DEVELOPING EIGHTH GRADE STUDENTS' MATHEMATICAL PRACTICES IN SOLIDS THROUGH ARGUMENTATION: A DESIGN-BASED STUDY

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Prof. Dr. Tülin Gençöz
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Doctor of Philosophy.

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Assoc. Prof. Dr. Elvan Şahin
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Doctor of Philosophy.

____

Assoc. Prof. Dr. Didem Akyüz
Supervisor

Examining Committee Members

Prof. Dr. Erdinç Çakıroğlu	(METU, MSE)	_______________
Assoc. Prof. Dr. Didem Akyüz	(METU, MSE)	_______________
Assoc. Prof. Dr. Çiğdem Haser	(METU, MSE)	_______________
Assist. Prof. Dr. Zeynep Sonay Ay	(Hacettepe Uni., MFE)	_______________
Assist. Prof. Dr. Mesture Kayhan Altay	(Hacettepe Uni., MFE)	_______________

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Name, Last name: Şule, Şahin Doğruer

Signature:

ABSTRACT

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Şahin Doğruer, Şule
Ph.D., Department of Elementary Education
Supervisor: Assoc. Prof. Dr. Didem Akyüz

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The purpose of this study was to obtain classroom mathematical practices of eighth graders' during the concept of solids and to test the effectiveness of this content in an eighth-grade mathematics classroom. In this respect, an instructional sequence was used with guidance of a hypothetical learning trajectory. The context was basic elements of prisms, their surface area, basic elements of cylinder, its surface area and its volume. The process continued through four and half weeks. Argumentations, dynamic geometry software and daily life examples supported the classroom activities. Pretest and posttest were applied to the students to obtain the development of students' understanding in related context.

The classroom mathematical practices were obtained and analyzed by using emergent perspective as a theoretical framework. This view asserts learning occurs with combination of individual working and social aspects of environment. Using Krummheuer's argumentation model which focus on taken-as-shared ideas, the mathematical practices were interpreted. Four mathematical practices were obtained as: (a) finding definition and properties of prisms, (b) finding surface area of prisms, (c) finding surface area of cylinder and (d) finding volume of cylinder. The results indicated that students' understanding of three-dimensional solids improved with support of argumentations and dynamic geometry software.

Key Words: Design-based research, Solids, Argumentation, Classroom mathematical practices, Hypothetical learning trajectory.

ÖZ

SEKİZİNCİ SINIF ÖĞRENCİLERİNİN SINIF TARTIŞMALARI KULLANILARAK KATI CİSİMLER KONUSUNDA MATEMATİK UYGULAMALARININ GELİŞTİRİLMESİ: TASARIM TABANLI BİR ÇALIŞMA

Şahin Doğruer, Şule
Doktora, İlköğretim Bölümü
Tez Yöneticisi: Doç. Dr. Didem Akyüz
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Bu çalışmanın amacı, katı cisimler konusu kapsamında sekizinci sınıf öğrencilerinin matematik uygulamalarını saptamak ve bu içeriğin etkililiğini sekizinci sınıf matematik dersinde test etmektir. Bu bağlamda, bir varsayıma dayalı öğrenme yörüngesinin rehberliği ile bir öğretim dizisi kullanılmıştır. Konu olarak prizmanın temel elemanları, yüzey alanı, silindirin temel elemanları, silindirin yüzey alanı ve hacmi belirlenmiştir. Süreç dört buçuk hafta boyunca devam etmiştir. Sınıf içi tartışmalar, dinamik geometri yazılımı ve günlük yaşam örnekleri sınıf etkinliklerini destekledi. Öğrencilerin ilgili bağlamdaki anlayışlarını geliştirmek için öğrencilere ön test ve son test uygulanmıştır.

Sınıf matematiksel uygulamalarını tespit etmek için teorik bir çerçeve olarak ortaya çıkan perspektif kullanılmıştır. Bu görüş, öğrenmenin, çevrenin bireysel çalışma ve sosyal yönlerinin kombinasyonuyla gerçekleştiğini ileri sürer. Paylaşılan fikirlere odaklanan Krummheuer'in argümantasyon modelini kullanarak matematiksel uygulamalar yorumlanmıştır. Dört matematiksel uygulama şu şekilde elde edilmiştir: (a) prizmaların tanımı ve özellikleri, (b) prizmaların yüzey alanı bulma, (c) silindirin yüzey alanı bulma ve (d) silindir hacminin bulunması. Sonuçlar, öğrencilerin üç boyutlu katı cisimleri anlamalarının, sınıf içi tartışmalar ve dinamik geometri yazılımlarının desteğiyle geliştiğini gösterdi.

Anahtar Sözcükler: Tasarım tabanlı çalışma, katı cisimler, matematiksel uygulamalar, tartışma, varsayıma dayalı öğrenme yörüngesi.

To My Three Little Princesses,

Zeynep Beyza, Elif İpek, Ayşe Begüm

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LIST OF ABBREVIATIONS

HLT	Hypothetical Learning Trajectory
DGS	Dynamic Geometry Software
NCTM	National Council of Teachers of Mathematics
KMA	Krummheuer’s Model of Argumentation

CHAPTER 1

INTRODUCTION

School mathematics course has several sub-domains and geometry is one of the most important ones among them. The most important part of geometrical thinking is about two or three-dimensional geometric shape in space and looking for various aspects of them (National Council of Teachers of Mathematics, 2000). In geometry classes, students evaluate the relationships between geometric shapes, structures, theorems, and formulas (Keşan & Çalışkan, 2013). For example, students should understand how to come up with a theory or a formula, according to geometric features of related shape. This requires an effective teaching and learning of geometry. In the opposite case, students prefer memorizing geometrical concepts and formulas rather than understanding them (Fuys, Geddes, & Tischler, 1988).

Baki (2001) states that students should learn geometry by understanding and explaining the physical world by using appropriate problem-solving strategies within. Our physical world cannot be explained just by two-dimensional Euclidean geometry. Because everything that we contact by using, seeing, producing, i.e. have a three-dimensional geometric shape (Güven & Kosa, 2008). In the same way, Pittalis and Constantinou (2010) state that this type of thinking is “a form of mental activity that enables individuals to create spatial images and to manipulate them in solving various practical and theoretical problems” (p. 191). Sack (2013) summarizes this statement as getting the meaning of any object or process in the shape, size, orientation, location, or direction. Therefore, many national documents (NCTM, 2000) have stated that all students should have opportunities to work with three-dimensional shapes by visualization to develop spatial skills since they are important and useful for everyday life and for many future careers. Moreover, the importance of three-dimensional thinking abilities has been expressed by researchers across mathematical and scientific disciplines. Despite its importance, solid shapes, polygons, triangles, geometrical ratio, geometrical transformation are defined as the most problematic ones in terms of teaching and learning. Students tend to define them as difficult to understand (Adolphus, 2011). Thus, three-dimensional solid shapes are among the challenging concepts for students. In this sense, the research has shown that those concepts should be learned through appropriate learning experiences (Alqahtania, & Powell, 2017; Ganesh, Wilhelm, & Sherrod, 2009; Marchis, 2012).

For instance, Yackel and Cobb (1996) claim learning mathematics includes both individual working, but also collaborative working by involving in whole class discussions and by explaining and justifying their works in a wider community. Moreover, in several studies (Bauersfeld, Krummheuer, & Voigt, 1988; Cobb, Boufi, McClain, & Whitenack, 1997; Giannakoulias, Mastorides, Potari, & Zachariades, 2010; Mueller, 2009), the importance of discussion and argumentation in mathematics classes, and the classroom norms are characterized by processes of explanation, justification, and argumentation. Thus, as a sub-area of mathematics, it is appropriate to adopt the argumentative classroom environment to the geometry classes. By this way, it might be useful for students to understand the structures and theorems, and their relations by exchanging ideas. Additionally, while discussing the scientific argumentation process, Driver, Newton, and Osborne, (2000) conclude that argumentation promotes deep conceptual understanding of the context. Moreover, various studies support that argumentation encourages conceptual understanding of mathematics and geometry by justifying and criticizing ideas (Abi-El-Mona & Abd-El- Khalick, 2011; Jonassen & Kim, 2010; Osborne, Erduran, & Simon, 2004; Zembaul-Saul, 2005). In this respect, it may be useful to include argumentation in geometry to increase conceptual understanding of students.

Aligned with the features of the designed based research, preparing an instructional sequence with a conjectured hypothetical learning trajectory (abbreviated as HLT in this study) for geometric concepts may provide benefits for students to think and learn that context effectively. Moreover, by supporting those activities with classroom discussions and argumentation, students would have a chance to communicate their ideas with others. Furthermore, argumentation on a specific context provides to transfer ideas among students to become taken-as-shared ideas which are a way of construction of mathematical practices (Cobb, Gravemeijer, Yackel, McClain & Whitenack, 1997). In this respect, in the current study, classroom mathematical practices formed by the classroom argumentations in the context of three-dimensional shapes were evaluated.

MoNE (2013) has stressed that the use of technology in mathematics and geometry lessons develop students' thinking and spatial abilities. Geometry instruction should include a specific attention on the three-dimensional figures. Especially, the visualization skills and representation of three-dimensional shapes should have a continuous development. Ben-Chaim, Lappan, and Hoaung, (1985) states that spatial thinking can be taught and developed successfully by using appropriate strategies in the middle and high school students. Relatedly, educators believe that use of technology as an appropriate strategy can effectively support teaching and learning mathematics and specifically geometry (McClintock, Jiang & July, 2002). There are various technological tools that can be used in geometry lessons such as word processor and spreadsheets. But, dynamic geometry software (abbreviated as DGS in this study) is a more effective tool to construct more student-centered learning environments (Hannafin, Truxaw, Vermillion, & Liu, 2008).

NCTM (2000) states that it is crucial in school mathematics to use concrete materials, drawings, and dynamic geometry software to provide an effective learning of geometry. By using DGS in the education field and by transferring them to the dynamic computer screen, it has become possible for students to evaluate the relations between structures, to develop a hypothesis, to test theorems without using papers and pens (Güven &Karataş, 2003). Researchers have shown that DGS gives students the opportunity to concentrate on much more abstract structures than widely used paper-pen studies with its dynamic features (Hollebrands & Okumuş, 2018). In this way, students' power of imagination increases. In mathematics, the increase in imagination opens the way of intuition, so the way of creation and discovery. When these ways are used, the student will be able to analyze, hypothesize, and generalize. This will directly develop student's problem-solving skills (Baki, 2001). DGS, with its features of supporting experience and teaching geometry through research, offers alternative possibilities to geometry which had been taught in the same way for years (Edwards, 1997). In geometry teaching, by using dynamic geometry software, students can create geometric drawings or do interactive investigations on the dynamic geometric shapes prepared by the teacher (MoNE, 2013); and in this way students' learning of geometry can be supported through mediating their activity in DGS environments (Alqahtania & Powell, 2017).

For an effective usage of DGS and to provide a student-centered inquiry, it should be created with a flexible instruction that is open to making conjectures to guide students (Hollebrands, 2007). For instance, the designers of Geometer's Sketchpad expected that by clicking and dragging geometric shapes, students would be able to make conjectures about the context through a series of designed activities (Hannafin, Truxaw, Vermillion, & Liu, 2008). Many educators who advocate the effectiveness of such learning environments suggest that in these settings, students can work together to develop theories and draw inferences (Battista, 2003; Cognition and Technology Group at Vanderbilt [CTGV], 1992; Sinclair & Crespo, 2006). In this respect, it is appropriate to conduct a design-based research to provide students a learning environment in which they would think about the context, discuss, express, and justify their ideas, accept, or refute others reasoning, with a planned, conjectured HLT and instructional sequence including series of activities.

Although there have been various researches conducted to evaluate learning environment in its natural settings, it seems like there is a gap in the literature on design-based research on geometry concepts that were with instructional sequence and also with DGS. With respect to explanations above, since the geometry is an important area of mathematics in which students have difficulties to learn and understand structures and context, it may be significant to evaluate and explore their classroom argumentations in the context of three-dimensional shapes and obtain the formed classroom mathematical practices. Accordingly, this study evaluated the argumentation and collaborative learning environment related to planned instructional content and activities. This process performed by application of planned instructional sequence with the support of HLT during the teaching-learning sessions. In the collaborative learning environment, eighth graders' understanding of three- dimensional shapes were examined. Also, their classroom discourse was important to evaluate their reasoning on the context as well as to identify the construction of classroom mathematical practices. Since, the formation of classroom mathematical practices is related to the social learning environment (Gravemeijer & Cobb, 2013), students' learning of three-dimensional shapes was evaluated through classroom mathematical practices.

Accordingly, the aim of this study was to develop technology-supported mathematical contents within an instructional sequence and apply them for a predetermined period in an eighth-grade class to investigate the effects of these contents on student success. The instructional sequence of the study was prepared based on Stephan's (2015) study named "surface area". The content was evaluated according to students' needs and the national curriculum. Also, appropriate questions supported with GeoGebra, questions that were inappropriate for the content of the study were removed and appropriate questions were added. According to the results, it was planned to evaluate and revise the content to make it available to use in other classrooms.

Generally, design-based studies are not formulated with a single question of purpose. Of course, a research question can be produced on how a topic can be learned or taught in the most effective way. However, it must be completed with several assumptions about what conditions affected the answer to this question should fulfill, and at the same time it should be noted what kind of innovations this study is expected to bring. In addition, new questions and new estimates may arise during the execution of the research project (Gravemeijer & Cobb, 2013).

In this context, the research questions that guide the study are as in the following:

1. What are the mathematical ideas that support the mathematical practices which students developed during this instructional sequence?

2. Are there any effects of this instructional sequence on the students' achievement by using argumentations and dynamic geometry software in that context?

1.1 Significance of the Study

Geometry as a study of space has an important place in mathematics lessons at all grade levels of education. It is important for students to have a deep understanding of geometry concepts. It is stressed in NCTM (2000) that spatial understanding and abilities are important to understanding our physical world. By having an in-depth understanding of spatial relations and relatedly geometric structures, students are expected to be ready for many careers including advanced mathematical topics.

Despite the importance of geometry, international assessment programs such as Program for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS) show that for many years Turkey has ranked at the bottom rows in among the participating countries. For instance, PISA (2015) report indicates that in Turkey, the level of mathematical literacy of the students is very low and going worse; furthermore, it can be deduced that students' skills of using of mathematical language and understanding are not enough. In the same way, TIMSS (2015) report shows that Turkey remains under the TIMSS average achievement score. In the same way, students have low mathematics achievement in national examinations like University Entrance Examination. Examining those exams, it has been observed that the most wrongly answered questions are coming from geometry. At this point, it turns out how important geometry is. However, despite many efforts, there are still problems that students having with geometry (Adolphus, 2011). To overcome these problems, there is a need to observe learning environments, plan and prepare instructional contents, perform those contents for a period and evaluate students' classroom practices for whether it has an effect on their achievement or not (Geraniou, Mavrikis, Hoyles, & Noss, 2009).

In geometry, the teaching process consists of series of rules and formulas, which causes the memorization of them. This process does not provide a conceptual understanding of content. The geometry lessons are full of ragged drawings that make students confuse the whole content (Keşan & Çalışkan, 2013; Sinclair & Bruce, 2015). Generally, geometry lessons include the teaching of geometrical concepts. Those concepts are taught in an order by giving a definition, talking about elements and characteristics of shapes, stressing important rules, giving the formulas. Students rarely involved in the processes in which they can produce the related knowledge. With these practices, it is not possible to expect students to show success in processes that they need to explain their own ideas on the context, justify those ideas with using appropriate mathematical language, and apply the produced formulas to solve conceptual problems (Adolphus, 2011; Cunningham, & Roberts, 2010).

As an anticipation, that kind of classroom environment does not provide a deep understanding of geometric concepts for students. Consequently, forcing students to imagine those content through their own mental process, makes them fail to develop insights into the concepts. In this sense, understanding the geometric content may be difficult for learners in the paper and pen environment and may prevent learning (Denbel, 2015). To overcome this problem, the curriculum has stressed the usage of DGS in geometry lessons for a time (MoNE, 2013). Relatedly, a conceptual understanding of geometric concepts can be provided by making them involve in instructional sequences with an addition to technological support. By operating those instructional sequences with mathematical classroom discourses, meaningful and deep understanding of geometric concepts can be provided. Yackel & Cobb (1996) believe that classroom discourse with classroom argumentation on the context has a positive effect on students learning of mathematics. Argumentations include mathematical communication in which students share ideas among students and teacher that shapes the learning environment. Krummheuer (2015) mentions the process of learning mathematics as argumentative and states that it is based on students' participation in practice by explanation and justification. Accordingly, the learning of mathematics may occur by participation (Krummheuer, 2011; Sfard, 2008). Thus, argumentation is a social phenomenon that occurs while interacting verbally with other members of the classroom by explaining and justifying their actions during the practice (Krumheuer, 2011). Relatedly, argumentation of mathematical ideas is considered to improve abilities of students' reasoning on mathematical concepts, their explanations about the context and expressing their justification about that ideas. Additionally, use of technology in geometry classes seems to increase collaboration and creative reasoning by providing an environment for students to exchange ideas with others. Furthermore, these collaborative activities enhance creative reasoning by getting them involved in whole class argumentations (Granberg & Olsson, 2015).

In our national mathematics curriculum, the use of DGS is offered in geometry lessons (MoNE, 2013), but in mathematics textbooks, there are not sufficient content to provide a source for students or teacher that explains how those technological tools can adapt into the lessons. The instructional sequence (Stephan, 2015) prepared for the current study may be useful both for students and teachers. Research also indicate that teaching geometry with the support of DGS have a positive effect on students' conceptual understanding and relatedly on their achievement (Goss & Bennison, 2008; Hannafin, Truxaw, Vermillian & Liu, 2008; Kalbitzer, & Loong, 2013; Kondratieva, 2013; Obara, 2009; Tayan, 2011; Yemen, 2009; Kutluca & Zengin, 2011). In the current study, students worked on the activities with the support of argumentations and DGS through an instructional sequence and HLT. The use of dynamic environments also may help students to develop their visualization, construction, and reasoning skills (Dixon, 1997).

In order to help students to get those skills, the lessons are planned and organized through an instructional sequence by a conjectured HLT and with an ongoing analysis of classroom process. Accordingly, conducting a design-based research may be beneficial since it's aimed "to develop a class of theories about both the process of learning and the means that are designed to support that learning, be it the learning of individual students, of a classroom community, of a professional teaching community, or of a school or school district viewed as an organization" (Cobb, Confrey, diSessa, Lehrer & Schauble, 2003, p. 10).

It is also stated that design research both has a theoretical orientation and also has a pragmatic feature with resulting collaborative learning environment. By connecting the theory and practice, it should have an ongoing changing process that is redesigned according to needs of students (Cobb, et al., 2003). Involving that kind of collaborative learning environment within an instructional sequence, students may construct a deep understanding of geometric concepts.

The hypothetical learning trajectory of the current study was prepared based on the geometric concept of three-dimensional shapes since it is seemed as difficult by students (Adolphus, 2011). For this concept, choosing an eighth-grade classroom was appropriate by considering the national curriculum and also the thinking levels of students.

Looking at the literature, it can be concluded that there is a need for a learning environment in which students can express and share their ideas freely, comment on others' works by accepting or refuting. Considering memorized learning environments, it can be argued that this may provide a more meaningful learning of geometry for students. In addition, it may be meaningful to add dynamic geometry software to the learning environment when considering the problems that students have in embodying the relationship between geometric structures in their minds.

In this respect, this study is expected to fill a gap in the related literature by planning and preparing lessons through an instructional sequence and HLT on the concept of three-dimensional shapes and by supporting the lessons with using argumentations during classroom activities, giving daily life examples of related context and supporting the instruction with one of the dynamic geometry software GeoGebra.

Additionally, this study aims to maintain an ongoing analysis and development process for obtaining classroom mathematical practices that occur during the classroom argumentations. With this aim, by adding technology supported activities in Stephan's (2015) work an instructional sequence was prepared. By this way, it is planned to evaluate students' ways of thinking about geometry concepts, their errors in those ideas, how geometry lessons should be designed and what kind of tools should support the instruction of the lesson.

1.2 Definition of Terms

Hypothetical Learning Trajectory(HLT) is a set of instructional activities designed to support students' mental processes like thinking and learning in a specific mathematical domain. It also aimed to support students' achievement in that specific domain (Clements & Sarama, 2004).

For the current study, an HLT was prepared as a pathway for related context including expected and actual mental processes of students and the ways to support students' learning the context through argumentation and dynamic geometry software.

Instructional Sequence includes set of tasks that are sequenced according to the developmental progressions for completing the hypothetical learning trajectory. Tasks are designed to promote students' conceptual learning of a particular content by requiring them applying the actions both by mentally and externally (Clements & Battista, 2000).

In the current study, the instructional sequence was prepared based on Stephan (2015)'s work and national curriculum.

Classroom Mathematical Practices focus on the taken-as-shared ways of reasoning, arguing, and symbolizing that occur while arguing on specific mathematical content. *Taken-as-shared ideas* indicates the social environment that includes discussions about specific mathematical ideas by using appropriate mathematical language (Cobb, Stephan, McClain, & Gravemeijer, 2011).

In the current study, classroom taken-as-shared ideas used as students' common understanding about a specific issue, and they produced the mathematical practice by constructing on each idea.

Argumentation is a way of expressing students' justifications of mathematical ideas through classroom communication (Lampert, 1990).

In the current study, argumentations included both pair-discussions and whole class discussions on a specific context. Students expressed their own ideas, justified their works, responded on other’s ideas by using argumentations.

Dynamic Geometry Software are computer programs by which geometry can be learned interactively. These softwares provide students opportunities rather than paper and pencil by making constructions and justifications of geometric concepts under various transformations (Denbel, 2015).

GeoGebra is a free dynamic geometry software for teaching and learning mathematics that can be used at all education levels beginning from elementary (Hohenwarter & Preiner, 2007). GeoGebra has many tools to help construction of geometric concepts. Users can construct many geometric concepts with their measurements. Also, it is possible to see various transformation of shpaes. This helps users to observe the relationship between geometric constructions and transformations dynamically.

CHAPTER 2

LITERATURE REVIEW

The National Council of Teachers of Mathematics (NCTM) (2000) emphasizes the importance of communication to develop students' mathematical understanding in *Principles and Standards for School Mathematics*. They state that the instruction should be designed to enable students to share their ideas in a mathematical community, evaluate and analyze others thinking in the classroom community. Students generally work together to construct their solutions while working on questions whose solutions require justifications (Mueller, 2009). Students should have opportunities to share and discuss their ideas with others to involve in mathematical discussions effectively and to reason about context (Lampert & Cobb, 2003). Ball and Bass (2003) assert that ". . . mathematical understanding is meaningless without a serious emphasis on reasoning" (p. 28). According to them, meaningful learning is possible by understanding the ideas of the other students and generating new ideas from it. In this context, through reasoning, students can reconstruct previous knowledge, which can be based on previous knowledge and create new insights. By giving the opportunity to reason about mathematical knowledge in a supportive environment like as young as primary school, young learners can create, reflect and evaluate assumptions and try to persuade others to accept these reasons (Maher & Davis, 1995; Yackel & Hanna, 2003). The instruction is created that allows students to share their ideas with others, participate in mathematical discussions and reasoning, students can present persuasive arguments that show various ways of reasoning in the development of solutions to problems (Maher & Davis, 1995; Maher & Martino, 2000; Mueller & Maher, 2008).

Researchers emphasize some characteristics of an effective classroom learning environment for mathematics and relatedly for geometry classes, such as task design, tools, representations, inviting children to explain and justify their reasoning and mathematical discussion (Davis & Maher, 1997; Francisco & Maher, 2005; Maher & Davis, 1995; Mueller, 2009; Mueller & Maher, 2008).

For some time, education researchers have recognized the potential for mathematics learning to be transformed by the availability of digital technologies such as computers, graphics calculators, and the Internet (Arnold, 2004; Forster, Flynn, Frid, & Sparrow, 2000; Lynch, 2006). For example, interactive whiteboards are predicted to be in at least one of every six classrooms around the world by 2012 (Bowers & Stephens, 2011). These technologies offer new opportunities for students to communicate and analyze their mathematical thinking by enabling fast, accurate computation, collection, and analysis of data, and exploration of the links between numerical, symbolic, and graphical representations (Hennessy, Fung, & Scanlon, 2001). Researchers state that if used in appropriate way, technology may be very effective in teaching and learning practices in classroom environment. Particularly for the mathematics education, technology has the potential to support the instruction (Connell, 1998; Roschelle, Pea, Hoadley, Gordin & Means, 2000).

In the same way, research support that usage of technology as an instructional tool provides an inquiry-based learning environment in which students communicate, argue, justify and explain their ideas to construct mathematical understanding (Chapman, 2011; Goos, Galbraith, Renshaw, & Geiger, 2003; Hähkiöniemi, 2013).

Since, studies stress the importance of collaborative learning environment and interaction of students, social constructivism has importance for the emergence of classroom mathematical practices (O'Donnell & King, 1998). For construction of knowledge, impacts of other people should be considered in terms of social interaction, classroom society (Jones & Brader-Araje, 2002; Palmer, 2005).

In this respect, this chapter reviews the studies about the issues aligned with the aim of the study. First, the main concepts of the study HLT, classroom mathematical practices and argumentation in mathematics are discussed. After that, the geometry education and usage of technology -specifically DGS- are discussed relatedly. Lastly, the philosophies who emphasize the current study social constructivism and RME are mentioned to explain the theoretical framework.

2.1 Teaching and Learning Geometry

Geometry provides opportunities for students develop their thinking and proving skills (NCTM, 2000). Thus, for many years, the teaching and learning of the geometry is an issue for educators and researchers (Adolphus, 2011; Baki, 2001, 2002). Thus, it is affected by many reforms -specifically by modern mathematics reforms- which emphasize avoiding the usage of diagrams in lessons since they make the geometry difficult for students. Accordingly, students have confused because of the knowledge provided by those diagrams since they guide students to deductive thinking (Laborde, Kynıgos, Hollebrands, & Strasser, 2006).

Many researchers viewed that the origin of the problem was in the absence of graphical representations associated with geometry as part of the repertoire for expressing mathematical meanings. They were expressing the absence of usage of diagrams a shortcoming for geometry teaching and learning. Freudenthal (1973) was among those researchers thinking as the same as the others; and he was followed by many researchers, which stressed the reintroducing of diagrams in geometry teaching. Despite the importance of usage of graphical representations in teaching and learning of geometry, it was not yet got the sufficient attention at those times.

Various theories and studies about the teaching and learning of geometry focus on the van Hiele model of geometrical thinking (van Hiele 1986), the theory about figural concepts (Fischbein 1993, Mariotti & Fischbein 1997), the theory about figural apprehension (Duval 1998), and as a more recent theory of geometric work (Kuzniak, 2014). Moreover, there are more general theories focused on the specifics of geometry education such as about the conception, knowing, concept (abbreviated as cKc) model (Balacheff 2013), as a more recent use of discursive, collaborative, and material perspectives (Ng & Sinclair 2015a, b; Owens 2014, 2015), and use of digital technologies for geometry education (Hegedus & Moreno-Armella 2010; Jagoda & Swoboda, 2011). Looking at the literature, it can be asserted that more recent studies preferred to evaluate students' reasoning about three-dimensional shapes by using dynamic geometry environments (Morgan & Alshwaikh, 2012), since three-dimensional thinking and understanding those concepts are labelled as difficult by learners (Adolphus, 2011).

Looking at the research on geometry teaching and learning, the most obvious theory can be asserted as Van Hiele 's model originated in 1950s that proposed five thinking levels for geometry (Sinclair, Bussi, de Villiers, Jones, Kortenkamp, Leung, & Owens, 2016). Then, theories emerged that those thinking levels may not be definite to obtain (Lehrer et al. 1998). For example, Wang and Kinzel (2014) evaluated use of mathematical terminology through parallelograms context. They studied with preservice elementary mathematical teachers and found that various reasoning types and differences emerged during participants' discourses. Forsythe (2015) investigated students' dragging strategies in a dynamic geometry environment and types of dragging modes through van Hiele levels. Using dynamic geometry software, it allows monitoring the change in the figures and increase the reasoning process to observe relations between the kite and rhombus.

In another perspective, studies were conducted about figural concepts, figural apprehension and their dimensional constructions. In this context, these researches support that students should learn beginning from one-dimension to two-dimension and later solids -that are three dimensional objects (Duval, 2000).

Another perspective supports existence of spaces for geometric work. In the same context, Duval (1998) offers three kinds of cognitive processes for a geometric activity that students involve in as; visualization, construction by tools, and the last one is reasoning. He states that each of those steps is connected to and supports each other. He also stresses the importance of the visualization process related to the solution processes of a geometry problem. He defined some different approaches related to visualization process. Fischbein (1993) considers geometrical concepts as they include two sub-components as the figural the conceptual. The relation between those two components cannot be separated and also students should ground on a mental construction process. In the same respect, Kuzniak (2014) mentioned two interconnected planes as epistemological and cognitive planes. Epistemological plane included support of materials, use of artefacts and geometric definitions and properties. Cognitive plane was defined as combination of visualization process, construction process (including use of materials) and discursive process supporting geometric argumentation. In a later research, Gomez-Chacon and Kuzniak (2015) focused on the effects of DGS on relationships between those three processes that visualization, instrumental and discursive. These studies exemplified use of combination of epistemological and the cognitive dimensions effected geometric reasoning.

One of the more recent models about teaching and learning geometry has developed as ckc (conception, knowing, concept) (Balacheff 2013). This is a perspective that focus on students' understanding by considering situational characteristics. In this respect, Gonzalez and Herbst (2009) focused on students' conceptions about congruency. They proposed four conceptions as perceptual, measure-preserving, correspondence, and transformation. The study conducted in a high school and dynamic geometry environment and authors concluded that concluded that measuring process did not supported transformation process and there was a need for a theoretical approach to highlight this issue.

Another recent understanding about geometry is discursive perspective including argumentations. Recently, many researchers have supported use of argumentations and discursive activities. In this respect, in Massarwe, Verner and Bshouty's (2010) study, prospective teachers worked about construction and analysis of geometric ornaments and taught geometry by using this context to middle school students. Middle school students' creations were observed when they constructed new ornament styles, worked on problems including ornaments and tried different ways to solve these problems. Rowlands (2010) offered a curriculum initiative including history of Greek geometry. The aim was to encourage discourse which could provide opportunities for students to understand abstract proof. Owens (2014, 2015) studied with different cultures about space and geometry. Data were collected by interviews, questionnaires, field experiences, focus groups and personal stories to provide a framework that is useful across a range of languages and cultures for teaching early mathematics education. Ng and Sinclair (2015a, b) used a communicational approach. For instance, Ng and Sinclair (2015a) investigated children's learning of reflectional symmetry by use of dynamic geometry software. They conducted a classroom-based instruction, they evaluated the changes in students' thinking about reflectional symmetry. Use of DGS and argumentations supported students' understanding of symmetry. Ng and Sinclair (2015b) investigated junior high school students' reasoning about area. They used shearing in dynamic geometry environment. The aim was moving students from formula-driven and computational conception of area to get conceptual understanding. They found that dynamic geometry technology that supported students' learning, as well as the teacher's role in orchestrating classroom argumentations.

Geometry interests in space and shapes (Clements, 1998). It studies spatial objects such as shapes, their edges, grids; relations such as equality, parallelism; and transformations such as reflection and rotation. To make these concepts clear for students, teachers use various representations, such as drawings, schemes, and graphs. These ways of representations give the contextual descriptions of geometric concepts, may support the conceptual understanding of students, and help them to develop their spatial reasoning (Hallowell, Okamoto, Romo, & La Joy, 2015).

The most emphasized geometric thought is spatial reasoning which is defined as the ability to "see, inspect and reflect on spatial objects, images, relationships, and transformations" (Battista, 2007, p.843). Spatial reasoning includes building and manipulating mental representations of these objects, relationships, and transformations, generating images, inspecting images to answer questions about them, transforming and operating on images, and maintaining images in the service of other mental operations (Clements & Battista, 1992; Clements, 1998; Battista, 2007). For example, we might see in our mind's eye what shapes would result from cutting a square from corner to corner. Thus, spatial reasoning provides not only an input for formal geometric reasoning, but also provides critical cognitive tools for it. But, many students have difficulties in geometric and spatial thinking (Mamolo, Ruttenberg-Rozen & Whiteley, 2015) These include, creating three-dimensional structures of unit-cubes, making, and working with two-dimensional representations of three-dimensional objects, including plans and isometric diagrams, using and making two-dimensional nets of three-dimensional objects and comparing mathematical properties of three-dimensional shapes. Therefore, it is necessary to use appropriate language to the level of students as well as various activities supporting geometric thinking and spatial skills (Kalbitzer & Loong, 2013).

As mentioned in the previous section, the reflections of the developments in technology have brought many changes to the classes. It is not expected that the geometry, which constitutes an important part of mathematics, is excluded from this effect. Geometry has a critical position in mathematics because of its contribution to the physical world. It has been used throughout history to explain much mobility from micro worlds to macro worlds. However, research has revealed that students do not develop strong conceptual understandings (Mistretta, 2000).

Denbel (2015) explains that, in traditional classrooms, geometry lessons are performed by paper and pen. Similarly, geometry textbooks that students use just give descriptions and figures afterwards. However, for some situations, those illustrations may not be much comprehensive for not providing a visual description of the geometric concept for the students' construction of it. Because geometry, in general, requires a dynamic visualization of figures or shapes, but textbooks have a static nature in themselves (Christou, Pittalis, Mousoulides, & Jones, 2005). By working with textbooks, students are left to complete the dynamic visualization of geometrical figures or shapes by their own mental processes (which can be impossible for many times). Textbooks provide only one ideal and most common form of any shape or figure, but students need to construct the whole forms of the figure or shape in their minds. Thus, it can be concluded that, in general, those textbooks are not appropriate with the construction process. In paper-and-pencil environment, it is possible to observe the last product of construction process on the textbooks; but it ignores students' mental process (Smith III, Males, & Gonulates, 2016). Reversely, to provide a conceptual understanding of geometric concepts, it is important for students to develop abilities for mental imagination of shapes and figures (Baki, 2001). Because to get a conceptual understanding of such as proofs, theorems and formulas, those require an insight and ability of mental imagination related to flexibility of shapes or figures (Kondratieva, 2013). Textbooks are far away from providing the dynamic nature of geometric concepts on paper (Hazzan & Goldenberg, 1997). Consequently, students often fail to understand or fail to develop a conceptual understanding for the taught concepts. Because, it is difficult for nearly all students to visualize for instance, how to produce the formula for the volume of a cylinder, to which knowledge they can relate it while finding. Thus, to understand conceptually and internalize the concept creates a mental challenge to students in the pencil and paper environment that is the point what makes learning geometry difficult for many of them (Baki, 2001, 2002).

In addition, the Euclidean geometry, which is being taught in our schools, cannot provide students with rich experiences and present research and exploration environments (Güven & Karataş, 2005). Students who cannot find themselves in enriched experiences choose to memorize the rules, associations, examples, and proofs when necessary. Many teachers avoid using pencil and paper to form and measure shapes in order to explore associations in geometry lessons (Goos & Bennison, 2008). Because it takes a lot of time to form these shapes, measurements do not give accurate results (De Villiers, 1996). In addition, it is an issue in traditional environments to create new forms for students to generalize through induction (Güven & Karataş, 2005). The restrictive structure of traditional school geometry has recalled the idea of teaching other geometries instead of Euclidean geometry in many countries, especially in America (Güven & Kosa, 2008). Perhaps, it was the dynamic geometry software, such as Cabri Geometry, Geometer’s Sketchpad and GeoGebra, that the technology has introduced to the field of education that saved the embedding of Euclidean geometry in the history (De Villiers, 1996).

In the same context, Goodson-Espy, Lynch-Davis, Schram, and Quickenton, (2010), studied with preservice teachers. By referring to Kennedy, Tipps, and Johnson’s (2004) explanation as elementary school geometry should be based on four basic areas including topological, Euclidean, coordinate, and transformational; they constructed and organized their study and context around those areas. They stressed that to be an effective teacher and to support their students in getting conceptual understanding of geometry, at first hand, preservice teachers should get that understanding themselves before they teach. Accordingly, they evaluated how can geometry method courses can be designed to help preservice teachers to get basic geometric concepts meaningfully; and how technological tools may be helpful in this way. They supported the instruction of the study with 3-D computer graphics. At the end of the study, the results showed that knowledge of preservice teachers increased in terms of basic geometry concepts. The participated preservice teachers' usage of geometric terminology improved, and they felt themselves more proficient especially in 2-D and 3-D geometry and ready to teach those contexts.

Reviewing literature, while some studies prefer to work with textbooks, drawings and concrete materials (Hallowell, Okamoto, Romo, & La Joy, 2015; Thom & McGarvey, 2015), it is seemed that various studies are conducted by using technology -specifically dynamic geometry environments- to evaluate students' learning of particular geometric concepts and improve their conceptual understanding (Zahner, Velazquez, Moschkovich, Vahey, & Lara-Meloy, 2012), to evaluate their visualization skills and their spatial reasoning (Ng & Sinclair, 2015b; Owens & Highfield, 2015; Sinclair & Moss, 2012), to enhance argumentations of mathematics (Morgan & Alshwaikh, 2012), and to evaluate effects of DGS on students' mathematizing (Greefrath, Hertleif, & Siller, 2018).

For instance, Morgan and Alshwaikh (2012) tried to understand the discursive resources may affect students' participation to mathematical activities. They gathered data from an experimental teaching program, conducted as a part of math project focusing on 3-D shapes. An instructional sequence was prepared including dynamic geometry software to provide students make connections between static and dynamic contexts of domain based on Stephan (2015)'s work. The study showed that supporting instructional activities with dynamic geometry environment supported students' participation to the mathematical discussions about related context and enhanced construction of argumentative classroom environment. Similarly, Granberg and Olsson (2015), investigated sup-port of GeoGebra on students' collaboration and creative reasoning during mathematical problem-solving activities. Students worked in pairs to solve a linear function using GeoGebra. For data collection they recorded conversations, and computer activities. Gathered data were analyzed using Lithner's (2008) framework of imitative and creative reasoning. The results of the study indicated that the use of GeoGebra supported collaboration by providing students a shared working area and relatedly to think more creatively by this sharing and exchanging ideas. Use of DGS as an instructional tool, enhanced students' collaboration and communication. In the same respect, Lai and White (2014) designed a different study. In their study, students worked in four groups collaboratively. The research was a part of a larger project and students dragged the four vertices of a quadrilateral by using mobile devices. The findings indicated that students' working collaboratively was also an indicator of their enhanced learning when compared to individual working.

Looking at the literature, it can be deduced that recent trends about geometry education has focused on use of collaborative learning environment including classroom argumentations. Moreover, as mentioned above, the introduce of Dynamic Geometry Software (DGS) to the teaching and learning of geometry has become a possible solution to the defined problem above. Use of argumentations which supports collaboration and communication among students and use of DGS together may provide dynamic and visual representations of geometric concepts for the students. The current research explores students' learning experiences with guidance of an instructional sequence and the conjectured HLT by supporting DGS and using it as an instructional tool. The detailed information will be provided in the following sections of this part.

2.2 Solids

Geometry plays an important role in making correlations between mathematical concepts and everyday life (NCTM, 2000). Therefore, the reason for overemphasizing geometry teaching can be asserted. Thus, there is a call for a comprehensive geometry teaching and spatial reasoning in mathematics curriculum (NCTM, 2006). The development and improvement of teaching and learning theories is one of the main objectives of research in education. Focusing on this process involves developing and refining theories, especially on geometry teaching and learning, and applying more general theories to the properties of geometry education.

Compared to the other fields of mathematics, geometry is a domain that contains more abstract concepts in particular three-dimensional shapes that require students to think comprehensively using their visualization skills (Yıldız, 2009). Most of the problems that students face with while teaching and learning are said as solid shapes, polygons, triangles, geometrical ratio, and geometrical transformation. They are generally identified as difficult concepts for students and teachers (Adolphus, 2011). Since the solid shapes (or three-dimensional shapes) are defined as problematic by students, it may be beneficial and significant to conduct a research and evaluate the lessons based on those three- dimensional shapes.

By reviewing the literature, studies on three-dimensional shapes, especially based on students' ability to establish links between the two-dimensional representations of three-dimensional solids and also focus on the ability of reasoning about those three-dimensional solids. The research's first part focuses on generally drawings of solids (Lehrer, Jenkins, & Osana, 1998), drawing their nets (Potari & Spiliotopoulo, 1992), recognition of nets (Bourgeois, 1986), description of nets (Lawrie, Pegg, & Gutierrez, 2000), construction of nets (Despina, Leikin, & Silver, 1999) were examined. Studies about judgement skills were especially based on examination of different structures formed with cubes (Battista & Clements, 1998; Ben-Chaim, Lappen, & Houang, 1985), students' reasonings are examined according to Van Hiele levels (Gutierrez, Jaime, & Fortuny, 1991), students' spatial thinking skills (Saads & Davis, 1997), and integration of the technology and software to the teaching three-dimensional shapes (Markopoulos & Potari, 2005; McClintock, Jiang & July, 2002). Also, a variety of studies examined the preservice teachers' understanding of visualization of solid shapes which is also important for teaching those concepts (Gökkurt, Şahin, Erdem, Başıbüyük, & Soylu, 2016; Markopoulos, Chaseling, Petta, Lake, & Boyd, 2015; Pittalis, Christou, & Pitta-Pantazi, 2012).

Potari and Spiliotopoulo (1992) aimed to explore the children's perceptions about nets of solids and relatedly, their ability of visualization of characteristics of solids according to their nets. The participant students were asked to draw the nets of the given objects as matchbox, toilet roll and sardine tin. Also, they were interviewed to explain their drawings. Moreover, the study included whole class discussions on relations between the objects and students' drawings. The study explored the ways of children's imagination and drawings of the nets of objects given to them that revealed children's understanding of space. They found that the physical objects and classroom discussion supported students understanding and drawings relatedly. In Gutierrez (1996), he outlined the importance of visualization in geometry learning, especially in three dimensional solids. He discussed about roles of mental images and ability of visualization in learning and reasoning on mathematics. He pointed that usage of technology would be helpful to gain those abilities.

Similarly, Lehrer, Jenkins, and Osana (1998) designed a three-year longitudinal study and examined the students' conceptions of two and three-dimensional shapes, the measurement of length and area, mental manipulation of drawings and graphing. For the study of three-dimensional shapes (solids), the data were collected through drawing and spatial visualization tasks. The study found that curricular practices promote the conceptual change. They suggested that for learning geometry, a systematic instruction should be provided especially for later years of students.

McClintock, Jiang and July (2002) reported the four studies were carried out for four years. Those related studies investigated the middle and high school students' development of geometric thinking and reasoning through three-dimensional visualization. The study was supported by Geometer's Sketchpad that is one of the DGS. They constructed the dynamic representation of those solids. The study followed a constructivist approach and found that DGS provided opportunities for students and has a positive effect on them.

Similarly, Marcopoulus and Potari (1999, 2000, 2005) studied on a part of the project, students' thinking about three-dimensional solids and properties of those solids. They used three different contexts for the study. First one was with the students' usage of physical materials, the second one was defined through students' interactions in a computer-based environment and the last one was formed by students' visualization abilities concerning dynamic transformations of the solids. Each report explained the one phase of the project. The project concluded the importance and support of the materials used in the geometry lessons related to students' conceptual understanding. Those materials were especially physical dynamic materials and dynamic software. As a result, although not all the students reached an advanced level of thinking, the context designed with the support of dynamic objects both physical or on computer increased the development of most of the students' geometrical thinking. In the same context, Presmeg (2006), evaluated the studies about the importance of visualization in understanding geometry, she discussed the importance of visualization skills especially in context of 3-D solids; as a last point the place of computer technology in geometry teaching and learning. She underlined the research state that in conceptual understanding of geometric concepts and relations, visualization is the critical point in the instruction. Especially, in learning of 2-D, 3-D and transformation geometry, those skills are very crucial, and usage of computer technology has a positive effect on students' learning.

Cheng Meng and Idris (2012) explored effects of phase-based instruction by using Geometer's Sketchpad (GSP) on students' geometric thinking and achievement in solid geometry. They used van Hiele's geometric thinking levels. The study was a case study. The illustrated that use of GSP thorough phase-based instruction could support the participants' geometric thinking and achievement in terms of solid geometry.

Marchis (2012) conducted a research on pre-service primary school teachers' mastering some notions and properties related with shapes and solids in elementary level. The research illustrated that there were students who could not recognize basic geometrical shapes or solids. Most of the students could not state correct definition for geometrical shapes and they could not explain the basic properties of the shapes. Regarding geometrical solids, most of the students couldn't draw the correct two-dimensional representation of the solids and most of them didn't know how to draw the net of them.

Huang (2012) examined effects of computer-based curricula in terms of volume measurement concepts in fifth-grade geometry lessons. The research also evaluated how did the computer-based curricula effect on students' ability to solve volume measurement problems that demand mathematical explanations. The instructional approach included an environment in which students could representing and communicating their solutions, reasoning about explanation, evaluating measurement claims, and clarifying their mathematical thinking. The context of the instruction was about volume of a unit cube, geometric properties of a 2-D shape and a solid, transformation of 2- and 3-D figures, differences between area and volume. Findings indicated that guided argumentative and computer-based instruction enhanced students' acquisition of volume of solids. Moreover, they were likely to show gains in explaining mathematical thinking for volume measurement when they exposed to that kind of enriched curriculum.

Latsi and Kynigos (2012) studied with six graders in a public school of Greece. The participating classroom included 23 students involved in 16 teaching sessions for two months. The students worked collaboratively on 3-D shapes through their dynamic manipulations and transformations by using 3D turtle geometry. The results indicated that use of turtle geometry provided more constructivist approaches for students and enhanced collaboration among them.

In Kalbitzer and Long (2013), they prepared open-ended tasks based on three dimensional solids. They used multiple representation methods to teach solids including computer applications. They taught year 5/6 mixed ability class by this way. The study showed that students like to engage in activities that differ from traditional methods. Also, they observed that usage of concrete or technological manipulatives and tools was directly related to the students' interest and understanding of three dimensional shapes since they provide students mental visualization of the context.

İncikabı and Kılıç (2013), conducted a study that aimed to analyze and evaluate the conceptual knowledge of three-dimensional solids in primary school level. For this reason, they prepared a diagnosis test that consist of three questions related to conceptual knowledge of cube, square prism, and rectangular prism. 272 students participated to the study and 12 of them were chosen for the interview. Data analysis were conducted both quantitively and qualitatively. The results showed that most of the students cannot name the solids and cannot tell their features, very few of them could. Additionally, it was obtained that students have some misconceptions about geometric concepts in solids. Students often confused three-dimensional objects with the names of two-dimensional shapes. Moreover, some participating students couldn't provide any explanation to the questions nor prove their own claims.

Güçler, Hegedus, Robidoux, & Jackiw, (2013) examined the experiences of fourth grade students. They involved in a dynamic geometry environment and explored the characteristics of 3D shapes. This dynamic multi-modal environment supported semiotic mediation and provided social interaction since students worked in groups. The researchers mainly focused on students' discourse on 3D shapes. Results showed that use of technology by combining inquiry environment "have the potential to present students with the opportunities to explore 3D objects through multiple perceptions, supporting meaningful discourse as students engage in mathematical activities such as exploring, conjecturing, negotiating meaning, and sensemaking" (p. 97).

Chang, Wu, Lai, and Sung (2014) developed a system to facilitate learning of 3-D geometry by supporting spatial thinking. They developed that system based on Duval's four critical elements of geometric learning that, perceptual apprehension, sequential apprehension, operative apprehension, and discursive apprehension. The idea of the system was based on supporting high school students learning of 3-D geometry problem-solving. Also, it offered an approach for manipulating spatial figures to develop the students' visualization skills and conceptualization of images. 58 students participated from different classes. The experimental group learned by mentioned system and the control group used traditional pencil-and-paper method. The findings indicated that proposed system increased students understanding of 3-D geometry and enhanced their spatial and visualization abilities.

In this respect, Markopoulos et all., (2015) examined primary and early childhood preservice teachers' geometric thinking and visualization processes on three dimensional shapes. Authors stated that 3-D shapes were very complex to visualize and require improved spatial abilities Researchers studied with 289 pre-service teachers. Results of study indicated that it was difficult for students to decode and encode the visual information. They found difficult to identify and understand the relationship between flat (two-dimensional) representation of solids and their 3-D mental constructions. Incorrect ideas were occurred incorrect ideas related to volumes of solids. Study was an indicator of preservice teachers' need for developing their visualization and conceptualization of 3-D objects. Moreover, two-dimensional learning was inadequate for teaching and learning of solids in terms of providing preservice teachers information and activities to help them develop their spatial abilities.

Kotsopoulos, Zambrzycka, and Makosz (2017) conducted a study whether there were visual-spatial gender differences in two-year-old children. They also evaluated environmental and cognitive factors that affect and make any contributions to children's visual-spatial skills. Moreover, they looked for gender differences for these factors. 63 children were assessed on their visual-spatial skills including works based on intelligence, quantitative reasoning, working memory, and home spatial activity engagement. Additionally, children's mothers were assessed in terms of mental rotation skills. The study questions were mainly about children's getting in touch with three-dimensional objects, toys, and shapes. Findings of the study indicated that there was no difference between boys' and girls' visual-spatial skills at age two.

When the national curriculum of mathematics course is examined, it is seen that besides geometrical shapes, geometrical objects are also included. The students have the knowledge of cube, rectangle prism, cylinder, sphere, cone, and pyramid beginning from first grade through fourth grade (MoNE, 2013). When the student reaches the fifth grade, it is expected that the students explain the properties by specifying the names of the geometric objects. Later, at the middle school level, students are expected to acquire deeper understanding of those shapes including their nets, surface areas, and volumes, since they move to higher-level thinking skills (MoNE, 2013).

In this respect, the current study studied students' understanding of three-dimensional solids -specifically prisms and cylinder- by conducting a design-based research. For this aim, an instructional sequence and HLT was prepared including basic features of prisms, their surface area, basic features of cylinder, its surface area and volume by supporting the process with argumentations and DGS.

2.3 Classroom Mathematical Practices

For many years, researches have focused on the sociological side of the teaching and learning of mathematics. Specifically, the focus is cooperative learning by forming classroom mathematical practices (Ball & Bass, 2000; Cobb & Bauersfeld, 1995; Cobb, Stephan, McClain & Gravemeijer, 2011; Stephan & Rasmussen, 2002). In general, they prefer to focus on the social side of the teaching and learning of the mathematics, since mathematics is considered to be learned in community by doing mathematics (Cobb, Yackel & Wood, 1992; Yackel & Cobb, 1996). The studies in the literature have focused on the different sides and definitions of classroom mathematical practices. For example, Bowers, Cobb and McClain (1999), defined the mathematical practices as "focuses on shifts in ways of acting and reasoning mathematically that become institutionalized and hence are beyond justification" (p.28).

There are some researchers that define and use the term mathematical practices from different perspectives. For example; Font, Godino and Gallardo (2013) defines the mathematical practices from two perspectives. First one is operative side which is the reading mathematical texts and production of mathematics, and the second one is discursive side which is about reflection on the former activities. Moschkovish (2002), brings a different point of view to the term and distinguishes it in two groups. First one is defined as activities such as shopping and ordering. The second one is academic practices which are about the academic side of the mathematic that occur in school environment such as performing mathematical talks, involving in mathematical activities like problem solving etc. In Moschkovich (2004), she describes goals, meanings and focus of attention of those practices. In Moschkovich (2007), she analyzes discourse practices of a third-grade classroom. In that study, she distinguishes school and professional (academic) practices. Because, she thinks that school mathematical practices do not reflect the practices that mentioned in the mathematical literature. She points out that most of the mathematical classrooms do not produce the practices that explained by mathematicians. As a last point, Godino, Batanero and Font (2007) states that mathematical practice is "any action or manifestation (linguistic or otherwise) carried out by somebody to solve mathematical problems, to communicate the solution to other people, so as to validate and generalize that solution to other contexts and problems" (Godino, Batanero, & Font, 2007, p. 129). In here, they highlight the role of mathematical activities by using in construction of mathematical practices.

Classroom mathematical practices occur while arguing specific mathematical ideas and it is a way of sharing, arguing, reasoning of those ideas (Cobb, Stephan, McClain & Gravemeijer, 2011). The definition that produced by Cobb et al. (2011) as "a conjectured learning trajectory as consisting of an envisioned sequence of classroom mathematical practices together with conjectures about the means of supporting their evolution from prior practices" (Cobb et al., 2011, p. 125). A similar definition is made by Bowers, Cobb and McClain (1999) as the ways that "the teacher and students discuss problems and solutions, and these practices involve means of symbolizing, arguing, and validating in specific task situations" (p. 28). The starting points of those definitions are the individual and social views of learning process. As it is stated in the definitions, the mathematical practices imply taken-as-shared ways of reasoning, discussing, and arguing mathematically. Cobb, Wood, Yackel and McNeal (1992) defined the taken-as-shared ways as a process that performed by arguing on mathematical explanations, justifications, symbolizations etc. which end up with emergence of classroom mathematical practices. Accordingly, it can be concluded that the emergence of the mathematical practices is strongly related to the social interaction among classroom members. By creating a socially active classroom environment, students can be motivated to involve in process of mathematics teaching and learning more voluntarily (Cobb & Yackel, 1996).

By reviewing the explanations and definitions, it can be understood that classroom mathematical practices are just formed by cooperative learning in the classroom environment. The formation of classroom mathematical practices is influenced by the individual studies and activities of students as well as by collective learning environment. Hence, it is not possible to ignore the individual work of students in the formation of classroom mathematical practices. The critical point in the process of development of classroom mathematical practices is to evaluate the ways of students' participation in the collaborative learning environment and trying to make some contributions to that environment (Cobb & Yackel, 1996).

The formation of classroom mathematical practices is mentioned as when the classroom practices become taken-as-shared (Cobb, Stephan, McClain & Gravemeijer, 2011). To make classroom practice taken-as-shared, there is a need for students should make some contributions such as sharing ideas, giving examples, making justifications, proving solutions etc. Those activities are products of students own mental processes and this is the point why the individual participation of students has that much importance. Cobb and Yackel (1996) underlined the same point by stating that there is an interrelation between those students' individual and social participation. They mentioned students make a permanent contribution to the classroom mathematical practices during they reorganize their own individual works and activities and participating to the classroom mathematical practices force them to reorganize their works permanently (Cobb & Yackel, 1996). Relatedly, Cobb and Bowers (1999) stated that to provide the individual learning of students, it should have provided them opportunities in which they can participate the social context of classroom by sharing their ideas (Cobb & Bowers, 1999).

As mentioned above, mathematical practices are the ways of students' understanding, explaining, justifying, refuting, reasoning of a specific mathematical context, and make them taken-as-shared by the classroom community (Bowers, Cobb & McClain, 1999; Cobb et al., 2011; Stephan, Cobb & Gravemeijer, 2003). To identify classroom mathematical practices, students' ways of reasoning and their reflections are taken as starting point. The reflections of students occur during the classroom argumentations and the activities on a specific content (Stephan, Bowers, Cobb & Gravemeijer, 2003). Thus, social learning including individual practices of students are the focus of the classroom mathematical practices. Accordingly, the data about the learning environment including classroom discourse and usage of the learning tools are collected by social side of the classroom which is the formation of classroom mathematical practices (Stephan & Rasmussen, 2002).

By participating to the classroom activities including classroom discussions, students are forced to develop social and socio-mathematical norms in the classroom environment which support the development of mathematical practices (Akyuz, 2014; Cobb et al., 1997; Stephan & Akyuz, 2012). Detailed information will be given about these norms in the following sections. These norms are important since they shape students' classroom mathematical practices that are constructed by taken-as-shared ways of ideas. Additionally, while these norms support the formation of classroom mathematical practices, they also provide information about the features of classroom interaction between participants of the classroom community (Cobb et al., 1997).

According to the information above, there are two critical elements of classroom environment in which learning take place and those social and individual sides of learning. This is the same perspective with the one that social constructivism states. Accordingly, learning take place in the classroom environment with the equal effect of those two sides of the community. In the current perspective, students' understanding, and development of mathematics are evaluated throughout both their individual works and their participation to the classroom discussions and activities in which classroom mathematical practices emerge (Cobb et al., 2001; Cobb et al., 2011). Additionally, this perspective embraces two consecutive parts that each student makes some contributions to the classroom community by their individual works and that classroom community forms the classroom mathematical practices by the support of taken-as-shared ways of students (Cobb et al., 2011). Thus, in the current study, those perspectives of social constructivism are considered as a path to obtain and evaluate the classroom mathematical practices since they emerge by the students social and individual contributions to the classroom community. Also, parallel to the current study, Cobb et al. (2011), it is stated that in the literature, the studies focusing on evaluation of classroom mathematical practices generally use a design-based approach to link the theory and practice. It is possible to see various studies conducted to evaluate the classroom mathematical practices in different contexts (Bowers, Cobb, & McClain, 1999; Cobb et al., 2011; Stephan & Akyuz, 2012; Stephan et al., 2003; Stephan & Rasmussen, 2002).

Bowers, Cobb, and McClain (1999) defined classroom mathematical practices as classroom mathematical practices have students' ways of interpreting and solving specific instructional activities. Specifically, they explained classroom mathematical practices include teacher and students' discussions on problems and their solutions. Additionally, those classroom practices should be constructed on explaining, justifying, symbolizing, questioning, and arguing about specific tasks or contexts. In their study, as an illustration, interpretations and solutions that involved counting by one's was established mathematical practices at the beginning of the school year in participating second-grade classrooms. During the experiment, some students from those classes could be able to develop solutions related to conceptual understanding of units of ten and one. After doing that, students were obliged to explain and justify their interpretations of number words and numerals. At the end of the school term, solutions based on such interpretations were taken as self-evident by the classroom community. Doing the interpretation of number words and numerals in the related context was beyond justification and accepted as a classroom mathematical practice. This example serves to illustrate that an analysis of classroom mathematical practices focuses on shifts in ways of acting and reasoning mathematically that become institutionalized and hence are beyond justification (Bowers, Cobb & McClain, 1999).

Bowers and Nickerson (2001) designed a study to establish preservice teacher's mathematical practices in a dynamic geometry environment. In the study, preservice teachers involved in teaching sessions by using Geometer's Sketchpad. In an undergraduate course, their social norms, socio-mathematical norms, and mathematical practices are evaluated. Learning environment was constructed on classroom discussions. The study was performed by designing, testing, modifying and retesting the conjectured learning trajectory. The students' individual and collective learning activities were examined through social and socio-mathematical norms and mathematical practices. The study obtained four mathematical practices by using framework of Cobb et al. (1997).

In the study of Stephan and Rasmussen (2002) the classroom mathematical practices were examined during 15-week classroom sessions. They used the RME theory for the study and the participants were university students. The instructional sequence was designed through the context of differential equations for engineers. Students' learning of differential equations was examined through the students' classroom argumentations which are designed and guided by a learning trajectory and an instructional sequence. Toulmin's argumentation model was used to obtain the structure of the classroom discussions. To determine the taken-as-shared mathematical ideas which form mathematical practices, emergent perspective and a three-phase scheme were used. There have been six mathematical practices obtained that formed during the experiment. The researchers state that according to the results of the study, it is critical to form the classroom mathematical practices through the time and structure concepts.

Andreasen (2006) conducted qualitative study at an undergraduate mathematics education course for 16 elementary school teacher candidates. The study investigated classroom mathematical practices on the concept of place value and whole number operations. A design-based research approach was used for formulating the study with an HLT and instructional sequence related to place value and operations. The emergent perspective that aim to coordinate both individual learning and the social aspects of the classroom was used for data collection and analysis. Data analysis for the establishment of classroom mathematical practices was conducted using Toulmin's argumentation model. A three-phase approach described by Rasmussen and Stephan (2008) and Stephan and Rasmussen (2002) was used to determine classroom mathematical practices. The study provided insights for the refinement of the HLT and in defining an instructional theory for preservice teachers' understanding of place value and whole number operations.

Roy (2008) conducted a design-based research to evaluate preservice teachers' classroom mathematical practices in whole number concepts and operations. For this study, the researcher used the revised learning trajectory of Andreasen (2006). To obtain and analyze the classroom mathematical practices the same methods were used as, Toulmin's argumentation model and Rasmussen and Stephan's three-phase methodology (2008). There have been Four classroom mathematical practices evaluated.

Similarly, Stephan and Akyuz (2012) examined the classroom mathematical practices of a seventh-grade classroom with a design-based research. They also used the RME theory for the framework of the study. Through the classroom sessions of the participating classroom, classroom mathematical practices were evaluated via testing and revising an HLT in the context of integer addition and subtraction. The instructional tools that used for the study were financial tables and vertical number lines. With the guidance of HLT and instructional sequence, classroom mathematical practices were evaluated through 19 class sessions in the context of addition and subtraction of integers. To analyze the experimental process, Krummheuer's (2015) adaptation of Toulmin's argumentation model was used. By this way, the obtained logs that were used to identify the collective activities of the students that form the classroom mathematical practices. To obtain the classroom mathematical practices, a three-phase approach was used which is described in Stephan and Rasmussen (2002). Students' taken-as-shared ideas in the context of addition and subtraction of integers and the argumentation process of the students' construction of the conceptual understanding of related context were revealed the classroom mathematical practices. Results showed that there have been five mathematical practices obtained through the classroom sessions of the addition and subtraction of the integers. Additionally, researchers applied pre-posttests to the participant students to obtain and evaluate the effectiveness of HLT on students' achievement. The quantitative data from those tests implicated that with the support of instructional sequence that are prepared in the integers concept, students developed and improved their conceptual understanding on the related context more effectively.

Akyuz (2014) examined the classroom mathematical practices under the framework of RME and by using a design-based research approach. The participants were ten students from university which were from department of mathematics teacher education program. Also, eight of them were juniors and two of them were senior grade students. The study conducted during "teaching experiment" course which is an elective course for teacher education programs. Through the concept of circle unit, the participants' classroom mathematical practices were evaluated with the guide of a conjectured HLT by testing and revising it. The instructional tool that used of the study was GeoGebra which is a dynamic geometry software. The experiment continued for five weeks and four hours in each week. The classroom environment was constructed as an inquiry-based and technology-supported. To analyze the classroom argumentation of students', the Toulmin's argumentation model was used. To obtain and determine the taken-as-shared ideas of students which form the classroom mathematical practices, emergent perspective and the scheme described in Stephan and Rasmussen (2002) were used. Findings from the study showed that there have been occurred three sequential classroom mathematical practices according to complexity levels.

Uygun (2016), documented preservice middle school mathematics teachers' (PMSMT) classroom mathematical practices on instructional sequence about triangles during six-week. A conjectured hypothetical learning trajectory and an instructional sequence were planned and prepared for the experiment. By considering both collective learning activity of whole class discussions which constructed the social side of the classroom and also individual learning of each students, classroom mathematical practices were evaluated and analyzed. To determine the mathematical practices, Toulmin's argumentation model was used for extracting taken-as-shared ideas of the participants. At the end of the study, three classroom mathematical practices were obtained based on the triangles concept. Results of the study showed that PMSMT improved their conceptual understanding of the triangles with whole class argumentations and also with the support of other geometry concepts such as transformation geometry and geometric constructions.

Moyer-Packenham, Bolyard and Tucker (2014), conducted a study to understand the nature of children's mathematical practices better with an exploratory examination of the practices of second-graders. The participant students were involved in activities based on rational number concepts. 25 second-grade students were asked to complete three fraction tasks during structured clinical interviews. Students' works, and interviews analyzed and interpreted to determine the data which is beneficial for explaining the classroom mathematical practices of students. Constructs, themes, and patterns were used for the analysis process. A variety of mathematical practices were obtained during the study. Classroom mathematical practices were formed by students as a product of efforts to solve specific mathematical situations and also developed during the classroom interactions. The study provided some insights about how mathematical practices occur and what kind of activities promote the development of those practices.

Similarly, Özdemir (2017) used RME theory in her study which aimed to evaluate classroom mathematical practices in an RME based learning environment. The study conducted with preservice teachers' learning and teaching cone and pyramid. A five-week instructional sequence which is designed by a hypothetical learning trajectory about cone and pyramid was applied to preservice teachers. Five preservice middle school mathematics teachers participated to the study. In this qualitative research, the social learning environment of the classroom evaluated by Rasmussen and Stephan's (2008) three phase methodology which was developed according to Toulmin's argumentation model. Four mathematical practices emerged during the instructional sequence. The study showed that emergence of those practices supported by RME based learning environment. In that kind of learning environment, participating teachers had chances to express, share, criticize their own and others' ideas to reach the appropriate and right mathematical idea. Additionally, study implicated that RME supported learning environment may be helpful for emergence of mathematical practices by developing conceptual understanding of content by providing a collective learning community.

Pei, Weintrop and Wilensky (2018) conducted a study in a low-income, urban public high school. They implemented a computational learning environment called as Lattice Land, evaluated effect of the microworld on students' mathematical practices and observed whether it promote computational thinking practices in high-school mathematics classrooms. Lattice Land was a program that provide students to explore geometrical concepts by manipulating polygons on a plane. The microworld provided opportunities for learners to use computational thinking practices and develop mathematical practices such as experimentation, pattern recognition, and formalizing hypothesis. This study was an indicator of designing computational learning environments can support meaningful learning and enhance students' production of mathematical practices.

By reviewing the literature, it can be observed that studies evaluated the classroom mathematical practices in different levels and on different contexts. Also, since students' having challenges in learning and understanding of geometry, it is critical to find out what kind of practices do the learners form during the geometry learning process. Additionally, as mathematics learning is considered to be a social activity, for determining the mathematical practices, their relation to social and socio-mathematical norms should be considered as other dimensions of interpretative framework. Moreover, with the support of technology in mathematics education, it becomes critical to understand how usage of technology affects the formation of classroom mathematical practices (Akyuz, 2014). The technological tools can make it easier for students to develop different practices than they do in pen and pencil environment. Furthermore, there are still gaps in literature designing studies about using technology as an instructional tool. Thus, there is a need for evaluating classroom mathematical practices with the support of technological instructional tools (Akyuz, 2014). Accordingly, the current study was conducted for evaluating classroom mathematical practices of eighth graders with an HLT and instructional sequence under the RME theory for teaching three-dimensional solids using DGS.

The research of Johnson (2013) examined mathematical practices through notations and symbols which were different from others mentioned above. In the study, students' learning was evaluated through mathematical practices as local changes and making implications. The context was symmetries of an equilateral triangle under the RME theory. Additionally, the students evaluated the notations and symbols. Analysis were made by Toulmin's argumentation model and Rasmussen and Stephan's (2008) three-phase methodology (2008).

As a different perspective, Font and Planas (2008) focused on mathematical practices by emphasizing meaning of mathematical practices explained by Godino, Batanero and Font (2007). In that sense, it is important to put forward efforts while working on mathematical problems by discussing. They used an onto-semiotic approach to evaluate mathematical practices, socio-mathematical norms and semiotic conflicts (as different from other research). They focused on cognitive conflicts while evaluating the mathematical practices through discussing about solution of a mathematical problem. Learning said to occurred related to changes positioning of participants'. Accordingly, while students were solving those conflicts, semiotic conflicts were explored. In terms of socio-mathematical norms and mathematical practices, learning emerged through efforts of understanding other's ideas.

Harel (2017) brought a viewpoint by studying cognitive and instructional analyses of mathematical practice through discussions about field-based activities with in-service secondary mathematics teachers and students. They defined specific field-based hypothesis to find answers to the research questions that aimed to observe learners' mathematical behaviors in natural classroom settings. Explanation of mathematical practices included cognitive and instructional analyses of teaching and learning sessions. In the study, researchers organized specific hypothesis around four focus practices and evaluated the mathematical practices of learners in this respect.

In the literature, it is examined that there are various perspectives about evaluation of mathematical practices. Moreover, related to difficulties that learners having with geometry in terms of understanding it, it is critical obtain how and what kind of practices can students produce in geometry concepts. While thinking about mathematical practices, it should be considered the close relationship between social and socio-mathematical norms since they emerge in a collaborative and social learning environment. Evaluation of classroom mathematical practices during the subject of three-dimensional shapes was aimed by using argumentations and dynamic geometry software as instructional tools. In this respect, it was also important to plan a hypothetical learning trajectory to organize instructional sequence including activities and those tools. Moreover, it is critical to observe the ongoing process in terms of its meeting the needs of learners.

2.4 Hypothetical Learning Trajectory

In a design-based research, a hypothetical learning trajectory (HLT) is used as a guide and basis for developing instructional sequences. Simon (1995) first introduced the term Hypothetical learning trajectory (HLT) as a tool that is helpful for planning and describing the pedagogical thinking for teaching mathematics meaningfully. According to him, the teacher's learning goal provides a direction for a hypothetical learning trajectory. He used this term as referring to the teacher's prediction of the path in which learning may occur. The reason for being hypothetical is of the unknown feature of the actual learning trajectory is in advance. Thus, it is about an expected plan. Individual students' learnings occur in similar ways in general. Accordingly, an individual's learning may have some regularity in a way that the classroom community often produces mathematical activities in a predictable way in which most of the students in the same class may benefit from the same mathematical task. Preparing a hypothetical learning trajectory is a good way to provide a rationale for the teacher with the choice of a particular instructional design; while preparing an HLT, Simon (1995) suggested to try to make best predictions for how can learning of a specific content may occur.

Although Simon used the term hypothetical, recently mathematics education researchers prefer to use learning trajectories. Addressing learning trajectories, Confrey, Maloney, Nguyen, Mojica, and Myers (2009) stated that "A researcher-conjectured, empirically supported description of the ordered network of constructs a student encounters through instruction" (p.347). Additionally, Corcoran, Mosher and Rogat (2009) mentioned that learning trajectories shows students' progression of cognition, and also actual research roots learning trajectories in terms of students learning and reasoning mathematically. They defined learning trajectories as "a hypothesized description of successively more sophisticated ways student thinking about an important domain of knowledge or practice develops as children learn about and investigate that domain over an appropriate span of time" (Corcoran, Mosher, & Rogat, 2009, p. 37).

In the body of research, there are various explanations and definitions of HLT (Clements & Sarama, 2004; Gravemeijer, 2004; Simon, 1995; Simon & Tzur, 2004), it can be deduced some common features for them. For example, some of them state that learning trajectories are constructed on a specific mathematics domain (Daro, Mocher & Corcoran., 2011), underline the importance of using tasks and tools for emergence of communication between students in terms of mathematical concepts (Battista, 2004; Wilson, Sztajn & Edgington, 2013b), include an ongoing revision and refinement process called validation (Confrey & Maloney, 2011; Duncan & Hmelo-Silver, 2009). Confrey and Maloney (2011) add that learning trajectories aim to evaluate development of students' mathematical understanding and thinking. They also examine the starting point of students mathematical reasoning and the point they have reached. To sum up whole explanations above, it can be deduced that most of them agree upon the HLT includes three aspects as the learning goals, the instructional sequence of tasks to support those learning goals, and the expected developmental progressions of students (Andreasen, 2006).

In contrast to Simon's (1995) approach, Clements and Sarama (2004) express that some of the researchers give importance to the developmental processes of learning which is called as hypothetical learning process by Simon (1995). Clements and Sarama (2004) believe that those aspects are very important for them, and they have power to inform mathematics education with studying appropriate research aims, studies and contexts. In this sense, they realize those aspects and different views have a strong interrelation. They define learning trajectories as a description of students' thinking and learning in a mathematical content. Additionally, they see it as a conjectured route constructed by a set of instructional tasks which are designed to support students' understanding and achievement on specific domains in mathematics (Clements, 2002; Clements & Sarama, 2004).

Some researchers (Carpenter & Moser, 1984; Griffin & Case, 1997) specify learning models that define developmental progressions in a limited age range and in a specific culture. That is what researchers build a cognitive model of students' learning that is sufficiently explicit to describe the processes involved in the construction of the goal mathematics across several qualitatively distinct structural levels of increasing sophistication, complexity, abstraction, power, and generality. That is what researchers constitute a cognitive model of the learning of the students, so that they are sufficiently clear to describe the process of establishing mathematical goals at qualitatively different levels of structure, such as complexity, abstraction, and generality (Clements & Sarama, 2004). This constructivist understanding of learning trajectory distinguishes it from previous instructional design models that used reductionist ways to divide an objective into according to an adult's perspective. Fuson's (1997) curriculum explained this model with a study constructed based on children's solving of word problems that were increasingly difficult types of word problems. This theory states that when learning occurs consistent with that kind of natural developmental processes becomes more effective and generative for learners when compared to the learning that does not follow these paths (Clements & Sarama, 2004).

Additionally, Wilson, Mojica and Confrey (2013), asserted that as used in designing learning environments for students, it also could be useful at the level of curriculum development, assessment design, and in teacher education. They reported about two studies investigating prospective elementary teacher's practicing uses of a learning trajectory to make sense of students' thinking about rational numbers. Findings indicated that designing a mathematics learning trajectory supported teachers in terms of understanding students' thinking and in restructuring their own understandings of mathematics.

Wilson, Sztajn, Edgington and Myers's (2015) study evaluated teachers' learning of two frameworks those were for students' thinking in a particular domain and for broad student-centered instructional practices. They analyzed 19 lessons in which teachers participated in for professional development that designed to support their understanding of learning trajectories and student-centered instructional practices. Findings of the study explained brought together these frameworks to construct and enact instructional practices and use students' mathematical thinking in classroom. Also, the results supported that learning trajectories could be a referent for student-centered instructional practices and students' thinking styles of specific domain.

For the current study, the HLT is prepared initially as a framework of instructional sequence with expectations of how the class may involve in thinking and learning with the participation to the instructional activities. The HLT provides a basis to make decisions about the instructional tasks of the content. The learning goals of HLT are helpful in determining the instructional tasks which may support those goals. During the implementation of the instructional sequence (Stephan, 2015), the content is determined or modified by considering the learning that occurred during the preceding content and whether they matched or did not match with the expectations. Completing the teaching experiment, the instructional sequence refined and revised through a cyclical process of analyzing used content, the teacher's role, students' individual and collective learning. The classroom environment is evaluated for the realization of the HLT during the instructional sequence and necessary changes and instructional sequence are done in HLT for later iterations. The HLT is not an exact and scripted lesson plan, it is accepted and suggested as a framework prepared for the usage of teachers by adopting instructional sequences that fit with their own conditions and needs of students (Andreasen, 2006; Clements & Sarama, 2004; Simon, 1995; Simon & Tzur, 2004). Thus, the latest version of the HLT of the current study can be used by revising, adopting according to different environments in later researches or teaching experiments.

As a first step of development of an HLT, the developmental progressions of the participating classroom were considered. The HLT for the three-dimensional solids was constructed related to prior research that is an indicator of mathematical development for the specific domain being evaluated. This feature distinguishes HLT from other instructional design models in a way that giving importance to students' developmental progressions rather than teacher's choices (Clements & Sarama, 2004). To reach this feature of the HLT, with the support of the knowledge of research team's insights of children's conceptual development and with prior research, an HLT was developed to support children's development of conceptual understandings of three-dimensional solids. These formed a basis for the development of the HLT used in the current study.

The research on the conceptual development of students' in the context of three-dimensional solids then was used for informing and designing the instructional sequence in determining the manner and sequence. Tasks of the instructional sequence are designed including tools (specifically dynamic geometry software for this study) and actions to support the mathematical practices in which students are expected to involve in during the instructional process. The sequence of the tasks is designed intentionally based on the expected developmental progression of students. During the implementation process of the instructional sequence and analysis of individual and collective learning of the students', the tasks are modified, and the sequence evaluated to determine whether any changes in the HLT that may have taken place as a result of the implementation of the sequence. The revised instructional sequence may be altered for future use. The HLT and instructional sequence were implemented in this cyclical manner, completed with constant revision and review, until a local instructional theory was developed (Clements & Sarama, 2004; Gravemeijer, 2004; Gravemeijer & Cobb, 2013). With this aspect, HLT can be claimed as a powerful tool for curriculum development by which various mathematical topics can be developed and tested in classroom using design-based research in this manner. By revising and refining these kind of learning trajectories and instructional sequences, it can be established mathematics curriculum by this way. This can take place in all education levels starting from elementary level to the university level (Andreasen, 2006).

Learning trajectories are defined as being very useful for assessment (Battista, 2004). Moreover, by evaluating the effects of argumentations, they are expected to provide information about the nature of classroom environment which is designed based on a social constructivist perspective to evaluate the classroom mathematical practices. Those mathematical argumentations provide opportunities to the researchers to obtain and analyze the way students share their ideas, accept, or refuse other's thinking in classroom learning community. Also, this kind of discourse may support the students' participation to the learning environment.

Accordingly, the lessons of the current study designed with the support of the HLT by considering the usage of mathematical argumentations during the instructional sequence. By testing the classroom sessions in a hypothetical manner, it was expected to create powerful learning environments for students. Moreover, mathematical argumentations were expected to support students' thinking about the concepts of three-dimensional solids more effectively. In other words, mathematical argumentations may direct the students in a way to make reasoning on three-dimensional solids. By argumentation, the students could reason on how properties of those solids were formed by relating reasons. In this respect, students could construct the conceptual understanding of three-dimensional solids in a social constructivist learning environment including an instructional sequence supported with a conjectured HLT. Also, by evaluating the nature of the mathematical argumentations occurred during the students' participations to the classroom learning activities, it can be provided critical data about the process of teaching and learning session and identifying the classroom mathematical practices relatedly (Stein, Engle, Smith & Hughes, 2008). By this way, this study may offer powerful and supportive learning designs to the literature to provide support for students' conceptual understanding of three-dimensional solids.

As a summary, in the current study, to evaluate the classroom mathematical practices an instructional sequence was prepared with the help of the HLT that is based on the social constructivist theory and RME. The social learning environment in which classroom mathematical practices were formed, also identified the social and socio- mathematical norms of the classroom and supported by mathematical argumentations. In the following section, the place of argumentation in mathematics classes will be mentioned.

2.5 Argumentation in Mathematics Classrooms

The interrelation between interaction in mathematics classroom and learning of mathematics has taken attention (Krummheuer, 2015). By conducting a design-based research, it becomes important to evaluate how learning occurs in a social community and interaction (Cobb, 2000). Participation to the classroom discourse, provide opportunities for students to think aloud and make explanations about the ways how they think (Yackel & Cobb, 1996). Argumentative learning environments support students to interact with other people in that environment to get a meaningful understanding and learning. Steffe and Tzur, (1994) explain this process as creating confusions during the interaction with other people in the community and make them modify their own thinking schemes and learning occurs. The positive effects of communication by interaction with teacher-student and student-student occur inevitably on students learning (Lampert & Cobb, 2003). With argumentative classroom both the teacher and the students may benefit from that environment. While the teacher can create multiple ways for construction of mathematical understanding of students, students have chances to explain, judge, challenge, clarify and justify their ideas in related topics (Yackel & Cobb, 1996).

There are various definitions of argumentation in the literature. van Eemeren et al. (1996) defined argumentation is “a verbal and social activity of reason aimed at increasing (or decreasing) the acceptability of a controversial standpoint for the listener or reader, by putting forward a constellation of propositions intended to justify (or refute) the standpoint before a rational judge” (p. 5). This definition involves construction of claims, providing evidence to support that claims, and evaluation of such evidence to judge the validity of claims (Schwarz, Hershkowitz, & Prusak, 2010). It is asserted that in the mathematics classroom, the acceptable justifications are formed by negotiating socio-mathematical norms (Yackel & Cobb, 1996). Specifically, studies about mathematics education suggest that students' participation in that kind of activities promote meaningful understanding and deep thinking of mathematical concepts (Douek, 1999; Weber, Maher, Powell, & Lee, 2008).

Researches in mathematics education support the importance of students’ participation to the classroom argumentations by generating and commenting on other ideas (Balacheff, 1991; Ball & Bass, 2003; Krummheuer, 2007; Yackel & Hanna, 2003).

In this respect, Yee, Boyle, Ko, and Bleiler-Baxter (2018), evaluated effects of university students’ critiquing, constructing, and revising on mathematical arguments. Fifty-seven students of secondary mathematics methods classroom from four universities participated in an instructional sequence to define a valid proof through argumentations. Participants completed a proof-related task before class sessions, worked in small groups to evaluate other students’ arguments on context, based on their evaluations, they agreed upon criteria for said arguments. After class, they discussed and revised original argument to satisfy the common criteria. Results showed that students’ self-rating positively correlated with the argument categories, which is an indicator of effects of involving in a communal argumentation, creating ideas, critiquing, and revising other’s opinions.

In learning process, specifically, regarding the classroom argumentation, both the students’ participation and teacher’s role have equal importance in terms of providing and producing quality arguments that enhance conceptual understanding of mathematics. While students form the structure of the classroom argumentation, teacher should guide the students in a way of supporting them to participate in mathematical discourse. In addition to studies that support student participation in the formation of the argumentative classroom environment and positive effects of this participation on student learning, there are studies specifically focused on teachers' knowledge and practice in terms of argumentation (Kosko, Rougee, & Herbst, 2014; Mueller et al., 2014). Those studies evaluate impacts of such knowledge of teacher on the construction of argumentation on mathematics (Kosko et al., 2014). Research have evaluated the aspects of argumentative classroom environment (Ayalon & Even, 2016; Conner et al.,2014) and how the teacher might facilitate such environment (Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Mueller et al., 2014). These studies assert that teachers have critical roles in establishing norms of mathematical argumentation in the classroom. Teachers' roles are defined as listening to students, encouraging students to provide claims and justifications, considering different ideas and arguments of others' (Kosko et al., 2014). In this respect, Stein, Engle, Smith and Hughes (2008) offered five practices to help teachers to establish mathematical argumentation in classroom and how maintain it effectively. They proposed teachers to anticipate, monitor, select, sequence, and make connections between student responses, to establish an effective argumentation about mathematics. These key practices were about the teachers' orchestrating role about argumentations.

Yackel and Cobb (1996) investigated the teacher's role by regarding the classroom argumentation. They stated that teachers have a critical role in argumentation process by organizing the learning environment in this way. Also, the results showed that the social and socio-mathematical norms of the classroom have a powerful effect on formation on structure of argumentation since they are important for students learning process. In this respect, for the current study while creating the argumentative learning environment, the importance of teacher's orchestrating role and students' participation to the classroom practices in terms of mathematical argumentation was equally considered.

Yackel (2002), also investigated the teacher roles in terms of argumentation starting from elementary to the college level. The results showed that the teachers should provide a good point to start classroom mathematical argumentations about related concepts with the support of appropriate instructional tools. Moreover, the teachers should orchestrate instructional sequence parallel with classroom argumentations that critical for students' conceptual understanding of related mathematical topic. The result of this study was a good source for understanding the teacher's role in terms of starting and guiding the classroom argumentations.

Similarly, Van Zoest, Stockero, Leatham, Peterson, Atanga, and Ochieng (2017) investigated attributes of 278 instances of students' mathematical thinking during whole-class discussions that were identified as having potential to foster students' understanding of critical mathematical ideas. The aim was to identify attributes that foster students' mathematical thinking and reasoning. They defined pedagogical competencies that teachers should have as Mathematically Significant Pedagogical Opportunities to Build on Student Thinking [MOSTs]. Findings of the study revealed that teachers should give opportunity to students for sharing their mathematical thinking in the classroom environment. MOSTs were stated as providing high opportunities to foster learners' understanding of important mathematical ideas. Additionally, high linear correlation between instances of student mathematical thinking and MOSTs illustrated that the importance of teachers' creating opportunities for students to share their thinking with the class.

Kosko, Rougee and Herbst (2014) stated that students' increased understandings of and achievement in mathematics is strongly related to argumentation in classroom. They asserted that teacher's effective use of questioning strategies is a key component of mathematical argumentation. More specifically, this type of questioning requires students to explain and justify their ideas and relatedly enhance their understanding. In the study, the researchers evaluated teachers' types of questioning that were effective in enhancing classroom mathematical argumentation and students' understanding relatedly. At first hand, they obtained three types of teacher actions as; teacher statements, generating discussions, and teacher silence. These types of teacher seemed to decrease mathematical argumentation. On the other hand, they obtained three types of actions as, probing, orienting, and focusing. These type of teacher actions were identified as supporting students' active participation to classroom argumentation and facilitative for argumentative classroom environment.

In the same way, Ayalon and Hershkowitz (2018) evaluated secondary school mathematics teachers' paying attention to potential teaching situations that encourage argumentation. For this aim, 17 seventh grade teachers were asked to choose three tasks from a textbook which they were using in teaching practices. The choices were done according to teachers' views about tasks that may had the potential to encourage argumentation. Then, the teachers were wanted to justify their choices. Analysis of the teachers' responses revealed that the teachers' attentions were fall into three dimensions of attention about argumentation; (1) mathematics in which the argumentation is constructed, (2) socio-cultural aspects; and (3) students' ways of thinking. Additionally, the researchers categorized the findings according to combination of those dimensions. Viewed collectively, the teachers' explanations revealed that teachers seemed to attend rich dimensions of argumentation. Moreover, those dimensions of teachers' attention reflected the complex process of construction of argumentation in the mathematics classroom and the teachers' roles to facilitate argumentation. Further, combination of dimensions of attention supported that use of argumentations in mathematics classroom promotes learning.

Regarding the mathematics learning, mathematical argumentation is both a pre-condition and also a desired outcome (Krummheuer, 2015). In this respect, mathematical learning can be claimed as argumentative process. This process is based on students' participation to the practices of explanation and justification in classroom environment which is supportive for learning of mathematics. Thus, learning mathematics is considered as learning-as-participation (Krummheuer, 2015; Sfard, 2008). In mathematics classrooms, explaining is found both an individual and also a collective activity of classroom community (Yackel, 1995). Students contribute to those collective activities in various ways and situations. Thus, argumentation in mathematics learning is interest of both structure of course and also the ways how the teacher and the students are involved in that collective activities (Krummheuer, 2015).

Relatedly, argumentation can be defined as a kind of mathematical discourse regarding their participation to the classroom communication by explanations, justifications and using them in classroom discussions. Thus, construction of a quality argumentation process in mathematics classroom is strictly related to conceptual understanding of mathematics (Lampert, 1990). By providing a quality argumentation, students can improve reasoning skills on mathematical concepts by engaging in classroom interactions actively. This active participation comes from the dynamic nature of argumentation that require students should mentally involve in learning process by expressing, explaining, justifying, or refuting ideas instead of memorizing the structures and rules (Jonassen & Kim, 2010).

In his book "The Uses of Argument" which was printed in 1958, Toulmin introduced a model for argumentation including its components -data, claim and warrant. Also, he proposed three more auxiliary elements as qualifiers, backing and rebuttal which are not accepted by some researchers who are more critical and think about essentials for an argumentation structure (Krummheuer, 1995; Rumsey, 2012).

There are various researchers that used Toulmin's (1958) argumentation model in their studies in terms of mathematics education field (Krummheuer, 2007, 2015; Pedemonte, 2007). Furthermore, some of them used the model for analysis and evaluation of the structure of classroom discussion (Forman et al., 1998; Krummheuer, 1995, 2007, 2015; Wood, 1999; Yackel, 2001, 2002).

Krummheuer was the first researcher that adopted Toulmin's model and used in his study. He explained the adopted version of Toulmin's model in his book entitled "The Ethnography of Argumentation" in 1995. He used only three basic elements claim/conclusion, data and warrant in his study. Krummheuer (1995) states that argumentation is a kind of social phenomenon in which students try to express their ideas and thinking ways verbally related to their actions. This is the social interaction said to be occurred in the classroom environment. In a later study, he stated that argumentation "could not be created solely by single participants, because they could not have produced the contributions of the other participants based on their idiosyncratic definitions of the situation" (Krummheuer, 2000, p.31).

There are various definitions for these terms in literature. Some of them will be provided in the Table 2.2.

Table 2.1 Definitions for Elements of Argumentation

Element	Definition
Data	Facts we appeal to as the foundation of the claim, or minor premise (Toulmin, 1958, p. 101) Undoubted statement (Krummheuer, 2015, p. 56) The facts that serve as the basis for the conclusion (Walter & Johnson, 2007, p. 708)
Claim/ Conclusion	Conclusion of the argument (Toulmin, 1958, p. 101) The statement of the speaker (Pedemonte, 2007, p. 27) The statement to be proven (Krummheuer, 2015, p. 56)
Warrant	The statement authorizing the move from the data to the claim, or major premise (Toulmin, 1958, p. 101) The inference rule that allows data to be connected to the claim (Pedemonte, 2007, p. 27) Inference of an argument (Krummheuer, 2015, p.56)
Backing	Further reason to believe the warrant (Toulmin, 1958, p. 101) The statement that attempts to establish the authority of the warrant (Walter & Johnson, 2007, p. 708) Permissibility of warrant (Krummheuer, 2015, p. 56)

That kind of mathematical argumentations are expected to construct a common-shared understanding of specific mathematical concepts in determined domains. Constructing common-shared understanding through argumentative process, the students produce justifications, modifications of the mathematical concepts, statements and ideas used in mathematical discussions (Forman et al., 1998). Krummheuer defined this kind of argumentation as collective activity (Krummheuer, 1995).

There are also several studies focused on this collective feature of argumentation (Rasmussen & Stephan, 2008; Yackel, 2002). Accordingly, argumentations are very critical in obtaining the classroom mathematical practices since they are useful to examine the students' understanding by determining classroom mathematical practices when the ideas become taken-as-shared way of understanding by using the previous argumentations as a claim of later argumentations (Cobb et al., 2011; Gravemeijer & Cobb, 2013).

Krummheuer's (2015) argumentation model provides a background for the current study. In that study, Krummheuer analyzes argumentation using Toulmin's scheme of data, conclusion, warrant, and backing. According to the scheme, the conclusion is a statement that needs to be proven. It is a claim. If one gives a support to the conclusion, that is data. Warrant is an explanation why the data are considered to provide support for the conclusion. Backing refers providing further support for the warrant with undoubtable convictions (Krummheuer, 2015; Yackel, 2001). By this way, Krummheuer explains how an argumentation occurs as an interactive construct of social community. According to him, argumentation "contains several statements that are related to each other in a specific way and that by this take over certain functions for their interactional effectiveness" (Krummheuer, 1995, p. 247). Those statements are occurred as a part of the interaction of the environment in which they are situated, so, it is not possible to predetermine the statements that constitute the data, conclusion, warrant or backing since they are negotiated by the discussions in which students involve in (Yackel, 2001).

This is considered as a beneficial and helpful approach for documenting the collective learning of a class in terms of argumentation because it provides a tool to show the changes that take place over time (Yackel, 1997; Yackel, 2001).

Furthermore, it is a good way to show the relationship between the individual and the collective participating. In more explicit words, it is helpful to clarify the relation between the explanations and justifications given by individual students in specific context and the taken-as-shared ideas which become the classroom mathematical practices (Yackel, 2001). As, classroom mathematical practices become taken-as-shared in classroom community, they need something more than justification as data, warrant and backing. Also, the statements given as data, warrants, and backing for explanations and justifications are constructed bases for the development of taken-as-shared ideas in the classroom community.

Yackel (2001) gave as a sample analysis about the issue. It is an example from a second-grade classroom's involving in thinking activities. For the problem of five plus six, students gave some explanations based on the five plus five makes ten. One student constructed her answer on six was one more than five, thus the answer should be 11. After that, one student only expressed his/her idea by stating five plus five was ten, thus the answer was eleven. She states that, since there was no questioning in the second type of explanation, it could be concluded that the warrant and backing were in the earlier explanation, so it become taken-as-shared in the classroom. Thus, through the explanations, the taken-as-shared understandings develop at the same time. As a final claim, this approach is useful and helpful for analyzing classroom discourse by evaluating the contributions made by participants during the process (Yackel, 2001).

There have been various studies related to classroom mathematical practices by considering argumentation as a tool (Andreasen, 2006; Stephan & Akyuz, 2012; Stephan & Rasmussen, 2002; Roy, 2008; Wheeldon, 2008). For example, in the studies of Akyuz (2014), Andreasen, (2006), Johnson (2013) and Roy (2008) the classroom mathematical practices of participants were evaluated in collective learning environment. In the analysis of collective learning environment and determination of classroom mathematical practices, Toulmin's model of argumentation was used. Those classroom mathematical argumentations illustrated how the students' ideas become taken-as-shared. Another research of Stephan and Akyuz (2012) examined the classroom mathematical practices of a seventh-grade classroom with a design-based research. They used the Krummheuer's (1995) adaptation of Toulmin's model of argumentation was used as a tool of analysis of students' taken-as-shared ideas.

In their study, Giannakoulias, Mastorides, Potari and Zachariades (2010) investigated teachers' argumentation. The aim of the study was to persuade students about their inappropriate claims about calculus. With this aim, the researchers gave 18 secondary school mathematics teachers three scenarios including a student's proof that constructed on invalid claims. The participating teachers obtained possible mistakes of students and their way of refuting them. Interviews conducted with two teachers. The data analysis was done according to the content and structure of argumentation. Also, the type of counterexamples was at the focus of the analysis. Results indicated that teachers used two approaches to refute students' inappropriate claims as using counterexamples and theory. For the analysis of these argumentation process Toulmin's model was used and they obtained three types of reasoning in terms of structure of argumentation. This study showed importance of teachers' orchestrating role during argumentation process.

In the same respect, Pedemonte and Balacheff (2016) evaluated students' conceptions in geometrical problem-solving through argumentations. The main aim was to show relationship between students' conceptions and argumentation. Additionally, they evaluated how students' conceptions affected construction of a proof. Data was collected from a teaching experiment and analyzed through argumentation of 15 pairs of students. Toulmin's model was used for analysis of the data set that combined with the ckc (conception, knowing, concept) model. They explained the reason for enriching Toulmin's model with ckc as, making explicit students' knowledge base during the argumentation process and making a better characterization for elements in mathematical argumentation. The results revealed that there was a continuity between argumentation and proof. Also, they identified that use of argumentations during classroom activities facilitated students' participation, collaboration, and communication, and relatedly their understanding of the context.

In the environments supported by argumentations, usage of the instructional tools is helpful on students' learning and understanding of the related concepts. By this way, it is possible to form quality and effective argumentations with looking and understanding of the other students' statements produced by also using tools. Moreover, in conceptual understanding and learning of mathematical concepts, DGS may be used as an effective tool (Athanasopoulou, 2008). In this respect, Akyuz (2014) used GeoGebra as a tool for documenting the classroom mathematical practices observed in a university teacher education course related to the circle topic. She used the Toulmin's model of argumentation to analyze the interactions occurred in the classroom environment. The study was an important one because it was providing information about learning in a social context with the support of technological tools.

Also, there has been a research in the literature that examined the effect of the usage of technological tools on the quality of argumentation formed by taken-as-shared ideas in classroom environment. DGS said to improve the peer interaction which is a critical element of argumentation. In this respect, Vincent, Chick and McCrae (2005) studied with eight-graders by focusing on peer interaction. They applied tasks to the students including working on proofs by using paper and pen, and by using DGS. They obtained the quality of social interaction and relatedly argumentation affected by quality of peer interaction in all tasks they applied.

Hollebrands, Conner and Smith (2010) evaluated the nature of the arguments with the support of DGS. They studied with college students in terms of hyperbolic geometrical tasks. The argumentations were examined in three groups related to features of the warrants of those arguments. The groups named as explicit warrants without technology, explicit warrant with technology and warrant on the screen. The results indicated that students who were working on tasks about justification and proof did not use technology, but in their arguments, warrants appeared explicitly. Reversely, students used technology in their works, did not provide explicit warrants in their arguments. Researchers pointed out that this indirect relationship occurred because of the use of technology were not widespread in the traditional classroom environment and that students were unfamiliar with that kind of environment. Additionally, the results showed that the properties of warrants were related to usage of technology. It was observed that in the technology supported environment, students could produce qualified and effective warrants when compared to non-technological environments.

Similarly, in Lavy (2006), lower grade students' argumentations were examined by regarding the effect of technological tools. The study used Toulmin's model of argumentation to analyze the content and structure of the arguments occurred during the classroom sessions with the support of technological tools. The findings of the study showed that technology had a positive effect on students' understanding of the related content and producing quality warrants including necessary statements.

Thus, it can be claimed that technological tools such as dynamic geometry software may have a supportive role in enhancing students' argumentations on related context. By using technological tools effectively in learning environments, it may provide students to produce clear warrant those understandable to others. Relatedly, the current study used GeoGebra as a technological tool to support learning environment in terms of enhancing the students learning of mathematics by making them involve in classroom mathematical activities.

In Prusak et al. (2012), they conducted a study focusing on argumentation with DGS support. They used Toulmin's argumentation model in the study. They evaluated interaction of two preservice teachers in reasoning process during their argumentation. They focused on obtaining the core elements of Toulmin's argumentation model as conclusion, data, warrant and backing. The design of the study included a conflict situation, a collaborative situation and a usage of DGS to check conjectures of participants. The results indicated that those study situations by checking hypothesis with DGS, promoted the argumentation in a productive way. Additionally, Toulmin's argumentation model was seemed to be supportive for observing dynamic chances in collective argumentation process. Hanna and De Villiers (2008) mentioned important of dragging option of DGS. They explained this option was critical since it provide opportunity to observe different positions of a geometric figure dynamically. Dragging is also important, because it is accepted to provide evidence for students' argumentation on context (De Villiers, 2003; Healy & Hoyles, 2000).

Conner, Singletary, Smith, Wagner & Francisco (2014) studied about collective argumentation and its effects on mathematical reasoning. Different from others, they asserted that one perspective is not sufficient to explain all mathematical conversations in classroom environment. Thus, they constructed a model by combining both Toulmin (1958) and Peirce (1956) argumentation structures. They believe this may provide an insight into classroom argumentations and in students' reasoning. Combining those two perspectives may help mathematics educator researchers to analyze various reasoning types emerge in mathematics classes, determine effects of different types of reasoning on students' learning of mathematics, and examine students' learning to construct mathematical arguments in classroom environment.

Brown (2017) studied on students' engagement with mathematics by using socio-cultural theory. In the study, researcher used collective argumentation as socio-cultural approach. The collective argumentation was based student engagement in the classroom discourse. The aim of the study was to explore the affordances that could enrich student engagement with mathematics by using collective argumentation. The design of the study was a teaching experiment and conducted with primary and secondary school teachers with their mathematics classes. Data collected through interviews, report writings, journal entries and observational records and were analyzed by using a participation framework. Findings illustrated that collective argumentation can be used by teachers to promote students' engagement with mathematics by giving opportunities explaining and justifying ideas in a wider community.

By reviewing the literature, there have been many powerful researches about argumentations in the mathematics classrooms. Most of them concluded the importance and positive effect of argumentations on students' by enhancing their learning of specific contexts and by improving their academic achievement. Still, there is need for evaluation of classroom mathematical practices formed with argumentation of classroom community on the context of geometric concepts. Specifically, the current study evaluated classroom mathematical practices formed with argumentation of classroom community by conducting a design-based research using RME and social constructivism as frameworks on the concept of three dimensional solids. Thus, it is expected that the current study may improve learning and understanding of eighth-graders through argumentations.

2.6 Learning Geometry by DGS

When technology is used in appropriate forms in mathematical classes, it deepens the mathematical understanding. Use of computers in mathematics education develops mental skills such as research, reasoning, assumption and generalization (Wiest, 2000).

Different computer tools play different roles in improving students' thinking skills. However, the main goal should be to recognize the opportunity to act as a mathematician (Noss, 1988). For this reason, the purpose of the computer is to use the computer as a tool that allows the student to make assumptions, test and generalize; provide them opportunities to get the idea of mathematical outcomes, as well as providing students with a unique style of thinking (Cuoco & Goldenberg, 1996).

Briefly, the proper use of a computer in mathematics education can be expressed as a computer that helps students achieve high-level cognitive skills development. It is envisaged that dynamic geometry software which reflect the rapid developments experienced in computer technology to geometry classes, can help mathematics education to achieve these goals (Choi-Koh, 1999; Oldknow & Tetlow, 2008).

In the literature, many studies stressed support of DGS in geometry lessons (Clark-Wilson, & Hoyles, 2017; Çetin, Erdoğan, & Yazlık, 2015; Hollebarands, 2007; Leung, 2011; Oldknow, & Tetlow, 2008; Sack, 2013). Currently, DGS seem to be one of the most popular types of software used by mathematics teachers (Mariotti, 2001) and investigated by researchers. Mariotti (2001) states that, “This software seems to make the exploration of geometrical configurations and the identification of meaningful conjectures more accessible to pupils” (p.257).

To give a definition for dynamic geometry software may mean imprisoning it today (Güven & Karataş, 2003). Because of the technology is growing with gigantic steps, it is inevitable to take place in this technology for the changes. Although avoiding giving definitions for DGS, researchers prefer to mention some properties that characterize DGS (Şimşek, & Yücekaya, 2014).

Geometric shapes can be created very easily, measurements can be made to determine the features of the shapes (angle, circumference, length, area measurements, etc.). Shapes can be dragged and rotated on the screen, which allows the student to change shapes while observing unchanging features and dynamically changing previously measured quantities when the structure is moved (Couco & Goldenberg, 1996). With the help of this feature, hypotheses about the structure can be established, hypotheses can be tested, and generalizations can be made while the change of structure is observed. All aspects of transformation geometry can be studied (Choi-Koh 1999; Hölzl, 1996). Since, it is difficult to develop students' visualization skills through traditional learning environment in which students get involve in classic board by trying to reflect 3-D objects on it. Because, classic classroom board is only appropriate for drawings of 2-D shapes. Trying to represent those 3-D objects on 2-D environment is very complicated and time consuming (Christou, Pittalis, Mousoulides, & Jones, 2005). Accordingly, the aim of DGS is to enable students to construct geometric figures by observing their various positions under different transformations in space; and also make them focus on modeling those geometric situations related to their observations (Christou, Pittalis, Mousoulides, & Jones, 2005). In their study, Laborde, Kynigos, Hollebrands & Strasser (2006), worked with secondary school students to evaluate their understanding of geometry concepts by using DGS. The results indicated that DGS supported students' understanding of specific geometry concepts with its' providing opportunity to create shapes in different sizes and detecting the traces and locus of them.

Similarly, Hanna and De Villiers (2008) mentioned importance of dragging option of DGS. They stressed that this option as critical since it provides opportunity to observe different positions of geometric figure dynamically. This is also as important feature since it provides evidence for students' argumentation (De Villiers, 2003; Healy & Hoyles, 2000).

Dynamic mathematics software such as GeoGebra, Cabri, and Geometer's Sketchpad at first was designed for secondary schools to support geometry learning at first. Those software materials give opportunities for students to discover relations and patterns in geometrical concepts, to explore and to test conjectures by their own mental construction process (Goldenberg & Cuoco, 1998; Hazzan & Goldenberg, 1997). Dynamic mathematics software is very powerful for teaching and learning activities in mathematics and geometry; and it has been reported as being supportive for students' conceptual development, enhancing mathematics teaching, making easier the visualization of geometrical concepts, providing opportunities for creative and high-level thinking (Sanders, 1998). School students can improve their understanding by using software because the dynamic environment improves visualization skills and also ability to focus on interrelationships of the parts of geometric shapes (Clements, Sarama, Yelland & Glass, 2008). With all these aspects, Battista (2007) argues that usage of DGS also in elementary level geometry lessons makes much richer and more powerful learning of geometry rather than paper-pencil method; gives chance students to explain and justify their thinking and reasoning which supports classroom mathematical practices; and how it effects students' geometric and spatial thinking in positive way which means an increase in their achievement at the same time.

Research has shown that geometric software with dynamic properties offers students the opportunity to focus on much more abstract structures than commonly used paper-pencil works (Hazzan & Goldenberg, 1997). As it is mentioned before, trying to represent solids on flat surface lacks the students' opportunities to visualize those solids. Those flat surfaces are static and do not have any spatial depth, thus they do not have any manipulation, adaptation features to provide effective learning environment for students (Christou, Pittalis, Mousoulides, & Jones, 2005). DGS provides those opportunities for students. In this way, the imaginative power of the learner increases. The increasing power of imagination in mathematics means the opening of the way of creation and exploration. When these paths are opened, the student will be able to analyze, hypothesize and generalize. This directly improves the problem-solving skills of the students (Baki, 2001). Dynamic geometry software offers geometry teaching by supporting and researching student experiences. It offers new possibilities for the geometry taught in the same way for many years (McClintock, Jiang & July, 2002).

Another research showed that dynamic software programs make students' connecting algebra with geometry and dynamic graphics much easier (Ferrara, Pratt, & Robutti, 2006). In another study, it was obtained that, specifically low-achieved students choose to use the charts in solving math problems and this dynamic software provided them to solve their algebra questions more quickly (Yerushalmy, 2006). Clark-Wilson and Hoyles (2017) stated that DGS supported learning environment supports students in a way to share, discuss or accept/reject others' ideas; clarifies mathematical concepts through a planned instruction and interaction; and also helps to develop usage of mathematical language to increase shared understandings of students.

In the literature, a variety of researches are studied and discussed on the effect of usage of technology and relatedly usage of DGS in geometry lessons on pre-service teachers' (Agyei, & Benning, 2015; Pittalis, Christou, & Pitta-Pantazi, 2012) or in-service teachers' pedagogical content knowledge and on their designing and providing effective learning environment for students (Altaylı, Konyalıoğlu, Hızarcı, & Kaplan, 2014; Clark-Wilson, & Hoyles, 2017).

With the help of dynamic geometry program, students are able to learn the geometry assumptions and produce mathematical results by testing these assumptions (Hadas, Hershkowitz, & Schwarz, 2000). Many studies compared the DGS and the traditional method of teaching and results showed that the students' academic achievement more increased with DGS (Healy & Hoyles, 2002; Hollebrands, 2003, 2007; Ubuz, Üstün, & Erbaş, 2009). In Moyer and Niezgoda (2003), kindergarten students worked on patterns using pattern blocks both by software and physical wooden pattern blocks, and with drawings. Results indicated that students were more creative in constructing patterns by using software when compared to the wooden blocks and drawings.

Hohenwarter, Hohenwarter, Kreis and Lavicza, (2008) evaluated that how a calculus course can be designed by using GeoGebra. They also stressed that these kinds of interactive applications would be helpful for students' development of critical calculus concepts by integrating dynamic visualization of those concepts.

Marrades and Gutierrez (2000) evaluated how a geometry lesson would be designed with Cabri application to have an effect on students' proof abilities. They stated DGS with its dragging feature let students see various examples in a short time and get feedback at that time. By these exercises, this helped students to look for links between shapes and properties, understand specific properties which is important for constructing a conjecture or justification for related issue. Similarly, Andraphanova (2015) evaluated the GeoGebra as a tool that involve didactical opportunities such as visualization and dynamics. She mentioned about the opportunities of DGS in terms of differences from traditional methods in learning and teaching geometry. She stated that features of GeoGebra would be helpful for understanding challenging concepts, giving opportunities to develop "active mathematic vision" (p. 127) of those concepts. In a similar study, Almeqdadi (2000) investigated students' understanding of some geometrical concepts by using DGS. The statistical results showed a significant difference between the means of the students' scores. The experimental group had higher mean score from the control group regarding the posttest scores. Usage of the Geometer's Sketchpad had a positive effect on students' understanding of the geometrical concepts. Technology helped to create more student-centered instruction, supports cooperative learning, and enhances teacher-student interaction (Almeqdadi, 2000).

Marinas and Furner (2006) conducted a study based on teaching geometric concepts with the support of DGS. The participant group was chosen from kindergarten to fourth grade. With the study, Geometer's Sketchpad was introduced to students and content designed with it. The results indicated that DGS may be an appropriate tool for K-4 levels as well as for higher grades. It seemed to be helpful for making students to understand the content meaningfully. The use of DGS at primary level ensures or encourages students to take an active role in their own learning. Such experiences form the basis for students' ideas for abstract mathematical relations for future mathematics lessons. DGS can be used at all levels of education, starting from primary school to university. The use of DGS by younger students plays a more prominent role in this progress, since students will be technologically advanced as they age when compared to five or ten years earlier (Marinas & Furner, 2006).

The study of Tutak, Türkdoğan and Birgin (2009), investigated fourth grade students' geometry levels by using Cabri. They used a semi-experimental method. In the experimental group, the participants learned the geometry lessons with usage of Cabri software. For data collection process, a multiple-choice pre-posttest were applied. Results indicated that there was no meaningful difference between students learning by regarding information level, but understanding, applying, and analyzing levels of the students showed statistically meaningful difference.

In the literature, DGS was seemed as a motivation tool for students in terms of its providing various examples and supporting improvement in classroom (Ruthven, Hennessy, & Deaney, 2005). Tabuk (2003) showed that at 7th grade geometry lessons based on the context of circle, spherical and cylinder, usage of DGS in geometry lessons has positive effects on students' achievement. Baki ve Özpınar (2007) showed that the use of DGS in geometry lessons increase and support achievement, attitude and understanding of student. Sulak and Allahverdi (2002) concluded the positive affect of DGS on students' success and attitudes in sixth grade classroom. Furthermore, Hollebrands (2007) showed the relationship between DGS usage and understanding of geometry concepts.

In Obara (2009), he stressed that it is important for students to observe the relation between the surface area of a three- dimensional solid and area of its net. To provide this understanding for students, he conducted a study with support of DGS in the context of the surface areas of square and rectangular pyramids and a cone. Students stated that it was a great experience for them trying to construct the formula of the surface area, and that was a new practice for them. They stressed Geometer's Sketchpad was very helpful in this work. Healy and Hoyles (2000) focused on DGS' enabling students to construct various reasoning by using another one, thus they can observe and understand interrelation between geometric concepts and objects. Relatedly, Gonzalez and Herbst (2009) stated that when compared to paper and pencil environment, DGS users involved in in-depth thinking of geometric relations between concepts by evaluating them dynamically.

Yanık (2013) studied with four prospective middle school mathematics teachers, to explore their understanding of geometric translations by using GeoGebra as a pedagogical tool. Findings of that study confirmed that usage of DGS supported the prospective teachers' understanding of geometric translations. More specifically, the study stated that dragging and measurement features of DGS program supported prospective teachers to evaluate the properties of geometric translations, make conjectures, apply various strategies, and construct new ideas by this way.

Perry and Steck (2015) conducted a study to evaluate the effect of using iPads in geometry education in terms of student engagement and discourse, their achievement, self-efficacy and meta-cognitive self-regulation. Students in the iPad-using classroom experienced get lower results in geometry scores but their engagement and discourse, self-efficacy and self-regulation were increased when compared to non-iPad users.

Greefrath, Hertleif and Siller (2018) investigated the competence of mathematising with 709 students. The test group worked with digital tools while control group worked with paper and pencil on the same tasks during a four-lesson intervention on geometric modelling tasks. Comparing results of two groups, they obtained a significant improvement of mathematising in both groups. This development was also investigated in terms of the used software's effects on attitudes and program-related self-efficacy. They found that program related self-efficacy was a significant predictor of gaining competency while attitudes did not.

In accordance with these findings, to remedy shortcomings in the field of education in our country, the curriculum was decided to renew in 2005. In this respect, the constructivist approach was taken as the focus, and aimed an education without memorization in which students could make connections between math subjects and daily life (MoNE, 2013). In addition to this change, the computer technology classes were established to increase student achievement as much as possible in schools to disseminate the technology with the whole class. Apart from this, it was added to the program as a preference of the usage of dynamic geometry software regarding their support for implementation of multiple representation approach (MoNE, 2013).

Accordingly, current study used GeoGebra as a tool for instructional sequence with argumentations in classroom sessions. The data collection process in context of three-dimensional solids was performed with the guidance of the prepared instructional sequence and HLT.

2.7 Social Constructivism and Emergent Perspective

Gravemeijer and Cobb (2013) state that conducting a design-based research requires scientific interpretations of the data translation ways of classroom observations. It is a necessity to use an interpretive framework to make sense of the collected data from classroom environment. They stressed the importance of explicating the data systematically based on the interpretive framework. There are two key elements of an interpretive framework of the study, one for interpretation of classroom learning environment, and one for interpreting students' reasoning and learning mathematics (Gravemeijer & Cobb, 2013). In the following section, the framework is explained that is used for interpretation of classroom communication and discourse, and later on, Realistic Mathematics Education that is the domain specific instruction theory which is used as a conceptual framework to interpret students' learning. Thus, social constructivist theory is clarified as a background to the current study.

The analysis of the mathematical practices formed by classroom activities and discussions and its effect on student learning was conducted through the emergent perspective (Cobb, 2000; Cobb & Yackel, 1996; Stephan & Cobb, 2003). Stephan (2003) states that the emergent perspective includes coordination of both social and individual perspectives on mathematics learning. Learning has a psychological side on the part of the individual learner and also has a social side on the part of the learning group or classroom environment (Stephan, 2003). Also, Cobb (2000) adds "a basic assumption of the emergent perspective is, therefore, that neither individual students' activities nor classroom mathematical practices can be accounted for adequately except in relation to the other" (Cobb, 2000, p. 310). The emergent perspective makes students learning of mathematics placed in the social context of the classroom (Cobb, 2003).

Additionally, the emergent perspective stresses the importance of the analysis of classroom mathematical practices as it is situated in classroom social context. Because the students' mathematical development may include both coordination of psychological analysis of their individual activities and also social analysis of norms and practices (Cobb, 2000). Thus, it is not possible to separate the individual and the classroom community from each other and “the existence of one depends on the existence of the other” (Stephan, 2003, p. 28). Therefore, both the social and psychological perspectives are equally important in organization of the analysis of collective mathematical learning of the classroom community (Andreasen, 2006). Table 2.1 shows both two perspectives.

Table 2.2 Emergent Perspective (Cobb & Yackel, 1996)

Social Perspective	**Psychological Perspective**
Classroom social norms	Beliefs about one’s own role, others’ roles, and the general nature of mathematical activity in school
Socio-mathematical norms	Mathematical beliefs and values
Classroom mathematical practices	Mathematical conceptions and activity

Teachers’ understanding of learning is a process of both individual and social construction that provides them a conceptual framework for understanding the learning of students (Simon, 1995). Wood, Cobb, and Yackel (1995) asserted that teachers expected to construct the mathematical practices according to the learning ways of mathematics. It is the main challenge that researchers and mathematics educators -especially the mathematics teacher educators- face with (Simon, 1995). It is critical to reconstruct what it means knowing and doing mathematics in school and accordingly how to teach mathematics in that way.

The importance of social constructivism comes from its being the emergent perspective for the current study that developed the classroom mathematical practices. Social constructivism is said to be a kind of constructivism that specifies the context socially and defines the culture and learning collaboratively (O'Donnell & King, 1998). According to social constructivism learning occurs with social interaction of learning environment since it has some socio-cultural aspects. The idea of social constructivism developed related to Vygotsky's ideas (Palmer, 2005) by considering the effect of language used between learner and other people, and also the effect of this interaction on the other people situated in that learning society (Jones & Brader-Araje, 2002). Thus, it is mainly interested in the effects of language, communication, and culture on the learning which is a continuous process (Fosnot, 1996). Vygotsky advocated that the level of individual learning can be increased by interacting with the other people on the related issue. Thus, the knowledge gained by interaction with other people may be much more than the knowledge gained by working alone (Liang & Gabel, 2005).

In studies conducted by taking the social constructivism as background, the teachers' role is defined as organizing learning environments to support learners. By this way, they can support and improve skills such as analysis, critical thinking, and deep understanding (Trigwell, Prosser & Waterhouse, 1999). Accordingly, social constructivist approach has positive effect on learners by providing powerful learning environments. Thus, it is beneficial to conduct design experiments by using social constructivism as a background (Woo & Reeves, 2007). In this way, design experiments were used in the present study by organizing various instruction and working by and on them systematically to provide a powerful learning environment for the students. Moreover, design experiments may be conducted to provide effective learning environments for the learners. By using this perspective, it was aimed to evaluate the students' construction of learning from various ways. Thus, whole instructional design of the current study was organized around this philosophy.

For the current study, social constructivism was used as framework. In other words, to examine the mathematical practices that formed during the classroom sessions, an interpretive framework was used for explanation of learning based on psychological perspective and social perspective as mentioned above (Cobb, 2000; Cobb & Yackel, 1996). According to emergent perspective, students determine their mathematical understanding during the community works by making contributions for the emergence of mathematical practices (Cobb & Yackel, 1996; Yackel & Cobb, 1996). In this respect, to identify and determine the classroom practices both the working of individuals and the groups are considered equally

(Cobb, 2000). For the analysis of classroom learning an interpretive framework offered by Cobb and Yackel (1996) used as illustrated in Table 2.1 above.

This study aimed to examine social aspects of the classroom which were supported by individual learning. As mentioned above, the social perspective interested in evaluation of the social norms, socio-mathematical norms, and mathematical practices of the classroom by considering the collective learning of students in mathematics classroom. The social norms of classroom refer to the taken-as-shared ways of students' participation to the whole class activities. The term taken-as-shared was defined by Cobb, Wood, Yackel, and McNeal (1992) as when students' understandings, explanations and interpretations become compatible with classroom dialogue and activities, this is what they called those interpretations and meanings become taken-as-shared. Social norms may include some processes such as students' developing meaningful solutions to problems, explaining or justifying solutions, trying to understand other student's ideas, and asking questions to the arising misunderstandings or disagreements (Yackel, 2001).

The psychological aspect of the emergent perspective focuses on the individual's reasoning on specific context and the student's ways of conducting interactions with the classroom community (Cobb & Yackel, 1996). The students' ways of interacting with other members of the classroom community and how that interaction supports and develops the individual learning is closely linked to the social and socio-mathematical norms of the classroom. As mentioned above, the social and individual aspects of the emergent perspective go parallel with in a way that during the examination of social aspects, each student's individual learning has a contribution to the development of taken-as-shared mathematical ideas since they formed in classroom community. Also, during the examination of the individual student's understandings of mathematical ideas, the social aspects enlighten the individual student's participation in the whole classroom activities. Therefore, for an appropriate and complete analysis process of classroom sessions, the social and the individual aspects should be coordinated through its support of individual student's learning and collective mathematical understandings (Cobb et al., 2001; Cobb & Yackel, 1996).

2.8 Realistic Mathematics Education (RME)

In order to produce the basic philosophy of a design-based research, you must understand the innovative forms of education that you may wish to bring (Gravemeijer & Cobb, 2006). Gravemeijer and Cobb (2006) accordingly states that this is consistent with the phrase, "If you want to change something, you have to understand it and If you want to understand something, you have to change it" (p.17). Therefore, the RME has emerged because of the need for a change in mathematics education.

Realistic mathematics education is a domain of specific instruction theory for mathematics (Van den Heuvel-Panhuizen & Drijvers, 2014). According to Gravemeijer and Cobb (2013), this theory has emerged as a response to teaching approaches in which mathematics is used as a ready-made product. Freudenthal (1973), argued that mathematics should be a series of activities for students. A re-invention period in the guidance of the teacher, and subsequently, this mathematical activity should ensure that students encourage the imagination of mathematics as a unit of knowledge. This requires that the starting points of instruction should be real for students. This means that problem situations must be presented to students as they can reason, and they can actively take part in the solution. The main aim in this process is that the mathematics developed by the students should be real for them. In other words, one's learning of mathematics really depends on how much it combines with real life (Gravemeijer & Cobb, 2013). In this study, the term Realistic Mathematics Education will be abbreviated as RME.

RME is offered to have rich and realistic context in mathematics learning processes. The context may be everything that construct a source for the development of mathematical tools and procedures; and a context in which students can progress step by step to another stage by applying the mathematical knowledge in following step and going from informal knowledge to formal one. The real-world situations are stressed in RME and realistic context has a broader meaning here. Realistic; it is not only the necessity to produce a problem from the things that come from real life, but the students should be able to visualize the problem situations in their minds which are presented to them. So, mathematical problems can come from real life as well as imaginary world, as long as they can revive students in their minds (Van den Heuvel-Panhuizen & Drijvers, 2014). In this study, the participant teacher used the instructional sequence (Stephan, 2015) that was prepared based on the RME.

As mentioned above, RME was formed as a reaction to traditional mathematics teaching approach in which students are static receivers of knowledge. But, Freudenthal, thought and considered that mathematics would be an activity in which students can actively participate in education process (Van den Heuvel-Panhuizen & Drijvers, 2014). Thus, according to RME, the mathematical teaching and learning process should include both individual and social participation rather than being taught in a closed system (Cobb & McClain, 2001; Van den Heuvel-Panhuizen & Drijvers, 2014).

Six principles of RME is offered in the literature (Van den Heuvel-Panhuizen & Drijvers, 2014). The first principle is *activity* principle. This feature stresses the active participation of students and the importance of learning mathematics meaningfully depends on doing mathematics (Van den Heuvel-Panhuizen & Drijvers, 2014). In the current study, students actively participated in each activity which were presented through instructional sequence both by individually or by whole class discussion, by verbally or by written works. The second principle is *reality* principle and it is about understanding what reality is. In this principle, the students should face with problems from real world, and also the instructional process should start from meaningful situation for students. For instance, rather than starting the teaching process with giving definitions, students may be put into informal process related to the context. In other words, opportunities should be given to students to mathematize the related context (Van den Heuvel-Panhuizen & Drijvers, 2014).

In the current study, each specific sub-domain on the instructional sequence was started with asking a question from the real world to the students; or by making them to think about the real-world examples of the related domain. For instance, while starting to the lesson of basic characteristics of prisms, teacher asked students whether they had heard the term of prism, and about the examples of prisms from the physical world around them. Students gave examples for prisms from the physical world around them and tried to mathematize them by using appropriate mathematical language. Third one is *level* principle which stresses the students passing through various levels while learning mathematics. Furthermore, these levels may be passing from the informal knowledge to the formal concepts, starting from concrete to abstract, making connections between concepts and strategies, developing some shortcuts or solutions for the problems etc. The content should be prepared according to this principle to provide students' mental participation to the educational process (Van den Heuvel-Panhuizen & Drijvers, 2014). For the current study, the content was prepared step by step to provide this principle. The students first faced with informal knowledge of prisms by giving daily life examples, then they defined the prisms and related concepts, after that they learned the basic features of prisms by discussing on the given examples of prisms from physical world. Later on, they related the concepts to the problems, used that concepts to solve problems by also developing different strategies. The fourth principle is *intertwinement and it* underlines that the mathematics have several domains as a science and those are inseparable in it. For instance, students operate estimation, do mental arithmetic, and use algorithms at a close connection (Van den Heuvel-Panhuizen & Drijvers, 2014). It the current study, for example, by formulating the surface area of the prisms, they needed to use the features of prisms as well as algebraic expressions at the same time. The fifth one is *interactivity* which is mentioned above in the characteristics of RME.

RME considers mathematics learning as a social activity and wants students to involve in whole class discussions or group works to share their ideas with others to develop their understanding (Van den Heuvel-Panhuizen & Drijvers, 2014). In the present study, each classroom session was performed with the active participation of students by verbally or by involving in group works. The last principle is *guidance* principle that underlines the proactive role of the teacher. According to RME, teachers operate the educational process by supporting students' development of meaningful understanding of mathematics. For this, it is offered that educational programs should be prepared as long-term learning trajectories (Van den Heuvel-Panhuizen & Drijvers, 2014). In the current study, the instructional sequence was performed by a conjectured HLT with the guidance of the participating teacher and the researcher.

Additionally, based on these principles of RME, various local instruction theories and instructional sequences have occurred for many years (Van den Heuvel-Panhuizen & Drijvers, 2014). Specifically, in the last years, most of them was developed by integrating the technology to the theory. Similarly, Doorman (2005) developed a local instruction theory in the context of early statistics by the support of DGS. Gravemeijer (1994) elaborated the development of local instruction theory, by forming with design-based research. Also, it included involving a cyclic process during the experiment, designing an instructional sequence, and a retrospective analysis of the process.

Considering the explanations above, in the present study, six principles of RME were used by the researcher and the participating teacher to prepare an instructional sequence with guidance of an HLT. Also, the current study aimed to develop a local instruction theory including a cyclic process and retrospective analysis during the experiment. Development of local instruction theory and retrospective analysis issues are mentioned in the method part.

2.9 Summary

By reviewing the literature, it can be concluded that learning mathematics is a social interaction process in which students are expected to do mathematics to get a conceptual understanding of related issue. Doing mathematics refer to involve in learning process by explaining, expressing, justifying, supporting, refuting ideas of their own or other's ideas by using appropriate mathematical language in classroom community. As those actions become taken-as-shared way of the classroom's expressing their thinking, classroom mathematical practices take place. To support the formation of classroom mathematical practices, an argumentative classroom environment is needed to be created in terms of sharing ideas in social context. The relationship between classroom mathematical practices and argumentation of students in terms of social interaction, it requires the formation of social and socio-mathematical norms of classroom which are the aspects of social constructivism. The necessity of starting and guiding those argumentations from a good and appropriate starting point, it is required to use the RME theory as a framework since RME offers to have rich and realistic context in mathematics learning processes. The evaluation of that kind of comprehensive environment is needed to conduct a design-based research by organizing an instructional sequence. A hypothetical learning trajectory guided the instructional sequence and implementation of the study. It is formed by considering the learning objectives, learning activities, and learning process. HLT is planned by considering students active participation to the classroom argumentations in terms of forming, analyzing, testing, and discussing their mathematical ideas and reasoning in related context. Dynamic geometry software is used as an instructional tool to support the students learning and understanding of three-dimensional solids. The DGS is expected to encourage students' participation to the classroom actions.

To sum up, the current study aims to contribute to the literature by providing data about the students learning of three-dimensional solids by involving in an instructional sequence guided by an HLT and by engaging in an argumentative classroom environment by expressing themselves verbally in terms of their reasoning and understanding of the concept. Moreover, this study is expected to provide contribution related to the effect of usage of dynamic geometry software as an instructional and a technological tool for enhancing the students' participation to the experimental process.

CHAPTER 3

METHODOLOGY

A design-based research approach was used for the current study to provide an accurate and deep understanding of the learning environment of eight graders within the context of three-dimensional shapes with the support of argumentation and DGS. This chapter includes several issues related to methodology of the study. First discussion was based on the characteristics and sequence of design-based research. By this way, detailed information was given about HLT and the instructional sequence of the study. Interpretive framework of the study was explained. Then the case of study type was discussed. After that data collection and data analysis process were described. Finally, trustworthiness issue and limitations were mentioned. This section was closed with a short summary.

3.1 Design-Based Research Approach

The emergence of design-based research as a new methodology for educational research appears on the first decades of current century (Anderson & Shattuck, 2012) and has shown a growing popularity throughout this time (Barab & Squire, 2004). Most of the well-known and qualified journals, respectable authors and educator researchers identified the potential of design-based research to increase the quality and leap for educational area (Anderson & Shattuck, 2012). By giving this attention to design-based research, this methodology has shown an increasing attention in mathematics education (Cobb, 2003).

Anderson and Shattuck (2012) defines design-based research as a methodology that aims to increase the effect and transformation of education research into practice. Moreover, there is a strong emphasis for both practice and research need building up the theory and developing some principles that guide them in educational contexts. Plomp (2013) defines design research as “aims to design and develop an intervention (such as programs, teaching-learning strategies and materials, products and systems) as a solution to a complex educational problem as well as to advance our knowledge about the characteristics of these interventions and the processes to design and develop them, or alternatively to design and develop educational interventions (about for example, learning processes, learning environments and etc.) with the purpose to develop or validate theories” (p.15).

Similarly, Anderson and Shattuck (2012) mentioned about characteristics of a quality design-based research by addressing the educational context and developing intervention. First, they emphasized that research should be conducted in an educational context which would- provide more valid results for research and ensures those results can be source for other context to assess and improve the practice. Secondly, they focused on the significant intervention. The authors noted by referring to Brown (1992), an effective intervention should be applicable by average classroom environments and should be supported by personal and technological tools. Producing an intervention should be done by both researcher and practitioner. First issue is assessing a local context. It should be supported and informed by other context with appropriate literature, theory, and practice. Moreover, it should be designed to find a solution to a problem or providing an improvement in that local practice. Many examples could be given to intervention as a learning activity, a different type of assessment or application of a technological tool (Anderson & Shattuck, 2012). In the current research, the study was conducted in an educational context by using an instructional sequence including usage of technological intervention regarding the quality of design-based research.

According to researchers, the lack of relevance between the educational research and educational practice brought out a need for design-based research (Plomp, 2013). An important determination from the Design-Based Research Collective (2003) was that “educational research is often divorced from the problems and issues of everyday practice – a split that resulted in a credibility gap and creates a need for new research approaches that speak directly to problems of practice and that lead to the development of 'usable knowledge'." (p.5). In van den Akker (1999), it is stressed that traditional research approaches such as experiment, questionnaire, and correlation analysis just provide prescriptions for design and development problems in education. He argues that an important reason of design research is rooted in the complex nature of educational reforms all over the world. Radical reforms cannot be developed on drawing sheets in government offices but calls for systematic research are made to support development and implementation processes in various contexts. Similarly, Reeves (2006) mentions about traditional research approaches as are studies that are poorly thought out and poorly conducted, resulting in no significant difference or, at best, average effect sizes. In fact, design-based studies should be conducted to investigate what is needed to solve the emerging problems, rather than to investigate whether a method is better than another method (Reeves, 2006). In this field, design-based research has been proposed to allow researchers to test and generate various concepts in their natural contexts (Brown, 1992). Barab and Sequire (2004) introduce the "design-based research is not so much an approach as it is a series of approaches, with the intent of producing new theories, artifacts, and practices that account for and potentially impact learning and teaching in naturalistic settings" (p. 2). Moreover, it is evaluated as a function study by concerning educational settings with their learning process designs considering the complexity of them. It is expected to understand the learning process and the main purpose should be to develop domain specific theories. In this context, DiSessa and Cobb (2004) states that design-based research should produce considerable theoretical inferences to sign the distances between the educational theory and practice. Also, they claim that for description and discussion of educational phenomena, design research may provide new constructs.

Design-Based Research Collective (2003) stated some basic characteristics of design-based research as; it is generally conducted in a single setting during a determined period; it includes cycles of design, application, analysis, and redesign; documentation and connection of outcomes to the whole study process; researcher and participant collaboration; development of knowledge that it can be used in practice. Stated phases of design research are discussed below.

3.1.1 Phases

Testing and revising the assumptions of phenomena and developing theories in this context can be defined as the basic characteristic of design-based research. (Cobb, 2003). In Cobb et al., (2003), they suggest five features for design-based research. First one is about developing theories about learning process as mentioned above. Second feature is about the interventionist feature which provides opportunities for researchers to evaluate educational improvements in their natural context. Third one is that design-based research is prospective and reflective. While prospective side takes account of the possible ways of learning accompanied with a hypothetical learning trajectory; reflective side is about the several stages of experiment like testing, refusing, generating, or testing again. These two characteristics make the methodology have a cyclic process. The fourth, iterative feature is composed of prospective and reflective features; and is about process of cycling. And the last feature is that developing the theory during the experiment as it should be applied in the real world (Cobb et al., 2003).

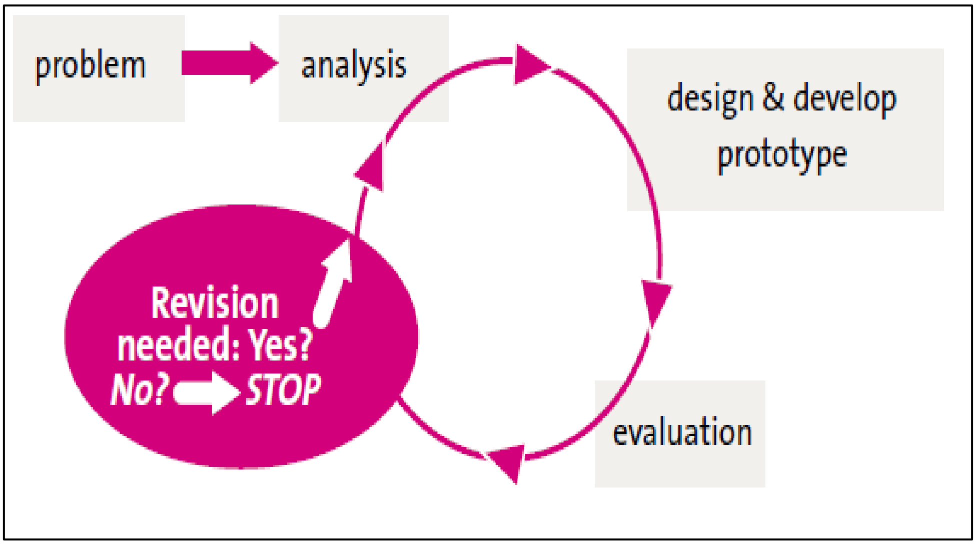


Figure 3.1 Iterations of systematic design cycles (Plomp, 2013., p.17)

All systematic education and training processes are cyclical because they involve design, analysis, evaluation, and revision activities; and this process continues until it reaches an appropriate balance of interest (Plomp, 2013). Also, there are authors illustrated this cyclic process in various ways (Bannan, 2013; Reeves, 2000, 2006). Authors may use variety of illustrations for picturing the details of design-based research, but they generally agree on that it has several phases (Plomp, 2013). For example, Cobb et al. (2003) mentions those phases as preparing for design experiment, conducting design experiment, and later retrospective analysis. Also, a variety of researchers used same categorization in their reports (Cobb, Gresalfi & Hodge, 2009; Gravemeijer & Cobb, 2006; Gravemeijer & Cobb, 2013). In this research, phases mentioned in Cobb et al. (2003) were chosen for design cycles and they were explained below in detail.

3.1.1.1 Phase 1-Preparing for the experiment

According to first phase of the design-based research, it is stressed that a local instruction theory can be evaluated and revised during the classroom experiment. In the ongoing process, learning goals should be clarified, instructional ending points and starting points should be determined. Determining the learning goals could be through assessment, tradition, or history. It is important not to get and use a school curriculum as it is given. Also, it should be examined, reorganized, and identified in a most useful way for students. The core idea of the content is also another important point in here (Gravemeijer & Cobb, 2013).

The present study designed around the context of the basic features and elements of prisms, the surface area of rectangular prism and the surface area and volume of the cylinder. Looking at the history of the classroom, they were eighth grade students and they had the knowledge of two dimensional shapes which they could relate the context to the three-dimensional shapes. Moreover, they had the knowledge of what a prism is and what a cube is and about its characteristics. This was an important issue for the current study since it was expected participating students to call back their knowledge about the context and provide dialogues for the classroom argumentation regarding data, warrant or claim.

In the literature, it is mentioned that traditional learning environment serves pencil-paper learning process regarding geometry and specifically for three dimensional shapes. Classical school textbooks provide just those “described illustrations above” (Denbel, 2015, p.23). But generally, because of their lack of the visualization of the concepts, they cannot provide comprehensive illustrations, since the textbook are static. For this reason, it is suggested to use dynamic geometry software like Geometer’s Skechpad, Cabri, Geogebra in lessons to provide accurate and comprehensive learning environment (Denbel, 2015).

Considering those issues, the learning goals which were already placed in national curriculum were revised, reorganized, and specified according to the domain of the current study. Here, another important issue was determining the starting points. For this determination, Gravemeijer and Cobb (2013) suggest making assessment like written tests, interviews, or performance assessments of whole class. For the current study, a pre-test was applied to the classroom aligned with the phase before the study started. After completing the identification of starting points and ending points of instruction, another step is to formulate a local instruction theory (Gravemeijer & Cobb, 2013).

Explaining the local instruction theory, Gravemeijer and Cobb (2013) states that kind of conjectured local instruction theory consists of assumptions about a possible learning process and assumptions about possible ways to support this learning process. Support tools include potentially productive instructional activities and (computer) tools, a predicted classroom culture and the proactive role of the teacher. The research team tries to predict how students will develop their thoughts and understandings in planned teaching activities. In this way, the research team tries to accommodate the need for planning in advance and the need to be flexible while developing the students' existing understanding as the design experience continues. Design based researchers are expected to get ideas from several sources while preparing an instructional sequence, but the important point is to work in an advanced way. They must use materials as much as possible those are available and adopt them to new applications (Gravemeijer & Cobb, 2013). Accordingly, for the preparation process of instruction of the current study, the classroom culture, available instructional tools that can be used in instructional process like smart boards, dynamic geometry software, concrete learning materials and worksheets were designed consistent with national curriculum according to the student needs and integrated into the instructional sequence. Also, the plans were left flexible that it could be possible to make any changes or developments in context if it were necessary. The classroom culture and the proactive role of the teacher were considered while formulating the design experiment. “What were the classroom norms, what kind of discussions could occur, what kind of activities could motivate students to participate in whole class argumentation, how to introduce the topic to the classroom by getting their attention, how to start and conduct classroom discussion” were the base questions for formulating the design of current study. To formulate the design of the study, also a hypothetical learning trajectory (HLT) was created to follow as a pathway. This HLT was planned for four and half weeks and seven lesson hours for each week.

3.1.1.2 Phase 2-Enactment of the design experiment

Second phase is to design experiment after completing the preparation of the study. After all the end points and starting points are defined, local instruction theory is formulated, design study can begin. This second phase starts with iterative process of the design cycles. This cycles and analysis are critical for process of testing, understanding, developing, and revising (Gravemeijer & Cobb, 2013). Design-based studies consist of a circular cycle involving testing and redesigning whole teaching activities. In fact, the research team evaluates how the interactions between the teacher and the students will occur aligned with the planned instructional activities during each lesson cycle.

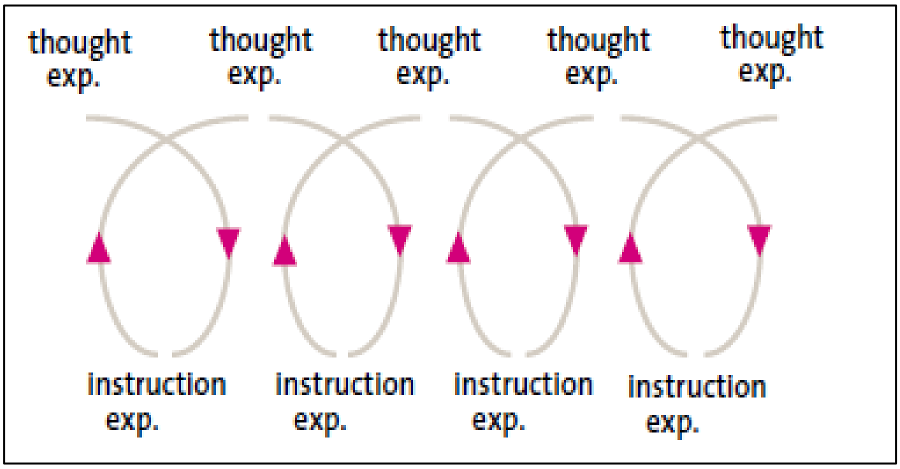


Figure 3.2 Cycling Process (Gravemeijer & Cobb, 2013)

Also, the research team tries to analyze the participation and learning of the students, considering both the progressive educational activities that are already taking place in the class and the retrospective activities. Based on these analyzes, the research team makes decisions such as the validity of the assumptions placed in the classroom activities, the formation of certain norms related to them, or the revision of the design from a specific perspective (Gravemeijer & Cobb, 2013). Therefore, this is a cyclic process (Figure 3.2) including whole experiments of thought and instruction (Cobb et al, 2003). As introduced in Simon (1995), a mathematical teaching cycle involves the learning objectives of the students, the planned teaching activities, and a predicted learning process. Thus, it is possible to associate this process to the Simon’s (1995) mathematical teaching cycle. The teacher observes the current understanding of the students during their activities and makes necessary revisions. Therefore, this cycle emphasizes the importance of anticipation and intervention in accordance with the design-based research (Akyüz, 2010, Cobb et al, 2003, Gravemeijer & Cobb, 2013). Relatively, a mathematical teaching cycle can be defined as a process of conjecturing, testing, and revising the hypothetical learning trajectory (Gravemeijer & Cobb, 2006).

3.1.1.2.1 Micro-macro cycles, local instruction theory and HLT

In design-based research, the micro cycles introduced above, support the development of local instruction theory. There is a reciprocal relationship between those two concepts. While the local instruction theory leads the micro cycles, those micro cycles of thought and instruction forms the theory (Gravemeijer & Cobb, 2013). This reciprocal relationship has been shown in Figure 3.3 that is adopted from Gravemeijer & Cobb (2013).

These micro cycles require that the research team involve in a continuous analysis process. These may be individual activities of the students, as well as social communication processes that will influence the thinking skills in the classroom. In this analysis process, short meetings with the participating teacher immediately following the completion of the classroom activities have a critical importance for the evaluation and reinterpretation of classroom sessions. It is also necessary to hold longer meetings, which should be repeated periodically in addition to short ones. Their overall focus is to evaluate the whole local instruction theory.

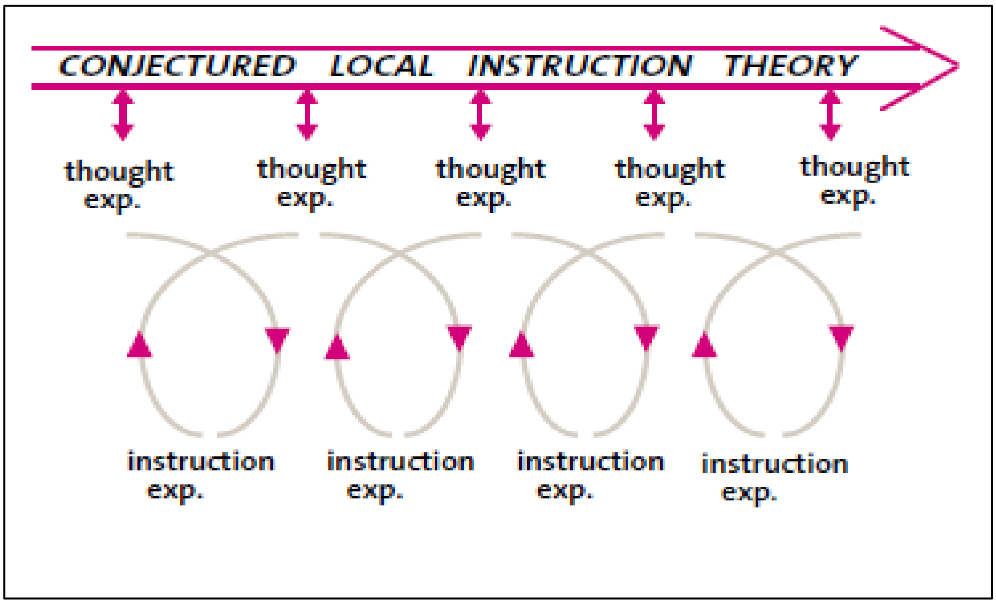


Figure 3.3 Reciprocal relation of local instruction theory and micro cycles (Gravemeijer & Cobb, 2013)

It was previously explained that the local instruction theory is a longer process involving all the learning processes and activities were planned to improve mental activities. Therefore, in a design-based study, two levels of conjecturing and revision process can be defined for each classroom session and for all instructional process. For the current study, during four and half week in instructional sequence, there were five micro cycles occurred for each phase of the designed HLT. Each micro cycle included necessary elements of cyclic process like holding small meetings immediately after completing daily classroom session to make an evaluation of that session.

Accordingly, in a design-based study, in addition to the adaptation of the general learning process, macro-design cycles can be defined that provide data from the retrospective analysis of the study to other studies (Cobb et al., 2003; Gravemeijer & Cobb, 2013; Gravemeijer & van Eerde, 2009). Therefore, combination of micro cycles formed macro cycle (Gravemeijer & van Eerde, 2009). Those macro cycles were shown in Figure 3.4 adopting from Gravemeijer & Cobb (2013).

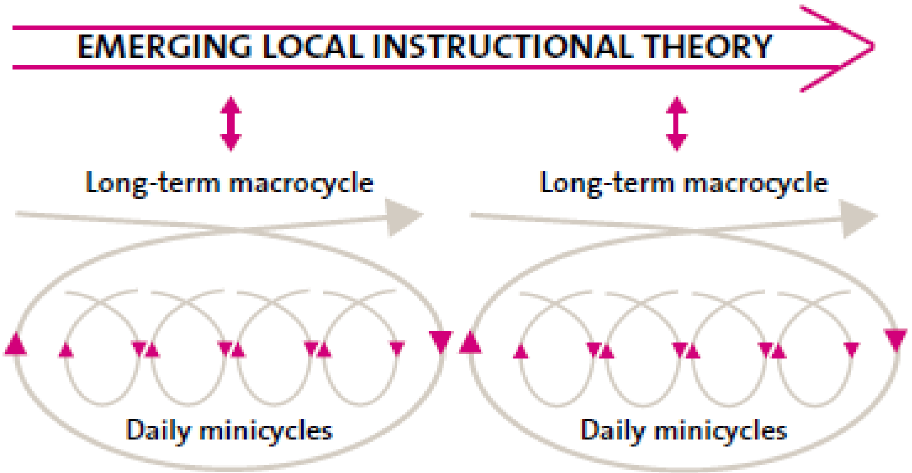


Figure 3.4 Micro and Macro Cycles (Gravemeijer & Cobb, 2013)

In the light of those explanations, in the current study, there was one macro cycle occurred and five weekly micro cycles were in it. The HLT was implemented in that macro cycle. Relatedly, the instructional sequence was examined and evaluated as weekly mini cycles, necessary revisions were made on the HLT. In everyday meetings conducted after completing each class sessions, the evaluation of the daily instruction was made and with guidance of those evaluations necessary revisions were made on the HLT for following classroom sessions. Also, after-class meetings provided data for the weekly long meetings. The process continued throughout the study and formed the long-term macro cycle.

For the current study, an HLT was planned on the basis of teaching prisms, their basic elements, and their nets, surface area and volume of the cylinder enriching the instruction with argumentations and GeoGebra. The main focus was to develop students' understanding of the concept by working on activity sheets and by involving in whole class discussions. Aligned with the requirements of the design research process, HLT were constructed on some conjectures of the researcher and the participating teacher. Those conjectures were drawn about the students' expected ideas, behaviors, claims, discussions related to the context of instruction.

In the preparation process of HLT, national mathematics curriculum for eighth graders, their textbooks used in lessons and literature review for the teaching-learning of three dimensional shapes were used for the current study. Activities were shaped simultaneously with HLT since they had a reciprocal relationship. While forming the activities, Stephan's (2015) work named "Surface Area", a website of the Ministry of National Education that includes tests for all classroom levels, and students' textbook were used as sources. While ordering the activity sheet, students' thinking levels and learning goals derived from national curriculum were considered.

HLT of the current study planned as a whole learning process by considering learning goals and teaching-learning activities. The learning practices proceeded during four and half weeks and seven lesson hours each week. Instructional learning cycle was constructed based on HLT, classroom sessions consisted of two-way interactions between teacher and students, and the researcher's and the instructor's knowledge. During the instructional sequence, revisions on the HLT were made if necessary which is nature of cyclical process. The responsivity of the teacher during the process was to select appropriate topics for whole class discussions and to provide an environment for those discussions aligning with the HLT. This was a requirement for proactive role of the teacher (Gravemeijer & Cobb, 2013) to develop students' understanding about the concept of three dimensional shapes. This study included four learning objectives derived from national curriculum and reorganized according to educational needs of the students. These four leaning objectives formed the five phases of the HLT. These learning objectives are defined in the National Mathematics Curriculum as:

- Identifies the right prisms and determines their basic features, elements, and draws the nets.
- Identifies the basic elements of a right circular cylinder and its net.
- Constructs the formula of surface area of right circular cylinder; solves related problems.
- Constructs the formula of volume of right circular cylinder; solves related problems (MoNE, 2013).

As, it is mentioned above, these national objectives were evaluated and reorganized according to students' needs (determined with small pilot study). Those phases were explained in detail below.

The first phase included two interrelated parts in it. First part was related to understanding of construction of prisms and determining its basic elements and the second part was related to displaying the surface nets of prisms. First phase of the HLT included 10 activity sheets to assist instructional sequence. Accordingly, first part of the first phase covered two worksheets and second part covered eight worksheets. The first part's activity sheets aimed the students to think about the common properties of prisms. The aim was to construct an understanding and identification of common properties of perpendicular prisms.

Constructing on the first part, second part of the HLT included activities that derived and reorganized from Stephan's (2015) work and from the website of Ministry of National Education. The order of activities designed step by step to provide an activity which would be the basis for the next one. It is important to improve mathematical reasoning; construction of the activities should be in an appropriate order. The aim of these activity sheets was to create a basis for the net of prisms by working with views of prisms from different ways.

Table 3.1 First Phase of HLT

Learning Objectives	Determining of basic elements of prisms Nets of prisms
Planned Period	4 lessons
Students' Prior Knowledge	Two dimensional shapes What a cube is. Common properties of cube
Context and Concepts	What is a prism? Common Properties of Prisms Types of Prisms Edge, Top-bottom bases, Height Different views of Prisms
Tools	Activity sheets GeoGebra File Unit cubes
Conjectured Classroom Discussion	Daily life examples for prisms Conclusions on the basic elements and common features of the prisms.

This part of the HLT was prepared under the concept of "candy wrapping company". Each shape was designed with unit squares on it to make students understand those unit squares are the same as the length of the shape. In this second part of the HLT, each activity assisted by a GeoGebra file and after working each question individually or in groups, classroom check was applied on the GeoGebra file to make student construct the conceptual understanding of the context. Moreover, the teacher gave unit cubes to students to construct the shapes given in the activity sheet for helping them to develop their three-dimensional imagining from various ways. In Table 3.1 first phase of the HLT was shown.

For the second phase of the HLT, six pages of activity sheet was prepared related the learning objective of "constructing the formula of the surface area of perpendicular prisms". For this part of the instructional sequence, Stephan's (2015) work was the primary source. By the end of this phase of the study, students are expected to understand the surface area of perpendicular prisms and could be able to solve related problems. Activities of this phase again designed step by step constructing on each other. For example, first shapes of the activities were given unit squares on them and wanted students to find out “how they can wrap those candies by using wrapper papers?” and “how many unit squares would be there?”. Working on those questions, the aim was to make an introduction to the surface area of the perpendicular prisms. After being asked various questions about unit squares, students worked on activities that included shapes without unit squares. With those questions students were expected to make connection between unit squares and length of the edge of shape. With the support of GeoGebra files for each activity sheet, students checked their solutions by discussing them in classroom environment.

At the end of the activity sheet, students were asked to work in pairs to produce a formula for perpendicular prisms. After giving a certain time period to them, they were asked to explain their answers with reasons and justifications. In here the important point was students’ abilities of transforming their numerical work into algebraic expressions. They worked on various activities by solving them and as a next step it was time to express the process by algebraically. This is a challenging process and an issue for students to transform that kind of numerical knowledge into algebraic expressions, also to understand “how to name any length with a letter or a character. For this process, the proactive role of the teacher and the ways of operating it comes forward. In here, teacher should be as much as supportive to make student overcome that transformation of knowledge in a mathematically meaningful way. For instance, she/he can remind the meaning of algebraic expressions by asking a question. For example, let’s assume the teacher asked a question as; "What is the perimeter of a square with an edge length of 5 cm?" Probably all students can answer such a question. The teacher can change it as; "How many centimeters is the perimeter when the edge length is “a” cm instead of 5 cm?" This discussion would be a guide for further steps to produce formula for surface area. The discussion can be developed with students’ answers. Students will probably find the perimeter of that a square is “4a” units that is given in length of each edge is “a” unit. What is important here is to organize the new knowledge they have gained with classroom activities by combining the former information. Second phase of the HLT was shown in Table 3.2.

Table 3.2 Second Phase of HLT

Learning Objectives	Constructing the formula of the surface area of perpendicular prisms
Planned Period	6 lessons
Students’ Prior Knowledge	Common Properties of Prisms Edge, Top-bottom bases, Height
Context and Concepts	Surface area
Tools	Activity sheets GeoGebra File
Conjectured Classroom Discussion	Wrapping equals to surface area Unit squares equals to length Process of finding the formula of surface area of perpendicular prisms

Third phase was planned related to the learning objective of “determining the basic elements of cylinder, constructing, and drawing the net of it”. This phase of the HLT was prepared with six pages of activity sheets. At the end of the phase, students were expected to construct the knowledge of basic elements and net of a cylinder. The process started with the teacher’s questioning students about what a cylinder is and asking them to give daily life examples of it. This process was to check the prior knowledge of students about cylinder.

Table 3.3 Third Phase of HLT

Learning Objectives	Determining of basic elements of cylinder Net of cylinder
Planned Period	6 lessons
Students' Prior Knowledge	Knowledge of rectangle Knowledge of circle Radius, diameter, PI number Circumference of circle
Context and Concepts	What is a cylinder? Common properties of a cylinder
Tools	Activity sheets GeoGebra File
Conjectured Classroom Discussion	Wrapping a cylinder equals to net of it. Is a cylinder also a prism? Circumference of one base cylinder equals to length of side edge on which it is wrapped.

The first question was about asking students to draw a wrapper of cylinder candy which tried to evaluate whether students understood the aim of the question which was "what the net of a cylinder is?" in real. Then, the teacher questioned the students about their answers with reasoning and justification ways. After whole class discussion session ended, the GeoGebra file was opened to check their answers and to evaluate the relationship between changes of lengths of closed shape and opened shape. After constructing the knowledge of basic elements and net of a cylinder, other step of the activity sheet was about constructing students' understanding on circumference of one base cylinder which equals to the length of the edge of its side surface on which it was wrapped. Actually, this was a conjectured classroom discussion about previous question. By constructing on it, students were expected to use that knowledge as data for classroom discussion. Another aim was to make students to transfer the knowledge between the different lengths of the cylinder. For example, they were expected to be able to find the length of the radius if they knew the length of the edge on which it was wrapped or vice versa. Throughout this phase students were expected to have knowledge base for the next phase which was about surface area of a cylinder. Activities of this phase were reorganized from the Stephan's (2015) work of surface area. The third phase of the HLT was shown in Table 3.3.

Fourth phase of the HLT was designed about the surface area of cylinder. It was prepared related to learning objective of "constructing the knowledge of the surface area of cylinder". This phase was composed of four pages. First activity sheet was critical for being the first step of the understanding of the surface area. The question was about wrapping cylinder shaped candies with their dimensions as they were given. Since, students constructed the knowledge of wrapping a cylinder equals to net of it from the prior phase, they were expected to understand they would need net of cylinder again. In here, there was an additional point as the students were given dimensions of cylinders. An expected whole class discussion that the teacher would conduct was about "what to do at this point?". Students were given a time period to work in pairs on the activities and the teacher started the argumentation process by questioning students with their justifications. Students' reasoning on finding and understanding how to wrap those cylinder-shaped candies was critical for the following step of forming a formula for surface area of cylinder.

Next worksheet was about students' abilities of transforming their numerical work into algebraic expressions again. It was expected that students would have had less difficulty in this process than they would have in finding the surface area formula of prisms in previous lessons. It was also expected that they should have structured the preliminary knowledge of how to transpose algebraic expressions from numerical data with classroom discussions which they do there. After completing these activities and constructed the knowledge of surface area of cylinder, following activities were based on strengthening this knowledge by solving additional questions. Moreover, those questions were prepared to practice both prisms and cylinder altogether. The activities of this phase were formed from students' textbook and Stephan's (2015) work of surface area. This phase did not include GeoGebra files. Table 3.4 shows the fourth phase of the HLT.

Table 3.4 Fourth Phase of HLT

Learning Objectives	Surface area of cylinder
Planned Period	5 lessons
Students' Prior Knowledge	Net of cylinder Basic elements of cylinder Area of circle Area of rectangle
Context and Concepts	Surface area Area of circle Area of rectangle PI number
Tools	Activity sheets GeoGebra File
Conjectured Classroom Discussion	Forming the formula of cylinder

Last phase of the HLT was based on the learning objective of "constructing the knowledge of the volume of the cylinder". While preparing the activities of this phase, students were expected to have the knowledge of "what volume is" and "volume of cube and rectangular prism" since they had learned those concepts in sixth grade.

By constructing those knowledge base, teacher started the process with a classroom discussion about "what is volume?". Also, another discussion task was about "how the volume of cube and rectangular prism can be calculated and what element do we need for those operations?". This process was conjectured to call back the students' former knowledge that they calculated how many of the unit cubes that actually filled the inner zone when they found volume, but while doing this calculation instead of counting the whole cubes, they multiplied the three dimensions of the prisms with each other. The critical questions were "how they can fill a cylinder with unit cubes since it does not have edges?" and "how they can find volume of the cylinder?" The answers that expected from the students here were the necessity to use circle segments instead of using unit cubes to fill the cylinder. Students would be able to figure out the volume of the cylinder by understanding how they could fill the cylinder by placing the circles via putting one on another one at the height of the cylinder. Additionally, students were expected to transfer the knowledge of the volume of cube and rectangular prism which can be formulated as “multiplication of base area and height”. By including the volume of cylinder by filling it with circle segments to the discussion and relating context about the volume of cube and rectangular prism, the students were asked to conclude that the volume of the cylinder is “multiplication of base area and height” with the guidance of whole class discussions. To support this phase, GeoGebra files were used to assist students’ understanding of volume of cylinder and three pages of activities were included to construct the conceptual understanding of volume task. Table 3.5 shows the fifth phase of the conjectured HLT.

Table 3.5 Fifth Phase of HLT

Learning Objectives	Volume of cylinder
Planned Period	4 lessons
Students’ Prior Knowledge	Volume of Cube Volume of Rectangular Prism
Context and Concepts	Base area Height
Tools	Activity sheets GeoGebra File
Conjectured Classroom Discussion	Forming the formula of cylinder

3.1.1.2.2 Data generation and implementation process

In design-based studies, data collection, generation, and procedural progress depend on the theoretical intent of the design-based study from the very beginning.

For example, if local instruction theory is being developed in a design-based study, it would be appropriate to record all classroom sessions with the video camera, get copies of the work of all students, and collect field notes while data is being collected and generated. Generally, a large amount of data will be needed because it is important and critical to document the mathematical development of the students, development of mathematical reasoning, and to evaluate the emerging learning ecology (Cobb, Gresalfi & Hodge, 2009; Gravemeijer & Cobb, 2013). It is also important to audio-record the research group meetings. Because these meetings provide one of the best opportunities for the research team to document the learning process. Therefore, data generation and collection are a mechanism consisting of processes such as review, interpretation, decision making and organizing which continues throughout the study (Gravemeijer & Cobb, 2013).

For this study, the data collection and generation process included the application of the phases of the hypothetical learning trajectory. This process consisted of a macro cycle containing weekly micro-cycles. After a total of four and half week and seven lessons per week, the study was completed. The literature, the thinking and the learning levels of the students were taken into consideration while preparing the instructional sequence and learning activities. The first form of activities was applied to ten randomly selected students from another non-participating eighth grade. With the direction of these collected data from ten students, the research team made revisions on the worksheets and instructional sequence and the main study had started with it. The revised HLT and content were applied in the main study. However, during the process, there were some changes that were done in instructional sequence, hypothetical learning trajectory and activities for the following courses in accordance with the needs of the students.

The research team, consisting of the researcher and the participating teacher, came together to form the macro cycle of the HLT. This process went parallel to the preparation of HLT. This process was completed in approximately in one week (throughout week-days). Throughout the instructional sequence, students went on working individually and sometimes in pairs. During these studies, the participating teacher and the researcher checked students or study groups to determine the progress of the studies, the way how students think differently, and the issues that may come up to discuss in class. After students' individual or dual group work had been completed, classroom discussions started and the different interpretations, demonstrations, questions of the students had been evaluated together with their reasons. This procedure had been followed at all stages of the HLT.

The first week of teaching began with the basic features and openings of prisms as it was the first stage of the HLT. This week was critical for the social and socio-mathematical norms of the class which would begin to emerge. With leading of the teacher, the first lesson started with the whole class discussion on “what the prism is” and “examples of prisms from daily life”. The aim was to examine students’ prior knowledge of the prisms and to make a beginning to the basic features of the prisms with the students' answers about the prism examples. Moreover, students were expected to make judgements about what kind of objects would be prisms and what could not be a prism by observing the examples of other students. Relatedly, they were expected to make conclusions about the basic features of the prisms. With the help of the worksheets, the teacher tried to lead them to think about “these shapes have a base, all of them has a height” etc. It was observed during the class discussion that students generally had some idea of what the prism was, but when looking at the prism examples, it was seen that some of the objects that were defined by the students as prisms, were not prisms in real. For example, an issue was about cylinder-shaped pencil cans and tin cans. Those were among the given examples of prisms.

Also, other given examples were camp tents and roofs of houses. Students were not sure about those were prisms or not. The problem with the tents and roofs were about their positions. Some students thought that they did not look like prisms. This was an unexpected issue for the researcher and the teacher. At that time, the teacher added the issue to the classroom discussion about those examples. During classroom discussions, the teacher asked the students to think about the basic features of the prisms and explain their thoughts along with the reasons, starting from the examples that are given. The features told by the students were noted on the board and then discussed. Examination of prism examples were written on the board and were done through classroom discussion. The compatible ones were chosen; and then the teacher arranged the definition about basic features of the prisms.

After writing those basic features of the prisms, a question aroused whether the cube was a prism or not. They discussed about the issue. Additionally, the teacher questioned about the camp tents and roofs of houses. They compared the features with other prisms; and decided tents and roofs were also prisms. The teacher explained the shapes that being slanted was not an obstacle to be prisms; and continued that when they change the position of any shapes they do not make any changes in their feature, length, height, i.e. She reminded the transformation of geometry and asked students whether they were doing any changes in a shape or not. With this discussion, students saw their own misconceptions in their minds and corrected themselves under the leadership of the teacher. After lesson, the researcher and the teacher talked about those unexpected questions and decided to add HLT of the study. The issue about tents and roofs would be added as a discussion issue for further studies; the cylinder was added to the phase of the HLT with the learning objective of determining the features of cylinder.

In the following process, the activity paper consisted of sections related to the nets of the prisms. At the beginning of the lesson, the teacher gave the first activity sheet of the second section and informed the students how to proceed. This paper was explaining that the context would continue by associating with the wrapping part of a candy factory. Then in the following pages, the students had to show how to make wrapping paper so that the candies in the given shapes could be covered. Given shapes were prepared by unit cubes, to make students understand the connection between those unit cubes and length of the edges. GeoGebra files were prepared for each of the activities there. The students were expected to work individually for the following three pages. Also, the drawings were evaluated during whole class discussions. The process progressed as it was planned. After the students worked on the questions for each page, they were asked to comment on the drawings and explain the reasons for the different ideas. There were some students that misunderstood the task. For example, they solved the questions by counting the cubes that constructed the shape as it was a way of finding the volume. This issue was noticed while discussing about the responses. To solve this misunderstanding, the teacher asked the class about their ways, and justified those. With students’ explanations, that problem was handled successfully. During the period of the students' work, the requested students were given unit cubes to see the concrete form of drawings. Later, the GeoGebra file on the smart board was opened to check the evolution of each problem and the process was completed.

Last three pages of this phase were based on the relationship between the closed and opened forms of prisms. Students were expected to evaluate and find out which point in the open form of the prism matches with the other point when the prism brought into closed form. This was an issue related with spatial thinking of students. There were ideas about the issue, but it was challenging for students to find out the matching points, so they needed to see the shapes on the GeoGebra file. For these pages of phase, there were not a proper GeoGebra file, thus the researcher opened the GeoGebra file that was prepared to show the net of rectangular prism and used that file to clarify the issue for students.

After seeing on the software, students could overcome the challenges using this way. When the classroom session was completed, the research team conducted a small after-lesson meeting and talked about the issue. This was a missing part in the instructional sequence and should be added, so the researcher and the teacher decided to prepare a GeoGebra file for those questions and added it into HLT of the study. Other activities were questioning the missing parts of a given prism in opened form. The students did not have any difficulties for those questions, they successfully found out the missing parts of the prisms without any need of a GeoGebra file. With these activities, first phase of the HLT was completed with a learning objective of determining the basic elements of prisms and their openings.

The second part of the HLT was related to the construction of the formula of the surface area of the prisms. This part started in the first week of the study and continued during the second week. The first page of the activity paper included questions about how many unit-square of wrapping paper should be used to pack the prisms formed by unit cubes. In these questions, the visual spatial thinking skills of students came forward. It is expected that the students would understand the incomplete or invisible parts of their minds. Moreover, they would understand that they cover the surfaces that actually appear while packaging. Thus, they made the procedure on the surface area. For this part, the teacher asked the students to think over the questions for some time after they got their working papers. Later, they started a class discussion here about what they were being asked. The question that starts the discussion was "how are you packing the candies given, what are the measures of the paper you need to use?" Students explained their ideas on the subject. When the statements made by the students were examined, it was observed that students in general understood what they would do and which way they would follow. When it came to the other page, the first few questions were formed by unit cubes, and the students again answered questions without any difficulties. The candy was given in the last question and the following two pages did not include unit cubes. What was expected from the students here was that they must define a unit length for each prism by relating with their previous experiences and continue their procedures accordingly. While passing to those questions, students had difficulty to understand how to do procedures without unit cubes. That was an issue and a required classroom discussion. The teacher questioned the students about how to transform the given data from unit cubes to edge length. For this process, some students wanted to see the a GeoGebra file or concrete materials. Thus, concrete unit cubes were used to show this transformation. From this discussion, it was obtained that some students needed to see a concrete material or a software to clearly understand the process. The Figure 3.5 is from the activity sheet in which students had difficulty to transfer the knowledge.

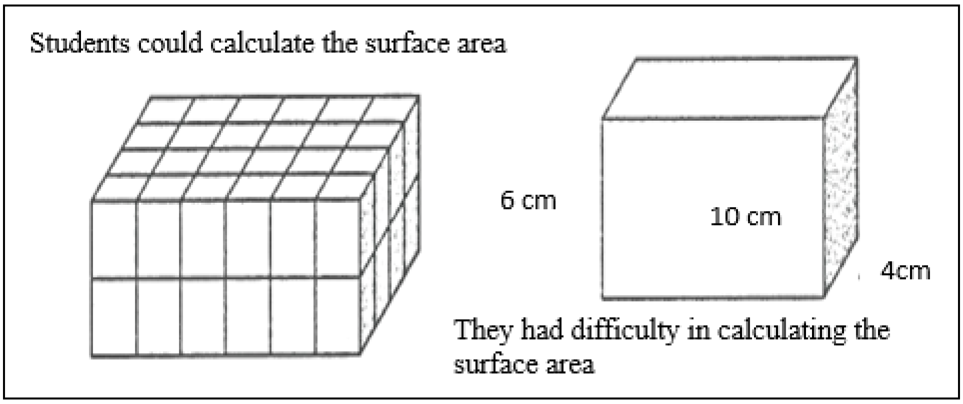


Figure 3.5 Students' difficulty

After overcoming this issue, the classroom continued to work on other page that was about forming a formula for surface area of prisms. This was a critical point for students because they needed to create the formula in a mathematically meaningful way in their minds. They were given a time period to work in pairs and discuss how to express the formula in algebraic version.

During that process, the teacher and the researcher guided the groups and listened their ideas and gave support to make them reach the result themselves. Moreover, during the process, if there were any critical questions arising, the teacher made a whole class discussion. Looking at the process in general, students were aware of what they were doing and what was the meaning of finding surface area. This issue was clear for them at the end. But the problem was about how to express their ideas or finding formulas. On the paper, there were clues for students as examples of given formulas. Some of them developed ways to follow, but some of them were not clear about naming the edges. Evaluating a few students' expressions in whole class discussion, they reached the final version of the formula of the surface area of prisms. After working on a few questions, this phase was completed with the surface area of prisms which was aimed to construct the knowledge.

The third phase of the HLT was about the learning goal which is about determining the basic elements of cylinder and net of it. This phase started at the second week of the study and continued at the third week. This phase of the HLT was included on the seven pages of activity sheet. At the beginning of the lesson, a discussion was planned about whether a cylinder is prism or not. As mentioned earlier, at the first lesson students started to talk about prisms, they gave examples of prisms in everyday life, some students gave examples to prisms as cylinder-shaped pencil boxes and tin drink boxes. Then, the teacher and researcher decided to add the HLT that was prepared for cylinder to the lesson by having a meeting after lesson. After the properties of the cylinders were discussed and samples were given, two separate columns were created on the board and the properties of the cylinders and prisms were written. By looking at what was written there, the students decided that the cylinder was not a prism. For example, the absence of the edges of the cylinder base was the most distinctive aspect that students perceived.

After a short discussion session, the issue was enlightened, and teacher continued with activity sheet. Teacher gave the first page of the activity sheets to the students and gave time to read and understand the first question. This page was questioning the how to wrap a given candy in cylinder shape with a wrapper. The teacher asked to the class what they understood from the expression. Whole class replied the question as it was asking the net of the cylinder. They did not have any difficulties in that question and they drew appropriate openings for the cylinder. The remaining pages of this section were related to matching the given parts of cylinder; for example, finding an appropriate circle base for a given rectangle side face, or vice versa; and finding other lengths of cylinder by giving a certain length. Basically, the framework required the students to think as the same way but by asking in different ways, it was questioned whether students would be able to connect with each other or not. On the first type of questions, the students worked in pairs. In this type of question, students were expected to understand that one length of the side face and circumference of the given base should be the same length or vice versa; and to act accordingly. During this process, a question occurred as whether they should use the long side of a given rectangle to wrap around a circle base. This was also an unexpected question and needed to be discussed. The teacher questioned the classroom in this way. There was not a proper GeoGebra file for this discussion, but one student used her notebook, tore a paper and by circling it around at one time from the long side and at one time from the short side, justified her answer, and completed the discussion. Through this process, there were not any other challenging issues for students to overcome and they completed the process successfully. Constructing on this part, in the second part students worked on missing lengths of the given cylinders. In this part, they worked individually without any questions and finished this phase.

The fourth phase of the HLT, was about the surface area of the cylinder and constructing the formula relatedly. This phase started at third week and continued through two lessons of last week. At the beginning of the lesson, teacher questioned about if there was any unrealized issue from previous task and started the session by giving the first page of the activity sheet. Students were asked to wrap the given cylinder-shaped candies with appropriate wrappers and their measures. This was an easy task for students because they did the similar activities at the previous lessons. They completed the page by working individually without any difficulty. The teacher asked about the activity they did in that page and the answers gave the same response as surface area. In the next page, it was time to produce formula for the surface area of the cylinder. The teacher gave some time for the class to discuss in pairs and try to produce a formula as they do while working on formula of the surface area of the prisms. During the working period, the teacher and the researcher supervised the groups and supported for their method. After working on the sheet, they started to talk about the responses. In general, similar to working on the formula of the prisms, the same problem occurred as transforming the numeric data to the algebraic data again. Most of the students were clear about what to do and how to find the surface area of a given cylinder, but they were again not sure about how to use letters to name it and construct a formula. By discussing it as whole class, the researcher gave a clue by reminding the formula of the area of the circle. After that time, most of the students could be able produce the formula of the cylinder. Actually, they knew it numerically, but they also constructed it algebraically. After-lesson discussion, the teacher and the researcher decided that it would be beneficial for the students to add a few examples with algebraic expressions to HLT for further studies. Accordingly, after students worked with numbers, and tried to find the surface area by that way, it would be a bit easier to transform the numerical data to the algebraic expressions to construct the formula. Having an overall look at the process, there were not any great challenges or any necessity to use a GeoGebra file except for producing the surface area formula.

The fifth and the last phase of the HLT was about the volume of the cylinder. This phase lasted during four lessons of process's fourth week. The teacher started the session by questioning about the knowledge of what volume is. The students had former knowledge of the volume from earlier grades. They had learned the volume of the cube and rectangular prism at sixth grade level, and during the discussion it was understood that they had the conceptual understanding of the volume that it means to fill inside of any shape. Moreover, to fill and find the volume, they successfully remembered the usage of unit cubes. Actually, this was an issue from the beginning of the process. As it was mentioned before, while they were working on the surface area of prisms and wrapping the candies that were given as constructed with unit cubes, some students misunderstood the issue and they calculated the volume of the given candies by counting the unit cubes. The teacher reminded the process and a GeoGebra file was opened to show how they were filling the inside of a cube or a rectangular prism and how they were transforming the operation from counting each unit cube to multiply the edges with each other. A whole class discussion started about what could be done to find the volume of the cylinder relatedly. There were different ideas that were suggested; but one of them was remarkable and it was saying "it is same as finding the volume of prisms since it is a three-dimensional shape". The idea was good but missing. The teacher went over that response and wanted that student to justify and prove his answer. The class thought about the issue, some offered to fill the cylinder with water, but again the volume of the water was rising as an obstacle. The researcher reminded the usage of unit cubes to fill prisms to show a different way and wanted them to think how they can fill it by using concrete materials as they do in prisms. One of the students offered the idea of using circles to fill it and got the point. The teacher went through that response by asking how to calculate the whole volume. The student responded as by using as many cycles as that would be able to fill the cylinder. The researcher questioned about how to find the number of those circles and another student responded as it was height of the cylinder. Those responses made the issue clear and the researcher opened a GeoGebra file and they evaluated how to fill a cylinder with circles to find the volume. This time it was easy for students to produce the formula of the volume except for a few of them. The teacher continued with following pages that required calculation of volume of the cylinder assisted with GeoGebra examples. After solving examples on GeoGebra file, there were not any questions left to ask about the task. This phase was the ending of the process.

3.1.1.2.3 Preparation of HLT

The instructional sequence of the current study which was an application of the planned HLT, continued during four and half weeks and seven lesson hours for each week. The research team was constructed as a school research team by the researcher and the instructor, and each lesson was observed by that research team. The data of the study were based on the understanding of solids and they were included the video-recordings of the lessons which include pair and whole class discussions, students' written works, after-class meetings, weekly research team meetings, pre-posttests, and the researcher's field notes. The data were collected from various sources to provide detailed and accurate knowledge of classroom sessions with the concept of three-dimensional shapes. Design of the current study was summarized on the following Figure 3.6 (Gravemeijer & Cobb, 2013).

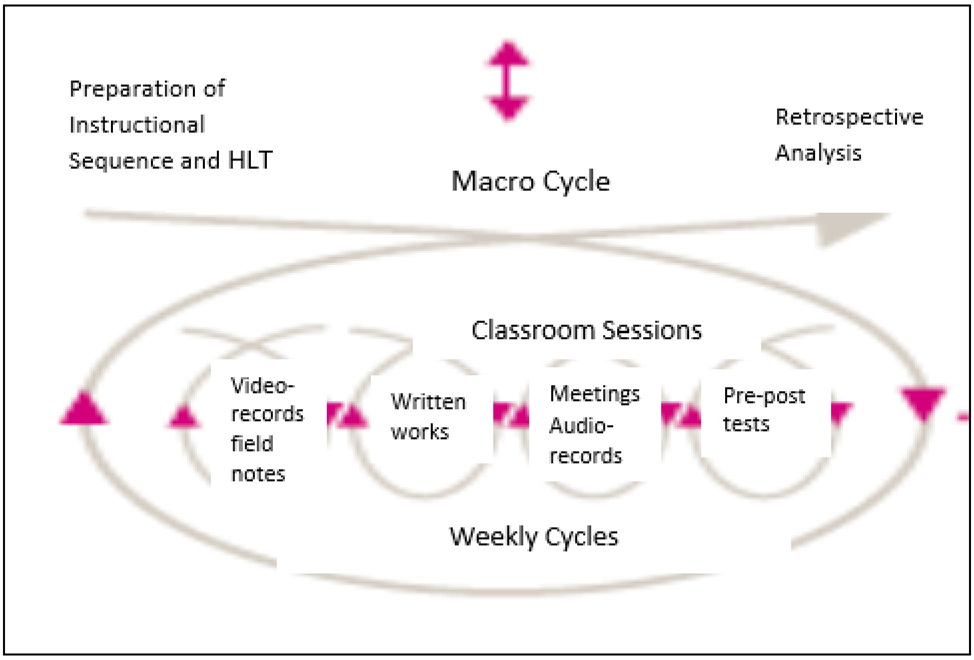


Figure 3.6 Design of the current study (Gravemeijer & Cobb, 2013)

The current study included one macro cycle and weekly cycles from each week. Throughout the study, five micro cycles occurred on the five phases of the designed HLT. To prepare the main instructional sequence which included the HLT and activities of the study, the prototype of the activity sheet was applied to the ten eighth-grade students from another non-participating classroom whom were randomly chosen. After this application, necessary revisions were made on the HLT and the activity sheet, and the last form of the instructional sequence was ready. Additionally, during the study process, necessary changes were made, or additions were done when it was needed according to students' needs related to the nature of designed based study. Thus, at the end of the study, a revision was made and adapted to HLT again which would be a source for further studies.

3.1.1.3 Phase 3-Retrospective analysis

This section explains the revisions that have emerged during the application of instructional sequence which have been done according to the needs of the students.

Since the aim of the design research study is to get information and to understand about the relationship between learning environment and students' learning, it is a necessity to collect various data set from various sources and evaluate the students' thinking process during the study by this way (Gravemeijer & Cobb, 2013). The main aim is to analyze the huge data set systematically and accurately. To provide the credibility of the data analysis process, all steps of the experiment need to be documented. Conjectures and claims should be done from the beginning of the study, throughout the study and at the end of the study as retrospective analysis (Gravemeijer & Cobb, 2013, Gravemeijer & van Eerde, 2009).

For the current study, as it's mentioned above, the data were collected through various sources and analysis of the collected data set was done during and at the end of the study. This data set constructed the macro cycle of the study that aimed to evaluate the mathematical practices emerging in an eighth-grade mathematics classroom in the context of solids which were supported by argumentations and DGS. With this process, some necessary changes were done on the instructional sequence and the HLT.

The learning objectives of the study did not change; the research team found it appropriate for the students' level and needs. In the first phase, a change was made in the activity sheet. Last three pages of this phase were based on the relationship between the closed form of prisms and nets of them. Student were expected to evaluate and find out which point in the net of the prism matches with other point when the prism brought into closed form. This was an issue related with spatial thinking of students. There were ideas about the issue, but it was challenging for students to find out the matching points, so they needed to see the shapes on the GeoGebra file. For these pages of the phase, there were not a proper GeoGebra file, for this reason the researcher opened the GeoGebra file that was prepared to show the net of rectangular prism and used that file to make the issue clearer for the students. After seeing on the software students could overcome the challenges by this way. When the classroom session was completed, the research team conducted a small after-lesson meeting and talked about the issue. This was a missing part in the instructional sequence and it should be added. So, they decided to prepare a GeoGebra file for those questions and add it into HLT of the study. Another change was about the conjectured classroom discussions during the process. There were unexpected questions and discussions occurred like "whether changing the position of any prism or its features; whether a cube is also a rectangular prism at the same time; whether it's a cylinder or a prism, i.e. These questions were decided to add to the HLT for the following lessons and also for further studies.

The second phase of the HLT was based on the surface area of the prisms, the students successfully completed process with the support of the GeoGebra file. But at the last page of the activity sheet, there was no GeoGebra file that would support the students' understanding, and students wanted to see the shapes on the dynamic environment, so the research team concluded that a GeoGebra file should be added to the activity related to those questions. The researcher prepared a GeoGebra file about one of the questions for the next day and students evaluated the question with support of that file, but because of the limited time, it was not possible to get prepared for the other questions. By adding it to the HLT, it would be suggested to use it for further studies.

The third phase of the HLT was related to the basic elements and net of the cylinder. The discussion about this phase came from the first phase of the HLT. The question was whether the cylinder was a prism also. In the meantime, the research team had decided to add the discussion to the HLT and that addition was directed the third phase. The research team also concluded that it should be added to HLT and to instructional sequence for further studies.

In the fourth and fifth phases of the HLT, there were not any necessary changes for research team, so they made a conclusion as the same as prepared before.

3.1.2 Interpretive framework

For design-based research, it is important to explain, how collected data can be transferred into scientific interpretations. Thus, researchers need to use an interpretive framework to make the data set scientifically meaningful starting from the beginning of the study, throughout the study that on progress and while doing the retrospective analysis. It is essential to maintain the process systematically to provide the data set to make sense while making scientific interpretations (Gravemeijer & Cobb, 2013). Gravemeijer and Cobb (2013) suggested some key elements for the interpretive framework of a designed based study. First one is suggested for interpretation of the learning environment in the classroom which is defined as emergent perspective (Cobb & Yackel, 1996; Yackel & Cobb, 1996), and the second one is suggested for students' reasoning on mathematics that is evaluated under the RME theory for the current study.

Current study used three domains of social aspect as interpretive framework. Those domains were social norms of classroom, socio-mathematical norms of classroom and as a last one mathematical practices of the classroom. As it was mentioned in the literature review of the current study, social norms of a classroom define the beliefs about the roles in the classroom and also about the general structure and nature of the activities of the instructional sequence. Moreover, these social norms refer to the communication between and the students by the way that teacher forces the students to explain their ideas, to justify those ideas with appropriate mathematical terminology, and to show their agreement or disagreement in classroom discussions (Gravemeijer & Cobb, 2013). Relatedly, the current study included some social interactions which occurred in the classroom environment as students' participation to the process. Students got involved in practices as individually, as working in groups and also as whole class discussions. Individually they worked on their work sheets, or other type of questions. Students worked in pairs by discussing and sharing their ideas with another peer. After that individual or peer works, whole class discussions were started in which students shared their ideas, solutions, explanations, justifications, i.e. For the transformation of these social norms to the scientific data set, Krummheuer's (2015) argumentation model was used which was developed by using Toulmin's argumentation model.

The second emergent perspective is socio-mathematical norms of the classroom which can be separated from social norms with being specific for mathematics (Gravemeijer & Cobb, 2013). For example, different and acceptable mathematical solutions, explanations, justifications, proofs, i.e. The teacher does not offer any ways students to follow, instead the teacher and the students develop the socio-mathematical norms of the classroom by participating in whole class discussions (Gravemeijer & Cobb, 2013). Thus, it is essential to obtain the mathematical practices that occur during the classroom sessions, since those socio-mathematical norms are base for the formation of the classroom mathematical practices (Cobb & Yackel, 1996). During the process of the current study, while involving in the whole class discussions, all the participants shared their ideas, solutions, explanations, justifications with others. For example, during the process, some socio-mathematical norms emerged from basic features of prisms, basic features of cylinder, producing the formulas of surface area of prisms and surface area of cylinder, and while discussing on the volume of cylinder.

Eventually social aspect of interpretive framework is offered as mathematical practices by Gravemeijer and Cobb (2013). As mentioned earlier, Cobb, Stephan, McClain, and Gravemeijer (2011) defined the mathematical practices "focus on the taken-as-shared ways of reasoning, arguing, and symbolizing established while discussing particular mathematical ideas" (p. 128) and also Gravemeijer and Cobb (2013) defined it as "the normative ways of acting, communicating and symbolizing mathematically at a given moment in time" (p.89). By considering the definitions, the classroom mathematical practices occurred from the multifaceted participation of the students. For the current study, to evaluate and obtain the classroom mathematical practices and to interpret them scientifically, Krummheuer's (2015) argumentation model was used with learning of the concept of three dimensional shapes supporting with DGS, argumentations and daily life examples.

For the current study, students' learning activities interpreted under RME theory as an interpretive framework. As it is explained in detail in the previous chapter, the RME theory allows the researcher to focus on various learning processes. It also examines whether students have produced their own solutions or not, or whether they imitate the methods used by the teacher or other students. In such a case, the student can look at the variety of solutions. In this case, students are expected to identify new routes when they have a solution. In addition, students can try other procedures that are not compatible with the reinvention process when they encounter a problem in the learning process. In this case, according to the RME theory, there will be a demonstration that the route they follow is not a natural reinvention process (Gravemeijer & Cobb, 2013).

RME guided the current study by looking for answers to the questions such as; whether the students create their own ideas during the learning activities, whether the instructional activities support students' reasoning and finding solution process.

3.2 A Case Study

Case study provides tools and opportunities for researchers to study complex phenomena within their natural environments and contexts (Baxter & Jack, 2008). It also allows researchers to explore individuals or communities with their relationships, communications, and programs (Yin, 2003). Thus, it seems that case study is one of the most preferred methodologies regarding those characteristics (Merriam, 2009). This research becomes a valuable method for educational research to evaluate programs and develop theories (Baxter & Jack, 2008). Qualitative case study is an approach that facilitates investigation of a phenomenon in its natural context using many kinds of data sources. By this way, researcher ensures that the research issue is explored through a variety of lenses. Thus, it will allow many facets of the phenomenon to be found out, evaluated and understood (Baxter & Jack, 2008).

Yin (2003) states that, a researcher should take a case study into consideration when the focus of the study is to find an answer to the “how” or “why” questions; if there are not clear boundaries between the context and the phenomenon; or to see the context in its natural conditions to find the relations between phenomenon and the context. Current research seeks for ways to find out the relations between students’ understanding of three-dimensional shapes in geometry and classroom mathematical practices that they developed during the instructional sequence. Moreover, this research wants to see how this process will support the students’ achievement. Thus, it can be concluded that the study is a case study.

While defining the case Miles and Huberman (1994) stated that, “a phenomenon of some sort occurring in a bounded context” (p. 25). The answer to the question “what I want to analyze?”, will determine the case of the study (Baxter & Jack, 2008). For the current study, the case is the process for development of mathematical practices in an eighth-grade class while practicing the instructional sequence that designed by researcher and the mathematics teacher.

Looking at the main approaches which guide the case study method, the most common ones are offered by Stake (1995) and Yin (2003, 2006). They proposed different types for the case study method. Stake (1995) defined case studies as instrumental, intrinsic, and collective; while Yin (2003) categorized it descriptive, explanatory, and exploratory. Intrinsic case study was explained by Stake (1995) that the researchers who want to understand the case better should use this type of case study. The case represents other cases, but also it has its own particularity. The case is at the center of the study itself.

Consequently, the aim of the current study is to develop content for three dimensional geometric concepts in eighth grade mathematics curriculum by using argumentations and GeoGebra dynamic geometry software, to develop an instructional sequence, to obtain mathematical practices during this process and, to test the effectiveness of this content; the case has its own particularity that the study conducted around it, so this study is an intrinsic case study.

3.3 Participants

Related to features of a qualitative research study, the number of participants was limited. Since the aim was not about generalizing the findings, the study was conducted in a public elementary school in Yenimahalle that is a town of Ankara city. The current study was conducted in the school that researcher has been teaching. This school and the participating teacher were chosen because of their voluntariness, their availability and ease of accessibility (Fraenkel, Wallen & Hyun, 2014).

For selection of the participating teacher, a purposeful method was applied. The participant mathematics teacher has seven-year teaching experience with a master's degree. She was also close to completing her PhD thesis. Moreover, her research area is similar to the researcher's. She has been working on RME and classroom practices like the researcher. Therefore, she is familiar with the research methodology and has some idea what the current study's aims. She placed in the research team of the study. The research team consisted of two participants; One of them is the researcher, other one is participating teacher.

The participating classroom consisted of 16 girls and 19 boys, 35 students in total. It was chosen purposefully by the participating teacher regarding their classroom communication skills and willingness for participating to classroom activities and argumentation. During the data collection process, students' participation to the lessons was high.

The study was conducted in four and half week instructional sequence and seven-class-hours in each week. Since a class-hour is 40 minutes, each weekly cycle got 280 minutes sessions for the classroom. The participating eighth-grade classroom learned in a social environment which is designed according to requirements of argumentative classroom environment throughout a proper instructional sequence in which they engaged in geometrical issues alone or with their peers in small groups; after that by participating in whole-class discussions.

3.3.1 Role of the participating teacher

The participating teacher was the main instructor of the classroom. She was responsible for leading the teaching-learning sessions. She acted as an orchestrator of the classroom activities including whole class and pair argumentations aligned with the instructional sequence and HLT. She directed the classroom argumentations to make students get the expected understanding of the context. Additionally, she made them involve in argumentations about unexpected ideas and situations to handle possible misconceptions.

3.3.2 Role of the researcher

Aligned with the nature of design-based research, the researcher had an interventionist role during the study. Thus, she was also a participant observer during the study. She was responsible for observing flow of the instructional sequence aligned with the HLT. Also, she opened GeoGebra files to show them to the students when necessary. During the classroom works, she interacted with all the participants of the study as much as possible in their natural settings (Fraenkel, Wallen & Hyun, 2014). She checked students' works with the teacher and provided feedbacks for them. Sometimes, she led classroom argumentations when students needed more explanations, when they had some missing points etc.

3.3.3 Physical setting of the classroom

The learning environment was the main classroom in where they were attending all other lessons during school time. The design of computer lab was not appropriate for using in a that kind of study. The classroom included a teacher desk and students desks in it. There was a smartboard on the wall. Figure 3.7 shows the physical setting of the classroom.

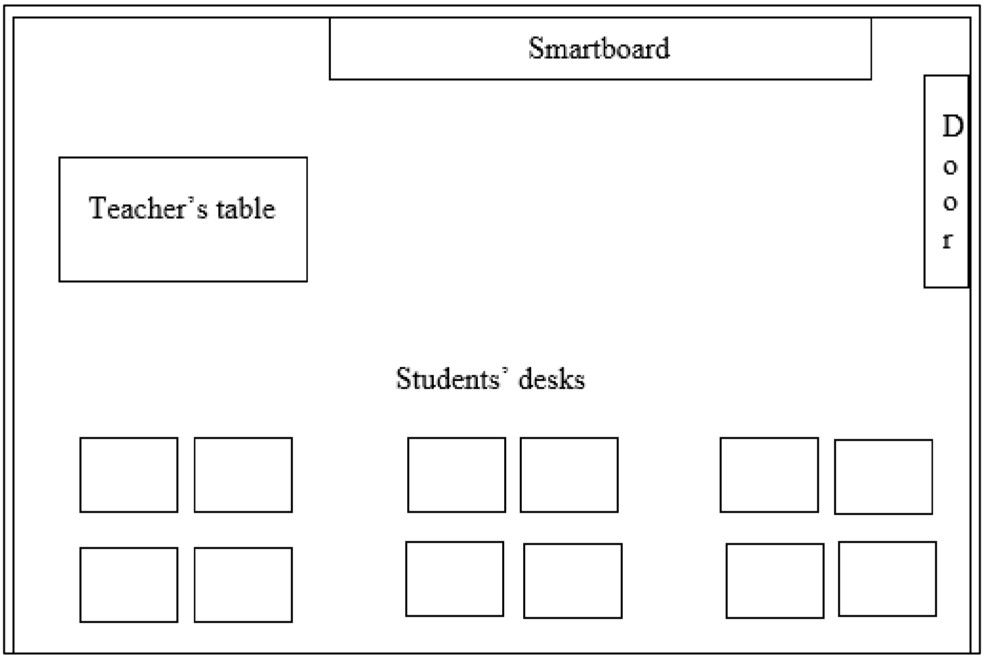


Figure 3.7 Physical setting of the classroom

3.4 Data Collection

The data corpus consisted of (a) classroom-based data, which include videotapes of all lessons, detailed field notes from the learning environment, and copies of all the students' written work; (b) audio-records of discussions from the meetings of school research team and (c) pre-posttests applied to the students before and after the study to obtain whether there were any changes in their achievement scores or not.

The school research team shared their ideas and experiences through the instructional sequence during the weekly meetings. Also, researcher and the participating teacher came together after completing the teaching-learning session of each course as in the objectives which was defined in the national curriculum. Those meetings were about what was happening in the classroom sessions regarding the instructional sequences; were there any problems, potential misconceptions, or any wrong-learnings in the students; what would be possible solutions to those; was is necessary to remove any content from the instructional sequence regarding the methodology of design-based research. These headlines were critical for revising the process of instructional sequence. In the current study, all the participants were mentioned by using pseudonyms.

The researcher started to prepare the instructional sequence nearly 6 months ago before the main data collection. The researcher and the participating teacher came together once or twice a week and talked about the content of the instructional sequence. They designed activities and arranged them in order according to the course-objectives of the national curriculum. While working on the worksheets, they discussed about if there were any shortcomings, something to add or any needs to change in the order etc. After the researcher and the teacher arrived at a consensus on the instructional sequence, four mathematics teachers from the participating school evaluated and investigated the results again. According to their claims, after the last form was given, the instructional sequence would be ready. The instructional sequence was applied to 10 eighth-grade students from different classrooms which are randomly selected, and their understanding of the activities was evaluated. The aim was to evaluate appropriateness of the content for the students' level. By evaluating the results from those ten students' works, the research team arrived at a consensus that it was ready for the experiment. When all the activities, instructional sequence and HLT were designed, the main study was started.

At the beginning of the data collection process, the pretests were conducted to the participating eighth-grade classroom. Also, after the application of the pre-test finished, the research team came together to talk about the tests and the first phase of the designed HLT. In the current study, to obtain and analyze the mathematical practices of an eighth-grade classroom; an instructional sequence designed for the basic elements and openings of 3-D shapes. Then, the data were collected throughout classroom observations and fieldnotes were recorded from classroom sessions, and meetings.

As Cobb, et al. (2003) states that design-based research has an active nature and to obtain the mathematical practices from classroom environment, it is required to connect socio-mathematical norms; the researcher tried to have detailed information and deep understanding of the content of the study both during and after the study. Thus, during all the processes of this research, it was critical to observe, collect, obtain, and analyze several types of detailed data for the requirements of design-based research (Cobb et al., 2003, Gravemeijer & Cobb, 2013). For the weekly cycles, the discussion issue was about the teaching-learning plans applied and also drawn consequences for following teaching-learning plans of the study. For the big macro cycle, a complete instructional process was evaluated.

Data collection started at the first week of May in 2016-2017 education year and it was completed after four and half weeks with participant and non-participant classroom observations, by taking fieldnotes from the classroom environment, video records of classroom activities, audio records of research team discussions and students written works. The researcher was both participant and non-participant observer of the study. She not only acted as a complete observer but also participated the instructional process. At first, she observed the classroom sessions, took notes about the classroom routines, behaviors of the teacher and the students, discussions, feedbacks, and tasks. Also, the researcher sometimes acted as a participant observer by joining the classroom sessions. She sometimes helped the main instructor while teaching tasks by using GeoGebra, supporting students or giving feedback, and starting a discussion about an important concept of the sequence.

To obtain the mathematical practices, video camera was used as a critical data collecting tool. Each lesson was recorded with a video camera. To capture accurate data from the learning environment, the camera was placed in several places in the classroom. Also, it sometimes carried by a school guard student -who do not attend any lesson that day- to get better video records from students' ideas, teacher instructions or peer discussions. Additionally, while peers were having discussion on activities and worksheets, the guard student brought the camera and captured the voices and written works.

Table 3.6 Data sources

Data Sources	
Classroom Observation	Field Notes
Whole Class Discussion	After-Lesson Meetings
Student Written Works	Weekly Research Team Meetings
Individual Works	Pre-Post Tests
Group Works	

In peer group discussions, the participating teacher followed the students' discussion process and she supported them and gave feedback to them. Thus, the data that came from the peer discussions provided information about how the teacher got interaction with those groups and how these short group discussions provided data to the whole classroom interaction throughout the instructional process. All the video and audio records that were collected from the classroom sessions, peer discussions and school research team discussions were transcribed by the researcher.

Another data from the study was the students' written works from instructional sequence. Worksheets were constituted of the written works in general. These worksheets were collected and evaluated to understand students' understandings of the whole instruction process. By doing this analysis, the aim was to see how the students worked together, how they discussed on issues, how they produced ideas or ways for problems or tasks. Additionally, the researcher watched the records of the classroom practices after each lesson and took notes about it to draw inferences for the following lesson plans and to discuss with the research team. As an example, in one of the lessons while working on the identifying different views of prisms that were made up of multiple cubes, a few examples did not include GeoGebra files of those shapes. This was an issue for the classroom because students wanted to see the example on the GeoGebra file and to work with the help of it. So, in the short meeting after-lesson, the research team decided to add a GeoGebra view for that kind of examples to provide accurate content for students.

In the meetings which were very critical for developing the instructional sequence, the school research team talked about content, accuracy, and order of the instructional sequence. Those meetings were done generally every Friday, after completing each week's teaching-learning practices. Throughout those meetings, the researcher and the participating teacher who was the instructor of the classroom came together and discussed the last week's general revision and drawn inferences for the following plans. Moreover, generally after completing each day's teaching-learning sessions, the researcher and the classroom teacher had short discussions about that day's performance, whether there were any difficulties, wrong learnings and any needs for change in instructional sequence. These small discussions also provided data and solved the issue for the weekly meetings of the research team. Moreover, those small discussions provided immediate feedback or solution for the issues which weren't clear. As mentioned above, those small meetings were held after lessons if it was necessary. Both meetings were audiotaped by the researcher and were transcribed. Those transcriptions were used for the evaluation of the classroom mathematical practices.

For the quantitative evaluation of the students' development, pre-posttests were applied to the students at the beginning and at the end of the study. Because of preparing those tests was time consuming and there were issues about validity and reliability; tests were derived from the website of the Ministry of National Education in an accordance with the level of students, learning objectives and were prepared according to the instructional sequence and HLT. A pretest was applied to the students at the first lesson of the process, and a posttest was applied at the last lesson of the instructional sequence. These tests were the same. The test included 11 questions. Ten of them were multiple choice questions and one of them was open-ended question. The pre-posttest was given in Appendix B.

3.5 Data Analysis

The data that were collected from the current study included qualitative and quantitative data. In this section, analysis of those data sets was explained in detail.

3.5.1 Analysis of qualitative data

In order to analyze and identify mathematical practices in the classroom, data analysis was done considering classroom discussions and how mathematical thinking was structured through these discussions. Regarding the nature of classroom mathematical practices, the main point of the study was the class discussions that took place through the collective participation of the whole class, even though individual studies and individual learning were included throughout the whole instructional sequence. For the analysis of the data set, two methods were followed as analysis way which were collected from the classroom observations, students' written documents and field notes from classroom environment.

Firstly, constant comparative method was used that was developed by Glaser and Strauss (2017). Researchers often reveal how their work is working, but they are insufficient to give information about the analysis. The systematic approach that can be used by researchers not only makes their work systematic, but also increases the traceability of their work when they explain how they use and apply this approach in research practice (Boeije, 2002). Constant comparative method is a cyclic method that evaluates what the data tell about the study process (Gravemeijer & Cobb, 2013) by making comparisons between old and new data. In this way, it is possible to answer questions that arise from the analysis and reflection of previous data. Then, the collected data is analyzed again and compared to the new data. The cycle of comparison and reflection is very old, so the new data can be repeated for several times. This process continues until the new cases do not provide new information to the categories. Gravemeijer and Cobb (2013) explained and used this method with two cycles process. They explained the first round as an explanation of what happened in the classroom and the second round as identification of pattern by constructing on the results of the first round.

Aligned with the explanations above, the current study aimed to describe the whole classroom learning process. A detailed analysis of classroom practices was conducted to identify this learning process. To determine how classroom mathematics applications were developed, the data were put in order chronologically. If a mathematical practice is formed, it means that students will not have a problem with it anymore, and there should not be any questions about that practice. If a student involved in a mathematical practice that is determined used the wrong explanation or argument for it, and if the other students in the class did not react to it or did not questioned, then it was necessary to revise the mathematical application that had been determined.

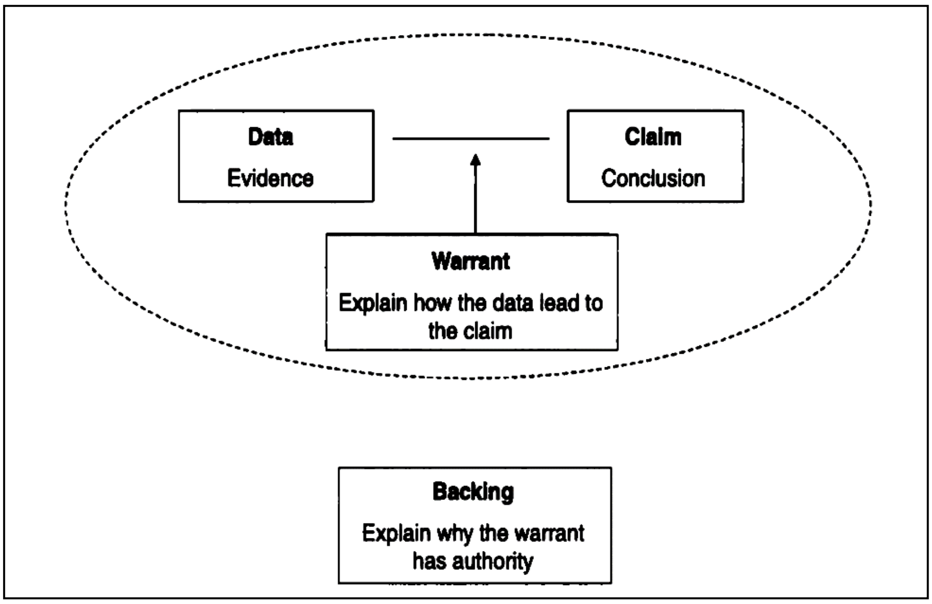


Figure 3.7 Krummheuer's model of argumentation (KMA)
(Rasmussen & Stephan, 2008)

It also demonstrated that practices evolved and replaced the considered mathematical practice. In the second round of analysis, these conjectures and refutations were treated as a new set of data that must be analyzed. When analyzing specific assumptions and confirmations in this section, some certain sections became important. In this view, two or more prominent assumptions were made to select the appropriate one (Gravemeijer & Cobb, 2013). Furthermore, to document and analyze classroom argumentation, Krummheuer's (2015) argumentation method was used that was adapted based on Toulmin's model. He stated that he used the Toulmin's argumentation method by confining it to four categories as data, conclusion, warrant and backing. Krummheuer (2015) defines the data as "undoubted statements" (p. 56), inference of the argumentation as warrant, and "permissibility of warrants" (p. 56) as backings based on the Toulmin's work. Figure 3.7 shows the Krummheuer's model of argumentation that was adapted from Rasmussen and Stephan (2008). To analyze classroom argumentation by means of this way, Rasmussen and Stephan (2008) have developed a three-phase method to document taken-as -shared ideas and mathematical practices. This method is useful for organizing the data set, and it reveals how the process's taken-as-shared ideas become mathematical practices. Each phase required different actions in themselves.

For the first phase, the process started by creating transcripts of each whole-class discussions. Then, the researcher watched all the video records and took notes for claims (conclusions) that were made by the teacher or any of the students. Then, KMA (2015) was used to form a scheme for each claim. To provide reliability, the participating teacher also produced her own argumentation log. Afterwards, the researcher and the teacher come together to discuss about their works of analysis and compared the two argumentation schemes. Then, they verified or refuted each other's analysis. By discussing on the data conclusion, backing and warrant issues, they came to agreement on the argumentation scheme at the end (Rasmussen & Stephan, 2008).

Second phase sees the argumentation log as a data set itself; and looks for whether the mathematical thinking become the groups' way of sharing their ideas normally. To understand this, Rasmussen and Stephan (2008) defined two criteria as; the first one is when any backing or warrant do not occur in the students' explanations, this means no one in the classroom have a challenge about that argument, the mathematical idea become a self-evident in the discussion; and the second one, the use of a previously justified conclusion or claim as data in subsequent discussions means that mathematical idea become the group's one of the ways of expressing thoughts (Rasmussen & Stephan, 2008). Then, they draw a chart to take notes about the mathematical ideas. This chart includes three columns that are (a) a column for the ideas that now function as if shared, (b) a column of the mathematical ideas that were discussed and that we want to pay attention to see if they function subsequently as if they were shared, and (c) a third column of additional comments, both practical and theoretical, or connections to related strands of literature (Rasmussen & Stephan, 2008; p. 200).

For the current study, with using this chart, it became more systematic to obtain the mathematical ideas which were needed to be discussed and to be taken-as-shared i.e. Moreover, with this chart, it was possible to see which ideas came to the first or second column from the second or third column by comparing the previous and current discussion dialogues. An example was provided for mathematical ideas chart in the Table 3.7 from current study. First column was about the mathematical ideas that they emerged during the whole class discussions. The second column was about the idea that emerged about cube and there was a need to pay attention to it. The third column indicates the practical actions about the context.

Table 3.7 An example of mathematical ideas chart

Ideas function as if shared	Ideas keep an eye on	Additional comments
Identification of basic features of prisms A cube is a rectangular prism	Whether a cube is a prism	By giving examples from real world, identified basic features of prisms was examined (RME).

This method also fitted with constant comparative method (Rasmussen & Stephan, 2008) as mentioned above. By this way, the research team could be able to make conjectures about current ideas whether they formed of as they were shared, and also look for following discussions if there were any data to construct on the previous one to make it taken-as-shared.

For the third phase of the analysis, by obtaining the taken-as-shared ideas, relatedly classroom mathematical practices were defined and produced. The ideas from the shared chart and the mathematical ideas were reorganized by labelling them as common mathematical activities if they occurred by the participation of the whole classroom; and they were named as classroom mathematical practices (Rasmussen & Stephan, 2008).

3.5.2 Analysis of quantitative data

Pretests and posttests were applied to the students. For the pre and posttests, results were constructed as quantitative analysis of the current study.

Test questions were derived from the website of General Directorate of Measurement, Evaluation, and Examination Service (which is a part of Ministry of National Education). The questions were selected in accordance with the HLT that was prepared for the current study. The questions on this website are constantly being updated in accordance with the national curriculum. Since, the conjectured HLT has already been prepared in accordance with the national curriculum, the questions have been adapted to the content of the study without deviation from the curriculum.

The test questions were prepared based on the concepts of general properties of prisms, their basic elements, understanding the relationship between open and closed states, surface area of prisms, general properties of cylinders, basic elements, surface area of cylinders and volume of cylinders. The number of questions was 11.

To provide reliability of the test, several ways were considered. For instance, as the number of questions used in an exam increases, in most cases the reliability of the total score obtained from that exam increases (Baykul, 1999). In this study, the pre-posttest included 11 questions which were focused on related content. Thus, it could increase the reliability of the test. Additionally, test questions were derived from web site of General Directorate of Measurement, Evaluation, and Examination Service (which is a part of Ministry of National Education). The questions on this website are constantly being updated in accordance with the national curriculum. Thus, those questions were expected to be checked and assessed by experts of Measurement and Evaluation. Also, if the questions are clearly understood and certainly answered, that increases the reliability of the score obtained from that exam (Baykul, 1999). The questions in the pre and posttest were prepared in a way that the students could easily understand. Moreover, each exam must be scored in objective ways. The answer key preparation increases the objective rating (Baykul, 1999). The tests were scored by an answer key that was prepared by the researcher. Furthermore, the duration of the test period should be balanced. More or less time should not be given. The time for the pre-posttests was 40 minutes which equals to one lesson hour. Those methods could increase the reliability of the test. Thus, by using those strategies the pre and posttests consired as reliable. For the analysis of pre-posttests of students' scores, paired-samples of t-test were applied to evaluate the difference.

3.6 Trustworthiness

To provide trustworthiness of the current study, several methods were considered. The first issue was about the triangulation which is gathering data from various sources like classroom observations, fieldnotes from the learning environment, meetings, i.e. (Creswell, 2009; 2012). The triangulation can give close or far-reaching results, whatever the case, it is a useful method for the researcher (Mathison, 1988). Denzin (2012) states that triangulation is not only a validation method, but also increases the generalizability of findings. Moreover, it is an approach to increase the confidence of data set, provides a clear understanding of the phenomenon, and opens new ways to get a deep and accurate understanding of the specific problem (Mok & Clarke, 2015; Thurmond, 2001).

Aligned with the explanations of data that were collected through several sources such as classroom observations and video-records of those observations, fieldnotes from the learning environment, pre-posttest results, meetings of research team.

By member-checking, the interpretations and transcriptions of data set went back to the participating teacher and provided her ideas and claims about those data. As a last issue, the study continued during four-week and it also provided reliability of the study for the researcher to gain patterns in data accurately by collecting data in a process (Creswell, 2009).

3.7 Limitations

There are some limitations about the current study, since a designed based method is used. For being a designed based research, the findings of the study are not much generalizable with the other contexts. Maybe, by developing and using the cycle of the study with other eighth graders from other schools can increase the generalizability of the study.

Also, another limitation of the study would be conducting the study to base on only one macro cycle. Before the main study, it would be appropriate to conduct a pilot study to get more accurate data set. However, even though the pilot study was not carried out, the instructional sequence of the study was prepared for a long time by discussing with other mathematics teachers and by getting their opinions. Then, the prepared content was applied to ten other non-participant students in order to measure the appropriateness, so that those work could fill the gap of a pilot study.

Moreover, the last version of HLT and instructional sequence from this study can provide a source for further studies and can be used to conduct a new design study with other environments which also would be able to increase the generalizability of the study.

Another limitation about the study is usage of the DGS on the smartboard by the participating teacher. During the instructional process, GeoGebra files was shown to students by the researcher or the participating teacher, because the school's computer lab was not suitable for that kind of study. It would be beneficial for students to evaluate GeoGebra files by individually or within groups to have stronger understanding. But during the study, they evaluated the shapes from the DGS on the smartboard as much as possible, and they did not have much challenge throughout the process.

Another limitation about the study would be the teacher's guiding the classroom discussions through the way she showed. This was to some degree shaped the emergence of the classroom mathematical practices, but that participating classroom was an eighth-grade classroom, students needed to be guided by an instructor since they did not have idea about how to operate those kinds of discussions.

CHAPTER 4

FINDINGS

The main focus of this research was to extract the eight graders' classroom mathematical practices in 3-D shapes during an instructional sequence and HLT. The instructional sequence was supported by an argumentative classroom environment and instructional activities designed with daily life examples and DGS to support instruction with the aim of developing students' understanding of geometric concepts. In this chapter, the answers were provided to the questions;

1. What are the mathematical ideas that support the mathematical practices which students developed during this instructional sequence?

2. Is there any effect of this instructional sequence on the students' achievement by using dynamic geometry software in that context?

The qualitative and quantitative findings were explained in this way. To explain qualitative findings, Krummheuer's (2015) model of argumentation which was developed from Toulmin's model was used with the aim of extracting the classroom mathematical practices in the context of three-dimensional solids. The quantitative findings demonstrated the scores obtained by pre-posttests results that were prepared to evaluate the students' understanding of three-dimensional solids. Pre-posttest results were analyzed by using paired samples t-test. First qualitative results and then quantitative results were explained in order.

Classroom mathematical practices are defined as takes-as-shared ways of students' ideas that occur during classroom processes in which students do not justify or prove the truth of the idea (Cobb & Yackel, 1996; Stephan & Cobb, 2003). In identification process of classroom mathematical practices, first the mathematical ideas' chart was evaluated to examine students' mathematical activities when the discussed mathematical ideas became taken-as-shared. Also, the emerged classroom mathematical practice should relate to the HLT of the study which guides the instructional sequence (Cobb et al., 2001). The HLT anticipates the process of learning mathematics in classroom by conjecturing in which and what kind of activities students may involve in that community. In this way, Cobb et al., (2001) states that "It is feasible to view a conjectured learning trajectory as consisting of an envisioned sequence of classroom mathematical practices together with conjectures about the means of supporting their evolution from prior practices" (p. 125).

Accordingly, the HLT of the current study was used as a basis to demonstrate the expectation of classroom mathematical practices that might occur in classroom community. The mathematical ideas chart used side-by-side (Andreasen, 2006) to analyze classroom mathematical practices that were formulated through classroom discussions. The tasks that support mathematical practices and changes in the instructional sequence as practiced were identified to determine the support to collective learning process. Identification of classroom mathematical practices may be helpful for the identification of actual HLT and with this respect, it could be possible to make further revisions and modifications for future implementations of the instructional sequence and HLT.

In this respect, the current study obtained four mathematical practices that occurred during the process were supported by this HLT and instructional sequence were (a) finding definition and properties of prisms, (b) finding surface area of prisms, (c) finding surface area of cylinder and (d) finding volume of cylinder. Additionally, it was explained that what kind of mathematical ideas made students to produce those mathematical practices.

More clearly, the taken-as-shared ideas that supported by related mathematical practices were explained. These classroom mathematical practices were produced by students and taken-as-shared ideas that supported those practices were illustrated in the Table 4.1.

Table 4.1 Four mathematical practices emerged from the study and the taken-as-shared ideas supported those practices

Classroom mathematical practices with supported mathematical ideas
Practice 1: Finding Definition and Properties of Prisms Idea 1: Understanding roof of buildings' and tents' shapes are prism Idea 2: Understanding a cube is a prism Idea 3: Understanding the relationship between base shape and other parts of a prism Idea 4: Understanding a cylinder is not a prism
Practice 2: Finding Surface Area of Prisms Idea 1: Understanding wrapping means drawing net of a prism Idea 2: Counting unit squares Idea 3: Transition from counting unit squares to calculating area Idea 4: Producing the formula for surface area of prisms
Practice 3: Finding Surface Area of Cylinder Idea 1: Structure of net of the cylinder Idea 2: Relation between the circumference of the circle base and edge of its side face Idea 3: Cylinder's surface area constructed by area of side face and area of circle bases
Practice 4: Finding Volume of Cylinder Idea 1: Volume is about third dimension Idea 2: Volume is about filling inside of a shape Idea 3: Calculation of volume requires the knowledge of width, length, and height Idea 4: Volume equals to the multiplication of base area and height.

4.1 Mathematical Practice 1: Finding definition and properties of the prisms

The first mathematical practice occurred during the instructional sequence on the concept of three-dimensional shapes; which determined the basic elements of prisms to reach a definition and to provide a meaningful understanding. The practice emerged with the guidance of conjectured HLT. This mathematical practice emerged through the concept of the basic elements of prisms. This practice emerged by the discussion of mathematical ideas from the first week and one day from the second week. During the first week, the issue was about the properties of prisms and their main elements and nets of the prisms. The process was based on the individual and peer works and also whole class discussions. After working individually and in pairs, students got involved in classroom discussions to construct mathematical practices. The process started with the teacher's questioning the students about types of daily life examples of prisms and relatedly their properties. The instruction continued with working on different views of prisms and relatedly understanding the nets of them.

4.1.1 Idea 1: Understanding roof of buildings' and tents' shape is a prism

The first mathematical idea emerged in the first week of the instructional sequence while the classroom was talking and learning about definition, types, and properties of prisms. The lesson started with the teacher's asking the students about "their ideas on what a prism is" and "what kind of things can be defined as prisms". Following dialogue happened at the first lesson of the instructional sequence.

Teacher: …. What does the prism mean? What comes to your minds when we say prism? I want you to think and explain your ideas about this issue. Yes, let's start with Zeynep?

Zeynep: Teacher, I think about it is a three-dimensional version of a geometric shape.

Teacher: You think about three-dimensional version of a geometrical shape. Yes, Buse.

Buse: A thing that has edge, corner, and faces.

Teacher: Ok, any other ideas? Yes.

Aydın: As I remember, it has bases at the top and at the bottom

Teacher: Yes. Another idea?

Hakan: Cardboard

Teacher: What do you mean by saying cardboard?

Hakan: We do it by using cardboard.

Teacher: What do we do by using cardboard?

Hakan: We do prisms.

Teacher: What kind of features do your shape have done by using cardboard?

Hakan: (No reply)

Teacher: Any other idea? Hakan says we do it by using cardboards.

Selma: We use plastic to make them.

Teacher: What kind of shapes do you make by using plastics or cardboards?

At the beginning of this section, teacher wanted to question students about their ideas on prisms. Zeynep explained her idea by stating that a prism was a three-dimensional form of a two-dimensional shape. Buse defined a prism with its basic elements. Aydın added the bases. Then other students stated their ideas by giving examples from daily objects. The section continued with teacher's asking students' explanations to make students find appropriate examples to express their own ideas. This dialogue demonstrated that the classroom had some idea about what a prism is, but they did not know how to explain their thinking about the properties of a prism and what kind of shapes could be defined as prisms. In other words, the class needed the support of the teacher while guiding them how and in what ways to think about context and to express those ideas verbally in classroom. In this dialogue, there was not a taken-as-shared idea, and the class continued to discuss the examples of prisms from daily life.

Teacher: Ok. Let's say what kind of things are prisms? I want you to think about examples from daily life about prisms? Yes, Hasan.

Hasan: Milk boxes.

Teacher: How milk boxes can be prisms?

Hasan: Rectangular prisms

Teacher: You say, it looks like rectangular prism.

Student: For example, the bookcase.

Teacher: The bookcase. Yes, you say for example, the bookcase in our classroom. Are there any other examples?

Kaan: Matchbox.

Teacher: Matchbox. Another one? Yes, Yağmur.

Yağmur: Roof of the buildings and camp tents.

Teacher: Roof of the buildings and tents. Another idea?

Harun: Cylinder-shaped pencil case.

Teacher: Yes, Mete.

Mete: Tin drink boxes.

Teacher: Tin boxes? Another idea?

With the section above, students tried to provide examples for prisms from daily life related to their prior knowledge. Looking at the examples, students seemed to provide appropriate examples for the prisms. This dialogue illustrated that students have the idea of prism and able to give examples from the physical world around them. Harun's example cylinder-shaped pencil case as a prism was an indicator of their lack of knowledge about the properties of prisms and relatedly confused with cylinder. A whole class discussion on cylinder will be mentioned in the next sections. These dialogues above, did not include any statements defined as claim, data, or warrant. Also, the example of buildings' roofs started another discussion.

Tuğçe: Can I ask a question?

Teacher: Yes.

Tuğçe: I think that roofs are not prisms, aren't they?

Teacher: Just a minute. Tuğçe asked a good question. She is not sure about whether the roofs are prisms or not.

Tuğçe: Because, if we remember Aydın's claim, they should have top and bottom bases, but roofs do not have that kind of equity.

Teacher: What do you say? What's your idea? She states based on what Aydın said, prisms should have equal bottom and top bases. But she says roofs do not look like that.

Tuğçe: Roofs have bottom bases but other edges merge at the top point of the roof, don't they?

Teacher: Yes, what do you think? Are roofs prisms or not?

Students: No.

Teacher: Why? Why do you think like that? Any ideas? Yes, Kerem.

Kerem: I agree with Aydın. As we learnt in previous years, prisms should have top and bottom bases. But when we look at the roofs, they are not appropriate with this definition.

Teacher: I guess everybody have the same idea.

Class: Yes.

At the beginning of this section, Tuğçe asked that whether a roof was a prism by referring Aydın's claim that prisms should have top and bottom bases, but roofs do not have that kind of equity. In general, students remembered from prior knowledge the prisms had equal top and bottom bases. But the problem was about the position of the shape. This section showed a visualization problem of prisms. Nearly, whole class was sure about roofs and camp tents were not prisms since they did not have top and bottom bases. But they did not consider the position of those objects. In this section, based on the Aydın's idea, Tuğçe and Kerem continued to explain their ideas but incorrectly. Students were having difficulty to understand a tent, or a roof was also a prism. This problem was based on the students' visualization problem about position of a prism. They could not visualize in their minds where the top and bottom bases while it was placed horizontally. Actually, the teacher was aware of the situation and continued as following to guide students' discussion to make them to see the position of roof and tents.

Teacher: Let's look at the common features of those examples you said. For example, looking at the bookcase or a matchbox, what can you say? Think about this. What are the common features?

Arda: They have corners.

Teacher: Good. They have corners. What else?

Berna: They have edges.

Teacher: Yes, they have edges.

Yalçın: They have faces.

Teacher: They have faces. Let's compare those faces. Where do you see those faces? Tuğçe.

Tuğçe: At the bottom and at the top. And also, they have side faces.

…

İpek: Those bottom and top bases are parallel to each other.

Teacher: Very good. She explained that top and bottom and top bases should be parallel.

Büşra: They have heights.

Teacher: Very good. They have heights. You said top and bottom bases. Side faces. Let's look at your example roof and camp tents. Do they have faces?

Aydın: Yes. They have side faces.

Teacher: What about top and bottom bases?

Aydın: They don't have those.

Büşra: But, why we cannot say tents or roofs are prisms? They have the same shape at both two sides.

Teacher: Yes, listen to Büşra, again please.

Büşra: I say, one side is a triangle in a roof and it also has same triangle other side.

Teacher: You say, it has two triangle faces. So, she asks why we cannot call it as a prism?

Class: (Silence)

Teacher: Ok. I want you to observe this illustration. (Teacher opens a GeoGebra file).

While discussing about the common features of the examples given by the students, they were able to express truly about the common features of prisms. İpek caught a good point as the parallelism of top and bottom bases, but the discussion did not continue, thus neither a student challenged the idea, nor the teacher continued the issue. But, by referring to previous discussion, Büşra challenged the idea of roofs' and tents' as not being prisms. She justified her idea by stressing the equity of top and bottom bases of those object as two equal triangles. At this point, the researcher opened a GeoGebra file that illustrates a roof -triangular prism-shape as in the following. The aim was to make students to understand the position of roof and tent and relatedly they're prisms. Actually, there was not a planned demonstration like this in the HLT of the study. During the whole-class discussions, the flow of the conversations required an illustration of roof and tent to clarify them about those shapes were prisms. In after-class meeting, the researcher and the teacher talked about the issue and decided to add demonstration of some prisms from the physical world around us to the HLT.

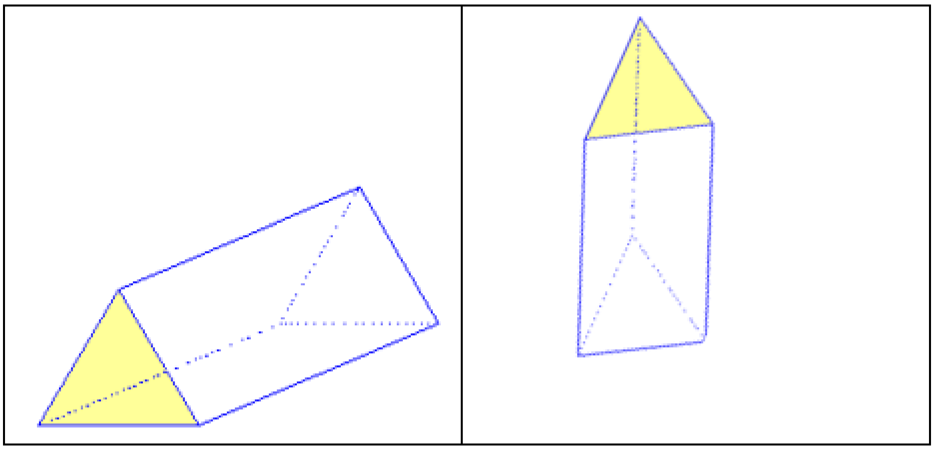


Figure 4.1 GeoGebra file showing two positions of a roof

Teacher: Now, what do you think about the issue? Do roofs and tents have top and bottom bases. Are there any changes in physical features of this shape when we turn it up?

Tuğçe: It's still a roof or tent, there is no change.

Teacher: Yes, Büşra.

Büşra: The shape has the same features with other examples that are on the board. So, it has the features of a prism. It has two equal bottom and top bases, it has height, and it has side faces.

Aydın: Those bases are parallel to each other.

Teacher: So, what is the decision about the roofs and tents?

Aydın: They are also prisms.

Teacher: Yes, they are prisms. What kind of prisms are they?

Class: Triangular prism.

Teacher: Are there any missing points here? Is there any one that did not understand?

Class: No.

The demonstration of the shape of a roof or a tent on the GeoGebra file made the students catch and fill the missing part of their viewpoint. By this way, they were clarified and confident about the roof and tent were prisms. In fact, students had the knowledge that tent, and roof were prisms, only they needed an assistant to help them realize that they had this knowledge. The GeoGebra file also undertook this task. Relatedly, by understanding the issue, they had the chance to use that knowledge for the following discussion which lead them to produce the mathematical practice. Also, that action changed the direction of the discussion in a positive way by making it easier for students to understand the common features of prisms and the importance of looking at other shapes from different perspectives. Another important point was, students' progression on the discussion by listening another one's idea and responding accordingly. Relatedly, after demonstration of the shape on the GeoGebra file, Aydın made the claim as, it was a prism with a data from Büşra and a warrant from Aydın. During the discussion process, the students formed the mathematical idea about "the definition of what a prism is", based on the examples given by them and based on the comparison of those examples according to their common features to find out the basic features of prisms. The process continued with the guidance of the classroom teacher and at the end of the process, there was nobody that challenged the idea again. The structure of the dialogue was made according to KMA (2015) and is illustrated as in the following.

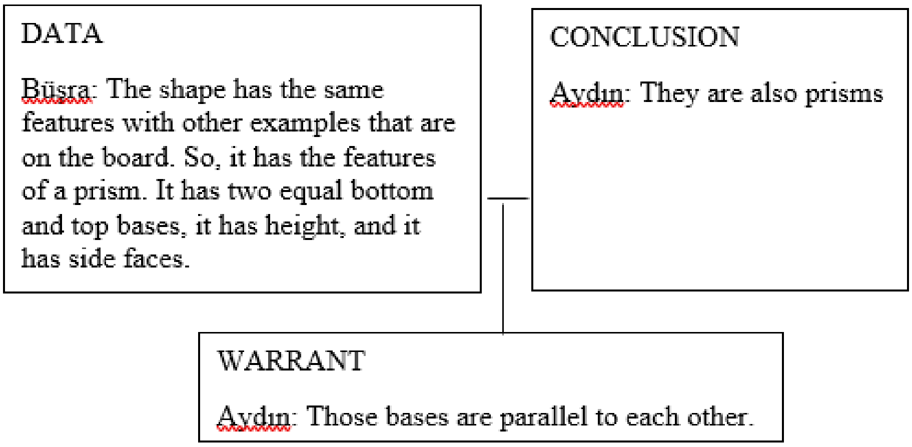


Figure 4.2 KMA on discussion of tents' and roofs' shape is a prism

After completing the discussion session, the teacher reorganized the definition of the prism as based on the examples and features that were given and told by students. By using this idea, students solved questions and involved in whole-class discussions without any challenge during the first week and second week of the instruction.

In the following lessons, a similar discussion was developed by students while evaluating the basic elements of the prisms on the GeoGebra file. They mentioned the position of the tent by stating that it was not a prism again. The teacher demonstrated the construction of the prisms on the smartboard. At this point, students were expected to relate the tent shape to the Figure 4.1. In the following, firstly the Figure 4.1 was shown from the GeoGebra file and the section was chosen from that discussion.

Teacher: …Look at this triangle prism. What does happen when do you tilt it to the one side?

Hakan: Teacher, when we do that, it becomes a tent and tent is not a prism.

Teacher: I think, we did a similar discussion in the first lesson. Let's remember it. Think about the features of the prisms, or any other shapes. Do you think the shape changes when you change the position of it? Is there any physical operation to it?

Mete: But, when we tilt it, it looks like a tent. It does not have the features of a prism.

Teacher: Doesn't it? What kind of features are absent in this tent?

Figure 4.3 A tent shape

At this part, the same issue arose related to the position of the prism. By relating to the prior discussion, teacher wanted students to think about their misunderstanding or missing the rule that changing the position of a shape does not affect the features of that shape. Thus, the aim of the teacher was to make students to understand that issue, so she tried to direct the discussion in that way by questioning students. Actually, students could observe a tent shape like in the Figure 4.3, by this way, they would understand the relationship between different positions of the same shape. Thus, the researcher and the teacher decided to show a tent or roof figure from internet and add this example to the instructional sequence and HLT at the after-lesson meeting. The section continued as following.

Mete: We defined that a prism has rectangle side faces. But a tent does not have those side faces.

Teacher: Do you agree with Mete? Let's remember the first day's discussion. I think, we talked about the same things.

Buse: When we tilt that shape, it looks like a tent or a roof. Those are not prisms.

Teacher: Are there any other ideas?

Selma: You demonstrated a triangular prism again. When we change the position of the shape that time, we saw that it was a triangular prism.

Teacher: Yes, we did this discussion at the beginning of the instruction. Remember the transformation geometry. If you turn a shape through any way, there is no change emerge in any edge, height, angle etc. of it. Actually, by cutting the shape vertically and horizontally, we observe some shapes. For instance, if you cut this triangular prism vertically, what shape do you observe?

Aydın: We see rectangle.

Teacher: Yes, exactly. Ok. If you cut it horizontally, what do you see?

Begüm: A triangle.

Teacher: A triangle. So, we stated before the way of naming a prism. How was it?

Aydın: Looking at bases. They are named according to the shapes at the bases.

Teacher: Yes. Actually, we look at the cut faces. If you see a rectangle on the vertically cut faces, it can be defined as a prism. So, I repeat that a shape does not change by changing its position.

Arda: They also have parallel bases. Then, a tent or a roof is a triangular prism.

This episode emerged in advancing hours of the instruction and demonstrated the usage of knowledge of features of a prism as data without any warrant in the whole class discussion. Mete stated that prisms should have rectangle side faces that were a data from previous discussion. Thus, he seemed to conclude that tent shape was a prism. Also, Aydın used the statement of parallel bases as data again and none of the students from the classroom challenged them. This discussion was shown according to KMA as following.

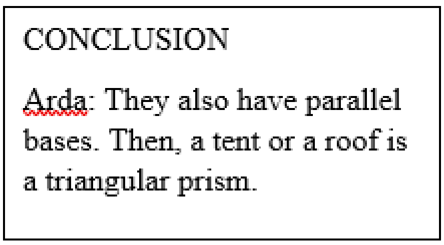


Figure 4.4 KMA on discussion of tents' and roofs' shape is a prism

Throughout the whole class discussions, the context of the arguments was appropriate for social and socio-mathematical norms in terms of involving by sharing ideas in mathematically meaningful way in the classroom environment. Thus, the idea was confirmed as became taken-as-shared.

4.1.2 Idea 2: Understanding a cube is a prism

This idea became taken-as-shared during the instruction in which the classroom continued after talking about the daily life examples of prisms. When the discussion process was completed, the teacher reorganized the definition of prism and students wrote it to their notebooks. After giving the definition of the prism based on the prior knowledge and daily life examples, the classroom continued with the first two pages of the activity sheet related to definition, types, and properties of prisms. Students were asked to complete the gaps in given questions. They worked individually on the papers. The following examples are given from the activity sheet that students used the mathematical idea of determining the basic elements of prisms. First question is about the basic elements of a prism such as edge, bases, height. Students used their knowledge that they developed during the whole-class discussion about properties of a prisms. During those two pages of the activity sheet, the teacher and the researcher visited the students to guide their works, but nearly none of them questioned or challenged about any missing points or misunderstanding of the issue. These two pages were generally, asking for basic features and elements of prisms. Question samples from these two pages were given in following parts. Figure 4.5 shows the first question of first page of the activity sheet.

Before starting the second question, the teacher asked the students about the relationship between the shape of the base of a prism and its name. The discussion was based on the first idea of the students' which developed the second idea by thinking on the concept of edge, face, height etc. Moreover, this process brought a new questioning of students about whether cube was a prism. The following section starts from the teacher's reorganizing the definition for students.

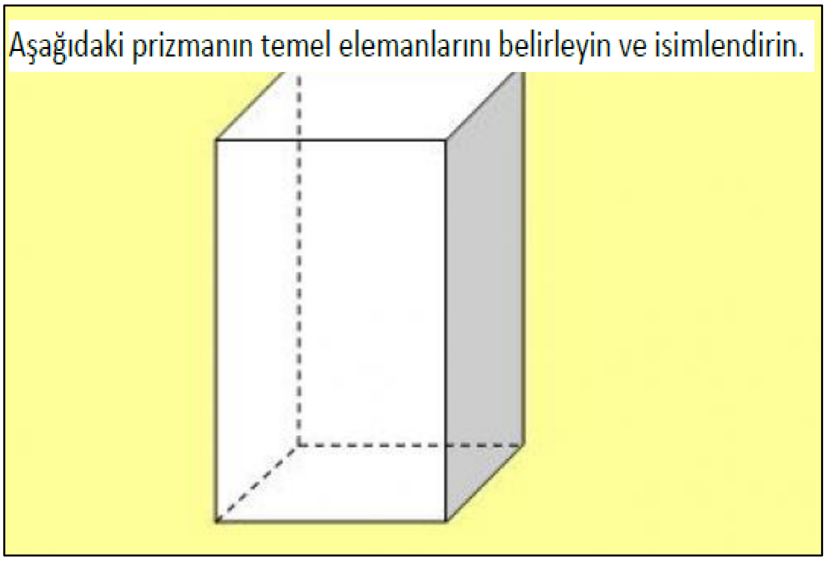


Figure 4.5 First question of the activity sheet about properties of prisms

Teacher: .... The geometric objects, whose side faces are made up of rectangular regions and whose bottom and top bases are made up of any polygonal regions, are called prisms. Do the side surfaces should be a rectangular?

Class: Yes.

Teacher: You say yes? So, let's remember what the cube is. You know cube from the 5th class. Is it a prism? Does it maintain the features of prism?

Beyza: It does not. It has all square sides, not rectangle. But, I'm not sure. It is also a three-dimensional shape.

Teacher: You say all sides are equal. So, all faces will be square, right? So, the cube is not a prism. Say Zeynep.

Zeynep: It is a prism because it's top and bottom bases are equal, and their side surfaces are equal.

Teacher: There is another important point. Let's remember.

Hakan: Its edges are equal.

Deniz: Also, all sides and faces are parallel to each other.

Teacher: Ok. But what does it say in the definition, the sides are made up of rectangular regions. Then, how the square can be a rectangle?

Aydın: The square is already a special rectangle, isn't it? So, a cube should be a prism also.

Teacher: Yes, did you remember from the 5th grade, you should have learned it?
Class: Yes.
Teacher: So, the square is a special rectangle, and the square should be a prism.
Class: Yes, …

In this debate, the teacher reorganized the definition of the prisms, and then started to question students about the side surfaces of a cube's being rectangle. The discussion was extended by talking about cube. Beyza was sure that the cube was a three-dimensional solid, but she confused the issue about side surfaces. At first, most of the students thought that cube was not a prism since side surfaces were not rectangle. But later, Aydın reminded that a square was a specific type of rectangle and that idea was accepted by the classroom. Thus, the idea became taken-as-shared and used several times in following weeks. For instance, while working on the surface area of prisms, there were some three-dimensional rectangular prisms constructed by unit cubes. At that time, the classroom used the idea of cube is a prism without any need for a warrant by the classroom. The structure of the idea of cube is a prism is shown in following Figure 4.6.

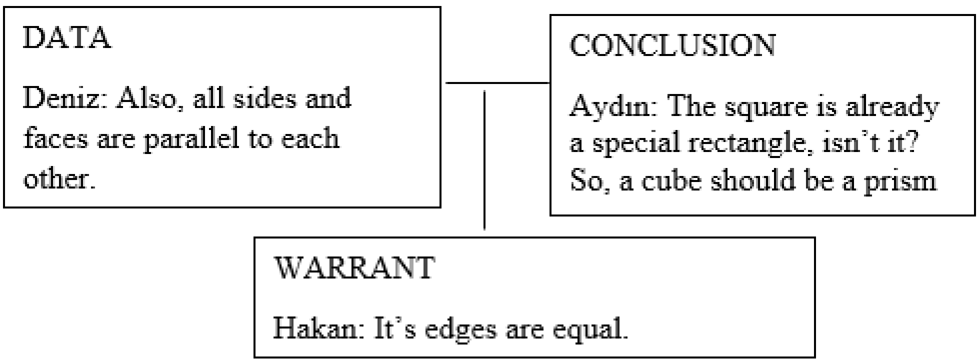


Figure 4.6 KMA by reasoning on definition, types, and properties of prisms.

In advancing lessons, while students were working on surface area of prisms, students worked on the cube as a prism and there was not any objection, or challenging idea for whether the cube was a prism. The context was based on the candy factory concept and students were expected to draw wrappers for candies in given shapes. The first shape was cube. The students were expected to draw a wrapper for a cube shaped candy. At the beginning, the context was introduced to the students.

Researcher: Yes, in this part, we have a factory concept. I want you to read the introduction part yourselves, and after we will talk. (Students read the introduction of this part). What do you want to say about your reading? What did you understand? Yes, Arda.

Arda: As I understand, there is a wrapping factory for candies and we produce wrappers for candies. Those wrappers have unit squares on them.

Teacher: Yes, any other idea?

Zeynep: Also, at the beginning, it gives us a cube as a basic prism to draw a wrapper.

Teacher: Yes, good. Is there any other idea?

In this section, the classroom read the beginning part of the surface area context. They started expressing their ideas about the part and Zeynep stated that a cube was given as a basic prism for them. After her explanation or idea, there was not any negative feedback, or any warrant, so it became a taken-as-shared idea among the classroom. This can be modelled as in the following according to Krummheuer's argumentation model.

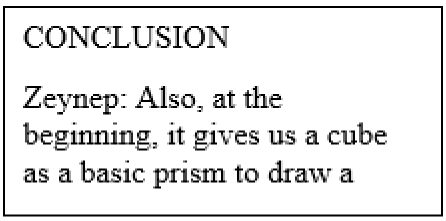


Figure 4.7 KMA by reasoning on definition, types, and properties of prisms.

In the following lessons, the classroom used the cube concept as a prism while they were talking about calculation of surface area of a rectangular prism and there was not any objection to the discussion. The class accepted the cube as a prism, and in the following lessons there were no discussions or questions about this topic. Additionally, the students used this idea in advancing hours of the instruction as data. For example, while they were working on the surface area of prism, one of the students supported her claim about cube is a prism, so finding the way of the surface area of a rectangular prism could be found by this way. At that conclusion, there was no warrant, or any opposite ideas, or misunderstanding for the issue. Thus, this idea became taken-as-shared and was not discussed anymore.

4.1.3 Idea 3: Understanding the relationship between base shape and other parts of a prism

The 3rd mathematical idea was emerged during the first week of the instruction and continued to be used in later practices of the instruction of the following weeks. This idea was constructed during the activities based on the basic elements of prisms such as edge, height, face etc. and based on the mathematical idea of one and two.

The following question in the Figure 4.8 is the second one of the activity sheets. It was prepared to obtain students' understanding of the elements of the prism and the ability of relating those elements to the name of the prisms. The students were asked to complete the missing parts of the given table related to faces, edges, etc. First, they worked individually and then the teacher started the classroom discussion. Following section illustrates the discussion conducted after completing the question.

Aşağıdaki tabloyu doldurunuz.

Geometrik cisim	Yüz sayısı	Köşe sayısı	Ayrıt sayısı	Tabanın benzediği çokgensel bölge
Küp				
Kare dik prizma				
Dikdörtgenler prizması				
Üçgen dik prizma				
Beşgen dik prizma				

Figure 4.8 Second question of the activity sheet about properties of prisms

Teacher: We talked about the elements of a prism. Let's continue with the second question. Is the first one cube? What do you say? How many faces does a cube have? Burcu.

Burcu: Six.

Teacher: Yes, six. One from bottom base, one from top base and four from sides so six in total. Ok. How many corners does it have? Tuna.

Tuna: Eight

Teacher: Eight. Very good. Yes, look at the cube here (by demonstrating a concrete cube). Four here at the top, four here at the bottom so eight in total. Ok. How many edges does it have?

Tuğçe: 12

Teacher: How did you find it? Did you count all the edges?

Tuğçe: Yes, I counted all of them.

Teacher: Is there another idea? What can you do instead of counting? Aydın.

Aydın: Top and bottom faces are equal, and they are squares. One square has four edges and two of them have eight. Also, it has four heights and 12 edges in total.

Teacher: Very good. Calculating the number of edges is easier. Are there any problems with Aydın's way?

Class: No.

Teacher: What about the square prism?

Buse: Number of the faces is six. Number of the corner is eight. Number of the edges is 12.

Teacher: What is the shape of the base?

Buse: It's square.

Teacher: Yes, very good. The following is rectangular prism. Say Hasan.

Hasan: Number of faces is six, number of corners is eight, number of edges is 12 and the base is rectangle.

First part of this section was about the completing the missing parts of the questions that were given in the activity sheet. The classroom successfully completed the missing parts by saying appropriate numbers with the given prism.

This example was important for students to understand the relation between edges, bases, heights and relatedly their names. The discussion continued as following.

Teacher: Exactly. Can you compare the three of those prisms?

Yağmur: They have the same number of the edges and faces.

Teacher: So, why do we name them by using different terms?

Mete: But, they have different bases. As we talked before, it is related to their bases. We name the prisms according to their bases. For example, cube is a special prism related to it has all square faces.

Teacher: Good. Let's continue with the following one. Kaan.

Kaan: Five faces, six corners, nine edges and it has a triangle base.

Teacher: Great. Say the following one. İpek.

İpek: It has seven faces, ten corners and 15 edges, it has a pentagon base.

Teacher: Yes, we completed this part. I think you understand how to calculate number of those elements of the prisms. Now, I want you to think about the relationship between the type of the prisms, their edges, faces and heights.

Hakan: We name the prisms according to the shapes of their bases.

Teacher: It's true that we said before. But, how the number of those elements changes related to the type of the prism? (After a while silence) Yes, Aydın.

Aydın: I think the number of the elements increases related to the shape of the base.

Teacher: Can you explain with an example?

Aydın: For example, a triangle prism has a triangle base. Relatedly, it has three bottom edges, three top edges and three heights.

Tuğçe: Also, side faces increase related to shape of the base.

Teacher: Yes. That is the point. …

This section was a good illustration of how students constructed the idea of the elements such as edge, height, and side face etc. At the beginning, they tried the way of counting to find the missing parts of the question, but later constructing the idea of relationship between the edges, faces and heights etc., they began to use that way easily. They began to understand the relationship between the base shape of a prism and other parts. At the end of the section, Hakan, Aydın and Tuğçe's ideas produced the third mathematical taken-as-shared idea about the relationship between base shape of a prism and other parts such as number of side faces, number of edges etc. The idea was accepted by the classroom without any challenge or any question. Also, they used this mathematical idea in the following parts of the instructional sequence such as surface area of the prisms as data for many times. The structure of the third idea was illustrated in Figure 4.9.

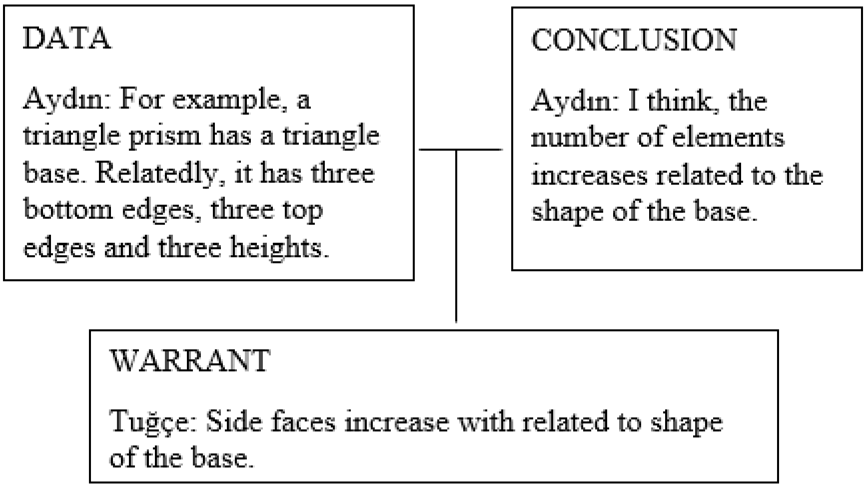


Figure 4.9 KMA on understanding the relationship between parts of a prism

In the discussion, they talked about the basic elements of a prism and relatedly they worked on the second question. The mathematical idea 3 was developed during this debate. The class used the mathematical idea while working on the following question. Moreover, Büşra extended the discussion based on a given example at the beginning of the instruction. In the following visual, the third question and the discussion that occurred continuously was illustrated.

Teacher: …Let's continue with the following question. What do you see in this question? What is it about?

Harun: It is about their open forms.

Teacher: Yes, the question wants you to guess the type of the prism by looking at its net. Look at the first one. What do you think?

Selma: It's a rectangular prism.

Teacher: Why did you think like that?

Selma: Because it has rectangle bases. We name the prisms according to their bases.

Teacher: Yes. As we said before, we look at the bases. Another one?

Hasan: It’s a cube. Because, it has all of the equal faces.

Teacher: Yes, it is a cube. Another one?

Selim: It is a rectangular prism again.

Teacher: That’s right. And the last one.

Begüm: It is a triangle prism. Here. We can also see Aydın’s idea. For example, this triangle prism (shown in the Figure 4.10) has two triangle bases and three side faces. It is related to base shape. It is easier to see those elements in their open forms.

Teacher: Yes, you are right. Can you say that again?

Begüm: Number of edges of the base shape determines the number of side faces.

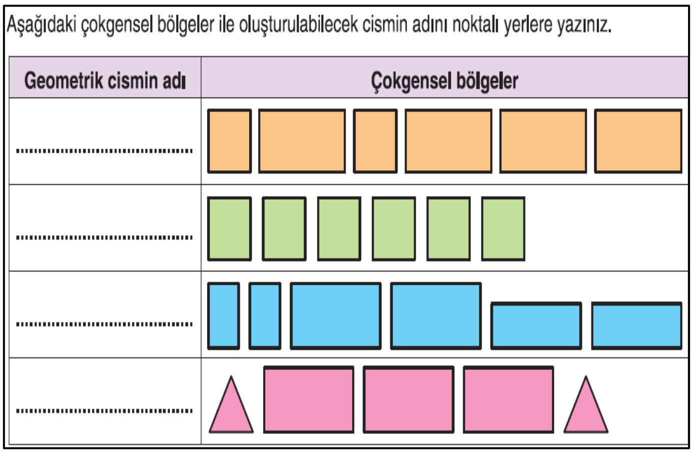


Figure 4.10 The third question of activity sheet about properties of prisms

This time, the subject was related to the given question on the activity sheet. The students completed the question as class discussion and there were not any challenges about naming the shapes that were given in open forms. Students successfully understood the relationship between base shape of a prism and its other parts.

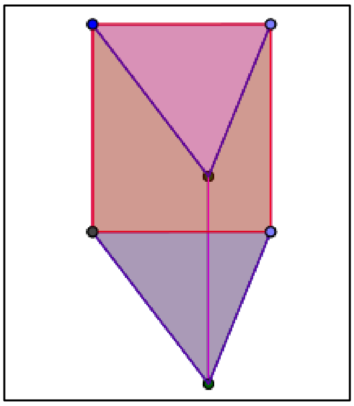


Figure 4.11 GeoGebra illustration of triangle prism

It is possible to show the structure of the debate according to Krummheuer's model of argumentation as following.

CONCLUSION

Begüm: Number of edges of the base shape determines the number of side faces.

Figure 4.12 KMA on understanding the relationship between parts of a prism

This idea became taken-as-shared and students used it two times in advancing hours of the instruction. Additionally, the structure of the classroom discussions was appropriate in terms of social norms and socio-mathematical norms. The students involved in class discussions by sharing their ideas and by using mathematically acceptable language. Thus, the mathematical idea can be concluded as-taken-shared in terms of constructing the mathematical practice of definition and properties of prisms.

4.1.4 Idea 4: Understanding that a cylinder is not a prism

The fourth taken-as-shared idea was occurred immediately after the third one; while the classroom was working on the first two pages of the activity sheet which was focused on basic elements and features of the prisms. At the end of the question, Zeynep shared her idea about an example that was given by a student in previous lessons. At the beginning of the instruction, the classroom was talking about examples for prisms from daily life. It was about an example that cylinder-shaped pencil box. The student had provided it as an example for prism, but at that time there was not a discussion occurred on that issue. Now, Zeynep seemed to have a challenge understanding the reason for it. So, she asked that issue and started a new topic to discuss.

Zeynep: Teacher. In the previous lesson, one of our friends said cylinder-shaped pencil box as an example for prisms. It is not aligned with the definition of prisms.

Teacher: Why? Listen to your friend. Do you think like her?

Kaan: It is a prism.

Zeynep: But we said the prisms have edges, cylinder does not have edges.

Teacher: What do you say? Look at our definition. We wrote the properties on the board.

Arda: Also, it does not have corners.

Teacher: Yes. We said it does not have corners.

Büşra: There are not side faces.

Teacher: Yes. There are not side faces.

Aydın: There aren't edges.

Teacher: So, what is the decision?

Class: It's not a prism.

Büşra: But, it is a three-dimensional solid.

Teacher: There is no doubt about it. But we say, it is not a prism. It's a three-dimensional solid. It's cylinder. That's it. Are there any problems with this issue?

In this debate, students completed the name of the prisms by looking at the nets of them that was given in the question. After completing their work on the paper, at the same time the researcher opened a GeoGebra file illustrating the nets of the prisms to make the content clearer for students. After seeing on the interactive-board screen, students became more confident with visualizing the prism in their mind. The following illustration was shown in Figure 4.11. In the later parts of the section, students tried to differentiate the cylinder from the prisms by using the first three mathematical ideas. In this sense, there was a common usage of those ideas that meant the ideas became taken-as-shared. To support their ideas, the students used the properties and basic elements of prisms.

At the end of the session, there was no one but Büşra challenged that the cylinder was not a prism. Büşra later constructed the idea while discussing the issue with her peer by comparing those shapes and features. One more discussion emerged related to differences between prisms and pyramids. The whole class discussion was started when one of the students asked a question about the issue.

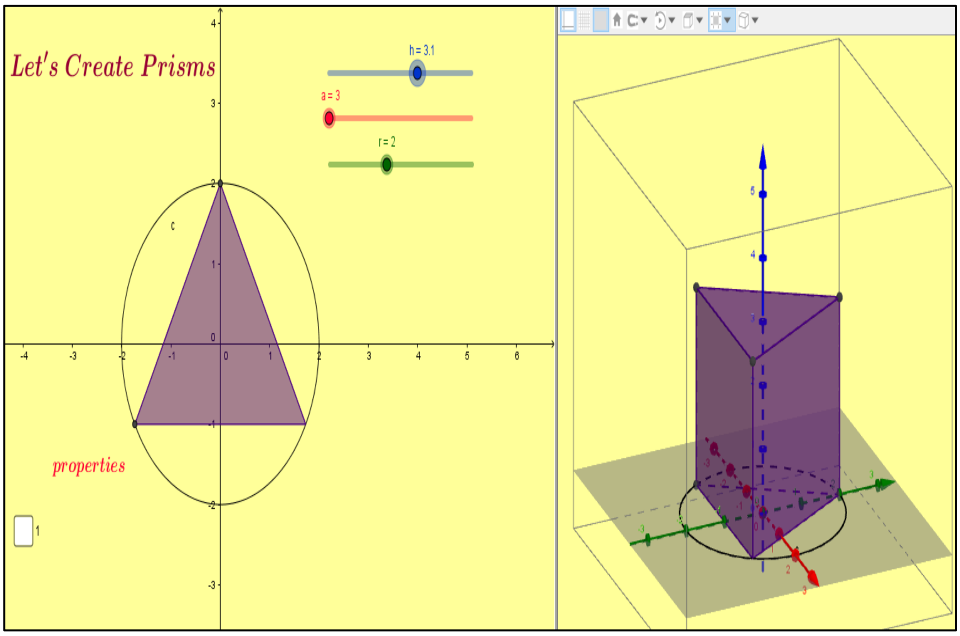


Figure 4.13 Illustration of a triangle prism from GeoGebra file

The classroom teacher guided the process and in a similar way by using the mathematical ideas that students produced, the problem was handled successfully. Also, the use of GeoGebra was a great support to make students to construct the conceptual understanding of the content. It was important for students to see and observe how the solids change by increasing or decreasing the number of the edges of the base. Also, how changing the position of a shape affects the features of that shape or whether that operation influences those features. Use of GeoGebra was critical for students to visualize the three-dimensional figures in their minds more easily, develop some ideas about the discussion and express their ideas about the subject related to those discussion issues. For example, by observing the GeoGebra file given in Figure 4.13, students' minds became clearer about how changing the number of edges of a prism also changes the number of side edges and number of side faces at the same time. It was not possible to show them those changes on the classic classroom board or by using any concrete material. Also, by evaluating the illustration, students could produce solutions to the challenges in their whole class discussions and reach the mathematical practice. For example, the illustration in the Figure 4.12 was helpful for students who had difficulty to understand why a cylinder was not accepted as a prism and showed the reasons practically. After evaluating a few prism types from GeoGebra file such as Figure 4.11 and Figure 4.12, the teacher wanted to ask students' ideas about the cylinder. The following debate occurred accordingly.

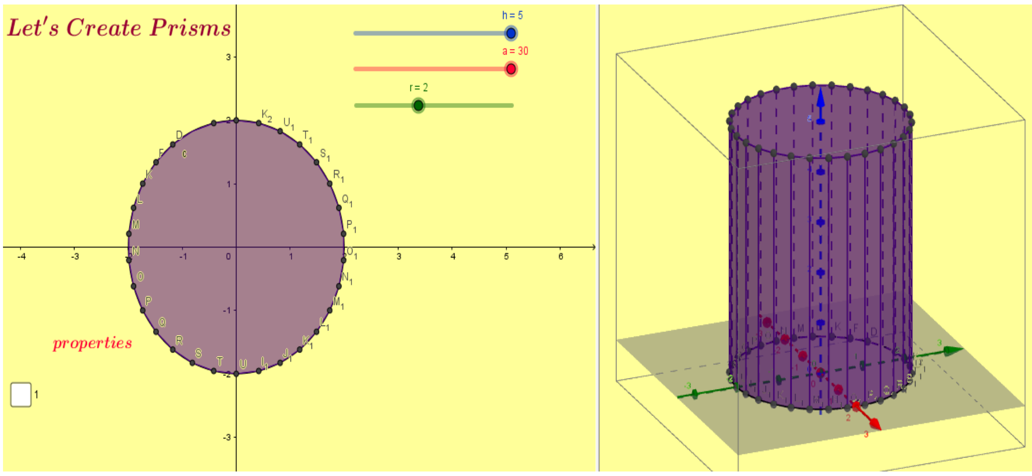


Figure 4.14 An illustration of cylinder on the GeoGebra file

Teacher: Now, what do you say about the cylinder? Is it a prism or not? Or do you understand the reason for it's not being a prism. Yes, Beyza.

Beyza: We learned the definition of the prism. We stated that prism has some basic elements such as height, edge, corner points, side faces. When you increase the number of edges of the base shape, number of side faces increased, and edges disappeared relatedly. Thus, if there is no edge, it cannot be a prism.

Teacher: Is there anyone who wants to add something?

Hakan: I agree. It is not a prism.

In this debate, the teacher wanted to see the possible changes of students' ideas about the cylinder. Beyza stated her idea as cylinder was not a prism. Also, Hakan stated his agreement. At this point, GeoGebra file was very helpful for students to make the reason clear for cylinder's not being a prism. In previous lessons, some of the students faced with some problems with understanding this issue. But observing the GeoGebra file helped them visualize the change of number of edges and their disappearing related to increase of number of edges.

Also, this debate was a good example of students' understanding of previous mathematical idea about understanding the relationship between base shape of a prism and other parts. Because, Beyza used the idea as data in her argument by stating the increase number of edges affected the number of side faces. Also, she stated that this increase caused side faces and edges to disappear. This was the acceptance of the idea of cylinder is not a prism and became taken-as-shared. The structure of the idea according to Krummheuer's argumentation model is shown as in the following.

CONCLUSION Beyza: … Thus, if there is no edge, it cannot be a prism.

Figure 4.15 KMA on understanding the cylinder is not a prism

These four mathematical ideas were mainly emerged through the discussion of basic elements of prisms and their properties; and constructed the basis for the first mathematical practices' definition and properties of prisms. Additionally, those ideas were important for being bases for the construction of the second mathematical practice which had taken-as-shared ideas in it. To provide a conceptual understanding of the nets of prisms and the surface area of them, students needed to have deep understanding of the definition and properties of a prism. Thus, for the second part of the instructional sequence in which students studied on nets of the prisms and their surface area, they used these mathematical ideas for several times as data or warrant during the whole class discussions. Additionally, the social and socio-mathematical norms of the classroom was supported the emergence of the first mathematical practice in terms of students' active participation to the activities individually or in peers or sharing their ideas in whole class discussions by using acceptable mathematical terminology. In the following section emergence of the second mathematical practice was explained with evidences and the related mathematical ideas that supported emergence of that practice.

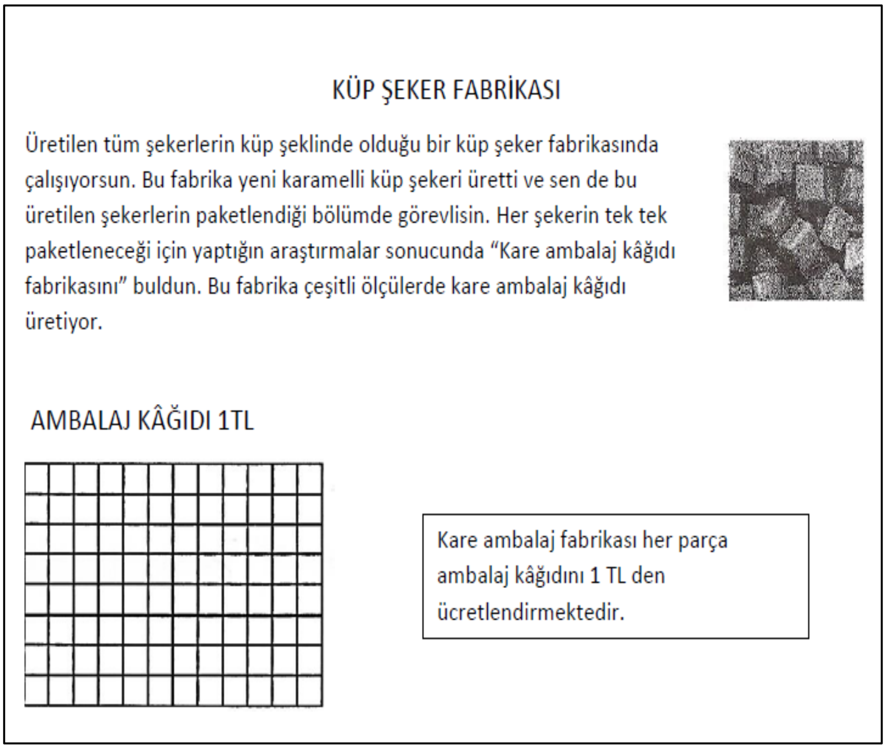
4.2 Mathematical Practice 2: Finding Surface Area of Prisms

The second mathematical idea was about surface area of prisms. It was emerged mainly during the second and third week of the instruction. During that process, students were involved in activities based on visualization of nets of prisms and construction of surface area of prisms by understanding formula of them. This part of the instructional sequence was constituted the long part of the study and continued during two weeks of the process. This was also related to importance of constructing the concept of understanding the formula of surface area of prisms instead of memorizing it. During this part, while using the knowledge from previous part as data for this context, students also produced and extended new ideas that became taken-as-shared and relatedly mathematical practice for the study.

For this part, the students worked both by individually and in groups. During the instruction, the GeoGebra files supported the progression of their understanding. Also, concrete unit cubes were used or given to the students that wanted to touch and see the shapes physically.

4.2.1 Idea 1: Understanding wrapping means that drawing net of a prism

This mathematical idea emerged during the second week of the instruction. For this part of the work, a wrapper factory concept was used to cover the candy produced in certain forms. In this factory, the produced candy wrappers were priced over the unit squares that were given, in fact, by this way both the nets of the prisms and the calculation of the surface area were introduced. On the first page which was given to the students, information was given on this subject. Figure 4.16 is from the first page of this part of the activity. It was an introduction to that part of the instruction by providing an introductory information about the progression. Students were given some time to read the given information in Figure 4.16 and then continued with the question that is shown in Figure 4.17. Students worked individually for the question. The researcher and the teacher visited the students and then the answers were checked on the GeoGebra file.


KÜP ŞEKER FABRİKASI

Üretilen tüm şekerlerin küp şeklinde olduğu bir küp şeker fabrikasında çalışıyorsun. Bu fabrika yeni karamelli küp şekeri üretti ve sen de bu üretilen şekerlerin paketlendiği bölümde görevlisin. Her şekerin tek tek paketleneceği için yaptığın araştırmalar sonucunda "Kare ambalaj kâğıdı fabrikasını" buldun. Bu fabrika çeşitli ölçülerde kare ambalaj kâğıdı üretiyor.

AMBALAJ KÂĞIDI 1TL

Kare ambalaj fabrikası her parça ambalaj kâğıdını 1 TL den ücretlendirmektedir.

Figure 4.16 Candy Wrapping Factory Concept

After reading and understanding the concept of this part, the classroom continued with the following question and the discussion related to it.

Size verilen kareli kâğıdı kullanarak, aşağıdaki tek parça karamelli şekeri kaplamak için kullanabileceğiniz ambalaj kâğıdını çizin. İsterseniz sağlama yapmak için çizimden sonra makasla keserek kontrol edebilirsiniz.

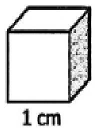


Figure 4.17 Wrapping the cube-shaped candy

Teacher: As you read, this question wants you to create a wrapping paper for the cube-shaped candy. What do you think about this?

Hakan: Actually, the question asks the net of the cube.

Beyza: Yes, it asks the net of the prism. We did it previous years.

Teacher: Exactly, it is about the net of the prism.

The section started with the teacher's questioning students whether they had the appropriate understanding about the question which was asking net of the cube. The debate demonstrated that students understood the context. In this context Hakan and Beyza replied the question in this way. There was not any challenging idea for the question. It was important for them to understand how to think about the question. For this question, students worked individually. During the process, the teacher visited the students to check and help if there was any challenge. For this part, there were not any discussions including data, conclusion or warrant of Krummheuer's model. The concept of cube was a known issue for students from previous years according to national curriculum. Thus, except for a few students, they successfully completed their drawings and all the students drew the net of the cube. One of the students' drawings is shown in Figure 4.18.

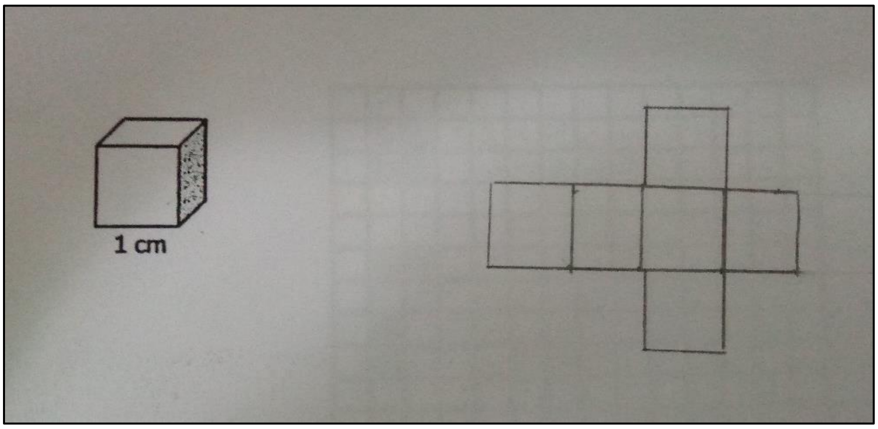


Figure 4.18 A sample of student drawing

While working on the question, a few students asked about the place of the top and bottom bases while drawing the net of the cube. To handle this issue, the researcher opened a GeoGebra file showing the net of the cube with different ways of open it (Figure 4.19).

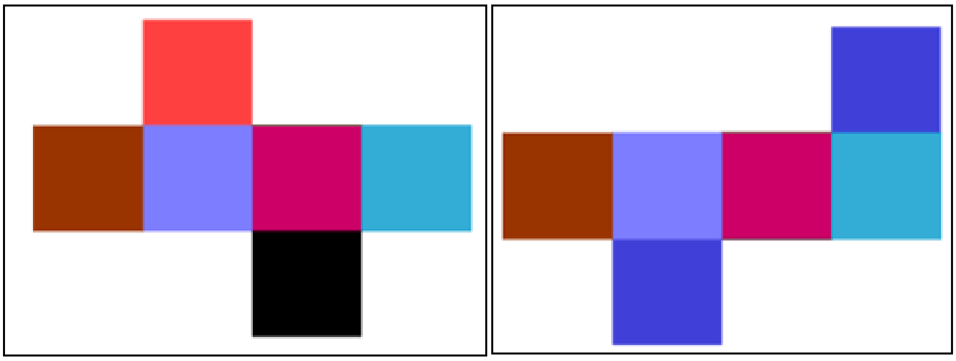


Figure 4.19 Different views of net of a cube from GeoGebra

Evaluating that illustration, the classroom understood the place of a base for the net of the cube. The illustration was showing the different views of the net of a cube and also closed form of it. As mentioned above, all of the students' drawings were correct. But, there were changes in places of bases at the open form. Although his drawing was true, one of the students seemed to have problem understanding those changes and asked questions about it.

Sude: Are there any differences when drawing those bases in different places?

Teacher: Ok. Let's look at the screen again. (Researcher moved the cube again). What do you say about your friend's question?

Arda: When we look at the shape, we got the same cube from different nets. So, I think, it is not important where to draw those bases.

Teacher: Sude, did you understand?

Sude: Yes.

Sude, asked whether any change of bases in net of a cube influenced its closed form. To handle this problem, the teacher wanted them to observe different views of its net and closed form. Students understood the missing point after viewing the GeoGebra file again. There was not any discussion about the issue anymore. The classroom continued the discussion with a related question. This section mainly focused on understanding to wrap a shape means that to draw its net.

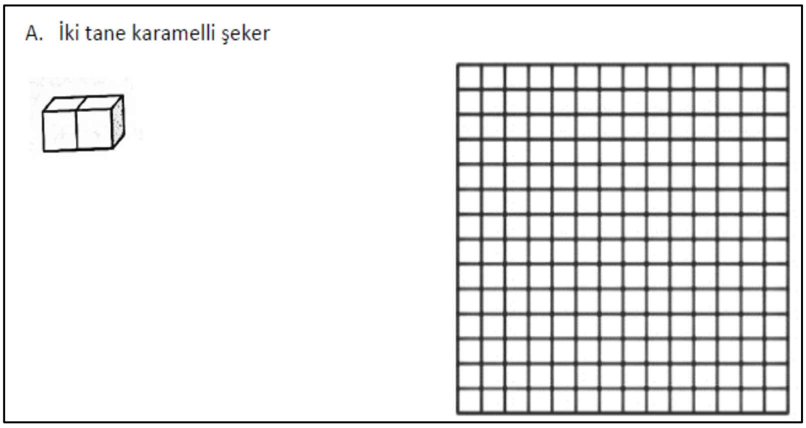


Figure 4.20 Second question of this part

Teacher: …Let's continue with the second question.

Begüm: Do we draw their nets again?

Teacher: What did you understand? It wants you to draw a wrapper for those two candies.

Begüm: It wants its net.

Teacher: What do you think about that shape? What type of prism is it?

Hakan: It looks like a square prism.

Teacher: Hakan says it is a square prism. What do you think?

Kaan: It is made up with unit cubes and a cube has square faces.

Begüm: By adding the two-unit cubes to each other, we have a square prism because, the whole shape's side faces are rectangle and bases are square.

Teacher: Yes, so you are expected to draw a wrapper for that square prism.

Zeynep: Its net, actually.

Teacher: These questions ask nets of the given prisms, you are right. I will check your drawings one by one.

During this section, students tried to understand the given shapes by relating the wrapping activity to their nets. Begüm wanted to teacher confirm that they were asked to draw nets of the given shapes. Actually, she used the knowledge as conclusion here. Structure of the discussion according to Krummheuer's (2015) argumentation model is shown in Figure 4.21.

CONCLUSION

Begüm: It wants its net.

Figure 4.21 KMA on understanding wrapping means that drawing net of a prism

The students were sure about working on nets of the given shapes, there was not any problem about that. The problems emerged during the teacher's visit of students. They weren't sure about how to draw those nets. They were trying to understand the shapes constructed by unit cubes. After the shapes became more complex, some students could not draw the nets of new shapes constructed by unit cubes. For instance, one of them was the shape which was shown in Figure 4.20. To handle those problems, each shape was checked on the GeoGebra file as mentioned in the instructional sequence. The net of the shape asked in Figure 4.20 is shown in the Figure 4.22. It was viewed in GeoGebra and was presented from top view here. Viewing GeoGebra files after each activity made students clear about their drawings and they had chance to check their works while the instruction is given. This is very essential for learning geometric concepts.

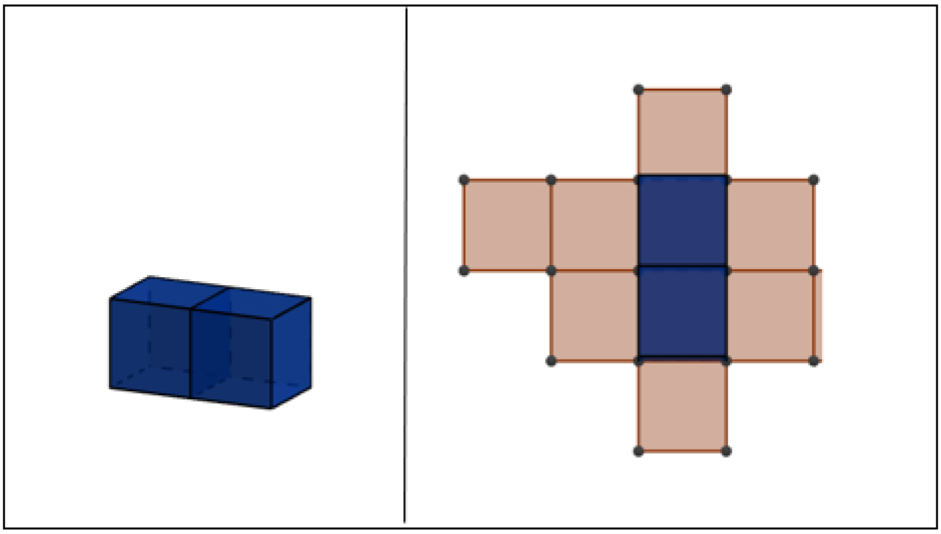


Figure 4.22 GeoGebra view of the question

Following questions were in the same context with the previous one. Those questions were prepared to construct the basis for the surface area of the prisms since that was related to the understand side faces of a prism. In general, the classroom completed the process successfully. They could draw the nets of the given shapes without any challenge except for a few students. The problem was solved by using GeoGebra file to show students the shapes on the interactive board. By this way, the students who had problems with the questions completed the missing parts of their drawings. During this process, they never questioned about the relationship between wrapping and net of shapes. Thus, the idea seemed to become taken-as-shared. Furthermore, they used this idea in the following context. After completing this part, the activity sheets were given to the students including questions about drawings of candy wrappers from only one side of the shape (view from top, from right side, from left side etc.). For that part, students stated that the questions were easier than the previous one. They stated that they knew this kind of drawings from the previous seventh grade classroom, but they were not aware of the aim of those drawings. Some of them asked the reason of doing the same procedures again. The researcher replied those students by stating that those were important steps for understanding the surface area of prisms.

Aydın: These are the same as we did previously. Why do we do these again?

Researcher: What did we do previously?

Aydın: We drew wrappers for the given prisms.

Researcher: What was the meaning of drawing a wrapper means mathematically? We talked about it.

Aydın: We said its net.

Researcher: So, these are again nets of the prisms, but by looking at different views.

Aydın: So, why do we do same things again?

Researcher: (By showing the shape in Figure 4.21 from GeoGebra) What do you see now?

Aydın: I'm looking from front.

Researcher: How many squares do you see?

Aydın: Two

Researcher: Think according to wrapping now. How many squares do you need to wrap that side?

Aydın: Two

Researcher: Do you see? Doing these practices helps you to observe each side particularly. This is a step we use for our other context.

In this debate, Aydın asked the reason for working on wrappers again by looking at different views of the given shapes. He thought that was unnecessary to do the examples. Researcher explained that both practices were important because they are preliminary steps in understanding the surface area and they are important to show the net of the prisms. Additionally, in his discussion Aydın used the idea of wrapping was meant to be net of a prism. The structure of the argumentation according to Krummheuer (2015) can be shown as in the following.

CONCLUSION

Aydın: We said its net.

Figure 4.23 KMA on understanding wrapping means that drawing net of a prism

According to students' explanations and feedbacks from them, the researcher and the teacher decided to change the place of this part and prior part in the HLT for following studies. That was because students easily completed the last part in which they drew the one side of the wrappers when compared to the prior one in which they drew a whole wrapper for each shape. Moreover, the teacher and the researcher found it more appropriate to work firstly on partial, one side drawings and then to continue with whole shape. By this way student would understand drawing of net of a prism by constructing on drawing each side of a whole shape. Following figure shows a sample from the students' drawings from the part when they drew one view of the given shape. After completing the session, view of each shape was controlled from the GeoGebra file to make students sure about whether their drawings were true.

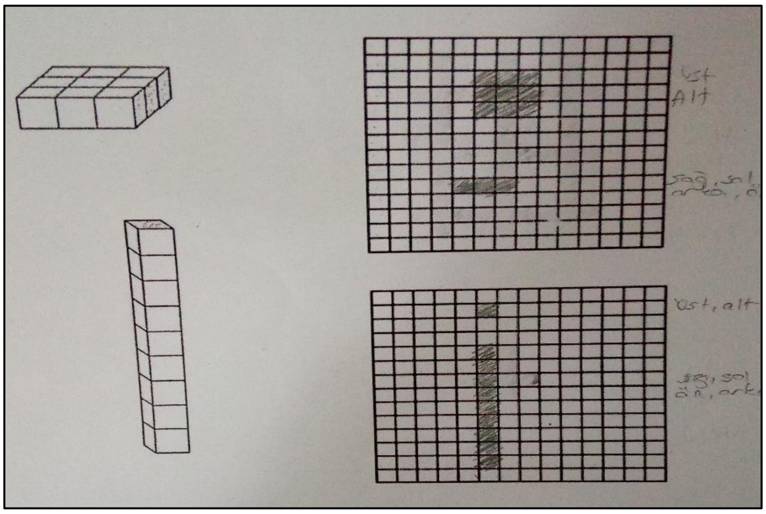


Figure 4.24 A sample drawing of students

For the following part of the activity sheet, students worked on a shape constructed by unit cubes and they were asked to draw from different views of sides again. While working on that question, students could successfully draw the asked views of the shape without any challenging idea. There was not any argumentation occurred during the process according to emergent perspective and argumentation model. GeoGebra was helpful again for students to catch a few missing points. It enabled the students to observe the given shape differentially. Figure 4.22 shows how GeoGebra helped the visualization of the given shape in the instructional sequence from different views.

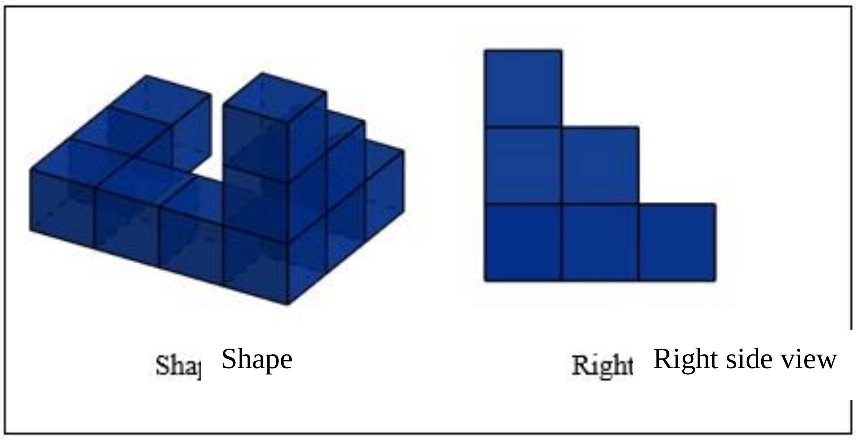


Figure 4.25 The given shape and its view from the right side

These kinds of questions constructed by unit cubes planned to be bases for understanding the nets of the prisms and their surface area directly related to the surface. Understanding these concepts requires understanding what the surface of a shape means and what it contains. So, those questions were expected to be helpful for students by construction of nets and surface area concepts.

Following step was about drawing wrappers for given candies in different shapes such as triangular prism, pentagonal prism and hexagonal prism. The students were given approximately ten minutes to draw the wrappers for those solids. The researcher and the teacher visited them while students were working on the question. After the classroom completed the process, they checked their drawings from GeoGebra (Figure 4.25). During the process the following conversation occurred between students and the teacher.

Teacher: In this page, you are expected to draw wrappers for candies in given shapes. So, what does it want you to draw?

Arda: Their nets again.

Teacher: Exactly, right. You are expected to draw their nets.

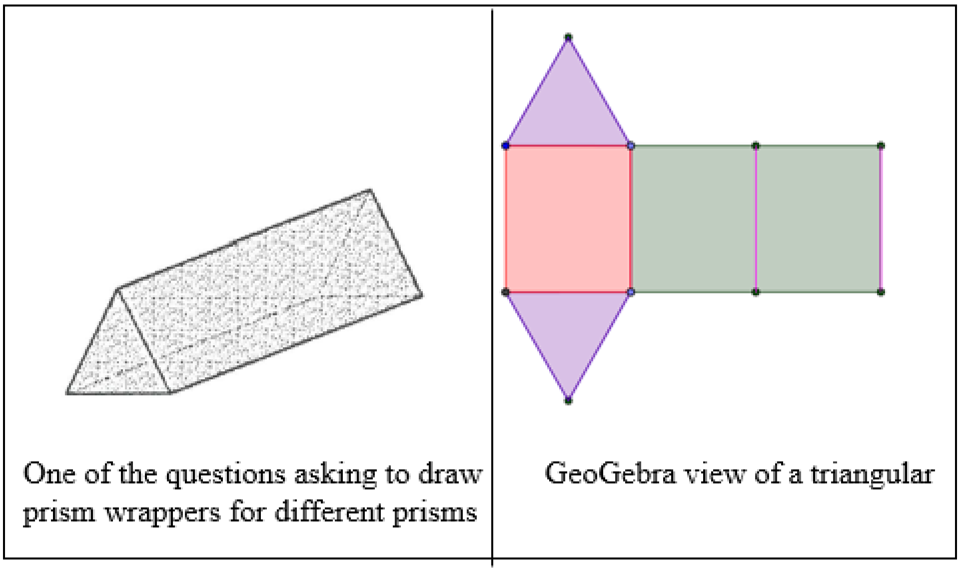


Figure 4.26 Triangular prism and GeoGebra file view

The section continued through the process. In this part, one student asked about the absence of unit cubes and their providing easiness for drawing the nets of the prisms. But this was not the critical point of the discussion. The teacher asked whether the classroom knew what to do. Arda stated that it was asking about the nets of the given shapes. Thus, the idea of wrapping a shape means to draw its net, became taken-as-shared among the classroom environment. By understanding this context, they are expected to construct the base for surface area. Figure 4.26 shows the analysis of the dialogue according to Krummheuer's argumentation model.

CONCLUSION

Arda: Their nets again

Figure 4.27 KMA on understanding wrapping means that drawing net of a prism

In following lessons, the classroom worked on activity sheet that included these kinds of questions again while the teacher introducing the surface area of the prisms. The content was about to understand the surface area by looking at the faces of candies constructed by unit cubes. Students were expected to calculate the area of the wrappers to cover the given candies. This was a transition to the surface area. While classroom was working on those candies, they used the concept of unit cubes as data and conclusion without any warrant or any challenging idea. This meant that students' understanding of the meaning of unit cubes in calculation of surface area was correct. The process was also constituted the second step of the mathematical practice by reasoning on area of rectangle.

4.2.2 Idea 2: Counting unit squares

This idea emerged during the third week of the instructional sequence and became taken-as-shared. The process was started with the teacher's questioning about the student's ideas about the meaning of surface area. Related to the previous work from the instruction, most of the students had the idea of what a surface area of a shape means. During the previous part of the instructional sequence, students worked on a wrapping factory concept aiming to introduce students with the surface area concept. This part of the activity sheet was constructed related to the same context. In this part of the worksheet, the aim was not only to design one piece of wrapping paper for candy, but also to calculate how many square units of the wrapping paper there was. The candies that were given to the students for this part were originally built using unit cubes. In later steps, it was asked the need of wrapping paper of unit square for each prism-shaped candy constructed without unit cubes. Students were expected to realize that they actually calculated 1 square meter of space for a unit cube, and from there they were expected to switch to calculation of surface area without using unit cubes. During this part of the instruction, students did not need to use or observe any GeoGebra file to provide help for questions, since this part did not include much questions or figures, they needed usage of visualization skills. GeoGebra files were opened for one or two times at the beginning of this part to show the students the back and side faces of prisms with unit cubes. The first page of this part was given to the students and they worked on it and then the classroom started to discuss on the page. The teacher wanted students to tell their answers and explain their reasons for those answers.

Teacher: Yes Aydın.

Aydın: I found 66.

Teacher: How did you arrive to that conclusion? What was your idea while doing your operation?

Aydın: I thought about the visible faces of the shape. I thought that we should count each unit squares of each face.

Teacher: So, you counted each face and each unit squares.

Aydın: Yes. I counted each of them.

Teacher: How did you count? What did you look for?

Aydın: Now. Can I show it on the board? (Aydın comes to the board and the GeoGebra file is ready on the smartboard). I counted this, this, and this side, and then I multiplied the result with two since there are two for each surface.

Teacher: Yes. You are right. Is there another idea or any different ways?

During this dialogue, the teacher wanted to get the ideas about the first question. Aydın explained his idea that he chose to count the number of each unit squares of each face. He counted each face and then he multiplied the result by two (Figure 4.28). Aydın's idea mathematically acceptable, but to become taken-as-shared it needed to be accepted and used normally by other members of the classroom.

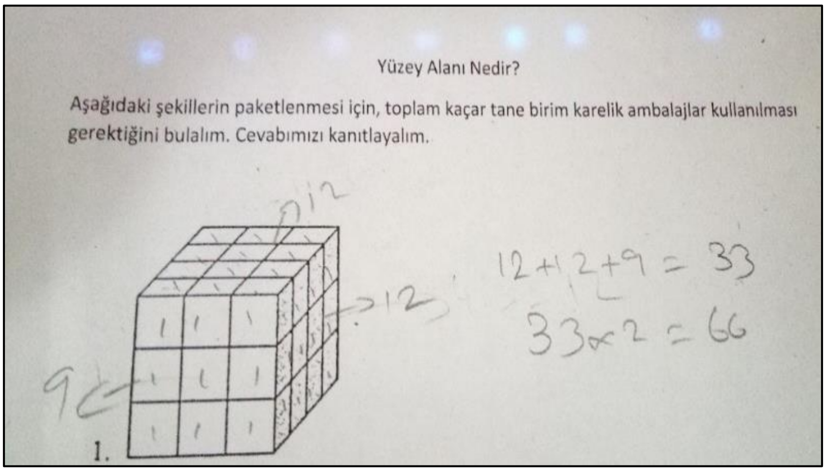


Figure 4.28 Aydın's solution to the first question

Aydın's solution was a way of calculating the surface area by counting each unit square. Teacher continued to ask about any other ideas. The dialogue did not include any element of Krummheuer's argumentation model, but it was a demonstration of students' understanding of the context. After Aydın explained his idea, most of the students agreed with the idea and still offered their own ideas. Most of the students stated that they counted the unit squares of each face and multiplied them by two since there were two for each face.

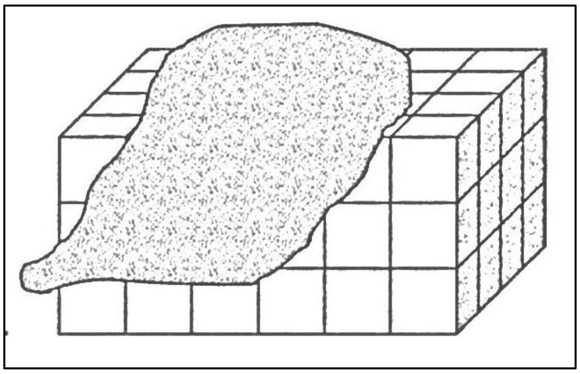


Figure 4.29 Another question of calculating unit squares of wrappers

Teacher: I want to listen your solutions. Yes, Arda.

Arda: I counted squares like Aydın. Here, six times three, there are 18-unit squares on the front side. And at the back side it's the same, 36 in total. On the top of the shape there are six times four there are 24 and 24 from the bottom. There are 48. On the right side, four times three, there are 12 and at the left as the same. 24 in total. And totally, there are 108-unit squares.

Teacher: Yes, you did the same as Aydın did. Do you want to add something? Is there anything wrong?

Selin: Also, I counted squares.

In this debate, for another question, the classroom started to talk about the solutions. Arda stated that he used the way as same as Aydın did for previous question. This was an indicator of that Aydın's idea started to become taken-as-shared according to emergent perspective and Krummheuer's (2015) argumentation model. This can be shown as in the Figure 4.30.

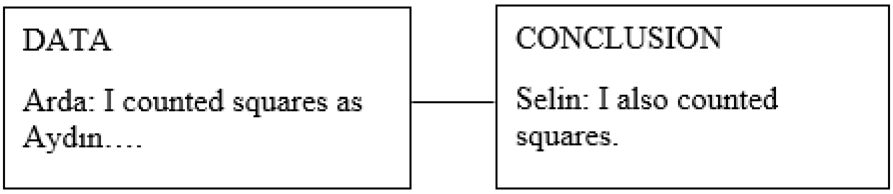


Figure 4.30 KMA on counting unit squares

This debate was not the last example of counting unit squares to calculate wrappers. In most activities of this section, students chose to use counting unit squares. Their ways of solving question were mathematically acceptable and this idea became taken-as-shared in classroom environment when evaluated according to emergent perspective. Thus, it was concluded that usage of counting unit squares, became a taken-as-shared idea (as a step for surface area of prims), while calculating area of wrappers. Following debate shows a different viewpoint of one the students.

Beyza: But, I found it 36.
Hakan: No, it is 66
Some students: It is 66.
Teacher: Listen, Beyza says something. Repeat please.
Beyza: I found the result 36.
Teacher: How did you do it?
Beyza: I counted one of the faces. There are 12-unit squares. Then, I multiplied that with three because there are three. So, the answer is 36.
Teacher: Can you explain again please?
Beyza: First, I counted the top face. There are 12-unit squares. There are three rows in the shape which is height. So, 12 times 3 makes 36.
Teacher: Why did you do that?
Beyza: Because, there are 12-unit squares and three rows, it makes 36-unit squares in total.
Teacher: Do you agree with Beyza?

This dialogue was a demonstration of misunderstanding of the context. Beyza explained her idea in return for Aydın's opinion, but she calculated the total number of the unit squares which was the volume of the prism. The teacher wanted her to understand her mistake with classroom discussion. To eliminate this misunderstanding the teacher wanted to show a concrete form of the main idea. She used a concrete cube and a squared paper. She wanted Beyza to wrap the cube by using that squared paper and after that Beyza understood the context. This small demonstration made most of the students got the deep understanding of what the surface area was.

After this practice, the classroom continued to discuss on following questions constructed with the same concept. As the process was going on, new solutions occurred in the classroom environment. Thus, related to new solution ways, new ideas emerged on the way to become taken-as-shared.

4.2.3 Idea 3: Transition from counting unit squares to calculating area

This idea started to emerge immediately after the previous idea. In previous section, classroom was working on finding area of wrappers that were appropriate for candies in given shapes. They preferred to use counting unit squares to find area. While the instruction and whole class discussions were going on, the students produced easier way of finding unit squares. Following section shows this process.

Teacher: Can you explain your way to your friends?

Mete: I thought that instead of counting all the unit squares, or as a shorter way, we can multiply edges with each other like we do while calculating area of a rectangle. This is easy and quicker. For instance, in this example, I multiplied three by five according to these edges (By showing his paper in Figure 4.18). It is 15. The result is the number of unit squares of this face, so it is the area of this face. And then, I multiplied 15 with two, because there are two same side faces, and it is 30. For the top and bottom faces, I multiplied five with six and I found 30, then I multiplied it with two, I found

60. Lastly, I multiplied three with six, I found 18 and I multiplied it with two, it is 36. When I summed up 30, 60 and 36, it makes 126.

Teacher: Yes, here what did he do? Who wants to repeat? What is the difference of Mete’s solution from the previous way. Remember most of you chose to count squares. Yes. Tuğçe.

Tuğçe: I think there is no difference. He only did the shorter way. Instead of counting, he multiplied according to faces.

Teacher: Yes, are there any other ideas about this solution? What does it remind to you?

Aydın: Actually, he found the area of one face. And then multiplied it with two.

Kerem: We can think like tiling on a ground. We said something like this. The number of tiles gives us the area of that ground. So, actually, we find the area of the rectangle on each side.

In this debate, Mete started to explain his way of solving the question by using the area of rectangle. He mentioned that he multiplied each edge by each other and multiplied with two and summed up those results to each other (Figure 4.31). During that time, everybody was doing the same thing but in a longer way. By this way, he reminded that they were actually trying to find the area of each face. He calculated the surface area of each face, but they did not name it as surface area at that time. But it was a step to capture the idea of surface area.

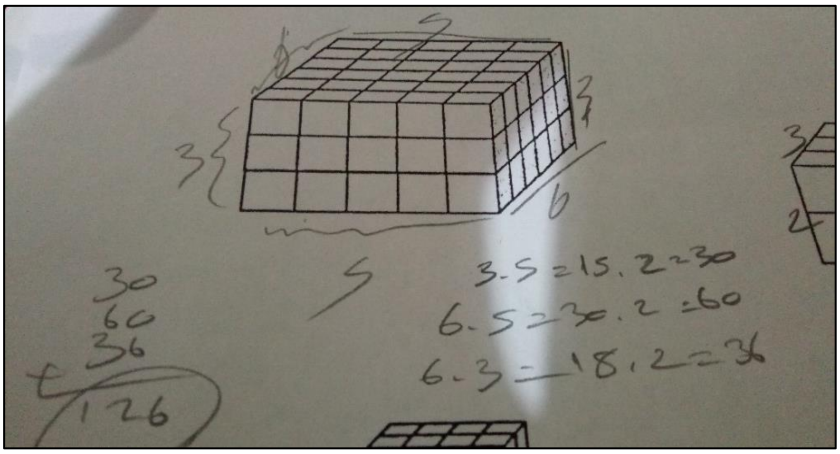


Figure 4.31 Mete’s solution to the question

This discourse of the students was obtained as suitable for Krummheuer's argumentation model. Moreover, those arguments were appropriate for social and socio-mathematical norms by exchanging ideas with appropriate mathematical terminology and using mathematically acceptable solutions. This structure can be summarized as following.

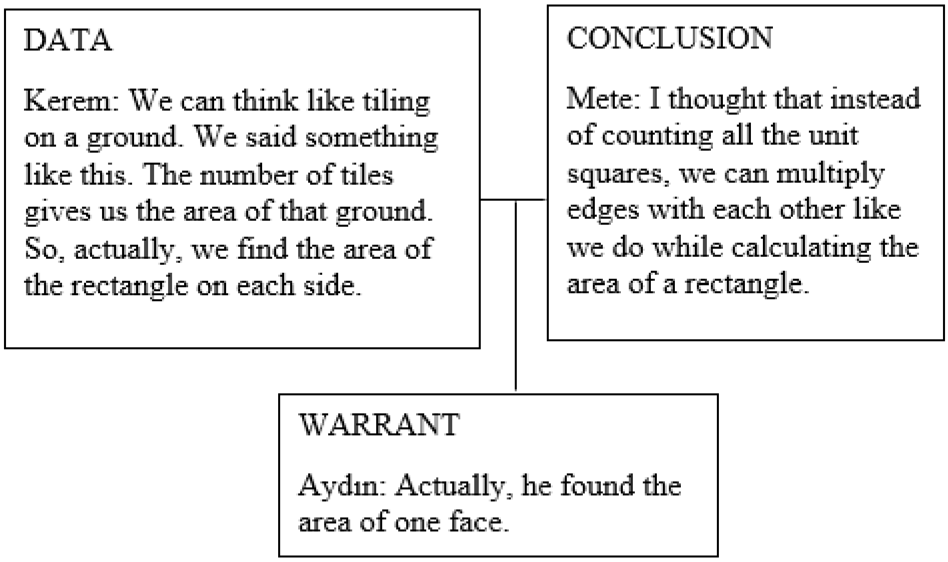


Figure 4.32 KMA on calculating surface area

After this time, in the following questions, the students often used the area of rectangle in their solutions. Thus, discussions mainly focused on that concept. Following section is chosen from the same lesson with the previous one. While students were working on the other questions, this dialogue occurred.

Teacher: Yes, who wants to solve the question?

Yağmur: To find the area of rectangle on front face, I multiplied two with six. It is 12. And then, I multiplied the two with four to find the area of another rectangle. It is eight. To find the area of bottom rectangle, I multiplied four with six. It is 24. Then, I summed up them, I found 44. But there are two for each side. I multiplied 44 with two and the answer is 88.

Teacher: Yes, good explanation. What do you think? Are there any other ideas?

Tuna: I did the same things. But, I first found the area of rectangles and then I multiplied each area with two. At the end, I summed them up.

Teacher: What are the differences between two solutions?

Kaan: Actually, there are no differences. They do the same thing but in different order.

In this debate, the question included a rectangular prism constructed by unit cubes. Yağmur explained her way of thinking for solution and Tuna accepted her solution and explained his way. Both of them did the same things but in a different order. Thus, Kaan confirmed that the solution was accepted by them. Yağmur used the data from the previous discussion about rectangles area as conclusion. Tuna added his explanation as data and then there were not any challenges or warrant for the discussion. Actually, the class started to understand that they were calculating the area of each face by doing those calculations that they referred in a short way. They started to transfer their thinking from counting to the calculating. Thus, the classroom seemed to accept the usage of rectangles area for finding the surface area. Accordingly, when the dialogue was evaluated according to Krummheuer's argumentation model the following figure could be drawn.

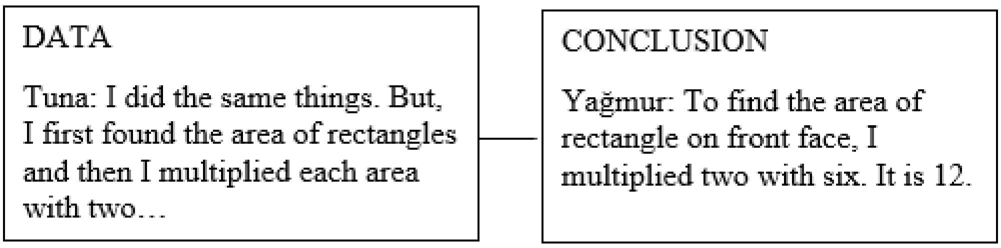


Figure 4.33 KMA on calculating area

In advancing lessons, the classroom continued to the instruction with calculation of surface area of given prisms without unit cubes. For that goal, students were expected to use the knowledge of area of rectangle again. After doing various examples, the classroom started to work on those kinds of questions. In following question, the classroom worked on the area of wrapper for candy which was constructed without unit cubes. After, the following discussion occurred during the solution process of mentioned question.

Teacher: …. In this question, yes. This time you are asked to solve the question without unit cubes. The shape is given in centimeter. Who wants to talk about this?

İpek: I thought about the area of each rectangle. First, I found the front face 60, back face is also 60. It is 120. Side face is 24, adding 24 to 24, it makes 48. For the bottom and top faces, I summed up 40 and 40, it is 80. It is 248 in total.

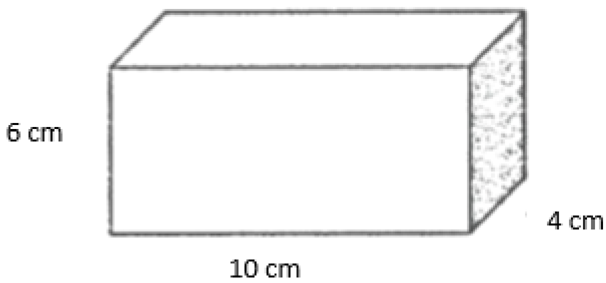


Figure 4.34 A question from surface area context

Teacher: Very good. In the question, the same thing is asked but by using numbers. Are there any problems?

Researcher: Can you explain again, why did you follow the same way as you did in previous ones with unit cubes? You could count unit cubes in those questions, but here there are none. What was your opinion?

İpek: I thought that each centimeter as one unit. I

Researcher: You thought 1 cm as 1 unit.

İpek: Exactly, So, here is 6, and here is 10 and this one is 4. By this way, I calculated the area of each face.

Teacher: Yes, Aydın. You said something.

Aydın: Actually, when we found the area, we found unit square. When we multiply base and height, we find the area.

Teacher: Which area?

Aydın: Rectangle’s area.

While talking on the question given in Figure 4.34, the classroom met a prism constructed without unit cubes. The teacher asked the ideas and İpek explained her way. She stated that she concluded that each one unit is equal to one centimeter and she continued her solution by this way. She stressed again the area of rectangle is calculated for each side and she reached the answer this way. After her explanation, Aydın stated that those calculations were all about the area of rectangle and accepted İpek's conclusion. Thus, following figure can be drawn for this discussion.

DATA	CONCLUSION
Aydın: Actually, when we found the area, we found unit square. When we multiply base and height, we find the area.	İpek: I calculated the area of each face.

Figure 4.35 KMA on calculating area

Following question and the related discussion shows another argument about using the idea of rectangle to calculate area.

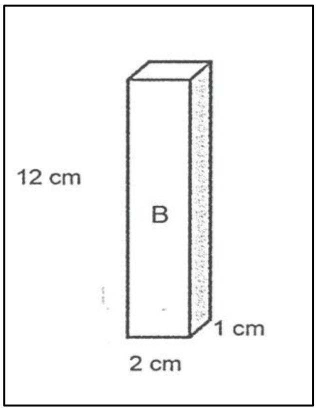


Figure 4.36 A question about calculating wrappers' area

Teacher: Let's look at another question. Who wants to solve? Yes, Kerem. Explain your solution at the same time.

Kerem: Aaaa. Ok. Now. I calculated the area of each rectangle. I found front side. I found the area of this rectangle. It is 12 times two equals to 24. With the other side, it is 48. Area of this top base is two times one, it is 2. With the other side, it is 4. And right side is 12 times 1, it is 12, with the left, it is 24. I summed up all faces and it is 76.

Teacher: Yes, that's good. Do you understand? Is there any other idea?

Class: No.

> CONCLUSION
>
> Kerem: Aaaa. Ok. Now. I calculated the area of each rectangle. I found front side. I found area of this rectangle.

Figure 4.37 KMA on calculating area

This debate was about the solution of the given question in Figure 4.36, Kerem explained his way by calculating the area of each rectangle on each face of the given shape. After he found the area of each face, he multiplied with two as many other students did. At the end, they summed up all of the results to reach total area. During the solution process or after it was completed, there was not any argument on the solution. This was because the classroom started to use the idea normally while working on calculating area of wrappers that produced for given candies.

After working on this question, the classroom worked on eight questions about the area of wrappers for candies that the area constructed without unit cubes. For discussions of each questions students used the area of rectangle as a way for solutions in reaching to the answers. Thus, the classroom concluded that the area of rectangle was used for surface area of rectangular prisms and/or area of cubes. Furthermore, this idea became taken-as-shared by involving in classroom activities and discussions by expressing ideas in a mathematically acceptable way. This was a requirement of social and socio-mathematical norms of emergent perspective. These questions were prepared to make them to be ready for reaching a generalization and producing a formula for the surface area of the prisms. Following step continued with producing the formula for prisms. More clearly, this time the students were calculating the surface area of shapes that were given to them. But they did not name it while calculating surface area of prisms. Now, it was time to express it in mathematical language.

4.2.4 Idea 4: Producing the formula for surface area of prisms

Following step of the instruction was working on producing a formula for surface area of prisms. The classroom was given fifteen minutes to think and work on the page which was constructed based on the thinking on formula of the surface area of the prisms. They worked in pairs and the teacher and the researcher visited them during this process. At the beginning of this section the teacher introduced the context to the students and explained what they were asked to do. This time, the classroom was good at finding the surface area of the given shape by using the area of rectangles of each side of the prism. But while working on finding a formula, it seemed challenging to them. They could not understand how to generalize this work into algebraic expressions. Following dialogue was from at the beginning of this part.

Teacher: Here you will think about a formula for surface area of the prisms. For example, think about the area of a square. What do you say for the area of square?

Class: a^2

Teacher: It is multiplication of two edges, isn't it? Now, you will find something like that. What did we do to this time? What were the questions that you worked on?

Tuna: Area.

Ayşe: Surface area

Teacher: Surface area, yes. Here, you are expected to produce a formula, a generalization for the surface area of prisms. So, related to the example of square, what can you use for that kind of procedure?

…..

Teacher: By looking your works, I see a misunderstanding for the process.

Researcher: We gave the square example before. Now, say the area of a circle.

Begüm: It is πr^2.

Researcher: How do you name it? You use letter, don't you? Here, you are expected to do same thing. You can use letters or symbols. This time, you proceeded with numbers and this time it wants you to express your procedure in letters or symbols. Look at the examples of three students. They are given to provide examples to you. You can get help from those examples. …

This debate was about the introduction to the producing formula to the surface area of prisms. At the beginning, students could not understand what to do and how to reach a formula. The teacher and the researcher tried to explain the way for them by relating the context to their prior knowledge. The students had already known the formula of square or circle from previous years. The teacher and the researcher tried to make them understand what to do by mentioning about the formula of area of those shapes. More clearly, these were (formula of area of a square and area of a circle) all well-known formulas by the students. The aim of the teacher and the researcher was to remind students the way to express a formula for a given shape. More clearly, how to use algebraic expressions while producing a mathematical formula. By this way, most of the students were clearer about the content, what to do and tried their way.

The students started to work on producing their formulas. During the process, the teacher and the researcher visited the pairs and helped them in their works. The teacher and the researcher tried to help students, how to express their ideas algebraically. Actually, they did many practices and solved many questions in the related context, but it was a new thing for students to produce a formula for a geometric structure. In traditional lessons, they usually get ready for formulas and apply them on the questions. The researcher and the teacher wanted them to think about those practices they did until that time. Especially, student formulas given on the activity sheet were helpful for students by providing examples for them. Additionally, there was a part below the question which was expected to provide a clue for students. This part showed some students' work on producing a formula for surface area of prisms. But not all of them were true. After students worked on their formula, they were also expected to discuss on those solutions (Figure 4.38). In general, pairs could produce some ideas (right or wrong) and tried to write something about formula. After working on the page, the teacher started the whole class discussion on it. Looking at the students' works, there were ideas mostly produced related to previous numerical questions. Some of the students used numbers to write formula again, some of them used letter but in wrong ways. But in general, they seemed to get support from the formula of rectangles area while working on the page.

Sezen'in formülü: t:taban, h: yükseklik ve g: genişlik olmak üzere | 2th+2tg+2hg |

Rıza'nın formülü: t:taban, h: yükseklik olmak üzere | 6th |

Ceren'in formülü: $A_{Ö}$: Ön yüzey alanı, $A_{Ü}$: Üst yüzey alanı ve A_{S}: Sağ yüzey alanı olmak üzere | $A_{Ö}+A_{Ü}+A_{S}$ |

Figure 4.38 The part that was expected to provide clue for students.

Moreover, they seemed to be aware of finding the surface area which requires finding area of whole faces of the prism. Accordingly, they focused on finding a formula for each face of the prism and tried to generalize it. In the following visual, samples from students' work and discussion on it are shown. (This dialogue occurred between the teacher and Zeynep while they were working on producing formula and discussing with peers. They were sitting on their desks and the teacher wanted Zeynep to explain her solution)

Zeynep: Teacher. I drew a square prism. I said these edges are *a* and *a*, and the area is a^2. Then, I said for this rectangle face, this edge is *b* and the area of this rectangle face is *ab*. This edge is *b* and this one is *a* again. The area is *ab* and *2ab* in total. The same thing is for square top base and there the same at the bottom and $2a^2$ in total. I summed them up and multiplied with two since there are two for each of them.

Teacher: You did it for square prism.

Zeynep: Here I drew a rectangular prism and I named the edges as *a*, *b* and *c*. This face's area is *ab*. This area is *bc*. And here is *a* and here is *c*, so the area is *ac*. I summed them up and multiplied with two since there are two for each face.

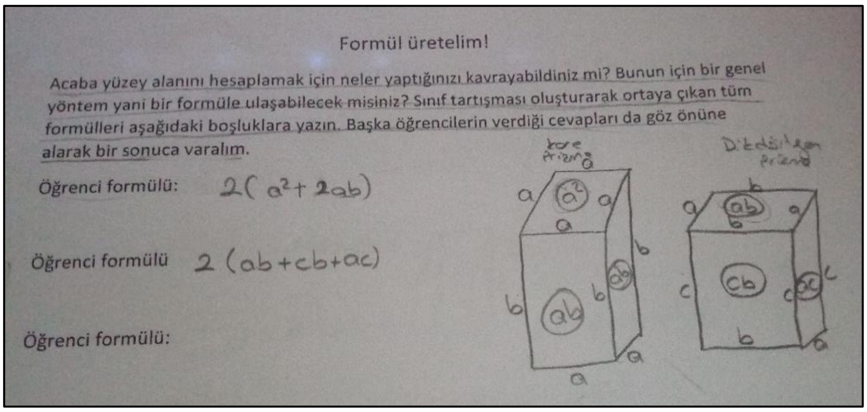


Figure 4.39 Zeynep's work for finding surface area of prisms

In this section, Zeynep explained her idea on how to produce formula for the area of a prism (Figure 4.39). She stated that she worked on square prism and on rectangular prism. She explained how she named the squared prism and how she found the area, and also the way how she worked for a rectangular prism. Her naming the solids and finding area of each face of them had made her to produce an accurate formula for prisms. After working process was completed, the teacher wanted students to explain their ideas on the board. First, Aydın explained his way. In the following section, his solution and his explanation were given.

Teacher: Yes, Aydın explain your way.

Aydın: I named the edges as a, b and c. Then, I found area of each rectangle as we did before. One face is ba, and there are two, so it is 2ba. Other face is cb, there are two, so it is 2cb. And this face is ca with the back side, it is 2ca.

Teacher: Then you summed all up.
Aydın: Yes. I summed up them like this.

Aydın followed the way as he did while they were working on numerical questions (Figure 4.40). He used the same things in the same order. He calculated area of each face, then he multiplied with two, and at the end he summed all them up to produce his formula. After his explanation, Zeynep came to the board and explained her way. The Figure 4.41 and the discussion is about that part.

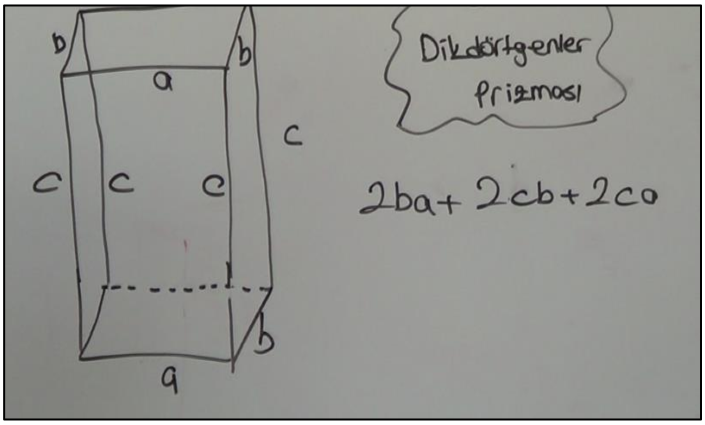


Figure 4.40 Aydın's formula for surface area of prisms

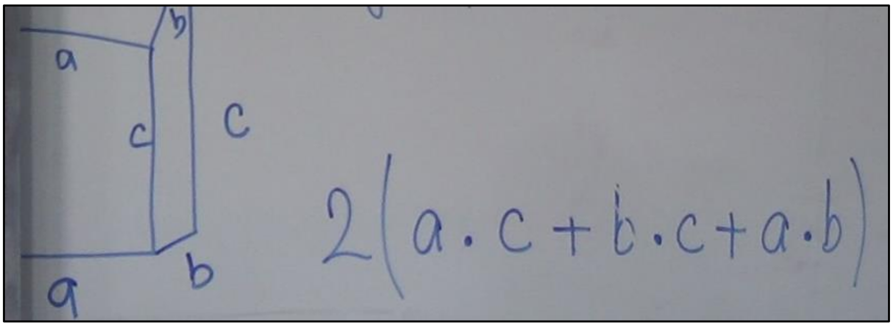


Figure 4.41 Zeynep's formula for the surface area of prisms

Teacher: Yes, Zeynep. We are listening to you.
Zeynep: I named these edges as a, b, c. For example, since these faces are rectangle, if this edge is a, this one is also a. This one is b, and this one also b. This

one is c and this one also c. And others are also like this. If I found this area, this is ac. This is bc and this is ab. There are two faces and for those faces I multiplied the result with two. We were doing this last year in algebraic expressions. I used common multiplier parenthesis.

Teacher: What is the difference between your formula and Aydın's formula? Or are there any differences?

Zeynep: Aydın's formula shows more clearly that there are two for each face and summed all them up. Maybe mine is confusing for some friends. I first found each area and then I thought that there are two for each face. Then I multiplied each multiplication with two. There is no difference. It's the same thing.

Teacher: Is there anyone who wants to add something?

Begüm: Actually, there are cross signs between each letter. I think some of our friends did not remember that point.

Teacher: Yes, this is an important point. Is there any problem with this point?

Class: No.

Teacher: Do you agree with these formulas? Is there any other comment?

Class: No.

Aydın's and Zeynep's explanation about how to use area seemed to be accepted by classroom since there were not any challenges, any warrant, or any question about it. She used the rectangles area as conclusion here and there were no objection to her. Most of the students tried to understand the solution (idea). For most of them, it was difficult to transfer the knowledge from numerical expressions to algebraic expressions. The discussion process was mathematically appropriate in terms of using mathematical language and offering acceptable solutions to the situations. Thus, it can be concluded that the idea became taken-as-shared. Actually, this was clear with later usages of students in their solutions.

CONCLUSION

Zeynep: … There are two faces and for those faces I multiplied the result with two. We were doing this last year in algebraic expressions. I used common multiplier parenthesis.

Figure 4.42 KMA on producing the formula for surface area of prisms

After completing and discussing on whole class study, the teacher reorganized the formula of surface area for prisms. Then, she started a whole class discussion for finding the surface area for different prisms such as square prism, triangular prism, pentagonal prism, hexagonal prisms etc., since they studied on cube and rectangular prism. The teacher wanted Zeynep to explain her work on square prism that she explained to the teacher while working in peers as in Figure 4.39. She came to the board and did same thing as she did in her paper. She explained her way one more time for the classroom.

Teacher: Yes, while you were working, Zeynep tried to work on a square prism and produced a formula for it. She will explain it to you. So, this may be a clue for you. Yes, Zeynep.

Zeynep: I said these edges are a and a, and the area is a^2. Then, I said for this rectangle face, this edge is b and the area of this rectangle face is ab. This edge is b and this one is a again. The area is ab and $2ab$ in total. The same thing is for square top base and there the same at the bottom and $2a^2$ in total. I summed them up and multiplied with two since there are two for each of them.

Teacher: Yes, as you see, we did the same things. The only difference is on the base shape. It is a square here. So, instead of finding the area of a rectangle, we work on the area of a square. Now for example, what do you think about the area of a triangular prism? Yes, Tuğçe.

After Zeynep explained her formula of square prism, they discussed on whether they should follow the same way for finding their surface area different from those cube and rectangular prism. The classroom was clear about what surface area was and they successfully produced ideas about the issue. For instance, they could think that finding surface area of a triangular prism requires that finding the area of triangle bases and area of rectangle side faces.

Tuğçe: It is related to base shape. We find area of side faces and we find area of top and bottom bases. And we sum them up.
Teacher: Very good. Yes.

Thus, it was clear that the classroom understood the idea of what a surface area was and how to calculate it. Tuğçe offered a good explanation to the surface of a triangular prism. Now, the whole-class discussions were constructed on social and socio-mathematical norms. So, Tuğçe's idea was a sign of conceptual understanding of what a surface area was and its calculation. The structure of this idea can be shown according to Krummheuer's argumentation model as in the following.

CONCLUSION

Tuğçe: It is related to base shape. We find the area of side faces and we find area of top and bottom bases. And we sum them up.

Figure 4.43 KMA on producing the formula for surface area of prisms

After they completed to work on producing the formula for surface area of prisms, they continued to solve the following questions about surface area. The following question was chosen from the questions about exercises after talking on surface area of prisms.

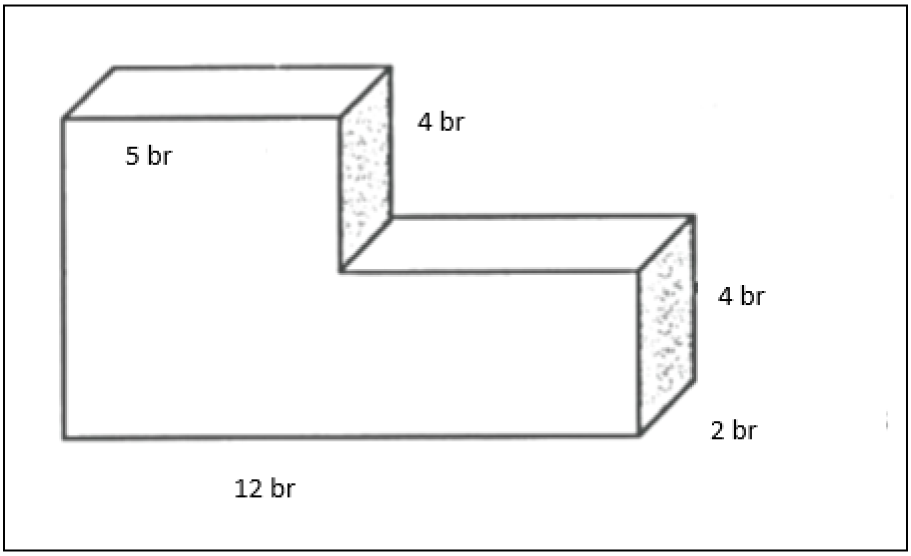


Figure 4.44 The question that the students confused.

The classroom started to work on those questions. These questions were a bit challenging from the previous ones. After they completed the process, the classroom started to talk about those. Especially, one of them was problematic for the students. During the discussion, students were clear about how to find the surface area, but the structure of the question confused them. Their usage of rectangles area stated that the idea became taken-as-shared, but the classroom needed to understand the construction of the shape which is shown in the Figure 4.44.

Some students tried to divide the shape vertically and some of them divided horizontally. But also, there were wrong solutions which meant that students were having visualization problems. Figure 4.45 shows a right approach but an incomplete solution. During the discussion about the question, the classroom used the idea of rectangle's area as solution way.

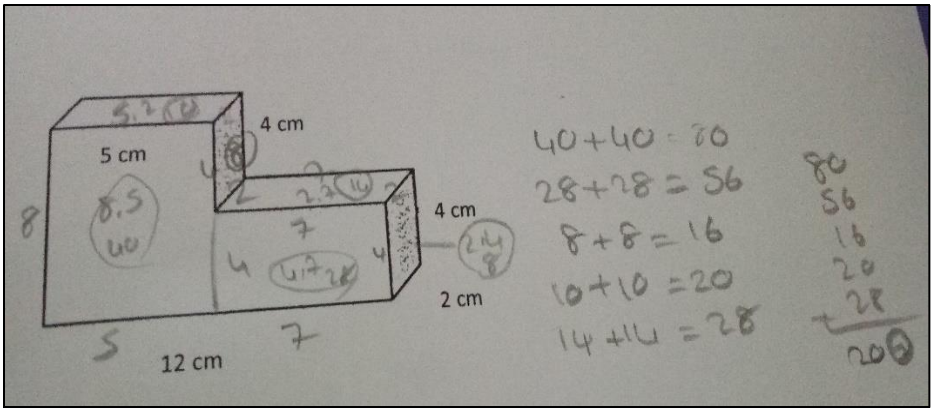


Figure 4.45 One of the student's solution of the question

Following section emerged during the solution process of the question. The shape was drawn on the board.

Beyza: I found each rectangle's area and summed up them at the end. I multiplied 5 with 2, 10.

Teacher: Write it on the board.

Beyza: I multiplied 8 with 5, 40.

Teacher: Yes.

Beyza: 4 times 2, 8

Teacher: Ok.

Beyza: 2 times 7, 14.

Teacher: Good.

Beyza: Here, 4 times 7, 28. There are two for each face, so I summed up these numbers for each one. It is 200.

Teacher: Yes. What do you think about Beyza's solution? Is it ok? Is there anything wrong?

Class: No.

Beyza divided the shape vertically and constructed her solution based on finding the area of each face. After that, she summed up each area for one time for another face, and at the end she summed up all numbers to find whole surface area. Here she used the idea of rectangle's area as conclusion and there was no response, any objection, or a warrant from the classroom. Thus, this dialogue can be evaluated according to Krummheuer's argumentation model as in the following.

CONCLUSION
Beyza: I found each rectangle's area and summed up them at the end. I multiplied 5 with 2, 10.

Figure 4.46 KMA on producing the formula for surface area of prisms

Her idea was good and appropriate, but there were missing parts. She forgot to consider left side of the shape. Thus, she did not add that side in the surface area calculation. Similar missing parts occurred in other students' solutions. They were clear about using the area of rectangles but there were missing parts in their procedures. The teacher and the researcher were aware of this situation, and also the teacher wanted them to notice this point by themselves. During the break time, the teacher and the researcher discussed about the students' visualization problems and their need for an illustration of the shape.

The researcher prepared a GeoGebra file during the break time and they decided to show it to the students in the next lesson. Figure 4.47 shows the GeoGebra file. Also, the research team decided to add it to the HLT and also to instructional sequence. After evaluating the illustration on the GeoGebra file, students became clearer about whole faces of the shape. GeoGebra provided an illustration of each unit square and relatedly the area of each face. They could clearly observe back, front, right, left, bottom and top faces. Moreover, they had a chance to compare the results from the procedures and the real shape.

In the following lesson, while talking on the illustration in Figure 4.47, following debate emerged in classroom environment. The issue was about the question in Figure 4.44 which students could not visualize the different views of the given shape. In the following debate, the researcher wanted to make them to handle that visualization problem by observing the GeoGebra file.

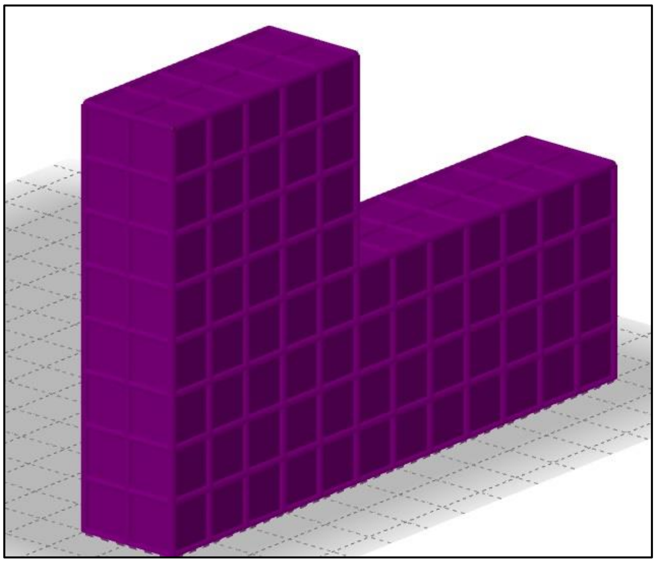


Figure 4.47 The GeoGebra file prepared to control the solution of the question.

Researcher: It was confusing for you to work on those questions. Thus, I prepared this GeoGebra file. Now, let's evaluate this shape and also follow it from your activity sheet. Here, this is the top base. Look from the top side. Can you see? Length of this edge is five, this one is two. Base is 12 units. From here, height is four units. Now, think according to this illustration. We will look at each side and calculate each surface area accordingly. Now. Let's look at top side (Figure 4.48).

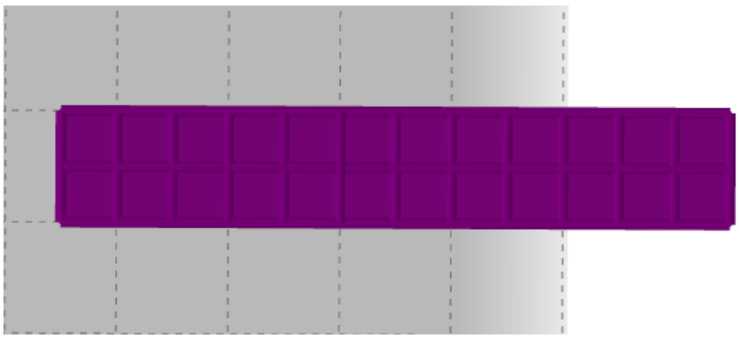


Figure 4.48 Top view of the illustration of the shape in Figure 4.45

Researcher: What do you say about this illustration? What would you do, if you tried to find area of this shape?

Kaan: We would only find the rectangles area. 12 times two, it is 24.

Researcher: Yes, this was the top view, think about the bottom view (the researcher shows the bottom view). What do you say?

Tuğçe: It is the same. Same view. They seem like the same shape's top and bottom views.

Researcher: Do you remember? At the beginning of this activity sheets, we were working on finding areas of different views of given shapes and some of you asked the reason for doing those practices. Do you understand the reason now? Thus, what do we do while finding the surface area of these kinds of shapes?

Burcu: Do we follow the way as we do for normal prisms?

Researcher: Normal prisms?

Burcu: Like we do for example for a rectangular prism. Because we will see the same view from right side and left side, and also the same for front and back sides. We will calculate each area that we see, and we will multiply with two.

The debate emerged after observing the illustration in Figure 4.47. GeoGebra file helped them to handle those visualization problems since it provided them to observe the same shape from different views at the same time. Thus, they realized the same view of two opposite sides (like right and left). By this way, they also noticed the relationship between calculating surface area of a rectangle prism and surface area of this kind of shape. Most of the students were aware of what to do and which way to follow on such a shape. Kaan stated that he saw a rectangle and to find the area of it, he would follow the same steps as they did before. Burcu and Tuğçe added that the opposite side of the same shape had same views. Thus, they stated they would calculate like they did for other shapes.

After this process, this part was completed. While working on remaining questions, the students often used the surface area as conclusion and data without any warrant. Thus, according to Krummheuer's argumentation model and emergent perspective the idea become taken-as-shared among classroom. Furthermore, those ideas that occurred during this part of instruction, supported emergence of the mathematical practice of surface area of prisms.

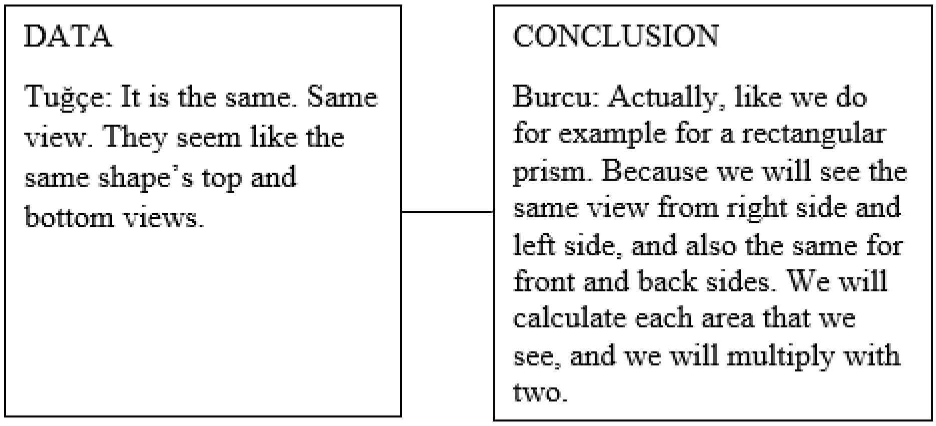


Figure 4.49 KMA on producing the formula for surface area of prisms

4.3 Mathematical Practice 3: Finding Surface area of cylinder

The third mathematical practice was about finding the surface area of cylinder. This practice emerged at the end of the third week and during the fourth week of the instruction. In this process, the classroom studied pages of the activity sheet that were based on the net of the cylinder, its basic features and elements and the surface area. The instruction was prepared according to construct a basis for students' understanding of surface area of cylinder. First exercises included drawing a wrapper for a cylinder-shaped candy. The aim was to make students get ready for the surface area. After this part, they worked and discussed on net of the cylinder and its parts. This step was important for students to understand the relation between the circle bases and rectangle side face of a cylinder. After working various examples and questions about this context, students continued to find area of the given cylinders separately. Then, they moved on to think about the surface area of a cylinder and to produce a formula for it. For all processes, students were given a time period to work and then they started to discuss on related issues to come up with a commonly shared idea which is taken-as-shared idea and relatedly a mathematical practice according to emergent perspective and argumentation model.

For this section, the students worked both individually and in groups. During the instruction, the GeoGebra files supported the progression of their understanding. By observing net of a cylinder on the GeoGebra, they could easily relate the circle base and rectangle side face. The detailed information is provided in the following parts.

4.3.1 Idea 1: Structure of net of a cylinder

In previous lessons, the classroom worked on surface area of prims and related questions. Now it was time to think about cylinder. The HLT and instructional sequence were prepared first based on basic elements and properties of it. The context was the same as with prisms. The factory concept continued in this part again. At the beginning of the first lesson of the cylinder, the teacher questioned classroom about daily life examples.

Teacher: Ok. I want you to give examples for cylinders from the physical world around us.

Mete: Bottle.

Zeynep: Pencil cases. At least, some of them.

Beyza: Glasses

Teacher: Yes, good.

Arda: Jar

Kaan: Bin

This section was about the daily life examples of cylinder and it was illustrated that students had the idea of cylinder's physical construction. The examples that were given by students were appropriate for cylinder. After this section, the students were given the first page of cylinder concept. It was about designing a wrapper for given cylinder-shaped candy box. Students were given approximately five minutes to work on the question. During the process, the researcher and the teacher visited the students and checked their drawings. In general, they were successfully completed drawing wrappers without any difficulty. Then samples from students' drawing were shown. After the students completed their work, they had a small classroom discussion on the context.

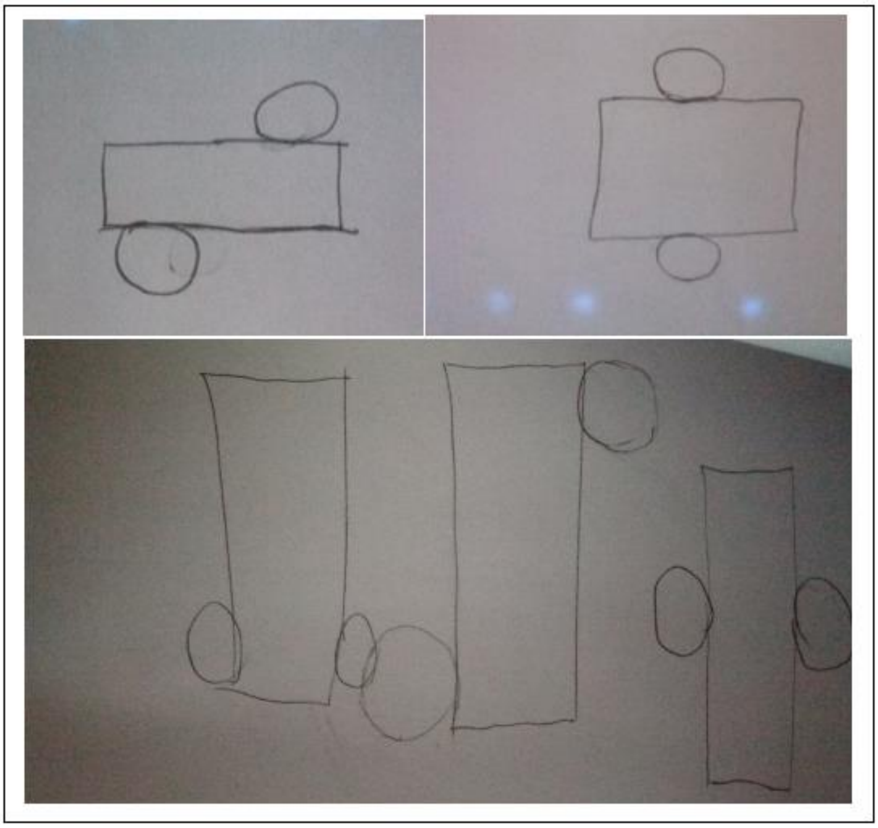


Figure 4.50 Samples of students' drawings for wrappers

Teacher: Your drawings were good. So, what did you understand from this question? What is the main idea?

Burcu: It is questioning us about which pieces a cylinder includes.

Teacher: Yes. Any other ideas? Say, Kaan.

Kaan: We can see that a cylinder constructed by circle and rectangle.

Teacher: Yes, anyone else?

Begüm: It is about the net of the cylinder.

Teacher: Yes. You are both right. The question wants you to see the parts of a cylinder and its net.

In this small section, the classroom got the main idea of this part of the instructional sequence. The students were clear about the drawing of a wrapper for a cylinder-shape candy box. Furthermore, they successfully completed the drawings without any error. This section provided data according to the Krummheuer's argumentation model as illustrated following.

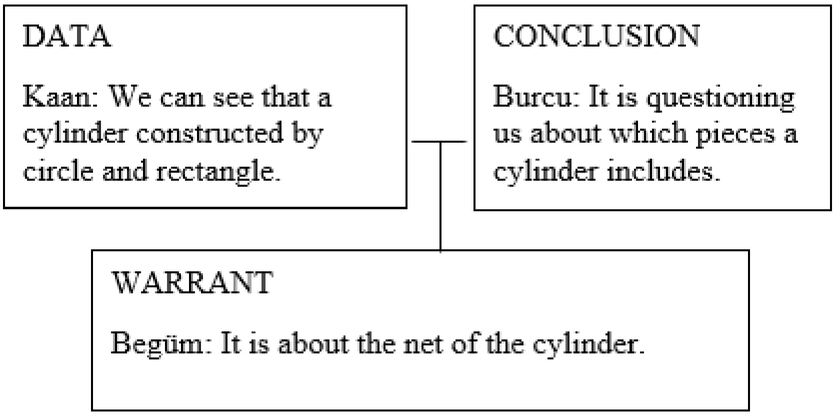


Figure 4.51 KMA on structure of net of a cylinder

After this session, the teacher continued with the definition, properties, and elements of a cylinder. A GeoGebra file was opened by the researcher including an illustration of cylinder (Figure 4.52). The following section is from that part.

Teacher: Yes, we have a cylinder shape on the GeoGebra file. We will talk about the properties and elements of a cylinder. This is the height of the cylinder. These are called as bases. Top base and bottom base.

Berna: Do we call the top one as a base again? Why? Isn't it a ceiling?

Teacher: In geometry, these kinds of solids such as prisms or a cylinder have top and bottom bases, not a base and a ceiling. Ok?

Berna: Yes.

Teacher: Yes, here are top and bottom bases. What are the shapes in the top and bottom bases?

Classroom: Circle

Teacher: So, what are the important parts of a circle?

Classroom: Radius.

Teacher: Yes, or diameter. So, here is the radius. And the same one is at the top base. Yes, now, write the definition of the cylinder….

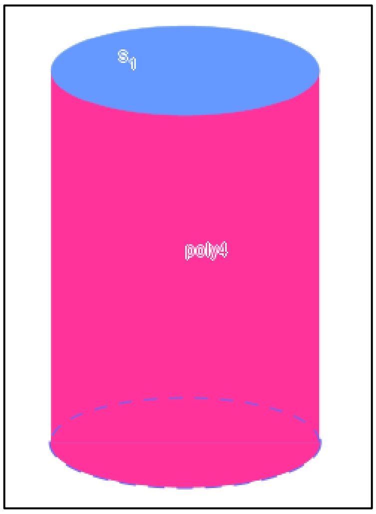


Figure 4.52 An illustration of cylinder from GeoGebra

The section showed that students had a prior knowledge about the cylinder, since they had some idea about their elements and properties. Some students confused naming top and bottom bases of the cylinder. Berna asked the reason for not calling the top base as ceiling. Most of the students were calling the top base of a prism and cylinder as ceiling. This could be related to their previous knowledge and/or developing some misconceptions about some terminology of geometric concepts. But teacher handled it by explaining. After they talked about the properties and elements of cylinder, the teacher told the definition of cylinder and students wrote it on their notebooks. This debate did not include any argumentation element of Krummheuer, but it was critical for the next step of the process which was based on understanding net of a cylinder. Because, understanding height of a cylinder means to understand an edge of the rectangle which constructs cylinder's side face. Moreover, understanding radius of circle base means to understand its circumference and relatedly another edge of the rectangle at the side face.

Later on, first page of the activity sheet of this part was given to students. The context was candy wrapping and the students were expected to draw a wrapper for a candy that was given in cylinder shape. The students worked individually and in peers, and they successfully completed this process. There wasn't anyone that have problem with drawing wrapper for the given cylinder candy. Moreover, they stressed that the question wanted them to draw net of cylinder. During the process, the teacher and the researcher visited the students and controlled their works. After the session, their drawings were controlled on the GeoGebra file. In the following visual, the GeoGebra file and related discussion is given.

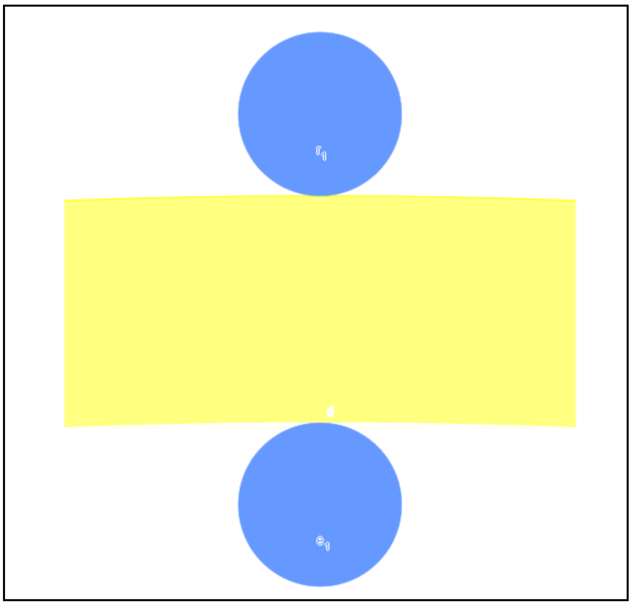


Figure 4.53 Net of the cylinder from GeoGebra

Teacher: In general, your drawings were right. So, what do you see as the basic elements of net of a cylinder?

Deniz: It has two circle bases and one face.

Selma: The face is rectangle.

Teacher: Yes, as we mentioned before, it has two bases and a side face. Does the side face have to be a rectangle? Can it be any other shape?

Tuğçe: Yes.

Hasan: No, while you were rolling the cylinder on the smartboard, I saw a square also.

Tuğçe: Likewise. I meant that it should be a quadrilateral.

Teacher: Why?

Tuğçe: When we roll it, the corner points should come together for bases. It is only possible with a quadrilateral shape.

Researcher: Very good point. Is there anyone who wants to talk about or add something to Tuğçe's idea? Or, Tuğçe can you explain your idea clearly? Why do you think like this?

Tuğçe: Hmmm. Aaaa. Can I explain it on GeoGebra?

Teacher: Of course.

Tuğçe: (By showing net of the cylinder) If we want to wrap something around these circles, we need two points that come together. Two points for top base and two points for bottom base and four points in total.

Teacher: Yes, and what do four points mean?

Tuğçe: If we combine four points, we get a quadrilateral.

Teacher: That's right. Good, Tuğçe. Is there any missing point? Is there anyone who does not understand?

Class: …. (Silence)

In this debate, the classroom was talking about net of cylinder after seeing the illustration of it on the GeoGebra file. First, the students completed their works and then the researcher opened the GeoGebra illustration on the smartboard. As it is cleared from the dialogue, most of the students were able to draw correctly. This may be related to their prior knowledge from earlier grades.

Observation of GeoGebra illustration on the smartboard provided a source for students to check their drawings in a meaningful way, since they could see the reason on the screen. The dynamic nature of the GeoGebra allowed students to follow the change from closed form and its net. Also, Tuğçe caught a very good and critical point for net of the cylinder. She explained the reason for side face's being a quadrilateral. She stated that a cylinder shape was constructed by two circle bases and if someone wanted to wrap those circles, he/she needed to combine two points. She stated that there were two circle bases and two points to wrap the top base, two points to wrap the bottom base, so it was four points in total. She stated that it was called as quadrilateral, formed by four joint points with the line segments. Thus, this was a taken-as-shared idea which emerged while the classroom was working on structure of net of a prism. Afterwards, the idea was normally used in questions and discussions which was an indicator of taken-as-shared idea. The examples were provided about this idea in the following sections. This explanation constituted an introduction for the following steps. In advancing lessons and in exercises, students worked on that concepts and discussed on it. This section was a good example of using social and socio-mathematical norms in whole-class discussion. Thus, according to Krummheuer's argumentation model and emergent perspective, this debate can be summarized as in the following section.

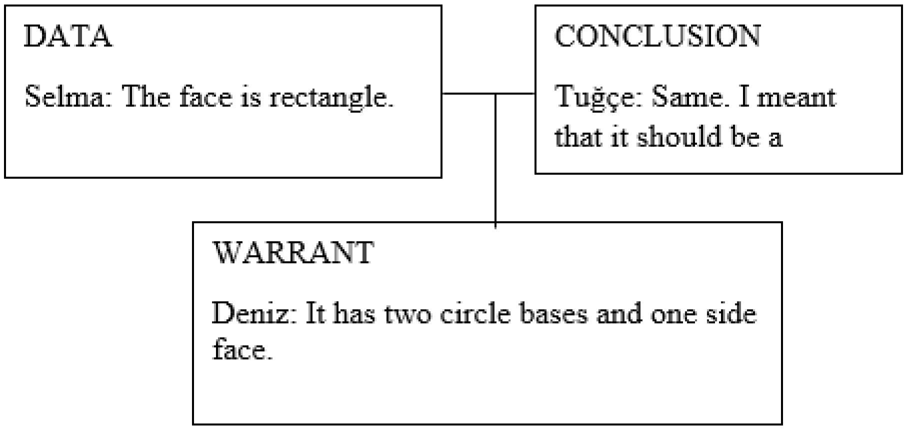


Figure 4.54 KMA about side face of a cylinder which should be quadrilateral.

Afterwards, the teacher questioned the classroom about height of cylinder. She tried to make them aware of how the height of a cylinder change related to the lengths of side face edges and also related to the circumference of circle top and bottom bases. The discussion about the following section is given below.

Teacher: Now, I want you to think about height of the cylinder. Our GeoGebra file will help you. Look at that opened form of the cylinder. Where is the height of the cylinder? Actually, which length indicates the height of cylinder? Yes, Büşra.

Büşra: Looking at the shape, we see that this edge is the height of the cylinder.

Teacher: What is that edge?

Büşra: It is the short edge of the rectangle. Actually, it is the height of the cylinder.

Researcher: Do the short edge need to be the height of the cylinder? Is it always okay? What do you think?

Zeynep: If we have a long cylinder, height will be the long edge. It depends on the given lengths and circle bases. We cannot conclude that the short edge is height of the cylinder.

Teacher: Yes, good. Did everybody understand? Are there any problems?

Class: No.

In this section, the classroom evaluated relation between closed and opened form of a cylinder by support of GeoGebra file. Moreover, they evaluated how one element of it placed in opened form, or vice versa. Students could easily understand the change of height as one edge of rectangle that is side face of cylinder. The use of GeoGebra in this process allowed students to observe the transition of the cylinder from opened to closed form. In this way, students had a chance to understand easily in which position an element of the cylinder placed in both cases. When Büşra explained change in the height of cylinder as one edge of rectangle in side face, the classroom did not react to that explanation positively or negatively. This was another idea that height of the cylinder is depends on the lengths of the side face. Thus, according to emergent perspective, it became taken-as-shared and when analyzed according to Krummheuer's argumentation model, it can be summarized as in Figure 4.55.

This part of the study was completed with this activity, but it was not the end, since the context had an interrelation with itself. For instance, the students often used the basic elements of a cylinder and ideas emerged during they were working on its net, its construction and also surface area. The usage of the ideas in the following lessons were indicator of that they became taken-as-shared. Thus, in following sections these relations will be mentioned accordingly.

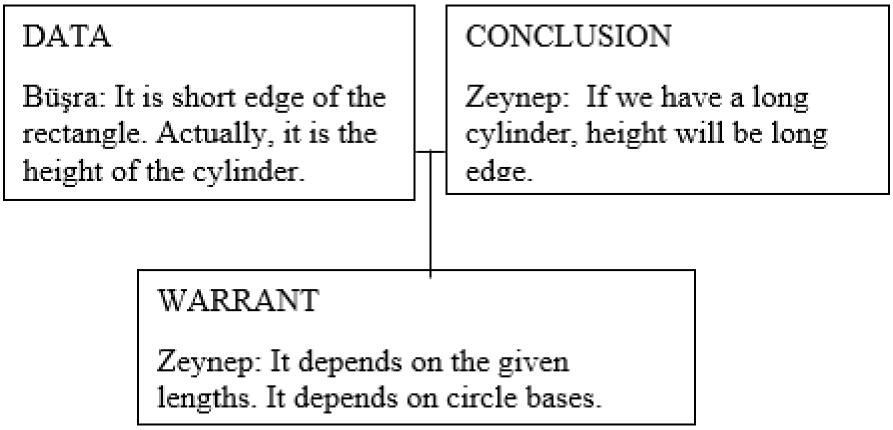


Figure 4.55 KMA on height of the cylinder depends on the lengths of the side face.

4.3.2 Idea 2: Relation between the circumference of the circle base and edge of its side face

This idea occurred after the classroom completed working and discussing the definition, basic elements, and structure of its net. The first part of this section was about thinking and talking about features and elements of the cylinder. At the beginning, the teacher asked the students about their prior knowledge about cylinder and about its daily life examples. Later on, the classroom discussed and learned the basic knowledge about this solid, then evaluated those elements both on closed form and opened form. The classroom discussions indicated that students had some prior knowledge thinking their examples from daily life and also from their reactions to the whole class discussions.

After completing this process, the classroom continued with questions about combining parts of net of given a cylinder. During the process, the students mainly worked on questions about net of cylinder. The context was based on producing candy boxes. This type of questions constituted the big part of this section; which was related to their importance for understanding surface area of cylinder. These were also important with their relation to the previous mathematical idea which was about the structure of net of the cylinder. For the construction of second mathematical idea of this section, first part constructed a basis for students' understanding. During the process, students often used the elements of a cylinder in their solutions or in their discussions, since it was inevitable to use these parts in the solution of questions.

The first question (Figure 4.56) of this process planned as whole class discussion to make students clear for the way to follow. This would make easier for them to understand the main aim of the questions. As it was mentioned above, the content was about understanding the relation between the circumference of the circle base of a cylinder and edge of its side face which is generally a rectangle. To construct this idea, the teacher started the whole class discussion and it continued approximately for ten minutes. They first talked about the content, the given data, the relation between given parts and the possible ways to follow. The question they worked on and a section from this whole classroom discussion is given in the following dialogue.

Teacher: Yes, you read the question. What did you understand? Who wants to explain it? Yes, İpek.

İpek: It asks how we can construct a cylinder by using given shapes.

Teacher: Good. Another idea? Yes, Tuğçe.

Tuğçe: Actually, it is about the parts of a cylinder. We should find which rectangle is appropriate for the given circles.

Teacher: Yes. Very good. You should find the appropriate rectangle for those circles. So, which way will you follow? Arda.

Arda. The area of the rectangle should be equal to the given circle's area.

Teacher: Arda says that the area of this rectangle should be equal to the area of circle base. What do you say? Is it right?

A few students: No

Beyza: The area is about whole shape. There is no relevance.

Researcher: Let's look at our GeoGebra file again. Look at the rectangle and the circle together. What is the relation between those two? Can you see Arda?

Arda: Yes, the side face is surrounding the circle.

Teacher: So, what can we say about that relation?

Kaan: Both two lengths should be equal. Circumference of one circle base should be equal to the length of the side face.

Teacher: Yes, we can conclude that both the circumference of the circle base and the length of the rectangle’s edge should be equal to be able to combine them together.

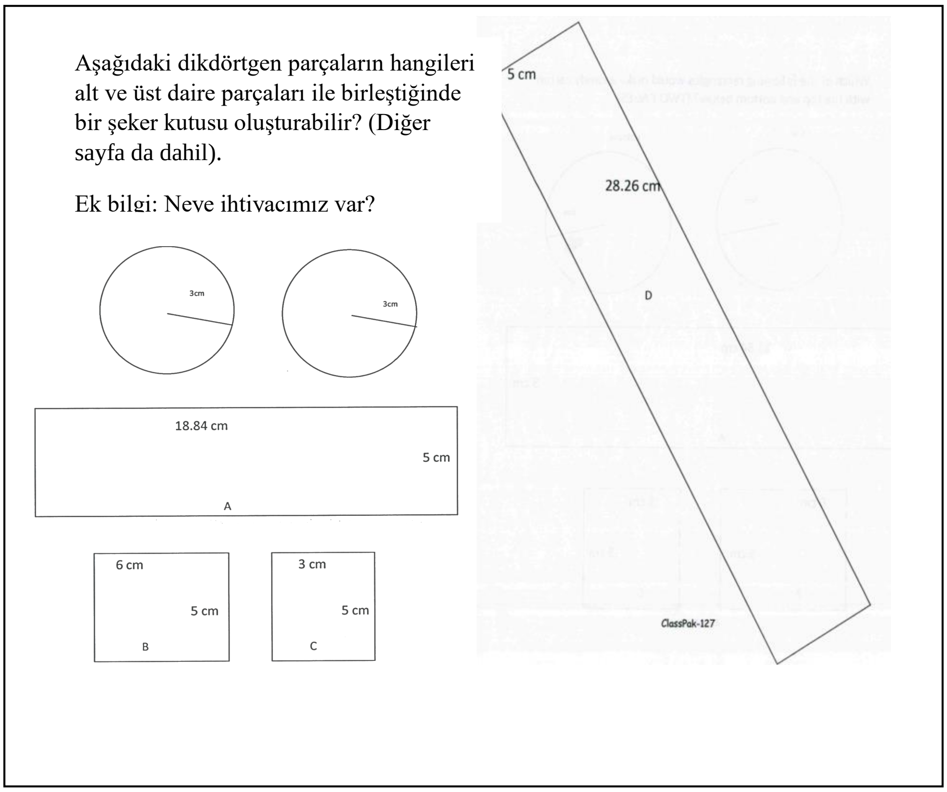


Figure 4.56 The first question of net of cylinder concept

In this debate, the issue of discussion was the first question that constructed on understanding its basic parts. Students thought on understanding those basic parts and the way to combine them. The idea which students were expected to catch was that the circumference of the circle base of a cylinder equals to the length of the one edge of the side face which is generally a rectangle. If they got the idea, they could solve the following questions easily without any challenge.

The teacher wanted to learn whether the students understand the main idea of question. İpek replied the teacher by stating it was about construction of a cylinder. It was a good idea since they were expected to combine those decomposed parts by finding appropriate shape for given circles. Actually, those given parts were net of the cylinder in real and according to their reaction, students were easily understood this issue.

The teacher continued with getting ideas of students. Arda's idea was that the area of circle base should be equal to the area of rectangle side face. The classroom showed a negative reaction towards his idea by stating that the idea was wrong. Beyza gave reaction to this explanation by stating that area of a shape was about its covered part. To help students to understand, the researcher opened the GeoGebra file that was illustrated in Figure 4.51 and Figure 4.52. By evaluating its motion between its net and closed form, most of the students handled the problem with this issue, including Arda. To show the main idea and reorganize it in appropriate words, the teacher directed the way of discussion by saying the last sentence. After all, the students started to solve the question.

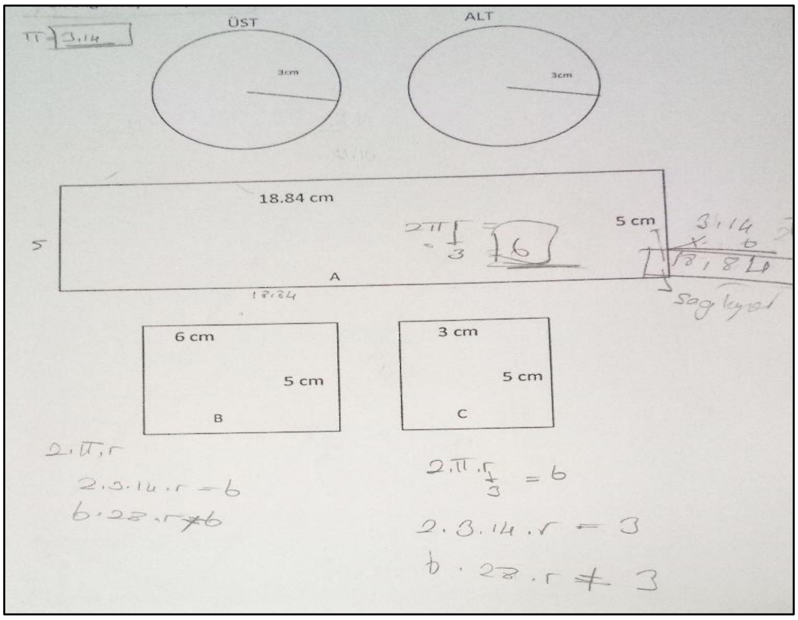


Figure 4.57 Zeynep's solution

According to argumentation model the section is evaluated as following. In this debate, there was not a warrant produced by students to support the conclusion of the Kaan. He constructed her claim on Arda's data by stressing that the side face was surrounding the circle base. Moreover, the students often used in their discussion the elements of the cylinder such as its bases and its height which is also one edge of side face. This is an indicator of they produced the mathematical idea about relation between the circumference of the circle base and edge of its side face, by constructing on the mathematical idea about structure of net of a cylinder.

DATA

Arda: Yes, the side face is surrounding the circle.

CONCLUSION

Kaan: Both two lengths should be equal. Circumference of one circle base should be equal to the length of the side face.

Figure 4.58 KMA about the relation between the circumference of the circle base and edge of its side face

After students worked on the question, the teacher started classroom discussion about solutions of each question.

Teacher: Let's start with talking about question A. Who wants to solve it? Come here Beyza. You will solve the question A. And Zeynep come here. You will solve the question B. Divide the board from the half.

Zeynep: I wrote that the circumference of the circle is $2\pi r$. We get π number 3,14. When we multiply the result is 6,28, and it is more than 6. Thus, the rectangle in B is not fits with the circle.

Teacher: Did you understand Zeynep's solution? Are there any problems? What did you do Beyza? Explain your solution.

Beyza: As Zeynep said, circumference of the circle should fit with one edge of the given rectangle. So, I said that circumference of the circle should be equal to the 6. The formula is $2\pi r$. π number is 3,14 and r is 3, when I multiply them it makes 18,84. This fits with A.

Teacher: Thank you. Your friends explained very good. Are there any problems?

Class: No.

This section was about students' solutions for the question in Figure 4.55. This part included the solutions about for only shapes in A and B. Also, Zeynep's solutions for the whole question were given in Figure 4.57 above. They explained their solution in order. Zeynep stated that she had found the circumference of the circle pieces to compare with length of the rectangle's edges. First, she calculated the circumference of the circle by using the formula $2\pi r$. She found the result 6,28 which was longer than 6. Thus, she stated that those two lengths did not fit with together. Then, Beyza explained her way for the shape in A. Beyza also mentioned about the necessity of equality for the circumference of the circle base and one edge of rectangle. She followed the same way to compare those two lengths. In this debate, Zeynep and Beyza's explanations for the solution of the question supported each other according to Krummheuer's argumentation model. Moreover, their discussion was an example of construction of social and socio-mathematical norms in terms of involving whole class discussions by using appropriate and acceptable mathematical terminology. Thus, the structure of argumentation can be shown as in the following.

DATA	CONCLUSION
Beyza: As Zeynep said, circumference of the circle should fit with one edge of the given rectangle.	Zeynep: I wrote that the circumference of the circle is $2\pi r$. We get π number 3,14. When we multiply the result is 6,28, and it is more than 6. Thus, the rectangle in B is not fits with the circle.

Figure 4.59 KMA the relation between the circumference of the circle base and edge of its side face

In another question, the concept was similar to the previous one. The students were again asked to find the appropriate circle base for given rectangle.

Researcher: You will get π value as 3.

Teacher: Yes, let's start to talk about your solutions. Yes, Begüm.

Begüm: As we talked before, the circumference of the circle should be equal to the one edge of the rectangle. First, I found circumference of each circle. For this circle (by showing the first one), r = 2.5 and the formula of circumference is 2πr. So, it is 2 times 2.5, it is 5. And the π number is 3, 5 times 3 is 15. For this circle (by showing second circle), the circumference is 45 and for the last circle the circumference is 60. When I look at the given rectangle, the appropriate one is the first one. Because its circumference is 15-units and the one edge of the rectangle is 15- units.

Teacher: Yes, that's right. Your friend explained very well. Is there anyone did not understand the solution? Or are there any different ways to reach result?

A few students: Same.

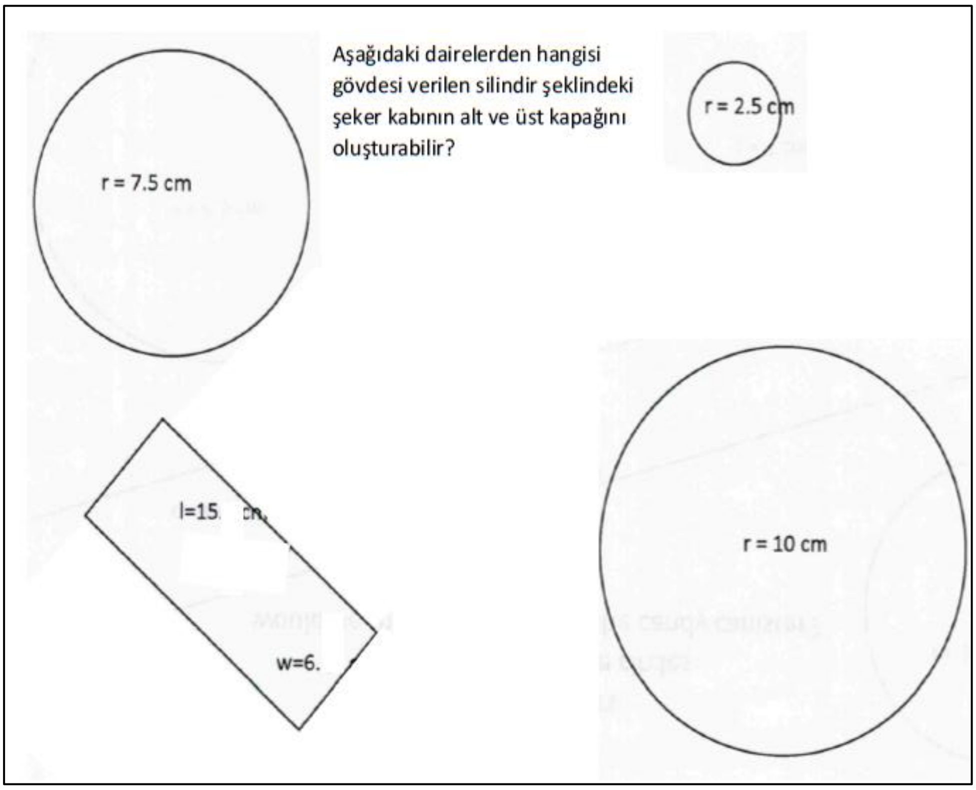


Figure 4.60 Question about the relation between circumference of circle bases and the side face

In this debate, the classroom was working on the second question. It was again prepared on the same concept. They tried to find the appropriate circle base for the rectangle to construct a cylinder. Begüm explained her way; by using the formula of circumference of a circle and by comparing the results with length of one edge of the given rectangle. While explaining her solution, Begüm used the idea as conclusion without any challenging idea or any warrant. Thus, her discourse can be analyzed as following.

CONCLUSION

Begüm: As we talked before, the circumference of the circle should be equal to the one edge of the rectangle. Both two lengths should be equal.

Figure 4.61 KMA about the relation between the circumference of the circle base and edge of its side face

After this time, students got the idea of circumference of the circle base should be equal to the one edge of side face in a cylinder by working and reasoning on questions about net of the cylinder. In advancing hours and in following questions, students often used this data for their solutions. It was not only in this same context, but also by working on surface area of cylinder. The classroom did not question or challenge the idea anymore. Thus, according to emergent perspective and Krummheuer's argumentation model, the mathematical idea about the relation between circumference of the circle base and the length one edge of side face, became taken-as-shared among classroom.

4.3.3 Idea 3: Cylinder's surface area constructed by area of side face and area of circle bases

In this section of the instructional sequence, the students produced three mathematical ideas to reach a mathematical practice. In the previous part, students worked on net of the cylinder and the way to construct it. They mainly focused on understanding equality of circumference of circle bases and rectangle side face. This was the critical point and main idea of the previous part.

While working and deliberating on area of rectangle and area of circle, students developed this idea to produce the mathematical practice of surface area of cylinder. For this part, the classroom mainly worked on finding area of circle bases and rectangle side face of cylinder. In this process, they reasoned both on net of the cylinder and on closed form of it. As a final point, they tried to produce a formula for surface area of cylinder. This section of the instructional sequence continued through fourth week of the study. In the initial questions, the closed form of the cylinder was given with height and radius information. By using that data, students tried to find the area of circle and rectangle, radius, or edge length. In this section questions were prepared based on the previous section which was about understanding the relation between circle bases and the side face. The questions had some given data on closed form and were asking about their nets and vice versa. They were given approximately ten minutes to work and then they started to talk about solutions. The following question and the part is from that section.

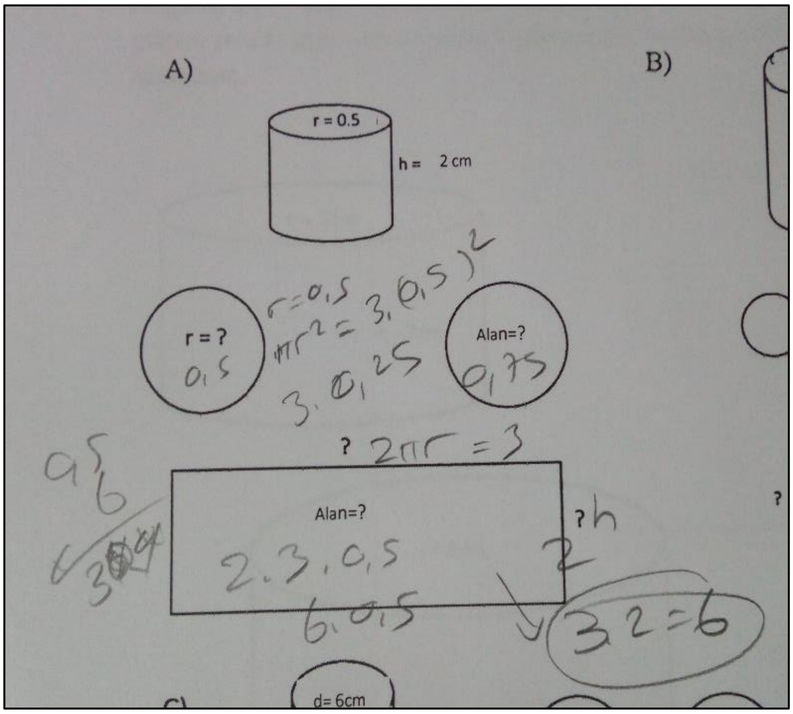


Figure 4.62 Deniz's solution to the question

Teacher: Deniz will come for the first one.

Researcher: Please explain your reasons while solving the question.

Deniz: We find the area of circle by using πr^2. π number is 3. And radius is 0,5. Then, r^2 is 0,25. Thus, when multiply them all, it is 0,75.

Teacher: And that area?

Deniz: And to find area of rectangle, we need the short and long edge. The area is found from their multiplication.

Researcher: So, what did you do?

Deniz: As we talked previous lessons, we find the long edge by using circumference of circle base. Thus, it will be find by 2πr. It is 3. Hight of the cylinder is short edge of this rectangle. Thus, 3 times 2 is 6.

Teacher: Is there any comment on Deniz's way?

Kerem: As we did in previous lessons, we have two circle bases and one quadrilateral side face. If we think like wrapping as candies, we need to find area of wrappers. Actually, we will find area of circle's and area of side face.

Teacher: Kerem supported Deniz's claim. Do you agree with them?

Class: Yes.

In this debate, the teacher chose a student to solve and explain the first question of this section. At first step, Deniz found the area of circle. And then, he stated that to find the area of a rectangle, he needed to find the short and long edges of it. Thus, by using the previous mathematical idea which was about the relation between circumference of circle base and length of the side edge, he calculated the long edge of the rectangle by circumference of the circle base. And he stressed that the short edge of the rectangle was the height of the cylinder. So, here he used the mathematical idea about the structure of net of a cylinder. Since one idea emerged based on a previous one, Deniz's argument provided a basis for following ideas. Furthermore, Kerem supported his claim by stating that a cylinder is constructed by two circles and one quadrilateral side face. Additionally, he used one of the previous idea as side face of a cylinder should be a quadrilateral shape. This usage was important to show that it was also used after the idea emerged. There were two more questions about the same concept. While working on those questions students used similar ways as explained above. They found area of circle bases and rectangle side face of cylinders given in opened forms (by working on their nets). The aim of these questions was to prepare (actually, they were calculating the surface area, but they did not name it at that time yet) students for the surface area of cylinder. Thus, they could evaluate the main idea of calculation of surface area of cylinder by deducing from its net. Also, in this process, the usage of previous ideas was important to get a conceptual understanding of what a surface area is and how to calculate it. After completing the problem-solving process, classroom continued to the discussion as in the following.

Researcher: Why did we do those things? Why did we calculate those parts of cylinder? Where do you use it?

Yağmur: They are basic elements of cylinder.

Hasan: Is it surface area?

Teacher: How it will be? Tell me.

Hasan: We calculate two circles' areas and one rectangle area and add them.

Teacher: Did you hear your friend? He says it is surface area. Other ideas? These are pieces of cylinder, aren't they?

Class: Yes.

Teacher: What do you actually find by finding these areas?

Kaan: Surface area of cylinder.

Beyza: Wrapping a cylinder.

Researcher: Actually, to wrap something means its surface area. Why do we continue step by step?

Metin: Volume.

Researcher: Metin says it is volume. What do you say?

Begüm: Volume is about filling something. This is about surface area.

Teacher: Metin. What do you say? Why did you think like that?

Metin: I don't know. I thought it should be volume.

Researcher: Wait. Please. Can you give us an example for volume? Not only you. Any of you can give an example. It can be a daily life example about volume. You know this issue. Yes. Ok.

Yağmur: For example, volume of this bottle of water (by showing her water bottle).

Teacher: Yes, are you okay now, Metin?

Metin: I see. That's ok.

Researcher: Let's look at it again. You found these areas. Circle's area and rectangle's area. When you know those, what do you actually know?

Akın: Surface area.

The teacher directed the discussion to make them to relate those areas to the surface area. Yağmur reminded that those were the basic parts of a cylinder and Hasan added that by finding those two circle areas and one rectangle area, they calculated the surface area of cylinder. Kaan stated they worked on surface area and Beyza supported by stating it is wrapping it. Metin expressed his idea by adding it was about volume of cylinder. Actually, this was an unexpected situation or claim about surface area. Because, the classroom worked about surface area about prisms and there was not any claim occurred in this way. After Metin's claim, Begüm corrected him by saying volume was about filling a shape, it is not about wrapping. The researcher wanted him to correct his wrong idea and made him to see his fault himself. Thus, the researcher directed the discussion in that way and wanted to think them about the daily life examples of volume. After Yağmur's example, Metin corrected himself. After teacher's explanation, as a last conclusion, Akın expressed his idea as it was a calculation of surface area. During this section, a new idea emerged and became taken-as-shared later about volume is about filling something and surface area is about wrapping a shape. This idea was important for understanding the meaning of surface area and relatedly for the calculation of it. In their discussion, students supported each other in context of surface area of cylinder. According to emergent perspective and Krummheuer's argumentation model this dialogue can be summarized as following.

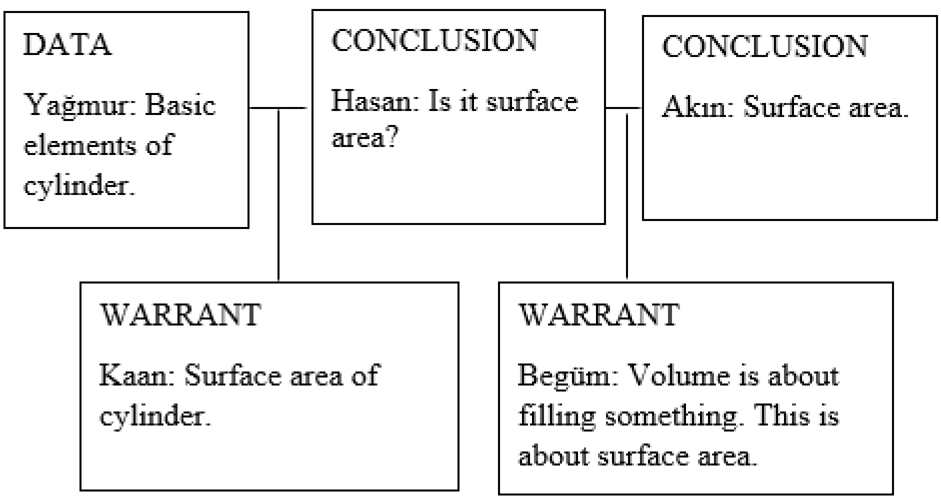


Figure 4. 63 KMA about surface area and volume

For the following part, students worked on closed form of cylinder (An example was provided in Figure 4.64). They were expected to design wrappers for cylinder-shaped candies given in closed forms. They were asked to calculate the surface area of each cylinder. In previous questions, they worked and reasoned on net of cylinder and for this part, they were expected to use that information in closed form. The process started with students' working on these questions individually. After they completed their works, the teacher started a class discussion on the context.

Teacher: Who will come for first question? Come here, Buse. Explain us your solution? What did you do? Yes, listen to your friend!

Buse: Teacher. I did this operation to find the long edge of rectangle. I used 2πr since it gives long edge, and it is 2 times 3, 6 and 6 times 5, 30. Height is short edge. By multiplying 30 and 7, it is 210.

Teacher: Please draw the rectangle and show the place of each number.

Buse: Here, it is 30. Height is 7, so to find area of rectangle, I multiplied two of them. It is 210. With using r^2 , I found circle.

Teacher: Draw the circle. Yes, we have two circles.

Buse: π'number is 3. r^2 is 25 and it is 75. There are two circles at the top and at the bottom. It is 150. Then, whole surface area is 360.

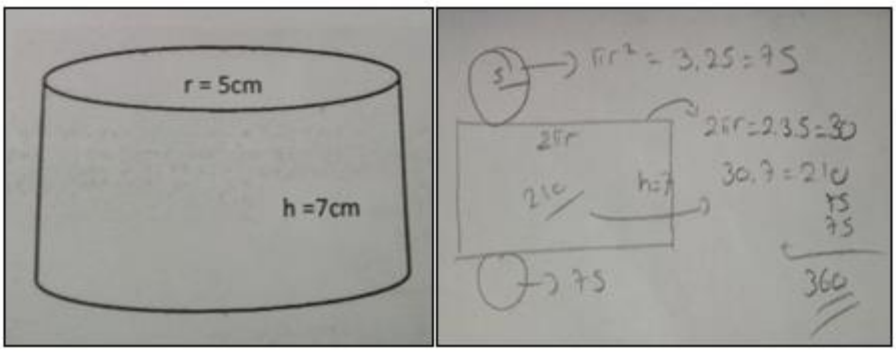


Figure 4.64 The question that Buse solved and her solution

In this debate, Buse explained how she solved the question given in Figure 4.62. She drew the net of the cylinder on the board. First, she calculated the area of the rectangle. By using circumference of the circle, she found the long edge of the rectangle. And then she stated that height of cylinder was short edge of rectangle. Thus, to find area of rectangle she multiplied those two lengths. For the following step, she calculated area of circle bases and at the end she summed all areas to find surface area of cylinder. In her discourse, she used mathematical idea which was about the structure of net of a cylinder, she used the idea that circumference of circle bases of a cylinder equals to the length of the side face. Additionally, she used the idea about the length of the height of the cylinder depends of one edge of side face. After she completed her explanation, there was no disagreement with her or any idea needed to be explained. Thus, she used her expressions as conclusion according to Krummheuer's argumentation model.

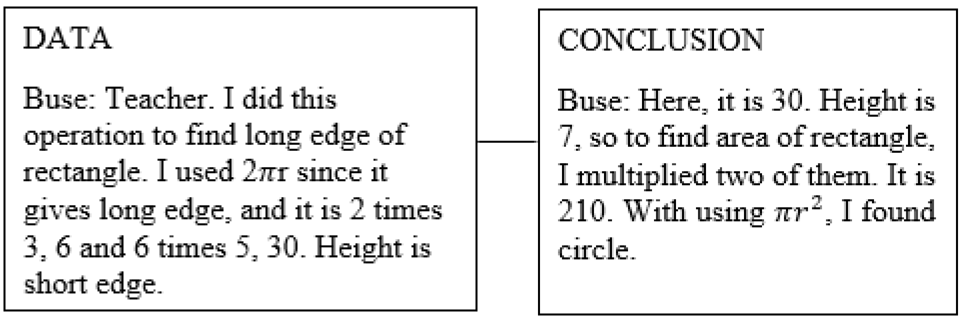


Figure 4.65 KMA on the discussion on cylinder's surface area

The instruction continued with two more similar questions. In general, the students used same ways while solving questions, and the whole class discussions focused on area of circle and rectangle. After this process was completed, it was time for working on formula of surface area of cylinder. The researcher gave the related page (Figure 4.66) to the students to think and try to produce a formula for surface area of cylinder. They were given approximately ten minutes for reason on context. After they completed their work, the teacher started whole class discussion. The following section is chosen from that part.

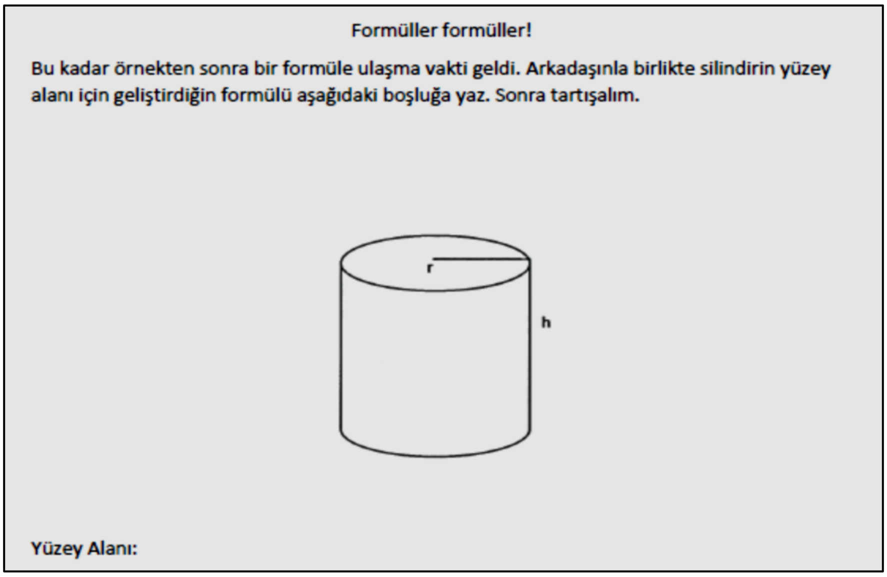


Figure 4.66 The page of activity sheet about producing surface area

Teacher: Yes, who wants to express his/her idea?

Aydın: I have an idea.

Teacher: You have an idea. Ok, Aydın. Come here, please. Tell us.

Researcher: Draw the shape and explain on it. Place the lengths on the shape.

Aydın: This length is 2πr. This is h. And this area is 2πr.h. Area of one circle is πr^2 and there are two, so it is 2 πr^2. And surface area is this.

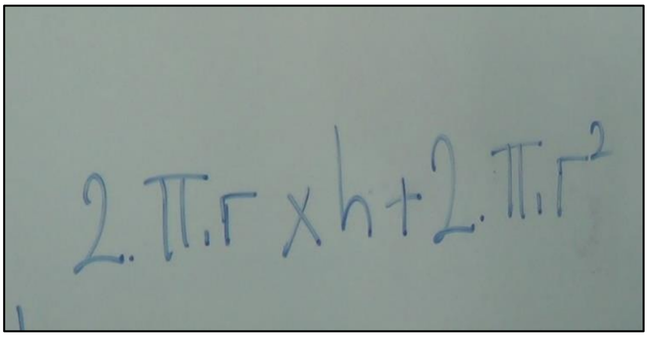


Figure 4.67 Aydın's formula for surface area of cylinder

In this debate, Aydın drew a rectangle and two circles to explain the way he thought about the formula of cylinder. He placed the algebraic expressions of each length on them. And then he organized his formula that he produced according to those lengths and according to problems they solved previously. During this discussion, Aydın used the previous mathematical ideas as data to explain equality of circumference of one circle base and long edge of rectangle, and also equality of height of cylinder and rectangle's short edge. Also, he stated that he produced his formula by reasoning on area of circle and area of rectangle. His discourse can be summarized according to Krummheuer's argumentation model as following.

CONCLUSION
Aydın: This length is 2πr. This is h. And this area is 2πr.h. Area of one circle is πr^2 and there are two, so it is 2 πr^2. And surface area is this.

Figure 4.68 KMA on area of circle and area of rectangle

But, there was an objection to Hasan to his explanation. The dialogue continued as in the following.

Hasan: Can I ask something?

Teacher: Yes.

Hasan: Teacher. He wrote 2 before π number. Doesn't he find area wrong by this way?

Teacher: Why do you think like this?

Hasan: He multiplied π number with 2. He multiplied 2π and r^2 .

Teacher: Is it wrong?

Hasan: Yes, teacher. Can we put π number in front of parenthesis?

Teacher: Now, I couldn't understand. What do want to say?

Hasan: I put π in front of parenthesis and summed up $r^2$and r^2 in the parenthesis.

Researcher: Come here Hasan. Show us your idea. Yes. Hasan also will tell us about his way.

According to Hasan, Aydın's multiplication of π and 2 was a meaningless operation. He could not understand the reason of Aydın's multiplication of 2π with r^2. Moreover, he offered to find summation of r^2 and r^2, and to place them in parenthesis. After, he multiplied that expression by π number. In the following figure, at the top, it is Aydın's formula and at the bottom Hasan's formula is shown.

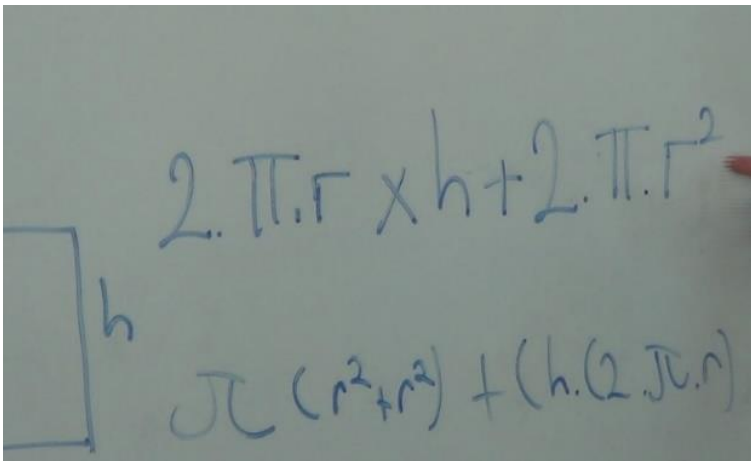


Figure 4.69 Aydın's and Hasan's formula together

After two ideas were written on the board, the classroom discussed on those two ways. The classroom was in a consensus about the place of algebraic expressions of each length. The problem was about expressing their way for formula. To handle this confusion, the teacher and the researcher directed the way of discussion to the formulas. They started to discuss about which one was true as surface area formula.

Zeynep: Can I say something? Aydın's operation is 2πr times h. It is area of rectangle, isn't it?

Teacher: Aydın. Is it area of rectangle?

Aydın: Yes. I deliberated according to our previous questions. We said that the circumference of circle equals to the long edge of rectangle. And cylinder's height equals to short edge of rectangle. We find the area of rectangle by multiplying short and long edge. Thus, I thought like this.

Zeynep: Then, he should be right. Because, we need to sum up whole areas to reach surface area of cylinder.

This debate was about Aydın's formula that he produces for surface area of cylinder. Zeynep asked whether he tried to find out each area and sum up hem at the end. After Aydın’s explanation, she supported his formula accordingly. This was the idea about producing formula by writing formula of area for circle bases and area of side face separately.

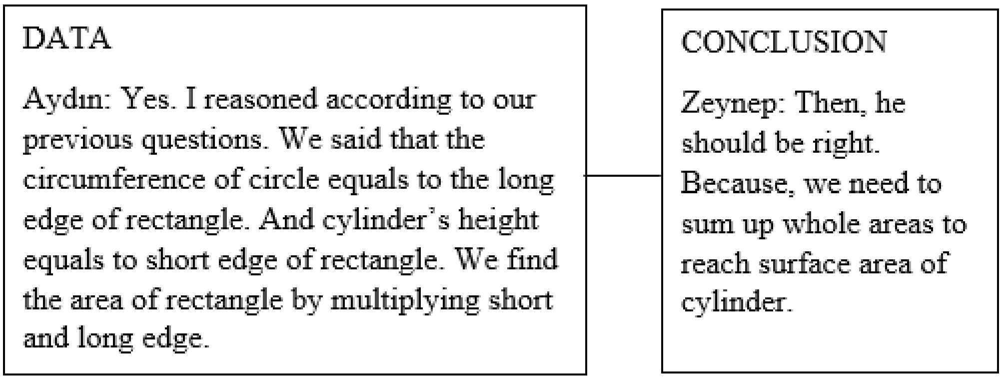


Figure 4.70 KMA on area of circle and area of rectangle.

After a few minutes more whole class discussion, Burcu offered to use distributive property over addition and stated that by this way they would have had two πr^2, and by summing them up they had 2 πr^2.

Burcu: Actually, if we use distributive property over addition in Hasan’s formula, we have $\pi r^2 + \pi r^2$. So, multiplying it by 2 is the same thing as summing two of them. They're the same thing.

Hasan: I tried to express, but I did not write like that. But I was thinking like Burcu. Here, I wanted to express that there were two circle bases and there should be two areas for those bases. Therefore, I wrote r^2+r^2 in the parenthesis.

Teacher: Exactly, this is the relation. Those formulas state the same things, they are the same algebraic expressions. Only, you needed to see the distributive property over addition in the second formula. Now, is it okay?

Class: Yes.

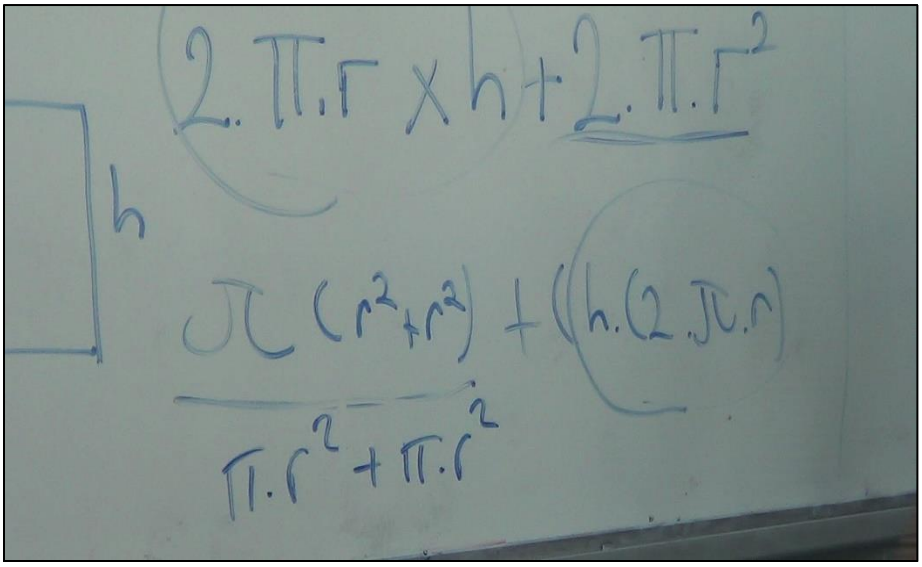


Figure 4.71 Burcu's reorganization to the Hasan's formula

After Burcu's explanation to the Hasan's formula, the classroom came to a consensus about both Aydın's and Hasan's formula was stating the same thing, actually, they were the same. Thus, the teacher recovered whole process and stated again formula of surface area of cylinder. Accordingly, this idea can be shown in terms of Krummheuer's argumentation model as following.

DATA	CONCLUSION
Hasan: Here, I wanted to express that there were two circle bases and there should be two areas for those bases. Therefore, I wrote r^2+r^2 in the parenthesis.	Burcu: if we use distributive property over addition in Hasan's formula, we have $\pi r^2 + \pi r^2$. So, multiplying it by 2 is the same thing as summing two of them.

Figure 4. 72 Discuusion on distributive property over addition

There were no challenging ideas, any missing point or any objection to the produced idea. Thus, the idea became taken-as-shared among classroom according to emergent perspective. The samples chosen from students work also showed that most of the students produced formula in similar ways which also illustrated that the idea became taken-as-shared. In advancing hours of instruction, students used the mathematical practices that emerged from the beginning of the instruction together while working on some questions. For example, at the end of the surface area part, there were questions forcing students to reason on both surface area of prisms and surface area of cylinder. In those questions, students' discourse included those mathematical practices together including the ideas that constructed those mathematical practices.

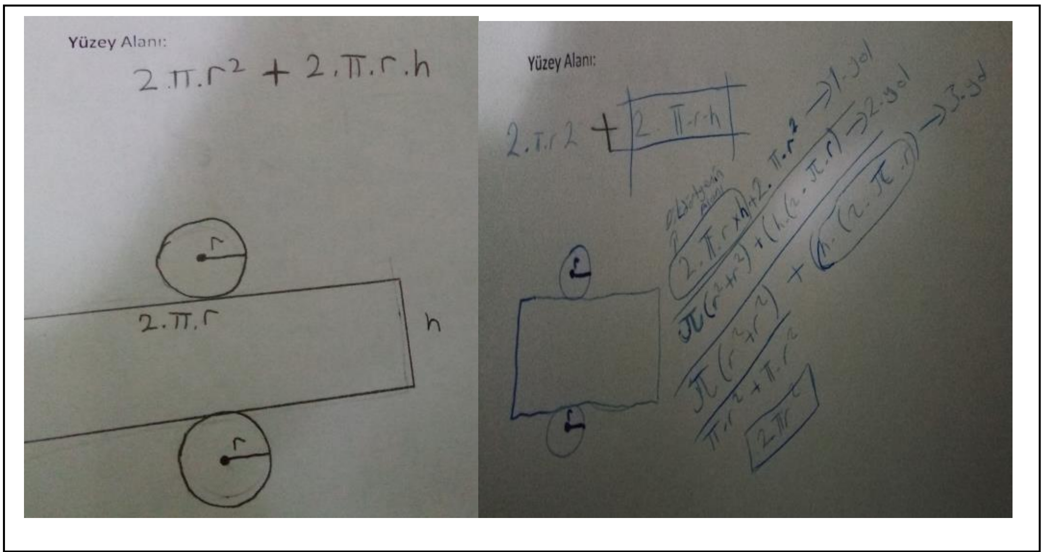


Figure 4.73 Students' work about formula of cylinder's surface area

The following part was chosen from that section. First, the related question was illustrated below. The question was about wrapping cost about a cube and a cylinder-shaped candy box. Students worked on the question for five minutes and then the classroom discussed it.

Teacher: Let's look at the question. It asks the cheapest wrapping cost. Who wants to talk about it? Yes, Yalçın. Come here. Explain your solution.

Yalçın: First, I found the surface area of cube. Cube is also a prism. As we talked before, we found the surface area by multiplying area of each face with two. Here, we can find it by finding area of one face. Each face of a cube is equal squares. Thus, I found area of one face by multiplying 3 with 3, it is 9. And there are 6 faces 9 times 6, it is 54.

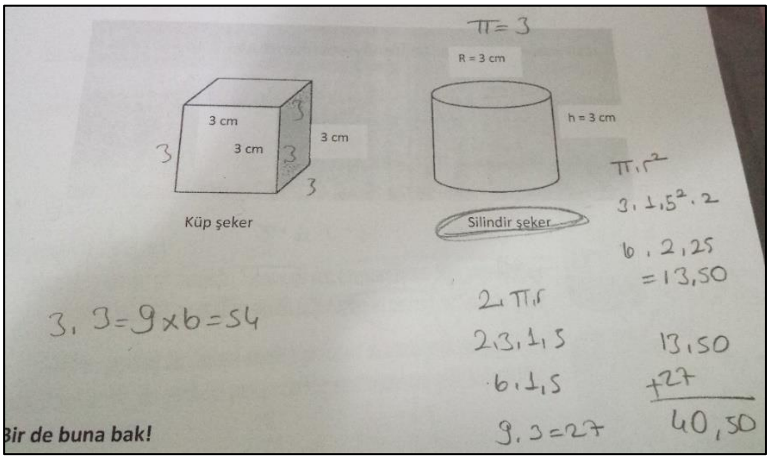


Figure 4.74 The question about surface area of cube and cylinder

This debate illustrated the usage of mathematical idea that cube is a prism and mathematical idea about transition from counting unit squares to calculating area. While talking on the question given in Figure 4.73, Yalçın explained his way of solving it. First, he stated that he could use the surface area of prisms, since cube was also a prism. He justified his solution by using the idea of cube was also a prism during the beginning parts of the instructional sequence. Yalçın used that idea as data for another step. In another step, he started to solve the question and he used the idea of calculating area of prisms by finding area of each face, summing them up and multiplying the result with two. In this way, he stated that, surface area of a prism can be found by 2(ab+bc+ac). Moreover, he stressed that cube has equal faces and it can be calculated by multiplying area of one face with 6. Thus, his discourse can be illustrated according to emergent perspective and Krummheuer's argumentation model as following.

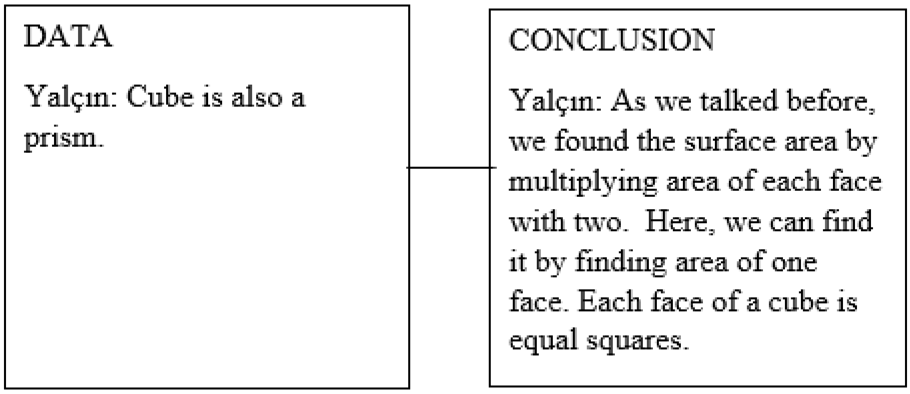


Figure 4.75 KMA on use of mathematical practice about surface area of prisms.

The discussion continued as following.

Teacher: Then, and what about the second one? Cylinder-shaped candy.

Yalçın: I found area of bottom base 2,25.

Teacher: How did you do it?

Yalçın: I used the area of circle. It is πr^2. Then, r=1,5 and square of it is 2,25. 3 times 2,25 is 6,75. There are two bases. Thus, 13,50 is total area of bases.

Teacher: Yes, bottom and top bases. Then?

Yalçın: There is a side face. It is a rectangle. I found the area of rectangle. Long edge of rectangle equals to circumference of circle. I used 2πr. It is 9. And by multiplying 9 with height, I found 27.

Teacher: Yes, there also a height. 9 times 3 is 27. Yes. What did you do later?

Yalçın: I summed up 13,5 and 27, it is 40,5.

Teacher: Yes, surface area of cylinder candy is 40,5. When we compare the two-surface areas, which one do you think cost less?

Yalçın: Cylinder costs less.

In this section of debate, Yalçın explained his way of finding the surface area of cylinder candy. He mentioned that he used the area of circle and area of rectangle to find the whole surface area. Thus, he used the third mathematical practice in his solution without any doubt. He used those practices as data for support for his solution. His discourse can be summarized according to emergent perspective and Krummheuer's argumentation model as in the following.

DATA

Yalçın: I used the area of circle. It is πr^2. Then, r=1,5 and square of it is 2,25. 3

DATA

Yalçın: There is a side face. It is a rectangle. I found the area of rectangle. Long edge of rectangle equals to circumference of circle.

Figure 4.76 KMA on use of mathematical practice about surface area of cylinder

After this process, the classroom continued to work on other questions about mixed questions including context of surface area of prisms and cylinder. They used the produced mathematical ideas relatedly mathematical practices without any doubt, any objection, or any questioning.

4.4 Mathematical Practice 4: Finding Volume of the Cylinder

Fourth mathematical practice was emerged about finding volume of the cylinder. This section of the instruction continued during three lessons of last half week of the process. During the instruction, the classroom worked on last two pages of the activity sheet. This section constituted the shortest part of the study in terms of both lesson hours and page number of activity sheets.

This part was prepared based on the learning objective of "constructing the knowledge of the volume of the cylinder". While preparing the activities of this phase, students were expected to have the knowledge of "what is volume" and "volume of cube and rectangular prism" since they had learned those concepts in sixth grade.

4.4.1 Idea 1: Volume is about third dimension

The first mathematical idea became taken-as-shared during the discussion of volume. The teacher started the session by questioning about the knowledge of what volume is. The students had the knowledge of the volume from earlier grades. They had already learned the volume of the cube and rectangular prism at sixth grade level, and they were expected to have the idea of volume and its conceptual understanding.

Teacher: Now, we will talk about volume. What do you understand when we say volume? Do you have any idea?

Kaan: The place that a shape covers on the earth or in space.

Teacher: The place that a shape covers on the earth. How? Give an example.

Kaan: Teacher. For example, a bottle of water. It has its own place in the space.

Teacher: So, you say it is volume. Any other ideas? Previously, we discussed about area. What is the difference between area and volume?

Zeynep: For example, we find the area of a rectangle. But, it does not have a volume. Because, it is flat.

In this section, the teacher asked students about the meaning of the volume. Their responses showed that they had some prior knowledge about volume, but they were having difficulty to express themselves. Kaan's example of bottle was a good example of volume, but he couldn't explain the reason of his thought. Also, Zeynep stated that it was possible to calculate the area of a rectangle since it was flat. Her idea was a step to understand the transition from 2-D calculations to the 3-D thinking. Thus, the teacher wanted to direct the discussion in that way. The discussion continued as following.

Teacher: You say, volume is about its place that cover in space, and Zeynep says the area is about flat shapes. I want you to make more clear explanations.

Mete: One of our friends had given an example while we were working on surface area of prisms. We can think like tiling on a ground. For example, this classroom's ground has an area and we can cover this place according to its area. We were saying something like this. The number of tiles gives us the area of that ground. This is area.

Zeynep: And also, we did wrapping the candies. They were about area. The volume includes the inside of the shape.

Kaan: For this reason, the area is about two-dimensional shapes. Volume is about three-dimensional shapes.

Teacher: Good. Yes, Tuğçe

Tuğçe: That's why we calculate area of a rectangle or a triangle. But we have volume of a cube, or a prism.

With this section, the students started to express their ideas more clearly. Mete reminded the example about tiling a ground that had been given while they were working about the surface area of prisms. This was an example of expressing the main idea of area. Also, Zeynep stated the work about wrapping candies that they practiced, was about the area, again. By discussing other's ideas, they grasped the idea of two-dimension and three-dimension. The idea of volume is about three-dimensional shapes emerged during this part but used in later sections of the instruction in students discourses normally. Thus, the structure of this section can be illustrated according to Krummheuer's argumentation model as in Figure 4.77.

The idea of volume is about third dimension supported emergence of other ideas relatedly and used as data or conclusion in places. Thus, it can be concluded that the idea became taken-as-shared. The examples were provided in following sections. For example, following section was provided from the later hours of the instruction.

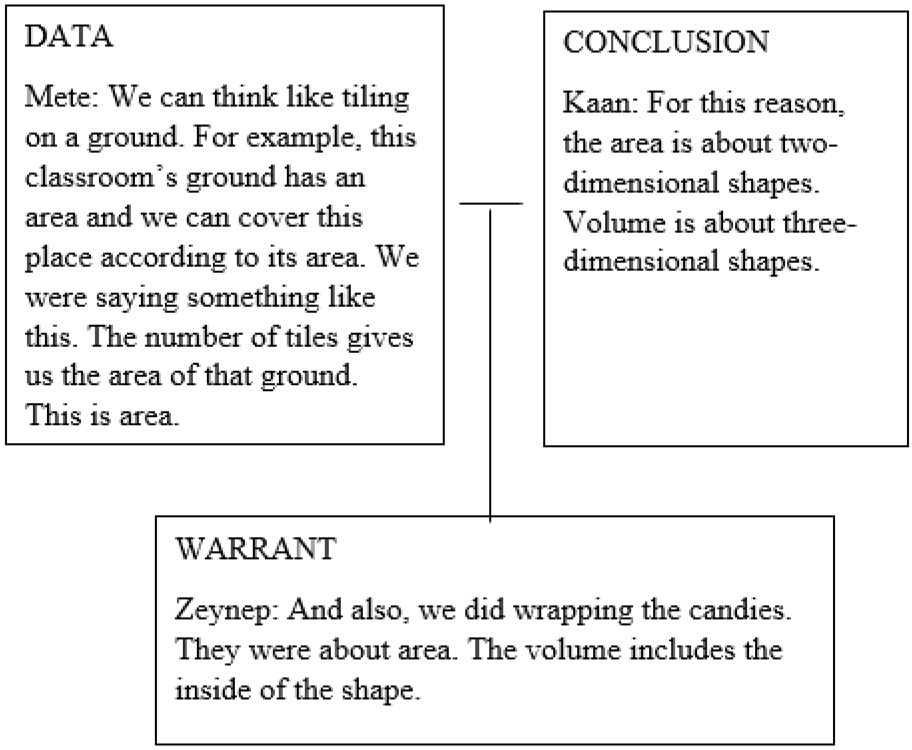


Figure 4.77 KMA on volume is about third dimension

Teacher: I want to ask you something. According to your explanations, what would you say about the volume of a piece of paper? Can we calculate it?

Melisa: No, it is two dimensional.

Teacher: Yes, it is two dimensional. What do we need?

Melisa: It should be three dimensional to have a volume. There is no height of it.

Teacher: Yes, good.

In this debate, teacher wanted to see whether the students understood the idea of volume and third dimension. She asked about some objects and shapes and about their dimensions. This part was chosen from that section of discussion. The teacher questioned that whether a piece of paper had a volume. Melisa responded by saying that a piece of paper was a two-dimensional shape, they needed a three-dimensional shape to calculate volume. Thus, the idea became taken-as-shared and can be illustrated as following figure.

CONCLUSION

Melisa: It should be three dimensional to have a volume. There is no height of it.

Figure 4.78 KMA on volume is about third dimension

4.4.2 Idea 2: Volume is about filling inside of a shape

This idea emerged immediately after the previous discussions in the same lesson. The classroom continued to talk about meaning of volume. The teacher directed the discussion in that way. Later on, she wanted the students to think about requirements of calculation of volume of a three-dimensional shape. Following discussion occurred in this process.

Teacher: Yes, as most of you stated the volume is about the whole shape with inside and surface. But area is about surface. As you mentioned, we worked previously about the surface area on your activity sheets. We were wrapping surface of candies with wrappers. Those were all about the calculation of surface area. Now, I want you to think about this classroom. How would you assess the volume of this classroom?

Aydın: We need to count the number of things that can fill this classroom. Those things should be equal.

Teacher: What are those things?

Aydın: I forgot their name. We used them in our activity sheets.

Teacher: Unit cubes.

Aydın: Yes. Unit cubes. If we find the number of unit cubes that fill this classroom, it gives us the volume of this classroom.

Kerem: This is why we call it three dimensional isn’t it?

Teacher: Yes. This is the meaning of the volume. This is the reason for saying that the place that a shape cover in space.

Begüm: This is similar to tiling a ground. That was area, filling here with cubes is about volume. Yes, I see.

Teacher: Did you see the relation now? Or difference?

Class: Yes.

In this debate, the issue of discussion was the meaning of volume again. The teacher directed their discourse to relate the first idea; which was about volume's relation to third dimension. At the beginning of the argument, the teacher confirmed the previous discourse that students produced related to the meaning of area. She reminded again their works about wrapping shapes were about calculation of surface area. Also, after understanding the difference between area and volume, she wanted students to grasp the idea of calculation of volume accordingly. She wanted them to think about finding the volume of the classroom. Aydın asserted a claim as filling inside of the classroom could give them the volume. He could not remember the name of the unit cubes and the teacher reminded him. Kerem supported Aydın's claim by providing evidence by reminding the third dimension was about that thinking a shape as a whole. In this way, Begüm supported that argument by reminding and connecting the examples of tiling a ground and filling inside of a classroom. Thus, students could relate their ideas constructing on their peers' and reached to the meaning of volume. When evaluated according to Krummheuer's argumentation model the structure of this discussion can be illustrated as following.

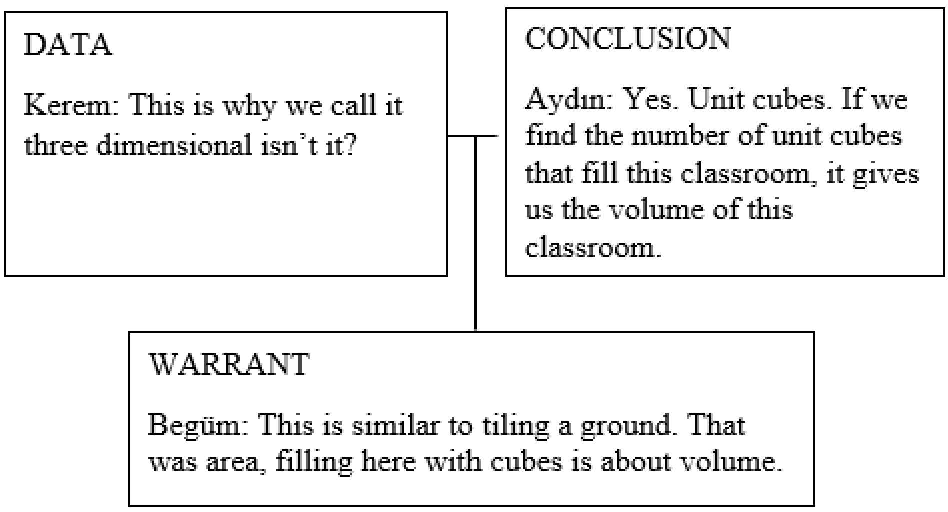


Figure 4.79 KMA on volume is about filling inside of a shape

The process was showed that students started to get the conceptual understanding of volume. This can be supported by observing students' usage of those ideas in their discourse while producing formula for volume of cylinder and working on related questions. In the following sections, the classroom started to think and discuss about the calculation of the cylinder's volume and producing a formula for that calculation.

4.4.3 Idea 3: Calculation of volume requires the knowledge of width, length and height

In the previous sections, students involved in whole class discussions related to the meaning of the volume. They produced ideas and those ideas became taken-as-shared by listening and commenting on other's thinking. This idea about calculation of volume requires the knowledge of width, length, and height, emerged after they completed to discuss on the meaning of volume. The teacher wanted the students to think about the requirements of calculation of the volume.

Teacher: Hmm. Ok. You got the idea of volume. Now. Let's think about the calculation of volume. What do you need to know? Yes, Beyza.

Beyza: We need width, length, and height.

Teacher: Why do you think that?

Beyza: Because, three dimension means that width, length, and height.

Teacher: What do you calculate with width and length?

Beyza: It gives us area. We calculate area of a shape by multiplying its width and length.

Teacher: What is the role of height? Yes, Arda.

Arda: It gives the third dimension. For example, if we multiply that area with height, it gives us the volume.

Teacher: Can any of you give an example for this?

Mete: For example, we can again think about this classroom. If we tile whole ground of the classroom, we find the area of ground. But if we multiply that result with height, it gives us volume.

In this debate, students realized the requirements of the calculation of volume. They easily caught the idea of multiplication of width, length, and height to calculate the volume. But, this would be related to their previous knowledge from previous years. Because, after the teacher's asking about the students' idea about the requirements of calculation for volume, Beyza asserted about the necessity of width, length, and height. The teacher wanted them to understand why they used them in the calculation. For this purpose, she wanted them to give an example for the explanation. Based on that, Mete provided an example about the volume of the classroom. He stated that after calculating the area of surface of ground, the multiplication that area with height would give them volume of the classroom. Moreover, Arda's explanation was important in terms of both providing a warrant for Beyza's claim and for also being an example of usage of the idea about the volume's relation to the third dimension. To evaluate whether the students grasped the conceptual understanding of the volume, there was a need to observe the usage of idea while the instruction was in progress. Thus, it can be illustrated as following.

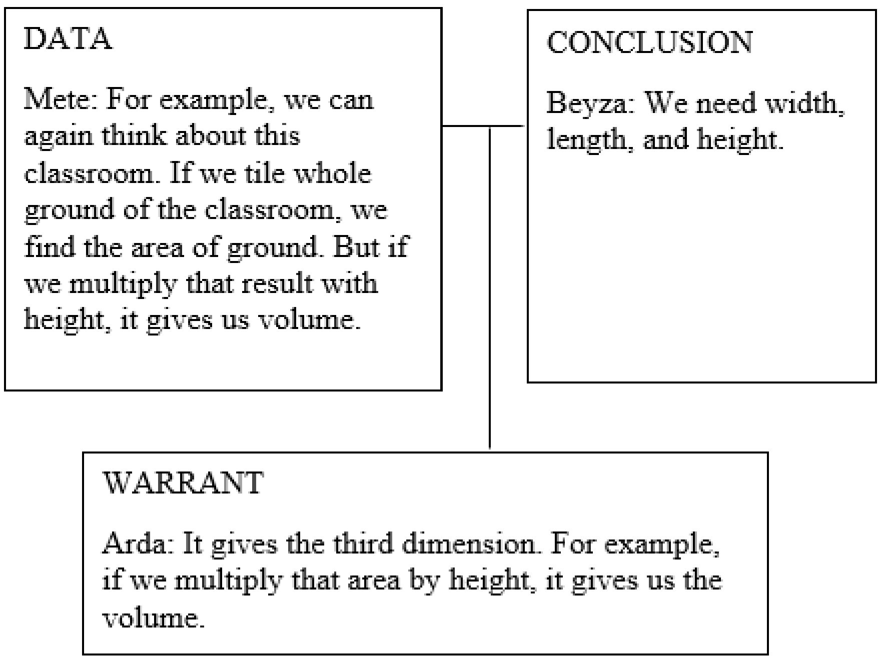


Figure 4.80 KMA on calculation of volume requires the knowledge of width, length, and height

After this section, the teacher started to direct the whole class discussion to the volume of cylinder. The aim was to make transition from calculation of volume of prisms to calculation of volume of cylinder. Students were expected to connect prior knowledge to current context. Following part is chosen from that section.

Teacher: Yes, we talked about the volume of the prisms and as we mentioned you had that knowledge from previous years. You learned that subject in the sixth grade. Now, let's think about the volume of the cylinder. You know the volume of prisms and you have the idea of volume. What do you want to say?

Kaan: We find the volume of the prisms by multiplying width, length, and height. So, we do the same thing for cylinder.

Teacher: What do you say for Kaan's claim?

Tuğçe: But, cylinder is not a prism and it does not have a width and length.

Teacher: Good point. A cylinder is not a prism and it does not have a width and length.

Kaan: Ahhh. Yes. Sorry.

Teacher: So, then, what do we do?

In this section, the classroom started to discuss the volume of cylinder. They constructed their discussion on their previous knowledge about the volume of prisms they learned in sixth grade. The teacher wanted the students to remind that knowledge again and wanted them to relate that knowledge with volume of cylinder. Kaan asserted that they found the volume of prisms by multiplication of width, length, and height of the given prism. Thus, he stated that they would follow the same way for calculation of cylinder. His claim about the calculation of volume of a prism was correct but his idea that using the same way for calculation of cylinder was wrong. Tuğçe identified this fault and refused that idea by stating that a cylinder did not have any width and length. The idea that calculation of volume requires to the multiplication of width, length, and height, Kaan's claim can be illustrated as following.

CONCLUSION

Kaan: We find the volume of the prisms by multiplying width, length, and height.

Figure 4.81 KMA on calculation of volume requires the knowledge of width, length, and height

4.4.4 Idea 4: Volume equals to the multiplication of base area and height

This idea emerged immediately after the discussion about the multiplication of width, length, and height. In the previous section, after the problem was handled about the Kaan's claim, the teacher wanted the classroom to think about the way of finding the volume of cylinder. Also, in advancing parts of the whole class discussion, GeoGebra was used to make students to understand the way of calculating volume of cylinder. So, the following part emerged in this process.

Teacher: Now, you said that while you are finding the volume of prisms, you use the multiplication of width, length, and height. What was the aim of multiplication of width and length?

Yağmur: Area.

Teacher: Which area?

Yağmur: The surface area.

Teacher: Yes, surface area. Remember we call it as base area. Okay. What is the later step, then? Mert?

Mert: Multiplying the surface area by height.

Teacher: Very good. So, we can say that, remember it. The volume is, we say, multiplication of base area with height. Now, is it the same for cylinder?

Zeynep: It should be the same.

Mete: Same. Because, it is three-dimensional.

Teacher: We will see. Let's look at this GeoGebra file.

This section was about relating the students' prior knowledge to the current situation. Students had the knowledge of volume of prisms from previous years. The teacher made them to remember this knowledge by discussing and reorganizing their knowledge in this way. In this discussion, the teacher made them to involve in this discussion to call back their knowledge about the knowledge about prisms volume can be find by multiplication of base area and height. Actually, the classroom knew this knowledge, but maybe related to the time passed, they forgot the way of expressing the formula. With support of the teacher's directions, Mert expressed the formula. The structure of this section can be illustrated as following.

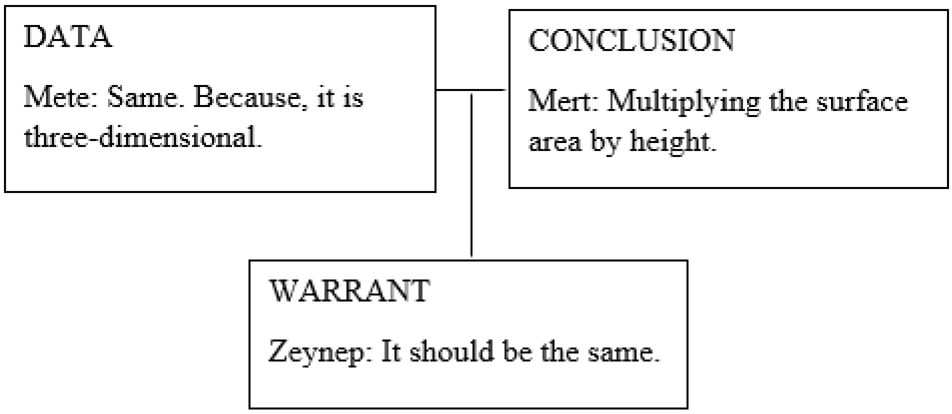


Figure 4.82 KMA on volume equals to the multiplication of base area and height

And then, the teacher asked about whether the same way could be followed for the volume of cylinder. The researcher opened a GeoGebra file (Figure 4.82) at the same time with the following discussion. This GeoGebra file was based on the idea of filling the cylinder. The goal was to show students how filling inside of a shape was related to the formula.

Teacher: You said, it is the same as finding the volume of prisms. We will multiply the three elements that width, length, and height. But then, you said there is not width and length of a cylinder. We can say there is no edge. So, what is the solution?

Kerem: πr^2.

Teacher: Why?

Kerem: To find that circle?

Teacher: Which circle?

Kerem: Cylinder's circle

Teacher: You mean base area.

Kerem: Yes. Base area.

Teacher: So, then why do you need to multiply with height?

Mete: To find volume.

Beyza: Because, height brings the volume concept, it provides third dimension.

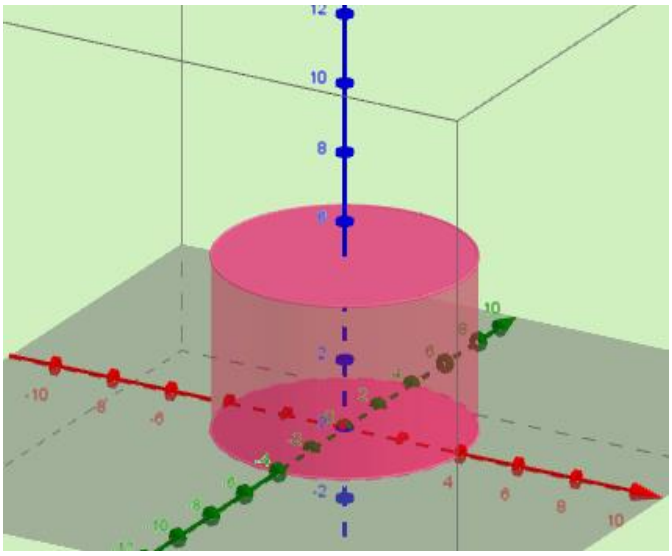


Figure 4.83 An empty cylinder illustration

This section was an introduction to whether the multiplication of base area and height gives the volume of cylinder as it gives in prisms. The teacher started the whole class discussion in this way. The GeoGebra was opened on the smartboard and the classroom discussed the issue. This section did not include any idea that became taken-as-shared, but it was critical in terms of being first step of understanding the volume of cylinder's formula. After that, the discussion continued as following. The researcher continued by indicating the GeoGebra file in Figure 4.83 and 4.84.

Researcher: Now. You said we need base area, but we need a height to get the third dimension. You can think here like, we were counting the number of unit cubes that fill the prism to find the volume of the given solid. So, here we can count the number of what?

Tuğçe: Circles.

Researcher: Yes, think about that what will happen when you place an infinite number of circles over on others?

Arda: Cylinder

Teacher: Yes. So, what do we count then, to find the volume of the cylinder?

Arda: Number of circles.

Teacher: Yes, the number of circles. Do we count the number of circles for every time when we want to calculate the volume of a given cylinder? What is the number of these circles? (By filling the cylinder on the GeoGebra-Figure 4.84)

Zeynep: It's height. So, volume is multiplication of base area and height.

Teacher: Exactly. So, this is the reason of the formula.

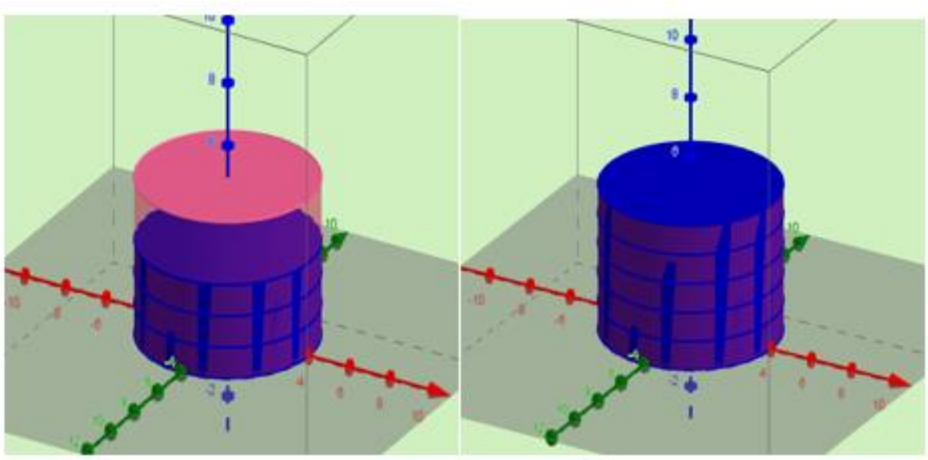


Figure 4.84 Observation of filling the cylinder o GeoGebra

The answers expected from the students here were the necessity to use circle segments instead of using unit cubes to fill the cylinder. Students would be able to figure out volume of the cylinder by understanding how they could fill the cylinder via placing the circles by putting one on another one at the height of the cylinder. Additionally, students were expected to transfer the knowledge of the volume of cube and rectangular prism which can be formulated as "multiplication of base area and height". With the discussion on volume of the cylinder by filling it with circle segments and the discussion about the volume of cube and rectangular prism, they wanted to conclude that the volume of the cylinder is "multiplication of base area and height" under the guidance of whole class discussion. Thus, after the teacher's and researcher's directions, Tuğçe stated that a cylinder could be fill with using circles. By putting her idea, Arda added that putting those circles was about construction of a cylinder. At the end, Zeynep obtained those number of circles that were put on each other, gave them the height of cylinder, and that was the idea of formula. The observation of GeoGebra file with filling the cylinder by using circles, provided students to observe the number of circles gives the height of the cylinder. Thus, illustration of this situation dynamically, supported the emergence of the idea that number of circles gives the height of the cylinder. The structure of this section can be illustrated as following.

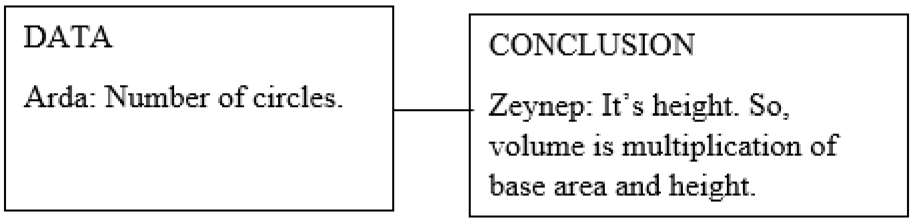


Figure 4.85 KMA on the idea volume equals to the multiplication of base area and height

In this way, the classroom produced the mathematical practice of finding the volume of cylinder. After whole class discussion completed, the teacher repeated the formula of cylinder and the reason and they started to work on the last two pages of the activity sheet. During their works, students used the mathematical practice of finding volume of cylinder including the taken-as-shared ideas which were supported the emergence of that practice. Following an example from questions about volume of cylinder was given.

Teacher: Let's look at the question. Height is 4 cm, diameter is 3 cm. Does it ask the volume?

Yağmur: If the diameter is 3 cm, radius is 1,5 cm.

Teacher: Yes.

Yağmur: Multiplying the base area and height, I found 9 π

Teacher: Yes. Wait. Okay. Yağmur again please.

Yağmur: Base area is $\pi . r^2$. π is π. So, r^2 is 1,5 times 1,5, it is 2,25. Height is 4. So, the result is 9 π.

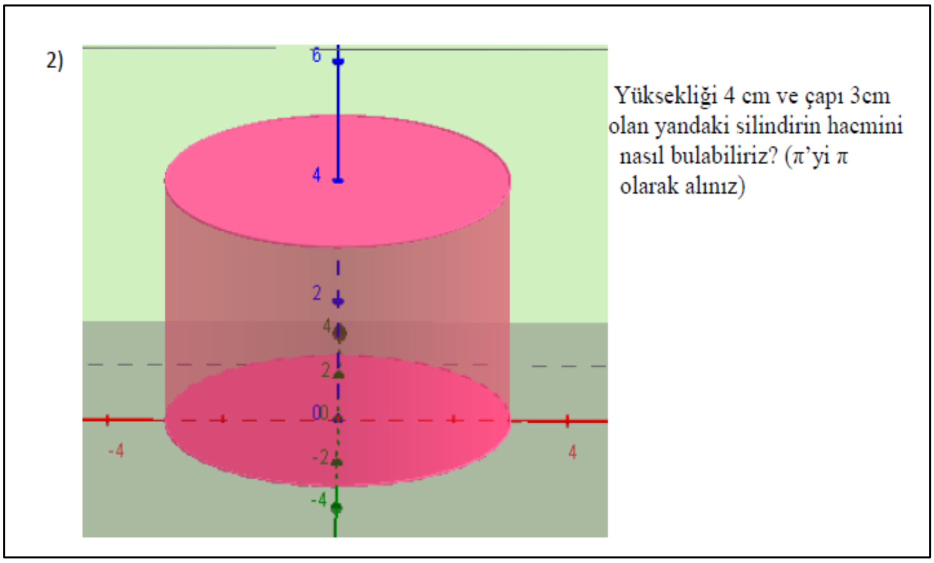


Figure 4.86 An example from activity sheet about volume of cylinder

In this section, they worked on the given question in Figure 4.81. It was a practice about application of the formula for the volume of cylinder. Yağmur explained her solution successfully and showed that the mathematical practice about finding the volume of cylinder. She used the idea of volume of cylinder can be found by multiplication of base area and height. There was not any objection or any comment about her claim and solution. Yağmur's discourse can be illustrated as following.

CONCLUSION
Yağmur: Multiplying the base area and height, I found 9 π

Figure 4.87 KMA on volume equals to the multiplication of base area and height

The qualitative part of the findings of the current study obtained four mathematical practices based on mathematical ideas that students produced during the instructional sequence. Those instructional sequences were carried out with support of a conjectured HLT. The instructional sequence continued through four and half week. The GeoGebra files supported the student's understanding and helped them to visualize the given three-dimensional shapes. Students did not use GeoGebra, but they observed prepared illustrations. Those illustrations provided support for students to get conceptual understanding of some concepts (especially the ones who need visualization abilities) and produce mathematical ideas relatedly. During the instructional sequence HLT was applied and discussed by the teacher and the researcher in terms of its missing parts or strong features. Those missing parts discussed to be changed for following applications. The quantitative findings of the study were mentioned following.

4.5 Quantitative Results

In this section quantitative findings of the study were explained. This study aimed to develop content for geometric concepts (solids) located in 8. Grade mathematics curriculum with the support of the GeoGebra dynamic geometry software, to develop an instructional sequence with guidance of a conjectured HLT, to obtain mathematical practices during this process in an argumentative classroom environment and, to test the effectiveness of this content in an eighth-grade math class. The content is expected to improve students' geometric thinking and learning related issues.

To test the effectiveness of the content on students learning of 3-D shapes, pre-posttests were applied to the students. Test questions were derived from web site of General Directorate of Measurement, Evaluation, and Examination Service (which is a part of Ministry of National Education). The questions were selected in accordance with the HLT prepared for the current study. The questions on this website are constantly being updated in accordance with the national curriculum. Since, the conjectured HLT has already been prepared in parallel with the national curriculum, the questions have been adapted to the content of the study without deviation from the curriculum.

The test questions were based on the concepts of general properties of prisms, their basic elements, understanding the relationship between open and closed states, surface area of prisms, general properties of cylinders, basic elements, surface area of cylinders and volume of cylinders. The number of questions was 11. Ten of them were test questions and one of them was an open-ended question.

For analysis of pre-posttests scores of students, paired-samples t-test was applied to evaluate the difference. Following tables shows the statistical analysis of the pre-posttest results.

Table 4.2 Paired sample statistics of pre-posttest results

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Pretest scores of students	45,83	35	21,809	3,686
	Posttest scores of students	66,23	35	23,662	4,000

"A paired-samples t-test was conducted to compare meaning of pretest results and posttest results of eight graders. There was a significant difference in the scores for pretest (M=45.83, SD=21.80) and posttest (M=66.23, SD=23.66) scores."

Table 4.3 Paired sample correlations of pre-posttest results

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	Pretest scores of students & Posttest scores of students	35	,911	,000

"A paired samples t-test found this difference to be significant, $t(34)=-12.34$, $p=0.000$"

These results suggest that application of instructional sequence which was prepared for the current study in guidance of a conjectured HLT, had a positive effect on students' achievement of 3-D shapes. Specifically, it can be concluded that if this instructional sequence is applied to the students in terms of understanding 3-D shapes, their conceptual understanding increase.

4.6 Summary of Findings

By conducting a design-based research, an instructional sequence was prepared with guidance of a conjectured hypothetical learning trajectory designed for eighth grade students in solids. Through the analysis of eight graders' classroom mathematic discussions emerged during this instructional sequence, to evaluate their geometrical understanding on 3-D shapes (specifically surface area of prisms and cylinder and volume of cylinder), the classroom mathematical practices were obtained in terms of students' taken-as-shared ways of thinking and communicating by using mathematical language. Also, GeoGebra used to support students' understanding of geometrical concepts, especially those required visualization of the given shapes. To identify classroom mathematical practices that emerged during the instructional sequence, Krummheuer's (2015) model of argumentation (which was adopted from Toulmin's (1969) work) and three-phase methodology of Rasmussen and Stephan (2008) was used as an interpretive framework of the study. Classroom mathematical practices documented classroom collective learning activities mostly included whole class discussions. During the application of the instructional sequence and throughout the whole class discussions, there were four mathematical practices that emerged related to solids.

The first mathematical practices emerged in terms of finding definition and properties of prisms. It was obtained from four taken-as-shared ideas which supported the emergence of this mathematical practice. Those were (a) understanding roof of buildings' and tents' shape is prism, (b) understanding a cube is a prism (c) understanding the relationship between base shape and other parts of a prism. And (d) understanding a cylinder is not a prism.

Second mathematical practice emerged about finding surface area of prisms and it was supported by four taken-as-shared ideas that are; (a) understanding wrapping is means that drawing net of a prism, (b) counting unit squares, (c) transition from counting unit squares to calculating area (d) producing the formula for surface area of prisms.

The third mathematical practice emerged about finding surface area of cylinder and it emerged around three taken-as-shared ideas that; (a) structure of net of the cylinder, (b) relation between the circumference of the circle base and edge of its side face and (c) cylinder’s surface area constructed by area of side face and area of circle bases.

And the last mathematical practice emerged about finding volume of cylinder and it was supported by four taken-as-shared idea among classroom that; (a) volume is about third dimension, (b) volume is about filling inside of a shape, (c) calculation of volume requires the knowledge of width, length, and height and (d) volume equals to the multiplication of base area and height.

Additionally, a pretest and posttest were applied to the students at the beginning of the instruction and at the end of the study. The statistical analysis of the pre-posttest results implicated that the instruction had a positive effect on students’ achievement on 3-D shapes.

CHAPTER 5

DISCUSSION, CONCLUSION, AND IMPLICATIONS

The overall purpose of this study was to evaluate the mathematical practices emerged in an eighth-grade mathematics classroom. More clearly, the aim was to obtain the mathematical practices which emerged in the social and collective learning environment of the classroom and the way those practices occurred and how they became taken-as-shared while the instruction was on progress. To obtain those practices, an instructional sequence was prepared with the guidance of a conjectured HLT in the context of three-dimensional shapes. The instructional sequence was put into practice with the support of argumentative classroom environment, dynamic geometry software GeoGebra, and daily life-based content (prepared based on RME theory). By this way, the current study aimed to provide a view to the geometry lessons in terms of three-dimensional shapes concept and to enlighten the possible ways to enhance students' learning and conceptual understanding of this content. Moreover, the study was conducted as design-based research in a natural classroom setting to make the participants involved in that learning community (Cobb, 2000).

Four major mathematical practices were determined including mathematical ideas which supported the emergence of those practices. (a) finding definition and properties of prisms, (b) finding the surface area of prisms, (c) finding the surface area of the cylinder and (d) finding the volume of the cylinder, were the determined mathematical practices of the current study. These were determined as mathematical practices after the students started to use those practices in their solutions or in their explanations while involving in whole class discussions without having any challenges. To transform the collected data into the scientific explanations and to extract classroom mathematical practices, Krummheuer's (2015) argumentation model was applied which was adopted from Toulmin's model of argumentation. This model helped to clarify the conclusions, the data and warrants were provided by the students.

5.1 Discussion of Social and Socio-mathematical Norms

The current study used the emergent perspective as one of the interpretive frameworks, the analysis of the mathematical practices was formed by classroom activities and discussions and its effect on student learning were conducted through the emergent perspective (Cobb, 2000; Cobb & Yackel, 1996; Stephan & Cobb, 2003). Stephan (2003) states that the emergent perspective includes coordination of both social and individual perspectives on mathematics learning. Learning has a psychological side on the part of the individual learner and also has a social side on the part of the learning group or classroom environment (Stephan, 2003). Also, Cobb (2000) adds "A basic assumption of the emergent perspective is, therefore, that neither individual students' activities nor classroom mathematical practices can be accounted for adequately except in relation to the other" (Cobb, 2000, p. 310). The emergent perspective makes students learning mathematics placed in the social context of the classroom (Cobb, 2003).

For the current study, the important social norms were; giving examples, explaining, or justifying those examples or solutions with using appropriate language, constructing the conceptual understanding of solutions or specific concepts, and asking questions. These norms were valuable, and the participating teacher was the main instructor of the study who supported to increase their development (Yackel, 2001). For example, while the classroom was working on a question as an introduction to the surface area that was including a rectangular prism constructed by unit cubes, they involved in a whole class discussion about the related question. One of the students explained her way of thinking for the solution and another student accepted her solution and explained his way. Both of them did the same things but in a different order. Thus, a third student confirmed that the solution was accepted by them. Consistent with the nature of argumentation, the students accepted/justified another's idea and constructed their own explanations on others. This process was about the construction of social norm of the classroom that explaining and justifying one's solution and solution processes, and also making sense of other students' solutions (Yackel, 2001). Actually, the class started to understand that they were calculating the area of each face by doing those calculations that they referred to in a short way. They started to transfer their thinking from counting to the calculating which was a step for construction of mathematical practice about finding the surface area of prisms. This finding was also consistent with Vygotsky's idea advocated that the level of individual learning can be increased by interacting with the other people on the related issue. Thus, the knowledge gained through interaction with other people may be much more than the knowledge gained by working alone (Liang & Gabel, 2005). Thus, it can be concluded that construction of classroom social norms during an instructional process may enhance students learning and understanding through producing mathematical practices.

The classroom socio-mathematical norms refer to the specific criteria for mathematical solutions that may be different or unique and also, what may be an acceptable mathematical explanation and justification (Yackel, 2001). In the present study, the valuable socio-mathematical norms included the development of different solutions and making acceptable mathematical explanations. Moreover, the participating teacher supported the development of those socio-mathematical norms in the classroom context. For example, as an introduction to the surface area, the instructional sequence was prepared to include shapes constructed by unit cubes as a first step. In later questions, the shapes were not constructed by unit cubes. The aim was to make students understand the idea of one edge of a unit cube equals to the one unit of measurement (such as one centimeter). By arguing and exchanging ideas which were critical to forming social norm of the classroom, students also made appropriate and acceptable explanations for mathematics. For instance, they stated that instead of counting all the unit squares, they could multiply each edge with other like they do while calculating the area of a rectangle. Thus, this was mathematically acceptable and an appropriate solution for formation of socio-mathematical norms as stated in the literature (Cobb & Yackel, 1996; Yackel, 2001).

Additionally, as mentioned before, the social and individual aspects of the emergent perspective go parallel with a way that during the examination of social aspects, each student's individual learning has a contribution to the development of taken-as-shared mathematical ideas since they formed in classroom community (Cobb & Yackel, 1996). Social and socio-mathematical norms were proceeded to be established during the study process.

5.2 Discussion of HLT

Consistent with the tenets of a design-based study, another important aspect of the study was to implement and modify the proposed HLT and instructional sequence and prepare next iteration of HLT and instructional sequence according to the students' needs. It was important to obtain the ways that the conjecture HLT and instructional sequence facilitated the students' learning and conceptual understanding of three-dimensional shapes. According to the collective learning activities that took place in the classroom environment and related to students' needs that emerged while the instruction was in progress, the revisions were obtained. These changes were done, while the instruction was in progress. There were some points needed to be revised that emerged during the application of the HLT and instructional sequence. These changes and the differences between conjectured HLT and the actual HLT will be discussed in the following sections.

The research team conducted their meetings at the end of each week, and after a class session in which they encountered an unexpected situation about the HLT and instructional sequence. During those meetings, the teacher and the researcher discussed the learning goals of the prior week, the determined problems, or missing parts to handle, and the possible changes for future learning goals related to HLT. The decisions were made at these research team meetings and they helped to develop a new learning trajectory that students created and to correct the hypothetical learning trajectory and instructional sequence for the future implementation of the teachers' goal of developing a teaching theory for learning situations. The research team discussed the areas in which social and socio-mathematical norms were established, as well as conceptual developments that were present or absent. The changes in the sequence were made on a daily basis when required and weekly meetings conducted at the end of each week. Aligned with the prior research, planning an HLT was effective in teacher's organizing her teaching, establishing student-centered (Wilson, Sztajn, Edgington & Myers, 2015) and argumentative environment and making sense of students' thinking (Wilson, Mojica & Confrey, 2013) to make necessary changes in her teaching ways and in instructional sequence.

The argumentations improved students' conceptual knowledge and understanding of three-dimensional shapes (Güçler et al., 2013). For example, at the beginning of the instruction, while discussing on and defining prisms, some students provided inappropriate examples and asserted mathematically unacceptable definitions for prisms. By involving in argumentation process during the instruction, they identified the missing points and inappropriate examples about prisms that they provided, and they corrected themselves by expressing, commenting on, and justifying or refusing other's ideas with the support of the teacher's directions. When the whole learning process of students' in the context of three-dimensional shapes was considered throughout producing mathematical practices, the students improved their conceptual understanding of solids by participating in argumentations through instruction. In the literature, there were researches that were consistent with these findings (Fukawa-Connelly, & Silverman, 2015; Kosko, Rougee, & Herbst, 2014; Mueller et al., 2014). Additionally, the research supported the conducted whole class discussions including argumentation between members of classroom (Abi-El-Mona & Abd-El-Khalick, 2011; Duschl & Osborne, 2002; Zembaul-Saul, 2005), improved participants' way of commenting on other's ideas more scientifically (Flores, Park, & Bernhardt, 2016; Osborne, Erduran, & Simon, 2004); the way of justifying and/or refusing those ideas and conceptual understanding of related issue (Cramer, 2011; Driver, Newton, & Osborne, 2000; Jim´enez-Aleixandre et al., 2000; Jonassen & Kim, 2010; Wheeldon, 2008). By supporting the instructional process with argumentation in whole class discussions, the students' understanding of three-dimensional shapes was enhanced. Since, it was supported in the literature that geometric concepts could be learned by argumentation (Kosko, Rougee & Herbst, 2014; Wiley & Voss, 1999; Prusak et al. 2012), skills such as arguing, supporting, justifying, and proving could be improved relatedly (Asterhan & Schwarz, 2007; Sadler & Fowler, 2006).

Consistent with the nature of argumentation and with the requirements of a collective learning environment, also, students were free to express themselves while working individually, in peers or while involving in whole class discussions. When students felt confident during the instruction, this supported the emergence of new and different ideas that were mathematically appropriate and acceptable. Thus, aligned with the literature, this kind of approach might be particularly effective for promoting student thinking (Boaler, 2016; Fujita, Kondo, Kumakura, & Kunimune, 2017; Yackel, et al, 1991).

Additionally, the teacher's role was important in terms of conducting classroom argumentations and enhancing students' participation in the classroom activities. During the study, the participant teacher tried to establish norms of mathematical argumentation by listening to students, encouraging students to provide claims and justifications, considering different ideas and arguments of others' (Kosko et al., 2014). Moreover, she started and directed classroom argumentations in a way of constructing mathematical practices in the related context. These activities of the teacher were consistent with the prior research (Conner et al.,2014; Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Mueller et al., 2014) in terms of her facilitating argumentative classroom environment and maintaining it. Additionally, prior research showed teachers' role in establishing argumentation in the mathematics classroom and how to facilitate argumentation (Asterhan & Schwarz, 2016). Consistent with Wood, Williams, and McNeal (2006), the current study implicated the importance of teachers' practice was to give students opportunities to share their mathematical thinking in the classroom.

The use of GeoGebra as an instructional tool supported students' conceptual understanding of three-dimensional shapes. During the study, students had the chance of visualizing the shapes from different views. This observation about the practices of shapes with the support of GeoGebra, provided students to catch the missing points that they could not get on paper and pencil environment, and by this way, they could produce ideas related to given context which would become taken-as-shared to produce classroom mathematical practices. For example, while the classroom was working on producing the formula for the volume of the cylinder, they were talking about the idea of multiplication of base area and height that came from the volume of prisms. They were discussing that whether it was possible to apply the same formula for the volume of the cylinder. So, to check and confirm the appropriateness of that idea, the researcher opened a GeoGebra file which was illustrating how to fill a cylinder. By observing that illustration, the students confirmed the idea of multiplication of base area and height would give the volume of the cylinder. Thus, they got the support of GeoGebra file in terms of producing the idea and relatedly mathematical practice of finding the volume of the cylinder. Thus, it is clear that usage of dynamic geometry software enhanced students' geometrical thinking and supported the emergence of mathematical ideas that constructed emergence of classroom mathematical practices (Pei, Weintrop & Wilensky, 2018). Moreover, consistent with the literature, usage of DGS made learning of geometry much richer and more powerful rather than paper-pencil method (Battista, 2007); gave chance students to explain and justify their thinking and reasoning which supports classroom mathematical practices (Wilson & Hoyles, 2017); and by this way it affected students' geometric and spatial thinking in positive way which provided an increase in their achievement at the same time (Ng & Sinclair, 2015b; Owens & Highfield, 2015; Sinclair & Moss, 2012). Furthermore, use of GeoGebra in lessons supported students' participation in the classroom activities. During each lesson, whole class discussions were conducted for a short or long time. Students were expressing their ideas in related context, justifying their solutions or refuting others' thinking. The use of GeoGebra allowed students to support their ideas as verbally expressing themselves and to feel more confident in this process. Thus, it was consistent with the prior research that use of DGS enhanced classroom mathematical argumentations by providing students visual proof for their ideas (Ng, 2015).

The context of the study was about the three-dimensional shapes and specifically, it was about the surface area of prisms, the surface area of cylinder and volume of the cylinder. The study was formed according to argumentations, but there were some errors or misunderstandings among students about definitions, properties of the prisms or cylinder. For example, they provided inappropriate shapes as examples for prisms such as pencil cases, or they could not understand the shape of a tent was a prism. Another example was a misconception about the orientation of the shape. It was a research that evaluated students' errors (Marchis, 2012). Also, studies state that students' prior experiences and knowledge construct their concept image about the related geometrical shape (Vinner & Hershkowitz, 1980). Students may have problems in recognizing different geometrical shapes if they are in a non-standard orientation (Marchis, 2012). For instance, as a most common misconception, a square is not a square if its base is not horizontal (Clements & Battista, 1992; Mayberry, 1983). Many students have difficulties with classifications of shapes (Feza & Webb, 2005; Mayberry, 1983). For example, a square is not rectangle (Marchis, 2008), a rectangle is not the parallelogram, and a square is not rhombus (Clements & Battista, 1992). Also, students may have problems with understanding of solids and relatedly they cannot draw nets of those solid shapes (Pittalis, Mousoulides & Christou, 2010).

The last version of the instructional sequence included some changes according to students' needs in order of context, tools, possible discourse between participants with the decisions of the research team that was constructed by the participant teacher and the researcher. To provide an effective instruction, the research team concluded to change the order of some content. For example, in the applied instructional sequence, the different views of the prisms were coming after drawing wrappers for them. According to the students' discourse, their places were decided to be changed. Also, there were not GeoGebra files ready for students' visualization and students stated their need to visualize that shapes from different views. In this respect, the research team decided to prepare and add to the next version of the instructional sequence. During the process, there were some errors of students related to their previous knowledge or lack of their visualization. In this respect, when the teacher and the researcher realized them, they directed the flow of the discussion in that way to make students to understand those errors themselves. In this way, new knowledge was constructed by correcting prior ideas, by putting on others' ideas under the guidance of the teacher and the researcher. For example, properties of prisms were understood in this way. So, it is important for researchers and teachers to realize students' errors and misunderstandings and correct them throughout and effective argumentative social learning environment (Gökkurt, Şahin, Soylu & Doğan, 2015). Students' errors may be used as a basis for construction of a new knowledge in a most effective way.

Thus, according to the findings which illustrated the emergence of taken-as-shared ideas and relatedly emergence of mathematical practices, the participant students improved their conceptual understanding of three-dimensional solids. This was supported by argumentations about related context and usage of GeoGebra as an instructional tool during the process. The results of this study can provide suggestions from the perspective of the study's content, which can help students to get a meaningful and conceptual understanding of three-dimensional solids.

5.3 Discussion of Classroom Mathematical Practices

By conducting a design-based research, an instructional sequence was prepared with the guidance of a conjectured hypothetical learning trajectory designed about three-dimensional shapes that eighth-grade students performed. Through the analysis of eight graders' classroom mathematics discussions emerged during this instructional sequence, to evaluate their geometrical understanding on 3-D shapes (specifically the surface area of prisms and cylinder and volume of the cylinder), the classroom mathematical practices were obtained in terms of students' taken-as-shared ways of thinking and communicating by using mathematical language. The classroom mathematical practices are defined as the content-specific mathematical ideas, the time when they become taken-as-shared for the classroom community (Cobb & Yackel, 1996; Yackel, 2001). In the current study, the classroom mathematical practices were established throughout the implementation of the instructional sequence in the context of three-dimensional solids. In this respect, the current study obtained four mathematical practices occurred during the process which were supported by this HLT and instructional sequence were (a) finding definition and properties of prisms, (b) finding surface area of prisms, (c) finding surface area of cylinder and (d) finding volume of cylinder. Additionally, it was explained that what kind of mathematical ideas made students to produce those mathematical practices. More clearly, the taken-as-shared ideas that supported the related mathematical practices were explained.

The first mathematical practices emerged in terms of finding definition and properties of prisms. It was obtained four taken-as-shared ideas which supported the emergence of this mathematical practice. Those were (a) understanding roof of buildings' and tents' shape is the prism, (b) understanding a cube is a prism (c) understanding the relationship between base shape and other parts of a prism and (d) understanding a cylinder is not a prism. This phase included two interrelated parts. The first part was related to the understanding of the construction of prisms and determining its basic elements and the second part was related to displaying the surface nets of prisms. In the conjectured HLT, learning objectives of the first phase were determining of basic elements of prisms and understanding nets of prisms. The mathematical ideas about the roof of buildings' and tents' shape is a prism, and the second idea was cube is a prism emerged during the first part of this phase of the HLT which was related to the understanding of the construction of prisms and determining its basic elements. While these two ideas emerged at the first week of the instruction, they were used from the beginning of that time to the end of the prisms, since they were the main knowledge about the context. During the process, the students discussed daily life examples of prisms, they asserted ideas related to features to be or not to be a prism, and by this way, they produced the definition of prisms and got the understanding of main elements and features of prisms. To produce a definition for prisms and understanding of other content, the whole class discussion including argumentation was effective on students' thinking. This was a finding consistent with the prior literature that mathematical argumentations enhanced and supported their knowledge about the definitions of prisms (De Villiers, Govender, & Patterson, 2009; Tsamir, Tirosh, Levenson, Barkai, & Tabach, 2014).

The second part of the first phase was about nets of the prisms and learning goal of the conjectured HLT was understanding nets of prisms. During this section, the third idea was, understanding the relationship between base shape and other parts of a prism, and understanding a cylinder is not a prism, and the fourth idea was, understanding a cylinder is not a prism was emerged related to this context. The aim of these activity sheets was to create a basis for the opening of prisms by working with the views of prisms in different ways. This part of the HLT was prepared under the concept of “candy wrapping company”. Each shape was designed with unit squares on it to make students understand those unit squares are the same as the length of the shape. In this second part of the HLT, each activity was assisted by a GeoGebra file and after working each question individually or in groups, classroom check was done on the GeoGebra file to make student construct the conceptual understanding of the context. Also, argumentations supported the construction of mathematical ideas through the emergence of mathematical practices. For example, they studied different views of given shapes which were constructed by unit cubes. The students tried to draw wrappers for those shapes which were actually about drawing their nets. At first, students worked individually, and in pairs, and then they checked the drawings on the GeoGebra files by argumentations. By this way, the students could visualize the shapes from different views which helped them to develop their three-dimensional thinking through drawing and understanding nets of prisms. By checking their drawings on the GeoGebra file, the students had a chance to control those solutions dynamically that could not be provided on paper and pen environment. Moreover, by reasoning on this second part which was prepared based on the first part, the students got the understanding of the definition, basic elements of prisms and their differences between other three-dimensional shapes. This was also important for the next stage which was about the surface area of prisms since the students needed to use the knowledge they got from this part in the following subject. Use of DGS and argumentations together was an issue of various researches in the literature (Hollebrands, Conner & Smith, 2010; Lavy, 2006; Prusak et al., 2012; Vincent, Chick & McCrae, 2005). Consistent with the prior research, the results of the study supported the use of DGS and argumentations together in geometry classes which enhance students’ geometrical thinking (Granberg & Olsson, 2015) and relatedly emergence of mathematical practices.

Second mathematical practice obtained from the study was finding the surface area of prisms. This practice emerged through the support of understanding that wrapping means drawing the net of a prism, counting unit squares, the transition from counting unit squares to calculating area and producing the formula for surface area of prisms. For the second phase of the HLT was prepared related to the learning objective of constructing the formula of the surface area of perpendicular prisms. Consistent with the proposed HLT, students produced mathematical ideas by putting one another through finding the formula for surface area. In this section, the classroom passed from views of prisms to the way of wrapping them which actually meant to be surface area. They first tried to wrap the given that was prepared with unit cubes. By discussing on the context, the students understood how to transfer the knowledge of unit squares on the measurement. Usage of unit cubes was beneficial for students to relate the unit squares of surface area. They used as an example tiling the ground of their classroom to get the understanding of the surface area. In the literature, there were examples that offered usage of unit squares and shapes constructed by unit cubes to teach the main idea of surface area (Ben-Chaim, Lappan, & Houang, 1985; Bonotto, 2003; Clements, 2003). Thus, in the light of literature and results of the current study, it can be concluded that usage of unit squares is beneficial in the teaching area. In addition to the example of tiling ground of the classroom surface, students tried to wrap a unit cube with a piece of paper to see a concrete experiment of wrapping. Both those examples were appropriate for nature of RME that was one of the theories underlying the instruction of the current study. The instructional sequence of the study was prepared aligned with the requirements of the RME theory. The questions or examples were chosen from real-life examples as much as possible. Thus, students' giving examples from daily life to construct the idea of surface area was an important finding in their learning. Usage of daily life-based examples in mathematics lessons was stressed in literature before (Bonotto, 2003; Van den Heuvel-Panhuizen, & Drijvers, 2014). Thus, it can be concluded that usage of daily life-based examples in geometry lessons are effective on students' conceptual understanding and learning of the related context.

The third mathematical practice was about finding the surface area of the cylinder. It emerged around three taken-as-shared ideas that; the structure of net of the cylinder, relation between the circumference of the circle base and edge of its side face and cylinder’s surface area constructed by the area of side face and area of circle bases. This practice emerged during the third and fourth phase of the proposed HLT. The third phase was planned related to the learning objective of determining the basic elements of the cylinder, constructing, and drawing the net of it, and the fourth phase was prepared related to learning objective of constructing the knowledge of the surface area of the cylinder. At the beginning of the section, the whole classroom discussion was conducted based on daily life examples of cylinder consistent with the RME theory. After, the activity sheet was asking the students to draw a wrapper for a cylinder-shaped candy which was actually net of the cylinder. At that example, nearly whole class successfully drew an appropriate drawing for the question. This would be related to their knowledge from previous years. Then, the content was based on understanding the relationship between elements of the cylinder. To make students understand that relation, GeoGebra was supportive again for them. For instance, to solve the problem about the shape of the side face of a cylinder, one student explained that when someone wanted to wrap something around these circles, there was a need for two points that come together. Two points for the top base and two points for bottom base and four points in total. Thus, the student concluded the side face of a cylinder should be a quadrilateral. GeoGebra was very effective to show students about the requirement of four points. By involving in a whole class discussion, they produced the idea of equality between the circumference of the circle base and edge of its side face by this way. Moreover, by producing this idea, they got a step into the surface area of the cylinder. As a result of the work done up to that time, students understand the idea of surface area was related to the net of the given shape. So, they were aware of calculation of surface area was related to its net. After producing the previous idea about equality of circumference of circle base and length of side base, it was time to produce the idea for calculation of the area of two circle bases and side base. Argumentations about this context made students produce the idea of calculation for the surface area of the cylinder. This finding was consistent with the prior research (Aktümen, Baltacı, & Yıldız, 2011; Hohenwarter, & Jones, 2007), in terms of usage of DGS in lessons and strengthen the instruction with real-life examples and questions. Thus, it can be concluded that argumentations, usage of DGS and supporting the instruction with real-life context are effective in the understanding surface area of the cylinder (Lai & White, 2014). Furthermore, while producing the formula for surface area of the cylinder, the classroom involved in a discussion based on the way to express the way of area calculation in algebraic expressions. Also, in GeoGebra file, students observed the formula of surface area of the cylinder. Additionally, they could observe changes in given length clearly in both dynamically and algebraically. There was a research in the literature that supported usage of DGS to enhance the understanding of the relationship between geometry and algebra (Atiyah, 2001; Davis, 1998; Edwards & Jones, 2006). Thus, usage of GeoGebra was effective in the understanding of algebraic expressions of formula (Erbas, Ledford, Orrill, & Polly, 2005) for the surface area of prisms and cylinder.

The fourth mathematical practice was about finding the volume of the cylinder. It was supported by four taken-as-shared ideas among classroom that; volume is about the third dimension, the volume is about filling inside of a shape, calculation of volume requires the knowledge of width, length, and height and volume equal to the multiplication of base area and height. This practice emerged during the last week of the instruction. The practice occurred parallel with the last phase of the HLT that was based on the learning objective of constructing the knowledge of the volume of the cylinder. The main process was started with the question "what volume is?". While this question was asked, another discussion emerged related to the differences between area and volume. To explain and understand these differences, the examples were provided by students about tiling ground of classroom and filling the classroom with unit cubes. With this discussion, the students understand the meaning of area and volume clearly. Also, another discussion task is about "how can the volume of the cube and rectangular prism be calculated and what element do we need for those operations?". This process was conjectured to call back the students' knowledge that when they calculate how many of the unit cubes actually fill the inner zone when they find the volume, but while doing this calculation instead of counting the whole cubes, they multiplied the three dimensions of the prisms with each other. These steps were also offered in the previous research teaching volume by using unit cubes and teaching area with using unit squares (Battista & Clements, 1996, 1998; Ben-Chaim, Lappan, & Houang, 1985; Cohen, Moreh & Chayoth, 1999; Fujita, Kondo, Kumakura, & Kunimune, 2017). Additionally, other critical questions were "how they can fill a cylinder with unit cubes since it does not have edges?" and "how they can find the volume of the cylinder?" The expected argumentation during the whole class discussion was about the usage of circle segments instead of using unit cubes to fill the cylinder. To make the issue clearer GeoGebra file opened that was prepared to illustrate filling of a cylinder shape. By observing the illustration of the cylinder, the students were able to figure out the volume of the cylinder by understanding how they could fill the cylinder by placing the circles by putting one on another one (that is the height of the cylinder). Additionally, students transferred the knowledge of the volume of the cube and rectangular prism which can be formulated as "multiplication of base area and height". Involving in the discussion about the volume of the cylinder by filling it with circle segments and by relating the context to the volume of the cube and rectangular prism, the last conclusion was made on that the volume of the cylinder is "multiplication of base area and height" under the guidance of whole class discussions. Also, filling of the inner zone of the shape was an appropriate example for daily life context. Moreover, there were researches that supported these findings (Enochs, & Gabel, 1984; Hirstein, 1981; Livne, 1996). In this respect, it can be stated that usage of DGS, argumentations and daily life examples are effective to teach the volume of the cylinder.

The aim of the current study aimed to evaluate the classroom mathematical practices emerged during an instructional sequence that directed by a conjecture HLT. The learning environment supported by GeoGebra file as instructional tools, argumentations in whole class discussions and daily life examples that consistent with the requirements of RME theory. According to the findings of the study, it can be stated that the participant students could involve in the instructional activities by reasoning, justifying, and commenting on other's ideas and produce new ideas by constructing on other ideas. By this way, they could develop a conceptual and meaningful understanding (Hallowell, Okamoto, Romo, & La Joy, 2015) of three-dimensional shapes (specifically for prisms and cylinder for this study). Also, results of the study supported that students' reasoning on the related issue can be improved with the support of DGS and argumentative classroom environment. The mathematical practices of this study can open a window for other researchers who want to study about surface area and/or volume of three-dimensional shapes in a similar learning environment.

5.4. Conclusion and Implications

The current study was conducted to make some contributions to the literature about eight grader's understanding of three-dimensional shapes and what kind of tools can be enhanced of this understanding. The study was conducted by using an instructional sequence with the guide of an HLT and with the support of argumentations, DGS, and daily life examples. This instruction can be used in any school while teaching eight graders three-dimensional solid shapes. Students' both correct and incorrect thinking ways that emerged during the study were obtained. Moreover, the solutions were explained clearly to handle their errors and wrong thinking. This can be helpful for teachers and teacher educators to have an idea about the reactions and thinking styles of their students about the content that they will teach.

To evaluate students' understanding and learning an instructional sequence was prepared with the guidance of an HLT. This context was applied to the students during four and half weeks by providing an argumentative collective learning environment that was supported by DGS and daily life examples. To evaluate students' understanding and learning about three-dimensional solid shapes, mathematical practices were determined including mathematical ideas that supported the emergence of those mathematical practices. Thus, the process brought some revisions according to students' needs. Those revisions were made in HLT and instructional sequence and were explained in the study. By considering those first and last versions of the HLT and instructional sequence, and also generalizing them according to their conditions and culture, teachers and researchers can design their research to evaluate participants' understanding about the related issue.

Use of argumentations are effective in students understanding of geometry and specifically in solids (Hollebrands, Conner & Smith, 2010; Lavy, 2006; Prusak et al., 2012; Vincent, Chick & McCrae, 2005). In the current study, the students learned the conceptual understanding of the surface area and volume of the three-dimensional shapes through argumentations (Latsi & Kynigos, 2012) by sharing ideas, justifying, commenting on other's ideas, or refusing them. When these positive effects of argumentations on students learning are considered, it can be used by teachers while designing lesson plans for geometry lessons. While considering this argumentative environment, it is important to guide those whole class discussions according to the aim of instruction. Thus, the role of the teacher is critical as an orchestrator of the flow of the discussion in terms of underlining important points, determining misconceptions or errors of students, and changing the direction accordingly. By this way, the teacher is also responsible for the construction of students understanding and learning of the related context. In this respect, the teacher's knowledge and role as an orchestrator are important (Yackel, 2002).

DGS is an effective instructional tool for teaching and learning of geometry (Agyei, & Benning, 2015; Pittalis, Christou, & Pitta-Pantazi, 2012). In the current study, the students did not use GeoGebra software individually, instead, they observed the ready files on the smartboard. Thus, another study can be conducted by providing opportunities to the students to use GeoGebra or any other dynamic geometry software individually, and in this way evaluate their learning and understanding. Moreover, an argumentative classroom environment can be added to that kind of study, and their effect can be evaluated together. Furthermore, students' mathematical practices can be determined while they use DGS by themselves.

This study used the emergent perspective as a framework that includes three dimensions as social, socio-mathematical norms and classroom mathematical practices. This study made a detailed analysis of mathematical practices, other dimensions were not the focus of the study. Thus, a research can be conducted to evaluate those dimensions of social and socio-mathematical norms in detail (Andreasen, 2006; Roy, 2008). By this way, a complete viewpoint may be provided for students' learning and understanding of three-dimensional shapes with the application of instructional sequence. Moreover, RME was one of the theories that underline the examples of the study but there were questions related to traditional techniques. It is possible to conduct a study based on RME theory that includes all content of the instructional sequence. Usage of DGS and argumentations can be adopted in that kind of study and by this way, students understanding of 3-D shapes can be evaluated and their mathematical practices can be obtained.

The findings of this study emerged from the setting in which this study was carried out. The study conducted in a public school in Turkey. Thus, it can be considered to be applicable for similar conditions. Some implications can be offered for teachers. The current study developed and tried an instructional sequence under the guidance of a conjectured HLT. Some changes were made in the content regarding the students' needs and their learning. The content can be applied in any eighth-grade classroom by doing appropriate changes according to the conditions. The mathematics teachers can use the instructional sequence and design their lessons accordingly. They can add any other instructional tools except for DGS and argumentations. The participating classroom included 35 students and argumentations could be constructed during the flow of the lessons. Thus, the crowd of the classroom was not an obstacle for usage of argumentations. Reversely, argumentations can make students involved in classroom activities more actively. When the teacher provides opportunities for students to express their ideas freely, students will have a chance to share their ideas in the classroom environment which construct a meaningful learning. Moreover, use of GeoGebra as an instructional tool can make them more interested in the context. In the current study, students did not use the GeoGebra individually, but only observed ready files of the smartboard. But even this made students give more interest lessons and better understanding. If the teachers have a chance to use a computer lab in their geometry lessons, they can use GeoGebra as a main instructional tool of the instruction.

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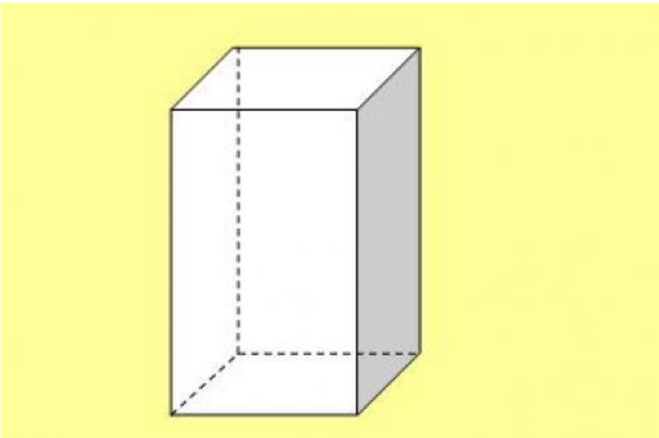
APPENDICES

A: INSTRUCTIONAL SEQUENCE

SURFACE AREA AND VOLUME OF SOLIDS

1. Name the following prism and determine the basic elements.



2.

Fill in the blanks below.

Geometrik cisim	Yüz sayısı	Köşe sayısı	Ayrıt sayısı	Tabanın benzediği çokgensel bölge
Küp				
Kare dik prizma				
Dikdörtgenler prizması				
Üçgen dik prizma				
Beşgen dik prizma				

3.

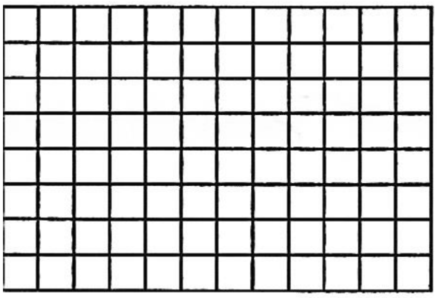
Write the appropriate name for each solid given in opened form.

Geometrik cismin adı	Çokgensel bölgeler
....	
....	
....	
....	

The “Cube – ilicious” Candy Company:

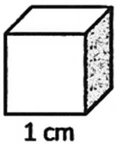
You work for the Cube-ilicious Candy Company, a candy company that packages all their candy in the shape of a cube. Cube-ilicious is ready to introduce a new Caramel Cube, and your department is in charge of wrapping the individual pieces of the candy. After much searching you find a company called “Square Paper Company” that supplies wrapping paper that is made up from individual unit squares of sizes of centimeters.

Wrapping Paper 1TL



The Square Paper Company charges you 1 TL for one sheet of their “square” wrapping paper!

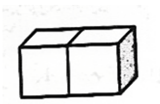
Using the squared paper, you received, draw the wrapper paper you can use to cover the following single piece caramel candy. If you want to check, you can check it by cutting it with scissors after drawing.

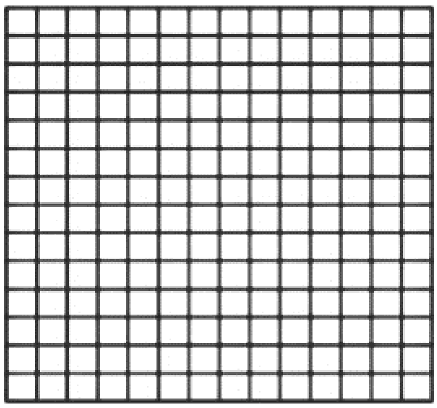


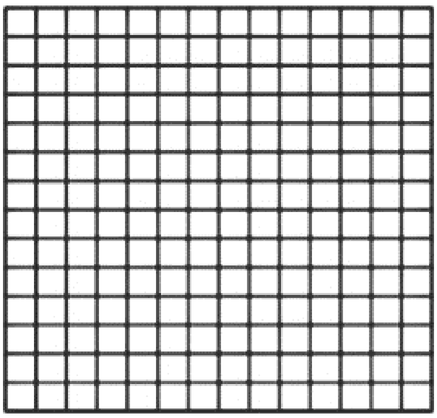
Your Wrappers

What is the cost of packaging the following sugars? Draw your own packaging design. If you want to build candies, you can ask for cubes from your teacher. Let's check in GeoGebra.

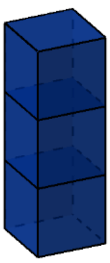
A. Two caramel candies



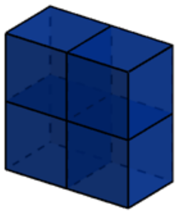


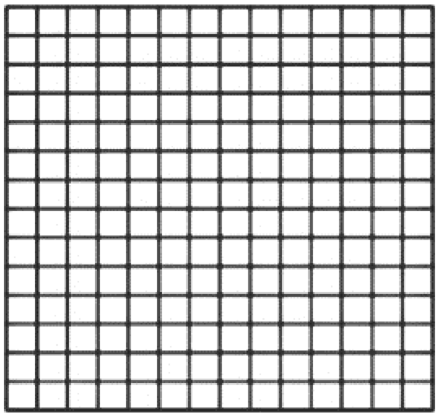


B. Three caramel candies



C. Four caramel candies

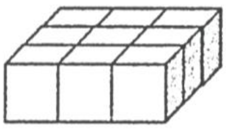


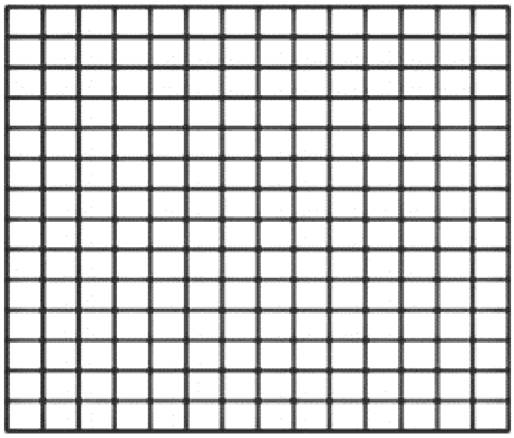


Let’s draw

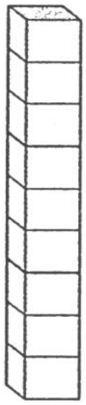
For each of the following candy packs, draw the wrapping paper covering the FRONT, BACK, RIGHT, LEFT, BOTTOM and TOP sides of the squares on the sides. Let's check in GeoGebra.

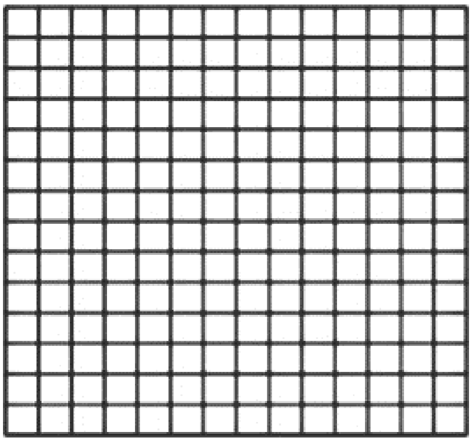
1.



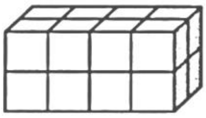


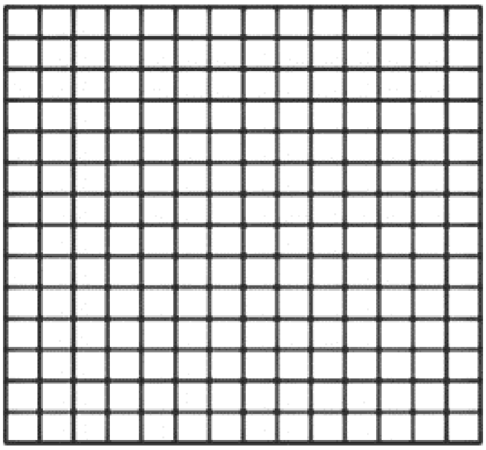
2.





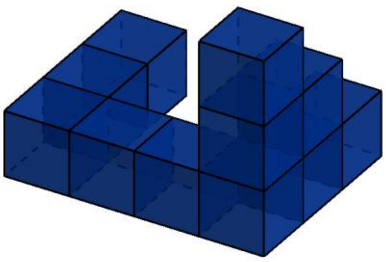
3.





Look at this!

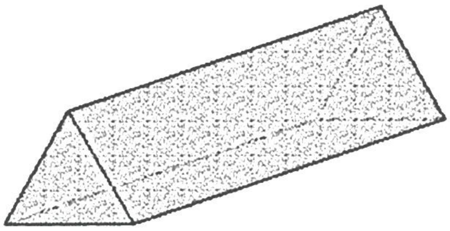
Draw the following shapes from the front, back, right, left, bottom and top. Let's check in GeoGebra.

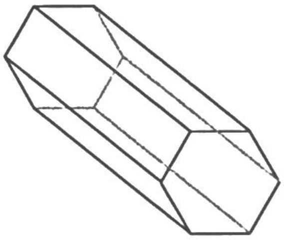


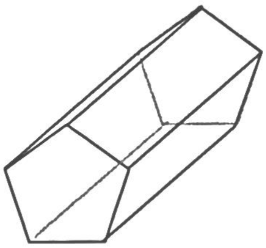
PACKING TIME

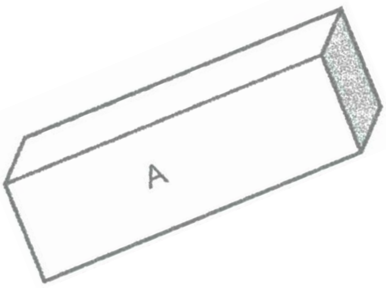
These candies are very cool!

Our cube sugar factory is now renewing itself and producing candies in different shapes. We have only one problem. The wrapping paper in this factory is for cube candy. We need to design new wrapping paper. Do you have new packaging papers for the following candies? Let's check in Geogebra.

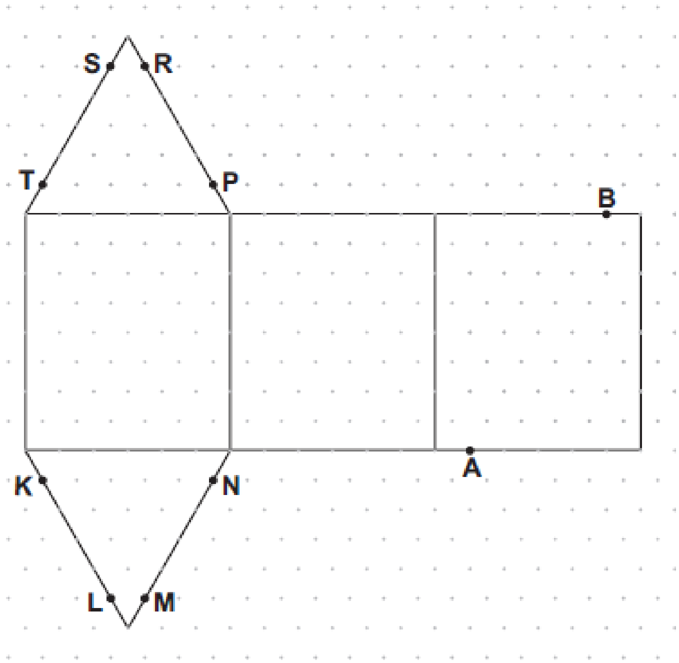




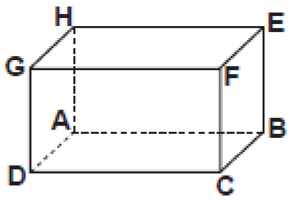


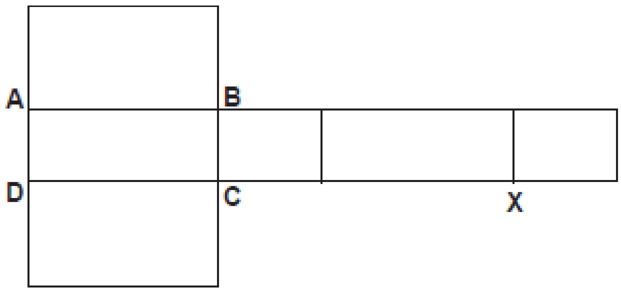


Find it!



If the given shape is closed as a triangular prism, which points point A and B match with?

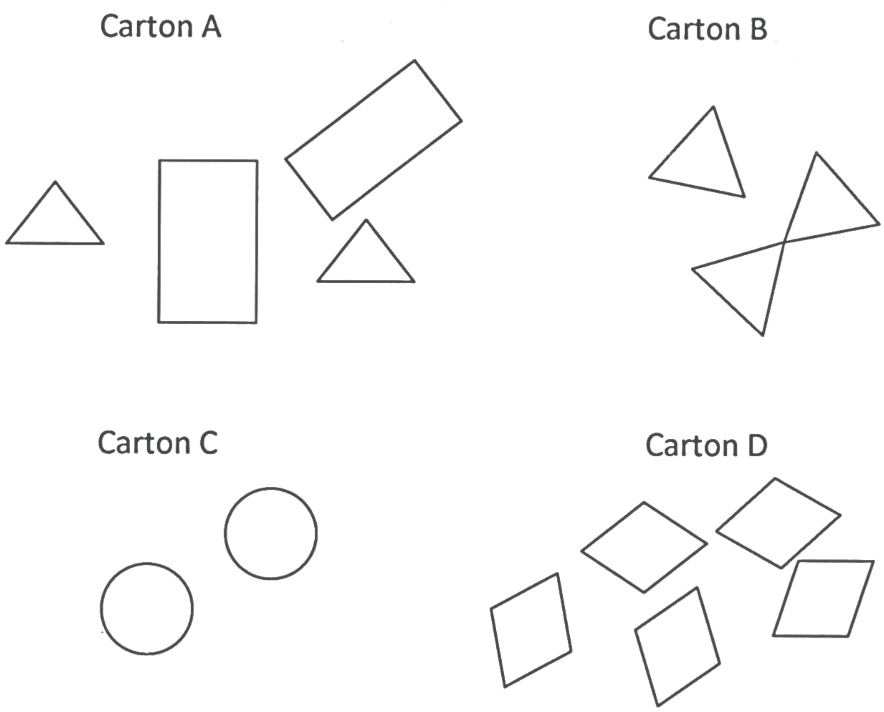




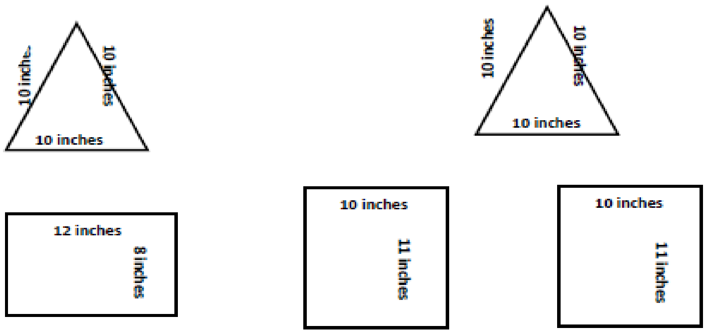
Which point, the point X given in the net of the prism match with in the closed form?

Lost Wrappers

Deniz, one of the employees at Cube-i-licious was making wrappers for these new candy cartons. The wrapper parts were on her desk, but when she went to lunch, Derya, the practical joker at Cube-i-licious, stole one of the wrapper parts from each candy carton. Can you figure out which wrapper part is missing from each carton below?



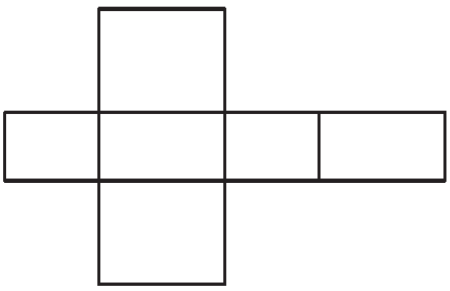
Sam and Sue's Dilemma

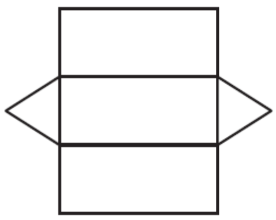


Sam said that these shapes could be used to wrap a triangular prism. Sue said that they could not. Who do you agree with and why?

Write the name of prisms given the nets below.

Aşağıda açınımları verilen geometrik cisimlerin adlarını altlarına yazınız.





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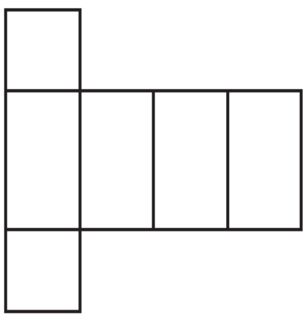

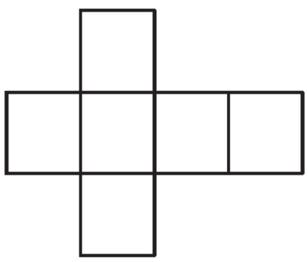

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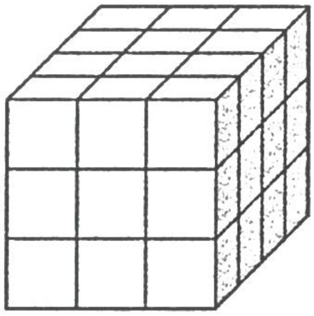
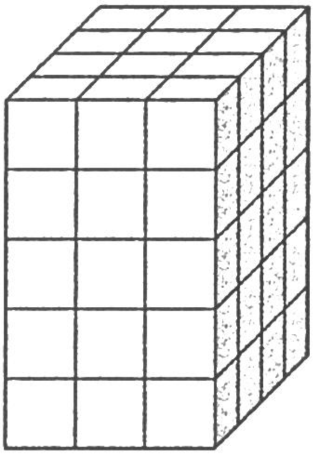
Surface Area

To package the following shapes, find out how many pieces of square-unit packages should be used. Let's prove our answer.

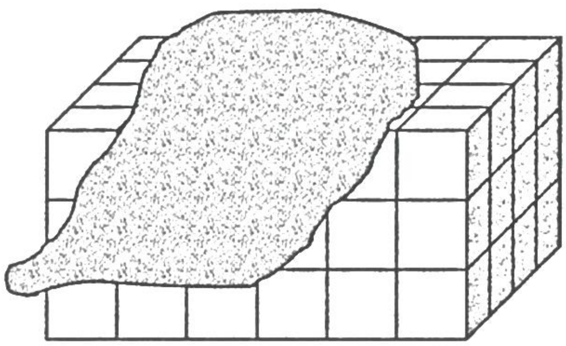
1.



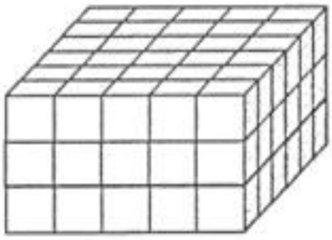
2.

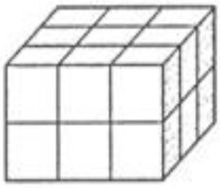


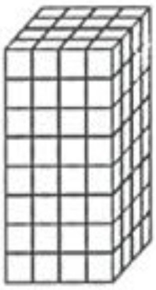
3.

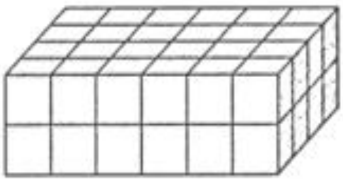


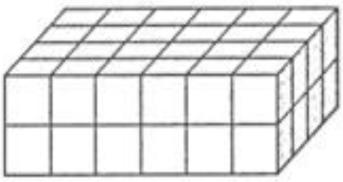
Calculate surface area of each shape below. Prove your answers.

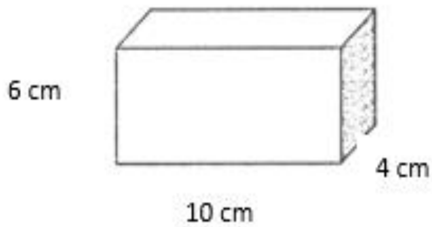






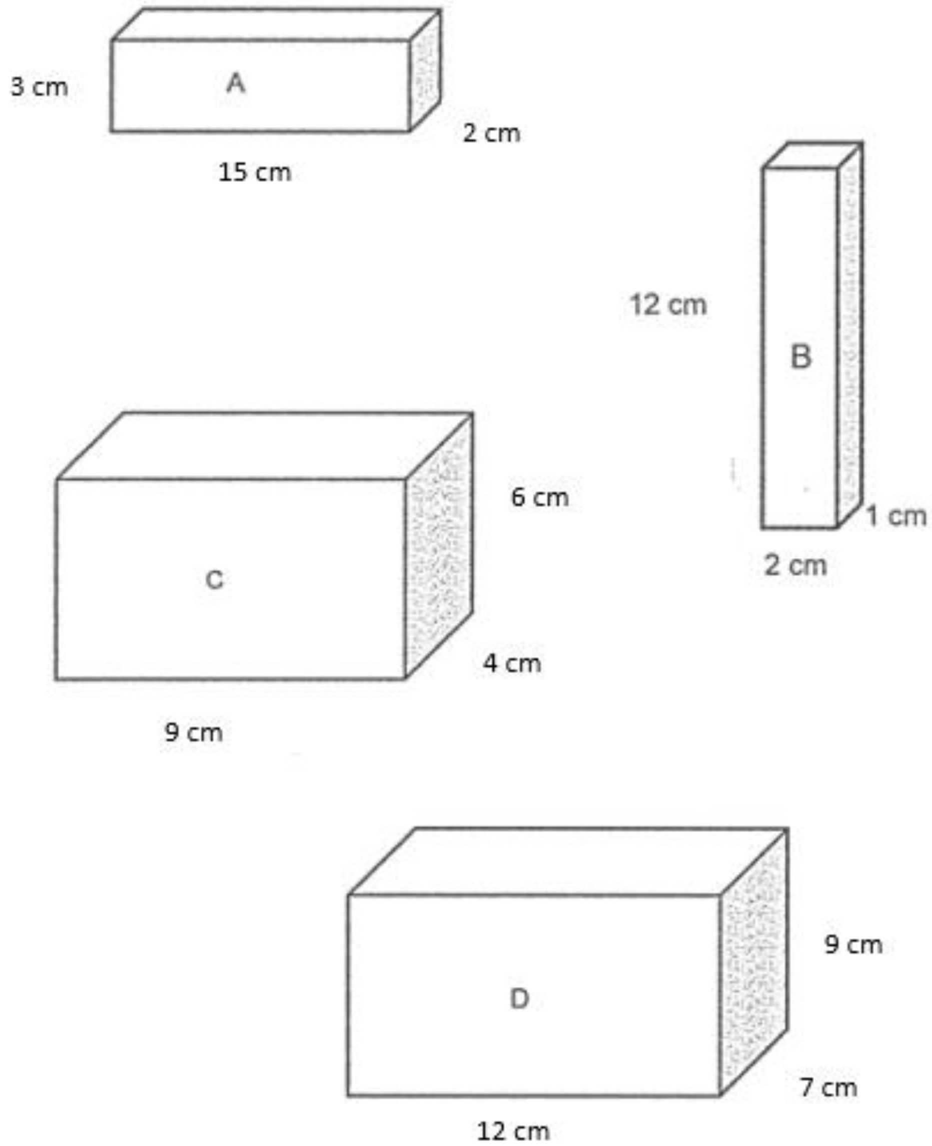




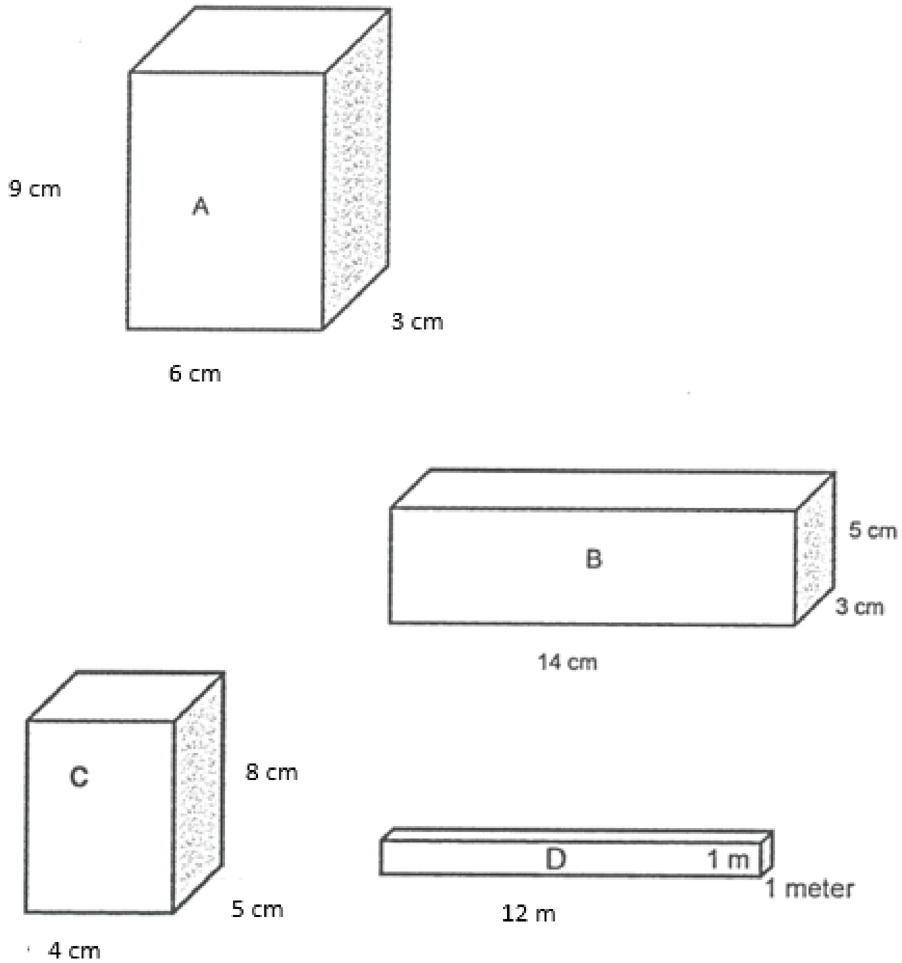


No unit squares

Calculate surface area of each shape below. Prove your answer.



Calculate surface area of each shape below. Prove your answer.



Fantastic Formulas

Have you discovered your own formula for surface area of rectangular prisms? Write down all your classmate's formulas below and decide which formulas are valid. Include the formulas that Mr. Klaus' class discovered.

Student formula:

Student formula:

Student formula:

Student formula:

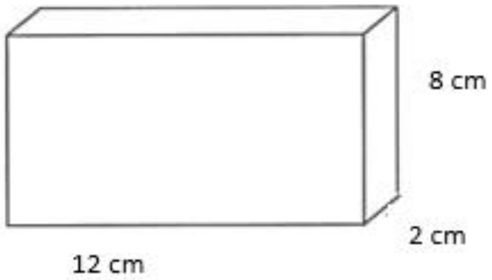
Polly's formula: 2bh + 2bw + 2wh where b stands for the length of the base, h the height, and w the width

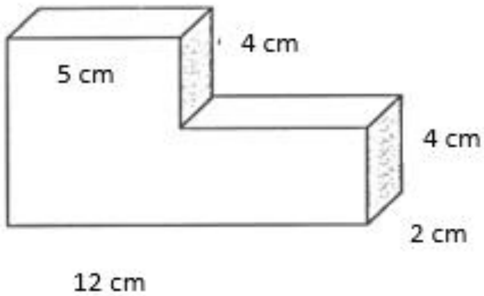
Richard's formula: 6bh where b stands for the length of the base, h the height

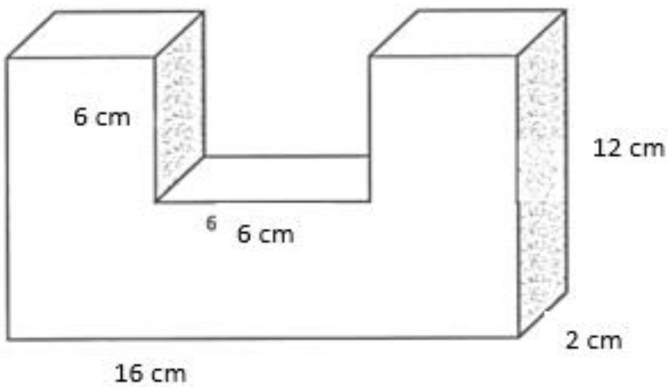
Carla's formula: BF + BT + BR where BF stands for the Area of the Front Face, BT stands for the Area of the Top Base and BR stands for the Area of the Right Face.

Result:

Find the surface area of each shape

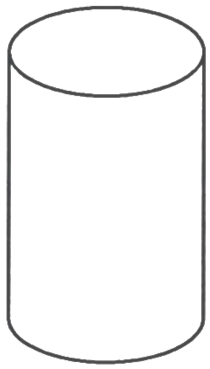




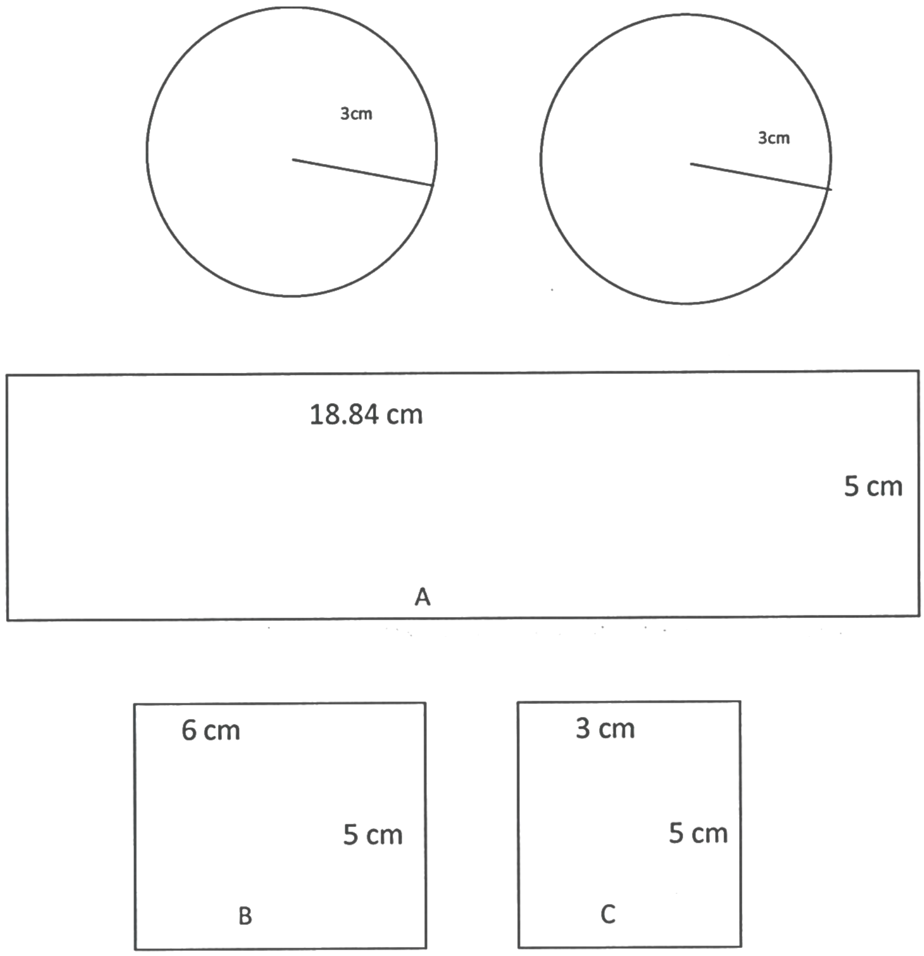


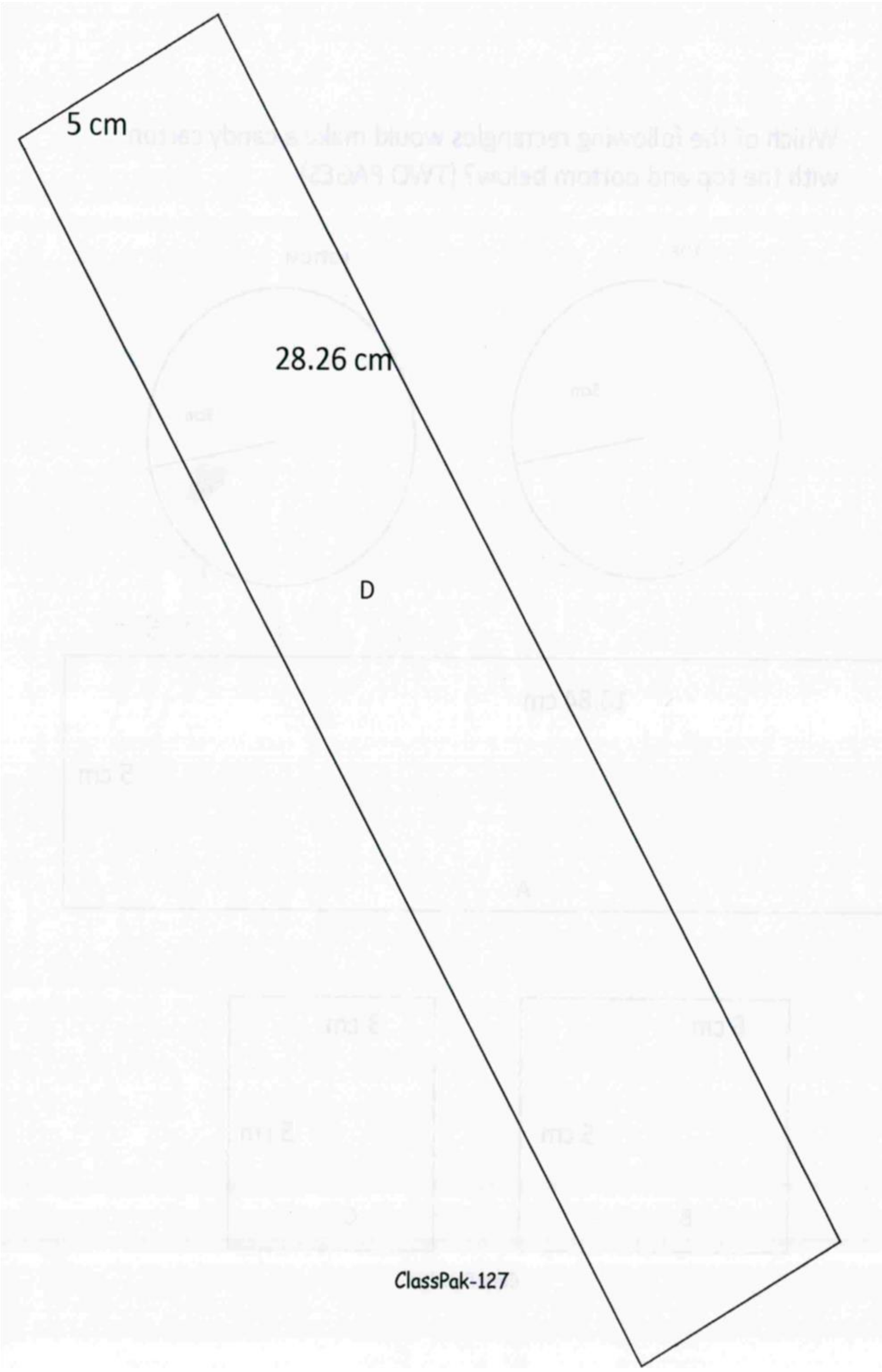
Net of cylinder and surface area

Below is a cylindrical candy box. You design a wrapping paper to cover this candy box. All parts of this box need a drawing for the design paper. Let's make a drawing of the packaging paper you designed. Let's check our results at Geogebra.



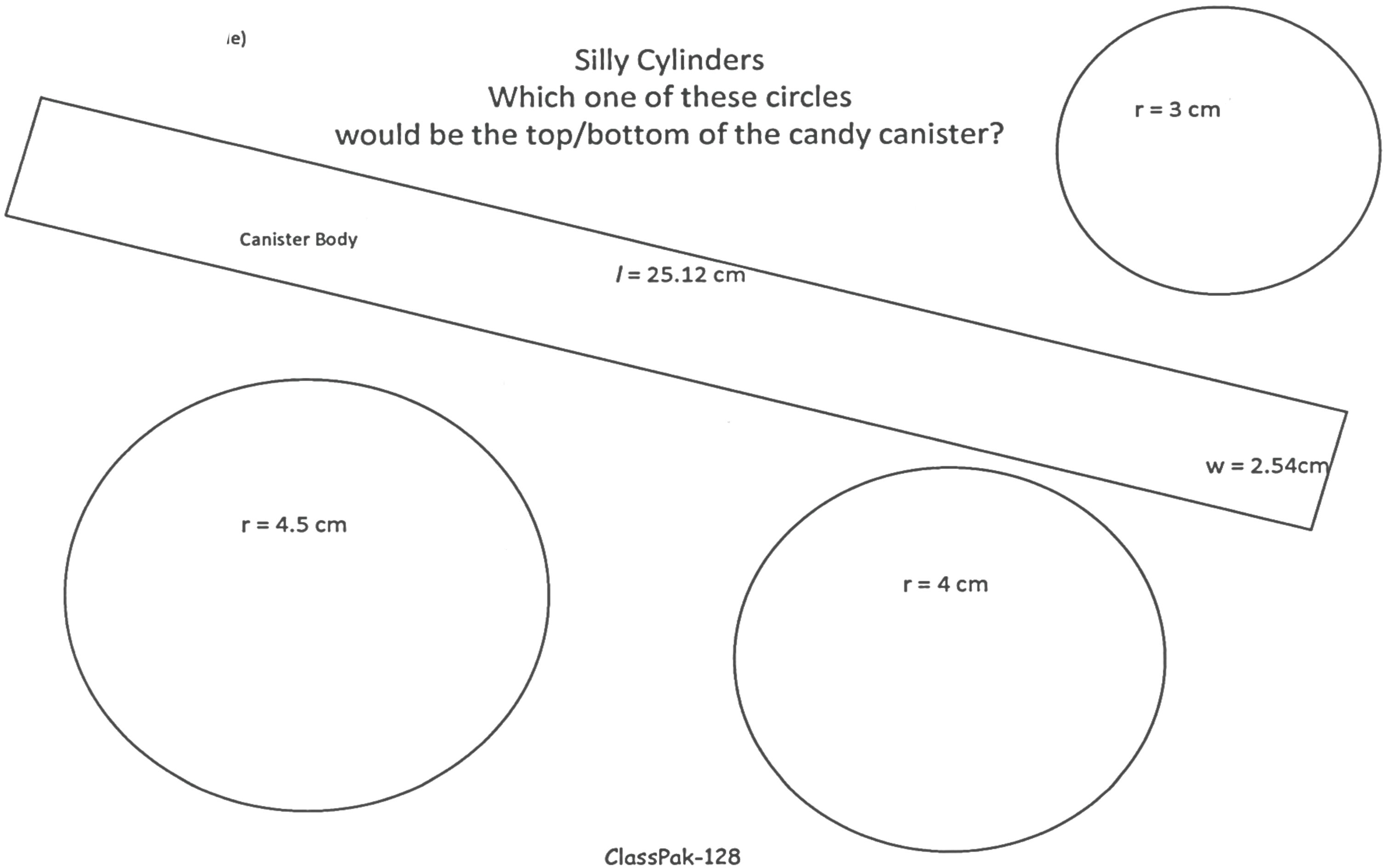
Which of the following rectangles would make a candy carton with the top and bottom below? (TWO PAGES)



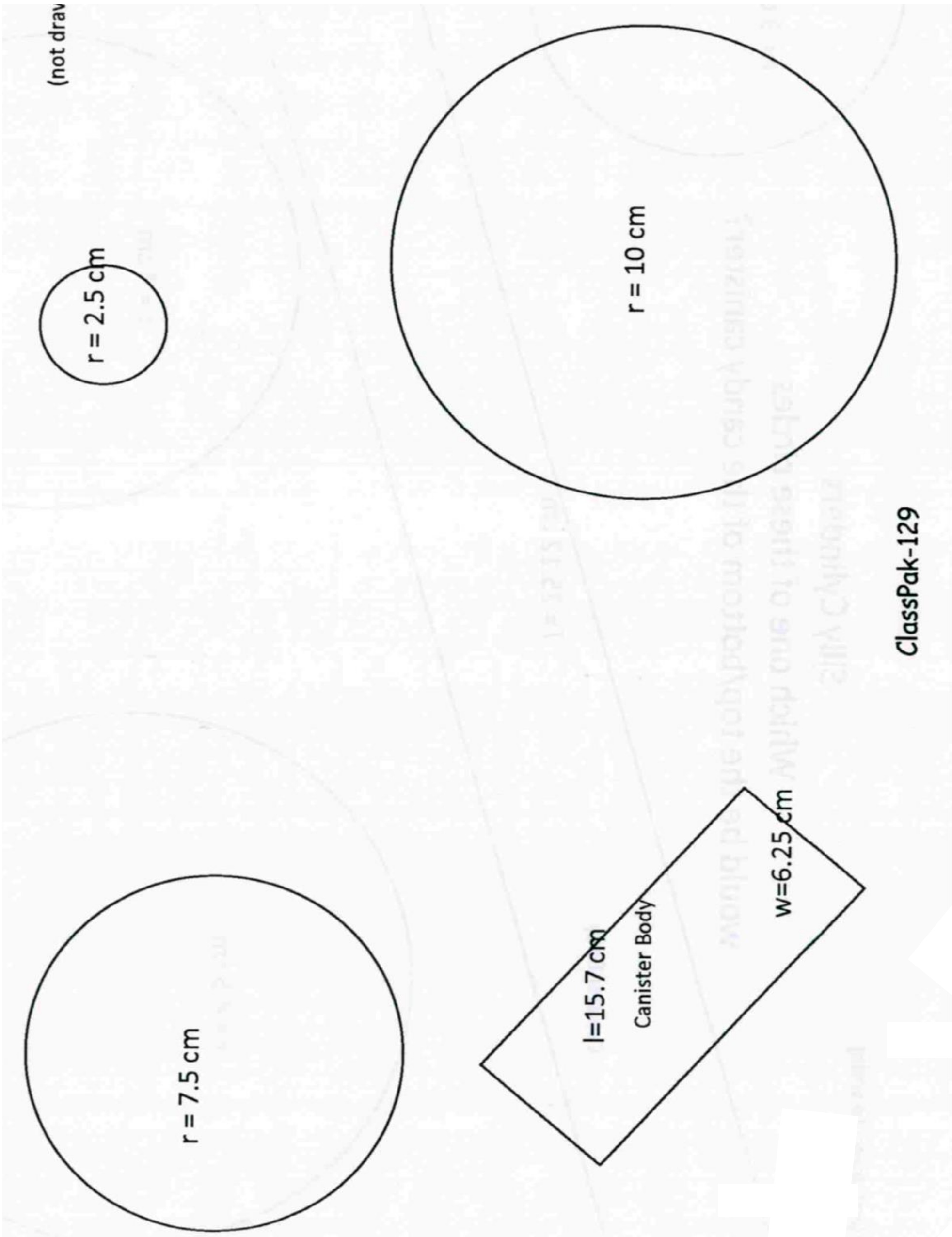

5 cm
28.26 cm
D
ClassPak-127

Silly Cylinders

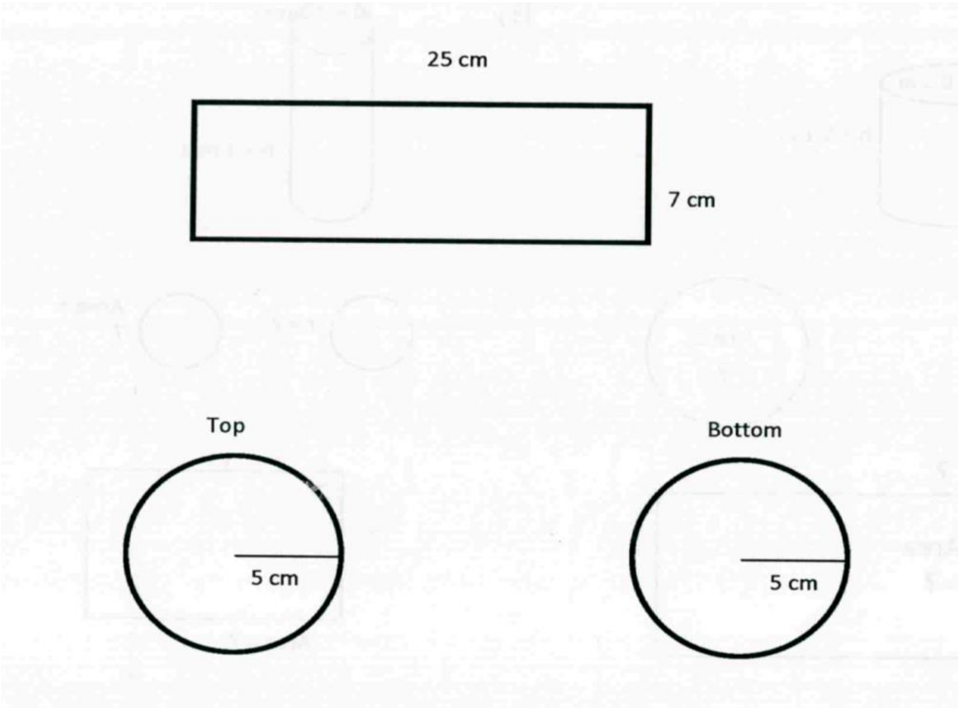
Which one of these circles would be the top/bottom of the candy canister?



ClassPak-128

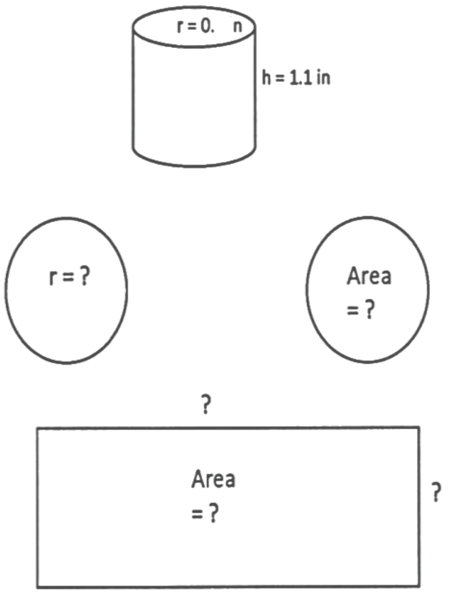

r = 2.5 cm
r = 10 cm
ClassPak-129
r = 7.5 cm
l=15.7 cm
Canister Body
w=6.25 cm

Jimmy said that the rectangle below will wrap around to make the body of a cylinder with top and bottom shown. Do you agree or disagree? What is your evidence?

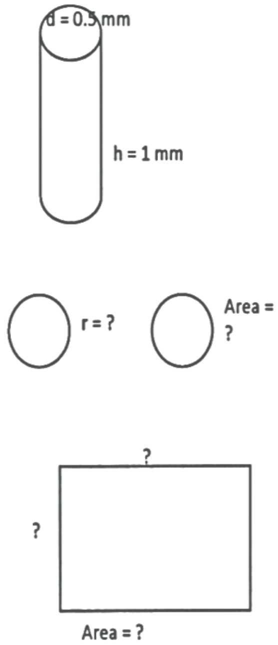


Find the missing parts.

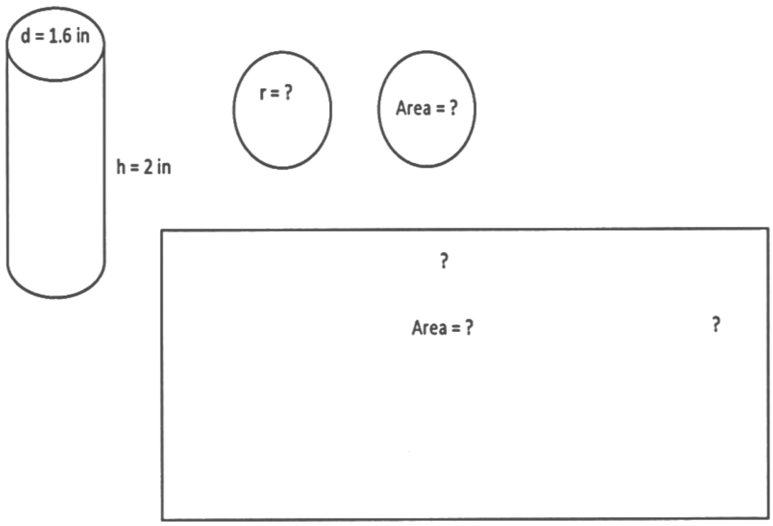
A)



B)

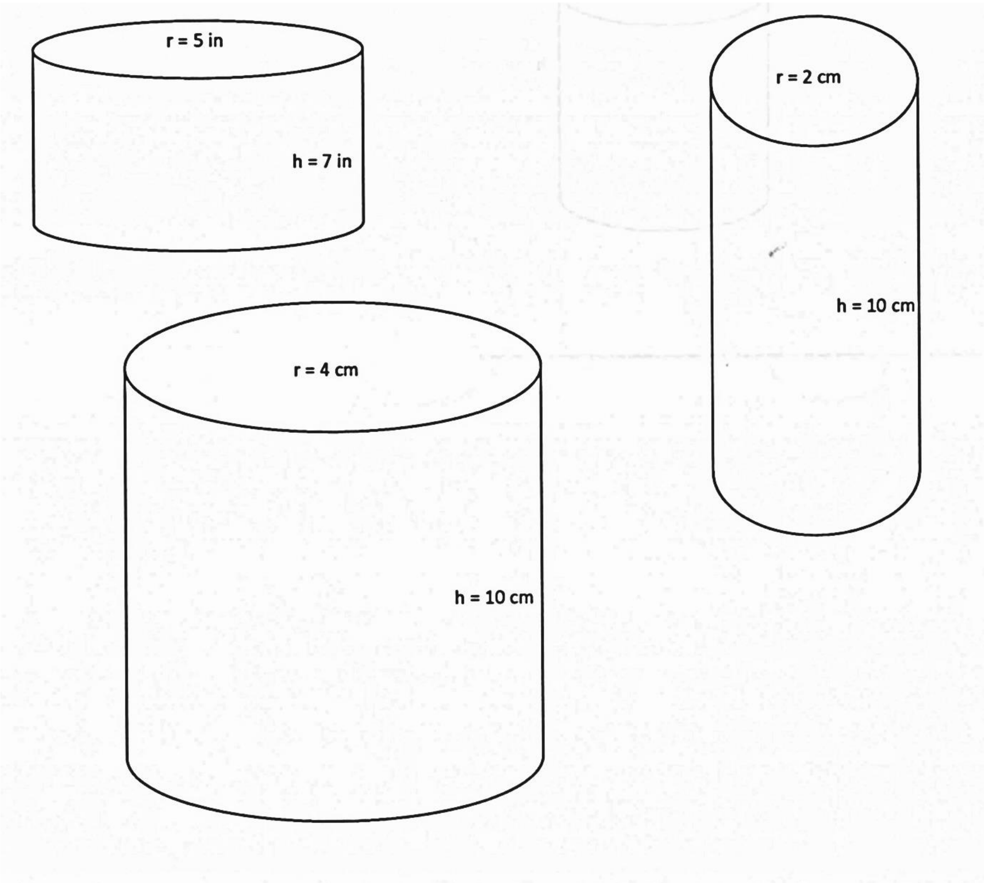


C)



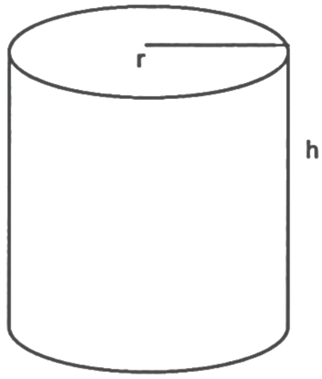
Surface area of cylinder

Draw and label all the parts of the wrapper for each shape below, including the dimensions! How many square units would it take to make each wrapper?



Formula page

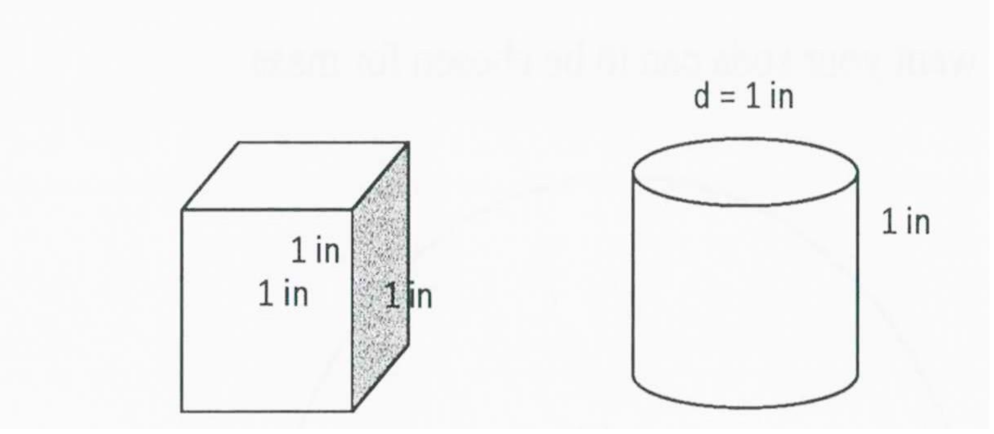
If you haven't already, create your own formula for the surface area of a cylinder.



Surface area:

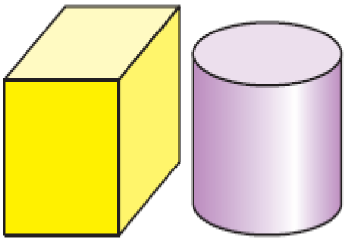
Choose the cheapest!

You are trying to decide which shape candy we want to make into a pack and sell. The business department has told us to make the one that will cost the least to distribute. Given that each pack will have the same number of pieces in it, which shape of the candy would require the least amount of material to wrap? Write your evidence below.

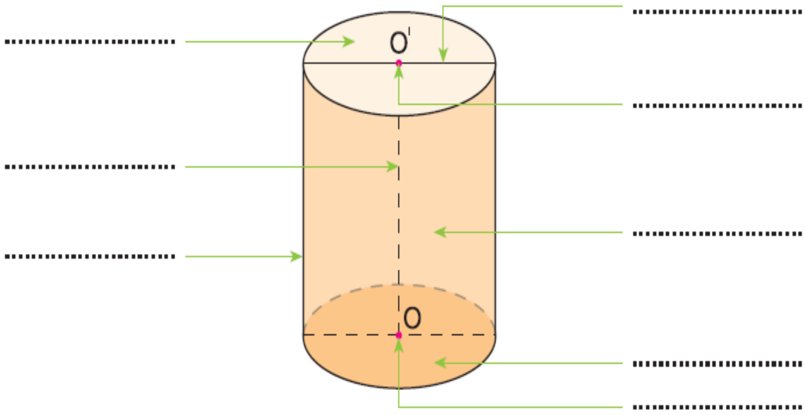


Look at this!

Surface area of a cube equalizes the side face area of the cylinder. The length of the cube is equal to the height of the cylinder and a height of 5 cm. Find the radius of the cylinder. ($\pi = 3$



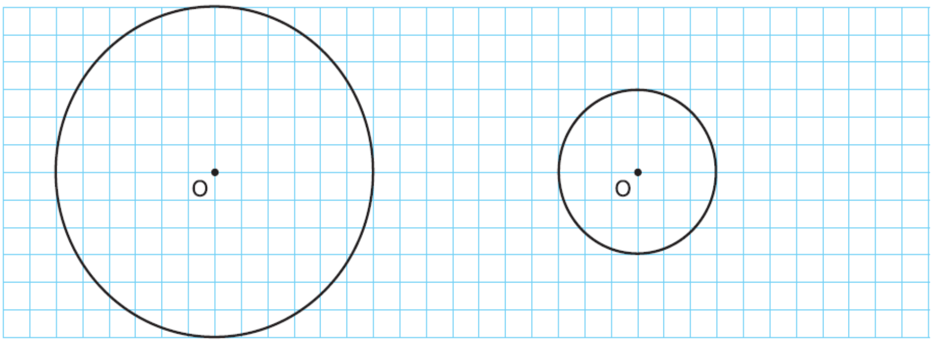
Find the name of indicated elements of cylinder below.



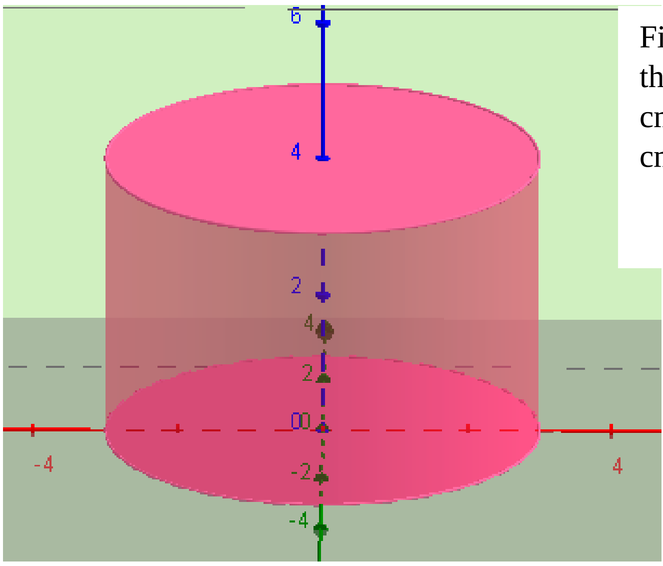
The cylindrical cupboard made of steel has a radius of 3 cm and a height of 9 cm. Let's find out how many square centimeters of steel is used to make this cup.

Volume of Cylinder

1) Let’s estimate the volumes of the cylinders below which are given the base circles and whose height is 12 br. Then calculate the volumes of these cylinders and compare the results to your estimates. ($\pi = 3$)



2)

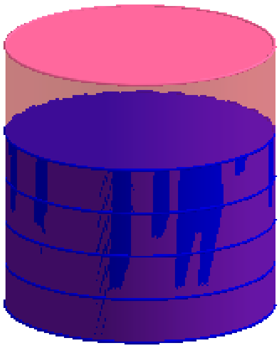


Find the volume of the next cylinder, 4 cm in height and 3 cm in diameter.

3) Find the volume of the cylinder with a base area of 4π cm2 and a height of 6 cm.

4) Ali will fill half of the side of the cup with water. If the diameter of the cup is 4 cm and the height is 6 cm, how much water will be used for this process?

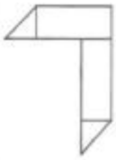
5) The height of the water in the cylinder is 4 cm. If the radius is 3 cm and the height is 6 cm, what is the volume of the void in the cylinder?

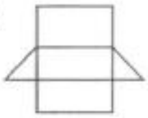


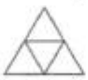
B: PRE-POSTTEST

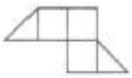
Prizmaların Temel Elemanları ve Açınımları Kazanım Testi

1. **Aşağıdakilerden hangisi bir dik üçgen prizmasının açınımı olabilir?**

A) 

B) 

C) 

D) 

2. **Tabanı düzgün beşgen olan dik prizmanın kaç yüzeyi vardır?**

A) 5 B) 6 C) 7 D) 8

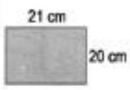
3. **Kare dik prizmanın kaç tane köşesi vardır?**

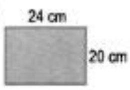
A) 6 B) 8 C) 10 D) 12

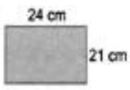
4. 

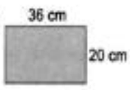
Şekilde bir ambalaj firması tarafından tasarlanan üçgen dik prizma şeklindeki kutu verilmiştir.

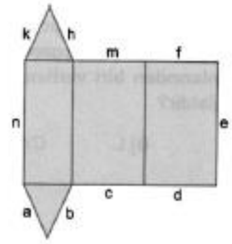
Bu kutunun yanal yüzeyini kaplamak için kullanılacak kağıt aşağıdakilerden hangisi olabilir?

A) 

B) 

C) 

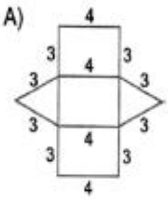
D) 

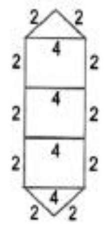
5. 

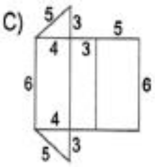
Yukarıda açınımı verilen üçgen dik prizmanın adlandırılmış kenarlarından hangisi aynı ayrıta ait <u>değildir</u>?

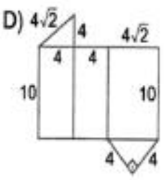
A) a ile d B) c ile k
C) e ile n D) m ile h

6. **Aşağıdakilerden hangisi bir üçgen dik prizmanın açınımı <u>olamaz</u>?**

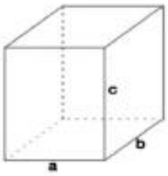
A) 

B) 

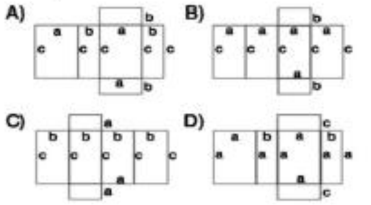
C) 

D) 

7.



Yandaki dikdörtgenler prizmasının açık şekli aşağıdakilerden hangisidir?



8.

I. 6 tane yüzeyi vardır.

II. Yan yüzeyleri birbirine eşittir.

III. 8 tane köşesi vardır.

Yukarıda özellikleri verilen geometrik cisim aşağıdakilerden hangisidir?

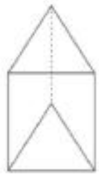
A) Kare dik prizma

B) Üçgen dik prizma

C) Dikdörtgenler prizması

D) Altıgen dik prizma

9.



Üçgen dik prizma için aşağıdakilerden hangisi yanlıştır?

A) 6 köşesi vardır.

B) 12 tane ayrıtı vardır.

C) Birbirine eş 2 üçgen ve üç tane dikdörtgenden oluşur.

D) Cisim köşegeni yoktur.

10. **Kare dik prizma için aşağıdakilerden hangisi yanlıştır?**

A) 8 köşesi vardır.

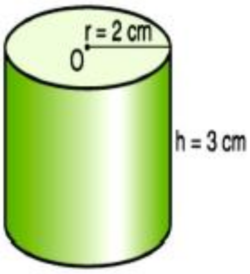
B) 12 ayrıtı vardır.

C) 6 yüzeyi vardır.

D) 8 tane yüzey köşegeni vardır.

11.

Yanda yarıçap ve yükseklik uzunlukları verilen dik dairesel silindirin açınımını çiziniz ($\pi = 3$ alınız.).



C: A SAMPLE PAGE FROM CONJECTURED HLT

Phase 1

Grade Level: 8

Materials: Paper, pencil, activity sheet, dynamic geometry files

Objectives: Students construct prisms, determine its basic elements.

Lesson Plan:

Starting: Before starting the lesson, teacher asks some questions.

-What does prism mean?

-Which shapes can be described as a prism in your home?

Expected answers: Aquarium, refrigerator etc.

Notes:

Middle:

Teacher gives the activity sheet to students.

First question is for showing the concepts perceptibly.

Students work on the main parts of a prism.

Second question is for helping them to develop their three dimensional imagining.

Students work on the opening prisms.

They try to understand the positions of base and lateral surfaces.

Teacher says that he/she is aware of three dimensional imagining is a little bit hard and it is easy to see this in a dynamic environment.

Teacher opens the geogebra file. This file shows them both close and open prisms at the same time.

Students can easily see the positions of lateral surfaces according to the base.

Notes:

End:

After these workings, students have more spesific ideas about prisms.

They learn their names, base components and their nets.

These activities are essential to give fundamental information about the subject.

Expected discussions

Teacher wants students to think about the common properties of these shapes.

Teacher tries to lead them to think about “these shapes have a base, all of them has an altitude” etc.

Unexpected situations (Fill during the lesson)

Students asked the difference between prisms and pyramid.

A discussion about camp tents and roofs of buildings

A discussion about whether a cube is a prism or not.

Additional comments about lesson

There are some misconceptions to handle about properties of prisms. Students don’t know some properties.

D: SAMPLE INFORMED CONCENT FORM-FOR TEACHER AND STUDENTS

ARAŞTIRMAYA GÖNÜLLÜ KATILIM FORMU

Bu çalışma, Orta Doğu Teknik Üniversitesi doktora öğrencisi Şule ŞAHİN DOĞRUER tarafından yürütülmektedir. Bu form sizi araştırma koşulları hakkında bilgilendirmek için hazırlanmıştır.

Çalışmanın Amacı Nedir?

Bu çalışmanın temel amacı 8. sınıf matematik müfredatında yer alan Geometrik kavramlar ve uzamsal düşünme ile ilgili içerikler geliştirmek, bu içerikleri Geogebra geometri yazılımı kullanarak uygulamak ve sınıf içi matematiksel pratikleri saptamak, bu içeriklerin etkililiğini test etmektir.

Bize Nasıl Yardımcı Olmanızı İsteyeceğiz?

Araştırmaya katılmayı kabul ederseniz, 2016/2017 eğitim öğretim yılının ikinci döneminde mayıs ayı süresince sürmesi planlanan Geometrik Cisimler ünitesinin dört kazanımı boyunca dersine girdiğiniz 8. sınıfın derslerini araştırmacı tarafından tasarlanan ve birlikte geliştireceğimiz etkinlikler çerçevesinde yürütmenizdir. Ayrıca araştırmacı sadece dersleri izlemekle kalmayıp, derslere aktif olarak katılacaktır.

Sizden Topladığımız Bilgileri Nasıl Kullanacağız?

Araştırmaya katılımınız tamamen gönüllülük temelinde olmalıdır. Çalışmada sizden kimlik veya çalıştığınız kurum/bölüm/birim belirleyici hiçbir bilgi istenmemektedir. Cevaplarınız tamamıyla gizli tutulacak, sadece araştırmacılar tarafından değerlendirilecektir. Katılımcılardan elde edilecek bilgiler toplu halde değerlendirilecek ve bilimsel yayımlarda kullanılacaktır.

Sağladığınız veriler gönüllü katılım formlarında toplanan kimlik bilgileri ile eşleştirilmeyecektir.

Katılımınızla ilgili bilmeniz gerekenler:

Çalışma, genel olarak kişisel rahatsızlık verecek sorular ya da etkinlikler içermemektedir. Ancak, katılım sırasında sorulardan ya da herhangi başka bir nedenden ötürü kendinizi rahatsız hissederseniz katılım işini yarıda bırakıp çıkmakta serbestsiniz. Böyle bir durumda çalışmayı uygulayan kişiye, çalışmadan çıkmak istediğinizi söylemek yeterli olacaktır.

Araştırmayla ilgili daha fazla bilgi almak isterseniz:

Bu çalışmaya katıldığınız için şimdiden teşekkür ederiz. Çalışma hakkında daha fazla bilgi almak için Şule ŞAHİN DOĞRUER ile sule_sahinn@hotmail.com adresi ile iletişim kurabilirsiniz.

Yukarıdaki bilgileri okudum ve bu çalışmaya tamamen gönüllü olarak katılıyorum.

(Formu doldurup imzaladıktan sonra uygulayıcıya ulaştırınız).

Adı Soyadı	Tarih	İmza
	---/----/-----	

E: SAMPLE INFORMED CONCENT FORM-FOR PARENTS

Veli Onay Formu

Sevgili Anne/Baba,

Bu çalışma Şüküfe Nihal Ortaokulu matematik öğretmeni ve aynı zamanda Orta Doğu Teknik Üniversitesi doktora öğrencisi Şule ŞAHİN DOĞRUER tarafından yürütülmektedir.

Bu çalışmanın amacı nedir?

Bu çalışmanın temel amacı 8. sınıf matematik müfredatında yer alan Geometrik kavramlar ve uzamsal düşünme ile ilgili içerikler geliştirmek, bu içerikleri Geogebra geometri yazılımı kullanarak uygulamak ve sınıf içi matematiksel pratikleri saptamak, bu içeriklerin etkililiğini test etmektir. Hazırlanan içeriklerin öğrencilerin geometrik düşünme ve ilgili konulardaki öğrenmelerini geliştireceği beklenmektedir. Ayrıca, bu çalışmada kullanılan yöntemin, matematik dersleri için içerikler geliştirmeyi hedefleyen diğer çalışmalara da model olması amaçlanmaktadır.

Çocuğunuzun katılımcı olarak ne yapmasını istiyoruz?

Bu amaç doğrultusunda, çocuğunuzdan yapmasını istediğimiz ekstra bir etkinlik yoktur. Onlar normal eğitim öğretimlerine devam edeceklerdir. Bu çalışma için gerekli veriler matematik dersleri süresince toplanacaktır. Mayıs ayı süresince matematik dersleri çalışmayı düzenleyen Şule ŞAHİN DOĞRUER tarafından izlenecek, notlar alınacak ve video kaydı yapılacaktır. Sizden çocuğunuzun katılımcı olmasıyla ilgili izin istediğimiz gibi, çalışmaya başlamadan çocuğunuzdan da sözlü olarak katılımıyla ilgili rızası mutlaka alınacaktır.

Çocuğunuzdan alınan bilgiler ne amaçla ve nasıl kullanılacak?

Derste toplanan veriler tamamen gizli tutulacak ve sadece araştırmacı Şule ŞAHİN DOĞRUER tarafından değerlendirilecektir. Elde edilecek bilgiler sadece bilimsel amaçla kullanılacak, çocuğunuzun ya da sizin ismi ve kimlik bilgileriniz, hiçbir şekilde kimseyle paylaşılmayacaktır.

Çocuğunuz ya da siz çalışmayı yarıda kesmek isterseniz ne yapmalısınız?

Katılım sırasında herhangi bir uygulama ile ilgili başka bir nedenden ötürü çocuğunuz kendisini rahatsız hissettiğini belirtirse, ya da kendi belirtmese de araştırmacı çocuğun rahatsız olduğunu öngörürse, çalışmaya sorular tamamlanmadan ve derhal son verilecektir. Şayet siz çocuğunuzun rahatsız olduğunu hissederseniz, böyle bir durumda çalışmadan sorumlu kişiye çocuğunuzun çalışmadan ayrılmasını istediğinizi söylemeniz yeterli olacaktır.

Bu çalışmayla ilgili daha fazla bilgi almak isterseniz: Çalışmaya katılımınızın sonrasında, bu çalışmayla ilgili sorularınız yazılı biçimde cevaplandırılacaktır. Çalışma hakkında daha fazla bilgi almak için Şule ŞAHİN DOĞRUER ile okulda ya da sule_sahinn@hotmail.com mail adresi yoluyla iletişim kurabilirsiniz. Bu çalışmaya katılımınız için şimdiden teşekkür ederiz.

Yukarıdaki bilgileri okudum ve çocuğumun bu çalışmada yer almasını onaylıyorum (Lütfen alttaki iki seçenekten birini işaretleyiniz.

***Evet onaylıyorum*___**   ***Hayır, onaylamıyorum*___**

Annenin (ya da Babanın) Adı-soyadı: _______________

Çocuğun adı soyadı: _________________

Bugünün Tarihi: _________________   İmza:

(Formu doldurup imzaladıktan sonra araştırmacıya ulaştırınız).

F: APPROVAL OF THE ETHICS COMMITE OF METU RESEARCH CENTER FOR APPLIED ETHICS

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

DUMLUPINAR BULVARI 06800
ÇANKAYA ANKARA/TURKEY
T: +90 312 210 22 91
F: +90 312 210 79 59
ueam@metu.edu.tr
www.ueam.metu.edu.tr

Sayı: 28620816 / 459

05 ARALIK 2016

Konu: Değerlendirme Sonucu

Gönderilen: Doç.Dr. Didem AKYÜZ

Eğitim Fakültesi

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Doç. Dr. Didem AKYÜZ;

Danışmanlığını yaptığınız doktora öğrencisi Şule ŞAHİN DOĞRUER "Tasarım Tabanlı Sınıf Ortamında Matematiksel Pratikleri Saptamaya Yönelik Teknoloji Destekli İçerikler Geliştirilmesi: 8. Sınıfların Üç Boyutlu Cisimlerde Uzamsal Düşünmelerine Yönelik Bir Durum Çalışması" başlıklı araştırması İnsan Araştırmaları Kurulu tarafından uygun görülerek gerekli onay **2016-EGT-164** protokol numarası ile **02.05.2017-31.07.2017** tarihleri arasında geçerli olmak üzere verilmiştir.

Bilgilerinize saygılarımla sunarım.

Prof. Dr. Canan SÜMER

İnsan Araştırmaları Etik Kurulu Başkanı

Prof. Dr. Mehmet UTKU
İAEK Üyesi

Prof. Dr. Ayhan SOL
İAEK Üyesi

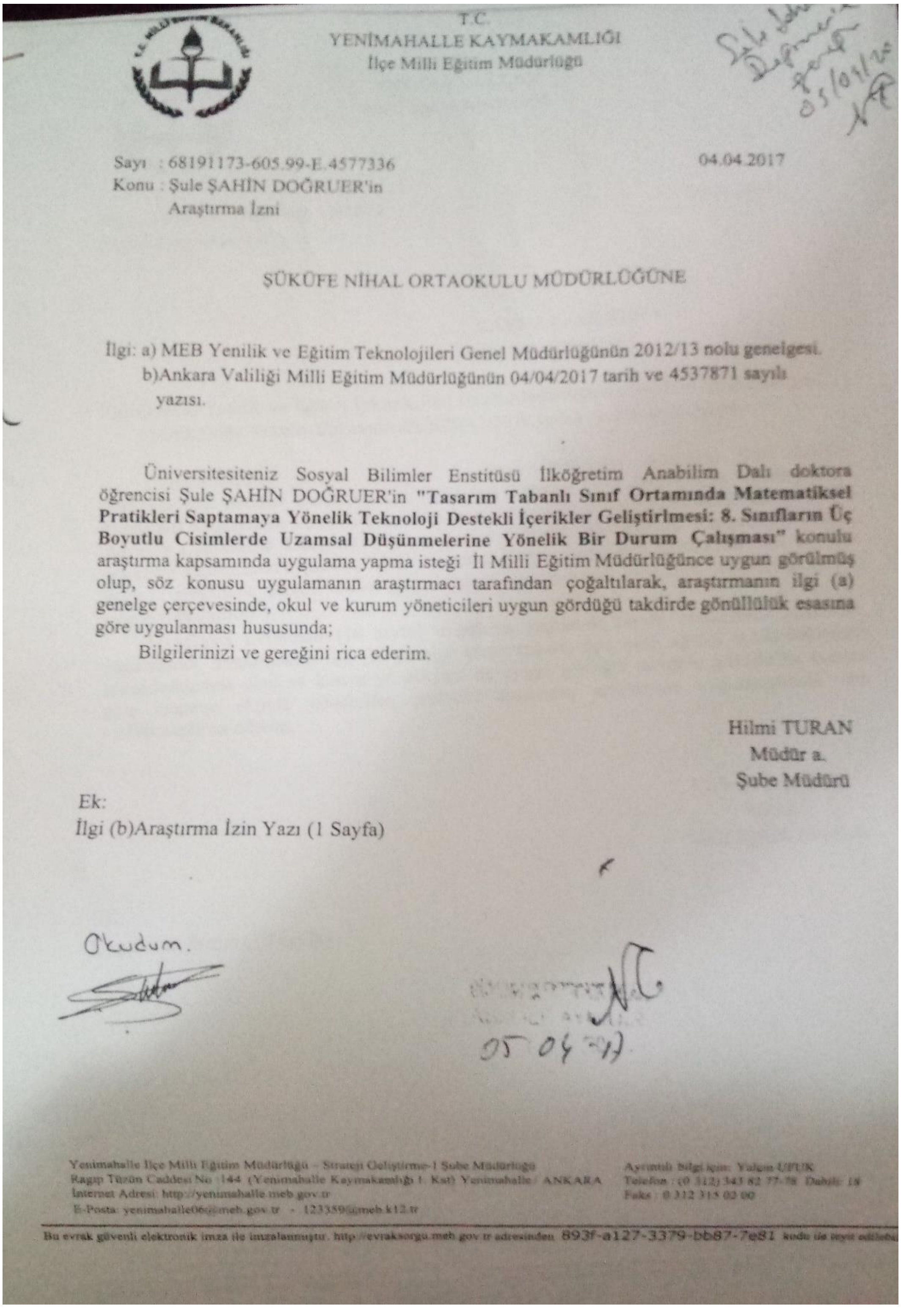
Prof. Dr. Ayhan Gürbüz DEMİR
İAEK Üyesi

Doç. Dr. Yaşar KONDAKÇI
İAEK Üyesi

Yrd. Doç. Dr. Pınar KAYGAN
İAEK Üyesi

Yrd. Doç. Dr. Emre SELÇUK
İAEK Üyesi

G: APPROVAL OF DIRECTORATE OF NATIONAL EDUCATION



T.C.
YENİMAHALLE KAYMAKAMLIĞI
İlçe Milli Eğitim Müdürlüğü

Sayı : 68191173-605.99-E.4577336 04.04.2017
Konu : Şule ŞAHİN DOĞRUER'in
Araştırma İzni

ŞÜKÜFE NİHAL ORTAOKULU MÜDÜRLÜĞÜNE

İlgi: a) MEB Yenilik ve Eğitim Teknolojileri Genel Müdürlüğünün 2012/13 nolu genelgesi.
b)Ankara Valiliği Milli Eğitim Müdürlüğünün 04/04/2017 tarih ve 4537871 sayılı yazısı.

Üniversitesiteniz Sosyal Bilimler Enstitüsü İlköğretim Anabilim Dalı doktora öğrencisi Şule ŞAHİN DOĞRUER'in **"Tasarım Tabanlı Sınıf Ortamında Matematiksel Pratikleri Saptamaya Yönelik Teknoloji Destekli İçerikler Geliştirilmesi: 8. Sınıfların Üç Boyutlu Cisimlerde Uzamsal Düşünmelerine Yönelik Bir Durum Çalışması"** konulu araştırma kapsamında uygulama yapma isteği İl Milli Eğitim Müdürlüğünce uygun görülmüş olup, söz konusu uygulamanın araştırmacı tarafından çoğaltılarak, araştırmanın ilgi (a) genelge çerçevesinde, okul ve kurum yöneticileri uygun gördüğü takdirde gönüllülük esasına göre uygulanması hususunda;

Bilgilerinizi ve gereğini rica ederim.

Hilmi TURAN
Müdür a.
Şube Müdürü

Ek:
İlgi (b)Araştırma İzin Yazı (1 Sayfa)

Okudum.

05.04.17

Yenimahalle İlçe Milli Eğitim Müdürlüğü – Strateji Geliştirme-1 Şube Müdürlüğü
Ragıp Tüzün Caddesi No :144 (Yenimahalle Kaymakamlığı 1. Kat) Yenimahalle / ANKARA
İnternet Adresi: http://yenimahalle.meb.gov.tr
E-Posta: yenimahalle06@meb.gov.tr - 123359@meb.k12.tr

Ayrıntılı bilgi için: Yalçın UFUK
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H: TURKISH SUMMARY/ TÜRKÇE ÖZET

Giriş

Okul matematiğinde geometri öğretimi önemli bir yer tutar. Geometrik düşüncenin en önemli parçası iki veya üç boyutlu uzayda geometrik şekiller ve bunları çeşitli yönlerden incelemektir (NTCM, 2000). Geometri derslerinde, öğrenciler geometrik şekiller ve yapılar arasındaki ilişkileri değerlendirir (Keşan ve Çalışkan, 2013). Geometriyi etkili bir şekilde öğrenmek ve öğretmek önemlidir. Tersi durumda, öğrenciler anlamaya çalışmak yerine geometrik kavramları ve formülleri ezberlemeyi tercih ederler (Fuys, Geddes ve Tischler, 1988).

Baki (2001), öğrencilerin uygun problem çözme stratejilerini kullanarak fiziksel dünyayı anlayarak ve anlatarak geometriyi öğrenmeleri gerektiğini belirtmektedir. Fiziksel dünyamız sadece iki boyutlu Öklid geometrisi ile açıklanamaz. Çünkü kullandığımız, gördüğümüz, ürettiğimiz, yani sahip olduğumuz her şey üç boyutlu geometrik bir şekle sahiptir (Güven ve Kosa, 2008). Aynı şekilde, Pittalis ve Constantinou (2010), bu tür düşüncenin "bireylerin mekânsal imgeler yaratmalarını ve çeşitli pratik ve teorik problemleri çözmede manipüle etmelerini sağlayan bir zihinsel aktivite biçimi" olduğunu belirtmektedir (s. 191). Sack (2013) bu ifadeyi şekil, boyut, yön, konum veya yönde herhangi bir nesnenin veya sürecin anlamını elde etmek olarak özetlemektedir. Bu nedenle, birçok ulusal belge (NCTM, 1989, 2000), tüm öğrencilerin günlük yaşamda ve gelecekteki kariyerlerinde önemli ve yararlı olduklarından mekânsal becerilerini geliştirmek için görselleştirme yoluyla üç boyutlu şekillerle çalışma fırsatlarına sahip olmalarının gerekliliğini belirtmiştir. Ayrıca, üç boyutlu düşünme yeteneklerinin önemi, araştırmacılar tarafından matematiksel ve bilimsel disiplinler arasında dile getirilmiştir. Bu öneme rağmen, katı cisimler, çokgenler, üçgenler, geometrik oran, geometrik dönüşüm konuları öğrenciler tarafından öğretme ve öğrenme açısından en sorunlu olanlar olarak tanımlanmaktadır. Dahası, öğrenciler bu kavramları anlaşılması zor olarak tanımlarlar (Adolphus, 2011). Bu anlamda, araştırmalar, uzamsal düşünme yeteneklerinin uygun öğrenme deneyimleri yoluyla öğrenilebileceğini göstermiştir (Alqahtania ve Powell, 2017; Ganesh, Wilhelm ve Sherrod, 2009; Marchis, 2012).

Bu bağlamda, örneğin, Yackel ve Cobb (1996) matematiğin hem bireysel çalışmayı hem de tüm sınıf tartışmalarına katılarak ve çalışmalarını daha geniş bir toplumda açıklayarak ve haklı göstererek işbirlikçi çalışmayı içerdiğini iddia ederler. Ayrıca, çeşitli çalışmalarda (Bauersfeld, Krummheuer, & Voigt, 1988; Cobb, Boufi, McClain ve Whitenack, 1997; Giannakoulias, Mastorides, Potari ve Zachariades, 2010; Mueller, 2009), matematik sınıflarında tartışma ortamının oluşturulmasının önemi ve sınıf normlarının açıklama, gerekçe gösterme ve argümantasyon süreçleriyle karakterize edildiğini belirtilir. Dolayısıyla, matematiğin bir alt alanı olarak, tartışmacı sınıf ortamını geometri sınıflarına uyarlamak uygundur. Bu sayede öğrencilerin geometrik yapı ve teoremleri arasındaki ilişkileri fikir alışverişinde bulunarak anlamaları yararlı olabilir. Ek olarak, bilimsel tartışma sürecini tartışırken, Sürücü, Newton ve Osborne, (2000) tartışmacı içeriğin derin kavramsal anlayışını desteklediğine karar vermiştir. Dahası, çeşitli araştırmalar tartışma ortamının başkalarının fikirlerini doğrulayarak ve eleştirerek matematiğin ve geometrinin kavramsal anlayışını artırdığını desteklemektedir (Abi-El-Mona ve Abd-El-Khalick, 2011; Jonassen ve Kim, 2010; Osborne, Erduran ve Simon, 2004; Zembaul-Saul, 2005). Bu bağlamda, öğrencilerin kavramsal anlayışını arttırmak için geometride tartışma ortamının dahil etmek yararlı olabilir.

Yine aynı kapsamda, uzun yıllar boyunca, araştırmalar matematik öğretimi ve öğreniminin sosyolojik yönüne odaklanmıştır. Spesifik olarak, odak noktası sınıf matematiksel uygulamaları oluşturarak iş birlikçi öğrenmeyi sağlamak olmuştur (Ball & Bass, 2000; Cobb & Bauersfeld, 1995; Cobb, Stephan, McClain ve Gravemeijer, 2011; Stephan ve Rasmussen, 2002). Bu çalışmalar, genel olarak matematik öğretimi ve öğreniminin sosyal yönüne odaklanmayı tercih ederler, çünkü matematiğin, matematik yaparak topluluk içinde daha iyi öğrenileceği savunulur (Cobb, Yackel, & Wood, 1992; Yackel & Cobb, 1996). Literatürdeki çalışmalar, sınıf matematik uygulamalarının farklı yönlerine ve tanımlarına odaklanmıştır. Örneğin, Bowers, Cobb ve McClain (1999), matematiksel uygulamaları “fikir birliğine varılmış ve dolayısıyla gerekçelendirmenin ötesinde matematiksel olarak hareket etme ve akıl yürütme biçimlerindeki değişimlere odaklanma” olarak tanımlamıştır (s.28).

Sınıf matematiksel uygulamaları belirli matematiksel fikirleri tartışırken ortaya çıkar ve bu fikirlerin paylaşılması, tartışılması ve akıl yürütmesinin bir yoludur (Cobb, Stephan, McClain ve Gravemeijer, 2011). Benzer bir tanım, Bowers, Cobb ve McClain (1999) tarafından “öğretmen ve öğrencilerin problemleri ve çözümleri tartıştıkları yollar” olarak tanımlanır ve bu uygulamalar belirli görev durumlarında simgeleştirme, tartışma ve doğrulama araçlarını içerir (s.28). Bu tanımların başlangıç noktaları, öğrenme sürecinin bireysel ve sosyal yönleridir. Tanımlarda da belirtildiği gibi, matematiksel uygulamalar, matematiksel olarak akıl yürütme, tartışma ve tartışmanın ortak yollarını içerir. Cobb, Wood, Yackel ve McNeal (1992), paylaşılan fikirleri, sınıfsal matematik uygulamalarının ortaya çıkmasıyla sonuçlanan matematiksel açıklamalar, gerekçeler, sembolleştirmeler olarak tanımlar. Buna göre, matematik uygulamalarının ortaya çıkmasının sınıf üyeleri arasındaki sosyal etkileşim ile güçlü bir şekilde ilişkili olduğu sonucuna varılabilir. Sosyal açıdan aktif bir sınıf ortamı yaratarak, öğrenciler matematik öğretimi sürecine katılmaya ve daha gönüllü olarak öğrenmeye motive olabilir (Cobb ve Yackel, 1996).

Belirli matematiksel fikirleri tartışırken matematiksel uygulamalar ortaya çıkmaktadır. Yukarıda bahsedildiği gibi, matematiksel uygulamalar, öğrencilerin belirli bir matematiksel bağlamı anlama, açıklama, haklı çıkarma, çürütme, mantık yürütme biçimleri ve onları sınıf topluluğu tarafından paylaşılmalarını sağlar (Bowers). & Cobb, 1999; Cobb ve arkadaşları, 2011; Stephan, Cobb & Gravemeijer, 2003). Sınıf matematik uygulamalarını tanımlamak için, öğrencilerin akıl yürütme yolları ve yansımaları başlangıç noktası olarak alınır. Öğrencilerin fikirleri ve akıl yürütmeleri, sınıf tartışmaları ve belirli bir içerikteki aktiviteler sırasında ortaya çıkar (Stephan, Bowers, Cobb ve Gravemeijer, 2003). Böylece, öğrencilerin bireysel uygulamalarını da içeren sosyal öğrenme, sınıf matematik uygulamalarının odak noktasıdır. Buna göre, sınıf içi söylem ve öğrenme araçlarının kullanımı da dahil olmak üzere öğrenme ortamı hakkındaki veriler, sınıf matematik uygulamalarının oluşturulması olan sınıfın sosyal yönünü oluşturur (Stephan ve Rasmussen, 2002).

Tasarım tabanlı araştırmaların özellikleri ile uyumlu olarak, geometrik kavramlar için varsayıma dayalı bir öğrenme yolu ile bir öğretim dizisinin hazırlanması, öğrencilerin içerik hakkında etkili bir şekilde düşünmelerine ve öğrenmelerine yardımcı olabilir. Dahası, bu etkinlikleri sınıf içi tartışmalarla desteklemek, öğrenciler fikirlerini başkalarıyla paylaşma şansına sahip olacaklardır. Yine, belirli bir içerik hakkındaki tartışmalar, öğrenciler arasında aktarılan fikirlerin, matematiksel uygulamaların inşasının bir yolu olan, paylaşılan fikirler olarak ortaya çıkmasını sağlar (Cobb, Gravemeijer, Yackel, McClain ve Whitenack, 1997). Bu bağlamda, mevcut çalışmada, sınıf tartışmalarının oluşturduğu matematiksel uygulamalar katı cisimler konusu kapsamında değerlendirilmiştir.

MEB (2013), matematik ve geometri derslerinde teknolojinin kullanımının öğrencilerin düşünme ve mekânsal yeteneklerini geliştirdiğini vurgulamıştır. Geometri öğretimi, üç boyutlu katı cisimlerin öğrenimine özel bir dikkat içermelidir. Özellikle, bu katı cisimleri görselleştirme becerileri ve temsili sürekli bir gelişime sahip olmalıdır. Ben-Chaim, Lappan ve Hoaung (1988), ortaokul ve lise öğrencilerine uygun stratejiler kullanarak uzamsal düşüncenin başarılı bir şekilde öğretilip geliştirilebileceğini belirtmektedir. Bu bağlamda eğitimciler, teknolojinin uygun bir strateji olarak kullanılmasının, matematiğin ve özellikle geometri öğrenim ve öğretiminin etkili bir şekilde destekleyebildiğine inanmaktadır (McClintock, Jiang ve July, 2002).

Geometri derslerinde kullanılabilecek, kelime işlemci ve elektronik tablolar gibi bir çok çeşitli teknolojik araçlar vardır. Ancak, dinamik geometri yazılımı (bu çalışmada DGY olarak kısaltılmıştır) daha öğrenci merkezli öğrenme ortamları oluşturmak için daha etkili bir araçtır (Hannafin, Truxaw, Vermillion, ve Liu, 2008). NCTM (2000), geometrinin etkili bir şekilde öğrenmesini sağlamak için somut nesneler, çizimler ve dinamik geometri yazılımlarını kullanmasının çok önemli olduğunu belirtmektedir. DGY'yi eğitim alanında kullanarak ve dinamik bilgisayar ekranına aktararak, öğrenciler için kâğıt ve kalem kullanmadan yapılar arasındaki ilişkileri değerlendirmek, hipotez geliştirmek, teoremleri test etmek mümkün hale gelmiştir (Güven ve Karataş, 2003). Araştırmacılar, DGY'nin öğrencilere dinamik özellikleriyle yaygın olarak kullanılan kâğıt-kalem çalışmalarından çok daha soyut yapılara odaklanma fırsatı verdiğini göstermiştir (Hollebrands ve Okumuş, 2018). Bu uygulamalar öğrencilerin zihinde canlandırma kullanımlarını arttırır. Bu artış sezgi yolunu açar ve bu yollar kullanıldığında, öğrenci analiz edebilir, hipotez ve genelleme yapabilir. Bu doğrudan öğrencinin problem çözme becerilerini geliştirecektir (Baki, 2001). DGY, araştırma yoluyla deneyim ve öğretme geometrisini destekleyen özellikleriyle, yıllar boyunca aynı şekilde öğretilen geometriye alternatif olanaklar sunmaktadır (Edwards, 1997). Geometri öğretiminde, dinamik geometri yazılımı kullanılarak, öğrenciler geometrik çizimler oluşturabilir veya öğretmen tarafından hazırlanan dinamik geometrik şekiller üzerinde etkileşimli araştırmalar yapabilir (MEB, 2013); ve bu sayede öğrencilerin geometri öğrenmeleri DGY ortamlarında faaliyetlerine aracılık ederek desteklenebilir (Alqahtania & Powell, 2017).

Gerçekçi matematik eğitimi, matematik alanına özel bir öğretim teorisidir (Van den Heuvel-Panhuizen ve Drijvers, 2014). Gravemeijer ve Cobb (2013) 'a göre, bu teori matematiğin hazır bir ürün olarak kullanıldığı öğretim yaklaşımlarına bir cevap olarak ortaya çıkmıştır. Freudenthal (1973), bu teoriyi savunarak, matematiğin öncelikle öğrenciler için bir dizi etkinlik olması gerektiğini savundu. Öğretmenin rehberliğinde yeniden bir buluş dönemi ve hazırlanan matematiksel etkinlikler öğrencilerin matematik hayal gücünü kullanmalarını teşvik edecek şekilde olmalıdır. Buna göre içeriğin başlangıç noktası öğrenciler için gerçekçi olmalıdır. Daha açık olarak, problem durumları öğrencilerin hayal edebileceği, mantık yürütebilecekleri ve çözümde aktif olarak yer alabilecekleri şekilde sunulmalıdır. Bu süreçteki temel amaç, öğrenciler tarafından geliştirilen matematiğin onlar için gerçek olması gerektiğidir. Başka bir deyişle, kişinin matematik öğrenmesi gerçek hayatla ne kadar birleştiğine bağlıdır (Gravemeijer ve Cobb, 2013). Bu bağlamda, öğrencilere, içerik hakkında düşünecekleri, tartışacakları, ifade ettikleri fikirlerini haklı çıkaracakları, başkalarının akıl yürütme biçimlerini kabul ettikleri veya reddettikleri, planlı ve öngörülen bir öğrenme yörüngesi ve etkinlik dizilerini içeren öğretim dizisi ile öğrencilere bir öğrenme ortamı sağlamak için tasarım temelli bir araştırma yürütmek uygun olacaktır.

Bu bağlamda, çalışmayı yönlendiren araştırma soruları aşağıdaki gibidir:

1. Öğrencilerin, hazırlanan öğretim dizisi sırasında geliştirdikleri matematiksel uygulamaları destekleyen matematiksel fikirler nelerdir?

2. Bu içerik kapsamında hazırlanan öğretim dizisinin, öğrencilerin bu içerikte dinamik geometri yazılımını kullanarak başarıları üzerinde herhangi bir etkisi var mıdır?

Yöntem

Mevcut araştırmada, argümantasyon ve DGS'nin desteğiyle katı cisimler kapsamında sekizinci sınıfların öğrenme ortamının doğru ve derin bir şekilde anlaşılması için tasarım tabanlı bir araştırma yaklaşımı kullanılmıştır.

Eğitim araştırması için yeni bir metodoloji olarak tasarım tabanlı araştırmanın ortaya çıkışı, mevcut yüzyılın ilk yıllarına denk gelmektedir (Anderson ve Shattuck, 2012) ve bu süre boyunca artan bir popülerlik göstermiştir (Barab ve Squire, 2004). Saygın dergilerin, saygın yazarların ve eğitimci araştırmacıların çoğu, eğitim alanlarındaki kaliteyi artırmak için tasarım temelli araştırma potansiyelini keşfetmiştir (Anderson ve Shattuck, 2012). Böylelikle bu metodolojinin kullanımına matematik eğitiminde giderek artan bir ilgi gösterilmiştir (Cobb, 2003).

Tasarım Tabanlı Çalışma Topluluğu (2003), tasarım tabanlı araştırmanın bazı temel özelliklerini şöyle ifade etmiştir; genellikle belirlenen bir süre boyunca tek bir ortamda yürütülür; tasarım, uygulama, analiz ve yeniden tasarım döngülerini içerir; tüm çalışma sürecine ilişkin belgelerin ve sonuçların bağlanması; araştırmacı ve katılımcı iş birliği ve pratikte kullanılabilecek bilgi birikimi.

Cobb ve arkadaşları, (2003), tasarım tabanlı araştırma için beş özellik önerirler. Birincisi, yukarıda belirtilen öğrenme süreci hakkında teoriler geliştirmektir. İkinci özellik, araştırmacıların eğitimsel gelişmeleri kendi doğal ortamlarında değerlendirebilmeleri için fırsatlar sunan müdahaleci özelliklerle ilgilidir. Üçüncüsü, tasarım tabanlı araştırmanın ileriye dönük ve yansıtıcı olmasıdır. İleriye dönük oluşu, varsayıma dayalı bir öğrenme yörüngesine eşlik ederek öğrenmenin olası yollarını dikkate alırken; yansıtıcı taraf, test, reddetme, üretme veya tekrar test etme gibi deneylerin birkaç aşamasıyla ilgilidir. Bu iki özellik metodolojinin döngüsel bir sürece sahip olmasını sağlar. Dördüncü, yinelenen özellik, ileriye dönük ve yansıtıcı özelliklerden oluşur ve döngüsel süreçle ilgilidir. Ve son özellik, uygulama sırasında teoriyi gerçek dünyada uygulanabilecek şekilde geliştirmektir (Cobb ve ark., 2003).

Tasarım araştırması, eğitim pratiğinde karmaşık problemler için araştırma temelli çözümlerin geliştirilmesi ile ilgilidir, çünkü öğrenme ve öğretme süreçleri teoriler geliştirmeyi veya doğrulamayı amaçlamaktadır. Tasarım çalışmasının amacı ne olursa olsun, araştırma süreci her zaman sistematik eğitim tasarım süreçlerini içerir (Plump, 2013). Yine, tasarım araştırması, araştırmaya dayalı çözümlerin geliştirilmesi ile ilgilidir. Yazarlar, tasarım temelli araştırmaların ayrıntılarını resmetmek için çeşitli gösterimler kullanabilirler, ancak genellikle çeşitli aşamalara sahip olduklarını kabul ederler (Plump, 2013). Örneğin Cobb ve ark. (2003), tasarım çalışmasını hazırlama, çalışmayı yürütme ve daha sonra geriye dönük analiz olarak bu aşamalardan söz etmektedir. Ayrıca, çeşitli araştırmacılar raporlarında aynı kategoriyi kullanmışlardır (Cobb, Gresalfi ve Hodge, 2009; Gravemeijer ve Cobb, 2006; Gravemeijer ve Cobb, 2013).

Tasarım temelli araştırmanın ilk aşamasına göre, yerel bir öğretim teorisinin sınıf uygulamaları sırasında değerlendirilip gözden geçirilebileceği vurgulanmaktadır. Devam eden süreçte öğrenme hedefleri netleştirilmeli, öğretim başlangıç ve bitiş noktaları belirlenmelidir. Öğrenme hedeflerinin belirlenmesi, değerlendirme veya tarih yoluyla olabilir. Bir okul müfredatının verildiği şekilde kullanılmaması önemlidir, öğrenciler için en iyi şekilde incelenmeli, yeniden düzenlenmeli ve tanımlanmalıdır. İçeriğin ana fikri burada da önemli bir noktadır (Gravemeijer & Cobb, 2013).

Bu çalışma, prizmanın temel özellikleri ve elemanları, prizmaların yüzey alanı, silindirin yüzey alanı ve hacmi bağlamında tasarlanmıştır. Sınıfın öğrenme geçmişine bakıldığında, sekizinci sınıf öğrencilerinin ve üç boyutlu şekilleri içerik ile ilişkilendirebilecekleri iki boyutlu şekiller konusu hakkında ön öğrenmeleri vardır. Dahası, bir prizmanın ne olduğu ve bir küpün ne olduğu ve özellikleri hakkında bilgi sahibidirler. Bu mevcut çalışma için önemli bir konudur, çünkü katılımcı öğrencilerden konu hakkında fikir üretip sınıf içi tartışmalara katılabilmeleri ve matematiksel uygulamalar üretebilmeleri için, önceden sahip oldukları bilgileri kullnabilmeleri beklenmiştir. Başlangıç noktasının belirlenmesi için Gravemeijer ve Cobb (2013) bütün sınıfın yazılı testleri, görüşmeleri veya performans değerlendirmeleri gibi değerlendirmeler yapmayı önermektedir. Mevcut çalışma için, çalışmaya başlamadan önce katılımcı sınıfa ön test uygulanmıştır.

Yine, mevcut çalışmanın hazırlanma süreci için, sınıf kültürü, akıllı tahtalar, dinamik geometri yazılımı, somut öğrenme materyalleri ve çalışma sayfaları gibi öğretim sürecinde kullanılabilecek mevcut öğretim araçları öğretim dizisine entegre edilerek, öğrenci ihtiyaçları ve göre ulusal müfredat ile tutarlı olarak tasarlanmıştır. Ayrıca, planlar, eğer gerekliyse, içerikte herhangi bir değişiklik veya gelişme yapmanın mümkün olabileceği şekilde esnek bırakılmıştır. Çalışma formüle edilirken sınıf kültürü ve öğretmenin proaktif rolü dikkate alındı. “Sınıf normları neler, ne tür tartışmalar olabilir, ne tür aktiviteler öğrencileri sınıf tartışmalarına katılmaya motive edebilir, konuyu dikkatleri üzerine çekerek, sınıf tartışmalarını nasıl başlatabilir ve uygulayabilir” mevcut çalışmanın tasarımını formüle etmek için oluşturulan temel sorulardı. Ayrıca, çalışmanın tasarımını formüle etmek için, bir yol olarak izleyebilmek için varsayıma dayalı öğrenme yörüngesi oluşturulmuştur. Bu öğrenme yörüngesi, toplamda dört buçuk hafta ve her hafta için yedi ders saati olarak planlandı.

Tasarım tabanlı araştırma modelinin ikinci kısmı oluşturulan öğretim dizisinin ve varsayıma dayalı öğrenme yörüngesinin uygulanma süreci gerçekleşir (Gravemeijer ve Cobb, 2006). Bu çalışma için, veri toplama ve üretim süreci, varsayıma dayalı öğrenme yörüngesinin aşamalarının uygulanmasını içermiştir. Bu süreç, haftalık mini döngüler içeren bir büyük döngüden oluşuyordu. Haftada yedi dersten ve toplam dört buçuk hafta sonra çalışma tamamlandı. Öğretim dizisi ve öğrenme aktiviteleri hazırlanırken yapılmış araştırmalar, öğrencilerin düşünme ve öğrenme düzeyleri dikkate alınmıştır. İlk hazırlanan aktiviteler, katılımcı olmayan sekizinci sınıftan on rastgele seçilmiş öğrenciye uygulandı. On öğrenciden toplanan bu veriler doğrultusunda, araştırma ekibi çalışma sayfaları ve öğretim dizileri üzerinde revizyonlar yaptı ve ana çalışma bununla başladı. Revize etme, öğrenme yörüngesi ve içerik ana çalışmada uygulanmıştır. Ancak, bu süreçte, öğretim dizisinde, varsayıma dayalı öğrenme yörüngesinde ve sonraki derslerin etkinliklerinde öğrencilerin ihtiyaçları doğrultusunda yapılan bazı değişiklikler olmuştur. Öğretim dizisi boyunca öğrenciler bireysel olarak ve bazen çiftler halinde çalışmaya devam ettiler. Bu çalışmalar sırasında, katılımcı öğretmen ve araştırmacı, çalışmaların ilerleyişini, öğrencilerin nasıl farklı düşündüklerini ve sınıfta tartışabilecekleri konuları belirlemek için öğrencileri veya çalışma gruplarını kontrol etmişlerdir. Öğrencilerin bireysel veya ikili grup çalışması tamamlandıktan sonra sınıf tartışmaları başladı ve öğrencilerin farklı yorumları, gösterileri, soruları nedenleriyle birlikte değerlendirildi. Bu süreç, tüm çalışma boyunca takip edilmiştir.

Tasarım tabanlı bir çalışmanın son aşamasında geçmişe yönelik analiz yapılır. Bu bölüm, öğrencilerin ihtiyaçlarına göre yapılan öğretim dizisinin uygulanması sırasında ortaya çıkan revizyonları açıklamaktadır. Tasarım tabanlı çalışmanın amacı, bilgi edinme ve öğrenme ortamı ile öğrencilerin öğrenmesi arasındaki ilişkiyi anlamaya yönelik olduğundan, çeşitli kaynaklardan çeşitli veri setlerini toplamak ve bu çalışma sırasında öğrencilerin düşünme sürecini değerlendirmek bir zorunluluktur (Gravemeijer ve Cobb, 2013). Ana amaç, büyük veri setini sistematik ve doğru bir şekilde analiz etmektir. Veri analizi sürecinin güvenilirliğini sağlamak için, deneyin tüm adımlarının belgelenmesi gerekir. Çalışmanın başlangıcından itibaren, çalışma boyunca ve geriye dönük analiz olarak çalışmanın sonunda değerlendirmeler yapılmalıdır. (Gravemeijer ve Cobb, 2013, Gravemeijer ve van Eerde, 2009). Buna göre, çalışmanın başlangıcında, çalışma boyunca ve bitişte geriye dönük olarak araştırmacı ve katılımcı öğretmen tarafından değerlendirmeler yapılarak öğrenci ihtiyaçları doğrultusunda gerekli değişiklikler yapılmıştır.

Katılımcılar

Nitel bir araştırma çalışmasının özellikleriyle ilgili olarak, katılımcı sayısı sınırlı kalmıştır. Amaç bulguların genelleştirilmesi ile ilgili olmadığından, çalışma Ankara ili, Yenimahalle ilçesinde bir devlet okulunda gerçekleştirilmiştir. Mevcut çalışma, araştırmacının çalıştığı okulda gerçekleştirilmiştir. Bu okul ve katılımcı öğretmen gönüllülüklerinden ve kolay erişilebilirlik nedeniyle seçildi (Fraenkel ve Wallen, 2014).

Katılımcı sınıf, toplamda 16 kız ve 19 erkek, 35 öğrenciden oluşuyordu ve katılımcı öğretmen tarafından, sınıf içi iletişim becerileri ve sınıf etkinliklerine ve tartışmalarına katılmaya istekli olmalarına göre seçilmiştir.

Veri Toplama

Toplanan veriler; (a) tüm derslerin video kasetlerini, öğrenim ortamından ayrıntılı alan notlarını ve öğrencilerin yazılı çalışmalarını içeren sınıf temelli veriler; (b) okul araştırma ekibi toplantılarından gelen tartışmaların ses kayıtları ve (c) başarı puanlarında herhangi bir değişiklik olup olmadığını öğrenmek için çalışma öncesi ve sonrası öğrencilere uygulanan ön test-son test sonuçlarıdır.

Veri Analizi

Sınıf tartışmasını belgelemek ve analiz etmek için, Toulmin'in modeline göre uyarlanmış Krummheuer'in (2015) argümantasyon modeli kullanılmıştır. Rasmussen ve Stephan (2008), sınıf tartışmasını bu yolla analiz etmek için, matematiksel fikirleri ve matematiksel uygulamaları belgelemek için üç aşamalı bir yöntem geliştirdiler. Bu yöntem, veri kümesinin düzenlenmesi için yardımcı olup ve paylaşılmış fikirlerin matematiksel uygulamalara nasıl dönüştüğünü ortaya çıkarır.

Verilerin analizinin geçerliği ve güvenirliği için çeşitli yöntemler kullanılmıştır. Veriler sınıf gözlemleri, video kayıtları, alan notları gibi çeşitli ve zengin kaynaklardan toplandı. Veri kodlaması için üye kontrolü ve karşılıklı kontrol yapılarak verilerin analizi neticesinde yapılan yorumlar tartışılmıştır. Ayrıca, analiz sonuçları ayrıntılı ve zengin açıklamalar kullanılarak sunulmuştur.

Öğrencilerin ön test-son test sonuçlarının analizinde, farklılıkları değerlendirmek için eşleştirilmiş t-testi uygulanmıştır.

Sınırlılıklar

Çalışma ile ilgili ilk sınırlılık, tasarım tabanlı bir çalışma olmasından dolayı bulguların fazla genelleştirilemiyor olmasıdır. Çalışmanın öğretim dizisini başka okullardaki sekizinci sınıflarda uygulanması genelleştirme düzeyini artırabilir. Ayrıca, çalışmanın bir başka sınırlaması da çalışmayı sadece bir makro döngüye dayandırmaktır. Ana çalışmadan önce, daha doğru veri seti elde etmek için bir pilot çalışma yapılması uygun olacaktı. Ancak, pilot çalışma yapılmamasına rağmen, çalışmanın öğretim dizisi, diğer matematik öğretmenleri ile görüşülerek ve görüşlerini alarak uzun bir sürede hazırlanmıştır. Daha sonra hazırlanan içerik, uygunluğunu ölçmek için katılımcı olmayan diğer bir sınıftan on öğrenciye uygulanmıştır, böylece bu çalışmalar bir pilot çalışmanın boşluğunu doldurabilir.

Sonuç ve Tartışma

Bu araştırmanın ana odak noktası, bir öğretim dizisi ve varsayıma dayalı öğrenme yörüngesinin uygulanması sırasında sekizinci sınıf öğrencilerinin katı cisimlerde matematik uygulamalarını çıkarmaktı. Öğretim dizisi, öğrencilerin geometrik kavramları anlamalarını geliştirmek amacıyla öğretimi desteklemek için argümantasyon ve DGy ile tasarlanmış tartışma ortamları ve öğretim etkinlikleri tarafından desteklenmiştir.

Buna göre, mevcut çalışmanın öğrenme yörüngesi sınıf ortamında meydana gelebilecek matematik uygulamalarının göstermek için bir temel olarak kullanılmıştır. Matematiksel fikir şeması, sınıf tartışmaları yoluyla formüle edilen sınıf matematik uygulamalarını analiz etmek için kullanılmıştır (Andreasen, 2006).

Bu bağlamda, bu çalışmada elde edilen dört matematiksel uygulama (a) prizmaların tanımı ve özellikleri, (b) prizmaların yüzey alanı bulma, (c) yüzey alanı bulma silindir ve (d) silindir hacmi bulmadır. Ek olarak, öğrencilerin matematiksel uygulamaları üretmeleri için hangi matematiksel fikirleri kullandıkları açıklandı. Bu matematiksel uygulamalar öğrenciler tarafından oluşturuldu ve bu uygulamaları destekleyen paylaşılan fikirler, aşağıdaki Tablo 1’de gösterilmiştir.

Tablo 1 Çalışmada ortaya çıkan dört matematiksel uygulama ve onların oluşumunu destekleyen fikirler

Matematiksel uygulamalar ve destekleyici fikirler
Uygulama 1: Prizmaların tanımının ve özelliklerinin bulunması
Fikir 1: Binaların çatılarının ve kamp çadırlarının prizma olduğunun anlaşılması
Fikir 2: Küpün prizma olduğunun anlaşılması
Fikir 3: Prizmaların taban şekli ve diğer elemanları arasındaki ilişkilerin anlaşılması
Fikir 4: Silindirin prizma olmadığının anlaşılması
Uygulama 2: Prizmaların Yüzey Alanını Bulma
Fikir 1: Bir prizmayı kaplamak, aslında açılım çizimini ifade eder.
Fikir 2: Birim kareleri sayma
Fikir 3: Birim kareleri sayımından alan hesaplamaya geçiş
Fikir 4: Prizmanın yüzey alanı için formül üretilmesi
Uygulama 3: Silindir Yüzey Alanı Bulma
Fikir 1: Silindirin açılımının yapısı
Fikir 2: Silindirin daire tabanının çevresi ve yan yüzünün kenarı arasındaki ilişki
Fikir 3: Silindirin yüzey alanı, yan yüz alanı ve daire taban alanı tarafından oluşur
Uygulama 4: Silindirin Hacminin Bulunması
Fikir 1: Hacim üçüncü boyutla ilgilidir
Fikir 2: Hacim bir şeklin içine doldurmakla ilgilidir.
Fikir 3: Hacim hesaplaması, genişlik, uzunluk ve yükseklik bilgisini gerektirir.
Fikir 4: Hacim, taban alan ve yüksekliğin çarpımına eşittir.

İlk matematiksel uygulama, prizmanın tanımı, prizma çeşitleri ve prizmaların genel özellikleri ile ilgiydi. Bunun için sınıf tartışmalarına öğrencilerin prizma hakkındaki görüşleri alınarak yola çıkılmış, sonrasında öğretim dizisi boyunca prizmaları tanımlamak için gerekli elemanlar, prizmanın tanımı ve genel özellikleri belirlendi. Bu aşama birbiriyle ilişkili iki bölümden oluşuyordu.

İlk kısım prizmaların yapısının anlaşılması ve temel elemanlarının belirlenmesi ile ilgilidir ve ikinci kısım prizmaların yüzey açılımlarının gösterilmesi ile ilgilidir. Bu bağlamda ortaya çıkan ilk fikir, binaların çatısının ve kamp çadırlarının şeklinin prizma olduğunun anlaşılması, ikinci fikir ise küpün prizma olduğunun anlaşılmasıdır.

Bu iki fikir uygulamanın ilk haftasında ortaya çıkarken, daha sonra süreç boyunca kullanıldılar, çünkü bunlar içerik hakkında temel bilgi idi. Süreç boyunca öğrenciler, prizmaların günlük yaşam örnekleri hakkında tartışmış, bir şeklin prizma olması ya da olmaması için sahip olması gereken özelliklerle ilgili fikirler öne sürmüşler ve bu şekilde prizmaların tanımını üretmiş ve prizmaların ana unsurları ve özelliklerinin anlaşılmasını sağlamışlardır. Prizmanın tanımını üretmek ve diğer içeriklerin anlaşılması için yapılan sınıf içi tartışmalar öğrencilerin düşüncesini yönlendirmede etkiliydi. Bu, önceki literatürle tutarlı bir bulgu idi ve matematiksel tartışmalar, prizmaların tanımı ve genel özellikleri ile ilgili olarak bilgilerini geliştirdi ve desteklediler (De Villiers, Govender ve Patterson, 2009; Tsamir, Tirosh, Levenson, Barkai ve Tabach, 2014).

İkinci kısım prizmaların yüzey açılımlarını anlama ile ilgiliydi. Bu bölümde, üçüncü fikir, bir prizmanın taban şekli ile diğer bölümleri arasındaki ilişkiyi anlamak ve dördüncü fikir olarak bir silindirin prizma olmadığının anlaşılması ortaya çıktı. Bu etkinlik sayfalarının amacı, prizmaların farklı şekillerde görünüşleriyle çalışarak prizmaların açılımı için bir temel oluşturmaktı. Öğrenme yörüngesinin bu kısmı "şeker paketleme fabrikası" konsepti altında hazırlandı. Sorulardaki her bir şekil birim küpler kullanılarak hazırlandı. Bunun sebebi öğrencilerin her bir birim karenin, şekil için kenar uzunluğu teşkil ettiğini anlamalarını sağlamaktır. Öğrenme yörüngesinin bu ikinci bölümünde, GeoGebra dosyası tarafından desteklenen her etkinlik bireysel veya grup halinde çalışıldıktan sonra, öğrencilerin içeriği kavramsal olarak anlamalarını sağlamak için GeoGebra dosyasında sınıf kontrolü yapıldı. Ayrıca, tartışmalarla oluşan matematiksel fikirler matematiksel uygulamaların ortaya çıkışını desteklemiştir. Örneğin, birim küpler tarafından oluşturulmuş verilen şekillere ait farklı görünüşler üzerinde, onlar için şeker paketi çizmeye çalışırken, öğrenciler aslında prizmaların açılımlarını çiziyorlardı. İlk başta, öğrenciler bireysel olarak ve çiftler halinde çalıştılar ve daha sonra sınıf içi tartışmalar ve GeoGebra dosyaları yardımıyla çizimleri kontrol ettiler. Bu sayede öğrenciler, öğrenciler şekillerin farklı yönlerden görünümleri üzerinde çalışırken, onların açılımlarını çizmeleri üç boyutlu düşünme becerilerinin gelişmesine yardımcı olmuştur. GeoGebra dosyasındaki görünümleri inceleyerek, öğrenciler kâğıt ve kalem ortamında sağlanamayan bu çözümleri dinamik olarak kontrol etme şansına sahip oldular. Ayrıca, ilk kısımdan yola çıkarak hazırlanan bu ikinci kısım üzerinde yapılan tartışmalar sonucunda, öğrenciler, prizmanın tanımı, prizmanın temel unsurlarını ve diğer üç boyutlu şekiller arasındaki farklılıklarını kavramışlardır. Bu, aynı zamanda, bu bölümden elde ettikleri bilgileri prizmanın yüzey alanı ile ilgili olan bir sonraki aşamada kullanmaları gerektiğinden önemlidir. DGyYve sınıf içi tartışmaların matematikte ve geometride kullanımı literatürde çeşitli araştırmalara konu olmuştur (Hollebrands, Conner & Smith, 2010; Lavy, 2006; Prusak ve ark., 2012; Vincent, Chick & McCrae, 2005). Önceki araştırmalarla tutarlı olarak, şimdiki çalışmanın sonuçları da geometri derslerinde DGY ve tartışmaların birlikte kullanılmasını öğrencilerin geometrik düşünme becerilerini (Granberg ve Olsson, 2015) ve matematiksel uygulamaların ortaya çıkışını desteklemiştir.

Çalışmadan elde edilen ikinci matematiksel uygulama prizmaların yüzey alanını bulmaydı. Bu uygulamanın ortaya çıkışı, bir prizma için paket üretilmesinin onun açılımını çizmek, birim karelerin sayılması, birim karelerden yola çıkarak alan hesabına geçilmesi ve prizmaların yüzey alanı için formül üretilmesi fikirleriyle desteklenmesiyle olmuştur. Önerilen öğrenme yörüngesi ile tutarlı olarak, öğrenciler, yüzey alanının formülünü üretmek için matematiksel fikirleri birbiri üzerine inşa etmişlerdir. Önce birim küplerle çalışan öğrenciler, bunlar için üretilen paketlerin alanının hesabının yüzey alanı hesabı olduğunu keşfettiler. İçerik üzerinde tartışarak, öğrenciler birim kare bilgisini ölçü birimlerine nasıl aktaracaklarını anladılar. Birim küplerin kullanımı, öğrencilerin yüzey alanın birim karelerini birbirleriyle ilişkilendirmeleri için faydalı olmuştur. Yüzey alanının anlaşılmasını sağlamak için sınıflarının zeminini fayans döşeme bir örnek olarak kullanmışlardır. Literatürde, yüzey alanlarının ana fikrini öğretmek için birim küplerin oluşturduğu birim karelerin kullanımını öneren örnekler vardı (Ben-Chaim, Lappan, & Houang, 1985; Bonotto, 2003; Clements, 2003). Bu nedenle, literatür ışığında ve mevcut çalışmanın sonuçları doğrultusunda, birim karelerinin kullanımının öğretim alanında faydalı olduğu sonucuna varılabilir. Sınıf yüzeyinin döşeme zemini örneğine ek olarak, öğrenciler paketleme kavramını somut olarak görmek için bir birim küpünü bir kâğıt parçasıyla sarmaya çalıştılar. Her iki örnek de mevcut çalışmanın altında yatan teorilerden biri olan gerçekçi matematik eğitiminin doğası için uygun olmuştur. Çalışmanın öğretim dizisi, gerçekçi matematik eğitiminin teorisinin gereksinimleri ile uyumlu olarak hazırlanmıştır. Sorular veya örnekler mümkün olabildiğince gerçek hayat örneklerinden seçilmiştir. Böylece, öğrenciler günlük yaşamdan örnekler vermeleri, yüzey alanı fikrini öğrenme konusunda önemli bulguydu. Matematik derslerinde günlük yaşam temelli örneklerin kullanımı literatürde daha önce vurgulanmıştır (Bonotto, 2003; Van den Heuvel-Panhuizen ve Drijvers, 2014). Bu nedenle, geometri derslerinde günlük yaşam temelli örneklerin kullanımının öğrencilerin kavramsal anlayışını ve ilgili içeriği öğrenmede etkili olduğu sonucuna varılabilir.

Üçüncü matematiksel uygulama, silindirin yüzey alanını bulmaktı. Bu uygulama paylaşılan üç fikir etrafında ortaya çıkmıştır; silindirin açınımının yapısı, daire tabanının çevresi ile yan yüzünün kenarı arasındaki ilişki ve yan yüzey alanı ile daire taban alanı tarafından oluşturulan silindirin yüzey alanı. Bölümün başlangıcı, tüm sınıf tartışması, gerçekçi matematik eğitimi teorisi ile tutarlı olarak silindir için verilen günlük yaşam örneklerine dayanılarak gerçekleştirildi. Daha sonraki aşamada etkinlik sayfası, öğrencilerden, aslında silindirin açılımı demek olan, silindir şekilli bir şekerleme için bir paket çizmelerini istedi. Bu örnekte, neredeyse bütün sınıf başarıyla uygun bir çizim yapmıştır. Bu, öğrencilerin önceki yıllardaki öğrenmelerine bağlı olabilir. Daha sonraki kısımlarda içerik, silindir unsurları arasındaki ilişkiyi anlamaya dayanıyordu. Öğrencilerin bu ilişkiyi kavramalarında GeoGebra dosyası yardımcı oldu. Örneğin, bir silindirin yan yüzünün şekliyle ilgili problemi çözmek için, bir öğrenci, bu yüzeyin etrafında bir şey sarmak istediğinde, bir araya gelen iki noktaya ihtiyaç olduğunu açıkladı. Üst taban için iki nokta, alt taban için iki nokta ve toplamda dört nokta. Böylece, öğrenci bir silindirin yan yüzünün dörtgen olması gerektiği sonucuna varmıştır. GeoGebra dosyasında silindirin açılır kapanır hareketli halini izlemek, öğrencilerin silindirin yan yüzeyinin neden dörtgen olması gerektiğini göstermesi açısından etkiliydi. Bütün sınıf tartışmasına katılarak, çember tabanı çevresi ve yan yüzünün kenarı arasındaki eşitlik fikrini bu şekilde üretmişlerdir. Dahası, bu fikri üreterek, silindirin yüzey alanına bir adım attılar. O zamana kadar yapılan çalışmalar neticesinde, öğrenciler yüzey alanının verilen şeklin açılımı ile ilgili olduğunu anladılar. Öncelikle, silindirin dairesel olan tabanının çevresi ve yan yüzey uzunluğunun eşitliği hakkında fikir üretildikten sonra, iki daire tabanı ve yan temel alanlarının hesaplanmasına geçildi. Bu bağlamdaki tartışmalar, öğrencilerin silindirin yüzey alanı için hesaplama fikrini üretmelerini sağlamıştır. Bu bulgu, önceki araştırmalarla (Aktümen, Baltacı, ve Yıldız, 2011; Hohenwarter ve Jones, 2007), DGY'nin derslerde kullanımı, içeriğin gerçek hayattaki örneklerle ve sorularla desteklenmesi bağlamında tutarlıdır. Bu nedenle, sınıf içi tartışmaların, DGY'nin kullanımının ve öğretimin gerçek yaşam bağlamıyla desteklenmesinin, silindirin yüzey alanını anlamada etkili olduğu sonucuna varılabilir (Lai ve White, 2014). Ayrıca, silindirin yüzey alanı için formül üretirken, alan hesaplama yolunu cebirsel olarak ifade etme yolunu temel alan bir tartışmada yer almıştır. Ayrıca, GeoGebra dosyasında, öğrenciler silindirin yüzey alanı formülünü gözlemlediler. Ek olarak, verilen uzunluktaki değişiklikleri hem dinamik hem de cebirsel olarak gözlemleyebildiler.

Literatürde, geometri ile cebir arasındaki ilişkinin anlaşılmasını geliştirmek için DGY kullanımını destekleyen araştırmalar vardı (Atiyah, 2001; Davis, 1998; Edwards ve Jones, 2006). Böylece GeoGebra'nın kullanımı prizma ve silindir yüzey alan formülünün cebirsel olarak ifade edilmesinin anlaşılmasında etkili olmuştur (Erbas, Ledford, Orrill ve Polly, 2005).

Dördüncü matematiksel uygulama, silindirin hacmini bulmaktı ve sınıf içi tartışmalarda paylaşılan dört fikirle desteklenmiştir; hacim üçüncü boyuttur, hacim bir şeklin içine doldurma ile ilgilidir, hacim hesaplaması, genişlik, uzunluk ve yükseklik bilgisini gerektirir ve hacim, taban alanı ve yüksekliğinin çarpılmasına eşittir. Uygulama, öğrenme yörüngesinin silindirin hacminin bilgisini inşa etme öğrenme hedefine dayanan son aşamasına paralel olarak gerçekleşmiştir. Ana süreç “hacim nedir?” sorusuyla başlatıldı. Bu soruya öğrencilere sorulduğunda, alan ve hacim arasındaki farklılıklar ile ilgili bir tartışma daha ortaya çıktı. Bu farklılıkları açıklayabilmek ve anlayabilmek için, öğrenciler sınıfın zemin döşenmesi ve sınıfın birim küpleriyle doldurulması konusunda örnekler sundular ve bunun üzerinde tartıştılar. Bu tartışma ile öğrenciler alan ve hacmin anlamını net olarak anladılar. Ayrıca, başka bir tartışma konusu da “küpün ve dikdörtgen prizmasının hacminin nasıl hesaplanacağı ve bu işlemler için nelere ihtiyacımız var?” şeklinde oluşmuştur. Bu süreç, öğrencilerin bir cismin içini birim küplerle doldurduklarında aslında bu birim küp sayısının o cismin hacmini verdiği konusundaki eski öğrenmelerini hatırlamaları planlanmıştır. Hatta bu hesaplamayı yaparken o birim küpleri saymak yerine cismin üç boyutun çarpımıyla elde edilebileceğini de hatırlamaları beklenmiştir. Bu şekilde birim kareler ve birim küpler kullanılarak alan ve hacim hesabının yapılmasını öngören çalışmalar literatürde de bulunmaktadır (Battista & Clements, 1996, 1998; Ben-Chaim, Lappan, & Houang, 1985; Cohen, Moreh & Chayoth, 1999). Buna ek olarak, diğer kritik sorular “kenarları olmadığına göre bir silindiri birim küplerle nasıl doldurulabileceği?” Ve “silindirin hacmini nasıl bulabilirler?” idi. Konuyu daha açık hale getirmek için silindir şeklinin doldurulmasını göstermek üzere hazırlanan GeoGebra dosyası açıldı. Bu hareketli görseli izleyerek, silindirin hacim bağıntısını oluşturmak için, silindirin yüksekliğinde daireleri üst üste yerleştirmeleri gerektiğini anlayabilmişlerdir. Bu sayede, öğrenciler “taban alanı ve yüksekliğinin çarpımı” olarak formüle edilebilen küp ve dikdörtgen prizmalarından gelen bilgilerini aktarmış oldu. Ayrıca, şeklin iç bölgesinin doldurulması günlük yaşam bağlamı için uygun bir örnektir. Dahası, bu bulguları destekleyen araştırmalar literatürde vardır (Enochs, & Gabel, 1984; Hirstein, 1981; Livne, 1996). Bu bağlamda, DGY, sınıf içi tartışmalar ve günlük yaşam örneklerinin kullanımının silindirin hacmini öğretmede etkili olduğu söylenebilir.

Araştırmanın bulgularına göre, katılımcı öğrencilerin akıl yürütme, gerekçelendirme, yorumlama ve diğer fikirlere dayanarak yeni fikirler üretme yoluyla öğretim faaliyetlerine katılabileceği belirtilebilir. Bu şekilde, üç boyutlu şekillerin (özellikle bu çalışma için prizmalar ve silindirler için) kavramsal ve anlamlı bir öğrenme gerçekleştirebilirler (Hallowell, Okamoto, Romo ve La Joy, 2015).

Ayrıca, çalışmanın sonuçları, öğrencilerin DGY ve tartışmacı sınıf ortamının desteğiyle ilgili konuya ilişkin gerekçelerini geliştirebilmelerini desteklemektedir. Bu çalışmanın matematiksel uygulamaları, benzer bir öğrenme ortamında yüzey alanı ve/veya üç boyutlu şekillerin hacmi hakkında çalışmak isteyen diğer araştırmacılar için bir pencere açabilir.

I: CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: ŞAHİN DOĞRUER, ŞULE
Nationality: Turkish (TC)
Date and Place of Birth: 12 October 1984, Gaziantep
Phone: +90 543 290 68 70
email: sule_sahinn@hotmail.com

EDUCATION

Degree	Institution	Year of Graduation
MS	METU Elementary Mathematics and Science Education	2009
BS	Hacettepe University Elementary Mathematics Education	2006
High School	Gölbaşı Anadolu High School, Ankara	2002

WORK EXPERIENCE

Year	Place	Enrollment
2007-Present	MoNE	Mathematics Teacher
2006-2007	Gölbaşı A Dersanesi	Mathematics Teacher

FOREIGN LANGUAGES

Advanced English

PUBLICATIONS

Articles Published in National Journals

Şahin Doğruer, Ş, Işıksal, M. & Koç, Y. (2015). Beşinci Sınıf Matematiksel Söylem Üzerine Bir Durum Çalışması. *Gaziantep University Journal of Social Sciences, 14* (1), 299-322. DOI: 10.21547/jss.256779

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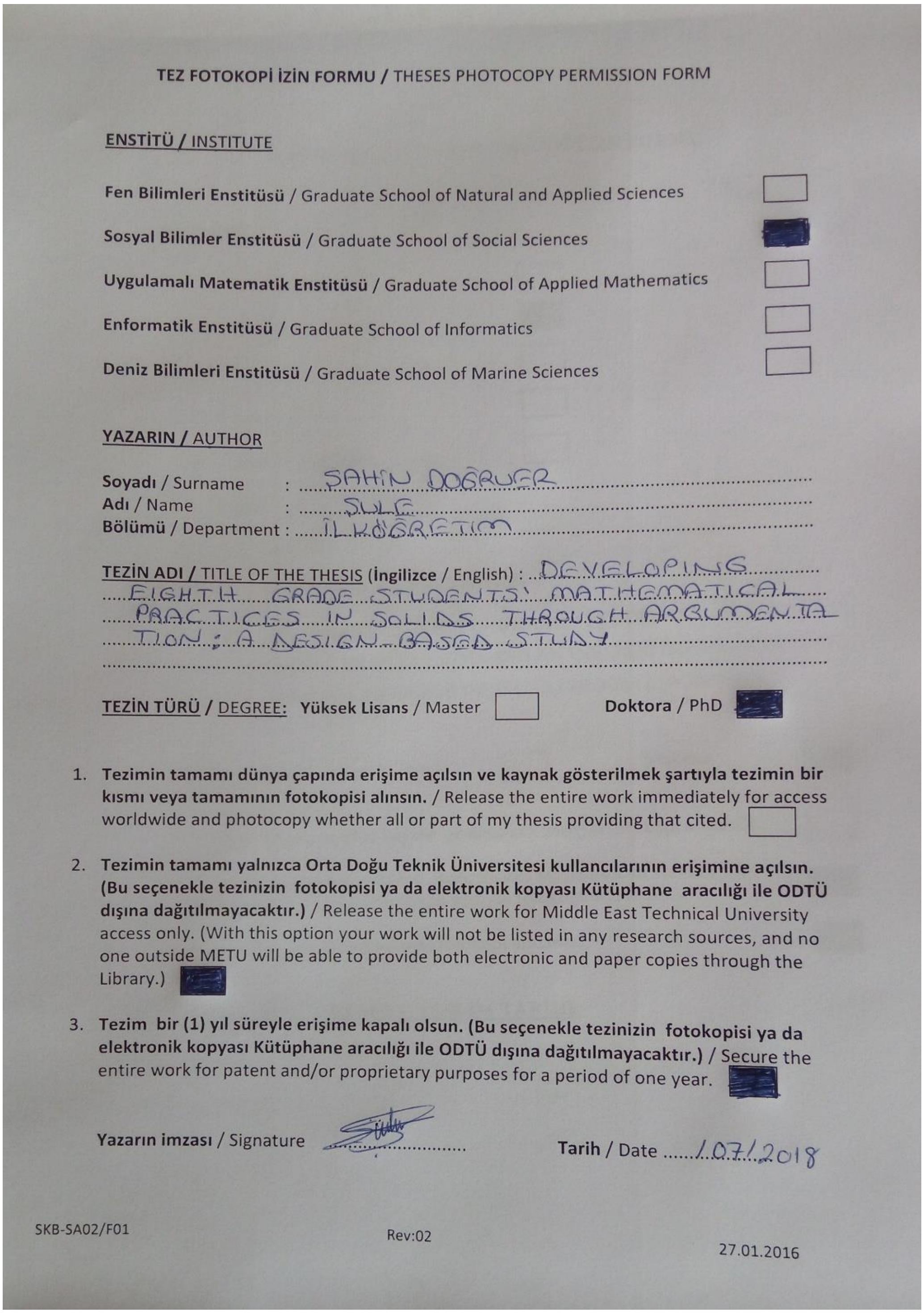
Şahin, Ş., Işıksal, M. & Koç, Y., (2011). Bir Beşinci Sınıf Öğretmeninin Matematiksel Sınıf Söylemi Kullanımı üzerine Bir Durum Çalışması. *"Matematikçiler Derneği 10. Matematik Sempozyumu"*, s.78-79.

J: TEZ FOTOKOPİ İZİN FORMU



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YAZARIN / AUTHOR

Soyadı / Surname : ŞAHİN DOĞRUER
Adı / Name : ŞULE
Bölümü / Department : İLKÖĞRETİM

TEZİN ADI / TITLE OF THE THESIS (**İngilizce** / English) : DEVELOPING EIGHTH GRADE STUDENTS' MATHEMATICAL PRACTICES IN SOLIDS THROUGH ARGUMENTATION: A DESIGN BASED STUDY

TEZİN TÜRÜ / DEGREE: Yüksek Lisans / Master [ ] **Doktora** / PhD [X]

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