A CASE STUDY ON MIDDLE GRADE MATHEMATICS TEACHERS' USE OF QUESTIONING IN TEACHING LINES AND ANGLES

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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ABSTRACT

A CASE STUDY ON MIDDLE GRADE MATHEMATICS TEACHERS' USE OF QUESTIONING IN TEACHING LINES AND ANGLES

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The purpose of the study was to identify tools that help teachers to use in teacher questioning in middle-grade mathematics classrooms. In addition to this, the study aimed to examine teachers' questioning behaviors concerning teachers' question types and the interaction among the tools for questioning and question types. This study was applied a multiple case with two middle grade mathematics teachers. The participant teachers were video recorded for the lines and angles topic. In one of the classroom, technology was included, and there was a non-technology enhanced classroom environment for the other classroom. The findings of the study showed that, in total, there were six categories of tools for questioning, which included information technology, printed supplementary materials, teacher drawings, student ideas, analogies, and real-life examples. Participant teachers used guiding, probing, and factual questions during their instructions. Participant teachers differed from each other in the types of questions and characteristics of the types of the questions they used throughout the lessons. The relations among the tools for questioning showed that for Teacher Caner, printed supplementary book was closely in relation to students' questions or ideas while for Teacher Barış, supplementary book was

closely related to his drawings while solving worked examples. The relations among the tools for questioning and question types of the teachers showed that Teacher Caner was used tools for questioning with all question types while Mr. Barış was only used gudinig questions with all his tools for questioning. The findings of the study were discussed and the further studies were suggested.

Keywords: Middle School Mathematics Teachers, Questioning, Question Types, Technology

ÖZ

ORTAOKUL MATEMATİK ÖĞRETMENLERİNİN DOĞRULAR VE AÇILAR KONUSU ÖĞRETİMLERİ SIRASINDAKİ SORU SORMA KULLANIMLARINA İLİŞKİN BİR DURUM ÇALIŞMASI

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Bu çalışmanın amacı, ortaokul matematik öğretmenlerinin matematik derslerindeki soru sormalarına yardımcı olan araçları ortaya koymaktır. Buna ek olarak, öğretmenlerin soru sorma davranışlarını, kullandıkları soru türlerine ve soru sorma araçları ile soru türleri arasındaki etkileşime göre incelemeyi amaçlamaktadır. Bu çalışma, iki ortaokul matematik öğretmeni ile yapılan bir durum çalışmasıdır. Katılımcılardan birinin sınıfında teknoloji kullanılıyorken, diğer katılımcı ders işlerken teknoloji kullanmamaktadır. Katılımcıların, doğrular ve açılar konusuna ait ders işleyişleri bu çalışmanın ana verisini oluşturmuştur. Çalışmanın sonuçları, öğretmenlerin toplam altı soru sorma aracı kullandıklarını göstermiştir: bilgi teknolojisi, basılı kaynak materyaller, öğretmen çizimleri, öğrencilerin fikirleri, analojiler, ve gerçek hayat örnekleri. Öğretmenler yönlendirici, sorgulayıcı, ve olgudal sorular kullanmışlardır. Öğretmenlerin derslerde kullandıkları soru tiplerine bakıldığında ise, soru kullanımları çeşit anlamında benzer olmakla birlikte, kullandıkları soruların karakteristikleri birbirinden farklı olabilmektedir. Soru sorma araçlarının kendi aralarındaki ilişkisine bakıldığında çalışmanın sonuçları, Caner

öğretmen için derste kullandığı kaynak kitaplar ile öğrencilerin soru ya da fikirlerinin yakın bir bağlantı içinde olduğunu gösterirken, Barış öğretmen için, derste kullandığı kaynak ders kitabı ile örnek soru çözerken yaptığı çizimlerin yakın bir bağlantı içerdiğini göstermiştir. Soru sorma araçları ve soru tiplerinin birbiriyle ilişkilerine bakıldığında, Caner öğretmenin tüm soru sorma araçlarıyla tüm soru tiplerini kullandığını, Barış öğretmenin ise tüm soru sorma araçlarını sadece yol gösterici (guiding) soru çeşidiyle kullandığını ortaya çıkmıştır. Çalışmanın bulguları tartışılmış ve çalışmalar önerilmiştir.

Anahtar Kelimeler: Ortaokul Matematik Öğretmenleri, Soru Sorma, Soru Tipleri, Teknoloji

to My Lovely Family

and

to My Dear Husband, Bilgi

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CHAPTER 1

INTRODUCTION

Questioning has a long history since Plato and Socrates (Ellis, 1993). Over a hundred years, it has had a growing interest in the literature (Ramsey, Gabbard, Clawson, Lee, & Henson, 1990). People learn what they think about each other or an issue through talking, asking questions, and giving answers (Christenbury & Kelly, 1983). In classroom learning, questioning is an essential tool that can be used as a teaching method or as a formative assessment technique for teachers (Jiang, 2014). In addition to using questions for inquiry, teachers can use questions for telling students what she wants to hear or to focus on to change their current undesirable behaviors (Mason, 2014). While teachers make questioning, students are not only expected to answer their teachers' or peers' questions; they are also expected to ask questions to them and themselves (Camenga, 2013; Mason, 2014).

Questioning is related to both questions and statements, which encourage students for clarifying complexities of the instruction, and it is a way of gathering student attention for their mathematical progress (Mason, 2014). There is a common point in the literature that when questions are formulated and appropriately posed, they can make positive changes in students' achievement (Redfield & Rousseau, 1981). Therefore, teachers' questioning is an essential component of student achievement (Redfield & Rousseau, 1981; Franke et al., 2009). Depending on the teacher's ability to use questioning, the development of student's mathematical thinking can change (Burns, 1985). To have active classroom learning through questioning, the teacher should integrate as many learners as possible into the course, regardless of whether students participate in the class. Teachers help students in reorganizing their ideas stimulating their thinking (Martino & Maher, 1994). What's more, teachers catalyze students' thinking by guiding them through questions.

Because of these, students can reexamine their ideas, and they can revise their original solutions (Martino & Maher, 1994).

Aizikovitsh-Udi and Star (2011) stated that good questions could be used as tools for teachers in the questioning process but they do not guarantee to implement good questioning. Therefore, questioning is a process for implementation of asking questions.

Research has made an effort to obtain more useful questioning behaviors for students' understanding since 1970 (Wilen & Clegg, 1986; Ellis, 1993). To use questioning productively, Wilen and Clegg (1986) suggested that teachers need to use clear questions and they should encourage their students for clarification. Additionally, teachers can use incorrect student responses as learning opportunities. While doing these, they should give students enough time, which increases students' productive thinking (Chin, 2006), to wait for their answers to questions. Moreover, teachers need to wait sometime after they took students' answers to give the learners opportunity to think on what their peers said. In this regard, teachers need to use questioning in an efficient way, which requires fast decision-making process of questioning (Zee & Minstrel, 1997). To conclude, teachers are required to manage the questioning process better.

1.1 Problem Statement

The middle school mathematics curricula in Turkey support a classroom environment where teachers construct mathematical knowledge by communicating with others (MoNE, 2013). MoNE (2013) expects teachers to create a rich classroom atmosphere where students can have a meaningful understanding of mathematical topics allowing them to communicate with each other. In this manner, reliable and in-depth knowledge about questioning behaviors of middle school mathematics teachers in classrooms need to be considered (Ryans, 1973) regarding teachers' creation of such an interactional environment. However, there are limited studies to describe teachers' questioning in mathematics classrooms in Turkey as compared to

international studies (Ong et al., 2010; Dillon, 1988; Heritage & Heritage, 2013; Wimer, Ridenour, Thomas, & Place, 2001; Aizikovitsh-Udi, Clarke, & Star, 2013).

Teachers' use of tools available in classrooms or integrated by the teacher encourages students' thinking in the instructional context and feed teachers' instruction as well (Gall, Dunning, Banks, & Galassi, 1972). For example, the use of manipulatives in mathematics lessons was one of tools of questioning which has an influence on the flow of the mathematical dialogues (Olkun & Toluk, 2004). The fact that such a tool has an effect on teacher's questioning, the lack of knowledge about what these tools are causes of addressing mathematics teachers' questions in a superficial way. We hypothesize that explaining the teacher's questioning behaviors without ignoring the presence and use of the tools helps mathematics teacher educators to understand teachers' questioning process in mathematics lessons and to explain teachers' questioning behaviors. Although questioning literature includes studies on teachers' questioning behaviors, little has been done to examine the tools that play a role in teachers' questioning process and their use of asking questions. That provides us the evidence of the classroom dynamics in middle grade mathematics classrooms concerning the use of tools playing a role in the questioning process. The use of the tools will provide insight to teacher educators with a deeper understanding of how questioning penetrates in mathematics classrooms.

The questioning behaviors of teachers in the literature has been examined quite frequently, especially in terms of question types. There are various types of questions studied in the literature (Ali, 2007; Shahrill & Clarke, 2014; Piccolo et al., 2008; Sahin & Kulm, 2008). To understand the meaning of the question types during the instruction, questions should be analyzed considering the instructional contexts of lessons (Carlsen, 1991; Şahin & Kulm, 2008). As the instructional context allows us to interpret the initiator of questioning, respondents to the questions, tools for the questioning, and the instructional content carried within the questioning process, we become knowledgeable about the process of questioning interaction in mathematics classrooms. In line with this, understanding teacher questioning depends on understanding these mentioned interpretations of instructional contexts. In available

literature, there are limited studies that encourage interpreting teacher questioning within instructional contexts (e.g.: Koizumi, 2013).

A teacher should provide classroom atmosphere in which students and the teacher can construct knowledge together (Hufferd-Ackles et al., 2004). In this kind of atmosphere, teachers are recommended to make changes in their lessons to create more dynamic learning environments where students and teacher interaction is in higher level (Acar & Kılıç, 2011; Piccolo et al., 2008). Although teachers could do this by questioning, teachers have difficulty in providing questions for getting students one-step further of their mathematical thinking (e.g., Franke, et al., 2009). To know the types of questions teachers' use helps to examine such difficulties teachers encounter together with the examination of how the question types were used (Koizumi, 2013). For this purpose, which question types teachers use tools for questioning when asking questions is essential. As we do not know the harmony and reflection of the tools for questioning to classroom teaching that contribute to teachers' asking questions, we are inadequate to interpret teachers' question types. That will be informative about how teachers manage and shape the questioning process. In available literature, there are limited studies (e.g.: Mitchell, 1994) that present a map of teachers' implementation of questioning.

Thus, the purpose of the study is to reveal tools that help teachers to use in teacher questioning in middle-grade mathematics classrooms. In addition to this, the study aims to examine teachers' questioning behaviors concerning teachers' question types and the interaction among the tools for questioning and question types. For these purposes, this research seeks answers to the following research questions:

1. What tools do middle grade mathematics teachers use in their mathematical questioning?
2. How do middle school mathematics teachers make use of tools in mathematical questioning during instruction?
3. How do middle grade mathematics teachers use factual, probing, and guiding questions during their instructions?

4. What is the nature of relationship among tools for questioning and question types in the teachers' questioning during the instruction?

a. How do the teachers' tools for questioning relate to each other?

b. How do the teachers' tools for questioning are related with their question types?

1.2 The significance of the Study

Teachers' questions and teachers' answers, students' questions, and students' answers need to be in harmony within the classroom communication dialogue to give students independence while constructing mathematical knowledge (Camenga, 2013). While providing the harmony, teachers need to manage the classroom environment for questioning (Darragh, 2005). That highlights how a teacher is constructing the way of teaching through questions and encourage learners to ask their questions in classroom contexts (Mason, 2002). In line with this, the management of the tools that help teacher to use questioning in classroom environment gives evidence about the quality and richness of the interaction between a teacher and learners, and help researchers to understand the structure of questioning regarding teacher and student interaction (Mitchell, 1994). Supported to this, as the related literature suggests, the usage of manipulative in lessons, which is a tool in teacher questioning, changes teachers' way of questioning as the way of utilizing manipulative supports learning through exploration (Olkun & Toluk, 2004). Therefore, in this study, to have a broader sense of questioning in mathematics classrooms, all the tools used for questioning in mathematics classroom were examined.

Teachers construct mathematical knowledge through the mathematical questions. The role of teachers' questions is a critical indicator to understand teachers' instructions through questioning (Barker, 1982). The use of different types of questions is related to elaboration on teachers' classroom discourse regarding questioning (Hufferd – Ackles, Fuson, & Sherin, 2004). The types of teachers'

questions and the use of the question types require to understand the instructional context (Wragg & Brown, 2001). In response to the research of Şahin and Kulm (2008), this study is significant to examine the usability of the characteristics of the question types Şahin and Kulm (2008) suggested for the analysis of classroom dialogues for questioning.

Teachers sometimes do not behave as if they have any theoretical consideration to support their questioning process (Delice, Aydın, & Çevik, 2013). On the other hand, there are implicit questioning theories of teachers that help us more accurately understand their questioning process about why they behave and follow such a way considering beliefs of teachers about questioning, pedagogical aspects of questions, and appropriateness of questioning in instructions (Mitchell, 1994). As there are limited studies focusing on mathematics teachers, with this study, this issue will allow us to examine how mathematics teachers behave in classrooms while questioning. In this regard, we will give attention to a fine-grained analysis of teachers' classroom teaching practice. For that purpose, the current study will help mathematics educators to shape and follow a roadmap of professional development on questioning for in-service teachers' understanding the nature of their questioning behaviors deeply.

This study is interested in questioning behaviors of teachers as the behavior is exhibited in classrooms many times (Shahrill & Clarke, 2014; Piccolo et. al., 2008; Sahin & Kulm, 2008; Fraenke et. al., 2009; Ali, 2007; Olkun & Toluk, 2004; Ong, Lim, & Ghazali, 2010), its acceptance as a teaching method of instruction in Turkish middle grade mathematics program (MoNE, 2013), and there are rare studies which point out teachers' questioning as a way of communication. Besides, we chose teacher questioning to examine in this study for the reason that how reform-based curriculum and necessary revisions in Turkey support interactional classroom environments where students should be given the autonomy to answer open-ended questions and should be given activities for doing mathematics.

As a last word, this study has a contribution to Turkish literature on questioning because there is a limited number of studies (Turgut, 2007; Kasar, 2013) which are interested in teacher questioning in mathematics classrooms in Turkey.

1.3 Definition of Important Terms

Based on the research questions and the title of the study, we utilized questioning and teacher questioning, tools for questioning, and question types. We clarified the terms by constitutive definition from the related literature, and operational definition, which is related to the context of the study. When necessary, the terms were clarified by example as well.

Mathematical Questioning: Specifically for mathematics, questioning is described as following:

> Questioning means here the use of questions and other prompts offered to students so as to help them get unstuck or to direct their attention in a potentially useful way so that they make mathematical progress (Mason, 2014, p. 514).

As the description emphasizes, questions and prompts are involved in questioning when a teacher uses them for students' mathematical progress. Therefore, this process focuses on questions and is shaped by how teachers use questions during their instructions. In light of the literature, the teacher's questioning behavior is a part of a questioning process in which teachers help their students through questions and prompts. At this point, Dillon's (1988) seven alternative statements to questions were considered in teacher's questioning as well: declarative statement, reflective restatement, state of mind, invitation to elaborate, speaker's question, class questions, and deliberate silence. In line with these (Mason, 2014; Dillon, 1988), prompts that organize the teacher's questioning were considered as questions as well as they contribute to teacher's questioning process regarding helping students in their difficulties or organizing their attention in a way that they get benefit from it for their mathematical progress. In this study, two middle grade mathematics teachers' verbal question statements in terms of question types, what

tools have contributions to the teachers' questioning, and the way that how teachers use the tools in instructional contexts were within the scope of teacher questioning.

Tools for questioning: Chapin et. al. (2009) described productive talk moves of classroom discussions as tools to support teachers for classroom talk. The talk moves have roles in classroom teaching in terms of improving mathematical thinking, orchestrating students' conversations, and providing an equal mathematical environment. As similar perspective, in this study, tools for questioning were considered as tools that provide teachers to create a learning environment to make questioning in the classroom. In line with this, the tools contribute to teachers' questioning shaping the instructional context through questioning. Interpretation of instructional role of tools in an instructional context reveals what a questioning tool is. For example, a printed supplementary book could be a tool for questioning as it has a role of providing teachers worked examples for questioning in the classroom.

Question type: Orrill (2013) described question types as "where type was an indicator of the way in which the question was posed (thus, a hybrid of form and purpose)" (p.288). Types of questions are varied in literature (Fraenke et al., 2009; Parks, 2010; Harbaugh, Carter, & Capraro, 2008; Way, 2008; Wragg & Brown, 2001; Ali, 2007). In the current study, three types of study were focused: probing, guiding, and factual questions (Şahin & Kulm, 2008).

Probing Questions: This type of questions refers to questions that probe students' prior knowledge or students' answers, ideas, or questions. They require students to make explanations, to make elaborations on their ideas, and encourage them to make deeper thinking (Şahin & Kulm, 2008).

Guiding questions: This type of questions refers to questions that help teachers to guide students while they encounter challenging situations, to open students' perspective in producing problem solving strategies, and to scaffold or lead students in completing a mathematical procedure, or understanding of big mathematical ideas (Şahin & Kulm, 2008).

Factual Questions: This type of questions refers to questions that provide teachers with a view to evaluating what a student knows about a subject. These

questions may be about definitions of concepts, may be a question for completing the procedure, or a result of an exercise (Şahin & Kulm, 2008).

CHAPTER 2

LITERATURE REVIEW

The purpose of the study was to reveal tools that help teachers to use in teacher questioning in middle-grade mathematics classrooms, to examine teachers' questioning behaviors concerning teachers' question types and the interaction among the tools for questioning and question types. In this chapter, the conceptual overview of questioning that identified description of questioning in literature and theoretical background of the study and in-service teachers' use of questioning that guided us to understand the questioning behaviors of teachers with different focuses were mentioned respectively.

2.1 Conceptual overview of questioning

This section examines how questioning was conceptualized in the literature. This conceptualization was made within the description of questioning and theoretical background of the study.

2.2.1 Description of Questioning

Questioning has a long history and it has various connections with contexts or disciplines in literature. It could be a communication skill (Carlsen, 1991), a teaching method of teachers during the instructions (Jiang, 2014), a way of assessment of students in their learning of big ideas to capture (Larson & Keiper, 2007), or a non-verbal behavior giving the responder a signal that makes him/her feel questioned (Dillon, 1988). Depending on these connections, questioning could be described in studies differently. For example, Christenbury and Kelly (1983)

emphasizes questioning as "in classroom practice, questioning is a skill, a process, a strategy, an attitude, an art." (p. 33). In the scope of this study, description of questioning was handled in related to educational perspective which aimed to reveal the value of questioning in classroom teaching, student learning, or other related issues with education and with a dictionary definition that could be interpreted by educational perspective.

Oxford Advance Learner Dictionary describes the word "questioning" as "the activity of asking somebody questions". This description includes what kind of situation we accept for asking questions and what a question is. This description takes questions not specific to education, but in genera, which is in contrast to Dillon (1983) who separates educative questions from everyday questions.

Christenbury and Kelly (1983) described questioning as a path to critical thinking and as a proposed strategy for logical-mathematical intelligence, which is a branch of the multiple intelligences theory. Taylor and MacKenney (2008) and Kurfiss (1988) emphasized the place of questioning in critical thinking process. According to them, in critical thinking process that includes suggesting hypothesis and an examination process intended to be ended by reaching a judgment, questioning is a strategy that requires teachers to scaffold students' learning in answering questions and ending the process with correct answers (Taylor, 2008; Kurfiss, 1988). In line with this, Larson and Keiper (2007) highlighted that questioning is a way of encouring students' thinking in a deeper way as well as a strategy of a teacher to provide students the flexibility of using thinking skills. According to Dillon (1988), questioning is a process in which "a person asks a question" (Dillon, 1987, p.17).

Koizumi (2013) has also drawn attention to questioning together with the attention to teacher questioning. Accordingly, questioning is a process of asking questions to a person who knows the answer to learn an unknown condition by the person who does not know the answer. Besides, he pointed out that, the word "Hatsumon" emphasizes teacher questioning in which the role of the questioner is to create a thinking process for students in order to provide the students the right answer,

which is already known by the teacher. Thus, it can be said that questioning in classroom settings or in everday settings could be different based on the knowledge of the person who asks questions. In addition, not only in asking questions, but also in answering them, teachers' role is not the same with the person who is answering a question. Meij (1994) emphasized when a teacher is a responder, it is different from a general person who is answering a question; a mathematics teacher considers pedagogical issues while questioning. Mason's (2014) following description of questioning in mathematics education pays attention to two pedagogical perspectives of a teacher while asking questions to students (p.514):

> Questioning means here the use of questions and other prompts offered to students so as to help them get unstuck or to direct their attention in a potentially useful way so that they make mathematical progress.

From this point of view, questioning is related to the ways of using questions and other prompts. Secondly, each use of them is not within the scope of questioning. In order to examine questioning, teachers need to use it to conclude the mathematical progress of students. During the process, students can have difficulty of comprehending mathematics and teachers might need to take students' attention to mathematical questions. The current study implemented the definition of Mason (2014) as the definition was for questioning in mathematics education specifically. The details of questions and other prompts and pedagogies of questioning which were essential to understand the Mason's (2014) definition were detailed below to understand the study.

2.2.1.1 Questions and other prompts

Although recognizing a question is easy in daily life, in research studies, to obtain question is difficult (Meij, 1994). For the purpose of understanding the description of Mason (2014), it is necessary to clarify what question and prompt is in educational settings by the help of related literature.

Conner et. al. (2014) described question in their research as 'a request for action or information, not simply an interrogative sentence'. (p.417). The fact that a

sentence could be represented as a question because of its grammatical form is not the only requirement for accepting that sentence as a question. While Mason (2010) acknowledges this information, he adds that some statements have a potential for being a question as they require giving a response. Those kind of statements could be used without question marks while they are questions. In addition to this, non-verbal signals are questions in contexts for the same reason that they require an answer (Dillon, 1988; Van der Meij,1994; Mason, 2014). According to Way (2008), questions can serve as prompts when children are in stuck, are required by their teachers to examine into what they said, or are invited to to participate in a mathematical discussion and so on. Prompts could be questions that teachers use for triggering what they want for questioning.

In this study, the question was approached in a way that they are sentences, which require a response from the interlocutor. In addition to this, the sentence might have a question mark or not. The requirement from the interlocutor is evaluated depending on the voice tone of the teachers and non-verbal behaviours of teachers. Teachers' questioning is examined in question types many times. Studies suggested that teachers can use the following question types to reveal students' thinking: general, specific, probing and leading questions (Fraenke et al., 2009); analytic questions and evaluative questions (Eldel & Paul, 2005), essential question, hook question, diagnostic question, probing question, inference question, interpretation question, transfer question, predictive question, and reflective question (Walsh & Sattes, 2011), self-answered, fill in the blank, who is this, follow-up, open-ended, assess idea, and justification/argument (Orrill, 2013), implicit and explicit questions (Parks, 2010), probing, guiding, and factual questions (Şahin & Kulm, 2008), clarification, extention, and guiding questions (Camenga, 2013), student generated and teacher generated questions (Harbaugh, Carter, & Capraro, 2008), starter questions, questions to stimulate mathematical thinking, assessment questions, and final discussion questions (Way, 2008), controlling questions, cloze technique, genuine-enquiry, meta-questions, and open and closed questions (Mason, 2002), conceptual questions, empirical questions, and value questions (Wragg & Brown,

2001); closed-ended questions specifying closed-procedural, closed-routine, closed-complete statement, closed-verification, closed-terminology, and closed-rhetorical etc. (Ali, 2007). Questions could be low or high cognitive level questions or yes/no questions (Shahrill & Clarke, 2014; Piccolo et al., 2008; Sahin & Kulm, 2008). Mason (2010) categorized questions in five: controlling, cloze technique, genuine-enquiry, meta-questions, open and closed questions while he characterized the way of asking questions that gives mathematics educators clue about the classification of questions: asking as telling and asking as enquiring. Questions may be used to resolve a student's unwanted behavior or to draw attention (asking as telling), as well as to explore a topic (asking as enquiring) (Mason, 2014). Walsh and Sattes (2011) categorized questions in ten types: essential question, hook question, diagnostic question, question to check for student understanding, probing question, inference question, interpretation question, transfer question, predictive question, and reflective question. The types of questions are divided into types according to their instructional functions.

As the literature points out, there are many question types which focus on different aspects of the questions. The current study utilized Şahin and Kulm's (2008) question types which were supported by real mathematics classroom dialogues and have certain criteria that were checked by the teachers' intentions. As the criteria were developed based on teachers' mathematical practices in classroom, this study used Şahin and Kulm's (2008) approach.

2.2.1.2 Pedagogies of questioning

The other essential part to understand Mason's definition (2014) requires understanding pedagogies of questioning. In literature, there are various reasons for using questioning depending on the disciplines. While some studies mention about the purpose of questioning in general, some of them are specific, explaining an area specifically.

Questioning could be used as kind of a formative assessment (Ginsburg, 2009), for supporting teaching method (Jiang, 2014; Wragg and Brown, 2001), or a skill providing mathematical communication (Ramsey et. al., 1990). According to Jiang (2014), while questioning is not going beyond taking attention of students and follow-up actions do not end with meaningful student learning, it can be labeled as a teaching method rather than formative assessment tool which requires teachers using instructional movements effectively for evidence of student learning (Jiang, 2014). Additionally, questioning is a skill that requires using mathematical content knowledge successfully, knowledge about questioning, and creativity for combination of them with a careful planning (Burns, 1985).

2.2.2 Theoretical Background of the Study

Vygotsky's theory takes attention to three interactional ways for human development which people interact; with each other (social interaction), within cultural-historical context where people are interacted with the world (person, object, or institution), and with their selves (personal factors). Classrooms are environment that provide those social, contextual and individual factors for interactions. Those interactions have an effect on students' mental structures. The part of this theory based on education says that learning can vary depending on the cultural-historical context where students construct knowledge. Referring to this theory in classrooms, students and teacher interact with each other, they interact within the culture of the classroom context, and they have individual thinking process while learning. In this manner, classroom interactions should be taken into consideration within the context.

Another related term applied to Vygotsky's perspective for this study is considered as the zone of the proximal development description that is

> The distance between the actual development level as determined by independent problem solving and the level of potential development and determined through problem solving and under adult guidance or in collaboration with more capable peers (Vygotsky, 1978, p.33).

In the ZPD, the classroom has a share that more knowledgeable ones collaboratively transfer the knowledge to the less ones. Teachers or students' more knowledgeable peers have a role of being a scaffolder by questioning (Way, 2008). Classroom interaction can be facilitated by teacher questioning and that provides instructional scaffolding for students while learning mathematics (Way, 2008). In this sense, in mathematics classrooms, students' zone of proximal development is supported by teachers' questions presented during the instruction while students have an interaction with their peers and his/her teacher (McLeod, 2012). As one of the aspects of the ZPD focuses on the ability of the person to accomplish number of tasks individually as compared to accomplishing them collaboratively, teachers help their students improve their school learning through teacher questioning. In order to understand the support of teacher questioning in the zone of proximal development of students, we need to understand actual development of students (Vygotsky, 1978). Vygotsky (19789) explain that children are tested through varying difficulty of tasks and their mental development is obtained by the specialists. However, he exemplifies that actual development of students are not accomplished by teachers' use of leading question, or the presentation of the solution way to be followed, or dependency of the members in collaboration because students cannot represent their individual performance on the tasks. With the support of teacher questioning, the peer interaction within the mathematical dialogues need to be made with the guidance rather than showing the target product or the memorizing the way to reach the product. Teacher has a role of assisting performance while improving students' level of performance within the ZPD (Scott, 1998, p. 48).

Questioning studies has a close relationship with classroom interaction. When students and teacher create a dialogue together, knowledge is built through many ways depending on the role of the teachers on questioning in mathematics classrooms. Interaction patterns in mathematics classrooms emerge between student(s) and a teacher while transferring the information among the participants of the conversation related to mathematics. That was documented as Initiation-Response-Feedback patterns or Initiation-Response-Follow Up in the literature

(Mehan, 1979; Sinclair & Coulthard, 1975). In these patterns, a teacher initiates the process by asking a question and the students' responses the teacher's question. Following this, the teacher follows up the response, sometimes specifically by giving feedback. In these I-R-E or I-R-F patterns, classroom interaction refers to a form of interaction named as 'funneling' and 'focusing' which are not the same in the opportunity of student's talk (Herbel-Eisenmann & Breyfogle, 2005). According to Mehan (1979), when teacher directs questions whose answer is already known by teacher, two kinds of interactional sequences are observed including basic elicitation questions and extended elicitation questions. Such conversations include initiation (I), response (R), evaluation (E) sequence. He revealed the sequential organization of the questioning as following:

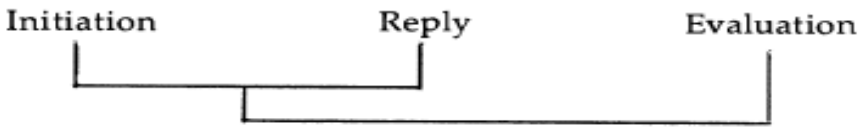


Figure 2.1 The sequential organization of a typical three part structure (Mehan, 1979, p.286)

When an initiation is followed by an expected correct response, basic elicitation sequence is ended with evaluation. However, when initiation is followed by no response, partially correct response, or incorrect response, IR and IRE pairs may be completed in further turns as teacher and student dialogs (see Figure 2.2).

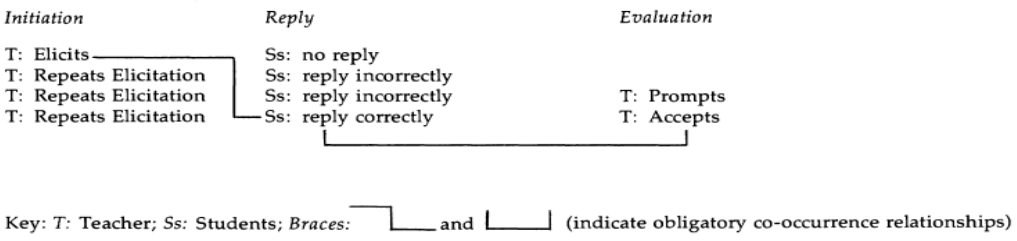


Figure 2.2 The sequential organization of a typical extended structure (Mehan, 1979, p.290)

In this extended elicitation sequence, teachers can use three strategies to make students reach the correct response: firstly teacher asks the question again (repeat elicitations), secondly she/he divides question into logical parts (simplifying elicitations), and thirdly she/he helps students in describing the answer of the question (prompting replies).

Carlsen (1991) explains the one step backward formula of interaction of Mehan (1979) in explaining Bellack's (1966) moves of teachers. According to this, there are four moves interacting with the students: structuring, soliciting, responding, and reacting. Mehan (1979) who is one of the pioneers of questioning, combines structuring and soliciting as initiation (I), responding as response (R), and reacting as evalution or feedback (E/F) (Carlsen, 1991).

Wells (1997) states that this triple sequence is closely related to how teachers use it and for what instructional purpose they use it. This kind of sequence needs to be evaluated considering the purposes of teachers for the sequence rather than evaluating the sequencing as solely good or not as it might depend on observing the whole picture of interaction.

Christenbury and Kelly (1983) mention that researchers hierarchically or non-hierarchically construct questioning (see Figure 2.3).

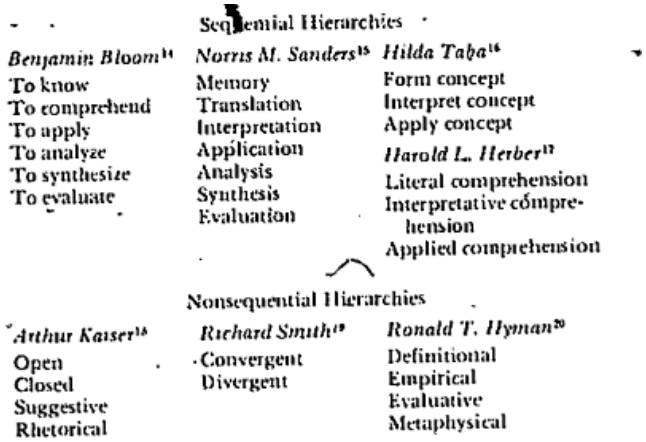


Figure 2.3 Questioning hierarchies of some researchers (cited in Christenbury, 1983; p.10)

In the hierarchical process, types of questions or behavior superior to each other are listed. For non-hierarchical sequence, it is observed that there is no level among the levels and there is diversification of questions. Those sequences or non-hierarchical models are cautiously interpreted because of not having evidence of cognitive level required for each steps in the hierarchy in students' mind and the possibility of making transitions between levels of each steps.

Researchers, on the other hand, have proposed a Venn diagram, which include three questioning circles consisting of the matter, personal reality, and external reality (see Figure 2.4) for conceptualizing questioning. The circle about matter represents what is discussed or making questioning as a subject. Another circle, personal reality relates to an individual's accumulation of background such as beliefs, experiences, or ideas. The other circle, external reality refers to the world consisting of the experiences, cultures, or historical processes out of the individual. Each of the circles are in relation to different cognitive domains and the intersection of the three domains represent "the union of the subject being explored, the individuals response and experience, and the experience of others" (Christenbury & Kelly, 1983; p.13).

This part includes *dense* of higher order questions as the part provides the most in-depth thinking on a subject. The researchers suggest that an instructional goal should include questions from each of the circles and the union of the intersection of each two circles and three circles as well. The way of circle intersections will provide an enriched teaching for students.

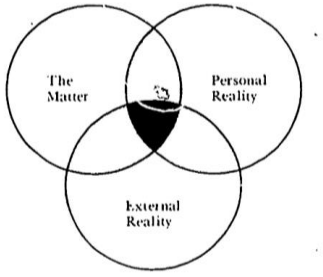


Figure 2.4 The questioning circle
(Christenbury & Kelly, 1983; p. 13)

According to Dillon (1990) and Meij (1994), questioning is a process in which there are three stages including *the onset of questioning, the development of a question (asking), and the search for, and processing of an answer (answering)* (p. 140). The researchers divide the process in terms of the how questions have a journey from the preparation to the presentation to the learners with ordered stages of the process. That can be thought as a further study that provides a detailed description of the questioning process suggested by Mehan (1979). Figure 2.5 illustrates the questioning process.

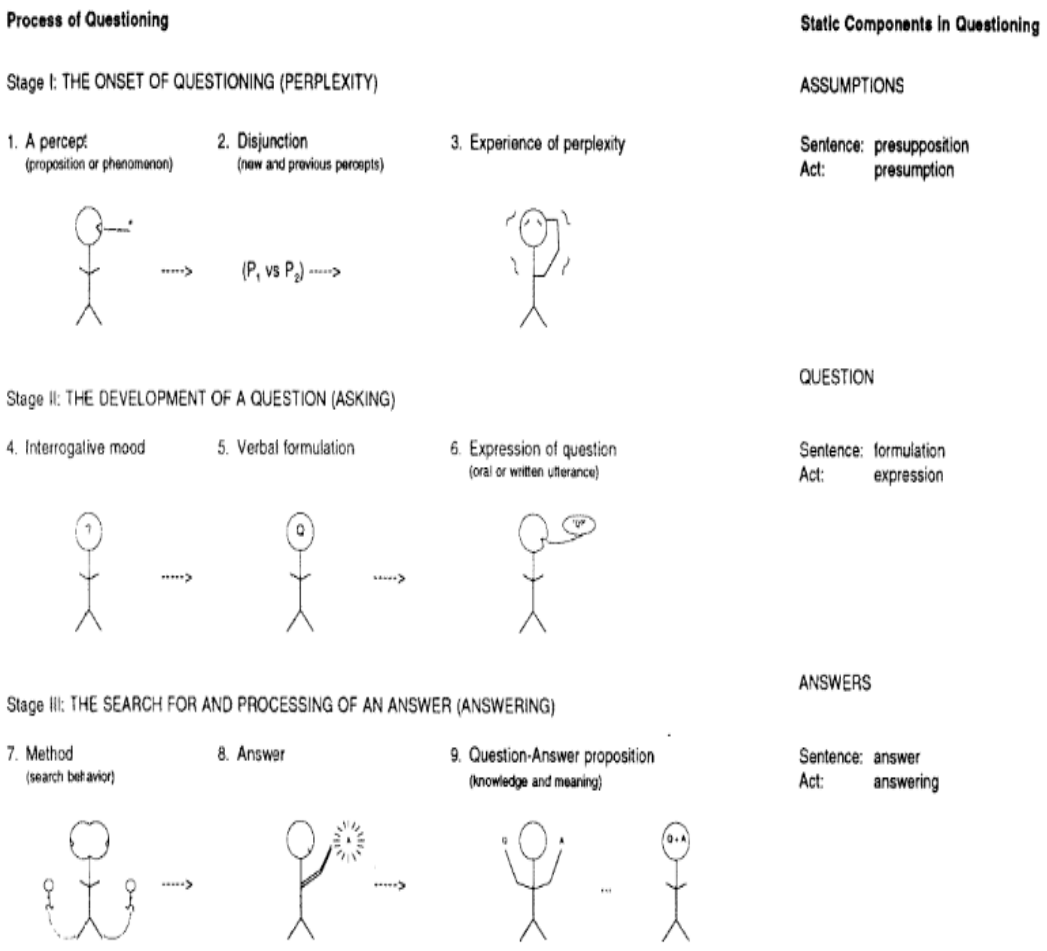


Figure 2.5 The model for the process of questioning
(Dillon, 1988, p.19)

Tanner, Jones, Kennewell, and Beauchamp (2005) addressed how interaction in whole class teaching occurs. The researchers conceptualize the interactivity in classrooms at varying levels of interaction. They described the whole class teaching in terms of the nature of interaction, the nature of the control, and the scaffolding during the interaction and control. According to the model, the nature of the interaction moves from lecture to collective reflection with more participation of the pupils.

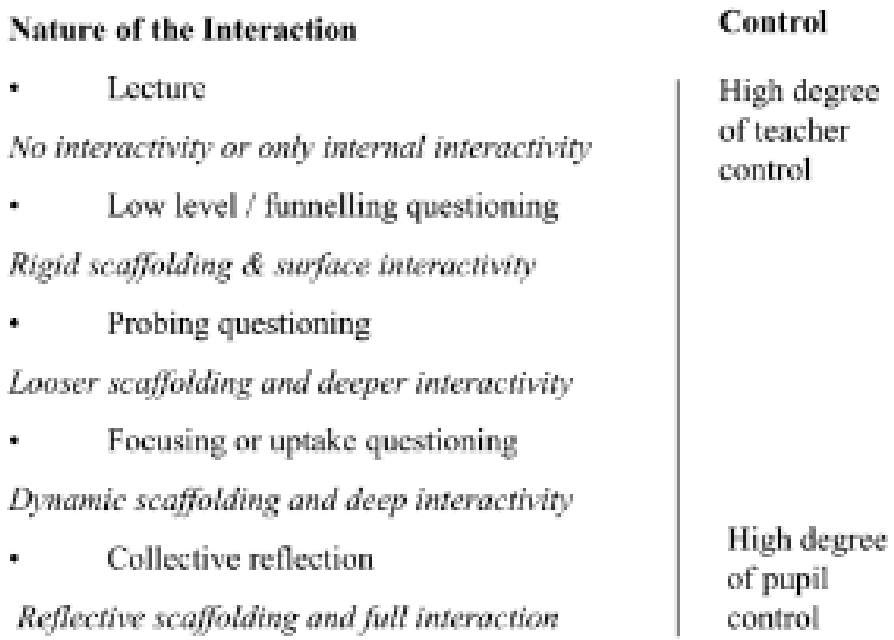


Figure 2.6 Nature of a classroom interaction involving questioning (Tanner et. al., 2005, p.723)

During the lecture type of the interaction, the teacher does not interact with the students and he/she decides on pedagogical movements in terms of representations, examples, and so on. For the nearest higher level, funneling kind of questioning includes teacher-student(s) interaction in which the teacher leads the discourse and decides the flow of the interaction. In probing questioning interaction, the teacher can evaluate a dialogue between a student and the teacher. In addition, the teacher can also ask questions to the students for formative assessment. When

the teacher and students evaluate the students' answers together and the flow of the interaction depends on the evaluation of the conjectures discussed in the classroom, there is a focusing or uptake questioning interaction during the instruction. The highest interaction is proposed as collective reflection in which evaluation and reflection appear on the foreground. In this case, it is important for the student to evaluate himself / herself. It is also relevant that the students have to reflect on the process and therefore reflect the community they are in.

Herbel-Eisenmann and Bregfogle (2005) describes three interactional patterns on the basis of Wood (1998): funneling, focusing, and turning the pattern one another. For the funneling pattern, the researchers stated that the way of a teacher's thinking about the solution of a problem through questioning is more prevalent in this interaction. It provides more opportunity for the statement of student responses clearly. Moreover, researchers emphasize that what this pattern means for the student is not known. In addition to this point, they highlight that funneling kind of teacher behavior might be derived from the need of scaffolding by the teacher. Even if that kind of pattern might be a scaffolding of teachers, the researchers think that the teachers should reduce the number of questions, and they should make students enquire on the questions posed by the teachers during the flow of the dialogue. Another pattern, focusing, is more related to focusing on student's ideas to articulate what the student is talking about. That is a more suggested way of managing questioning as students' ideas are valued and encouraged by the classroom. The transition from funneling to focusing helps revealing student thinking and emphasizes that it is important in terms of discovery learning.

2.2 Studies Conducted with In-Service Teachers' Use of Questioning

In this part, studies conducted with in service teachers' use of questioning were mentioned. The studies mentioned in this chapter showed how teachers' questioning were discussed in the literature. The results of the mentioned studies guided us to think about teachers' use of questioning. Accordingly, the studies helped

understanding the role of teacher questioning in classroom practice, addressed the importance of questioning in different educational levels, allowed mathematics educators to transfer findings belonging to teachers who teach courses different from mathematics, and gave insight about professional development of teachers' use of questioning.

Hattie (2008) stated that teachers' questioning behavior is one of the school predictors about the contribution of teaching approaches for students' achievement. His meta-analysis works showed that questioning is the second mostly used time activity for the teachers. Results of the studies showed that the effect size of questioning on contribution of teaching approaches for students' achievement equals to 0.46, which means that when all outcomes are assessed, teachers' questioning behaviors works in practical sense in medium (Cohen, 1988).

Hufferd – Ackles, Fuson, and Sherin (2004) proposed an action trajectory indicating levels of discourse in a mathematics classroom. This model focuses on teachers' and students' reactions during instruction and it assesses whole classroom discourse rather than bringing it into each student as an individual. According to the trajectory, classroom discourse in mathematics lessons can be ranged in level 0 to 3, and while the level increases, the quality increases as well. One of the components of classroom discourse is questioning in addition to explaining mathematical thinking, sources of mathematical ideas, and responsibilities for learning. The levels are described based on the quality of interaction between students and teacher in terms of questioning. For level 0, teacher-student interaction is obviously low and teachers use closed-ended questions. Students do not interact or share ideas with each other whereas in level 3 classrooms, students express their confusions with the class and they continue interaction until they are convinced with the right answer. During this process, the teacher guides the classroom discourse (Hufferd-ackles, Fuson, & Sherin, 2004) (see Figure 2.8). Conner et. al. (2014) criticizes the framework as it does not mention the 'actual ideas and reasoning being used' at the levels (p. 403). In line with this, Conner et. al. (2014) suggested a framework examining the way of

the support of argumentation of secondary mathematics teachers to provide collective argumentation.

Overview of Shift over Levels 0–3: The classroom community grows to support students acting in central or leading roles and shifts from a focus on answers to a focus on mathematical thinking.

A. Questioning	B. Explaining mathematical thinking	C. Source of mathematical ideas	D. Responsibility for learning
Shift from teacher as questioner to students and teacher as questioners.	Students increasingly explain and articulate their math ideas.	Shift from teacher as the source of all math ideas to students' ideas also influencing direction of lesson.	Students increasingly take responsibility for learning and evaluation of others and self. Math sense becomes the criterion for evaluation.
Level 0: Traditional teacher-directed classroom with brief answer responses from students.			
A. Questioning	B. Explaining mathematical thinking	C. Source of mathematical ideas	D. Responsibility for learning
Teacher is the only questioner. Short frequent questions function to keep students listening and paying attention to the teacher. Students give short answers and respond to the teacher only. No student-to-student math talk.	*No or minimal teacher elicitation of student thinking, strategies, or explanations; teacher expects answer-focused responses. Teacher may tell answers.* No student thinking or strategy-focused explanation of work. Only answers are given.	*Teacher is physically at the board, usually chalk in hand, telling and showing students how to do math.* Students respond to math presented by the teacher. They do not offer their own math ideas.	*Teacher repeats student responses (originally directed to her) for the class. Teacher responds to students' answers by verifying the correct answer or showing the correct method.* Students are passive listeners; they attempt to imitate the teacher and do not take responsibility for the learning of their peers or themselves.

Figure 2. 7 An action trajectory in a math-talk learning community
(Hufferd – Ackles et. al., 2004, p. 88–90)

In their research, prospective secondary mathematics teachers were the participants of the study. One teacher's two classes of ninth grade was the representative data of the study. The participants' teaching in field experiences were video recorded and the research team took field notes. Part of the results of the study indicated that prospective teachers' asking questions in order to elicit arguments are one of the supports that provide collective argumentation. There are five kinds of questions they use while they support the collective argumentation process: 'requesting a factual answer, requesting an idea, requesting a method, requesting elaboration, and requesting evaluation' (p. 419).

Piccolo, Harbaugh, Carter, Capraro, and Capraro (2008) focused on student-teacher interaction in mathematics classrooms and the study seeks answers whether rich and meaningful discourse is related to increasing number of questions which require description and decreasing number of yes/no questions. The nature of classroom discourse in the algebra, number, and data analysis from grades 6th to 8th in five school districts was examined. Dynamic Student-Teacher Communication Pathways map (DSTCP) (see Figure 2.9) showed that rich mathematical discourse includes many questioning types not only higher cognitive questions but also fill in blanks, open-ended, procedural questions, follow-up questions, or guiding questions. Results showed that interaction may be initiated by the students or the teacher and teachers mainly initiate interaction via questions and students mostly tend to answer them rather than suggesting questions.

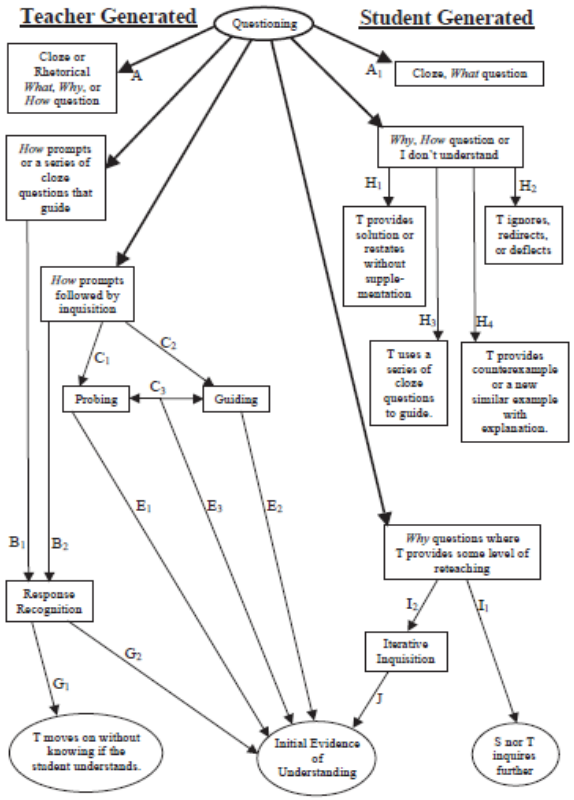


Figure 2.8 Dynamic student-teacher communication pathways (Piccolo et. al., 2008, p. 388)

Fraenke et. al. (2009) conducted a study with three elementary grade teachers (two second grade teachers and one third grade teacher) who have a one-year professional development experience on algebraic reasoning and whose classes have demonstrated different posttest results in algebraic thinking scores though the classrooms are similar in teaching style, concepts, or schools. The study categorized questions as probing sequence of specific questions, general question, specific question, leading question, and other questions. Results of the study showed that there is no relationship between the nature of teacher questioning and whether students' initial explanations are correct and complete or not. What's more, teachers ask questions to students even if students' explanations are correct and complete or not. Students tend to elaborate on explanations when teachers ask questions, though not in all cases. When teachers use leading question and other question types, students mostly avoid detailing explanations. There is a significant difference among question types of teachers in students' detailing explanations. When teachers use probing sequence of specific questions, students are likely to reach the correct and complete results. Although the prompts provide not missing learning opportunities and enable teacher understanding of student thinking (Fraenke et. al., 2009), students and teachers rarely used "why" and "how" prompts. Besides, it enables teachers and students to have a longer interaction. Otherwise, when teachers tend to use direct teaching giving limited chance to students for discussing or explaining, students mostly give short answers as yes or no (Shahrill & Clarke, 2014).

Ali (2007) studied with two primary teachers while they were teaching fractions where the lessons were divided into consolidation, core-content, a rehearsing, and a closure phases. Teachers' questions were categorized as closed-procedural, closed-routine, closed-complete the statement closed-verification, closed-terminology, and closed-rhetorical. Both of teachers initiated classroom dialogues in every way. In this regard, the teacher is questioner and students are responder all the time. No open-ended questions were used whereas a higher number of closed-procedural questions were used even though they believe that questioning

help conceptual understanding. Therefore, teachers had not such an aim that students' explanations need to be elaborated.

Shahrill and Clarke (2014) conducted a study with four eight-grade mathematics teachers to examine participating opportunities for students and emerging issues on student interactions while they were teaching topics. Those teachers were on the competent criteria in their educational system. Results of the study showed that students and male teachers used short utterances (1-4 word) in classrooms in a very apparent way whereas long utterances (25+ word) were quite low for all students and teachers. While the teachers use mathematical questions to check whether the students understand the lesson, students give yes-no answers chorally. Two emerging issues were obtained that yes-no questions were accepted by the teachers for public interaction and the students were accepted for the chorus of answers.

Şahin and Kulm (2008) conducted a study with two teachers whose experiences are quite different from each other. The study investigated a novice and an experienced teacher's three types of questions including probing, guiding, and factual and their intentions for using them regarding different parts of lessons. The study mentioned that although both of teachers are aware of higher order questions are essential in meaningful understanding of mathematical topics, they tend to use more factual questions whatever their related strategy in the lessons is. In addition to this, teachers tend to use more probing questions when they summarize the lesson. In this regard, teachers' questioning behaviors may change in different parts of lessons, however using factual question dominance remain stable except for the summary part. The researchers developed a criterion, which represent the reasons of why teachers use each question type (see Table 2.1). Teachers tend to use factual questions in different sections of lessons (introduction, development, practice and summary) except for the summary part. In the summary part, probing questions are mostly used whereas guiding questions are not used much. Exceptionally, usage of manipulative in one lesson supported increasing probing questions. As seen in the

following table, mathematics teachers could use probing, guiding, and factual questions for the represented intentions.

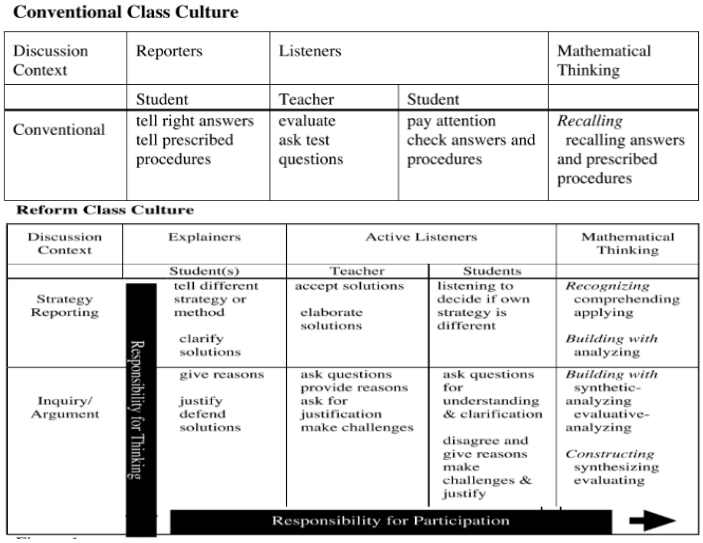
Table 2. 1

Criteria for identifying the question types

criteria for identifying probing questions	criteria for guiding questions:	criteria for factual questions:
Ask students to explain or elaborate their thinking.	Asks for a specific answer or asks for the next step of solution when students are confused or stuck.	Asks student for a specific fact or definition (Vacc 1993).
Ask students to use prior knowledge and apply it to a current problem or idea.	Ask students to think about or recall a general heuristic or strategy (Polya 1947).	Asks a student for an answer to an exercise.
Ask students to justify or prove their ideas.	Asks a sequence of factual questions that provides ideas or hints that scaffold or lead toward understanding a concept or completing a procedure.	Ask students to provide the next step in a procedure.

Koizumi (2013) compared competent 8th grade mathematics teachers' questioning in German and Japanese classrooms while teachers from two countries introduce a new content. The study compared and contrasted the teachers' way of questioning and their behaviors through the lessons. The results of the study showed that in both countries, student responses have an important place in the course. However, these classes were separated in terms of teachers' questioning behavior. According to this, while German teachers allow math as much as they want their students to discover, Japanese teachers teach through students' exploration of mathematical ideas towards the activity. Another important finding is that the study emphasizes that low cognitive level questions (eg, recall, procedural) have a key qualifier for introducing new content in this study. Therefore, questions are at appropriate cognitive levels depending on the context in which the questions are used. Besides, there are different roles of asking questions (eg, recalling, clarifying, and paying attention) depending on the topic being explained. This study has also shown us that the behavior of teachers as a cultural origin can change and that can have an influence on questioning behavior of teachers.

Wood and McNeal (2001) conducted a study that focuses on the types of teaching during the instructions and students' mathematical thinking in five elementary classes (four of them reform classes; one of them conventional). The following figure represents the framework that was used by the researchers for the analysis of teaching:



Conventional Class Culture

Discussion Context	Reporters	Listeners		Mathematical Thinking
	Student	Teacher	Student	
Conventional	tell right answers tell prescribed procedures	evaluate ask test questions	pay attention check answers and procedures	*Recalling* recalling answers and prescribed procedures

Reform Class Culture

Discussion Context	Explainers	Active Listeners		Mathematical Thinking
	Student(s)	Teacher	Students	
Strategy Reporting	tell different strategy or method clarify solutions	accept solutions elaborate solutions	listening to decide if own strategy is different	*Recognizing* comprehending applying *Building with* analyzing
Inquiry/ Argument	give reasons justify defend solutions	ask questions provide reasons ask for justification make challenges	ask questions for understanding & clarification disagree and give reasons make challenges & justify	*Building with* synthetic-analyzing evaluative-analyzing *Constructing* synthesizing evaluating

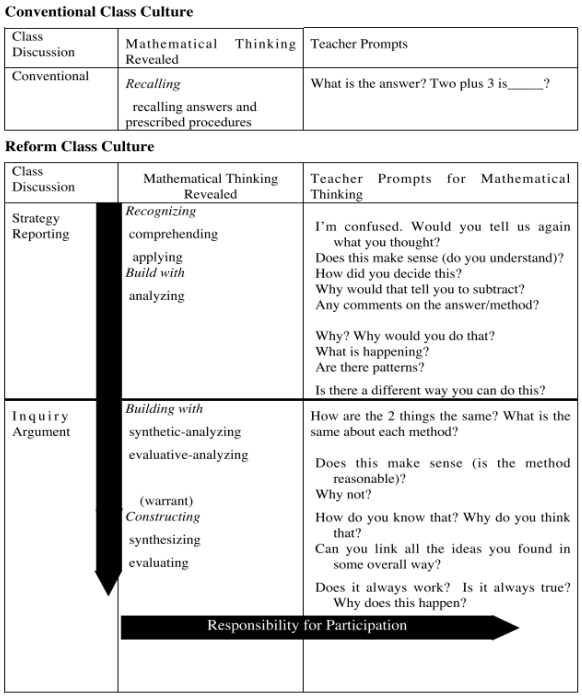
Responsibility for Thinking

Responsibility for Participation

Figure 2.9 Questioning in different class cultures
(Wood & McNeal, 2001, p. 436 – 437)

Their theoretical framework of the study emphasized that there are three classroom cultures in terms of thinking and student participation: conventional, strategy reporting, and inquiry/argument. These cultures represent the varying degree of student participation and mathematical thinking. As the vertical line goes down in the figure, it indicates that classroom cultures get deepening knowledge together with the class and what kind of knowledge the class constructs and how the class integrates the knowledge is informative. In addition to this, students' participation to the instruction increases deepening the mathematical thinking. As the horizontal line goes right, it shows that teacher and student interaction provide students to involve in discussions in the lessons more and they have more control on the lesson in terms of the content discussed. The student participation and mathematical thinking is not the same thorough the class cultures. Parts of the study showed that according to

changing teaching styles, questioning in the classroom is changing. For example, in conventional teaching, the teacher leads students to the desired answers while questioning changes to make students explore ideas and verify their discoveries. The researchers exemplified the teachers' questions in these class cultures as following:



Conventional Class Culture

Class Discussion	Mathematical Thinking Revealed	Teacher Prompts
Conventional	*Recalling* recalling answers and prescribed procedures	What is the answer? Two plus 3 is_____?

Reform Class Culture

Class Discussion	Mathematical Thinking Revealed	Teacher Prompts for Mathematical Thinking
Strategy Reporting	*Recognizing* comprehending applying *Build with* analyzing	I'm confused. Would you tell us again what you thought? Does this make sense (do you understand)? How did you decide this? Why would that tell you to subtract? Any comments on the answer/method? Why? Why would you do that? What is happening? Are there patterns? Is there a different way you can do this?
Inquiry Argument	*Building with* synthetic-analyzing evaluative-analyzing (warrant) *Constructing* synthesizing evaluating	How are the 2 things the same? What is the same about each method? Does this make sense (is the method reasonable)? Why not? How do you know that? Why do you think that? Can you link all the ideas you found in some overall way? Does it always work? Is it always true? Why does this happen?

Responsibility for Participation

Figure 2.10 Questioning in different class cultures
(Wood & McNeal, 2001, p. 440)

In addition to this, teachers expect different kinds of answers in reform classes. While there are students who are required to make proof of their correct answer in strategy reporting classes, in inquiry/argument classes students focus on the explanation behind the different reasoning strategies they have in terms of their justifications.

Parks (2010) conducted a study with an elementary mathematics teacher, student teacher, and five focal third grader students to examine the role of questions in classrooms in minority-majority urban school with equity concerns. The researcher classified question types in terms of traditional or reform oriented questions as well as the state to ask for reasoning in a more open or closed ways.

Results of the study showed that students had difficulties in reform implicit question type while explicit questions overcome the difficulties that reform implicit questions created. Reform implicit questions create difficulties for students, as the intention of the teacher is unclear for students; therefore, students hesitate to answer the teacher's questions. Because they are hesitant, whether students answer the teacher's question in a desired way or not is unclear. Explicit questions support revealing students' thinking, because the teacher asks specifically what the students learn. Students tried to convince the teacher and their friends by answering the teacher's questions. The conclusion of the study highlighted implicit questions could be more beneficial for some students but not be functional for students who have language barrier. Therefore, the students might not get benefit from the use of the reform implicit question as expected because they need to be capable of understanding the mathematics and the language as well. Therefore, the study suggested that while providing equity, race or culture need to be cautiously interpreted by mathematics educators; otherwise, the friends of a student might interpret race or culture as an indicator to students' achievement in the classroom speech.

	Implicit	**Explicit**
Reform	Why? What do you notice about this? Why does this make sense? What's a prediction you could make? What can you tell me about this? What do you think?	Caitlin, can you say why you disagree with Sienna's answer? Tell me why you're adding 32 and 33. Why would 26 not make any sense as an answer?
Traditional	What do you do to add two-digit numbers with regrouping? If you haven't memorized your facts. what can you do to get the answer?	What is four groups of two? What digit is in the one's place, everybody? What do we call the name of this coin? Okay, in Celsius, what temperature does water freeze at?

Figure 2.11 Question types in terms of communicating with reasoning and multiple answering points (Parks, 2010, p. 1884)

Mitchell (1994) conducted the study with two secondary social science teachers who are considered to be successful in their education system and have at least five years teaching experience. Moreover, those teachers' way of teaching is more interactive than traditional teaching. They have no training opportunity about

questioning in their educational career. The results of the data collected by interviews are cetagorized under the following titles: "1. teachers' 'beliefs' about questioning; 2. the pedagogical 'functions' the teachers saw questioning serving; and 3. 'strategies for implementation' in the form of 'rules' and 'principles'" (p. 73). One of the teacher's implicit theories of questioning was described as 'general' theory, which indicates that he adapts himself and his class according to changing situations. In this regard, the teacher manages the process flexibly and dynamicly and each teacher statements were supporting each other in a harmonious way. However, in some situations, the teacher seems to be behaving in a static way. Such teachers appear to shape the teaching of mathematics supported with questioning skills depending on the context, which can include students, subjects, or technological equipment and so on. The other teacher's theory was named as 'dynamic', which indicates that classroom is a dynamic environment where any factors like humans in classroom environment change every moment. In this regard, questioning behavior needs to be shared with the teacher where students feel themselves comfortable expressing their opinions. That study indicated that there is not a hierarchy among elements in implicit theories of teachers, which include beliefs, pedagogical functions, and questioning strategies. In addition to this, elements of beliefs and pedagogical functions cannot be changed according to context whereas questioning strategies are context-dependent. With respect to this, it was concluded that there was not a single theory, instead, it was seen that some elements of the theories can be same for both of teachers but some elements can change from teacher to teacher considering the context. Therefore, theoretical point of view for each of elements is not the same in different contexts.

As similar to Mitchell's (1994), Nisa and Khan (2012) conducted a study in a social studies secondary classroom in order to examine classroom questioning practices in Pakistan. A classroom, which was in low level in achievement, was selected together with six students who are thought as representative of the class for interviews and informal discussions. The data were collected by the researchers' observations focusing on the teacher's questions, students' answers, teacher's feedback, and the students' questions. For the teacher's questions, results of the study

showed that teachers use questions more frequently than students and their questions are low level. Additionally, students produce low level questions more frequently rather than producing high order or non-mathematical questions. While the teacher answered her question, mostly students answered the teacher's questions chorally. One of the conclusions of the study suggests that students' responses interact with the teachers' questions. In line with this, the same behavioral pattern is seen in teacher and students' questioning that both of them ask low-level questions.

Carlsen (2013) investigated how a kindergarten teacher uses a fairy tale to create learning opportunities for kids in learning mathematics consisting of counting, adding, and words of opposite meanings like big-small. The kindergarten teacher supported her telling the fairy tale with concrete materials like teddy bears, chairs, bowls with spoons, beds, and a table to investigate the issue of creating a mathematical discussion and argumentation environment. Results of the study showed that one of the learning opportunities provided by the teacher was the use of mathematical questioning after the teacher's telling of the fairy tale. The teacher used questions with specific purposes and used revoicing while interacting with students. One of the characterization of the orchestration includes questioning in addition to conscious use of voice, face and equipments and emphasizing contrasting words and comparisons.

Jurik, Gröschner, and Seidel (2014) conducted a study with 79 randomly selected schools with 1335 high school physics student participats in Germany and Switzerland. Part of the results of the study showed that deep-reasoning teacher questions and teacher feedback predicted the students' cognitive learning activity of a physic unit and intrinsic learning motivation on the same unit positively.

Hunter (2008) investigated four teachers' instructional strategies in mathematics lessons to scaffold students' inquiry in a primary school in New Zealand. In the study, teachers encouraged their students to ask questions to each other and to the teacher to understand the way of the peer that she/he followed while solving a problem better and to use questioning for mathematical explanations and for explanatory justification of mathematical conjectures in the instruction. In

addition to this, with this encouragement, students are initiated to discuss about the generalization of the conjectures as a result of teachers' and students' use of questions and prompts to each other while questioning. Results of the study indicated that teachers' design of a classroom environment can provide students justifying, generalizing and reasoning while questions and prompts are appropriately used for scaffolding.

Craig and Cairo (2005) suggested QUILT framework in which 5 steps are needed to perform professional development in the classroom questioning: (1) question preparation, (2) presentation of questions, (3) prompting, (4) processing of student responses, and (5) reflection on questioning practice (p.1). The researchers continue developing framework since 1990. This work focuses on the development of two aspects: teacher questioning behavior and the roadmap to educate qualified people about the implementation of questioning. For teacher questioning behavior, the framework focuses on improving the following questioning behaviors: wait time 1, wait time 2, asking questions at all cognitive levels, redirecting questions, designating a respondent, repeating student answers (p.2). This framework has been used for different purposes. For example, researchers examined the improvement of teachers while they took training for the use of QUILT. Teachers have developed themselves in terms of knowledge, understanding and practice as a result of one year of training. In addition to this, the researchers conducted a study for examining the effectiveness of the framework on student achievement. Participants of the study were 28 fifth and sixth grade elementary school teachers in a rural school in Kentucky. They have training experiences of QUILT lasted for 3 to 4 days. The school has fully embraced this framework so that the technique of asking questions has been adopted from the framework. The teachers' three videotaped lessons were coded considering QUILT Coding System (e.g., who asks the question, what the teacher's response is, and so on.). Participant students' mathematics achievement was measured by Measures of Academic Progress. Part of the results showed that teachers' questioning behaviors leading by QUILT framework are not in correlation with student achievement.

Walsh and Sattes (2012) mention about professional learning on questioning. According to the professional learning way of the researchers, there are five things that represent quality questioning: frame quality questions, strengthen thinking-to-learn behaviors, use formative feedback, promote response-ability, and nurture a culture for thinking (p.5). According to researchers, the teacher needs to be prepared to ask questions. It is important to include and reflect on it when preparing the lesson plan. If not prepared, quality questioning may emerge at any time or not. The teacher should plan the process so as to support student learning after asking questions. The teacher thinks that it is not enough for the student to give a correct answer. He chooses and uses the questions to understand why the student really thinks so. After the student answers the teacher's question, the teacher can give feedback. These feedbacks should be informative in terms of student learning and classroom teaching. Otherwise, there may be no alternative to classify the answer of the student, right or wrong. It is also important that students learn to respond so that every student will be responsible for their own learning. This is a skill at the same time and the teacher can improve this skill in the classroom. It is necessary for the teacher to ask the question of the learning culture that he creates in the classroom. In line with this, the teacher should create an environment in which there are appropriate norms, student and teacher behaviors, a language that is used consciously, and the relationship between the students and the teacher with students.

Widjaja, Dolk, and Fauzan (2010) conducted a design study about division of fractions examining the role of real life contexts and how teacher questioning could be used to improve students' thinking. A fifth grade teacher who was trained with the realistic mathematics education theory was the participant of the study. For the design phase, the research team of the study decided to represent a contextual problem to students as the problem is the teacher's own problem and to think about students' possible way of understandings of the problem. While the teacher was implementing her experiment, the very initial attempts of students showed that they used procedural algorithms which were not meaningful for the contexts. There have been cases where it is not enough for the teacher to give the students the real life

situation. The teacher asked students probing questions to better understand the meaning of the division. She did not change her behavior according to the students' answers, whether are right or not. A classroom environment in which students are required to make decisions about the correctness of their answers had been established. In line with this, it was concluded that the combination of meaningful context with teachers' probing questions provided to establish a productive discussion environment in which students were required to make justifications and explanations for understanding mathematics.

According to Ong, Lim, & Ghazali (2010), another way to change teachers' questioning behaviors is to involve teachers and mathematicians in a lesson study process. That process leads to the change of teachers' behaviors while collaboratively reviewing positive and negative aspects of the course and reteaching the same lesson considering the aspects.

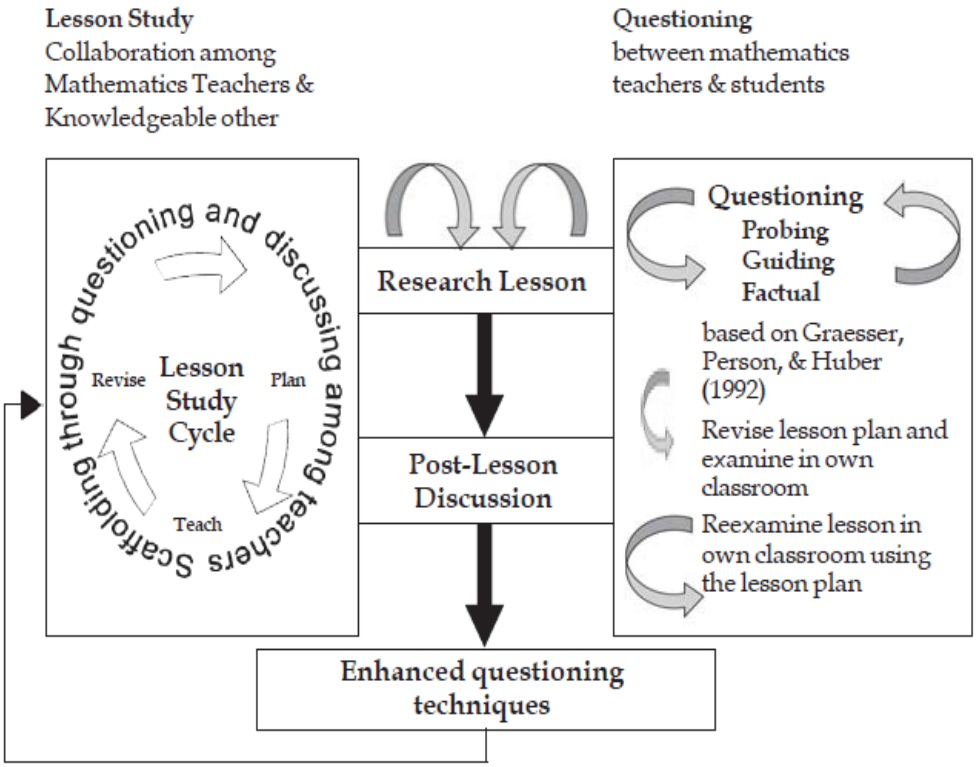


Figure 2.12 A conceptual framework for changing questioning technique of novice and experienced teachers (Ong et. al., 2010, p.94)

Findings of the study showed that it is observed that not all novice and experienced teachers are changed, but those teachers who were willing to change took the students' thinking in their restructured courses into account. Novice teachers predominantly used procedural based low-level cognitive questions and the important thing for those teachers was to reach the correct answer of the question as soon as possible whereas experienced teachers used probing questions for generalizing.

There are rare studies conducted related to in-service mathematics teachers' questioning in Turkey as compared to international studies. They were about evaluating the effect of questioning method and analogy technique in middle grade students' mathematics achievement (Turgut, 2007), teachers' questioning in classroom practice (Kasar, 2013), comparison of the questions of 8^{th} grade textbooks in United States, Singapure and Turkey (Özer & Sezer, 2014), and evaluation of mathematics teachers' use of questions as a formative and summative assessment tool (Delice, Aydın, & Çevik, 2013).

Delice, Aydın, and Çevik (2013) conducted a study with 86 high school mathematics teachers who are varied in school type (public and private schools, university entrance exam centers) and their years of experience (up to 20 years) to evaluate teachers' use of questions as a formative and summative assessment tool. Results of the study showed that teachers' reasons for asking questions differ in terms of the school type and the teaching experience. While experienced teachers in public schools consider class level and curriculum relevance for main reasons of asking questions, less experienced ones are interested in originality of the question, real life context, in addition to curriculum relevance factor. Private school teachers use questions for measurement of knowledge and real life context. Teachers in university entrance exam centers ask questions for measurement of knowledge and textbook without depending on their experience years in that institution. In exams, teachers in public school pay the most attention to previous knowledge questions and the least to curriculum relevance whereas private schools and exam centers use critical thinking questions and quite various types of questions. Textbook questions were

used during classroom practice and Bloom's taxonomy is given less importance in public schools whereas textbook questions are not anymore used in private schools and exam centers. Rather than this, quite various types of questions were used in private schools and previous knowledge connection questions were required in exam centers. The common thing is that questions for developing critical thinking were mostly required from students in homework questions in both public, private, and exam centers. The other result revealed that except from experienced teachers in exam centers, teachers in public, private schools, and exam centers were not using questions depending on contexts which were described as part of classwork. When teachers marked exam questions, it was seen that in general they made appropriate scoring regardless of experience and institution (ie: 8% for 0-10 experience years, there is not any inconsistent marking for 11-20 years teachers). Teachers usually have an explanation as to why they generally considered in that way. They give partial points or no points for students' incomplete answers. In conclusion, the working environments by preparing questions were differed.

Turgut (2007) examined the effect of two different methods (questioning method vs. analogy technique) on 7th grade students' mathematical achievement in lines and angles and polygons topics. Each of the methods were applied to one classroom. Findings of the study showed that there was not a significant difference between those two methods for the achievement test that was applied at the end of the teaching and both of the methods increased the student achievement.

Kasar (2013) applied a study focusing on the use of close and open-ended questions of primary and middle grade mathematics teachers and their approaches to alternative solutions. The study was conducted with four primary and four middle grade mathematics teachers. Findings of the study showed that they had a tendency of using close-ended questions and not giving chance to students for alternate solutions.

Özer and Sezer (2014) examined the questions of 8^{th} grade textbooks in United States, Singapure and Turkey in terms of mathematical features, contextual features, and performance requirements of the questions in them. One of the results

of the study indicated that less number of high-level questions were used in Turkish textbooks than the other countries. One of the suggestions of this study is to improve textbooks in the number of cognitive level of questions.

CHAPTER 3

METHODOLOGY

The purpose of this study was to examine the questioning behaviors of in-service middle school mathematics teachers in 7th-grade classrooms. In line with this aim, this chapter comprises of the design of the study, participants and context of the study, data collection tools, procedures, and data analysis. In addition, trustworthiness, limitation, and delimitations of the study were also explained.

3.1 The Design of the Study

In this study, qualitative research design (Creswell, 2007) was utilized to examine the questioning behaviors of in-service middle school mathematics teachers in 7th-grade classrooms. The features of qualitative research design that assist readers understanding the appropriateness of my preference for this study was mentioned briefly to understand this study from the qualitative design.

Qualitative research explores the participants in their everyday settings where the participants are in their natural settings (Hatch, 2002; Berg, 2000). The examination of people in their natural settings provides data which are "sensitive to people and places under study" (Creswell, 2007; p.37). Realities emerging from those settings are taken into consideration rather than basing the studies on objective realities of the world (Hatch, 2002). Therefore, researchers interpret the settings on the basis of their background such as views, beliefs, and so on (Creswell, 2007). Considering this, the aim is to investigate "What is happening here, specifically? What do these happenings mean to the people engaged in them?" (Erickson, 1986, p. 124). Researchers are in process of giving meaning to the settings (Yin, 2011). Therefore, the settings should be divided into parts that do not distort the meanings

taken from the natural settings (Hatch, 2002). The related frameworks and interpretive lenses of the study guide this process (Yin, 2011). Qualitative researchers have subjective glasses which are shaped by their reflective thinking. The way that a qualitative researcher follows for subjectivity while conducting research studies is controlled by the researcher's awareness of his/her personal states during the research process (Creswell, 2007; Fraenkel, Wallen, & Hyun, 2011). At every stage of qualitative research, reflectivity is concerned about providing the researcher viewthe study with critical lenses (Hatch, 2002). The researcher uses these lenses because he needs to isolate himself from the research setting and his background (Creswell, 2007; Fraenkel, Wallen, & Hyun, 2011). Related to this, qualitative research designs focus on and figure out the meaning of the phenomenon to explore (Yin, 2011). In this exploration process, researcher constructs the meaning of the research setting in line with the research questions. The settings include variables which are required to be examined and which needs to be separated into connected parts to figure out the complex nature of the setting in a systematical way (Hatch, 2002).

Researchers are instruments that are collected depending on the researchers' way of collecting data (Hatch, 2002). Qualitative research design consists of variety of methods to collect data (Berg, 2000; Creswell, 2007). Classroom observations, video and audio recordings, field notes, and other sources could be researchers' data gathering instruments while conducting research (Creswell, 2007; Fraenkel, Wallen, & Hyun, 2011). Researchers need to spend adequate time within the natural setting to explore the incidents confidently (Hatch, 2002; Fraenkel, Wallen, & Hyun, 2011). Depending on the nature of the phenomenon, the time spent within the setting could change and research evidence for a study is taken in parallel with the time spent within the setting (Hatch, 2002).

Qualitative research studies are emergent research designs. During the research process of a social phenomenon, the way of searching the phenomenon can change while engaging with the research setting. In line with this, the study can change until the research questions, methodology, and findings of the studies become mature enough (Hatch, 2002). Depending on the nature of the study, qualitative

research studies follow inductive and deductive ways of analysis to understand the research data (Fraenkel, Wallen, & Hyun, 2011).

From this point of view, qualitative research design was appropriate for this study, as the intention of the research was to explore and to reveal middle grade mathematics teachers' questioning behaviors through in depth analysis of their questioning during their instructions. This study was designed as acase study which is one of the qualitative research approaches (Hatch, 2002). Creswell (2007) describes case study that "involves the study of an issue explored through one or more cases within a bounded system"(i.e., a setting, a context) (p. 73). Specifically, a case can be an individual, an event, a specific organization, classroom, and so on. Researchers focus on investigation of cases since understanding cases is essential to understand the phenomenon to be studied (Fraenkel et. al., 2011). The cases of the present study were two in-service middle grade mathematics teachers. Educational case study (Stenhourse, 1979) was utilized to examine teachers' questioning in two mathematics classrooms. According to this, cases are bounded in the time and the context, and are mentioned descriptively in order to give the readers insight about understanding of a phenomenon. Cases are representative or exemplary of the other cases. Researchers strive to put their cases in relation to the population cases in a meaningful way. Educational case study focuses on understanding educational actions and their practical impact on teachers' instructions. In this study, as represented in the Figure 3.1, the case boundaries were the types of the schools (private vs. public school classroom context), the topic of lines and angles, using technology in mathematics lessons, and active interaction between the teachers and the student(s). Teacher questioning was the phenomena to be examined in this study.

This study meets the qualities of multiple case study strategy described by Yin (2003) which includes more than one case where each of the cases needs to be taken place in the research studies by virtue of a purpose in mind. There are two points emphasized for multiple case study strategy. The first one is that this study enables literal replication which makes the same study one more time, the latter is theoretical replication where research studies reach different points of views based

on the predictable results. In this study, participants are similar to each other at certain points and they are separated on certain points as well. The similar points among participants are considered to support literal replication and the differed characteristics could help for supporting predicted contrasts for theoretical replication.

This study interests the multiple cases with an embedded perspective with multiple unit of analysis, which refers to making multiple experiments with multiple units of analysis (Yin, 2003). The following figure represents the case study design of the study:

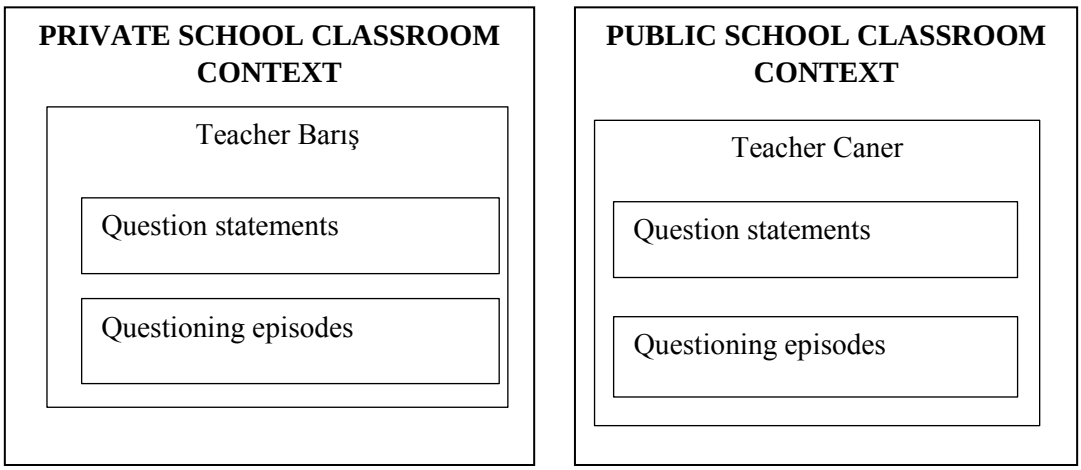


Figure 3.1 Case study design of the study (adapted from Yin (2003), p. 40).

As seen in Figure 3.1, two middle grade teachers were the cases of the studies in which multiple units of analysis existed.

The reason for studying multiple case study with embedded design was to be able to explore and identify how the participants use questions, which question types they use, and the relation among them through analyzing parts of lessons in which the teachers make questioning in a specific subject and analyzing the teachers' question statements as multiple units of analysis. According to this, mathematical dialogues were analyzed in terms of questioning episodes and question statements in both of the classrooms.

3.2 The Research Context

A public and a private middle grade school were chosen for the study as they were purposefully appropriate, accessible for the researcher, and included volunteering participants. The private school provided primary, middle, and secondary grade education in two different buildings and the public school provided primary and middle grade education in one building.

In the private school there were four 7th grade classes of the school while the public school had five 7th grade classes. One of the classrooms from each of the schools participated to the study. The classroom of the private school had 20 students and included available technology in the classroom such as smartboard, a supplementary book compatible with the smartboard, and an overheadprojector. The instruction was presented on the smartboard and, in some occasions, on a white board. Each of the students had their own desk and the desks were arranged to see each other from behind. Based on the classroom observations of the researcher and the mathematics teachers' comments on his lessons, the classroom had an ongoing interaction between the teacher and the students. The school had mathematics club and the school had a policy of allowing students to improve their mathematical understandings after class hours with extra courses. To do this, he assigned different grade levels whose students participated to the courses for individual feedback by the head of the middle grade department of the school.

The other classroom of the public school had 27 students and several technological tools were available including interactive screen and a projector. The teacher did not have adequate technological knowledge to use the devices as he did not have a chance to have training about this. Therefore, in these classrooms, the instructions continued on the white board and with supplementary materials. Similar to the other class, each of the students had their own desk and the desks were arranged to see each other from behind. Based on the classroom observations of the researcher and the mathematics teachers' comments on his lessons, the classroom had an ongoing interaction between the teacher and the students. Similar to Teacher Barış's

school, the school had a policy about improving their mathematical understandings after class hours with extra courses. To do this, he assigned a grade level (for example 7th grade) and students of this grade level participated voluntarily to the courses, which had a similar kind of an instruction he did in his classrooms. The research context of the study was two middle grade mathematics classrooms

3.3 Participants of the Study

In this study, convenience sampling method was employed. Both of the schools were convenient for the researcher and the teachers participated in the study on a voluntary basis. The middle school teachers in this study had research boundaries, which might create differences in terms of questioning of the teachers. First of all, the participants were working in public or private schools. Secondly, both of the teachers were making instruction on the same content, which was lines and angles. Thirdly, the availability of technology were not the same. One of the teachers was using technology in his lessons by smartboard while the other teacher did not apply technology in any of his instructions. Lastly, the nature of the interaction was high which referred to that students were not only respond but also ask questions to their teachers in both of the classrooms.

3.3.1 Teacher Barış

One of the participants of the study was Teacher Barış. He had been working in the same private school for four years and he was a PhD student in mathematics education. He was graduated from the department of middle grade mathematics teacher education. He had a master degree about the effect of using dynamic geometry software on students' geometry achievement and attitudes. He specifically knew how to use GeoGebra. In addition to this, the teacher followed an online learning platform for assigning homework to his students and used internet sources

when necessary. He did not use the national textbooks; instead, he used a supplementary book, which was applicable to and compatible with the smartboard.

3.3.2 Teacher Caner

Teacher Caner had been working in the public school for more than 5 years. He had twenty years of experience in teaching. He was graduated from mathematics department in a public university. He had been the teacher of the same class for three years. Therefore, he was familiar with the students and knew students' background well. The classrooms consisted of 27 students and included a white board, projector, and a interactive screen which was not used. The reason of not using the smartboard was explained by Teacher Can during an informal talk. He expressed that he did not have an opportunity to take training about utilizing the smartboard. He rarely used the national textbook and supported his lesson with printed supplementary materials such as supplementary books or practice sheets. The following table represented the similaries and the differences of the cases of this study:

Table 3.1

The similaries and the differences of the cases

Case of boundaries	Similarities	High questioning interaction Use of questioning in lines and angles topic
	Differences	Years of experience Use of technology Use of printed supplementary materials

3.4 Data Collection Procedures

Before the data collection process, I observed some classrooms in order to understand to what extend the teachers and the students interact during the instructions, how teachers ask questions, and how students participate to the instructions in the classroom environment. Following to this, I selected one classroom for each of the teacher.Before the data collection process, I was a non-

participant observer for approximately two weeks in the classrooms. In this context, observations were made in participant teachers' classes and the flow of the lessons were monitored. Besides, informal conversations were made with the teacher about the course flow. The pre-study was recorded for the teachers and the students. When collecting data, the position of the camera and the position of the researcher where the students and the teacher felt comfortable was decided. Possible problems were discussed with the teachers. I conducted the pre-study during the period when the Ratio and Proportion and Percentage topics were taught before the Lines and Angles topics. The pre-study was considered to be useful in terms of possible changes in resources and managing data collection process.

After getting familiar with the classrooms, I started the data collection process. To collect in-depth information from the participant teachers, classroom observations and classroom video-recordings were utilized as the main data collection tools. I took video recordings of mathematics lessons of both teachers throughout the semester. I observed the lessons of the two participant teachers at the same time and took observation notes about the classroom contexts, students' behaviors, and teacher behaviors.

The class hours take 40 minutes formally. In these cases, the start and the end of the lesson were obtain based on the teachers' opening and closing statements of the instructions. In Teacher Barış's classroom, the students have their individual desks. There were two white boards available to use for teaching. I positioned the video camera on the right back corner of the classroom in order to observe and record the teacher's behaviors on the stage of the classroom. The observation and the recordings were not appropriate for the students and the teacher because of the interruption of the camera in front of the class while the position of the camera was put on the left front side of the classroom where all the students and the teacher were observed as a whole. In this classroom, the instruction took 35 minutes in generally.

Teacher Caner's classroom included 27 students who were sitting in desks designed for one person. In this classroom, I placed the video camera on the right back side of the classroom where I can follow the teacher's movements without

interruption of students' desks. I started recording when the teachers'signals about the initiation of the lesson and finished recording when the teacher ended the lesson. During the video recordings, I did not take notes as students' attention might be on what I was writing. I took some notes about students' behaviors, teachers' behaviors related to questioning, and other related issues at the end of every recording.

The lines and angles topic was the focus for the analysis. Table 3.2 presents the data collection timeline of the study.

Table 3.2

The timeline of data collection procedure

Duration	Action
September 2015 – December 2015	Designing data collection process
Before the semester holiday	Non-participant observer
January 23 – February 7	Semester holiday
One week (5 class hours) for Teacher Barış and 2 class hours for Teacher Caner	Pre study of data collection (Percentages) and making necessary changes or revisions on the learning environment
February 2016 – June 2016	Data collection process

According to this, I designed data collection process through negotiations conducting with mathematics teachers and schools. Following to this, after taking the permission from MoNE, I initiated making observations of some mathematics classrooms which were convenient to me. At the end, I conducted pre study on problem solving process of percentages topic.

The duration of the topic was fourteen class hours according to the curriculum. In this study, I started recording video five class hours before the main study for Teacher Barış, and two class hours before the main study. What I did in this stage for pre study was that the teacher and the students were got engaged to me in the learning setting and allowed me to initiate the video recordings.

As seen in Table 3.2, data gathering was conducted during one semester from February to June in 2016. The details of the schedules of the courses were represented in Table 3.3.

Table 3.3

Course schedule of a week

February 17 – February 19 (Teacher Caner) February 22 – February 29 (Teacher Barış)				Main study Lines and Angles		
Teacher Barış		Monday	Tuesday	Wed.	Thur.	Fri.
	Morning			2 hrs.		
	Afternoon	2 hrs.			2 hrs.	
Teacher Caner	Morning					
	Afternoon			2 hrs.	2 hrs.	1 hrs.

The researcher participated in each of 5-hour lesson of the teacher and received video recordings every week. The time distribution of the topic for each teacher is shown below:

Table 3.4

Units and time distribution of lines and angles topic for the participants of the study (MoNE, 2013, p. XVII)

# of objectives		Class hours* allocated for these objectives	
Lines and Angles	3	Teacher Barış	Teacher Caner
7.3.1.1. A student should be able to draw equivalent angles to each other [Bir açıya eş bir açı çizer.]		2	1
7.3.1.2. A student should be able to describe bisector line separating an angle into two equal angles [Bir açıyı iki eş açıya ayırarak açıortayı belirler].		1	1
7.3.1.3a. A student should be able to examine the properties of the opposite, inverse, interior inverse, exterior inverse angles formed by intersecting lines and a line intersected with the other intersected pairs of the lines [İki paralel doğruyla bir keseninin oluşturduğu yöndeş, ters, iç ters, dış ters açıları belirleyerek özelliklerini inceler] 7.3.1.3b. A student should be able to solve problems related to angles which are equivalent to each other and are complementary to each other [oluşan açıların eş veya bütünler olanlarını belirler ilgili problemleri çözer]		4	3
Total hours spent by each teacher		7	5

*Note.*Class hours allocated for these objectives are ten class hours*

Depending on the aim of the study, the participants and the teachers were aware of the fact that they were being observed about teacher's questioning in the classroom. I explained the aim of the study to the teachers and the students at the beginning of the study. For this study, I focused on the observations for 12 class hours in total. The focus of the observation was on teachers' routine and non-routine behaviors in the classrooms for questioning.

3.4.1 Data Sources

Data sources of the study were classroom observation and video recordings. I explained how I used each of the data collection tool in the research study in the following titles.

3.4.1.1 Researcher's Observation of the Classroom Settings

Observation is an action of a researcher about 'observing how people act or how things look'. (Fraenkel et. al., 2011, p.445). While observing the research setting, researcher takes different roles. The role of the researcher was to involve in the classroom settings and take a valid data that represent the teachers' use of questioning. In order to do this, the researcher was a non participant observer throughout the study. A non-participant observer do not participate a research setting and only observes without intervening in the research environment (Creswell, 2007). In this study, classroom observation with non-participant observer role was used for obtaining moments that were critical for the research. The researcher made notes about the teachers' use of board, their use of books, their approaches to the students regarding questions, and students' participation to the class discussions. Additionally, teachers' nonverbal behaviors were observed in order to understand the teachers' style in classroom dialogues. Moreover, the researcher also made notes about the teachers' use of questions and the instructional contexts while watching the videos after the teachers had finished teaching.

In addition to this, teachers' general behaviors about these mentioned issues and their changing behaviors were obtained through the classroom observation.

3.4.1.2 Classroom Videos

According to Fraenkel et. al. (2011), recording an observable behavior allows a researcher to code the behavior watching repeatedly in a more available time. When the video recordings are one of the main data collection tools in research studies, a researcher has two responsibilities to provide a qualified data for analysis (Yin, 2011). One of them is to make recordings by a person who has a technical skill and familiar with the content. The other one is related to the verbatim transcription of the videos.

In this study, I recorded the lessons to examine questioning behaviors of teachers in a specific mathematics content. Two 7^{th} grade classrooms were recorded on a regular basis (5-hour video recording per week) by the researcher. The purpose of using classroom videos as a data source was to understand teachers' behaviors in their natural settings. While video recording, I as a researcher recorded the classroom setting, and that provided me to be familiar with the classroom settings. In addition to this, I transcribed and added necessary details about teachers' and students' behaviours within the classroom on the transcription of the videotapes.

In addition to this, after completing the verbatim transcriptions of the video recordings, I watched them over again for evidence. The video-based observation enabled me to capture verbal and non-verbal behavioral movements in the research settings in addition to the opportunity to watch the video over and over again, to re-examine the classroom environment, and to compare it with the observation notes during the instant observation in the research settings (Maxwell, 2009; Yin, 2011). In line with this, the transcriptions and, the observation notes taken during the classroom teaching of the the teachers and while watching the video recordings were the data sources of the study.

3.4.2 Data Analysis

For the data analysis, verbal communication between the teachers and students was focused. I examined the verbal activity focusing on questioning in classroom communication based on questioning approaches in literature (e.g. Mehan, 1979; Şahin & Kulm, 2008; Carlsen, 1991; Dillon, 1988). In this study, one of the data sources was verbatim transcripts of the study of which I conducted a content analysis. The other source was the observation notes, which was used to interpret the verbatim transcripts considering its natural setting at different times.

Analyzing the classroom videos involved multiple stages. In the first stage, I made verbatim transcripts, re-watched the videos, and took notes on the transcripts. In the second stage of the analysis, I separated the instruction into questioning episodes, which are specific to questioning a mathematical concept, procedure, or an idea. Next, I examined the episodes regarding question types of Şahin and Kulm (2008). I explained how I followed these stages in detail.

In order to identify questioning episodes in classroom dialogues, I separated each of the the classroom dialogues in parts that include questioning of different or similar mathematical ideas in separate worked examples, real life examples, or verbal questions related to a mathematical idea. When the parts are linked to each other, in other words, the idea is continuing to be discussed, I took the episode as a whole. Student questions that were not linked to the questioning episodes were evaluated as separate questioning episodes for this study. I made the data analysis based on these episodes. There were 93 questioning episodes and 54 questioning episodes for Teacher Barış and Teacher Caner respectively. Each of the questioning episode included a questioning sequence and an instructional content. For example, classroom talk about equivalent angles included different instructional contents for questioning. Therefore, it was divided in two questioning episodes in which description of what equivalent angles was questioning and the strategy or procedure of creating equivalent angles were questioning by the class, respectively in Episode 1 and Episode 2. In Table 3.5, sample of questioning episodes were represented:

Table 3.5

An example of the focus of the mathematical dialogues in the first lesson of the teachers

Lessons	Questioning episodes
Lesson 1 of Teacher Barış	Episode 1: Questioning the description of equivalent angles
	Episode 2: Questioning the strategy or procedure of creating equivalent angles
	Episode 3: The first worked example about creating equivalent angle
	Episode 4: The second worked example about creating equivalent angle
Lesson 1 of Teacher Caner	Episode 1 Questioning what the angle is
	Episode 2 Questioning the naming of an angle
	Episode 3: The first worked example about creating equivalent angle
	Episode 4: The second worked example about creating equivalent angle

As seen in Table 3.5, the focus of the dialogues was different from each other for both of the teachers. Episode 1 included questioning what the angle is, Episode 2 was related to naming of an angle. Therefore, they were separated to each other in terms of the instructional content. Each of the episodes included questioning sequences in which there were question statements following to each other for a specific instructional purpose. This stage was a general but a detailed one for the following stage of analysis. These episodes helped to identify tools used in questioning.

In order to identify tools used in questioning, I analyzed the questioning episodes in order to find out the presence and the use of physical equipment or thinking tools. I examined what the teachers benefit from while asking those questions and the way the teachers integrated the equipment or instructional contexts in these questioning episodes. Related literature provided tools for teaching (Gross Davis, 1993) and tools for thinking (Harrison & Treagust, 2006) which guided me at the beginning of the analysis. According to this, instructional media and technology, textbooks, students' ideas or questions (Gross Davis, 1993) and analogies (Harrison & Treagust, 2006) were evaluated as tools for questioning for the current study. The

rest of the tools were generated by the researcher. Each of these codes were defined in the findings section.

Following to the identification of the tools, I obtained question statements in each of the questioning episodes. Considering the definition of Mason (2014) and Dillon (1988), question statements were obtained. According to this, not only statements with a question mark but also statements that require students to answer were also coded as questions. For this purpose, statements, which require prompting and probing, were also evaluated as questions. In the following, you can see sample of examples for question statements:

Table 3.6

Sample statements representing the approach of whether a statement is a question

	Question	Example
Statements with a question mark	Yes	'What do you mean exactly by …?') (Wragg & Brown, 2001, p. 33)
Statements without a question mark	Might not be a question	'We learned the topic of angles in previous years.'
	Might be a question	'Think back to what we learned about…') (Wragg & Brown, 2001, p. 33)

Following to this, I looked for the characteristics for the question statements consisting of probing, guiding, and factual questions considering the contexts of the questioning episodes. I got benefit from Şahin and Kulm's (2008) characteristics of question types represented in the following figure and made some adaptations for using the framework (see Figure 3.2).

At the beginning of the analysis, question statements investigated by using Table 3.6 were categorized whether the questions were probing questions. Considering the context of the questioning episodes, the characteristics of the probing questions were assigned to each of the probing questions. Following to this, questions which were not probing questions were categorized whether they were used while students were confused or stuck. If it was used for that purpose, the question was

assigned to the related characteristics of the guiding questions. If questions were not used related with that purpose, the context of the questioning episodes were considered and the questions were assigned to the characteristics related to factual or guiding questions.

When teachers want students to give details about his/her answer, want students to explore the answer sharing by the class deeply, or probe students' prior knowledge, they used probing questions. In this kind of questions, teachers continue their instructions on the basis of the student's answer or probe class's or students' prior knowledge. While doing this, if teachers probe the students previously stated ideas or want them to dig into their thinking, the questions were categorized as the characteristics of 'ask students to explain or elaborate their thinking'.

If teachers probed students' prior knowledge and they required students to use the prior knowledge in learning new mathematical ideas, these questions were categorized as the characteristic of 'ask students to use prior knowledge and apply it to a current problem or idea'. While the teachers were probing students' ideas by requiring them justification or proof of what the students said, these kinds of questions were categorized as 'ask students to justify or prove their ideas'.

Another type of question was guiding questions. In such questions, the teacher tells the whole class and during the course of the subject, a dialogue with a particular student or with the entire class can take place. When questions help teachers to guide students while they encounter challenging situations, to open students' perspective in producing problem solving strategies, and to scaffold or lead students in completing a mathematical procedure, or understanding of big mathematical ideas, they were categorized as guiding questions.

If the teachers' questions guide students as they help completing the procedures step by step in case they were troubled with getting the mathematical idea, they were categorized as 'asks for a specific answer or asks for the next step of solution when students are confused or stuck'. If teachers' questions required students to use strategy in the way that teachers applied during the instruction or to produce strategy to use towards understanding of a mathematical idea or a procedure,

they were categorized as ‘ask students to think about or recall a general heuristic or strategy (Polya 1947)’. Teachers can guide their students through asking one or more than one questions successively serving to the purpose of understanding a concept or of completing a necessary procedure. When the way of asking the questions was leading students’ answers or improving their way of thinking, they were categorized as characteristics of ‘asks a sequence of factual questions that provides ideas or hints that scaffold or lead toward understanding a concept or completing a procedure’. If the questions were asked in succession, serving to the purpose of leading or scaffolding of students, each of these questions was coded into a related factual characteristic as well. However, if these questions do not form a sequence, it was only coded into this characteristic of guiding questions. The sequence was decided considering the context in the questioning episodes. If there were procedures that need to be implemented step by step, and if the teacher followed it, this was considered as a sequence. Nevertheless, if the teacher did not follow any order or if there wasn’t a procedure to follow, it was only coded in the characteristic of guiding question.

The other question type is factual questions. This type of questions refers to questions that provide teachers with a view to evaluating what a student knows about a subject. These questions may be about definitions of concepts, may be a question for completing the procedure, or a result of an exercise.

If teachers ask which had the characteristics of ‘asks student for a specific fact or definition (Vacc 1993)’ or ‘asks a student for an answer to an exercise’ or ‘ask students to provide the next step in a procedure’, then they were coded as factual questions. During the classification of the question types and assigning the characteristics of the question types, the video recordings of the study were rewatched in order to understand and give meaning to the questions within the instructional contexts. The type of the questions and their characteristics was determined. The following stage was related to identifying the tools for questioning. As similar to the analysis of the question types, the identification of the tools for questioning was the next stage for the data analysis.

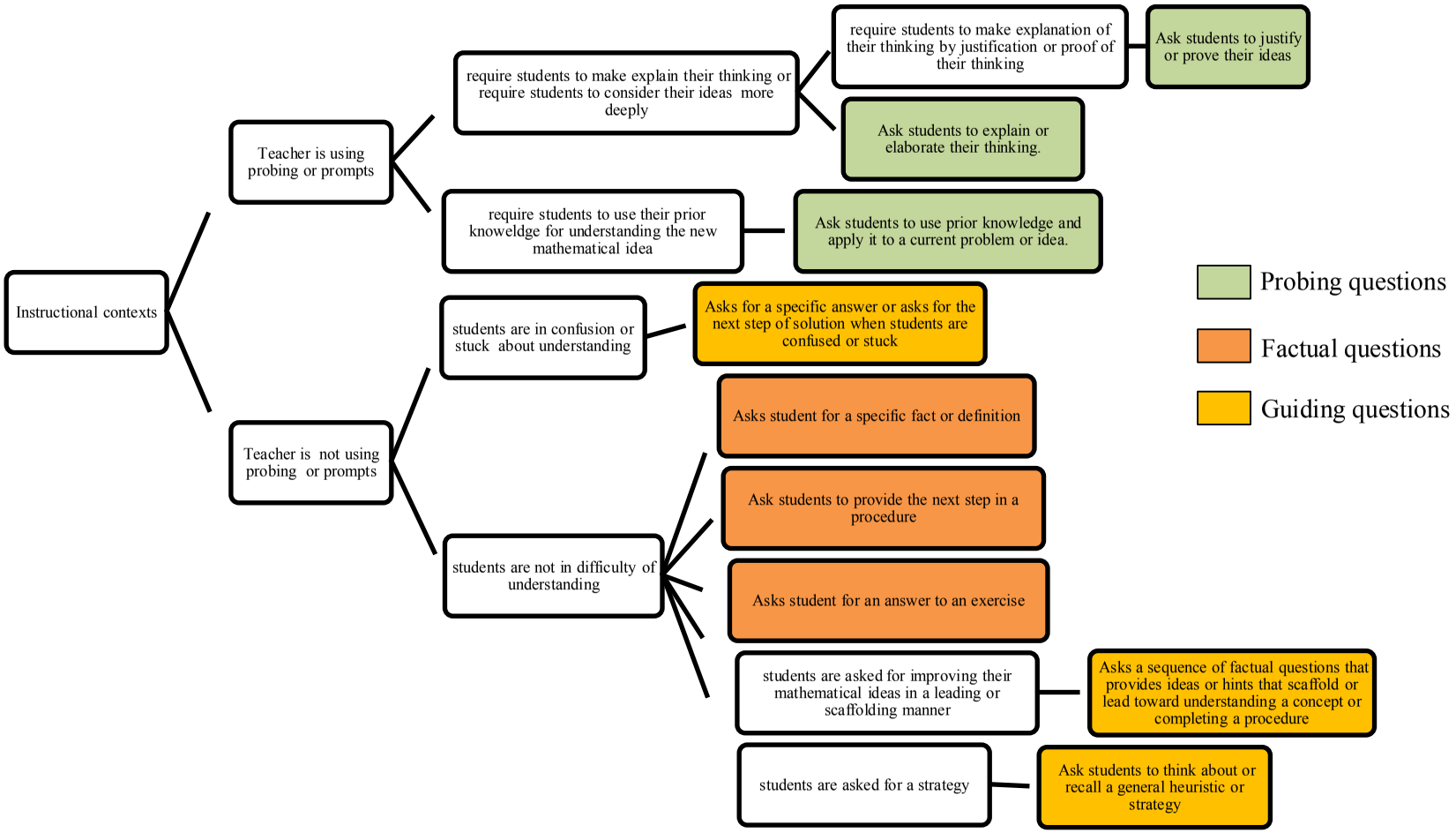


Figure 3.2 The focus of the characteristics of question types in this study

As a last stage, in order to identify the relations among the tools for questioning and the question types of the teachers, I got help from a qualitative data analysis software. According to this, 'Maxmaps' and 'Complex Coding Inquiry' options were appropriate for obtaining the intersection of the codes which represented the relation among the codes each other.

'Maxmaps' was an option of the software that provided all the binary relations between each of the codes in case of the use of Co-occurance model option of the Maxmaps. The option provided the frequency of the relations as the thickness of the codes (for more information please see VERBI, 2018).

After creating all the co-occurrence models for each of the codes (tools for questioning and question types), I found the frequency of the relations using complex coding inquiry option of the software. The 'intersection' function of the complex coding inquiry option allowed me to to detect the codes, which were observed in the same questioning episode. According to this, questioning episodes included tools for questioning and question types were involved in those questioning episodes (for more information please see VERBI, 2018).

3.5 Generalization in Qualitative Research

Fraenkel et. al. (2011) states that the nature of generalization in qualitative research design is not the same with qualitative research studies. In qualitative research designs, the 'seldom methodological justification for generalizing the findings of a particular study' results as a limitation derived from the nature of the qualitative research studies (p. 437). Therefore, generalization in qualitative research is not carried out by the researcher due to these methodological limitations. Instead, the findings and the conclusions of the study are generalized to the readers themselves by the same readers who are interested in that research study. Therefore, 'transferability' is preferred to be used instead of saying generalizability in a qualitative study (Maxwell, 2009). In line with this, I, as a researcher of this study do not have an aim of generalizing the findings or conclusions of the study for the other

mathematics classrooms, but the readers of the study can transfer the findings, conclusions, or discussion parts of the study to their own classroom settings.

3.6 Validity and Reliability of the Study

For qualitative research designs, researchers need to be careful of the inferences of the study and overcome possible threads deriving from the interpretation of results and conclusion to provide a valid study (Maxwell, 2009; Yin, 2011). According to Maxwell (2009), validity issues can be checked by the validity of data sources and the data analysis process, and the reflections of the conclusions could be conducted with intensive-long term involvement, "rich" data and triangulation strategies.

Intensive long-term involvement strategy recommends researcher involving the research setting several times. I spent a considerable amount of time in the research setting getting to know the culture of the schools and classrooms and the teachers' approach to their students. In this study, the researcher involved in the pre-study and the main study as a non-participant observer. I spent one semester in the same classrooms that means I participated each of the mathematics lessons in each of the weeks in the half of the school year. During this process, I conducted informal interviews with the teachers, and that gave me a chance to get to know the teachers. In line with this, while describing contexts of the study, the characteristics of the teachers guided me about understanding the data and about reflecting their behaviors into the analysis as much as possible. In addition to this, getting to know the culture of the classrooms, participant teachers' approaches to their students, observing each of the lessons provided me to understand the study together with the contexts. For example, the long-term involvement enabled to obtain participants' routine or non-routine behaviors related to questioning. Therefore, the verbatim transcripts were more than a written text for me. They represented part of the culture and practices of the teachers as well.

Rich data and triangulation strategy interest in data, which get richer through observation, interview, or other sources give us a more accurate picture about what is happening in related situations (Maxwell, 2009). The researchers are expected to make inferences less biased by enriching those multiple data sources. The data were collected mainly through transcripts of the video recordings and researchers' observational notes obtained by watching the lecture videos more than once. Therefore, the data got richer and triangulated with these data sources using these different data sources. Moreover, the encoding of the data was saturated through the lessons of lines and angles. Therefore, the data of the study were rich enough for encoding of the data.

Reliability refers to the consistency of the results of the studies in varied times or settings (Fraenkel et. al., 2011). More valid studies have the potential to produce more reliable results and to increase the reliability of the study. Peer briefing was used as a strategy for providing reliability of the study (Fraenkel et. al., 2012). According to this, a part of the data of the study was discussed with colleagues regarding evaluating the results, the conclusions and the way of the analysis.

3.7 Ethics

Merriam (2009) emphasizes that 'the validity and reliability of a study depend upon the ethics of the investigator' (p. 228). The ethics of the work is a thought that must be done from beginning to end at each stage of the study (Maxwell, 2009). The ethical concerns are related to the protection of the rights of the participants (Fraenkel et. al., 2011). I as the researcher of the study took the ethical concerns while conducting my research. In this study, I took into consideration of the following three concerns through the complement of the study and after the study as well.

At the very beginning of the research, I applied to ethical board of the university (see Appendix A) and following to the approval of the study by that committee, I took the permission from the Ethical board of the Minister of Education (see Appendix B). I applied to the two participated schools with these ethical forms

being aware of the fact that participant teachers need to be volunteer together with the middle grade students and their parents. I took a permission from each of the two teachers about their volunteer participation (see Appendix C) and from the parents of the students and the students themselves (see Appendix D). I was introduced to the classrooms by the participant teachers. I gave information about the purpose of the study. Therefore, when I initiated data collection procedure, each of the person knew me. In line with these efforts, I emphasized in these classrooms that the data would not be watched by anybody else. They would be protected by me and if they realize that any part of the video was available from anywhere, they can use their official rights. I was careful about the protection identities of the participants in the verbatim transcripts using nicknames for them, not only for the teachers but also for the students in the classrooms.

3.8 Limitations of the Study

This study was limited by the number of teachers participated to the study. The study was only conducted in two schools including one private and public schools in Ankara.

One of the participant teachers was following to a supplementary book which had much worked examples and had an effort to solve all the examples in the book. Similarly, the other teacher scheduled individual problem solving sessions for his students, which were shaped by student's supplementary books. Probably because of the national wide examination, the instructions of both of the teachers were affected. To decrease the affect of this, the research was conducted with 7^{th} graders. Since the participant teachers' classes they would teach were 7^{th} grades and the researcher conducted to the study for the same content for both of the teachers, the content of the current study was limited to the lines and angle topic.

There are two threats for the observer effects (Fraenkel et. al., 2011). One of them was that participants' non routine because of the presence of the researcher in the research setting. The other one was that the participants might be influenced by

the awareness of the purpose of the study. The researcher informed the participant teachers and the students because of their curiosity of the study. I explained that I was wondering classroom interaction in mathematics classrooms and teachers' questioning behaviors. That might influence the way of the teacher and students' way of behaviors. I tried to overcome the observer effects with the pre-study in which the classroom got used to me during the four weeks before the main study.

CHAPTER 4

FINDINGS

The aims of this study were to reveal the tools that middle grade teachers use while asking questions and how the teachers used them during the mathematics instructions, to identify question types middle grade mathematics teachers used, and to explore the relation within the tools for questioning, and between the tools for questioning and the question types. In addition to this, the relation within the tools for questioning and between tools for questioning and question types were examined.

The findings of the current study were summarized in three sections. In the first section of the study, I analyzed what tools middle-grade teachers were using for questioning in their mathematics classrooms and how they used the tools in their instructions using the observation of the video recordings. In the second section, I examined the teachers' mathematical question types based on the verbatim transcripts of the recordings. In the third section, I analyzed the relation between the teachers' tools for questioning and their question types through focusing on questioning episodes that included evidence from tools for questioning and question types at the same time.

4.1 Middle Grade Mathematics Teachers' Tools while Asking Questions

This section provided findings of the descriptions of the learning environments in two mathematics classes of middle-grade mathematics teachers, regarding the tools they used while questioning through the video recordings. Each of the situations, which had a role in asking questions were tools for questioning. This role could be related to provide content in question, to use a tool as a source for asking questions, or to establish a physical environment to ask questions etc. The

focus was to understand the contribution of the tools in the two teachers' ways of asking questions. In line with this, I explained the descriptions of the tools as well as the way of using the tools in teachers' questioning during this process in related titles. The findings were presented by comparing the two cases.

The findings of the study showed that, in total, there were six categories of tools for questioning, which included information technology (IT), printed supplementary materials (PSM), teacher drawings (TD), students (S), analogies (A), and real-life examples (RLE). I explained each of these tools below.

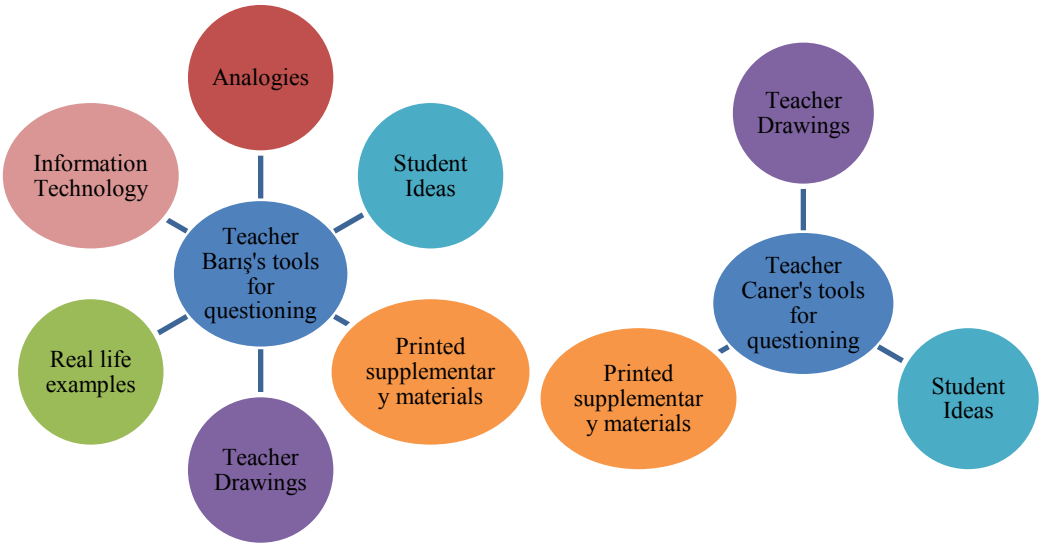


Figure 4.1 Middle grade mathematics teachers' tools for questioning

According to Figure 4.1, participant teachers did not use each of the tools. Teacher Barış used the mentioned six tools for questioning while Teacher Caner Caner used three of the tools for questioning including teacher drawings, student ideas, and printed supplementary materials. The rest of the tools which included information technology, analogies, and real life examples were different for both of the teachers. As seen in the figure, technology supported classroom included more diversity in terms of the tools for questioning. The tools could be used in questioning in diverse ways and in diverse frequencies.

For Teacher Caner, teacher drawings, student ideas, and printed supplementary materials were the main tools for his questioning while information technology, analogies, and real-life examples did not have a role on his questioning. According to the Table 4.1, his drawings helped him to explain a procedure through questions in most of the times and they helped him to solve a worked example through questions in less frequent. In line with this, the teacher was responsible for explaining a procedure of the mathematical content. Table 4.1 also showed that students' questions or comments that provided teachers to clarify or detect problematic aspects of students' mathematical thinking through asking questions were the other mostly used way of questioning tool compared to the use of student drawings that guided teachers for eliciting student thinking while solving worked examples. In line with this, student ideas were involved in teacher questioning by their questions, or comments and they were responsible for solving worked examples. Teacher Caner who had no use of technology in his lessons got benefit from printed supplementary materials to support his questioning. Printed supplementary books guided the teacher in terms of questioning sequence of worked examples. However, the printed supplementary materials were not compatible with smartboard; therefore, it had a different nature from the supplementary books of Teacher Barış. Teacher Caner applied a way of using student drawing (T11) and a way of use of printed supplementary book in less time (T15).

For Teacher Barış, the supplementary book, teacher drawings, and student ideas were the main questioning tools for his questioning. Specifically some uses of the tools for his questioning were more frequent.

The supplementary book itself guiding the teacher's questioning in terms of the sequence of the questions to be asked, teacher drawings to solve a worked example through questions, and students' questions or comments were frequent for the use of his questioning. The guidance of real life examples and some uses of supplementary book compatible with smartboard were the least used questioning tool for Teacher Barış. Table 4.1 represented the summary of the frequency of the use the tools for questioning for the participant teachers:

Table 4.1

The summary of the frequency of the use the tools for questioning for the participant teachers

Tools for questioning		The way of using the tools	# of obser. quest. ep.	
			Barış	Caner
Information Technology (IT)	Dynamic geometry software (DGS): Geogebra	Using DGS to build questioning sequence in response to student's questions. (T1)	1	-
		Using DGS to ask questions based on dynamic figures created by the teacher. (T2)	7	-
	Supplementary Book compatible with smartboard	The supplementary book itself guided the teacher's questioning in terms of the sequence of the questions to be asked.(T3)	48	-
		Educational animation in the textbook running with the help of smartboard was utilized to question a mathematical procedure. (T4)	1	-
		Part of the supplementary book was utilized to question student's performances. (T5)	1	-
		Dynamic shapes were utilized to question a mathematical procedure. (T6)	2	-
Analogies (A)		Analogies provided teachers to ask questions visualizing mathematical concepts. (T7)	6	-
		Analogies helped teachers to refer them while questioning a worked example. (T8)	9	-
Student Ideas (S)		Students' questions or comments provided teachers to clarify or detect problematic aspects of students' mathematical thinking through asking questions. (T9)	30	28
		Student drawings guided teachers for eliciting student thinking about mathematical procedures or concepts with questions. (T10)	2	-
		Student drawings guided teachers for eliciting student thinking while solving worked examples. (T11)	12	6
Teacher Drawings (TD)		Teacher drawings helped teachers to solve a worked example thorugh questions. (T12)	42	7
		Teacher drawings helped teachers to explain a procedure through questions. (T13)	22	23
Real life examples (RE)		Real life examples helped teachers to question mathematical procedures or concepts. (T14)	1	-
Printed supplementary materias (PSM)		Printed supplementary books guided the teacher telling about a mathematical procedure or concept with questions. (T15)	2	6
		Printed supplementary books guided the teacher in terms of questioning sequence of worked examples. (T16)	-	27
		# of tools for questioning	184	97
		Total questioning episodes	93	54

*Note.*The way of using the tools were represented in the form of abbreviations in Table 4.1 as shown in the parentheses in this table (e.g; T1, T2)*

Table 4.2 represented the presense of the use of the tools throughout the courses for Teacher Caner:

Table 4.2

The overview of the use of the tools in terms of lessons for Teacher Caner

TOOLS FOR QUESTIONING	The flow of the lessons				
	1st	2nd	3rd	4th	5th
PSM					
Printed supplementary books guided the teacher telling about a mathematical procedure or concept with questions.	■	■			
Printed supplementary book guided the teacher in terms of questioning sequence of worked examples.			■		■
Student Ideas					
Students' questions or comments provided teachers to clarify or detect problematic aspects of students' mathematical thinking through asking questions.	■	■	■		■
Student drawings guided teachers for eliciting student thinking while solving worked examples			■		
Student drawings guided teachers for eliciting student thinking about mathematical procedures or concepts with questions.					
Teacher Drawings					
helped teacher to solve the worked examples through questions.			■		■
helped teachers to explain a procedure through questions.	■	■	■		■

Individual problem solving session

According to this table, through all lessons except the 4th one, the teacher used printed supplementary materials. In the first two lessons, printed supplementary books were used to explain a mathematical procedure or concept with questions, while the other lessons, except 4th, any of the printed supplementary books guided the teacher in terms of questioning sequence of worked examples. While the teacher somehow was using the printed supplementary materials, the student ideas were involved in his questioning process by their questions or comments, which guided the teacher for eliciting student thinking through asking questions. Student ideas were also involved in the questioning process by drawings, which guided the teacher for eliciting student thinking while solving worked examples initiating from the 3rd lesson. Throughout all the lessons, he made drawings that helped the teacher to explain a procedure through questions and served for the solution of worked

examples initiating from the 3rd lesson. He did not expect his students to make drawings to explain mathematical procedures or concepts with questions.

Teacher Barış made the main difference from Teacher Caner by additional use of information technology tools, analogies, and real-life examples. Information technology was used more than analogies and real-life examples. It provided Teacher Barış to utilize information technology tools, which provided him to enrich his questioning by dynamic geometry software and supplementary book compatible with smartboard. The supplementary book itself, which guided the teacher's questioning in terms of the sequence of the questions to be asked, was used more than dynamic geometry software to ask questions based on dynamic figures created by the teacher. During the lectures of Teacher Barış, real-life examples and analogies were rarely used but real-life examples provided the teacher to question mathematical procedures or mathematical concept and analogies enabled him to visualize the procedures and concepts and to refer them while solving worked examples.

The main tools for questioning for Teacher Barış were teacher drawings, student ideas, and supplementary book. Teacher drawings helped teachers to solve a worked example thorugh questions, and the supplementary book guided the teacher's questioning in terms of the sequence of the questions to be asked in most of the times. In line with this, Teacher Barış was responsible for solving worked examples during the lectures. As similar to Teacher Caner, students' questions or comments that provided teachers to clarify or detect problematic aspects of students' mathematical thinking through asking questions were the other mostly used way of tools for questioning to clarify or detect problematic aspect of student mathematical thinking through asking questions.

Teacher Barış, deriving from the fact that he used supplementary book compatible with smartboard, did not use printed supplementary materials frequently. Additionally, many uses of information technology such as dynamic geometry software (especially T1) and uses of supplementary book compatible with smartboard including T4, T5, and T6, were used rarely. The overview of the use of the tools for Teacher Barış was represented in Table 4.3:

Table 4.3

The overview of the use of the tools in terms of lessons for Teacher Barış

TOOLS FOR QUESTIONING	The flow of the lessons						
	1st	2nd	3rd	4th	5th	6th	7th
Information technology							
Supplementary Book compatible with smartboard							
Dynamic shapes were utilized to question a mathematical procedure.				■	■		
Part of the supplementary book was utilized to question student's performances.							■
Educational animation in the textbook running with the help of smartboard was utilized to question a mathematical procedure.			■				
The supplementary book itself guided the teacher's questioning in terms of the sequence of the questions to be asked.	■	■	■	■	■	■	■
Dynamic Geometry Software (DGS)							
Using DGS to ask questions based on dynamic figures created by the teacher.		■	■				
Using DGS to build questioning sequence in response to student's questions.		■					
Student Ideas							
Student drawings guided teachers for eliciting student thinking about mathematical procedures or concepts with questions.							■
Student drawings guided teachers for eliciting student thinking while solving worked examples	■		■		■	■	■
Students' questions or comments provided teachers to clarify or detect problematic aspects of students' mathematical thinking through asking questions		■	■	■	■	■	■
Teacher Drawings							
helped teacher to solve the worked examples via questioning	■	■	■	■	■	■	■
helped teachers to explain a procedure through questions.		■	■	■	■	■	■
Real life examples							
Real life examples helped teachers to question mathematical procedures or concepts							■
Analogies							
Analogies helped teachers to refer them while questioning a worked example.				■	■	■	
Analogies provided teachers to ask questions visualizing mathematical concepts.	■			■		■	■

As Table 4.3, the teacher started solving worked examples and allowed his students to solve the examples at the beginning of the lesson. Additionally, the teacher questioned the worked examples in each of the lessons with their drawings. During all the instructions, the supplementary book was used to guide the teacher's questioning in terms of the sequence of the questions. Depending on the inclusiveness of a flash icon, which allowed working on dynamic shapes on the smartboard, in 4th and 5th lessons, dynamic shapes were utilized to question a mathematical procedure.

Additionally, an educational animation in the textbook running with the help of smartboard was utilized to question a mathematical procedure as the teacher followed the supplementary book systematically. Similarly, depending on the supplementary book, which included a part for student's work, the teacher utilized it to question student's performances at the end of the instruction.

Another use of information technology tool was DGS. DGS was utilized to ask questions based on dynamic figures created by the teacher in the second and third lessons. It was utilized to build questioning sequence in response to students' questions in the second lesson as well. It was clearly seen that even though student questions started to be integrated from the second lesson to the end of the lesson, DGS was utilized only in the second lesson. In line with this, DGS was not used for answering all student questions or comments.

Student ideas had a role in teacher questioning in three ways. According to this, firstly, student drawings guided teachers for eliciting student thinking about mathematical procedures or concepts with questions in the last lesson. Secondly, depending on the permission of the teacher about student involvement of solving worked examples on the smartboard, student drawings guided teachers for eliciting student thinking while solving worked examples in all lessons except the second and fourth. Lastly, all the lessons except the first one, students' questions or comments provided teachers to clarify or detect problematic aspects of students' mathematical thinking through asking questions. In line with this, students were active with their drawings and questions or ideas throughout the lessons.

Almost in each of the lessons, the teacher made drawings both to explain procedure and to solve worked examples. They helped the teacher implement questioning while utilizing them. Besides, the teacher started his lecture by a worked example, in line with this, his drawings helped him to solve the worked examples via questioning initiated from the first lesson.

Real life examples to question mathematical procedures or concepts were used only at the end of the last lesson. Analogies were used more than the real life examples. They were used in the very first lesson for visualizing mathematical

concepts. Then, starting from the 4th lesson to the 7th except from the fifth one, the teacher used analogies to ask questions visualizing mathematical concepts. In 4th, 5th, and 6th lessons, analogies helped teachers to refer them while questioning a worked example. In the following titles, I explained each of the tools for questioning by giving examples from the classroom dialogues.

4.1.1 Information Technology Tools

One of the tools for questioning was information technology tools whose was used by one of the teachers. This code emerged as a result the teachers' actions specific to questioning of mathematical concepts or procedures while using smartboard during the instruction. I presented the information technology tools describing their nature and demonstrating their role in teachers' questioning. The participant teacher, Teacher Barış, used Dynamic geometry software (DGS) and supplementary book compatible with smartboard as information technology tools in the instructions. In the following, I explained both of the tools explaining the way of using the tools for his questioning below.

4.1.1.1 Dynamic Geometry Software (DGS): Geogebra

DGS, specifically Geogebra, was one of the information technology tools utilized by Teacher Barış in his questioning. Observation data indicated that this tool contributed to the teacher's questioning mainly in two ways. First, in several occasions, teachers used DGS to build questioning sequence in response to students' mathematical questions. Second, he used DGS to ask questions based on dynamic geometric figures created by the teacher. In the following, I explained and gave two example uses of DGS in the teacher questioning by sample excerpts. This part included examples of Teacher Barış as the other teacher, Teacher Caner, did not use DGS for his questioning.

4.1.1.1.1 Using DGS to build questioning sequence in response to student's questions.

In this use of tools for questioning, the teacher created a questioning episode which initiated with student questions and the teacher benefited from DGS to respond the student questions. The following classroom dialogues exemplified how DGS assisted the teacher's questioning concerning students' questions. The student asked a question to the teacher. After the teacher told the class about the procedure of applying multiplicative reasoning for creating a congruent angle, one of the students questioned whether the procedure could give the same result by using additive reasoning to create the vertical and horizontal distances of a point. The teacher encouraged the class to question whether the use of additive reasoning is appropriate while creating a congruent angle:

> Student: Teacher, do we have to increase [the length of the line segments] one by one? Don't we do that by adding [numbers to the length of the line segments] (Öğretmenim illa kat kat arttırmamız mı lazım, ekleyerek olmaz mı?)
> Teacher: Hımm, she says,for instance, that is five, let's increase by one. Let's increase here by one as well, and six.. It is impossible. Let's try if you want. (Ha şunu diyor. Mesela şurası 5 diyor, 1 arttıralım diyor. Burayı da bir arttırıp altı.. (see Figure 4.3) İmkanı yok. Deneyelim istiyorsan.)
> Student: Let's try. (Deneyelim.)
> Teacher: Yes. We are going to test Damla's idea. Yes, I am drawing an angle. What are the coordinates, two and two [the distance of point C to the point B], what are the coordinates, let's do two and three [the distance of point A to the point C] (see Figure 4.4).
> I am doing like that. Check me if I am measuring [the angle] correctly? The distance.. three… is there any problem? All right. How many [units] to increase? (Evet. Damla'nın fikrini test edeceğiz şimdi. Evet bir tane açı çiziyorum. Kaça kaç ikiye üç olsun Görüyorsunuz ikiye üç. Hatta onuda şöyle yapayım ben. Doğru mu ölçüyorum bak. Uzaklık üçe... Var mı sıkıntı? Tamam. Kaç arttıralım bunları?)
> Student: Two (İki).
> Teacher: Let's increase by two. Would that be four, and would that be six?. (İki arttıralım. Bu dört olcak bu altı mı olacak?)
> Student: Teacher, in that case they will be congruent [to each other]. (Öğretmenim öyle eş olur.)

Teacher: Let's increase by three. I am increasing by three. That [distance of a point to the axis of ordinate] would be five, and that [distance of a point to the axis of apssisca] would be six. That was five and six? What I did is, I increased that one by three and that one by three as well. We are checking if they are in equal [to each other] about the measurement of the angle? I am selecting the points. The measurement of the angle is 50,19° and the measurement of the angle is 50,31°. Let's see. Is there something like that? Is the rate important? Okay, I'm passing. (Üç arttıralım. Üç arttırıyorum. Beşe altı olacak. Beşe altı mı oldu ne yaptım bunu üç arttırdım, bunuda üç arttırdım. Bakıyoruz eşit çıkıyor mu? Seçiyorum noktaları. Elli nokta on dokuz. Bakalım kaç çıkıcak? Elli nokta on dokuz elli nokta otuz bir Var mıymış öyle bir şey? Oran mı önemliymiş? Tamam geçiyorum.

Teacher Barış, Lesson 2, Line 216-232

The teacher gave the opportunity to the entire class by integrating the student question about constructing a congruent angle to a given angle procedure by himself. At the beginning of the dialogue, a student asked a question. He rephrased the student's question by giving an example of what the student said. Following this, he answered her question explicitly saying that 'That's impossible' (see Figure 4.2 and Figure 4.3).

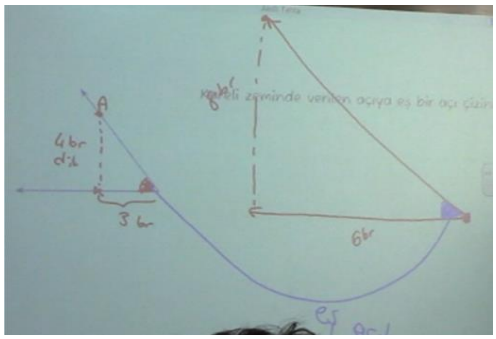


Figure 4. 3 Constructing a congruent angle by multiplicative reasoning

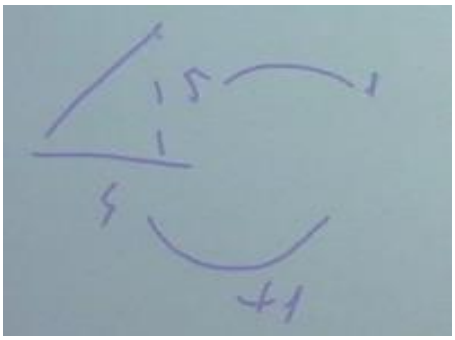


Figure 4. 2 Creating a congruent angle by additive reasoning

However, he gave another opportunity to her to test her idea. After the student agreed on testing her idea, the teacher initiated creating a dynamic figure on smartboard. The teacher found the measurement of the angles in both of the figures

and proved that the figures would not be equal if students applied additive reasoning while constructing a congruent angle (see Figure 4.4).

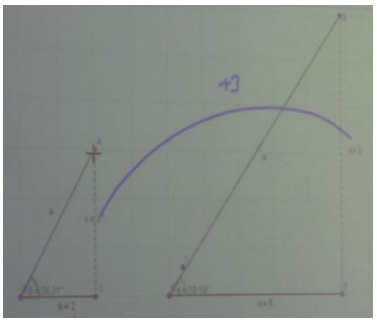


Figure 4. 4 Creating an incongruent measurement of angles by additive reasoning

The frequency of the use of the questioning tool in questioning episodes was as follows:

Table 4.4

The frequency of using DGS to build questioning sequence in response to student's questions

The nature of questioning episode	# of questioning episodes	% of questioning episodes
	Teacher Barış	
Questioning episodes including student questions	30	30
Questioning episodes in which student questions were responded via DGS	1	1
Total questioning episodes	93	100

In line with the Table 4.4, even though there were thirty audible student questions, which were one-third of the total questioning episodes during the instruction, the teacher did not answer the students' questions by using DGS except one situation, which was represented in the dialogue.

4.1.1.1.2 Using DGS to ask questions based on dynamic figures.

Another way of using DGS by the teacher in his questioning was about questioning dynamic figures. Independent from the situations using DGS to build questioning sequence in response to student's questions, the teacher used DGS to ask questions based on dynamic figures he created. Teacher Barış created dynamic figures to his students in any time during the instruction. While he was telling about a procedure, he was solving a worked example of a procedure, or wanted to represent a mathematical concept through a dynamic figure, he posed questions to the class based on dynamic figures. Creating dynamic figures were not required in the supplementary book they followed, and the supplementary book did not guide the teacher to ask questions based on dynamic figures. He presented students with a chance to understand mathematical aspects in a dynamic environment and encouraged students to make sense of the explanations in the book through dynamic figures. These dynamic figures helped the teacher to ask questions about the learning content or made explanations on the form of questions. The teacher created dynamic figures which was represented in the following dialogue and the figures provided students to see the intersection of pairs of lines, which were in a static form in the supplementary book. While the teacher was constructing Figure 4.7, he asked questions utilizing the Figure 4.4, 4.5, and 4.6:

> Teacher: Yes. [The definition says pairs and pairs. So one of them is this one. The other one is this one. Are they intersected to each other?] (Evet. İkişerli ikişerli diyor. O zaman biri bu olsun. Diğeri bu olsun. Kesişti mi?) (Figure 4.5)
> Student:Yes.(Evet.)
> Teacher: All right. This one, another line is intersected the other line? (Tamam. Şu, burdan geçen başka bir doğru da bununla mı kesişsin?) (Figure 4.6).
> Student: Yes. (Evet.)
> Teacher: Are the lines intersected by pairs? (İkişerli kesişti mi?) (Figure 4.6).
> Student:Yes.(Evet.)
> Teacher: Did a triangle emerge? (Üçgen oluştu mu?) (Figure 4.6).
> Student:Yes.(Evet.)

Teacher: We have a triangle here. Is it true? (Burada bir üçgenimiz oluştu. Doğrumu?)
Student:true.(Doğru.)
Teacher: Look, kids, when you change them, there's no such thing that these two lines should intersect to each other. For example, this one could be that one as well (Figure 4.7). This does not matter. It is important that here are two pairs that the intersection of these two are important. The intersection of this [showing a line] with that one [showing another line] is important. Okay? The intersection of pairs is important (Figure 4.8). (Bakın çocuklar bunları değiştirdiğinizde illa mesela bu bunu kescek diye birşey yok. Bu mesela şu da olabilirdi fark etmez Burada ikişer ikişer mesela şununla şunun kesişme durumu önemli ikişer. Bununla şunun kesişme durumu önemli. Tamam mı? Ikişer ikişer kesişmesi önemli.)

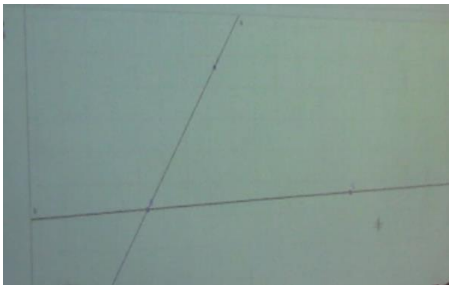


Figure 4. 5 The representation of the intersection of two lines

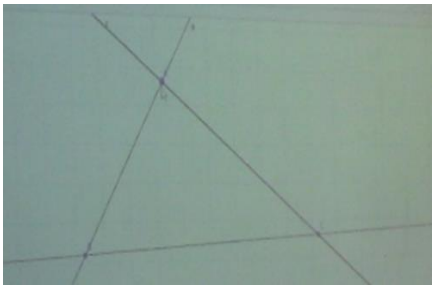


Figure 4. 6 The representation of the intersection of each of the two lines

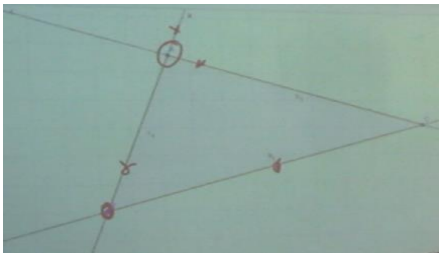


Figure 4. 7 The representation of the triangle formed by the intersection of each of the two lines

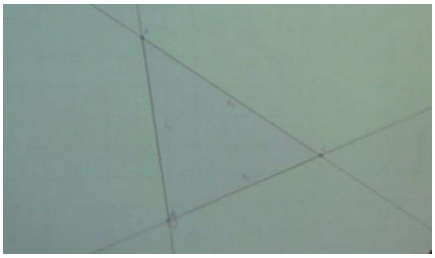


Figure 4. 8 The dynamic figure to show the intersection

The dynamic figure guided students toward an understanding of the mathematical content of the intersection of three lines by the teacher questions. Based on the dynamic figure, the teacher questioned the pairs of the intersection of two lines and intersection of three lines. The questions were supporting the teacher in providing students' understanding of the procedure of creating the intersection of three lines.

The teacher followed such a way to construct the figure and ask questions based on it. The frequency of questioning episodes related to this use was represented in Table 4.5:

Table 4.5

The frequency of questioning episodes in which teacher used DGS to ask questions based on dynamic figures

The nature of questioning episode	# of questioning episodes	% of questioning episodes
	Teacher Barış	
Using DGS to ask questions based on dynamic figures	7	8
Total # of questioning episodes	93	100

As seen in the table, the teacher created dynamic figures not in many occasions to ask questions. These two uses of DGS enabled Teacher Barış to question mathematical procedures or concepts with the class and to make explanations via questions. He used DGS as a tool to answer students' questions and a tool to question mathematical content with dynamic figures. He integrated DGS in his questioning using the features of DGS such as measurement of an angle or distance of a line segment, dynamicity of shapes and that provided the teacher to create questioning episodes from his drawings. In line with this, both of the uses produced additional questioning episodes for the class and gave the classroom a chance to improve their learning in such a dynamic environment.

4.1.1.2 Supplementary Book Compatible with Smartboard

Another information technology tool was supplementary book compatible with the smartboard. This tool was revealed in Teacher Barış's classroom, which was supported by technology. The book was used in compatible with smartboard and each of the pages of the supplementary book was seen on the smartboard. The teacher started each of his lessons opening the last page he did. The supplementary book had roles in the teacher's questioning. It had four contributions to the teacher's

questioning: The textbook itself guided the teacher's questioning in terms of the sequence of the questions to be asked, educational animation in the textbook running with the help of smartboard provided teacher an instructional context for asking questions, part of the supplementary book was utilized to question student's performances, and dynamic shapes compatible with smartboard were utilized to question student performances. Each of the contributions was explained below.

4.1.1.2.1 The supplementary book itself guided the teacher's questioning in terms of the sequence of the questions to be asked.

One of the ways of using the supplementary book as a tool for the teacher's questioning is the guidance of the textbooks regarding the sequence of the questions to be asked. Teacher Barış followed the supplementary book step by step and utilized each of the information such as concept explanations, concept definitions, or worked examples in it during the instructions. He got to benefit from the information and asked questions according to this and the teacher's questions that were posed related to the part in the supplementary book. Therefore, the sequence of the way of questioning in the supplementary book was in the same order with the sequence of the teacher's questions. The supplementary book was used as a source that managed the sequence of questions for the teacher. In line with this, the tool helped the teacher in organizing which questions the teacher would ask and in which sequence he would use the information in his questioning during the instruction. However, occasionally, when the teacher gave real life examples, created figures, or used analogies, the sequence of the questions was broken down. In the following Table 4.6, the frequency of the use of the supplementary book regarding the sequence of the questions to be asked was represented. The table indicated that the supplementary book guided the teacher in the sequence of questions to be asked in more than half of the questioning episodes. The supplementary book was a resource that the teacher frequently utilized information in it to initiate the questioning process and therefore

had a significant role in teacher questioning about the sequence of the questions to be asked.

Table 4.6

The frequency of questioning episodes where the supplementary book itself guided the teacher's questioning in terms of the sequence of the questions to be asked

The nature of questioning episode	# of questioning episodes	% of questioning episodes
	Teacher Barış	
Including the questions in the supplementary book	48	49
Total questioning episodes	93	100

Even though the teacher used each of the information in the supplementary book in his questioning, the total number of questioning episodes were not equal to this use. There were other tools used together in the remaining of the questioning episodes. For instance analogies, and real-life examples created independent questioning episodes of the supplementary book.

4.1.1.2.2 Educational animation in the supplementary book running with the help of smartboard guided the teacher to ask questions about mathematical procedures.

Another way of using the supplementary book in the teacher's questioning was the use of educational animation. The supplementary book was supported with educational animations which were represented on the title of a page as a flash icon and run by flash player. The supplementary book was supported with educational animations which were represented on the title of a page as a flash icon and run by flash player. The supplementary book was supported with educational animations which were represented on the title of a page as a flash icon and run by flash player.

This educational animation included the procedure about how to draw a bisector with a compass. The animation was represented in Figure 4.9.

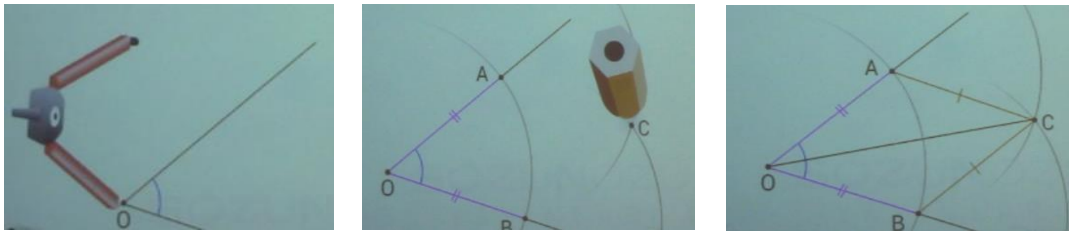


Figure 4.9 Educational animation that represent the procedure about how to draw a bisector with compasses

He watched the animation with the class at the same time. During the playing, the teacher asked students the procedure of drawing a bisector in a leading manner like 'It drew a ray with one of the leg of the compasses. It drew the other ray with the leg of the compasses. There is an angle there, right? [Bir kolu ışın çizdi. Diğer ışını çizdi, kolu. Bir açı oldu değil mi orada?]', 'It opened the legs of the compasses, put the corner of the angle, and it drew an arch, didn't it? [Pergeli açtı, açının dirseğine koydu bir yay çizdi, doğru mu?]'After the end of the playing, the teacher imitated the construction using DGS on the smartboard and while doing this, the teacher was talking about the procedure by posing questions together with the class. In this regard, the teacher used educational animation in the supplementary book to ask questions about a mathematical procedure.

The frequency of the use of the educational animation in the questioning episodes were represented in Table 4.7. According to the table, the teacher used educational animations in only one questioning episode. The teacher rarely used educational animations. That might be because the supplementary book did not require more than one animation. In line with this, using the educational animation depended on whether the supplementary book included animations. It was a minor tool for the teacher's questioning.

Table 4.7

The frequency of the questioning episodes in which educational animation in the supplementary book was used in teacher questioning

The nature of questioning episode	# of questioning episodes	% of questioning episodes
	Teacher Barış	
Using educational animation in the supplementary book	1	1
Total questioning episodes	93	100

4.1.1.2.3 Part of the supplementary book was utilized to question student's performances

Another use of the supplementary book in teacher's questioning was that the teacher used the supplementary book to question student performance. Teacher Barış applied a part of the supplementary book at the end of the chapter, which included a part related to students' performances named 'this is your turn.' That part of the book was utilized to question students' performances concerning the problems in the 'part of this is your turn'. Although the teacher was questioning students' performances by asking questions every second during the instruction, that part of the supplementary book gave special attention to students' performances. The frequency of using the tool was represented in Table 4.8 below:

Table 4.8

The frequency of the use of part of the supplementary book was utilized to question student's performances

The nature of questioning episode	# of questioning episodes	% of questioning episodes
	Teacher Barış	
Part of the supplementary book was utilized to question student's performances	1	1
Total questioning episodes	93	100

As seen in the table, the teacher used the tool rarely. Only one questioning episode was related to this use of questioning. The tool helped the teacher to question student's performances while it had a limited role in teacher's questioning. That part was rarely used as a way to ask students allowing them to solve the problems in that part on the board. That might be because the teacher often used that part as a part of giving homework. One percent of the questioning episodes showed that the book was used to question student performances in related worked examples written in the book and the teacher asked mathematical question to make questioning about the worked example .

4.1.1.2.4 Dynamic shapes supported by the supplementary book were utilized to question a mathematical procedure.

The other tool that supported teacher's questioning was about dynamic shapes that were represented in the supplementary book in a flash icon. As similar to educational animation which was represented as a flash icon in the supplementary book, the teacher noticed an icon together with his students. Different from the educational animation, this icon was dynamic and it could not be watched. The icon represented a procedure about the sum of the inside angles when two lines parallel to each other were intersected by a third line (see Figure 4.10).

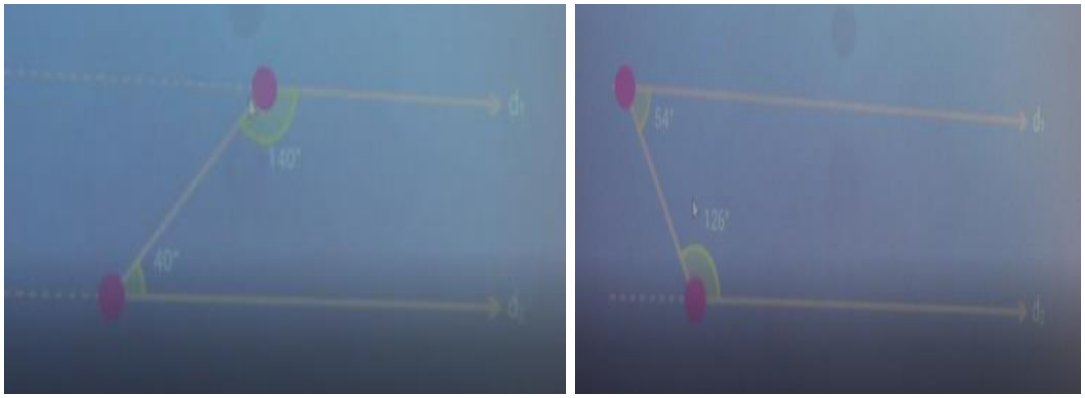


Figure 4.10 Dynamic shape that represent the sum of the interror angles when two lines parallel to each other were intersected by a third line

It had a dynamic nature as similar to dynamic figures created by using DGS. The teacher encouraged students to move the shape from the purple corners and to check the application of the procedure on the shape. The teacher rarely used the DGS tool in the way that dynamic shapes supported by the supplementary book were utilized to question a mathematical procedure.

As seen in Table 4.9, it was observed in two questioning episodes. As similar to the use of educational animation, this tool was utilized during the instruction very rarely because there was a small number of the icon in the supplementary book, which included dynamic shapes to question.

Table 4.9

The frequency of the use of dynamic shapes that were utilized to question a mathematical procedure

The nature of questioning episode	# of questioning episodes	% of questioning episodes
	Teacher Barış	
Use of dynamic shapes to question a mathematical procedure	2	2
Total questioning episodes	93	100

4.1.2 Analogies as a Tool for Questioning

Another tool that one of the teachers, Teacher Barış, used in his questioning was analogies, through which he made connections between mathematical ideas and other phenomena from daily life. In some of his questions, there was a need to use an analogy to make mathematical concepts more understandable to the students. In such cases, one of the participant teacher, Teacher Barış benefited from analogies that resemble mathematical concepts. The results of the study showed that he utilized analogies in his questioning in two ways: analogies provided teachers to ask questions visualizing mathematical concept and helped the teachers to refer them while asking questions about a worked example. In the following, each of the ways was described and analyzed regarding the frequency of the use of the tool.

4.1.2.1 Analogies Provided Teachers to Ask Questions Visualizing Mathematical Concepts.

One of the use of analogies for a teacher's questioning was related to the role of analogies in visualizing mathematical concepts. In this kind of use, the teachers asked questions about the mathematical concepts as the way that students resembled the concept on the analogies. Therefore, asking questions that provide a similarity to another phenomenon can make it easier for students to imagine the mathematical idea. For example, at the beginning of the instruction, Teacher Barış used an analogy requiring students to give meaning to point, ray, and line as seen below:

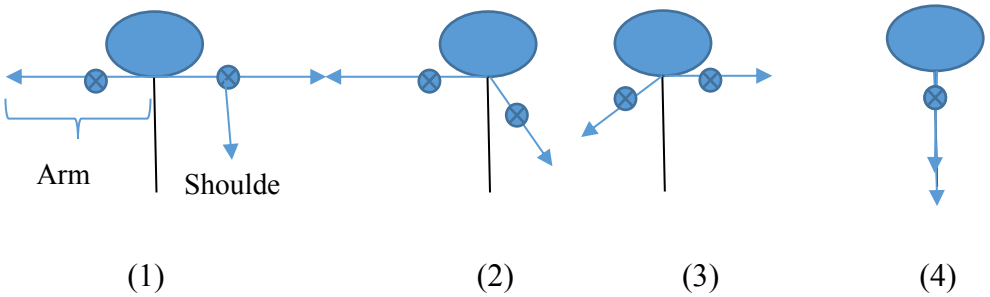


Figure 4.11 Body analogy

He related these mathematical concepts with specific places on his body:

> Teacher: Okey, tell me that what am I doing now? Think these (showing the shoulder and the part between the shoulders) as two points. What is this? (Peki şunu bana söyleyin bakayım. Ben şu an neleri yapıyorum. Şunları iki tane nokta gibi düşünün. (omuzlarını ve arasında kalan kısmı göstererek)Bu ne şuan?)
> Student: Line segment. (Doğru parçası.)
> Student: A ray. (Işın.)
> Barış: (opening the arms) these are the points. Is that a ray? What is this? (kollarını iki yana doğru açarak) Buralar geçtiği noktalar. Bu ışın mı? Bu ney?)
> Öğrenci: Line segment (Doğru parçası.)
> Barış: Just a second. What is this? (Bir dakika ne?)
> Öğrenci: Line segment (Doğru parçası.)

Barış: That one? You mean the line between the two points. These are the points [that the shape passess] My shoulders. (Şu? İki nokta arasında ki çizgiyi diyosun. Geçtiği noktalar bunlar. Omuzlarım.)
Öğrenci: That is a line in that case (O zaman doğru.)
Barış: That one? (Şu?)
Öğrenci: Line (Doğru.)
Barış: (Like in Figure 4.11-(2) This?(Bu?)
Öğrenci: A ray (Işın.)
Barış: (Like in Figure 4.11-(3) This?(Bu?)
Öğrenci: A ray (Işın.)
Barış: This? (Bu?)
Öğrenci: Doğru.
Barış: (Like in Figure 4.11-(4)) This? (Bu?)
Öğrenci: A line segment (doğru parçası.)
Barış: Ayşe is confused about line segment and line. (Ayşe doğru parçası ile doğruyu karıştırıyor.)

Teacher Barış, 1st lesson, Line 41-49

As seen in the dialogue, that analogy helped the teacher in concreteness of the mathematical concepts with the body analogy and in detecting students' misconceptions or errors about the concepts during his questioning. Teacher Barış considered that Student Ayşe had a problem with the concepts of line and segment. The frequency of using analogies that provided teachers to ask questions visualizing mathematical concepts was represented in Table 4.10 below:

Table 4.10

The frequency of the use of analogies that provided teachers to ask questions visualizing mathematical concepts

The nature of questioning episode	# of questioning episodes	% of questioning episodes
	Teacher Barış	
Use of analogies that provided teachers to ask questions visualizing mathematical concepts	6	7
Total questioning episodes	93	100

As seen in Table 4.10, analogies were not used in six questioning episodes. In line with this, they were not the main tool for the teacher's questioning. Even though that tool was not information technology related tools for questioning,

Teacher Caner did not use the tool in his questioning. In this regard, information technology tool was not the only tool that differed from the teachers.

Analogies were not frequently used by the teacher but the tool for questioning provided teachers to ask questions visualizing mathematical concepts.

4.1.2.2 Analogies Helped Teachers to Refer Them While Asking Questions about a Worked Example.

The other use of analogies during the questioning process was that analogies helped the teacher to refer them while questioning a worked example. One of the teachers, Teacher Barış, used the tool in his questioning. In his use of analogy, the teacher did not go into a questioning process as before to visualize the procedure or concept. The teacher referred to the analogy while solving a worked example by himself or guided student in solving the example. Therefore, the analogy played a reminder role for the procedure for students in the questioning process.

In the following example, one of the students was on the board to solve a worked example and Teacher Barış guided the student including questions referring to the river analogy:

> Teacher: For example, here… Now, here is 116, look at there. Is that right? Here, 116 is looking at the right side and it is on the same side with the river. Right? (Mesela burası...şimdi burası 116ysa burası bak şurası. Doğru mu? Bakıyorsun burası 116 sağa bakıyor ve nehir kısmında kalıyor doğru mu?)
> Student: Yes. (Doğru.)
> Teacher: We are thinking it [the parallel two lines] as a river, right? (Şunu bir nehir gibi düşünüyorduk doğru mu?)
> Student: Yes. (Evet.)
>
> Teacher Barış, 4th lesson, Line 107-111

Analogies helped the teachers to remind students of the procedure of alternate interior and alternate exterior angles emerged from the intersection of three lines.

Therefore, the use of the analogy was used to activate previously talked issues while solving a worked example related to that content.

Table 4.11 represented the frequency of the use of analogy in the questioning episodes while the teacher used different tools in the remaining of the questioning episodes. It showed that the teacher utilized analogies in ten percent of the questioning episodes, which was quite a less use compared to the use of other tools. While analogies were used to question a worked example, teachers got to benefit from their drawings questioning the same worked example as well. Therefore, the use of the analogies while questioning a worked example showed that teachers sometimes apply analogies to refer them but they do not frequently do that. They made questioning from their drawings and student drawings more than the use of analogies to support the questioning process during the solution of a worked example.

Table 4.11

The frequency of the use of analogies helped teachers to refer them while questioning a worked example

The nature of questioning episode	# of questioning episodes	% of questioning episodes
Use of analogies that helped teachers to refer them while questioning a worked example	9	10
Use of teacher drawings to solve worked examples via questioning	42	45
Use of student drawings to solve worked examples via questioning	12	13
Total questioning episodes	93	100

4.1.3 Student Ideas as a Tool for Teacher Questioning

Another tool that contributed to teacher questioning was student ideas. Student ideas can be a tool for questioning while they are involved in teacher's questioning through questions or comments. In the learning environment, instead of directly answering students' questions, both of the teachers used student questions as opportunities to new questioning dialogues. The student ideas were involved in the

teachers' questioning in the following three ways: students' questions or comments provided teachers to clarify or detect problematic aspects of students' mathematical thinking through asking questions, student interpretations guided teachers to elicit student thinking about mathematical procedures or concepts with questions, and student drawings guided teachers to elicit student thinking while solving worked examples. Each of them was explained below.

4.1.3.1 Students' Questions or Comments Provided Teachers to Clarify or Detect Problematic Aspects of Students' Mathematical Thinking Through Asking Questions.

The use of student questions or comments by teachers was that teachers used student questions or ideas to clarify or to explore possible problematic aspects of students' mathematical thinking by asking follow-up questions. In this way, teachers integrated the questions or comments into their instructions. The student questions actually informed the teachers about the students' thinking and teacher questioning helped students to think further about the unclear parts of their thinking. The mathematical dialogue given under the title of *'using DGS to build questioning sequence in response to student's questions. (p.62)* was an example for this use. In that dialogue, one of the students asked a question about whether drawing a congruent angle is possible by making additive reasoning between the horizontal and vertical distance of the points located on a ray. The student question was used in teacher's questioning as a teacher question and was clarified by using DGS. Another example was represented below:

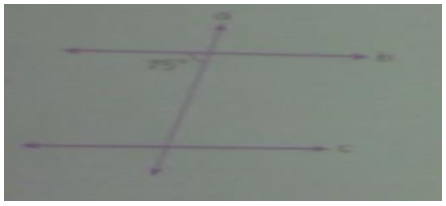


Figure 4. 12 The situation discussed in the following dialogue

Teacher: Look, if here is 75, there is 75? It is because of the exterior angles, right? (Şimdi bak burası 75 ise şurası 75 mi? Ters açılardan doğru mu?)
Student: Yes (hıhı.)
Teacher: Well, the 75 degrees is out of the two parallel lines, isn't it? Well, what is looking between these two lines looking at the same direction ... and looking in this direction? Take a look. (Peki bu iki tane paralel doğrunun dışında kalıyor değil mi şu 75 derece. Peki aynı yöne bakan bu iki doğrunun arasında kalan ... ve bu yöne bakan kim var? Bir bak bakalım.)
Student: That one (Şu var.)
Teacher: But this one looks that side (E bu bu tarafa bakıyor.)
Student: That one (bu.)
Teacher: Yes. Then this place is 75? (Evet. O zaman burası da mı 75 oluyor.)
Student: Yes. Now it's here and there. (hıhı. şimdi burası ve şurası var.)
Student: Teacher, doesn't this side have to be 75? This place is looking outward, but why is it 75? I don't get it. (Öğretmenim bu tarafta da 75 olması gerekmez mi? Birde bir şey diyeceğim; burası dışa doğru bakıyor ama burası niye 75? onu anlamadım.)
Teacher: But it looks this side, it looks that side. How can they be equal to each other? Is here 105 because of supplementary angles? (İyide bu bu tarafa bakıyor, bu bu tarafa bakıyor. bunlar nasıl birbirine eşit olabilir? Bura 105 mi bütünler açıdan?)
Student: okey. (tamam.)

Teacher Barış, 7th lesson, Line 47-57

In these questioning episodes, the teacher established a questioning episode based on the student's idea that said she did not understand the mathematical procedure about the placement of interior angles. Following this, the student asked questions about the way of placement of the alternate interior angles and alternate exterior angles depending on the situation. The student questions gave the teacher a clue about what the problematic aspects of student's mathematical thinking were. The teacher answered the student's questions by clarifying through asking questions to the student.

Teacher Caner, used a student idea in his questioning. According to the student, the sum of the internal opposite angles is 180 degrees. In the following dialogue, the student idea was being questioning:

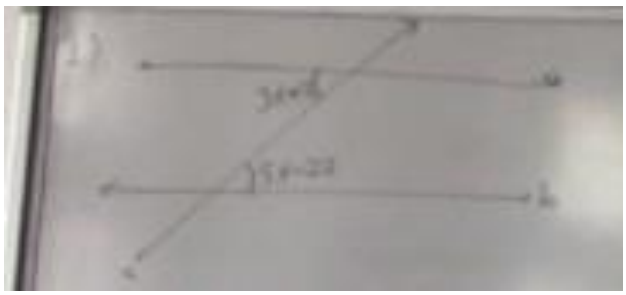


Figure 4.13 The situation discussed in the following dialogue

Student: Teacher, I solved it in a different way. (Öğretmenim ben farklı yolla yaptım.)
Teacher: What kind of a way do you mean? (Farklı yol dediğin nasıl bi şey kızım?)
Student: I mean 3x plus 12 plus 5x minus 20 is 180. (Yani 3x artı 12 artı 5x eksi 20 eşittir 180 dedim.)
Teacher: Why did you sum up two [3x+12 and 5x-12]? (Niye topladın ikisini?)
Student: Isn't it a full angle? (Tam şey oluşturmuyor mu?)
Teacher: No, I do not understand why you sum up them. (Hayır neden topladın onu anlamadım.)
Student: If this side is 3x plus 12 times, two of them are 180 degrees (Ya şimdi şu taraf 3x artı 12 siyse, ikisi 180 derece yapıyor.)
Teacher: No, the two don't make 180 degrees. The two would be 60, 60, and the sum maybe 120. (Hayır ikisi 180 derece etmezz. Ikisi 60 60 olur belki toplamları 120 olur.)
Student: It depends on the question. (Soruya göre değişir.)
Teacher: Changes. (değişir.)

Teacher Caner, 3rd lesson, Line 23-32

In this dialogue, the teacher wrote a worked example to practice the procedures of interior and corresponding angles. After one of the students solved the worked example in a correct way, another student wanted to share her ideas about the solution of the same worked example. The teacher questioned why the student followed such a way to solve the worked example and encouraged the student to think of the reason of the way she followed such a way.

As seen in the dialogues above, student questions/ideas opened a way to question student's question or ideas for both of the teachers. The frequency of the

use of student questions/ideas to clarify or detect problematic aspects of students' mathematical thinking through asking questions was shown below:

Table 4. 12

The frequency of the use of students' questions or comments through asking questions

The nature of questioning episode	# of questioning episodes	# of questioning episodes
Use of students' questions or comments	Teacher Barış	Teacher Caner
	30 (30%)	28 (50%)
Total questioning episodes	93 (100%)	54 (100%)

Table 4.12 showed that students asked questions or shared their ideas during questioning episodes in both of the classes in many times. The students initiated new questioning episodes by asking questions or making comments about problematic aspects where they did not understand. Students' of both of the teachers asked questions or mentioned about their ideas, which provided the teachers to clarify or detect problematic aspects of students' mathematical thinking through asking questions. The use of the tool was important in terms of understanding the difficulties of students deeply and of creating learning opportunities for the class. Problematic aspects were questioned by teachers' questions and the teachers guided them about the aspects that were not understood. The teachers concerned with the instructional decision whether students' questions were valuable to be utilized in their questioning. Examination of the questioning episodes showed that not every student question was embedded to the instruction as questioning episodes. The teachers ignored some student questions, which did not serve the purpose of the instruction. Student questions as a tool for questioning was in relation to the scope of the student's question or ideas. The way of using the tools was dependent on the teacher while the creation of the tool was dependent on student ideas. Therefore, the integration of the tool depended on the collaboration with students and teachers.

Another tool related to students was student drawings. Student drawings represented student markings in worked examples, about a mathematical procedure, or their additional works for solving the worked examples or mthematical

procedures. Both of the teachers used students' drawings as a tool for their questioning. They used student drawings through questions to elicit students' thinking by benefiting from such drawings that were done for the solution of worked examples and for explaining mathematical procedures. Based on the students' actions on these worked examples or teachers' questioning utilizing student drawings, student drawings were used as tools for teacher questioning. The tool guided teacher questioning for *eliciting student thinking about mathematical procedures or concepts with questions* or *for eliciting student thinking while solving worked examples.* In the following, each of them were explained.

4.1.3.2 *Student Drawings Guided Teachers for Eliciting Student Thinking about Mathematical Procedures or Concepts with Questions.*

One of the use of student drawings in teachers' questioning is that *student drawings guided teachers for eliciting student thinking about mathematical procedures or concepts with questions.* During the classes, students occasionally made drawings about mathematical procedures or concepts. Based on these drawings, teachers asked questions to elicit their thinking about the related procedure or the concept. For example, Teacher Barış asked a student to draw a figure that represented the definition about corresponding angle mentioned in the supplementary book. The teacher questioned the drawing of the student, which was formed to represent a corresponding angle concept:

> Teacher Barış: I am reading (the text about a definition). Aylin will draw what she understood. A thing is parallel between two lines, the other thing is not between two parallel lines but they both face to the same direction. What do you understand from that? [Okuyorum, Ayşe anladığını çizecek. Biri paralel iki doğru arasında olan, diğeri paralel iki doğru arasında ve olmayan ve aynı yönlere bakan. Biri paralel iki doğru arasında olan, diğeri paralel iki doğru arasında olmayan ancak aynı yönlere bakan. Bundan ne anlıyorsun?]
> Student: That is what I understood (from the text). Well.. For example, the angle here.. [Şöyle. Benim anladığım.şimdi..mesela burdaki açı..]

Teacher Barış: That is one of them. that is between the (lines), right? That is between the two parallel lines. Where is 'not between the two parallel lines?' [Biri o olsun. Arasında olan oluyor o değil mi? İki paralel doğru arasında olan oluyor. Olmayan neresi olabilir.]
Student: 'not' is here. Can be (that one)? [Olmayan ise bura. olabilir mi?]
Teacher Barış: Can't be the other one? [Diğer taraf olamaz mı?]
Student: It can be that one. [Bura da olabilir.]
Teacher Barış: What are they, which ones are corresponding angles? [Kim bunlar sence, yöndeş olanlar hangileri?]
Student: corresponding angles are this one and that one. [Yöndeş olanlar bu ve bu.]
Teacher Barış: Why are they corresponding angles? [Neden onlar yöndeş?]
Student: Because they face to the same direction. [Çünkü aynı yöne bakıyorlar.]
Teacher Barış: Did you understand? [Anlaşıldı mı?]
Students: Yes. [Evet.]

Teacher Barış, 7th lesson, Line 7 – 26

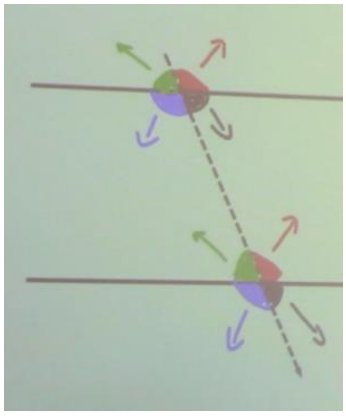


Figure 4. 14 The situation discussed above

As seen in the dialogue, the student made a marking about the corresponding angle and that drawing guided the teacher about what the student's thinking about corresponding angle concept. During this, teacher questions helped to reveal the student's thinking. Student drawing provided the student to explain her thinking and that provided the teacher to make questioning of the drawing. In the following, the frequency of using student drawings as a tool for teacher questioning were represented:

Table 4.13

The frequency of the use of student drawings in terms of eliciting student thinking about mathematical procedures or concepts with questions

The nature of questioning episode	# of questioning episodes	# of questioning episodes
	Teacher Barış	Teacher Caner
Use of student drawings eliciting student thinking about mathematical procedures or concepts with questions	2 (3%)	-
Total questioning episodes	93 (100%)	54(100%)

As seen from the table, student drawings were one of the tools Teacher Barış rarely applied their questioning. Teacher Caner did not expect students to make drawings about mathematical procedures or concepts the students would learn.

4.1.3.3 Student Drawings Guided Teachers for Eliciting Student Thinking while Solving Worked Examples

The other use of student drawings in teacher questioning was that s*tudent drawings guided teachers for eliciting student thinking while solving worked examples.* This use had a role in the teachers' questioning in terms of eliciting student thinking through asking questions while solving worked examples. In this use, students made drawings towards the solution of worked examples and that provided the teachers to question student thinking about worked examples.

In the following example, Teacher Barış followed what the student did to solve the worked example which was about putting corresponding and interior angles in right places. Based on the student's placement of the angles on the worked example, the teacher asked questions related to these placements. For example, after the student found the measurement of an angle as 94, the teacher probed the student to find out what places were representing 94 for the measurement of the angle:

Student: We are subtracting 86 from 180. Here is 94. [180'den 86'yı çıkartıyoruz. Burası 94.]
Teacher Barış: Where is 94, show with the pencil with different color. Choose blue one. Where is 94? [Neresi 94, kalemle göster bakalım farklı renkle. Maviyi falan seç. Alttan seç alttan alttan. Görünmeyecek o şimdi. 94 neresi?]
Student: 94, just a second, this one. [94, bir saniye; burası.]
Teacher Barış: Where is (94) at the same time? [Aynı zamanda neresi?]
Student: This one at the same time. [Aynı zamanda burası.]
Teacher Barış: Where is (94) at the same time? [Aynı zamanda bir yer daha?]
Student: This one is 94 at the same time. [Aynı zamanda 94 burası.]
Teacher Barış: Great. Congratulations. [İşte bu ya. Tebrik ediyorum.]
Teacher Barış, 6th lesson, Line 187-193

As seen in the dialogue, the teacher asked questions about the student's markings on the worked examples. The teacher questioned whether the student noticed the interior angles and moved the angles to the places they should be. Student drawings helped the teacher what to ask to the student. In line with this, the teacher's questioning depended on the student's solution. In the following example, one of the students wanted to share his solution way in front of the class. The student made a marking to transfer an angle and a drawing to create a line whose starting point was D on the worked example. The teacher asked questions to the student about the reason of why he put an angle in the place he displayed on the worked example:

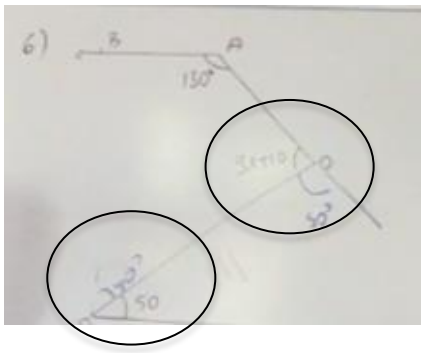


Figure 4.15 The situation discussed in the following dialogue

Student: Teacher, can I come and show? [Hocam gelip gösterebilir miyim?]

Teacher: Come. [Gel.]
Student: Here is 40 degrees. This one should be 50 degrees. I stretched it and then created a figure. [Şimdi burası 40 derece dedim. Bunun burası da 50 derece olur. Bunu uzatıp şekil çizdim hocam..]
Teacher Caner: Why is that 50? [Orası niye 50 oldu?]
Student: Because… [Çünkü…]
Teacher Caner: Hey, you need to use parallelism in order to make opposite angles. Which ones are parallel? [Çocuklar bakın ters açı olması için paralelliği kullanman lazım. Şimdi bu çizdikleri ya da bunların hangisi parallel?]
Student: no one is parallel (to each other) because not 90 degress, but that is 180 degrees. [Hiçbiri parallel değil çünkü hocam 90 derece değil de 180 derece oluyor, nasıl parallel.]
Teacher Caner: Okey, wait a minute, you drew a right line here. Okey. 40 degrees. What are you going to do (with that 40 degrees)? Wait a minute I am asking him. So? [Şimdi bi dakika buraya dik çizdin tamam. 40 derece. Ne işine yarayacak? Ama bi dakika arkadaşına soruyorum. Evet?]
Student: …
Teacher Caner: Well, if the line was parallel to that line, you can say that they are interior angles. But there is not such a situation here. [Şimdi bak Eğer şuna parallel bir doğru şöyle olsaydı o zaman iç ters açı diyebilirdin. Öyle bi şey yok şu anda çizdiğinde.]

Teacher Caner, 3rd lesson, Line 234-243

As seen in the dialogue, student drawing of the worked example about applying the mathematical procedures about interior angles and corresponding angles gave a chance to the teacher to understand the student's mathematical thinking. There was a problematic aspect of the student's mathematical thinking about moving angles in wrong places on the given figure of the worked example and about additional drawings. During the process, teacher questions guided the student to realize the problem in his drawings was the operations which were not performed based on two parallel lines. Therefore, as seen in the dialogues, students' drawings supported and changed both of the teachers' way of questioning. The frequency of this use was represented in Table 4.14. According to this, both of the teachers used the way of questioning. Teacher Barış and Teacher Caner used the tool in sixteen and ten percent of their questioning episodes.

Table 4.14

The frequency of the use of student drawings in terms of eliciting student thinking while solving worked examples

The nature of questioning episode	# of questioning episodes	# of questioning episodes
	Teacher Barış	Teacher Caner
Use of student drawings eliciting student thinking while solving worked examples	12 (16%)	6 (10%)
Total questioning episodes	93 (100%)	54 (100%)

4.1.4 Teacher Drawings as a Tool for Teacher Questioning

Sometimes teachers made markings or drew on a shape which were in a static form represented in a worked example or they sometimes created drawings for the solutions of the worked examples. These drawings consisted of required steps for a solution of a worked example or they might be a static shape. While doing these, teachers questioned the worked example by questions or made explanations about a mathematical procedure or a concept to students in the form of asking questions.

The teachers used their markings or drawings as a tool for their questioning in two ways: *teacher drawings helped teachers to solve a worked example via questioning* and *teacher drawings helped teachers to explain a procedure through questions.* In the following, each of the use of the tools were explained below with the sample excerpts. .

4.1.4.1 Teacher Drawings Helped Teachers to Solve a Worked Example via Questioning

Both of the teachers draw figures representing the solution of worked examples or they wrote on the figure about the solution steps. While doing these, teachers posed questions about the figures, which helped the students to understand mathematical procedures on worked examples. In the following example, the teacher directed questions to the students simultaneously with his drawings or through the

drawings. He told a procedure of drawing a congruent angle benefiting from a worked example in the supplementary book. As seen in the dialogue, deriving from the requirement of the worked example. The teacher used his drawings to question the procedure of creating a congruent angle. He obtained vertical and horizontal distance of a point so that he could create rays of an angle. He provided students to follow the way of the procedure with his questions.

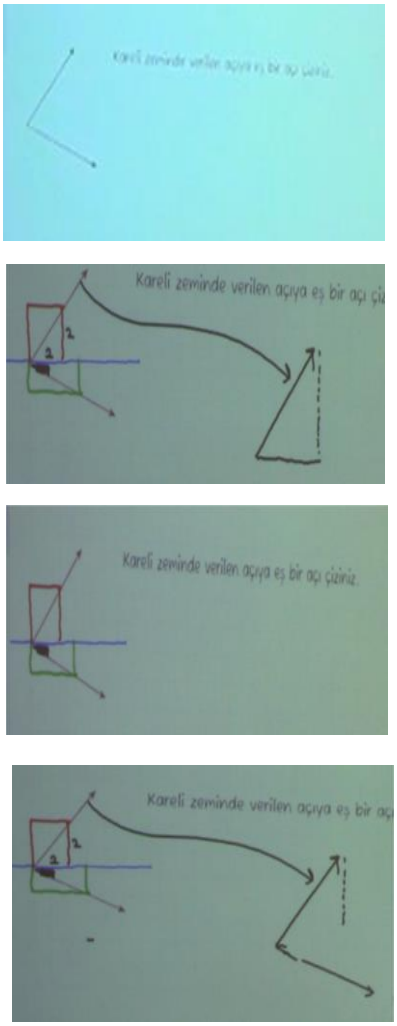


Figure 4. 16 The teacher's drawing helping teachers to solve a worked example via questioning

Teacher Barış: One (line) is that one. That is that (line), right? [Kolun biri doğru. Şu kırmızı kol bu değil mi?]
Student: Yes [Evet.]

Teacher Barış: Well Selma, I am not seeing (the distances here). [Şimdi Selma bura kaç ben göremiyorum.]
Student: Three and one. One and three [Üçe bir. Bire üç.]

Teacher Barış: Well, let's look at the bottom side. We need to combine them. Horizontal is three, vertical.. [Peki alt tarafa bakalım; birleştireceğiz bunları. Yatayda üç dikeyde...]
Student: Two [İki].
Student: No, one. [Hayır bir.]
Teacher Barış: One.. If it is three in horizontal and vertical in two, let's make as horizontal is six and vertical is two. Is it possible? Okey. It was six in horizontal line and what was the vertical? [Bir hıı. Yatayda üç dikeyde birse; dikeyde iki yatayda altı olsun. Olur mu? Tamam. Yatayda altı. Dikeyde kaçtı?]
Student: That must be two. [İki olacak.]
Teacher Barış: Like this? [Şöyle mi?]
Student: Yes. [Evet.]

Teacher Barış, 3rd lesson, Line 18-35

As similar to the teacher, Teacher Caner used drawings to solve a worked example via questioning. Different from Teacher Barış who used smartboard for his drawings, Teacher Caner made all his drawings on a traditional board but drawings of both of the teachers were in a static form. In the following dialogue, Teacher Caner solved a worked example through questioning the solution procedure about interior angles. Because the drawings including M rule and putting angles in correct places guided students to understand the mathematical procedure, he asked some questions through drawings on the worked example. His drawing helped him to emphasize the essential points about the procedure and it contributed his questioning by providing an atmosphere for the students to practice of interior angles through asking questions:

> Teacher Caner: Look at here, stretch this, strect that one, right? Yes. Now what is the measurement of that (angle)? [Şimdi bakın, şurayı uzatın, burayı uzattınız mı, evet. şimdi şurası kaç derece olur?]
> Student: 50.
> Teacher Caner: Now, hide here, there is a M rule here, right? [Şimdi şurayı görmeyin, şimdi şurda bi M kuralı yok mu? Ha?]
> Student: Yes. [Evet]
> Teacher Caner: That one is the sum up of these two. [Şimdi şu ikisinin toplamı bunu verecek.] Sum up 50 and 50? [50 ile 50 nin toplamı?] The sum up 50 and 50 will be this one. Right? 3x plus 10 equals to 50 plus 50. 3x equals to 90, right? X equals to 30. [50 ile 50 nin toplamı şurayı verecek. Di mi? 3x artı 10 eşittir 50 artı 50. 3x eşittir 90 olur mu. X eşittir 30.]
>
> Teacher Caner, 3rd lesson, Line 285-292

For both of the teachers, the frequency table of this use of the tool was represented in Table 4. 15. The table showed that teacher drawings for a worked example were utilized more than half of the episodes in Teacher Barış's questioning and not used much by Teacher Caner. 56% of questioning episodes of Teacher Barış and 13% of questioning episodes of Teacher Caner included the use of teacher drawings to solve a worked example via questioning:

Table 4. 15

The frequency of the use of teacher drawings to solve a worked example via questioning

The nature of questioning episode	# of questioning episodes	# of questioning episodes
	Teacher Barış	Teacher Caner
Use of teacher drawings to solve a worked example via questioning	52 (56%)	7 (13%)
Total questioning episodes	93 (100%)	54 (100%)

4.1.4.2 Teacher Drawings Helped Teachers to Explain a Procedure Through Questions.

Another case that the teacher drawings were used as a tool for questioning was the use of teacher drawings to explain a mathematical procedure through questions. This use of the tool showed that the teachers not only used questions to solve worked examples but also to explain procedures. In this use of teacher drawings as a tool, teachers created static figures and used them to explain a procedure. They asked questions about the drawings or they posed questions at the same time when they were drawing. In the following dialogue, Teacher Barış was questioning the reason of selecting an appropriate point on a ray in order to create a congruent angle in an easier way. While doing this, the teacher made drawings representing the procedure:

> Student: Teacher, how can we obtain points? [Öğretmenim biz neye göre nokta belirliyoruz?]
> Student: I agree. [Aynen.]
> Student: According to what (we obtain points?) [Neye göre öğretmenim o?]
> Teacher Barış: The corner of the square units. [Karelerin köşelerine.]
> Student: Okey. [tamam.]
> Teacher Barış: Listen to me, why do we (apply the procedure), because here, it is easier to detect the vertical distance. Right? If you want you can choose from here or from here (showing any points on the line). But if you select this point, is it possible to find out the distance as integers? 1,2,3, .. do you know what is the exact coordinates of the point? [Çocuklar dinle neden öyle yapıyoruz; çünkü burada dikey mesafeyi tespit edebilmemiz daha kolay. Doğru mu? İstersen şuradan seç herhangi bir yerden seçebilirsin. Ama bak

burayı seçtiğin zaman şuranın uzunluğunu tam tespit etme şansın var mı? 1..2..3... ee şurası ney biliyo muyuz?]
Student: Hımm.

Teacher Barış, 2nd lesson, Line 83-89

Figure 4. 17 The teacher's drawing explaining a procedure through questions

The frequency of the use of the tool was represented for both of the teachers below:

Table 4. 16
The frequency of the use of teacher drawings to explain a procedure through questions

The nature of questioning episode	# of questioning episodes	# of questioning episodes
	Teacher Barış	Teacher Caner
Use of teacher drawings to explain a procedure through questions	22 (24%)	23 (43%)
Total questioning episodes	93 (100%)	54 (100%)

Table 4.16 showed that both of the teachers used drawings to explain a procedure through questions in 24% and in 43% of questioning episodes of Teacher Barış and Teacher Caner, respectively. The following table represented the use of teacher drawings without considering whether they were used for worked example

or for the explanation of mathematical procedures. As seen in Table 4.17, the teachers benefited from drawings while asking questions very often.

Table 4. 17

The frequency of the use of teacher drawings in terms of the teachers' use of them

The nature of questioning episode	# of questioning episodes	# of questioning episodes
	Teacher Barış	Teacher Caner
Questioning episodes representing the use of teacher drawings	64 (69%)	30 (56%)
Total questioning episodes	93 (100%)	54 (100%)

4.1.5 Real Life Examples Helped Teachers to Question Mathematical Procedures or Concepts

Another tool that one of the teachers, Teacher Barış, used in his questioning was real life examples. When the teacher made a relation between a mathematical concept and procedure with a real-life example and supported the relation with questions, real-life examples were used as a tool for questioning. For example, in the following, Teacher Barış utilized spirit level as a real-life example of the topic to relate it with the interior angle procedure:

> Teacher Barış: Where do we use spirit level? [Su terazisi nerede kullanılıyor?]
> Student: To balance [Dengeyi sağlamak için.]
> Teacher Barış: Which balance? [Ne dengesi?]
> Student: Balance of a base [Zeminin dengesini..]
> …..
> Teacher Barış: Let's say we make a shelf. One of them is like this, the other one is like that, right? [Evet şimdi mesela raf yapıyoruz, tamam mı çocuklar? biri böyle biri böyle durdu.]
> Student: But they are not parallel. [Ama paralel değil.]
> Teacher Barış: Not parallel.In that case, if I intersect that line like this, z rule is applicable? [Paralel değil. hıh. o zaman ben şunu böyle kessem, "z"

kuralı işler mi?]
Student: No. [Hayır.]
Teacher Barış, 7th lesson, Line 111-131

As mentioned in the dialogue, the teacher emphasized the mathematical background of an example he saw in real life so that the mathematical procedure became meaningful on it. In this process, the teacher asked students question about whether z rule is applicable in that example. The question was related to the mathematical procedure of interior angles and the function of the mathematical procedures in real life. The following table represented the frequency of using real life examples as tools for questioning:

Table 4. 18

The frequency of the use of real life examples as a tool for questioning

The nature of questioning episode	# of questioning episodes	# of questioning episodes
	Teacher Barış	Teacher Caner
Use of real life examples as a tool for questioning	1(1%)	-
Total questioning episodes	93(100%)	54 (100%)

As seen in the Table 4.18, real life examples was used very rarely. In line with this, real life example was not a main tool to utilize for both of the teachers' questioning and a teacher dependent tool for teacher questioning. This tool was independent from student questions/ideas, student drawings, teacher drawings, or information technology tools.

4.1.6 Printed Supplementary Materials as a Tool for Teacher Questioning

The other tool for both of the teachers' questioning was printed supplementary materials. These materials were different from supplementary books mentioned under the title of information technology tools. Here, the printed supplementary materials were not compatible with smartboard and provided

guidance for the teachers to select some worked examples or to organize the flow of his instructions with explanations written in the text. The materials could be various including supplementary books, chapters, or worksheets including the explanations and worked examples about the mathematical content. The printed supplementary materials guided the teachers' questioning in two ways: *the supplementary materials guided the teacher telling about a mathematical procedure or concept with questions* and *printed supplementary book guided the teacher in terms of questioning the sequence of worked examples.* Each of the use of the printed supplementary materials were mentioned below.

4.1.6.1 Printed Supplementary Books Guided the Teacher Telling about a Mathematical Procedure or Concept with Questions.

One of the use of printed materials in the teachers' questioning was related to the use of printed supplementary books to tell students about mathematical procedures or concepts. These printed materials helped the teacher to design the flow of the instructions of mathematical content in terms of talking about mathematical procedures or concepts with questions. The teacher enriched this content with his questions and presented it to his students. The following classroom dialogue represented that the teacher got benefit from the text in which some rules were written in it:

> Teacher Caner: Here are rules here that I don't agree (on teaching in this way). You suppose you have to memorize it when you write the rules; if we don't write the rules you think we didn't learn them. I'll tell you how to give meaning to them. so that you won't have to memorize. [burada çocuklar bir de belli başlı çok taraftarı olmadığım kurallar var, o kuralları yazdığınız zaman ezberlemek zorunda kalıyorsunuz, vermediğimiz zaman farklı kaynaklarda görüyorsunuz, hocam bunu biz öğrenmedik diyorsunuz, bi şekilde vericez ama o kuralları nerde çıkaracağımızı da anlatıcam, ezberlemenize gerek kalmayacak.]
>
> Teacher Caner, 2nd lesson, Line 74

As seen in the dialogue, the teacher intentionally utilized a part of a supplementary book in which the rules he mentioned were given in it. He mentioned about the rules with the reasons of them. The frequency of this use was represented in Table 4.19:

Table 4. 19

The frequency of the use of printed supplementary books in terms of questioning mathematical procedure or a mathematical concept

The nature of questioning episode	# of questioning episodes	
	Teacher Barış	Teacher Caner
Use of printed supplementary books in terms of questioning mathematical procedure or a mathematical concept	2 (2%)	6 (12%)
Total questioning episodes	93 (100%)	54 (100%)

Table 4.19 showed that the teachers applied printed supplementary books in terms of questioning mathematical procedure or mathematical concept quite a few. The procedure of creating a congruent angle, was one of the uses of the printed supplementary book in Teacher Barış's lesson while Teacher Caner utilized them to support his questioning on teaching of the concept of intersection of three lines in a plane and the sequence of writing the titles of the lesson in students' notebooks.

4.1.6.2 Printed Supplementary Books Guided the Teacher in terms of Questioning the Sequence of Worked Examples.

The other use of printed materials in teacher questioning was that *printed supplementary books guided the teacher in terms of questioning the sequence of worked examples.* One of the teachers, Teacher Caner supported his questioning with this use while Teacher Barış did the same thing with the supplementary book compatible with the smartboard. Teacher Caner followed a way of instruction while solving worked examples based on printed supplementary books including the teacher's supplementary books and students' workbook in which there were worked

examples and tests to practice. Beginning from the third lesson, the teacher sometimes followed the examples in the student workbook. The sequence of the worked examples to be solved were the questioning sequences settled by these printed supplementary books. Throughout the fourth lesson, the teacher used questioning for helping each of their students individually. The teacher allowed the students to practice from the student workbook as a printed supplementary book and allowed them to ask their questions when they had difficulties on practice.

In Table 4.20, the frequency of the use of the printed supplementary book in terms of the support of questioning sequence of worked examples was represented:

Table 4. 20

The frequency of the use of printed materials in terms of questioning sequence of worked examples

The nature of questioning episode	# of questioning episodes
	Teacher Caner
Use of printed materials in terms of questioning sequence of worked examples	27 (50%)
Total questioning episodes	54 (100%)

As seen in Table 4.20, printed materials in terms of questioning the sequence of worked examples was observed in half of the questioning episodes.

4.1.7 Summary of Teachers' Tools for Questioning

The frequency of the use of the tools showed that for Teacher Barış, the supplementary book and teacher drawings were used very frequently. Moreover, the guidance of real life examples and some uses of information technology tools were not used very frequently. For Teacher Caner, teacher drawings, students, and printed supplementary materials shaped his questioning while information technology, analogies, and real-life examples did not have a role on his questioning.

Details of how teachers use these tools showed that Teacher Barış used the tools for questioning as following: the supplementary book compatible with

smartboard book organized the sequence of questions to be asked (47%), teachers drawings helped the teacher to solve a worked example through questions (45%), student questions or ideas for questioning (32%), and analogies to refer while questioning a worked example (10%). For the other teacher, the frequency was in the following: student questions or ideas for questioning (52%), teacher drawings helped teachers to explain a procedure through questions (43%), and printed supplementary book organized the sequence of questions to be asked (50%).

Findings of the study showed that analogies provided teachers to question the mathematical procedures on worked examples (10%) and to visualize the mathematical procedures or concepts (6%). That means analogies were utilized in both teaching of mathematical concepts or procedures and applying the procedures on worked-out examples. The frequency of the observed episodes showed that analogies were used in 16% of questioning episodes while real life examples were used in 1% of total questioning episodes. Analogies were utilized more frequently than real life examples while questioning mathematical procedures, concepts, or worked examples.

4.2 Teachers' Mathematical Question Types

The purpose of this section was to present the findings of the analysis of verbatim transcripts of the video recordings of the classes and observation notes regarding teachers' mathematical question types. In order to understand which question types teachers used and how the teachers used the questions in the questioning episodes, I examined teachers' mathematical question types by using the content and context of the questioning episodes (Şahin & Kulm; Mason, 2002; Carlsen, 1991). This part included the analysis of the classroom video recordings from the verbatim transcripts and observation notes of the video recordings by using Mason's approaches to open and close-ended questions as well as Şahin and Kulm's approaches to classification of guiding and probing questions. In order to analyze how the teachers used questions, Hancock's (1995) openness of questions and

Mason's (2002) descriptions for open and close-ended questions were used to make such a classification in this study. While explaining the question types of mathematical teachers during their instructions, teachers' question statements were given with the context.

The findings of the study showed that middle school mathematics teachers' questions concluded based on guiding, probing, and factual questions. As represented in Figure 4.18, for Teacher Barış, guiding questions were observed in 265 question statements, which were the most commonly used type of question by him. Next, he used factual questions 67 times and probing questions 45 times. Teacher Barış used probing and factual questions in close frequency while guiding questions were observed quite a lot than the others. Different from Teacher Barış, Teacher Caner, used probing questions 76 times when the probing questions were the mostly observed question type. Guiding questions were observed in 34 question sentences and factual questions were observed 43 times. In line with this, guiding questions were the least used question type in his class.

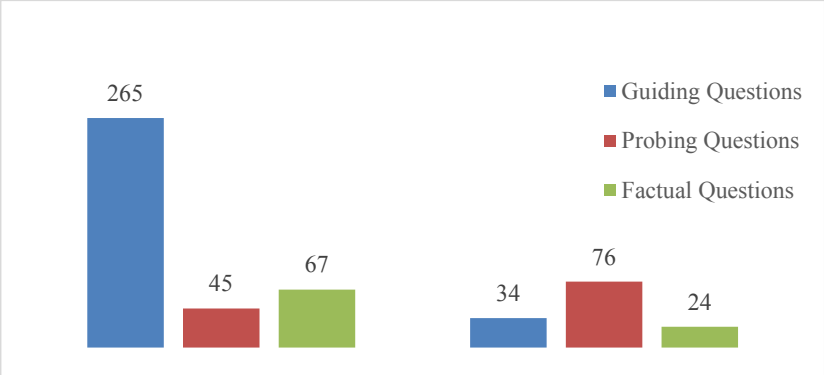


Figure 4. 18 The frequency of mathematical question types of middle school teachers

Considering the use of each of the question types in terms of the lessons, Figure 4.19 and Figure 4.20 the following two figures were represented for each of the teachers. It was seen that the teachers used each of the question types almost in each of their lessons.

As it can be seen in Figure 4.19, Teacher Caner did not use each of the question types in each of the lessons. He asked all three types of questions in the first three lessons while he did not used factual questions in the last lesson. When factual questions were observed in the first lesson mostly, the teacher did not use any factual questions in the last lesson. There was not a frequency pattern of question types in the teacher's lessons throughout the lessons. There were changes in the frequency of probing questions regarding the lessons. According to this, in Lesson 1, the teacher used factual question most of the times, while the second lesson included probing questions mostly.

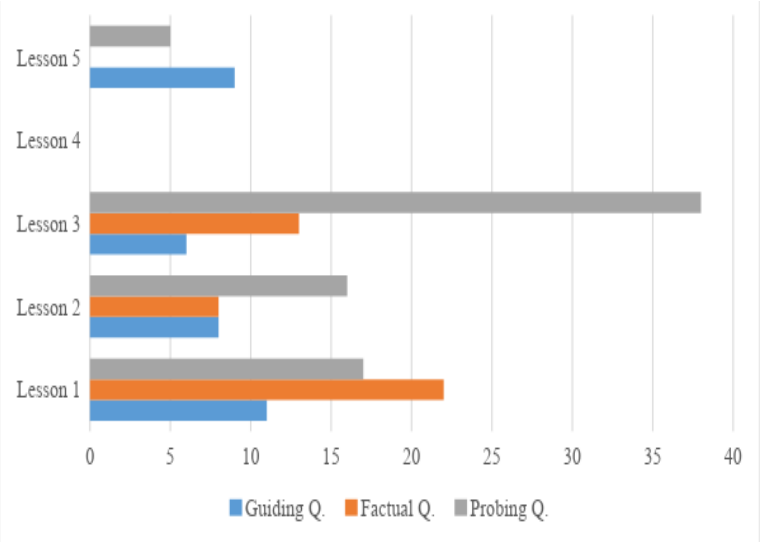


Figure 4. 19 The overview of the percentage distributon of the questions in terms of the lessons for Teacher Caner

As the Figure 4.20 showed, Teacher Barış used each of the question types in each of his lessons. Also, the teacher asked guiding questions more often than probing and factual questions through all the lessons. In other words, different from Teacher Barış, the teacher followed a frequency pattern of using guiding question, factual question, and probing question from more to less except from the last two lessons.

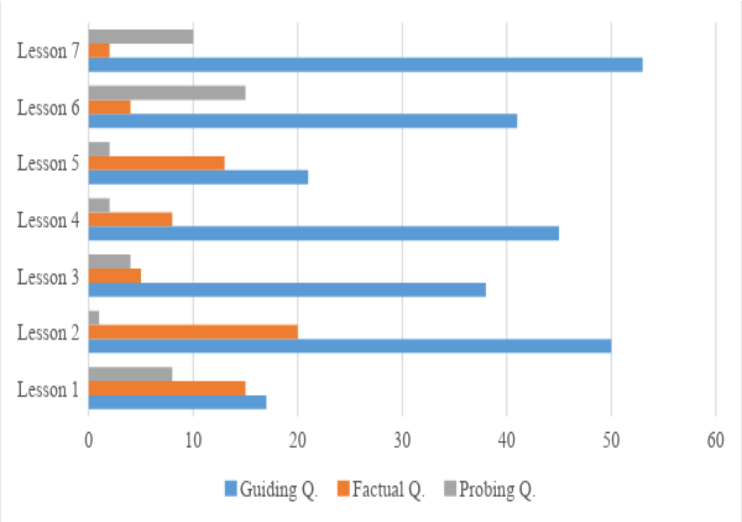


Figure 4. 20 The overview of the percentage distribution of the questions in terms of the lessons for Teacher Barış

In the following part, sample mathematical dialogues were represented for each of the question types. Additionally, the characteristics of the question types of both of the teachers were revealed.

4.2.1 Guiding Questions

One of the question types that participant teachers used in each of their lessons were guiding questions. Findings of the study showed that both of the teachers utilized guiding questions in each of their lessons and they were utilized in three ways. The ways included helping students when they have difficulty in their work or are in a process of learning a new content, in solving a problem or in producing solution strategies for a problem, and in scaffolding or leading learners for improving their mathematical progress through a series of factual questions. Within this part, a sample of guiding questions and characteristics of guiding questions of participant teachers were exemplified. In the following figure, the frequency of the characteristics of guiding questions in terms of the teachers were represented below:

Table 4. 21

Characteristics of guiding questions for the participant teachers

Characteristics of guiding questions	Number of Questions	
	Teacher Barış	Teacher Caner
asks for a specific answer or asks for the next step of solution when students are confused or stuck	33	13
ask students to think about or recall a general heuristic or strategy	12	2
asks a sequence of factual questions that provides ideas or hints that scaffold or lead toward understanding a concept or completing a procedure	220	19
Total	265	34

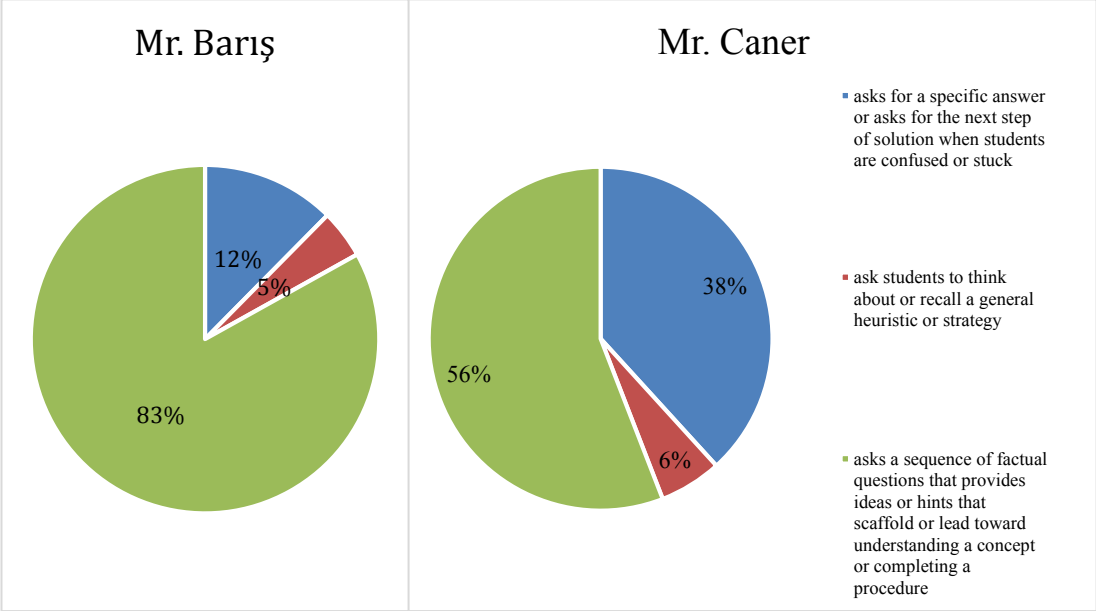
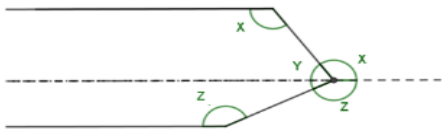


Figure 4. 21 The frequency of the use of characteristics of guiding questions

Throughout the observation data, instances of guiding questions were identified. As seen in the figure, Teacher Barış used 221 question sentences (83%) in 378 questions as guiding questions which had the characteristics of *asks a sequence of factual questions that provides ideas or hints that scaffold or lead toward understanding a concept or completing a procedure.* As similar to Teacher Barış, the same characteristics was observed in 19 question sentences (56%) of Teacher Caner.

In this use, teachers use factual questions sequencing a query aligned with reaching a learning objective. The use of the factual questions in a series guide students to understand a concept and complete a procedure. The following example showed that a guiding question helped the teacher to lead students in understanding a mathematical procedure that emphasized the sum of angles were full angle when angles were transferred by the z rule:

> Teacher Caner: Look over there. The angle and the angle x is alternate interior angles so, this is also x. Did you understand? Similarly, this Z and that one are alternate interior angles from the (rule of) angles emerging from intersection of two paralel lines with a non parallel third line. Does the sum of three make full angle? [bakın şurdaki açıyla, x açısıyla, şurdaki açı içters açılar, dolayısıyla burası da x olur. Anladık mı? aynı şekilde, paralel iki doğrunun bir kesenle yaptığı açıdan şu Z ile şurası içters açıdır, şurası da z olur. *bakın üçünün toplamı bir tam açı yapmadı mı?]*
> Students: Yes. [evet.]
>
> Teacher Caner, 2nd lesson, Line 94



$x+y+z=360^0$

Figure 4. 22 The worked example in the dialogue above

The question leaded students in understanding about the angles formed by two parallel lines, which are intersected by a third line. He wanted students to realize the structure of the intersection of three lines on his drawings for completing the procedure. Different from the transcripts above, in the following, the teacher used a sequence of questions that was serving as guiding questions in order to either lead or scaffold the class toward understanding of a procedure of creating an equal angle:

Teacher Barış: I am drawing from here, what is the vertical distance (of the point) Kenan? [Burdan çiziyorum dikey kaç Kenan?]
Student: 3..4...
Teacher Barış: Yes. What is the horizontal distance (of the point)? [Evet. Peki indiğimiz yerden açıya kadar yatay mesafe kaç?]
Student: 3
Teacher Barış: 3. Then we need to make our 4 to 3 ratio constant to create the same angle? Is it correct? What will be the points? 8 to 6? OK. I'm drawing it like this. 6 ... How can I draw from here? [O zaman 4'e 3 oranımızın korunması mı gerekiyor eş açı olması için? Doğru mu? Kaça kaç yapalım... 8'e 6 yapalım mı? Tamam. Şöyle çiziyorum. 6.. şurdanda kaç çizmem gerekiyor?]
Student: 8.
Teacher Barış: 8. That ends here. I will unite those. That is the starting point of the angle. Is it okey? [Şurada bitsin. Şunları birleştiricem açının başlangıç noktası. Oldu mu?]
Student:...(no answer)

Teacher Barış, 2nd lesson, Line 183 -191

The sequence of the factual questions gave hints about the steps required for the mathematical procedure of creating a congruent angle including selecting an appropaite point on a ray and taking attention to the horizontal and vertical distances of the selected point. He scaffolded and leaded the students by completing the procedure and understanding the concept of congruent angle. As a whole, each of the questions were a guiding question.

Another characteristics of guiding questions aimed to *ask for a specific answer or for the next step of solution when students are confused or stuck.* It was observed in 33 question sentences (12%) of Teacher Barış and 13 question senteces (38%) of Teacher Caner. According to this, for both of the teachers, the mentioned characteristic was the secondly most used characteristics of guiding questions. This use of guiding questions was asking questions about the steps of a solution while students were in confusion. In such cases, students were asked a question with a specific answer or they were asked a question to seek information about the problem solving process, specifically about the next step in the solution process. In the following dialogue, the teacher asked a guiding question, which required a specific answer when the student was confused about the equality of the angles. Following

the student's difficulty in applying interior angles in worked examples, the teacher asked a guiding question about checking the determination of the interior angles on a teacher drawing:

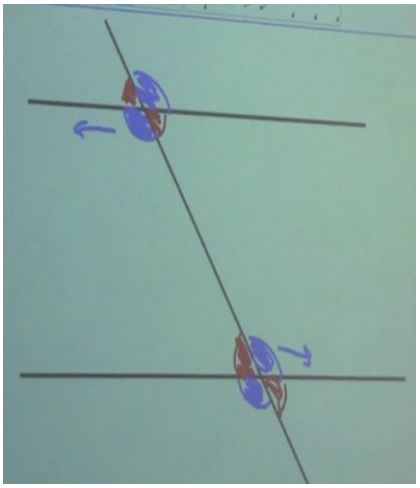


Figure 4. 23 The representation of the equality of corresponding angles

> Teacher Barış: *Here, 1, 2, 3, 4, here are 5, 6, 7, 8 angles, or indeed most of them are equal. Which are equal? [Burada 1, 2, 3, 4, işte 5, 6, 7, 8 tane açı oluşturuyor ya aslında çoğu birbirine eşit bunların. Kimler eşit?]*
> Student: Why are they like this? [Niye böyle yapıyorlar?]
> Teacher Barış, 4th lesson, Line 150-153

With this question, the teacher guided the student in understanding the equality of measurement of angles that was a precondition of understanding to solve the worked example.

The least frequent observed characteristics of guiding questions were about *asking students to think about or recall a general heuristic or strategy.* The characteristic was seen in 12 question sentences of guiding questions (5%) of Teacher Barış and 2 question sentences of guiding questions (6%) of Teacher Caner. The following sample from the transcript was an example that Teacher Barış was helping students in recalling a strategy of creating a congruent angle which was discussed in the previous class and about the application of the strategy for creating a congruent

angle procedure in a worked example given in their supplementary book. The teacher gave students clues about the way of solving the worked example (see Figure 4.24) which was related to selection of appropriate points on the rays given on the worked examples through a guiding question. In line with this, the question required students to recall the strategy the students applied before. The teacher posed a guiding question that required the students to recal or think about an appropritate strategy at the beginning of the solution of the worked example:

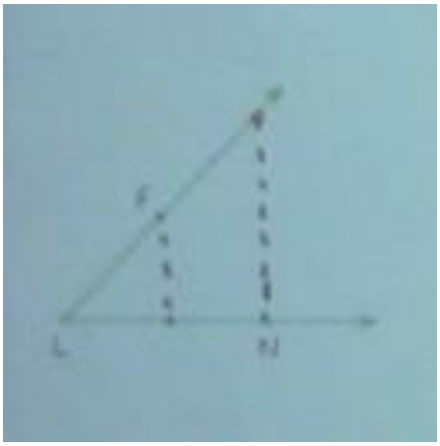


Figure 4. 24 The worked example in the supplementary book

Teacher Barış: (The worked example) set the point (F) there. This one is a ray LM. This is a unite of both of the rays, a point (L) is common. They are rays whose common point is L. You can choose this one (the red point in the figure), it has a corner. You can choose that one as well.*Which one you like. [*Işının geçtiği bir noktayı belirlemiş orada. Bu da LM ışını. Işınların birleşimi değil mi, noktası ortak. Bir L noktası ortak olan ışınlar. İsterseniz burayı seçin. Bakın tam köşeye geliyor. İsterseniz şurayı seçin. Hangisini isterseniz.]
Student: F.
Teacher Barış: You choose F. Okey, what is the vertical distance of F? [F'yi istiyorsunuz. Peki, F'nin dikey mesafesi kaçmış?]
Student: 2.
Teacher Barış: 2? Okey 2 units, vertical. (What is) horizontal? [2 mi? tamam 2 br. dikey. Yatay?]
Student: that is also 2. [o da 2.]
Teacher Barış: *2 units, horizontal (distance). We say 2 units, horizontal (distance), right? So when I choose two-two, three-three, five-five, one hundred-one hundred, and one thousand-one thousand, do I get an angle*

equal to that angle?[2 birim yatay. 2 birim br diyoruz yatay mı? [O zaman ben ikiye iki, üçe üç, beşe beş, yüze yüz, bine bin olduğu zaman her türlü bu açıyla; şu açı ölçüsüyle eş bir açı mı elde ederim?]
Student: Hıhı.

Teacher Barış, 2nd lesson, Line 74 – 83

The teacher initiated the dialogue giving specific hints about the selection of an appropriate point on a ray and horizontal and vertical distance of them by asking '*Which one you like'.* In this example, the teacher asked their students to recall the strategy they discussed before but he gave freedom to them in the way of application of the strategy. While in this example the teacher preferred strategies to solve by a guiding question, in the following transcript Teacher Caner posed a guiding question, which provided them to produce a strategy and talk about the strategy in the class in the same content with Teacher Barış:

Teacher Caner: How can I draw an angle so that it is equal to the angle of this angle? Yes? [*Ben bu açının eş açısı olacak şekilde buna eşit olacak şekilde bir açıyı nasıl çizebilirim?* Evet?]
Students: *No voice.*
Teacher Caner: I want to draw an angle using that ray. That ray. Get P, R. How can I draw the angle that is equal to this one? Emir? [*Şu ışını kullanarak bir eş açı çizmek istiyorum. Şu ışını. Şurasıda P, R olsun. Şimdi buna eş olan açıyı nasıl çizebilirim?* Efe.]
Student: Two units up one unit right. [İki birim yukarı bir birim sağa.]

Teacher Caner, 1st lesson, Line 47-50

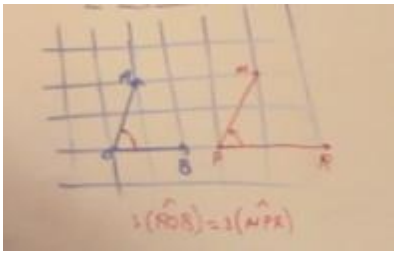


Figure 4. 25 The question in the dialogue above

The teacher asked the students to think about a strategy with the guiding question inviting the students to produce a strategy about creating a congruent angle.

The students answered the question based on the teacher's placement of the rays on the isometric drawing on the board.

4.2.2 Probing Questions

Another question type for both of the teachers was probing questions. Teachers posed probing questions in three ways: to ask details about student's answer, to request a defence of their own idea, or to ask to use their prior knowledge in order to solve a problem. All the three uses belonged to probing questions, but each of them served to different characteristics in the dialogues. Throughout this part, each of the use of probing questions were exemplified. The following table represented the use of the probing questions in terms of the characteristics:

Table 4. 22

The frequency of the characteristcis of probing questions (that) middle school teachers' use

Characteristics of probing questions	Number of questions	
	Teacher Barış	Teacher Caner
Ask students to explain or elaborate their thinking.	31	36
Ask students to use prior knowledge and apply it to a current problem or idea.	6	26
Ask students to justify or prove their ideas.	8	14
Total	45	76

Both of the teachers used each of the characteristics of probing questions in their lessons. The most frequently used characteristic was about *asking students to explain or elaborate their thinking.* 31 sentences of Teacher Barış and 36 question sentences of Teacher Caner had that characteristic. In other words, 68 percent of probing questions of Teacher Barış and 47 percent of probing questions of Teacher Caner had the characteristic. In this use of probing questions, teachers posed questions requiring the students to explain or elaborate their thinking according to students' responses stated before.

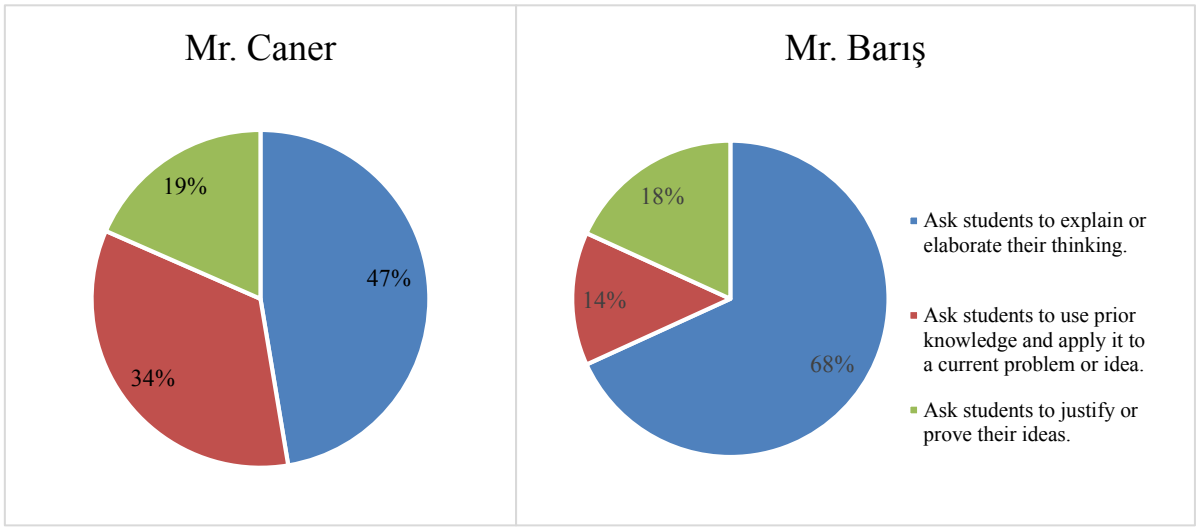


Figure 4. 26 The frequency of the use of characteristics of probing questions

As seen in the following transcript, Teacher Caner posed a question which required the student to explain his thinking about the situations emerged by the intersection of three lines in space. When the teacher was collecting student responses about these situations, one of the students proposed that three lines could be located as 'independent' of each other. The teacher asked the student to explain his thinking about 'independent':

> Teacher Caner: Yes, (three lines) can be intersected like a triangle. Let's say this one is d1, that one is d2, and this one is d3. They might be (intersected) like this. *What else? [*Evet, yani üçgen oluşturacak şekilde kesişebilirler. şuraya d1, şuraya d2, buraya d3dersem, böyle olabilir. *Başka?]*
> Student: Can they be independent of each other? [Birbirinden bağımsız olabilir mi?]
> *Teacher Caner: What do you mean by independent? [bağımsız derken?]*
> Student: one of them is like this, another one is like this, the other one is like that. [biri şöyle, biri böyle, biri de şöyle.]
> Student: But (the three lines) all intersect. [ama onlar her türlü kesişir ki.]
> Teacher Caner: They are intersected somewhere [onlar her türlü kesişir biryerlerde.]
>
> Teacher Caner, 1st lesson, Line 80-85

With the question of 'What else?' the teacher wanted his students to elaborate their thinking in addition to the situations stated. Also, the teacher requested the student to explain what he intended to mean by saying 'independent' through the probing question of 'What do you mean by independent?' He did not give an immediate feedback about the student's response and he stated the correctness of it as a result of another studen's explanation that rejected the idea. Teacher Barış used this characteristic of probing question serving to the purpose of elaborating student's thinking.

In the following dialogue, the teacher asked the student a probing question about determination of alternate exterior angle while the student was solving a worked example on the board:

> Teacher Barış: Here is the river, right? What are the angles between the (lines of) the river? 1,2,72, and 3, right? Then, which ones are looking in opposite directions? [Şimdi burası nehrimiz değil mi? Arasında kalan açılar kimler? 1, 2, 72 ve 3 değil mi? Peki, 1, 2, 72 ve 3 açılarından kimler zıt yönlere bakıyor?]
> Student: 72 is looking the right direction. [72 sağa bakıyor.]
> Teacher Barış: *To the left, when you look as to the left.* [şöyle sola, şöyle yan baktığında.]
> Student: 2, as well. 2 is looking to the right side as well.[2 de, 2 de sağa.]
> Teacher Barış: So, both of them have the same (measurement of angle.) *What else?* [O zaman bunlar aynı. Başka?]
> Student: We are going to subtract 72 from 180. [Öğretmenim şimdi 180 den 72 yi çıkaraçağız.]
> Teacher Barış: *What are you going to find out?* [Neyi bulucağım?]
> Student: 3 and 1. [3 ve 1 i.]
> Teacher Barış: She is ging to find, 3, 108, she found 3. Congratulations. [3 ü bulucak, 108, buldu bile 3 ü. Aferin kızıma. 108].
>
> Teacher Barış, 4th lesson, Line 186-195

With the questions 'What else?' and'*To the left, when you look as to the left'* the teacher elaborated the student's thinking inviting the student to think more deeply about the way of her solution. In addition to this, with the question 'What are you going to find out?' the teacher elaborated the student's thinking about the awareness of the way of her solution.

Another characteristic of the probing question was that teachers *ask students to use prior knowledge and apply it to a current problem or idea*. 6 sentences of Teacher Barış's questions (14%) and 26 question sentences of Teacher Caner represented that characteristic (34%). The following dialogue is an example of this use. In this example, when the student described an obtuse angle, the teacher posed a question requiring the student to use prior knowledge about the description of obtuse angle:

> Teacher Caner: Obtuse angle? Elif? [Geniş açı? Elif?]
> Student: Measurement of angle above 90 degrees. [90 derecenin üstünde olan açı.]
> Teacher Caner: Over 90 degrees, for example, 270 degrees? [*90 derecenin üstünde mesela, 270 derece?]*
> Student: No, between 90 and 180. [hayır, 180 le 90 arası.]
>
> Teacher Caner, 1st lesson, Line 35-38

With this question, the teacher probed the students' prior knowledge which was about the obtuse angle. As a result of this, the teacher wanted the student to use her prior knowledge about the obtuse angle and to apply the knowledge of range of the measurement of obtuse angle into the current problem which was about the description of the obtuse angle.

The other use of probing question was teachers *ask students to justify or prove their ideas*. Both of the teachers used this characteristic of the probing question in their lessons. Teachers wanted students to defend their ideas by justifying or proving their responses. Teacher Barış's 8 questions (18%) and Teacher Caner's 14 questions (19%) had this characteristic. In the following, related parts in class dialogues were shared. The following dialogue took place when Teacher Barış wanted a student to interpret the definition of corresponding angle written in the supplementary book compatible with smartboard. The student interpreted what she understood from the description of the corresponding angle. Based on the student's responses about corresponding angles, the teacher wanted the student to justify her answer by her drawings:

Teacher Barış: I am reading all (of the text). Aylin will draw what she understand. One is parallel between two lines, the other one is not between two parallel lines but they both face to the same direction. What do you understand from that?
[Şimdi herşeyi okuyorum, Aylin anladığını çizecek. Biri paralel iki doğru arasında olan, diğeri paralel iki doğru arasında ve olmayan ve aynı yönlere bakan. Biri paralel iki doğru arasında olan, diğeri paralel iki doğru arasında olmayan ancak aynı yönlere bakan. Bundan ne anlıyorsun?]
Student: That is what I understood (from the text). Well.. For example, the angle here.. [Şöyle. Benim anladığım.şimdi..mesela burdaki açı..]
Teacher Barış: That is one of them. That is between the (lines), right? That is between the two parallel lines. Where is 'not between the two parallel lines?'
[Biri o olsun. Arasında olan oluyor o değil mi? İki paralel doğru arasında olan oluyor. Olmayan neresi olabilir.]
Student: 'not' is here. Can be (that one)? [Olmayan ise bura. olabilir mi?]
Teacher Barış: Can't be the other side? [Diğer taraf olamaz mı?]
Student: It can be that one. [Bura da olabilir.]
Teacher Barış: What are they, which ones are corresponding angles? [Kim bunlar sence, yöndeş olanlar hangileri?]
Student: corresponding angles are this one and that one. [Yöndeş olanlar bu ve bu.]
Teacher Barış: Why are they corresponding angles? [Neden onlar yöndeş?]
Student: Because they face to the same direction. [Çünkü aynı yöne bakıyorlar.]
Teacher Barış: Did you understand? [Anlaşıldı mı?]
Students: Yes. [Evet.]

Teacher Barış, 7th lesson, Line 14-24

In this example, it can be seen that the question 'why' or 'are they corresponding angles' required the student to justify or prove of her thinking.

During the lessons, Teacher Caner explicitly stated that he wanted students to follow such a way that students could justify or prove and support their ideas while solving worked examples. In the following example, he wanted a student to justify or prove her reasoning of the operation done during the solution of the following problem represented in Figure 4.27:

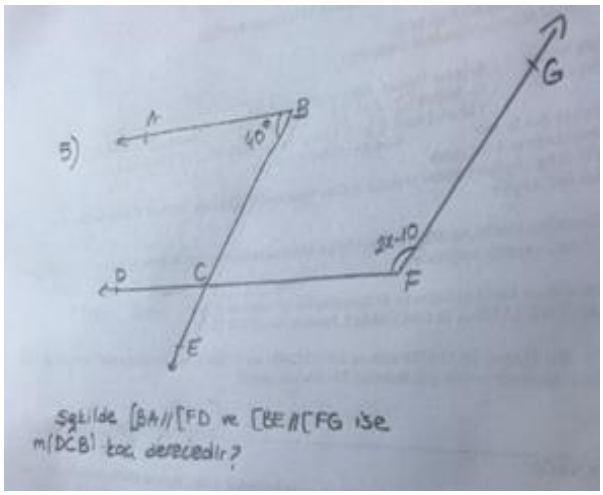


Figure 4. 27 The situation discussed in the following dialogue

Student: I subtracted 40 from 180. [Ben 180 den 40 I çıkardım.]
Teacher Caner: *Why did you subtract?* [*Niye çıkardın?]*
Student: Because that is obtuse angle [Çünkü hocam ora geniş açı.]
Teacher Barış: 91 degree is an obtuse angle, it is impossible [Geniş açıda 91 derecede geniş açı canım benim olur mu öyle şey.]
Teacher Caner, 3rd lesson, Line 180-183

In this example, one of the students suggested a solution way for the problem. The teacher criticized the student's way and asked the student why she did it. Thus, the student had to say the reason of why she thought so. In this case, the student explained his reasoning by the characteristic of probing question which asked the student to justify or prove their ideas about putting interior reverse angles in correct places. Even though the student applied a correct step, the stated reason was not appropriate for solving the worked example.

4.2.3 Factual Questions

The other question type both of the teachers used was factual questions. The question type had the characteristics of posing questions to students about facts, definition, and answer of an exercise and about necessary steps while trying to

complete a mathematical procedure. In the following table, each uses of factual questions were represented. As the table shows both of the teachers used factual questions representing all the characteristics of factual questions:

Table 4. 23

The frequency of the characteristcis of factual questions (that middle school teachers' use

Characteristics of Factual questions	Number of questions	
	Teacher Barış	Teacher Caner
Asks student for a specific fact or definition	8	27
Asks student for an answer to an exercise	12	10
Ask students to provide the next step in a procedure	47	6
Total	67	43

As seen in the table and figure, Mr Barış used most of the factual questions in terms of the characteristics of factual question which require the students to say the next step in a procedure. 47 question statements (70%) of Teacher Barış represented the characteristics.

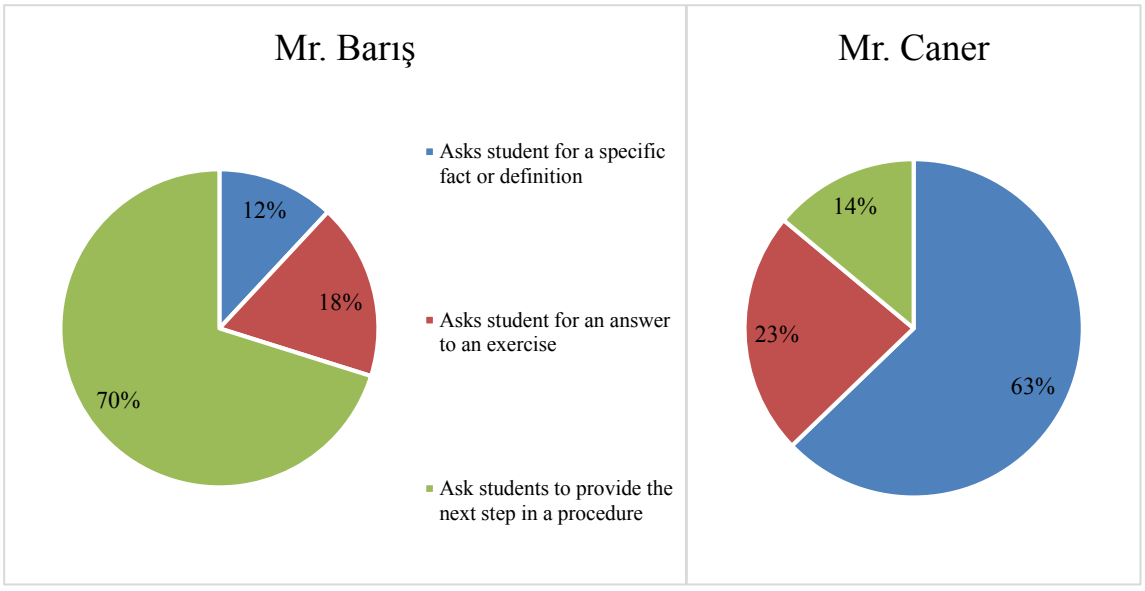


Figure 4. 28 The frequency of the use of characteristics of probing questions

In addition to this, Teacher Barış asked factual questions, which require students to talk about a specific fact or definition rarely (12%) while Teacher Caner posed factual questions related to a specific fact or definition more than half of the factual questions (63%). The other characteristic that required students to give answers to an exercise was the secondly prefereed characteristic utilized by both of the teachers.

One of the characteristics of factual questions both teachers used was that a teacher *'asks student for a specific fact or definition'.* In this use, the teachers posed students a question that focuses on facts about lines and angles and definitions of mathematical concepts related to this content. In the following dialogue, one of the teacher's factual question which asks for definition of a mathematical concept, angle, was posed to students at the very beginning of the lesson:

> Teacher Caner: Do you remember what the angle was? Tell me, Emel?[açı neydi hatırlayanınız var mı? söyle Emel.]
> Student: A figure formed by the unification of the two rays. [iki tane ışının birleşmesiyle oluşan şekil.]
> Teacher Caner: Yes, the figure formed by the unification of two rays. Is there anyone else who wants to express? or anyone else who want to say something different from the others? Yes Emir? [Evet, iki tane ışının birleşmesiyle oluşan şekil. başka türlü ifade etmek isteyen var mı? yada hayır arkadaşımın söylediğinden farklı bir şey söylemek istiyorum diyen var mı? evet emir.]
> Student: The space between the two edges. [Iki kenarın arasında kalan boşluktur.]
> Teacher Caner: yes, the space between the two edges. Is it correct?[evet, iki kenar arasında kalan boşluktur. doğru mu?]
> Student:Correct. [doğru.]
>
> Teacher Caner, 1st lesson, Line 4-9

This was the very first time of the first lesson. The teacher collected student ideas about the definition of angle. He asked the students same question three times by giving students a chance for different definitions. The other teacher, Teacher Barış used a factual question to ask for the specific fact of interior angles:

Teacher Barış: If these two lines are parallel, we are strecthing those two lines. How are the angles? f and g? [şimdi canım bu iki doğru paralelse. Şöyle nehrimizi zatıyoruz. Bunlar nasıl açılar sizce? F ile g?]
Student: Parallel [Paralel.]
Teacher Barış: f and g? [f açısı ile g açısı?]
Student: The same. [aynı mı?]
Teacher Barış: hıh?
Öğrenci: Alternate interior angles [iç ters].
Teacher Barış: Alternate interior angles [iç ters].

Teacher Barış, 4th lesson, Line 224-230

While the teacher was solving a worked example about the intersection of three lines, he asked about the fact of two angles which were interior angles to each other through recalling this knowledge.

Another characteristic of factual question was that a teacher *asks student for an answer to an exercise*. In this use of the factual questions, teachers asked the class what they found as a result of worked examples. For example, in the following, Teacher Caner asked the class for the answer of a worked example about a figure representing the intersection of three lines and the students shared the results of the worked example with the class:

Teacher Caner: Yes, these two lines are parallel. Everybody do it, then we're gonna ask someone for answers.[Evet bu iki doğru birbirine paralel. Herkes yapsın sonra birinden cevap istiyeceğiz.] What is the result? [Kaç?]
Student: 17
Student: 17.
Student: 17.
Teacher Caner: Is there anyone who find different result? Do you have a different value than 17? Okey, 17. [Farklı bulan var mı? 17 nin dışında farklı bir değer bulan var mı? Peki 17.]

Teacher Caner, 3rd lesson, Line 34-38

The teacher asked the class what they found about the solution of the worked example. With this question, the teacher realized the answers they reached as a result of the solution. The other teacher's factual question was questioning about how to move the interior opposite angles on a worked example represented in Figure 4.29:

Teacher Barış: Which shape will I put 100 degrees? [100 dereceyi hangi şekle koyacağım Aylin?]
Öğrenci: That shape. [Şu şekil.]
Teacher Barış: This one? Correct. [Bu mu? Doğru.]

Teacher Barış, 5th lesson, Line 6-11

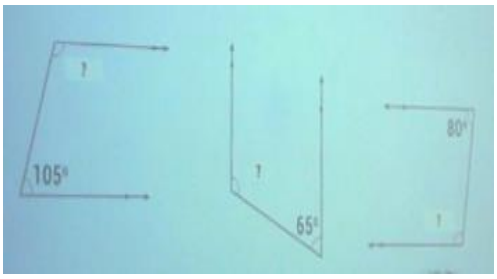


Figure 4.29 The situation discussed above

The other characteristic of factual question was *'ask students to provide the next step in a procedure'*. During the solution of worked example, teachers posed questions related to required steps to accomplish the solution of the examples. These questions provided teachers to complete procedures by making sure of the following correct way to reach the solution.

> Teacher Barış: What is the perpendicular distance did I have until I strech (the lines of angle) vertically? [Kaç dikey mesafe almışım bu açıyı dikey olarak açana kadar?]
> Student: 4 units [4 kare.]
>
> Teacher Barış, 2nd lesson, Line 8-14

With these questions, the teacher followed steps to complete the mathematical procedures of creating a congruent angle initiating with the horizontal and vertical distance of an appropriate point on a ray. In line with this, these questions provided students to understand the procedure and to complete it correctly.

4.2.4 Summary of Teachers' Question Types

Participant teachers differed from each other in the types of questions they used throughout the lessons. While Teacher Barış was using guiding question, factual question, and probing questions from more frequent to less, the other teacher used probing, factual, and guiding questions.

Question types in terms of the lessons do not have a pattern for both of the teachers throughout the lessons. In other words, the tendency of using types of questions in each of the lessons were not the same in lessons. When factual questions were the highest one in third lesson of Teacher Caner, it might be the least one in the second lesson.

Examination of the use of the characteristics of the question types showed that the teachers used characteristics of the question types in similar or different ways. To exemplify, for guiding questions, both of the teachers represented similar behaviours in terms of the characteristics of the questions. They mostly used 'asks a sequence of factual questions that provides ideas or hints that scaffold or lead toward understanding a concept or completing a procedure' and they limitedly used 'ask students to think about or recall a general heuristic or strategy'. For probing questions, both of the teachers mostly used the characteristics of 'ask students to explain or elaborate their thinking'. However, the characteristic of the least use was not the same. Teacher Caner's probing questions had the characteristics of 'ask students to justify or prove their ideas' in least questions while 'ask students to use prior knowledge and apply it to a current problem or idea' was the least one for Teacher Barış. For factual questions, they represented opposite kinds of behaviours. Teacher Caner's factual questions were mostly related to 'asks student for a specific fact or definition' while Teacher Barış used the characteristics the least. Teacher Barış's factual questions were mostly related to 'ask students to provide the next step in a procedure'.

4.3 The Relation within Tools for Questioning and between the Tools for Questioning and Teachers' Mathematical Question Types

This part aimed to represent the relation within tools for questioning as well as between tools for questioning and teachers' mathematical question types. The relation clarified the questioning practice of the participant teachers in terms of the use of tools for questioning in a questioning episode simutaneously and the use of question types and tools for questioning in the same questioning episodes together. In other words, it provided detailed information on how teachers use the kinds of questions. In the following titles, each of them were explained.

4.3.1 The Relation within the Tools for Questioning

In a questioning episode, two tools for questioning were used together. The overlapping of the codes related to the use of the tools for questioning in a questioning episode at the same time provided evidence of implementation of the teachers' questioning practices. The relations between the tools for questioning of Teacher Caner were represented in Table 4.24. According to the table, T9 and T19 were highly related to each other.

Table 4. 24

The relations between the tools for questioning of Teacher Caner

Tools for questioning	T9	T11	T12	T13	T15	T16
T9		2	5	5	1	19
T11						6
T12				1		7
T13					6	1

According to this, students' questions or comments providing teachers to clarify or detect problematic aspects of students' mathematical thinking through asking questions (T9) and student drawings guiding teachers for eliciting student

thinking while solving worked examples (T11) were observed in two questioning episodes together. In this kind of episodes, it was seen that students were asking questions about their own drawings or ideas on the basis of the worked examples. The following dialogue represented that situation:

> Teacher Caner: What your friends said are right. If you strect the line, what were the two angles, they were opposite angles, therefore the sum up the two angles were 180 degrees, did you understand? [Bakın arkadaşınızın söylediği doğru, şurdaki doğruyu biraz uzatırsanız eğer şu açıyla şu açı noldu iç ters açı oldu, o yüzden toplamları 180 derece, anladık mı? çünkü onların ikisi ne oluşturuyor doğru açı oluşturuyor, toplamları 180 derece olacak evet.]*Student is solving on the board.*
> Teacher Caner: What did you find out? [Şimdi sen neyi buldun?]
> Student: X value. [X in değerini.]
> Teacher Caner: What does the worked example require to do? [Soruda senden ne isteniyor?]
> Student: What is the measurement of DCA angle? [Dca, dca kaç derecedir?]
> Teacher Caner: Yes. What is the measurement of DCA angle? [Evet, dca açısı kaç derecedir?]
> Student: DCA is there, right? [DCA, şura mı?]
> Teacher Caner: So is that DCA? [Yani orası mı DCA?]
> Student: Isn't it? [Değil mi?]
> Teacher Caner: Yes, it is there. If it is there, find it. [Orası. Hıh, orasıysa bul onu.]
>
> Teacher Caner, 3rd lesson, Line 74-84

Another relation was between T9 and T12. Students' questions or ideas providing teachers to clarify or detect problematic aspects of students' mathematical thinking through asking questions (T9) and teacher drawings helping teachers to solve a worked example via questioning (T12) were observed in 5 questioning episodes together. Based on the student's ideas or questions to the teacher, Teacher Caner supported his questioning with his drawings. The following dialogue represented that one of the students got stuck during the problem solving process and asked the problem to the teacher during the individual problem solving process. The teacher preferred to solve the problem on the board by his drawings:

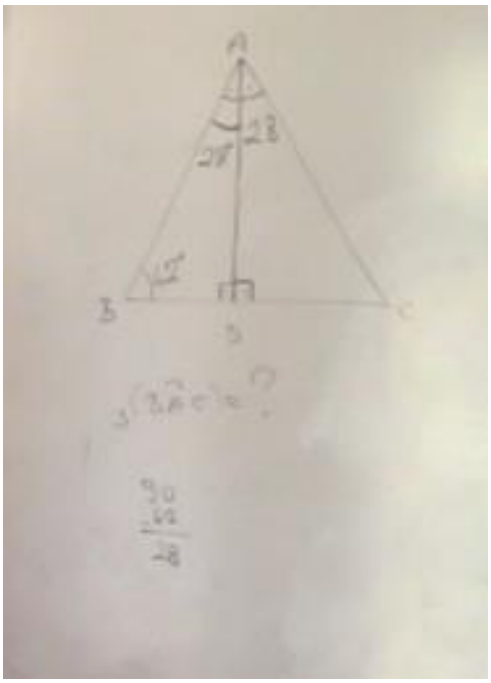


Figure 4.30 The situation discussed in the following dialogue

Teacher Caner: Look at the triangle. We have that kind of a triangle. The height AG is the bisector of it (A). So both of the angles are equal to each other. Right? [Kızım bak, şu üçgen, üçgene bak, şöyle bir üçgenimiz var, şurdan çizilen yükseklik şurdaki AG yüksekliği buranin açirotayiymış, yani şu iki açi birbirine eşit. Doğru mu?]
Student: Yes. [Evet.]
Teacher Caner: One of the measurement of an angle was given, as 62 degree. What is required to find is what is the degree of BAC, right? Then, in ADB angle, here is a right angle, I know two angles, can I find the other one, how I find is that I should subtract 62 from 90. Why do I subtract? Because the sum of the angles of a triangle is 180 degree. So if one of them 90 degress, the left two angles should be 90. Is that right? [Şu açıdan bir tanesi de verilmiş 62 derece, benden istenen burası bc dimi, bac açısı kaç derecedir diyor soru. Ozaman bakın şu üçgende adb üçgeninde şurası dik açı, iki tane açıyı biliyorum, şu açıyı bulabilirmiyim, nasıl bulucam doksandan 62 yi çıkararak bulucam. Neden doksandan çıkarıyorum. Çünkü bir üçgenin iç açıları toplamı 180 derece, e açının biri doksan derecryse, diğer iki açının toplamı 90 derece olur. Doğru mu]
Student: Right. [Doğru]
Teacher Caner: Therefore, I subtracted (62) from 90. What did I find? [O yüzden 90 dereceden çıkarttım. Kaç buldum?]
Student: 28.

Teacher Caner: So, the measurement of the angle is 28. If the line is a bisector, is it 28, right? What I need to find is BAC there, right? 28 and 28 is? 56. [Demek ki bu açı 28 derece. Bu doğru parçası açırortay olduğuna göre burası da 28 olmaz mı? Şimdi benden istenen BAC açısı bulundu mu? 28 28 daha ne yapar?56.]

Teacher Caner, 5th lesson 5 - 14

The relation showed students participated teacher questioning by asking questions about teacher drawings that were represented for the solution of a worked example. Teacher drawings provided students to make questioning of worked examples in an instructional context to be learned.

Another relation was between T9 and T13. Students' questions or ideas, which provided teachers to clarify or detect problematic aspects of students' mathematical thinking through asking questions (T9) and teacher drawings helped teachers to explain a procedure through questions (T13) and were observed in 5 questioning episodes at the same time. The teacher questioning was initiated by either student questions, or comments or teacher drawings. In both of the situations, the questioning tool provided an instructional context for questioning mathematical procedures for the other questioning tool. For example, in the following dialogue, teacher drawings about M rule initiated the questioning and one of the student asked a question based on the teacher drawing:

Figure 4. 31 The situation discussed in the following dialogue

Student: So, the measurement of x and a do not matter? There are more number of angles (a and b, 2 angles) on the right side than number of angles on the left side (x, 1 angle).[… yani x ile a nın açıları farketmiyor mu? ? yani, sağ tarafa bakanlar sol tarafa bakanlardan daha çok.]
Student: Why does the problem ask if they are both the same? [aynı olsa niye sorsun ki?]
Teacher Caner: The sum of the two is important [olsun, toplamları öyle.]
Student: so, they (a and b) are not the same? [yani eşit değiller.]
Teacher Caner: They might be equal or not. The important thing is that the sum of the two (a and b) is equal to the other one (x) [oladabilir, olmayadabilir. önemli olan toplamlarının birbirine eşit olmasıdır.]

Teacher Caner, 2nd lesson, Line 113 – 118

Another relation was between T9 and T15. Students' questions or ideas providing teachers to clarify or detect problematic aspects of students' mathematical thinking through asking questions (T9) and printed supplementary books guiding the teacher to talk about a mathematical procedure or concept with questions (T15) were observed in only one questioning episode together. That relation showed while the teacher was using a printed supplementary book for teaching mathematical procedure or concepts by asking questions, students' questions or ideas were integrated to the questioning episode. In the following example, while the teacher was talking about "pen (*kalem ucu)*" rule, one of the student asked a question about the generalizability of the rule:

Student: Teacher, the pen rule says that in all situations the sum up angles are 360 degress? What else can it be? [hocam şu kalem ucu kuralında var ya, her türlü 360 derece mi olacak? başka olabilir mi?]
Teacher Caner: do you mean that 270, 540 degrees? No, it is not possible. [başka derken 270, 540 derece filan mı? yok. Olmaz.]

Teacher Caner, 2nd lesson, Line 103-104

Another relation was between T9 and T16. Students' questions or ideas providing teachers to clarify or detect problematic aspects of students' mathematical thinking through asking questions (T9) and printed supplementary materials, which guided the teacher in terms of questioning the sequence of worked examples (T16)

were observed in 19 questioning episodes together. The following dialogue is an example where the teacher wrote a worked example from the supplementary book and students participated to teacher questioning with their ideas about the solution of the worked example:

> Student: Teacher, is that the pen rule? [Hocam bu da mı kalem ucu?]
> Teacher Caner: Does it look like the pen rule? [Benziyor mu kalem ucuna?]
> Student: No. [hayır]
> Teacher Caner: Yes, then it does not look like pen, we need to think other ways to solve. [Evet, benzemediğine göre başka bir şey düşünmemiz gerekiyor.]
>
> Teacher Caner, 3rd lesson, Line 53 – 56

That relation showed that the teacher used the worked examples in the printed supplementary book and that provided the class an instructional context for asking questions, explaining their ideas, or participating to teacher questioning.

Another relation was between T11 and T16. Student drawings guiding teachers to elicit student thinking while solving worked examples (T11) and printed supplementary book guiding the teacher in terms of questioning sequence of worked examples (T16) were observed in 6 questioning episodes together. That relation showed that students' drawings had a role in solving worked examples while the teacher selected a worked example to solve. Printed supplementary book supported teacher questioning in a way that the content of questions were related to the worked example in the book. Student drawings allowed the teacher to ask questions related to the drawings and organize his questioning as compatible with the drawings. In the following example, the teacher wrote the worked example from the supplementary book and asked students about the solution of the example. He gave a right to a student who raised her hand. She mentioned about her drawing that represented the part of the solution of the worked example. Following the details of the student's drawings, extending a ray on the left side (see the following picture) and making additional drawings on the original example were emphasized:

> Teacher Caner: Emel, how did you find it? [Ezgi sen nasıl buldun?]

Student: I strected a line from the broken part of the line [Ya ben o kırık noktadan bir tane çizgi çizdim.]
Teacher Caner: You can do two kinds of questions in this kind of way, draw a parallel from that point to that ray, solve it, or extend that short one. You find the same result. After drawing this parallel line, as in the previous question, the sum of the two angles are 180. So this place is 50 degrees. Again using the same strategy, the angle will be 180, what will be this angle, 40. And now x plus 90 is equal to 180 degrees, right? Look, this is the right angle. Then x is equal, 90. [Aferim. Şimdi bakın bu tür sorularda iki türlü yapabilirsiniz, ister şu noktadan şu ışınlara bir paralel çizin, öyle çözün, ya da şu kısa olanı uzatın. Aynı sonucu bulursunuz. Şimdi bakın bu paraleli çizdikten sonra, izliyor musunuz, az önceki sorduda olduğu gibi, şurdaki açıyla şuranın toplamı kaç olur, 180. Dolayısıyla burası 50 derece. Yine aynı mantıkla şu açıyla da burdaki açı 180 olacak, burası nolur, 40. Ve şimdi x artı 90 eşittir 180 derece oldu mu. Bakın bu bi doğru açı artık. O zaman x eşittir nolur, 90.]

Teacher Caner, 3rd lesson Line 121 – 124

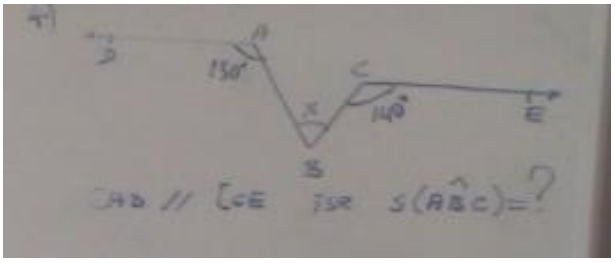


Figure 4. 32 The situation discussed in the following dialogue

Another relation was between T12 and T13. Teacher drawings helping teachers to solve a worked example via questioning (T12) and teacher drawings helping teachers to explain a procedure through question (T13) were observed in 1 questioning episode. In the following example, the teacher asked a question about an application of a mathematical procedure calling M rule and the teacher drawing provided an instructional content to ask a question:

Teacher Caner: Is there an M rule here? [Şurada bi M kuralı yok mu?]
Student: Yes. [Evet]

Teacher Caner, 3rd lesson 287 – 288

That relation showed the way that teacher drawings were used for questioning of a worked example as well as questioning of a mathematical procedure.

Another relation was between T12 and T16. Teacher drawings which helped teachers to solve a worked example via questioning (T12) and printed supplementary book which guided the teacher in terms of questioning sequence of worked examples (T16) were observed in 7 questioning episodes together. The relation showed that the printed supplementary book provided the teacher to organize his questioning in terms of worked examples and teacher drawings supported the solution of worked examples. In line with this, the printed supplementary book was a resource of worked examples while teacher drawings were the resource of themselves for questioning worked examples.

Another relation was between T13 and T15. Teacher drawings, which helped teachers to explain a procedure through questions (T13), and printed supplementary books, which guided the teacher telling about a mathematical procedure or concept with questions (T15), were observed in 6 questioning episodes together. The relation showed that the teacher benefited from the printed supplementary books while questioning mathematical procedures or concepts and during the use of the books, he supported his questioning with his drawings which were helpful for the questioning of the mathematics he was explaining. In line with this, both of the tools for questioning served for the same purpose and they had a role of talking about mathematical procedures or concepts in the teacher's questioning. Teacher drawings helped the procedures to be represented visually while the printed supplementary books guided the teacher about the procedures or concepts to be talked about. The following dialogue represented that printed supplementary book guided the teacher about creating an instructional context and teacher drawings supported the context visualizing the mathematical procedures or concepts. Teacher Caner visualized the mathematical procedure of creating equal angles to each other on the board. While doing that, the teacher examined the supplementary book, which mentioned about the procedure:

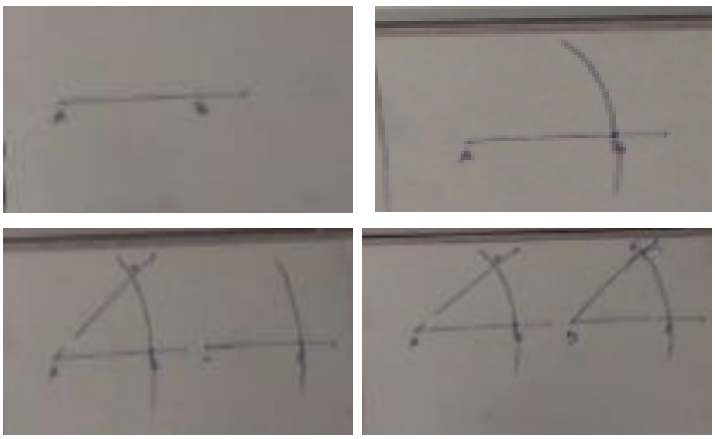


Figure 4.33 The mathematical procedure of creating equal angles

The last limitedly relation was observed between teacher drawings helping teachers to explain a procedure through questions (T13) and printed supplementary books guiding teacher in terms of questioning the sequence of worked examples (T16). It shows that the teacher did not make explicit explanations about the procedures while following the worked examples in the printed supplementary books. The other teacher, Teacher Barış utilized tools for questioning as represented in Table 4.25. According to this, one of the tools for questioning that was guiding the teacher's questioning was the supplementary book itself which guided the teacher's questioning in terms of the sequence of the questions to be asked (T3). The relation between the supplementary book itself guided the teacher's questioning in terms of the sequence of the questions to be asked (T3) and DGS to ask questions based on dynamic figures (T2) were observed in 5 questioning episodes together.

Table 4. 25

The relations between the tools for questioning of Teacher Barış

Tools for questioning*	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13	T14	T15
T1			1						1						
T2			5									1			
T3				1		1	1	6	15	1	11	35	13		1

Table 4.25 (cont’d)

T5									1				
T6						1							
T7							1	1			2		
T8							2			7	2		
T9								1	2	9	8		1
T10											2		
T11										5	1		
T12											8		
T13												1	2

The relation showed that while the teacher was using the book for worked examples or explanations in the book, he created a dynamic figure to explain the situation in the book. For example, in the following example, the teacher used the explanation written in the supplementary book and represented the explanation as a dynamic figure he created (see p.67, Figure 4.5, 4.6, 4.7, and 4.8). The dynamic figure showed the intersection:

> Teacher Barış: Yes, (the supplementary books says that) three lines could be intersected by pairs. [Evet üç doğru ikişer ikişer kesişebilir diyor çocuklar.]
>
> Teacher Barış, 3rd lesson 339 – 339

In 6 questioning episodes, the supplementary book in terms of the sequence of questions to be asked (T3) and analogies while questioning a worked example (T8) were utilized together. Considering that the supplementary book did not include analogies, he decided to use the analogies independently from the supplementary book while solving worked examples. The following dialogue exemplified the the use of analogies while questioning a worked example:

> Teacher Barış: 80, right? There are two parallel lines. We have 80 degrees. Does it stay inside the river? Which fish are left in the interior of the river and floating to the other side of the river? [Mesela 80, doğru mu? Bakıyorsun iki tane parelel doğru var. 80 derece açımız var. Bu nehrin iç kısmında mı kalıyor? Nehrin iç kısmında kalan ve akıntının diğer tarafına yüzen hangi balık var?]
>
> Teacher Barış, 4th lesson Line 83 - 83

The supplementary book (T3) and and student questions or comments (T9) were observed together in 15 questioning episodes. In these episodes, the students asked questions or shared their ideas within the instructional context of the supplementary book. Students participated to the teacher's questioning of their own will.

Following to this relation, there was another relation between the supplementary book (T3) and student drawings (T10, T11). In 11 questioning episodes, supplementary book guiding the teacher's questioning in terms of the sequence of the questions to be asked (T3) and student drawings for eliciting student thinking while solving worked examples (T11) were observed together. The relation showed the teacher gave the students chance to solve worked examples whose were preferred to be solved by student drawings. In line with this, the frequency showed us the students' involvements of teacher's questioning.

The supplementary book which itself guided the teacher's questioning in terms of the sequence of the questions to be asked during the instruction (T3) and teacher drawings, which helped teachers to solve a worked example via questioning (T12) were observed at the same time in 35 questioning episodes. That relation showed the teacher supported his questioning with his drawings for the solution of a worked example, which were taken from the supplementary book. Therefore, the supplementary book itself helped the teacher's questioning as it was a source of worked examples.

In 13 questioning episodes, the supplementary book (T3) was in relation to teacher drawings explaining a mathematical procedure through questions (T13). The relation showed the supplementary book was a guide for the teacher to ask questions about mathematical procedures or concepts while the teacher did the same thing with his drawings. In the classroom, while the teacher was following the supplementary book step by step about teaching mathematical procedures or concepts, he represented the mathematics on his drawings as well. The relation showed the teacher was lecturing the content of the book by his drawings.

Teacher drawing which helped teachers to explain a procedure thorugh questions (T13) was the tool that was used together with student questions or comments (T9) as a questioning tool in 8 questioning episodes. The relation showed that teacher drawings could be a questionable point for the students as well as being a way of clarification corresponding to a student's questions or ideas. In both of the situations, students were involved in the teacher questioning unexpectedly. Teacher drawings were sometimes used to ask questions and sometimes when giving answers. In the following example, based on the teacher's instruction about the way to find an appropriate point to create equal measurement of angles, the student asked a question about the same content that the teacher had mentioned.

> Student: Teacher, how can we obtain points? [Öğretmenim biz neye göre nokta belirliyoruz?]
> Student: I agree. [Aynen.]
> Student: According to what (we obtain points?) [Neye göre öğretmenim o?]
> Teacher Barış: Corner of the square. [Karelerin köşelerine.]
> Student: Okey. [tamam.]
> Teacher Barış: Listen to me, why do we (apply the procedure), because here, it is easier to detect the vertical distance. Right? If you want you can choose from here or from here. But if you select this point, is it possible to find out the distance as integers? 1, 2, 3 ,.. Do you know what is hthe coordinate of the point? [Çocuklar dinle neden öyle yapıyoruz; çünkü burada dikey mesafeyi tespit edebilmemiz daha kolay. Doğru mu? İstersen şuradan seç herhangi bir yerden seçebilirsin. Ama bak burayı seçtiğin zaman şuranın uzunluğunu tam tespit etme şansın var mı? 1..2..3... ee şurası ney biliyo muyuz?]
> Student: Hımm.
>
> Teacher Barış, 2nd lesson, Line 83 – 89

As seen in the dialogue, the teacher clarified the student's question making markings on a worked example. Another relation was seen between the two uses of teacher drawing (T12, T13). Teacher drawings helping teachers to explain a procedure through questions (T13) and teacher drawings helping teachers to solve a worked example via questioning (T12) were observed in 8 questioning episodes at the same time:

Teacher Barış: Look. Let's solve it by another strategy. We can stretch it a little. Right? That's right. What is the complementary of the angle of 145 degrees?
[Bak. Bunu da değişik bir yöntemle çözelim hadi. Şunu uzatalım birazcık. Olur mu? Olur bence. 145 in bütünleri kaç yapıyor?]
Student: I would solve it. [Ben çözecektim.]
Teacher Barış: 35 degress? I have a triangle including 35, 45 degrees. I don't know the degree of the angle. That one is 100 degrees? [35 derece mi? 35 45 derecelik bir üçgenim var şu açıyı bilmiyorum ben. Doğru mu? Bak aynısını kopyalıyorum. Şurası 35 burası 45. Burası kaç derece oluyor 100 derece mi?]
Student: Yes. [Evet.]
Teacher Barış: If this is 100, this one is 100, if this one is 100, that one is 80. Right? [Burası 100 ise burası da 100 bura 100 ise bura 80. Tamam mı?]

Teacher Barış, 5th lesson Line 239 – 244

The dialogue represented that the teacher used his drawings in order to explain the procedure about the correct positioning of angles and extension of parallel rays as well as solving the worked examples by applying the procedures on the examples.

4.3.2 The Relation between Question Types and Tools for Questioning

Tools for questioning were used with question types. In order to understand how the question types penetrated into the tools for questioning, intersection between the tools for questioning and the question types were examined. For that purpose, the relation between the use of the tools for questioning and the characteristics for each of the question types were examined. In particular, the tools for questioning were used more or less together with some question types. Table 4.26 represented which tools for questioning were used with which question types. Accordingly, Teacher Caner used each of his tools for questioning (T9, T11, T12, T13, T15, and T16) with each question type more or less. However, Teacher Barış did not use each of his tools for questioning with each question type. As the table indicated, Teacher Barış used each of his tools for questioning (from T1 to T15) with guiding questions while he did not use T1, T4, T6, T14, and T15 with probing questions.

Table 4. 26

The relation between the tools for questioning and question types of the teachers

Tools for questioning	Question Types					
	GUIDING		PROBING		FACTUAL	
	Teacher Barış	Teacher Caner	Teacher Barış	Teacher Caner	Teacher Barış	Teacher Caner
T1	*	NA	NO	NA	*	NA
T2	*	NA	*	NA	*	NA
T3	*	NA	*	NA	*	NA
T4	*	NA	NO	NA	NO	NA
T5	*	NA	*	NA	NO	NA
T6	*	NA	NO	NA	NO	NA
T	*	NA	*	NA	*	NA
T8	*	NA	*	NA	*	**NA**
T9	*	*	*	*	*	*
T10	*	NA	*	NA	NO	NA
T11	*	*	*	*	*	*
T12	*	*	*	*	*	*
T13	*	*	*	*	*	*
T14	*	NA	NO	NA	NO	NA
T15	*	*	NO	*	NO	*
T16	NA	*	NA	*	NA	*

*Note.**: observed NA: not applicable NO: Not observed

Additionally, he did not use T4, T5, T6, T10, T14, and T15 with factual questions.

4.3.2.1 The relation between guiding question and tools for questioning

Teachers benefited from tools for questioning while asking guiding questions. There were different relations between characteristics of guiding questions and the tools for questioning for each of the teachers. In the following section/part, each of the relations for the participant teachers were exemplified. The relations between the tools for questioning and characteristics of guiding questions for Teacher Caner were shown below:

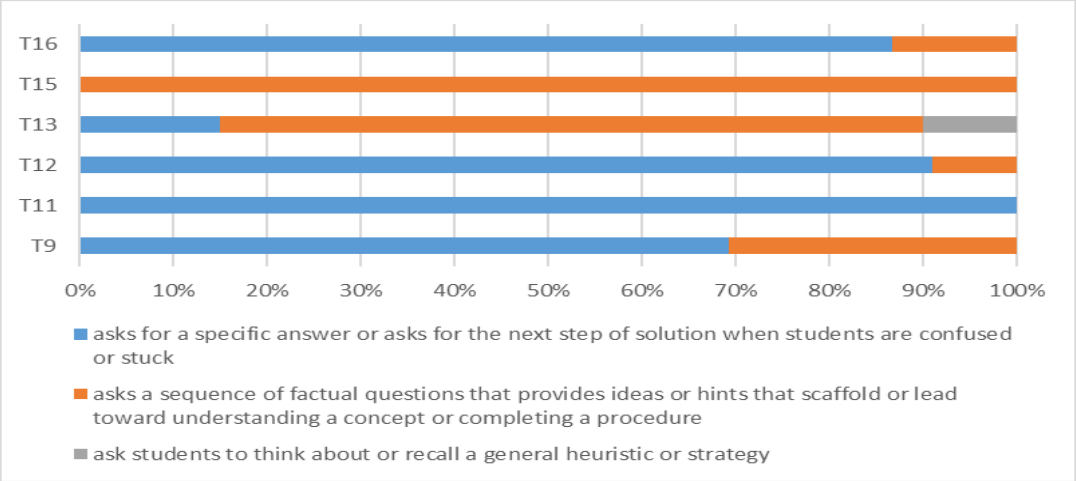


Figure 4.34 The relations between the tools for questioning and characteristics of guiding questions for Teacher Caner

The findings of the study showed that as seen in the Figure 4.33, characteristics of guiding questions were in relation to specific tools for questioning. According to this, Teacher Caner utilized tools for questioning including teacher drawing, supplementary book, and student questions, or comments with guiding questions. Teacher drawings with both of uses, student questions or comments, and printed supplementary book in terms of questioning sequence of worked examples were mostly in relation to two characteristics of guiding questions. Those characteristics of guiding questions were to overcome students' specific difficulties and to scaffold or lead students in understanding related procedures or concepts about lines and angles. The other characteristic of guiding questions which asks students' thinking about strategies or heuristics was only used together with teacher drawings helping teachers to explain a procedure through questions.

Considering each of them in detail, guiding questions were involved in questioning episodes including student questions or comments providing the teacher to clarify or detect problematic aspects through asking questions (T9). According to this, there were more than sixty percent of these situations in which T9 was intersected with 'asks for a specific answer or asks for the next step of the solution when students are confused or stuck'. The remaining situations (more than 30%)

included ‘asks a sequence of factual questions that provides ideas or hints that scaffold or lead toward understanding a concept or completing a procedure’. The following dialogue represented the relation:

Table 4. 27

Sample questioning dialogues representing the relation between T9 and different characteristics of guiding questions

Characteristics of guiding questions	
asks for a specific answer or asks for the next step of solution when students are confused or stuck	asks a sequence of factual questions that provides ideas or hints that scaffold or lead toward understanding a concept or completing a procedure
Sample dialogues	
Now, you sum up 90 and 62, subtract from 180, what did you find? [Sen şimdi burdaki 90 la 62 yi topladın, 180 den çıkardın, kaç buldun burayı?[Sen şimdi burdaki 90 la 62 yi topladın, 180 den çıkardın, kaç buldun burayı?] Teacher Caner, 5.lesson Line 30 – 30	Student: ikisi paralel biri dik kesen olabilir mi? Teacher Caner: bak ikisi paralel biri dik söyledik zaten. Student: aynısı oluyor di mi? Teacher Caner: ikisi paralel biri dik. şimdi şu ikisinin dik olması demek, şu ikisinin paralel olması demek. öyle değil mi? Teacher Caner, 1st lesson, Line 93-96

Another relation was observed between student drawings guding teachers for eliciting student thinking while solving worked examples (T11) and the characteristics of guiding questions of ‘ask for a specific answer or asks for the next step of solution when students are confused or stuck’. In the following dialogue, a student already solved the worked example on the board, but another student had a confusion about the application of moving interior angles. In all the situations, T11 was used with that characteristic of the guiding questions:

> Student: Teacher, I did not understand 5x plus 60, why 5x plus 60.
> [Hocam, ben şu 5x artı 60 ı anlamadım, neden 5x artı 60.]
> Teacher Caner: The summation of these two is 180, isn’t it?
> [Kızım bunların ikisinin toplamı 180 derece değil mi?]
> Teacher Caner, 3rd lesson, Line 94-95

The teacher used a guiding question that clarified the way of the solution and required student to the specific answer 'yes'. More than 90% of guiding questions in which teacher drawings helping teacher to solve the worked examples via questioning (T12) was involved supported by the mentioned characteristic of guiding question. The remaining 10% of teacher drawings (T12) used together with the characteristic of 'asks a sequence of factual questions that provides ideas or hints that scaffold or lead toward understanding a concept or completing a procedure'. From this point of view, we can say that the teacher used guiding questions to encourage students to understand the mathematical content more procedurally. The following dialogue represented each of the relations (see Table 4.28):

Table 4. 28

Sample questioning dialogues representing the relation between T12 and different characteristics of guiding questions

Characteristics of guiding questions	
asks for a specific answer or asks for the next step of solution when students are confused or stuck	asks a sequence of factual questions that provides ideas or hints that scaffold or lead toward understanding a concept or completing a procedure
Sample dialogues	
Yes. Listen. 9th question. Your friend asks 9th question. Now, here is 70 degree, these are again parallel to each other, if you extend this part, these are corresponding, thus, this angle becomes 70 degree, this agle 151 degree. Then, this agle becomes 29 degree, you may compute from the sum of angles in a triangle. Right? [Evet. Dinle. 9. Soru. Dinle burayı. Arkadaşınız 9 u soruyor. Şekli çiziyorum. Şimdi şurası 70 derece, bunlar parallel yine birbirine, şurayı uzatırsan kızım, şunların ikisi yöndeş olduğu için, şurası da 70 olur, şurası 151 derece, o zaman şurası 29 derece olur, üçgenin iç açıları toplamından da burayı bulursun. Değil mi?] Teacher Caner, 5th lesson Line 58 - 58	Look, when you extend like this, did you realize a triangle? [Bakın şöyle, uzattığınızda bir üçgen oluştu mu karşınızda?] Teacher Caner, 3rd lesson Line 132 - 132

Similarly, another use of teacher drawing helping the teacher to explain a procedure through questions (T13) supported 75% of the use of T13 with guiding questions. The teacher used the characteristic of guiding question requiring 'ask students to think about or recall a general heuristic or strategy' in 10% of the use T13 with guiding questions. In similar percent, the teacher used the characteristic of 'asks for a specific answer or asks for the next step of solution when students are confused or stuck' with T13 in 15% of guiding questions. This showed the teacher did not give much chance to students in understanding mathematical procedures or concepts in a conceptual way of learning. The following dialogues represented the relation of teacher drawing in terms of helping teachers to explain a procedure through questions with the characteristics of guiding questions, asks for a specific answer or asks for the next step of solution when students are confused or stuck, ask students to think about or recall a general heuristic or strategy, asks a sequence of factual questions that provides ideas or hints that scaffold or lead toward understanding a concept or completing a procedure (see Table 4.29):

Table 4. 29

Sample questioning dialogues representing the relation between T13 and different characteristics of guiding questions

Characteristics of guiding questions		
asks for a specific answer or asks for the next step of solution when students are confused or stuck	ask students to think about or recall a general heuristic or strategy	asks a sequence of factual questions that provides ideas or hints that scaffold or lead toward understanding a concept or completing a procedure
Isn't there an M rule there? Isn't there? Ha? [şurda bi M kuralı yok mu? Ha?] Teacher Caner, 3rd lesson Line 287 – 287	How can I draw an angle, which equals to this angle? Yes? [Ben bu açının eş açısı olacak şekilde buna eşit olacak şekilde bir açıyı nasıl çizebilirim? Evet?] Teacher Caner, 1st lesson 47 - 47	For instance, can it be like this? Suppose these two are parallel like this, is the other one intercept (with that one) like this? [Şöyle olabilir mi mesela? Diyelim ki ikisi şu şekilde paralel, diğeri de şöyle kesebilir mi?] Teacher Caner, 1st lesson Line 90 - 90

The teacher used printed supplementary books guiding the teacher to talk about a mathematical procedure or concept with questions (T15) with the characteristics of guiding question of 'asks a sequence of factual questions that provides ideas or hints that scaffold or lead toward understanding a concept or completing a procedure' in 100%. The teacher used the same characteristic in approximately 12% of intersection of printed supplementary materials which guided the teacher in terms of the questioning sequence of worked examples (T16) and guiding questions. The remaining percent of the use of the material (more than 80%) used the characteristic of 'asks for a specific answer or asks for the next step of solution when students are confused or stuck'. In line with these, the teacher's questioning was closely related to procedural understanding of the students. The following dialogues represented the relations between T16 and different characteristics of guiding questions (see Table 4.30).

Table 4. 30

Sample questioning dialogues representing the relation between T16 and different characteristics of guiding questions

Characteristics of guiding questions	
asks for a specific answer or asks for the next step of solution when students are confused or stuck	asks a sequence of factual questions that provides ideas or hints that scaffold or lead toward understanding a concept or completing a procedure
Look, these two segments, ad segment and bc segment are parallel to each other. Isn't it? Since they are parallel, this angle and that angle are alternate interiar angles. Hence, equal to each other. Right? [Bakın şu iki doğru parçası, ad doğru parçası ile bc doğru parçası birbirine parallel di mi parallel olduğu için burdaki açı ile şurdaki açı içters açıdır ve birbirine eşit olur. Doğru mu?] Teacher Caner, 5th lesson 50 – 50	Thus, here becomes 140 doesn't it? [Değil mi o yüzden burası 140 olur.] Teacher Caner, 3rd lesson 189 - 189

While Teacher Caner was using printed supplementary book, benefiting from their drawings constructed during the solution process of worked examples, and

students' questions/ideas were involved in teacher's questioning while teaching content, two characteristics of guiding questions were used. As similar to this, the same two characteristics of guiding questions were frequently used in relation to the tools for questioning by Teacher Barış. The teacher used the other characteristic of guiding questions in addition to those two while using some tools for questioning. Figure 4.35 represented all those relations.

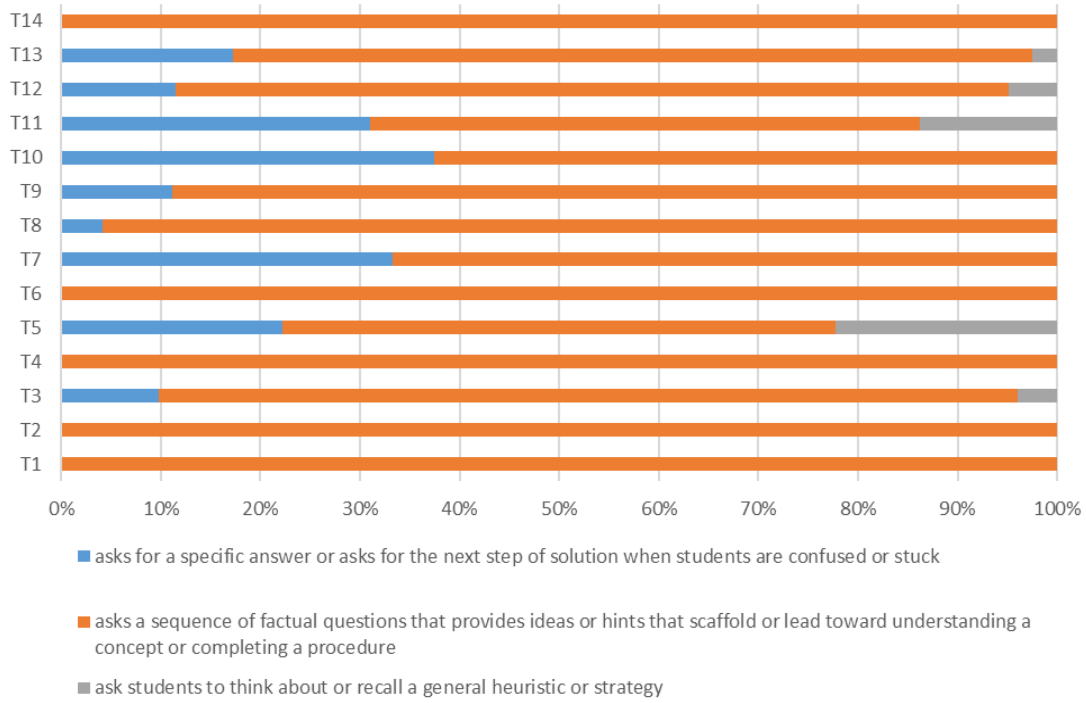


Figure 4.35 The relations between tools for questioning and characteristics of guiding questions for Teacher Barış

One of the prominent relation was between DGS (T1 and T2) and the characteristics of guiding question which was asked as a sequence of factual questions that provides ideas or hints that scaffold or lead toward understanding a concept or completing a procedure. According to this, the teacher used that characteristic of guiding questions in all the questioning episodes including DGS. In line with this, in 100% of the questioning episodes, the teacher used the characterstic of guiding questions. Similarly, in all the questioning episodes related to educational

animation which guided the teacher to ask questions about mathematical procedures (T4), dynamic shapes utilized in response to student questions (T1), and real life examples used to question mathematical procedures (T14), the teacher used the mentioned characteristics of guiding questions in a hundred percent. Considering the use of the information technology tools, the teacher used the characteristic of guiding questions in more than 80 percent. More than fifty percent of T3, T5, T7, T8, T9, T10, T11, T12, and T13 with guiding questions, the teacher used *a sequence of factual questions that provides ideas or hints that scaffold or lead toward understanding a concept or completing a procedure.* In Table 4.31, sample questioning dialogues represented the relation between the characteristic of the guiding question and tools for questioning (T2, T4, T6, and T14):

Table 4. 31

Sample questioning dialogues representing the relation between one characteristic of the guiding question and tools for questioning

Characteristic of guiding question	
asks a sequence of factual questions that provides ideas or hints that scaffold or lead toward understanding a concept or completing a procedure	
Tools for questioning	Sample dialogues
Using DGS to ask questions about dynamic figures (T2)	Teacher Barış: Yes, guys, it says three line may intercept pairs by pairs. We just saw the case that three of them intercept. We said they are intercepted lines, so they have common point. Right? [Evet üç doğru ikişer ikişer kesişebilir diyor çocuklar. Az önce üçünün beraber kesiştiği durumu gördük. Dedik ki bunlar noktadaş doğrulardır dedik yani bir noktası ortak doğru mu?] Teacher Barış, 3rd lesson Line 339 - 339
Educational animation which guided the teacher to ask questions about mathematical procedures (T4)	(The animation) drew it. There is an angle there, right? [Diğer ışını çizdi, kolu. Bir açı oldu di mi orada?] Teacher Barış, 3rd lesson Line 96 - 96

Table 4.31 (cont'd)

Dynamic shapes which were utilized in response to student questions (T6)	Student: Teacher, you can move it (the slider)?[Öğretmenim onu hareket ettirebiliyorsunuz.] Teacher Barış: Yes, I can. Look, I am changing. 33; when you sum 15 with 18. 33 isn't it? [Evet ettiriyorum. Bakın değiştiriyorum. 33; 15 ile 18'i topladığında 33 değil mi?] Student: Yes. [Evet.] Teacher Barış: The angle looking at right is 15 and 18 the angle looking at left, is 33 degree. [Bak sağa bakanlar; 15 ile 18. Sola bakan 33 derece.] Teacher Barış, 5th lesson Line 361 - 368
Real life examples which were used to question mathematical procedures (T14)	If (the spirit level) looks like a trapezoid, the spirit level is in balance? [Bu yamuk olsa dengede olabilir mi?] Teacher Barış, 7th lesson 126 - 126

Another relation showed that the characteristic of guiding question that *asks students to think about or recall a general heuristics or strategy* was used only together with supplementary book which organizes the sequence of questions to be asked (T3), part of supplementary book to question student's performances (T5), student drawings guiding teachers to elicit student thinking while solving worked examples (T11), and with both of the uses of teacher drawings (T12 and T13). The Table 4.32 represented the relations:

Table 4. 32

Sample questioning dialogues representing the relation between one of the characteristic of guiding question and tools for questioning

Characteristic of guiding question	
asks students to think about or recall a general heuristics or strategy	
Tools for questioning	Sample dialogues
The supplementary book guided the teacher in sequence of the questions to be asked (T3)	Teacher Barış: Now, which point do we choose? [Şimdi hangi noktayı seçelim?] Teacher Barış, 2nd lesson 66 – 66)

Table 4.32 (cont'd)

Part of supplementary book to question student's performances (T5)	Teacher Barış: What this figure reminds you? [Sana neyi hatırlatıyor bu şekil?] Teacher Barış, 7th.lesson 298-298
Student drawings guiding teachers for eliciting student thinking while solving worked examples (T11)	Teacher Barış: What will we do here? How bisector line can be drawn? [Burada ne yapacağız. Nasıl açıortay çizilir buna?] Öğrenci: With the compasses. [Pergelle.] Teacher Barış, 3rd lesson, Line 256-257
Teacher drawings helped teachers to explain a procedure through questions (T13)	Teacher Barış: If here is 70 decree, why here is 110? Why? [Örnek veriyorum burası 70 ise neden burası 110? Neden?] Teacher Barış, 5th.lesson 16 – 16
Teacher drawings helped teachers to solve a worked example via questioning (T12)	Teacher Barış: What is the thing that we called "m"rule? ["m" kuralı dediğimiz şey neydi?] Teacher Barış, 6th lesson 29 – 29

The other relationship was about the relation between information technology in terms of the use of the supplementary book compatible with smartboard (T3, T4, T5, andT6) and use of guiding questions. The Figure 4.33 showed that the supplementary book guided the teacher's questioning in terms of the sequence of the guiding questions more than 60% of the question episodes in which the information technology tools were used with guiding questions. That provided us to understand that during the implementation of the book, the teacher used a sequence of factual questions that provides ideas or hints that scaffold or lead toward understanding a concept or completing a procedure' very frequently.

4.3.2.2 The Relation between Probing Questions and Tools for Questioning

As seen in Figure 4.35, for Teacher Caner, some tools for questioning somehow supported some characteristics of probing questions. According to this figure, Teacher Caner used the tools with the probing questions in diverse percentages. While using T9 and T13, the teacher used the three charactrstics and T15 was used with one characteristic of probing questions. The rest of the tools (T11,

T12, and T16) was used by the two characteristics of the probing questions as represented in Figure 4.36:

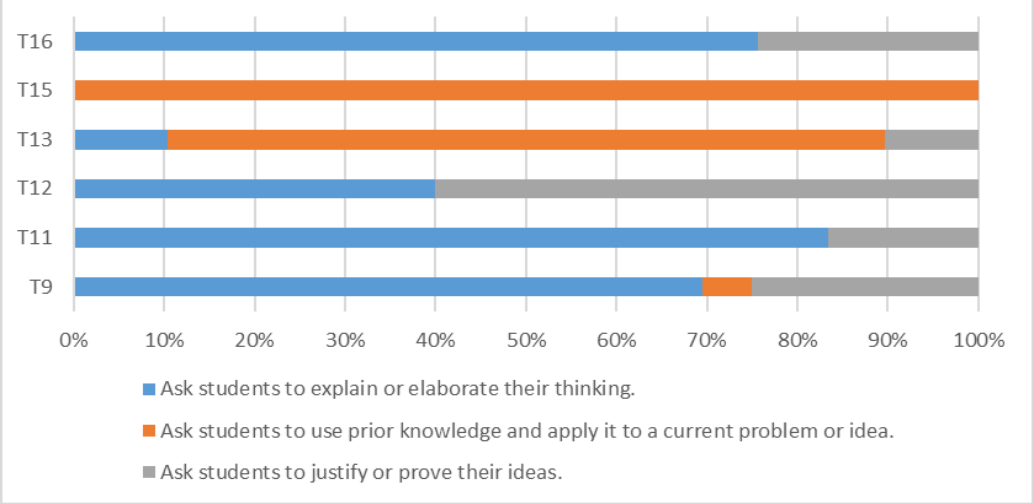


Figure 4.36 The relations between the tools for questioning and characteristics of probing questions for Teacher Caner

Findings of the study showed that T9 was used with all the three characteristics of probing questions. Approximately 70% of probing questions involved in T9 was used with the characteristic of 'ask students to explain or elaborate their thinking' and 25% of probing questions with T9 was used to ask students to justify or prove their ideas. Following this, approximately 5% of probing questions with T9 was used to ask students to use their prior knowledge and apply it to a current problem or idea. In contrast to this, while using T13, the teacher used the same characteristic approximately 80% of the questions with T13. The other two characteristics were used approximately the same percentage.

T11 and T16 were similarly used with probing questions. Both of them were used with the characteristic of 'ask students to explain or elaborate their thinking' more than the characteristic of 'ask students to justify or prove their ideas' (more than 80%, and more than 70%, respectively). In contrast to these, T12 was used with the characteristic of 'ask students to justify or prove their ideas' (60%) more than the

characteristic of 'ask students to explain or elaborate their thinking'. Table 4.33 included the sample classroom dialogues:

Table 4. 33

Sample questioning dialogues representing the relation between two characteristics of probing questions and the tools for questioning

	Characteristics of probing questions	
	ask students to justify or prove their ideas	ask students to explain or elaborate their thinking
Tools for questioning	Sample dialogues	
Table 4.33 (cont'd) ...led ...ent thinking while solving worked examples (T11)	Teacher Caner: Why it becomes 50? [Orası niye 50 oldu?] Teacher Caner, 3rd lesson Line 236 – 236	Teacher Caner: Now, what did you find? [Şimdi sen neyi buldun?] Teacher Caner, 3rd lesson Line 76 – 76
Teacher drawings helped teachers to solve a worked example thorugh questions. (T12)	Teacher Caner: Look, it is not related to triagle structure, (the teacher drew on the board). As soon as you say why it is equal, I will accept the solution. [Bakın Üçgenin yapısından değil, neden eşit olduğunu söylediğin anda, ben çözümü Kabul edicem.] Teacher Caner, 5th lesson Line 21 - 21	Teacher Caner: We sum up interior of that triagle. Student: Triangle? [O üçgenin içini topladık. Üçgenin?] Teacher Caner, 3rd lesson 135-136
Printed supplementary books guided the teacher in terms of questioning the sequence of worked examples (T16)	Teacher Caner: Why did you multiplied with 2? That is important. [Niye 2 yle çarptın? Önemli olan bu] Teacher Caner, 5th lesson Line 17 - 17	Teacher Caner: Here, there are 3 angles. When there are 4 angles, still can you say 360 degree? [Burada 3 tane açı var. 4 tane açı olunca yine 360 derece diyebilecek miydin?] Teacher Caner, 3rd lesson 148 – 148

The other relation was between T9, T13 and probing questions. According to the figure, teacher drawings, which helped teachers to explain a procedure through questions (T13) was in relation to all the characteristics of probing questions. T9 was used with the characteristic of 'ask students to explain or elaborate their thinking' and T13 was used with the characteristic of 'ask students to explain or elaborate their thinking' mostly. The following dialogues represented the relations between teacher drawings (T13) and students' questions or comments (T9), and the characteristics of probing questions, ask students to justify or prove their ideas, ask students to explain or elaborate their thinking, ask students to use their prior knowledge and apply it to a current problem or idea:

Table 4. 34

Sample questioning dialogues representing the relation between T13, T9, and different characteristics of probing questions

	Characteristics of probing questions		
Tools for questioning	Sample dialogues		
	ask students to justify or prove their ideas	ask students to explain or elaborate their thinking	ask students to use their prior knowledge and apply it to a current problem or idea.
Students' questions or comments provided teachers to clarify or detect problematic aspects of students' mathematical thinking through asking questions (T9)	Look, it is not related to the structure of the triangle, why I will accept the solution as soon as they are equal. [Bakın Üçgenin yapısından değil, neden eşit olduğunu söylediğin anda, ben çözümü Kabul edicem.] Teacher Caner, 5th lesson 21 - 21	How did you find this? [Nasıl buldun kızım?] Teacher Caner, 3rd lesson Line 14 – 14	How many different kind of angles do we have? Said 3 yes. Said 5, tell them. [kaç çeşit açımız vardı? üç dedi evet. 5 dedi, söyle onları.] Teacher Caner, 1st lesson Line 27 – 27

Table 4. 34 (cont'd)

Teacher drawings which helped the teacher to explain a procedure thourgh questions (T13)	It is time to mention why the angles that we mentioned at first hour are equal. For instance we said 1 and 5. Why 1 and 5 are equal? [Birinci derste söylemiş olduğumuz eşit açıların neden eşit olduğunu söylemenin zamanı geldi. Mesela 1 ve 5 demiştik, 1 ve 5 neden eşit?] Teacher Caner, 2nd lesson Line 4 – 4	What do you mean by the intersection? [Kesişebilirlikten kastın nedir?] Teacher Caner, 1st lesson 77 - 77	Now, the question is this.I wonder which of these angles are equal to each other? [Şimdi soru şu, acaba bu açılardan hangileri birbirine eşittir?] Teacher Caner, 1st lesson 107 - 107

The other teacher, Teacher Barış, used probing questions in relation to tools for questioning as represented Figure 4.37 below.

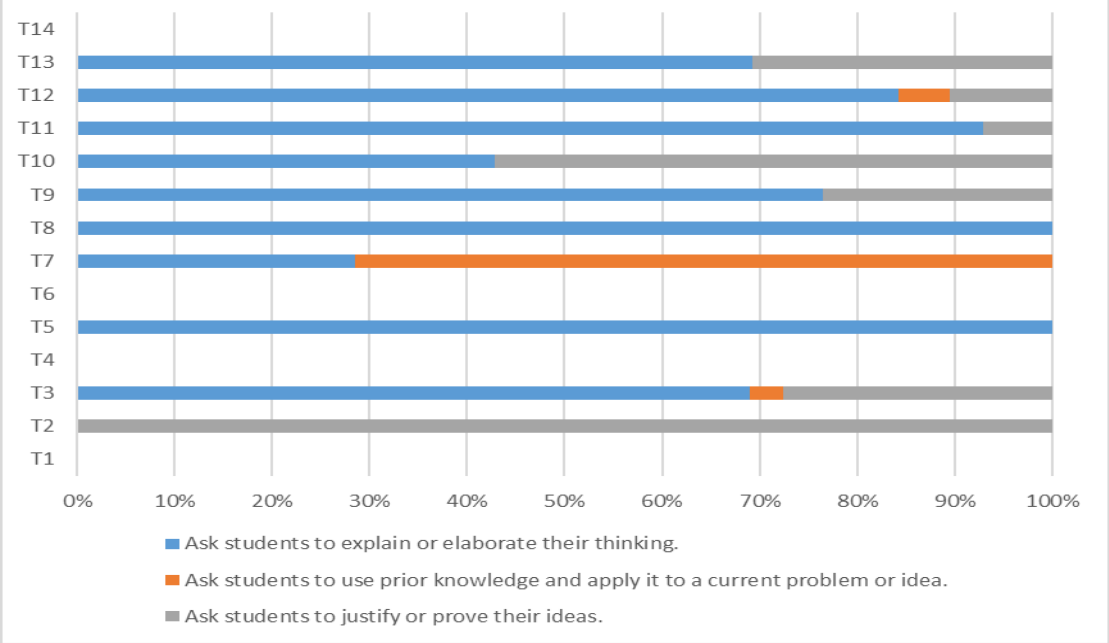


Figure 4. 37 The relations between the tools for questioning and characteristics of probing questions for Teacher Barış

As the figure showed that T1, T4, T6, T14, and T15 were not used with any characteristics of probing questions while T3 and T12 were used with all three characteristics of probing questions. T2, T5, and T8 were used with one characteristic of probing questions. The rest of the tools were used with two characteristics of probing questions.

Starting from T2, all the probing questions in which T2 was involved had the characteristic of 'ask students to justify or prove their ideas'. Following this, the same characteristic was used while using T10 in more than 50% of the probing questions interacted with T10.

The supplementary book which guided the teacher in sequence of questions to be asked (T3) and T12 had all the characteristics of probing questions. Both had the same tendency that 'ask students to explain or elaborate their thinking' was the mostly used characteristic (more than 60%, more than 80%, respectively), followed by 'ask students to justify or prove their ideas' (close to 30%, approximately 10%, respectively), and the least frequently used one was 'ask students to use their prior knowledge and apply it to a current problem or idea' (app. 5%, 6%, respectively). T5 and T8 were used with the characteristic of probing questions of 'ask students to explain or elaborate their thinking' in 100%.

Table 4. 35

Sample questioning dialogues representing the relation between T12 and different characteristics of probing questions

Characteristics of probing questions		
ask students to justify or prove their ideas	ask students to explain or elaborate their thinking	ask students to use their prior knowledge and apply it to a current problem or idea.
Well, if I want to draw an equal angle to this; for example 5 units like this, 10 units like this and if I intercept this and draw an angle with this, will they be same? [Peki ben buna eş açı çizmek istersem;	Student: Well, is it always 180? [Yani hep 180 mi?] Teacher Barış: Which of angles you suppose to sum up as 180, are you asking the angles 95 and 42, 95 and 42, hıım, 153	Barış: Howmuch degree does it looks like? [Kaç derece gibi görünüyor?] Öğrenci: Ninty. [Doksan.] Teacher Barış, 3rd lesson Line 208 - 209

Table 4.35 (cont'd)

örnek veriyorum şöyle 5 birim, şöyle 10 birim olsa ve bunu birleştirsem şöyle bir açı çizsem bununla aynı olur mu?] Öğrenci: It will. [Olur.] Barış: Why? [Neden?] Öğrenci: because they are the same. 5 times. It is enlarging with a proportion. [Çünkü aynı. 5 katı. Oranlı bir şekilde büyütüyor.] Teacher Barış, 1st lesson 93 - 96	hıım 137 plus 53, is it 180 according to you? I am passing. [hangilerinin toplamının 180 olduğunu söylüyorsun sen, açı söyle bana. soru işareti, 95 ve 42. 95, 42, şey, 153 şey 137 artı 53'ü mü soruyorsun sen? 53, daha 180 mi yapıyor sence? geçiyorum.] Teacher Barış, 6th lesson 92 - 95

Analogies, which provided teachers to ask questions visulizing mathematical concepts (T7) were used with the characteristics of ask students to use their prior knowledge and apply it to a current problem or idea (more than 70%) and ask students to explain or elaborate their thinking (close to 30%). Analogies which helped teachers to refer to them while asking questions about a worked examples (T8) were utilized only with one characteristic, which is *'ask students to explain or elaborate their thinking'*, of probing questions with a hundred percent, Analogies were not utilized by probing questions requiring justification and proving their ideas from students. Analogies did not have a role in teacher questioning about justification and proof.

Table 4. 36

Sample questioning dialogues representing the relation between T7, T8 and different characteristics of probing questions

	Characteristics of probing questions	
The use of the tools for questioning	ask students to explain or elaborate their thinking	ask students to use their prior knowledge and apply it to a current problem or idea.
Analogies provided teachers to ask questions visualizing mathematical concepts (T7)	Teacher Barış: Show me the angles within the river and out of the river. Which angles? [Nehrin dışında kalan ve içinde kalan açıları göster bana. Nereler?] Student: Right there.[İşte şurası.] Teacher Barış: Is it only one? [*Bir tane mi sadece?]* Student: No. here. No that is out (of the river). Wait a minute. [Hayır. Bura. Hayır ora dışı. bi dakika] Teacher Barış, 7th lesson 249 - 252	Teacher Barış: There is a dot at one side, at the other side? (referring to the body analogy) What is this? [Bir tarafta nokta var diğer taraf.. *Ne bu?]* Teacher Barış, 1st lesson 51 - 51
Analogies helped teachers to refer to them while asking questions about a worked example (T8)	Teacher Barış: Now, here is not the river isn't it? Who are the angles stay within the river? 1, 2, 72, and 3 aren' they? Well, which of the angles of 1, 2, 72 and 3, are looking opposite directions? [Şimdi burası nehrimiz değil mi? Arasında kalan açılar kimler? 1, 2, 72 ve 3 değil mi? Peki 1, 2, 72 ve 3 açılarından kimler zıt yönlere bakıyor?] Student: 72 is looking at right. [72 sağa bakıyor.] Teacher Barış: When you look at like this. [şöyle sola, şöyle yan baktığında.] Student: 2 looks at right side [2 de, 2 de sağa.] Teacher Barış: Then, these two are the same? What else? [O zaman bunlar aynı. *Başka?]* Teacher Barış, 4th lesson, Line 186 - 191	-

Student drawings, which guided teachers for eliciting student thinking while solving worked examples (T11) and teacher drawings, which helped teachers to explain a procedure through questions (T13), were used with two characteristics of probing questions. In both of them, the characteristic of 'ask students to explain or elaborate their thinking' was used by these tools more than 70% and close to 70%, respectively, while the characteristic of 'ask students to justify or prove their ideas' was used with less than 10% and 30%, respectively. 55% of probing questions which had the characteristic of 'ask students to justify or prove their ideas' and 45% of probing questions which had the characteristic of 'ask students to explain or elaborate their thinking'.

In Table 4.37, sample questioning dialogues representing the relation between the two characteristics of probing questions, ask students to explain or elaborate their thinking and ask students to justify or prove their ideas, student drawings (T11) and teacher drawings (T13) were represented with sample excerpts from Teacher Barış's classroom:

Table 4. 37

Sample questioning dialogues representing the relation between the two characteristics of probing questions and student drawings

	Characteristics of probing questions	
The use of the tools for questioning	ask students to explain or elaborate their thinking	ask students to justify or prove their ideas
Student drawings guided teachers to elicit student' thinking while solving worked examples (T11)	Teacher Barış: I could not see a "z" there. [Ben orada bir "z"yi göremiyorum ama.] Teacher Barış, 6th lesson 171 - 171	Teacher Barış: Okey. Why did you choose F while selecting a point on a ray? [Neden F noktasını seçtin?] Teacher Barış, 1st lesson 105 - 105

Table 4.37 (cont'd)

Teacher drawings, which helped teachers to explain a procedure through questions (T13)	Student: Teacher, it is a square. The angle did not pass through the middle of the angle, it does not complete(ly pass right through the angle). How can be it possible? [Öğretmenim bu bir kare ya. Açı böyle ortasından geçmiş, tamamlamıyor nasıl oluyor?] Teacher Barış: Yavrum onun tamamlaması bizim için önemli değil ki. Hayır bizim için önemli olan şey şu ya. Şu açımız ya. Açının gördüğü dikey mesafe bu seni ilgilendiren şey. Tamam mı? Ben F noktasını seçtim. İstesem bunu seçerdim. Şurada köşeden geçiyor. Kaç birim 4 birim. Buradan itibaren yatay uzunluğuna bak o noktanın; açının köşesine. Buda dört birim. Demek ki dörde dört olan herşey eşit olacak doğru mu? [The completion of the angle is not important for us. The important thing for us is that here is the angle, you need to focus on the vertical distance of the point, right? I chose F point. If I want to choose that one, I can. That one also pass though the corner (on the grid). What is the distance? 4 units. Check the horizontal distance of it, the corner of (the point belonging to the ray of the angle). That is 4 units. That means 4 (as a horizontal distance) and 4 (as a vertical distance) would be equal (to what I want to create as an equal angle), right?] Teacher Barış, 2nd lesson, 112-113	Teacher Barış: Esin, come to the stage, please. Come. There is nothing to afraid. [Esincim seni piste davet ediyorum. Gel. Korkacak bir şey yok. Zaten şimdi şu makası biliyoruz değil mi biz?] Student: I did not undertand the topic (angles). [Öğretmenim ben açılardan hiçbir şey anlamadım.] Teacher Barış: You already know the vertically opposite angles, right? [Zaten şimdi şu makası biliyoruz değil mi biz?] Teacher Barış, 7th lesson 43-43

4.3.2.3 The Relation betwen Factual Questions and Tools for Questioning

In the following table, the use of factual questions with tools for questioning of Teacher Caner was represented.

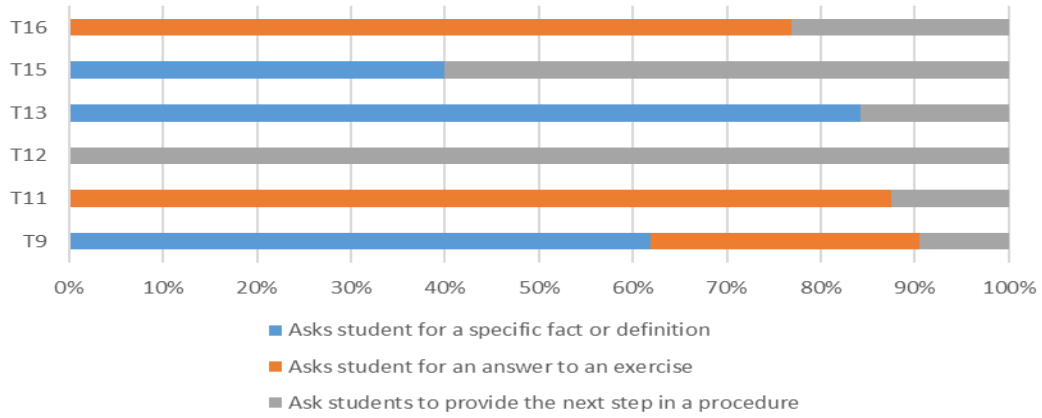


Figure 4.38 The relations between the tools for questioning and characteristics of factual questions for Teacher Caner

As represented in Figure 4.38, the teacher used some tools for some characteristics of the factual questions. According to this figure, T12 was used with one characteristic of the factual questions while T9 was used with all three characteristics of factual questions. The rest of the tools (T11, T13, T15, and T15) were used with two characteristics of the factual questions.

T9 was used with the three characteristics of factual questions in which 'asks student for a specific fact or definition' was used mostly (more than 60%) and 'ask students to provide the next step in a procedure' was used the least (approx. 10%). The characteristic of asking students the next step in a procedure was used with the factual questions with T12 in hundred percent. However, T11 and T16 mostly used the characteristic of 'ask students to provide the next step in a procedure' the least one. The following dialogues represented the relations between T9, T11, T12, and T16 and different characteristics of factual questions:

Table 4. 38

Sample questioning dialogues representing the relation between T9, T11, T12, and T16 and different characteristics of factual questions

	Characteristics of factual questions	
The use of the tools for questioning	asks student for an answer to an exercise	ask students to provide the next step in a procedure
Printed supplementary books guided the teacher in terms of questioning sequence of worked examples (T16)	Teacher Caner: Is there anyone who find (the worked example) different? Is there anyone who find a value different than 17? [Farklı bulan var mı? 17 nin dışında farklı bir değer bulan var mı?] Teacher Caner, 3rd .lesson 38 -38	Student: Teacher, I did not understand that 5x plus 60, why 5x plus 60. [Hocam, ben şu 5x artı 60 ı anlamadım, neden 5x artı 60.] Teacher Caner: The sum of these two is 180 degree isn't it? [Kızım bunların ikisinin toplamı 180 derece değil mi?] 4x plus one x is 5x, 14 plus 46 is 60. Ok. [4x bir tane x daha 5x, 14 46 daha 60. Tamam.] Teacher Caner, 3rd lesson 95 – 95
Students' questions or ideas provided teachers to clarify or detect problematic aspects of students' mathematical thinking through asking questions (T9)	Teacher Caner: How did you find? [Nasıl buldun kızım?] Student: As the angles are alternate interior angles, 3x plus 12 is equal to 5x minus 20. [Iç ters açı oldukları için 3x artı 12 eşittir 5x eksi 20 den.] Teacher Caner: Come and solve. She is right. [Gel. Doğru yaptı.] Teacher Caner, 3rd lesson 14 - 16	
Student drawings guided teachers to elicit students' thinking while solving worked examples (T11)	Teacher Caner: What did you find out? [Şimdi sen neyi buldun?] Student: X value. [X in değerini.] Teacher Caner: What does the worked example require to do? [Soruda senden ne isteniyor?] Student: What is the measurement of DCA	-

Table 4.38 (cont'd)

	angle? [Dca, dca kaç derecedir?] Teacher Caner: Yes. What is the measurement of DCA angle? [Evet, dca açısı kaç derecedir?]	
Teacher drawings helped teachers to solve a worked example via questioning (T12)	-	Look, when you extend like this, you construct a triangle didn't you? [Bakın şöyle, uzattığınızda bir üçgen oluştu mu karşınızda?] Teacher Caner, 3rd lesson 132 - 132

T13 and T15 were used with the two characteristics of factual questions including 'asks student for a specific fact or definition' (more than 80% of factual questions with T13) and 'asks students to provide the next step in a procedure' (40% of factual questions with T15). Table 4.39 represented the relations between T13, T15 and different chacracteristics of factual questions:

Table 4. 39

Sample questioning dialogues representing the relation between T13, T15 and different chacracteristics of factual questions

Characteristics of factual questions		
The use of the tools for questioning	asks student for a specific fact or definition	ask students to provide the next step in a procedure
Teacher drawings helped teachers to explain a procedure through questions (T13)	How many name can I assign to this angle? Tell me. [Ben bu açıyı kaç değişik biçimde isimlendirebiliyordum, söyle kızım,] Teacher Caner, 1st lesson 20 - 20	Look, isn't the summation of these three is a full angle? [Bakın üçünün toplamı bir tam açı yapmadı mı?] Teacher Caner, 2nd lesson 94 – 94
Printed supplementary books guided the teacher to talk about a mathematical	By this parallelism will this angle and that angle be equal? [Bu paralellikte bakın	Thus, we will write that from zigzag rule. Look, assum that thi is like this. Is it possible? [İşte zigzag

Table 4.39 (cont'd)

procedure or concept with questions (T15)	şurdaki açıyla şurdaki açı içters olur mu?] Teacher Caner, 2nd lesson 78 - 78	kuralında onu yapacağız. Bakın diyelim ki şurası şu şekilde böyle bi şey, olabilir mi?] Teacher Caner, 2nd lesson 109 – 109

The relations for the other teacher, Teacher Barış, was represented in Figure 4.39. According to the figure, T4, T5, T6, T10, T14, and T15 were not used with factual questions any more while T3 was used with the three characteristics of factual qestions. T2, T7, T11, and T13 were only used with one characteristic of factual questions. T1, T8, T9, and T12 were used with two characteristics of factual questions. For T1 and T2, the teacher behaved differently. The teacher used 60% of factual questions with the characteristic of 'ask students to provide the next step in a procedure' and 40% of factual questions with the characteristic of 'asks student for a specific fact or definition' interacting with T1. All factual questions interacting with T2 represented the characteristic of 'ask students to provide the next step in a procedure'.

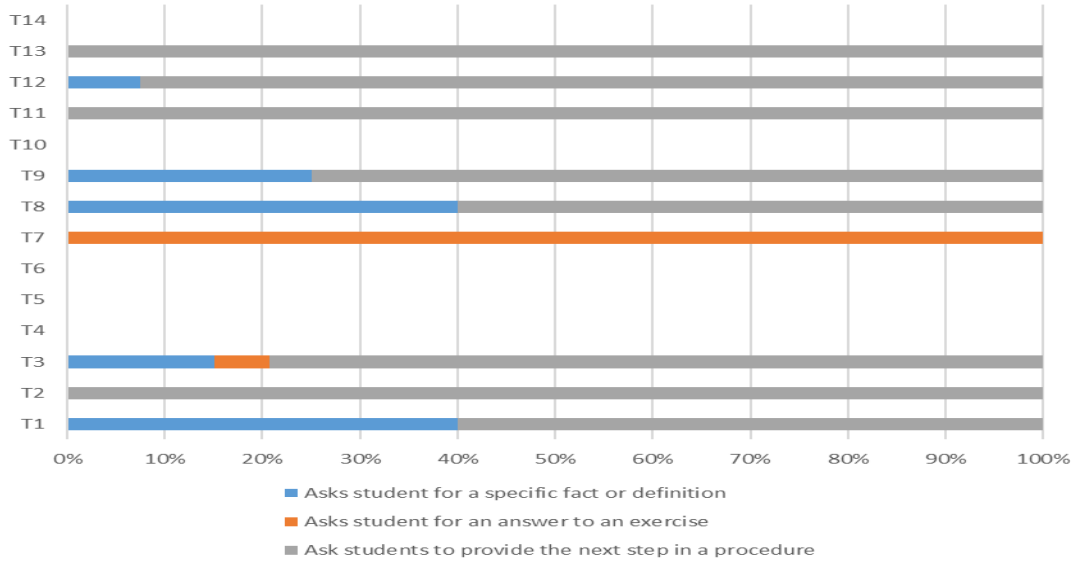


Figure 4. 39 The relations between the tools for questioning and characteristics of factual questions for Teacher Barış

As seen in Figure 4.39, T3 was used with all the three characteristics of factual questions. Factual questions related to T7 had the characteristic of factual questions 'requiring students to answer an exercise' in hundred percent. Additionally, 60% of factual questions with T8 was used with the characteristic of 'ask students to provide the next step in a procedure' and 40% of the questions with the characteristic of 'asks student for a specific fact or definition'.

Table 4.40 represented some of the relations berween the tools for questioning and two characteristics of factual questions. The following table represented the relations between T1, T2 and two chacracteristics of factual questions, ask students to provide the next step in a procedure and asks student for a specific fact or definition:

Table 4. 40

Sample questioning dialogues representing T1, T2 and different characteristics of factual questions

	Characteristics of factual questions	
Tools for questioning	Ask students to provide the next step in a procedure	Asks student for a specific fact or definition
Using DGS to ask questions based on dynamic figures (T2)	Teacher Barış: What did we find (by measuring the angle with geogebra)? [Kaç çıktı?] Teacher Barış, 3rd lesson 179 - 179	-
Using DGS to build questioning sequence in response to student's questions (T1)	Teacher Barış: We are checking, are they equal? [Bakıyoruz eşit çıkıyor mu?] Teacher Barış, 2nd lesson 228 - 228	Teacher Barış: Is the ratio important? [Oran mı önemliymiş?] Teacher Barış, 2nd lesson 231 - 231

In the following table, sample questioning dialogues represented the relation between the three characteristics of factual questions and analogies (T7 and T8):

Table 4. 41

Sample questioning dialogues representing the relation between the characteristics of factual questions and analogies (T7 and T8)

	Characteristics of factual questions		
Tools for questioning	Asks student for an answer to an exercise	Ask students to provide the next step in a procedure	Asks student for a specific fact or definition
Analogies provided teachers to ask questions visualizing mathematical concepts (T7)	Teacher Barış: What is this? You are saying the line between two dots. These are the intercept points. My shoulders. [Barış: Şu? İki nokta arasında ki çizgiyi diyosun. Geçtiği noktalar bunlar. Omuzlarım.] Student: Line segment. Teacher Barış, 1st lesson 29 - 29	-	-
Analogies helped teachers to refer to them while asking questions about a worked example (T8)	-	Teacher Barış: Which fish stays in the river and swim opposite direction to the river? [Nehrin iç kısmında kalan ve akıntının diğer tarafına yüzen hangi balık var?] Teacher Barış, 4th lesson 83 - 83	Teacher Barış: Now, if these two lines are parallel, we extend our river. What kind of angles are they? f with g? [Şimdi canım bu iki doğru paralelse. Şöyle nehrimizi zatıyoruz. Bunlar nasıl açılar sizce? f ile g?] Teacher Barış, 4th lesson 223 - 223

As similar to T8, T9 and T12 were recorded similarly. All factual questions related with T11 and T13 were used with 'asks for the next step of a procedure' in hundred percent.

In the following table, sample questioning dialogues represented the relation between the characteristics of factual questions and student drawings and teacher drawings. Table 4.42 represented the relation between student drawings guided teachers to elicit student thinking while solving worked examples (T11), teacher drawings helped teachers to solve a worked example via questioning (T12), and teacher drawings helped teachers to explain a procedure through questions (T13), and two characteristics of the factual questions, ask students to provide the next step in a procedure and asks student for a specific fact or definition. Sample questioning dialogues were represented in the following table:

Table 4. 42

Sample questioning dialogues representing the relation between the characteristics of factual questions and drawings

	Characteristics of factual questions	
Tools for questioning	Ask students to provide the next step in a procedure	Asks student for a specific fact or definition
Student drawings guided teachers to elicit student thinking while solving worked examples (T11)	Teacher Barış: Show the angle. Where is the angle, equal angle? [Açıyı tara bana. Neresi açı, eş açı?] Teacher Barış, 1st lesson Line 117 - 117	-
Teacher drawings helped teachers to solve a worked example via questioning (T12)	Teacher Barış: You chose F (to create an equal angle), well, what is the vertical distance of F? [Barış: F yi istiyorsunuz peki Fnin dikey mesafesi kaç mış?] Teacher Barış, 2nd lesson Line 76 - 76	Teacher Barış: How can they (the two angles) be equal? [Bunlar nasıl birbirine eşit olabilir?] Teacher Barış, 7th lesson Line 57 - 57

Table 4.42 (cont'd)

Teacher drawings helped teachers to explain a procedure through questions (T13)	Teacher Barış: Let's solve this with a different approach. Extend this a little bit. Is it ok? To me, it is ok. What is the complementary angle of 145? [Barış: Bak. Bunu da değişik bir yöntemle çözelim hadi. Şunu uzatalım birazcık. Olur mu? Olur bence. 145 in bütünleri kaç yapıyor?] Teacher Barış, 5th lesson Line 239 - 239

4.3.3 Summary of the Relations

According to the tables in this section, the relation within the tools for questioning showed that for Teacher Caner, the mostly observed relation was between T9 and T16 (n=19) and the least observed relation was between T9 and T15 (n=1). No relation was obaserved between T11-T12, T11-T13, T11-T15, T12-T 15, and T15-T16 (n=0). For Teacher Barış, the mostly observed relation was between T3 and T12 and the least observed relations were between T1-T3; T1-T9; T2-T12; T3-T4; T3-T6; T3-T7; T3-T10; T3-T15; T5-T11; T6-T8; T7-T9; T7-T10; T9-T10; T9-T15; T11-T12; and T13-T14 (n=1). No relation was observed between many tools such as T1-T2, T1-T4, T1-T5, T1-T6, or T1-T7.

The relation between question types and tools for questioning showed that all the tools were used with guiding questions by Teacher Caner. T2, T3, T7, T8, T9, T11, T12, and T13 were used with all the question types by Teacher Barış. For Teacher Barış, most of the tools for questioning were used with the three types of questions. Some tools were not used with probing and factual questions. According to this, T1 was not used with a probing question, T4 was not used with probing and factual questions, T5 was not used with factual questions, T6 was not used with probing and factual questions, T10 was not used with factual questions, and T14 was not used with probing and factual questions.

For Teacher Caner, the results of the relation between guiding question and tools for questioning showed that in questioning episodes in which guiding question and tools for questioning were used together, T9, T11, T12, and T16 were mostly used with the characteristics that are used when students or confused or stuck while T13 and T15 were used mostly with the characteristics that has a sequence of factual questions for completing a procedure or a concept to understand. Except from T13, students were not guided to think on strategies with the tools. For Teacher Barış, all the tools were mostly used with one characteristics of guiding questions that has a sequence of factual questions for completing a procedure or a concept to understand. Except from T3, T5, T11, T12, and T13, students were not guided to think on strategies with the tools.

For probing questions of Teacher Caner, results of the study showed that T9, T11, and T16 were mostly used with the questions require students to explain or elaborate their thinking. For Teacher Barış, T3, T5, T8, T9, T11, T12, and T13 were used with the same characteristics of probing questions. T13 and T15 were used with the characteristics of probing questions that require students to apply their prior knowledge to a problem or idea for Teacher Caner. For Teacher Barış, T7 was used mostly with the same characteristic. For Teacher Caner, only T12 was used by probing questions mostly that require students to make justification of their ideas while Teacher Barış used T2 and T10 mostly with the same characteristic.

For factual questions of Teacher Caner, T9 and T13 were mostly used with the characteristic of factual question that ask for fact or definition mostly. T11 and T16 were used with the characteristic that ask for an answer to an exercise mostly while T7 was used with the same characteristic in Barış's lessons. T12 and T15 were used with the characteristic that ask for the next step in a procedure mostly by Teacher Caner while T1, T2, T3, T8, T9, T11, T12, and T13 were observed with the same characteristic mostly in Teacher Barış' lessons.

CHAPTER 5

DISCUSSION & CONCLUSION

The aim of the current study was to examine the nature of middle grade mathematics teachers' tools for questioning, the teachers' question types, and the relation between the tools and question types through the lessons. In order to do this, socio cultural perspective and questioning were considered as a theoretical background of the study. In line with this, mathematics classrooms were examined in their natural contexts. The results of the study showed that middle grade mathematics teachers used variety of tools while asking guiding, probing, and factual questions. The relation within the tools showed that supplementary books had a prominent relationship with some tools for questioning. The relation between the tools for questioning and question types showed that both of the teachers used guiding questions with all kind of tools for questioning. Some tools for questioning (T1, T4, T6, T14, and T15; T4, T5, T6, T10, T14, and T15) were not used with probing questions and factual questions, respectively.

In this chapter, the findings of the study were discussed in the light of the related literature. In addition, the conclusions and recommendations of the study were mentioned with suggestions for future work.

5.1 The Tools for Questioning in Middle Grade Mathematics Classrooms

The first question in this study sought to determine the tools middle grade mathematics teachers used while asking questions. The tools for questioning in this study exemplified the way of using questions or other prompts revealed in mathematical questioning episodes that Mason (2014) described questioning in mathematics education. Even in the same content, teachers followed different ways

for integrating tools for questioning. Six tools for questioning including information technology tools, analogies, real life examples, student ideas, teacher drawings, and printed supplementary material were observed in the teachers' instructions. The teacher who used technology in his lessons utilized information technology tools, analogies, and real life examples different from the other teacher who did not use technology anymore. The information technology tools aroused from the use of the technology in the course. Therefore, technology enabled learning settings had information technology tools deriving from the mathematical technology that the teacher used. He used supplementary book compatible with smartboard and DGS. The uses of the tools for questioning showed that supplementary book helped the teacher to question a mathematical procedure, to question student performances, and to manage the sequence of questions to be asked. He used DGS to ask questions based on dynamic figures created by the teacher and to build questioning sequence in response to student's questions. Therefore, the technology use helped the teacher to make communication and to collaborative work with his students through questioning. This finding confirms that the association between the teachers' questioning and different media has a potential to change the way of teachers' questioning (Akkoç, 2013).

Teacher Caner was in a more traditional nature where there was a board, boardmarker, and some printed supplementary materials with him. Teachers' way of questioning was expected to differ deriving only from the learning opportunities that the technological tools provide the learner, considering the existence of the variety of mathematical software, online tools, or the usage of technology with the variety of purposes during the instruction. However, analogies and real life examples were emerged independently from the information technology tools which were used for scaffolding students' cognitive process as mentioned in previous research studies (e.g., Tanner et. al., 2005). From this perspective, questioning is a teacher dependent action (Mitchell, 1994). The present findings seem to be consistent with Fox (1983) who claimed that teachers' personal teaching theories could vary in terms of experience years. In this study, Teacher Caner had more than twenty years and

Teacher Barış had five years of teaching experiences. There might be teacher related factors, like experience, that are related to the teachers' own instructional decisions. In line with this, the results supported the idea that questioning is a personal action which was shaped by implicit questioning theories of teachers (Mitchell, 1994) or by personal teaching theories (Fox, 1983).

The presence of the tools through the lesson might depend on the teachers' questioning behaviors of their practice (Mitchell, 1994). According to this, the frequency of the tools for questioning might be a clue for modeling of Teacher Barış's questioning discourse on the basis of supplementary book compatible with smartboard, and teacher and student drawings as a tool for teacher questioning while Teacher Caner's questioning discourse could have elements of students, teacher drawings and printed supplementary materials. However, it is important to bear in mind the possibility of the dependency of the questioning discourse of the participants to the mathematical content itself. For example, it is very likely that the teacher drawings and student drawings as tools for questioning emerge depending on the content itself. As the mathematical content was lines and angle, the teachers or students had to use some markings or drawings, or DGS deriving from the support of the software to the content.

As Bills, Dreyfus, Mason, Tsamir, Watson, and Zaslavsky (2006) reported, examples provide mathematics communication between teacher and the students. Bills et. al. (2006) emphasized that *example of a concept* and *example of the application of a procedure* had different pedagogical aspects. Worked examples as *examples of the application of a procedure* were observed more frequently in this study. Moreover, the researchers gave details about worked-(out) examples 'in which the procedure being applied is performed by the teacher, textbook author or programmer, often with some sort of explanation or commentary, and 'exercises', where tasks are set for the learner to complete' (Bills et. al., 2006, p. 127). In this study, mostly both of the teachers used examples; namely, worked-out examples. Both of the teachers used worked-out examples which were taken from the supplementary book and printed supplementary materials most of the times in order

to practice the mathematical procedures about lines and angles. However, in this study, it was observed that because of the supplementary books, teachers expected students to solve worked-out examples before the student reaches the cognitive level to complete the exercise given to them. Therefore, the leveling was based on those books. In line with this, in this study, teachers' questioning were based on worked-out examples more. On the basis of the description, the teacher used the example for the application of procedures of intersection of two parallel lines in real life which were generated himself without depending on the other tools for questioning. The teacher used real life examples at the end of the last lesson and the example was used after almost all of the questions in the book were completed. Therefore, it seems that real life examples were helpful to question mathematical procedures rather than question concepts. During the instruction, Teacher Barış gave only one real life example in order to make a relation between 'spirit level' and the rules between two parallel lines (especially Z rule). The real life example required students to make sense of the mathematical procedures hidden between the two parallel lines. Even though the presence of the real life contexts provided students to understand the mathematics background of the contexts and to be more motivated (Boaler, 1993), teacher dependency of the books have a barrier for questioning for mathematical procedures. In other words, it might be explained that the supplementary book did not support real life examples. Therefore, the teacher did not integrate real life examples into his questioning.

In total, Teacher Barış used three analogies including body analogy, river analogy, and fish analogy. Because while students were working on the worked example, the teacher told the way of the solution of the worked example by analogies which were not easy to be understood by the students. Students asked the teacher some questions to make sense of the analogies and they limitedly made explanations using the teacher's logic of analogies. The findings supported the idea that analogies are not easily understood by the students and have a potential to create alternative conceptions for students (Harrison & Treagust, 2006). While the teacher generated and utilized analogies during the lecture and solving worked examples, there were

no student-generated analogies in this study. These results are consistent with the findings of other studies (Harrison & Treagust, 2006) in which teacher-generated analogies are more frequent than those of students. The reason for this finding might be that Teacher Barış did not invite student to generate analogies but he expected students to utilize his generated analogies while solving worked examples or visualizing mathematical concepts. As Harrison and Treagust (1994) suggested, students are involved by the teacher through questioning and discussing the analog, talking about the similarities between the analog and the target concept or procedures, and through detecting differences that have potential to create alternative conceptions. Also, analogical instructions need to be carefully planned depending on the role of the analogy in the instruction and that is possible by a systematic approach (Harrison & Treagust, 1994). Considering the requirements of effective analogical instruction for this study, the use of analogies showed that the teacher utilized to activate prior knowledge by visualizing mathematical concepts in the first lesson, and by solving worked examples for the remaining of the lessons without questioning the analogy itself.

Findings of the study showed that in both of the mathematics classrooms, number of tools were more than the number of the questioning episodes. That showed us in one questioning episode in which the classroom was discussing mathematics; the teachers applied more than one questioning tool and the tools for questioning were having interaction with each other. Considering the presence of the number of instructional tools used and the relation between these tools, teacher questioning is a content and context dependent discourse (Carlsen, 1991; Koizumi, 2013; Nisa & Khan, 2012).

In both of the classrooms, there were common tools for questioning including printed supplementary book, teacher drawings, and students that both of the teachers used in similar ways. According to this, there was a classroom routine of using those tools during the questioning for both of the teachers. While printed supplementary book and teacher drawings might depend on the instructional content, the integration of student drawings and student questions or comments into teacher questioning

represented classroom norms specific to questioning. According to these social norms, students were involved in the teacher's questioning at their own will and they were allowed to explain their own ideas whenever they want. Therefore, classroom norms encourage students to learn together (Cobb & Yackel, 1996) through questioning. That is essential because the classroom environment for learning mathematics requires students to share responsibility of it. In such classroom cultures in which there was such norms and a high interaction between teacher and students, new tools for questioning could be established. In line with this, there might be an interaction between the classroom norms for questioning and the presence of the tools for questioning.

The current study showed that tools for questioning had different uses while asking questions. Frequency of the use of the tools for questioning indicated that the teachers had different way of instructional practices. Questioning differed in general for Teacher Barış and Teacher Caner, as asking questions about mathematical procedures or concepts (%30; 46%), asking questions via worked example (%68; 74%), questioning the student ideas or questions (%30; 50%), asking questions by using supplementary books (%56; 61%), questioning real life examples (%1, 0%), and asking questions by adapting analogies to content (%16, 0%). DGS, analogies, real life example, and students provided the teachers to make questioning to create a new questioning context for the opportunity of student learning while supplementary book compatible with smartboard or printed supplementary materials provided teachers the opportunity to follow the way of questioning as a preplanned instructional practice. The created questioning episodes changed the way of teacher's questioning as well as preplanned instructional practices, which included unexpected situations as well (i.e., unknown flash icon in the textbook). Therefore, teachers should have noticing skills to integrate the unexpected situations for their questioning (Jacobs, Lamb, & Philipp, 2010) and to integrate student contributions in case of unexpected events (Rowland, Huckstep, Thwaites, 2005). By doing these, the use of tools for questioning manage teachers' questioning practice.

‘Digital competence’, was represented as one of the requirements of teacher competencies in middle grade mathematics curriculum of Turkey for teaching mathematics (MoNE, 2013). The use of information technology tools for teachers questioning is related to digital competence of teachers. For middle grade teachers, the teaching style that inludes using the information technologies to construct knowledge in a meaningful way was encouraged by The Ministry of Education (MoNE, 2013). MoNE (2013) encouraged teachers to create highly interacted math talking learning communities. This study revealed the nature of the whole class teaching in a technology supported and a non-technology supported class. In this study, interactive whiteboard created a learning environment for the use of the information technology tools including DGS and supplementary book compatible with the smartboard. Prior studies (e.g., Tanner et. al., 2005) have noted the importance of teaching with interactive whiteboard and the necessity of guidance with the support of pedagogy while using the tool. As Tanner et. al. (2005) reported, interactive whiteboards (IWB) do not provide pedagogy alone, but the interaction between a teacher and the board determines this. The current study suggested that the pedagogy of teaching with interactive whiteboard required understanding of IWB specific to teachers’ questioning behaviors and the contribution of the tools in their questioning. DGS represented pedagogical goals of questioning like making discussions on the frame of students’ questions and dynamic figures, the teacher created. Additionally, deriving from the supplementary book, which was compatible with smartboard, the teacher had different uses but one pedagogical goal as organizing questioning keeping in step with the supplementary book. The pedagogical goal emphasized that the questioning practice of the teacher as a way of management of questioning. The frequency of the observed situations showed that the supplementary book was used to organize the sequence of the questions of the teacher’s. Therefore, the information technologies were used either little in unexpectedly created situations or most in preplanned instructional contexts. In line with this, the use of the supplementary book was independent from the context. Therefore, information technology tools were integrated pedagogically by supporting

students' learning in a pre-planned context. As technology provides teachers to make questioning mathematical big ideas through interpreting and exploring of mathematical concepts, or applying mathematics in real life examples (Arbaugh et al., 2010), technology should be integrated strategically while teaching (Heid, 2005) depending on the context.

Another finding of the study indicated that independent from the teachers' use of printed supplementary books or supplementary book compatible with smart board, supplementary materials guided both of the teachers while asking questions in all their lessons. The frequency of the use of supplementary book showed that for both of the teachers, it was the mainly used questioning tool. It guided teachers' questioning. These findings supported a research study which revealed that middle grade mathematics teachers preferred to use supplementary books as they provide the teachers variety of worked examples with sufficient number of the examples (Özmantar, Dapkın, Çırak-Kurt, & İlgün, 2017).

Findings indicated that teacher drawings were one of the tools for questioning through which Teacher Barış practised mathematical procedures using worked examples. Doing this, he asked questions or encourage students to question the solution of a worked example. As the teacher took a role of representing the solution of worked examples accurately on the smartboard, he solved almost half of the worked examples on the smartboard asking questions. However, the other teacher did not utilize his drawings so much. That might be because he gave importance to his drawings on worked examples in classroom sessions when students were individually solving the worked examples. In addition to this, the way Teacher Caner followed for the solution of the worked examples was that he started with talking about the procedures or mathematical concepts with students, and following this, he gave the students the right to make their drawings to solve the worked examples. Therefore, he did not take a role of solving worked examples in a correct way; instead, he created an atmosphere that gave the students the opportunity to share their ideas and to make drawings for the solution of the worked examples. In other words, the teacher did not attribute himself a role in solving worked examples; he was a

guide for the students. The teachers' authority while solving the worked examples were not the same. Both of the teachers' use of their drawings showed that the use of the tool depended on the teachers' attribution of the responsibility of solving worked examples with questioning. When a teacher takes this responsibility, to use the teacher drawings as a tool for questioning becomes inevitable.

According to Davis (2009), there are tools for teaching. One of the tools for teaching is discussion strategies, which is related to questioning. In this study, questioning as a tool in classroom teaching was detailed. For this purpose, the tools for questioning represented ways to be integrated into teaching. One of the questions under the third research question in the current study was looking for the relations between tools for questioning. Findings of the study showed that in these ways, there were some main tools for questioning and some of them were used limitedly. According to this, T9 and T16 were in relation to each other for Teacher Caner while T3 and T12 were observed together mostly. Both of the relations highlighted that use of supplementary books has a main role in both of the classrooms. Accordingly, the relation between T9 and T16 revealed the role of student while using the supplementary book in questioning episodes, while T3 and T13 gave a clue about the use of teacher drawing in teacher questioning together with the supplementary book. In line with this, teachers' classroom behaviors represent relationship among strategies building a learning environment to encourage student learning (Chapin et. al., 2009) and to apply for questioning in this study.

5.2 The Mathematics Teachers' Question Types and Its Relation to the Tools

Findings of the study revealed that the participant teachers utilized the question types including probing, guiding, and factual questions consistent with the related literature (Camenga, 2013; Piccolo et. al., 2008; Şahin & Kulm, 2008; Ong et. al., 2010). Both teachers used all kinds of questions in each lesson except that Teacher Caner did not use factual question in the last lesson. While talking about a mathematical procedure and a concept, or study in worked examples, the teachers

used all the question types. This study confirms that question types are not associated with the parts of the lessons (Şahin & Kulm, 2008).

Contrary to the related literature (Faruji, 2011; Jiang, 2014), one of the findings is that both of the teachers did not use factual questions in most of their lessons. Teacher Barış used guiding questions quite much (77%) compared to probing (12%) and factual questions (10%) which were used in similar frequencies. The other teacher used probing question mostly (%51) followed by the guiding questions (%31) and factual questions (%17). This situation might be expectable if a teacher does not prefer to explain their lessons by associating them with facts. Also, a possible explanation for these results may be related to the teachers' different instructional strategies while questioning. For example, in this study, for one of the participant teachers, Teacher Barış, the flow of the instructions were mostly related to completing a procedure and he had an authority of posing questions based on his way of thinking. The teacher tended to speak more than his students did during the instruction as he used his thinking rather than using student thinking while asking questions. While the other teacher, Teacher Caner gave his students a voice about their thinking many times and the teacher used student thinking in the instruction in most of the times. As Koizumi (2013) stated, experienced teachers, Teacher Caner who was the experienced teacher in this study, gave more importance to students' creative thinking and questioning practice improves by practice (Ramsey at. al., 1986).

Considering the characteristics of the questions, both of the teachers represented some similarities and some differences of using various characteristics of the question types. For factual questions, Teacher Caner posed questions requiring students to provide the next step in a procedure with the least number of questions, but Teacher Barış used mostly that characteristic while asking questions. One of the reason might be that it is related to teachers' differences of the questioning practices. Teacher Caner limitedly used that characteristic because he did not feel necessary to ask students to provide the next step in a procedure. Rather than this, he probed his students to make their explanations and tried on creating such a classroom

atmosphere for questioning. It is therefore possible that he focused on specific facts or definitions to highlight the students' way of thinking and their solutions. However, Teacher Barış, followed such a way that he taught the mathematical content through solving worked examples in the supplementary book. Therefore, he might prefer to construct the student learning based on asking questions about procedures involved in the worked examples.

For guiding questions, both of the teachers represented the same tendency of using the characteristics of guiding questions, which emphasized the teachers' help about understanding a concept, or completing a procedure in most of the times. These factual questions were used by the teachers in a way that students were leaded to reach a desired point or in a way that students were more open to divergent thinking. The fact that teachers guided students in understanding of the procedures or concepts by the help of factual questions showed that the teachers supported procedural understanding about the related mathematics content while they both asked questions to recall a strategy or invented strategies very little. Therefore, in both of the classrooms, independent from the classroom norms, experience, or technology, students were little encouraged to involve in questioning of mathematical procedures or concepts. In line with this, the reflection of the characteristic in classroom environment is essential to promote students to reason (Conner et. al., 2009) and to create a focusing pattern of interaction (Wood, 1980).

In probing questions, both of the teachers asked questions requiring explaining or elaborating students' thinking in most of the times. However, in using probing questions, the teachers differed in using the characteristic of asking students to use their prior knowledge and applying the prior knowledge to a current problem or idea. The use of that characteristic was consistent with the use of a factual characteristic about using specific facts or definitions. In both of the uses, prior knowledge should be activated. Therefore, it might be hypothesized that characteristics of different question types have interaction with each other.

The other question under the third research question was to examine which tools for questioning were used with question types. Based on this, it was found that

Teacher Caner used each of his tools for questioning with all the question types, however, the other teacher was using guiding question with all the tools for questioning and T1, T4, T6, T14, and T15; T4, T5, T6, T10, T14, and T15 were not used with probing questions and factual questions, respectively. In this way, a guiding question could not be used with a specific tool. In this study, as it is understood from the names of the tools, some of which were represented in different parts of the lessons, the tools for questioning could be used any parts of the lesson with any kind of questions.

Another important finding of the study showed that some information technology tools (i.e.; T2, T3) were used with factual, guiding, and probing questions. Probing questions or factual questions were not used with the tools (i.e.; T1, T4, T5, and T6). It can therefore be assumed that the technology integration do not have a strong influence of the types of questions to be used and it provided a teacher to use all three types of questions.

5.3 Implications and Recommendations for Further Research

Previous studies classified questions in mathematics education in different ways. Types of questions were analyzed based on explicitness of student thinking (Franke et al., 2009), openness of the question statements (Ali, 2007), cognitive demand of the questions (Shahrill & Clarke, 2014; Smith & Stein, 1998; Piccolo et. al., 2008). In available literature, there were limited explanations for the descriptions of the question types with the notable exception of Şahin and Kulm's study (2008). Şahin and Kulm's study (2008) developed criteria for the types of questions including probing questions, factual questions, and guiding questions, and therefore, the framework played a key role in classifying questions in this study. Question types and the characteristics of these question types were analyzed based on that literature (Sahin & Kulm, 2008). Responding the call of Sahin and Kulm (2008), the characteristics of the question types were tested in the learning environments. In addition to this, classroom talk of teachers in a technology integrated classroom

environment as well as a non-technological one were used in order to test the usability of the criteria. There were some difficulties and facilities during the analysis and related to this, the reflections of the framework for each of the question types were shared.

The results of the study showed that both of the teachers used factual questions while they were asking a mathematical fact of definition, for asking answer to an exercise and for asking the next step of a procedure. In addition to this, in this study, factual questions were also used in a higher level of problem solving process, for example, while a written question was a problem of students rather than an exercise or drill for the students. Depending on the flow of the course and the timing of the given information, the classification of problem, exercise, or worked example and where the problem, exercise or worked examples start and end can be interpreted differently. Therefore, in this study, the word *exercise* was considered in a broader sense in which it requires of an answer related to students' existing knowledge and of the general call of a teacher about an answer for a worked example, problem, or exercise.

Another findings of the study showed that both of the teachers used guiding questions when they required students to use strategy in the way that teachers applied in solving worked examples. In addition to this, teachers involved students' confusions or stuck with guiding questions. For example, Teacher Barış helped the student in a step-by-step process when the students had confusions about creating equal angles. Although teachers tried to help them when the students had difficulties, sometimes Teacher Barış was ready to help the students without giving students the opportunity to solve problems with guiding questions. In these situations, the teacher did not give wait time for students to solve problems and required the students to observe the solution of the teachers. In line with this, guiding questions were not only used when students confused or stuck but also they were used while students had potential to make confusions or stuck. Especially for that characteristics of guiding questions teachers sometimes tended to answer their own questions (Ramsey et. al., 1990) and that also decrease the quality level of the responses (Dean, 1986).

Findings of the study showed that both of the teachers used series of factual questions orchestrating to serve understanding a concept or completing a procedure, that was not easy to separate *scaffolding* and *leading* way toward understanding a concept or completing a procedure. In line with this, the related characteristic of guiding question in two separate ways in which one of them requires teachers to make scaffolding which give student opportunity to make discussions or questioning, while the other one is leading students' way of thinking which give students information in a funneling manner could be divided in two. In this way, the separate characteristics would enable that teachers provide students leading to accept the teachers' way of thinking with funneling questioning and that focusing questioning provides scaffolding with in depth understanding of student thinking (Herbel-Eisenmann, 2005; Wood, 1998).

In the current study, one of the characteristics of probing questions was related to students' requirement of making explanations and elaborations of their thinking. In contrast to combining both of explanation and elaboration as a way of using probing questions, the classroom dialogues of both of the teachers showed that the requirement of student explanation could be related to clarification questions for understanding what students say, in other words, a repetition of what students' talk, while elaboration of student thinking is related to making sense, critique, or reflect on what the student said. For example, students could be required to make explanation based on their previously stated idea and that explanation might be related to make clarification of what the student stated. However, making elaboration was requiring students to reflect on what students say further, rather than waiting for explanation for clarification. As those two uses were different, separation of the uses could clarify the teachers' talk moves better, especially for analysis of interactional patterns of Wood (1998).

The other findings of the study showed that teachers probed students' prior knowledge and they required students to use the knowledge in learning new ideas. The instructional context that kind of the probing questions were involved in was similar to the involvement of factual questions into the same instructional context in

which the teachers were asking students questions about the next step of a mathematical procedure. When questioning episodes include a problem (Polya, 1943) for students, and students need help to use their prior knowledge or current knowledge to solve or complete a procedure related to the problem, in contrast to the separate characteristics of probing and factual questions, both of the characteristics seemed to serve to the same question statement. In this case, the interpretation of the instructional context in which students' prior knowledge helps to interpret the knowledge they will learn, not the prior knowledge the students needed to complete a procedure they had learned before, was the solution for separating the characteristics to each other. In other words, while students had necessity of using prior knowledge to provide the next step of a procedure, that was evaluated a part of a procedure not part of a prior knowledge of students.

These question types and characteristics for each question type were illustrated with classroom dialogues in the mathematics classrooms. This work contributes to existing knowledge about question types in Turkish contexts by providing examples for each of the question types and the characteristics of them together with tools for questioning. This study was also realistic in terms of showing what was happening in practice in terms of questioning.

One of the issues that emerged from the findings of the study is that in a technology enhanced learning environment, the teacher used the probing, guiding, and factual questions with its specific characteristics. These findings were corroborated by one of the participant teacher’s classroom dialogues that the use of technology does not directly cause that teachers can use characteristics of questions, which guide students to make explorations by DGS more. The participant, who used technology in his class, depended on to supplementary books mostly and used a sequence of factual questions that provides ideas lead students toward understanding a concept or completing a procedure very frequently. The narration of the supplementary book and worked examples in that book may have guided the teacher to ask verbal questions in a way that is more factual. Therefore, this procedural way of instruction was likely to be related to saving time than the pedagogical integration

of technology. Considering that technology integration requires teachers to have knowledge about technology, pedagogy, and mathematical content, and the integration of those three (Koehler & Mishra, 2005), there could be different levels of integration in classrooms in which supplementary books has a leading role in providing teachers using the characteristics of questions in a more productive way of thinking for students. Therefore, this study concluded that for the mathematics lessons where the technology was used, examples of tasks in supplementary books which provide guidance to teachers about integrating pedagogy and mathematical knowledge with sample verbal questions that make the use of technology in the way that mathematical aspects could be discussed through technological tool are necessary.

One of the question type Mason (2010) suggested was that open and close questions. They were handled as if the open and close questions have two dimensions for each of the category: open-ended and open fronted for open questions and closed ended and close-fronted for close questions. The author classified the open and close end questions together with the person who asked the question, and the person who answered it. Accordingly, the question can be open-ended because it contains multiple answers, and it is close-fronted as there is an expected answer by the questioner. It may be open-ended because it may have multiple answers, but it may also be open-fronted when there is no definite answer expected by the person who asked the question. All of the types of the questions need to be used as long as they are used in useful form for students. He suggested that in order to use the questions effectively, teachers should draw attention to their own questioning. It is also very important for teachers to reduce their questions that make students feel authority, to enable students to produce answers, to direct students testing their arguments, and to teach their students self-questioning. Turkish mathematics curriculum encouraged open-ended questions for using during the instruction (MoNE, 2013). However, in the present study, characteristics of questions in instructional contexts became prominent rather than requiring an open or closed-ended question. For example, while factual questions were expected to be closed-ended, it was observed that they

could play as open-ended question within a context as similar to what Mason (2010) emphasized. Supported to this, as Koizumi (2013) suggested factual questions which have certain answers are essential especially while introducing a content, therefore, the curriculum might have a more flexible point of view supporting question types that are used in an organizational way.

According to William (1999), teachers' questions are essential so that students' conceptions could be revealed by rich questions. Good questions are a way of eliciting student thinking and overcoming student misconceptions (William, 1999). As suggested in the literature, professional development is required for teachers for improving questioning behaviors in practice (Ong et. al., 2010; Craig & Cairo, 2005; Walsh & Sattes, 2012; Widjaja et. al., 2010). These sample of dialogues, were a resource that can be utilized in the training of mathematics teachers and in gaining awareness of mathematics teacher candidates about their questioning behaviors. Question types exemplified by this study could be integrated to a professional development pack that can be prepared to make teachers aware of their instructional moves for questioning. The professional development for questioning could be used in-service training organized by Ministry of education (MoNE) or by a private institution.

The evidence from this study suggested that there were teacher-dependent tools for questioning, including analogies and real life examples. There might be other tools in other lessons or in mathematics lessons of different contents as well. Question types of both of the teachers had different uses through instructions. Therefore, questioning is a way of practice specific to teachers and it is difficult to identify a general or appropriate general pathway for teachers about questioning. Teachers may act according to their individual theories when asking questions (Mitchell, 1994), for example, they can be differed in tools for questioning and types of questions of teachers. Considering that teacher candidates' implicit questioning theories are in the stage of maturation with the courses about teaching methods of mathematics, the tools for questioning and their use in this study could guide mathematics educators in terms of how teacher candidates could act in these two

environments. Considering the lack of teaching experience of prospective teachers, the real classroom dialogues can be used for educating prospective teachers for improving noticing of their questioning behaviors in the lesson planning stage. Accordingly, pre-service teachers' microteaching or practice teaching could guide mathematics educators reflecting on their real classroom dialogues and give opportunity to evaluate their questioning in practice. As the tools for questioning were described representing pedagogical purposes of integrating technologies in the current study, this study will raise awareness of how mathematics educators want a picture of their teacher candidates' questioning behaviors.

This study focused on lines and angles. The subject of angles contained definitions that are open to questioning in different perspectives since the historical processes (Keizer, 2004). It has been in the middle grade Turkish mathematics curriculum for a long time at the level of middle school and there has been a need for changes in the educational objectives from time to time (Uysal & Inckabı, 2017). This study was important in terms of revealing the way that middle grade teachers use the questions in this specific topic and the types of the questions while applying their instructions. Therefore, the findings of the study might be explained within the limitation of the content. Considering that prospective teachers need getting familiarity of real classroom experiences, tools for questioning and corresponding real classroom dialogues could serve a real classroom environment for their training in method courses for teaching mathematics or informing them about instructional principles and methods. Although this study focused only on lines and angles topic, that give insight to about common and uncommon potential teacher behaviors of different mathematical topics considering the nature of the topic of the study. The framework of the study (Şahin & Kulm, 2008) was applicable to analyze classroom dialogues and provide practice-based evidences from these two cases, however, depending on the cases; some modifications, which were mentioned under the previous title (5.3.1), were suggested for better classification of the question types in terms of characteristics. Considering that prospective teachers' practice teaching experiences in hypothetical teaching environments or real classroom environments

as a requirement of practice teaching course, the same framework could be tested or use as a guide for analyzing prospective teachers' question types in their practice teaching.

This study clarified what tools for questioning in-service teachers have for questioning during the mathematics instructions. The tools for questioning in varied classroom settings would open the ways for improving teachers' questioning in practice. Considering the tools for questioning as an initial step, this process can be repeated with getting larger number of teachers and increasing the number of mathematical topics, and to understand the questioning interaction as detailed as possible for training prospective teachers accordingly.

In order to observe that technology changes teachers' way of questioning, teachers need to be aware of information technology tools in terms of how they integrate questioning into their practice and they might be guided about technological pedagogical content knowledge which is specific to content to be taught for strategical use of technology. In line with this, in-service teachers could be encouraged to receive training in this direction by carrying out their awareness by the tools revealed in the current study about information technology tools in mathematics education. In this perspective, considering the tools for questioning specific to technology can guide researchers to understand and obtain middle grade teachers' attitudes towards the use of technology in mathematics lessons while improving a scale as similar to Technology Use in Mathematics Lessons Attitude (TMLA) Scale (Aytekin & Işıksal-Bostan, 2018).

This study revealed the questioning behaviors of teachers including tools for questioning in relation to question types middle grade teachers used in their teaching. According to this, for instance, teachers used probing questions having the characteristics of justification or proof used in little times. Another example is that Teacher Caner did not use real life examples or analogies anymore for questioning mathematical topics even though real life provides students to make connections between mathematics and the real life (Sawatzki & Sullivan, 2017). In order to understand teachers' questioning behaviors related to more deeply, it is necessary to

find out the reasons of why teachers follow such a way in terms of questioning. With doing this, why the characteristics of questions types were used or their beliefs about questioning would be revealed. The results of that kind of study support the roadmap of teacher training about questioning.

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APPENDICES

APPENDIX A: HUMAN SUBJECTS ETHICS COMMITTEE

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
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03 KASIM 2015

Gönderilen: Prof.Dr. Erdinç ÇAKIROĞLU
İlköğretim Bölümü

Gönderen: Prof. Dr. Canan SÜMER
İnsan Araştırmaları Komisyonu Başkanı

İlgi: Etik Onayı

Danışmanlığını yapmış olduğunuz İlköğretim Bölümü Doktora öğrenciniz Ayşenur KUBAR'ın "Ortaokul Matematik Öğretmenlerinin Soru-Cevap Davranışlarının İncelenmesi " isimli araştırması İnsan Araştırmaları Komisyonu tarafından uygun görülerek gerekli onay 19.11.2015 -17.06.2016 tarihleri arasında geçerli olmak üzere verilmiştir.

Bilgilerinize saygılarımla sunarım.

Prof. Dr. Canan SÜMER
Uygulamalı Etik Araştırma Merkezi
İnsan Araştırmaları Komisyonu Başkanı

Prof. Dr. Meliha ALTUNIŞIK
Etik Komitesi Üyesi

Prof. Dr. Aydan BALAMİR
Etik Komitesi Üyesi

Prof. Dr. Mehmet UTKU
Etik Komitesi Üyesi

Prof. Dr. Ayhan SOL
Etik Komitesi Üyesi

ODTÜ 2015

BU BÖLÜM, İLGİLİ BÖLÜMLERİ TEMSİL EDEN İNSAN ARAŞTIRMALARI ETİK ALT KURULU TARAFINDAN DOLDURULACAKTIR.

Protokol No: 2015-EGT-137

İAEK DEĞERLENDİRME SONUCU

Sayın Hakem,

Aşağıda yer alan üç seçenekten birini işaretleyerek değerlendirmenizi tamamlayınız. Lütfen "Revizyon Gereklidir" ve "Ret" değerlendirmeleri için gerekli açıklamaları yapınız.

Değerlendirme Tarihi: 29.10.2015

Ad Soyad:

☒ **Herhangi bir değişikliğe gerek yoktur. Veri toplama/uygulama başlatılabilir.**

☐ **Revizyon gereklidir**

- ☐ Gönüllü Katılım Formu yoktur.
- ☐ Gönüllü Katılım Formu eksiktir.

  Gerekçenizi ayrıntılı olarak açıklayınız:
- ☐ Katılım Sonrası Bilgilendirme Formu yoktur.
- ☐ Katılım Sonrası Bilgilendirme Formu eksiktir.

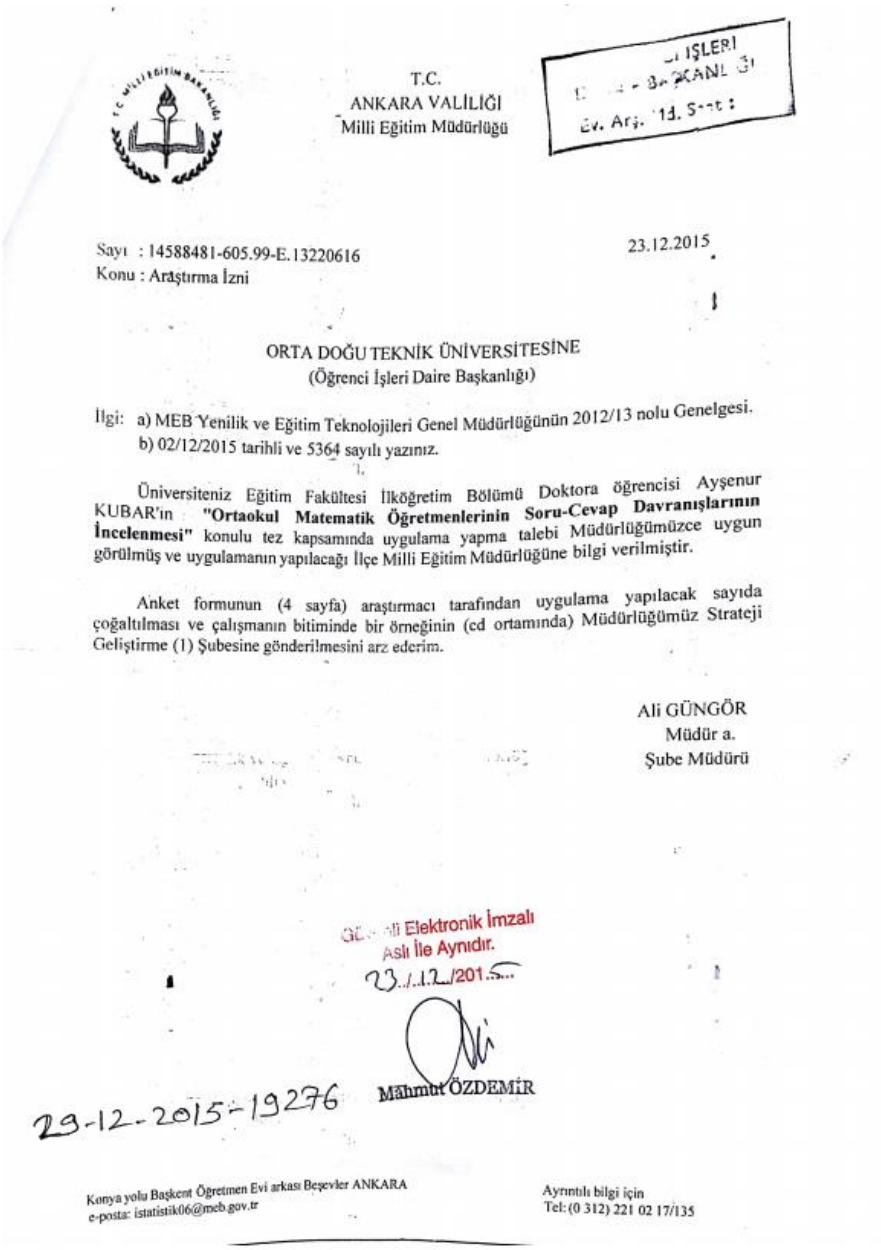
  Gerekçenizi ayrıntılı olarak açıklayınız:
- ☐ Rahatsızlık kaynağı olabilecek sorular/maddeler ya da prosedürler içerilmektedir.

  Gerekçenizi ayrıntılı olarak açıklayınız:
- ☐ Diğer.

  Gerekçenizi ayrıntılı olarak açıklayınız:

☐ **Ret**

APPENDIX B: PERMISSION FROM MoNE



T.C.
ANKARA VALİLİĞİ
Milli Eğitim Müdürlüğü

...I İŞLERİ
...BAŞKANLIĞI
Ev. Arş. ...lı. Sınıf :

Sayı : 14588481-605.99-E.13220616
Konu : Araştırma İzni

23.12.2015

ORTA DOĞU TEKNİK ÜNİVERSİTESİNE
(Öğrenci İşleri Daire Başkanlığı)

İlgi: a) MEB Yenilik ve Eğitim Teknolojileri Genel Müdürlüğünün 2012/13 nolu Genelgesi.
b) 02/12/2015 tarihli ve 5364 sayılı yazınız.

Üniversiteniz Eğitim Fakültesi İlköğretim Bölümü Doktora öğrencisi Ayşenur KUBAR'ın **"Ortaokul Matematik Öğretmenlerinin Soru-Cevap Davranışlarının İncelenmesi"** konulu tez kapsamında uygulama yapma talebi Müdürlüğümüzce uygun görülmüş ve uygulamanın yapılacağı İlçe Milli Eğitim Müdürlüğüne bilgi verilmiştir.

Anket formunun (4 sayfa) araştırmacı tarafından uygulama yapılacak sayıda çoğaltılması ve çalışmanın bitiminde bir örneğinin (cd ortamında) Müdürlüğümüz Strateji Geliştirme (1) Şubesine gönderilmesini arz ederim.

Ali GÜNGÖR
Müdür a.
Şube Müdürü

...li Elektronik İmzalı
Aslı İle Aynıdır.
23../12./201.5...

Mahmut ÖZDEMİR

29-12-2015-19276

Konya yolu Başkent Öğretmen Evi arkası Beşevler ANKARA
e-posta: istatistik06@meb.gov.tr

Ayrıntılı bilgi için
Tel: (0 312) 221 02 17/135

T.C.
ÇANKAYA KAYMAKAMLIĞI
İlçe Milli Eğitim Müdürlüğü

Sayı :78520003/605.99/13256785 24.12.2015
Konu :Araştırma İzni -
Ayşenur KUBAR

İLGİLİ OKUL MÜDÜRLÜKLERİNE

İlgi :a) MEB Yenilik ve Eğitim Teknolojileri Genel Müdürlüğünün 2012/13 nolu Genelgesi.
b) ODTÜ'nün 02/12/2015 tarihli ve 5364 sayılı yazısı.
c) İl Milli Eğitim Müdürlüğü'nün 23/12/2015 tarihli ve 14588481/605.99/E. 13220607 sayılı yazısı.

ODTÜ Eğitim İlköğretim Bölümü Doktora öğrencisi Ayşenur KUBAR'ın "**Ortaokul Matematik Öğretmenlerinin Soru-Cevap Davranışlarının İncelenmesi**" konulu tezi kapsamında okulunuzda anket uygulaması İl Milli Eğitim Müdürlüğünün ilgi (c) yazıyla ile uygun görülmüştür.

Anket formunun(4 sayfa) uygulama yapılacak sayıda araştırmacı tarafından çoğaltılarak, araştırmanın ilgi (a) Genelge çerçevesinde, okul müdürlüğünün sorumluluğunda okul ve kurum yöneticileri uygun gördüğü takdirde gönüllülük esasına göre uygulanmasını rica ederim.

Hasan Hüseyin ÖZİPEK
Müdür a.
Şube Müdürü

Ekler :
1-Anket formu (4 sayfa)
2-Okul Listesi (1 sayfa)
3-İlgi yazı (1 sayfa)

APPENDIX C: CONSENT FORM

Orta Doğu Teknik Üniversitesi İnsan Araştırmaları Etik Kurulu Gönüllü Katılım (Bilgilendirilmiş Onay) Formu

Sevgili katılımcı,

Orta Doğu Teknik Üniversitesi İlköğretim Bölümü Matematik Eğitimi alanında "Ortaokul Matematik Öğretmenlerinin Soru-Cevap Davranışlarının İncelenmesi" isimli doktora tezim için bir çalışmayı yürütmekteyim. Araştırmamızın amacı, öğretmenlerin öğretim öncesinde, sırasında ve sonrasında kendi soru-cevap sürecini izleyerek, kendi süreçlerine dair bireysel teorilerini ortaya çıkarmaktır. Bu amaçla "ortaokul matematik öğretmenlerinin soru-cevap süreçlerine ilişkin bireysel teorileri nelerdir?" araştırma sorusuna cevap aranmaktadır. Bu araştırma sorusunun aydınlatılması için okul ders saati içinde öğretmen odaklı ve sınıfın tümünü içerecek şekilde video çekimi yapılacaktır. Bu çalışmada, 2 ortaokul matematik öğretmeni yer almaktadır. Öğretmenlerle uygun oldukları vakitlerde görüşmeler yapılarak ses kaydı alınacaktır. Hazırlanan görüşme soruları Ek 1, Ek 2, ve Ek 3 sunulmuştur. Pilot çalışma kapsamında bu sorular üzerinde değişikiğe gidilebilir.

Çalışmadan elde edilen video kayıtları ve ses kayıtları, yazıya dökülerek analizi yapılacaktır. Hiçbir şekilde herhangi bir yerde yayınlanması söz konusu değildir. Çalışma içerisinde öğretmen isimleri takma isimler kullanılarak payalşılacaktır. Okul ismi çalışma içerisinde kullanılmayacaktır, onun yerine çalışma içerisinde 'devlet okulu', 'özel okul' tabirleri kullanılacaktır.

Bu çalışmaya katıldığınız için şimdiden teşekkür ederiz. Araştırmamıza yönelik sorularınız olması durumunda ya da çalışma hakkında daha fazla bilgi almak için İlköğretim Bölümü araştırma görevlisi Ayşenur Kubar (Tel: 210 7505; E-posta: akubar@metu.edu.tr) ile iletişim kurabilirsiniz.

Bu çalışmadaki görüntü kaydı ve ses kaydı örnekleri hiçbir şekilde herhangi bir yerde yayınlanmayacağını teyit ederim. Aksi durumda gönüllü katılımcı hukuki yollara başvurabilir.

Ad-Soyad: Ayşenur Kubar **İmza:**

Tarih:

Bu çalışmaya gönüllü olarak katılmayı kabul ediyorsanız, lütfen aşağıda belirtilen yere isminizi ve tarihi yazarak imzalayınız.

Katılımınız için teşekkür ederim.

Ad-Soyad:

İmza:

Tarih:

APPENDIX D: PARENT APPROVAL FORM

1956

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY
06531 ANKARA-TURKEY

Department of Elementary Education

Tel: 90 (312) 210 40 54
Faks:90 (312) 210 79 84

Veli Onay Mektubu

Sayın Veliler,

Orta Doğu Teknik Üniversitesi İlköğretim Bölümü Matematik Eğitimi alanında "Ortaokul Matematik Öğretmenlerinin Soru-Cevap Davranışlarının İncelenmesi" isimli doktora tezim için bir çalışma yürütmekteyim. Araştırmamızın amacı, öğretmenlerin öğretim öncesinde, sırasında ve sonrasında kendi soru-cevap sürecini izleyerek, süreçlerine dair bireysel teorilerini ortaya çıkarmaktır. Bu amaçla, "ortaokul matematik öğretmenlerinin soru-cevap süreçlerine ilişkin bireysel teorileri nelerdir?" araştırma sorusuna cevap aranmaktadır.

Bu araştırma sorusunun aydınlatılması için öğretmen davranışlarına odaklanılacak şekilde sınıf videosu çekilmesine ihtiyaç vardır. Okul ders saati içinde öğretmen odaklı ve sınıfın tümünü içerecek şekilde video çekimi yapılacaktır. Çocuğunuzun sınıfın olağan ortamında video çekimine katılmasının onun psikolojik gelişimine olumsuz etkisi olmayacağından emin olabilirsiniz. Bu formu imzaladıktan sonra çocuğunuz araştırmaya katılmaktan ayrılma hakkına sahiptir. Araştırma sonuçlarının özeti tarafımızdan okula ulaştırılacaktır.

Araştırmayla ilgili sorularınızı aşağıdaki e-posta adresini veya telefon numarasını kullanarak bize yöneltebilirsiniz.

Saygılarımızla,

Araş. Gör. Ayşenur KUBAR

Orta Doğu Teknik Üniversitesi, Ankara
Tel: (0312) 210 7505
e-posta: akubar@metu.edu.tr

Lütfen bu araştırmaya çocuğunuzun katılım durumunu aşağıdaki seçeneklerden size en uygun gelenin altına imzanızı atarak belirtiniz ve bu formu çocuğunuzla okula geri gönderiniz.

A) Bu araştırmaya tamamen gönüllü olarak çocuğum .....'nın okul saati içinde yapılacak video çekiminde yer almasına izin veriyorum.

Velinin Adı-Soyadı: .....

İmza: .....

B) Bu çalışmada çocuğum .....'nın okul saati içinde yapılacak video çekiminde herhangi bir şekilde görünmesine izin vermiyorum, çalışma için video üzerinde karartılma yapılması halinde izin veriyorum.

Velinin Adı-Soyadı: .....

İmza: .....

APPENDIX E: CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: YILMAZ, Ayşenur
Nationality: Turkish
Date and Place of Birth: 9 March 1987
Marital Status: Married
Phone: +90 312 210 7506
0545 268 3187
email: akubar@metu.edu.tr
aysenurkubar@gmail.com

EDUCATION

2012 Spring - 2018 Fall	Middle East Technical University Ph.D Student, Dept. of Elementary Mathematics Education (Passed Ph.D Qualification Examination in November 2013)
2014-2015 Fall	Erasmus Student, Faculty of Psychology and Educational Sciences, Ghent University, Belgium.
2009 Fall – 2012 Fall	Middle East Technical University Department of Elementary Mathematics Education
2005 Fall- 2009 Spring	M. S., Dept. of Elementary Mathematics Education Başkent University

WORKING EXPERIENCE

2012 January – 2019 January Research Assistant
Middle East Technical University
Faculty of Education
Department of Elementary Education
(In division of Mathematics Education)

2011 October- 2012 December Research Assistant
Kahramanmaraş Sütçü İmam Üniversitesi
Faculty of Education
Department of Elementary Mathematics Education

2010 September- 2011 September Research Assistant
Başkent University
Faculty of Education

INTERNATIONAL PRESENTATIONS

Yilmaz, A. & Cakiroglu, E. (2018). Towards A More Contextual Approach For Analyzing Teachers' Mathematical Questioning. ERPA International Congresses on Education 28 June-1 July 2018, İstanbul, Turkey. (p. 211).

Yilmaz, A. & Isiksal-Bostan, M. (2018). Middle Grade Students' Relational Thinking About Ordering Two Negative Integers Within The Context Of Money. International Conference on Research in Education and Science (ICRES) April 28 - May 1, 2018, Marmaris, Turkey. (p. 294).

Yilmaz, A. & Cakiroglu, E. (2018). Middle School Mathematics Teachers' Uses of Tools for questioning. International Conference on Education in Mathematics, Science & Technology (ICEMST) April 28 - May 1, 2018, Marmaris, Turkey. (p.164)

Dilberoğlu, M., **Yilmaz, A.,** Sevinç, S. & Akyuz, D. (2018). Pre-Service Teachers' Goals for Integrating Technology Into Teaching Middle School Mathematics. International Conference on Education in Mathematics, Science & Technology (ICEMST) April 28 - May 1, 2018, Marmaris, Turkey. (p. 163).

Coskun-Tuncay, T., Alan, H. A.,Yılmaz-Tüzün, Ö., Karacı, G., Namlı, Ş., **Yilmaz, A.** (2017). Who Decide What To Buy in Toystores? 5. Uluslararası Okul Öncesi Eğitim Kongresi, 347-348. Ankara, TURKEY

Yilmaz, A. & Cakiroglu, E. (2017, May). A mathematics teacher's use of questions while teaching the concepts of lines and angles. Educational Researches and Publications Associations (ERPA), International Science and Mathematics Education Congress, Budapest, HUNGARY.

Yilmaz, A. & Isiksal-Bostan, M. (2017, February). How middle grade students explain ordering statements within real life situation? An example of temperature context. Tenth Congress of the European Society for Research in Mathematics Education (CERME 10), Dublin, IRLAND.

Yilmaz, Aysenur & Cakiroglu, E. (2016, August). Questioning behaviours of teachers in middle grade classrooms. 8th Yerme Summer School, Faculty of Education, Charles University in Prague, Podebrady, CZECH REPUBLIC.

Kubar, A. & Cakiroglu, E. (2014, May). Prospective teachers' ideas about where children are confused and why: the case of describing integers, International Conference on Education in Mathematics, Science And Technology (ICEMST), Konya, TURKEY.

Kubar, A. & Cakiroglu, E. (2014, September). Pre-service Elementary Mathematics Teachers' Definitions of Integers and Their Interpretations of Quoted Definitions of Integers. The European Conference on Educational Research (ECER), Porto, PORTUGAL.

NATIONAL PRESENTATIONS

Kubar, A. & Akyüz, D. (2014, September). Ortaokul 7. Sınıf Öğrencilerinin Sayı Doğrusu Modellerinin İncelenmesi. XI. Ulusal Fen Bilimler ve Matematik Eğitim Kongresi (UFBMEK), Adana, TURKEY.

Kubar, A. & Işıksal-Bostan, M. (2016, September). Ortaokul Öğrencilerinin Bağlam İçerisinde Verilen Negatif Sayıların Sıralanmasına İlişkin Durumları Anlamlandırırken Kullandıkları Stratejiler. XII. Ulusal Fen Bilimler ve Matematik Eğitim Kongresi (UFBMEK), Trabzon, TURKEY.

TRANSLATION OF BOOK CHAPTERS

Küçük Çocuklar İçin Matematiği Anlama. Bölüm 11. Problem Çözme ve Akıl Yürütmeyi Anlama. Yazarlar: Derek Haylock ve Anne D.Cockburn - Çeviri: **Kubar, A.** Ed. Zuhal Yılmaz. Nobel AkademikYayıncılık, 2014, 285 – 313.

Lise Matematik Öğretimi. Giriş. Yazarlar: David Rock, Douglas K. Brumbau – Çeviri: **Yılmaz, A.** Ed. Zuhal Yılmaz, Savaş Baştürk, Hülya Kılıç. Nobel AkademikYayıncılık, 2017, 3 – 20.

ARTICLES

Kubar, A. & Cakiroglu, E. (2017). Prospective teachers' knowledge on middle school students' possible descriptions of integers. *International Journal of Education in Mathematics, Science and Technology (IJEMST), 5*(4), 279-294. DOI:10.18404/ijemst.75211.

Kostur, M. & **Yilmaz, A.** (2017). Technology Support For Learning Exponential and Logarithmic Functions. *Ihlara Eğitim Araştırmaları Dergisi, 2*(2), 50-68.

Yilmaz, A., Akyuz, D., & Stephan, M. (2019). Middle Grade Students' Evoked Concept Images of Number Line Models and Their Calculation Strategies with Integers on These Models. *International Journal of Education in Mathematics, Science and Technology (IJEMST), 7*(1), 93-115.

Ms. Thesis

Kubar, A. (2012). Pre-service elementary mathematics teachers' knowledge about definitions of integers and their knowledge about elementary students' possible misconceptions and errors in describing integers. Unpublished Master's Thesis, Middle East Technical University, Ankara.

WORKSHOPS AND SUMMER SCHOOLS

Playing with Mathematics. Marmaris, International Conference on Education in Mathematics, Science & Technology, 29.4.2018 (Uluslararası)

Qualitative Data Analysis with MAXQDA, Ortadoğu Teknik Üniversitesi Eğitim Fakültesi, Kurs, 31.10.2016 -02.01.2017 (Ulusal)

Sosyal ve Beşeri Bilimlerde TUBİTAK Araştırma Projesi Yazma Eğitimi, Hasan Kalyoncu Üniversitesi, Çalıştay, 01.06.2015 -03.06.2015 (Ulusal)

8th Yerme Summer School (YESS-8)., Poděbrady, Yazokulu, 13.08.2016 - 20.08.2016 (Uluslararası)

H16 - Science and Mathematics Education course, Freudenthal Institute Science and Mathematics Education Department, Yazokulu, 18.08.2014 -29.08.2014 (Uluslararası)

AWARDS

Başkent University Student Achievement Award (Second Prize, 2009)

2210 TUBITAK MSc Scholarship (2009 – 2011)

2211 TUBITAK Phd Scholarship, (2013 – 2017)

TEACHING ASSISTANTSHIPS

ELE 221 Instructional Principles And Methods

ELE 301 Research Methods

ELE 341 Methods Of Teaching Mathematics I

ELE 342 Methods Of Teaching Mathematics II

ELE 419 School Experience I

ELE 420 School Experience II

ELE 430 Exploring Geometry With Dynamic Geometry Applications

ELE 465 Nature Of Mathematical Knowledge For Teaching

APPENDIX F: TURKISH SUMMARY / TÜRKÇE ÖZET

1. Giriş

Soru sormanın Platon ve Sokrates'ten bu yana uzun bir geçmişi vardır (Ellis, 1993). İnsanlar birbirleriyle anlaşırken veya bir konu hakkında ne düşündüklerini konuşarak, sorular sorarak ve cevap vererek öğrenirler (Christenbury ve Kelly, 1983). Soru sorma, öğretmenler için bir öğretim yöntemi veya biçimlendirici bir değerlendirme tekniği olarak kullanılabilen önemli bir araçtır (Jiang, 2014). Öğrencilere ne duymak istediklerini söyletmek veya mevcut istenmeyen davranışlarını değiştirmek için de sorular kullanabilir (Mason, 2014). Öğretmenler soru sorduklarında öğrencilerden sadece öğretmenlerinin veya akranlarının sorularını cevaplamaları beklenmez; ayrıca kendi kendilerine soru sormaları beklenir (Camenga, 2013; Mason, 2014).

Soru sorma, öğrencilerin zihin karmaşıklığını netleştirmeye teşvik eden hem sorular hem de ifadelerle ilgilidir ve öğrencilerin matematiksel ilerlemelerine dikkat çekmenin bir yoludur (Mason, 2014). Alan yazınında sorular formüle edildiğinde ve uygun şekilde yönlendirildiğinde öğrencilerin başarısında olumlu değişiklikler yapabileceğine dair ortak bir nokta vardır (Redfield ve Rousseau, 1981). Bu nedenle, öğretmenlerin soru sorması öğrenci başarısının temel bir bileşenidir (Redfield ve Rousseau, 1981; Franke ve diğerleri, 2009). Öğretmenin soru sormayı kullanma yeteneğine bağlı olarak, öğrencinin matematiksel düşüncesinin gelişimi değişebilir (Burns, 1985). Öğretmenler, öğrencilerin fikirlerini soru sorma yoluyla yeniden düzenlemelerine yardımcı olur (Martino ve Maher, 1994). Böylece, öğrenciler fikirlerini yeniden değerlendirebilir ve özgün çözümlerini gözden geçirebilirler (Martino ve Maher, 1994).

Aizikovitsh-Udi ve Star (2011), soru sorma sürecinde öğretmenlerin iyi soruları bir araç olarak kullanalabileceğini, ancak iyi soruların iyi soru sorma uygulamalarını garanti etmediğini belirtmiştir. Bu nedenle, soru sorma, soru sormanın uygulandığı bir süreçtir. Alan yazını, 1970'den beri öğrencilerin

anlamalarına yönelik daha faydalı soru sorma davranışları elde etmek konusunda çaba sarf etmiştir (Wilen ve Clegg, 1986; Ellis, 1993). Soru sormayı verimli bir şekilde kullanmak için, Wilen ve Clegg (1986) öğretmenlerin açık uçlu sorular kullanmaları gerektiğini ve öğretmenlerin öğrencilerini açıklama yapmaları için teşvik etmeleri gerektiğini önermiştir. Ayrıca, öğretmenler yanlış öğrenci cevaplarını öğrenme fırsatlarına çevirebilirler. Bunu yaparken, öğrencilere, öğrencilerin üretken düşüncelerini arttıran (Chin, 2006) yeterli zaman vermelidirler. Ayrıca, öğretmenlerin, ne söylediklerini düşünme fırsatı bulması için öğrencilerin cevaplarını aldıktan sonra da bir süre beklemeleri gerekir. Bu bağlamda, öğretmenlerin soru sormayı etkili bir şekilde kullanmaları gerekir ki bu da soru sorarken hızlı karar vermelerini gerektirir (Zee ve Minstrel, 1997). Sonuç olarak, öğretmenlerin soru sorma sürecini yönetmeleri kolay değildir.

Türkiye'deki ortaokul matematik müfredatı öğretmenlerin öğrencileriyle iletişim kurarak matematik bilgilerini yapılandırdığı bir sınıf ortamını desteklemektedir (MEB, 2013). MEB (2013), öğretmenlerden öğrencilerin birbirleriyle iletişim kurmalarını sağlayan matematiksel konularını anlamlandırabilecekleri iletişim açısından zengin bir sınıf ortamı yaratmalarını beklemektedir. Bu şekilde bakıldığında, ortaokul matematik öğretmenlerinin böyle bir etkileşim ortamını yaratmasıyla ilgili olarak sınıflardaki soru sorma davranışları hakkında güvenilir ve derinlemesine bilgi sahibi olmaya ihtiyaç vardır (Ryans, 1973). Ancak, uluslararası çalışmalarla karşılaştırıldığında (Ong ve diğerleri, 2010; Dillon, 1988; Heritage & Heritage, 2013; Wimer, Ridenour, Thomas ve Place, 2001; Aizikovitsh-Udi, Clarke ve Yıldız, 2013) Türkiye'deki matematik derslerinde öğretmenlerin soru sorma kullanımlarını tanımlayan sınırlı sayıda çalışma bulunmaktadır.

Öğretmenlerin sınıflarda mevcut olan veya öğretmen tarafından entegre edilen eğitim araçlarını kullanmaları, öğrenci düşünmesinin öğretim bağlamında değerlendirilmesini teşvik eder (Gall, Dunning, Banks ve Galassi, 1972). Matematik derslerinde manipülatiflerin kullanımı, matematiksel diyalogların akışını etkileyen bir soru sorma aracından biridir (Olkun ve Toluk, 2004). Böyle bir aracın öğretmenin

soru sormasına etkisi olduğu gerçeği, bu araçların ne olduklarını bilmeyişimizin matematik öğretmenlerinin soru sormalarını yüzeysel bir şekilde ele almamıza neden olabilir. Öğretmenlerin soru sorma kullanımlarını, bu araçların varlığını ve kullanımlarını göz ardı etmeden açıklamanın, matematik öğretmen eğitimcilerinin öğretmenlerin matematik derslerinde soru sorma sürecini derinlemesine anlamalarına ve öğretmenlerin soru sorma kullanımlarını açıklamalarına yardımcı olduğunu iddia ediyoruz. Öğretmenlerin soru sorma sürecinde rol oynayan bu araçların kullanımı ortaokul matematik sınıflarındaki sınıf dinamiklerini anlamamızı sağlayacaktır. Ayrıca bu araçların kullanımı, öğretmen eğitimcilerine soru sormanın matematik derslerine nasıl nüfuz ettiğine dair daha derin bir anlayışla içgörü sağlayacaktır. Her ne kadar alan yazını, öğretmenlerin soru sorma davranışları ile ilgili çalışmaları içeriyor olsa da, öğretmenlerin soru sorma sürecinde hangi araçlardan yararlandıkları ile birlikte soru sorma kullanımlarını incelemek için çok az şey yapılmıştır. Bu bize,

Alanyazınında öğretmenlerin soru sorma kullanımları, özellikle soru tipleri açısından oldukça sık incelenmiştir. Soru türlerinin öğretim sırasındaki anlamı, öğretim bağlamı dikkate alınarak analiz edilmelidir (Carlsen, 1991; Şahin ve Kulm, 2008). Öğretim bağlamı, soruyu soran kişiyi soru sorarken kullandığı araçlar ve soru sorma sürecinde anlatılan ders içeriğini yorumlamamıza izin verdiğinden, matematik sınıflarındaki etkileşimin soru sorma yönü hakkında bilgi sağlar. Mevcut alan yazınında, öğretmen sorgulamasını öğretim bağlamında yorumlayan sınırlı çalışmalar bulunmaktadır (örneğin: Koizumi, 2013).

Öğretmenlerin kullandıkları soru türlerini bilmek, öğretmenlerin soru türlerinin nasıl kullanıldığının incelenmesiyle birlikte karşılaştıkları zorlukları incelemeye de yardımcı olmaktadır (Koizumi, 2013). Bu amaçla, öğretmenlerin soru sormak için hangi soru türlerini hangi araçlarla kullandıklarını bilmek gereklidir. Öğretmenlerin soru sormasına katkıda bulunan soru sorma araçlarının uyumu ve yansımalarını bilmediğimiz için öğretmenlerin soru türlerini yorumlamakta yetersiz kalıyoruz. Bu çalışma, öğretmenlerin sorgulama sürecini nasıl yönettiği ve şekillendirdiği konusunda bilgilendirici olacaktır. Mevcut alan yazınında,

öğretmenlerin soru sorma uygulamalarına ilişkin bir uygulama haritası sunan sınırlı sayıda çalışma vardır (örneğin, Mitchell, 1994).

Bu çalışmanın amacı, ortaokul matematik öğretmenlerinin matematik derslerinde öğretmen soru sormasına yardımcı olan araçları ortaya çıkarmaktır. Buna ek olarak bu çalışma, öğretmenlerin soru tipleri ile ilgili soru sorma kullanımlarını ve soru türleri ile soru sorma araçlar arasındaki etkileşimi incelemeyi de amaçlamaktadır. Bu amaçlara yönelik, bu araştırma aşağıdaki araştırma sorularına cevap aramaktadır:

1. Ortaokul matematik öğretmenleri matematiksel soru sorarken hangi araçları kullanırlar?
2. Ortaokul matematik öğretmenleri matematiksel soru sorarken bu araçlardan nasıl yararlanırlar?
3. Ortaokul matematik öğretmenleri öğretimleri sırasında olgusal, sorgulayıcı ve yönlendirici soruları nasıl kullanır?
4. Öğretmenlerin soru sormaları sırasında soru sorarken kullandıkları araçlar ile kullandıkları soru türleri arasındaki ilişkinin niteliği nedir?
    a. Öğretmenlerin soru sorarken kullandıkları araçlar birbirleriyle nasıl ilişkilidir?
    b. Öğretmenlerin soru sorarken kullandıkları araçlar soru türleriyle nasıl ilişkilidir?

Matematiksel bilgiler inşa ederken öğretmenlerin soruları ve öğretmenlerinin cevapları, öğrencilerin soruları ve öğrencilerin cevapları, uyum içinde olmalıdır (Camenga, 2013). Uyumun sağlanmasında öğretmenlerin sınıf ortamını soru sorarak yönetmeleri önemli bir yer tutar (Darragh, 2005). Bu, bir öğretmenin sorular yoluyla öğretme şeklini nasıl oluşturduğuna dikkat çeker ve öğrencileri soru sormaya teşvik eder (Mason, 2002). Buna paralel olarak, öğretmenlerin sınıf ortamında soru sorma kullanımlarına yardımcı olan araçların yönetimi, öğretmen ve öğrenciler arasındaki etkileşimin kalitesi ve zenginliği hakkında kanıtlar sunmaktadır (Mitchell, 1994). Buna, ilgili alanyazınında bahsedildiği gibi, öğretmenin soru sormasında bir araç olan somut materyal kullanımının keşfederek öğrenmeyi sağlayarak öğretmenlerin

soru sorma kullanımlarını değiştirebilmesi örnek olarak verilebilir (Olkun ve Toluk, 2004). Bu çalışmada matematik derslerinde soru sormada kullanılan tüm araçlar incelenmiştir.

Öğretmenler matematiksel bilgileri matematiksel sorularla yapılandırırlar (Mason, 2000). Öğretmen soruları, öğretmenlerin öğretimlerini anlamada kritik bir göstergedir (Barker, 1982). Farklı türden soruların kullanımı, öğretmenlerin soru sorma söylemleriyle ile ilgilidir (Hufferd - Ackles, Fuson ve Sherin, 2004). Öğretmenlerin soru tipleri ve soru tiplerinin kullanımı öğretim bağlamını anlamayı gerektirir (Wragg ve Brown, 2001). Şahin ve Kulm'ın (2008) araştırmasına cevap olarak, bu çalışma, sınıf diyaloglarının öğretim bağlamı içerisinde anlamızı sağlayacağından ve Şahin ve Kulm'un geliştirdiği (2008) soru türlerinin özelliklerinin kullanılabilirliğini incelemek için de önemlidir.

Öğretmenler bazen soru sorma sürecini desteklemek için teorik bir düşünceleri varmış gibi davranmazlar (Delice, Aydın ve Çevik, 2013). Öte yandan, öğretmenlerin, soruların pedagojik yönleri ve öğretimin uygunluğa ilişkin inançlarını göz önünde bulundurarak soru sorma süreçlerini daha doğru anlamamıza yardımcı olan örtülü soru sorma teorileri vardır (Mitchell, 1994). Matematik öğretmenlerine odaklanan sınırlı çalışmalar olduğu için, bu çalışma matematik öğretmenlerinin öğretim sırasındaki soru sorma süreçlerini incelememize olanak sağlayacaktır. Bu amaçla, bu çalışma matematik öğretmenlerinin, soru sorma kullanımlarının niteliğini derinlemesine anlamamıza ve soru sorma konusunda profesyonel bir gelişim yol haritası oluşturulması ihtiyacına yardımcı olacaktır.

Son olarak Türkiye'deki matematik derslerinde öğretmen sorgulamasıyla ilgilenen sınırlı sayıda çalışma (Turgut, 2007; Kasar, 2013) var olduğundan dolayı bu çalışmanın Türk alan yazınında öğretmenlerin soru sorma kullanımları açısından katkısı vardır.

2. Soru Sormaya Kavramsal Bakış

Mason (2014) matematik eğitiminde soru sormanın tanımını aşağıdaki şekilde yapmaktadır (s.514):

> Soru sorma, burada öğrencilere, sorularının çözülmesine yardımcı olmak veya dikkatlerini potansiyel olarak yararlı bir yolla matematiksel ilerlemelerini sağlamak amacıyla yönlendirmek için sunulan soruların ve diğer istemlerin kullanılması anlamına gelir.

Bu tanıma bakıldığında soru sorma, soruları ve diğer istemleri kullanma yöntemleri ile ilgilidir. İkincisi, bunların her kullanımı soru sorma kapsamında değildir. Soru sormayı inceleyebilmek için, öğretmenlerin öğrencilerin matematiksel gelişimini tamamlamak amacıyla soru sormayı kullanması gerekir. Bu süreçte, öğrenciler matematiği kavramada zorluk çekebilir ve öğrencilerin öğretmenlerin matematiksel sorularına dikkat çekmesi gerekebilir. Bu çalışma Mason tanımını (2014) kullanmıştır.

Alanyazınının işaret ettiği gibi, soruların farklı yönlerine odaklanan birçok soru türü vardır. Bu çalışma, Şahin ve Kulm'un (2008) gerçek sınıf diyalogları tarafından desteklenen ve her soru türü için kriterler öneren soru türlerini kullanmıştır.

Vygotsky'nin teorisi, insanların etkileşime girdiği, insan gelişimi için üç etkileşimli yola dikkat çekiyor; birbirleriyle (sosyal etkileşim), insanların etkileşime girdikleri kültürel-tarihsel bağlamda dünyayla (insan, nesne veya kurum) ve kendileriyle (kişisel faktörler) iletişim. Soru sorma çalışmaları sınıf etkileşimi ile yakın bir ilişki içindedir. Sınıflar, etkileşimler için bu sosyal, bağlamsal ve bireysel faktörleri sağlayan ortamdır. Bu etkileşimlerin öğrencilerin zihinsel yapıları üzerinde etkisi vardır. Öğrenciler ve öğretmen birbirleriyle etkileşime girerler, sınıf kültürü içinde etkileşirler ve öğrenirken bireysel düşünme süreçleri vardır. Sınıf içi etkileşimler bu bağlamda dikkate alınmalıdır. Öğretmenler veya öğrencilerin daha bilgili akranları, soru sorarak birbirlerine rehberlik etme rolüne sahiptir (Way, 2008). Sınıf etkileşimi öğretmenin soru sorması ile kolaylaştırılabilir ve bu da matematik öğrenirken öğrencilere yardımcı olur (Way, 2008).

3. Yöntem

Öğretmenlerin soru sormalarını iki matematik öğretmeni ile incelemek için eğitimsel vaka çalışması kullanılmıştır. Bu durumlar diğer durumların temsili veya bir örneğidir. Buna göre, bu vakalar zaman ve bağlamla sınırlandırılmıştır ve okuyuculara bir olgunun anlaşılması hakkında fikir vermek için tanımlayıcı olarak belirtilmiştir. Araştırmacılar, durumları anlamlı bir şekilde ortaya koymak için çabalarlar. Eğitimsel vaka çalışması, eğitim eylemlerini ve öğretmenlerin öğretim sırasındaki pratiklerini anlamaya odaklanır. Bu vakaların sınırları, okul türleri (özel ve devlet okulu bağlamı bağlamında), doğrular ve açılar ile ilgili konular, matematik derslerinde teknolojiyi kullanma ve öğretmenler ile öğrenciler (ler) arasındaki aktif etkileşimdir. Öğretmenin soru sorması bu çalışmada incelenecek olgudur.

Bu çalışmada, iki ortaokul matematik öğretmeninin soru cümleleri çoklu analiz birimleriyle incelenmiştir. Buna göre, katılımcıların soruları nasıl kullandıklarını, hangi soru tiplerini kullandıklarını ve öğretmenlerin belirli bir konuda soru sormayı nasıl kullandıklarıyla ilgili ders bölümlerini analiz ederek aralarındaki ilişki araştırılıp ortaya çıkarılmıştır.

Araştırmanın amacına bağlı olarak, katılımcı öğretmenler, öğretmenlerin sınıfta soru sormaları konusunda gözlemlendiklerinin farkındaydılar. Çalışmanın amacı çalışmanın başında öğretmenlere ve öğrencilere anlatılmıştır. Bu çalışma için toplam 12 ders saat süreli gözlemlere odaklanılmıştır. Gözlemin odağı, öğretmenlerin soru sorarken kullandıkları rutin ve rutin olmayan davranışlarıdır.

Çalışmanın veri kaynakları sınıf içi gözlem ve video kayıtlarıdır. Bu çalışmada, katılımcı olmayan gözlemci rolü ile yapılan sınıf içi gözlem, araştırma için kritik olan anların elde edilmesinde kullanılmıştır. Öğretmenlerin kitap kullanımı, öğrencilere sorular konusundaki yaklaşımları ve öğrencilerin sınıf tartışmalarına katılımları hakkında notlar alınmıştır. Buna ek olarak, öğretmenlerin sınıf diyaloglarındaki stilini anlamak için öğretmenlerin sözel olmayan davranışları da gözlenmiştir.

Video kayıtlarının sözlü yazımları tamamlandıktan sonra, tekrar tekrar izlenmiştir. Videoya dayalı gözlem, videoyu tekrar tekrar izleme, sınıf ortamını

tekrar inceleme ve gözlem notlarıyla karşılaştırma fırsatına ek olarak, araştırma ortamlarında sözel ve sözel olmayan davranışsal hareketlerin yakalanmasına olanak vermiştir (Maxwell, 2009; Yin, 2011).

Sınıf videolarını analiz etmek, birçok aşamayı içermiştir. İlk aşamada, sözlü transkriptler yapılmış, videolar tekrar izlenmiş ve transkriptler üzerine notlar alınmıştır. Analizin ikinci aşamasında, öğretmenin öğretimi matematiksel bir kavram, prosedür veya bir fikri sorgulamaya özgü olan soru sorma bölümlerine ayrılmış ve tarif edilmiştir. Daha sonra Şahin ve Kulm (2008) 'un soru tipleri ile ilgili bölümler incelenmiştir.

4. Bulgular

Çalışmanın bulguları, toplamda, bilgi teknolojisi (BT), basılı yardımcı materyaller (BYM), öğretmen çizimleri (ÖÇ), öğrencilerin düşünceleri (ÖD), analojiler (A) ve gerçek hayattan örnekler (GHÖ) içeren soru sormaya yardımcı olan altı araç olduğunu göstermiştir. Öğretmen Barış, söz konusu altı soru sorma aracını kullanırken, Öğretmen Caner, öğretmen çizimleri, öğrenciler ve basılı ek materyaller olmak üzere üç sorgulama aracını kullanmıştır.

Buna göre, öğrencinin sorularına cevap olarak soru sorma sırasını oluşturmak için DGS'yi kullanma (T1), öğretmen tarafından oluşturulan dinamik figürlere dayalı sorular sorarak DGS'yi kullanma (T2), ek kitabın soruların sırasına rehberlik etmesi (T3), bir matematiksel prosedürü sorgulamak için akıllı tahta uyumlu ders kitabındaki eğitim animasyonunun kullanılması (T4), ek kitabın bir kısmının öğrencinin performanslarını sorgulamak için kullanılması (T5), matematiksel bir prosedürü sorgulamak için dinamik şekiller kullanılması (T6), analojilerin öğretmenlere matematiksel kavramları görselleştiren sorular sormalarını sağlaması (T7), analojilerin örnek soruyu hakkında sorular sorarken öğretmenlerin onları yönlendirmesine yardımcı olması (T8), öğrencilerin soruları veya yorumlarının, öğretmenlere öğrencilerin matematiksel düşüncesinin problemli yönlerini sorular sorarak açıklığa kavuşturmalarını veya tespit etmelerini sağlaması (T9), öğrenci

çizimlerinin öğretmenleri öğrencinin matematiksel prosedürler veya kavramlar hakkında düşünmesini sağlamaya yönlendirmesi (T10), öğrenci çizimlerinin, öğretmenlerin örnek soruları çözerken öğrenci düşüncesini ortaya çıkarmaları için rehberlik etmesi (T11), öğretmen çizimlerinin örnek soruların sorgulanarak çözülmesinde öğretmenlere yardımcı olması (T12), öğretmen çizimlerinin öğretmenlere bir prosedürü sorularla açıklamasında yardımcı olması (T13), gerçek hayattan örneklerin, öğretmenlerin matematiksel prosedürleri veya kavramları sorgulamalarına yardımcı olası (T14), basılı ek kitapların, öğretmene matematiksel bir prosedür veya kavram hakkında bilgi vermesine rehberlik etmesi (T15), ve basılı ek kitapların öğretmene örnek soruların sorgulanma sırasına rehberlik etmesi (T16) şeklindedir.

Soru sorma araçlarının kullanım sıklığına bakıldığında, Öğretmen Barış ek kitap ve öğretmen çizimlerini çok sık kullanmıştır. Fakat, gerçek hayattan örneklerin rehberliği ve bilgi teknolojisi araçlarının bazı kullanımları oldukça nadirdir. Öğretmen Caner soru sorarken basılı ek materyalleri, öğrenci düşünceleri, ve öğretmen çizimlerinden oldukça sıklıkla yararlanırken, bilgi teknolojisi, analojiler ve gerçek hayattan örnekler soru sormasında hiçbir şekilde kullanmamıştır.

Öğretmenlerin bu araçları nasıl kullandıklarına ilişkin detaylar, Öğretmen Barış'ın soru sorma araçlarını şu şekilde kullandığını göstermiştir: akıllı tahta kitabıyla uyumlu ek kitabın sorulacak soru sırasını düzenlemesi (% 47), öğretmen çizimleri öğretmene örnek soruların çözülmesine soru sorma yoluyla yardımcı olması (% 45), soru sormak için öğrenci soruları veya fikirlerinin kullanılması (% 32) ve örnek soruya dair soru sorarken analojilerin referans gösterilmesi (% 10) şeklindedir. Öğretmen Caner için ise sıklık şu şekildedir: soru sormak için öğrenci soruları veya fikirlerinin kullanılması (% 52), öğretmen çizimlerinin öğretmenlere matematiksel prosedürü sorular aracılığıyla açıklamalarında yardımcı olması (% 43) ve basılı ek kitabın sorulacak soru sırasını düzenlemesi (% 50).

Araştırmanın bulguları analojilerin öğretmenlerin örnek sorular üzerindeki matematiksel prosedürleri sorgulamaları (% 10) ve matematiksel prosedürleri veya kavramları (% 6) görselleştirmelerini sağladığını göstermiştir. Bu durum, analojilerin

hem matematiksel kavramların ya da prosedürlerin öğretilmesinde hem de örnek sorulara uygulanmasında kullanıldığı anlamına gelir. Gözlemlenen soru sorma diyalog bölümlerinin % 16'sında analojilerin, gerçek yaşam örneklerinin bu bölümlerin % 1'inde kullanıldığını göstermiştir. Analojiler, matematiksel prosedürleri, kavramları veya örnek soruları sorgularken gerçek hayattan daha sık kullanılmıştır.

4.1 Öğretmenlerin Soru Tipleri

Katılımcı öğretmenler, ders boyunca kullandıkları soru türlerinde birbirlerinden ayrılmışlardır. Öğretmen Barış yol gösterici soru tipini daha sıklıkla kullanırken, sorgulayıcı soruları en az sıklıkta kullanmıştır. Öğretmen Caner ise sorgulayıcı soruları en fazla sıklıkta kullanmış ve yol gösterici soruları ise en az sıklıkla kullanmıştır.

Soru tiplerinin kullanılma sıklığı ders boyunca her iki öğretmen için de bir örüntü içermemektedir. Örneğin, Öğretmen Caner üçüncü dersinde olgusal soruları en sık olarak kullanıyorken, ikinci derste bu soru tipini en az sıklıkta kullanıyor olabilmektedir.

Soru tiplerinin karakteristiklerinin incelenmesi, öğretmenlerin soru tiplerinin karakteristiklerinin benzer veya farklı şekillerde kullanıldıklarını göstermiştir. Örnek vermek gerekirse, yönlendirici soruları kullanırken her iki öğretmen de bu tip soruların karakteristikleri açısından benzer davranışlar göstermiştir. Çoğunlukla bir kavramı anlama ya da bir prosedürü tamamlama yolunda ilerleyen, rehberlik eden ya da yönlendiren fikirleri ya da ipuçlarını barındıran bir dizi olgusal soru soruyorlar ve sınırlı bir şekilde öğrencilerden sezgisel ya da genel bir strateji hakkında düşünmelerini ya da hatırlamalarını istediler. Sorgulayıcı sorular içinse her iki öğretmen de çoğunlukla öğrencilerden düşüncelerini açıklamalarını veya ayrıntılandırmalarını isteyen özelliğe sahip soruları kullanmıştır. Bu tür soruların en az kullanılan karakteristikleri ise aynı değildir. Öğretmen Caner'in sorgulayıcı soruları öğrencilerden fikirlerini haklı çıkarmalarını ya da kanıtlamalarını isteme özelliklerine sahipken, öğrencilerden önceki bilgileri kullanmalarını ve bunu güncel bir soruna ya da fikirlere uygulamalarını istemek özelliği, Öğretmen Barış için en az

sıklıkta kullanılıyor olanıydı. Olgusal sorular için, her iki öğretmen de farklı şekilde davranmışlardır. Öğretmen Caner'in olgusal soruları, çoğunlukla öğrenciden belirli bir gerçek veya tanım isteme ile ilgili olurken, Öğretmen Barış bu özelliği en az sıklıkta kullanmıştır. Öğretmen Barış'ın olgusal soruları çoğunlukla öğrencilerden bir prosedürün bir sonraki adımını isteme ile ilgilidir.

4.2 Öğretmenin Soru Sorarken Kullandığı Araçlar Ile Kullandıkları Soru Tipleri Arasındaki Ilişki

Çalışmanın bulguları, Öğretmen Caner için en çok gözlenen ilişkinin T9 ile T16 (n = 19) arasında olduğunu ve en az gözlenen ilişkinin T9 ile T15 (n = 1) arasında olduğunu göstermiştir. T11-T12, T11-T13, T11-T15, T12-T15 ve T15-T16 (n = 0) arasında ilişki bulunamamıştır. Barış öğretmen için en çok gözlenen ilişki T3 ile T12 arasında, T1-T3; T1 T9; T2 T12; T3-T4; T3-T6; T3, T7, T3-T10,; T3-T15; T5-T11; T6-T8; T7 T9; T7 T10; T9 T10; T9-T15; T11-T12; ve T13-T14'dür (n = 1). T1-T2, T1-T4, T1-T5, T1-T6 veya T1-T7 gibi birçok araç arasında ise ilişki gözlenmemiştir.

Soru tipleri ve soru sorma araçları arasındaki ilişki, tüm araçların Öğretmen Caner tarafından yönlendirici sorularla kullanıldığını göstermiştir. Öğretmen Barış ise T2, T3, T7, T8, T9, T11, T12 ve T13 araçlarını tüm soru tipleriyle birlikte kullanmıştır. Öğretmen Barış, soru sorma araçlarının çoğunu üç tip soru ile birlikte kullanmakla beraber bazı araçlar sorgulayıcı ve olgusal sorularla hiç kullanılmamıştır. Buna göre, T1 sorgulayıcı soru tipiyle birlikte kullanılmamıştır. T4 sorgulayıcı ve olgusal sorularla kullanılmamıştır. T5 olgusal sorularla kullanılmamıştır. T6 sorgulayıcı ve olgusal sorularla kullanılmamıştır. T10 olgusal sorularla kullanılmamıştır. T14 sorgulayıcı ve olgusal sorularla kullanılmamıştır.

Öğretmen Caner için, yönlendirici soru tipi ile soru sorma araçları arasındaki ilişkinin sonuçları, T9, T11, T12 ve T16' nın çoğunlukla öğrencilerin kafası karıştığı ya da bir yere takıldıkları durumlarda kullanıldığını göstermiştir. T13 ve T15 araçlarının ise çoğunlukla anlaşılması gereken bir kavramı veya bir prosedürü tamamlamak için bir takım olgusal sorular dizisi ile birlikte kullanılmıştır. T13 aracı dışında öğrenciler herhangi bir soru sorma aracıyla stratejileri düşünmeye

yönlendirilmemiştir. Öğretmen Barış için ise, T3, T5, T11, T12 ve T13 araçları dışında, öğrenciler herhangi bir araçla stratejileri düşünmeye yönlendirilmemiştir.

Öğretmen Caner T9, T11 ve T16 araçlarını öğrencilerin düşüncelerini açıklamalarını veya ayrıntılandırmasını gerektiren sorularla kullanıldığını göstermiştir. Öğretmen Barış ise T3, T5, T8, T9, T11, T12 ve T13 araçlarını aynı soru sorma özelliğiyle birlikte kullanılmıştır. T13 ve T15 araçları, öğrencilerin önceki bilgilerini bir probleme veya düşünceye uygulamalarını gerektiren soru sorma özelliğiyle birlikte kullanılmıştır. Öğretmen Barış ise, T7 aracını çoğunlukla aynı özellikle kullanılmıştır. Öğretmen Caner için, çoğunlukla öğrencilerin fikirlerini haklı göstermelerini gerektiren soruları sorma özelliğini yalnızca T12, Öğretmen Barış ise T2 ve T10 araçlarıyla aynı özellikte kullanıyordu.

Öğretmen Caner'in olgusal soruları T9 ve T13 araçlarıyla çoğunlukla olgu veya tanım isteyen özelliği ile birlikte kullanılmıştır. Öğretmen Barış derslerinde en çok T7 aracını bir alıştırmaya cevap isteyen karakteristik ile ve bu karakteristik ile en az T11 ve T16 araçlarını kullanmıştır. T12 ve T15 araçları, Öğretmen Caner tarafından çoğunlukla bir prosedürde bir sonraki adımı isteyen olgusal soru tipinin özelliği ile kullanılırken, Öğretmen Barış'ın derslerinde T1, T2, T3, T8, T9, T11, T12 ve T13 araçları, çoğunlukla aynı özellik ile gözlenmiştir.

5. Tartışma ve Öneriler

5.1 Matematik Sınıflarında Soru Sormak İçin Kullanılan Araçlar

Bu çalışmanın ilk araştırma sorusu, ortaokul matematik öğretmenlerinin soru sorarken kullandıkları araçları belirlemektir. Bu çalışmada soru sorma araçları, matematik eğitiminde soru sormayı açıklayan Mason'un (2014) matematiksel soru sorma diyaloglarında ortaya çıkan soruları ya da diğer soruları kullanma şeklini örneklemiştir. Aynı içerikte bile, öğretmenler soru sorma araçlarını entegre etmek için farklı yollar izlemiştir. Buna göre, ortaokul matematik öğretmenlerinin derslerinde bilgi teknolojisi araçları, analojiler, gerçek yaşam örnekleri, öğrenci

fikirleri, öğretmen çizimleri ve basılı ek materyal içeren altı soru sorma aracı gözlemlenmiştir. Teknolojiyi derslerinde kullanan katılımcı bir öğretmen, teknolojiyi kullanmayan öğretmenden farklı olarak bilgi teknolojisi araçlarını, analojilerini ve gerçek yaşam örneklerini soru sorarken kullanmışlardır. Bilgi teknolojisi araçları, teknolojinin derste kullanılmasından kaynaklanmıştır. Teknoloji kullanımı, öğretmenin iletişim kurmasına ve sorgulama yoluyla öğrencileriyle birlikte çalışmasına yardımcı olmuştur. Bu bulgu, öğretmenlerin sorgulanması ile farklı medya arasındaki ilişkinin, öğretmenlerin sorgulama şeklini değiştirme potansiyeli olduğunu doğrulamaktadır (Akkoç, 2013).

Katılımcı Caner Öğretmen, Barış öğretmene kıyasla daha geleneksel bir yapıdaydı; bir tahta, tahta kalemi ve onunla birlikte basılmış bazı ek materyallerle öğretimini gerçekleştirdi. Öğretmenlerin soru sorma kullanımlarının, teknolojinin kullanımını göz önünde bulundurarak, teknolojik araçların öğrenciye sağladığı öğrenme fırsatlarından kaynaklanması beklenmekteydi. Bununla birlikte, analojiler ve gerçek yaşam örnekleri, önceki araştırma çalışmalarında belirtildiği gibi öğrencilerin bilişsel süreçlerini desteklemek için kullanılmış (örneğin, Tanner ve diğ., 2005) ve bu çalışmada bilgi teknolojisi araçlarından bağımsız olarak ortaya çıkmıştır. Bu açıdan, soru sorma öğretmene bağımlı bir eylemdir (Mitchell, 1994). Bulgular, sorgulamanın öğretmenlerin örtülü sorgulama kuramları (Mitchell, 1994) veya kişisel öğretim kuramları (Fox, 1983) tarafından şekillendirilen kişisel bir eylem olduğu fikrini desteklemektedir.

Öğretmenlerin uygulama yaparken gözlemlenen soru sorma davranışları, soru sorma araçlarının varlığnaı bağlı olabilir (Mitchell, 1994). Buna göre soru sorma araçlarının sıklığı, Öğretmen Barış'ın soru sorma söyleminin akıllı tahta ile uyumlu ek kitap, öğretmen çizimleri ve öğrenci çizimleri temelinde modellenebileceğini, Öğretmen Caner'in ise soru sorma söyleminin öğrenci fikirleri, öğretmen çizimleri ve basılı ek materyal öğelerine sahip olabileceğini göstermektedir. Bununla birlikte, katılımcıların soru sorma söylemlerinin matematiksel içeriğe bağlı olma ihtimalinin akılda tutulması gerekir. Örneğin, öğretmen çizimlerinin ve soru sorarken kullanılan öğrenci çizimlerinin içeriğe bağlı olarak ortaya çıkması çok muhtemeldir.

Matematiksel içeriğin doğrular ve açılar olması nedeniyle, öğretmenler veya öğrenciler bazı işaretlemeler veya çizimler kullanmak durumunda kalmış ya da DGS yazılımının geometri içeriğini desteklemesinden dolayı bu yazılım tercih edilip kullanılmış olabilir.

Bills, Dreyfus, Mason, Tsamir, Watson ve Zaslavsky'nin (2006) bahsettiği gibi, örnekler öğretmen ve öğrenciler arasında matematiksel iletişimi sağlar. Bills ve diğerleri (2006), bir kavram örneğinin ve bir prosedür uygulamasının örneğinin farklı pedagojik yönlere sahip olduğunu vurgulamıştır. Bu çalışmada bir matematiksel prosedür uygulamasının örnekleri daha sık gözlenmiştir. Her iki öğretmen de ek kitaptan alınmış örnek sorular kullanmış ve dolayısıyla çoğu zaman doğrular ve açılar ile ilgili matematiksel prosedürleri uygulamak için basılı ek materyaller kullanılmıştır. Bununla birlikte, bu çalışmada öğretmenler ek kitapları takip ettiklerinden dolayı, öğrencilerin bilişsel seviyrsinin üstünde örnek sorular çözmelerini bekledikleri görülmüştür. Bu nedenle, örnek soruların seviyesi bu kitaplara dayanmıştır. Öğretmen Barış, son dersin sonunda gerçek yaşam örneklerini kullanmış ve bu örnek, kitaptaki örnek soruların neredeyse tamamının ardından kullanılmıştır. Bu nedenle, gerçek hayat örneklerinin, matematiksel kavramlardan ziyade matematiksel prosedürleri sorgulamakta yardımcı olduğu görülmektedir. Gerçek yaşam örneği, öğrencilerin iki paralel çizgi arasında oluşan açılardaki matematiksel prosedürleri anlamalarını gerektirmiş, Öğretmen Barış, "su terazisi" ile iki paralel çizgi arasındaki kurallar (özellikle Z kuralı) arasında bir ilişki kurmak için gerçek yaşam örneği vermiştir. Gerçek yaşam bağlamlarının varlığı, öğrencilere bağlamların matematiksel arka planını anlama ve daha motive olmalarını sağlasa da (Boaler, 1993), öğretmenlerin kitapları takip ediyor olmaları, gerçek yaşam örnekleriyle matematiksel prosedürleri sorgulamaları konusunda bir engel teşkil etmektedir. Ek kitapların gerçek hayattan örnekleri desteklemediği görülmüştür. Bu nedenle, öğretmen gerçek yaşam örneklerini soru sormasına ek materyallerden bağımsız olarak entegre etmiştir.

Öğretmen Barış vücut analojisi, nehir analojisi ve balık analojisi olmak üzere üç analoji kullanmıştır. Öğrenciler örnek sorular üzerinde çalışırken, öğretmen bu

örnek soruların çözülmesinin bir yolu olarak öğrenciler tarafından anlaşılması pekte kolay olmayan analojiler kullanmıştır. Öğrencilerin, öğretmene analojileri anlamak için bazı sorular sordukları ve öğretmenin analojilerin mantığını açıklamak için sınırlı açıklamalar yaptıları gözlemlenmiştir. Bulgular, analojilerin öğrenciler tarafından kolayca anlaşılmadığı ve öğrenciler için alternatif kavramlar oluşturma potansiyeli olduğu fikrini desteklemektedir (Harrison ve Treagust, 2006). Öğretmen ders sırasında analojileri oluştururken ve kullanırken ve örnek soruları analojiler yoluyla çözerken, öğrencilerin analoji oluşturmadıkları gözlemlenmiştir. Bu sonuçlar, öğretmen tarafından üretilen analojilerin öğrencilerinkinden daha sık olduğu diğer çalışmaların (Harrison ve Treagust, 2006) bulgularıyla da tutarlıdır. Bu bulgunun nedeni, Öğretmen Barış'ın öğrencilerinden analoji üretmelerini beklememesi ama buna karşılık örnek soruları çözerken ve matematiksel kavramları görselleştirirken öğretmenin kendi oluşturduğu analojileri kullanmasını beklemesi olabilir. Harrison ve Treagust'un (1994) önerdiği gibi, öğrenciler analog ile hedef kavramı veya matematiksel prosedürler arasındaki benzerliklerden bahsetmek ve alternatif kavramlar yaratma potansiyeli olan farklılıkları tespit etmek yoluyla analojiyi sorgulama ve tartışmalarına dahil etmelidir. Ayrıca, analojinin öğretimdeki rolüne bağlı olarakta sistematik bir yaklaşımla ve dikkatlice planlanarak öğretime dahil edilmesi gerekmektedir (Harrison ve Treagust, 1994). Bu çalışmada bahsedilen etkili analoji öğretiminin gereklilikleri göz önüne alındığında, analojilerin kullanımı, öğretmenin önceden öğrenilen matematiksel kavramları görselleştirerek harekete geçirmek ve analojinin kendisini sınırlı olarak sogulayarak örnek sorular çözmede kullanıldığını göstermiştir.

Çalışmanın bulguları her iki matematik dersinde de, soru sormak için kullanılan araç sayısının soru sorma diyaloglarının oluşturduğu bölümlerden daha fazla olduğunu göstermiştir. Buna göre, her bir bölümde birden fazla soru sorma aracı kullanılmış ve bu araçlar birbiriyle etkileşime girmiştir. Kullanılan soru sorma araçlarının sayısının ve bu araçlar arasındaki ilişkinin varlığına bakıldığında, öğretmenin soru sorması, konu içeriğine ve öğretim bağlamıyla ilişkili bir söylemdir (Carlsen, 1991; Koizumi, 2013; Nisa ve Khan, 2012).

Bu çalışma, soru sorma araçlarının soru sorarken farklı kullanımları olduğunu göstermiştir. Buna göre Öğretmen Barış ve Öğretmen Caner için soru sorma, matematiksel işlemler veya kavramlar hakkında sorular sorma (%30; %46), örnek sorular yoluyla sorular sorma (%68; %74), öğrenci fikirlerini veya yorumlarını sorgulama (%30; %50) ek kitap kullanarak soru sormak (%56; %61), gerçek hayattan örnekler sormak (%1, % 0) ve analojileri içeriğe uyarlayarak soru sormak (%16,% 0) kullanımlarını içerir. Soru sorma araçlarının kullanım sıklığı öğretmenlerin farklı öğretim uygulamalarına sahip olduğunu göstermiştir. DGS, analojiler, gerçek yaşam örneği ve öğrencilerin fikirleri öğretmenlere yeni soru sorma bölümleri oluşturmayı sağlarken, akıllı tahta veya basılı ek materyallerle uyumlu ek kitaplar öğretmenlere soru sorma yollarını takip etme imkanı sağlayarak planlanmış soru sorma bölümleri oluşturmaktadır. Bazen planlanmış soru sorma bölümleri, öğretmenin sorgulama şeklini ve beklenmedik durumları da içeren (örneğin, ders kitabındaki bilinmeyen flaş simgesi) önceden planlanmış öğretim uygulamalarını da içerebilir. Bu nedenle, öğretmenler beklenmedik durumları soru sormalarına entegre etme ve beklenmedik olaylar sırasında öğrenci katkılarıyla süreci yönetme becerilerine sahip olmalıdır (Rowland, Huckstep, Thwaites, 2005).

Dijital yeterlilik, Türkiye'nin ortaokul matematik müfredatında matematik öğretimi için öğretmen yeterliklerinin gereklerinden biri olarak bahsedilir (MEB, 2013). Bilgi teknolojisi araçlarının kullanımı, öğretmenlerin dijital yeterlilikleri ile ilgilidir. Ortaokul matematik öğretmenleri için, bilgiyi anlamlı bir şekilde oluşturmak için bilişim teknolojilerini kullanmayı içeren öğretim yöntemi, Milli Eğitim Bakanlığı tarafından önerilmiştir (MEB, 2013). MEB (2013), öğretmenleri etkili etkileşimle matematik konuşan öğrenme toplulukları oluşturmaya teşvik etmiştir. Bu çalışma, teknoloji destekli ve teknoloji destekli olmayan bir sınıfta gerçekleşen bir ders öğretiminin doğasını ortaya koymuştur. Bu çalışmada interaktif beyaz tahta, DGS ve akıllı tahta ile uyumlu ek kitap dahil olmak üzere bilgi teknolojisi araçları, öğrenme ortamı yaratmıştır. Önceki çalışmalar (örneğin, Tanner ve diğerleri, 2005), etkileşimli beyaz tahta ile pedagojiyi dahil ederek öğretmenin önemini belirtmiştir. Tanner ve diğ. ark. (2005) belirttiği gibi interaktif yazı tahtalarının (IWB) tek başına

pedagoji sağlamadığını, ancak öğretmen ile tahta arasındaki etkileşimin pedagoji sağladığını bildirmiştir. Bu çalışma, interaktif beyaz tahta ile öğretim pedagojisinin, öğretmenlerin soru sorma davranışlarına özgü interaktif beyaz tahta kullanımlarını ve bu araçların soru sormalarına katkısını anlamamızı sağlamıştır. Çalışmanın bulgularından biri, Barış öğretmenin kullandığı dinamik geometri yazılımının, öğrencilerin soruları ve öğretmenin oluşturduğu dinamik figürler temelinde tartışmalar yapmak gibi pedagojik hedefleri temsil etmiş olmasıdır. Buna ek olarak, akıllı tahta ile uyumlu olan ek kitaptan kaynaklı olarak, öğretmenin soru sormasını düzenlemesi anlamında farklı kullanımları olmuştur, ancak ek kitapla öğretmenin kitabı adım adım takip etmesi soru sormasını düzenlemiştir ve bu araç tek bir pedagojik amaç için kullanılmıştır. Bu pedagojik amaç, soru sormayı düzenlemede bir yöntem olarak kitap kullanımını vurgulamıştır. Gözlemlenen durumların sıklığı, ek kitabın öğretmenin sorularının sırasını düzenlemek için kullanıldığını göstermiştir. Bu yönüyle bilgi teknolojileri çoğu önceden planlanmış öğretim bağlamlarında kullanılmıştır. Teknoloji, öğretmenlerin matematiksel kavramları yorumlayarak ve keşfederek matematiksel fikirleri sorgulamalarını sağladığı veya gerçek hayattaki örneklere matematik uygulamayı sağladığı için (Arbaugh ve diğerleri, 2010), teknolojinin stratejik olarak entegre edilmesi önerilmektedir.

Bulgular, Barış öğretmenin çizimlerinin, matematiksel prosedürleri uyguladığı soru sorma araçlarından biri olduğunu göstermiştir. Bunu yaparken, öğretmen sorular sormuş veya öğrencileri örnek soruların çözümünü sorgulamaya teşvik etmiştir. Bu öğretmen, örnek soruların çözümünü akıllı tahtada doğru bir şekilde çözme rolünü üstlenirken, örnek soruların neredeyse yarısını akıllı tahtada soru sorarak çözmüştür. Ancak, diğer öğretmen çizimlerini çok fazla kullanmamıştır. Bunun nedeni, kendi çizimlerine dördüncü derste öğrencilerin bireysel olarak çözdükleri örnek soruların çözümlerinde önem vermiş olması olabilir. Buna ek olarak, Öğretmen Caner'in örnek soruların çözümü için izlediği yol, öğrencilerle prosedürler veya matematiksel kavramlar hakkında konuşmaya başlaması ve bunu takiben öğrencilere örnek soruları çözmek için kendi çizimlerini yapma fırsatını vermesidir. Bu nedenle, öğretmen, örnek soruları doğru şekilde çözme rolünü

üstlenmemiştir; bunun yerine, öğrencilere çözüme dair fikirlerini paylaşma ve örnek soruların çözümü için çizim yapma fırsatı veren bir atmosfer yaratmıştır. Başka bir deyişle, öğretmen örnek soruları çözmede kendine bir sorumluluk atfetmemiştir, öğretmen öğrenciler için bu şekilde bir rehber olmuştur. Her iki öğretmenin örnek soruları soru sorarak çözme yaklaşımları aynı değildir. Her iki öğretmenin de kendi çizimlerini farklı sıklıkta kullanması, öğretmen çizimlerinin bir soru sorma aracı olarak kullanımının öğretmenlerin kendilerine atfettikleri örnek soruları çözme sorumluluğuna bağlı olabileceğini göstermiştir. Bir öğretmen bu sorumluluğu aldığında, öğretmen çizimlerini soru sorma aracı olarak oldukça sık kullanılması kaçınılmaz hale gelebilir.

Davis'e (2009) göre, öğretim için araçlar vardır. Öğretme araçlarından biri sorgulama ile ilgili tartışma stratejileridir. Bu çalışmada soru sormada kullanılan araçlar detaylandırılmıştır. Bu amaçla soru sorma araçları, öğretime entegre olmanın yollarını temsil etmiştir. Üçüncü araştırma sorusu altındaki sorulardan biri, soru sorma araçları arasındaki ilişkileri araştırmaktır. Buna göre Öğretmen Caner için T9 ve T16 araçları ilişkiliyken, Öğretmen Barış için T3 ve T12 çoğunlukla birlikte gözlemlenmiştir. Her iki ilişki de ek kitap kullanımının her iki sınıfta da ön plana çıkan bir rolü olduğunu vurgulamıştır. Buna göre, T9 ve T16 arasındaki ilişki, öğrenci fikirleri ile basılı ek kitabın soru sormada kullanımı arasındaki ilişkiyi ortaya koyarken, T3 ve T13, öğretmen çizimleri ile ek kitabın yakın ilişki içinde olduğunu göstermiştir. Bununla ilgili olarak, öğretmenlerin sınıf içindeki bu tür davranışları, öğrencinin öğrenmesini teşvik etmek için kullanılan stratejiler arasındaki ilişkiyi de temsil etmiştir (Chapin et. al., 2009).

5.2 Matematik öğretmenlerinin soru tipleri ve soru sorma araçlarıyla ilişkisi

Araştırmanın bulguları, katılımcı öğretmenlerin ilgili literatürle uyumlu sorgulayıcı, yönlendirici ve olgusal sorular içeren soru türlerini kullandıklarını ortaya koymuştur (Camenga, 2013; Piccolo ve diğerleri, 2008; Şahin ve Kulm, 2008; Ong

ve diğerleri, 2010). Her iki öğretmen de, her derste her türlü soruyu kullanmıştır, ancak Öğretmen Caner, son derste olgusal soruyu kullanmamıştır. Matematiksel bir prosedür ve bir kavramdan bahsederken ya da çalışılmış örneklerle çalışırken öğretmenler tüm soru türlerini kullanmışlardır. Bu çalışma, soru türlerinin ders bölümleriyle ilişkili olmadığını doğrulamaktadır (Şahin ve Kulm, 2008).

İlgili literatürün aksine (Faruji, 2011; Jiang, 2014), bulgulardan biri, her iki öğretmenin de derslerinin çoğunda olgusal soruları kullanmadığıdır. Öğretmen Barış, yönlendirici sorularını oldukça sık (% 77), ardından ise sorgulayıcı (% 12) ve olgusal (% 10) soruları benzer sıklıkta kullanmıştır. Öğretmen Caner ise çoğunlukla sorgulayıcı (% 51) ve ardından yönlendirici sorular (% 31) ve olgusal sorular (% 17) kullanmıştır. Bir öğretmen derslerini olgularla ilişkilendirerek açıklamayı tercih etmezse bu durum beklenebilir. Ayrıca, bu sonuçların olası bir açıklaması, soru sorma sırasında öğretmenlerin farklı öğretim stratejileriyle ilgili olabilir. Örneğin, bu çalışmada, katılımcı öğretmenlerden biri olan Öğretmen Barış'ın öğretim akışı çoğunlukla bir prosedürü tamamlamakla ilgiliydi ve kendi düşünme biçimine dayanarak soru sorma yetkisini kullanmıştı. Bu öğretmen, soru sorurken öğrenci düşüncesini kullanmak yerine kendi düşüncesini kullandığı için öğretim sırasında öğrencilerinden daha fazla konuşma eğilimindeydi. Diğer öğretmen, Öğretmen Caner ise öğrencilerinin düşüncelerini çoğu zaman öğretimine entegre etti. Koizumi'nin (2013) de belirttiği gibi, deneyimli öğretmenler, öğrencilerin yaratıcı düşünme ve soru sormalarına daha fazla önem vermiştir.

Soruların özellikleri göz önüne alındığında, her iki öğretmen için de soru türlerinin özelliklerini kullanmada bazı benzerliklerin ve bazı farklılıların olduğunu göstermiştir. Öğretmen Caner için bir prosedürde öğrencilerin bir sonraki adımı atmalarını isteyen olgusal soru özelliğini en az sayıda soru kullanımıyla ortaya koyarken, Öğretmen Barış soru sorurken çoğunlukla bu özelliği kullanmıştır. Bunun sebeplerinden biri, öğretmenlerin soru sorma uygulamalarındaki farklılıklar olabilir. Öğretmen Caner bu özelliği sınırlı bir şekilde kullanmıştır, çünkü öğrencilerden prosedürde bir sonraki adımı atmalarını isteme gereği duymamıştır. Bundan ziyade, Öğretmen Caner, öğrencilerinden kendi açıklamalarını oluşturmalarını istemiş ve

böyle bir sınıf atmosferi yaratmaya çalışmıştır. Bu nedenle, öğrencilerin düşünme tarzını ve çözümlerini vurgulamak için belirli gerçeklere veya tanımlara odaklanması olasıdır. Ancak, Öğretmen Barış, ek kitaptaki örnek soruları çözerek matematiksel içeriği öğretme yolunu izlemiştir. Bu nedenle, örnek soruların içindeki prosedürlerle ilgili sorular sormaya dayanan bir yol izlemiş olabilir.

Her iki öğretmen de, yönlendirici soruları öğrencilerin bir kavramı anlama ya da çoğu zaman bir prosedürü tamamlama konusunda yardımcı olma özelliğini vurgulamıştır. Bu süreçte kullanılan olgusal sorular, öğrencilerin istenen bir noktaya ulaşmalarını sağlayacak şekilde veya öğrencileri farklı düşüncelere daha açık hale getirecek şekilde kullanılmıştır. Öğrencilerin matematiksel prosedürleri veya kavramları sorgulamaya katılmaları ise çok az teşvik edilmiştir.

Sorgulayıcı sorularda her iki öğretmen de çoğu zaman öğrencilere düşünmelerini açıklamayı veya detaylandırmayı gerektiren sorular sormuştur. Bununla birlikte, sorgulayıcı soruları kullanırken öğretmenler, öğrencilerden önceki bilgilerini kullanmalarını isteme ve önceki bilgileri yeni bir probleme ya da düşünceye uygulama özelliğini kullanma konusunda farklılaşmıştır. Bu özelliğin kullanımı, belirli gerçeklerin veya tanımların kullanılmasıyla ilgili olgusal soruların bir özelliğinin kullanılmasıyla tutarlıdır. Her iki kullanımda önceden öğrenilmiş bilginin etkinleştirilmesini gerektirmektedir. Bu nedenle, farklı soru türlerinin özelliklerinin birbiriyle etkileşimi olabilir.

Üçüncü araştırma sorusunun alt sorularından biri de soru tipleriyle hangi soru sorma araçlarının kullanıldığını incelemektir. Buna dayanarak, Öğretmen Caner'in her soru sorma aracını tüm soru türleriyle kullandığı, ancak diğer öğretmenin soru sorma araçlarını sırasıyla T1, T4, T6, T14 ve T15 ile T4, T5, T6, T10, T14, ve T15 araçlarıyla sorgulayıcı soruları ve olgusal soruları kullanmadığı bulunmuştur. Çalışmanın bir diğer önemli bulgusu bazı bilgi teknolojisi araçlarının (örneğin; T2, T3) olgusal, yönlendirici, ve sorgulayıcı sorular ile kullanıldığını göstermiştir. Sorgulayıcı sorular veya olgusal sorular, bazı soru sorma araçlarıyla kullanılmamıştır (örneğin; T1, T4, T5 ve T6). Barış öğretmenin bazı bilgi teknoloji soru sorma araçlarıyla üç soru türünü de kullanabildiği gözlemlenmiştir. Bu nedenle, teknoloji

entegrasyonunun, kullanılacak soru türlerini güçlü bir şekilde etkilemediği söylenebilir.

5.3 Öneriler

Bu soru tipleri ve her soru tipinin özellikleri, matematik sınıflarındaki sınıf diyalogları ile ayrı ayrı gösterilmiştir. Bu çalışma, her bir soru türüne örnek teşkil ederek ve soru sorma araçlarıyla birlikte özelliklerinden bahsederek Türk bağlamındaki soru tipleri hakkında mevcut bilgilere katkıda bulunmaktadır. Bu çalışma aynı zamanda öğretmenlerin soru sorma davranışlarının uygulamalarını gerçekçi olarak göstermiştir. Bu çalışma, iki matematik öğretmeni için doğrular ve açılara özgü soru sorma araçlarını araştırmıştır. Bu nedenle, çalışmanın bulguları içeriğin sınırlılığı içinde açıklanabilir. Öğretmen adaylarının gerçek sınıf deneyimlerine aşina olmaları gerektiğine dikkat çekmek, soru sorma araçlarını sorgulamak ve bunlara karşılık gelen gerçek sınıf diyalogları, matematik öğretimi için metot derslerindeki eğitimleri ve öğretim ilkeleri ve yöntemleri hakkında bilgilendirmeye hizmet edebilir.

Her ne kadar bu çalışma sadece doğrular ve açılar konusuna odaklanmış olsa da, çalışmanın konusunun niteliği göz önüne alındığında potansiyel öğretmen davranışları hakkında fikir vermektedir. Çalışmanın çerçevesi (Şahin ve Kulm, 2008) sınıf diyaloglarını analiz etmek ve bu iki durumdan uygulamaya dayalı kanıtlar sağlamak için uygulanabilir. Öğretmen adaylarının varsayımsal öğretim ortamlarında veya gerçek sınıf ortamlarında öğretmenlik deneyimleri göz önünde bulundurularak, aynı çerçeve test edilebilir veya öğretmen adaylarının pratik uygulamalarındaki soru türlerini analiz etmek için bir rehber olarak kullanılabilir.

Bu çalışma, ortaokul matematik öğretmenlerinin matematik dersi sırasında soru sorarken kullandıkları araçları açıklamaktadır. Çeşitli sınıf ortamlarındaki soru sorma araçları, öğretmen adaylarının soru sormalarını geliştirmenin yollarını açacaktır. Soru sorma araçlarını başlangıç adımı olarak kabul ederek, bu süreç daha fazla öğretmen, daha fazla matematiksel konu sayısı, ve öğretmen adaylarını buna

göre eğitmek için ayrıntılı soru sorma etkileşimini anlamaya ihtiyaçları gözününde bulundurularak tekrarlanabilir.

Teknolojinin öğretmenlerin soru sorma kullanımlarında farklılıklar oluşturabileceğini gözlemlemek için öğretmenlerin, soru sormayı uygulamalarına nasıl entegre ettikleri konusunda bilgi teknolojisi araçlarının farkında olmaları ve öğretilecek içeriğe özgü teknolojik pedagojik içerik bilgisi konusunda yönlendirilmeleri gerekir. Buna paralel olarak, öğretmen adaylarının matematik eğitiminde bilgi teknolojisi araçları ile ilgili bu çalışmada ortaya konan araçlarla farkındalıklarını oluşturarak bu yönde eğitim alabilirler. Teknolojiye özgü soru sorma araçlarının dikkate alınması, araştırmacılara ortaokul öğretmenlerinin matematik derslerinde teknoloji destekli matematik derslerinde tutum (TMLA) ölçeğine (Aytekin ve Işıksal-Bostan, 2018) benzer bir ölçek geliştirerek teknoloji kullanımına yönelik tutumlarını anlamalarına yardımcı olabilir.

Bu çalışma, öğretmenlerin öğretmenliklerinde kullanılan soru tiplerinin soru sorma araçlarını içeren davranışlarını ortaya koymaktadır. Öğretmenlerin soru sorma davranışlarını daha derinlemesine anlamak için, öğretmenlerin soru sormada neden böyle bir yol izlediğinin nedenleri araştırılabilir. Bunu yaparak, öğretmenlerin soru türlerini ve alt karakteristiklerini kullanma nedenleri veya öğretmenlerin soru sorma konusundaki inanışları ortaya çıkarılarak hizmet içi eğitimin kalitesi artırılabilir.

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TEZİN ADI / TITLE OF THE THESIS (**İngilizce** / English): A CASE STUDY ON MIDDLE GRADE MATHEMATICS TEACHERS' USE OF QUESTIONING IN TEACHING LINES AND ANGLES

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