PERFORMANCE OF ENSEMBLE FORECASTING TOOLS FOR ANALYSIS TURKISH CONSUMER PRICE INDEX

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ABSTRACT

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Major challenge in time series analysis is to get reasonably accurate forecasts of the future data from the analysis of the previous records. Accurate forecasting of inflation has great importance in the market economies, the policymakers and the monetary system since the inflation rate determines the cost and standard of living. Also, it affects the decision on investments. In Turkey, the inflation rate is measured by the consumer price index (CPI). There exist many methods to predict the future values of the CPI. In this study, six individual models were applied to forecast the Turkish CPI. Those are Seasonal Autoregressive Integrated Moving Average Model with Exogeneous variables (SARIMAX), Holt-Winters Exponential Smoothing, Trigonometric Exponential Smoothing State Space model with Box-Cox transformation, ARMA errors, Trend and Seasonal Components (TBATS) model, Artificial Neural Network (ANN), Theta Model, Seasonal Trend Decomposition with LOESS (STL). Then, ensemble model was constructed to improve the forecast performance. Ensemble model is combination of the several forecasting models to improve the performance of the forecast. The forecast accuracy of all models is evaluated by the Root Mean Square Error and Mean Absolute Percentage Error. Our findings show that $SARIMAX(4,1,4)(2,0,1)_{12}$ and ensemble model composed of auto.arima, and neural network produce the best forecasts for 12 month Turkish CPI.

Keywords: Time Series Analysis, Forecasting Inflation, Ensemble Model

ÖZ

TÜKETİCİ FİYAT ENDEKSİ ANALİZİ İÇİN TOPLU ÖNGÖRÜ ARAÇLARININ PERFORMANSI

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Zaman serileri analizindeki en büyük zorluk, geçmiş değerleri kullanarak gelecek değerlere ait öngörüleri mümkün olduğunca doğru bir şekilde elde etmektir. Enflasyonun doğru öngörülmesi, piyasa ekonomilerinde, politika yapıcılarda ve parasal sistemde büyük bir öneme sahiptir, çünkü enflasyon oranı insanların yaşam maliyetini ve standardını belirler. Ayrıca, yatırımcıların kararlarını da etkiler. Türkiye'de enflasyon oranı tüketici fiyat endeksi (TÜFE) ile ölçülmektedir. Bu çalışmada TÜFE'nin öngörü değerlerini elde etmek için altı bireysel modele başvurulmuştur. Bunlar, dışsal değişkenli mevsimsel otoregresif tamamlanmış hareketli ortalama (SARIMAX) modeli, Holt-Winters Üstel Düzgünleştirme, TBATS modeli, Yapay Sinir Ağları modeli, Theta Modeli ve STL modelidir. Daha sonra, TÜFE'nin öngörü performansını artırmak amacıyla birkaç metodun birleştirilmesi ile elde edilen topluluk modeli kullanılmıştır. Kullanılan bireysel modellerin ve topluluk modelinin öngörü performansı Hata Kareler Ortalamasının Kare Kökü ve Mutlak Hata Ortalama Yüzdesi kullanılarak

değerlendirildi. Bulgularımız, $SARIMAX(4,1,4)(2,0,1)_{12}$ modeli ve yapay sinir ağları ve auto.arima'dan oluşan topluluk modeli 12 aylık TÜFE için en iyi öngörüleri elde ettiğini gösteriyor.

Anahtar Kelimeler: Zaman Serisi Analizi, Enflasyon Öngörüsü, Topluluk Modeli

To my family

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LIST OF ABBREVIATIONS

ADF	Augmented Dickey Fuller
AIC	Akaike Information Criteria
ANN	Artifical Neural Network
AR	Autoregressive
ARCH	Autoregressive Conditional Heteroscedasticity
ARFIMA	Autoregressive Fractionally Integrated Moving Average
ARIMA	Autoregressive Integrated Moving Average
BIC	Bayesian Information Criteria
BMA	Bayesian Model Average
BVAR	Bayesian Vector Autoregressive
CBRT	Central Bank Of Republic Of Turkey
CCF	Cross Correlation Function
CPI	Consumer Price Index
CumRAE	Cumulative Relative Absolute Error
CV	Cross Validation
DF	Dickey Fuller
DMA	Dynamic Model Averaging
DMS	Dynamic Model Selection
ETS	Exponential Smoothing
FAVAR	Factor Augmented VAR
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
HEGY	Hylleberg, Engle, Granger, and Yoo

HW	Holt Winters
HWT	Taylors Adaptation of Holt Winters
IPG	Industrial Production Growth
KPSS	Kwiatkowski, Phillips, Schmidt and Shin
LASSO	The Least Absolute Shrinkage and Selection Operator
LOESS	Locally Weighted Scatterplot Smoothing
MA	Moving Average
MAE	Mean Absolute Error
MAPE	Mean Absolute Percentage Error
MASE	Mean Absolute Scaled Error
ME	Mean Error
MLE	Maximum Likelihood Estimation
MPE	Mean Percentage Error
MSE	Mean Squared Error
MSFE	Mean Squared Forecast Error
NN	Neural Network
NNAR	Neural Network Autoregressive
OECD	Organisation For Economic Cooperation and Development
OLS	Ordinary Least Square
PCA	Principle Component Analysis
PPI	Producer Price Index
PVAR	Panel Vector Autoregressive
RMSE	Root Mean Squared Error
RW	Random Walk
sACF	Sample Autocorrelation Function
SARIMA	Seasonal Autoregressive Integrated Moving Average

SARIMAX	Seasonal Autoregressive Integrated Moving Average With Exogeneous Variables
SES	Simple Exponential Smoothing
SIC	Schwarz Information Criterion
sPACF	Sample Partial Autocorrelation Function
STL	Seasonal Trend Decomposition with LOESS
SVR	Support Vector Regression
TBATS	Trigonometric Exponential Smoothing State Space model with Box-Cox trans-formation, ARMA errors, Trend and Seasonal Components.
TURKSTAT	Turkish Statistical Institute
TVAR	Time Varying Parameter VAR
UCM	Unobserved Component Model
UCSV	Unobserved Component Stochastic Volatility
VAR	Vector Autoregressive

CHAPTER 1

INTRODUCTION

The main objective of time series analysis is to produce accurate forecasts of future values of the series from the analysis of the past values. Forecasting the future is the most attractive field for researchers and experts. In today's world, although there exist various advanced forecasting tools such as time series models and econometric models, obtaining precise forecast of time series is challenging task since the series have some undesirable characteristics which are nonstationarity, seasonality, irregular fluctuation, cyclical variations and multicollinearity. Moreover, because almost all countries have undergone the economy policy and structural changes for twenty and thirty years, the political elements, economic activities and external factors affect the future values of the target macroeconomic variables. The most widely used macroeconomic variable for forecasting is an inflation rate. The inflation rate measures the changes of general level of prices of services and products, and it determines the cost and the standard of living. It is an important element for economical activities because unless it can be brought under control, it may lead to unfair distribution of income and uncertainity. This uncertainity situation influences the decision of investors. Also, the inflation rate is determinant for pensions and wage bargaining among employees and employers. In addition, most of the decision makers and the financial institutes make choice according to the inflation rate. Predictions provide clarity and confidence in an economy, and the decision based on these predictions is important and efficient factor to earn profit to overcome bankruptcy. Hence, forecasting inflation is crucial issue for market economies, policymakers and monetary system in Turkey. There exist several studies on modelling inflation in Turkey. The most common studies on inflation are done by the Central Bank of the Republic of Turkey (CBRT). The main objective of the CBRT is to maintain and achieve price stability via an inflation targeting regime. The inflation targeting is a type of monetary policy that Central Banks determine acceptable inflation rate for a given period by taking into consideration the general economical variables and their realizations, and they conduct their monetary systems according to these regimes.

Many economists and statisticians try to develop a method for forecasting inflation. Some of them apply econometric models such as Phillips curve. The others implement time series techniques which are univariate seasonal autoregressive integrated moving average (SARIMA) model, vector autoregressive (VAR) model, Bayesian VAR model and machine learning techniques like artificial neural network (ANN) and support vector regression (SVR). Apart from econometric models, time series models and machine learning techniques, ensemble model which has been trendy in recent years is alternative approach for forecasting variables. Ensemble model is a method that combines the forecasts obtained by several models. The idea of combination forecasts was originally proposed by Bates and Granger [4]. Their findings showed that combined forecasts produced smaller error than any of the individual forecasts. The advantage of ensemble model is that the best possible combination reduces the forecast error and the risk of the forecasting failure. Because of the structure of the target variables and the external factors, individual model may not be enough to predict their future values, and the effect of more than one model on forecast values may be significant. In contrast, in ensemble model, assigning optimal weights is a problematic issue. Although there exist many studies on decision of optimal weights, there are several methods to find optimal weights. Hence, it may need time to obtain optimal weights for accurate forecasting.

Despites the advantages and drawbacks of the ensemble model, many researchers have been interested in ensemble model building for forecasting the target variable. For instance, Günay [23] did the study on forecasting Turkish inflation and industrial production with individual models and combined model. He displays that the individual models obtain better results than the combined models for forecasting Turkish inflation rate. Hibon and Evgeniou [25] did scientific study on whether forecasts should be combined or not. Their findings show that the best potential combination is not always better than the best potential individual forecasts. It depends on the selected methods and their combinations. Similarly, Ang et al. [2] examines the individual models and ensemble model for forecasting inflation rate in the U.S. Their computational analysis concludes that ensemble model does not give superior performance than the individual models.

Terui and Van Dijk [51] investigated the combined forecasts of the nonlinear model. It is indicated that the ensemble model performs well for forecasting some US and Canadian macroeconomic variables. Öğünç et al. [38] studied on short time forecast of Turkish inflation with model of forecast combination declares that the model which integrates more than one individual methods gives outstanding performance in comparison to the individual methods. Similarly, Huong et al. [27] built ensemble model for forecasting consumer price index (CPI). They indicate that the ensemble model has the lowest MSE for in-sample and out-of-sample forecast performance. Ouyang and Yin [41] did a study on multi-step forecasting of time series with ensemble model. It is concluded that the ensemble model has higher accuracy than the used separate methods after many experiments on real foreign exchange rate and weather temperature.

As it is understood from the literature review on the ensemble models, some of the researchers find that the ensemble models have the best forecast performance. In contrast, the others show that the individual methods have the lowest error for forecasting the target variables. Therefore, in this study, our aims are to produce accurate forecasts of the Turkish inflation rate by modelling the CPI and to evaluate the forecast performance of the ensemble model for Turkish inflation rate. In this approach, we construct individual models and the ensemble model. The individual models are seasonal autoregressive integrated moving average with exogeneous vairables (SARIMAX), HW Exponential Smoothing model, Trigonometric Exponential Smoothing State Space model with Box-Cox transformation, ARMA errors, Trend and Seasonal Components (TBATS), Artificial Neural Network (ANN), Theta model and Seasonal

Trend Decomposition based on LOESS (STL). Later, the ensemble model is built by using "forecastHybrid" package in R studio with version 1.1.383 for forecasting Turkish inflation rate. The ensemble model is composed of auto.arima, exponential smoothing, theta, TBATS, neural network and STL models. The Root Mean Squared Error (RMSE) and the Mean Absolute Percentage Error (MAPE) of all models are computed to assess the forecast performance of the models. We examine whether the performance of the ensemble model is higher than the individual models or not. Due to the fact that the inflation rate is measured by the CPI in Turkey, we use Turkish CPI series between 2005 and 2016 received from Turkish Statistical Institute (TURKSTAT). Some exogeneous variables are added into the models to enhance the forecast performance. These exogeneous variables are Turkish producer price index (PPI), seasonally adjusted monthly Turkish unemployment rate, monthly Turkish deposit interest, monthly real effective exchange rate of Turkey, monthly export unit value index of Turkey and monthly import unit value index of Turkey between January 2005 and December 2016 received from TURKSTAT.

The rest of this thesis is designed as follows: The Chapter 2 reviews the literature on forecasting inflation. The Chapter 3 describes the univariate time series models, nonlinear models and ensemble model. The Chapter 4 contains empirical analysis of Turkish CPI data with individual models and ensemble model and comparison out-of-sample performance of all models. Finally, the Chapter 5 includes conclusion and further discussion.

CHAPTER 2

LITERATURE REVIEW ON FORECASTING INFLATION

In this section, we will review the studies on forecasting inflation rate and investigate the types of methods to predict the future values of the inflation rate. The literature consists of two part. Firstly, the studies on the inflation rate in Turkey will be examined. Then, inflation forecast studies in the world will be reviewed.

2.1 Studies on Inflation Forecast in Turkey

The first main article on the inflation forecasts in Turkey is proposed by Öğünç et al. [38]. Öğünç et al. employ the CPI data without tobacco and unprocessed food for short period ahead inflation forecasts. They use series of methods which are univariate techniques, nonlinear models, Phillips curve motivated time-varying parameter method, approaches based on decomposition, Vector Auto-regressive (VAR) process, Bayesian VAR process (BVAR), dynamic factor analysis. These methods are applied to estimate the CPI and obtain short period inflation forecast in Turkey. They conclude that the models with more economic activities predominate the benchmark random walk and the performance of the first two quarters ahead forecasts are averagely 30% better. Later, they integrate their forecasts by assigning weights to each model in order to enhance the predictive performance of the short period forecasts. The overall result shows that BVAR model and combinations models with RMSE weight more accurately predict the future values of inflation.

The newest study on inflation forecast in Turkey is proposed by Günay [23]. Günay implements two methods which are forecast combination and factor models in the article in order to forecast the industrial production growth (IPG) and core inflation in Turkey. The factor models are suitable for large data sets to obtain reliable forecasts. In factor models, information of the data is learned by several underlying factors, then these factors are utilized in the forecasting procedure. He applys principal component analysis (PCA) to estimate the factors. He analyzes the influences of the factor numbers, aggregation level of data, factor extraction technique on the forecast accuracy. His results indicate that best forecast performance hinges on the variable types that he wants to be forecasted, time period and forecast horizons. When he compares both models in terms of forecast performance, factor models outperform the combination models.

Soybilgen and Yazgan [47] implement the auto-regressive (AR) model to predict the inflation in Turkey. The AR model is compared with the inflation expectation of CBRT which is based on the survey of decision makers and experts with respect to root mean square errors (RMSEs). They find that the expectation of current inflation gives more accurate results than the AR model, whereas; the AR model more accurately predicts the two and twelve months step ahead forecasts of the inflation.

Önder [40] studies on forecasting inflation. He compares several forecasting techniques for prediction of future values of inflation in Turkey. She suggests most widely used method, the Phillips curve for five step ahead forecasts of the inflation in Turkey. Specifically, the Phillips curve is an economical analysis which shows the negative relationship between the expectation of inflation and past values of the unemployment rate. She also applies the univariate ARIMA, the vector error correction models, the vector autoregressive and naive no-change models to be able to make comparison with the Phillips curve for five-step-ahead forecast of inflation. The results of her empirical works indicate that according to RMSE and MAE, the Phillips curve provides more accurate forecasts based on some macroeconomic activities, namely, interest rates and money in Turkey.

Saz [45] proposes the use of the seasonal autoregressive integrated moving average (SARIMA) models for inflation forecast in Turkey. In the forecasting procedure, he firstly applies the stationarity test and seasonality test for inflation series from 2003 to 2009 in Turkey. He shows that Turkish inflation rate series have both deterministic and stochastic seasonality. The SARIMA model identification and estimation procedures are conducted via the Box-Jenkins methodology, Akaike information criterion (AIC) and Schwarz/Bayesian information criterion (SIC/BIC) after stationarity condition is satisfied. His suggested SARIMA model is selected by step-by-step procedure of the Hyndman-Khandakar algorithm. The best forecast model is $SARIMA(0,0,0)(1,1,1)_{12}$ for inflation rates in Turkey.

Domaç [18] focuses on three models; monetary models, Phillips curve and mark-up models. According to monetary models, the inflation is virtually monetary phenomenon. According to mark-up models, the price levels are calculated by given mark-up and costs. In model estimation part done by training set of the inflation, He adds two dummy variables to clarify the sharp rise in inflation.The forecasts of the inflation in Turkey are obtained from the estimated model for three techniques. The RMSE, the cumulative relative absolute error, mean absolute error (MAE), and Theil's Inequality Coefficient measures of the three models are evaluated. He finds that monetary models and the Phillips curve have higher out of sample predictive performance than the mark-up models. Therefore, these findings show that Phillips curve enlarged with money models and exchange rates has a quite effective approach for Turkish inflation rates forecasts.

Çatık and Karaçuka [10] study on forecasting Turkish inflation. Monthly seasonally adjusted Turkish CPI series are used for forecasting inflation. The used forecasting techniques are artificial neural network (ANN) with backpropagation technique and unobserved components models (UCM) and linear time series models such as ARFIMA, ARIMA, random walk without drift models and fractionally integrated generalized autoregressive conditional heteroscedasticity (GARCH) model. The forecast performance of both models are evaluated with respect to dynamic and static which are multi-step-ahead forecasts and one-step-ahead forecast respectively. According to MAE, RMSE and Theil's U statistics for each model, the linear models are more accurate forecasting tools for short term. However, ANN and UCM give more accurate forecasts for long term period.

2.2 Studies on Inflation Forecast in the World

Huong et al. [27] suggest a way of building an ensemble model for forecasting CPI in order to improve the predictive performance. The proposed ensemble model is a hybrid model of ANN with assigned weights for forecasting CPI of Spain and OECD countries. They construct a set of M neural networks consisting same output neurons, input neurons and hidden neurons. However, the weights of each neural network model are different. Then, M individual neural network models are combined according to assigned weights. In order to optimize the performance of the ANN ensemble model, multi-objective evolutionary algorithm is implemented. They split the data into train and test set to evaluate the forecast performance of the ensemble model. Comparing the ARIMA, ANN and ensemble model, the least MSE for train and test set of CPI belongs to proposed ensemble model.

McAdam and McNelis [36] propose linear and neural network (NN)-based "thick" models for prediction of the future values of the inflation based upon formulation of Phillips curve in the Euro areas, USA and Japan. Thick models stand for forecasts obtained from trimmed mean of various NN models. Their findings indicate that thick model has the best bootstrap and "real time" predictive performance for the CPI in the Euro areas. The success ratio and RMSE values are very close for both linear and "NN" thick model. Therefore, they conclude that thick model gives accurate forecasts for CPI like linear models.

Filippo [17] proposes approaches for forecasting CPI inflation in the United States and Euro Areas. The proposed approaches are Dynamic Model Averaging (DMA) and Dynamic Model Selection (DMS). According to his findings, DMS model performs better than DMA model, and both DMA and DMS provide more accurate forecasts than random walk model. Due to the fact that he uses various sample periods and diverse forecast horizons, the results of DMA and DMS models are robust and DMA and DMS provide the best CPI forecasts.

Stock and Watson [48] studies on forecasting US inflation. They apply generalized Phillips curve based on aggregate activities instead of the unemployment rate. They predict 12 month step ahead forecasts of the US inflation. According to the authors, the Phillips curve plays an important role in short term inflation forecasting, and Phillips curve is able to define the relation among the future inflation and the current economic activities. Their finding indicates that Phillips curve obtains the most accurate and reliable short term forecast of inflation rate in US based on interest rates, commodity prices and monetary aggregates.

Wright [55] forecasts inflation in US via Bayesian model average (BMA) model based on 93 financial and macroeconomic predictors for the period from 1971:Q1 to 2003:Q2. The logic behind BMA is to get forecasts from several models, but which model is the true model is not known. Hence, the prior probabilities are determined and used to calculate the posterior probability under the condition which each model is the correct one. Then, all model forecasts are assigned weights by their posterior probabilities. His empirical study shows that BMA model has higher forecast performance than the simple average the forecasts from other models used in the study.

Ang et al. [2] analyze the forecast power of several alternative techniques for the prediction US inflation. These techniques are ARIMA process, term structure models, survey-based measures and regression hinged on real activity indicators. Also, they investigate forecast combination methods. The forecast accuracy of the used models are evaluated by RMSE of the each model. Their results indicate that model based on survey outperforms in terms of obtaining reliable US forecasts, whereas the model using term structure series has poor forecast performance. The combination methods of average or medians do not perform better than the survey forecasts.

Reigl [44] considers the inflation rate in Estonia in his analysis. He applies forecasting using factor models. In the factor augmented vector autoregressive (FAVAR) model building procedure, the factors are built with PCA and they merge into VAR forecast method. The results indicate that VAR models integrated factor model outperform simple univariate AR model. For inflation forecasting, a few number of factors taken from big datasets are more efficient to forecast. In addition, various factors taken from small datasets perform better than the benchmark model, univariate AR process of order p in order to predict the future values of inflation.

Dees and Günthner [16] study on forecasting inflation by applying a panel vector autoregressive (PVAR) method. They suggest PVAR model to estimate and predict the inflation dynamics in services, agricultural, industry and construction sectors in France, Spain, Italy and Germany. While the inflation series are modelled, real activity, wages and unemployment at the sectoral level are also taken into account. According to out-of-sample forecast performance, the PVAR model outperforms autoregressive process and random walk with drift model for particularly short term forecasts. The predictive performance of PVAR model starts to decline for long term forecast. The proposed model gives reliable forecasts for inflation in the event of Great Recession.

Garcia et al. [21] apply factor models with main predictors, adaptive the least absolute shrinkage and selection operator (adaLASSO) method, random forests, complete subset regression with main predictors for forecasting Brazilian inflation which is measured by CPI in Brazil. Also, they include forecasts obtained by AR process and random walk. In order to compare the forecast accuracy of each model, they prefer to use RMSE and MAE. Their findings show that the LASSO and the adaLASSO model have the smallest forecast errors for one and two step ahead forecasts, while the complete subset regression model provides more reliable forecasts for more than two step ahead forecasts of Brazilian inflation. Moreover, they combine all forecasts by averaging the results. This combination method gives the best forecast for Brazilian inflation. When they implement Bayesian VAR model, their finding shows that BVAR generates accurate forecasts for short term horizon.

Plakandaras et al. [43] implement one of the machine learning techniques; support vector regression (SVR) and LASSO in US inflation forecasts based on term structural and autoregressive models. In addition, ordinary least squares regression and random walk (RW) models are used as benchmark. They calculate MAPE to make comparison the predictive performance of each model. After the forecast analysis of all models, their empirical results conclude that RW has higher MAPE value than the other models, the predictive performance of structural and autoregressive models are similar, irrespective of forecasting terminology. The structural LASSO model produces the most reliable US inflation forecasts for three month ahead period; whereas, the autoregressive OLS method has the lowest MAPE values for one month step forecast period.

Omane et al. [39] examine time series methods for prediction of Ghana's inflation. In order to evaluate the performance of the models, they split the series into train sample and validation sample. Train sample is used to build model, and validation sample is used to assess the predictive performance of the models. The proposed models are SARIMA and HW based on multiplicative seasonality. The accuracy measure of validation sample forecasts is calculated using RMSE, MASE, MAE and MAPE. Their findings emphasize that $SARIMA(2,1,1)(0,0,1)_{12}$ model produce best forecasts compared to HW technique.

Moshiri and Cameron [37] recommend Artificial Neural Network (ANN) model for prediction of inflation. They make comparison performances of ANN model with classical time series techniques which are ARIMA process, structural model, VAR model and BVAR model. RMSE and MAE are preferred to examine whether the models generate precise forecasts or not. Also, inflation series were splitted into train and test set for robustness of the models. The authors apply two types of forecasts; dynamic and static forecasts. In the static forecast, values of lagged response variables are used. In contrast, in dynamic forecast, previous forecast of response variables is used. Their empirical results show that for dynamic forecast, all models produce similar one and three period ahead forecasts and BVAR model performs well for 12 period step ahead forecast of inflation. Moreover, for static forecast, VAR model and ANN with ARIMA produce close one and three period forecast of inflation and ANN with ARIMA outperforms for 12 period forecasts of inflation.

Binner et al. [5] apply both nonlinear and linear forecasting approaches in order to predict Euro inflation. The univariate and multivariate ARIMA processes are preferred as traditional linear methods and Neural Network (NN) model is implemented as nonlinear techniques. The main goal of authors is to display how several time series forecast techniques compare in forecast accuracy of Euro inflation. Their empirical results display that the best NN model has superior performance to linear ARIMA and VAR models for forecasting Euro inflation based on the out of sample MAPE, RMSE and MAE values.

Lipovina et al. [34] mention the Montenegrin economy structure and internal factors which affect the Montenegro's inflation. They test the practicality and utility of ARIMA models for prediction of Montenegro's inflation. In the forecasting preprocess, the unit root test is checked by Dickey Fuller (DF) test and the inflation series have trend-stationarity. Then, AR model of order 1 is implemented to estimate model and forecast the inflation. It is concluded that ARIMA model based solely on own past values are not enough to predict the future values of Montenegro's inflation since various external conditions affect the inflation rate in Montenegro.

Carlo and Marcal [9] examine different forecasting techniques using aggregated and disaggregated Brazilian CPI series for 12 month ahead forecasts. Seasonal ARIMA process estimates the disaggregated models, and SARIMA, Markov switching and state-space models estimate aggregated models. Test sample performance is assessed by model confidence set and mean square forecast error (MSFE) for one to 12 month period. Their findings display that forecasts produced from disaggregated models are closer to the actual values of the inflation in comparison with forecasts of aggregated models. In addition, the SARIMA-52 which inflation series are divided into 52 items had minimum MSFE. The maximum MSFE are calculated in aggregated models that are SARIMA, state space and Markov switching. Later, they merge the estimated models to increase the accuracy. The weights are assigned with respect to regression performance of the estimated models. Their empirical results show that combination of Markov switching and SARIMA highly improve the prediction performance. In contrast, combination of disaggregated models does not enhance the forecasting performance for 12 month ahead.

Mandalinci [35] conducts diverse forecasting exercises to evaluate the test sample predictive performance of several models including a benchmark AR, ARMA, recursive and rolling Bayesian VAR, unobserved component stochastic volatility (UCSV) model, FAVAR model, time varying parameter VAR (TVAR) model, a factor augmented TVAR model and TVAR with Bayesian variable selection model. The study concentrates upon inflation forecasts in nine developing countries. It is concluded that the predictive performances of all used models change remarkably across countries and time. The UCSV model outperforms in considering the countries. However, when he takes into account time period, the UCSV model has low accuracy for forecasting in the period of global financial crisis, and TVAR model has high accuracy for forecasting at that time.

CHAPTER 3

METHODOLOGY

In this chapter, the forecasting methodologies will be investigated and introduced. Forecast is the prediction of the future value of the time series data through the analysis of the past values of the data. The forecast methods enable to obtain the point forecast of the variables. In this study, seven forecasting models are implemented. The first six models are individual models. These models are the SARIMAX, HW Exponential Smoothing, TBATS, ANN, Theta and STL models. The last model is the combination of the six models which is called the ensemble model.

3.1 Seasonal Autoregressive Integrated Moving Average Model with the Exogeneous Variables (SARIMAX)

In time series analysis, the stationarity property is the most significant assumption to draw a statistical inference about the structure of the time series. The core idea behind the stationarity is that the probability laws that the mean, the variance, the covariance and the joint distribution of the time series do not change over the time. The most commonly used stationary process in time series forecasting is Autoregressive Moving Average (ARMA) process proposed by Box and Jenkins [6]. If the stationarity condition is not satisfied, ARMA process is insufficient to identify the pattern of the nonstationary time series. In this situation, Statisticians Box and Jenkins proposed the most powerful and frequently used forecast approach for nonstationarity time series data [6].This proposed approach is the Autoregressive Integrated Moving Average (ARIMA) framework. In the ARIMA model, the future values of the variables are linear function of several past observations and random error. The difference between ARMA and ARIMA model is the number of regular differencing affecting the ARIMA model. The most important factor in ARIMA model is to derive stationary time series by applying differencing and forecast future values. ARIMA model has three parts. The components of the ARIMA model are Autoregressive (AR), Integrated and Moving Average (MA). Autoregressive part shows that the future values of the time series data are the function of the past values of the series. Integrated part exhibits the number of differences needed to obtain stationary time series. Moving Average part is the function of past forecast errors of the future values of the series.

The general ARIMA(p, d, q) process y_t is:

$$\phi_p(B)(1-B)^d y_t = \theta_0 + \theta_q(B)\varepsilon_t \tag{3.1}$$

where θ_0 is the drift term, *B* is backshift operator, $\phi_p(B)$ is an autoregressive polynomial of order *p* and $\theta_q(B)$ is an moving average polynomial of order *q* and ε_t is distributed $WN(0, \sigma_a^2)$, and

$$\phi_p(B)y_t = 1 - \phi_1 y_{t-1} - \phi_2 y_{t-1} - \cdots - \phi_p y_{t-p},$$

$$\theta_q(B)\varepsilon_t = 1 - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q}.$$

A large number of real time series show seasonal behaviour. Seasonality means the series repeating itself after a regular period of time. If there is a seasonal pattern in time series data, ARIMA method fails to obtain accurate forecasts. In this situation, the Seasonal Autoregressive Integrated Moving Average (SARIMA) model was proposed by Box and Jenkins [6]. The seasonal ARIMA is the extension of the ARIMA process. Seasonal ARIMA denotes SARIMA $(p, d, q)(P, D, Q)_s$. In here, (p, d, q) represents non seasonal components, (P, D, Q) represents seasonal components and s is the seasonal period.

The mathematical form of the Box-Jenkins multiplicative seasonal ARIMA model of y_t is:

$$\phi_p(B)\Phi_P(B^s)(1-B)^d(1-B^s)^D y_t = \theta_0 + \theta_q(B)\Theta_Q(B^s)\varepsilon_t \tag{3.2}$$

where θ_0 is a drift term, ε_t is a white noise sequence with 0 mean and variance σ_a^2 and the respective polynomials are given by

$$\phi_p(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p,$$

$$\Phi_P(B^s) = 1 - \Phi_1 B^s - \Phi_2 B^{2s} - \cdots - \Phi_P B^{Ps}$$

$$\theta_q(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q,$$

$$\Theta_Q(B^s) = 1 - \Theta_1 B^s - \Theta_2 B^{2s} - \cdots - \Theta_Q B^{Qs}.$$

The ordinary AR and MA polynomials are represented by $\phi_p(B)$ and $\theta_q(B)$, respectively. The seasonal AR and MA polynomials are represented by $\Phi_P(B^s)$ and $\Theta_Q(B^s)$, respectively.

Using only the past observations of the series is sometimes inadequate to predict accurately the future values of the series. The future values of the series may be influenced by the past observations of the external variables. In this situation, the SARIMA and the ARIMA model may not be enough to produce the forecast so it is needed to add the covariates which helps to identify the main variable. In the time series model building, adding external inputs is called utilization of exogeneous variables. If the exonegenous variables are added into SARIMA model, the model is replaced as SARIMA Model with exogeneous variables (SARIMAX). The mathematical form of SARIMAX model is represented as:

$$\phi_p(B)\Phi_P(B^s)(1-B)^d(1-B^s)^D z_t = \theta_0 + \theta_q(B)\Theta_Q(B^s)\varepsilon_t, \tag{3.3}$$

where

$$z_t = y_t - \beta_1 x_1 - \beta_2 x_2 - \cdots - \beta_b x_b. \tag{3.4}$$

In the SARIMAX model, the parameters; $\phi_p(B)$, $\theta_q(B)$, $\Phi_P(B^s)$,$\Theta_Q(B^s)$, are same with the parameters of the SARIMA model. $(1-B)^d$ and $(1-B^s)^D$ are defined as regular differencing and seasonal differencing of the series, respectively. z_t is the autocorrelated regression residuals, x_k is k^{th} exogeneous variable, *b* is the number of the exogeneous variables and y_t is the response variables at time *t*. In the parameter estimation, mostly maximum likelihood estimation (MLE) is used.

3.1.1 Diagnostics of the SARIMAX Model

There exist three diagnostics of the SARIMAX models. These are:

- The errors are normally distributed with mean 0.
- The errors are not serially correlated with their own lagged values.
- The error variance is constant (homoscedasticity).
- No multicollinearity.

3.2 Holt Winters Exponential Smoothing Method

Exponential smoothing was originally introduced by Brown in 1950s [8]. Exponential smoothing method has been one of the most popular and flexible technique for forecasting univariate time series since 1950s. It is popular because it is a deterministic model and easy to obtain forecast with high accuracy. It is flexible as it is not necessary to fit the parametric model [22]. Forecasts of exponential smoothing method are described as weighted averages of recent observations, whose weights are exponentially decreasing towards past values [28]. There exist many methods in exponential smoothing family. If the time series data exhibit linear trend and single seasonality pattern, the Holt Winters Exponential Method is preferred to get reliable forecast. The Holt Winters (HW) method is simple to conduct and perform well in practice.

The HW method is proposed by Winter [54]. HW Exponential Smoothing method was an extension of Holt methods of exponential smoothing by Holt [26]. The HW method is a forecasting technique for the series which has seasonality and trend. In the HW method, time series data are modelled by local level, local trend and local seasonality factor that are updated by exponential smoothing. The local trend gives the change in the underlying level expected between period t and period $t + 1$. The local level gives the exponentially smoothed series. The seasonal factor gives seasonal term of the HW model. The HW method is designed for seasonality types which are multiplicative and additive way.

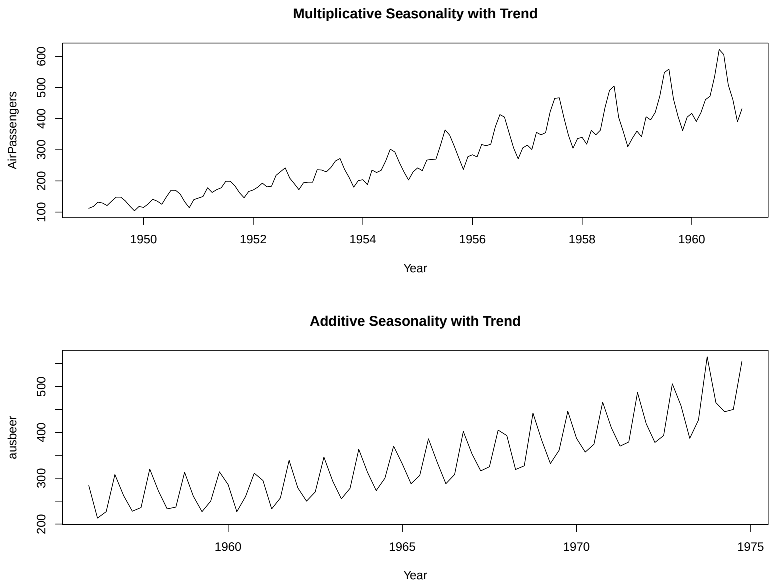

Figure 3.1: The time series plot of Additive and Multiplicative Seasonality

Figure 3.1 demonstrates the graph of additive and multiplicative seasonality model. As it is seen from Figure 3.1, the additive seasonality means that seasonal component is stable in time series data and time series plot gives same seasonal pattern in all periods. Differently, the seasonal component is inconstant and seasonal period shows increase and decrease in time.

3.2.1 Multiplicative Seasonality

The basic mathematical representation for the Holt Winters' multiplicative method are followed by [28]:

The h-step ahead forecast:

$$\hat{y}_{t+h} = (l_t + b_t h) s_{t-m+h_m^+} \tag{3.5}$$

where

Level: $l_t = \alpha \frac{y_t}{s_{t-m}} + (1-\alpha)(l_{t-1} + b_{t-1})$

Trend: $b_t = \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1}$

Seasonal: $s_t = \frac{\gamma y_t}{l_{t-1}+b_{t-1}} + (1-\gamma)s_{t-m}$.

m is period of seasonality, l_t represents the level of the series at time *t*, b_t denotes the local trend at time *t*, s_t is the seasonal component at time *t*, $\hat{y}_{t+h}$ is the forecast for *h* period ahead and $h_m^+ = [(h-1) mod m] + 1$. The smoothed parameters are denoted by α, β, γ and these smoothed parameters take values between 0 and 1. The values of the smoothing paramaters are incremented until the smallest MSE value is obtained.

3.2.2 Additive Seasonality

Although additive seasonality model is seen rarely in the real data, the mathematical representation of *h*-step-ahead forecast is followed by :

$$\hat{y}_{t+h} = l_t + b_t h + s_{t-m+h_m^+} \tag{3.6}$$

where

Level: $l_t = \alpha(y_t - s_{t-m}) + (1-\alpha)(l_{t-1} + b_{t-1})$

Trend: $b_t = \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1}$

Seasonal: $s_t = \gamma(y_t - l_{t-1} - b_{t-1}) + (1-\gamma)s_{t-m}$.

Like multiplicative seasonal model, *m* is a period of seasonality, l_t represents the level of the series at time *t*, b_t denotes the trend trend at time *t*, s_t is the seasonal component at time *t*, $\hat{y}_{t+h}$ is the forecast for *h* period ahead and $h_m^+ = [(h-1) mod m] + 1$. The trend term is same for both additive and multiplicative models. However, the level and seasonal equations change as the seasonal factor. That is, seasonal indices are added or substracted rather than multiplication or ratio [28]. The smoothed parameters are denoted by α, β, γ and these smoothed parameters take values between 0 and 1. The values of the smoothing paramaters are incremented until the smallest MSE value is obtained.

Taylor extended the HW model as Taylor's adaptation of Holt Winters (HWT) method by introducing second seasonal component and error term in the model [50]. Hyndman expresses HWT model as a statistical model to generate prediction intervals and developed and ets() function in the *forecast* package [30].

3.3 TBATS Model

Most of the time series data display complex seasonal behaviour like multiple seasonal periods, high frequency seasonality and non-integer seasonality. It is hard to model this type of seasonality in time series data by using traditional statistical methods such as Exponential Smoothing and SARIMA. Hence, Livera introduces a new alternative approach which is the modification of the ETS model based on trigonometric formulation in order to deal with the seasonal complexities in 2011. The new alternative approach is Trigonometric Exponential Smoothing State Space model with Box-Cox transformation, ARMA errors, Trend and Seasonal Components (TBATS) [15]. TBATS model consists of Box-Cox transformations, Fourier representations with time varying coefficients, and ARMA error correction.

Firstly, the original series y_t is transformed by using Box-Cox transformation to overcome the heteroscedasticity in the model.

$$y_t^{(w)} = \begin{cases} \frac{y_t^w - 1}{w} & w \neq 0 \\ log y_t & w = 0. \end{cases} \tag{3.7}$$

$y_t^{(w)}$ represents Box Cox transformed observations with the parameter w, where y_t is the observation at time *t* [15].

$$y_t^{(w)} = l_{t-1} + \phi b_{t-1} + \sum_{i=1}^{T} s_{t-m_i}^{(i)} + d_t. \tag{3.8}$$

The equation shows that the Box-Cox transformed observations is the function of the level, trend and seasonality.

Level equation is illustrated as:

$$l_t = l_{t-1} + \phi b_{t-1} + \alpha d_t, \tag{3.9}$$

where l_t is local level in period t, ϕ is a damping parameter on the trend. α is the smoothing parameter on the ARMA error component.

Trend equation is illustrated as:

$$b_t = (1-\phi)b + \phi b_{t-1} + \beta d_t, \tag{3.10}$$

b is a long term trend in the data. If we ignore βd_t , the current trend b_t is the weighted function of the long term trend (b) and short term trend b_{t-1} with the weight ϕ [33]. β is the smoothing parameter on the ARMA error component.

Error component is illustrated as:

$$d_t = \sum_{i=1}^{p} \phi_i d_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t \tag{3.11}$$

where d_t denotes an *ARMA(p,q)* process and ε_t is a Gaussian white noise process with mean equal to 0 and constant variance equal to σ^2 [15].

Trigonometric Seasonal component is illustrated as:

$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)}, \tag{3.12}$$

$$s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \cos\lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin\lambda_j^{(i)} + \gamma_1^{(i)} d_t, \tag{3.13}$$

$$s_{j,t}^{*(i)} = -s_{j,t-1} \sin\lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos\lambda_j^{(i)} + \gamma_2^{(i)} d_t, \tag{3.14}$$

$$\lambda_j^{(i)} = \frac{2\pi_j}{m_i}. \tag{3.15}$$

The above equations are the trigonometric representation of seasonal components based on Fourier series. $\gamma_1^{(i)}$ and $\gamma_2^{(i)}$ are the smoothing parameters. The stochastic level of the ith seasonal component is described by $s_{j,t}^{(i)}$. The stochastic growth in the level of the ith seasonal component essential to identify the change in the seasonal component in progress of time by $s_{j,t}^{*(i)}$.

3.4 Artificial Neural Networks (ANNs)

Artificial neural networks (ANNs) is a computing system inspired by biological neural networks sending signals by way of neurons and synapses. Structurally, in the neural network model, interconnection of many autonomous individual processing units behave similarly in certain respects to the interconnections of individual neurons in the brain [7]. The method generates correlation between input and output information with network system. In order to learn ANNs comprehensively, it is necessary to understand biological neural network structure.

3.4.1 Biological Neural Network Structure

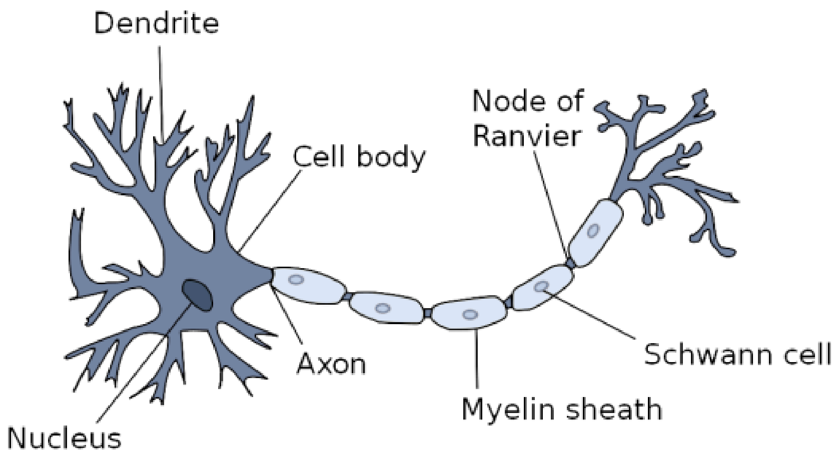

Figure 3.2: Structure of biological neural network [32]

Figure 3.2 shows the biological neural network. Dendrites transmit the signals which are conveyed from other neurons to nucleus. The nucleus is the center which aggregate all signals transmitted by dendrites. The nucleus transmits an aggregate signal to axons. Axons are transmission lines that convey the information to synapses which are located on the margin of the axons. Synapses is capital importance to convey all the information to next neurons since before conveying, synapses is preprocessor that if the received signal is higher than the threshold value, it fires and is transmitted to next neurons' dendrites. By this way, biological neural network is constructed.

3.4.2 Artificial Neural Network Structure

The ANN is a nonlinear forecasting method for time series. The Artificial Neural Networks have fairly similar components as the biological network. Figure 3.3 give the architecture of the ANNs.

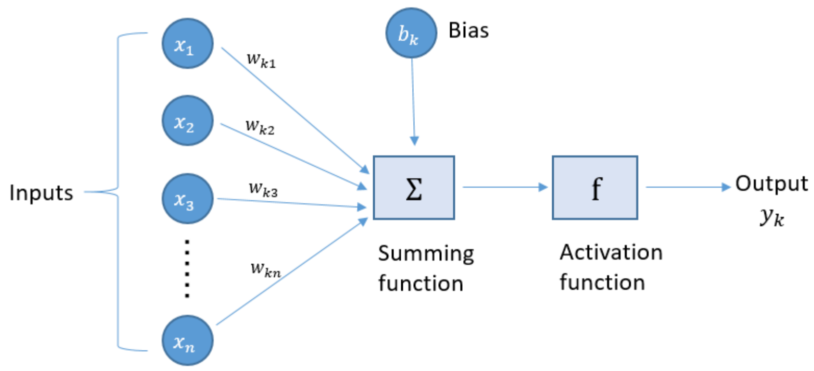

Figure 3.3: The General Concept of Artificial Neural Network

In the structure of ANNs, neurons are the basic building blocks that is fundamental to the operation of the neural networks [24]. As it is seen from Figure 3.3, the ANN model contains a bias, denoted by b_k. x_n is an input signal, w_n is a weight of the input signal. The summation function obtains the weighted sum of all input signals. Activation function states the conditions for the decision of the output neuron. Activation function is used to decide whether the neuron should be activated or not. Decision of the output neuron is the condition related to whether the weighted sum with a bias fires or not. Activation function is really important for the ANN model because it enables to learn and make sense of nonlinear complicated functional mappings between the inputs and the target variable. If the weighted sum with the bias is higher than threshold value, it fires. The fired weighted sum with bias is the output signal of neurons, denoted by y_k.

The mathematical representation of neuron *k* illustrated in Figure 3.3 is defined as:

$$u_k = \sum_{j=1}^{m} w_{kj} x_j \tag{3.16}$$

and

$$y_k = f(u_k + b_k). \tag{3.17}$$

The most commonly used activation functions in time series analysis is the sigmoid (logistic) function. The mathematical equation of sigmoid function is illustrated by:

$$f(\sum_{j=1}^{m} w_{kj} x_j) = \frac{1}{1 + e^{-x}}. \tag{3.18}$$

There are two major types of neural networks in terms of connections between neurons and direction of data propagation: feed-forward and recurrent networks.

3.4.3 Feed-forward Neural Networks

The feed-forward structure shows that the network intelligence starts from the input and continues towards the output [7]. In feed-forward neural network, there exist hidden layers. The computation part of hidden layer is named hidden neurons or hidden units; hidden means the fact that the network does not flow directly from either input or the output of the network. By adding one or more than hidden layers, they intervene between the input layers and output of the network to capture the nonlinear structure of the data [24].

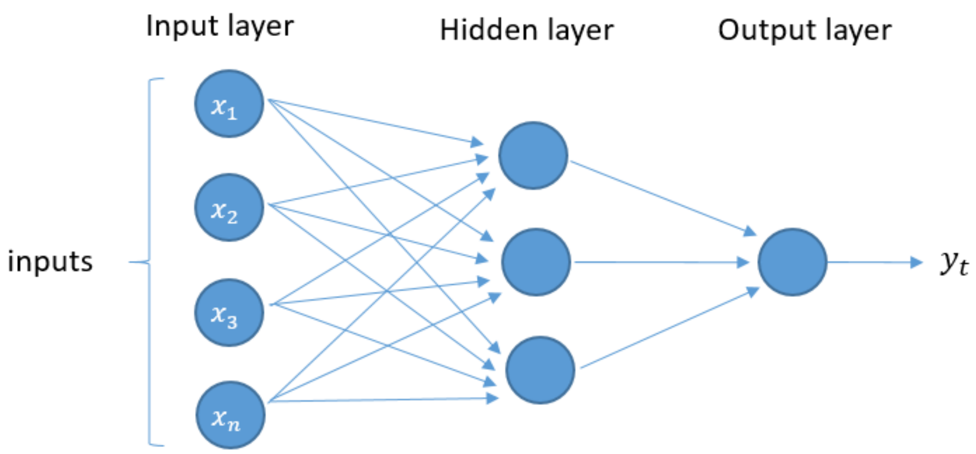

Figure 3.4: Feed-forward Neural Network with Single Hidden Layer

Figure 3.4 indicates the architecture of three-layered feed-forward neural network. The input layer passes the data received to the hidden layer. Then, hidden layer transmits them to the output layer. In the structure of feed-forward, there exists network with interconnections, but these interconnections do not form any loops [20].

3.4.4 Recurrent Neural Networks

The second neural network model is the recurrent neural network. The recurrent neural network was proposed by Elman [19]. The difference between the feed-forward neural network and the recurrent neural network is "feedback loop". That is, each neuron feeds its output, then the output signal turns back as the inputs of all the other neurons so the output get involved in the flow of the network. The structure of recurrent network is illustrated in Figure 3.5 .

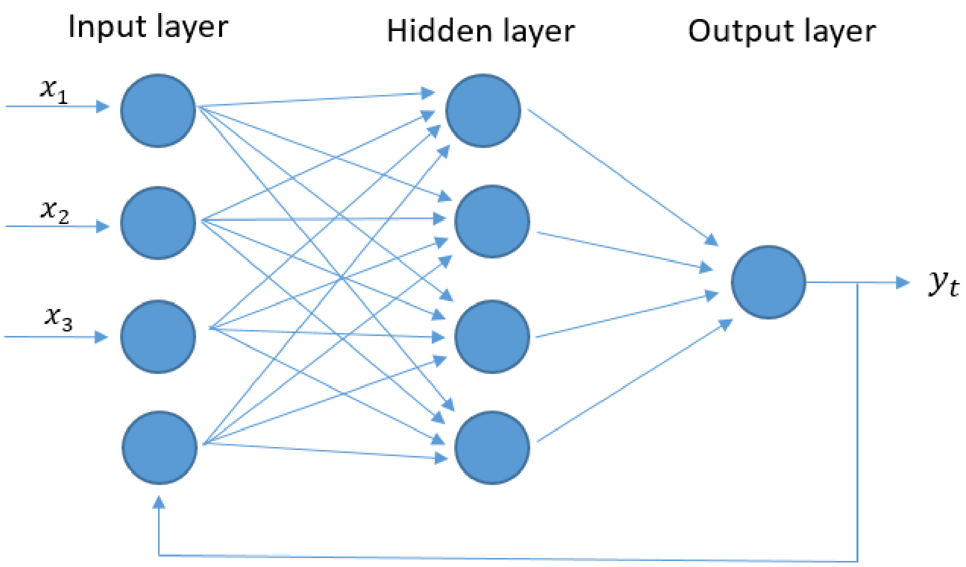

Figure 3.5: Recurrent Neural Network with Single Hidden Layer

As it is seen from Figure 3.5, there exist one or more loops of interconnections in the recurrent neural network. The nodes receive feedback from the other nodes. The input signal is the combination with the former activation by way of adding weight layer. Therefore, the feedback connections are occurred after updating the network.

3.5 Theta Model

There exist wide variety of patterns in time series data and it is useful to classify some of the patterns and behaviours seen in the time series data by splitting a time series into several components, each representing one of the underlying categories of the pattern. This phenomenon is described as time series decomposition. Decomposed parts are trend, seasonality and the irregular part which are designed individually into the future and recombined to create a forecast of the underlying series. Assimakopoulos and Nikolopoulos [3] introduced Theta model inspired by time series decomposition. The benefit of the Theta Model is that there is no need to know comprehensive statistics. The logic behind Theta model is to extrapolate the useful information embedded in the data which has short and long term components. Then, the integration of the components improves the accuracy of the forecast in time series. The model is related to the modification of the local curvatures of the time series through the theta coefficient (θ) which is carried out directly to the second differences of the time series [3]. For example, if the original series are represented as y_t, a theta line can be computed as [33]:

$$Z_t(\theta) = \theta y_t + (1 - \theta)(\hat{\alpha} + \hat{\beta}t). \tag{3.19}$$

In the equation, the fitted time trend is $(\hat{\alpha} + \hat{\beta}t)$, the estimated intercept parameter is $\hat{\alpha}$ and the estimated slope parameter is $\hat{\beta}$. θy_t is the curvature of the data.

If the θ is equivalent to 0, the model is represented by:

$$Z_t(0) = (\hat{\alpha} + \hat{\beta}t). \tag{3.20}$$

This equation gives the linear regression of the time trend. It is necessary to compute the long term trend by taking out the curvature of the data.

If the θ is other than zero, the theta lines are a weighted combination of y_t and linear regression time trend. If θ is higher than 1, the line increases the short term dynamics of the time series. If θ is less than 1, the line increases the long term trend of the series. Whether the local curvatures are removed or enlarged is determined by the θ value.

Theta method is a dynamic forecasting approach that the number of theta lines are decided by the user and the individual forecasts are combined with equal and unequal weights. In the Standard Theta model, there exist two theta lines used with θ. The steps of the Standard Theta model is defined as [33]:

Step 0: Whether the seasonality exists in time series data is determined by applying autocorrelation function and statistical test of the autocorrelation coefficients.

Step 1: If the seasonality occurs, a seasonal decomposition of data is applied under the multiplicative relationship for the seasonal part.

Step 2: Time series decomposition is performed by splitting the series into two lines. The first line is a linear regression time trend $Z_t(0)$ and the other line is theta line of the series $Z_t(2)$.

Step 3: Linear regression is applied to estimate $Z_t(0)$ and simple exponential smoothing is used to estimate $Z_t(2)$.

Step 4: When the estimation of the two lines are integrated with equal weights, the forecasts are obtained.

Step 5: Lastly, if the forecasts are seasonally adjusted in step 1, the seasonal part is incorporated.

3.6 STL Decomposition Method

STL method is a Seasonal-Trend Decomposition Procedure Based on the locally weighted scatterplot smoothing (LOESS). The concept of the STL method is based on the decomposition of time series into three components: trend, seasonal, and remainder. If time series data (Y_v) have the trend part, the seasonal part, and the remainder part denoted by T_v, S_v, R_v, respectively, for $v = 1$ to *N*, then the mathematical form of the method is given by [13]:

$$Y_v = T_v + S_v + R_v.$$

According to Cleveland, STL method is such a filtering procedure that it decomposes the data via a sequence of applications of smoothing operations using the LOESS [14]. Locally weighted polynomial regressions at each point in the dataset are implemented by a LOESS smoother. LOESS is the estimation technique for regression surface by means of a multivariate smoothing procedure. This procedure is fitting a function of explanatory variables locally, which enables to estimate a broader class of regression surface than the usual classes of parametric function [14].

In the concept of STL, eigenvalues and frequency response analysis of a given time series are used to get the parameter of the STL in order to understand whether the variation comes from either seasonal component or trend component. The procedure is performed in an iterated cycle composed of two recursive procedures, which are the inner and the outer loop [52]. In each pass of the inner loop, the seasonal and trend components are updated by seasonal smoothing and trend smoothing. There exist six steps in the inner loop:

Step 1: *Detrending.* $S_v^{(k)}$ and $T_v^{(k)}$ are the seasonal and trend components at the end of the k^{th} pass. At $(k+1)^{th}$ iteration, detrended series is computed by the formula

$$Y_t - T_v^{(k)}.$$

Step 2: *Cycle-subseries Smoothing.* LOESS smoother is implemented to each cycle-subseries of the detrended series obtained in the first step. Then, temporary seasonal series; $C_v^{(k+1)}$, is calculated.

Step 3: *Filtering of Cycle-subseries.* The filtering procedure relates to a simple moving average process. The simple moving average is implemented to the temporary seasonal component obtained in the second step, pursued by the practice of the LOESS smoother to describe any remaining trend; $L_v^{(k+1)}$.

Step 4: *Detrending of Smoothed Seasonality.* The difference between temporary seasonal component; $C_v^{(k+1)}$, computed in step 2 and temporary trend component; $L_v^{(k+1)}$, estimates the additive seasonal component.

$$S_v^{(k+1)} = C_v^{(k+1)} - L_v^{(k+1)}.$$

Step 5: *Deseasonalizing.* Substracting additive seasonal component calculated in the fourth step from the original data gives a seasonally adjusted series. The mathematical equation is followed by:

$$Y_t - S_v^{(k+1)}.$$

Step 6: *Trend Smoothing.* The deseasonalized series is smoothed again. Then, trend component denoted by $T_v^{(k+1)}$ is estimated. In each pass of the inner loop, seasonal smoothing is implemented to update the seasonal component, pursued by trend smoothing implemented to update the trend component.

In an iteration of the outer loop, estimates of the seasonal and trend components calculated in the inner loop are used to compute the irregular component

$$R_v^{(k+1)} = Y_v - T_v^{(k+1)} - S_v^{(k+1)}.$$

If the irregular component has a large value, it is described as an extreme value and a weight is computed. Then, outer loop ends. In the next iteration of the inner loop, the weights are used to reduce the effect of the extreme value. Hence, implementation of the method within the algorithm for the application of STL procedure gives STL returns, T_v, S_v and R_v for every time series Y_t such that $Y_t = T_t + S_t + R_t$ where T_t is the trend part, S_t is the seasonal part and R_t is the residual part [52].

3.7 Ensemble Model

The traditional forecast methods in time series have been popular for many years. There exist various statistical methods to predict the future values of the time series variable. However, in real life, most of the time series data exhibit complex structure, so the individual methods may have less predictive performance in time series forecasting. In this situation, the concept of "combination of the forecasts" has more remarkable performance than individual forecast methods. The ensemble model is based on the combination of several individual forecasts with the weights. The idea of combining forecasts is not a new approach. The combining forecasts was first introduced by Bates and Granger in 1969 [4]. According to Bates and Granger, combining forecast is an alternative approach and linear combination of forecasts give more accurate result than the single forecasting method. Many researchers have preferred to use ensemble model to get forecast which is close to real values instead of individual method recently. The main question in ensemble model is how different forecasting methods are combined. The logic behind the ensemble model is that the several forecasting methods are trained separately, then the forecast of each method are combined to increase the performance of the forecast. The combination of the forecasts depends on the weight of each model.

Let $Y = [y_{t+1}, y_{t+2}, ..., y_{t+h}]^T$ be the actual values of testing data. The actual values are predicted by using the past observations via forecasting methods.

Let $\hat{Y}^i = [\hat{y}^i_{t+1}, \hat{y}^i_{t+2}, ...\hat{y}^i_{t+h}]^T$ be its forecast through the i^{th} model $i = 1, 2, ...c$. Then, the general form of linear combination of these c forecast is mathematically described as [1]:

$$\hat{y}_{t+h} = w_1\hat{y}^{(1)}_{t+h} + w_2\hat{y}^{(2)}_{t+h} + ... + w_c\hat{y}^{(c)}_{t+h} = \sum_{i=1}^{c} w_i\hat{y}^{(i)}_{t+h} \tag{3.21}$$

where *i=1,2,...c*. w_i is the weight value for i^{th} forecasting model and usually, the weights are generally assumed to be higher than zero. The summation of all weights is generally equivalent to 1. h stands for the number of forecasts for each model and c represents the number of the forecasting models which are used in the ensemble model. Then, the forecast vector for Y is $\hat{y} = [\hat{y}_{t+1}, \hat{y}_{t+2}, ..., \hat{y}_{t+h}]^T$.

Obtaining the best combination of forecasts depends upon the weight of each model. Therefore, the main factor in the ensemble model is to specify the weights because the values of the weights help to achive reliable results in ensemble model building. In this study, two weight assignment techniques are discussed. These are simple average and cross validated errors.

3.7.1 Simple average

In the simple average, equal weights are assigned to all forecasting methods. In this procedure, each individual method is trained and the prediction of future values are found by each method. Then, the forecasts are combined with equal weights. For example, when *c* forecasting methods are used, the weights of each model is $w_i = \frac{1}{c}$ where *i=1,2,..,c* .

The mathematical form of the ensemble model is followed by:

$$\hat{y}_{t+h} = w_1\hat{y}^{(1)}_{t+h} + w_2\hat{y}^{(2)}_{t+h} + ... + w_c\hat{y}^{(c)}_{t+h} = \sum_{i=1}^{c} w_i\hat{y}^{(i)}_{t+h} = \sum_{i=1}^{c} \frac{\hat{y}^{(i)}_{t+h}}{c}. \quad (3.22)$$

3.7.2 Cross Validated Errors

Cross validation proposed by Stone is the most commonly used technique to evaluate the generalizability of the statistical prediction [49]. The concept of the cross validation is on the basis of dividing the sample into two sets which one is trained to estimate the model and the other one is used for the measure the accuracy of the model. In time series analysis, cross validation is quite complicated due to the serial correlation between the observations of the time series data. To deal with the problem about the dependence of the observations, rolling cross validation is applied. The rolling cross validation is an alternative approach that increase the length of the training sets used to evaluate the sampling variation effect [42].

The rolling cross validation needs many iteration and takes a long time for computation. For example, supposing that there exist 10 years monthly time series data starting from 1990 to 1999. In the rolling cross validation procedure, 5 fold framework is applied. The rolling means that the in-sample periods are enlarged sequentially beginning from 1990. One additional year is added to the in-sample period in each successive fold. In this example, the first in-sample period starts from 1990 to 1994, the second from 1990 to 1995, and the fifth from 1990 to 1998. Test set period is the year after the training set period in each fold. Therefore, the 5 out of samples are used as in the moving test procedure.

In the ensemble model based on cross validated errors building process, the errors of the cross validated forecast models are firstly computed. Later, according to the cross validation error, the weights of each model are assigned. The model with the highest forecast error is assigned to have a less weight.

3.7.3 The Proposed Ensemble Model Methodology

In this study, the ensemble model building is based on the cross validated errors for optimal weights. The model procedure is followed by:

Step 1: Splitting the original time series data $Y = [y_1, y_2, ..., y_n]^T$ into the train set and validation set.

Step 2: For *c* component forecast models, rolling cross validation is applied to the train set.

Step 3: The forecasts are predicted by fitting the model for each individual method. The forecast values of i^{th} model are $\hat{y}^i = \hat{y}_1^i, \hat{y}_2^i, ...\hat{y}_h^i$, *i=1,...,h*.

Step 4: The optimal weight is determined by the errors of the cross validated forecasting model. If the error is high in the i^{th} model, the weight of the i^{th} model is small.

Step 5: The linear combination of the individual models are constructed with the weights based on the cross validated errors. The *h*-step ahead forecasts are

$$\hat{y}_{t+h} = w_1\hat{y}_h^{(1)} + w_2\hat{y}_h^{(2)} + ... + w_c\hat{y}_h^{(c)} = \sum_{i=1}^{c} w_i\hat{y}_h^{(i)}.$$

Step 6: Forecast performance of the ensemble model is assessed by measuring the accuracy to determine how well the model predict the actual values of the time series.

3.8 Measuring Forecast Performance

The forecast performance means how well the forecast methods predict the future values. The main goals for measuring forecast performance are (1) to calculate the accuracy of how well we predicted the actual values, and (2) to compare the different forecast techniques to specify the best forecast model [12]. In this study, seven forecast techniques are implemented. To compare the performance of all methods, there are several forecast accuracy measures used to evaluate the error size. The error is the difference between the actual and the predicted values. The less the error values are, the higher performance of the model is. Most commonly used accuracy measures in time series analysis are the scale-dependent errors and percentage errors.

The h-step-ahead forecast error is :

$$e_{t+h|t} = y_{t+h} - \hat{y}_{t+h}$$

where y_{t+h} is the original time series and $\hat{y}_{t+h}$ is the forecast of original data in time t.

3.8.1 Scale-Dependent Errors

The most widely used scale-dependent measures are the absolute error and squared errors. However, according to Hyndman, mean absolute error (MAE) may not give reliable results to compare the forecast methods [28]. The root mean squared errors (RMSE) is the most preferable measure in time series analysis. In addition, evaluation of the reliability of the macroeconomic series is based on the measure of RMSE, following Cecchetti et al. [11]. The mathematical form of the RMSE is represented by:

$$RMSE = \sqrt{\sum_{t=n+1}^{n+h} (y_t - \hat{y}_t)^2/h} \tag{3.23}$$

where h is the number of forecasts that we want to obtain. If the RMSE value is low, the forecast technique has high accuracy.

3.8.2 Percentage Errors

The advantage of the percentage errors is a scale-independent approach. The most commonly used percentage errors in time series forecasting is the mean absolute percentage error (MAPE). The MAPE takes into consideration to the effect of the magnitude of the original values [31]. MAPE is calculated by the average absolute percent error for each time period. The mathematical form of the MAPE is represented by:

$$MAPE = h^{-1} \sum_{t=n+1}^{n+h} \mid \frac{y_t - \hat{y}_t}{y_t} \mid . \tag{3.24}$$

MAPE is preferred many times because it is useful to make comparisons between forecasts from different scenarios [12]. As with RMSE performance, if the MAPE value is low, the forecast technique has high accuracy.

CHAPTER 4

EMPIRICAL ANALYSIS

In this chapter, the datasets used in the analysis will be introduced firstly. Secondly, preprocess of the data will be conducted. Then, the numerical results of 12 month ahead forecasts for each selected model will be graphically displayed. Lastly, the forecast accuracy of all selected models will be evaluated.

4.1 Data Description

The inflation measures the changes of general level of prices of services and products. It is vital factor as it determines cost and standard of living. The inflation are measured by the CPI in Turkey. Hence, in this study, we used the data of the annual change of the Turkish CPI from January 2005 to December 2016 in order to produce 12-month ahead forecasts of the Turkish inflation. The CPI data were received from TURKSTAT. According to the literature and opinions of the experts at TURKSTAT, the CPI is influenced by some macroeconomic variables. In the forecasting procedure, adding these variables into the models helps obtaining more accurate results. These variables are the annual change of Turkish producer price index (PPI), seasonally adjusted monthly Turkish unemployment rate, monthly Turkish deposit interest, monthly real effective exchange rate of Turkey, monthly export unit value index of Turkey and monthly import unit value index of Turkey between January 2005 and December 2016. The time period of Turkish CPI is equivalent to 144 months. In this study, we applied R-Studio with version 1.1.383 to estimate the model and predict the Turkish CPI from January 2017 to December 2017.

4.2 Pre-processing of the Data

Before the application of forecasting techniques to the Turkish inflation series, it is needed to look the data structure and conduct model building process. The general procedure in constructing time series model was described by step-by-step:

Step 1: Conduct explanatory data analysis to look the general overview and control the existence of the abnormalities in the series.

Step 2: Transform the data if the assumption of variance stabilization is not satisfied.

Step 3: Examine the stationarity condition of the data, and test the trend and seasonality condition of the data by Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test, Augmented Dickey Fuller (ADF) test and Hylleberg, Engle, Granger, and Yoo (HEGY) test. If the series are not stationary, take difference of the series to make it stationary.

Step 4: Investigate the overall view of the exogeneous variables and test whether the exogeneous variables are stationary or not.

Step 5: Examine the cross correlation function (CCF) between the exogeneous variables and the response variable and add the exogeneous variables and their lags into the model based on the CCF structure.

Step 6: Identify the model and estimate the model parameters.

Step 7: Check the model diagnostic.

Step 8: Calculate the forecast of the series.

Step 9: Examine the forecast performance of the model.

4.2.1 Exploratory Data Analysis

Firstly, the descriptive statistics and the overall view of the Turkish CPI data were examined. The overall view of the series enable to detect the trend, seasonality and stationarity of the series. Table 4.1 shows the descriptive statistics of the series.

Table 4.1: Descriptive Statistics of the Consumer Price Index

Min	Mean	Median	Std Dev	Variance	Max
3.990	8.254	8.170	1.634	2.671	12.06

As it is seen from Table 4.1, the mean and median are very close to each other. The range is not large so there is no outlier in the Turkish CPI series. The variance is low compared to its mean. In addition, there are no missing values in the series. The general overview of annual change of the Turkish CPI between January 2005 and December 2016 is illustrated by Figure 4.1:

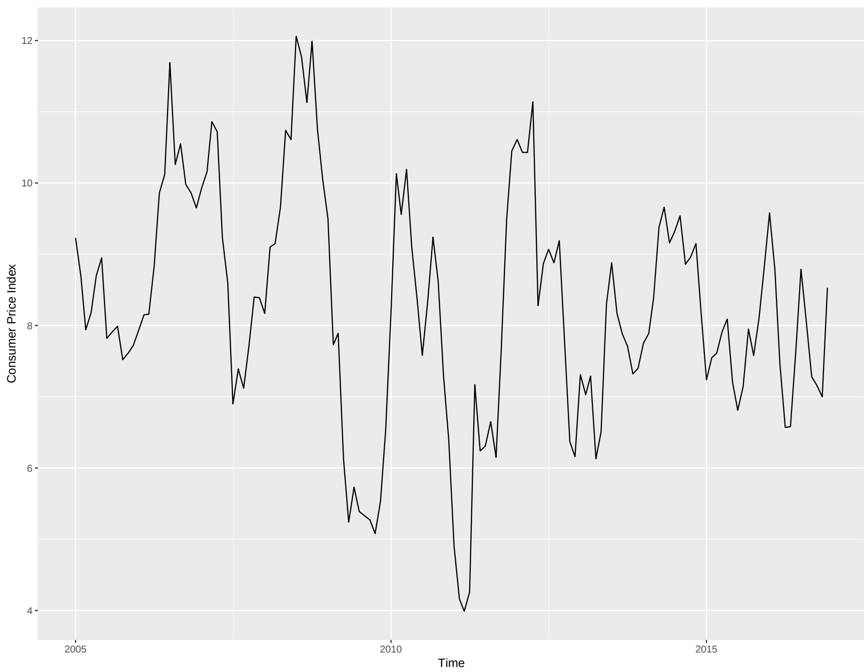

Figure 4.1: The overview of consumer price index

Figure 4.1 shows that the series have seasonal behaviour. Also, the mean term is not constant. It seems that there exists decreasing trend in the series. The definite judgement can not be reached from the graphical display. Hence, it needs to test for the existence of the regular unit root and the seasonal unit root by examining the sample autocorrelation function (sACF) and the sample partial autocorrelation function (sPACF), and conducting statistical tests.

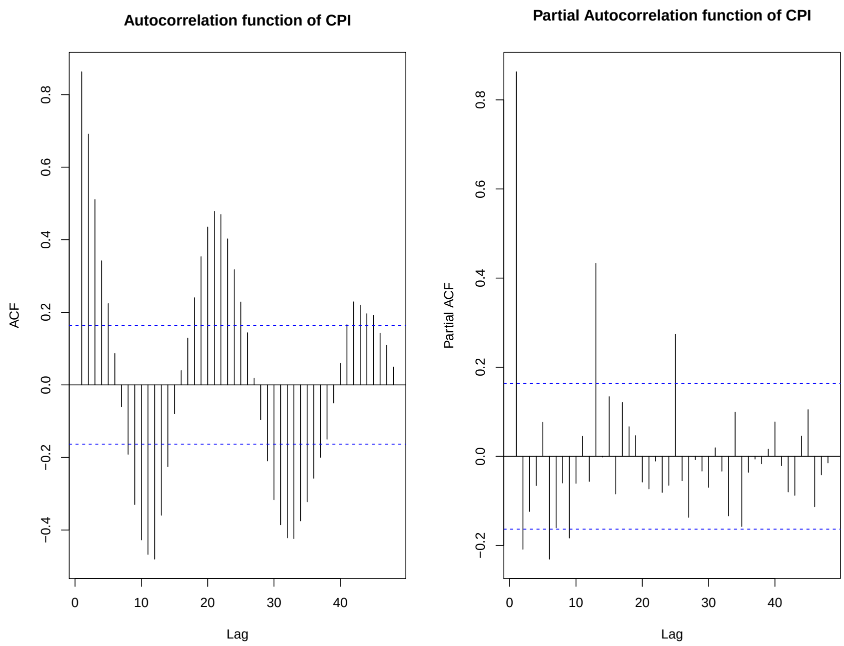

Figure 4.2: The sACF and sPACF of The Turkish CPI

The left side of Figure 4.2 shows the sACF of the data and the right side shows the sPACF of the data. The autocorrelation function calculates the correlation between Y_t and Y_{t+k} from the same process, and the partial autocorrelation function calculates the correlation between Y_t and Y_{t+k} after the effects of the intervening variables Y_{t+1}, Y_{t+2},....,Y_{t+k-1} are removed. By sACF and sPACF plots, we can understand whether the series are stationary and have seasonality. Also, we can identify the model order for SARIMA. If the lags of the series periodically increase and decrease at lags of multiples of seasonal period, the series have seasonal behaviour. The sACF and sPACF plots exhibit that the series have seasonal pattern and the stationarity condition of the series seems to be not satisfied.

4.2.2 Variance Stabilization and Stationarity Condition

Before starting forecasting the Turkish CPI, the series are standardized because there are six exogeneous variables and the base year of these variables and CPI variable are different. To overcome the problem, it is necessary to the standardization for all variables used. After standardizing the variables, assumptions of the time series data were checked to estimate the model. The first assumption is the variance stabilization. If the variance is not constant, Box-Cox transformation is implemented to the series. After applying BoxCox.ar() in R, it was decided that the Box-Cox transformation was not needed for the Turkish CPI. Hence, the first assumption of the series is satisfied.

The second main assumption is stationarity. If the time series data have unit root, the stationarity condition is not satisfied. In this study, KPSS test and ADF test were applied for checking whether the series have trend or not by using *tseries* and *fUnitroot* packages in RStudio. Test results showed that the series do not have trend so the series are stationary. However, as it is seen from Figure 4.1 and Figure 4.2, the series exhibit seasonal behaviour. Hence, the HEGY test is utilized for testing the existence of the regular and the seasonal unit root by using *uroot* package. Based on the HEGY test results, we can say that there is no seasonal unit root, but there exists regular unit root in the series. Therefore, it is necessary to take difference of the series. After differencing, the stationarity condition is satisfied. Figure 4.3 illustrates the time series plot of the differenced Turkish CPI data.

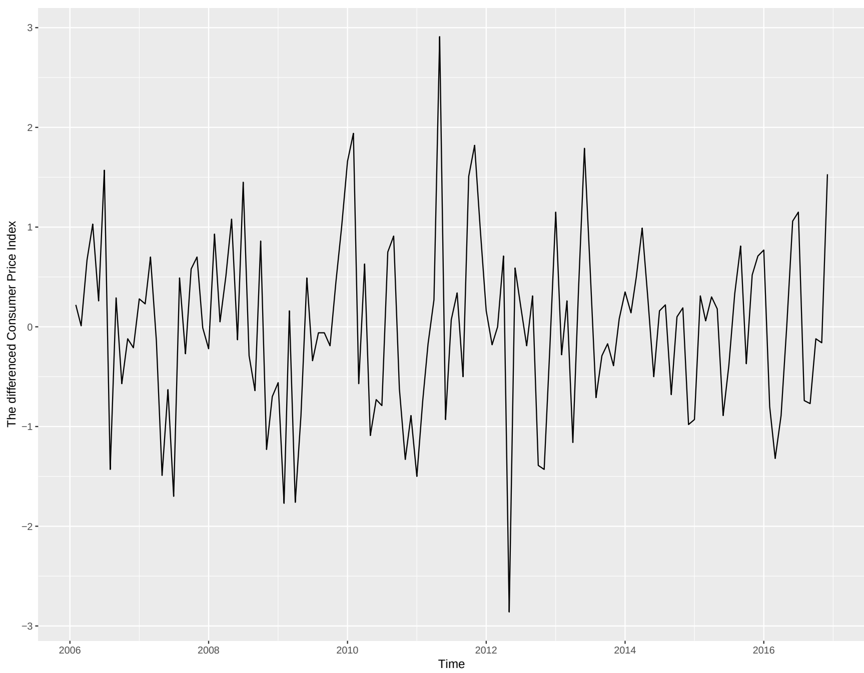

Figure 4.3: The overview of differenced consumer price index data

4.2.3 Exogeneous Variables

The annual change of the Turkish PPI, the seasonally adjusted monthly Turkish unemployment rate, monthly Turkish deposit interest, monthly real effective exchange rate of Turkey, monthly export unit value index of Turkey and monthly import unit value index of Turkey are used as the exogeneous variables as mentioned in data description. However, adding these variables into analysis is not simple because the structure of the CPI has to be similar with the structure of the explanatory variables. From the previous analysis, differencing is applied to the CPI series to obtain the stationary series. This condition of the CPI data has to be valid for the exogeneous variables too. When the general overview and the stationarity condition of the each exogeneous variable are examined, our findings are given below.

When the time series plots of the each exogeneous variable are examined, the stationarity condition of the each variables is not satisfied. For example, the Turkish PPI series are trend-stationary and have a seasonal behaviour, the unemployment rate series have fluctuations, the real effective exchange rate series show the decreasing trend and the seasonal behaviour. There is no seasonal behaviour in the deposit interest series, and the export and the import unit value index series have trend. Therefore, KPSS test, ADF test, and HEGY test were implemented to test the existence of the unit root for all exogeneous variables. Our findings showed that there exists stochastic trend in all exogeneous variables. To handle this problem, differencing is needed. After taking difference of all exogeneous variables, they become stationary.

Our aim is to produce accurately 12-month ahead forecasts of the Turkish CPI by building the appropriate model with exogeneous variables. In accordance with this purpose, it requires obtaining 12-month ahead forecasts of the exogeneous variables. The forecast values of the exogeneous variables were predicted by *auto.arima()* and *ets()*. The exponential smoothing forecast values were used for the PPI and the exchange rate because of the higher accuracy. The ARIMA forecast values were used for the unemployment rate, the deposit interest, the import and the export unit value due to the higher accuracy. The actual values from January 2005 to December 2016 and the forecast values were combined for each exogeneous variable. The cross correlation function (CCF) is calculated to decide which lags of these variables affect the CPI series. The CCF gives the direction and strenght of the correlation between two series [53]. The CCF takes value between -1 and +1. If the value is -1 or +1, there is a strong relationship between two variables. If the value is 0, there is no relationship between these variables. In this study, the CCF plot was drawn to give the correlation between the Turkish CPI and each exogeneous variable. The negative lag part in the CCF plot shows the effect of the Turkish CPI on the exogeneous variables. The positive lag part in the CCF plot indicates the effect of the exogeneous variables on the Turkish CPI. Since we consider the effect of those variables on the Turkish CPI, the positive lag part of the CCF is taken into consideration. The CCFs are illustrated in Figure 4.4.

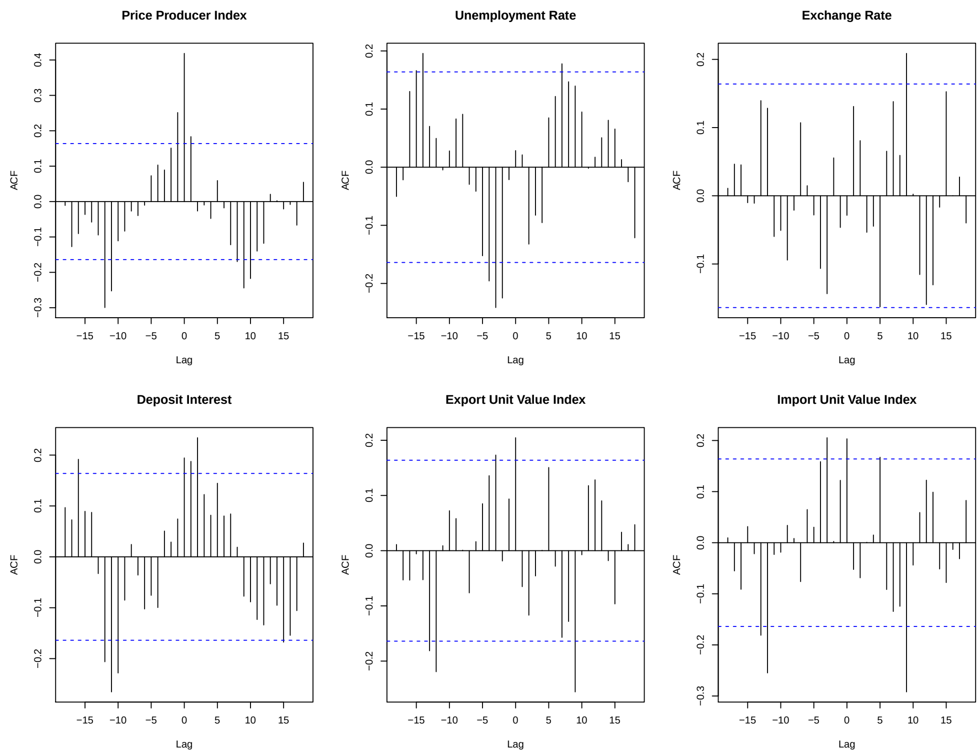

Figure 4.4: The CCF between Turkish CPI and each exogeneous variables

As it is seen from the CCF, the current, 1^{st}, 9^{th} and 10^{th} lag values of the PPI series are significant; the 7^{th} lag value of the unemployment rate is significant, 9^{th} lag value of the exchange rate is significant; the current, 1^{st} and 2^{nd} lag values of the deposit interest are significant; the current and 9^{th} lag values of the export unit values are significant, and the current, 5^{th} and 9^{th} lag values of the import unit values are significant. These lag and current values of each exogeneous variable were combined to be utilized for model estimation. However, when the lag values of the exogeneous variables were obtained, the first 12 observations were unobserved. To handle this problem, the first 12 values of all exogeneous variables were dropped from the model. Hence, the time period used in the analysis is from January 2006 to December 2016. Also, the time period of the Turkish CPI is from January 2006 to December 2016. In the forecasting procedure, the last 12 values of the combined data were used.

4.3 Forecasting Methods

In this study, our objective is to find the best model which produces 12-month ahead forecasts of the Turkish CPI. The best model accurately predict the actual CPI series by the least number of parameters and the exogeneous variables. In this purpose, we implemented six different individual models and the ensemble model for 12-month ahead forecasts of the inflation. The individual forecast methods are the SARIMAX model, additive HW model with additive error, the TBATS model, the Artificial Neural Network model, the Theta model and the Seasonal Trend Decomposition with LOESS method. Ensemble model combines forecasts of the individual methods with optimal weights. Then, the RMSE and MAPE values of all forecast models were calculated. According to the RMSE and the MAPE values of all models, the best model for the forecast analysis of the Turkish CPI was selected.

4.3.1 SARIMAX Model

In constructing the SARIMAX model, we applied *forecast* package in RStudio. The first step of the model building is model identification after the stationarity condition of the target variable is satisfied. The Turkish CPI series became stationary after taking difference. In model identification, the sample ACF and the sample PACF of the differenced series were computed to determine the orders of the model.

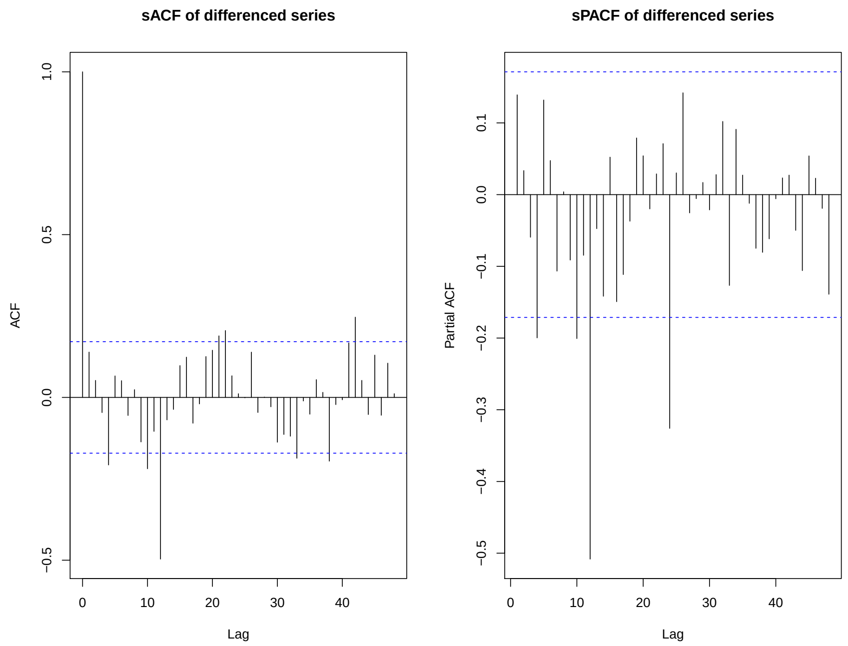

Figure 4.5: The sACF and sPACF of the differenced series

It is evident from Figure 4.5 that the orders of the seasonal ARIMAX model are $SARIMAX(4,1,4)(2,0,1)_{12}$. The h-step ahead forecast of Y_{n+h} using the SARIMAX model is defined as:

$$\hat{Y}_n(h) = E[Y_{(n+h)} \mid Y_1, Y_2, ..., Y_n, X_1, X_2, ..., X_n, \hat{X}_n(1), \hat{X}_n(2), ...\hat{X}_n(h)] \quad (4.1)$$

where Y_i is the past values of the variables, X_i is the past values of the i^{th} exogeneous variable and the $\hat{X}_n(i)$ is the forecast values of i^{th} exogeneous variable.

After specifiying a tentative model, the parameters of the model which are ϕ_i, Φ_i, θ_i, Θ_i were estimated according to the maximum likelihood estimation method. The maximum likelihood estimation is a statistical method which estimates the model parameters from the sample data, and those parameter values maximize the probability of obtaining the observed data. The model selection criteria is based on the Akaike Information Criteria (AIC). This criteria is utilized to evaluate the quality of the model fitting. The AIC is calculated :

$$AIC(M) = -2ln(maximumlikelihood) + 2M \quad (4.2)$$

where *M* is the number of parameters in the model.

The best forecast model is determined according to the significance of the exogeneous variables and forecast performance. The best forecast model for 12-months ahead forecasts of the Turkish CPI is represented in Table 4.2.

Table 4.2: The Estimates and Standard Errors of the $SARIMAX(4, 1, 4)(2, 0, 1)_{12}$

Component	AR1	AR2	AR3	AR4	MA1	MA2
Coefficient	0.5756	-0.2686	-0.2923	0.4102	-0.6711	0.1870
S.E	0.3682	0.4643	0.4284	0.2644	0.3446	0.4601
Component	MA3	MA4	SAR1	SAR2	SMA1	PPI.0
Coefficient	0.1519	-0.6678	-0.2959	-0.1380	-0.5811	0.4545
S.E	0.4216	0.2770	0.2391	0.1864	0.2640	0.1116
Component	PPI.1	INT.1	INT.2	EXP.0	IMP.5	
Coefficient	0.1206	0.2131	0.1727	0.3446	-0.1436	
S.E	0.1039	0.1765	0.1908	0.1514	0.1724	

The model parameters are estimated by the MLE. As it is seen from Table 4.2, coefficients give the parameter estimates and standard errors give their standard errors. The model is $SARIMAX(4, 1, 4)(2, 0, 1)_{12}$ which has the highest out of sample forecast performance. In this model, not all components are significant. The AR with order 4, the seasonal AR with order 2, the first lag of the PPI, the first lag of the deposit interest, the second lag of the deposit interest and the fifth lag of the import unit values are not significant. However, the forecast values of the $SARIMAX(4, 1, 4)(2, 0, 1)_{12}$ are very close to the actual values of the CPI.

Table 4.2 also shows that the current value of the Turkish PPI and the first lag of the Turkish PPI affect positively the Turkish CPI. The first and second lag of the deposit rates have a positive effect on the Turkish CPI. The current export unit value index positively influences the Turkish CPI. However, the fifth lag of the import unit value index has a negative influence on the Turkish CPI. In brief, the used exogeneous variables except for import unit value impact positively the CPI. The forecast plot of the model is illustrated by Figure 4.6.

In Figure 4.6, the dark line gives the actual values of the CPI between January 2006

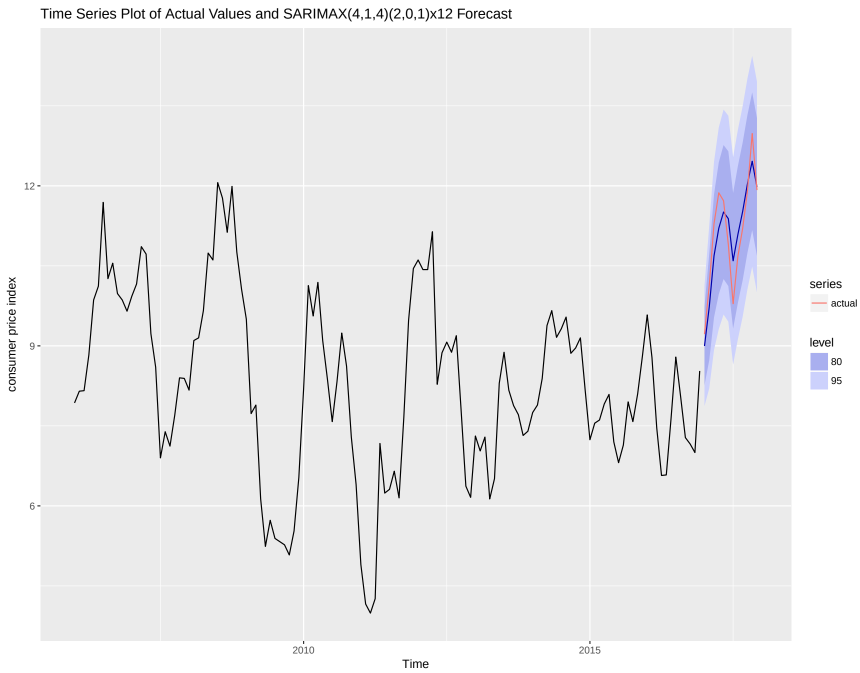

Figure 4.6: The SARIMAX forecasts of the CPI giving the best forecasts

and December 2016. The dark blue line gives the forecast values of the CPI series. The red line gives the actual values of the CPI series. The forecast plot shows that 12 month ahead forecasts of the Turkish CPI are very close to the actual Turkish CPI values from January 2017 to December 2017. Moreover, the CPI forecasts have same seasonal pattern with the actual values of the CPI.

Table 4.3: The Forecast Performance of the $SARIMAX(4,1,4)(2,0,1)_{12}$

ME	RMSE	MAE	MPE	MAPE	MASE	Correlation
0.0345	0.4566	0.4026	0.1817	3.6731	0.1669	0.8937

Table 4.3 shows the forecast accuracy of the $SARIMAX(4,1,4)(2,0,1)_{12}$. The RMSE is 0.4566 and the MAPE is 3.6731. Both accuracy measures are low.

The second model is $SARIMAX(3,1,3)(0,0,1)_{12}$ which has the best train set performance for 12-month ahead forecasts of the Turkish CPI. The optimal orders of the SARIMAX model are selected by the number of parameters so that AIC is minimum. After the checking the significance of the components, the coefficients of the model are given in Table 4.4.

Table 4.4: The Estimates and Standard Errors of the $SARIMAX(3,1,3)(0,0,1)_{12}$

Component	AR1	AR2	AR3	MA1	MA2
Coefficient	1.0961	-0.9433	0.4875	-1.2613	1.0784
S.E	0.2457	0.2295	0.1438	0.1996	0.1717
Component	MA3	SMA1	PPI.0	INT.2	EXP.0
Coefficient	-0.8171	-0.7516	0.5228	0.3270	0.1567
S.E	0.1389	0.0833	0.0934	0.1134	0.0996

Table 4.4 shows the parameter estimates and their standard errors of the each component in the $SARIMAX(3,1,3)(0,0,1)_{12}$ model. All components of the model are significant. The forecast plot of the $SARIMAX(3,1,3)(0,0,1)_{12}$ is demonstrated in Figure 4.7:

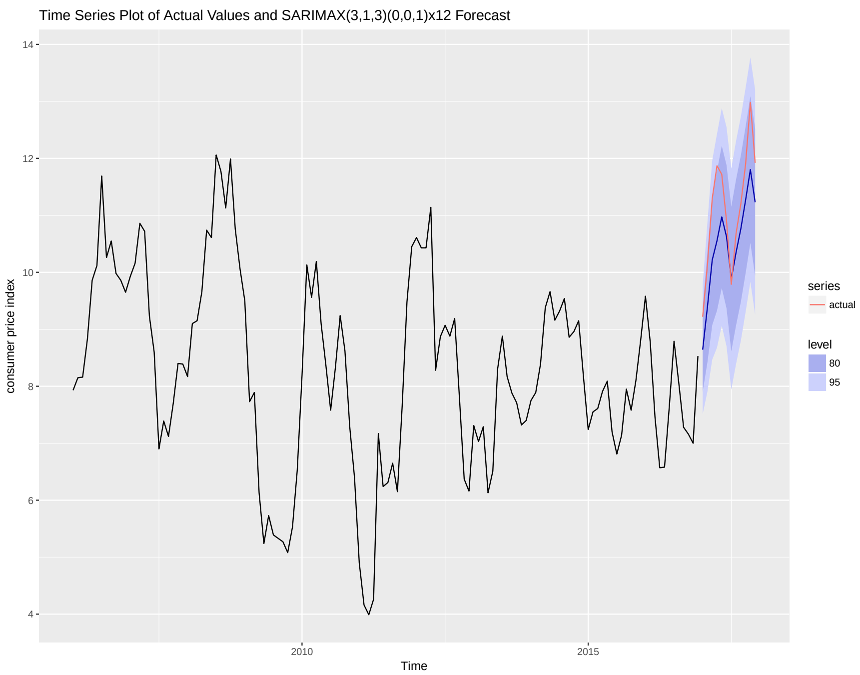

Figure 4.7: The SARIMAX forecasts of the CPI given the best train set performance

The dark line gives the actual values of the CPI between January 2006 and December 2016. The dark blue line gives the forecast values of the CPI series. The red line gives the actual values of the CPI series. The forecast plot of the second SARIMAX model shows that forecast line catches the pattern of the actual values. However, forecast values are lower than the actual values of the CPI series. Table 4.5 exhibits the forecast accuracy of the model. The forecast performance of the model is evaluated by the RMSE and the MAPE values. The RMSE value is 0.7620. The MAPE value is 5.9286. The accuracy measures of the model are low. However, the performance of the first SARIMAX model is better than the $SARIMAX(3,1,3)(0,0,1)_{12}$.

Table 4.5: The Forecast Performance of the $SARIMAX(3,1,3)(0,0,1)_{12}$

ME	RMSE	MAE	MPE	MAPE	MASE	Correlation
0.6584	0.7620	0.6738	5.7716	5.9286	0.2794	0.9317

When the diagnostics of the models were tested, For $SARIMAX(3,1,3)(0,0,1)_{12}$ and $SARIMAX(4,1,4)(2,0,1)_{12}$, the p values of the Jarque Bera test were 0.9422 and 0.8342, respectively. We cannot reject the null hypothesis which is the normality of the residuals. These results showed that the errors of the models are normally distributed. For $SARIMAX(3,1,3)(0,0,1)_{12}$ and $SARIMAX(4,1,4)(2,0,1)_{12}$, the p values of the Ljung Box test were 0.7256 and 0.9993, respectively. We cannot reject the null hypothesis which is "no serial correlation". These results showed that there is no serial correlation in both SARIMAX models. For $SARIMAX(3,1,3)(0,0,1)_{12}$ and $SARIMAX(4,1,4)(2,0,1)_{12}$, the p values of the ARCH test were 0.0009 and 0.0305, respectively so we can reject the null hypothesis which is "constant error variance". These results indicated that there exists heteroscedasticity in both SARIMAX models. The variation in errors can be modelled by GARCH type model but it only affects the prediction interval of the forecasts, which is not the aim of the study. Lastly, the Pearson Correlation showed that there exists no multicollinearity.

4.3.2 Additive Holt Winters with Additive Error

The second method to produce 12-month ahead forecasts of the Turkish CPI is additive Holt Winters method with additive errors. We used *ets()* function in *forecast* package in RStudio to build the model. In the additive Holt Winters with additive error model building, the initial values of the parameters were automatically calculated in *ets()* command in RStudio. Then, the smoothing parameters (α, β, γ) were incremented until having the lowest mean squared error (MSE). The smoothing parameters of the final model for 12-month ahead forecasts of the Turkish CPI are given in Table 4.6:

Table 4.6: Smoothing Parameters of the HW Model

Alpha	Beta	Gamma
0.9991	0,0067	0.0007

The alpha value is close to 1. This means that recent observations are assigned higher weight than the past observations. In the fact that the beta value is close to 0 means that the slope between the past consecutive observations is weighted higher than the slope between the recent consecutive observations. Because the gamma is close to 0, the past seasonal offsets have higher weights.

The coefficient of the model components are given in Table 4.7.

Table 4.7: Coefficients of the HW Model

Component	Level	Trend	Season.1	Season.2	Season.3
Coefficient	0.4106	-0.0328	-0.2799	-0.2439	-0.1359
Component	Season.4	Season.5	Season.6	Season.7	Season.8
Coefficient	0.0911	0.1513	0.222	0.2223	0.0777
Component	Season.9	Season.10	Season.11	Season.12	
Coefficient	0.1015	0.0341	-0.1156	-0.1247	

Because the seasonal period of the model is 12, 12 seasonal components were calculated. The level term is equal to 0.4106 and trend term is equivalent to -0.0328. Then, the mathematical form of the *h*-step ahead forecast is calculated as:

$$\hat{y}_h = 0.4106 - 0.0328 * h + s_h \tag{4.3}$$

where *h=1,2,...,12*. As it is understood from the mathematical model, the time period negatively affects the forecast values of the Turkish CPI. The seasonal components from April to November positively influence the forecast values of the Turkish CPI. The forecast plot of the Holt Winters Method is illustrated in Figure 4.8.

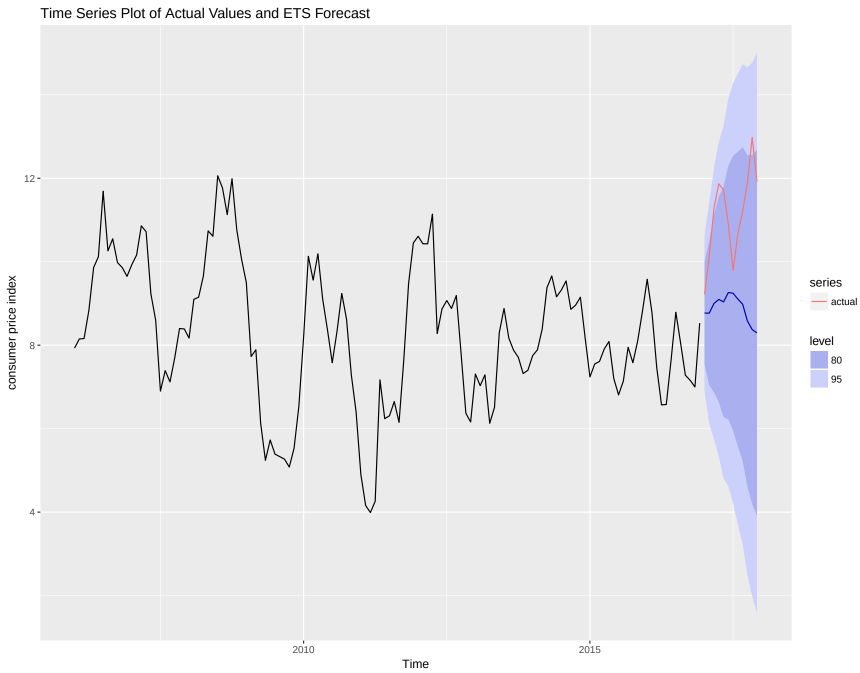

Figure 4.8: The Holt Winters forecasts of the CPI

Like SARIMAX model, the dark line gives the actual values of the CPI between January 2006 and December 2016, the dark blue line gives the forecast values of the CPI series and the red line gives the actual values of the CPI series. The forecast plot shows that the forecast values of the CPI are far away from the actual CPI values. The actual and forecast values of the CPI series increase until the May 2017, but the actual values increase much more than the forecasts. After May 2017, both the actual and forecast values are decreasing. The origin CPI values again increase after July 2017, but the forecast values continue to decrease. In addition, The Holt Winters forecasts do not catch the seasonal behaviour of the CPI series. The forecast performance of the Holt Winters model is displayed in Table 4.8.

Table 4.8: The Forecast Performance of the Holt Winters Method

ME	RMSE	MAE	MPE	MAPE	MASE	Correlation
2.2566	2.5482	2.2566	19.4790	19.4790	0.9357	-0.4562

The RMSE value of the HW model is 2.5482 which is high. The MAPE value is 19.4790 which is quite high. These accuracy measures show that the predictive performance of the Holt Winters model is low for forecasting the Turkish CPI.

4.3.3 TBATS Forecast with Regressors

In the TBATS model building, regression with ARIMA errors was implemented to add useful covariates because the results of the both model are nearly similar. The Fourier terms are used as additional covariates when the multiple seasonal period exists in data. The number of Fourier terms and the order of the ARIMA model were selected by minimizing the AIC. The best fitted model was obtained according to the AIC. Then, 12-month ahead forecasts of the Turkish CPI were obtained from the best fitted model. The forecast plot is illustrated in Figure 4.9.

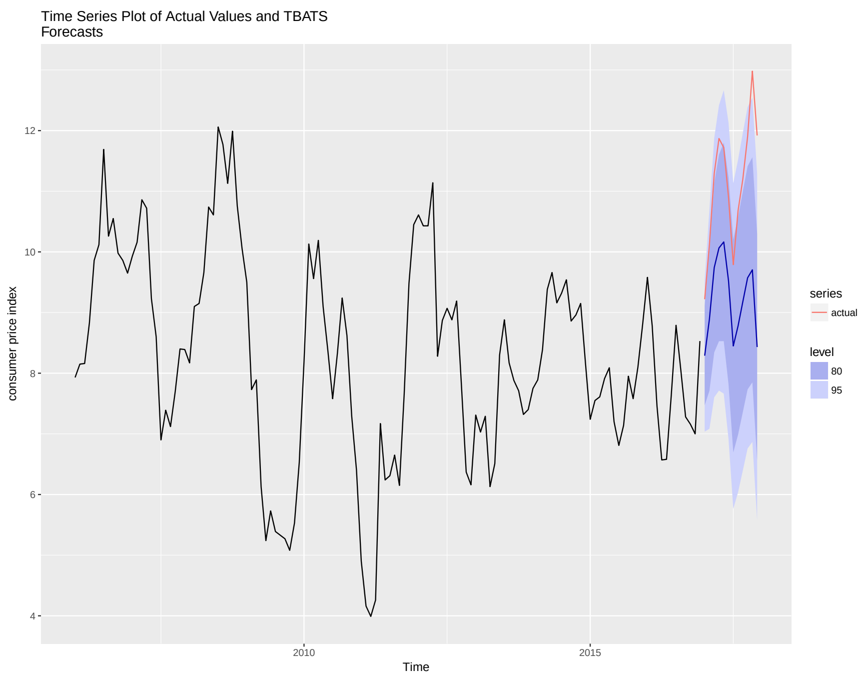

Figure 4.9: The TBATS forecasts of the CPI

The actual values of the Turkish CPI series is higher than the forecast values. The forecast values of the CPI series have the same pattern with the original values of the CPI series. Both actual and forecast values of the CPI have similar seasonal behaviour. That is, both actual and forecast values increase until May 2017, then both decrease until July 2017 and again rise until September 2017 and finally go down in December 2017. The fluctuation of the actual CPI values are larger than the fluctuation of the forecast values of the CPI series. To decide if the model produces accurate forecasts or not, it requires to look forecast performance. Table 4.9 gives the accuracy measures of the model. When we look the RMSE and MAPE, the RMSE

Table 4.9: The Forecast Performance of the TBATS Method

ME	RMSE	MAE	MPE	MAPE	MASE	Correlation
1.9005	2.0448	1.9005	16.7265	16.7265	0.7880	0.6674

is 2.0448 and the MAPE is 16.7265. The RMSE and MAPE of the model are lower than the RMSE and MAPE of the additive HW model with additive error. However, the performance of the TBATS model is lower when we compare the TBATS and SARIMAX model.

4.3.4 Neural Network Autoregressive (NNAR) Forecast

In the forecasting procedure of the Turkish CPI, the Neural Network Autoregressive (NNAR) model was implemented. In this approach, we used *forecast* package in RStudio for NNAR model building. As mentioned in the Chapter 3, there exist three layers which are input, hidden and output in the neural network model. The NNAR model in RStudio is based on the feed-forward neural networks with a single hidden layer. The lagged inputs and the exogeneous variables in the input layer were used for forecasting the Turkish CPI data. After applying nnetar() function in R, the appropriate model was automatically found and the model is NNAR(4,2,7)[12]. The model parameters are 4, 2 and 7. The number of the non-seasonal lags used as inputs is 4 which is AR order, the number of the seasonal lags used as inputs is 2 which is SAR order and the number of the nodes in the hidden layer is 7. In addition, we add 6 exogeneous variables as the inputs. These exogeneous variables are the current value of the PPI, the first lag of the PPI, the first lag of the deposit interest, the second lag of the deposit interest, the current of the export unit values and the fifth lag of the import unit values.

The network stucture of the model is "12-7-1". 12 gives the number of the inputs which are composed of the nonseasonal lags, the seasonal lags and exogeneous variables. 7 is the number of the nodes in the hidden layer. The number of the hidden nodes is equal to the half of the number of the input nodes plus 1 which is the output node. The random starting weights were automatically assigned in *nnetar()* function. Because the weights are assigned randomly, the different forecast values are obtained for every NNAR model building process. Hence, after we repeated the model function in 30 40 times, we chose the best model which had the least RMSE and MAPE values. After model building, the forecast plot of the NNAR model is demonstrated in Figure 4.10.

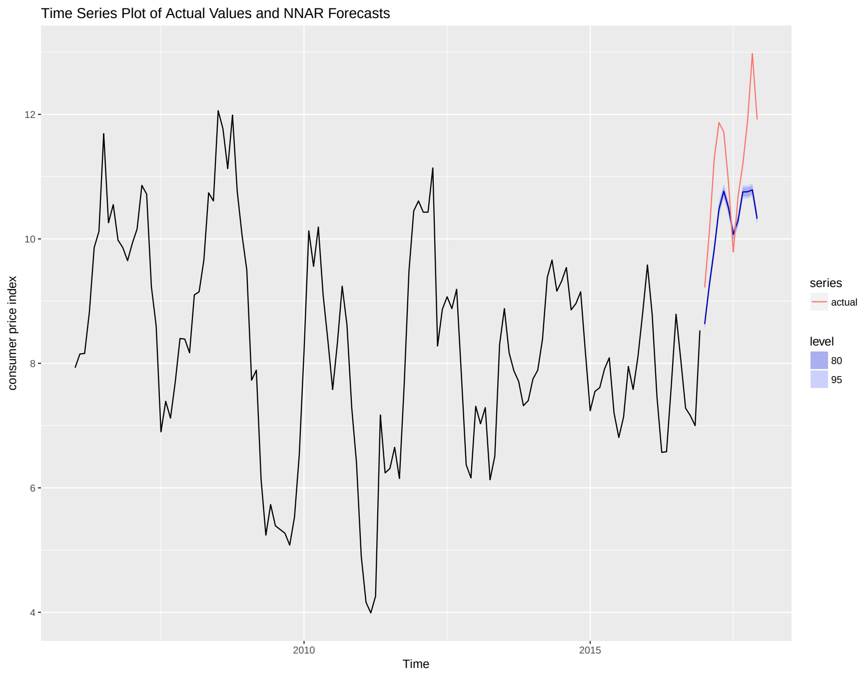

Figure 4.10: The ANN forecast of the CPI

Figure 4.10 shows that the forecast values of the Turkish CPI are approximately equal with the actual values for January and February 2017. Later, both values increase but the actual values increase further until April 2017. Later, the original values more sharply decrease and increase than the forecast values. It is understood from the forecast plot, the forecast values do not exactly catch the seasonal behaviour of the original CPI data. To prove this, we need to get accuracy measures for predictive performance of the NNAR model. Table 4.10 shows the performance of the NNAR model.

Table 4.10: The Forecast Performance of the NNAR Model

ME	RMSE	MAE	MPE	MAPE	MASE	Correlation
0.9277	1.1295	0.9751	7.9922	8.4765	0.4043	0.7850

Table 4.10 shows that the RMSE and MAPE of the NNAR model is not high. The RMSE values is 1.1295 and MAPE is 8.4765. The forecast performance of the NNAR model is lower than both SARIMAX models. However, the forecasts of the NNAR model are closer to the actual values of the Turkish CPI than forecasts of the additive HW with additive error model and the TBATS model.

Actually, it is hoped that the neural network model obtains more accurate forecasts of the CPI series. However, RStudio limits the function of the neural network. It permits applying feed-forward neural network with a single hidden layer. If we used more than one hidden layer in the neural network structure, our forecast performance could improve and the neural network model could produces more accurate forecasts of the Turkish CPI.

4.3.5 Theta Forecast

The Theta model proposed by Assimakopoulos and Nikolopoulos [3] were implemented to produce 12 month ahead forecasts of the Turkish CPI series. According to Hyndman and Billah [29], the Theta model consists of many pages of algebraic operations which are complex and confused, so they developed simpler method for Theta model and they found that the forecasts of the Theta model are equivalent to the forecasts obtained by the simple exponential smoothing (SES) with drift method. Hence, in the application of the Theta model for forecasting the Turkish CPI, the *thetaf()* function in the *forecast* package was used. The model results are equivalent to SES with drift.

The function automatically obtains 12 month ahead forecasts. The smoothing parameter; α, is 0.0001. Due to the fact that the smoothing parameter is close to 0, the distant values are given more weights than the recent values. The forecast plot of the Theta model is demonstrated in Figure 4.11.

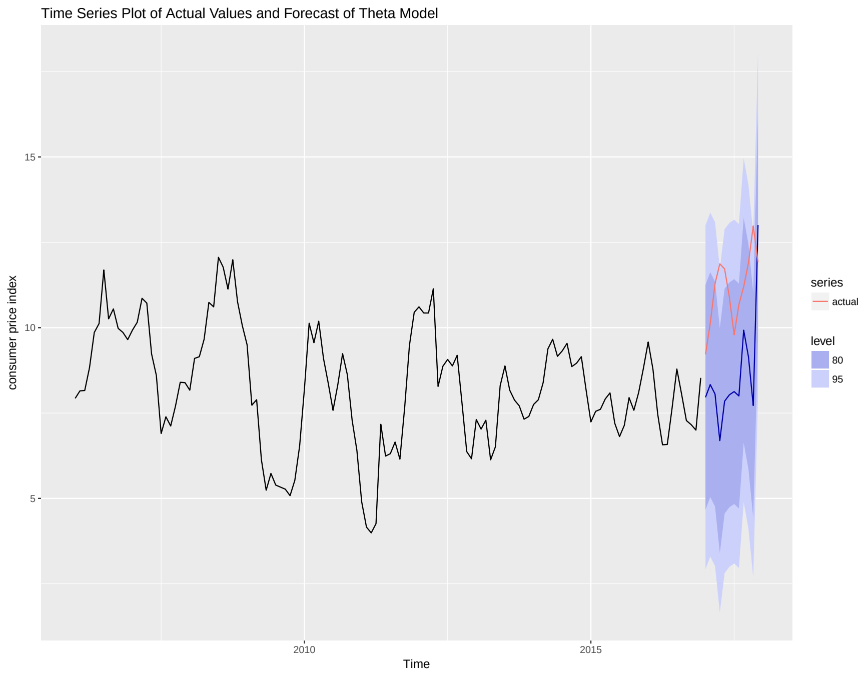

Figure 4.11: The Theta model's forecasts of the CPI

Figure 4.11 shows that the forecast values of the CPI series are blue line and the actual values of the CPI series are red line. The actual values and forecast values of the CPI data from January 2017 to December 2017 do not share similarity. While the actual CPI series increase, the forecast values of the CPI series decrease. Similarly, when the actual CPI values go down, the forecast values rise. The forecast performance of the model is described in Table 4.11.

Table 4.11: The Forecast Performance of the Theta Method

ME	RMSE	MAE	MPE	MAPE	MASE	Correlation
2.5628	3.0730	2.7440	22.5860	24.1053	1.1378	0.1586

The RMSE and MAPE values of the Theta model are 3.0730 and 24.1053, respectively. The predictive performance of the Theta model is quite low. The RMSE and MAPE values of the Theta model are higher than the SARIMAX models, additive HW with additive error model, TBATS model and NNAR model. These results showed that the Theta model is not appropriate for modelling the Turkish CPI series.

4.3.6 STL Decomposition Forecast

The final individual forecast method is the Seasonal-Trend Decomposition procedure based on LOESS (STL). We used *stats* package and *forecast* package in RStudio to construct the STL model and obtain 12 months ahead forecasts of the Turkish CPI series. In the model constructing procedure; firstly, the original values of the Turkish CPI from 2006 to 2016 were decomposed into trend, seasonal and remainder components and the seasonally adjusted series of the Turkish CPI data were obtained. After applying nonseasonal forecasting methods as either ARIMA or ETS to the seasonally adjusted series and deseasonalizing using the last year of the seasonal components, the forecast values of the STL model were obtained. In this study, we tried both models to forecast the Turkish CPI series, and ARIMA model gave better results than the ETS model. Therefore, the ARIMA model was selected for forecasting of the Turkish CPI. In addition, in the model building and forecasting procedure, we added the lagged of the exogeneous variables into model. These exogeneous variables are the current value of the PPI, the first lag of the PPI, the first lag of the deposit interest, the second lag of the deposit interest, the current of the export unit values and the fifth lag of the import unit values. The forecast plot of the STL Decomposition model is illustrated in Figure 4.12.

As it is seen from Figure 4.12, both the actual and forecast values of the Turkish CPI series increase. However, after a certain time, the forecast values of the Turkish CPI series consistently increase whereas the actual CPI values start decreasing. Then, the actual CPI values increase again. The forecast values of the CPI are lower than the original values of the CPI. In addition, the forecast values of the Turkish CPI show increasing trend. Therefore, the STL forecast method did not catch the seasonal pattern of the Turkish CPI data.

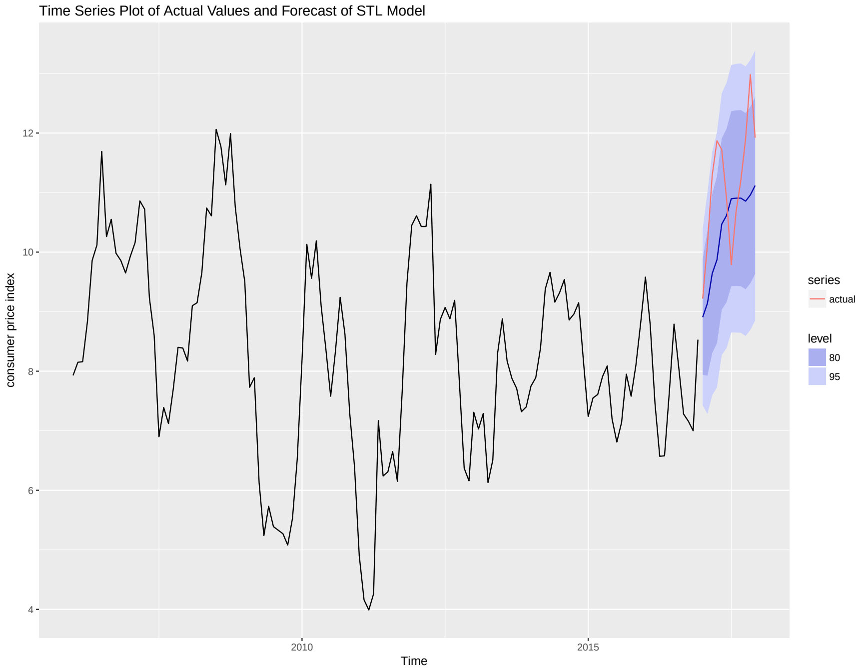

Figure 4.12: The STL forecast of the CPI

The accuracy measures of the model are shown in Table 4.12.

Table 4.12: The Forecast Performance of the STL Method

ME	RMSE	MAE	MPE	MAPE	MASE	Correlation
0.6422	1.0977	0.9172	5.3257	8.0794	0.3803	0.5315

The RMSE and MAPE of the model is 1.0977 and 8.0794, respectively. The performance of the model is better than all other individual forecast methods except the SARIMAX model.

4.4 Ensemble Model

The ensemble model is the combination of the forecasts obtained by the several forecasting methods with the optimal weights. We used the "*forecastHybrid*" package written by Shaub in RStudio to construct the ensemble forecast model[46] . The ensemble model in "*forecastHybrid*" package is composed of the auto.arima, the additive HW model with additive error, the Theta model, the NNAR model, the STL model and the TBATS model. Two methods were used to assign the weights of the ensemble model. First one is equal weight assignment. Second one is the weight based on CV error assignment. Both methods were implemented. As can be seen from Appendix A, the ensemble model with the weights based on the CV error have the lower RMSE and MAPE values and the combination of the forecasts with the weight based on the CV error provide to obtain more accurate forecasts. Thus, in this study, the weights based on the CV error were preferred.

4.4.1 Ensemble Model with auto.arima, HW, TBATS, NNAR, Theta and STL Models

In this study, in the ensemble model building, the forecasts are generated from the combination of the auto.arima model, additive HW model with additive error, the Theta model, NNAR model, STL model and the TBATS model with the weights based on the CV error. In model building, the optimal weights assigned according to the CV error of the each individual model. That is, the individual model with high CV errors is assigned lower weight in the ensemble model. In this framework, we constructed the first ensemble model which comprises of all individual methods used before. The *h*-step-ahead forecasts of the Turkish CPI are calculated by :

$$\hat{y}_h = 0.21\hat{y}_h^{auto.arima} + 0.227\hat{y}_h^{hw} + 0.012\hat{y}_h^{theta} + 0.104\hat{y}_h^{nnetar} + 0.227\hat{y}_h^{stl} + 0.22\hat{y}_h^{tbats} \tag{4.4}$$

where *h=1,2,...,12*.

$\hat{y}_h$ is the *h*-step ahead forecast of the ensemble model and $\hat{y}_h^i$ is the forecast values of the i^{th} model. As it is seen from the function, the forecasts of the exponential smoothing model and the STL model were assigned high weights than the other models. The lowest weight was assigned into the forecast of the Theta model. The forecast plot of the ensemble model is illustrated in Figure 4.13.

Figure 4.13 indicates that the forecast values predicted from the ensemble model are not similar to the actual values of the Turkish CPI series. While the actual CPI series

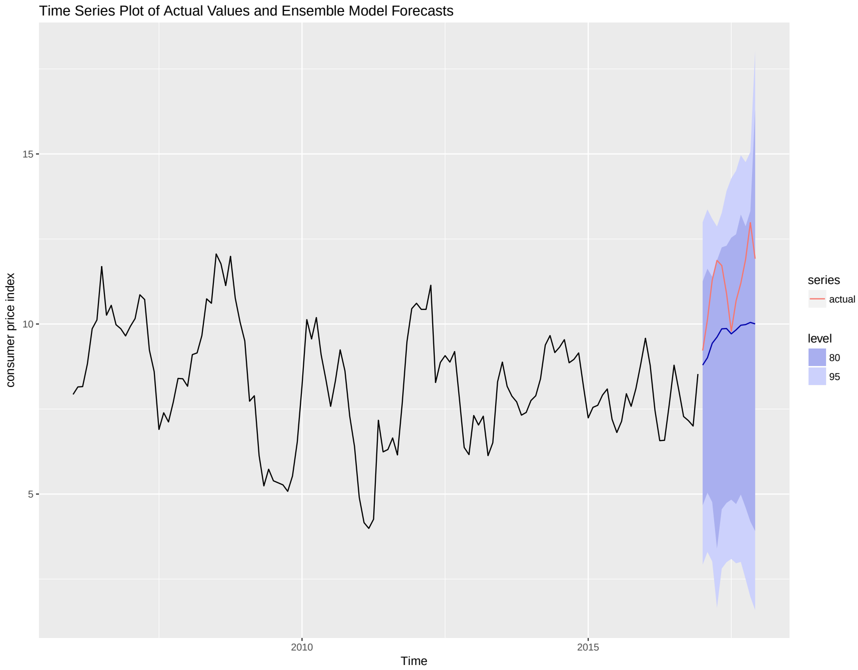

Figure 4.13: Ensemble Model of auto.arima, HW, TBATS, NNAR, Theta and STL Models with CV error

have seasonal behaviour, the forecast values of the Turkish CPI have increasing trend and no seasonal pattern. Firstly, both actual and forecast values of the Turkish CPI series increase, but actual CPI values increase a lot. Later, actual values of the Turkish CPI decrease more than the forecast values. Finally, the original values increase highly. Generally, the forecast values are quite lower than the original Turkish CPI series. The forecasts values do not catch the seasonal pattern of the CPI series.

The accuracy measures of the ensemble model are shown in Table 4.13. When the predictive performance of the ensemble model are considered, the RMSE and MAPE values are high in comparison with individual models so this result shows that the model does not give better results than the individual models.

Table 4.13: The Forecast Performance of the Ensemble Model based on CV error

ME	RMSE	MAE	MPE	MAPE	MASE	Correlation
1.4578	1.6514	1.4578	12.5964	12.5964	0.6045	0.7275

4.4.2 Ensemble Model with auto arima, Theta, NNAR, STL and TBATS

Although the forecast performance of the individual additive Holt Winters model with additive error is highly low, the HW forecasts were assigned the high weights in the ensemble model building process. Therefore, the Holt Winters forecasts were dropped from the ensemble model to enhance the performance. Then, the h-step-ahead forecasts are computed as:

$$\hat{y}_h = 0.275\hat{y}_h^{auto.arima} + 0.016\hat{y}_h^{theta} + 0.127\hat{y}_h^{nnetar} + 0.296\hat{y}_h^{stl} + 0.286\hat{y}_h^{tbats} \quad (4.5)$$

where h=*1,2,...,12*.

The forecast plot of the ensemble model is demonstrated in Figure 4.14.

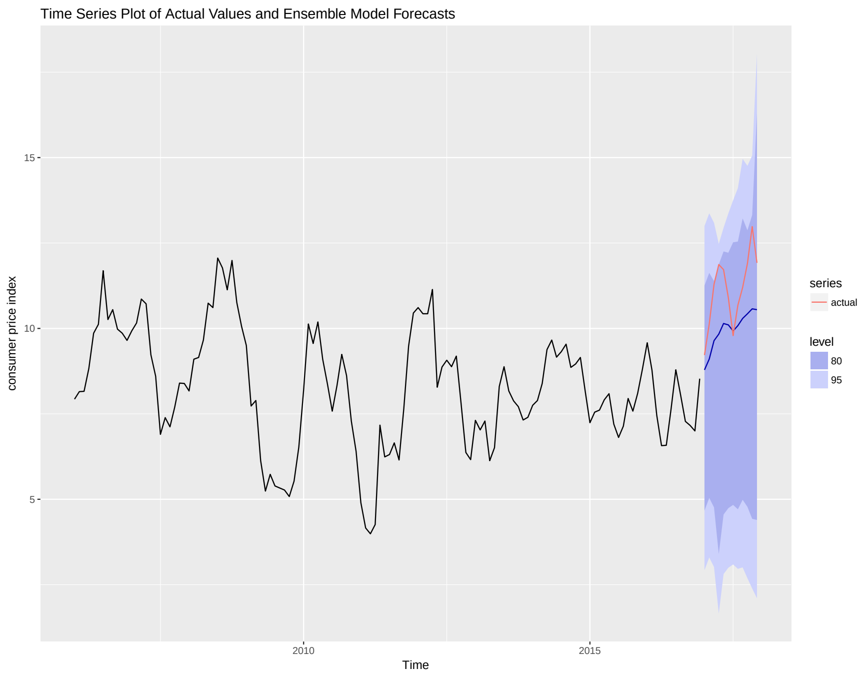

Figure 4.14: Ensemble Model of auto.arima, TBATS, NNAR, Theta and STL Models

As it is seen from Figure 4.14, there is no strong change in the forecast plot of ensemble model with auto arima, Theta, NNAR, STL and TBATS, but the forecast values of the CPI series catch the breakdown of the actual CPI values. In addition, the actual values of the Turkish CPI are higher than the forecasts of the ensemble model. The predictive performance of the ensemble model based on CV error is illustrated in Table 4.14.

Table 4.14: The Forecast Performance of the Ensemble Model based on CV error

ME	RMSE	MAE	MPE	MAPE	MASE	Correlation
1.1780	1.3619	1.2005	10.1722	10.4020	0.4978	0.7807

The RMSE and MAPE are 1.3619 and 10.4020, respectively. The forecast accuracy of the ensemble model without Holt Winters forecasts increase compared to the first ensemble model composed of auto.arima, HW, TBATS, NNAR, Theta and STL Models.

4.4.3 Ensemble Model with auto arima, Theta, NNAR and STL

In the second ensemble model, the weight of the TBATS model is quite high although the individual forecast performance of the TBATS model is highly low. Therefore, to improve the predictive performance of the ensemble model, the TBATS model was dropped from the ensemble model. The *h*-step-ahead forecasts are obtained by:

$$\hat{y}_h = 0.396\hat{y}_h^{auto.arima} + 0.023\hat{y}_h^{theta} + 0.154\hat{y}_h^{nnetar} + 0.427\hat{y}_h^{stl} \tag{4.6}$$

where *h=1,2,...,12*. The forecast plot of the third ensemble model is demonstrated in Figure 4.15.

The forecast plot shows that after dropping the HW and TBATS model, the forecast values start approaching to actual values of the Turkish CPI but the forecast values have not catched the seasonal pattern of the actual values yet. In addition, the accuracy measure of the model shows that there is enhancement for the forecasting performance of the ensemble model because of the decrease in the RMSE and MAPE values of the ensemble model.

Table 4.15: The Forecast Performance of the Ensemble Model based on CV error

ME	RMSE	MAE	MPE	MAPE	MASE	Correlation
0.7226	0.9652	0.8156	6.1896	7.1397	0.3382	0.7750

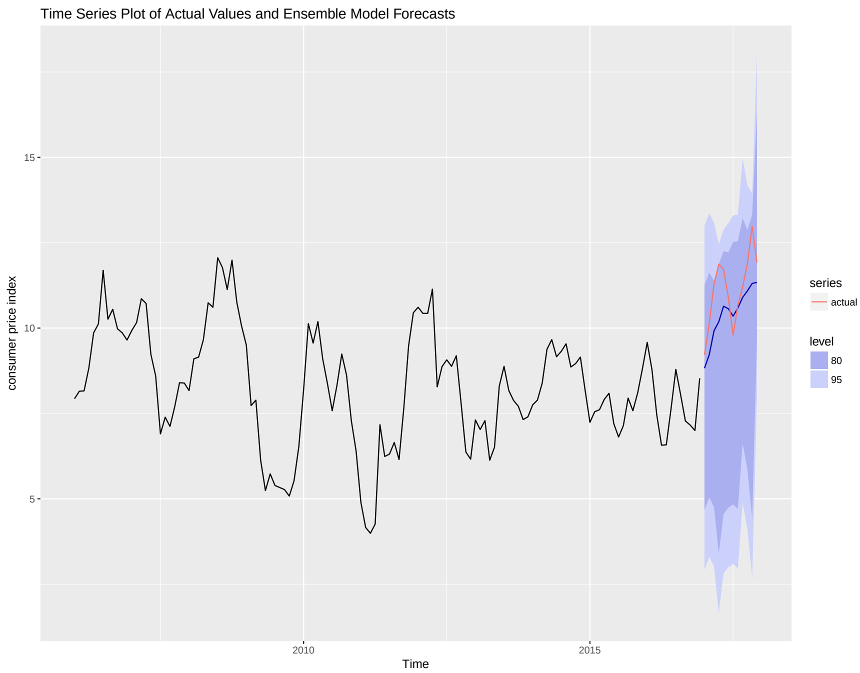

Figure 4.15: Ensemble Model of auto.arima, NNAR, Theta and STL Methods

4.4.4 Ensemble Model with auto.arima, NNAR and Theta

To improve the predictive performance of the ensemble model, we built ensemble model composed of three methods and compare the performance of all ensemble model composed of three components. The results showed that the ensemble model composed of auto.arima, Theta and NNAR models had higher accuracy. Hence, the STL method was dropped from the ensemble model. Then, the *h*-step-ahead forecasts of the ensemble model is calculated by:

$$\hat{y}_h = 0.726\hat{y}_h^{auto.arima} + 0.232\hat{y}_h^{nnetar} + 0.042\hat{y}_h^{theta}. \tag{4.7}$$

where *h=1,2,...,12*

The forecast plot of the ensemble model is demonstrated in Figure 4.16. The forecast plot of the combination of the auto.arima, Theta and NNAR models shows that the forecast values of the Turkish CPI series are close to actual values of the data. In addition, the forecast values catch the seasonal behaviour of the original series. However, the actual values more sharply go up and down than the forecast values of the CPI series.

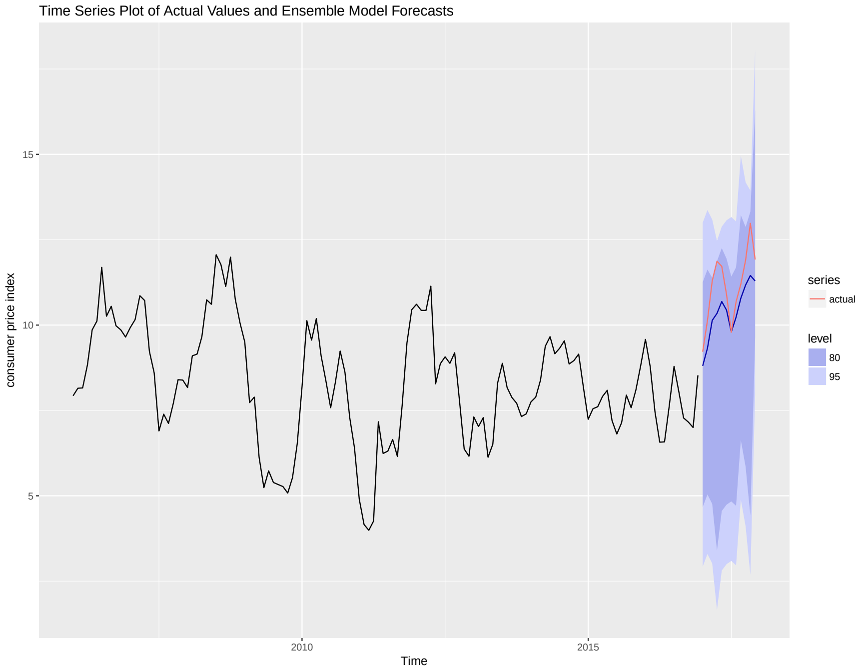

Figure 4.16: Ensemble Model of auto.arima, Theta and NNAR Methods

The forecast performance of the ensemble model is described in Table 4.16. The RMSE and MAPE values of the ensemble model decrease.

Table 4.16: The Forecast Performance of the Ensemble Model

ME	RMSE	MAE	MPE	MAPE	MASE	Correlation
0.7613	0.8849	0.7627	6.6310	6.6458	0.3163	0.9090

4.4.5 Ensemble Model with auto.arima and NNAR

Because the forecast performance of the individual Theta model was the lowest, the Theta model was dropped from the ensemble model. The final ensemble model is the combination of the auto.arima model and NNAR model. The model weights are given in the formula of the *h*-step-ahead forecasts:

$$\hat{y}_h = 0.723\hat{y}_h^{auto.arima} + 0.277\hat{y}_h^{nnetar}. \tag{4.8}$$

where *h=1,2,...,12*

The weights of the auto.arima forecasts are higher than the NNAR model. The final ensemble model forecasts are demonstrated in Figure 4.17.

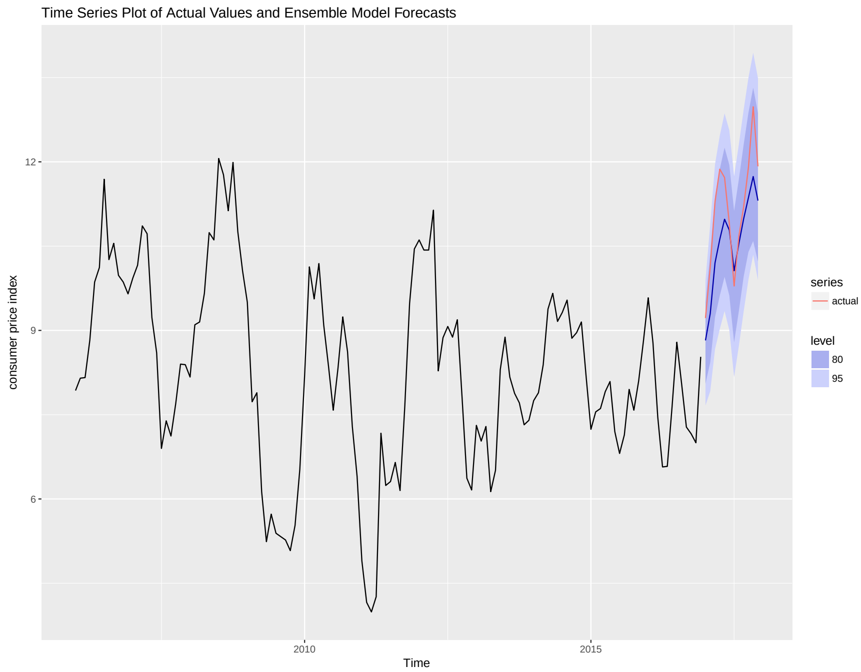

Figure 4.17: Ensemble Model of auto.arima and NNAR Methods

Forecast plot of the final ensemble model indicates that the forecast values of the CPI series are closer to the actual values of the CPI series, but actual CPI series are higher than the forecast values of the CPI series. In addition, the forecast values of the CPI series catch the seasonal behaviour of the actual CPI series.

Table 4.17: The Forecast Performance of the Ensemble Model based on CV error

ME	RMSE	MAE	MPE	MAPE	MASE	Correlation
0.5740	0.7356	0.6192	4.9706	5.4323	0.2567	0.8947

The forecast accuracy of the final ensemble model is evaluated in Table 4.17. The final ensemble model has lower RMSE and MAPE. That means that the combination of auto.arima and NNAR forecasts obtain more accurate forecasts in comparison with the other ensemble models.

4.5 Assessment of Forecast Performances of All Models

Table 4.18 gives out of sample forecast performances of both the individual models and the ensemble models. Table 4.18 shows that the $SARIMAX(4,1,4)(2,0,1)_{12}$ model has the lowest RMSE and MAPE so the best model for 12 month ahead forecasts of the Turkish CPI is the $SARIMAX(4,1,4)(2,0,1)_{12}$. The second best model is final ensemble model composed of auto.arima and NNAR forecasts. The $SARIMAX(3,1,3)(0,0,1)_{12}$ model is the third best model for 12 month ahead forecasts of the Turkish CPI. The worst forecast models are Holt Winters and Theta model due to having the highest MAPE and the RMSE. Our findings show that the ensemble model improves a bit the forecast performance of the individual models. That is, the combination of auto.arima and NNAR forecasts produce more reliable forecasts than the individual models except for the $SARIMAX(4,1,4)(2,0,1)_{12}$ models. It is concluded that the combination of the several models do not always give the best forecasts, the individual models may have high predictive performance.

Table 4.18: The RMSEs and MAPEs for All Models

MODEL	RMSE	MAPE
SARIMAX$(4,1,4)(2,0,1)_{12}$	**0.4566**	**3.6731**
SARIMAX$(3,1,3)(0,0,1)_{12}$	0.7620	5.9286
HW with additive error	2.5482	19.4790
TBATS	2.0448	16.7265
NNAR (4,2,1)[12]	1.1295	8.4765
Theta	3.0730	24.1052
STL Decomposition	1.0977	8.0794
Ensemble 1 (auto.arima, hw, nnar, stl, tbats, theta)	1.6514	12.5964
Ensemble 2 (auto.arima, nnar, stl, tbats, theta)	1.3619	10.4020
Ensemble 3 (auto.arima, nnar, stl, theta)	0.9652	7.1397
Ensemble 4 (auto.arima, nnar, theta)	0.8849	6.6458
Ensemble 5 (auto.arima, nnar)	0.7356	5.4323

CHAPTER 5

CONCLUSION AND FURTHER RESEARCHES

The inflation rate is simply identified as a rise in the general level of the prices. The inflation rate is one of the most vital element for the governments as high inflation may lead the uncertainity which affects the economical activities in the countries. That is, investors do not want to invest, and they save their money. Therefore, forecasting inflation is so crucial factor that many governmental and economical decision makers and policymakers conduct their system and take precautions according to the reliable inflation forecasts. Though many forecasting approaches are implemented, obtaining best forecasts of inflation rate is challenging task because the inflation series may have nonstationarity, irregular fluctuation, multicollinearity and seasonality, and also there exist many external factors which influence the inflation rate. One of the methods for forecasting inflation is the ensemble model which merges the results of the several forecast techniques.

There are numerious studies on forecasting inflation, coping with aforementioned issues. To forecast inflation, many methods are implemented in the literature. Some of those are the Phillips curve, ARIMA model, seasonal ARIMA model, vector autoregressive model, the factor models and combination models. Some of them show that ensemble model produces more reliable forecasts, the others find that the individual models have higher forecast accuracy.

In this study, seven forecasting techniques were methodologically introduced. These methods, SARIMA with exogeneous variables, Holt Winters Exponential Smoothing, TBATS model, ANN, Theta model, STL and the ensemble model. Then, RMSE and

MAPE were identified to evaluate the forecast performance.

In computational analysis, we performed seven forecasting techniques in R-Studio with version 1.1.383 for forecasting Turkish inflation rate. Since the Turkish inflation rate is measured by the Turkish CPI, we apply modelling of the Turkish monthly CPI in order to predict the future values of the inflation. In addition, six exogeneous variables which were Turkish monthly producer price index, unemployment rate, deposit interest, reel effective exchange rate, import unit value index and export unit value index were used for the forecasting procedure. Firstly, all individual methods were implemented. Then, all individual forecasts were combined by giving weights based on their CV performance. Our findings showed that $SARIMAX(4,1,4)(2,0,1)_{12}$ model has the lowest RMSE and MAPE. The second best model for forecasting the inflation rate is the ensemble model composed of auto.arima and NNAR forecasts. The Theta model and Holt Winters model have the lowest forecast performance. Although the ensemble model composed of auto.arima and NNAR has high accuracy, $SARIMAX(4,1,4)(2,0,1)_{12}$ model more accurately predicts future values of Turkish CPI compared to the ensemble models.

By getting this thesis as an initial point in forecasting Turkish inflation, further researches can improve ensemble modelling of the monthly Turkish CPI by making some simulation study to predict the future values of the CPI. To see the overall performance of ensemble models and under which condition they are superior, detailed simulation study is needed. In addition, some machine learning methods such as the Boosting, the SVR can be implemented and their results can be compared with traditional time series model according to the forecast performance of the Turkish CPI data.

REFERENCES

[1] R. Adhikari, G. Verma, and I. Khandelwal. A model ranking based selective ensemble approach for time series forecasting. *Procedia Computer Science*, 48:14–21, 2015.

[2] A. Ang, G. Bekaert, and M. Wei. Do macro variables, asset markets, or surveys forecast inflation better? *Journal of Monetary Economics*, 54(4):1163–1212, 2007.

[3] V. Assimakopoulos and K. Nikolopoulos. The Theta model: A decomposition approach to forecasting. *International Journal of Forecasting*, 16(4):521–530, 2000.

[4] J. M. Bates and C. W. Granger. The combination of forecasts. *Journal of the Operational Research Society*, 20(4):451–468, 1969.

[5] J. M. Binner, R. K. Bissoondeeal, T. Elger, A. M. Gazely, and A. W. Mullineux. A comparison of linear forecasting models and neural networks: An application to Euro Inflation and Euro Divisia. *Applied Economics*, 37(6):665–680, 2005.

[6] G. E. Box and G. M. Jenkins. *Time Series Analysis: Forecasting and Control*. Holden-Day, San Francisco, 1976.

[7] P. L. Brockett, W. W. Cooper, L. L. Golden, and U. Pitaktong. A neural network method for obtaining an early warning of insurer insolvency. *Journal of Risk and Insurance*, pages 402–424, 1994.

[8] R. G. Brown. *Statistical Forecasting for Inventory*. McGraw-Hill, New York, 1959.

[9] T. C. Carlo and E. F. Marçal. Forecasting Brazilian inflation by its aggregate

and disaggregated data: a test of predictive power by forecast horizon. *Applied Economics*, 48(50):4846–4860, 2016.

[10] A. N. Catik and M. Karaçuka. A comparative analysis of alternative univariate time series models in forecasting turkish inflation. *Journal of Business Economics and Management*, 13(2):275–293, 2012.

[11] S. G. Cecchetti, R. S. Chu, and C. Steindel. The unreliability of inflation indicators. *Current issues in economics and finance*, 6(4):1, 2000.

[12] C. W. Chase. *Demand-Driven Forecasting: A Structured Approach to Forecasting*. John Wiley & Sons, 2013.

[13] R. B. Cleveland, W. S. Cleveland, J. E. McRae, and I. Terpenning. STL: A seasonal-trend decomposition. *Journal of Official Statistics*, 6(1):3–73, 1990.

[14] W. S. Cleveland and S. J. Devlin. Locally weighted regression: An approach to regression analysis by local fitting. *Journal of the American Statistical Association*, 83(403):596–610, 1988.

[15] A. M. De Livera, R. J. Hyndman, and R. D. Snyder. Forecasting time series with complex seasonal patterns using exponential smoothing. *Journal of the American Statistical Association*, 106(496):1513–1527, 2011.

[16] S. Dées and J. Güntner. Forecasting Inflation Across Euro Area Countries and Sectors: A Panel VAR Approach. *Journal of Forecasting*, 36(4):431–453, 2017.

[17] G. Di Filippo. Dynamic model averaging and CPI inflation forecasts: A comparison between the euro area and the United States. *Journal of Forecasting*, 34(8):619–648, 2015.

[18] İ. Domaç. Explaining and Forecasting Inflation in Turkey. *Policy Research Working Paper*, 2004.

[19] J. L. Elman. Distributed representations, simple recurrent networks, and grammatical structure. *Machine Learning*, 7(2-3):195–225, 1991.

[20] E. Erişen. *On the parametric and nonparametric prediction methods for electricity load forecasting*. PhD thesis, METU, 2013.

[21] M. G. Garcia, M. C. Medeiros, and G. F. Vasconcelos. Real-time inflation forecasting with high-dimensional models: The case of Brazil. *International Journal of Forecasting*, 33(3):679–693, 2017.

[22] S. Gelper, R. Fried, and C. Croux. Robust forecasting with exponential and Holt-Winters smoothing. *Journal of Forecasting*, 29(3):285–300, 2010.

[23] M. Gunay. Forecasting industrial production and inflation in Turkey with factor models. Technical report, 2018.

[24] S. S. Haykin, S. S. Haykin, S. S. Haykin, and S. S. Haykin. *Neural Networks and Learning Machines*, volume 3. Pearson Upper Saddle River, NJ, USA:, 2009.

[25] M. Hibon and T. Evgeniou. To combine or not to combine: Selecting among forecasts and their combinations. *International Journal of Forecasting*, 21(1):15–24, 2005.

[26] C. Holt. Forecasting trends and seasonals by exponentially weighted averages. carnegie institute of technology. Technical report, Pittsburgh ONR memorandum, 1957.

[27] D. T. T. Huong, V. Van Truong, and B. T. Lam. Forecasting of consumer price index using the ensemble learning model with multi-objective evolutionary algorithms: Preliminary results. In *Advanced Technologies for Communications (ATC), 2015 International Conference on*, pages 337–342. IEEE, 2015.

[28] R. Hyndman, A. B. Koehler, J. K. Ord, and R. D. Snyder. *Forecasting with Exponential Smoothing: The State Space Approach*. Springer Science & Business Media, 2008.

[29] R. J. Hyndman and B. Billah. Unmasking the theta method. *International Journal of Forecasting*, 19(2):287–290, 2003.

[30] R. J. Hyndman and Y. Khandakar. Automatic time series forecasting: the forecast package for R. *Journal of Statistical Software*, 26(3):1–22, 2008.

[31] R. Klimberg, G. Sillup, K. Boyle, and V. Tavva. Forecasting performance measures - what are their practical meaning? *Advances in Business and Management Forecasting*, 7:137–147, 2010.

[32] D. Kriesel. *A Brief Introduction to Neural Networks*. 2007.

[33] N. Lewis. *Automated Time Series Forecasting Made Easy with R*, volume 1. CreateSpace Independent Publishing Platform, North Charleston, 2017.

[34] M. Lipovina-Božović, J. Cerović, and S. Vujošević. Forecasting inflation in Montenegro using univariate time series models. *Business and Economic Horizons*, 11(1):51–63, 2015.

[35] Z. Mandalinci. Forecasting inflation in emerging markets: An evaluation of alternative models. *International Journal of Forecasting*, 33(4):1082–1104, 2017.

[36] P. McAdam and P. McNelis. Forecasting inflation with thick models and neural networks. *Economic Modelling*, 22(5):848–867, 2005.

[37] S. Moshiri and N. Cameron. Neural network versus econometric models in forecasting inflation. *Journal of Forecasting*, 19(3):201–217, 2000.

[38] F. Öğünç, K. Akdoğan, S. Başer, M. G. Chadwick, D. Ertuğ, T. Hülagü, S. Kösem, M. U. Özmen, and N. Tekatlı. Short-term inflation forecasting models for Turkey and a forecast combination analysis. *Economic Modelling*, 33:312–325, 2013.

[39] M. Omane-Adjepong, F. T. Oduro, and S. D. Oduro. Determining the better approach for short-term forecasting of Ghana 's inflation: Seasonal arima vs Holt-Winters. *International Journal of Business, Humanities and Technology*, 3(1):69–79, 2013.

[40] A. Ö. Önder. Forecasting inflation in emerging markets by using the Phillips curve and alternative time series models. *Emerging Markets Finance and Trade*, 40(2):71–82, 2004.

[41] Y. Ouyang and H. Yin. Multi-step time series forecasting with an ensemble of varied length mixture models. *International Journal of Neural Systems*, 28(04):1750053, 2018.

[42] H.-T. Pao. Forecasting electricity market pricing using artificial neural networks. *Energy Conversion and Management*, 48(3):907–912, 2007.

[43] V. Plakandaras, P. Gogas, T. Papadimitriou, and R. Gupta. The informational content of the term spread in forecasting the US inflation rate: A nonlinear approach. *Journal of Forecasting*, 36(2):109–121, 2017.

[44] N. Reigl. Forecasting the Estonian rate of inflation using factor models. *Baltic Journal of Economics*, 17(2):152–189, 2017.

[45] G. Saz. The efficacy of SARIMA models for forecasting inflation rates in developing countries: The Case for Turkey. *International Research Journal of Finance and Economics*, 62, 04 2011.

[46] D. Shaub and P. Ellis. *forecastHybrid: Convenient Functions for Ensemble Time Series Forecasts*, 2018. R package version 2.2.12.

[47] B. Soybilgen and E. Yazgan. An evaluation of inflation expectations in Turkey. *Central Bank Review*, 17(1):31–38, 2017.

[48] J. H. Stock and M. W. Watson. Forecasting inflation. *Journal of Monetary Economics*, 44(2):293–335, 1999.

[49] M. Stone. Cross-validatory choice and assessment of statistical predictions. *Journal of the Royal Statistical Society. Series B (Methodological)*, 36(2):111–147, 1974.

[50] J. W. Taylor. Exponentially weighted methods for forecasting intraday time series with multiple seasonal cycles. *International Journal of Forecasting*, 26(4):627–646, 2010.

[51] N. Terui and H. K. Van Dijk. Combined forecasts from linear and nonlinear time series models. *International Journal of Forecasting*, 18(3):421–438, 2002.

[52] M. Theodosiou. Disaggregation & aggregation of time series components: A hybrid forecasting approach using generalized regression neural networks and the theta method. *Neurocomputing*, 74(6):896–905, 2011.

[53] W. William and S. Wei. *Time series analysis: Univariate and multivariate methods*. Pearson Addison Wesley, Segunda edicion. Cap, USA, 2006.

[54] P. R. Winters. Forecasting sales by exponentially weighted moving averages. *Management science*, 6(3):324–342, 1960.

[55] J. H. Wright. Forecasting US inflation by bayesian model averaging. *Journal of Forecasting*, 28(2):131–144, 2009.

APPENDIX A

APPENDIX

A.1 Decomposition of the Turkish CPI

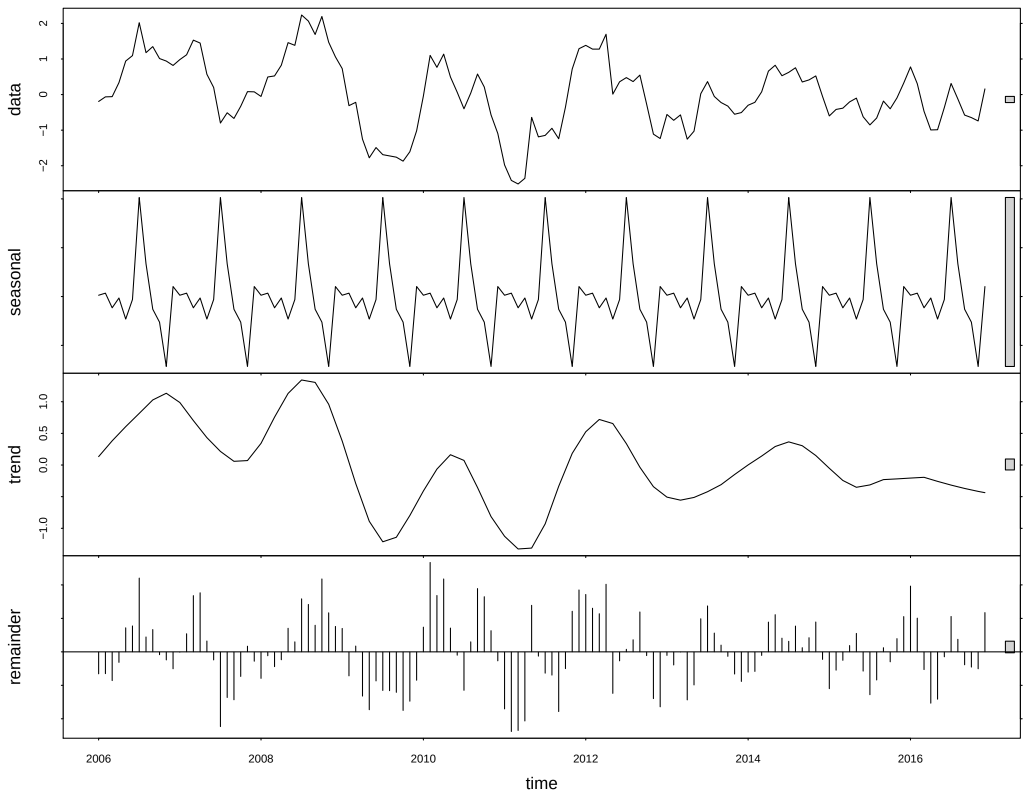

Figure A.1: STL Decomposition of the Turkish CPI

A.2 Time series plots of Exogeneous Variables

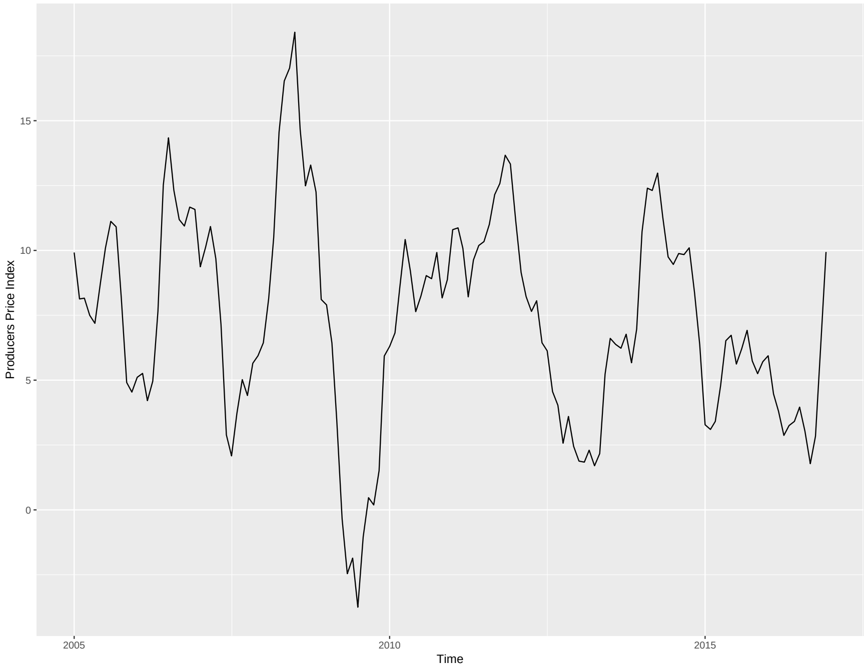

Figure A.2: Time Series plot of Producer Price Index

Figure A.3: Time Series Plot of Unemployment rate

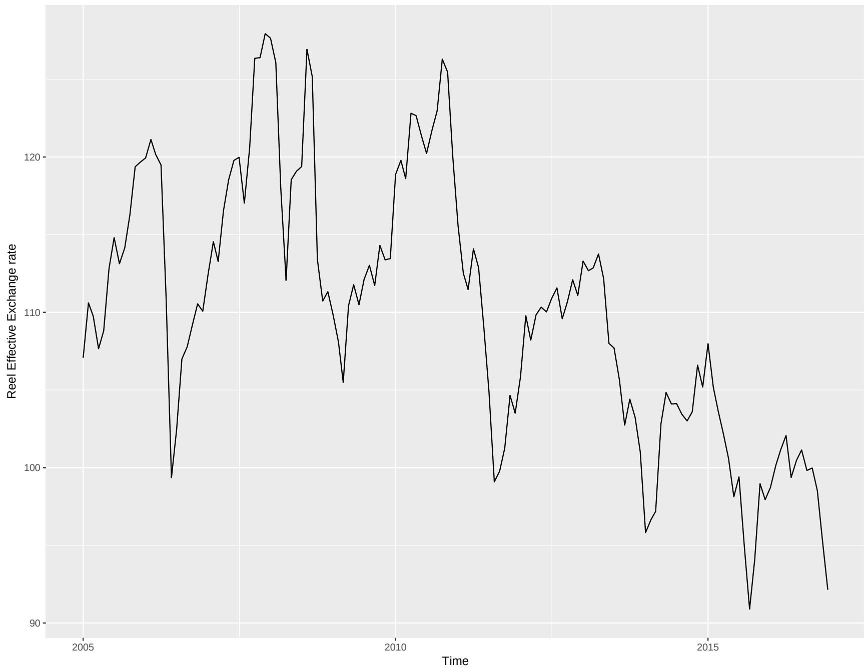

Figure A.4: Time Series Plot of Reel Effective Exchange Rate

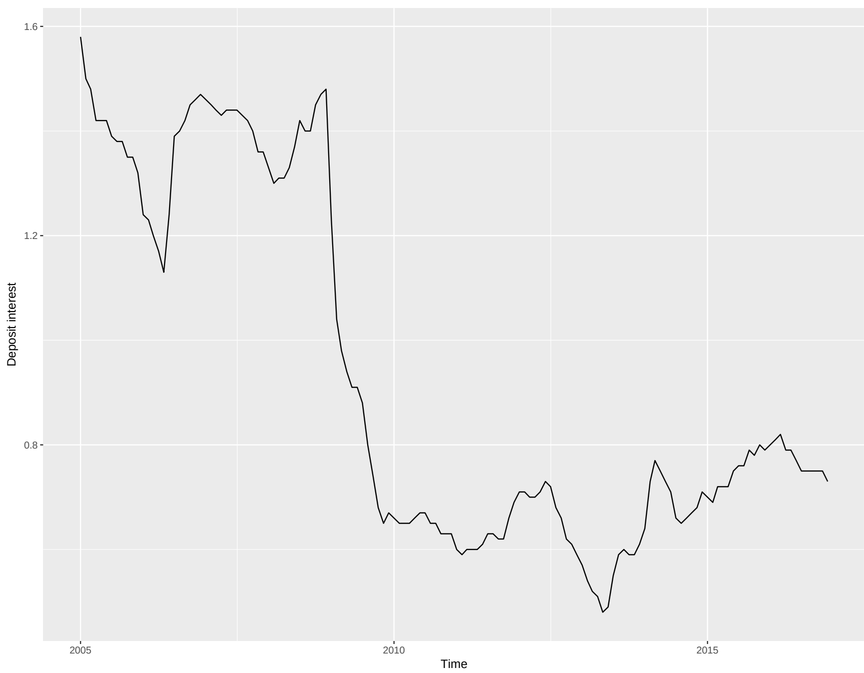

Figure A.5: Time Series Plot of Deposit Interest

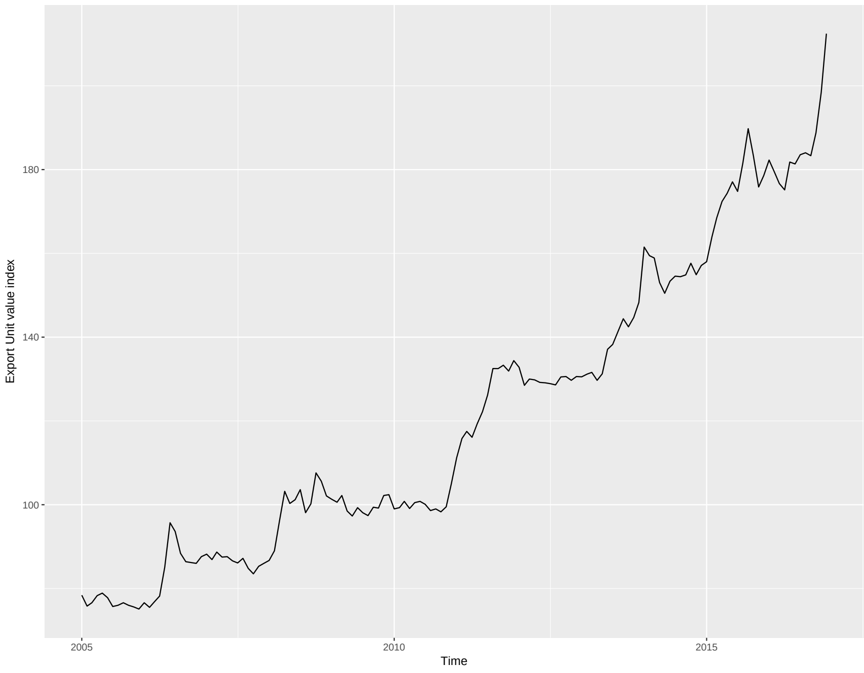

Figure A.6: Time Series Plot of Export Unit Value Index

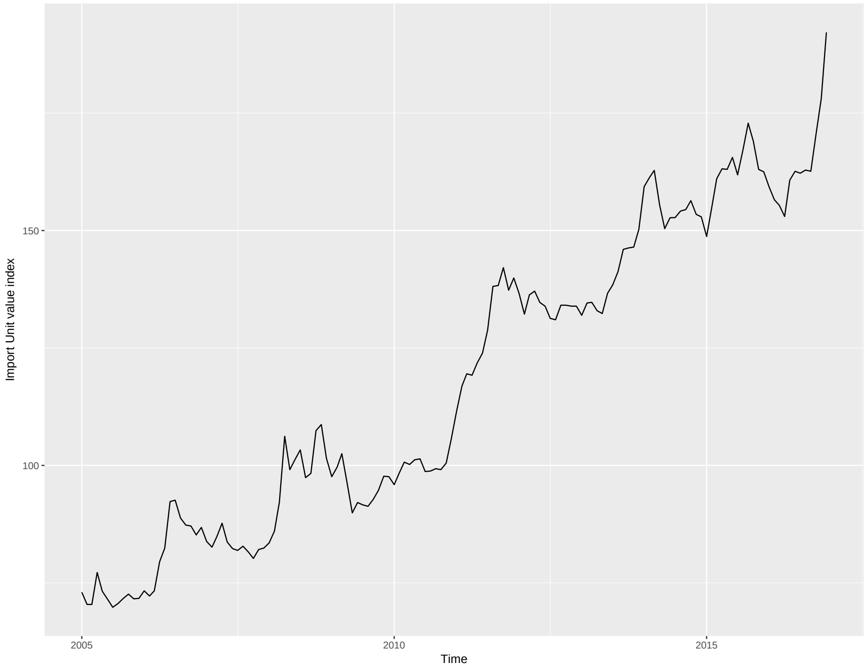

Figure A.7: Time Series Plot of Import Unit Value Index

A.3 SARIMAX Model with All Parameters and Exogeneous Variables

Table A.1: The Coefficient of the $SARIMAX(4,1,4)(2,0,1)_{12}$

Component	AR1	AR2	AR3	AR4	MA1
Coefficient S.E	-0.0672 0.0290	-0.4926 0.0160	0.4895 NaN	-0.0271 0.0081	-0.0567 0.0190
Component	MA2	MA3	MA4	SAR1	SAR2
Coefficient S.E	0.2660 0.0363	-0.8417 0.0580	-0.3662 0.0197	-0.5989 0.0129	-0.2875 0.0244
Component	SMA1	PPI.0	PPI.1	PPI.9	PPI.10
Coefficient S.E	-0.3313 0.0217	0.3855 0.0996	0.0733 0.0927	0.0310 0.0884	-0.1095 0.0874
Component	UNEM.7	EXC.9	INT.0	INT.1	INT.2
Coefficient S.E	-0.2097 0.0720	-0.1519 0.1085	-0.0633 0.2024	0.0855 0.2553	0.2789 0.2130
Component	EXP.0	EXP.9	IMP.0	IMP.5	IMP.9
Coefficient S.E	0.5023 0.2993	-0.2353 0.3882	-0.0578 0.2919	-0.1543 0.2032	-0.1183 0.3658

A.4 The forecast plot of SARIMAX Model with All Parameters and Exogeneous Variables

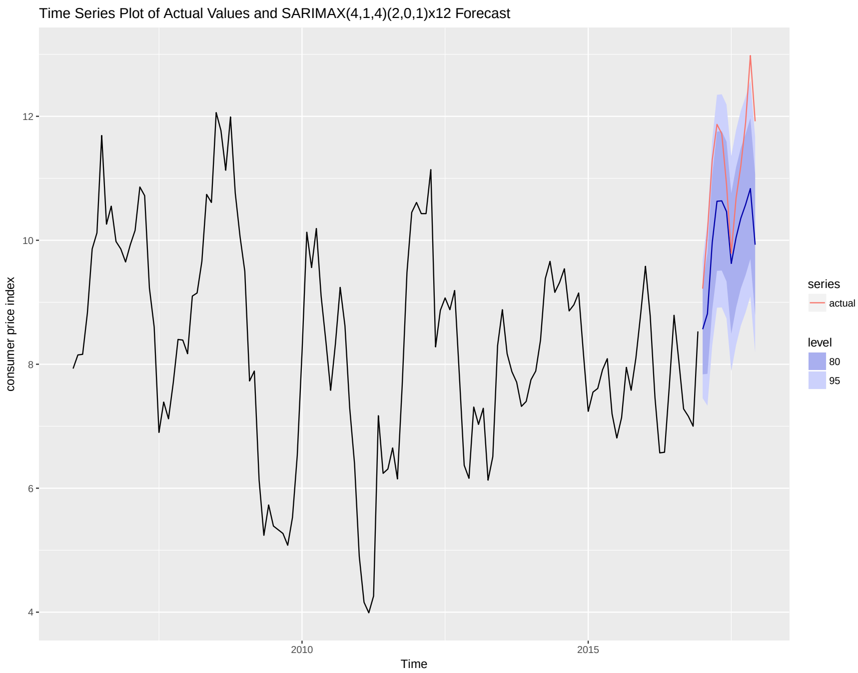

Figure A.8: Forecast Plot of $SARIMAX(4, 1, 4)(2, 0, 1)_{12}$ with all exogeneous variables

A.5 Assessment of Forecast Performance of Ensemble model based on CV error and Ensemble Model with Equal Weight

Table A.2: The RMSEs and MAPEs for Ensemble Model based on CV Error

MODEL	CV ERROR	
	RMSE	MAPE
Ensemble 1 (auto.arima, hw, nnar, stl, tbats, theta)	1.6514	12.5964
Ensemble 2 (auto.arima, nnar, stl, tbats, theta)	1.3619	10.4020
Ensemble 3 (auto.arima, nnar, stl, theta)	0.9652	7.1397
Ensemble 4 (auto.arima, nnar, theta)	0.8849	6.6458
Ensemble 5 (auto.arima, nnar)	0.7356	5.4323

Table A.3: The RMSEs and MAPEs for Ensemble Model with Equal Weight

MODEL	SIMPLE AVERAGE	
	RMSE	MAPE
Ensemble 1 (auto.arima, hw, nnar, stl, tbats, theta)	1.8744	14.7285
Ensemble 2 (auto.arima, nnar, stl, tbats, theta)	1.7280	13.3348
Ensemble 3 (auto.arima, nnar, stl, theta)	1.5411	11.5839
Ensemble 4 (auto.arima, nnar, theta)	1.5887	12.0546
Ensemble 5 (auto.arima, nnar)	1.0878	8.2288