraic expressions meaningfully by using algebra tiles. By means of the present study, I expect to find the answers of these questions.

CHAPTER 2

LITERATURE REVIEW

2.1 Theoretical Background

Theoretical foundation for the use of manipulatives dates back to developmental theorists Piaget (1926) and Bruner (1966). According to them, children are not born with the capacity for abstract thought. Instead, they form abstract concepts by interacting with objects in their environment. Therefore, children should be physically involved in hands-on experiences with manipulatives to add new ideas to their cognitive structure (Fennema, 1973).

Learning theories developed by Dienes, Piaget, Skemp, and Brownell assert that children might bridge the gap between the world where they live and abstract mathematics, if their mathematical learning is based on experiences with manipulatives (Kennedy, 1986). That is, when the children use manipulatives, they can understand mathematical ideas meaningfully and transfer these ideas to real life situations easily (Yıldız, 2012).

Piaget (1973), in his Theory of Cognitive Development, described four stages of children's cognitive development: Sensorimotor Stage (birth to age 2), Preoperational Stage (ages 2 to 7), Concrete Operational Stage (ages 7 to 11), and Formal Operational Stage (age 11 onwards). While going through these stages, children first use physical actions and then use symbols to create schemas. In concrete operational stage, children can organize data only if concrete objects are presented. Piaget (1952) stated that children cannot understand abstract mathematics through explanations and lectures and they should have experiences with models and materials.

According to Sowell (1989), children understand mathematical ideas by having concrete, concrete-abstract, and pictorial-abstract learning experiences before strictly abstract experiences. Hence, learning experiences should be planned according to the order of cognitive development stages. Cognitive development theory implemented to classroom practice requires both concrete and symbolic models be included in the learning environments so that children with different levels of development can benefit (Fennema, 1972).

2.2 Students' Learning of Algebra

Algebra is one of the important areas in school mathematics because it improves the critical thinking skills of the students. In addition, algebra gives students an opportunity to solve real life problems and reach various solutions in a logical way (Brian, 2010 as cited in Anthony, Michael & Victoria, 2012). Being successful in algebra during middle school years results in higher scores on mathematics tests, understanding of advanced mathematics better and higher enrolment in high school (Wang & Goldschmidt, 2003). Therefore, learning and understanding algebra in middle school years is important.

"Algebraic thinking is the capacity to represent quantitative situations so that relations among variables become apparent" (Driscoll, 1999, p. 1). Similarly, Kieran (1996) defines algebraic thinking as "the use of any of a variety of representations that handle quantitative situations in a relational way" (p. 4). Developing algebraic thinking leads to meaningful understanding of algebra rather than focusing on procedures. In addition, development of algebraic thinking at early ages promotes long term learning of many students (Windsor & Booker, 2010). Algebraic thinking can be facilitated in a classroom context where collaborative learning is valued and encouraged and students have opportunities to communicate their mathematical ideas and assumptions (Windsor, 2010).

Algebraic thinking can be maximized simply by making necessary changes in teaching methods instead of making changes in mathematics curriculum

(Lawrence & Hennessy, 2002). Teachers should design learning environments that support students' algebraic thinking by "modeling, exploring, arguing, predicting, conjecturing, and testing their ideas, as well as practicing computational skills" (Blanton & Kaput, 2003, p. 75). In addition, teachers should create algebraic activities by using existing materials. They can transform arithmetic activities and single-answer word problems to support students' algebraic thinking by making them find patterns, make conjectures and generalizations, and justify mathematical facts and relationships (Blanton & Kaput, 2003).

According to Kieran (1989, as cited in Girit & Akyüz, 2016), using algebraic symbols is an essential part of algebraic thinking. However, development of algebraic thinking does not occur rapidly. It firstly requires experience with concrete materials and then with pictorial, tabular, graphic and finally with symbolic representations. Presenting situations including relationships in contexts and pictures to elementary and middle school students supports development of their algebraic thinking (Lawrence & Hennessy, 2002). Especially, using concrete models helps middle school students establish meaningful connections in algebraic thinking (Chappell & Strutchens, 2001).

In algebra, it is necessary to think not only a few numbers, but sets of numbers. For this reason, algebra seems more abstract than arithmetic for the students (Palabıyık & Akkuş, 2011). The primary reason of students' difficulties in algebra is not understanding the underlying logic (MacGregor, 2004). Similar to learning arithmetic, in learning algebra, students are inclined to calculate at first. However, algebra requires recognizing, constructing and manipulating algebraic expressions before computation. If students do symbolic manipulation without conceptual understanding, they will only do mechanical manipulation (Kirshner & Awtry, 2004).

Early research studies have shown some shortcomings students have about algebra. These are incomplete understanding of equal-sign (Booth, 1984, 1988; Kieran, 1981, 1985; Vergnaud, 1985), misconceptions related to letters which represent variables (Kieran, 1985; Küchemann, 1981; Vergnaud, 1985), rejecting that algebraic expression, such as 3a+7, is an answer of the problem (Sfard & Linchevski, 1994), and having difficulty in solving one-variable equations where the variable appears on both sides of the equals sign (Filloy & Rojano, 1989; Herscovics & Linchevski, 1994).

Jupri, Drijvers and van den Heuvel-Panhuizen (2015) address difficulties in initial algebra into five categories. The first difficulty is applying arithmetical operations in algebraic expressions such as adding or subtracting like terms. The second difficulty is understanding the multiple functions of letters because a letter can have different roles as a placeholder, a generalized number, an unknown, or a variable in expressions. Ely and Adams (2012) explained the distinction between these four different uses of letters in algebraic expressions. Unknown is a specific value (or a few values) that can be found from the given information. For instance, while in "4+x=9," the unknown is x which stand for a unique value, in "$x^2 - 3x = 6$," the unknown is x which represents two values. Variable is "the letter seen as representing a range of unspecified values, and a systematic relationship is seen to exist between two such sets of values" (Küchemann, 1981, p. 104). In other words, variable does not represent a unique value or a few values. Instead, it represents a set of values. For example, in "$y = -\frac{1}{2}x + 6$," the variable is both x and y. Placeholder is a letter which represents a number in a specific problem or context. It can also be called given, constant, parameter or coefficient. For example, in the equation "$ax^2 + bx + c = 0$," a, b, and c are placeholders (particularly, coefficients). Generalized number refers to "the use of literal symbols when all replacement values of the literal symbols will result in a true statement, as with identities" (Philipp, 1992, p. 160). For instance, the letters in "a(b + c) = ab + ac" function as generalised numbers. Thus, it can be said that several usages of letters and how to distinguish between them are among the important issues that teachers should help students make sense (Ely & Adams, 2012).

The third difficulty is the acceptance of lack of closure or the expected answer obstacle (Tall & Thomas, 1991). When the algebraic expressions do not have equal sign or something on the right side, they do not make sense for some students, because they tend to give specific numerical answers. The fourth difficulty is understanding the different meanings of equal sign because the equal sign means calculation in arithmetic but it means equivalence in algebra (Bush & Karp, 2013; Herscovics & Linchevski, 1994; Kieran, 1981). The fifth and last difficulty is mathematization, which means converting real life problems to the world of mathematics, and vice versa (Jupri et al., 2015).

There are also several research studies addressing middle school students' difficulties related to variables in Turkey (such as Akgün & Özdemir, 2006; Dede & Argün, 2003; Soylu, 2008) and they indicate similar results with the studies abroad. Thus, it can be said that students have difficulty in understanding multiple meanings of letters in general according to the research studies (Ulusoy, 2013).

In addition, research studies indicated that arithmetic and algebraic concepts are connected to each other. For instance, the reason of misconceptions while operating algebraic expressions having integers (particularly negative integers) and overgeneralizing the notion of cancelling arises from having arithmetic misconceptions (Norton & Irvin, 2007; Stacey & Chick, 2004; Stacey & MacGregor, 1994; Wu, 2001). As a result of lack of arithmetic understanding, students have difficulty in transmission of this understanding to algebraic contexts. Since most of the algebraic tasks include fractions, decimals, negative numbers, equivalence, ratios, percentages or rates concepts, students need to have a conceptual understanding of these concepts to solve algebraic tasks (Norton & Irvin, 2007; Stacey & Chick, 2004; Stacey & MacGregor, 1994).

Unfortunately, in middle schools, algebra is taught as only applying a set of rules and following some steps. Moreover, it is taught as not much related with real life, independent from the other subjects and have no connection to arithmetic (Kaput, 1999). Therefore, many students do not see algebra as an extension of arithmetic and they cannot make a connection between algebraic concepts and previously learned arithmetic concepts. For example, while 58 percent of the eighth graders were able to identify that m + m + m + m is equivalent to 4m; seventh graders had more difficulty in recognizing it with 47 percent based on the results from TIMSS (Beaton et al., 1996). In Turkey, Şengül and Erdoğan (2014) found that 6^{th} grade students have low performance in solving declarative, procedural, and conditional problems about algebra.

Although mathematics teachers think that students can easily understand that variable represents any number, students have a difficulty in understanding it (Roberts, 1989). Most students see algebra as “little more than many different types of rules about how to write and rewrite strings of letters and numerals, rules that must be remembered for the next quiz or test” (Kaput, 1995, p. 4). However, learning algebra is more than only memorizing a list of rules. To make algebra learning meaningful, it is required to understand the meaning of symbols, properties and techniques (Thornton, 1995).

When overreliance on textbook with procedural focus and teacher-centred instruction exist, students are not able to make transition from arithmetic to algebra. Therefore, they complain that they do not deeply understand mathematical concepts and they think it is not worth the effort to learn (Watt, 2005). Students learn algorithms rote and they do not know why they apply that algorithm. They cannot visualize the solutions. This causes instrumental learning where students employ rules without thinking about the reason (Skemp, 1976 as cited in Roberts, 1989). Therefore, it is necessary to provide students an opportunity to visualize their solutions in algebra teaching in order to make them understand algebraic concepts so that they will know how to deal with algebra topics with the reasons.

Although there are many studies about learning/teaching algebra, there are not many studies about especially how teachers can teach algebra and which factors should the effective algebra learning environments have (Kaya &

Keşan, 2014). Thus, teaching algebra in schools remains loyal to traditional instructions (Doerr, 2004).

2.3 Manipulative Use in Algebra and Algebra Tiles

“Math manipulatives are physical objects that are designed to represent explicitly and concretely mathematical ideas that are abstract” (Moyer, 2001, p. 176). Manipulatives can be bought as ready-made materials, prepared by teachers, or students can create themselves with the help of their teachers. Cuisenaire rods, tangrams, geoboards, pattern blocks, algebra tiles, fraction strips, and base-ten blocks can be given as examples of manipulatives (Furner & Worrell, 2017). They can “introduce, practice, or remediate a math concept” (Boggan, Harper & Whitmire, 2010, p. 2).

Students cannot learn mathematics by only listening to the teacher. Through the use of manipulatives, students become active participants rather than passive bystanders (Carbonneau, Marley & Selig, 2013). Students can discover the patterns themselves and make generalizations with the help of manipulatives (Roberts, 1989). Furthermore, research showed that most effective learning takes place when the students build mathematical understanding themselves with the use of manipulatives (Boggan et al., 2010). In this way, instruction will be student-centred instead of teacher-centred and the teacher will be the facilitator when students discover the mathematical concepts and relationships (Fletcher, 2009).

In mathematics teaching, students should be encouraged to learn by doing and to have experience with mathematical manipulatives that help development of cognitive, affective and psychomotor domains (Okpube, 2016). Manipulatives also generate motivation for students to engage in learning process and enable students to understand and visualize concepts more clearly (Bruins, 2014). Therefore, teachers should begin mathematics lessons with concrete manipulatives, and then pass to the representational models such as pictures, diagrams, and figures. Finally, at the end of the lessons, students learn symbols and operations at the abstract level and they will not need manipulatives eventually (Furner & Worrell, 2017).

Using manipulatives can be useful for all students with different achievement levels. Cooper (2012) stated that manipulatives are beneficial even for students who are proficient in symbolic procedures because they enhance students' conceptual understanding by providing a different perspective for mathematics. Furthermore, manipulatives are effective tools for kinesthetic learners at the elementary and secondary levels because they learn better when they touch or are physically involved in what they are studying (Corrales, 2008; Gage, 1995).

Manipulatives provide numerous benefits for students. However, there are several issues that need attention while using manipulatives in the mathematics classes. First, there are certain difficulties related to using manipulatives in the classroom. Students can use manipulatives to play games rather than complete their assignments. Moreover, distributing and collecting manipulatives result in considerable loss of time. For these reasons, before implementing manipulatives in the classroom, the teacher should consider the amount of time and be aware of the possibility that students can use manipulatives as toys (Magruder, 2012).

Second, the correct use of manipulatives is also important. If manipulatives are not used appropriately, using manipulatives does not guarantee meaningful learning. Therefore, appropriate use of manipulatives is necessary for effectiveness (Furner & Worrell, 2017). Such usage requires that the manipulative connects informal and formal school mathematics, be appropriate for the students' developmental level (Smith, 2009), and be at the level of students' mathematical ability (Boggan et al., 2010). Students must understand the mathematical concepts behind the manipulatives rather than seeing them only as toys. Therefore, teachers should give students time to work with manipulatives before starting to teach the concepts (Boggan et al., 2010). Furthermore, mathematics manipulatives should be selected in line with the goals and objectives in the mathematics curriculum (Smith, 2009) and teachers should be aware of when, why and how to use manipulatives in an effective way in the class (Kelly, 2006). They should help students establish correct connections between manipulatives and the meaning they represent (Ball, 1992).

While learning algebra, students need a connection between concrete and abstract concepts. This connection can be provided by manipulatives (Bruins, 2014). Piaget (1952) believed that due to the fact that students cannot understand abstract mathematics only with the explanations and instructions, they should have experiences with models and materials. Similarly, according to Bruner (1960), students' early experiences and interactions with concrete objects provide a basis for their future abstract learning. Using manipulatives helps students make a transition between concrete and symbolic representations of the concepts (Fennema, 1972). Research suggest that manipulatives should be used when the basic concepts of algebra (equal sign, variable and unknown) are introduced to students in order to help them comprehend these concepts easier, followed by pictures and figures, and finally mathematical symbols (Akkaya, 2006 as cited in Çağdaşer, 2008).

One of the materials that can be used in algebra teaching process is algebra tiles. Algebra tiles are manipulatives that are used to visualize operations with mathematical expressions including variables and numbers (Karakırık & Aydın, 2011). Figure 2.1 shows a set of algebra tiles.

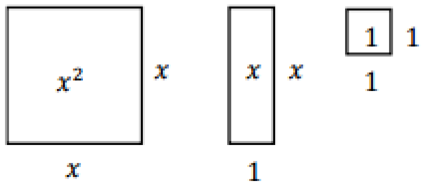


Figure 2.1 A set of algebra tiles. Adopted from "Supporting Students' Understanding of Linear Equations with One Variable Using Algebra Tiles," by S. Saraswati, 2016, *Journal on Mathematics Education*, *7*, p.24. Copyright 2010 by the American Psychological Association.

"Algebra tiles usually come with a small square, an oblong-rectangular strip, and a larger square. The tiles are purposely designed so that the side length of the larger square is not an integral multiple of the side length of the smaller square" (Chappell & Strutchens, 2001, p. 20). They can be used to model several mathematical processes in algebra concepts and help students visualize and conceptually understand these processes (Brahier, 2016). Moreover, adding, subtracting, multiplying and dividing integers; completing the square; factoring and distributive property can be taught by using algebra tiles (Leitze & Kitt, 2000).

Algebra tiles are effective manipulatives because they enable students to make sense of mathematical problems algebraically (Heddens & Speer, 2001 as cited in Saraswati et al., 2016). By using algebra tiles, students can explore algebraic expressions in a visual and hands-on way. Thus, students can learn the rules of algebra from their own experiences (Okpube, 2016). Furthermore, using algebra tiles helps students avoid making mistakes and eliminate students' confusion between expressions such as "2x" and "2+x" (Picciotto & Wah, 1993) and they provide better understanding of zero principles (Sibbald, 2009). Students can create varied pairs of zero while simplifying algebraic expressions and generate different expressions without changing their values (Chappell & Strutchens, 2001).

Students generally tend to use symbols such as "x" and "y" to represent variables because of the common usage of these symbols and forget that different symbols can also be used. Algebra tiles enable students to understand the arbitrary nature of the variable concept. On the other hand, algebra tiles have some limitations. Polynomials beyond first and second degree cannot be modelled with algebra tiles (Smith, 2017). Besides, modelling complicated examples with algebra tiles is difficult. Therefore, rules to complicated examples can be extended by using the symbolic form. Algebra tiles cannot represent fractions. For this reason, it is difficult to represent division equations by using algebra tiles (Magruder, 2012). Furthermore, in modelling algebraic expressions with algebra tiles, one color of the rectangle algebra tile represents –x but area cannot have a negative value in reality. Hence, this can lead students to a misconception (İşleyen, 2012).

Algebra tiles have been used in several research studies. Sobol (1998) found that using algebra tiles had significant effect on 7^{th}, 8^{th}, and 9^{th} grade students' learning of algebraic concept of zero and operations with integers and polynomials. Use of algebra tiles increased treatment group students' understanding in mathematics learning process compared to control group in Larbi (2011)'s experimental study. Saraswati et al. (2016) found that algebra tiles helped students find the formal solution of linear equation with one variable. Using algebra tiles have also been found to assist students when they make geometric connection to factoring polynomials (Schlosser, 2010). In the same way, while teaching solving quadratic equations by completing a square, using algebra tiles helped students build connections between algebraic and geometric concepts (Vinogradova, 2007). In addition, high school students expressed meaningful and easy learning through algebra tiles in Sharp (1995)'s study although there was not any difference between the test scores of students who used algebra tiles while factoring and those who did not. Similarly, students who used algebra tiles expressed the process of polynomial multiplication better (Goins, 2001). Johnson (1993) found that when the algebra tiles were used, not only students, but also teachers understood multiplication of polynomial concept much better. Using algebra tiles increased treatment group’s scores of students with learning disabilities in Castro’s (2017) pretest-posttest control group design experimental study. When algebra tiles were used to improve senior high school students’ conceptual understanding of a system of two linear equations, Akpalu, Adaboh and Boateng (2018) found that there was a statistically significant improvement in the experimental group’s posttest scores.

2.4 Group Work in Middle Schools

Collaborative learning means the grouping and pairing of students working together to achieve a common academic goal (Gokhale, 1995). The students are responsible not only for their own learning, but also for other students’ learning. Hence, the success of one student assists the others to become a successful and this situation ensures individual responsibility (Gokhale, 1995). Exchange of thoughts in small groups increases interests among the students and promotes critical thinking (Gokhale, 1995). Working in groups provides some benefits that teacher centered instruction does not always provide (Koblitz & Wilson, 2014). For example, when students work in groups, they remember information longer than students who work individually (Johnson & Johnson, 1986). According to Bruner (1985), cooperative learning helps students develop problem solving strategies because they encounter different explanations of the given task and internalize external knowledge. Small group work increases conceptual understanding and development of mathematical reasoning skills. In addition, it promotes positive dispositions towards mathematics and procedural fluency (Jansen, 2012). While students in a group are learning a new concept, they might realize what other students in the group do not understand and they can explain that concept to them and correct their misconceptions (Webb & Farivar, 1994). Explaining to peers also enables students to fill in the gaps in their minds and develop their understandings (Fuchs et al., 1997). Furthermore, group work improves students’ social interaction and positive feelings towards peers (Hammond & Barron, 2008). In addition, working in groups enables development of the students’ teamwork skills (Felder & Brent, 1996). According to Fletcher (2008), when the group work is used, algebra learning increases and students' self-efficacy in algebra improves. However, not using group work in the classroom results in decline in confidence, lack of motivation, anxiety towards algebra and passive learning.

Balt (2017) found that there was an increase in 7^{th} grade students' pretest scores to the posttest scores when the small group math instruction was used and most of the students stated that working in small group affected them in a positive way. Jones (2008) found that after 7^{th} grade students engaged in a group work over nine weeks, their conceptual thinking in mathematics improved. In Ünlü and Aydıntan (2011)'s research study, it was concluded that cooperative learning was more effective than traditional instruction on 8^{th} grade students' mathematics achievement and recalling the concept longer. Varank and Kuzucuoğlu (2007) found that posttest scores of 5^{th} grade students who were taught mathematics operations with natural numbers by cooperative learning method were higher than students who were taught by regular instruction. Similarly, Bilgin (2004) found that there was a statistically significant difference between mathematics performances of 7^{th} grade students who received instruction by cooperative learning method and those who received regular instruction. In Hinzman (1997)'s research study, middle school students made comments that using manipulatives in small group work enabled them to work without being embarrassed when they have a difficulty while learning algebraic concepts.

2.5 Research Studies in Turkey

In this section, studies conducted in Turkey about students' learning of algebra topics by using manipulatives and algebra tiles in specific are summarized.

A quasi-experimental research design study conducted to investigate the effects of multiple representations-based instruction on 7^{th} grade students' algebra performance, attitudes toward mathematics, and representation preference (Akkuş, 2004). Participants were 131 7^{th} grade students in two public schools in Ankara in 2003-2004 academic year. While two experimental groups received multiple representations-based instruction, two control groups received regular instruction. During the multiple representations-based instruction, algebra tiles, balance, pattern blocks, marbles, cartoons, cotton buds and activity sheets were used. In order to evaluate students' algebra performance; algebra achievement test, translations among representations skill test, and Chelsea diagnostic algebra test were administered. To learn students' attitudes towards mathematics; mathematics attitude scale and to determine students' representation preferences; representation preference inventory were implemented. In addition, interviews were made with students from experimental and control groups. Results of the study showed that students who took multiple representations-based instruction had higher algebra performance than students who took traditional instruction (Akkuş, 2004).

Palabıyık and Akkuş (2011) conducted a research study to investigate the effects of pattern based and non-pattern based algebra instruction on 7^{th} grade students' algebraic thinking and attitude towards mathematics in a public school in Eskişehir. During the instruction of the experimental group, pattern based activities including algebra tiles, matchstick and pattern blocks as manipulatives were conducted. Control group had the regular instruction based on the Elementary Education Mathematics Curriculum. Researchers implemented Conceptual Algebra Test, Procedural Algebra Test and Attitudes Towards Mathematics Scale on 40 students before and after the instruction. They found that pattern-based instruction had a significant effect on experimental group students' conceptual algebra development. On the other hand, there was no significant difference between the groups in terms of procedural algebra achievement and attitudes towards mathematics.

In a pretest-posttest experimental study conducted to investigate the effects of visualization approach on the 8^{th} grade students' attitudes towards and achievements in mathematics, researchers implemented Mathematics Attitudes Scale and Algebraic Expressions and Equations Achievement Test on students

(Koğ & Başer, 2012). Participants were 43 8th grade students of a middle school in İzmir in the 2010-2011 academic year. In the experimental group, factorisation and first degree equations with one and two unknowns were taught with the help of visualization approach. While factorizing algebraic expressions and modelling them, algebra tiles, computer-assisted visual materials, concept cartoons, metaphors and activity sheets were used. On the other hand, control group students took traditional instruction of the same topic. Researchers found that visualization approach not only affected the students' attitudes towards mathematics but also their mathematics achievement positively (Koğ & Başer, 2012).

Yıldız (2012) conducted a qualitative case study to investigate the views of middle school teachers and students about the use of manipulatives in teaching and learning mathematics. In this study, base-ten blocks, fraction bars, pattern blocks, geoboards, four-pan balance and algebra tiles were used as manipulatives. Participants were four middle school mathematics teachers in a private school and their 6th, 7th, and 8th grade students. Data were collected through one-to-one interviews, observations and analysis of annual plan, daily plan, notebooks of students, and the field notes. According to the findings of the study, most of the middle school students expressed that they desire to learn mathematics by using manipulatives and they stated that in this way they both played and learned. In addition, students claimed that using manipulatives enabled them to have positive attitudes toward mathematics and learn the concepts much better.

Akyüz and Hangül (2013) conducted a research study to investigate and eliminate 6th grade students' misconceptions about first degree equations with one unknown. Participants were 25 6th grade students in a public school in Balıkesir in the spring semester of the 2011-2012 academic year. Researchers implemented a test including 20 open-ended items to detect the misconceptions, and conducted interviews with the students. After that, activity-based instruction was given to students for eight hours and then posttest was given. During the activity-based instruction, algebra tiles, colored papers, balance, seesaw and model plane were used. Researchers found that activity-based instruction was effective in overcoming students' misconceptions. In addition, they suggested that algebra instruction should first begin with concrete materials, and then move towards symbols in order to make students understand algebra concepts better.

Gürbüz and Toprak (2014) conducted a research study to design, implement and evaluate activities that enable 7^{th} grade students to make transition from arithmetic to algebra. Participants were 58 7^{th} grade students in a public school in Gaziantep in 2010-2011 academic year. While activity-based instruction was carried out in the experimental group, regular instruction was given to the control group. During the activity based instructions, materials such as balance, counters and algebra tiles were used. A test consisting of 10 open-ended questions was administered to students before and after the treatment. Results of the study showed that activity-based instruction was more effective than regular instruction.

2.6 Summary of the Literature Review

The review of the literature indicated that students have some difficulties while learning algebra. They have difficulties in understanding the underlying logic in algebra, why they apply an algorithm and therefore, they learn without meaning. Furthermore, students could not visualize their solutions. They have difficulties in understanding multiple functions of letters. Students are not able to make transition from arithmetic to algebra due to overreliance on textbook with procedural focus and teacher-centred instruction. They are inclined to calculate the variables as in arithmetic at first, when they learn algebra. Algebra seems more abstract than arithmetic for the students because they need to think not only a few numbers, but sets of numbers in algebra. Manipulatives can be used in algebra learning process to eliminate these difficulties. Particularly, with the help of algebra tiles, students can make a transition between concrete and symbolic representations of the concepts. By using algebra tiles, students can explore algebraic expressions in a visual and hands-on way and learn the rules from their own experiences. Besides, modelling with algebra tiles enhances students' visualization skills and promotes conceptual understanding. Algebra tiles help students avoid making mistakes and eliminate their confusion between expressions.

In the international literature, studies were conducted with middle school students to teach solving linear equations with one variable or high school students to teach factoring by using algebra tiles. The findings of these studies revealed that using algebra tiles helped students learn the concepts quicker and meaningfully. Moreover, students could build connections between algebraic and geometric concepts. On the other hand, there are not sufficient studies related to using algebra tiles of the students who encounter algebra for the first time. In Turkish literature, there are some studies related to the use of manipulatives in teaching algebra concepts in middle school level. These studies address use of algebra tiles, pattern blocks, balance, colored papers, seesaw, matchstick, and computer-assisted visual materials. The findings of the studies revealed that using these materials positively affected not only mathematics achievement, but also students' conceptual development in algebra and elimination of their misconceptions. However, there are not sufficient studies which examined the effectiveness of use of algebra tiles in the accessible literature.

CHAPTER 3

METHODOLOGY

The aim of the present study was to investigate the effects of using algebra tiles on sixth grade students' algebra achievement and algebraic thinking, and to investigate their views about using algebra tiles. The following research questions were sought through this aim:

1. Is there a statistically significant mean difference between posttest scores of algebra achievement test for 6^{th} grade students who use algebra tiles and those who do not use algebra tiles?
2. How does students' algebraic thinking differ in the algebra achievement test for those who use algebra tiles and who do not use algebra tiles?
3. What are 6^{th} grade students' views about use of algebra tiles in mathematics lessons?

This chapter presents the processes of sampling, data collection and analysis. Details are given below.

3.1 Research Design

In order to find answers to the research questions, both quantitative and qualitative methods were used. The first research question was sought through pretest-posttest control group design because this study mainly investigated the cause–effect relationship (Fraenkel, Wallen, & Hyun, 2011). The Prior Algebra Knowledge Test was implemented as a pretest to see students' existing knowledge about algebraic expressions. Then, students were involved in group work where they experienced algebra topics with algebra tiles. The Algebra Achievement Test was implemented as the posttest at the end of the activities.

Qualitative methodology was used to investigate the second and the third research questions. Both experimental group and control group students' responses in the Algebra Achievement Test were examined in detail to answer the second research question. Since random assignment of the subjects to the groups was impossible in the schools, already existing classes constituted the experimental and control groups which were being taught by the same mathematics teacher, who was the mathematics teacher of the both classes. The pretest was different from the posttest in terms of the included topics and the possible effects of algebra tiles were investigated through only posttest. The third research question was investigated through Views about Algebra Tiles Questionnaire.

3.2 Population and Sample

In this study, target population was all 6^{th} grade students in Sakarya. All 6^{th} grade students who attended public schools in Hendek, Sakarya were the accessible population because Hendek was an accessible area for the researcher. Since reaching all these students and collecting data from them might require considerable time and effort that was not much possible for the researcher at the time of the study, convenience sampling method was used. For this reason, first, the researcher chose one school from 5 public schools in Hendek according to her convenience. In the chosen school, there were six sixth grade classes and 50 sixth grade students from two classes were chosen in the spring semester of 2017-2018 academic year. The classes chosen were the ones that were taught by the same mathematics teacher. One class was assigned as the experimental group and the other class was assigned as the control group randomly. There were one inclusive student in experimental group and two inclusive students in control group and there were also 4 immigrant students in control group. Although these students were implemented the pretest and posttest, they were not included in the sample of the study considering that they were either subject to additional training or that they have not received the previous mathematics education properly due to the language barriers. After these students were removed from the data set, there were 26 students with 12 girls and 14 boys in experimental group and 24 students with 10 girls and 14 boys in control group. These two classes consisted of students of similar mathematics achievement level as the mathematics teacher indicated based on the school mathematics examinations. Students' ages in both groups ranged from 11 to 12. The number of students who took the pretest and the posttest in experimental and control groups is given in Table 3.1. The table shows that a total of 40 students composed the sampling of the study.

Table 3.1 The Number of Students who took the Pretest and the Posttest in Experimental and Control Groups

Groups	Pretest	Posttest	Pretest ∩ Posttest
Experimental	23	23	22
Control	21	20	18
Total	44	43	40

3.3 Context of the Study

The school was a public school which was located in city center of Hendek district of Sakarya. The school was a double-shift school, that is, middle school students (Grades 5 through 8) were at school from morning till noon and primary school students (Grades 1 through 4) were at school from noon till early evening. The school had a population of 501 students. The majority of students were coming from middle socioeconomic level families as indicated by the teacher. The average class size in the school was 25. There were 26 teachers in this school at the time of the study and four of them were mathematics teachers. There were 20 classrooms and their physical conditions were similar. There were double seat desks in the classrooms and each of the class had a smart board and two white boards. There was one science laboratory in the school.

In Turkish Middle Grades Mathematics Curriculum, objectives related to algebra learning area take part at the 6th grade level for the first time and there are several algebra-related objectives in curriculum until the end of the middle school. Thus, algebra is one of the important learning areas of mathematics for the middle school students (MoNE, 2013, 2018).

By the recent revision of middle grades mathematics curriculum in Turkey, it was seen that there are some changes in algebra topics among the grade levels. In 2013 Turkish Middle Grades Mathematics Curriculum, at the 6^{th} grade level, finding asked term in arithmetic sequences, interpretation of algebraic expressions and making operations with algebraic expressions are aimed. At the 7^{th} grade level, it is expected from students to understand equal concept and solve first degree equations in one variable and related problems. In addition, students should be able to learn coordinate system with its properties and linear relationships between variables in different settings, and to draw graphs of linear equations. Algebra learning area takes part at the 8^{th} grade level more than the other grade levels in 2013 curriculum. In this grade level, students learn algebraic expressions and identities, linear equations, systems of equations and inequalities. Moreover, understanding algebraic expressions and identities, and factorising algebraic expressions are expected from students. Examination of linear relationships between variables and solving equations are also included. Middle school algebra topics end with solving systems of equations in two variables and investigating one variable inequalities (MoNE, 2013).

When 2018 Turkish Middle Grades Mathematics Curriculum is compared to 2013 curriculum in terms of algebra learning area, the most striking change is that some of the objectives related to algebraic expressions at the 6^{th} grade level were transferred to the 7^{th} grade level. That is, while at the 6^{th} grade level, students are expected to interpret algebraic expressions; at the 7^{th} grade level, they are required to learn making operations with algebraic expressions and find asked term in arithmetic sequences in 2018 curriculum. Another change is that linear equations and coordinate system topics were transferred to 8^{th} grade level from 7^{th} grade level in 2018 curriculum. Thus, at the 7^{th} grade level, students are expected to understand only equal concept and solve first degree equations in one variable and related problems. In addition to topics in 2013 curriculum, students learn coordinate system and linear equations for the first time at the 8th grade level in 2018 curriculum. Moreover, it is noticed that systems of linear equations in two variables topic was removed from this grade level and it is not included in Turkish Middle Grades Mathematics Curriculum any longer (MoNE, 2018).

In the present study, data collection tools and lesson plans were prepared according to the 2013 curriculum because the 2018 curriculum is implemented to only the 5th grade students. The 6th grade students have been learning mathematics based on the 2013 curriculum at the time of the present study.

3.4 Data Collection Tools

The purpose of this study was to investigate the effects of using algebra tiles on 6th grade students' algebra achievement, algebraic thinking and views about using algebra tiles. A Prior Algebra Knowledge Test, an Algebra Achievement Test and a Views about Algebra Tiles Questionnaire were used in order to gather data.

3.4.1 Prior Algebra Knowledge Test

The Prior Algebra Knowledge Test (PAKT) was an essay type test constructed to learn students' prior knowledge about algebraic expressions by the researcher according to the literature and objectives in Turkish Middle Grades Mathematics Curriculum (MoNE, 2013) (See Appendix A for Turkish version of the questions). The 6th grade objectives in the mathematics curriculum related to algebraic expressions which were covered in the PAKT are given in Table 3.2.

Table 3.2 Sixth Grade Objectives in the Mathematics Curriculum related to Algebraic Expressions which were covered in the PAKT

Algebraic Expressions
1. Students write a phrase as an algebraic expression and write a phrase for a given algebraic expression.
2. Students evaluate an algebraic expression for different values of variable.

PAKT included 4 essay type questions, all with sub-questions. The test including 15 questions altogether was administered to both experimental and control groups as a pretest allowing 40 minutes to learn whether there was an existing difference between the groups in terms of prerequisite knowledge or not before the treatment started.

The objectives of each question in Prior Algebra Knowledge Test are given in Table 3.3.

Table 3.3 Objectives of Each Question in Prior Algebra Knowledge Test

Question	Objectives: Students should be able to
1	write a given phrase as an algebraic expression
2	write a phrase for a given algebraic expression
3	evaluate algebraic expressions for different values of variables
4	a. write a given phrase as an algebraic expression b. evaluate it for a given value of variable

For the construct-related validity of the test, questions were reviewed by two researchers studying in the mathematics education field, one with more than 10 years of experience in teaching middle school mathematics. Their opinions were taken and the test was revised according to these opinions. Each question in the test was analyzed by giving 1 for each correct answer and 0 for each incorrect answer. SPSS 20.0 program was used for the analyses.

3.4.1.1 Pilot Study of Prior Algebra Knowledge Test

The pilot study version of PAKT included a total of 17 sub-questions under 5 main questions. This version of PAKT was piloted on 55 7^{th} grade students in one of the middle schools in Hendek, Sakarya during the 1^{st} semester of 2017-2018 academic year. One question was removed from the test (which is explained below) and the analysis was conducted for the remaining 15 sub-questions under 4 main questions. The sub-questions were scored as 1 for each correct response and 0 for each incorrect response. Therefore, the maximum score that one can have in PAKT is 15 and the minimum score is 0.

The descriptive statistics of the pilot study data is given in Table 3.4. As seen from the Table 3.4, students' mean score in PAKT is 10.96 with standard deviation 3.07. Minimum and maximum scores were computed as 4 and 15 respectively.

Table 3.4 Descriptive statistics of the pilot study of PAKT

	Statistics
N	55
Mean	10.96
Std. Error of Mean	0.41
Median	12
Mode	12
Std. Deviation	3.07
Variance	9.44
Skewness	-.44
Kurtosis	-.82
Range	11
Minimum	4
Maximum	15

The histogram of the pilot study involving the normal curve is given in Figure 3.1. The shape of the distribution was normal for the pilot study of PAKT, as Figure 3.1 shows.

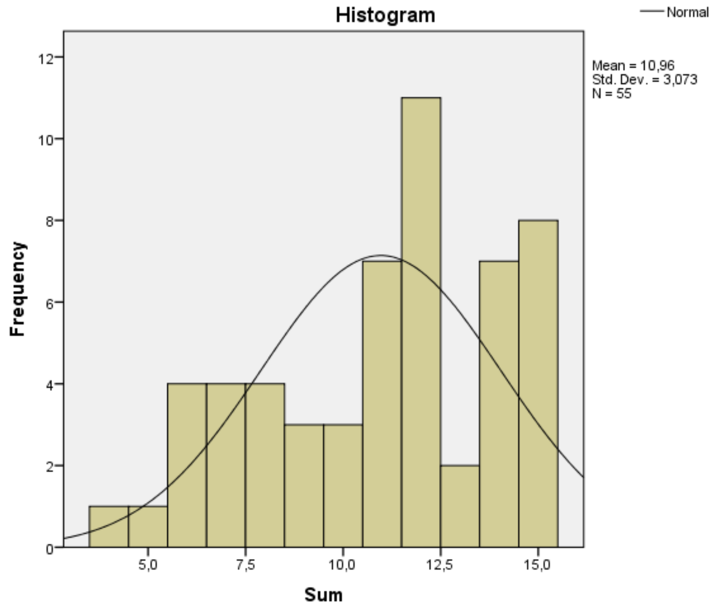
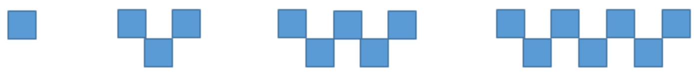


Figure 3.1 Histogram of the pilot study of PAKT

The internal consistency reliability estimate of the test was computed by Kuder-Richardson (KR-21) formula as 0.73 by assuming that all sub-questions were in equal difficulty. Common answers of students in each question are given below in detail along with the questions and sub-questions.

The removed question was about finding the general rule for the given sequence and the number of squares used in the 13th step. The question is given in Figure 3.2 below.

i. Find the general rule for the given sequence including identical squares.



1st step **2nd step** **3rd step** **4th step**

ii. Find the number of squares used in the 13th step of the sequence.

Figure 3.2 Removed Question in PAKT

The question was removed from the test based on the observations made by the researcher while conducting the test in the classrooms, because none of the students could write the general rule of sequence algebraically. Instead, they only wrote "increasing by 2 each time". In addition, to find the number of squares in the 13^{th} step, most of the students wrote the number of squares for each step one by one until the 13^{th} step.

Question 1

Question 1 was about writing a given phrase as an algebraic expression. The question is given in Figure 3.3 below.

1) Write each phrase as an algebraic expression.

3 less than twice a number of candies in the jar ……………………….

12 TL more than half of Aslı's money…………………………………

13 less than a number of Efe's marbles times five……………………..

2 less than a number plus twice the same number……………………..

The amount of remaining time of the exam when 15 minutes of the time completed ……………………

If the sum of two numbers is 80 and one of the numbers is m, the other number is…………………….

Figure 3.3 1^{st} Question in PAKT

There were six sub-questions in question 1. The performance of students in percentage and frequency for each sub-question are given in Table 3.5.

Table 3.5 Frequency and percentage of correct and incorrect answers, and empty responses of question 1 in the pilot study of PAKT

	Correct (%)	**Incorrect (%)**	**Empty (%)**
Q1a	54 (98.2)	1 (1.8)	-
Q1b	35 (63.6)	20 (36.4)	-
Q1c	32 (58.2)	23 (41.8)	-
Q1d	27 (49.1)	28 (50.9)	-
Q1e	46 (83.6)	8 (14.5)	1 (1.8)
Q1f	14 (25.5)	30 (54.5)	11 (20)

Students performed differently in items of question 1. Although the items involved the same objective, the context of the questions seemed to affect students' performances. They had the most difficulty in item Q1f. Most of the students left the item blank or wrote "m" or "40" as an answer. Some students wrote "m+x=80" as an equation form, but they could not write the answer as "80-m."

Question 2

Question 2 was about writing a phrase for a given algebraic expression. The question is given in Figure 3.4 below.

2) Write a phrase for each algebraic expression given below.

$5(c-2)$……………………………………………………..

$\frac{m+1}{2}$…………………………………………………………

$7k-6$……………………………………………………….

$\frac{x}{2}+5$……………………………………………………..

Figure 3.4 2nd Question in PAKT

There were four sub-questions in question 2. The performance of students in percentage and frequency for each sub-question are given in Table 3.6.

Table 3.6 Frequency and percentage of correct and incorrect answers, and empty responses of question 2 in the pilot study of PAKT

	Correct (%)	Incorrect (%)	Empty (%)
Q2a	42 (76.4)	11 (20)	2 (3.6)
Q2b	35 (63.6)	18 (32.7)	2 (3.6)
Q2c	42 (76.4)	11 (20)	2 (3.6)
Q2d	47 (85.5)	7 (12.7)	1 (1.8)

In the 2nd question, among four items, it was seen that the fewest correct answers were given to item Q2b because some students wrote the phrase "half of a number, plus 1" instead of "half of the sum of a number and 1" for $\frac{m+1}{2}$.

Question 3

Question 3 was about evaluating algebraic expressions for different values of variables. The question is given in Figure 3.5 below.

3) Evaluate each algebraic expression given below for a given value of variables.

- $\frac{2(n-3)}{5}$ **for n=13**
- $\frac{3x+4}{2}$ **for x=6**
- $\frac{85}{y} + 1$ **for y=5**

Figure 3.5 3rd Question in PAKT

There were three sub-questions in question 3. The performance of students in percentage and frequency for each sub-question are given in Table 3.7.

Table 3.7 Frequency and percentage of correct and incorrect answers, and empty responses of question 3 in the pilot study of PAKT

	Correct (%)	Incorrect (%)	Empty (%)
Q3a	47 (85.5)	8 (14.5)	-
Q3b	43 (78.2)	11 (20)	1 (1.8)
Q3c	46 (83.6)	7 (12.7)	2 (3.6)

In question 3, among three items, the fewest correct answers were given to item Q3b. While evaluating the algebraic expression $\frac{3x+4}{2}$ for x=6, some students have failed to realize that 3 is a coefficient of x, and so, it is required to multiply 6 by 3. Instead, they put 6 in the place of x and wrote 36. Then, they added 4 to 36 and got 40. After that, they divided 40 by 2 and found the answer as 20.

Question 4

Question 4 was about writing a given phrase as an algebraic expression and evaluating it for a given value of variable. The question is given in Figure 3.6 below.

4)

i. **Write an algebraic expression for the phrase "7 more than 3 times a number of fishes in the aquarium."**

ii. **Evaluate the algebraic expression you wrote in i, when the variable is equal to 15.**

Figure 3.6 4th Question in PAKT

There were two sub-questions in question 4. The performance of students in percentage and frequency for each sub-question are given in Table 3.8.

Table 3.8 Frequency and percentage of correct and incorrect, and empty responses answers of question 4 in the pilot study of PAKT

	Correct (%)	**Incorrect (%)**	**Empty (%)**
Q4a	51 (92.7)	3 (5.5)	1 (1.8)
Q4b	41 (74.5)	9 (16.4)	5 (9.1)

In question 4, items Q4a and Q4b were dependent on each other. That is, to give an answer to item Q4b, students need to answer item a first. Most of the students answered item Q4a correct. Some of the students who answered item Q4a, left item Q4b blank. A few students wrote "3x+7=15" instead of evaluating the algebraic expression 3x+7 for x=15. This can be due to the fact that these students did not know the meaning of the word *variable* which took place in the question.

The detailed analysis of the responses to the tasks in the pilot study of PAKT showed that 7th grade students could not perform well in general because of lack of their prior knowledge in algebraic expressions. After the pilot study, one question was removed from the test and no changes were made in the rest of the questions in PAKT.

3.4.2 Algebra Achievement Test

Algebra Achievement Test (AAT) was developed to investigate 6th grade students' algebra achievement and algebraic thinking. Two questions (7th and 10th) in the test were taken from"Chelsea Mathematics Diagnostic Tests – Algebra" developed by Hart, Küchemann, Brown, Kerslake and Ruddock (1985) and adapted to Turkish by Altun (2005), and they were modified by the researcher for the purposes of the study. Other questions were developed by the researcher according to the literature and objectives in the Turkish Middle Grades Mathematics Curriculum (MoNE, 2013) (See Appendix B for Turkish version of the questions). The 6th grade objectives in the mathematics curriculum related to algebraic expressions which were covered in AAT are given in Table 3.9.

Table 3.9 Sixth Grade Objectives in the Mathematics Curriculum related to Algebraic Expressions which were covered in AAT

Algebraic Expressions
1. Students express the meaning of simple algebraic expressions.
2. Students make addition and subtraction in algebraic expressions.
3. Students multiply an algebraic expression with a natural number.

The test consisted of 11 essay type questions, 6 of which included sub-questions. The test including 35 questions altogether was administered to both experimental and control groups as a posttest allowing 40 minutes. The objectives of each question in AAT are given in Table 3.10.

Table 3.10 Objectives of Each Question in AAT

Question	Objectives: Students should be able to
1	a. determine whether given representations are correct or incorrect b. rewrite incorrect representations as correct representations
2	write algebraic expressions for the given models
3	determine variable, term, constant term, coefficients and sum of coefficients of given algebraic expressions
4	find the perimeter of a given rectangle in terms of algebraic expressions
5	a. write algebraic expressions for the given models b. perform operations with algebraic expressions c. model the results of operations
6	perform operations for the given algebraic expressions
7	find the perimeter of the polygon in terms of algebraic expressions whose number of the side is unknown
8	explain which representation is correct
9	write given algebraic expressions as multiplication of a natural number and an algebraic expression
10	explain which algebraic expression is greater
11	find the length of one side of the square in terms of algebraic expressions

Questions in the AAT included both sixth grade objectives related to algebraic expressions in Turkish Middle Grades Mathematics Curriculum (MoNE, 2013) and items targeting algebraic thinking in ways that are not covered in the curriculum objectives. In addition, objectives in the curriculum were divided into sub-objectives. Accordingly, 1^{st} and 2^{nd} questions were for the objective "Students express the meaning of simple algebraic expressions." The 3^{rd}, 4^{th}, 5^{th}, and 6^{th} questions were for the objective "Students make addition and subtraction in algebraic expressions." The 8^{th} question was for the objective "Students multiply an algebraic expression with a natural number." The 7^{th}, 9^{th}, 10^{th}, and 11^{th} questions were not directly placed under the algebraic expressions objectives in the curriculum. They addressed students' algebraic thinking based on their existing knowledge in algebraic expressions.

For the construct-related validity of the test, questions were reviewed by two researchers studying in the mathematics education field, one with more than 10 years of experience in teaching middle school mathematics. Their opinions were taken and test was revised according to these opinions. Each question in the test was analyzed by giving 1 for each correct answer and 0 for each incorrect answer. SPSS 20.0 program was used for the analyses.

3.4.2.1 Pilot Study of Algebra Achievement Test

Prior to the main study, the test was piloted on 52 7^{th} grade students in one of the middle schools in Hendek, Sakarya during the 1^{st} semester of 2017-2018 academic year because 7^{th} grade students had already learned the 6^{th} grade topics in the previous academic years. The analysis was conducted for 35 sub-questions under 11 main questions. The sub-questions were scored as 1 for each correct response and 0 for each incorrect response. Therefore, the maximum score that one can have in AAT is 35 and the minimum score is 0.

The descriptive statistics of the pilot study data is given in Table 3.11. As seen from the Table 3.11, students' mean score in Algebra Achievement Test is 14.92 with standard deviation 3.47. Minimum and maximum scores were computed as 8 and 24 respectively.

Table 3.11 Descriptive statistics of the pilot study of AAT

	Statistics
N	52
Mean	14.92
Std. Error of Mean	0.48
Median	15
Mode	14
Std. Deviation	3.47
Variance	12.07
Skewness	.15
Kurtosis	-.39
Range	16
Minimum	8
Maximum	24

The histogram of the pilot study involving the normal curve is given in Figure 3.7. The shape of the distribution was normal for pilot study of AAT, as Figure 3.7 shows.

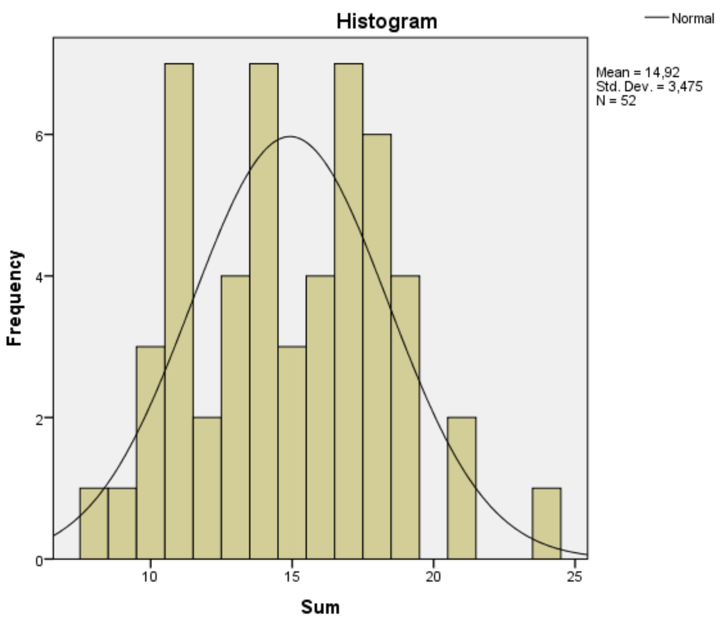


Figure 3.7 Histogram of the pilot study of AAT

The internal consistency reliability estimate of the test was computed by Kuder-Richardson (KR-21) formula as 0.62 by assuming that all sub-questions were in equal difficulty. Common answers of students in each question are given below in detail along with the questions and sub-questions. Although the questions were not given with sub-question indicators such as 1a and 1b, the findings below are given with these indicators in the order of the sub-questions in order to report the findings more clear.

Question 1

Question 1 was about determining whether given representations are correct or incorrect and rewriting incorrect representations as correct representations. The question is given in Figure 3.8 below.

1) Determine whether given representations are correct or incorrect and rewrite incorrect representations as correct representations.

….. $y + y + 1 = 3y$

….. $x + x - 1 = -1 + 2x$

….. $\frac{a}{2} + \frac{a}{2} = 2a$

….. $5 - c - c + c = 5 - 3c$

Figure 3.8 1st Question in AAT

There were four sub-questions in question 1. The performance of students in percentage and frequency for each sub-question are given in Table 3.12.

Table 3.12 Frequency and percentage of correct and incorrect answers, and empty responses of question 1 in the pilot study of AAT

	Correct (%)	**Incorrect (%)**	**Empty (%)**
Q1a	40 (76.9)	12(23.1)	-
Q1b	49 (94.2)	2 (3.8)	1 (1.9)
Q1c	26 (50)	23 (44.2)	3 (5.8)
Q1d	38 (73.1)	12 (23.1)	2 (3.8)

In the1st question, among four items, the fewest correct answers were given to item Q1c. It can be said that half of the students had difficulty in solving problems involving fractions because given algebraic expression involved fraction in item Q1c.

Question 2

Question 2 was about writing algebraic expressions for the given models. The question is given in Figure 3.9 below.

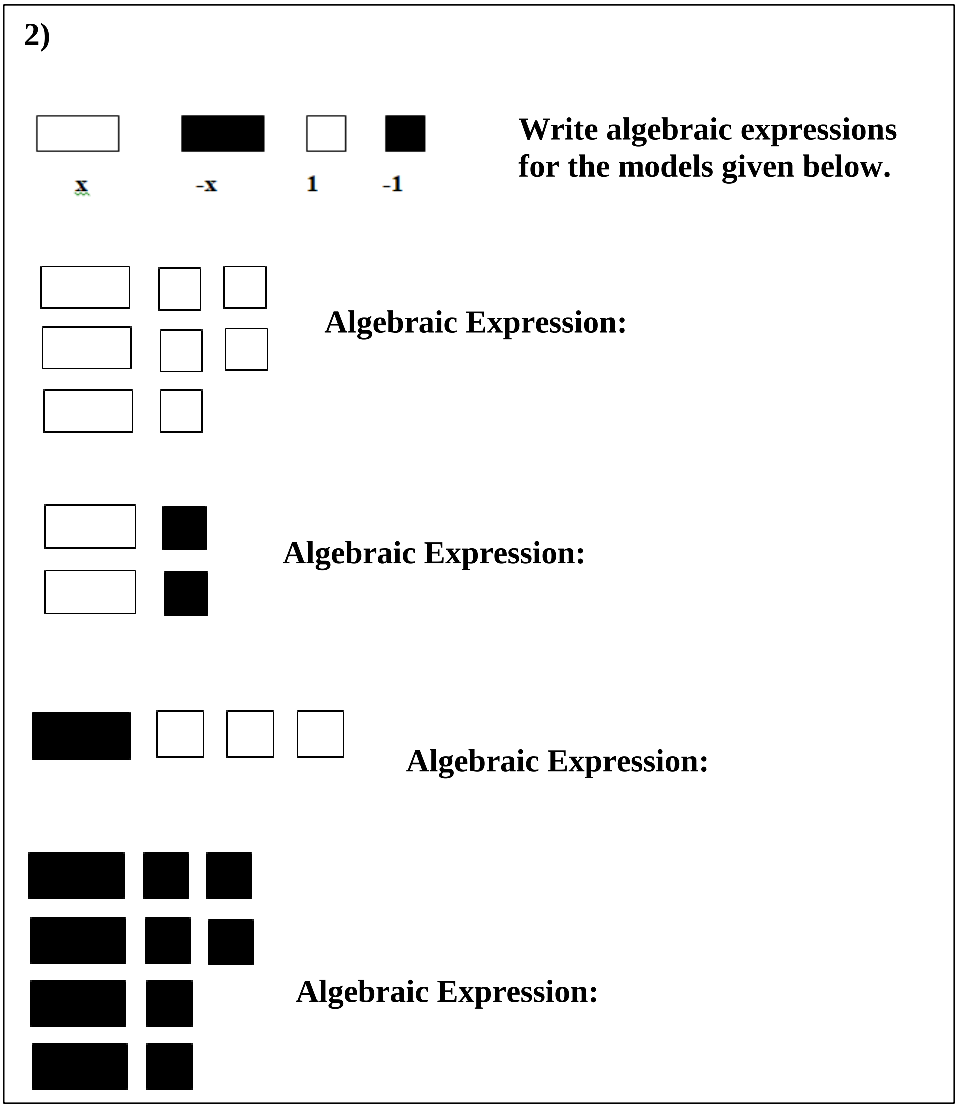


Figure 3.9 2nd Question in AAT

There were four sub-questions in question 2. The performance of students in percentage and frequency for each sub-question are given in Table 3.13. Based on Table 3.13, it can be said that students performed better in writing algebraic expressions for the given models.

Table 3.13 Frequency and percentage of correct and incorrect answers, and empty responses of question 2 in the pilot study of AAT

	Correct (%)	Incorrect (%)	Empty (%)
Q2a	51 (98.1)	1(1.9)	-
Q2b	44 (84.6)	8 (15.4)	-
Q2c	50 (96.2)	2 (3.8)	-
Q2d	48 (92.3)	4 (7.7)	-

Question 3

Question 3 was about determining variable, term, constant term, coefficients and sum of coefficients of given algebraic expressions. The question is given in Figure 3.10 below.

3) Determine variable, term, constant term, coefficients and sum of coefficients of each algebraic expression given below.

Algebraic Expression	Variable (s)	Term(s)	Constant Term (s)	Coefficient (s)	Sum of Coefficient (s)
3k					
–6xy +1					
2a + 5b – 8					

Figure 3.10 3rd Question in AAT

There were fifteen sub-questions in question 3. The performance of students in percentage and frequency for each sub-question are given in Table 3.14.

Table 3.14 Frequency and percentage of correct and incorrect answers, and empty responses of question 3 in the pilot study of AAT

	Correct (%)	**Incorrect (%)**	**Empty (%)**
Q3aa	30 (57.7)	11 (21.2)	11 (21.2)
Q3ab	18 (34.6)	18 (34.6)	16 (30.8)
Q3ac	8 (15.4)	28 (53.8)	16 (30.8)
Q3ad	26 (50)	6 (11.5)	20 (38.5)
Q3ae	22 (42.3)	8 (15.4)	22 (42.3)
Q3ba	28 (53.8)	12 (23.1)	12 (23.1)
Q3bb	3 (5.8)	33 (63.5)	16 (30.8)
Q3bc	14 (26.9)	25 (48.1)	13 (25)
Q3bd	8 (15.4)	22 (42.3)	22 (42.3)
Q3be	8 (15.4)	24 (46.2)	20 (38.5)
Q3ca	26 (50)	14 (26.9)	12 (23.1)
Q3cb	4 (7.7)	31 (59.6)	17 (32.7)
Q3cc	14 (26.9)	24 (46.2)	14 (26.9)
Q3cd	8 (15.4)	20 (38.5)	24 (46.2)
Q3ce	9 (17.3)	20 (38.5)	23 (44.2)

It was seen that many students had difficulty in determining terms in algebraic expressions (items Q3ab, Q3bb and Q3cb). Some students confused terms with variables. Some students did not accept constant term as a term. For example, for the algebraic expression -6xy+1, they wrote "-6xy" as a term. On the other hand, some students wrote coefficients in the algebraic expressions as terms. In addition, it was noticed that most of the students failed to write constant terms (items Q3ac, Q3bc and Q3cc). Some students wrote coefficients as constant terms. For instance, for the algebraic expression 2a+5b-8, they identified 2, 5, -8 as constant terms. Or, some students wrote variables as constant terms. Moreover, some students did not take constant terms as coefficients (items Q3bd, Q3be, Q3cd and Q3ce). Or, some students who admitted constant terms as coefficients did not pay attention to the negative sign of the numbers. For example, for the algebraic expression 2a+5b-8, they wrote 2, 5, 8 as coefficients. Since they could not identify coefficients correctly, they could not calculate the sum of coefficients correctly. Results from the 3rd question show that students could not completely understand variable, term, constant term and coefficient concepts in the pilot study.

Question 4

Question 4 was about finding the perimeter of a given rectangle in terms of algebraic expressions. The question is given in Figure 3.11 below.

4) Find the perimeter of a rectangle whose length is 3cm less than the width in terms of algebraic expression.

Figure 3.11 4^{th} Question in AAT

There were not any sub-questions in question 4. The performance of students in percentage and frequency for the question are given in Table 3.15.

Table 3.15 Frequency and percentage of correct and incorrect answers, and empty responses of question 4 in the pilot study of AAT

	Correct (%)	Incorrect (%)	Empty (%)
Q4	5 (9.6)	24 (46.2)	23 (44.2)

Only 5 students responded the 4^{th} question correctly. Some students could write x-3 as one side of a rectangle but they could not find the perimeter of the rectangle.

Question 5

Question 5 was about writing algebraic expressions for the given models, performing operations with algebraic expressions, and modeling the results of operations. The question is given in Figure 3.12 below.

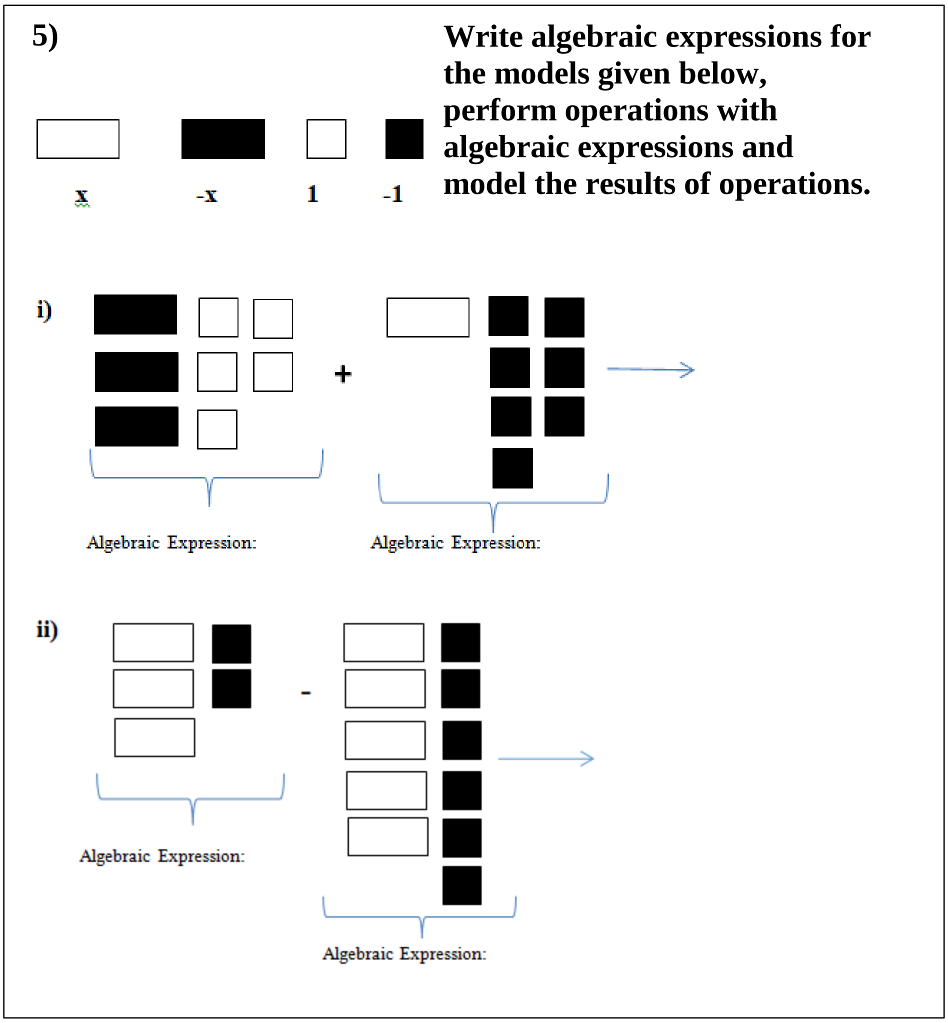


Figure 3.12 5th Question in AAT

There were two sub-questions in question 5. The performance of students in percentage and frequency for each sub-question are given in Table 3.16.

Table 3.16 Frequency and percentage of correct and incorrect answers, and empty responses of question 5 in the pilot study of AAT

	Correct (%)	Incorrect (%)	Empty (%)
Q5a	22 (42.3)	26 (50)	4 (7.7)
Q5b	2 (3.8)	47 (90.4)	3 (5.8)

Although most of the students wrote algebraic expressions for the given models, they failed to perform addition and subtraction with algebraic expressions. In particular, in item Q5b, only two students performed subtraction correctly. The reason for students' incorrect answer seemed to be ignoring distributing negative sign.

Question 6

Question 6 was about performing operations for the given algebraic expressions. The question is given in Figure 3.13 below.

6) Perform operations for the algebraic expressions given below.

i) (4x–5) + (–2x+3)

ii) (x+3) – (–2x – 1)

Figure 3.13 6^{th} Question in AAT

There were two sub-questions in question 6. The performance of students in percentage and frequency for each sub-question are given in Table 3.17.

Table 3.17 Frequency and percentage of correct and incorrect answers, and empty responses of question 6 in the pilot study of AAT

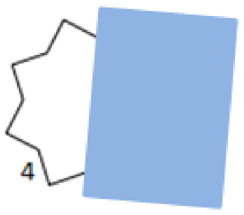
	Correct (%)	Incorrect (%)	Empty (%)
Q6a	33 (63.5)	18 (34.6)	1 (1.9)
Q6b	7 (13.5)	42 (80.8)	3 (5.8)

In 6^{th} question, in item Q6a, the majority of students performed addition for given algebraic expressions correctly. On the other hand, in item Q6b, most of the students could not perform subtraction correctly. Some students distributed first negative sign but they ignored distributing second negative sign while subtracting. They wrote "(x+3)-(-2x-1) = x+3+2x-1" and found "3x+2".

Question 7

Question 7 was about finding the perimeter of the polygon in terms of algebraic expressions whose number of the side is unknown. The question is given in Figure 3.14 below.

7)



Assume that one part of the regular polygon, whose the length of one side is 4 unit and number of the side is unknown, is covered by paper. Find the perimeter of the polygon in terms of algebraic expression.

Figure 3.14 7^{th} Question in AAT

There were not any sub-questions in question 7. The performance of students in percentage and frequency for the question are given in Table 3.18.

Table 3.18 Frequency and percentage of correct and incorrect answers, and empty responses of question 7 in the pilot study of AAT

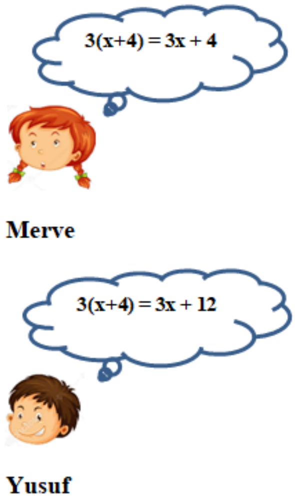
	Correct (%)	**Incorrect (%)**	**Empty (%)**
Q7	12 (23.1)	21 (40.4)	19 (36.5)

Some students identified the number of side of a polygon as x or n, but they could not write the perimeter of the polygon. Since eight sides of the polygon were apparent, some students wrote 32 by multiplying 8 and 4. Some of the students tried to draw sides to complete a given polygon.

Question 8

Question 8 was about explaining which representation is correct. The question is given in Figure 3.15 below.

8)



Merve and Yusuf says the equivalent algebraic expression of 3(x+4) as shown in the picture. Explain which representation is correct.

Figure 3.15 8th Question in AAT

There were not any sub-questions in question 8. The performance of students in percentage and frequency for the question are given in Table 3.19.

Table 3.19 Frequency and percentage of correct and incorrect answers, and empty responses of question 8 in the pilot study of AAT

	Correct (%)	Incorrect (%)	Empty (%)
Q8	41 (78.8)	7 (13.5)	4 (7.7)

Most of the students responded 8th question correctly. The reason of why seven students thought that Merve represented 3(x+4) = 3x+4 correctly might be having difficulty in applying distributive property when there were variables instead of numbers inside the parentheses.

Question 9

Question 9 was about writing given algebraic expressions as multiplication of a natural number and an algebraic expression. The question is given in Figure 3.16 below.

9) Write each algebraic expression given below as multiplication of a natural number and an algebraic expression.

- **6x + 8**
- **9 – 3x**
- **–2x – 10**

Figure 3.16 9^{th} Question in AAT

There were three sub-questions in question 9. The performance of students in percentage and frequency for each sub-question are given in Table 3.20.

Table 3.20 Frequency and percentage of correct and incorrect answers, and empty responses of question 9 in pilot study of AAT

	Correct (%)	Incorrect (%)	Empty (%)
Q9a	21 (40.4)	26 (50)	5 (9.6)
Q9b	18 (34.6)	26 (50)	8 (15.4)
Q9c	17 (32.7)	28 (53.8)	7 (13.5)

Some of the students had difficulty in understanding what was asked in the 9^{th} question. These students either left the question blank or tried to multiply the given algebraic expressions with any arbitrary number. In addition, some students wrote “6x+8=6(x+8)” probably because they thought that the number outside the parenthesis was multiplied by only the first term inside the parenthesis and it can be said that these students did not know distributive property well. Few students wrote variable outside the parenthesis and coefficients inside it such as “9-3x = x(9-3).” In item Q9c, some students ignored writing given algebraic expressions as multiplication of a natural number and an algebraic expression and they wrote “-2x-10= -2(x+5).” Results showed that distributive property concept was not completely comprehended by the students.

Question 10

Question 10 was about explaining which algebraic expression is greater. The question is given in Figure 3.17 below.

> **10) When you compare 3n and (n+3) algebraic expressions for different values of n, which algebraic expression is greater? Explain.**

Figure 3.17 10th Question in AAT

There were not any sub-questions in question 10. The performance of students in percentage and frequency for the question are given in Table 3.21.

Table 3.21 Frequency and percentage of correct and incorrect answers, and empty responses of question 10 in pilot study of AAT

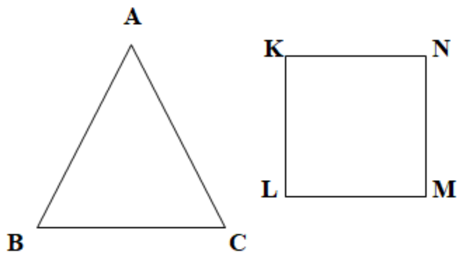
	Correct (%)	Incorrect (%)	Empty (%)
Q10	4 (7.7)	27 (51.9)	21 (40.4)

Only 4 students responded the 10th question correctly. Most of the students wrote that 3n was greater than n+3 because the operation in 3n was multiplication, but the operation in n+ 3 was addition. Few students wrote that they were equal to each other but they could not explain why it was so. Some students evaluated given algebraic expressions for only one value and according to the result of this evaluation, they wrote one was greater than other.

Question 11

Question 11 was about finding the length of one side of the square in terms of algebraic expressions. The question is given in Figure 3.18 below.

11)



The perimeters of ABC equilateral triangle and KLMN square are equal to each other. If the length of one side of the triangle is 8a, find the length of one side of the square.

Figure 3.18 11th Question in AAT

There were not any sub-questions in question 11. The performance of students in percentage and frequency for the question are given in Table 3.22.

Table 3.22 Frequency and percentage of correct and incorrect answers, and empty responses of question 11 in pilot study of AAT

	Correct (%)	Incorrect (%)	Empty (%)
Q11	24 (46.2)	23 (44.2)	5 (9.6)

In the 11th question, most of the students who gave incorrect answer wrote only 6 as an answer instead of 6a.

The detailed analysis of the responses to the tasks in the pilot study of AAT showed that 7th grade students had lower performance in AAT than PAKT. It was seen that since 7th grade students learned algebraic expressions topic in the 6th grade, approximately 8 months before implementing the test, they seemed to have forgotten the concepts. Particularly, most of the students could not perform subtraction with algebraic expressions. In addition, the reason of students' difficulty can be lack of their prior knowledge in fractions, geometry concept and distributive property. After the pilot study, none of the questions were removed or changed in AAT.

3.4.3 Views about Algebra Tiles Questionnaire

The Views about Algebra Tiles Questionnaire (VATQ) included 5 open-ended questions and was developed to learn students' views about using algebra tiles during the instructions (See Appendix C). The questions in the VATQ were related to the materials students have used so far, whether using algebra tiles helped them understand the topic, the difficulties they encountered while using algebra tiles during the instructions, whether group work helped them learn algebraic expressions, and their comments about and suggestions for the instructions including use of algebra tiles.

The items in the questionnaire were shared by mathematics teachers and mathematics education researchers to ensure the content validity. Although there was no pilot study for this instrument, two sixth grade students who were not in the experimental and control group were asked to read the questions in VATQ to determine the clarity of the questions. The questionnaire was administered to only experimental group students after the treatment, allowing 20-30 minutes.

3.5 Procedure and Treatment

Prior Algebra Knowledge Test was administered to both experimental control groups before the treatment. Students were implemented this test after they learned writing a phrase as an algebraic expression, writing a phrase for a given algebraic expression, and evaluating algebraic expressions for different values of variables.

During the treatment, while the experimental group learned expressing the meaning of simple algebraic expressions, addition and subtraction in algebraic expressions and multiplying an algebraic expression with a natural number by using algebra tiles throughout seven class hours in three weeks; algebra tiles were not used in the control group for the same objectives. Algebra tiles are rectangle and small square with two different colours, one represents positive and the other represents negative.

In the experimental group, instruction about how the algebra tiles can be used was given to the teacher by the researcher before the study. Then, the researcher shared the lesson plans she prepared with the teacher. For the appropriateness of the lesson plans, lesson plans were reviewed by two researchers studying in the mathematics education field, one with more than 10 years of experience in teaching middle school mathematics. In the experimental group, students discovered the rules in operations with algebraic expressions themselves with the help of algebra tiles. For this purpose, first of all, students modeled algebraic expressions by using algebra tiles. They used algebra tiles in groups in order to make effective use of algebra tiles to provide as many pieces for each of 1, x, -1 and –x as possible. Then, they drew pictures that represented algebra tiles. Finally, they wrote their work by using algebraic notation and reached the rules. The purpose of this instruction was to make students perform a transition from concrete representations to abstract concept. In addition, they did not depend on the tiles to perform operations with algebraic expressions at the end of the lesson. In addition, exit cards were given to the students for each of three objectives after they achieved them at the end of the lessons. Questioning, discussion, cooperative learning and individual work were used as instructional techniques. For the objectivity of the study, instruction by using algebra tiles was delivered by the mathematics teacher in the school. However, during the instruction, the researcher was in the classroom and she observed the class to ensure that the treatment proceeded as intended in the lesson plans that the researcher prepared.

On the other hand, in control group, algebra tiles were not used as concrete manipulatives and regular instruction took place in the classroom. However, the teacher sometimes drew algebra tiles on the board while explaining the topics. The teacher used direct instruction, drill and practice and questioning as instructional techniques. After she explained the topics, she solved examples related to them. When the students did not understand, she explained again. Then, she wrote some questions on the board and asked students to solve them.

The researcher also observed this class in order to make sure that the teacher was conducting the regular instruction.

At the end of the treatment, Algebra Achievement Test was administered to both experimental and control groups. Furthermore, the views of students in experimental group about use of algebra tiles were gathered by the questionnaire. The treatment in both experimental and control groups were explained in detail below.

3.5.1 Experimental Group Treatment

In experimental group, three objectives were accomplished with the help of algebra tiles and lesson plans prepared by the researcher were used (See Appendix D). These objectives were that students should be (i) able to express the meaning of simple algebraic expressions, (ii) make addition and subtraction in algebraic expressions and (iii) multiply an algebraic expression with a natural number respectively.

3.5.1.1 The First Objective

The first objective was that students should be able to express the meaning of simple algebraic expressions. For this objective, two class hours were allocated. Lessons included five phases as engagement, exploration, explanation, elaboration and evaluation respectively.

At the beginning of the lesson, in the engagement part, the teacher said that expressing the meaning of simple algebraic expressions would be learned. Then, she introduced algebra tiles to the students by sticking them on the board. Algebra tiles were described as rectangle and small square with two different colours. Rectangle represented x and square represented 1, and the red ones represented positive and the blue ones represented negative. She stated that the blue ones were additive inverses for their counterparts and a zero pair was created when used together. She emphasized that algebra tiles included both algebra and geometry because area of rectangle was x and area of square was 1. She said that algebra tiles would be used to understand algebraic expressions better by visualizing.

After the algebra tiles were introduced, in exploration part, model of algebra tiles was stuck to the board by the teacher and students were expected to write an algebraic expression for the given model. Students raised their hands and gave different answers such as 3x, 3. Then, they discussed answers as a whole class and agreed that answer was 2x+1. The same process was repeated for another model of algebra tiles. Next, students were invited to work in pairs and algebra tiles were distributed to the pairs. The teacher wrote algebraic expressions (3x-2 and -5x+6) on the board and told students to model the given algebraic expressions by using algebra tiles. After groups finished modelling, they showed the models to the teacher and one group for each of algebraic expression came to the board and stuck algebra tiles on the board to show their answers. She asked students to think about 3x-2 algebraic expression again by showing modelling of it on the board and she asked "*How can we represent this algebraic expression differently?"* Students discussed and some of them gave answers such as 3n-2, 4x-3.

In the explanation part, the teacher explained that 3x-2 algebraic expression can also be written as x+x+x-1-1. After the students understood, she asked students to represent other algebraic expressions on the board differently. For instance, for algebraic expression -5x+6; students said that it comprised of five times minus x and six times plus one.

In the elaboration part, algebra tiles were collected from the students and the activity sheet was distributed to them. The teacher asked students to write different representations of given algebraic expressions individually. Students had difficulty in representing algebraic expressions including fractions. For this reason, she gave some clues and asked leading questions. For example, *"How do we subtract factions with same denominator?"* and *"How do we multiply fractions?"* After the students completed the activity, different students came to the board and explained results for each item in the activity sheet. For some of the items, students gave different answers and they were all correct. For instance, students represented the algebraic expression $\frac{2a}{3}$ as $\frac{2.a}{3}$, $2.\frac{a}{3}$, $\frac{2}{3}$.a.

In the evaluation part, the exit card was distributed to the students. Since they have encountered such kind of material for the first time, they were surprised. They asked questions such as *"Are we going to write an arbitrary algebraic expression?"* and *"Can we colour algebra tiles that we drew?"* The teacher explained what was expected from them. After they completed, they gave exit cards to the teacher while leaving the classroom.

3.5.1.2 The Second Objective

The second objective was that students should be able to make addition and subtraction in algebraic expressions. For this objective, three class hours were allocated. Lessons included five phases as engagement, exploration, explanation, elaboration and evaluation respectively.

In the engagement part, the previous lesson was reviewed and algebra tiles were stuck on the board by the teacher. By showing one rectangle and one square piece in a different color, the teacher asked *"Do these two pieces cancel each other and create zero pair?"* Students said *"No because one represents a variable and the other represents a number, and they are different from each other."* The teacher formed groups in the way that there were four students in each group and groups were heterogeneous in terms of ability level of students. Next, algebra tiles were distributed to the groups.

In the exploration part, the teacher stuck the algebra tiles modelling (2x+3) + (x+1) operation on the board and asked groups *"How can we make this operation?"* Students responded that *"We will add similar shapes together."* While groups were making addition with algebra tiles, she walked around the desks and helped students if they needed. After the groups finished, the teacher showed the result of operation by sticking algebra tiles. One student from each group came to the board and explained how they performed the addition as a group. Most of them explained that *"We added xs between each other and ones between each other."* Later, the teacher drew a rectangle with the length of long side as 2x+1 and length of short side as 1 on the board. She said *"An ant started walking from one corner around the rectangle and then it returned to that corner. How much distance did the ant walk?"* Some of the students raised their hands and explained their answers to the class. One student said *"I will add 2x+1 and 2x+1, 1 and 1, then I will add all of the similar ones."* Another student said *"I will multiply 2x+1 by 2 and 1 by 2, then I will add all of the similar ones."* After these explanations, the teacher asked students to find an answer of the problem as a group. After the groups finished, one student from each group explained. One student said *"Firstly, we added four xs and got 4x, then we added ones."* Another student said *"Firstly, we added the length of long sides, and then we added the length of short sides. Finally, we added all of the similar ones."* Then, the teacher stuck the algebra tiles modelling (-2x+5) + (x-4) operation on the board and asked groups to model this operation by using algebra tiles and find the result. After the groups finished, they showed models to the teacher and one student from each group explained how they made addition as a group. One student said *"+x and –x cancelled each other and –x remained. +4 and -4 cancelled each other and +1 remained. Therefore, result is –x+1."* She also showed the result of operation by sticking algebra tiles.

Next, the teacher asked *"If the operation was subtraction, how would we perform it by using algebra tiles?"* After the groups discussed, only one group could give an answer. They said *"We will convert subtraction into addition. Then, in second algebraic expression, red ones will be blue and blue ones will be red."* The teacher stuck the algebra tiles modelling (2x+2) - (x+1) operation on the board and asked groups to model this operation by using algebra tiles and find the result. After the groups finished, they showed models to the teacher and all groups could perform correctly. She showed once again how to perform the operation by using algebra tiles on the board. This time, the teacher stuck the algebra tiles modelling (2x-3) - (-3x+2) operation on the board and asked groups to model this operation by using algebra tiles and find the result. The groups wanted to find result immediately. The teacher said *"You are not required to find the result immediately. Firstly, you should convert subtraction into addition by using algebra tiles."* After the groups finished, they showed models to the teacher. She showed once again how to perform operation by using algebra tiles on the board and also represented operation algebraically. Finally, she stuck the algebra tiles modelling (2x-3) - (2x+1) operation on the board and asked groups to model this operation by using algebra tiles and find the result. While the groups were working, some of the students asked *"Can we explain to our group mates who did not understand?"* and the teacher said *"yes."* Then, she asked *"Who wants to come to the board and show?"* and one student among the students who raised their hands came to the board and showed by explaining. The teacher also showed how to represent operation algebraically.

Next, the teacher asked students *"How did you make addition and subtraction in algebraic expressions?"* One student said *"I brought together and added."* Another student said *"When I added positive and negative, they cancelled each other."* Another student said *"I converted subtraction into addition."* Then, the teacher asked *"Can we make a connection between addition-subtraction in algebraic expressions and addition-subtraction in integers?"* One student said *"We convert subtraction into addition in both algebraic expressions and integers."* Then, she asked *"How can we make addition-subtraction in algebraic expressions without using algebra tiles?"* One student answered *"We convert subtraction into addition and change the signs of subtrahend."*

In the explanation part, the teacher explained how to make addition and subtraction in algebraic expressions and gave the definition of term, like term, constant term, and coefficient. She wrote 2x+3 on the board and showed the terms, coefficients, constant term and variable of this algebraic expression. One student asked *"Is 3 both a coefficient and a constant term?"* The teacher said *"Yes because it is the coefficient of itself."* Then, the teacher asked *"While* *writing coefficients should we write negative signs in front of the coefficients?"* Some students said *"No"* and the teacher asked *"What is the coefficient of -3x?"* Some of them said 3 and she asked *"What is the coefficient of 3x?"* They said 3 again. The teacher asked *"Are algebraic expressions -3x and 3x same algebraic expressions?"* Students said *"No"* and they understood that the coefficient of -3x is -3.

In the elaboration part, algebra tiles were collected from the students and the activity sheet was distributed to them, and the teacher asked students to fulfill given table individually. After the students completed, the teacher asked students for correct answer in each blank provided. Students said what they wrote and they discussed. According to the students' answers, teacher wrote the correct answers on the board. Then, another activity sheet was distributed to the students and they were expected to make addition and subtraction for given algebraic expressions without using algebra tiles individually. After the students finished, the teacher showed how to make operations on the board. She explained by referring to the algebra tiles. For example, for the operation (5x-10) + (-2x+7), she said *"There are 5 red rectangles and 2 blue rectangles. 2 red rectangles and 2 blue rectangles created zero pair. Now, I have 3 red rectangles. In addition, I have 10 blue squares and 7 red squares. 7 blue squares and 7 red squares created zero pair. Now, I have 3 blue squares. Therefore, the result is 3x-3."* She emphasized performing the operations by combining like terms.

In the evaluation part, the exit card was distributed to the students. After they completed, they gave exit cards to the teacher while leaving the classroom.

3.5.1.3 The Third Objective

The third objective was that students should be able to multiply an algebraic expression with a natural number. For this objective, two class hours were allocated. Lessons included five phases as engagement, exploration, explanation, elaboration and evaluation respectively.

In the engagement part, the activity sheet was distributed to the students and the teacher asked students to read the first question and think about it. After finding an answer, students raised their hands and one student answered as 6x. The teacher also wrote the answer as 6.x=6x on the board. Next, they read the second question and raised their hands to give an answer. One student said *"We will add the length of all sides."* The teacher said *"You are expected to find an area in the question, not perimeter."* Then, students said *"We will multiply the length of long side by the length of short side"* and the teacher wrote 2.(x+3) on the board.

In the exploration part, students were asked to work in pairs and algebra tiles were distributed to the pairs. The teacher stuck algebra tiles on the board to remind them and told students to model 3x by using algebra tiles. After the pairs modelled 3x, the teacher also showed the model on the board by sticking three red rectangle pieces. Next, students were asked to model multiplication of (x+1) by 2 by using algebra tiles. While pairs were modelling with algebra tiles, the teacher walked around the desks and helped students if they needed. After the groups finished, the teacher showed how to multiply by sticking algebra tiles and she also represented operation algebraically as 2.(x+1) = 2x+2. Next, the teacher told students to model multiplication of (x-2) by 3 by algebra tiles. After they finished modelling, the teacher showed how to multiply by sticking algebra tiles and wrote 3.(x-2) = 3x-6. Finally, the teacher asked students to model multiplication of (-x-1) with 4. After the pairs modelled, the teacher also showed modelling on the board and wrote 4.(-x-1) = -4x-4. This time, model of algebra tiles was stuck to the board by the teacher and students were expected to write the given model as the multiplication of an algebraic expression with a natural number. After the students found the correct answer, the teacher emphasized the commutative property of addition and wrote on the board as 3.(-3x+2) = -9x+6 = 6-9x = 3.(2-3x) by explaining. Then, the teacher showed all the models and their algebraic expressions on the board and asked students *"How did we perform multiplication while modelling* *these?"* and *"How can we perform multiplication in algebraic expressions without using algebra tiles?"* Students discussed and some of them said *"We multiply the number outside the parenthesis with inside the parenthesis."*

In the explanation part, teacher explained how to multiply an algebraic expression with a natural number. She said *"While multiplying an algebraic expression with a natural number, each term of the algebraic expression is multiplied with the natural number."* In addition, she explained multiplication by drawing arrows on the algebraic representations near the algebra tiles on the board again.

In the elaboration part, algebra tiles were collected from the students and the activity sheet was distributed to them, and the teacher asked students to perform given multiplications individually without using algebra tiles. In addition, students were expected to determine whether given representations were correct or not, and to correct the incorrect ones. After the students completed activity, for each item in the activity sheet, different students came to the board and explained results.

In the evaluation part, the exit card was distributed to the students. After they completed, they gave exit cards to the teacher while leaving the classroom. The treatment process in the experimental group is summarized in Table 3.23.

Table 3.23 Experimental Group Process

Objectives	Time	Tools	Group Work in Exploration Phase	Instructional Techniques	The Flow of the Lesson
Students should be able to express the meaning of simple algebraic expressions.	2 class hours	Algebra Tiles (as concrete material) Activity sheets Exit cards	Pairs (Groups of 2 students)	Questioning Discussion Cooperative Learning Individual work	Engagement Exploration Explanation Elaboration Evaluation
Students should be able to make addition and subtraction in algebraic expressions.	3 class hours	Algebra Tiles (as concrete material) Activity sheets Exit cards	Groups of 4 students	Questioning Discussion Cooperative Learning Individual work	Engagement Exploration Explanation Elaboration Evaluation
Students should be able to multiply an algebraic expression with a natural number.	2 class hours	Algebra Tiles (as concrete material) Activity sheets Exit cards	Pairs (Groups of 2 students)	Questioning Discussion Cooperative Learning Individual work	Engagement Exploration Explanation Elaboration Evaluation

3.5.2 Control Group Treatment

In the control group, three objectives were accomplished by regular instruction without using algebra tiles. These objectives were the same as the objectives covered in the experimental group.

3.5.2.1 The First Objective

First objective was that students should be able to express the meaning of simple algebraic expressions. For this objective, two class hours were allocated.

At the beginning of the lesson, the teacher asked "*What is the necessary condition for an expression to be an algebraic expression?"* One student said *"There must be a letter and operation."* The teacher wrote some expressions on the board and asked whether these expressions were algebraic expressions or not. After that, she wrote modelling of algebraic expressions as a title on the board. She drew rectangles and squares and used black pencil for positive ones and blue pencil for negative ones. She drew three black rectangles and asked algebraic expression for this modelling. One student said 3x. Then, she drew two rectangles and one square with black pencil and asked algebraic expression for this modelling. One student said 2x+1 and the other said "*Can we write 1+2x?"* She said "*Yes, you can because of the commutative property of addition." The* same process was repeated for another algebraic expressions (-2x-1 and 3x-2). Next, the teacher wrote 3x-5 and -4x+2 on the board and asked students to model these algebraic expressions by drawing on their notebooks. Students drew and showed their modelling to the teacher. Afterwards, the teacher asked *"Who wants to come to the board and show?"* and two students among the students who raised their hands came to the board and drew modelling for each. Then, the teacher wrote -2x-1 algebraic expression as -x-x-1 and she did the same thing for other algebraic expressions 3x-2, 3x-5 and -4x+2. Next, the teacher asked students to draw modelling of 2x+4 algebraic expression and wrote the expansion of it. She asked *"Who wants to come to the board and show?"* and one student among the students who raised their hands came to the board and drew modelling of it and wrote expansion. Then, the teacher wrote some algebraic expressions on the board such as 4b+2, $\frac{a}{4}+\frac{3}{4}$, $\frac{1}{5}$.x and asked students to represent these algebraic expressions differently. After the students finished, she explained the results on the board.

3.5.2.2 The Second Objective

The second objective was that students should be able to make addition and subtraction in algebraic expressions. For this objective, three class hours were allocated.

At the beginning of the lesson, the teacher reminded the previous lesson and wrote 4x+3 as x+x+x+x+1+1+1 on the board. Then, she wrote x+3x on the board and asked *"What is this operation equal to?"* One student said 4x. Next, she wrote 5x-2x and one student said 3x. The teacher asked *"Can we make* *operation by adding and subtracting the coefficients?"* Students responded yes. The teacher wrote -2x-5x and asked its equivalent expression. One student said -7x. Then, she wrote x+3+x+1. One student said 4x, the other student said 4+2x. Another student said 2x+4. The teacher said *"There is a commutative property of addition, 4+2x and 2x+4 are the same."* She showed the same operation by drawing the model. She asked *"Did we add x and 3? No, we added x and x; 3 and 1. That is, we added like terms."* Afterwards, the teacher asked the equivalent of (3x+4) + (x-1). One student said 4x+3. The teacher drew the model of same operation and showed that two squares cancelled each other. Then, she wrote (5x+4) + (-2x-2) and asked the result of this operation. One student said 3x+2. She explained it by combining like terms.

Next, the teacher asked the result of (-4x-6) + (-2x+4). One student said -6x-2. Then, the teacher wrote (4x+2) + (-4x-2) and one student said 0. She explained it by combining like terms and wrote (4x-4x) + (2-2). Afterwards, students took notes. Then, the teacher asked *"How do you make subtraction in integers?"* One student said *"We convert negative to positive"*. The teacher said *"We will also convert while making subtraction in algebraic expressions."* She wrote (3x+2) – (2x+1) on the board, converted it to addition and explained how to subtract. She said *"We will convert it to addition first and then, we will make addition."* Then, the teacher wrote (3x+2) – (x-1) on the board and showed its solution. She wrote *"While making subtraction in algebraic expressions, first, we convert it to addition like in integers because subtraction means the adding minuend with the opposite sign of subtrahend"* on the board and students took notes. Then, the teacher asked students to find the result of (3x+2) – (4x+6). After the students found, she showed its solution on the board. Finally, the teacher wrote some algebraic expressions on the board and asked students to find terms, variables, coefficients and constant terms of them. For each algebraic expression, one student said its terms, variables, coefficients and constant terms and she wrote on the board.

3.5.2.3 The Third Objective

The third objective was that students should be able to multiply an algebraic expression with a natural number. For this objective, two class hours were allocated.

At the beginning of the lesson, the teacher wrote multiplying an algebraic expression with a natural number as a title on the board. She asked *"What is a natural number?"* Students said 0,1,2,3… She asked *"What is algebraic expression?"* Students said *"There must be a letter and operation."* Then, the teacher drew three rectangles where each of them representing x, and asked algebraic expression of this modelling. Students said 3x. She said *"We multiplied 3 and x"* and wrote 3.x = 3x. She asked the result of 5.2x and one student said 10x. The teacher explained it as 5 times 2x by drawing 10 rectangles each of them representing x, and said that coefficients could be directly multiplied. Then, she asked the result of 3.5x and one student said 15x. One student asked *"If there are big numbers, how will we draw?"* The teacher said *"You will not draw, you will multiply the coefficients. For example, you cannot draw 15x.20 = 300x"* Next, she asked the result of 3.(x+1) and some students said 3x+4 or 4x. She explained it by drawing algebra tiles and drawing arrows that shows multiplication of 3 with both x and 1. Then, she asked the result of several multiplication such as 5.(2x+1), 3.(x-2), 2.(2b+3k), (6-2m).3, 4.(m-2n+6), 5.(-4k+20+5x). Students raised their hands and for each of them one student said the result. Afterwards, the teacher asked whether they understood multiplication or not and students said yes. Then, she allowed them to take notes. Afterwards, she drew a rectangle with length 12k+7-16t and width 3 and asked its area. After the students found the result, one student came to the board and showed the solution. Next, the teacher wrote 2x+2 on the board and asked *"Multiplication of which natural number and algebraic expression is equal to 2x+2?"* One student said it is equal to the multiplication of 2 and x+1 and the teacher explained. The same process was repeated for

2x+4, 2x-8 and -4x+16. The treatment process in the control group is summarized in Table 3.24.

Table 3.24 Control Group Process

Objectives	Time	Tools	Instructional Techniques	The Flow of the Lesson
Students should be able to express the meaning of simple algebraic expressions.	2 class hours	Drawing of algebra tiles on the board	Direct Instruction Questioning Drill and Practice Individual work	Explaining the topic Solving examples Writing questions on the board and asking students to solve them
Students should be able to make addition and subtraction in algebraic expressions.	3 class hours	Drawing of algebra tiles on the board	Direct Instruction Questioning Drill and Practice Individual work	Explaining the topic Solving examples Writing questions on the board and asking students to solve them
Students should be able to multiply an algebraic expression with a natural number.	2 class hours	Drawing of algebra tiles on the board	Direct Instruction Questioning Drill and Practice Individual work	Explaining the topic Solving examples Writing questions on the board and asking students to solve them

3.6 Data Collection

Data were collected in the spring semester of 2017-2018 academic year because algebra topics were taught in the spring semester to the sixth grade students according to the Turkish Middle Grades Mathematics Curriculum. Pilot studies of Prior Algebra Knowledge Test and Algebra Achievement Test were conducted in the fall semester of the same academic year in order to refine the instruments for the students.

Before the data collection process, necessary permissions were taken from the Ethics Committee of METU Research Center for Applied Ethics. In addition, since data were collected from the students in a public school, permission from Ministry of National Education was also taken. Data were collected before the treatment to learn students' prior knowledge about algebraic expressions and after the treatment to examine the effects of using algebra tiles.

Data collection tools were administered to students in their classes by the researcher. During the administration of instruments, I, the researcher ensured that there would be no interaction between the students. For the completion of Prior Algebra Knowledge Test and Algebra Achievement Test, one class hour was given to the students. Views about Algebra Tiles Questionnaire was implemented to the experimental group students to learn the students’ views about using algebra tiles in mathematics lessons and it took 20-30 minutes to conduct it. Before the administration of the questionnaire, students were informed that there was no right or wrong answer for the questions in the questionnaire. Furthermore, it was announced that their answers would not be graded and shared with anybody.

3.7 Analysis of Data

In this study, both quantitative and qualitative research methodologies were used. The first research question was sought through quantitative methods, and the second and the third research questions were sought through qualitative methods.

Rubrics were prepared by the researcher and used to evaluate students’ responses in the tests (See Appendix E and Appendix F). In the rubrics, correct and incorrect answers were written and students’ answers were coded as “1” if their answers were correct, and coded as “0” if their answers were incorrect.

After the pilot study, analyses of data were made to check the reliability of the tests. To determine internal consistency of the tests, Kuder-Richardson approach, particularly formula KR21 was used assuming that the items were in equal difficulty.

After the main study, to answer the first research question, data were analyzed by using SPSS 20.0 software program. As descriptive statistics, means and standard deviations of pretest and posttest scores of both experimental and control group were computed. As inferential statistics, independent samples t-test and Mann-Whitney U test was conducted to compare the scores of the experimental group and the control group. Before conducting independent samples t-test, the researcher ensured the assumptions that independence of observations, level of measurement and normality. To deal with missing values in statistical analyses, exclude cases pairwise option was selected.

For the second research question, I analysed responses to each question in the posttest in-depth and tried to detect students' common mistakes, possible misconceptions and/or alternative solutions. In order to investigate the third research question, I went through the responses given by the students by carefully reading the responses several times and identified two major categories in their responses: using algebra tiles and group work. Some of the students referred to these two implementations together. I grouped these responses separately to reflect their ideas better. Then, I regrouped their responses for using algebra tiles and group work under major subgroups based on their reference to their experiences in the class. Using algebra tiles had two subcategories as effective understanding of algebra and enjoying the class. Group work had two subcategories as learning easier and enjoyment. Finally, students' comments which reflected their ideas more comprehensively or in a more integrated way were grouped separately. I presented the findings based on major and sub categories. For example, students referred to the benefits of using algebra tiles mostly in terms of understanding and learning well. I grouped these findings as effective understanding of algebra.

3.8 Assumptions and Limitations

In this study, it is assumed that participants reflected their own opinions and they were not affected by anyone. Moreover, it is assumed that treatment in the experimental group and instruction in the control group were conducted as intended.

In this study, non-random sampling method was used. That is, the school was not selected randomly. Instead, it was selected according to the convenience of the researcher. This situation creates a major limitation of the present study because sample might not be a representative of the population and thus, generalizability might be limited.

As a limitation, it can also be said that results of the study was limited with the data provided by the participants through the instruments prepared by the researcher. In addition, the length of the treatment was only seven class hours because the teaching duration of the content was limited to seven class hours in the Middle Grades Mathematics Curriculum, and it was not possible to extend the time.

3.9 Internal and External Validity of the Study

3.9.1 Internal Validity

According to Fraenkel, Wallen and Hyun (2011), internal validity is ensured when “observed differences on the dependent variable are directly related to the independent variable and not due to some other unintended variable” (p. 166). In this study, pretest-posttest control group design was preferred. In this design, some issues can be threats to internal validity and the result of the study can be affected by these threats. In this section, the internal validity threats for this study were evaluated.

Subject characteristics might be a threat to internal validity when already existing groups were used. In this study, all participants were at the same age and classes were heterogeneous with respect to ability level. Therefore, the effects of subject characteristics threat were reduced.

In this study, students’ pretest and posttest scores were not compared. In addition, day and time of the testing was not announced beforehand. Hence, the absence of participants could be incidental rather than intentional and mortality (loss of subjects) was not a threat for this study.

Unexpected events can become a threat if they affect participants’ responses. To prevent this, researcher was alert to the extraneous events that may occur in the school during the data collection and asked the school administrator to inform her beforehand if the lessons in experimental and control groups would be disrupted. No disruption occurred during the instruction of the algebra topics.

In this study, giving treatment and data collection processes together took approximately one month and this time period was short for the maturation of the participants. Furthermore, participants were 11-12 years old students and students at this age do not mature rapidly. Moreover, control group was included in the study and the content was taught over the same time period to both experimental and control group. Therefore, maturation was not a problem.

Essay type questions were used in data collection procedure. For scoring of these questions, rubrics were prepared and scoring was performed by using these rubrics for all students by the researcher. Sample size was small and data were collected only two times. Therefore, data collection and scoring procedures did not cause any change in the instrument and instrument decay was not a threat to internal validity.

In this study, data were collected by the researcher in the classrooms for both experimental and control groups. Therefore, data collector characteristics were the same for all participants.

The researcher implemented the instruments in the classrooms and instruments did not include an interview protocol. Thus, students were not asked leading questions. Nevertheless, researcher could distort the data unconsciously by favoring one method over other. Therefore, to control data collector bias threat, all procedures were standardized. Data collection tools were administered to students in their classes by the researcher and both students in experimental and control groups were allowed equal time on tests. In addition, they were not allowed ask questions and the researcher ensured that there was no interaction between the students in both groups. Rubrics were used to score the PAKT and AAT responses.

Testing was not a threat to internal validity. Because pretest was administered to learn students' prior knowledge and it was different from posttest.

Hawthorne effect is a positive effect of an intervention resulting from the subjects' knowledge that they are involved in a study or their feeling that they are in some way receiving "special" attention (Frankel, Wallen & Hyun, 2011). Students who learned algebra by using algebra tiles might have positive attitudes toward algebra. On the other hand, students in the control group might have negative attitudes toward algebra since they did not use algebra tiles. In addition, if the students in experimental group knew that they took part in the study and received a different treatment, their feelings could have been improved. In order to limit this effect, first, the mathematics teacher of the experimental group conducted the treatment. Then, the teacher announced, in both groups, that the researcher was in the class to observe the classroom. Therefore, the researcher's presence in both groups were assumed to affect the groups in the same way. The researcher was in the classroom one week before the study and during the study in both groups. In this way, students got used to the researcher. In addition, after the treatment ended, activities in the experimental group were also conducted in control group by using algebra tiles and the effects of subject attitude threat were reduced in these ways.

In this study, data were collected in one school and classroom environments in the school were similar. Hence, location threat was eliminated to a great extent.

Implementation could be a threat to internal validity in this study because it was an experimental study. To prevent this bias, the researcher did not conduct the instruction in the experimental group. Instead, instruction was delivered by the same mathematics teacher, who was the mathematics teacher of the both classes, in both experimental and control groups in the school. Furthermore, the researcher observed both groups during the instructions. In these ways, implementation threat was eliminated to a great extent.

Participants of this study were not chosen according to their extreme scores. Already existing classes constituted the experimental and control groups. Therefore, regression was not a threat to internal validity.

3.9.2 External Validity

“External validity is the extent to which the results of a study can be generalized” (Fraenkel, Wallen & Hyun, 2011, p. 103). While the target population of this study was all sixth grade students in Sakarya, accessible population was all sixth grade students who attended to public schools in Hendek, Sakarya. The school was selected according to researcher’s convenience. Since non-random sampling method was used, generalization of the findings to the population might be limited.

“Ecological generalizability refers to the degree to which the results of a study can be extended to other settings or conditions” (Fraenkel, Wallen & Hyun, 2011, p. 105). The results of the study can be generalized to other public schools in the district which have the similar conditions and then to similar public schools.

CHAPTER 4

RESULTS

This chapter presents the results of the descriptive and inferential statistics analysis to and findings in detail to respond to the research questions below.

1. Is there a statistically significant mean difference between posttest scores of algebra achievement test for 6th grade students who use algebra tiles and those who do not use algebra tiles?

   H_0: There is no significant difference between posttest scores of algebra achievement test for 6th grade students who use algebra tiles and those who do not use algebra tiles.

   H_1: There is a significant difference between posttest scores of algebra achievement test for 6th grade students who use algebra tiles and those who do not use algebra tiles.

2. How does students' algebraic thinking differ in the algebra achievement test for those who use algebra tiles and who do not use algebra tiles?
3. What are the 6th grade students' views about use of algebra tiles in mathematics lessons?

Although the questions were not given with sub-question indicators such as 1a and 1b, the findings below are given with these indicators in the order of the sub-questions in order to report the findings more clear.

4.1 Effects of Using Algebra Tiles on Sixth Grade Students' Achievement in Algebra

In order to respond to the first research question, the descriptive and inferential statistics of the Prior Algebra Knowledge Test (PAKT) and Algebra Achievement Test (AAT) were presented. Additionally, in-depth analysis of students' responses to the PAKT questions were given to document the initial status of the EG and CG students in terms of prior algebra knowledge.

4.1.1 The Results of PAKT

In order to investigate whether there was a significant mean difference between the experimental group and control group before the treatment in terms of pretest scores in PAKT, firstly, assumptions were checked and reported in the following sections. Since normality assumption could not be ensured, Mann-Whitney U test, which is a non-parametric technique, was conducted instead of independent samples t-test.

4.1.1.1 Assumptions of T-Test for PAKT

Before conducting the analysis, assumptions for independent samples t-test which were level of measurement, independence of observations, and normality of the dependent variable (Pallant, 2011) were checked.

4.1.1.1.1 Level of Measurement

Pallant (2011) stated level of measurement as "the dependent variable is measured at the interval or ratio level; that is, using a continuous scale rather than discrete categories" (p. 205). In this study, dependent variable was the Prior Algebra Knowledge Test scores and it was a continuous variable.

4.1.1.1.2 Independence of Observations

Pallant (2011) explained that "data must be independent of one another; that is, each observation or measurement must not be influenced by any other observation or measurement" (p. 205). In this study it was assumed that the measurements were not influenced by each other.

4.1.1.1.3 Normality

According to Pallant (2011), to ensure normality assumption, the populations from which the samples were taken should be normally distributed. In this study, sample size were smaller than 30 for both groups. Therefore, in order to check this assumption, Shapiro-Wilk Test was conducted. The result of Shapiro-Wilk test for pretest is given in Table 4.1.

Table 4.1 Result of Shapiro-Wilk Test for Pretest

	Statistic	df	Sig.
Experimental Group	0.888	23	0.014
Control Group	0.881	21	0.015

As seen from the Table 4.1, the significance values for both groups for pretest were 0.014 and 0.015 violating the normal distribution assumption. Therefore, a non-parametric technique, Mann-Whitney U test was used.

4.1.1.2 Mann-Whitney U Test Results

Prior Algebra Knowledge Test (PAKT) which included 15 questions was implemented to 23 students in the experimental group and 21 students in the control group as a pretest before the treatment. Maximum score that a student could get from PAKT was 15. Table 4.2 shows the descriptive statistics of both groups in PAKT.

Table 4.2 Descriptive Statistics of Scores in PAKT for Both Groups

	Experimental Group	Control Group
N	23	21
Minimum	0	0
Maximum	15	15
Mean	8.61	6.95
Standard Deviation	5.42	5.56

As seen from the Table 4.2, experimental group students' mean score in PAKT (Mean = 8.61, SD = 5.42) was higher than control group students' mean score in PAKT (Mean = 6.95, SD = 5.56).

To investigate whether there was a significant mean difference between the experimental group and control group before the treatment in terms of pretest scores in PAKT, Mann-Whitney U Test was conducted. The result of Mann-Whitney U test for pretest is given in Table 4.3.

Table 4.3 Result of Mann-Whitney U Test for Pretest

	Mann-Whitney U	Sig.
Pretest	205.500	0.395

$p>0.05$

As seen from the Table 4.3, there was no statistically significant mean difference between experimental and control groups in terms of pretest scores. Therefore, pretest scores were not taken as covariate. The students in the control and experimental group classes were considered as having equal prior algebra knowledge based on the statistical results.

4.1.1.3 The Detailed Findings of PAKT

Although there was no statistically significant difference between the PAKT scores of experimental and control groups, a further detailed analysis was carried out in order to reveal the nature of students' prior algebra knowledge. Answers of both experimental (EG) and control (CG) group students for each question in PAKT are given below in detail along with the questions and sub-questions.

Question 1

Question 1 was about writing a given phrase as an algebraic expression. The question is given in Figure 4.1 below.

1) Write each phrase as an algebraic expression.

3 less than twice a number of candies in the jar ……………………….

12 TL more than half of Aslı's money…………………………………

13 less than a number of Efe's marbles times five……………………..

2 less than a number plus twice the same number……………………..

The amount of remaining time of the exam when 15 minutes of the time completed ……………………

If the sum of two numbers is 80 and one of the numbers is m, the other number is…………………….

Figure 4.1 1st Question in PAKT

There were six sub-questions in question 1. The performance of students in percentage and frequency for each sub-question are given in Table 4.4.

Table 4.4 Frequency and percentage of correct and incorrect answers, and empty responses of EG and CG in question 1 in PAKT

	Experimental Group (N=23)			**Control Group (N=21)**		
	Correct (%)	**Incorrect (%)**	**Empty (%)**	**Correct (%)**	**Incorrect (%)**	**Empty (%)**
Q1a	16 (69.6)	7 (30.4)	-	15 (71.4)	5 (23.8)	1 (4.8)
Q1b	15 (65.2)	5 (21.7)	3 (13)	7 (33.3)	7 (33.3)	7 (33.3)
Q1c	15 (65.2)	6 (26.1)	2 (8.7)	15 (71.4)	3 (14.3)	3 (14.3)
Q1d	9 (39.1)	13 (56.5)	1 (4.3)	8 (38.1)	9 (42.9)	4 (19)
Q1e	12 (52.2)	6 (26.1)	5 (21.7)	12 (57.1)	2 (9.5)	7 (33.3)
Q1f	8 (34.8)	8 (34.8)	7 (30.4)	4 (19)	8 (38.1)	9 (42.9)

In the 1st question, although the objectives of items were the same, context of the items affected the performance of students. Both EG and CG students had a difficulty in item Q1d and Q1f because these items were different than the examples that they saw in the lessons. Some of the students showed acceptance of lack of closure in their responses and they tried to equalize algebraic expression to an arbitrary number. Moreover, in item Q1e, a few students wrote 25 as the amount of remaining time of the examination when 15 minutes of the time completed by thinking that examination duration can only be 40 minutes. In item Q1f, although algebraic expression is required in terms of m, some students wrote algebraic expression including x.

Question 2

Question 2 was about writing a phrase for a given algebraic expression. The question is given in Figure 4.2 below.

2) Write a phrase for each algebraic expression given below.

5(c – 2)……………………………………………………..

$\frac{m+1}{2}$………………………………………………………….

7k – 6……………………………………………………….

$\frac{x}{2}$ + 5……………………………………………………..

Figure 4.2 2nd Question in PAKT

There were four sub-questions in question 2. The performance of students in percentage and frequency for each sub-question are given in Table 4.5.

Table 4.5 Frequency and percentage of correct and incorrect answers, and empty responses of EG and CG in question 2 in PAKT

	Experimental Group (N=23)			**Control Group (N=21)**		
	Correct (%)	**Incorrect (%)**	**Empty (%)**	**Correct (%)**	**Incorrect (%)**	**Empty (%)**
Q2a	15 (65.2)	7 (30.4)	1 (4.3)	9 (42.9)	9 (42.9)	3 (14.3)
Q2b	12 (52.2)	10 (43.5)	1 (4.3)	9 (42.9)	7 (33.3)	5 (23.8)
Q2c	12 (52.2)	6 (26.1)	5 (21.7)	9 (42.9)	5 (23.8)	7 (33.3)
Q2d	15 (65.2)	7 (30.4)	1 (4.3)	7 (33.3)	8 (38.1)	6 (28.6)

In the 2^{nd} question, both EG and CG students preferred "a number" form such as "*six less than seven times a number*", "*five more than half of a number*" while writing a phrase for a given algebraic expression instead of associating to the real life. It can be said that students had mathematization difficulty that is, converting mathematics to the real life problems. In addition, some of the students did not know the order of operations or they did not pay attention to it. For example, for item Q2a, they wrote two less than five times a number. Moreover, it was seen that some students confused addition and multiplication. For instance, for item Q2c, they wrote six less than a number plus seven.

Question 3

Question 3 was about evaluating algebraic expressions for different values of variables. The question is given in Figure 4.3 below.

3) Evaluate each algebraic expression given below for a given value of variables.

- $\frac{2(n-3)}{5}$ **for n=13**
- $\frac{3x+4}{2}$ **for x=6**
- $\frac{85}{y} + 1$ **for y=5**

Figure 4.3 3rd Question in PAKT

There were three sub-questions in question 3. The performance of students in percentage and frequency for each sub-question are given in Table 4.6.

Table 4.6 Frequency and percentage of correct and incorrect answers, and empty responses of EG and CG in question 3 in PAKT

	Experimental Group (N=23)			**Control Group (N=21)**		
	Correct (%)	**Incorrect (%)**	**Empty (%)**	**Correct (%)**	**Incorrect (%)**	**Empty (%)**
Q3a	15 (65.2)	4 (17.4)	4 (17.4)	10 (47.6)	10 (47.6)	1 (4.8)
Q3b	10 (43.5)	8 (34.8)	5 (21.7)	6 (28.6)	12 (57.1)	3 (14.3)
Q3c	13 (56.5)	4 (17.4)	6 (26.1)	9 (42.9)	5 (23.8)	7 (33.3)

Students in both EG and CG gave similar answers to the 3rd question. Students gave incorrect answers because of not taking order of operations into consideration. In addition, in item Q3b, some of them taught that x was a unit digit and 3x was two-digit number rather that 3 and x was multiplied in 3x algebraic expression. The reason of students' difficulty can be lack of their arithmetic understanding.

Question 4

Question 4 was about writing a given phrase as an algebraic expression and evaluating it for a given value of variable. The question is given in Figure 4.4 below.

4)

i. Write an algebraic expression for the phrase "7 more than 3 times a number of fishes in the aquarium."

ii. Evaluate the algebraic expression you wrote in i, when the variable is equal to 15.

Figure 4.4 4th Question in PAKT

There were two sub-questions in question 4. The performance of students in percentage and frequency for each sub-question are given in Table 4.7.

Table 4.7 Frequency and percentage of correct and incorrect answers, and empty responses of EG and CG in question 4 in PAKT

	Experimental Group (N=23)			**Control Group (N=21)**		
	Correct (%)	**Incorrect (%)**	**Empty (%)**	**Correct (%)**	**Incorrect (%)**	**Empty (%)**
Q4a	17 (73.9)	1 (4.3)	5 (21.7)	14 (66.7)	3 (14.3)	4 (19)
Q4b	13 (56.5)	4 (17.4)	6 (26.1)	11 (52.4)	1 (4.8)	9 (42.9)

In the 4th question, EG and CG students gave similar answers and they preferred to write variable before coefficient such as E.3 instead of 3E while writing a given phrase as an algebraic expression. In addition, students in both groups used different letters such as A, c, b, s, m, L, E, G instead of the commonly used letters like x, y and n.

These findings showed that EG and CG students did not differ in prior algebra knowledge. Their responses to the questions were similar in the PAKT in terms of preferences and mistakes.

4.1.2 The Results of AAT

In order to investigate the first research question, independent samples t-test was conducted for AAT scores and hypotheses were tested at the 0.05 level of significance. Before conducting the analysis, assumptions were checked and reported in the following sections.

4.1.2.1 Assumptions of T-Test for AAT

Before conducting the analysis, assumptions for independent samples t-test which were level of measurement, independence of observations, and normality of the dependent variable (Pallant, 2011) were checked.

4.1.2.1.1 Level of Measurement

Pallant (2011) stated level of measurement as “the dependent variable is measured at the interval or ratio level; that is, using a continuous scale rather than discrete categories” (p. 205). In this study, dependent variable was the Algebra Achievement Test scores and it was a continuous variable.

4.1.2.1.2 Independence of Observations

Pallant (2011) explained that “data must be independent of one another; that is, each observation or measurement must not be influenced by any other observation or measurement” (p. 205). In this study it was assumed that the measurements were not influenced by each other.

4.1.2.1.3 Normality

According to Pallant (2011), to ensure normality assumption, the populations from which the samples were taken should be normally distributed. In this study, sample size was smaller than 30 for both groups. Therefore, in order to check this assumption, Shapiro-Wilk Test, skewness and kurtosis values, and histograms were examined. Table 4.8 presents the result of skewness and kurtosis values of posttest.

Table 4.8 Result of Skewness and Kurtosis Values of Posttest

		Standard Deviation	Skewness	Kurtosis
Posttest	Experimental Group	8.51	-.23	-.48
	Control Group	10.32	.22	-1.34

Skewness and Kurtosis values of scores on posttest were in acceptable range (between -2 and +2) for a normal distribution (Pallant, 2011) as seen from the Table 4.8. In addition to skewness and kurtosis values, Shapiro-Wilk test was conducted. Table 4.9 presents the result of Shapiro-Wilk test for posttest.

Table 4.9 Result of Shapiro-Wilk Test for Posttest

	Statistic	df	Sig.
Experimental Group	0.977	23	0.858
Control Group	0.930	23	0.154

As seen from the Table 4.9, significance values for both groups for posttest as 0.858 and 0.154 indicate normal distribution.

In addition, histograms with normal curves supported the normality assumption for posttest scores. Figure 4.5 shows the histogram of posttest scores for experimental group.

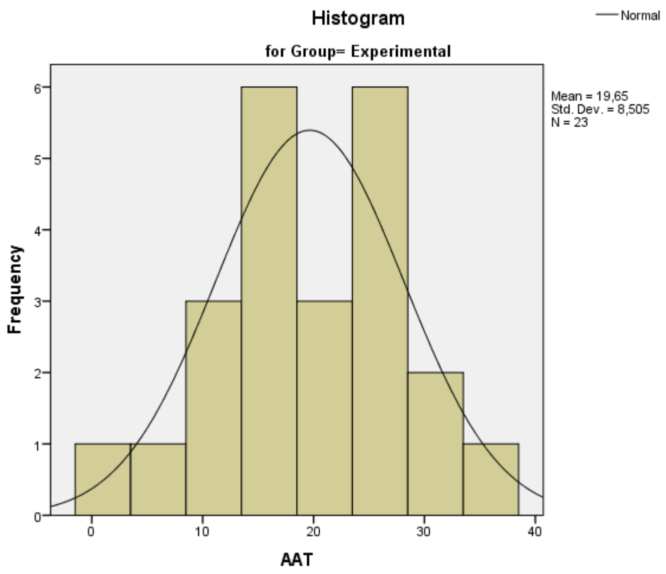


Figure 4.5 Histogram of posttest scores for experimental group

These results showed that the AAT scores satisfied the normality assumption.

Figure 4.6 shows the histogram of posttest scores for control group.

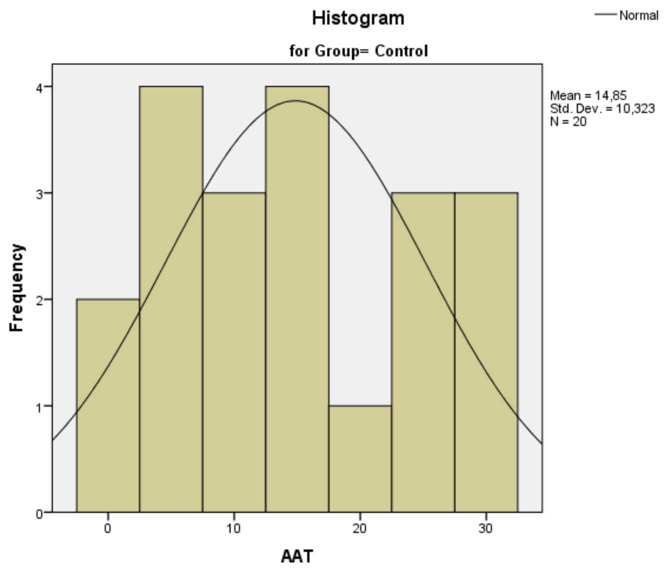


Figure 4.6 Histogram of posttest scores for control group

4.1.2.1.4 Homogeneity of Variances

“Samples are obtained from populations of equal variances. This means that the variability of scores for each of the groups is similar” (Pallant, 2011, p. 206). To test this, Levene’s test for equality of variances was performed. Results showed that homogeneity of variances assumption was not violated for posttest ($p = .311$) and both samples had equal variances.

4.1.2.2 T-Test Results

Algebra Achievement Test (AAT) including 35 questions was administered to 23 students in the experimental group and 20 students in the control group as a posttest after the treatment. The maximum score that a student could get from AAT was 35. Table 4.10 shows the descriptive statistics of both groups in AAT.

Table 4.10 Descriptive Statistics of Scores in AAT for Both Groups

	Experimental Group	Control Group
N	23	20
Minimum	1	0
Maximum	34	32
Mean	19.65	14.85
Standard Deviation	8.51	10.32

As seen from the Table 4.10, experimental group students’ mean score in AAT (Mean = 19.65, SD = 8.51) was higher than control group students’ mean score in AAT (Mean = 14.85, SD = 10.32).

The first research question was “Is there a statistically significant mean difference between posttest scores of algebra achievement test for 6^{th} grade students who use algebra tiles and those who do not use algebra tiles?” For the first research question the following null hypothesis was tested:

There is no significant difference between posttest scores of algebra achievement test for the 6^{th} grade students who use algebra tiles and those who do not use algebra tiles.

In order to test the hypothesis, independent samples t-test was performed. Independent samples t-test results of AAT are given in Table 4.11.

Table 4.11 Result of T-Test of Posttest Scores

	Experimental Group		Control Group		t value
	Mean	SD	Mean	SD	
AAT	19.65	8.51	14.85	10.32	1.67

$p>0.05$

As seen from the Table 4.11, there was no statistically significant mean difference between the groups who received instruction with algebra tiles and who received regular instruction in terms of posttest scores.

4.2 Effects of Using Algebra Tiles on Sixth Grade Students' Algebraic Thinking

In order to investigate the second research question, both experimental group and control group students' answers in Algebra Achievement Test were examined in detail. Answers of both experimental and control group students for each question in AAT are given below in detail along with the questions and sub-questions.

Question 1

Question 1 was about determining whether given representations are correct or incorrect and rewriting incorrect representations as correct representations. The question is given in Figure 4.7 below.

1) **Determine whether given representations are correct or incorrect and rewrite incorrect representations as correct representations.**

….. $y + y + 1 = 3y$

….. $x + x - 1 = -1 + 2x$

….. $\frac{a}{2} + \frac{a}{2} = 2a$

….. $5 - c - c + c = 5 - 3c$

Figure 4.7 1st Question in AAT

There were four sub-questions in question 1. The performance of students in percentage and frequency for each sub-question are given in Table 4.12.

Table 4.12 Frequency and percentage of correct and incorrect answers, and empty responses of EG and CG in question 1 in AAT

	Experimental Group (N=23)			**Control Group (N=20)**		
	Correct (%)	**Incorrect (%)**	**Empty (%)**	**Correct (%)**	**Incorrect (%)**	**Empty (%)**
Q1a	17 (73.9)	6 (26.1)	-	16 (80)	4 (20)	-
Q1b	20 (87)	3 (13)	-	18 (90)	2 (10)	-
Q1c	13 (56.5)	10 (43.5)	-	13 (65)	7 (35)	-
Q1d	13 (56.5)	10 (43.5)	-	10 (50)	10 (50)	-

In the 1st question, the number of EG and CG students who gave correct and incorrect answers were close to each other. It can be said that most of the both EG and CG students could determine whether given representations are correct or incorrect. The percentages address that EG and CG students had similar performances in this question. A few students in the groups determined item Q1b as incorrect and wrote 2x-1 as a correct answer. They thought that -1+2x and 2x-1 are not equal to each other. This might show students' lack of arithmetic understanding, particularly the commutative property.

Question 2

Question 2 was about writing algebraic expressions for the given models. The question is given in Figure 4.8 below.

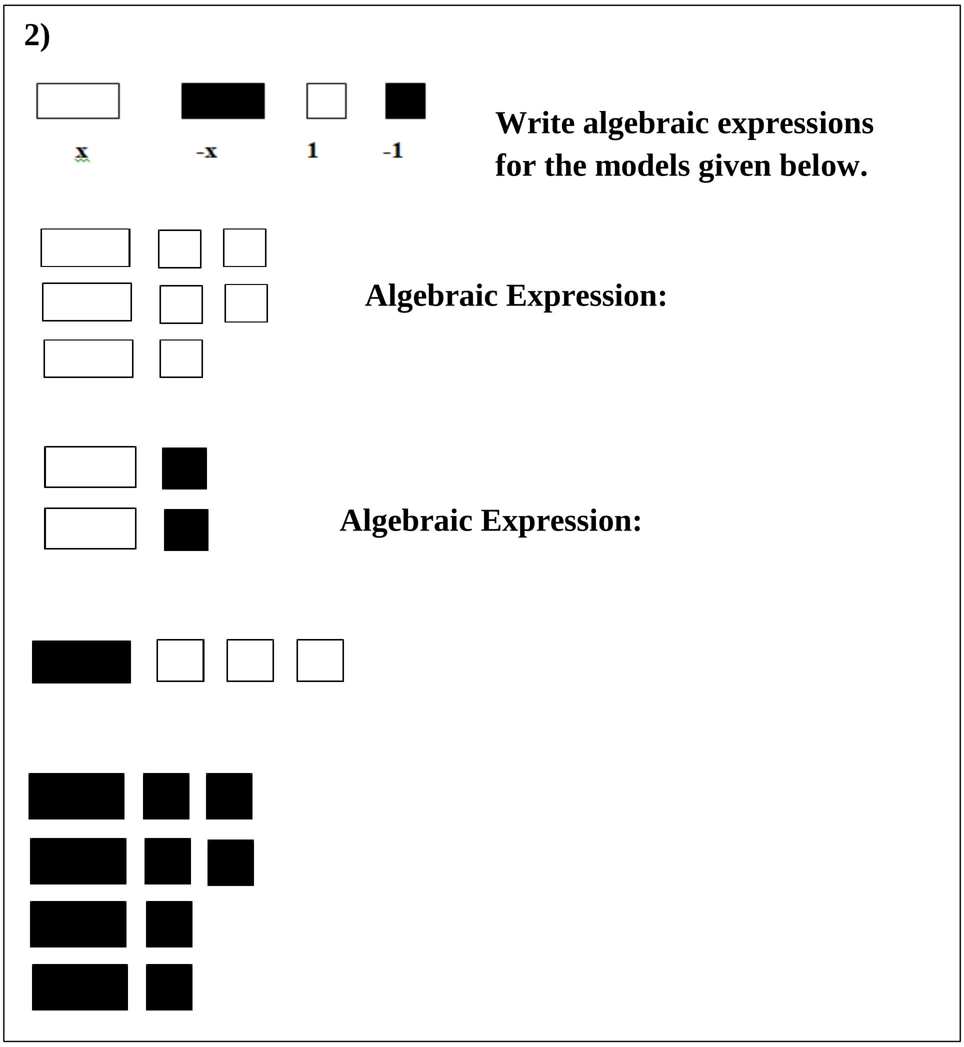


Figure 4.8 2nd Question in AAT

There were four sub-questions in question 2. The performance of students in percentage and frequency for each sub-question are given in Table 4.13.

Table 4.13 Frequency and percentage of correct and incorrect answers, and empty responses of EG and CG in question 2 in AAT

	Experimental Group (N=23)			Control Group (N=20)		
	Correct (%)	Incorrect (%)	Empty (%)	Correct (%)	Incorrect (%)	Empty (%)
Q2a	19 (82.6)	3 (13)	1 (4.3)	11 (55)	9 (45)	-
Q2b	20 (87)	2 (8.7)	1 (4.3)	12 (60)	8 (40)	-
Q2c	19 (82.6)	3 (13)	1 (4.3)	11 (55)	9 (45)	-
Q2d	19 (82.6)	3 (13)	1 (4.3)	11 (55)	9 (45)	-

In the 2nd question, EG students performed better than the CG students while writing algebraic expressions for the given models. Tasks Q2b, Q2c and Q2d seemed to be performed better more by EG students than CG students. For example, Figure 4.9 shows an illustrative example of an EG students' responses for the first two tasks in this question, which are correct.

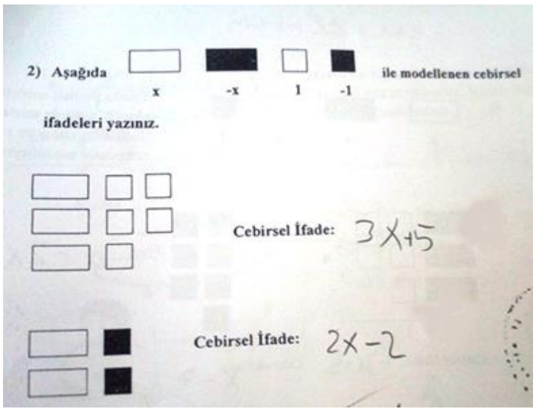


Figure 4.9 One EG student's answer to 2nd question

On the other hand, an illustrative example of one CG students' answer to the same tasks in Figure 4.10 shows that the CG student could not make sense of the expressions in these tasks.

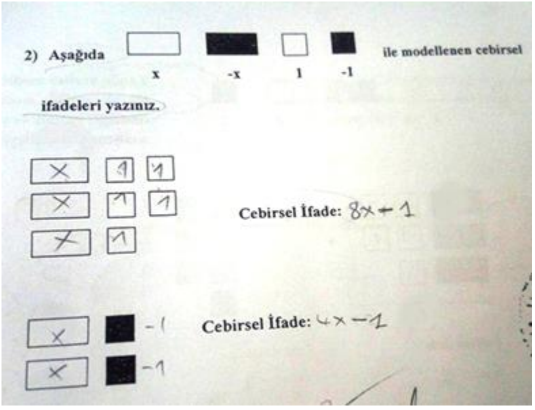


Figure 4.10 One CG student's answer to 2nd question

In addition, although one student in both groups wrote minus signs, they did not write plus signs. For example, for item Q2b, they wrote 2x-2, but for item Q2a, they wrote 3x 5. This can be due to the conception that it is not necessary to put + sign in front of the number for positive integers.

Question 3

Question 3 was about determining variable, term, constant term, coefficients and sum of coefficients of given algebraic expressions. The question is given in Figure 4.11 below.

3) Determine variable, term, constant term, coefficients and sum of coefficients of each algebraic expression given below.

Algebraic Expression	Variable (s)	Term(s)	Constant Term (s)	Coefficient (s)	Sum of Coefficient (s)
3k					
–6xy +1					
2a + 5b – 8					

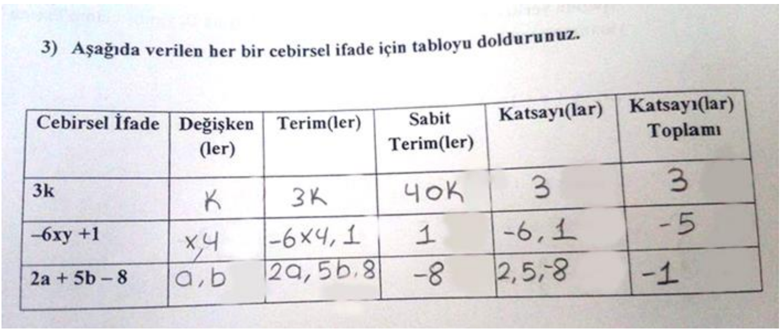
Figure 4.11 3rd Question in AAT

There were fifteen sub-questions in question 3. The performance of students in percentage and frequency for each sub-question are given in Table 4.14.

Table 4.14 Frequency and percentage of correct and incorrect answers, and empty responses of EG and CG in question 3 in AAT

	Experimental Group (N=23)			**Control Group (N=20)**		
	Correct (%)	**Incorrect (%)**	**Empty (%)**	**Correct (%)**	**Incorrect (%)**	**Empty (%)**
Q3aa	11 (47.8)	9 (39.1)	3 (13)	12 (60)	4 (20)	4 (20)
Q3ab	14 (60.9)	6 (26.1)	3 (13)	12 (60)	4 (20)	4 (20)
Q3ac	9 (39.1)	11 (47.8)	3 (13)	8 (40)	7 (35)	5 (25)
Q3ad	13 (56.5)	6 (26.1)	4 (17.4)	9 (45)	4 (20)	7 (35)
Q3ae	15 (65.2)	5 (21.7)	3 (13)	12 (60)	2 (10)	6 (30)
Q3ba	11 (47.8)	9 (39.1)	3 (13)	8 (40)	8 (40)	4 (20)
Q3bb	12 (52.2)	8 (34.8)	3 (13)	8 (40)	7 (35)	5 (25)
Q3bc	13 (56.5)	7 (30.4)	3 (13)	8 (40)	6 (30)	6 (30)
Q3bd	14 (60.9)	6 (26.1)	3 (13)	10 (50)	3 (15)	7 (35)
Q3be	14 (60.9)	6 (26.1)	3 (13)	9 (45)	4 (20)	7 (35)
Q3ca	11 (47.8)	9 (39.1)	3 (13)	13 (65)	2 (10)	5 (25)
Q3cb	14 (60.9)	6 (26.1)	3 (13)	7 (35)	7 (35)	6 (30)
Q3cc	13 (56.5)	7 (30.4)	3 (13)	7 (35)	6 (30)	7 (35)
Q3cd	14 (60.9)	6 (26.1)	3 (13)	9 (45)	4 (20)	7 (35)
Q3ce	16 (69.6)	4 (17.4)	3 (13)	7 (35)	5 (25)	8 (40)

In general, EG students were better than CG students in determining variable, term, constant term, coefficients and sum of coefficients of given algebraic expressions. CG students confused between variable, term and constant term. Figure 4.12 presents the responses of a student in EG for the tasks in question 3.

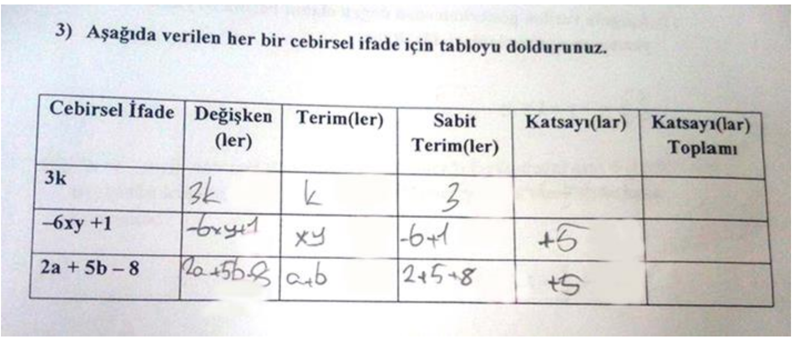

3) Aşağıda verilen her bir cebirsel ifade için tabloyu doldurunuz.

Cebirsel İfade	Değişken (ler)	Terim(ler)	Sabit Terim(ler)	Katsayı(lar)	Katsayı(lar) Toplamı
3k	K	3K	yok	3	3
–6xy +1	xy	-6xy, 1	1	-6, 1	-5
2a + 5b – 8	a, b	2a, 5b, 8	-8	2, 5, -8	-1

Figure 4.12 One EG student's answer to 3rd question

Table 4.14 showed that the third task and its subtasks in the question seemed especially difficult for CG students compared to the EG students. Figure 4.13

illustrates one CG student's responses to question 3 which also showed the low performance in the third task.



3) Aşağıda verilen her bir cebirsel ifade için tabloyu doldurunuz.

Cebirsel İfade	Değişken (ler)	Terim(ler)	Sabit Terim(ler)	Katsayı(lar)	Katsayı(lar) Toplamı
3k	3k	k	3		
–6xy +1	-6xy+1	xy	-6+1	+6	
2a + 5b – 8	2a+5b-8	a+b	2+5+8	+5	

Figure 4.13 One CG student's answer to 3rd question

Question 4

Question 4 was about finding the perimeter of a given rectangle in terms of algebraic expressions. The question is given in Figure 4.14 below.

4) Find the perimeter of a rectangle whose length is 3cm less than the width in terms of algebraic expression.

Figure 4.14 4th Question in AAT

There were not any sub-questions in question 4. The performance of students in percentage and frequency for the question are given in Table 4.15.

Table 4.15 Frequency and percentage of correct and incorrect answers, and empty responses of EG and CG in question 4 in AAT

	Experimental Group (N=23)			Control Group (N=20)		
	Correct (%)	Incorrect (%)	Empty (%)	Correct (%)	Incorrect (%)	Empty (%)
Q4	3 (13)	6 (26.1)	14 (60.9)	4 (20)	3 (15)	13 (65)

Most of the EG and CG students responded to the 4th question incorrectly or they left the question blank. Thus, it can be said that both regular instruction and use of algebra tiles did not help students find the perimeter of a given rectangle in terms of algebraic expressions. One student in each group determined the length of the rectangle as 6 cm and the width of the rectangle as 3 cm. It can be said that these students did not consider that algebraic expression was an answer of the question and they tended to give specific numerical answers.

Question 5

Question 5 was about writing algebraic expressions for the given models, performing operations with algebraic expressions, and modeling the results of operations. The question is given in Figure 4.15 below.

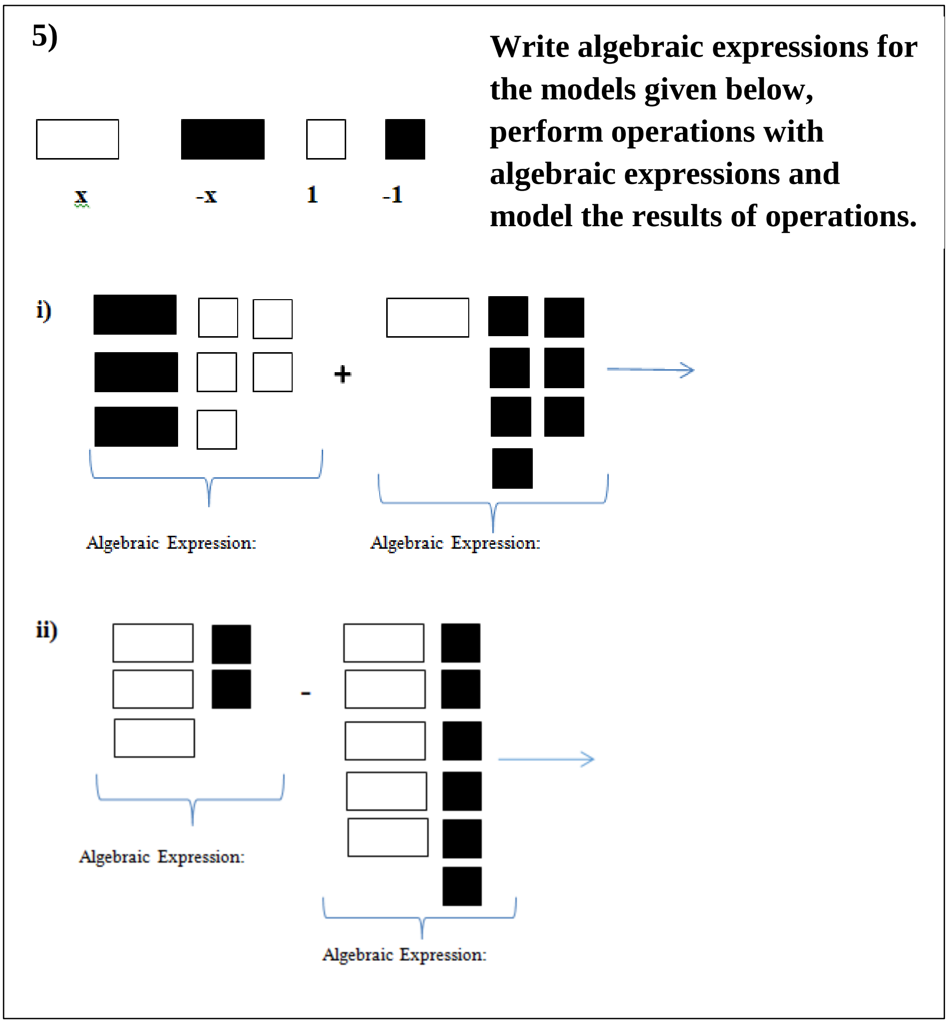


Figure 4.15 5th Question in AAT

There were two sub-questions in question 5. The performance of students in percentage and frequency for each sub-question are given in Table 4.16.

Table 4.16 Frequency and percentage of correct and incorrect answers, and empty responses of EG and CG in question 5 in AAT

	Experimental Group (N=23)			**Control Group (N=20)**		
	Correct (%)	**Incorrect (%)**	**Empty (%)**	**Correct (%)**	**Incorrect (%)**	**Empty (%)**
Q5a	17 (73.9)	4 (17.4)	2 (8.7)	5 (25)	10 (50)	5 (25)
Q5b	7 (30.4)	11 (47.8)	5 (21.7)	1 (5)	14 (70)	5 (25)

EG students performed considerably better than CG students in the 5th question. Most of the students in the EG responded to the first task in the question correctly and more EG students than CG students were able to respond correctly to the second task. Figure 4.16 shows one EG student's responses to question 5.

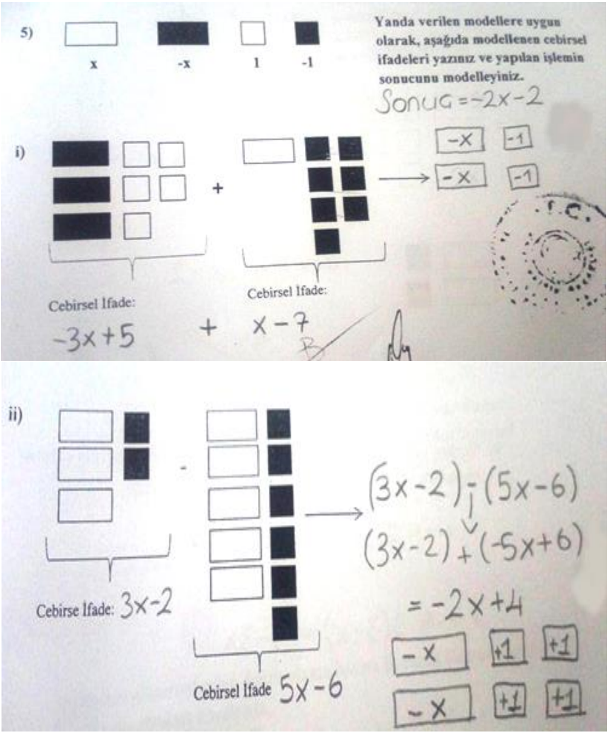


Figure 4.16 One EG student's answer to 5th question

Only one CG student was able to respond to the second task in the question. Figure 4.17 shows one CG student's responses to the tasks in question 5. It seems that this CG student was not able to fully conceptualize –x and -1 in algebraic expressions. Some students in CG performed operations between unlike terms and they added or subtracted like in integers. This might show that these students had difficulty in applying arithmetical operations in algebraic expressions particularly adding or subtracting like terms.

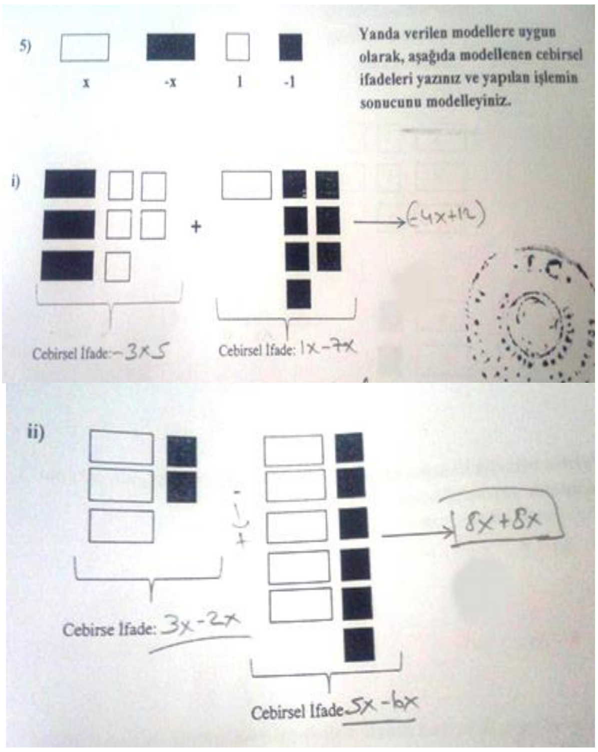


Figure 4.17 One CG student's answer to 5th question

It can be said that using algebra tiles had a positive effect on performing operations with the given models of algebraic expressions.

Question 6

Question 6 was about performing operations for the given algebraic expressions. The question is given in Figure 4.18 below.

6) Perform operations for the algebraic expressions given below.

ii) (4x–5) + (–2x+3)

ii) (x+3) – (–2x – 1)

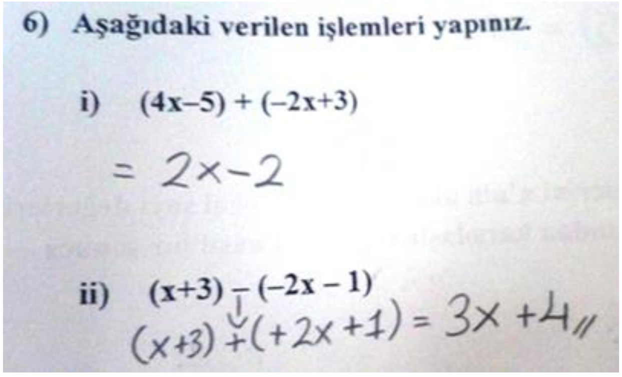
Figure 4.18 6th Question in AAT

There were two sub-questions in question 6. The performance of students in percentage and frequency for each sub-question are given in Table 4.17.

Table 4.17 Frequency and percentage of correct and incorrect answers, and empty responses of EG and CG in question 6 in AAT

	Experimental Group (N=23)			**Control Group (N=20)**		
	Correct (%)	**Incorrect (%)**	**Empty (%)**	**Correct (%)**	**Incorrect (%)**	**Empty (%)**
Q6a	14 (60.9)	6 (26.1)	3 (13)	8 (40)	8 (40)	4 (20)
Q6b	12 (52.2)	7 (30.4)	4 (17.4)	3 (15)	13 (65)	4 (20)

The results showed that using algebra tiles seemed to help EG students when they performed addition and subtraction with the given algebraic expressions. Although the number of incorrect responses was high in EG, students in this group performed considerably better than the CG students as seen in Figure 4.19.



6) Aşağıdaki verilen işlemleri yapınız.

i) (4x–5) + (–2x+3)

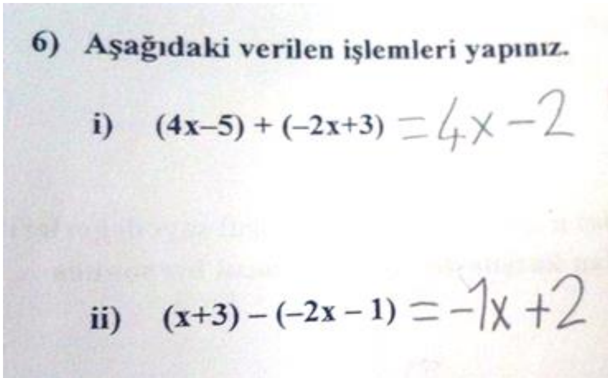
$= 2x-2$

ii) (x+3) – (–2x – 1)

$(x+3) + (+2x+1) = 3x+4$

Figure 4.19 One EG student's answer to 6^{th} question

Figure 4.20 shows one CG students' response to the tasks in question 6.



6) Aşağıdaki verilen işlemleri yapınız.

i) (4x–5) + (–2x+3) $= 4x-2$

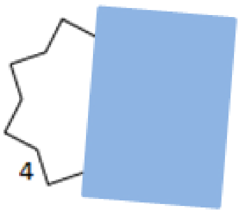
ii) (x+3) – (–2x – 1) $= -1x+2$

Figure 4.20 One CG student's answer to 6^{th} question

Question 7

Question 7 was about finding the perimeter of the polygon in terms of algebraic expressions whose number of the side is unknown. The question is given in Figure 4.21 below.

7)



Assume that one part of the regular polygon, whose the length of one side is 4 unit and number of the side is unknown, is covered by paper. Find the perimeter of the polygon in terms of algebraic expression.

Figure 4.21 7th Question in AAT

There were not any sub-questions in question 7. The performance of students in percentage and frequency for the question are given in Table 4.18.

Table 4.18 Frequency and percentage of correct and incorrect answers, and empty responses of EG and CG in question 7 in AAT

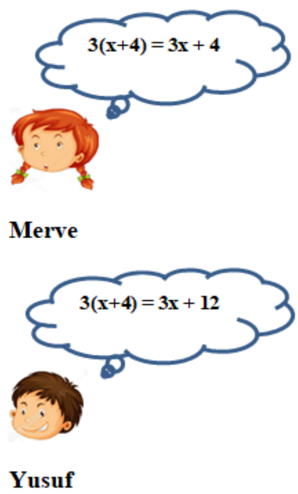
	Experimental Group (N=23)			**Control Group (N=20)**		
	Correct (%)	**Incorrect (%)**	**Empty (%)**	**Correct (%)**	**Incorrect (%)**	**Empty (%)**
Q7	5 (21.7)	5 (21.7)	13 (56.5)	2 (10)	5 (25)	13 (65)

Both EG and CG students had a difficulty in finding the perimeter of the polygon in terms of algebraic expressions whose number of the side is unknown because most of the EG and CG students responded 7th question incorrectly or they left the question blank. A few students in both groups wrote 4+x as an answer instead of 4x, or tried to added apparent sides of the polygon.

Question 8

Question 8 was about explaining which representation is correct. The question is given in Figure 4.22 below.

8)



Merve and Yusuf says the equivalent algebraic expression of 3(x+4) as shown in the picture. Explain which representation is correct.

Figure 4.22 8th Question in AAT

There were not any sub-questions in question 8. The performance of students in percentage and frequency for the question are given in Table 4.19.

Table 4.19 Frequency and percentage of correct and incorrect answers, and empty responses of EG and CG in question 8 in AAT

	Experimental Group (N=23)			**Control Group (N=20)**		
	Correct (%)	**Incorrect (%)**	**Empty (%)**	**Correct (%)**	**Incorrect (%)**	**Empty (%)**
Q8	16 (69.6)	6 (26.1)	1 (4.3)	12 (60)	3 (15)	5 (25)

The number of EG and CG students who gave correct and incorrect answers to the 8th question were close to each other. Most of the both EG and CG students could explain which representation is correct. One student in EG explained the correct representation by assigning an arbitrary value to the x in both Merve's and Yusuf's responses as a different solution than other students.

Question 9

Question 9 was about writing given algebraic expressions as multiplication of a natural number and an algebraic expression. The question is given in Figure 4.23 below.

9) Write each algebraic expression given below as multiplication of a natural number and an algebraic expression.

- **6x + 8**
- **9 – 3x**
- **–2x – 10**

Figure 4.23 9^{th} Question in AAT

There were three sub-questions in question 9. The performance of students in percentage and frequency for each sub-question are given in Table 4.20.

Table 4.20 Frequency and percentage of correct and incorrect answers, and empty responses of EG and CG in question 9 in AAT

	Experimental Group (N=23)			**Control Group (N=20)**		
	Correct (%)	**Incorrect (%)**	**Empty (%)**	**Correct (%)**	**Incorrect (%)**	**Empty (%)**
Q9a	12 (52.2)	8 (34.8)	3 (13)	5 (25)	4 (20)	11 (55)
Q9b	10 (43.5)	9 (39.1)	4 (17.4)	5 (25)	6 (30)	9 (45)
Q9c	8 (34.8)	10 (43.5)	5 (21.7)	4 (20)	6 (30)	10 (50)

Frequencies and percentages on Table 4.20 showed that EG students performed better than CG students in writing given algebraic expressions as multiplication of a natural number and an algebraic expression. Figure 4.24 shows one EG student’s responses to the tasks in question 9.

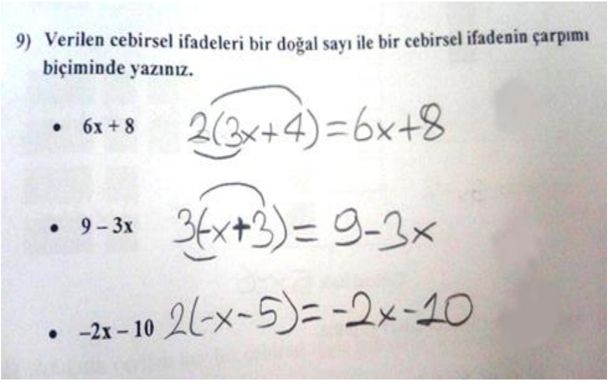


Figure 4.24 One EG student's answer to 9th question

One CG student's responses to the 9th question's tasks are given in Figure 4.25. It seemed that the student did not fully comprehend the transition from algebraic expression to the multiplication of a natural number and an algebraic expression. In addition, a few students in CG thought that the number outside the parenthesis was multiplied by only x because they wrote 6(x+8), 3(9-x), 2(-x-10) respectively as answers. This might show that these students had difficulty in transmission of arithmetical understanding to algebraic contexts because this question included distributive property.

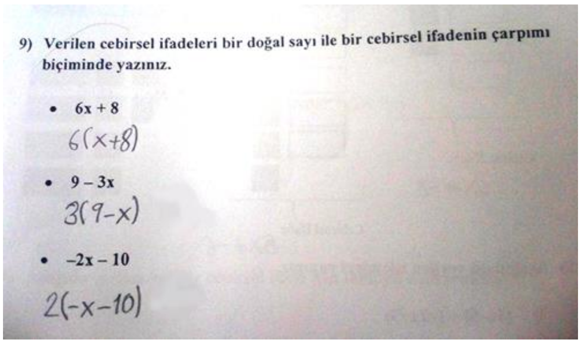


Figure 4.25 One CG student's answer to 9th question

A few students in both groups multiplied the terms of given algebraic expressions. For example, for item 9a, they wrote 48x as an answer by multiplying 6x and 8.

Question 10

Question 10 was about explaining which algebraic expression is greater. The question is given in Figure 4.26 below.

> **10) When you compare 3n and (n+3) algebraic expressions for different values of n, which algebraic expression is greater? Explain.**

Figure 4.26 10th Question in AAT

There were not any sub-questions in question 10. The performance of students in percentage and frequency for the question are given in Table 4.21.

Table 4.21 Frequency and percentage of correct and incorrect answers, and empty responses of EG and CG in question 10 in AAT

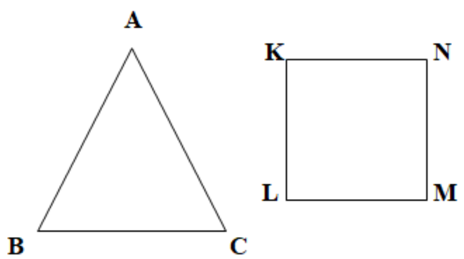
	Experimental Group (N=23)			**Control Group (N=20)**		
	Correct (%)	**Incorrect (%)**	**Empty (%)**	**Correct (%)**	**Incorrect (%)**	**Empty (%)**
Q10	2 (8.7)	14 (60.9)	7 (30.4)	1 (5)	7 (35)	12 (60)

Only a few students in both EG and CG could explain that which algebraic expression is greater. Most of the EG and CG students responded 10th question incorrectly or they left the question blank. Some students in both groups evaluated given algebraic expressions for only one value and according to the result of this evaluation, they wrote one was greater than other. Using algebra tiles did not make difference in favour of EG for this question.

Question 11

Question 11 was about finding the length of one side of the square in terms of algebraic expressions. The question is given in Figure 4.27 below.

11)



The perimeters of ABC equilateral triangle and KLMN square are equal to each other. If the length of one side of the triangle is 8a, find the length of one side of the square.

Figure 4.27 11th Question in AAT

There were not any sub-questions in question 11. The performance of students in percentage and frequency for the question are given in Table 4.22.

Table 4.22 Frequency and percentage of correct and incorrect answers, and empty responses of EG and CG in question 11 in AAT

	Experimental Group (N=23)			**Control Group (N=20)**		
	Correct (%)	**Incorrect (%)**	**Empty (%)**	**Correct (%)**	**Incorrect (%)**	**Empty (%)**
Q11	12 (52.2)	6 (26.1)	5 (21.7)	6 (30)	2 (10)	12 (60)

EG students performed better in 11th question than CG students. Most of the CG students left the question blank. It can be said that using algebra tiles made difference in favour of EG for this question. Figure 4.28 shows one EG student's answer to the task in question 11.

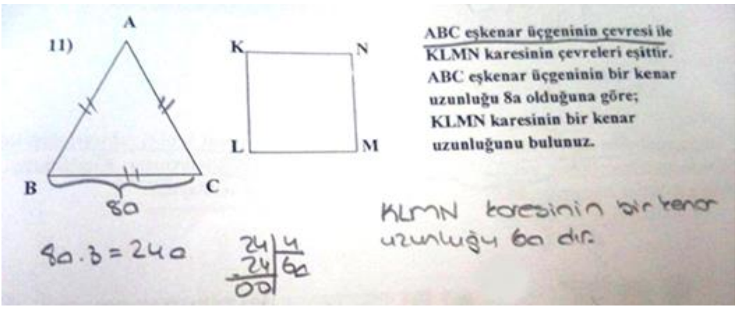


Figure 4.28 One EG student's answer to 11th question

The detailed analysis of the responses to the tasks in AAT by EG and CG students showed that more EG students responded to the questions correctly than CG students. However, students in both groups could not perform well in some of the tasks such as finding the perimeter of a given rectangle in terms of algebraic expressions; finding the perimeter of the polygon in terms of algebraic expressions whose number of the side is unknown; and explaining which algebraic expression is greater. These findings might show that algebra tiles might have limited but positive effect on the 6^{th} grade students' algebraic thinking.

4.3 Students' Views about Using Algebra Tiles

The third research question was "What are the 6^{th} grade students' views about use of algebra tiles in mathematics lessons?" To investigate this research question, responses to the questions given by the experimental group students in Views about Algebra Tiles Questionnaire were examined.

Most of the students indicated that they have used counters and fraction tiles as materials so far. In addition, students expressed that algebra tiles helped them "*learn better*", "*understand better*", "*remember easily*", "*make complicated operations easier*", "*learn faster*", and "*make lessons enjoyable*". Moreover, students generally stated that they did not have any difficulties while using algebra tiles and learning with them. One student mentioned that "*I had a difficulty at the beginning, but now, I understand better*."

Students referred to the enjoyment and learning easier with their group friends about group work. One student stated "*I helped my group mates for their understanding.*" In addition, some of the students commented on using algebra tiles in group works. They stated that algebra tiles and group work together facilitated their understanding. The response of one student illustrates this:

> "*In group work, students who understood explained others who did not understand. Before the group work, we were confused about what to do. We had some questions such as "how will we do?" in our*

minds. However, when we used algebra tiles, we understood immediately."

Students made comments on lessons in which they used algebra tiles. The following excerpts illustrate their positive comments:

"*While using algebra tiles, we did not only have fun, but we also comprehended topic. I have already liked mathematics, now, I began to like much more. I thanks to the teachers who developed this idea*."

"*Using algebra tiles helped me find more tricks, tactics, and methods and improved my perception*."

"*At the beginning, since I did not understand, it was boring. However, after I learned, it was funny. If we use algebra tiles again, it will be funny again*."

4.4 Summary of the Findings

The aim of this study was to investigate the effects of using algebra tiles on sixth grade students' algebra achievement and algebraic thinking, and to investigate their views about using algebra tiles. There was no significant difference in terms of students' prior algebra knowledge in EG and CG. Although the difference was not significant between groups, experimental group (M = 8.61) had higher mean score than control group (M = 6.95). In addition, it was concluded that there was not a difference between the answers of experimental group and control group students in Prior Algebra Knowledge Test. After the treatment, t-test result showed that there was no statistically significant mean difference between the groups in terms of posttest scores. Although the mean difference between groups was not significant, experimental group (M = 19.65) had higher mean score than control group (M = 14.85). In addition, when both EG and CG students' answers were examined in detail, it was concluded that EG students performed better than CG students in Algebra Achievement Test in writing algebraic expressions for the given models; determining variable, term, constant term, coefficients and sum of coefficients of given algebraic expressions; performing operations with the given models of algebraic expressions; performing addition and subtraction with the given algebraic expressions; writing given algebraic expressions as multiplication of a natural number and an algebraic expression; and finding the length of one side of the square in terms of algebraic expressions tasks. However, students in both groups did not perform sufficiently in tasks about finding the perimeter of a given rectangle in terms of algebraic expressions; finding the perimeter of the polygon in terms of algebraic expressions whose number of the side is unknown; and explaining which algebraic expression is greater.

The findings of the study showed that most of the students indicated positive effects of using algebra tiles and group work in mathematics lessons. They stated that using algebra tiles helped them learn and understand better and made lessons enjoyable. In addition, they stated that working in groups led to enjoyment and learning easier with group mates.

CHAPTER 5

DISCUSSION AND CONCLUSIONS

The purpose of the present study was to investigate the effects of using algebra tiles on sixth grade students' algebra achievement, algebraic thinking and views about using algebra tiles. In this chapter, findings are summarized and discussed. In addition, recommendations and implications for the future studies are presented.

5.1 Effects of Using Algebra Tiles on Algebra Achievement

When the mean scores of groups in AAT were compared, t-test result showed that there was no statistically significant mean difference between the experimental and control groups in terms of posttest scores. Hence, using algebra tiles in algebraic expressions did not lead to significantly better results than regular instruction. Although no significant effect was found by the statistical analysis, experimental group had higher mean score than the control group. The instructional method, use of algebra tiles, might have played a role in this score difference.

The result of this study for algebra achievement was consistent with those of similar research studies conducted by Sharp (1995) and Schlosser (2010). While differences existed in the students' grade levels and algebraic concepts in the mentioned studies, the present study confirmed the results of those previous studies.

The duration of the treatment was limited to seven class hours in this study and limited exposure to algebra tiles can be the reason for non-significant results. The study conducted by Larbi and Okyere (2016) showed significant result in favour of experimental group who used algebra tiles when the duration of treatment was over a period of four weeks.

In addition, although control group students did not use algebra tiles as concrete manipulatives and did not experience algebra tiles themselves as experimental group students did, the teacher drew algebra tiles on the board to familiarise students. This situation can also be the reason for non-significant results.

5.2 Algebraic Thinking and Using Algebra Tiles in Groups

Although no significant difference was found between the groups according to the students' mean scores, qualitative differences were found in students' learning in about half of the questions in AAT.

In the 1st and 8th questions in AAT, the number of both experimental group and control group students' correct and incorrect answers were close to each other and they gave similar answers to the questions. Both experimental and control group students were able to determine whether given representations were correct or incorrect; and they were able to explain which representation was correct by using multiplication of a natural number and an algebraic expression. This can be due to the fact that algebra tiles were not used in experimental group and similar examples were given in both groups while both groups were learning different representations of given algebraic expressions. In addition, both experimental group and control group students knew distributive property before the study because they learned it at the beginning of the sixth grade in number and operations learning area and operations with natural numbers sub-learning area through the objective of *"students make operations related to taking the common multiple parenthesis and applying distributive property."* Therefore, they could transform their knowledge of distributive property in natural numbers to in algebraic expressions.

In the 4th, 7th, and 10th questions, both experimental group and control group students had a difficulty in finding the perimeter of a given rectangle in terms of algebraic expressions; finding the perimeter of the polygon in terms of algebraic expressions whose number of the side was unknown; and explaining which algebraic expression was greater. It can be said that neither regular instruction nor use of algebra tiles made a difference in students' responses to these tasks. The reason of students' difficulty can be lack of their prior knowledge in related geometry concepts. Additionally, the fact that students were not familiar to that kind of tasks can be the reason of students' lower performance in these questions. This might also show that students cannot perform when they were asked to combine their knowledge and skills of different concepts in a single task.

Experimental group students performed better than control group students in the rest of the questions in AAT (2^{nd}, 3^{rd}, 5^{th}, 6^{th}, 9^{th}, and 11^{th} questions). Experimental group students were better in writing algebraic expressions for the given models; determining variable, term, constant term, coefficients and sum of coefficients of given algebraic expressions; performing operations with the given models of algebraic expressions; performing addition and subtraction with the given algebraic expressions; writing given algebraic expressions as multiplication of a natural number and an algebraic expression; and finding the length of one side of the square in terms of algebraic expressions.

It can be concluded that although there was no statistically significant difference between experimental and control groups, using algebra tiles during the instructions made a qualitative difference between students' learning. Experimental group students were able to make transition between representational models and symbolic representations of the algebraic expressions. In addition, they were able to analyze given algebraic expressions and determine their parts. They also learned performing addition and subtraction in algebraic expressions meaningfully. Moreover, experiences with algebra tiles as concrete manipulative helped students develop algebraic thinking.

Students perform better when they use multiple materials while learning algebra (Koğ & Başer, 2012). However, the experimental group students indicated that they have only used counters and fraction tiles in the mathematics lesson before they used the algebra tiles. If these students had used more manipulatives in mathematics lessons especially while learning algebra topics, it would be possible that they would benefit more from the treatment. Therefore, algebra instruction should employ more manipulatives to make students' learning more meaningful and algebraic thinking better.

Although group work is a beneficial method in general (Koblitz & Wilson, 2014), limited exposure to group work may have prevented seeing its benefits in the present study. The teacher indicated that experimental group students have not worked in a group in the mathematics lessons before. Since they worked in a group while learning algebraic expressions for the first time in the exploration phase, this can be considered as adaption period. Thus, if the students worked in a group longer time or if they worked in a group before this study, different results would have been obtained.

5.3 Students' Views about Instructions with Algebra Tiles and Group Work

Students in the present study had never seen or used algebra tiles before. For this reason, they were surprised at first. At the beginning, some of them confused which color-tile represented negative and which color-tile represented positive, and also which piece represented x and which piece represented 1. After they learned these representations, they did not have any difficulty in using them. In addition, students were willing to participate in the lessons and they were active while using algebra tiles. Even rather passive students tried to model given algebraic expressions by using algebra tiles. These students participated in group works and pair works, and they also showed their models on the board. Most of the students seemed to be having fun while using algebra tiles and they enjoyed through lessons. None of them considered algebra tiles as toys or played with them. They concentrated on learning algebraic expressions with algebra tiles.

Previous studies have found that even though there were not statistically significant differences in students' performance in algebra tasks when they used algebra tiles, students expressed the effect of algebra tiles in their meaningful learning of the algebra concepts (Schlosser, 2010; Sharp, 1995). In Yıldız (2012)'s qualitative case study, students stated that they felt both playing and learning better the concepts. These findings were all consistent with the present study's findings. Experimental group students in the present study also stated that with the help of algebra tiles, they learned concepts faster and remembered concepts easily.

Students claimed that group work facilitated their understanding, similar to previous results (Balt, 2017). While students in a group are learning a new concept, they might realize what other students in the group do not understand and they can explain that concept to them and correct their misconceptions (Webb & Farivar, 1994). This was observed during the treatment in the experimental group and some of the students also expressed that they helped their groupmates for their understanding during the group works. Good cooperative learning occurs when the students solve tasks involving use of manipulatives while working in group because they motivate and entertain students (Mulryan, 1994). Students in the present study also referred to enjoyment about both using algebra tiles and working in groups.

5.4 Recommendations for Further Studies

In the present study, there were one experimental group and one control group in one school. In the future studies, there can be three groups (classes) that one group uses algebra tiles in group work, other group uses algebra tiles without group work and the other group uses neither algebra tiles nor group work to increase the generalizability. Teaching duration of the content was limited to seven class hours in this study. The length of the treatment can be increased in future studies. In this study, data were collected from only students. Future research studies can collect data from both students and their teachers about how algebra tiles increase students' algebra achievement. In the present study, effects of using algebra tiles on students' algebra achievement and algebraic thinking were investigated. Studies investigating the effects of using algebra tiles should focus more on students' conceptual understanding of algebra. In future studies, interviews can be carried out with students for an in-depth examination. Furthermore, students' attitudes and motivations can also be investigated. While algebra tiles were used in the lessons during the study, students were not permitted to use them in posttest. It is suggested that algebra tiles can also be used during the assessment part.

5.5 Implications

The findings of this study addressed that a focus on the qualitative benefits of the manipulatives, such as meaningful learning of the concepts, should be carefully investigated when there is no statistically significant effect of the treatment including manipulatives. Similarly, the nature of the group work and how group work helped students learning the mathematical concepts even when students did not have any group work experience before should be explored in rather qualitative ways. The findings of this study showed that students can recognize the benefits of a new approach to learning including the manipulatives and the group work and studies should consider gathering students' views when they are subject to a new treatment.

This study presents some implications for middle school mathematics teachers, teacher educators, program makers and MoNE. Mathematics teachers can use lesson plans in the present study, activity sheets in the lesson plans, PAKT and AAT in their lessons or they can prepare their own resources by benefiting from these resources. MoNE should provide training and seminars to in-service teachers in order to make them familiar with the manipulatives and to encourage them to use manipulatives. MoNE should also provide more manipulatives for the schools. Although the findings of the study do not specifically address this issue, students should be given an opportunity to create manipulatives with the help of their teachers before the related concepts, if possible. This would familiarize students with manipulatives. In addition, mathematics laboratories consisting of different manipulatives can be set up in the schools.

In the questionnaire, students agreed the idea that they liked using algebra tiles during the instructions about algebraic expressions. Therefore, teachers can integrate algebra tiles to the lessons including algebra concepts. In addition, cooperative and discovery learning methods can be used while teaching algebra concepts in the middle schools.

Pre-service teachers should be given an opportunity to use manipulatives during their teaching practice in real classroom environments and gain experience about use of manipulatives in order to use them in their teaching career. Teacher educators should enable pre-service teachers to prepare lesson plans and activities including appropriate teaching methods and they should provide an environment for pre-service teachers to engage in using manipulatives especially in methods of teaching mathematics courses. There should be courses that promote pre-service teachers' skills of creating and using algebra tiles as well as other manipulatives in teaching algebra topics in the middle schools.

Program makers, book authors and researchers can take into consideration the results of this study while writing textbooks for students and teachers. They can include more tasks encouraging the teachers and the students to use algebra tiles in algebra concepts.

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APPENDICES

Appendix A: Prior Algebra Knowledge Test

CEBİR ÖN BİLGİ TESTİ

Sevgili öğrenciler;

Bu testin amacı cebirsel ifadeler konusuna ilişkin ön bilgilerinizi ölçmektir. Sizden beklenen aşağıda verilen 4 soruyu cevaplandırmanızdır. Cevaplarınız herhangi bir şekilde not ile değerlendirme amacıyla kullanılmayacaktır. Süreniz 40 dakikadır.

Ad Soyad:

Sınıf/Şube:

1) Aşağıda sözel olarak verilen durumlara uygun cebirsel ifadeleri yazınız.

Bir kavanozdaki şekerlerin 2 katının 3 eksiği……………………….

Aslı'nın parasının yarısının 12 TL fazlası…………………………...

Efe'nin bilyelerinin 13 eksiğinin 5 katı……………………………..

Bir sayının 2 eksiği ile 2 katının toplamı……………………………

15 dakikası geçen sınavın kalan süresi………………………………

Toplamları 80 olan iki sayıdan biri m ise diğeri…………………….

2) Aşağıda verilen cebirsel ifadelere uygun sözel durumlar yazınız.

$5(c-2)$…………………………………………………………………

$\frac{m+1}{2}$…………………………………………………………………..

$7k-6$…………………………………………………………………..

$\frac{x}{2}+5$…………………………………………………………………...

3) Aşağıda verilen cebirsel ifadelerin değerini değişkenin alacağı doğal sayı değerleri için hesaplayınız.

- $\frac{2(n-3)}{5}$ **cebirsel ifadesinin n=13 için değerini bulunuz.**

- $\frac{3x+4}{2}$ **cebirsel ifadesinin x=6 için değerini bulunuz.**

- $\frac{85}{y}+1$ **cebirsel ifadesinin y=5 için değerini bulunuz.**

4)

i. "Bir akvaryumdaki balıkların sayısının 3 katının 7 fazlası" ifadesine uygun bir cebirsel ifade yazınız.

ii. Yazdığınız cebirsel ifadenin değerini değişkenin 15 olması durumunda hesaplayınız.

Appendix B: Algebra Achievement Test

CEBİR BAŞARI TESTİ

Sevgili öğrenciler;

Bu testin amacı cebirsel ifadeler konusuna ilişkin bilgilerinizi ölçmektir. Sizden beklenen aşağıda verilen 11 soruyu cevaplandırmanızdır. Cevaplarınız herhangi bir şekilde not ile değerlendirme amacıyla kullanılmayacaktır. Süreniz 40 dakikadır.

Ad Soyad:

Sınıf/Şube:

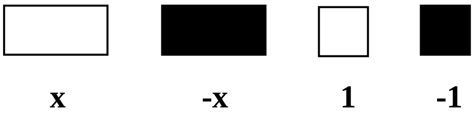
1) Aşağıda verilen gösterimlerden doğru olanın başına D, yanlış olanın başına Y yazınız ve yanlış olanları düzeltiniz.

….. $y + y + 1 = 3y$

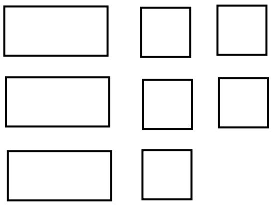
….. $x + x - 1 = -1 + 2x$

….. $\frac{a}{2} + \frac{a}{2} = 2a$

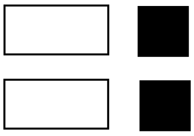
….. $5 - c - c + c = 5 - 3c$

2) Aşağıda 

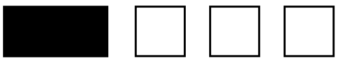
ile modellenen cebirsel ifadeleri yazınız.



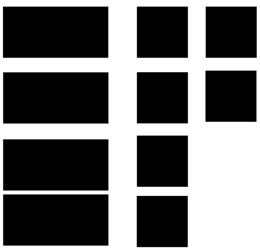
Cebirsel İfade:



Cebirsel İfade:



Cebirsel İfade:



Cebirsel İfade:

3) Aşağıda verilen her bir cebirsel ifade için tabloyu doldurunuz.

Cebirsel İfade	Değişken(ler)	Terim(ler)	Sabit Terim(ler)	Katsayı(lar)	Katsayı(lar) Toplamı
3k					
–6xy +1					
2a + 5b – 8					

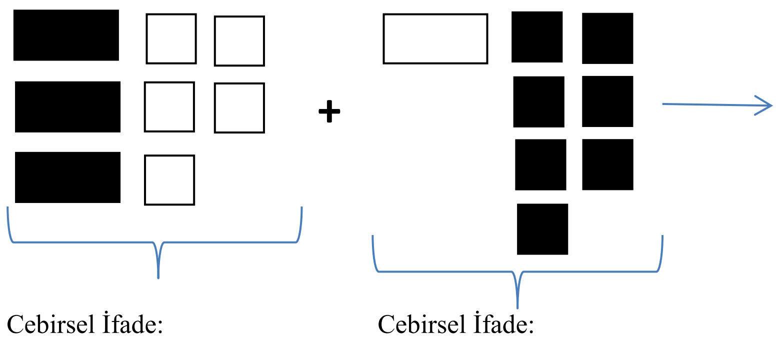
4) **Bir dikdörtgenin kısa kenarı uzun kenarından 3cm eksiktir. Dikdörtgenin çevresini veren cebirsel ifadeyi yazınız.**

5)

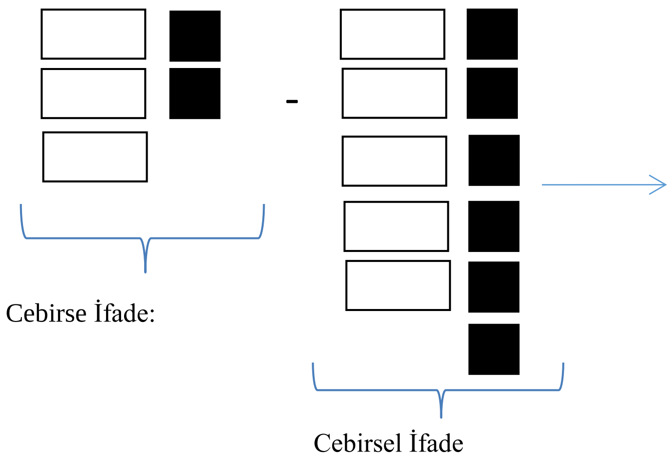
x **-x** **1** **-1**

Yanda verilen modellere uygun olarak, aşağıda modellenen cebirsel ifadeleri yazınız ve yapılan işlemin sonucunu modelleyiniz.

i)



ii)

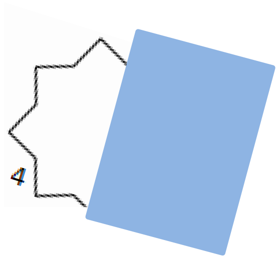


6) **Aşağıdaki verilen işlemleri yapınız.**

i) **$(4x-5) + (-2x+3)$**

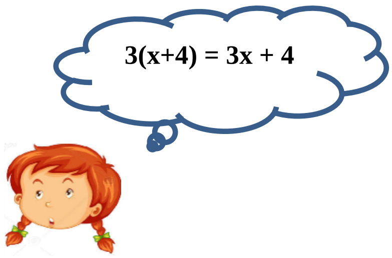
ii) **$(x+3) - (-2x - 1)$**

7)

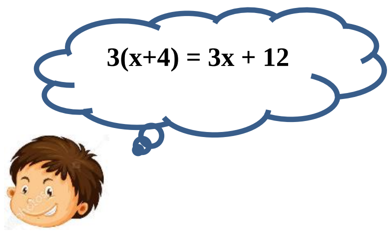


Bir kenarının uzunluğu 4 birim olan ve kenar sayısını bilmediğimiz düzgün bir çokgenin bir tarafına kâğıt kapandığını varsayalım. Bu durumda çokgenin çevresini veren cebirsel ifadeyi yazınız.

8)



Merve



Yusuf

Merve ve Yusuf 3(x+4) cebirsel ifadesinin eşitini yandaki gibi söylüyorlar. Kimin doğru söylediğini açıklayınız.

9) **Verilen cebirsel ifadeleri bir doğal sayı ile bir cebirsel ifadenin çarpımı biçiminde yazınız.**

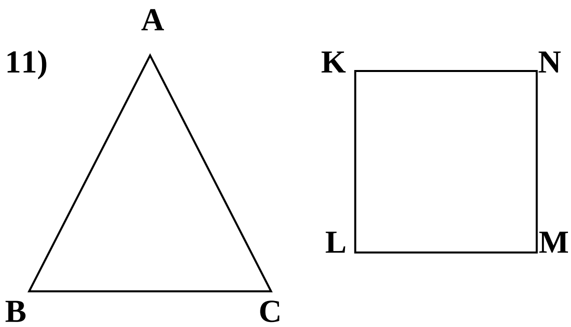
- **6x + 8**

- **9 – 3x**

- **–2x – 10**

10) **3n ve (n+3) cebirsel ifadelerini n’nin alacağı farklı doğal sayı değerleri için büyüklük-küçüklük bakımından karşılaştırdığınızda nasıl bir sonuca varırsınız?**

11)



ABC eşkenar üçgeninin çevresi ile KLMN karesinin çevreleri eşittir. ABC eşkenar üçgeninin bir kenar uzunluğu 8a olduğuna göre; KLMN karesinin bir kenar uzunluğunu bulunuz.

Appendix C: Views about Algebra Tiles Questionnaire

CEBİR KAROSU KULLANIMINA İLİŞKİN ÖĞRENCİ GÖRÜŞ FORMU

Sevgili öğrenciler;

Bu formun amacı, sizin cebir karosu kullanımınıza ilişkin görülerinizi almaktır. Lütfen elinizden geldiğince aşağıdaki soruları cevaplandırmaya çalışınız. İsim belirtmenize gerek yoktur. Cevaplarınız notlandırma amacıyla kullanılmayacaktır. Bu yüzden lütfen gerçek görüşünüzü belirtiniz.

1) Matematik derslerinde ya da ders dışı etkinliklerde daha önce herhangi bir materyal (kesir çubukları, geometri tahtası, sayma pulları vb.) kullandınız mı?

☐Evet ☐Hayır

Yanıtınız evet ise, hangi materyalleri kullandınız? Belirtiniz.

2) Cebir karoları konuyu anlamanızı etkiledi mi?

☐Evet ☐Hayır

Nasıl etkiledi? Açıklayınız.

3) Cebir karolarıyla öğrenirken herhangi bir zorlukla karşılaştınız mı?

☐Evet ☐Hayır

Yanıtınız evet ise, nasıl bir zorluk yaşadınız? Açıklayınız.

4) Grup çalışmaları cebirsel ifadeleri öğrenmenizi etkiledi mi?

☐Evet ☐Hayır

Nasıl etkiledi? Açıklayınız.

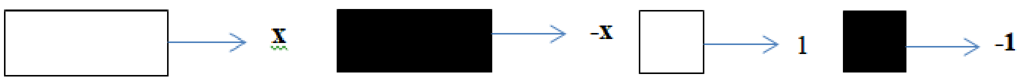
5) Cebir karolarını kullandığınız derslere ilişkin yorumlarınızı ve önerilerinizi lütfen belirtiniz.

Appendix D: Experimental Group Lesson Plans

Sınıf:	6 **Süre:** 2 ders saati
Öğrenme Alanı:	Cebir
Alt Öğrenme Alanı:	Cebirsel ifadeler
Kazanım(lar):	6.2.1.4. Basit cebirsel ifadelerin anlamını açıklar.
Gerekli Ön Bilgiler:	Cebirsel ifade ve değişken kavramlarını bilir.
Öğretme-Öğrenme Yöntem ve Teknikleri:	Soru-cevap Tartışma Grup Çalışması Bireysel Çalışma
Beceriler:	İletişim, ilişkilendirme, akıl yürütme ve psikomotor beceriler
Materyaller:	Cebir Karoları Etkinlik Kâğıdı Çıkış Kartı (Exit card)

Giriş (Engagement)

Öğretmen, derste öğrencilerin basit cebirsel ifadelerin anlamını açıklamayı öğreneceklerini söyledikten sonra, cebir karolarını aşağıdaki gibi tahtaya yapıştırır ve kısaca tanıtır.

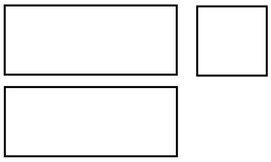


Öğretmen: "Matematik materyallerinden biri olan cebir karoları, dikdörtgenler ve karelerden oluşmaktadır. (Kırmızı cebir karoları gösterilerek) 1 karelerle, x dikdörtgenler ile ifade edilir. Mavi cebir karoları ise kırmızı olanların toplama işlemine göre tersi yani zıt işaretlisidir. Kırmızı ve mavi cebir karoları birlikte kullanıldığında birbirini götürürler ve 0 elde edilir.

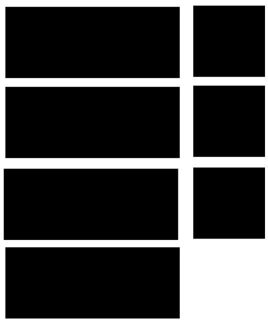
Ayrıca uzun kenarı x olan dikdörtgenin kısa kenarı da küçük karenin bir kenarına yani 1′e eşittir. Cebir karoları işleyişinde hem cebir hem geometri yer alır. Cebir karoları matematiğin birçok alanında kullanılmaktadır. Biz, cebirsel ifadeleri daha iyi anlamamız amacıyla görselleştirmek için kullanacağız."

Keşfetme (Exploration)

Öğretmen aşağıda verilen modelleri tahtaya yapıştırır ve öğrencilerden gösterilen modelleri cebirsel ifade olarak yazmalarını ister. Öğrencilerin verdiği cevaplar sınıfça tartışılır ve ortak bir sonuca varılır.



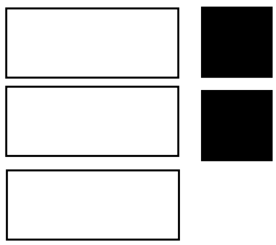
Cebirsel İfade: 2x+1



Cebirsel İfade: -4x-3

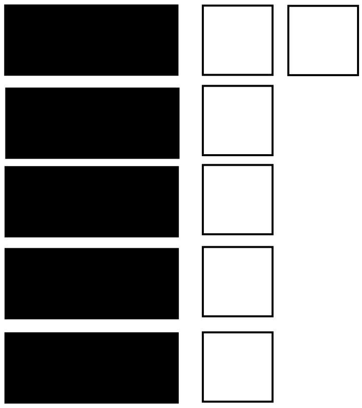
Ardından, öğrencilere ikişerli gruplar halinde çalışmaları söylenir ve gruplara cebir karoları dağıtılır. Öğretmen tahtaya 3x-2 cebirsel ifadesini yazar ve öğrencilerden cebir karoları ile göstermelerini ister. Gruplar çalışırken, öğretmen gruplar arasında dolaşarak gözlem yapar, gerektiğinde destek verir. Gruplar modellemeyi tamamladıktan sonra, bir grup tahtaya gelerek buldukları modeli tahtaya yapıştırır. Doğruluğu sınıfça tartışılır.

Cebirsel İfade: 3x-2



Bu defa, öğretmen tahtaya -5x+6 cebirsel ifadesini yazar ve öğrencilerden cebir karoları ile göstermelerini ister. Gruplar çalışırken, öğretmen gruplar arasında dolaşarak gözlem yapar, gerektiğinde destek verir. Gruplar modellemeyi tamamladıktan sonra, bir grup tahtaya gelerek buldukları modeli tahtaya yapıştırır. Doğruluğu sınıfça tartışılır.

Cebirsel İfade: -5x+6



Tahtada yukarıdaki iki model ve cebirsel olarak yazılımları yer alırken, öğretmen öğrencilerden tekrar 3x-2 cebirsel ifadesini düşünmelerini ister ve şu soruyu sorar:

- 3x-2 cebirsel ifadesini daha açık nasıl yazabiliriz / Başka nasıl ifade edebiliriz?

Öğrenciler düşündükten ve beyin fırtınası yaptıktan sonra verdikleri cevaplar sınıfça tartışılır.

Açıklama (Explanation)

Ardından öğretmen, "3x-2 cebirsel ifadesini x+x+x -1-1 şeklinde de yazabiliriz" diye açıklar.

Derinleştirme (Elaboration)

Bu aşamada öğrencilerden cebir karoları toplanır ve her bir öğrenciye EK 1'deki etkinlik kâğıdı dağıtılır. Öğrencilerden verilen cebirsel ifadelerin farklı gösterimlerini yazmaları istenir. Öğrenciler bireysel olarak çalışırlar. Öğrenciler tamamladıktan sonra, her biri için bir öğrenci tahtaya çıkarak nasıl yaptığını sınıfa açıklar.

Değerlendirme (Evaluation)

Değerlendirme için, öğrencilere aşağıdaki çıkış kartı dağıtılır. Öğrenciler verilen soruyu bireysel olarak cevaplandırarak, kartı sınıftan çıkarken öğretmene teslim ederler.

Çıkış Kartı (Exit Card)

ÇIKIŞ KARTI
Derste gördüklerinizden farklı olarak bir cebirsel ifade yazınız ve cebir karolarıyla gösterilmiş halinin şeklini çiziniz.   **Adı Soyadı:**

EK 1

ETKİNLİK KÂĞIDI

3x-2 = x+x+x -1-1 gösteriminden yola çıkarak, aşağıda verilen cebirsel ifadelerin farklı gösterimlerini yazınız.

1) b+b+b+b

2) -2m-6

3) 5c+3

4) $\frac{k}{15} - \frac{4}{15}$

5) $\frac{1}{4} \cdot y$

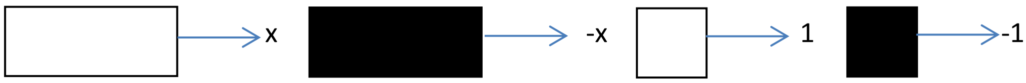
6) $\frac{2a}{3}$

7) $\frac{3+2x}{5}$

Sınıf:	6 **Süre:** 3 ders saati
Öğrenme Alanı:	Cebir
Alt Öğrenme Alanı:	Cebirsel ifadeler
Kazanım(lar):	6.2.1.5. Cebirsel ifadelerle toplama ve çıkarma işlemleri yapar.
Gerekli Ön Bilgiler:	Cebirsel ifade ve değişken kavramlarını bilir. Cebirsel ifadenin değerlerini değişkenin alacağı farklı doğal sayı değerleri için hesaplamayı bilir. Basit cebirsel ifadelerin anlamını açıklamayı bilir.
Öğretme-Öğrenme Yöntem ve Teknikleri:	Soru-cevap Tartışma Grup Çalışması Bireysel Çalışma
Beceriler:	İletişim, ilişkilendirme, akıl yürütme ve psikomotor beceriler
Materyaller:	Cebir Karoları Etkinlik Kâğıtları Çıkış Kartı (Exit card)

Giriş (Engagement)

Dersin giriş kısmında, cebir karoları tahtaya öğretmen tarafından aşağıdaki gibi yapıştırılır ve öğrencilere hatırlatılarak bir önceki ders tekrar edilir.

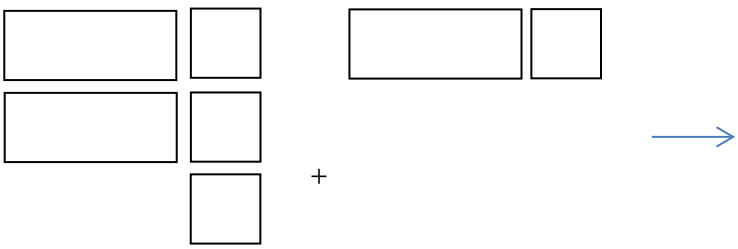


Öğrencilere dörderli gruplar halinde çalışmaları söylenir ve gruplara cebir karoları dağıtılır.

Keşfetme (Exploration)

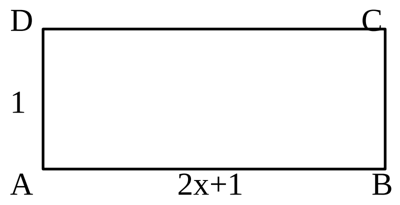
Öğretmen toplama işleminin aşağıdaki gibi cebir karolarıyla modellenmiş halini tahtaya yapıştırır. Öğrencilerden bu işlemi yine cebir karoları kullanarak yapmaları ve sonuçları da cebir karolarıyla göstermeleri istenir. Ayrıca, yapılan işlemleri cebirsel olarak da ifade etmeleri söylenir.

i)



Gruplar çalışırken, öğretmen gruplar arasında dolaşarak gözlem yapar, gerektiğinde destek verir. Gruplar tamamladıktan sonra, öğretmen cebir karolarıyla tahtada nasıl yapılacağını bir kez daha gösterir. Ardından, öğretmen tahtaya aşağıdaki dikdörtgeni çizerek öğrencilere şu soruyu sorar:

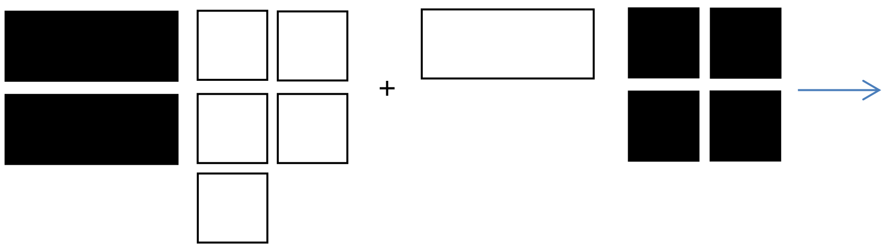
Dikdörtgenin A noktasında bulunan bir karınca, şeklin çevresini bir tam tur dolanıyor. Karıncanın gittiği yolu veren cebirsel ifadeyi nasıl yazabiliriz?



Öğrencilerden sorunun nasıl yapılacağına dair tahminleri alınır. Daha sonra, grup olarak sorunun cevabını bulmaları istenir. Gruplar tamamladıktan sonra, her gruptan bir öğrenci nasıl yaptıklarını sınıfa açıklar.

Öğretmen toplama işleminin aşağıdaki gibi cebir karolarıyla modellenmiş halini tahtaya yapıştırır. Öğrencilerden bu işlemi yine cebir karoları kullanarak yapmaları ve sonuçları da cebir karolarıyla göstermeleri istenir. Ayrıca, yapılan işlemleri cebirsel olarak da ifade etmeleri söylenir.

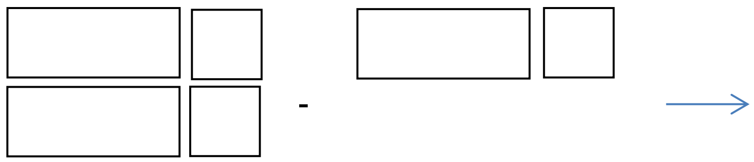
ii)



Gruplar çalışırken, öğretmen gruplar arasında dolaşarak gözlem yapar, gerektiğinde destek verir. Gruplar tamamladıktan sonra, her gruptan bir öğrenci nasıl yaptıklarını sınıfa açıklar. Ayrıca, öğretmen cebir karolarıyla tahtada nasıl yapılacağını bir kez daha gösterir.

Öğretmen "eğer işlem çıkarma işlemi olsaydı cebir karolarıyla nasıl yapardık?" diye sorar. Gruplar tartışır ve tahminleri alınır. Daha sonra, öğretmen çıkarma işleminin aşağıdaki gibi cebir karolarıyla modellenmiş halini tahtaya yapıştırır. Öğrencilerden bu işlemi yine cebir karoları kullanarak yapmaları ve sonuçları da cebir karolarıyla göstermeleri istenir. Ayrıca, yapılan işlemleri cebirsel olarak da ifade etmeleri söylenir.

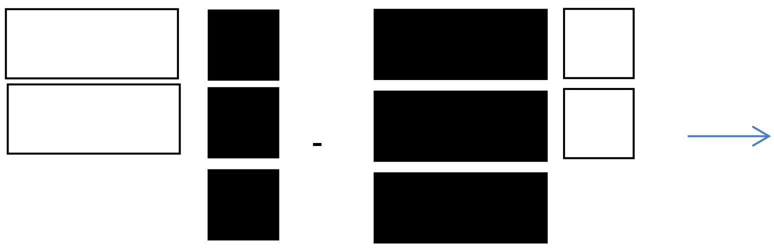
iii)



Gruplar çalışırken, öğretmen gruplar arasında dolaşarak gözlem yapar, gerektiğinde destek verir. Gruplar tamamladıktan sonra, öğretmen cebir karolarıyla tahtada nasıl yapılacağını bir kez daha gösterir.

Öğretmen çıkarma işleminin aşağıdaki gibi cebir karolarıyla modellenmiş halini tahtaya yapıştırır. Öğrencilerden bu işlemi yine cebir karoları kullanarak yapmaları ve sonuçları da cebir karolarıyla göstermeleri istenir. Ayrıca, yapılan işlemleri cebirsel olarak da ifade etmeleri söylenir.

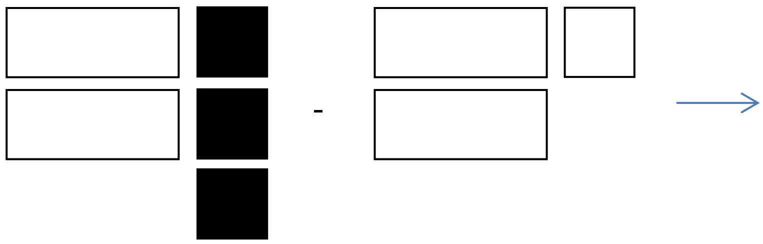
iv)



Gruplar çalışırken, öğretmen gruplar arasında dolaşarak gözlem yapar, gerektiğinde destek verir. Gruplar tamamladıktan sonra, öğretmen cebir karolarıyla tahtada nasıl yapılacağını bir kez daha gösterir.

Öğretmen çıkarma işleminin aşağıdaki gibi cebir karolarıyla modellenmiş halini tahtaya yapıştırır. Öğrencilerden bu işlemi yine cebir karoları kullanarak yapmaları ve sonuçları da cebir karolarıyla göstermeleri istenir. Ayrıca, yapılan işlemleri cebirsel olarak da ifade etmeleri söylenir.

v)



Gruplar çalışırken, öğretmen gruplar arasında dolaşarak gözlem yapar, gerektiğinde destek verir. Gruplar tamamladıktan sonra, bir öğrenci cebir karolarıyla nasıl yaptıklarını tahtada açıklayarak gösterir.

Yapılanlardan hareketle, öğrencilere aşağıdaki sorular yöneltilir:

- Toplama ve çıkarma işlemlerini yaparken nasıl bir yol izlediniz?
- Cebirsel ifadelerdeki toplama ve çıkarma işlemleriyle; tamsayılardaki toplama ve çıkarma işlemleri arasında bir bağlantı kurabilir miyiz?
- Cebir karoları olmadan cebirsel ifadelerde toplama ve çıkarma işlemlerini nasıl yaparız?

Öğrenciler yöneltilen sorularla ilgili beyin fırtınası yaptıktan ve tartıştıktan sonra açıklama kısmına geçilir.

Açıklama (Explanation)

- ✓ Cebirsel ifadelerde toplama ve çıkarma işlemleri yapılırken tam sayılardaki toplama ve çıkarma işlemleri yöntemleri kullanılır.
- ✓ Cebirsel ifadelerle çıkarma işlemi yaparken önce çıkarma işlemi toplama işlemine çevrilir. Sonra toplama işlemi yapılır.
- ✓ Bir cebirsel ifadede değişkenleri ve bu değişkenlerinin üsleri aynı olan terimlere **benzer terim** denir.

- ✓ Cebirsel ifadelerde toplama ve çıkarma işlemleri benzer terimlerin toplanıp çıkarılması ve sabit terimlerin toplanıp çıkarılması olarak ifade edilir.
- ✓ Bir cebirsel ifadede “+” veya “-“ lerle ayrılan kısımların her birine **terim**, terimlerin sayısal çarpanlarına **katsayı** denir. Değişken içermeyen terime ise **sabit terim** adı verilir.

Derinleştirme (Elaboration)

Bu aşamada, öğrencilerden cebir karoları toplanır ve EK 1’deki etkinlik kâğıdı dağıtılır. Öğrencilerden verilen her bir cebirsel ifade için tabloyu <u>bireysel olarak</u> doldurmaları istenir. Etkinlik tamamlandıktan sonra, öğretmen her bir istenilen cevap için, öğrencilerin ne cevap verdiğini sorar ve ortak bir karara varılarak öğretmen doğru cevapları tahtaya yazar.

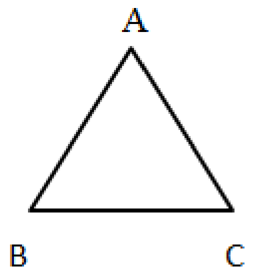
Ardından, EK 2’deki etkinlik kâğıdı öğrencilere dağıtılır. Öğrencilerden verilen toplama ve çıkarma işlemlerini <u>bireysel olarak</u> <u>cebir karosu kullanmadan</u> yapmaları istenir**.** Öğrenciler tamamladıktan sonra, her işlem için bir öğrenci tahtaya çıkarak nasıl yaptığını sınıfa açıklar.

Değerlendirme (Evaluation)

Değerlendirme için, öğrencilere aşağıdaki çıkış kartı dağıtılır. Öğrenciler verilen soruyu cevaplandırarak, kartı çıkarken öğretmene teslim ederler.

Çıkış Kartı (Exit Card)

ÇIKIŞ KARTI



Yanda verilen ABC üçgeninde
|AB| = 2b+3 cm, |BC| = b-4 cm,
ve Ç(ABC)= 5b+4 cm
olduğuna göre |AC| kenarının
uzunluğu kaç cm'dir?

Adı Soyadı:

EK 1

ETKİNLİK KÂĞIDI

Aşağıda verilen her bir cebirsel ifade için tabloyu doldurunuz.

Cebirsel İfade	Değişken (ler)	Terim (ler)	Sabit Terim (ler)	Katsayı (lar)	Katsayı (lar) Toplamı
-6a+6b					
3x-2y-5					
4m+7mn-n+9					

EK 2

ETKİNLİK KÂĞIDI

Aşağıda verilen toplama ve çıkarma işlemlerini cebir karosu kullanmadan yapınız.

1) $(5x - 10) + (-2x + 7)$

2) $(8x - 15) - (9x - 15)$

3) $(3t + 21) + (-2t + 11) + (t - 33)$

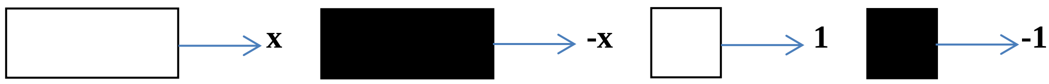
Sınıf:	6 **Süre:** 2 ders saati
Öğrenme Alanı:	Cebir
Alt Öğrenme Alanı:	Cebirsel ifadeler
Kazanım(lar):	6.2.1.6. Bir doğal sayı ile bir cebirsel ifadeyi çarpar.
Gerekli Ön Bilgiler:	Cebirsel ifade ve değişken kavramlarını bilir. Basit cebirsel ifadelerin anlamını açıklamayı bilir. Cebirsel ifadelerle toplama ve çıkarma işlemleri yapmayı bilir.
Öğretme-Öğrenme Yöntem ve Teknikleri:	Soru-cevap Tartışma Grup Çalışması Bireysel Çalışma
Beceriler:	İletişim, ilişkilendirme, akıl yürütme ve psikomotor beceriler
Materyaller:	Cebir Karoları Etkinlik Kâğıtları Çıkış Kartı (Exit card)

Giriş (Engagement)

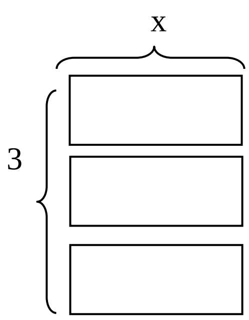
Öğrencilere EK 1’deki etkinlik kâğıdı dağıtılır ve sorularla ilgili 5 dakika düşünmeleri istenir. Ardından öğrencilerin buldukları sonuçlar sınıfça tartışılır.

Keşfetme (Exploration)

Öğrencilere ikişerli gruplar halinde çalışmaları söylenir ve gruplara cebir karoları dağıtılır. Ayrıca, cebir karoları tahtaya öğretmen tarafından aşağıdaki gibi yapıştırılır.

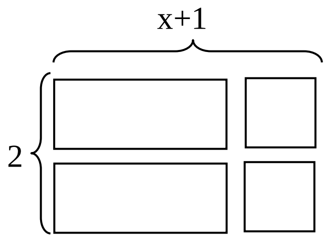


Öğrencilerden 3x cebirsel ifadesini cebir karolarıyla göstermeleri istenir. Öğrenciler gösterdikten sonra, doğru model öğretmen tarafından tahtaya yapıştırılarak gösterilir. İşlem sonucu da cebirsel olarak aşağıdaki gibi yazılır.



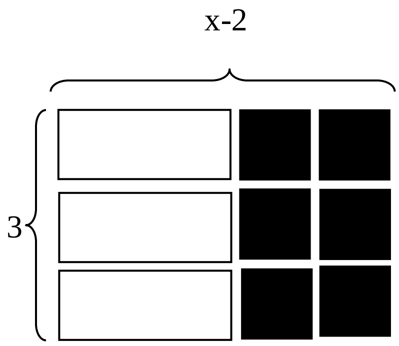
$3.x = 3x$

Öğrencilerden 2 ile (x+1) cebirsel ifadesinin çarpımını cebirsel karolarıyla göstermeleri istenir. Öğrenciler gösterdikten sonra, doğru model öğretmen tarafından tahtaya yapıştırılarak gösterilir. İşlem sonucu da cebirsel olarak aşağıdaki gibi yazılır.



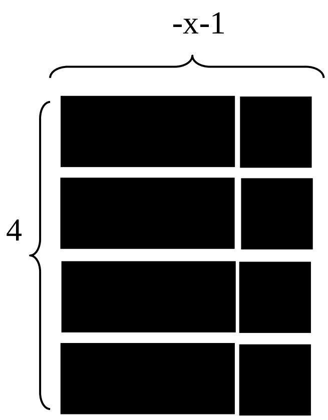
2.(x+1) = 2x + 2

Öğrencilerden 3 ile (x-2) cebirsel ifadesinin çarpımını cebir karolarıyla göstermeleri istenir. Öğrenciler gösterdikten sonra, doğru model öğretmen tarafından tahtaya yapıştırılarak gösterilir. İşlem sonucu da cebirsel olarak aşağıdaki gibi yazılır.



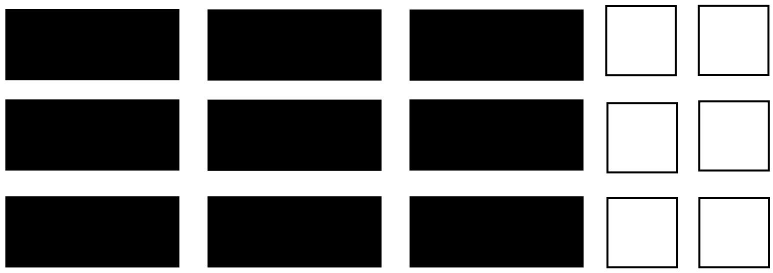
3. (x-2) = 3x-6

Öğrencilerden 4 ile (-x-1) cebirsel ifadesinin çarpımını cebir karolarıyla göstermeleri istenir. Öğrenciler gösterdikten sonra, doğru model öğretmen tarafından tahtaya yapıştırılarak gösterilir. İşlem sonucu da cebirsel olarak aşağıdaki gibi yazılır.



4.(-x-1) = -4x-4

Bu defa öğretmen aşağıda verilen modeli tahtaya yapıştırır. Öğrencilerden, verilen modeli bir doğal sayı ile bir cebirsel ifadenin çarpımı biçiminde yazmaları istenir. Öğrenciler yazdıktan sonra, tahtaya da aşağıdaki gibi öğretmen tarafından yazılır. Toplama işleminin değişme özelliği vurgulanarak, her iki şekilde de yazılabileceği söylenir.



$-9x + 6 = 3.(-3x+2) = 3.(2-3x) = 6-9x$

5 farklı model cebirsel gösterimleriyle birlikte tahtada yer alırken, yapılanlardan hareketle, öğrencilere aşağıdaki soru yöneltilir:

- Cebir karoları olmadan cebirsel ifadelerde çarpma işlemini nasıl yaparız?

Öğrenciler yöneltilen sorularla ilgili beyin fırtınası yaptıktan ve tartıştıktan sonra açıklama kısmına geçilir.

Açıklama (Explanation)

"Bir doğal sayı bir cebirsel ifade ile çarpılırken; doğal sayı cebirsel ifadenin her terimi ile ayrı ayrı çarpılır" ifadesi öğretmen tarafından vurgulanır. Tahtada cebir karolarının yanında yer alan cebirsel gösterimleri, öğretmen aşağıdaki gibi oklar çizerek tekrar açıklar.

$2.(x+1) = 2.x + 2.1 = 2x + 2$

$3.(x-2) = 3.x - 3.2 = 3x - 6$

$4.(-x-1) = 4.-x - 4.1 = -4x - 4$

$3.(2-3x) = 3.2 - 3.3x = 6 - 9x$

Derinleştirme (Elaboration)

Bu aşamada, öğrencilerden cebir karoları toplanır ve EK 2'deki etkinlik kâğıdı dağıtılır. Öğrencilerden verilen çarpma işlemlerini bireysel olarak cebir karosu kullanmadan yapmaları istenir. Ayrıca, verilen gösterimlerden doğru ve yanlış olanları belirlemeleri ve yanlış olanları düzeltmeleri istenir.

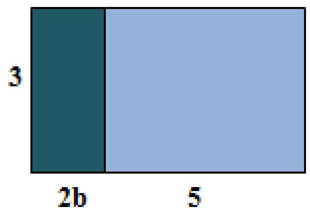
Öğrenciler etkinlikleri tamamladıktan sonra, her işlem için bir öğrenci tahtaya çıkarak nasıl yaptığını sınıfa açıklar.

Değerlendirme (Evaluation)

Değerlendirme için, öğrencilere aşağıdaki çıkış kartı dağıtılır. Öğrenciler verilen soruyu cevaplandırarak, kartı çıkarken öğretmene teslim ederler.

Çıkış Kartı (Exit Card)

ÇIKIŞ KARTI



Yanda verilen dikdörtgenin kenar uzunluklarına göre, dikdörtgenin alanını veren cebirsel ifadeyi yazınız.

Ad Soyad:

EK 1

ETKİNLİK KÂĞIDI

Şeker dolu bir kavanozun içinde kaç tane şeker olduğunu bilmediğimizi ve bu kavanozlardan 6 tane olduğunu düşünelim. Bu durumda toplam şeker miktarını nasıl ifade ederiz?

Arif Bey'in yeni aldığı halının kısa kenarı 2 metre, uzun kenarı x+3 metre olduğuna göre, halının yerde kapladığı alanı cebirsel olarak nasıl ifade ederiz?

EK 2

ETKİNLİK KÂĞIDI

1) Aşağıda verilen çarpma işlemlerini yapınız.

3.(8 – 2m)

6.(-3x + 2y – 4)

(10a + 8+b). 2

2) Aşağıda verilen gösterimlerden doğru ve yanlış olanları belirleyiniz ve yanlış olanları düzeltiniz.

4(x-2) = 4x – 2

3(-2x + 4) = -6x + 7

5(6-x) = -5x + 30

Appendix E: Rubric for Prior Algebra Knowledge Test

Cebir Ön Bilgi Testi İçin Puanlama Anahtarı

1.Madde

0. Yanlış cevaplar

1. Doğru cevaplar

   Örneğin: 2x-3, 2y-3, 2a-3, 2k-3…

   $\frac{x}{2}+12$, $\frac{y}{2}+12$, $\frac{z}{2}+12$, $\frac{b}{2}+12$…

   5(x-13), 5(t-13), 5(c-13)…

   x-2+2x, 3x-2…

   x-15, s-15, a-15…

   80- m

2.Madde

0. Yanlış cevaplar

1. Doğru cevaplar

   Örneğin: Bir sayının 2 eksiğinin 5 katı

   Ahmet'in bilyelerinin 1 fazlasının yarısı

   Yolcuların 7 katının 6 eksiği

   Bir sayının yarısının 5 fazlası

3.Madde

0. Yanlış cevaplar

1. Doğru cevaplar: 4

   11

   18

4.Madde (i)

0. Yanlış cevaplar

1. Doğru cevaplar: 3x+7, 3b+7, 3a+7…

4.Madde (ii)

0. Yanlış cevaplar

1. Doğru cevap: 52

Appendix F: Rubric for Algebra Achievement Test

Cebir Başarı Testi İçin Puanlama Anahtarı

1.Madde

0. Yanlış cevaplar

1. Doğru cevaplar: Y
   D
   Y
   Y

- y + y + 1 = 2y + 1
- $\frac{a}{2} + \frac{a}{2} = a$
- 5 – c – c + c = 5 – c

2.Madde

0. Yanlış cevaplar

1. Doğru cevaplar: 3x + 5
   2x – 2
   - x + 3
   - 4x – 6

3.Madde

0. Yanlış cevaplar

1. Doğru cevaplar

Cebirsel İfade	Değişken(ler)	Terim(ler)	Sabit Terim(ler)	Katsayı (lar)	Katsayı (lar) Toplamı
3k	k	3k	-	3	3
-6xy +1	x, y	-6xy, 1	1	-6, 1	-5
2a + 5b - 8	a, b	2a, 5b, -8	-8	2, 5, -8	-1

4. Madde

0. Yanlış cevaplar

1. Doğru cevaplar:

Örneğin: Dikdörtgenin kısa kenarı a-3 ve uzun kenarı a olursa;

Çevre= a+a+a-3+a-3 / 2a+2(a-3) / 2(a+(a-3))

2a+2a-6 / 4a-6

Ya da

Dikdörtgenin kısa kenarı a ve uzun kenarı a+3 olursa;

Çevre= a+3+a+3+a+a / 2(a+3)+2a / 2(a+(a+3))

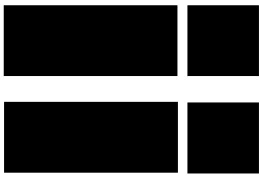
2a+6+2a / 4a+6

5.Madde (i)

0. Yanlış cevaplar

1. Doğru cevap:

(-3x + 5) + (x-7) = -2x-2

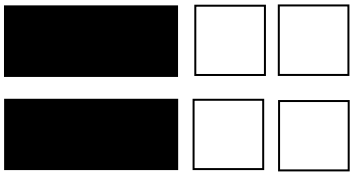


5.Madde (ii)

0. Yanlış cevaplar

1. Doğru cevap:

(3x-2) - (5x-6)= -2x+4



6.Madde (i)

0. Yanlış cevaplar

1. Doğru cevap: 2x-2

6.Madde (ii)

0. Yanlış cevaplar

1. Doğru cevap: 3x+4

7.Madde

0. Yanlış cevaplar

1. Doğru cevaplar: 4n, 4x, 4a, 4k…

Kenar sayısına n diyelim.

Bu durumda düzgün çokgenin çevresi n.4 / 4.n / 4n

8.Madde

0. Merve

1.Yusuf

Açıklama: Doğal sayı ile cebirsel ifade çarpılırken, doğal sayı ile cebirsel ifadenin bütün terimleri çarpılır
(çarpma işleminin toplama işlemi üzerine dağılma özelliği)

Ya da

3 defa (x+4) ifadesini toplarsak;

$(x+4)+(x+4)+(x+4) = 3x+12$

9.Madde

0. Yanlış cevaplar

1. Doğru cevaplar: $6x+8 = 2(3x+4)$

$9-3x = 3(3-x)$

$-2x-10 = 2(-x-5)$

10.Madde

0. Aynı, 3n büyük, n+3 büyük

1. n'nin aldığı farklı sayı değerlerine göre değişir

$n= 0$ ve $n=1$ için $n+3>3n$

$n\geq 2$ için $3n>n+3$

11.Madde

0. Yanlış cevaplar

1. Doğru cevap: 6a

ABC eşkenar üçgeninin bir kenar uzunluğu 8a ise çevresi 24a'dır. ABC eşkenar üçgeni ile KLMN karesinin çevreleri eşit olduğuna göre, KLMN karesinin de çevresi 24a'dır. Bu durumda, KLMN karesinin bir kenar uzunluğu 6a'dır.

Appendix G: Ethical Approval

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

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www.ueam.metu.edu.tr

Sayı: 28620816 / 576

15 ARALIK 2017

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Doç.Dr. Çiğdem HASER ;

Danışmanlığını yaptığınız yüksek lisans öğrencisi Büşra ÇAYLAN'ın **"Cebir Karosu Kullanımının 6. Sınıf Öğrencilerinin Cebir Başarısı ve Cebir Karosu Kullanımına İlişkin Görüşleri Üzerindeki Etkileri"** başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülerek gerekli onay **2017-EGT-188** protokol numarası ile **01.01.2018-30.12.2018** tarihleri arasında geçerli olmak üzere verilmiştir.

Bilgilerinize saygılarımla sunarım.

Prof. Dr. Ş. Halil TURAN
Başkan V

Prof. Dr. Ayhan SOL
Üye

Prof. Dr. Ayhan Gürbüz DEMİR
Üye

Doç. Dr. Yaşar KONDAKÇI
Üye

Doç. Dr. Zana ÇITAK
Üye

Yrd. Doç. Dr. Pınar KAYGAN
Üye

Yrd. Doç. Dr. Emre SELÇUK
Üye

Appendix H: Permission Obtained from Ministry of Education

T.C.
SAKARYA VALİLİĞİ
İl Milli Eğitim Müdürlüğü

Sayı : 10284503-605.01-E.1969506 29.01.2018
Konu: Araştırma İzinleri

MÜDÜRLÜK MAKAMINA

Orta Doğu Teknik Üniversitesi, Matematik ve Fen Bilimleri Eğitimi Bölümü Yüksek Lisans Programı öğrencisi Büşra ÇAYLAN'ın Doç. Dr. Çiğdem HASER'in danışmanlığında yürütmekte olduğu *"Cebir Karosu Kullanımının 6.Sınıf Öğrencilerinin Cebir Başarısı ve Cebir Karosu Kullanımına İlişkin Görüşleri Üzerindeki Etkileri"* konulu anket çalışması uygulama talebi, adı geçen Üniversitenin 17.01.2018 tarih ve 290 sayılı yazıları ile bildirilmiştir.

Söz konusu anket çalışmasının, İlimiz, Hendek İlçesine bağlı aşağıda belirtilen okullarda öğrenim gören 6. sınıf öğrencilerine, 2017-2018 eğitim öğretim yılında, öğretimin aksamasına mahal vermeden, gönüllülük esasına dayalı olarak, okul yönetiminin belirleyeceği zaman ve şartlarda uygulanması, yasal gerekliliğin ilgili Okul Müdürlüklerince yerine getirilmesi kaydıyla Müdürlüğümüzce uygun mütalaa edilmekte ise de;

Makamlarınızca uygun görülmesi halinde olurlarınıza arz ederim.

Osman TERZİ
İl Milli Eğitim Şube Müdürü

Eki: İl MEM Değerlendirme Onayı.

OLUR
29.01.2018

Ergüven ASLAN
İl Milli Eğitim Müdür V.

S.No	İLÇESİ	ÇALIŞMA YAPILACAK OKULUN ADI
1	Hendek	Şehit Ali Gaffar Okkan Ortaokulu
2	Hendek	Atikehanım Ortaokulu
3	Hendek	Şehit Mahmutbey Ortaokulu
4	Hendek	Ziya Gökalp Ortaokulu

Resmi Daireler Kampüsü
B Blok 54290 Adapazarı / SAKARYA
http://sakarya.meb.gov.tr /temelegitim54@meb.gov.tr

Ayrıntılı Bilgi İçin :Memur : İsa GEM
Tel : 0 264 251 36 14-15-16
Fax :0 264 251 36 04

Appendix I: Turkish Summary / Türkçe Özet

CEBİR KAROSU KULLANIMININ 6. SINIF ÖĞRENCİLERİNİN CEBİR BAŞARISI, CEBİRSEL DÜŞÜNMELERİ VE CEBİR KAROSU KULLANIMINA İLİŞKİN GÖRÜŞLERİ ÜZERİNDEKİ ETKİLERİ

GİRİŞ

Matematikteki önemli alanlardan biri de cebirdir. Cebirde sadece birkaç sayıyı değil, sayılar kümesini düşünmek gerekir (Palabıyık ve Akkuş, 2011). Bu nedenle, cebir aritmetikten daha soyut görünmektedir ve öğrenciler cebir öğrenme sürecinde çeşitli zorluklarla karşılaşmaktadırlar (NCTM, 2000). 2013 Ortaokul Matematik Öğretim Programı'nda cebir öğrenme alanına ilişkin kazanımlar ilk kez altıncı sınıf seviyesinde yer almaktadır ve ortaokulun sonuna kadar cebirle alakalı çeşitli kazanımlar mevcuttur. Bu sebeple, cebir ortaokul öğrencileri için matematiğin önemli öğrenme alanlarından biridir (MEB, 2013).

Cebirsel düşünme ise değişkenler arasındaki ilişkiyi açık hale getirecek şekilde nicel durumları gösterebilme kapasitesidir (Driscoll, 1999). İşbirlikli öğrenmeyi teşvik eden, öğrencilerin matematiksel söylemleri ile matematiksel fikir ve tahminlerini iletebilme fırsatları sunan sınıf ortamları cebirsel düşünmeyi daha çok geliştirir (Windsor, 2010). Cebirsel düşünmenin gelişimi bir anda olmaz. Öncelikle somut materyalleri, ardından resimsel, grafiksel ve son olarak sembolik gösterimlere ilişkin çeşitli gösterimleri anlamlı bağlamlarda deneyimlemek öğrencilerin cebirsel düşünmelerinin gelişimine katkı sağlar (Lawrence ve Hennessy, 2002).

Cebir ortaokullarda birtakım kuralların uygulanması ve belirli adımların izlenmesi olarak öğretilmekte, sadece ders kitabına bağlı kalınmakta ve öğretmen merkezli öğretim benimsenmektedir. Bunların sonucunda, öğrenciler cebiri ezbere öğrenmekte ve anlatılanları görselleştirememektedir. Ayrıca, aritmetik ve cebir konuları arasında bağlantı kurmakta da zorlanmaktadırlar (Kaput, 1999; Watt, 2005).

Cebir öğrenme sürecindeki zorlukları ortadan kaldırmak ve öğrencilerin cebirsel düşünmelerinin gelişimini sağlamak için somut materyaller kullanılabilir. Matematik Öğretmenleri Ulusay Konseyi (NCTM) birçok öğrencinin manipülatiflerin kullanıldığı uygulamalı işbirlikli öğrenmeden yarar sağladığını belirtmiştir (NCTM, 2000). İşbirlikli öğrenme öğrencilere farklı fikirler ve sorular ile karşılaşma, kendi anlamalarını kontrol etme ve kavramları daha derinlemesine algılama olanağı sunar (Mercier ve Higgins, 2013). Ayrıca, grup çalışması ile işbirlikli öğrenmede öğrenciler manipülatif kullanarak problemleri çözdüğünde motive olurlar ve eğlenirler (Mulryan, 1994).

Özellikle cebir karoları ile öğrenciler kavramların somut ve sembolik gösterimleri arasında geçiş yapabilirler. Cebir karolarını kullanarak cebirsel ifadeleri görsel ve uygulamalı olarak keşfederler ve kurallara kendi deneyimleri sonucu ulaşırlar (Okpube, 2016). Ayrıca, cebir karolarıyla modelleme öğrencilerin görselleştirme becerilerini geliştirir ve kavramsal anlama düzeylerini arttırır (Brahier, 2016).

Bu çalışmanın amacı, cebir karosu kullanımının altıncı sınıf öğrencilerinin cebir başarısı, cebirsel düşünmeleri ve cebir karosu kullanımına ilişkin görüşleri üzerindeki etkilerini incelemektir. 2013 Ortaokul Matematik Öğretim Programı'nda cebir öğrenme alanına ilişkin kazanımlar ilk olarak altıncı sınıf seviyesinde yer almaktadır ve eğer öğrenciler temel cebirsel kazanımları bu sınıf seviyesinde iyi öğrenemezlerse, gelecek yıllarda diğer cebirsel kavramları anlamada zorlanacaklardır. Araştırmalar, öğrencilerin ortaokulda cebir konularını iyi bir şekilde öğrendiklerinde, matematik testlerinde daha yüksek performans gösterdiklerini ve ileri matematiği daha kolay anladıklarını göstermektedir (Wang ve Goldschmidt, 2003). Bu yüzden materyal kullanımının altıncı sınıf öğrencilerinin cebir başarısına etkisinin araştırılması gelecek yıllardaki matematik başarısı için de bilgi verecektir.

Alan yazına bakıldığında, uluslararası çalışmalarda ortaokul öğrencilerine bir bilinmeyenli lineer denklemleri çözmeyi öğretirken (Magruder, 2012; Saraswati vd., 2016) ya da lise öğrencilerine çarpanlara ayırmayı öğretirken (Sharp, 1995; Thornton, 1995) cebir karolarının kullanıldığı görülmektedir. Ayrıca, polinomların çarpımında cebir karosu kullanımıyla ilgili çalışmalar da vardır (Goins, 2001; Johnson, 1993). Ancak, cebirle ilk defa karşılaşan öğrencilerin cebir karosu kullanımı ile ilgili çalışmalar oldukça azdır. Diğer taraftan, Türkiye'de cebir konularını öğretirken cebir karoları da dâhil olmak üzere çeşitli materyallerin kullanıldığı çalışmalar yer almaktadır (Akyüz ve Hangül, 2013; Gürbüz ve Toprak, 2014; Işık ve Çağdaşer, 2009; Koğ ve Başer, 2012; Palabıyık ve Akkuş, 2011; Yıldız, 2012). Ancak, cebir karolarının tek başına etkisinin incelendiği yeterli çalışma bulunmamaktadır. Bu sebeple, bu çalışmanın alan yazına katkı sağlayacağı düşünülmektedir.

Çalışmanın Amacı ve Araştırma Soruları

Çalışmanın amacı, cebir karosu kullanımının altıncı sınıf öğrencilerinin cebir başarısı ve cebirsel düşünmeleri üzerindeki etkilerini incelemektir. Ayrıca, bu çalışma cebir karosu kullanımına ilişkin öğrenci görüşlerini incelemeyi de hedeflemektedir. Çalışmada aşağıdaki araştırma sorularına cevap aranmıştır:

Cebir karosu kullanan altıncı sınıf öğrencileri ile kullanmayan altıncı sınıf öğrencilerinin sontest puanları arasında istatistiksel olarak anlamlı bir fark var mıdır?

Cebir karosu kullanan öğrenciler ile kullanmayan öğrencilerin cebirsel düşünmeleri Cebir Başarı Testi'nde nasıl farklılık göstermektedir?

Altıncı sınıf öğrencilerinin matematik derslerinde cebir karosu kullanımına ilişkin görüşleri nelerdir?

YÖNTEM

Araştırma Deseni

Araştırma sorularına cevap bulabilmek için nicel ve nitel yöntemler kullanılmıştır. Bu çalışma neden-sonuç ilişkisi araştırdığı için (Fraenkel, Wallen, ve Hyun, 2011), ilk araştırma sorusu için öntest-sontest kontrol gruplu araştırma deseni gerçekleştirilmiştir. Cebir Ön Bilgi Testi öğrencilerin cebirsel ifadelere ilişkin var olan bilgilerini belirlemek amacıyla öntest olarak uygulanmıştır. Ardından, öğrenciler cebirsel ifadelere ilişkin diğer kazanımları grup çalışması ile cebir karolarını kullanarak öğrenmişlerdir. Etkinliklerin sonunda Cebir Başarı Testi sontest olarak uygulanmıştır. İkinci ve üçüncü araştırma soruları için nitel yöntemler kullanılmıştır. İkici araştırma sorusuna cevap bulabilmek için deney ve kontrol grubu öğrencilerinin Cebir Başarı Testi'ndeki yanıtları detaylı bir şekilde incelenmiştir. Okullarda öğrencilerin gruplara rastgele atanması mümkün olmadığı için, aynı matematik öğretmeni tarafından okutulan var olan sınıflar deney ve kontrol gruplarını oluşturmuştur. Öntest içerdiği kazanımlar bakımından sontestten farklı olup, cebir karosunun etkileri sadece sontest ile incelenmiştir. Üçüncü araştırma sorusu ise Cebir Karosu Kullanımına İlişkin Öğrenci Görüş Formu ile araştırılmıştır.

Örneklem

Bu çalışmada, hedeflenen kitle Sakarya'daki bütün altıncı sınıf öğrencileri iken, erişilebilir kitle Hendek ilçesindeki bir devlet okuluna giden bütün altıncı sınıf öğrencileridir. Bu çalışma 2017-2018 eğitim-öğretim yılında Sakarya'nın Hendek ilçesinde bulunan ve araştırmacıya uygunluğu açısından seçilen bir devlet okulunda gerçekleştirilmiştir. Okuldaki altı şubeden aynı öğretmen tarafından okutulan iki şube rastgele olarak deney ve kontrol gruplarını oluşturmuştur. Deney grubunda 12 kız ve 14 erkek öğrenci, kontrol grubunda ise 10 kız ve 14 erkek öğrenci vardır. Seçilen sınıflardaki öğrencilerin matematik başarıları birbirine yakındır. Öğrencilerin yaşları 11-12 arasında değişmektedir. Tablo 3.1 deney ve kontrol grubundaki öntest ve sontest uygulanan öğrenci sayılarını göstermektedir. Öntest ve sontest uygulanan toplam 40 öğrenci çalışmanın örneklemini oluşturmaktadır.

Tablo 3.1 Deney ve Kontrol Grubundaki Öntest ve Sontest Uygulanan Öğrenci Sayıları

Gruplar	Öntest	Sontest	Öntest ∩ Sontest
Deney Grubu	23	23	22
Kontrol Grubu	21	20	18
Toplam	44	43	40

Veri Toplama Araçları

Çalışmanın verileri araştırmacı tarafından geliştirilen Cebir Ön Bilgi Testi, Cebir Başarı Testi ve Cebir Karosu Kullanımına İlişkin Öğrenci Görüş Formu aracılığıyla toplanmıştır.

Cebir Ön Bilgi Testi

Cebir Ön Bilgi Testi altıncı sınıf öğrencilerinin cebirsel ifadelere ilişkin ön bilgilerini öğrenmek amacıyla Ortaokul Matematik Öğretim Programı'ndaki kazanımlara (MEB, 2013) ve alan yazına uygun olarak araştırmacı tarafından geliştirilmiştir. Cebir Ön Bilgi Testi'nde yer alan cebirsel ifadelere ilişkin altıncı sınıf kazanımları Tablo 3.2 de belirtilmiştir.

Tablo 3.2 Cebir Ön Bilgi Testi'nde Yer Alan Kazanımlar

Cebirsel İfadeler
1. Sözel olarak verilen bir duruma uygun cebirsel ifade ve verilen bir cebirsel ifadeye uygun sözel bir durum yazar.
2. Cebirsel ifadenin değerlerini değişkenin alacağı farklı doğal sayı değerleri için hesaplar.

Cebir Ön Bilgi Testi 4 açık uçlu soru ve bu 4 sorunun da alt soruları ile birlikte toplam 15 sorudan oluşmaktadır. Test deney ve kontrol grubu öğrencilerine 40 dakikalık süre içerisinde gruplar arasında önbilgileri açısından fark olup olmadığını belirlemek amacıyla öntest olarak uygulanmıştır.

Testin geçerliliği için sorular matematik eğitimi alanında çalışan, biri ortaokul matematik öğretmeni olarak 10 yıldan fazla deneyime sahip olan, iki araştırmacı tarafından incelenmiş ve araştırmacıların görüşlerine göre tekrar düzenlenmiştir. Testin pilot uygulaması 40 dakikalık süre içerisinde Hendek ilçesinde bulunan başka bir ortaokuldaki 55 7. sınıf öğrencisine 2017-2018 eğitim-öğretim yılının güz döneminde uygulanmıştır. Testteki sorular doğru cevaplara 1, yanlış cevaplara 0 verilerek analiz edilmiştir. Cebir Ön Bilgi Testi'nden alınabilecek en yüksek puan 15 ve en düşük puan 0 olarak hesaplanmıştır. Testin güvenirlik katsayısı Kuder-Richardson 21 formülü ile hesaplanarak 0.73 bulunmuştur.

Cebir Başarı Testi

Cebir Başarı Testi altıncı sınıf öğrencilerinin cebirsel ifadeler konusundaki başarısını ve cebirsel düşünmelerini belirlemek amacıyla hazırlanmıştır. Testteki iki soru (7 ve 10) Hart, Küchemann, Brown, Kerslake ve Ruddock (1985) tarafından geliştirilen ve Altun (2005) tarafından Türkçe' ye uyarlanan "Chelsea Matematik Tanı Testleri-Cebir" den alınmış ve araştırmacı tarafından modifiye edilmiştir. Testteki diğer sorular ise Ortaokul Matematik Öğretim Programı'ndaki kazanımlara (MEB, 2013) ve alan yazına uygun olarak araştırmacı tarafından geliştirilmiştir. Cebir Başarı Testi'nde yer alan cebirsel ifadelere ilişkin altıncı sınıf kazanımları Tablo 3.9 da belirtilmiştir.

Tablo 3.9 Cebirsel İfadelere İlişkin Cebir Başarı Testi'nde Yer Alan Kazanımlar

Cebirsel İfadeler
1. Basit cebirsel ifadelerin anlamını açıklar.
2. Cebirsel ifadelerle toplama ve çıkarma işlemleri yapar.
3. Bir doğal sayı ile bir cebirsel ifadeyi çarpar.

Cebir Başarı Testi 11 açık uçlu soru ve bunlardan 6 tanesinin alt sorularıyla birlikte toplam 35 sorudan oluşmaktadır. Test deney ve kontrol grubu öğrencilerine 40 dakikalık süre içerisinde gruplar arasında cebir başarısı ve cebirsel düşünmeleri açısından fark olup olmadığını belirlemek amacıyla sontest olarak uygulanmıştır.

Cebir Başarı Testi, hem 2013 Ortaokul Matematik Öğretim Programı cebirsel ifadelere ilişkin altıncı sınıfta yer alan kazanımlara ilişkin soruları hem de öğretim programındaki kazanımlarla direk bağlantılı olmayıp cebirsel düşünmeyi belirlemeyi hedefleyen soruları içermektedir.

Testin geçerliliği için sorular, matematik eğitimi alanında çalışan, biri ortaokul matematik öğretmeni olarak 10 yıldan fazla deneyime sahip olan, iki araştırmacı tarafından incelenmiş ve araştırmacıların görüşlerine göre tekrar düzenlenmiştir. Testin pilot uygulaması 40 dakikalık süre içerisinde Hendek ilçesinde bulunan başka bir ortaokuldaki 52 7. sınıf öğrencisine 2017-2018 eğitim-öğretim yılının güz döneminde uygulanmıştır. Testteki sorular doğru cevaplara 1, yanlış cevaplara 0 verilerek analiz edilmiştir. Cebir Başarı Testi'nden alınabilecek en yüksek puan 35 ve en düşük puan 0 olarak hesaplanmıştır. Testin güvenirlik katsayısı Kuder-Richardson 21 formülü ile hesaplanarak 0.62 bulunmuştur.

Cebir Karosu Kullanımına İlişkin Öğrenci Görüş Formu

Cebir Karosu Kullanımına İlişkin Öğrenci Görüş Formu 5 açık uçlu sorudan oluşmaktadır ve öğrencilerin derslerde cebir karosu kullanımına ilişkin görüşlerini öğrenmek amacıyla araştırmacı tarafından hazırlanmıştır. Görüş formunda öğrencilere daha önce materyal kullanıp kullanmadıkları, cebir karosu kullanmanın konuyu anlamalarını etkileyip etkilemediği, cebir karosu kullanırken karşılaştıkları zorluklar ve grup çalışmalarının cebirsel ifadeleri öğrenmelerini etkileyip etkilemediği sorulmuştur. Ayrıca, dersle ilgili yorumları ve önerileri de alınmıştır. Görüş formunun geçerliliği için formda yer alan sorular matematik öğretmenleriyle ve matematik eğitiminde çalışan araştırmacılarla paylaşılmıştır. Uygulamadan sonra, görüş formunda yer alan sorular 20-30 dakikalık süre içinde deney grubunda yer alan öğrenciler tarafından cevaplandırılmıştır. Görüş formu için pilot çalışma yapılmamış ancak formdaki sorular deney ve kontrol grubunda yer almayan iki öğrenciye okutulmuştur.

Uygulama

Deney ve kontrol gruplarına cebirsel ifadelere ilişkin kazanımlar (basit cebirsel ifadelerin anlamını açıklama, cebirsel ifadelerde toplama ve çıkarma işlemleri yapma ve bir doğal sayı ile bir cebirsel ifadeyi çarpma) aynı matematik öğretmeni tarafından 7 ders saati boyunca kazandırılmaya çalışılmıştır. Bu süreçte, deney grubunda cebir karoları somut materyal olarak kullanılırken, kontrol grubunda kullanılmamış ve öğretmen olağan dersini yapmıştır. Ancak öğretmen kontrol grubundaki öğrencilere tanıtmak amacıyla cebir karolarını tahtaya çizerek göstermiştir. Uygulamanın ortaokul matematik öğretim programındaki karşılığı 7 ders saati olarak belirlendiğinden araştırmacı buna sadık kalmıştır. Araştırmacı matematik derslerinde her iki grup için de sınıflarda bulunarak gözlem yapmıştır.

Deney Grubu Uygulaması

Öğretmen, araştırmacı tarafından hazırlanan ders planlarına göre hareket etmiştir. Deney grubu öğrencileri keşfetme kısmında cebir karolarını 2-4 öğrenciden oluşan gruplarda somut materyal olarak kullanmışlardır. Cebir karoları öğrencilere dağıtılmış ve öğretmen tarafından da tahtaya yapıştırılmıştır. Ayrıca, derinleştirme için etkinlik kâğıtları ve değerlendirme için çıkış kartları öğrencilere dağıtılmıştır. Öğrenciler etkinlik kâğıtlarındaki ve çıkış kartlarındaki soruları ise bireysel olarak cevaplandırmışlardır. Öğretim yöntem ve tekniklerinden soru-cevap, tartışma, grup çalışması ve bireysel çalışma kullanılmıştır.

Kontrol Grubu Uygulaması

Kontrol grubu öğrencileri uygulama boyunca cebir karolarını somut materyal olarak kullanmamışlar ve öğretmen olağan dersini yapmıştır. Ancak öğretmen kontrol grubundaki öğrencilere tanıtmak amacıyla cebir karolarını tahtaya çizerek göstermiştir. Öğretim yöntem ve tekniklerinden anlatım, soru-cevap, alıştırma-uygulama ve bireysel çalışma kullanılmıştır. Öğretmen konuyu anlattıktan sonra konuyla ilgili örnek sorular çözmüştür. Öğrenciler tarafından anlaşılmayan noktalar olduğunda öğretmen tekrar anlatmış, ardından da tahtaya sorular yazarak öğrencilerin cevaplamalarını istemiştir.

Veri Analizi

Bu çalışmada nicel ve nitel araştırma teknikleri kullanılmıştır. İlk araştırma sorusuna cevap bulabilmek için nicel veri analizi kullanılırken, ikici ve üçüncü araştırma sorusuna cevap bulabilmek için nitel veri analizi kullanılmıştır. Puanlama anahtarları araştırmacı tarafından hazırlanmış ve öğrencilerin testlerdeki cevaplarını değerlendirmek için kullanılmıştır. Puanlama cetvellerinde doğru ve yanlış cevaplar yazılmış; öğrencilerin doğru cevapları "1" olarak kodlanırken yanlış cevaplar "0" olarak kodlanmıştır. Pilot uygulamadan sonra, testlerin güvenirliğini test etmek için veriler analiz edilmiştir. Testlerin iç tutarlılığını belirlemek için soruların eşit zorluk derecesinde olduğu kabul edilerek KR21 formülü kullanılmıştır. Ana uygulamadan sonra, ilk araştırma sorusunu cevaplandırmak için veriler SPSS 20 programı kullanılarak analiz edilmiştir. Betimsel istatistik olarak deney ve kontrol gruplarının öntest ve sontest ortalama puan ve standard sapma değerleri hesaplanmıştır. Çıkarımsal istatistik olarak, deney ve kontrol gruplarını karşılaştırmak için Mann-Whitney U testi ve bağımsız örneklemler t-testi kullanılmıştır. İkinci araştırma sorusu için posttesteki her bir soruya verilen yanıtlar derinlemesine analiz edilmiş ve öğrencilerin hataları, kavram yanılgıları ve alternatif çözümleri belirlenmiştir. Üçüncü araştırma sorusuna cevap bulabilmek için görüşme formundaki öğrenciler tarafından verilen cevaplar detaylı bir şekilde incelenmiş ve verdikleri cevaplar kategorilere ayrılmıştır.

Varsayım ve Sınırlılıklar

Bu çalışmada, katılımcıların kimseden etkilenmeyerek kendi gerçek fikirlerini yansıttıkları varsayılmıştır. Ayrıca, deney grubundaki uygulamanın ve kontrol grubundaki öğretimin planlandığı gibi gerçekleştiği varsayılmıştır.

Çalışmada rastgele olmayan örneklem yöntemi kullanılmış ve okul araştırmacıya uygunluğuna göre seçilmiştir. Bu durum, örneklemin popülasyonu tam olarak temsil etmesini ve bulguların genellenebilirliğini sınırlamaktadır. Ayrıca, çalışmanın bulguları katılımcılar tarafından veri toplama araçları aracılığıyla sağlanan veriler ile sınırlıdır. Milli Eğitim Bakanlığı tarafından Ortaokul Matematik Öğretim Programı'nda uygulamanın süresi 7 ders saati ile sınırlandırılmış ve araştırmacı buna sadık kalmıştır.

SONUÇ

Cebir Karosu Kullanımının Altıncı Sınıf Öğrencilerinin Cebir Başarısına Etkisi

İlk araştırma sorusuna cevap bulabilmek için Cebir Ön Bilgi Testi ve Cebir Başarı Testi'nin betimsel ve çıkarımsal analizleri sunulmuştur.

Cebir Ön Bilgi Testi Sonuçları

15 sorudan oluşan Cebir Ön Bilgi Testi, deney grubundaki 23 öğrenciye ve kontrol grubundaki 21 öğrenciye uygulamadan önce öntest olarak uygulanmıştır. Bu testten alınabilecek en yüksek puan 15 olarak hesaplanmıştır. Tablo 4.2 her iki grubun Cebir Ön Bilgi Testi'nden almış oldukları puanları göstermektedir.

Tablo 4.2 Her İki Grubun Cebir Ön Bilgi Testi'nden Almış Oldukları Puanlar

	Deney Grubu	Kontrol Grubu
S	23	21
En Düşük	0	0
En Yüksek	15	15
Ortalama	8.61	6.95
Standard Sapma	5.42	5.56

Tablo 4.2 de görüldüğü üzere deney grubu öğrencilerinin öntestte almış oldukları puanların ortalaması (Ort = 8.61, SS = 5.42) kontrol grubundaki öğrencilerin öntestte almış oldukları puanların ortalamasından (Ort = 6.95, SS = 5.56) yüksektir.

Uygulamadan önce deney grubu ve kontrol grubu öğrencilerinin Cebir Ön Bilgi Testi öntest puanları arasında anlamlı bir fark olup olmadığını belirlemek için Mann-Whitney U Testi kullanılmıştır. Tablo 4.3 öntest puanları Mann-Whitney U testi sonuçlarını göstermektedir.

Tablo 4.3 Öntest Puanları Mann-Whitney U Testi Sonuçları

	Mann-Whitney U	Sig.
Öntest	205.500	0.395

$p>0.05$

Tablo 4.3 te görüldüğü gibi deney ve kontrol grubu öğrencilerinin Cebir Ön Bilgi Testi öntest puanları arasında istatistiksel olarak anlamlı bir fark yoktur.

Ayrıca, deney ve kontrol grubu öğrencilerinin Cebir Ön Bilgi Testi'ndeki sorulara verdikleri cevaplar detaylı bir şekilde incelendiğinde, cebir ön bilgileri arasında fark olmadığı görülmüştür. Her iki gruptaki öğrencilerin de sorulara verdikleri cevaplar tercih ve hata açısından benzerdir.

Cebir Başarı Testi Sonuçları

35 sorudan oluşan Cebir Başarı Testi deney grubundaki 23 öğrenciye ve kontrol grubundaki 20 öğrenciye uygulamadan sonra sontest olarak uygulanmıştır. Bu testten alınabilecek en yüksek puan 35 olarak hesaplanmıştır. Tablo 4.10 her iki grubun Cebir Başarı Testi'nden almış oldukları puanları göstermektedir.

Tablo 4.10 Her İki Grubun Cebir Başarı Testi'nden Almış Oldukları Puanlar

	Deney Grubu	Kontrol Grubu
S	23	20
En Düşük	1	0
En Yüksek	34	32
Ortalama	19.65	14.85
Standard Sapma	8.51	10.32

Tablo 4.10 de görüldüğü üzere deney grubu öğrencilerinin sontestte almış oldukları puanların ortalaması (Ort = 19.65, SS = 8.51) kontrol grubundaki öğrencilerin sontestte almış oldukları puanların ortalamasından (Ort = 14.85, SS = 10.32) yüksektir.

İlk araştırma sorusu "Cebir karosu kullanan altıncı sınıf öğrencileri ile kullanmayan altıncı sınıf öğrencilerinin sontest puanları arasında istatistiksel olarak anlamlı bir fark var mıdır?" Araştırma sorusuna ait şu hipotez test edilmiştir:

Cebir karosu kullanan altıncı sınıf öğrencileri ile kullanmayan altıncı sınıf öğrencilerinin Cebir Başarı Testi sontest puanları arasında istatistiksel olarak anlamlı bir fark yoktur.

Hipotezi test etmek için bağımsız örneklemler t-testi kullanılmıştır. Tablo 4.11 sontest puanları t-testi sonuçlarını göstermektedir.

Tablo 4.11 Sontest Puanları T-Testi Sonuçları

	Deney Grubu		Kontrol Grubu		t değeri
	Ort	SS	Ort	SS	
Sontest	19.65	8.51	14.85	10.32	1.67

$p>0.05$

Tablo 4.11 de görüldüğü gibi deney ve kontrol grubu öğrencilerinin Cebir Başarı Testi sontest puanları arasında istatistiksel olarak anlamlı bir fark yoktur.

Cebir Karosu Kullanımının Altıncı Sınıf Öğrencilerinin Cebirsel Düşünmesine Etkisi

İkinci araştırma sorusuna cevap bulabilmek için deney ve kontrol grubu öğrencilerinin Cebir Başarı Testi'nde verdikleri yanıtlar detaylı bir şekilde incelenmiştir.

Deney grubu öğrencilerinin kontrol grubu öğrencilerinden daha fazla soruya doğru yanıt verdikleri görülmüştür. Verilen model gösterimleri için cebirsel ifadeleri yazma; verilen cebirsel ifadeler için değişken, terim, sabit terim, katsayılar ve katsayılar toplamını belirleme; verilen modellere göre cebirsel ifadelerde toplama ve çıkarma işlemleri yapma; verilen cebirsel ifadelerde toplama ve çıkarma işlemleri yapma; verilen cebirsel ifadeleri bir doğal sayı ile bir cebirsel ifadenin çarpımı biçiminde yazma ve karenin bir kenar uzunluğunu cebirsel ifade olarak bulma sorularında deney grubu öğrencileri kontrol grubu öğrencilerinden daha iyi performans göstermiştir. Ancak, verilen dikdörtgenin çevresini cebirsel ifade olarak bulma; kenar sayısı bilinmeyen düzgün çokgenin çevresini cebirsel ifade olarak bulma ve verilen cebirsel ifadelerden hangisinin daha büyük olduğunu açıklama sorularında her iki gruptaki öğrenciler de yeterince iyi performans gösterememiştir. Bu durum cebir karolarının altıncı sınıf öğrencilerinin cebirsel düşünmesinde sınırlı ama pozitif bir etkiye sahip olduğunu göstermektedir.

Öğrencilerin Cebir Karosu Kullanımına İlişkin Görüşleri

Üçüncü araştırma sorusu "Altıncı sınıf öğrencilerinin matematik derslerinde cebir karosu kullanımına ilişkin görüşleri nelerdir?" Bu araştırma sorusuna cevap bulmak için Cebir Karosu Kullanımına İlişkin Öğrenci Görüş Formu'ndaki sorulara deney grubu öğrencilerinin verdikleri cevaplar incelenmiştir.

Öğrencilerin büyük çoğunluğu daha önce materyal olarak sayma pulları ve kesir çubukları kullandıklarını belirtmişlerdir. Ayrıca, öğrenciler cebir karolarının "daha iyi anlama ve öğrenme", "daha kolay hatırlama", "karmaşık işlemleri basitleştirme", "daha hızlı öğrenme" ve "dersleri eğlenceli hale getirme" gibi etkilerinden bahsetmişlerdir. Öğrenciler genel olarak cebir karolarını kullanırken ve cebir karolarıyla öğrenirken herhangi bir zorlukla karşılaşmadıklarını ifade etmişlerdir.

Öğrenciler grup çalışması ile ilgili olarak ise grup çalışmasının eğlenceli olduğunu ve grup arkadaşlarıyla daha kolay öğrendiklerini belirtmişlerdir. Bazı öğrenciler ise grup çalışmasında cebir karosu kullanımı ile ilgili yorum yapmışlar, cebir karoları ve grup çalışmasının birlikte öğrenmelerini kolaylaştırdığını ifade etmişlerdir.

TARTIŞMA

Araştırmanın amacı cebir karosu kullanımının altıncı sınıf öğrencilerinin cebir başarısı, cebirsel düşünmeleri ve cebir karosu kullanımına ilişkin görüşleri üzerindeki etkilerini incelemektir.

Cebir Karosu Kullanımının Cebir Başarısına Etkisi

Grupların Cebir Başarı Testi'ndeki ortalama puanları karşılaştırıldığında, t-testi sonuçlarına göre deney grubu ile kontrol grubu öğrencilerinin sontest puanları arasında istatistiksel olarak anlamlı bir fark bulunmamıştır. İstatistiksel analizler sonucu anlamlı bir etki bulunmamasına rağmen, deney grubunun ortalama puanı kontrol grubunun ortalama puanından yüksektir. Bu puan farkında cebir karosu kullanımının etkisi olabilir.

Bu çalışmanın cebir başarısına ilişkin sonuçları Sharp (1995) ve Schlosser (2010) tarafından yapılan çalışmaların sonuçlarıyla tutarlılık göstermektedir. Mevcut çalışmalarla bu çalışmanın arasında öğrencilerin sınıf seviyeleri ve cebirsel konular açısından farklılıklar olsa da, bu çalışma önceki çalışmaların sonuçlarını doğrulamaktadır.

Bu çalışmada uygulamanın süresi 7 ders saatiyle sınırlıdır ve cebir karolarıyla olan bu sınırlı etkileşim istatistiksel olarak anlamlı olmayan sonuçlara neden olmuş olabilir. Larbi ve Okyere (2016) tarafından yapılan çalışmada, uygulama dört haftadan fazla sürmüş ve cebir karosu kullanan deney grubu lehine istatistiksel olarak anlamlı bir fark bulunmuştur.

Kontrol grubu öğrencileri cebir karolarını somut materyal olarak kullanmamalarına rağmen, öğretmen kontrol grubunda cebir karolarını tahtaya çizerek göstermiştir. Bu durum istatistiksel olarak anlamlı olmayan sonuçların nedeni olabilir.

Cebirsel Düşünme ve Gruplarda Cebir Karosu Kullanımı

Öğrencilerin puanlarına göre gruplar arasında istatistiksel olarak anlamlı bir fark bulunmamasına rağmen, Cebir Başarı Testi'ndeki soruların yaklaşık yarısında öğrencilerin öğrenmeleri açısından nitel bir fark bulunmuştur. Derslerde cebir karosu kullanımının bu farkı yarattığı söylenebilir. Deney grubu öğrencileri cebirsel ifadelerin modelle gösterimleri ve sembolik gösterimleri arasında geçiş yapabilmişlerdir. Verilen cebirsel ifadeleri analiz edebilmişler ve cebirsel ifadelerle anlamlı bir şekilde toplama-çıkarma yapabilmişlerdir. Cebir karolarını somut materyal olarak kullanmaları öğrencilerin cebirsel düşünmelerinin gelişimine katkı sağlamıştır.

Deney grubu öğrencileri matematik derslerinde cebir karosu kullanmadan önce sayma pulları ve kesir çubuklarını kullandıklarını belirtmişlerdir. Eğer bu öğrenciler, matematik derslerinde özellikle cebir konularını öğrenirken daha fazla materyal kullansalardı, uygulamadan daha fazla yararlanabilirlerdi.

Grup çalışması genel olarak yararlı bir yöntem olmasına rağmen (Koblitz ve Wilson, 2014), grup çalışmasının kısa süreli kullanımı bu çalışmada etkilerini görmeyi engellemiş olabilir. Öğretmenin belirttiğine göre, deney grubu öğrencileri matematik derslerinde daha önce grup çalışması yapmamışlardır. Öğrenciler cebirsel ifadeleri öğrenirken ilk kez keşfetme aşamasında grup çalışması yaptıkları için, bu süre alışma süreci olarak düşünülebilir. Bu nedenle, öğrenciler daha uzun süre ya da bu çalışmadan önce grup çalışması yapsalardı, farklı sonuçlar elde edilebilirdi.

Öğrencilerin Cebir Karosu Kullanımına ve Grup Çalışmasına İlişkin Görüşleri

Bu çalışmadaki öğrenciler daha önce cebir karosu görmemiş ve kullanmamışlar ve bu sebeple, ilk gördüklerinde şaşırmışlardır. Başlangıçta, bazı öğrenciler hangi rengin pozitif, hangi rengin negatif olduğunu ve hangi parçanın x'i hangi parçanın 1'i temsil ettiğini karıştırmışlar ancak öğrendikten sonra cebir karolarını kullanırken zorlanmamışlardır. Ayrıca, öğrenciler derslere katılmaya istekli olmuş ve cebir karosu kullanırken aktif olmuşlardır. Pasif öğrenciler bile cebirsel ifadeleri cebir karolarıyla modellemeye çalışmış, grup çalışmaları ve ikili çalışmalara katılmış, modellerini tahtada göstermişlerdir. Öğrencilerin çoğu cebir karosu kullanırken eğlenmişlerdir. Hiçbir öğrenci cebir karolarını oyuncak olarak görüp oynamamış, cebir karolarıyla cebirsel ifadeleri öğrenmeye yoğunlaşmışlardır.

Önceki çalışmalarda, öğrenciler cebir karolarını kullandıklarında cebir konularındaki performanslarında istatistiksel olarak anlamlı farklılıklar bulunmamasına rağmen, cebir konularını anlamlı öğrenmelerinde cebir karolarının etkilerinden söz etmişlerdir. Yıldız (2012)'ın nitel durum çalışmasında, öğrenciler hem oynadıklarından hem de konuları daha iyi öğrendiklerinden bahsetmişlerdir. Adı geçen çalışmaların bulgularının tamamı bu çalışmanın bulgularıyla tutarlılık göstermektedir. Bu çalışmada da deney grubu öğrencileri cebir karoları yardımıyla konuları daha hızlı öğrendiklerini ve daha kolay hatırladıklarını belirtmişlerdir.

Balt (2017)' ın çalışmasında da bahsedildiği gibi, öğrenciler grup çalışmasının anlamalarını kolaylaştırdığını söylemişlerdir. Webb ve Farivar (1994) öğrencilerin grup çalışması ile yeni bir konu öğrenirken, grup arkadaşları anlamadıklarında fark ettiklerinden ve onlara konuyu açıklayarak kavram yanılgılarını düzeltebileceklerinden bahsetmişlerdir. Bu durum deney grubundaki uygulama sırasında gözlenmiş ve bazı öğrenciler de grup çalışması sırasında grup arkadaşlarına anlamaları için yardım ettiklerini söylemişlerdir. Grup çalışması ile işbirlikli öğrenmede öğrenciler manipülatif kullanarak problemleri çözdüğünde motive olurlar ve eğlenirler (Mulryan, 1994). Bu çalışmada öğrenciler hem cebir karosu kullanırken hem de grup çalışması yaparken eğlendiklerinden bahsetmişlerdir.

Öneriler

Bu çalışmada bir okuldaki bir deney ve bir kontrol grubu yer almıştır. İleride yapılacak çalışmalarda, bir grup grup çalışmasında cebir karosu kullanacak şekilde, bir grup grup çalışması olmadan cebir karosu kullanacak şekilde ve bir grup da hem cebir karosu kullanımı hem de grup çalışması olmayacak şekilde üç gruba yer verilebilir. Bu çalışmada konunun öğretim süresi 7 ders saatiyle sınırlıdır. İleride yapılacak çalışmalarda uygulama süresi arttırılabilir. Bu çalışmada veriler sadece öğrencilerden toplanmıştır. İleride yapılacak çalışmalarda cebir karolarının öğrencilerin cebir başarısını nasıl arttıracağına ilişkin veriler hem öğrencilerden hem de bu öğrencilerin öğretmenlerinden toplanabilir. Bu çalışmada cebir karosu kullanımının öğrencilerin cebir başarısı ve cebirsel düşünmeleri üzerindeki etkileri incelenmiştir. Cebir karosu kullanımının öğrencilerin cebire ilişkin kavramsal anlama düzeyleri üzerindeki etkileri de incelenebilir. İleride yapılacak çalışmalarda öğrencilerle görüşmeler yapılabilir ve tutum ve motivasyonları da incelenebilir. Bu çalışmada cebir karoları sadece keşfetme aşamasında kullanılmış olup, ileride yapılacak çalışmalarda değerlendirme aşamasında da kullanılabilir.

Matematik öğretmenleri bu çalışmadaki ders planlarını, ders planlarında yer alan etkinlik kâğıtlarını ve Cebir Ön Bilgi Testi ile Cebir Başarı Testi'ni kullanabilir ya da bunlardan yararlanarak kendi kaynaklarını oluşturabilirler. Görüş formunda öğrenciler cebirsel ifadeleri öğrenirken cebir karosu kullanmayı sevdiklerini belirttikleri düşünüldüğünde, öğretmenler cebir konularını içeren derslere cebir karolarını entegre edebilirler ve ortaokullarda cebirsel kavramların öğretiminde buluş yoluyla öğrenme ve işbirlikçi öğrenme yöntemlerini kullanabilirler. Ayrıca, matematik öğretmenliği programında, öğretmen adaylarının ortaokulda cebir konularını öğretirken cebir karosu ve diğer manipülatifleri hazırlama ve kullanma becerilerini geliştirmeye yönelik derslere yer verilebilir. Öğrenciler ve öğretmenler için hazırlanan kitaplarda cebir konularında cebir karosu kullanmayı teşvik edecek etkinliklere yer verilebilir.

Appendix J: Tez Fotokopisi İzin Formu

ENSTİTÜ

Fen Bilimleri Enstitüsü

Sosyal Bilimler Enstitüsü

Uygulamalı Matematik Enstitüsü

Enformatik Enstitüsü

Deniz Bilimleri Enstitüsü

YAZARIN

Soyadı : Çaylan
Adı : Büşra
Bölümü : İlköğretim Fen ve Matematik Alanları Eğitimi

TEZİN ADI (İngilizce) : The Effects of Using Algebra Tiles on Sixth Grade Students' Algebra Achievement, Algebraic Thinking and Views about Using Algebra Tiles

TEZİN TÜRÜ : Yüksek Lisans Doktora

1. Tezimin tamamı dünya çapında erişime açılsın ve kaynak gösterilmek şartıyla tezimin bir kısmı veya tamamının fotokopisi alınsın.
2. Tezimin tamamı yalnızca Orta Doğu Teknik Üniversitesi kullancılarının erişimine açılsın. (Bu seçenekle tezinizin fotokopisi ya da elektronik kopyası Kütüphane aracılığı ile ODTÜ dışına dağıtılmayacaktır.)
3. Tezim bir (1) yıl süreyle erişime kapalı olsun. (Bu seçenekle tezinizin fotokopisi ya da elektronik kopyası Kütüphane aracılığı ile ODTÜ dışına dağıtılmayacaktır.)

Yazarın imzası .... Tarih ....

RB-SA01/F01 Rev:0 26.10.2011