PLAYING WITH MATHEMATICS IN THE ARTS STUDIO: STUDENTS' VISUAL-SPATIAL THINKING PROCESSES IN THE CONTEXT OF A STUDIO THINKING BASED-ENVIRONMENT

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ABSTRACT

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The aim of the study was to investigate how students make use of visual-spatial thinking processes in a Math-Art Studio Environment in which students are engaged in geometry-rich artworks through Studio Thinking Framework, which describes the nature of learning and teaching in visual art courses (Hetland, Winner, Veneema, & Sheridan, 2013). To achieve this aim, a case study method was employed. Participants of this environment were six seventh grade students enrolled in a public middle school. Data sources of the study were stimulated recall interviews, observation of video recordings of students' verbal expressions and behaviours in studio environment, and students' documents (written notes, sketches, and artworks). Data were analysed through qualitative methods to search for indicators of visual-spatial thinking.

Analysis of students' visual-spatial thinking processes indicated that students made use of four major visual-spatial thinking processes in Studio Thinking Based-Math-Art Studio Environment, which were recognizing geometric shapes, decomposing and composing shapes, patterning, and transforming geometric shapes. These processes of visual-spatial thinking were interrelated to each other, which required students to use them in a coordinated manner. Findings of this study also indicated that this studio thinking based-environment had a potential to elicit different processes of visual-spatial thinking.

Keywords: Visual-Spatial Thinking, Studio Thinking, Mathematics and Visual Arts

ÖZ

SANAT STÜDYOSUNDA MATEMATİK İLE OYNAMAK:
STÜDYO DÜŞÜNME TABANLI ORTAM BAĞLAMINDA ÖĞRENCİLERİN
GÖRSEL-UZAMSAL DÜŞÜNME SÜREÇLERİ

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Bu çalışmada, geometri yönünden zengin sanat çalışmaları yaptıkları Stüdyo Düşünme tabanlı bir Matematik-Sanat Stüdyosu Ortamında öğrencilerin görsel-uzamsal düşünme süreçleri incelenmiştir. Stüdyo Düşünme, görsel sanatlar stüdyo ortamında, öğrenme ve öğretmenin doğasını açıklayan bir teorik çerçevedir (Hetland, Winner, Veneema, & Sheridan, 2013). Bu çalışmada durum çalışması yöntemi kullanılmıştır. Matematik-Sanat Stüdyosu Ortamının katılımcıları, bir devlet okuluna kayıtlı olan altı 7. sınıf öğrencisidir. Çalışmanın veri toplama kaynaklarını; uyarılmış hatırlama görüşmeleri, stüdyo ortamında öğrencilerin sözel ifadeleri ve davranışlarına yönelik video kayıtlarının gözlemi ve öğrencilerin belgeleri (yazılı notlar, eskizler ve sanat çalışmaları) oluşturmaktadır. Görsel-uzamsal düşünme süreçlerinin göstergelerini aramak üzere veri, nitel yöntem ile analiz edilmiştir.

Görsel-uzamsal düşünme süreçlerinin analizi sonucunda, Matematik-Sanat Stüdyosu Ortamında öğrencilerin dört temel görsel-uzamsal düşünme sürecinden yararlandıkları bulunmuştur: Geometrik şekilleri tanıma, şekil oluşturma ve parçalarına ayırma, örüntüleme, ve şekilleri dönüştürme. Bu düşünme süreçlerinin birbirleriyle bağlantılı olduğu ve koordineli bir şekilde kullanılmayı gerektirdiği ortaya çıkmıştır. Bu çalışmanın bulguları, Stüdyo Düşünmesine dayanan bu Matematik-Sanat Atölye Ortamının, öğrencilerin farklı görsel-uzamsal düşünme süreçlerini ortaya çıkarma potansiyeline sahip olduğunu göstermiştir.

Anahtar Kelimeler: Görsel-Uzamsal Düşünme, Stüdyo Düşünme, Matematik ve Görsel Sanatlar

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LIST OF ABBREVIATIONS

OECD	Organization for Economic Co-operation and Development
STEM	Science, Technology, Engineering, and Mathematics
STEAM	Science, Technology, Engineering, Arts, and Mathematics
2D	Two-Dimensional
3D	Three-Dimensional

CHAPTER 1

INTRODUCTION

Recent approaches to mathematics education emphasize applying mathematical knowledge into a variety of real life issues. The reason of such an emphasize is that students should use what they learned in mathematics classrooms in their careers or jobs in the future such as engineering, science, business, and architecture (Quinn & Bell, 2013). This is a future oriented approach through which students practice exercises that put strong emphasis on the aim of transfer of learning to the future tasks. This approach towards education reveals the fact that current education tends to delay the use of knowledge in their current tasks. However, students could use their knowledge in their current practices with bearing future view in mind (Perkins, 2013). In other words, students could use mathematics as a tool to make something through applying knowledge to current practices that serve as a mirror for future undertakings (Papert & Harrel, 1991).

How this study was shaped on the basis of this problem was explained through describing the overall picture. When zooming out to see overall picture, it is seen that how this study was shaped within the perspective of constructionism. Constructionism rooted in the work of Papert and Harrel (1991) be used as a lens to design learning environment and to interpret how students construct meaning. In constructionist learning environment, students learn through making in personally meaningful activities or projects. This philosophy of learning provides a new vision to mathematics education (Papert & Harrel, 1991). This new vision could be visible in the places like studio or atelier in which students are encouraged to learn thinking and have opportunity to make use of a variety of materials to express their ideas, feelings or opinions. They could import their knowledge into what they are making through thinking with their hands and learn from their experiences.

When zooming in to see the details of the picture, it is seen how arts education could be one of the contexts that are compatible with constructionism (Papert & Harrel, 1991) and might provide a new vision to mathematics education by engaging students to work on their projects so that they use their knowledge of mathematics and discover new ideas in the studio environment (Shaffer, 2005). While visual art is used as one of the contexts for mathematics education to motive students and engage students to learn mathematics, it could also be used as crucial part of "a new line of work that would explore possible synergies in the development of visual-spatial thinking in the visual arts and STEM (Science, Technology, Engineering, Mathematics) domains" (Goldsmith, Hetland, Hoyle & Winner, 2016, p.67).

How visual arts could be integrated with mathematics or other learning fields has been debated in research community. There have been several studies on integration of visual arts with mathematics or transfer of learning in arts to mathematics, particularly geometry. While some of the studies (Hanson, 2002; James, 2011; Marino, 2008) were experimental and indicated positive effects of visual arts on students' performances in mathematics, some of them (Ben-Chetrit, 2010; Walker, Winner, Hetland, Simmons & Goldsmith, 2011.) were quasi-experimental or correlational and found the difference between students who took visual art courses and students who did not take it was either significant or not significant in terms of their performances in mathematics. They were neither true experimental studies nor included random assignment of subjects. In addition to methodological concerns, these studies mostly provide lack of information about how the art-based activities were designed and at what conditions they observed specific outcomes of visual arts and mathematics integration (Winner, Goldstein, & Vincent-Lancrin, 2013). This is what it is seen when zooming in on the picture.

These findings make researchers sceptical about the way of integration of arts with other domains. Most of the studies about art integration and STEAM (Science, Technology, Engineering, Arts, and Mathematics) studies are anecdotal, superficial and mostly popularized the role of arts in other domains (Burton, Horowitz & Abeles, 2000). Researchers need to gain strong evidences of the outcomes of arts integration to other domains with a theoretical basis, which involves specifying the lens through which we are looking at the design of tasks or interpreting students' thinking processes or learning at what conditions. In OECD (Organization for Economic Co-operation and Development) report on the impact of arts education, Winner et. al. (2013) argued this problem and showed promising approach to solve this problem, as stating "The claims for the transformative effects of the arts on non-arts outcomes often exceed the evidence. This does not mean that the claims are false. Rather, they have not yet been shown to be true." (p.41). In this regard, Goldsmith, Hetland, Hoyle and Winner (2016) provided evidences for the relationship between geometric reasoning, spatial reasoning, and artistic envisioning. Visual-spatial thinking could be addressed as a common element between visual arts and mathematics and considered as thought processes that emerge during engagement with tasks involving arts and mathematics integration. Supportively, Newcombe (2010; 2013) pointed out that visual-spatial thinking is crucial for STEM and as well as art and architecture.

In order to understand students' visual-spatial thinking in the context of visual arts and mathematics, particularly geometry, it is important to think about how students thinking processes could be made visible in the environments that involve art making. In the current study, Math-Art Studio Environment was designed by the researcher to achieve this goal. Math-Art Studio Environment was considered as an an environment in which researcher introduced minimalist artworks to students, asked students to observe them and create their own artworks, and critique their own and their friends' artworks. This environment was basically designed on the basis of Studio Thinking Framework (Hetland, Winner, Veneema, & Sheridan, 2013) and studies on visual-spatial thinking (Newcombe & Shipley, 2015; Sarama & Clements, 2009) with the taking into consideration of minimalist artworks that involve explicit use of geometric shapes and forms (Meyer, 2000). Studio Thinking Framework describes the nature of learning and teaching in visual art studios and could be used as a base to design studies on integration of arts with other learning domains (Sheridan, 2011). It basically defines several habits of mind that are taught in the arts studio (e.g. observe, envision, explore, understand art world) and describes three structures of the studio: (1) demonstration (giving lecture and/or introducing artworks), (2) students-at-work (creating artworks), (3) critique (explaining and evaluating artworks). Since it describes nature of the arts studios in a comprehensive manner, it was used to provide a base for Math-Art Studio Environment. After the design of Math-Art Studio Environment based on these previous studies, it was used as a tool to understand how students make use of visual-spatial thinking processes in the context of visual arts and mathematics, particularly geometry.

1.1 The Rationale for the Study

The purpose of this study is justified by explaining the background of the study on the basis of prior research studies through taking into consideration of two main issues: the role of math-art studio environment on interplay between visual arts and mathematics, and identification of students' ways of visual-spatial thinking in such an environment.

Researchers have conducted art integration studies and have popularized the role of arts in learning other domains such as mathematics, science, and history (Burton, Horowitz, & Abeles, 2000). However, discussion is going on about whether art education affect learning in these domains. In the context of visual arts and mathematics, there have been the studies that advocate examining congruent elements of visual arts and mathematics to integrate arts with mathematics (Bickley-

Green, 1995; Burton, Horowitz & Abeles, 2000). In line with this argument, Goldsmith et. al. (2016) discussed this congruence between visual arts and geometry and suggested that visual-spatial thinking could be overlap between visual arts and mathematics in their correlational study. They found students' drawing performances that require visual-spatial thinking was significantly related with their performances in geometry test. They focused on the question of whether students at visual art major transfer what they learned in art courses to the context of geometry. Such an examination of transfer could be affected by the facts that the visual-spatial thinking test that it is assumed to represent the content of art course might not be representative and the results could be mediated by the students' prior abilities rather than learning in art courses (Goldsmith et. al., 2016).

This study suggests looking at this issue from different perspective by proposing to create and examine new synergies between visual arts and mathematics besides investigation of transfer of learning from one domain to another. Perkin and Salomon (1989; 1992) suggested that transfer of thinking skills to another learning domain become rich if teacher deliberately aims to transfer by establishing specific conditions such as searching for connections between two disciplines and direct engagement of students in the integration of two disciplines. This study provides foundation for the current study. It triggers to think about the conditions to be established in integration of visual arts and mathematics even though the aim of the study is not to transfer of thinking skills from one domain to another. Rather, it is to investigate how students think in the environments that integrate visual arts and mathematics. In the current study, this environment is called as Math-Art Studio Environment.

On the basis of this background, this study examined students' visual-spatial thinking processes in the Math-Art Studio Environment that was regarded as an ecology that involves organic relations of nature of the tasks (minimalist artworks with geometry-rich context), implementation of tasks through Studio Thinking, teacher/researcher's role as a coach, and physical environment of an arts studio. It is an environment that maximizes the probability of connection between visual arts and mathematics with the direct use of artworks with geometric shapes based on minimalist art movement (Meyer, 2000) and critical features of art education (Studio Thinking Framework by Hetland et. al. (2013)). The maximization of connection is also increased by investigating visual-spatial thinking that is regarded as an overlap between visual arts and mathematics (Goldsmith et. al., 2016).

This study could be considered as a starting step to establish connection between visual arts and mathematics while recognizing and appreciating other possible connections that one could establish to integrate visual arts and mathematics. It is hypothesized that if Studio Thinking (e.g. observing artworks, envisioning, exploring) is embedded into the tasks with geometric and spatial content, it would result in eliciting and interpreting diverse visual-spatial thinking processes in the Math-Art Studio Environment (see details of embedding studio thinking into the spatial tasks in the part of 3.7.2 in the method chapter and its rationale in the part of 2.3 in the literature review chapter). In this regard, this study aimed to examine how students make use of visual-spatial thinking in a Math-Art Studio Environment in which students are basically asked to observe famous minimalist artworks, create/copy the artworks and critique their own and their friends' artworks. It is assumed that this study would provide strong evidences for students' thinking processes and at what conditions they become visible. In line with the purpose of the study, the main research question of the study is:

- How do seventh grade students make use of visual-spatial thinking in a Math-Art Studio Environment in which students are engaged in geometry-rich artworks through Studio Thinking?

1.2 Definition of the Terms

Math-Art Studio Environment was designed by the researcher as a Studio-Thinking Based-Environment to examine students' visual-spatial thinking processes. This environment was defined as an ecology that involve organic relations between studio works (tasks with geometry-rich and spatial content), implementation of these studio works through Studio Thinking, reactions of students to this environment, teacher/researcher' role as coach, and flexible physical structure of the arts studio. It was used as a tool to make students' thinking visible and examine their thinking processes in the context of visual arts and mathematics.

Studio Thinking is described as dispositional approach to learning and teaching in arts education proposed by Hetland et. al. (2013) in the Projects Zero of Harvard University. They identified eight thinking dispositions that visual art educators intend students to learn: Developing craft, Engaging and Persisting, Expressing, Reflecting, Observing, Envisioning, Stretching and Exploring, Understanding Art World. These dispositions are used to analyse and design of studio art environments and to make students' thinking visible. They also identified three main structures of a studio environment that teachers use to teach these thinking dispositions: (1) demonstration (teacher introduce artists' artworks, shows some techniques), (2) students-at-work (students create their own artworks), and (3) critique part (students explain and evaluate their artworks and their friends' artworks). Both studio thinking dispositions and three structures of studio environment describes Studio thinking in visual art courses. These thinking dispositions are interconnected rather than hierarchical. Their detailed descriptions and how they were used in this study were explained in the part of 2.3 in the literature and the part of 3.7.2 in the method. In the current study, the studio works (tasks in the study) and structure of the environment were designed on the basis of these thinking dispositions to examine students' thinking processes.

Visual-Spatial Thinking is basically described as "thinking about the shapes and arrangements of objects in space and about spatial processes, such as the deformation of objects, and the movement of objects and other entities through space" (Hegarty, 2010, p. 266). There are different types of visual-spatial thinking on the basis of categorization of Newcombe and Shipley (2015). These categories are encoding intrinsic and extrinsic, static and dynamic information. Intrinsic information is related to characteristics of objects (e.g. shapes, arrangements of parts of object, sizes, and orientation). On the other hand, extrinsic information involves the relation between and among objects with respect to each other, or other frames of reference (e.g. locating an object relative to other objects). Another categorization is between static and dynamic information. Static information are related to intrinsic characteristics of objects shape and the relation among objects (e.g. defining objects in terms of their shapes, arrangements, sizes, and orientations or locating an object with regard to another object). Dynamic properties are related to changing or transforming these properties of objects with regard to other objects, frame of reference or to self (e.g. rotating, bending, scaling, relating 2D views to 3D views, and cross-sectioning of objects, perspective taking). Conceptualization of spatial thinking is described in the literature in detail. In the current study, these descriptions were used as a base to search for indicators of students' visual-spatial thinking**.**

Art Studio refers to a flexible physical environment in which students work on their projects during a period of time and is a dynamic place whose arrangements could change depending on the nature of the projects. It was called as atelier or the arts studio in previous studies (Gandini, Hill, Cadwell, & Schwall, 2005; Shaffer, 2005). Arts studio involves a variety of materials to enable students to construct and express their thoughts and ideas. It also involves a smart board to enable students to share their artworks with friends and make a search for their projects, cupboards to keep their materials in it and a wall area that students put their artworks on it.

1.3 Significance of the Study

The significance of the study is explained in terms of contribution to literature (how researchers could benefit from this study) and in terms of its contribution to educational settings (how teachers, students and educational material developers for school and out-of-school contexts could benefit from the findings of the study).

The first aspect that points out the significance of the study is related to its contribution to the literature. Studies on the relation of visual arts and mathematics focused on a wide range of topics (e.g. symmetry, golden mean, pattern, transformations) by designing courses (Kappraff, 1986, Marino, 2008; Shaffer, 1997), infusing arts into mathematics education (James, 2011); or investigating the transfer of learning from one domain (visual arts) to another (mathematics) (Ben-Chetrit, 2010; Goldsmith et. al, 2016). Neither experimental studies nor non-experimental studies did provide enough information about the nature of tasks and specify learning outcomes of art-based activities and at what conditions or at what type of tasks learning outcomes were observed and how transfer of learning occur (Winner et. al., 2013). Whether mathematics education could benefit from arts education is still a controversial issue and further evidence is needed to support such integration. Regarding this issue, as a starting step, this study would contribute to literature by providing evidences of students' visual-spatial thinking processes in an environment that links visual arts with mathematics through Studio Thinking. The findings of the study could provide insights into the discussions among the educational research community about in what way visual arts and mathematics are integrated and whether this integration triggers students to think spatially, if so, how it does trigger visual-spatial thinking.

Another contribution to the literature is that this study attempts to adapt conceptualization of visual-spatial thinking frameworks proposed in cognitive science and psychology domains (Newcombe & Shipley, 2015) to the context of arts and mathematics and also adapt Studio Thinking (Hetland et. al., 2013) to mathematics education context. It might enrich theoretical models regarding studio thinking and visual-spatial thinking by validating them at a different context and revise them through providing a variety of examples of visual-spatial thinking. Because of its interdisciplinary nature, this study not only contributes to the mathematics education literature, but also contributes to the psychology and visual arts literature in terms of investigation of visual-spatial thinking in Math-Art Studio Environment apart from use of factor analytic tests of visual-spatial thinking.

Moreover, development of tasks in relation to visual-spatial thinking could be helpful for future research to develop tests to measure individual differences in students' visual-spatial thinking, which is considered as crucial ability for STEAM careers (Newcombe, 2013; Uttal & Cohen, 2012). Also, documentation of thinking patterns could be important to design of learning environments to improve students' visual spatial thinking. What type of tasks triggers what kind of visual-spatial thinking is a crucial question to thinking about the design of a studio environment, especially for studies on STEAM.

The second aspect that points out the significance of the study is related to contribution of the findings to the educational settings. The possible contributions to educational settings are described in terms of students, teachers, and educational material developers' perspectives. From students' perspective, they could get opportunities to use their knowledge of mathematics and make their own decisions in order to make artifacts that are meaningful to them if such integration programs could exist; particularly, for students who are interested in visual arts and mathematics (Papert & Harrel, 1991). There could be students who uses imagistic or analytic or both of them (Cohen & Hegarty, 20102). Students with different characteristics could get opportunities to express their ideas through diverse ways such as through words in speaking and writing, gestures, drawings (sketches), communicating with themselves and others. Findings of this study might contribute to the efforts for intuitively raising students' awareness regarding the fact that mathematics involves not only making computations but also spatial representations and transformations of shapes, and regarding the fact that visual arts involves not only affective components (expression of feelings) but also cognitive component (representing (drawing) what they imagined in their minds in a coherent and accurate way) (Efland, 2002).

When the possible contributions of this study are interpreted from teachers' perspective, this study could inform teachers about how, when, and where to use Studio Thinking to make students' thinking visible and understand students' difficulties and needs through identification of their thinking patterns. Both mathematics and art teachers could use the studio works that were designed to make students' visual-spatial thinking visible in the current study. This study also becomes a guide for teachers to understand their role in such an environment and to overview their own practices. With regard to visual-spatial thinking, teachers could better understand what visual-spatial thinking is and its use in the context of visual art and mathematics integration since the findings of this study would provide particular cases of visual-spatial thinking. It is crucial to make sense of students' actions and expressions in such a context, especially making sense of students' drawings, and understand individual differences between students' visual-spatial thinking processes at different tasks such as observing artwork, creating artworks, and critiquing artworks. This, in turn, would provide new methods of assessment that is made during actively engaging with the tasks (Tishman & Palmer, 2006).

From the perspectives of educational material developers, this study could provide sample studio works to design educational materials for out-of-school programs (e.g. summer school), for art and science centers, museums or other types of educational settings. Particularly in the art and science centers in Turkey, there have been courses for students who are talented at specific domains such as mathematics, science, visual arts, and music. This study would lead curriculum developers to thinking about the needs of students who are talented at both mathematics and visual arts. Besides out-of-school contexts, the studio works used in the current study could be considered as education materials to develop curriculums with interdisciplinary vision in the schools such as art-based curriculum (Marshall, 2015) or visually-oriented mathematics curriculum (Rivera, 2011). This study could also shed light on the nature of mathematics classrooms in public schools in the future by suggesting to consider mathematics classrooms as arts studios in which students have active role in using and learning mathematics to create meaningful artworks for themselves through imaginative process.

CHAPTER 2

LITERATURE REVIEW

In this chapter, grand theory of this study (constructionism), previous studies on approaches to art integration including frameworks in arts education, potential of Math-Art Studio Environment, visual arts and mathematics, characterization of visual-spatial thinking, studies on visual-spatial thinking, and national studies were examined respectively. At the end of the chapter, summary of these studies and the place of this study in the literature were presented.

2.1. Constructionism

In the current study, constructionism was regarded as a lens for understand students' visual-spatial thinking processes in the Math-Art Studio Environment and to design tasks for eliciting students' thinking processes. Constructionism was proposed by Seymour Papert who worked with Piaget between 1950s and 1960s and, then became cofounder of Artificial Intelligence Lab at Massachusetts Institute of Technology (Kafai, 2006; Kafai & Resnick, 1996). In the 1970s, Papert expanded Piaget's constructivism and proposed a learning theory, called as constructionism.

Papert defined constructionism as "learning by making" that refers to constructing knowledge especially when students are making or building artifacts that are personally meaningful for them. These artifacts have potential for enabling students to think and learn in a self-directed way on the basis of their' conversation with artifacts (Ackermann, 2001). Papert and Harrel described constructionism by explaining its difference from Piaget's constructivism:

> Constructionism—the N word as opposed to the V word— shares constructivism's view of learning as "building knowledge structures" through progressive internalization of actions... It then adds the idea that this happens especially felicitously in a context where the learner is consciously engaged in constructing a public entity, whether it's a sand castle on the beach or a theory of the universe (Papert & Harrel, 1991, p.1)

The description of constructionism above indicates that constructionism and constructivism have common points that they both emphasize constructing knowledge by connecting old and new knowledge. Both of them have agreement on which knowledge is not memorized; rather it is constructed through interaction with the world. While Piaget explain the ultimate goal of education as abstract thinking and referred to concrete thinking as lower level of thinking, Papert claimed concrete thinking could also be advanced thinking. Constructionism also differs from constructivism in term of its emphasize on people's expressing their feeling and ideas with a media or a tool. People make their feeling and ideas concrete and communicate it with other people by giving importance on their own preferences (Kafai, 2006).

The story behind this learning theory was based on Papert' observation of an art course. He explained how he inspired from this art course in which students were making their own soap sculptures and worked on their project throughout several weeks. What attracted him in this course was that students created personally meaningful and desirable artifacts by continuing their project for a period of time, shared with other people their artworks and observed their works. He dreamed that students in mathematics courses could be like those in the art course:

> …the art room I used to pass on the way. For a while, I dropped in periodically to watch students working on soap sculptures and mused about ways in which this was not like a math class. In the math class students are generally given little problems which they solve or don't solve pretty well on the fly. In this particular art class, they were all carving soap, but what each student carved came from wherever fancy is bred and the project was not done and dropped but continued for many weeks…An ambition was born: I want junior high school math class to be like that. I didn't know exactly what "that" meant but I knew I wanted it. I didn't even know

> what to call the idea. For a long time it existed in my head as “soap-sculpture math” (Papert, 1991, p.5).

On the basis of his dream, in one of his speeches to Japan educators, he suggested to use mathematics as a tool to make meaningful objects rather than doing mathematics like in the mathematics courses (Papert, 1980s). These objects could be “a robot, a poem, a sand castle, or a computer program” (Kafai & Resnick, 1996, p.1).

In conclusion, in the current study constructionism could provide a lens for interpreting students’ thinking processes in the Math-Art Studio Environment in which students use mathematics as a tool, create and share their artworks with their friends. To interpret students’ thinking and design Math-Art Studio Environment, approaches to arts integration and studies that integrated arts and mathematics were examined in the following parts.

2.2. Education through Arts: Approaches to Arts Integration

Education through arts has been a considerable attention among educational research community. Several researchers have pointed out the role of arts in education and how arts could be integrated with other subjects such as mathematics, science, and history (Bresler, 1995, Hetland et.al, 2013; Marshall, 2010; Tishman & Palmer, 2006). In one of these studies, Bresler (1995) described four approaches for integrating art; subservient, co-equal, affective, and social integration approaches. Subservient approach is used to create a joyful learning environment. For example, drawing some geometrical shapes or singing a song in a unit could be activities for subservient approach. Co-equal approach requires integration of cognitive skills of both disciplines. Thus, this approach is mostly conducted with art experts collaboratively. The affective approach emphasizes the role of arts as a tool for self-expression and expose own creative ideas. Lastly, the social integration approach is used to support for sociality of school community. The co-equal approach seems too challenging, but it supports higher order thinking skills.

In line with the approach of co-equal proposed by Bresler, Marshall (2010) conceptualized art integration as an approach to learning in arts and other domains rather than just as a strategy for learning in other domains. She describes art as a lens for looking into and exploring content rather than as just aesthetic object. Art provides a different lens to other domains through imaginative inquiry. She proposed several strategies for art integration; depiction (reproduction of the object), projection (imagination of what something might be), reformatting (portraying something in new context), mimicry (experiencing the process or methods of experts in other disciples to make art), and metaphor (describing something through other things) (p. 14).

While these studies provide us approaches and strategies to integrate arts with other learning domains, some other studies described the nature of art courses in terms of thinking dispositions or routines that become visible during art making or analyzing artworks, which are transferable to other learning domains. There are two well-defined and interrelated thinking approaches in art education context: Artful thinking (Tishman & Palmer, 2006) and Studio Thinking (Hetland et. al., 2013). Artful Thinking approach was characterized based on research in Harvard Zero Project. Researchers suggested artful thinking as thinking dispositions that could be used for identifying students' thinking and transferable to other learning contexts. They are: Questioning & investigating (asking questions about complex situations, finding out new problems to this situation through wonder and curiosity), observing & describing (looking carefully and making detailed descriptions), reasoning (building claims with evidences), exploring viewpoints (considering different perspectives or views), comparing & connecting (exploring connections or relationships between things, making connection between diverse things), and finding complexity (recognizing parts or relationships between parts, dimensions of complex things).

Studio Thinking is also described as thinking dispositions or habits of mind that emerged in visual art studios (Hetland et. al., 2013). It involves main eight thinking dispositions: Developing craft (learning to use tools and care of them), engaging and persisting (working on a task or project a period of time rather than giving up), expressing (conveying meaning regarding feelings, ideas or thoughts), reflecting (describing own working process and making a judgement about it), observing (looking at something closely, seeing what is seen and not seen), envisioning (mental depiction of something that is not seen directly, and imagine possibilities or further steps while constructing an art work), stretching and exploring (making attempts to do new things and discover what might happen), understanding art world (learning history of art, artworks from past to contemporary time and learning to become a part of community of art).

These studies suggest several approaches for learning through arts and enlighten about how to design tasks or courses with visual arts in various educational domains. In this regard, these thinking approaches could be adapted to the context of mathematics education. To be precise, in the current study artful thinking and studio thinking could be used to design the tasks, called as studio works in the Math-Art Studio Environment. This, in turn, would allow to examine students' thinking processes in such an environment. In the next part, details of these thinking approaches were described.

2.2.1. Frameworks of Thinking Approaches in Art Education

This part explains the thinking approaches used in the art education. There are two well-defined and interrelated thinking approaches emerged in art education context, namely, Artful thinking (Tishman & Palmer, 2006) and Studio thinking (Hetland et.

al., 2013). They put emphasis on dispositional approach for learning and teaching in the context of arts that aims not only to develop students' skills, but also inclination and alertness to use particular skill. Although two frameworks share common points, they differ from each other from several aspects. It is discussed at the end of this part.

The first framework is Studio Thinking developed by Hetland et. al. (2013) in Harvard University Project Zero. Studio thinking is described as thinking dispositions or habits of mind that teachers tried to teach in the visual art studios, identified by Hetland et. al (2013) based on their observations of studios. They identified main eight thinking dispositions: Developing craft, Engaging and Persisting, Expressing, Reflecting, Observing, Envisioning, Stretching and Exploring, Understanding Art World. These thinking dispositions are interconnected rather than hierarchical. These thinking dispositions could be used to analyse and design of art studio environments and could be transferable to other learning domains. Each of studio thinking dispositions was briefly described in the Table 1. In addition to thinking dispositions, they also identified three structures of studio environments: demonstration, students-at-work, and critique. Each of the studio structures was described in the Table 2.

Table 1. *Studio Thinking Dispositions (Hetland et. al., 2013)*

Studio Thinking Dispositions	Descriptions
Develop Craft	Using tools (technique) and considering careful usage of tools & materials and having a sense of which tools and materials to use (studio practice)
Engage and Persist	Working on a task or project a period of time rather than giving up.
Express	Conveying meaning regarding feelings, ideas, or thoughts in the artworks
Reflect	Describing working process (e.g. what kind of difficulties students have, what and why they did something and what they are planning to take further steps) and making a judgment about their own art works and working process, and as well others' artworks.

Table 1 (Continued)

Observe	Looking at something closely, seeing what is seen and is not seen.
Envision	Mentally depicting something that is not seen directly and imagination of possibilities or further steps while constructing an art work.
Stretch and Explore	Making attempts to do new thing, discovering what might happen, finding out new possibilities
Understand Art World	Learning the history of art, artworks from past to contemporary time (understand art world as domain) and learning to become a part of the art community (understand art world as community)

Table 2. *Three Structures of Studio Environment (Hetland. et. al., 2013)*

Structures in the Studio	Description
Demonstration	Teacher presents visual contexts such as artworks to engage students into the making art-work, shows some techniques that helps students in making art, and explains the assignments
Students-at-Work	Students work independently and create their own artworks. Students have an opportunity to share their ideas or thoughts in an informal way and start using their ideas or plans to carry out them. Teacher communicates with the student through one-to-one conversation.
Critique	Students examine their own and their friend's works. Students explain their artworks to others. It involves mostly students' interaction with each other and with teacher.

The second framework is Artful Thinking proposed by Tishman and Palmer (2006). Artful Thinking was suggested as a dispositional approach to art education that could be used by teachers for making students' thinking visible, identifying students' thinking routines, and designing instructional process. They described six dispositions of thinking that could be used both in art classes or transferable to other learning contexts: Questioning & investigating, observing & describing, reasoning, exploring viewpoints, comparing & connecting, and finding complexity (p. 8). These thinking dispositions are interconnected rather than hierarchical. For example, observing is inherently related to reasoning, which might in turn lead to questioning or connecting & comparing. Each of thinking dispositions are briefly described in the Table 3.

Table 3. *Artful Thinking Dispositions (Tishman & Palmer, 2006)*

Artful Thinking Dispositions	Description
Reasoning	Building arguments or claims based on evidences and make logical and coherent interpretations about own or others' art-making process.
Exploring Viewpoints	Considering different perspectives/views and looking at things through these viewpoints during discussion of a topic.
Finding Complexity	Recognizing parts or relationships between parts of a topic or artwork and unfolding complex things.
Comparing and Connecting	Exploring connections or relationships between things. It also involves metaphorical thinking and making analogies or comparing previous knowledge with new ideas.
Questioning and Investigating	Asking questions with the consideration of complexity of the situation and finding out new problems relevant to this situation through the power of wonder and curiosity.
Observing and Describing	Looking at the artworks carefully and making detailed descriptions.

In summary, Artful Thinking and Studio Thinking Frameworks put importance on dispositional approach to teaching that is also used to make students' thinking visible in art courses or in other domains. They involve similar thinking dispositions. For example, they both involve observing, envisioning (creative questions in Artful Thinking), exploring (questioning and investigating in Artful Thinking), reflecting (reasoning in Artful Thinking), understanding artwork (investigating artworks in Artful Thinking). Although they share common habits of mind, they also differ from each other. While Studio Thinking Framework was described for studio art classes and suggested to be used in other domains, Artful Thinking was designed for all teachers to make connection between art and other domains. Artful Thinking only focuses on investigating and acknowledging artworks. Unlike Artful Thinking, Studio Thinking involves both investigating and making artworks within three structures of studio: demonstration, students-at-work and critique parts. In the current study, Studio Thinking was used dominantly in designing studio works to make students' visual-spatial thinking. It also shares several habits of mind with Artful thinking as described above. In the following part, the reason why Studio Thinking was used was explained and crucial elements of Math-Art Studio Environments was determined on the basis of previous studies.

2.3. Potential of Math-Art Studio Environment on Eliciting Students' Thinking Processes

This section presents how the Math-Art Studio Environment rooted in the previous studies. In order to understand students' visual-spatial thinking processes in geometrically-rich context when students are engaged in artful activities in an art studio, it is important to take into account of the questions of "How do we make students thinking visible in the arts studio" and "How do we understand students' visual-spatial thinking processes in geometrically-rich context". They are basic and crucial questions that help to design Math-Art Studio Environment. Each of them was interrogated on the basis of previous studies.

In order to make students' thinking visible, it was designed on the basis of two interrelated thinking approaches emerged in art education context, namely, Studio thinking (Hetland et. al., 2013) and Artful thinking (Tishman & Palmer, 2006). Studio Thinking was dominantly used in the current study. In order to make students' visual-spatial thinking processes visible, previous studies on visual-spatial thinking were examined and Studio Thinking was embedded into the spatial content of studio works. In the literature the researchers emphasized several visual-spatial thinking processes (Clements, 1998, Linn & Peterson, 1986; Newcombe & Shipley, 2015). The spatial content of the study was mostly determined on the basis of these studies. Six studio works were designed as a part of larger project and the first three studio works were the focus of this study (see Table 9 in the method chapter).

To conclude, in order to elicit students' visual-spatial thinking processes in Math-Art Studio Environment, artful thinking/studio thinking was embedded into the studio works that has spatial content. In other words, it became a tool to understand students' thinking processes. To understand students' visual-spatial thinking processes, studio works were designed so that it involves spatial and geometric-rich content, which resulted in identification of several pedagogical principles of Math-Art Studio Environment. Details of these principles were explained in the part of 3.7.2 in the method chapter.

2.3.1. Elements of Math-Art Studio Environment

This part explains the rationale behind the embedding of Studio Thinking into the studio works that has spatial content to understand students' visual-spatial thinking processes. It also provides a basis for pedagogical principles of Math-Art Studio Environment in the current research (3.7.2 in the method). On the basis of previous studies several elements were identified: Studio thinking, teacher/researcher role, structure of the art studio, and physical environment (Figure 1).

Each element of Math-Art Studio Environment was described by justifying how it has a role in making students' thinking visible in the next part. Some of these elements would directly be related with visual-spatial thinking such as observing and envisioning. It is assumed that synergy between these elements might result in understanding of students' thinking processes in a comprehensive manner.

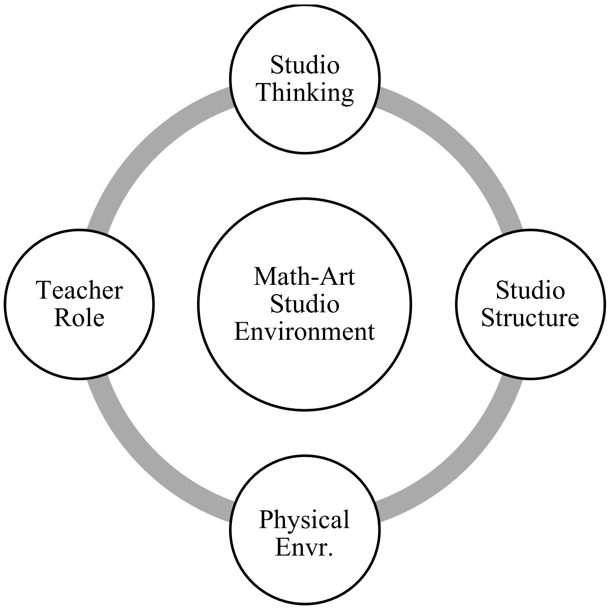


Figure 1. Elements of math-art studio environment to make students' thinking visible, mostly based on the study of Hetland et. al. (2013)

2.3.1.1. Studio Thinking Dispositions

There are several thinking dispositions that are used in the art education context (Hetland et. al, 2013; Tishman et. al., 2006). They are understanding art world, observing, envisioning, reflecting, stretching and exploring, developing craft, and finding complexity. The rationale of using each thinking disposition in the current study was explained in the following.

First of all, understanding art world is one of the aims of art education (Hetland et. al, 2013). It involves learning the history of art, artworks from past to contemporary time. To help student understand art world, art teachers introduce specific paintings of artists to observe, to understand that they might have similar problems or difficulties with those of students, and point out the similarities between the techniques used by artists and students. In the current study, artworks of minimalist artists with different styles were used as a tool to elicit students' visual-spatial thinking (see artworks in Table 10 and Table 11 in the method). The reason of such a focus on minimalism is that examining artworks in minimal art might elicit students' visual-spatial thinking processes in geometrically rich contexts since minimalist artworks consist of single or repeated geometric shapes or forms (Meyer, 2000).

The second key element is the act of observing (Hetland et. al., 2013; Tishman et al., 2006). The origin of observing could be explained by the term of visual perception. According to Arheim (2007), as a prominent researcher in visual thinking, visual perception begins with encoding remarkable arrangement of objects. As an observer carefully looks at an object, his eyes become more equipped to see the details of object. Eyes as an invisible finger touch the space around objects and explore the features of objects and relation between objects. Thus, observation of a visual context could be a tool to identify and making sense of spatial information within an object and among objects. In the current study, it is hypothesized that observing artworks with geometrical shapes and reflecting on them could be a way to elicit students' visual-spatial thinking process. In other words, it serves a tool to explore how students make use of visual-spatial thinking.

The third key element is envisioning. Envisioning is described as ''Learning to picture mentally what cannot be directly observed and imagine possible next steps in making a piece.'' (Sheridan, 2011, p.22). Envisioning could be an important property of a task to elicit students' visual-spatial thinking since envisioning is intrinsically related with visual-spatial thinking. It involves imagination of properties of objects or transformation of objects that are not seeing directly (Tversky, 2005). Thus, the tasks requiring envisioning could tap the use of visual-spatial thinking.

The four key element is reflecting. There could be three ways of reflecting; speaking, writing, and drawing. Speaking is one of the ways of reflecting own thinking process. Students talk about what they are doing, explain why, and make judgments. This helps teachers to see students' thinking process (Hetland et. al., 2013). This reflection process is also explained with the thinking routine of reasoning in the artful thinking framework proposed by Tishman et al. (2006), in which students are encouraged to reflect thinking process on the basis of reasonable evidence. Writing notes is also another way of reflecting that helps to understand how they think. Students are asked to write their ideas and what is on their mind (Tishman et. al., 2006). Sketching is referred as another way of eliciting students' visual-spatial thinking. Sketching or drawing serves as a tool for presentation of structural properties of objects and relations between them. It also helps to examine, reflect and making corrections on it (Goldsmith et. al., 2014; Clements, 1998). Supportively, Suwa (2003) suggest that sketching serves a means not for reflecting ideas but also discovering new ideas with the re-examination of sketches. This helps designers to detect new perceptual cues in the sketches. In the current study, it is hypothesized that sketching could be a tool to elicit students' visual-spatial thinking. The reason behind such an assumption is that representation of structural features in the space is one of visual-spatial thinking processes. In addition, as students revise their sketches, they could detect and reflect new spatial cues.

Stretching and exploring is another tool to elicit students thinking process. In the arts education students are asked to try to do new things and discover what might happen (Hetland et. al., 2013). Each try of students might have elicit different students' visual-spatial thinking processes and find out new possibilities of spatial information. Supportively, Clements (1998) suggest that students should explore geometric shapes and their properties by their hands, bodies, or eyes rather than looking in a passive way. Students can explore shapes through drawing, using concrete materials such as sticks to build geometric shapes. Thus, it is assumed that tasks involving experimenting new possibilities though drawing, touching, moving around an object could elicit students' different visual-spatial thinking processes and their difficulties and strengths in encoding and representing spatial information.

Developing craft is one of the crucial habits of mind in arts education. Developing craft through technique refers to learning to use apparatus or tools such as brushes, clays, cutting tools, paint and pencils in the arts education. Through developing technique, students can learn fundamental ways of perspective drawing, shading, and combining colors (Hetland et. al., 2013). In this study it is assumed that students might elicit some ideas about the relationship between shapes when they developed a technique for drawing shapes or using specific materials such as ruler and protractor to measure the lengths and angle. For example, Leon Battista Alberti as an architect found a technique for perspective based on his geometrical explorations.

Finding Complexity is another thinking routine in the art education. In the arts education, the routine of finding complexity is used for detecting parts of or pieces of a topic or artwork or objects (parts) and understand how it works (purposes), and describing complexity of the things with the consideration of relationship between parts. Students are asked to place their observations, facts or ideas abut a topic to the complexity scale that involves a rating from simple to complex, and explain why they put it that point in the scale. This routine might be adapted to the context of spatial education. Spatial thinking involves finding basic shapes that are embedded in a complex figure (Clements, 1998; Kastens & Ishikawa, 2006). Thus, use of tasks that requires to find simple shapes in embedded figures might elicit students' visual-spatial thinking. In this study, in order to elicit students' thinking process about hidden figures, researcher prepared tasks that require finding geometric shapes and forms that are difficult to see at first glance and asked students place several artworks on the complexity scale to understand their perceived difficulties.

In summary, there are several thinking dispositions that are used in the art education context. In the current study, they were adapted to the context of the study. Even though there is not any scientific evidence that they are directly related to visual-spatial thinking, they could be used as a way of making students thinking visible and transferable to the other contexts (Hetland et. al., 2013). On the basis of this assumption, several pedagogical principles were designed to elicit students' visual-spatial thinking processes (see the part of 3.7.2 in the method chapter).

2.3.1.2. Teacher/Researcher Role

Teacher role is an important factor to gain insight into students' thinking process. In the arts education teacher behaves like a coach. He/she gives demonstrations, provides suggestions and does evaluations to help students develop their artworks (Hetland et al., 2013). The role of a coach might help students to reflect on his/her performances, weaknesses and strengths during a tasks such as creating artwork. When teacher has a role as a coach, students has an opportunity to control over their own work and but also to get coach's help when they were stuck. Thus, reflective practices with teacher might help students think new ways of thinking or elicit what students already thought (Hetland et al, 2013; Schön, 1988).

2.3.1.3. Studio Structure

Studio structure is one of the important elements that might have a role in eliciting students' thinking process. Studio structure shows in what ways teacher and students interact in art education. Hetland et al. (2013) identified four studio structures observed in art studios: demonstration, students-at-work, critique, and exhibition. In the current study, the first three structures were taken into consideration.

First of all, during demonstration process, teacher presents visual contexts such as artworks to engage students into making artwork and shows some techniques that helps students create artwork. Students make use of these artworks to create their own work by inspiration rather than directly use them. The emphasis of visual contexts in demonstration part could encourage students to make observation of spatial relations in the artworks carefully and reflect on them. It could also help students relate this visual information to their performances or thought process in the students-at-work and critique parts, and to their future artworks. Thus, it might result in eliciting new processes of students' thinking.

The second structure of the arts studio is the part of students-at-work. At this part students work independently and create their own artworks. Students have an opportunity to share their ideas or thoughts in an informal way and starts using their ideas or plans to carry out them. During this part teacher communicates with the student one by one. Thus, teacher can be able to observe students' thinking paths: what they imagine to do, how they do it, what kind of changes they make, what kind of difficulties they have over a period of time. Thus, this structure of the studio work could be one of the milestones of the design to elicit students' thinking.

The third structure of the arts studio is the critique part. It involves mostly students' interaction with each other and with teacher. Students examine their own and their friend's works. In this part, students are asked to express their ideas and thinking process verbally and using body language. Hetland et al (2013) describes this process as a reflective process in which students describe and evaluate their own work and/or others' work. Thus, in the current study it is assumed that this structure of the arts studio could be driving factor to express thinking process.

2.3.1.4. Physical Environment

Physical environment could be a crucial element for students to reflect on their performances. In the studio-based environments students feel flexible to sit where they prefer to work, to make use of different materials, eat and have a break whenever they want, to make changes on their projects on the basis of their decisions. Students can listen music to encourage themselves to work on their artworks. Such a flexibility in physical environment might result in students' intensive engagement on their projects (Cossentino & Shaffer, 1999; Hetland et. al., 2013), students' interconnected use of hand mind, and use different ways of expressivity (Gandini et. al., 2005). Supportively, Cadwell, Geismar-Ryan and Scwall (2005) give importance on the nature of studio-based environments which provides organic, living, and complex relations between students, and between students and teacher and opportunities for express thoughts and ideas.

In summary, there four main elements of Math-Art Studio Environment to make students' thinking visible: studio thinking dispositions that were embedded within spatial content, teacher role, studio structure, and physical environment. The synergy between these elements would allow us to gain insight into students' thinking processes in a comprehensive way.

2.4. Current Art Integration Programs

Recently several researchers have worked on the transdisciplinary educational reform of STEAM (Science, Technology, Engineering, Arts and Mathematics). They attempted to conceptualize STEAM, what kind of conclusions arises from the teaching practice of STEAM with its difficulties and affordances and the role of Arts in STEAM practices.

How STEAM practices are addressed among the researchers is the central question to understand the nature of STEAM. Various researchers referred to arts as a catalyzer to promote students creative and innovative skills (Clapper and Lafratte, 2015, Connor et. al., 2015, Ghanbari, 2015, Land, 2013, Madden et. al, 2013). Connor et. al. (2015) worked with engineering students in their STEAM projects. They addressed art to promote creativity and innovation. They integrated art-based pedagogies such as studio-based learning, inquiry, problem, and project-based learning, formative assessment and considering students' autonomy and focused on engineering process. However, how they integrated art-based pedagogies is not clearly described in their study.

Ghanbari (2015) investigated two STEAM programs implemented in a university. He referred art as a tool to foster creativity and broaden students' perspectives. They group these programs in two types; art-science program and art-technology program in which they integrated one of the arts disciplines with one of the STEM disciplines. These programs were organized on the based of sociocultural theory and experiential learning theory. Various scholars were asked to give lecture on the arts and science disciplines. Similarly, Clapper and Lafratte (2015) identified the role of art and design in STEM practices for college students as a tool to foster engagement of students, students' interest, develop their problem solving skills and creative thinking. They analysed two approaches to STEAM projects; separate-courses and same-course method. Students' task was to develop mobile web applications for clients. STEM students and arts students participated to these courses. They uncover some challenges regarding admistrative and multidisciplinarity issues. Although separate-course method has more advantages in terms of administrative aspects, same-course method provides more opportunity for multidisciplinary learning even though it is challenging.

While art is addressed in terms of instrumental approach, some of the studies also emphasize arts is for its own sake rather than just a catalyzer to promote learning in other domains (Guyotter, Sochacka, Constantino, Walther, & Kellam, 2014, Quigley and Herro, 2016, Sochacka, Guyotte & Walther, 2016). Quigley and Herro (2016) examined the implementation of STEAM practices in the middle schools and described STEAM as a transdisciplinary approach to learning with a focus on problem solving. Thus, STEAM practices are mostly related with problem-based learning and project-based learning. In their analysis of teachers' practices, they identified key elements in STEAM practices; instructional approaches, student interest, student choice, technology integration, problem-based, authentic assessment, transdisciplinary teaching, arts integration and collaboration (p.417). They addressed art to foster motivation, engagement, imagination, critical thinking and creativity. They approached to art in two ways; creative and expressive art

("How does the life of sea turtles change when they migrated from one place to another" and "how do you feel when you change your home or school due to migration?) and design & technical art (Designing organ models on technological software) (p. 422). The conclusions of the study indicate that teachers need to be engaged in interdisciplinary and multidisciplinary approaches rather than transdisciplinary approaches. Most of the teacher mostly focused on arts integration as design arts rather than expressive arts and they needed to work with art experts.

Sochacka et. al. (2016) reported their experiences in a collaborative project in the context of art and environmental engineering. They conceptualized STEAM education as a process-oriented approach to problem solving in a transdisciplinary context. They explained the aim of STEAM education as a combination of several type of goals; foundational goals (creative thinking), application goals (exploring different tools and media), integration goals (connection between different disciplines), human dimension goals (knowing yourself), caring goals (value and consider environmental problems and appreciating arts role), learning to lean goals (metacognitive process). In their projects, students in art, landscape architecture, and civil and environmental engineering departments worked together.

In conclusion, these current art integration studies used arts an instrument or appreciated arts as a major discipline on its own. They mostly put emphasize on the studio-based learning, project-based-learning, inquiry-based learning, or problem-based learning. However, most of them provided lack information about how they designed these learning environments. This study examined a possible synergy between visual arts and mathematics by providing detailed information about the study while recognizing and appreciating visual arts and mathematics as major disciplines. To investigate possible synergy between visual arts and mathematics, it is important to investigate how previous studies examined the integration of visual arts and mathematics. The studies on visual arts and mathematics were examined in the next part.

2.5. Visual Arts and Mathematics in Education

Studies on integration of visual arts and mathematics in the context of education focused on a variety of topics: designing educational materials or activities (Frantz, Crannell, Maki & Hodgson, 2006; Hart & Heathfield, 2017; Jarvis & Adams, 2007; Kappraff, 1986; O' Dell, 2014; Wilcock, 2014), and examining the effect of art-based instruction on mathematics performances (Ben-Chetrit, 2010; Hanson, 2002; James, 2011; Marino, 2008), engagement in mathematics (Hart &Heathfield, 2017; James, 2011) and attitude towards mathematics (Healy, 2004; Marino, 2008); aesthetics and mathematical problem solving (Sinclair, 2006); the relation between visual arts and geometry (Goldsmith et. al., 2016; Walker et. al., 2011).

These studies mostly focused on different concepts of mathematics and geometry such as symmetry (Shaffer, 1997), space filling, similarity and proportions, golden mean, transformations (Kappraff, 1986), tessellation, origami, Islamic pattern, op-art, quilt patterns (Ugurel Okbay, 2013), polyhedra (Hart & Heatfild, 2017; Morgan, Sack, & Knoll, 2010); fractal geometry, spirals, and golden ratio (Boles & Newman, 1988), perspective drawing and algebra (Frantz et. al., 2006), anamorphosis (Fenyvesi & Hähkiöniemi, 2015), tessellation (Hähkiöniemi et. al., 2016; Marino, 2008).

While most of the experimental studies on visual arts and mathematics integration found positive effect of art-based learning environment on mathematics or geometry performance (Hanson, 2002; James, 2011; Marino, 2008), there was a study that did not find significant effect of visual art courses on geometry performances (Ben-Chetrit, 2010). In one of the studies which found significant positive effect of art-based instruction, James (2011) examined students' (from third to fifth grades) mathematics performances and engagement in the mathematics courses when arts was infused into mathematics classrooms. He conducted an experimental study by forming two groups: experimental and control group. Students in the experimental group were taught multiplication concept of mathematics through arts by a teacher who were educated in art infusion. Students in the control group were taught the concept of multiplication without using arts. The researcher explained objectives of each courses in which they both used visual arts and music. However, it is not clear how he designed the content of course and how the teachers implemented them. He found significant positive effect of art infused course on students' performance in a test of multiplication for each grade level in the experimental group (based on pre-test and post test). On the other hand, Ben-Chetrit (2010) investigated effect of visual art courses on geometry and measurement concepts by comparing geometry scores of 10th grade students who took visual art courses and students who did not take it. They did not find significant difference between students' scores on the basis of two-way ANOVA analysis. The researcher suggested to investigate an art course with infusion of mathematics and art course without infusion of mathematics for further studies.

Despite of the high number of quantitative studies, some studies was conducted with qualitative analysis (Shaffer, 1997) and with correlational analysis (Goldsmith et. al., 2016; Walker et. al., 2011). Regarding qualitative studies, for example, Shaffer (1997; 1999; 2005) worked with twelve students at Grade 9 and Grade 10 in a mathematics studio. He investigated students' understanding of symmetry, visual thinking in mathematics and their attitudes towards mathematics through the project combining art and mathematics, called as The Escher's World project. One of the most striking conclusions is that expressive problems and studio learning environment encouraged students to control their own learning. The nature of expressive problems in art making process suggests multiple solutions and these solutions could be evaluated regarding their appealing, aesthetic, or desirability in social, economic or political domains. Shaffer (2005) described students' conceptual insights of mathematics under three categories; (a) students' statements about underlying mathematical concepts of activities (b) about properties of mathematical concepts (c) about students' recognition of these properties in their designs. The results of the study indicate that students develop mathematical understanding about symmetry and transformations by using design strategies and with the desire of exactness and fitting when they make something wrong and not appealing.

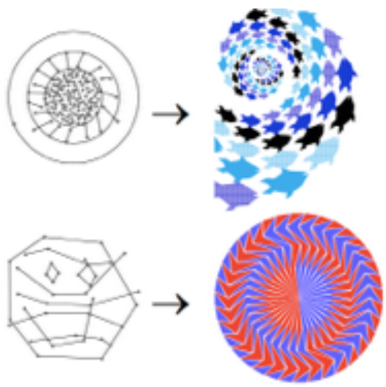


Figure 2. Two students' first sketches and final artworks in Escher World Projects. Adapted from "Studio Mathematics: The Epistemology and Practice of Design Pedagogy as a Model for Mathematics Learning" by D. W. Shaffer, 2005.

Supportively, Sinclair (2006) argues that aesthetically rich learning environments might play important role in mathematical problem solving. For example, a student is asked to construct Theo van Doesburg's work called as Arithmetic Composition using Geometers' Sketchpad (see Figure 3). The student approached to the problem by using approximation as a mean for heuristic thinking and with the aesthetic experiences of exactness and fitting.

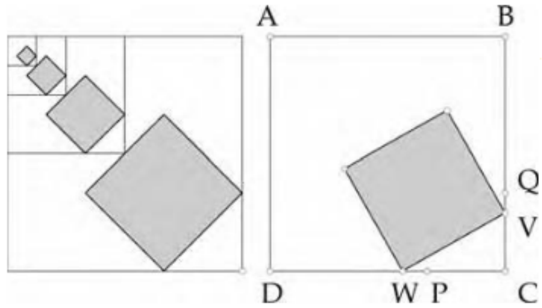


Figure 3. Theo van Doesburg's artwork (Arithmetic Composition) and a student' reconstruction of the artwork. Adapted from "Mathematics and Beauty: Aesthetic Approaches to Teaching Children" by N. Sinclair, 2006, p.104.

Some of the studies were correlational studies that investigated the relation between visual arts and mathematics (Goldsmith et. al, 2016; Walker et. al, 2011). They investigated the transfer from arts education to geometry and investigate correlation relation between visual arts and geometry. Walker et. al. (2011) found that students at visual art department had higher performance on geometric reasoning compared to the student of psychology department. In the further study, Goldsmith et.al. (2016) investigated students (from start of 9th grade to end of 10th grade) at visual arts and theatre departments of an Arts Academy. They investigated two groups' artistic envisioning, geometric reasoning, and visual-spatial thinking. They found that the scores in artistic envisioning test predicted the scores in geometric reasoning. They suggested visual-spatial thinking could be intersection of visual arts and geometry. These studies provided valuable contributions to the literature. On the other hand, how does transfer occurred still remains questionable. There could be mediating factors that affect this relationship (Winner et. al., 2013).

Another important point arising from the literature was that while some of studies (Hanson, 2002; James, 2011) focused on integration of arts into mathematics education for all students; some of them integrated mathematics into art, design or architecture courses for students at art and design departments (Marino, 2008; Kappraff, 1986). For example, Marino (2008) focused on tessellation concept even though the researcher also included other topics of mathematics related with art such as symmetry, golden ratio, fractals, and solids. He designed materials to integrate mathematics into art and design course on the basis of van Hiele levels of thinking and investigated the change in their knowledge regarding the topic of tessellations and in attitude towards mathematics. The course included the use of technology and manipulatives and was conducted on the basis of methods of discourse method and problem solving. Students were asked to construct artwork by using these concepts of mathematics. Pre-test and post test was implemented before and after the course. The test for measuring tessellation was prepared by the researcher.

Similarly, Kappraff (1986) used a different approach to arts integration. He questioned what if mathematics is used as a tool to foster learning in nonmathematical domains rather than arts as a tool to support other learning domains. He designed a course for design education students. He integrated some mathematical concepts (exp. platonic solids, graph theory, tiling, similarity, proportion, transformations, and symmetry) into architecture students' course. Kappraff used mathematics as a tool to foster architecture students' mathematical thinking and imaginative thinking and appreciation of mathematics' role in their works. He advocated that mathematics provides a new insight into non-mathematical learning domains such as art, design, architecture. He provided valuable contributions to the education in architecture in terms of designing activities that integrates design and mathematics.

Other important point arising from the literature was that some of the studies were related to the implication of arts and mathematics integration in out-of-school contexts (Hart & Heathfield, 2017; Hodzhev & Chernev, 2018). For example, Hart and Heathfield develop activities that focus on visualization of mathematics in order to make people comfortable with mathematics. They suggested the use of activities that involved a wide range of mathematical topics from constructing polyhedron from wood, cardboard, pattern, to combinatorics. On the basis of their personal observations of students' reactions to such activities, they concluded students enjoy mathematics through creating visually appealing mathematical artifacts and these activities could also be implemented for public to change their perception of mathematics. Although they provided valuable and rich contribution to visual arts and mathematics courses, they did not give details regarding how to design such tasks and what kind of thinking process was evolved.

Lastly, some researchers examined the potential in relation between arts and mathematics (Bickley-Green, 1995; Hickman & Huckstep, 2003). Bickley-Green (1995), for example, suggested developing a curriculum in art education that involves integration of arts and mathematics on the basis of their congruent elements. She examined the relation between arts and mathematics on the basis of theories of Bruner, Lowenfeld, and Piaget mainly. She suggested that integration of arts and mathematics has potential in supporting learning in both disciplines with coordination between intuitive and analytic thinking. However, she also suggested that there is need for examining the mental structures or patterns of thoughts that are common in both domains.

In summary, there have been a variety of studies on integration of visual arts and mathematics ranging from experimental studies to qualitative studies in the school context and out-of-school contexts. However, there are a few studies that put emphasize on investigation of the interplay between visual arts and mathematics on the basis of a theoretical background. They suggested to investigate the overlap between visual arts and mathematics (Bickley-Green, 1995; Goldsmith et. l., 2016) and to provide detailed information on learning outcomes and at what conditions they were observed (Winner et. al., 2013). Visual-spatial thinking was explicitly described as an overlap between visual arts and mathematics (Goldsmith et. al., 2016) and also implicitly described as thinking processes uncovered in the integration of visual arts and mathematics (Hart & Heathfield, 2017). These studies provided foundation for design of the current study in which students' visual-spatial thinking processes were examined in a studio environment designed on the basis of a theoretical framework (Studio Thinking). To identify and interpret students' visual-spatial thinking, previous studies on visual-spatial thinking were examined in the next part.

2.6. Characterization of Visual-Spatial Thinking

This part presents information about how visual-spatial thinking is conceptualized in different contexts such as cognitive science and psychology, art education, and mathematics education. First of all, in the domains of cognitive science and psychology, there have been several characterizations of visual-spatial thinking from early years to date. In the early years, researchers conducted factor analytic research that lead to development of tests on spatial ability. In the 1960s, researchers attempted to identify components of visual-spatial ability rather than considering it as a single factor. However, factor analytic studies did not find consistent results regarding components of visual-spatial thinking due to several reasons such as the nature of test (e.g. dynamic structures was examined in the static environment) or scaling factor (e.g. investigation of spatial ability in only small scale) (Hegarty & Waller, 2005).

Hegarty and Waller summarized descriptions of spatial thinking identified by previous studies. For example, McGee (1979) characterized spatial ability under two major factors: spatial visualization and spatial orientations. Spatial visualization was described as the ability to mental manipulation of objects such as rotation, bending, twisting without referring to own frame of reference and measured by paper folding tests. Spatial orientation was described as imagining the relation between elements in the visual stimuli when the observer changed the orientation and measured by Guilford-Zimmerman Spatial Orientation Test and Cube Comparison Test (as cited in Hegarty & Waller, 2005). On the other hand, Lohman (1979) identified spatial ability under three major factors: spatial visualization, spatial relations and spatial orientation. While Lohman described spatial visualization similar with McGee, he put emphasis on the complexity of the stimuli that require several steps of transformations and suggested to measure by paper folding test, mental rotation test regarding three-dimensional shapes, and form board test. Spatial relations were described as the ability to mentally rotate two-dimensional shapes during a limited time and were measured by Card Rotation test. He described spatial orientation as how a visual stimulus was seen from another point of view.

In a more comprehensive study, Carroll (1993) identified five major factors of spatial ability: spatial visualization, spatial relations, closure speed, perceptual speed, flexibility of closure. Spatial visualization was defined as the ability to solve more difficult spatial problems compared to spatial relations was measured by tests of paper folding, form board test, comparison of cubes and spatial orientation test. Spatial relations were measured by card rotation test. He also defined close speed (examining an object in noisy picture that subjects do not know it), flexibility of closure (examining a target object in noisy picture known by subjects as in the hidden figures test), perceptual speed (identifying identical shapes by comparing their visual appearance).

In a further study, Linn and Peterson (1985) conceptualized spatial ability in three categories; spatial perception, mental rotation, and spatial visualization. He defined spatial perception as locating the horizontal and the vertical despite of disturbing situations. He used rod and frame test, and water level tasks to measure spatial perception. He referred mental rotation as rotating two and three dimension figures in a quick and accurate way, which is different from categorization of Carroll that separated rotation of two and three-dimensional shapes. Lastly, spatial visualization is described as manipulating spatial representations that require multi-step solutions. It is measured with the tests of embedded figures, hidden figures, and paper folding.

In addition to this identification of factors related spatial ability, due to inconsistent results of these factor analytic studies, current researchers attempted to characterize spatial ability or spatial reasoning in a different way by investigation of the relation between them and how they differ from each other (Hegarty, 2014). Newcombe and her colleagues, currently, conceptualized visual-spatial thinking in a different and more comprehensive way (Newcombe & Shipley, 2015; Newcombe, Uttal, & Sauter, 2013; Uttal, Meadow, Tipton, Hand, Alden, Warren & Newcombe, 2013). As an initial step in conceptualization, Newcombe, Uttal and Sauter (2013) suggested two different skills of spatial thinking: inter object (within object) representations and transformations and intra object (between object) representations and transformations. In particular, they described inter object relations in terms of tool making skill, involves depicting and transforming internal properties of objects through several acts such as sliding, rotating, and cross-sectioning. On the other hand, they described intra object relations in terms of navigation skill, which is related to representation of objects location and environmental properties in a landscape with respect to moving self or each other.

In their further study, they developed their categorization under four sub-categories. Newcombe and Shipley (2015) proposed a new categorization of spatial thinking with the consideration of the fact that there are several types of spatial thinking. These different types of spatial thinking are grounded in the works of various fields such as mathematics, engineering, science, technology, design, and art. These categories are intrinsic and extrinsic, static and dynamic. Intrinsic information is related to defining objects in terms of their shapes, arrangements, sizes, and orientation, and transforming these properties of objects such as rotating, bending, scaling, relating 2D views to 3D views, and cross-sectioning of objects. On the other hand, extrinsic information involves the relation between and among objects with respect to each other, or other frames of reference such as locating an object relative to other objects. Another categorization is between static and dynamic properties of spatial tasks. Static properties are related to intrinsic characteristics of objects such as shape, size, and orientation, location with regard to other objects or with regard to a frame reference. Dynamic properties are related to changing or transforming these properties of objects with regard to other objects, frame of reference or to self. Table 4 presents this categorization of spatial thinking.

Table 4. *Categorization of Spatial Thinking (Newcombe and Shipley, 2015)*

	Intrinsic	Extrinsic
Static	Representing shape of objects, identifying visual properties of objects such as size, shape, texture, color, determining parts and their relations between parts, and recognizing the hidden figures from a complex structure.	Determining relations between and among objects (e.g. distance and angles), determining the location of an object with respect to other objects or to reference frame.
Dynamic	Transforming properties of object through rotating, bending, folding, slicing (cross-section), visualizing how an object changes when transformed, and projecting three-dimensional objects onto two-dimensional flat.	Perspective taking, navigation (e.g. visualizing an environment (large scale) from different vantage point, making a connection between different vantage points of an environment to make inference about it, building a view or landscape from someone else's perspective)

Similar identification of static and dynamic relations, Tversky (2005) summarized fundamental properties of representations ((visual properties such as shape, size, color, distance, direction, path, movement) and transformations (change of visual properties of the objects) used in visuospatial reasoning. Tversky summarized elementary properties of representations and transformations through five key items: Identifying static properties such as shape, size, symmetry, color, and texture, identifying the relationship between static objects regarding a reference frame (direction, distance, and location) or other objects (comparison with other objects in terms of several properties such as size, shape, location, and so on), identifying the links between static and dynamic objects such as speed, speed-up, and collision, transforming properties of objects (e.g. making changes in location, size, shape, perspective, rearrangement of the parts), transforming properties on self (e.g. making changes in location, perspective, size, shape etc.)

On the basis of these studies, in the context of art education, Goldsmith, Hetland, Hoyle, and Winner (2016) are among the pioneer researchers who investigate the relationship between visual arts, geometric reasoning, and visual-spatial thinking. In their recent research in 2016, they investigated the relationship between geometric reasoning, artistic envisioning, and spatial reasoning of students in the visual art program and students in the theatre program. They described visuospatial thinking in visual arts, named as artistic envisioning (Table 5). They conducted interviews with artist and art educators to describe visual-spatial thinking in visual arts. Here are the factors that are related to artistic envisioning:

Table 5. *Artistic Envisioning as Visuospatial Thinking in Visual Arts (Goldsmith et. al., 2016, p.59)*

Ways of Artistic Envisioning	Descriptions
Flattening the space	Representing three-dimensional objects on the two dimensional flat through making deformations through perspective drawing
Abstraction	Simplification of a form through imagining its basic structure and direction
Mental Rotation	Observing an object from a particular point of view and mentally rotate it to see it from another point of view, rather than physical rotation of the object
Shadow Projection	Representing light and shadow through imagination of where the light source is and how it affects the appearance of the object.
From 2D to 3D	Constructing 3D objects with the use of their 2D images

In the mathematics education context, Clements (1998) describe spatial sense in two main spatial abilities; spatial orientation and spatial visualization. Spatial orientation requires reading and making maps and navigation skills through the ideas of perspective (e.g. identifying various views from different perspectives, matching different perspectives of the same thing, finding our from which perspective a photographer took a photo), direction (ability to understand ideas of navigation such as above, over, left, right, north, and west) and measurement (to construct and read maps of their environment), location (identifying location of objects on the map and its change on the map; understanding concepts of coordinate grids). On the other hand, spatial visualization is defined as understanding and imagining two and three-dimensional objects movements and their transformations (e.g.) comparing images of shapes that is rotated, drawing objects, seeing combination of geometric forms differently).

In summary, several researchers conceptualized visual-spatial thinking. Currently, Newcombe and Shipley (2015) have suggested a new categorization of visual-spatial thinking including diverse visual-spatial thinking processes. In this study, examples regarding categorization of visual-spatial thinking in the work of Newcombe and Shipley (2015) could be used as a base for interpreting students' visual-spatial thinking processes. Besides conceptualization of visual-spatial thinking, it is also important to take into consideration the previous studies on visual-spatial thinking in the mathematics education context. In the next part, previous studies on visual-spatial thinking and mathematics were examined.

2.7. Studies on Visual-Spatial Thinking and Mathematics

From the early years to date, researchers discussed the relation between visual-spatial thinking and mathematics, how they are related to each other, and how visual-spatial thinking is improved (Bishop, 1986; Clements & Battista, 1992; Hawes, Tepylo, and Moss, 2015; Mulligan, 2015). First of all, researchers seemed to have a consensus on that spatial thinking is closely related with mathematics (Clements & Battista, 1992; Young, Levine, & Mix, 2018; Presmeg 1986, Tartre, 1990; Verdine, Golinkoff, Hirsh-Pasek, & Newcombe, 2017). For example, Clements and Battista explained the domains of geometry that requires spatial thinking on the basis of Usiskin's conceptualization of geometry. These are “visualization, drawing, and construction of figures, study of the spatial aspects of the physical world, use as a vehicle for representing nonvisual mathematical concepts and relationships” (as cited in Clements & Battista, 1992, p.2). In addition to geometry, quantitative relations in mathematics such as arithmetic and number line involve geometric and spatial relations (Clements & Sarama, 2011; Newcombe & Booth, & Gunderson, in press). Supportively, Hawes, Tepylo, and Moss (2015) explained that measurement, patterning, algebra, fractions are among the mathematical topics, in addition to geometry, that involve spatial relations.

The relation between visual-spatial thinking and mathematics might not be a straightforward (Clements, 1998) and directly apparent (Hawes et. al, 2015). There is a need for investigating underlying mechanism of this relation. There are very few studies that notably well investigated spatial thinking in mathematics education (Bruce & Hawes, 2015). Some of the studies investigated the relation between spatial thinking through quantitative methods by using tests in spatial thinking literature and tests in mathematics education for measuring geometric and mathematical knowledge (Goldsmith et. al., 2016; Pitttalis & Christou, 2010). Investigation of this relation is beyond the context of this study. However, it is important to examine how the concepts in spatial thinking literature would have a place in mathematics education.

Researchers has explained either the role of geometry in spatial thinking or the role of recognizing spatial relations in geometry explicitly or implicitly by focusing on particular concepts of spatial thinking such as scaling (Möhring, Frick, & Newcombe, 2018; Vasilyeva & Bowers, 2006); cross-sectioning (Cohen & Hegarty, 2012) and mental rotation (Bruce & Hawes, 2015); recognizing shapes and patterns (Craine, 1994; Gal & Linchevski, 2010; Mulligan & Mitchelmore, 2009; Pittalis & Christou, 2013), decomposition and composition of shapes concerning geometric transformations (Clements, Wilson & Sarama, 2004; Spitler, 2009); disembedding and embedding of shapes (Sarama & Clements, 2009; Liu & Toussaint, 2011);

representation of geometric shapes or unit of cubes through perspective and orthogonal drawing (Mitchelmore, 1978, 1980; Olkun, 2003; Pittalis & Christou, 2013).Some of these studies was explained in detail.

Regarding scaling, researchers explained the role of geometric cues in mapping tasks (Vasilyeva & Bowers, 2006; Uttal, 1996) and the relation between proportional reasoning and scaling (Möhring, Frick, & Newcombe, 2018). For example, Vasilyeva and Bowers (2006) investigated the role of geometric cues in scaling tasks. They conducted a study with young children from 3 to 6 years old. They aimed to examine whether they could gather geometrical properties of layout (relative angles and lengths of a triangle) to locate objects in a mapping task. Young children were asked to find the correct location of a dot in a layout with a shape of isosceles triangle after they were shown a picture of the layout that involves the dot placed on a corner of the triangle. They conducted several experiments: The isosceles triangle was constructed through continuous lines, a number of dots, and only three dots respectively. The authors found that children showed a higher performance on the experiment involving the isosceles triangle with continuous lines. Their performance also differed in terms of geometrical feature of triangle. They were more successful in the tasks that involve the dot on the unique corner of isosceles triangle rather than equal-sized corners. They also showed developmental progress across age levels. The findings of the study highlighted the crucial role of geometric properties in identifying locations of the objects as an individual entity and as a part of a pattern.

From a different perspective, some researchers have also focused on the relationship between proportional reasoning and spatial scaling skills (Newcombe, Booth, & Gunderson, in press; Möhring, Frick, & Newcombe, 2018; Möhring, Newcombe, Levine, & Frick, 2016; Möhring, Newcombe, & Frick, 2015). Newcombe, Booth and Gunderson (inpress) investigated the possible relations between mathematical thinking and spatial thinking. The link between proportional reasoning and spatial scaling is one of the relations between mathematics and spatial thinking. While proportional reasoning is defined as understanding the part-whole and part-part relations, spatial scaling is defined as reasoning about the relations between a referent space and its representation (e.g. map) that differs in size from its referent space. Both involve thinking about preserving the proportion between different scales. For example, if the length of a path is ½ of the length of another path in a map, the proportion between two paths will be same in the referent space. They argued that spatial scaling and proportional reasoning are highly related. Their argument relies on the evidences of studies with young children that found proportional reasoning in a non-symbolic sense is significantly related with spatial scaling ability (Möhring, Newcombe, & Frick, 2015).

Regarding cross-sectioning, Cohen and Hegarty (2012) investigated how undergraduate students imagine transformations in geometric shapes such as slicing and identify their cross-sections that is crucial ability for learning mathematics and STEM education. They developed a test that involves thirty items whose focus on determining two-dimensional cross sections of three-dimensional geometric forms. The difficulty of items changes in terms of the complexity of combination of geometric forms (simple, joined, and embedded solids) and orientation of plane that cuts the solid (orthogonal and oblique). The results of the study indicate that students outperformed on the tasks with orthogonal cutting plane rather than oblique plane. They found significant interaction between orientation of plane and complexity of figures' combination. The study also provided some clues about participants' thinking processes. They might have used both analytic and imagistic way of thinking. Students who are successful in difficulty tasks might have used analytic strategies such as decomposing shapes or matching the properties of shapes.

Regarding decomposing and composing shapes, Clements, Wilson and Sarama (2004) analysed 3 to 7 years old children's decomposition and composition of geometrical shapes in a software including pattern blocks. They identified seven developmental levels for composition of shapes: Precomposer, Piece Assembler, Picture Maker, Shape Composer, Substitution Composer, Shape Composite Iterater, and Shape Composer with Superordinate Units. They identified these levels in relation to recognizing shapes' properties and using transformations such as rotation. Children at the Precomposer level can not match simple shapes with a frame and consider each shape individually rather than composing them. Children at the Piece Assembler level can fill the frame with shapes by using the method of trial and error. They do not often consider the properties of shapes and transformations such as flip and turn. Children at the Picture Maker level combine more than one shape to form a figure; but they use the trial and error method by considering some simple properties of shapes such as side length and corner. However, they do not conceptualize angles as a quantitative property. Children at the Shape Composer level intentionally chose the shapes to form a figure by considering their lengths and angles. They rotate or flip the shape with a purpose through imagining what kind of shape will be constructed. Children at the Substitution Composer level chose shapes on purpose that are combined to construct another figure. For example, they know that a rhombus consists of two triangles. Thus, two triangles could be placed instead of a rhombus. Children at the Shape Composite Iterater level iterate combined shapes intentionally. Children at the Shape Compose with Superordinate Units level coordinate units of units of composite shapes. To be precise, they combine units of shapes and form a pattern. As they continue to the pattern, they perceive a new unit that is formed through combining units of shapes and iterate it deliberately. This study contributed to the literature by examining how to identify individual differences between children regarding decomposing and composing shapes, which could be related with decomposing and composing of numbers.

Regarding mental rotation, it is one of the concepts that has been mostly studied in mathematics education as transformational geometry, that is highly related with spatial ability (Bruce & Hawes, 2015). Bruce and Hawes developed activities for early grade students including use of pattern block, tangrams, and pentaminoes.

These activities engaged students in rotating two and three-dimensional shapes mentally by asking them to identify similarities and differences between them or reproduce a shape by looking its photograph. At the end of the intervention, they found that students had higher performance on mental rotation compared to their performance before the intervention. In higher grades, researchers investigated three transformations of shapes; turn, flip and slide. Students from four to eight years old had more difficulty in turning compared to slide and flips (Clements & Battista, 1992; Moyer, 1978). Students from nine to 13 years old had difficulty in conceptualizing transformations or compositions of Euclidean transformations (Kidder, 1976).

Regarding disembedding and embedding of shapes, there have been rare studies that investigated disembedding in mathematics education unlike psychology literature (Sarama & Clements, 2009). In the field of psychology, embedded or hidden figures tests have been used to measure the ability of recognizing shapes as a one component of spatial thinking (Oltman, Raskin, & Witkin, 1971). Participants were asked to find a simple shape in the complex shape. One of the examples from Group Embedded Figures Test is presented in the following. For example, x named shape is hidden in the complex figure next to it (Figure 4).

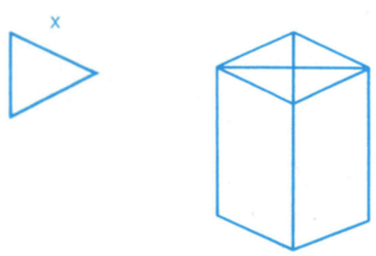
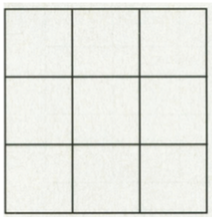


Figure 4. Sample item for embedded figures test. Adapted from "Group Embedded Figures Test" by P. K. Oltman, E. Raskin, & H. A. Witkin, 1971, p. 1.

In the mathematics education, it was related with identifying geometric shapes that are nested to each other or perceiving reversible figures by discriminating the figure from the ground (Sarama & Clements, 2009). For example, In the context of mathematics education, Craine (1994) proposed activities on recognizing geometrical shapes and patterns (Figure 5). For example, students are asked to count squares that are embedded and differ in terms of size. A sample task is presented in the following: Students are asked to construct their own embedded-figures problem by using geometrical shapes such as squares, rectangles, and triangles. They also aimed to encourage students to think algebraically by recognizing the pattern in the number of squares.



The 3x3 square:

a) How many 1x1 squares?
b) How many 2x2 squares?
c) How many 3x3 squares?
d) Find the total number of squares.

Figure 5. Sample item from activity sheet for embedded figures. Adapted from "Counting Embedded Figures" by T. V. Craine, 1994, p. 528.

Another example is related with reversible figures. Some pictures and geometrical forms can be seen differently when someone look at them for a while (Attneave, 1971). The necker cube is one of the prominent examples of multi-stable perception. A necker cube could be perceived as multistable. It depends on the point that one stare at. For example, when one stared at a point steadily, one can perceive the cube so that its top is viewed. If he/she changes the point of fixation, the depth is reversed (see Figure 6). It might also happen regardless of eye movement (Attneave, 1971).

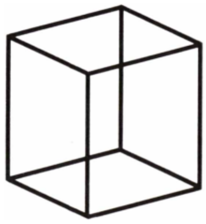


Figure 6. The Necker cube. Adapted from “Multistability in Perception” by F. Attneave, 1971, p. 63.

In the mathematics and arts literature, Liu and Toussaint (2011) in their article presented examples of geometrical structures underlying the ornaments in the Siena Cathedral in Italy. One of the examples that presented is related to multi-stable perception of geometrical figures. The Figure 7 illustrates different interpretations of an ornament pattern constructed by simple geometrical shapes. One might perceive it as a triangular pyramid that seen from the top or from the front point of view, as a pattern of cubes, or as a six pointed three-dimensional star.

Figure 7. One of the geometrical ornaments in the Siena Cathedral (Photo by Yang)

Regarding representation of three-dimensional geometric shape, Mithelmore (1978; 1980) classified students' drawings of three-dimensional shapes, which might provide cues regarding students' lack of representing shapes and their transformations mentally beyond the motor skills (Sarama & Clements, 2009). Mithelmore described four stages: stage 1: preschematic (representing a shape with a single face), stage 2: schematic (representing a shape with more than one faces; but without depth), stage 3: prerealistic (representing shape with little depth (3A) or with depth (3B) and with only visible faces), and realistic (accurate perspective drawing). An example of drawing a triangular pyramid was shown in the following figure 8.

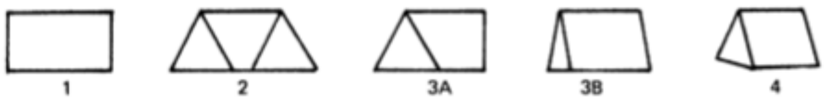


Figure 8. Students' drawings of a triangular prism for each stages of representation. Adapted from "Prediction of Developmental Stages in the Representation of Regular Space Figures, 1980, p.88.

In a further study, Pittalis and Christou (2013) investigated how students (from fifth to ninth grades) represent three-dimensional shapes on plane. They found two processes that were reflected during representation of three-dimensional shapes: decoding (identifying geometric properties of shapes) and coding (making transition between orthogonal and perspective drawing of shapes, constructing nets to represent them). These abilities are closely related rather than separate constructs, which require understanding and manipulating spatial relations in the shapes.

In summary, these studies give clues about examination of visual-spatial thinking in mathematics education, particularly geometry. They focused on a variety of concepts such as recognizing shapes and patterns, rotation of shapes, scaling, cross-sectioning, representation of geometric shapes. In the current study, these studies could provide a base for examining and interpreting students' visual-spatial thinking processes in the contexts of visual arts and mathematics even though they did not include integration of visual arts and mathematics.

2.8. National Studies on Visual-Spatial Thinking, Visual Arts and Mathematics

In Turkey, research on students' visual-spatial thinking in mathematics education focused on several topics: the relationship spatial thinking with several factors such as performance in mathematics and gender (Turğut, 2007; Turğut & Yılmaz, 2012), geometry knowledge, gender, and type of school (Eryılmaz-Çevirgen, 2012), mathematical reasoning (Gürbüz, Erdem, &Gülburnu, 2018); levels of thinking regarding decomposition and composition of shapes (Gündoğdu-Alaylı & Türnüklü, 2013), students' strategies in spatial visualization tasks (Kaplan, 2012); effect of particular activities on development of spatial abilities such as drawing (Olkun, 2003; Olkun & Sinoplu, 2008), use of concrete materials and computer applications (Yolcu & Kurtulus, 2010), use of dynamic geometry software (Kösa, 2011; Şimşek& Koru-Yücekaya, 2014), use of manipulatives (Enki, 2004); augmented reality environment (Özçakır, 2017), origami-based instruction (Arıcı & Aslan-Tutak, 2015; Çakmak, İsiksal & Koç, 2014).

Regarding correlational studies, Turğut and Yılmaz (2012) investigated seventh and eight grade students' spatial abilities and its relationship with the gender, achievement in mathematics and early childhood education background. They collected the data from 674 students who are enrolled in nine public middle schools. They used the test Spatial Visualization Test prepared by Middle Grades Mathematics Project (MGMP). They conducted descriptive analysis and t-test. The results of the study indicated that most of the students had a low performance in spatial ability test. There was not significant relation between spatial ability and the factor of gender. However, there was a significant relation between students' achievement levels in mathematics and spatial ability. Furthermore, it was found that spatial visualization ability was significantly related to the ability of spatial relations. Moreover, spatial ability of students who had early childhood education background was higher than those who did not have a background of early childhood education in the past. Similarly, Gürbüz, Erdem, and Gülburnu (2018) found significant relationship between eight grade students' mathematical reasoning and spatial ability. They used the test of Mathematical reasoning test developed by authors and spatial ability test developed by Turğut (2007) including views of unit cubes from several points of views. They indicated that student who had higher performances in spatial ability test also had higher performance in mathematical reasoning test. They suggested developing students' spatial abilities should be one of the crucial aims of mathematics education.

Regarding qualitative studies, Gündogdu-Alaylı and Türnüklü (2013) investigated sixth to eight grade students' decomposition and composition of shapes on the basis of level of thinking proposed by Clements, Wilson, and Sarama (2004). They conducted clinical interview with six students. Students were asked to solve problems prepared on the basis of levels of thinking. These problems included solving problems with pattern blocks, solving problems mentally, solving problems with pencil or scissor. They found that students who outperformed in the tasks deliberately decided how to use geometric motions such as turn, slide and flip. They mostly took into consideration not only lengths but also angles of shapes during decomposition and composition of shapes. On the other hand, other students mostly combined shapes by trial and error. They focused on only lengths of shapes. During the tasks that require mental transformation, most of the students tended to use pattern blocks. They had more difficulty in imagining transformations of shapes compared to students who had a higher performance. In a similarly way, student who had higher performance drew shapes deliberatively and provided alternative solutions to the tasks.

There are also other qualitative studies on detailed analysis regarding components of spatial thinking such as students' naming and identifying geometric shapes (Türnüklü & Ergin, 2016; Ubuz & Gökbulut, 2015; Ulusoy & Cakiroglu, 2017). They emphasized the role of prototypes in students' identification of geometric shapes. For example, Türnüklü and Ergin (2016) investigated eight grade students' identification of prisms, pyramids, cylinder and cone. Analysis of semi-structured interview indicated that students mostly identified them on the basis of their visual similarity to the real-life objects and identified them on the basis of their non-critical attributes. Supportively, in the study of Ulusoy and Cakiroglu (2017) middle school students made errors of either underspecification or overgeneralization to discriminate examples and non-examples of parallelogram.

Regarding experimental studies, Olkun (2003) proposed several engineering drawing activities to develop students' spatial abilities that is described as manipulating images mentally. He explained two types of spatial abilities that are required in drawing process of engineering: spatial relations (mental rotation of shapes) and spatial visualization (relating different views of a shape and join them to make a single shape). Thus, he asserted that engineering drawing could be a tool for improving students' spatial abilities. Engineering drawing involves drawing top, front, and right-side views (orthographic) and isometric views. In the activities that he proposed students can use concrete materials such as cube and triangular prism and use dot paper to draw their different views. Students are asked to build the real-life objects such as car and ship on the basis of their drawings. They can also be asked to think how such a change has effect on their orthogonal and perspective view the if one part of the figure is changed or removed. In the further study, Olkun and Sinoplu (2008) investigated the effect of the use such activities on students' understanding of rectangular solids. Participants of the study were 121 fourth and fifth grade students. Pre-test-post test experimental design was used to explore the research question. The results of the study revealed that the effect of this instruction has significantly effect on students' understanding of spatial structuring (constructing a figure so that it is composed of units) regarding rectangular solids composed by unit of cubes.

In addition to use unit cubes as one of the manipulatives, Yolcu and Kurtulus (2010) used computer applications to explore their effect on sixth-grade students' spatial visualization ability. They conducted action research with twenty students in a public school. They measured students' spatial visualization ability through Block of Cubes Test before and after the experiment. The tasks in the activities involved constructing three-dimensional model of two-dimensional representation of a figure composed by unit cubes through concrete manipulatives, drawing units of cubes from different perspectives, identifying number of cubes in that figure. Students made use of concrete materials; then they practiced similar activities in a virtual learning environment in the computer. The results of the study indicated that there was significant effect of such activities on students' development of spatial visualization ability.

While these studies focused on the role of spatial ability in mathematics education and development of students' spatial thinking, there are also studies on arts and mathematics education (Erdogan-Okbay, 2013; Ugurel, Tuncer, & Toprak, 2012). There is also a study on investigation of visual-spatial thinking in visual arts and mathematics integration (Alyeşil Kabakçı & Demirkapı, 2016). Regarding arts and mathematics integration, few studies were conducted in Turkey. Ugurel, Tuncer and Toprak (2012) investigated pre-service mathematics teachers' design of lesson plan to integrate arts and mathematics. Pre-service teachers are selected among those who take the courses of Mathematics and Art in a public university. Through content analysis, researchers investigated to what extent pre-service teachers design a lesson plan to integrate math into art. They analysed the documents on the basis of three categories (good, average, and inadequate). Most of the lesson plan was coded as average to integrate arts and mathematics. Very few (16 percent) lesson plans were coded as the category of good. They also pointed out that pre-service teachers mostly preferred to use the topics of golden mean, artworks of Escher, architectural context and Fibonacci series. They concluded that only providing information about art and mathematics integration in a course is not sufficient to design instructional plans regarding arts and mathematics.

Erdogan-Okbay (2013) examined the effect of art-based mathematical activities on seventh grade students' motivation towards mathematics (enjoyment, self-efficacy, and academic effort). Art-based activities involve tessellations, origami, pattern in Islamic Art, op-art, animation of snowflakes, and pattern in quilts. She used both quantitative and qualitative methods. Students' enjoyment level, self-efficacy and academic efforts were measured through pre- and post-tests. The researcher also conducted focus group discussions with participants during the course of mathematics and art. In addition to focus group discussion, participants were also interviewed. The quantitative analysis of data indicated that there was not a significant change in three constructs of motivation when attending to such a course that integrates arts and mathematics. On the other hand, qualitative analysis of the data revealed that students' motivation who had a tendency to think analytically decreased after the intervention. However, students' motivation who are interested in art became more motivated at the end of the intervention. Moreover, students started to perceive mathematics from different perspective after intervention.

Differently, Alyeşil Kabakçı and Demirkapı (2016) explored the effect of a course that integrates mathematics and arts on students' spatial thinking abilities. Participants were selected among those who are in the mathematics group of the Arts and Science Center in İzmit. Researchers conducted static-group pre-test-post-test design. There were 22 participants. Half of them were educated in the experiment group and half of them in the control group. The course involved activities related to golden mean, fractals, isometric drawing of unit cubes, cross-sectioning of objects, transformational geometry, drawings of Escher, patterns, and perspective. They conducted independent-t-test to investigate students' spatial thinking in both groups. The findings of the study revealed that students' spatial thinking abilities in experimental group increased after the experiment compared to the control group. They could not find significant effect of the course on students' spatial thinking abilities on the control group. They suggested the use of concrete materials and such activities that integrate mathematics and arts to improve students' spatial thinking skills.

In summary, researchers mostly investigated spatial thinking in the mathematics education context through quantitative methods rather than qualitative methods. There has been a variety of studies on visual spatial thinking, changing from investigation of particular concepts of spatial thinking to development of spatial thinking. On the other hand, visual arts and mathematics integration was one of the rare topics that were investigated by the researchers in Turkey, especially investigation of visual-spatial thinking in the context of visual arts and mathematics integration. Researcher rarely explained how the tasks or activities were designed with a theoretical framework and what kind of factors could affect students' spatial thinking that was measured after instruction.

2.9. Summary and the Place of Current Study in the Literature

In this chapter, the research on visual arts and mathematics integration, and visual-spatial thinking were investigated. This investigation reveals that researchers conducted art integration studies from early years to date. On one hand, there have been some controversial findings regarding its effect on learning in other learning domains such as mathematics. Researchers mostly did not give information about theoretical background of the study and what factors might have affected the results of the study in both national and international studies. In this regard, although researchers have been interested in integrating arts into other learning domains, there is a need for conducting studies that provide concrete evidences of outcomes of arts and mathematics integration with a theoretical background.

On the other hand, investigation of visual-spatial thinking in mathematics education was mostly based on the factor analytic tests, which lead to conflicting findings due to the difficulty in discriminating the concepts of spatial thinking and static nature of tests. Current studies attempted to revise conceptualization of visual-spatial thinking. There are rare studies that relate visual-spatial thinking with arts and mathematics integration. There is a study that advocates examining congruent parts of visual arts and mathematics (Bickley-Green, 1995). In one of the current studies, Goldsmith et. al. (2016) suggested that visual-spatial thinking could be overlap between visual arts and mathematics in their correlational study. They have been interested in the transferring of learning in arts to other learning domains.

This study suggested looking this issue from a different perspective by investigating how students make use of visual-spatial thinking in a Math-Art Studio Environment in which students are deliberately engaged in art-making with geometric shapes through Studio Thinking. To achieve this goal, a Math-Art Studio Environment was designed on the basis of Studio Thinking Framework and previous studies on visual-spatial thinking. Crucial elements of this environment were determined to describe the nature of this environment (see the part of 2.3 in the literature).

In conclusion, it is assumed that this study would fill the gap in the literature by providing detailed information regarding the nature of Math-Art Studio Environment with a theoretical background and at what conditions students' visual-spatial thinking processes were observed.

CHAPTER 3

METHOD

The purpose of the study was to investigate how students make use of visual-spatial thinking in a Math-Art Studio Environment based on Studio Thinking that involves studio works with geometric-rich content. This chapter presents design of the study, participants of Math-Art Studio Environment, research context of the study, data collection and analysis processes, trustworthiness of the study, and ethical issues, researcher role, and limitations of the study.

3. 1 Design of the Study

The aim of the study was to explore how students make use of visual-spatial thinking processes in a Math-Art Studio Environment based on Studio Thinking Framework. To achieve this goal, qualitative research was conducted since it aims to understand what meanings people form, how they think, make meaning, experience in a particular setting (Bogdan & Biklen, 2007; Merriam, 2009).

A case study method was employed to investigate research questions of this study. Case study methods are used to investigate a particular setting, a participant, an event, or a program in depth (Merriam, 2009; Stake, 2005; Yin, 2009). Yin (2009) describes case study as “empirical inquiry that investigates a contemporary phenomenon in depth and within its real-life context” (p. 18). What is important in the case studies is to define the case of the study. Researcher defined the case in a bounded system (Merriam, 2009; Stake, 2005). This bounded system is described as boundaries of the research. In other words, it makes explicit what is studied or what is not studied in the research.

In this study, the case of the research is a Math-Art Studio Environment that was purposively designed by the researcher to integrate the visual art and mathematics, particularly geometry. This environment is an ecology that involves organic relations of nature and structure of the tasks (studio works), implementation of tasks through Studio Thinking Framework, reactions of students to such an environment, teacher/researcher's role, and physical structure of the environment. In this study, the unit of analysis was seventh grade students' visual-spatial thinking processes in the Math-Art Studio Environment. This environment was used to identify the units of analysis of the study. The boundaries of the environment are the use of minimalist artworks involving simple geometric shapes (Table 10 and Table 11), spatial content of the tasks (studio works) (Table 9), use of Studio Thinking as a way of eliciting students' thinking (see the part of 3.7.2).

In this regard, the case serves as a secondary role in this study. It was used as a tool to understand another phenomenon: students' visual-spatial thinking processes as a common point between visual art and mathematics education, specifically geometric thinking. To be more precise, it was used for investigating visual-spatial thinking processes that students make use of when they are engaged in studio works with a focus on geometric shapes. It provides insight into research studies that investigate how visual arts and mathematics are related. Therefore, this study was regarded as a kind of instrumental case study proposed by Stake (2005) since in an instrumental case study the case becomes a tool to better understand something else (Grandy, 2010, p.473). The purpose of this study was not to understand this environment. Rather it was to understand students' visual-spatial thinking processes in such an environment.

It is also important to emphasize that such an environment could become as a natural setting for students to investigate their thinking processes since such an environment does not exist in the current middle schools of Turkish educational system. If students had been just interviewed on a specific task, it would not be realistic and natural for students; for example, creating an artwork for several hours by alone. Thus, this environment provides a naturalistic setting in which students observe artworks, create, evaluate artworks, and interact with each other. In this way, the researcher could gather information about students' visual-spatial thinking processes most efficiently.

3.2 Participants of the Math-Art Studio Environment

Participants are crucial part of Math-Art Studio Environment. To describe the Math-Art Studio Environment, it is necessary to explain how the participants were selected and provide information about their backgrounds regarding visual arts and mathematics. There were six seventh grade students, two males and four females, participated to Math-Art Studio Environment. They were students in a public middle school in Ankara. Two of them (two female students) left the study due to the personal reasons after the second studio work. These students' thinking process were also included into the study since they might have had a potential role in eliciting and affecting other students' thinking process.

Participants were selected through purposeful sampling strategy since it aims to select participants who provide rich data to explore the issue in-depth, rather than making a generalization (Patton, 2002). The school in which participants enrolled was convenience in terms of location and its opportunities such as having an art studio, teacher and school management's willingness to conduct the study. In addition to convenience, it was assumed that inclusion of students with different backgrounds into the study would result in exploring different visual-spatial thinking processes. To identify students with different backgrounds, the researcher took opinions of their mathematics and visual art teachers. Teachers were asked to think about students who have interest in mathematics and/or visual arts, their performances in mathematics and visual art courses, their use of different approaches or strategies in mathematics and/or visual arts courses. On the basis of teacher's view, seven students were firstly invited to the study. They were given a parental consent form to be signed by their parents (see Appendix C). On the basis of parents' confirmation, six of them voluntarily participated to the study.

3.2.1 Participants' Background

This part presents information regarding participants of the environment on the basis of mathematics and visual art teachers' opinions and interviews on students' experiences and their interests regarding visual arts and mathematics before the study. The researcher described students with pseudonymous names. Pseudonymous names of the students are Fatma, Emre, Ali, Melek, Burcu, and Esra. Two of them (Burcu and Esra) left the study after two studio works.

Teachers' opinions indicated that three students (Fatma, Melek, and Burcu) have interest in arts education and are successful in their arts performances, and have potential to use creative approaches in arts while three other students (Emre, Ali and Esra) have interest in mathematics and are successful in mathematics performances. Differently, it was told that Emre have potential to use different approaches in mathematics. Also, only Burcu was characterized as both having interest in art and mathematics, being successful and having potential to use different approaches in both disciplines. Fatma and Melek's performances in mathematics based on their written exams were respectively low (50 out of 100) and medium (70 out of 100) compared to others even though they were relatively better in visual art courses.

After students were selected, pre-implementation interviews were conducted before the study. The reason of these interviews was to provide information regarding students' backgrounds such as their experiences and interests in visual arts and mathematics. Teacher opinions and the main points arising from the interviews were explained for each student in the Table 6.

Table 6. *Characteristics of the Participants*

Participants	Description of Students' Characteristics	Teacher Opinions
Fatma	Has experiences in visual art and interested in visual art; difficulties in mathematics	Creative and successful in visual arts
Emre	Not very interested in visual art and has not experiences in visual art; Interested in mathematics and feel confident in mathematics courses	Use different approaches and successful in mathematics
Ali	Has a few experiences in drawing; Interested in mathematics and feel confident in mathematics courses.	Successful in mathematics
Melek	Has experiences in visual art and interested in visual art; Feel more confident in visual art; Mathematics score is relatively medium	Creative and successful in visual arts
Burcu	Has experiences in visual arts and mathematics; Interested and feel confident in both visual arts and mathematics.	Creative and successful, use different approaches in both disciplines
Esra	Interested in visual arts and mathematics; Feel confident in mathematics courses	Successful in mathematics

Fatma had several experiences regarding drawing. She usually spends her spare time drawing pictures. She enjoys exploring artworks of artists, drawing human figures, real-life objects, cartoon characters, clothes, and flowers. She has a sketch book that she always carries in her bag. While she draws the objects by looking at them, she sometimes draws them by imagination without seeing them. She had gone an art course when she was almost four years old. She enjoys visiting art galleries. She attended to an art competition when she was at the fifth grade. She had a dream of being an art teacher. She thinks she could do most of the requirements of the art course except for drawing human figure. Regarding mathematics, she is not interested in mathematics during her spare time. She perceives mathematical problems as challenging. Thus, when mathematical problems become challenging and hard, it becomes boring for her. She does not have any experiences in participating to the math competition.

Emre is not so much interested in visual arts. He thought visual art as necessary only if one wants to be an artist. He just enjoys doing painting with his friends. He does not enjoy making art during his spare time. He does not have any experiences in visiting art galleries and participating to an art competition. His most favourite lesson is mathematics. Constructing equations and solving them are among the activities that he does during his spare time. He thinks he is successful in mathematics class and wants to be a successful student in mathematics classrooms.

Ali has some previous experiences in drawing such as drawing the draft of the projects in project competition he attended to, drawing house mentally, drawing happy moments with his family. He appreciated the role of visual arts in real life since he thinks it is highly related with other disciplines such as engineering and architecture. It is used to make sketches of objects in these disciplines. He likes drawing figures. However, he thinks he is not good enough at visual art. He feels strong himself in drawing geometrical figures rather than organic figures. He dreams making inventions and drawing them in the future. He stated that he is interested in math, and likes it so much, especially finding unknowns in equations. He also expressed that he does not any difficulty in any of the topics of mathematics. Even though mathematical problems are hard, he really enjoys solving them. He also appreciated the role of mathematics as a necessary discipline to be a scientist in the future.

Melek had also several experiences in visual art: spending her spare time drawing figures; participating in art competitions and getting award in them; being interested in exploring artists' artworks. She appreciated the role of the visual arts as relaxing component of life and as a professional discipline. When compared to mathematics, she feels stronger in visual arts courses than in mathematics courses. She seems to find dealing with shapes easier than the dealing with numbers.

Burcu is very interested in visual arts. She has experiences in visual art: drawing with or without imagination in her spare time, participating in art competitions and getting awards; visiting art galleries. She has a sketch book that she always carries in her bag. She appreciated the role of visual arts as making relaxing our life and as a hobby. She dreams to be a math teacher and organize her own art exhibition in the future. She finds mathematics enjoying and loves it. She thinks she only had difficulty in probability in mathematics. She spends her spare time doing tests in mathematics. She had an experience in participating in a mathematics competence as well.

Esra stated that she is interested in visual arts. She spends her spare time drawing figures such as portrait. She draws them with imagination rather than seeing it. She appreciates the role of visual arts as crucial element of the soul and helping to develop imagination. She also expressed how she likes mathematics. She feels confident in solving mathematical problems, especially, doing the tests in mathematics. She feels ambitious in math and wants to be a successful student in mathematics classrooms.

To conclude, on the basis of interviews and teachers' opinions, it seems that three students (Fatma, Burcu, Melek) become more prominent in visual arts compared to others; other three students (Ali, Emre, Esra) become more prominent in mathematics courses. It is only Burcu who is seen as a prominent student in both visual art and mathematics courses.

3.3. Research Context of the Study

The research context of the study consists of what students are expected to learn in regular visual art and mathematics courses in public schools, number of hours per week for each courses, and the nature of places in which these courses are taught, how students were taught in the school where the study was conducted. First of all, what students are expected to learn in visual arts courses is explained. Then, what students are expected to learn in mathematics courses is described on the basis of national curriculum of Turkey.

Firstly, visual arts course in a public school takes one hour in a week from the first grade to eight grades while technology-design course takes two hours in a week from seventh grade to eighth grade. Students are taught at the art studio or in a classical classroom setting (students' desks, teacher's desk, and board) depending on the opportunities of schools. In the visual arts (1 to 8 grades) and technology design courses (7 to 8 grades), they learn to use different materials and techniques. They are expected to identify and use elements (line, color, texture, size, and value) and principles of art or design (contrast, balance (symmetric vs asymmetric), harmony, scaling (proportion), rhythm (pattern), and emphasis). They learn to use both geometric and organic figures to construct an artwork. In the first grades, they learn the relations among objects in terms of proximity and size relations. They are expected to draw an object through careful observation, construct both two-dimensional and three-dimensional artworks. In the further grades, they learn to envision geometry underlying the figures, learn proportion concept, form a depth in the two-dimensional surface, shading, and perspective (MONE, 2018a). Visual art teachers of participants in the current study confirmed that seventh grades students were taught about these concepts.

Secondly, mathematics courses in public schools take five hours in a week and they are compulsory. There are also elective courses such as Mathematics Applications courses, which takes two hours in a week. Students are taught in a classroom-based environment that involves students' desk, teacher desk, and board. They learn basic of concepts of geometry up to seventh grade. They learn basic concepts of geometry such as point, line, angle and basic two-dimensional shapes such as square, triangle, rectangle, circle, parallelogram, rhombus, and trapezoid. They are expected to name and identify those shapes and their properties, and classify them. In addition to two-dimensional shapes, they are expected to identify basic three-dimensional shapes such as cube, rectangular prism, square prism, triangle prism, cylinder, cone and identify their properties. Besides identification of shapes, they should be able to draw basic shapes such as square, rectangle, triangle, and angle.

Regarding spatial relations, they are expected to learn relative position and direction of shapes, symmetry concept, constructing geometrical pattern, decomposition and composition of shapes. Regarding measurement, they also should have knowledge of measuring and comparing lengths, area, and volume. They learn the nets of basis three-dimensional shapes such as cube and rectangular prism. They should know to identify a shape from different perspectives and should be able to draw its different views (MONE, 2018b).

In the elective course of Mathematics Application, students mostly practice to solve problems that are closely related to national exams. Moreover, students in public schools could take additional courses of mathematics, visual arts, and other disciplines at the weekends. These courses are not compulsory.

In the school where the study was conducted, students were educated both in the mornings and afternoons separately depending on their grade levels. Seventh grade students' classes were in the afternoons. They took an elective course named Application of Mathematics in addition to their regular mathematics courses. Optional weekend classes were also available for students.

In summary, in the current education system, visual arts and mathematics are taught separately. There is not any course that integrates arts and mathematics even though each of them includes some contents of the other. Thus, a new environment in arts studio was designed so that visual arts and mathematics is connected, named as Math-Art Studio Environment in the current study. This environment was designed by the researcher since there is not a course that is deliberately designed to integrate arts and mathematics in public schools. If it had been integrated into existing courses of visual arts or mathematics, it might have prevented the flow of the course and effect its schedule. Furthermore, it was not designed to attain objectives of the curriculum even though involves some of them. This environment is deliberately used as an instrumental case to understand how students make use of visual-spatial thinking in such an environment (see for details of the Math-Art Studio Environment in the part of 3.7).

3.4. Overall Process of the Study

Overall process of the study is illustrated in the Figure 9. In order to understand students' visual-spatial thinking processes in the contexts of visual arts and mathematics, the first thing to do was to design such Math-Art Studio Environment that integrated visual arts and mathematics in a particular way. On the basis of literature review and experts' opinions (two artists in visual arts department of a public university), crucial elements of such a design is determined to elicit students visual-spatial thinking processes (see the parts of 3.7.2.). On the basis of these elements and the researchers' views and experiences, six studio works were designed so that it would result in the specific visual-spatial thinking processes (see table 9). After initial drafts of studio works, an art teacher examined the studio works. On the basis of her recommendations, minor revisions were made on some aspects of the studio works. Then, pilot study was conducted to understand what works or what does not work. The details of the pilot study were explained in the section of 3.6. in the method. After pilot study, to examine what did not work in the pilot study the researcher consulted an expert, who is a professor in a university and conducts studies on visual-spatial thinking. On the basis of her views, several aspects of studio works were revised (see part of 3.6.3). Then, tentative pedagogical principles for the study became more structured (see part of 3.7.2). On the basis of these principles, the main study was conducted.

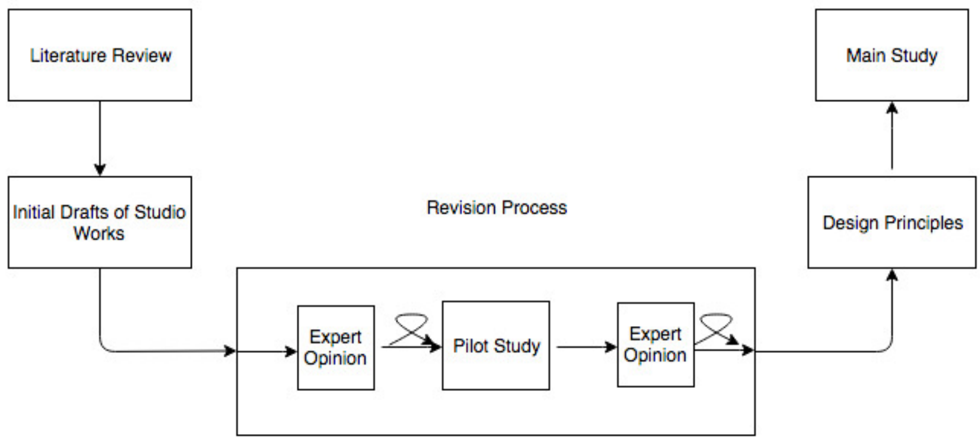


Figure 9. Overall process of the study

3.5. The Main Study

This part describes the setting of the main study, studio works used in the main study, data collection process, and data sources used in the main study. Each of them was explained in detail.

3.5.1. Setting of the Math-Art Environment in Main Study

The main study was conducted in the arts studio of a public school in Ankara. The studio involves tables and chairs for students, and a teacher desk, and two cupboards to keep students' materials and works, and a smart board. There is also a wall area to put the works on the wall and take notes.

Students sit next to each other; but they were not so close that they would copy others' observation notes or artworks. Studio works of students were both audio and video recorded. Two voice recorders were placed on the table to record what participants talk in case the cameras would not record the voices with a good quality. Four cameras were used to record students' actions. The setting of the cameras and seating arrangement of students were presented in the following illustration (Figure 10). One of those cameras was sometimes used for recording group observation of art-works and the critique of students' art-works.

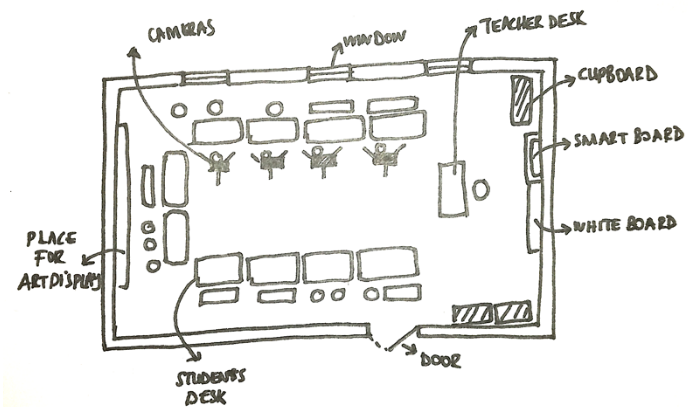


Figure 10. Sketch of art studio in the main study

The necessary materials for each studio work were provided by the researcher. Some of these materials were: Drawing pencil, sketch book, glue, different type papers, colorful dry paint, pastel, miter-ruler, eraser, compass, cartons, model carton, and pencil sharpener. The researcher also brought a computer to encourage students to make research for their artworks. Music (classical piano music) was used to motivate students during their work. In the studio, students can make break whenever they need. Smart board was used to display students' and artists' artworks to observe and critique them.

3.5.2. Data Collection Process of the Main Study

Students were expected to participate to each studio work; but it was not compulsory since they have rights to leave the study if they do not want to participate any more. Four students participated to the all studio works. Of those students, only one student did not attend to a part of the fourth studio work. Two students left the study after the second student work with personal reasons.

There were six studio works. Studio works was scheduled to finish in two weeks depending on the students' performances. Students participated to the studio in the morning from 9:00 to 12:45 each day since they are educated at the same school afternoon. This continued through eight days. After six days, there was a break for a day. Then next two days they participated to the study in a similar way. The reason such an intensive studio works was that students might forget what they did previous day since students' artworks was critiqued in the following day and they might not effectively get involved in the study. Each day it was assumed to finish a studio work. In case it does not finish, remaining part of the studio work continued next day. Mostly, the critique part of the studio works was implemented on the following day.

Stimulated recall interviews with each student were conducted after each studio work. In addition to stimulated recalls interviews, before and after implementation, students also were interviewed to learn about their prior experiences and interests in visual arts and mathematics and their experience in the current study (see part of 3.5.4.)

3.5.3. Researcher and Teacher Role in the Main Study

Researcher and teachers have different roles in the main study. While researcher participated and directed all processes of the study, visual art teachers and mathematics teachers of students only attend to critique parts of the study. Their roles are described respectively.

The researcher acted as a coach and directed all studio works to experience close interaction with students. It is important to understand students' spontaneous performances on-the-spot (Hetland et. al., 2013). The researcher as a coach demonstrates, advises, questions, and criticizes. The student tries to strike a balance between taking responsibility for self-education in designing, and remaining open to the coach's help. Students learn by doing and also learn through reflective practices with researcher (Hetland et al, 2013; Schön, 1988). To what degree the researcher made demonstrations or help students depended on the level of individual students' struggles. When a student had more difficulty than the others, the researcher asks questions step by step to prompt students' thinking. If she/he could not manage to solve the problem and feel frustrated, the researcher demonstrated how to draw a shape or transformations on shape. This kind of help is important for eliciting students' thinking process in further tasks of the studio work and provides motivation for the student even if he/she had struggle in one part of the tasks in studio works.

Teachers also have a role in the study as experts who made comments and suggestions on students' artworks during the critiquing part. Before the implementation of the study, teachers were informed about the content of all studio works. Two teachers of students (a mathematics teacher and visual arts teacher) were invited to the critique parts of the studio works. Teachers only attended to only critique parts of the studio work due the fact that they also had courses in the school during the implementation of this study and during this process, it was difficult for them to stay three hours in the studio. Thus, they were only invited to the part of critique in which their experiences were mostly needed to make comments on students' artworks. Teachers did attend to different studio works depending on their available time. Involvement of the teachers into study is important to overcome the limitation of researcher's experience in visual art and to help students to feel on their natural environment and to provide them opportunities of explaining their artworks to someone else. During critiquing parts, teacher reflected her opinions regarding students' artworks and made suggestions for development of students' works. Other parts of the studio were led by the researcher, who also some experiences in both visual arts and mathematics (see researcher background/role in section of 3.10 in the method).

3.5.4. Data Sources of the Main Study

The data sources of the study are interviews with participants, observation notes, students' documents such as sketches or other types of works that were produced in the process of art-making. Each data sources are explained in the Table 7.

Table 7. *Data Sources of the Study*

Data Sources	**Purpose for Data Sources**
Interviews	To describe students characteristics and support other sources of data (documents, observation)
Pre-Implementation	To learn about students' feelings and opinions about visual arts and mathematics and their previous experiences in both disciplines
During-Implementation (Stimulated-Recall)	To examine students' thinking processes by asking to recall their particular past experiences and to explain why they did them.
After-Implementation	To learn about the experiences of students in the current study

Table 7 (Continued)

Observation of Video Recordings	To take note of the critical actions regarding visual-spatial thinking (verbal expressions, gestures, order of actions, communication between researcher and students)
Documents (written notes, sketches, artworks)	To learn about students' visual thinking processes by supplementing other data sources of the study

3.5.4.1. Interviews

Interviewing is one of the methods for data collection. It is important to understand how participants think and feel and to support or refute the data obtained from other data sources such as observation and documents of participants (Fraenkel, Wallen, & Hyun, 2011). In this study, the researcher conducted three interviews with each participant: (1) Pre-implementation interview, (2) During-implementation interview (stimulated recall interview), and (3) Post-implementation interview. While during-implementation interviews (stimulated recall interview) were the major data source of this study that is used to support other sources of data such as documents, and observations, pre-and post-implementation interviews was used to learn about students' opinions and prior experiences in visual arts and mathematics, rather than as major data source of the current study.

The first type of interview conducted by the researcher was pre-implementation interview. The purpose of pre-implementation interviews was to learn about the students' feelings and opinions about visual arts and mathematics and their previous experiences in visual arts and mathematics in order to describe their characteristics in the current study. They were audio-recorded and lasted between fifteen and twenty minutes. Interviews were conducted in a private room so that the researcher and interviewee were not distracted from other people. The questions were basically related with their experiences regarding visual arts and mathematics in school and out-of-school contexts, their opinions about the necessity of visual arts courses, their strengths and weaknesses in visual arts and mathematics, their opinions on the relation between visual arts and mathematics (see Appendix D for interview protocol).

The second type of interview conducted by researcher was during-implementation interviews, called as stimulated recall interview. Stimulated recall interviewing is considered as a crucial method for understanding individual's decision making process such as what they do and why they do so, and cognitive process underlying their actions. In the stimulated recall interviews, participants are asked to recall of their cognitive process through replaying video records of their behaviours or examining non-video materials (De Smet, Van Keer, De Wever, & Valcke, 2010). In the current study, non-video stimulated recall interviews were conducted even though stimulated recall interviews were mostly conducted through replaying video records (Lyle, 2003). Non-video stimulated recall interviews involved recalling students' actions on their artworks, sketches, and written notes. The purpose of these interviews was to examine the way students think about selected critical points by asking students to recall their particular past experiences and to explain why they did them. These interviews took place after each studio work in both pilot and main study. Interviews were conducted either in the studio or in a private room after each studio work was completed. They lasted between twenty and thirty minutes approximately for each studio work. They were both audio and video recorded. During interviews students were asked to reflect on *general* and *specific issues* related to their own experiences. General issues involve perceived complexity level of studio work; students' enjoyment during the studio work, students' suggestions regarding current studio work. In addition to general issues, the researcher also asked specific points in their written notes, artworks, and sketches so that they remember how they did it (see table 8 for sample questions, and see Appendix F for specific questions regarding each studio work).

Table 8. *The General Structure of Stimulated Recall Interviews*

Question types	Questions
General issues	How difficult was the requirements of the studio work? Is there any task that you thought it was very easy and then it became difficult? Or Is there any task in which you have any difficulties from the beginning and then you made it easier later? Could you give examples? What made you enjoy? What are your suggestions regarding studio works? What worked? Or What did not work?
Specific issues	Where did you start from? Why did you choose these shapes? What was your first idea to do it? What kind of changes did you make? Is there anywhere you deleted and modified in your artwork? Why did you give up making it? What did you do to achieve it (draw shapes)? Why did you place each shape in this way? How did you embed the shape into other shapes?

The third type of interview conducted by the researcher was post-implementation interview. The purpose of post-implementation interviews was to learn about the experiences of the students during studio works, what they learned; their views on visual arts and mathematics after the study, which could shed light on the future studies (see Appendix E for interview protocol). They were audio and video-recorded and lasted approximately ten minutes in the main study and about twenty minutes in the pilot study. Interviews were conducted in a private room so that the researcher and interviewee were not distracted from other people. The interview protocol for post-implementation interviews included questions about their perceived difficulties in the tasks and their opinions after implementation regarding the connection of visual arts and mathematics, and their feelings about the activities. After the study, all interviews were transcribed by the researcher through recording what participants and researcher said exactly in a dialogue. In addition, the researcher noted where the participant or researcher pointed at the documents, particularly during stimulated recall interviews. After transcription, stimulated recall interviews were analysed.

3.5.4.2 Observation Notes

Observation notes are written reports of what the researcher sees, think, make inferences about participants' actions. It is an important way of collecting data to monitor participants' process over time and support other sources of data such as documents and interviews. It could be both descriptive and reflective. While descriptive notes involve objective records of setting, participants, and their actions, reflective notes involve researcher' subjective opinions, impressions, and inferences about participants' actions (Bogdan & Biglen, 2007).

In this study, observation notes were written both during the studio work and after each studio works by watching the videos. However, most of them were taken after the studio works since it was very difficult to take notes about students' actions when the researcher directs studio works and acts as a coach. The purpose of observation notes is to take note of the critical actions of students' visual-spatial thinking in the context of studio thinking. At the same time, verbal and visual communication between the researcher/teacher and the students or between the students is very important in order to define the studio atmosphere.

Each studio works is both audio and video recorded. The duration of videos for each student in a studio work last between three and five hours. Each video was observed to search for instances of visual-spatial thinking. Descriptive notes were taken by the researcher to describe the setting, activities, processes and non-verbal language. In addition to descriptive notes, reflective notes were taken to record researcher's opinions and thoughts regarding explanation of students' thinking processes and what works or not work during studio works, noteworthy events during studio works. Observation notes were recorded on three critical aspects to describe studio environment and students' thinking process. Each was explained in the following. Observation form and guiding questions are presented at the Appendix G.

- **Context**: Information about the physical setting (mapping the layout of seats, desks and other objects in the studio), name and number of students, materials in the studio, roles of researcher, teacher, and students during each studio work and scheduling of activities during each studio work.

- **Students' actions at three phases of studio thinking**: This aspect explains students' general actions and gestures with regard to phases of studio thinking in each studio work. Students were observed during demonstration, students at work and the critique phases separately. How students react in each process and what kind of visual-spatial thinking students make use of during each process are critical questions of this observation dimension.

- **Students' actions with regard to studio thinking**: This aspect searches for instances of students' in-depth thinking processes and gestures regarding visual-spatial thinking through studio habits of mind such as observing, exploring, envisioning, and reflecting. Students' visual-spatial thinking processes were observed with regard to each habits of mind. The following questions guided to collect data regarding this dimension: In what circumstances they have difficulties; How do students start to their assignments; In what circumstances students make changes in their art making process; When researcher asks students to observe famous artists' works, what kind of things do they focus on, what do they see?, What are their justifications regarding their opinions? How do they explain and evaluate other students' works? How students react to others artworks? (See Appendix G for more specific guiding questions.)

After the study, what participants and researcher said during studio works was recorded by watching the videos. The researcher typed all of the dialogues after the study. During and after typing transcripts, the researcher took notes about students' critical actions regarding visual-spatial thinking and what and where students pointed on the smart board or on their documents.

3.5.4.3. Students' Documents

Documents are used to supplement other data sources of the study such as interviews and observations (Bogdan & Biklen, 2007). In this study, documents refer to the materials that participants produced through writing and drawing. Students' documents involve their *written explanations* (notes about their strategies, their evaluation of own artworks, notes about famous artworks), *sketches or drawings* and *art-works* (Figure 11).

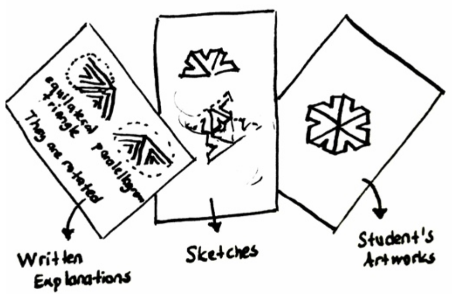


Figure 11. Three different students' documents: Written explanations, Sketches/Drawings, Final Artworks

The written explanations involve students' notes about the shapes that they see in famous artists' artworks when they were asked to observe it, and notes regarding their own artworks (angle and length sizes of shapes, the shapes that they drew, what did not work). *The sketches or drawings* refer to visual forms of students' thinking process. Especially during students-at-work part, students are encouraged to make sketches and re-examine them. A sketch book and pencils were given each student to draw what they think and take notes on it. Sketches provide information about how one thinks and changes his/her ideas (Suwa, 2003). *Students' artworks* refer to last versions of students' sketches. After a few sketches they make a copy of the last version of the sketch in a paper larger than their sketch books. It enables researcher to compare the sketches with the final artworks and record what kind of changes they made or their difficulties.

3.6. Pilot Study

The pilot case studies are important to clarify research design and revise the plan of data collection such as data collection procedures and tools (Yin, 2009). In the current study the pilot study was conducted to make revisions and modifications regarding studio works. It was conducted at one of the Science and Arts Centers in Ankara. Participants of the pilot study were three students at seventh-grade. They were voluntarily participated to the study on the basis of parental consent. They participated to the study shortly after the the semester at seventh grade ended.

At the Science and Arts Centers, students are enrolled with regard to their abilities. There are three main ability groups such as general mental ability group, group with visual arts ability and the group with music ability. They are selected on the basis their primary teachers' views and their performances on the ability tests. Firstly, primary teachers nominate students as candidates of ability group in Science and Art Centers. Students who are nominated by their teachers participate to general screening test. Students who get a score above a specified score—it changes depending on ability groups, the test are invited to individual screening process for each group of ability. On the basis of their performances in these evaluations they are selected (Kanlı & Özyaprak, 2015; MONE, 2017).

While two of the students were in the general mental ability group, the other student was in the visual-art ability group. Students with different background were involved into study to elicit different ways of visual-spatial thinking. These students are educated both in public school and at Science and Art Centers. They came to the Science and Arts during off-hours of the public school.

3.6.1. The Setting of the Pilot Study

The pilot study was conducted in the arts studio of Science and Arts Center. The studio involves tables and chairs for students, and a teacher desk, and a cupboard to keep students' materials and works. Three cameras were used to record students' actions. Each student's detailed work process was recorded with a camera. One of the cameras was used for recording the interactions between researcher, teacher, and students during critiquing part. There is a wall area to put the works or their notes on the wall.

The necessary materials for each studio work were provided by the researcher. Some of these materials are: Drawing pencil, sketch book, glue, different type papers, colorful dry paint, pastel, miter-ruler, eraser, compass, cartons, model carton, pencil sharpener. The researcher also brought a computer to encourage students to make research for their artworks. Music (classical piano music) was used to motivate students during their work. In the studio, students could make break whenever they need.

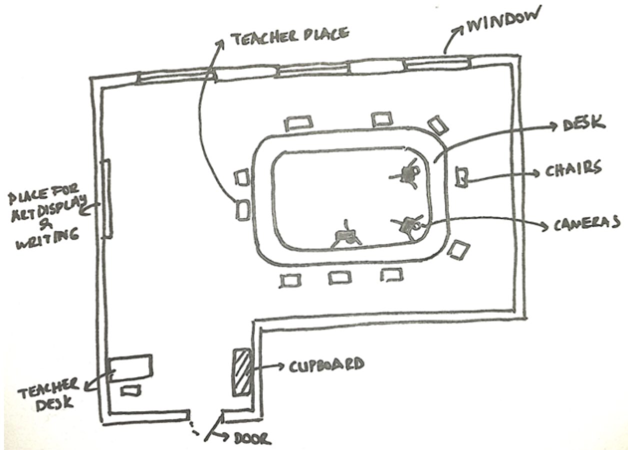


Figure 12. Sketch of the art studio in the pilot study

3.6.2. Data Collection Process of Pilot Study

Pilot study was conducted at the beginning of the summer semester in 2017. Implementation of studio works lasted in almost ten days. The study was carried out with a one or two day breaks. Six studio works were implemented. Each studio work lasted between four hours and six hours. It depended on time period that students finish their artwork. Some students sometimes finished their artwork earlier or later than their friends. When they finish their art-works, they were allowed to leave the studio. On the next day, students' artwork was evaluated at the critique part. In other words, critique part of previous studio work has been implemented on the next day.

Visual art teacher of the Science and Art Center involved into the study mostly during critiquing part. During this process, she made comments on students' artworks. She also sometimes visited the art studio to observe students' making art process and to demonstrate how to use colors.

Interviews about students' critical actions during studio works were conducted after each studio work. It lasted between twenty and thirty minutes. In addition to these interviews, before and after implementation, students also were interviewed to learn about their prior experiences visual arts and mathematics, their feelings towards visual arts and mathematics (interviews in pre-implementation) and to learn how they experienced the studio works and changed their view towards visual arts and mathematics (interviews in post-implementation). All interviews were both audio and video recorded.

3.6.3. Revisions After Pilot Study and Expert Opinion

In the current study, the purpose of the pilot study was to review the content of studio works and procedures used during the data collection. In addition, the researcher consulted experts to examine studio works before and after pilot study. Experts were a visual art teacher and a professor who studies development of visual-spatial thinking. On the basis of pilot study, reviews of experts, and researcher's experiences in the pilot study, some changes were made. These changes are categorized in seven main topics as follows:

- **The order of tasks in studio works:** In the pilot study, imagination process was not emphasized even though students are asked to imagine the given task. Thus, they had to tendency to solve the problem by trial and error or using tangible materials. This process might be helpful to imagine the given task. But, we firstly need to understand whether students could mentally imagine the situation without any tangible material or trial an error. If they could not do it, we could understand they have a difficulty. Then we could encourage them to use tangible materials to explore the situation. Thus, in the main study students were firstly asked to imagine the event. If they have difficulty in envisioning, they could use tangible materials to achieve the goal. For example, regarding studio work 3, students had a tendency to use pencil box to measure the lengths of shapes even though they are not allowed to use ruler. The reason such as restriction was to understand how they place each shape mentally with respect to each other. If they had only used the ruler, they just measure the lengths of the shapes and place them with regard to absolute lengths in the corresponding space. Thus, it was decided that students should first envision where they put each shape to the paper without using any materials, and then they control it by using a ruler.

Another change in order of tasks was related to observation of artworks. In the pilot study it was observed that while students observe artworks, they share their ideas with their friends. This resulted in affecting their friend's view and students sometimes felt weak themselves in finding shapes compared to his/her friend. Therefore, observation of famous artworks during demonstration part was organized in two parts: individual observation and group observation. During individual observation, students took notes of what kind of shape they saw and are not allowed to share it with their friends. After all students observed the artworks individually, they are asked to share what they see during group observation.

- **Additional artworks to observe:** During studio work 2 in the pilot study, students did not have a tendency to imagine the rotations of shapes mentally. There was a symmetrical series of an artwork by Frank Stella. On the basis of expert opinion analogous series of a different artwork was added to encourage students to imagine rotations mentally. These series were not symmetrical compared to the first series. In conclusion, it was added to encourage students to envision rotations of shapes and to observe their thinking process in rotations of symmetrical and asymmetrical shapes.

- **Encouragement for writing as an additional reflection tool:** During pilot study students were asked to take notes about their observations of artworks. In addition to taking notes during observation, after completing their own art-work students were asked to write the description of their artworks. The reason of such a change was that students sometimes had a tendency to randomly compose shapes or rotate them without using their geometrical or mathematical knowledge. To encourage students to use at least informal spatial ways of thinking, students were asked to write the description of their artworks. Students were guided by several questions regarding how they created their artwork. For example, regarding studio work 1, students were asked to answer the questions of what shape(s) did you hide? What else shape did you use hide that shape? What kind of strategies did you use to hide it? How did you start and continue drawing shapes, their sizes, and angles? Regarding studio work 2, students were asked to answer the questions of which shape did they rotate? How did you rotate? (e.g. the angle of rotation, direction, the point of rotation) Regarding studio 3, students were asked to evaluate their reproductions of artworks of famous artists and take notes about mistakes or the points to be revised. The reason of such a change was that during the pilot study, students evaluated their own works during scaling. However, the researcher did not directly understand or observe what they thought. Their notes could be evidence for their thinking process.

- **Encouragement for sketching:** During pilot study, sketch books were given to students to make drawings. The researcher told students they could make drawings on their sketch books to create their own artworks. It was observed that one of the participants had a tendency to sketch his ideas and explore his sketches and make relations between them. His sketching process frequently elicited his' difficulties and their strengths in visual-spatial thinking. Thus, in the main study, the researcher put more emphasis on sketching and asked to students draw re-examine their old sketches, draw new sketch, or draw new possibilities.

- **Encouragement for exploration:** In the pilot study of studio work 2, it was observed that students had difficulty in rotating the shapes mentally. To encourage for exploration, art teacher suggested using a piece of paper for rotation. This encouragement helped to students to mentally imagine the rotation of shapes, which might potentially elicit how students think during rotating a piece of paper. Thus, in the main study, in case that they had difficulty in rotating, students are asked to draw it to the dot paper to see what happens or they are given concrete materials such as a piece of paper to rotate. But they should first rotate it mentally to see whether they need a material for rotation or they can mentally rotate the shapes without a physically experiencing the rotation.

- **Time limitation for completion of tasks:** In the pilot study, studio works lasted between four hours and six hours in a day. It was very long for students. Thus, some tasks were time-limited in the main study. For example, time for individual observation for each artwork was limited. The researcher reminded that students have three minutes to observe each artwork. However, this time limitation was not strict to avoid pressure on students because students sometimes might not finish it in that time period. In addition to observation tasks, during studio work 3 students were asked to copy each artwork in five minutes. This yielded students to explore different ways of reconstructing artworks (from small spare to larger space; 1:4 scale). Otherwise, they would consider each side lengths of each shape one by one and multiply by four to transfer each shape from the smaller space to larger space.

- **Complexity level of tasks:** In the pilot study, during studio work 3 students were asked to copy four artworks created by different artists with the scaling factor of 1:4. In other words, they had to draw each shape in larger space and place each of them in a correct place in that space. In the pilot study, students were not given a paper that was exactly four times the size of the artworks.

Students are asked to find the exact size of the paper so that its size is four times the size of artworks. However, it increased complexity for placing each shape to the correct location. Thus, students became disappointed. Therefore, in the main study, to decrease complexity level of the task, students were given a paper that was exactly four times the size of the artworks.

In addition, before pilot study, on the basis of art teacher's comment, an artwork (Tony Smith, Untitled (Louisenberg), 1953-1968) was removed since it could be very easy for seventh grade students and several artworks (e.g. Sol LeWitt, Cube Circle 4; Frank Stella, River of Ponds; Robert Mangold, Three Color + series) added to provide diversity, which elicit students' different visual-spatial thinking processes. Pilot study indicated that observations of artworks with different geometrical configurations elicited students' difficulties and different processes of thinking. Lastly, artwork with pale color (Agnes Martin, Harbor Number 1, 1957) was recolored to make understanding the relation between shapes in the artwork easy because students had difficulty in encoding relations between shapes due to the color of the shapes.

To conclude, studio works were reviewed on the basis of pilot study and experts' opinions. In this way, the last version of pedagogical principles was formed (see part of 3.7.2.). The last version of the studio works is presented in the Appendix I.

3.7. Description of Studio Works and Pedagogical Principles in the Math-Art Studio Environment

In this part, studio works were described firstly. Then, the pedagogical principles used in the Math-Art Studio Environment are presented on the basis of previous studies in the literature, expert opinions, and pilot study.

3.7.1 Description of Studio Works

In the current study, three art works were analysed to understand students visual-spatial thinking processes (see table 9). Six studio works were designed and implemented as part of larger project. While the first three studio works were about two-dimensional artworks, last three studio works were about three-dimensional artworks. The focus of this study was on studio works with two-dimensional artworks so that students' visual-spatial thinking in two-dimensional artworks could be analyzed in-depth and in a consistent way. They were the first three studio works; thereby, students' thinking processes would not be affected by other studio works. Each studio work is explained in the following parts.

Table 9. *Description of Studio Works and Their Spatial Content**

	Description	**Spatial Content of Studio Works**
Studio Work 1	-Finding embedded figures in artworks -Creating an artwork that hide geometrical shapes and forms.	To describe shapes To specify parts, relation between parts, shapes, orientation and their size To pick shape out from overlapping objects
Studio Work 2	-Performing key elements of transformational geometry (rotation, flip, reflection): The case of Frank Stella's artworks and The case of Robert Mangold's artworks -Completing an artwork (one of Frank Stella's V series) that is considered as a beginning of another artwork	To recognize geometric shapes To predict the image of an object when it is rotated, flipped, or reflected. To predict the resulting shape when nested triangles are combined on the basis of different combinations.
Studio Work 3	-Drawing the given artworks at 1:4 scale: The cases of Robert Mangold, Mel Bochner, Agnes Martin's artworks (categorized as ordered versus scattered, decomposed versus composed)	To distinguish basic geometric shapes To recognize the hidden geometric shapes in the nested figures & shapes To change the size of the shapes proportionally To determine angular and length relations between shapes To place the shapes correctly in a larger paper

*Note: These three studio works were implemented to understand students' visual-spatial thinking processes in two-dimensional artworks in depth that was the focus of this study. Then, it continued by implementing studio works with three-dimensional artwork.

3.7.1.1. Description of Studio Work 1

The focus of the studio work 1 is basically on recognition of geometric shapes. It also has potential to elicit other visual-spatial thinking processes such as spatial proportional reasoning, perspective taking, and mental rotation of shapes depending on the students' processes of thinking. It involves three main parts: demonstration part in which the researcher/teacher presents visual contexts such as artworks to observe both individually and in the group (observing with friends), creating artwork, and critiquing artworks. Even though there are three main parts in the studio work 1, they, in fact, are interrelated to each other. For example, depending on the students' art-making process, researcher/teacher can make critiques on students' artworks or demonstrate some techniques regarding using materials when students are in stuck.

At the first part, students were firstly asked to watch a video to warm-up students to the study. This video was about the process of a group of artists restructure one of the wall drawings of Sol LeWitt on a wall surface. After warm-up, students observed several artworks with different properties such as artworks with embedded geometric shapes, artworks with reversible geometric shapes (perceived as both two-dimensional and three-dimensional) (see table 10). Students were firstly asked to observe them individually and take notes about what kind of geometrical shapes they see. After students observed each artwork individually, the researcher selected some artworks in which students' identification of shapes differed from each other. Selected artworks were presented on the smart board. Then, students were asked to observe again with their friends again (group observation) and explain what they saw during individual observation and what they saw that they had not seen before.

In the second part, students were asked to create an artwork through inspiration from the artworks observed in the demonstration part. Students created artworks with the purpose of hiding shapes that are difficult to be perceived by someone else.

This could be achieved through embedding geometric shapes and/or creating a composition that is perceived as both two-dimensional and three-dimensional. During this process, students were encouraged to take risks, represent what they imaged on the paper through sketching.

In the last part, students were asked to explain their artworks to their friends, teacher and researcher. Their artworks were imported into smart board; thereby, each student could see others' artworks. Students reflected about how they did it. Students and teacher also made some evaluations regarding artworks. In the current study, evaluations were not often made due to the grade level of students. Also, students were not getting used to being evaluated and making evaluations (see for detailed plan of studio work 1 in Appendix I).

3.7.1.2. Description of Studio Work 2

The focus of the studio work 2 is mainly on transformation of geometric shapes mentally such as rotation, flip, and reflection. It also has potential to elicit other visual-spatial thinking processes such as recognition of shapes, identifying congruence and similarity between shapes, recognition and envisioning of spatial patterns, and representing/drawing transformations depending on the students' processes of thinking. It involved three main parts: individual and group observation of artworks (in the demonstration part), creating artwork (in the students-at-work part), critiquing artworks. Even though there were three main parts in the studio work 2, they, in fact, were interrelated to each other. For example, depending on the students' making art process, researcher/teacher made critiques on students' art work or demonstrated some techniques regarding using materials when students were in stuck.

In the first part, students were asked to observe two different series of artworks: symmetrical series and non-symmetrical series of artworks involving different transformations of shapes such as rotation and flip. Students were firstly asked to observe them individually and take notes about what kind of geometrical shapes they see and the differences and similarities between these artworks (individual observation). After students observed each artwork individually, the researcher selected the first series of art wok (symmetrical) and asked students to reflect what they saw on the smart board and think about how the artist could rotate one of the artworks to make the next one (group observation). The researcher asked students to estimate the number of degrees required to rotate the artwork in order to make the next artwork. When students had difficulty in estimating it, the researcher asked them to explore on the dot paper and try to rotate the shape physically that is made of paper. After students had previous experiences in rotation of shapes and reflected how they rotated a shape, students were asked to create an artwork through rotation.

In the second part, students were asked to create an artwork. Researcher asked students to think about the question of "If this artwork is only the beginning of whole artwork, what might happen next?" and create an artwork that complete one of the artworks of Frank Stella (see table 11). The researcher encouraged students to envision a variety of possibilities to create an artwork.

In the last part (critique part), researcher asked students to talk about what strategies they used and how they did it. Each students' artworks were imported into the smart board to enable to students to reflect on artworks easily. The researcher and teacher made comments on their artworks. In this way, students realized different organization of the same artwork. (See for detailed plan of studio work 2 in Appendix I)

3.7.1.3. Description of Studio Work 3

The focus of the studio work 3 is mainly on scaling that requires transforming sizes of geometric shapes mentally. It also has potential to elicit other visual-spatial thinking processes such as recognition of shapes, recognition of spatial patterns, and spatial proportional reasoning. It involved three main parts: demonstration, copying artwork and critiquing artworks. Even though there were three main parts in the studio work 3, they, in fact, were interrelated to each other. For example, depending on the students' making art process, researcher/teacher made critiques on students' art work or demonstrated some techniques regarding using materials when students were in stuck.

In the first part, the researcher gave students four different artworks of famous artists (see table 11). She introduced the task in which students copy four artworks with scaling factor of 1:4. In other words, they had to draw each shape larger and place each of them in a correct place in a larger paper. Firstly, they were asked to order each artwork in terms of their complexities from 1-10 to understand their perceived difficulties. This was important to understand how students perceive the difficulty of scaling transformation in each artwork and what kind of factors they considered in deciding difficulty level of the artworks.

In the second part, students restructured each artwork in a paper that is four times larger than the original artworks. During this process, the researcher encouraged students to predict what each object (shape) could be located in a larger-scale painting. She also reminded students to observe their drawing again, think about what kind of problems exists, and take notes. After students completed each artwork, they were asked to check their drawing of an artwork by using ruler whether it was correct or not. This was a part of critiquing their drawings. It was the interrelated process of students-at-work part and critiquing part.

In the last part, the focus was on critiquing of drawings of students. Some of artworks were chosen to describe and critique. Each students' copies of artworks were imported into the smart board to enable to students to reflect on them. Students described how they put each shape in a larger paper. After students' description, the researcher and their friends evaluated the drawing and suggested ways to revise it (see for detailed plan of studio work 3 in Appendix I).

In summary, each part of the studio works was interrelated to each other depending on students' performances. The researcher/teacher made critiques during creating artwork (students-at-work part). During this process, the researcher made some demonstrations regarding how to use materials and how to draw geometric shapes if they were in stuck and if students were not familiar with geometric shapes.

Table 10. *Description of Artworks Used in This Study: Studio Work 1*

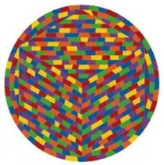
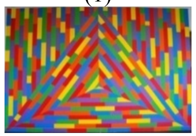
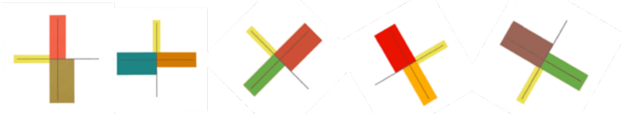
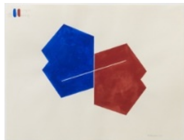
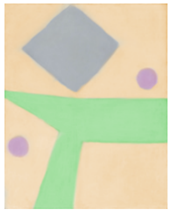
Artworks in Studio Work 1		Description of Artworks
 (1)  (2)	 (3)	**(1)[Sol LeWitt, Cube Circle 4].** It involves isometric drawing of a cube. The cube is embedded into circle. It also could be interpreted as two-dimensional such as hexagon consisting three rhombi. **(2) [Sol LeWitt, Wall Drawing #1113].** It involves two-dimensional representation of a triangular pyramid that can be perceived from the top view. This shape is embedded within very small quadrilaterals. It is also perceived as two-dimensional such as an equilateral triangle consisting of three isosceles triangles or two trapezoids and an isosceles triangle. **(3)[Sol LeWitt, name unknown].** It involves two-dimensional representation of a triangular prism whose lateral faces and front face is seen. It is also perceived as half of a rectangular prism. When it is perceived as two-dimensional, it consists of two congruent parallelograms, three right triangles.
 (4)	 (5)	**(4)[Frank Stella, Tomlinson Court Park from Black Series, 1967].** It is perceived as both two and three-dimensional. It is seen as nested rectangles when it is perceived as two-dimensional. It is seen as truncated square pyramid from top view or bottom view when it is perceived as three-dimensional. **(5)[Frank Stella, Hampton Roads, 1961].** It is perceived as both two-and three-dimensional. It is seen as nested squares when it is perceived as two-dimensional. Their sizes change proportionally It can also perceived as a square pyramid from top view or top view from the left side depending on the position of observer and obliquity of the pyramid.
 (6)	 (7)	**(6)[Mel Bocher, Four Shapes, 1973/1976].** In this artwork, there are four figures that consist of hidden geometric shapes. Each shape involves a combination of a regular pentagon, triangle, and square so that there is not a space between them. **(7)[Frank Stella, River of Ponds].** It consists of embedded geometric figures. Different sized squares and sectors are embedded in the artwork. In other words, it involves nested squares and sectors whose sizes is getting smaller or bigger.

Table 11. *Description of Artworks Used in This Study: Studio Work 2 and Studio Work 3*

Artworks in Studio Work 2 and Studio 3	Description of Artworks
 (1)	**(1) [Frank Stella, V series].** It involves symmetrical analogous series of an artwork that are rotated. The last artwork is not identical to others. Students are asked to find the differences and similarities between these artworks and think about how each artwork are rotated to make the next one.
 (2)	**(2) [Robert Mangold, Three Color + series].** It involves non-symmetrical series of artworks. The first, the second and the fourth artworks are congruent in terms of shape structure and they are rotated. The third and the last artwork is flipped version of the first artwork. Students are asked to find the differences and similarities between these artworks and think about how each artwork are rotated to make the next one. Colors and direction of some artworks were adapted to the context of the study.
 (3)  (4)	**(3) [Robert Mangold, Four Square Within a Square (Light Blue), 1974].** At this painting, there are four squares that is located in the corners. Each squares' sizes differ from each other proportionally. The spaces between squares are also located proportionally. (ordered layout with decomposed shapes) **(4) [Mel Bochner, First Fulcrum, 1975].** It has a symmetrical configuration formed by a line segment located at the center of the painting. It involves two different colored objects with the same shape. Two regular pentagons are located as nested inside each shape. (ordered layout with composed shapes (nested and hidden))
 (5)*  (6)	**(5) [Agnes Martin, Harbor Number 1, 1957].** It has an asymmetrical configuration of shapes similar to non-regular geometrical shapes such as different sized quadrilaterals, non-regular pentagon including deltoid and a circle. They are not placed in a geometrical pattern and geometrical shapes are not hidden. *Colors of the artwork was changed to increase the visibility of the shapes. (scattered layout with decomposed shapes) **(6) [Mel Bochner, Two Shapes, 1976].** It has an asymmetrical configuration of regular polygons that are hidden inside two colored asymmetrical shapes. The hidden geometrical shapes are square, pentagons, and equilateral triangles that are connected to each other with a common line segments. Thus, each length of regular polygons is equal. (scattered layout with composed shapes (hidden))

3.7.2 Pedagogical Principles of the Studio Works in the Math-Art Studio Environment

On the basis of previous studies in the literature (Hetland et. al., 2013; Tishman & Palmer, 2006), pilot study, and experts' opinions, several principles were derived for designing the studio works of the current study. It is important to note that these principles do not focus on examining students' artistic abilities, creativity or students' understanding of aesthetics. Principles were organized under two categories: principles regarding overall characteristics of the studio works in the current study, principles regarding specific characteristics of the studio works. It is also worthy writing down that these principles have tentative nature. These principles might be revised and changed at the end of the main study or other future studies.

3.7.2.1. Overall Characteristics of the Studio Works

There were four general properties of the studio works in the current study. These characteristics of studio works were regarded as driving forces to elicit students' thinking process even though they were not directly related with visual-spatial thinking. These properties were explained as follows:

1. Each studio work consisted of three steps; demonstration, students-at-work and critique.
   - In the demonstration part, students observed famous artworks individually (individual observation) and with their friends (group observation) and teachers introduced what students would do at the students-at-work part.
   - In the students' artwork part, students created their own artworks.
   - In the critiquing art, students described and evaluated their own and friends' artworks.

2. The studio works started with famous artists' works in the demonstration phase and generally ended with critiquing phase. However, when it was necessary, teacher demonstrated some techniques or students critiqued their artworks during students-at-work part depending on the nature of the studio works and students' needs.

3. Teacher had a role as a coach who demonstrated, advised, questioned and criticized.

4. The studio works were implemented in the arts studio in which students had physical freedom, used a variety of materials, took breaks, went to the bathroom, and played music (classical piano music).

3.7.2.2. Specific Characteristics of the Studio Works

There were five specific principles of the Math-Art Studio Environment that explains characteristics of the studio works. These principles were directly related with understanding of students' visual-spatial thinking processes. These principles were explained through specific examples respectively.

1. Students were given opportunities to observe and reflect upon famous artists' artworks that were rich in geometric shapes.

1a. Students were asked to observe famous artists' artworks that were rich in geometric shapes: For recognizing shapes, students observed the artworks that involved nested two polygons that form a new polygon, overlapped geometric shapes that required completing them, the art works that could be perceived both as two-dimensional and three-dimensional objects or perceived from different point of views. For transforming shapes*,* for example rotating shapes, students observed analogous series of an artwork that were rotated. However, one of them was not identical to others in terms of rotation. Also, students observed both symmetrical and non-symmetrical series of artworks involving rotation of shapes. For scaling, they observed a variety of artworks with different properties such as different layouts of shapes such as ordered and scattered and shapes such as nested/hidden and discrete (see table 10 and table 11).

1b. Researcher asked questions during observation in order to encourage students to reflect on geometrical aspects of the artworks. Here are several sample questions: What shapes/colors/lines do you see? Take notes of at least five words or descriptions about the shapes and forms in the artwork. Look at again and take notes about the similarities and differences between artworks; what kind of mathematical or geometrical strategy does the artist might have used to make the second artwork different from the first artwork? [In case of series of an artwork]; what shape is that? What makes you say that? [e.g. Why is equilateral triangle or a prism or a pyramid?]; From which perspective do you perceive it? Could you describe how do you imagine it? You can explain by drawing as well; what else do you see? Do you notice something that you're not used to paying attention to, what do you see that you have not seen before? Do you perceive any three-dimensional forms? [In case that they do not realize two-dimensional representations of three-dimensional shapes in a painting].

1c. Students firstly were asked to to observe individually (individual observation) and then to observe with their friends so that they share what they see and realize (group observation). The reason such an order was to learn about each student's thinking process without interrupted by someone else and then learn about how they make use of visual-spatial thinking with others and elicit new processes of thinking. Here are some questions to probe their thinking: Could you see the shape that your friend saw? Do you agree with your friend? How do you agree?

2. Students experienced active art-making during students-at-work phase in which they were involved with *reproducing artworks of artists* or *creating their own work* to elicit a variety of visual-spatial thinking skills such as recognizing shapes, mental transformations such as rotation, scaling, and

cross-sectioning, perspective taking, making relationship between 2D and 3D.

2a. Students were asked to make artworks in line with demonstration phase in which students generally observed artworks with specific properties (e.g. hidden/embedded shapes, rotated or flipped shapes)

2b. Students were asked to reproduce artworks of artists that involve geometric shapes (e.g. making artworks in a larger scale (e.g. at the scaling factor of 1:4) through using their strategies such as proportional reasoning, considering relationship between shapes and their size and geometric properties (see studio work 3).

2c. Students were asked to create their own artworks. This required students to think creatively rather than copying an artwork. There could be two types of creating artworks: Completing an artwork and creating an original artwork. They are explained through examples as follows:

- **Completing an artwork:** Students were given an artwork or part of an artwork and asked to complete missing its parts in a creative way or continue to artwork as if it is just a beginning of another artwork (e.g. Completing an artwork though transformation strategies such as rotation, flipping by thinking as if it was just a beginning of an artwork (studio work 2))

- **Creating an original artwork:** Students were asked to create their own work with a purpose or on a topic through inspiration from the artworks observed in the demonstration part (e.g. creating artworks with the purpose of hiding shapes and forms that are difficult to be perceived by someone else (studio work 1))

3. **Students were encouraged for envisioning at all phases of the studio works by using a variety of tasks regarding different visual-spatial thinking skills, using additional warm-up tasks, and asking questions to encourage students to envision.**

3a. Researcher used a variety of tasks in different contexts such as observing famous artworks and encouraging students to make artworks that require envisioning. Through these tasks students were asked to envision the shapes that are not directly seen or are seen partially, transformations on their properties (e.g. rotation, scaling, slicing, folding), the position of shapes and relationship between them in a larger space, changes in own perspective and position or direction of objects. Students were also encouraged to observe the artworks with different properties that could result in envisioning different spatial properties (see table 10 and table 11).

3b. Researcher asked the questions that prompt students to envision: What if questions, what would happen …if, imagine that…, how do you envision in your mind, could you draw it? [e.g. Imagine what kind of shape you want to draw?; Imagine where the object/shape is positioned when it is drawn in larger paper. What if you changed the position or direction of the shape, how would the relation between the shapes change? How do you envision that shape from the top view? What would happen if repeat rotating that shape?]

3c. Students were involved into the studio works through additional warm-up tasks if it was necessary: The researcher used warm-up tasks before students created their own artworks. The reason of the use of such tasks was to help students get involved into the task. Students sometimes had difficulty in thinking mentally and were not get used to doing such artworks. Thus, she asked students to think on small warm-up tasks. For example, students were asked to draw the rotation of shapes in the art work of Frank Stella by using dot paper after observation of the artwork and before they were asked to create an artwork. They were asked to imagine how they rotated and the degree of rotation.

3d. **Students were asked to imagine and sketch firstly; then experiment by using trial and error during creating artwork.** This principle was important to understand whether they could imagine mentally without trial and error or to understand to what extent they consciously know what to do or how to imagine. Students had a tendency to use trial and error to understand how something works and they did not imagine whole process at first glance. Students were firstly asked to imagine the situation. If they had difficulty, researcher asked them to use trial-error. However, it did not mean that they were not simultaneous processes. Students imagined a transformation through trial and error as well.

4. **Students were given opportunities to stretch and explore by using the tasks that require thinking about possibilities, asking students to sketch and revise their ideas and to play with tangible materials during art making, asking them to make mistakes and take risks.**

4a. **Researcher used the tasks that required students to think about possibilities such as relationship between different geometric shapes, different versions of the same shape, different compositions or juxtapositions of the shapes, different views of geometric forms and the relative positions of shapes** (e.g. hiding basic geometric shapes in a painting with different compositions that students would create, rotating a particular shape at different angles and directions around different points in the plane; exploring the position of a particular shape with taking account of its possible relationships with other shapes when scaling transformation is required)

4b. **Students were asked to sketch/draw what they imagined during creating their own geometric artworks or to sketch/draw directly what saw when**

they were asked to reproduce or copy famous artist's geometrical artworks.

- **Students were asked to re-examine old sketches, reinterpret what they did, regroup parts, draw new elements, and focus different parts of their sketches.** For example, students were encouraged to make transition from one composition of shapes to another composition of the same shapes (e.g. change pattern of rotation, changing relative positions of shapes in an artwork), to elaborate a particular composition with including additional shapes (e.g. hiding geometric shapes into other shapes), make transition from a drawing to a new drawing that is completely different from the first one.

4c. Students were encouraged to play with tangible materials such as clay, paper, and three-dimensional objects to explore new possibilities. For example, students used a geometric shape out of paper to visualize the rotation of the shape and identify congruence between rotated shapes and to experience different types of rotation and different compositions of rotated shapes. Student also used three-dimensional mathematical materials such as a pyramid to visualize and explore how it is seen from different perspectives.

4d. Students were encouraged to make mistakes and take risks: Researcher gave students support to keep work, try new things, and to feel comfortable in expressing their ideas. She also reminded students that it is natural to have difficulty. In this way she encouraged students to make mistakes. She also encouraged students to pursue art-making; thereby, students avoided giving up the work.

5. Students were encouraged to reflect upon mathematical/geometric properties of geometric artworks at all phases of studio works through speaking, writing, and showing their ideas on smart board.

5a. Students were asked to talk about their works: Students were asked to put their thinking process into word through verbal language. This principle was related to the "reflecting" habits of mind in the arts education that involves two types of reflecting: describing and evaluating. It was also related to the thinking routine of reasoning in the artful thinking framework.

- **Students were prompted to explain their own working process or works and their friend's artworks during one-to-one conservations with students during students-at-work and critiquing phases** (e.g. *The researcher asked for describing their plan*: what kind of composition of shapes they wanted to create? what shape(s) they wanted to hide in recognizing shape/form task? *asked for describing their ways of creating artwork*: how they were placing each shape in a larger canvas in scaling task, why did you give up to make this artwork? how he/she constructed each square with increasing lengths? how he/she related between the shapes of triangle, square, and pentagon in hiding shape task? *asked for observe and explain what their friends did*: What shape might your friend have hidden? In your friends' art work you see the geometric shape that is perceived both two-dimensional and three-dimensional. How could it be perceived as three-dimensional?)

- **Researcher asked questions to enable students to justify their thinking with evidences such as "What makes you say that? Or Why did you it that way**": Specific examples of the questions were "Why do you think that shape is a rhombus?", "Why did you place the circle close to the edge of the rectangular shape in the scaling task?", "Why do you consider it as a prism rather than a pyramid?", "What makes you say that two parts of the painting is symmetrical?

- **Students were encouraged to evaluate own work and their friends' work during students-at-work phases and critique phases**. The researcher asked the questions of what is working and what is not and why, how do you solve this problem? What do you suggest for your friend? Why did you erase this shape too much during drawing? Are the rotated shapes identical in terms of their lengths? Which part of the painting did you have more difficulty in during copying art? Why did you have difficulty in drawing this shape? What do you suggest your friend to solve the problem in drawing in a larger paper [scaling task]?

- **Students were asked to observe own artwork regularly and pausing & thinking about their artwork** to identify problems in drawing of geometrical shapes/forms in terms of their geometrical properties such as angle and lenght (e.g. to identify scaling problems regarding difference between shapes, the angular relation between shapes, length of shapes the problems)

- **Students were given opportunities through additional tasks to evaluate their own work** (e.g. *deciding whether their drawings are accurate in the scaling task (studio work 3)*: after students finished their drawings, students evaluated their drawings through checking their mistakes with a ruler and compared their drawings with the original artworks, *deciding whether their drawings of rotation of shapes are accurate in the studio work 2* :after students represented the rotation of shapes, some of the students checked the rotation of shapes by testing it physically with a tangible material).

5b. Students were asked for writing: Students were asked to take notes regarding observation of artworks, problems in their own artworks, computations they made while they were creating their own artworks, or descriptions of their own artworks.

- **Students were given opportunities to keep notes in their sketch books or on worksheets and write about the points needed to be revised at all phases of studio work** (e.g. students took notes of what shapes they observed in the famous artists' artworks on their worksheets during individual observation; the changes regarding compositions of artworks and the steps in making transformations in the mental rotation task; further steps to remember during creating artwork such as what the lengths of the shapes and angles between shapes would be; the problems in the artwork that should be refined).

- **Students were asked to complete complexity scale from 1-10 to understand their perceived difficulties.** This principle was related to thinking routine of finding complexity in artful thinking framework. For example, in the scaling task [studio work 3], students gave numbers from 1 to 10 when they were asked to order four different artworks in terms of their complexities. Also, after some studio works, researcher used this scale so that students compare the complexities of each studio work.

- **Students were asked to write final description of their work by using mathematical language to share with others at the end.** For the studio work 1, students were asked to answer some specific questions regarding their artworks such as What shape(s) did they hide? What other shapes did they use to hide it? In what order they placed shapes? What were their sizes or angles between them? For the second artwork, students were asked to take notes of which shape did they rotate? How they rotated? What degrees and the size of the shapes? At which direction? Around which point? Steps in making this art.

In summary, several pedagogical principles were determined to design studio work to elicit students' visual-spatial thinking processes. Studio works were described on the basis of its overall and specific characteristics. The description of characteristics of studio works through several pedagogical principles is important to decide applicability of them in other similar studies.

3.8 Data Analysis

Data analysis is conducted to make meaning from the data to answer research question of study by reducing data into manageable parts (Bogdan & Biklen, 2007;

Merriam, 2009). To conduct data analysis, all data sources of the study were organized firstly. The data sources of the study were interviews with participants, observation notes regarding video recordings, and documents of students. Transcripts of interviews and video records, students' documents and observation notes were brought together in the MAXQDA 12 software so that it become organized and easily accessible.

After importing data into MAXQDA software, the transcripts of data were analysed through open coding through which the data are interpreted and questioned with constant comparative analysis (Strauss & Corbin, 1998). The instances of students' visual-spatial thinking processes were analysed by looking for similarities and differences between them. To analyse data through constant comparative analysis, each source of data concerning each student were revised respectively in each studio work. Videos and transcripts were first examined holistically before starting to analytic coding to make sense of the overall process in each studio work. Then, videos were watched by looking back and forth repetitively. Then, students' actions in the video were related with the transcripts of videos and interviewing and students' documents such as written explanations and artworks. During this process, researcher assigned names or labels to a collection of words and sentences, called as tentative codes, as early step in analysis (Miles & Huberman, 1994). Tentative codes were restated or revised up to they become saturated. Sub-codes are formed as instances of its general code by providing different perspective for explanation of that code (Creswell, 2007). The saturation was formed through comparative analysis of data between each students' own data sources and across students' data sources, and and data across different studio works.

The sources of the names concerning each code were driven from the data and literature. Although a particular framework of spatial thinking was not used directly, some studies on spatial thinking, especially the typology of spatial thinking proposed by Newcombe and Shipley (2015) and the review in the work of Sarama and Clements (2009), provide basis for initial coding in the current study (see literature part for detailed information). During coding process, a coding booklet was formed. It is important to record the definitions of codes and examples related to codes in a booklet since it allows researcher to revise and make clear the definitions of codes by comparing them (DeCuir-Gunby, Marshall, & Mcculloch, 2011). After analysis of the data, the codebook was shared with a second coder, who is a doctoral student in mathematics education. The final codes were determined on the basis of negotiation between the second coder and researcher. The researcher and the second coder discussed the codes by examining the transcriptions of the data. When there has been disagreement regarding the codes between researcher and the second coder, the meanings of the codes were revised by looking for its instances in the literature again and making its meaning its explicit. It was conducted until having a consensus on them. After all, final codes and their descriptions are determined. The Table 12 presents codes and descriptions used in the current study.

Table 12. *Indicators of Students' Visual-Spatial Thinking in the Current Study*

Codes and Sub-codes	**Description**
Recognizing geometric shapes	Identifying two-dimensional shapes and two-dimensional representations of three -dimensional geometric shapes in the artworks on the basis of their visual appearance or properties
Identification of geometric shapes as real-world objects	Associating geometric shapes with real-world objects on the basis of their visual appearance.
Identification of basic geometric shapes	Naming two-dimensional shapes, two-dimensional representations of three-dimensional shapes on the basis of properties at different conditions (change in orientation of the shape or viewpoint of the observer, embedded in the artwork)
Identification of shapes on the basis of properties	Identifying geometric shapes based on lengths relations (number of length, lengths size), arrangement of parts of shapes (symmetric), angular relations, number of vertices, faces, edges

Table 12 (Continued)

Identification of shape through disembedding & embedding shapes	Picking out a shape that are embedded into other shapes by ignoring them (disembedding) and nesting shapes into each other (embedding).
Identification of shapes from different orientations & perspectives/viewpoint	Identifying identical 2D geometric shapes even though they are rotated and identification of two-dimensional representation of 3D shape by imagination of a shape's view when one changed the view point.
Decomposing and Composing Shapes	Putting shapes together to produce new shapes (composition of shapes) or taking apart shapes into small shapes (decomposition of shapes)
Decomposing Shapes	Partitioning a whole shape into smaller shapes (dividing a shape into polygons based on their properties of length and angles or partitioning a whole shape into equal parts (slicing a shape into same-sized units).
Composing Shapes	Producing a new whole shape by combining individual units or units of units repeatedly or combining different geometric shapes to make a coherent whole.
Spatial Patterning	Identifying repeating and growing visual geometric patterns in the artworks. It involves identifying the parts of a spatial pattern (segmentation) and combining the parts based on a rule (integration)
Transforming Geometric Shapes	Identifying manipulations of shapes rigidly or non-rigidly that preserve the properties of shape.
Scaling Transformations	Identifying transformations in size of shapes and changing the size of shapes mentally by preserving their properties and the relation within shape or between shapes. It involves spatial proportional reasoning.
Mental Rotation and Flip	Identifying transformations in orientation of shapes and changing orientation of shapes mentally that preserves shapes and sizes.
Comparison of shapes though rotation & flip	Deciding whether shapes are identical or not through mental transformations of shapes.
Identification of congruence between shapes	Mapping the relation between shapes and their rotated images by considering visual aspects of shapes
Identification of angle, center and direction of rotation	Identifying angle of rotation based on visual appearances of angles and benchmark angles (45, 90, 180); Identifying movement of rotation around a center through change in direction.

3.9 Trustworthiness of the Study

Trustworthiness refers to the extent to which researcher convince the readers that the findings of the study are crucial and reasonable to take notice (Lincoln & Guba, 1985). They identified four major trustworthiness criteria that the researchers should take account of in a qualitative study. They are credibility, transferability, dependability, and conformability. How the researcher established each criteria is explained in detail.

3.9.1. Credibility

The third criteria to establish trustworthiness is dependability. Dependability corresponds to the term of reliability in quantitative studies. Reliability is defined as the extent of replicability and reaching same results in quantitative studies. In qualitative studies it refers to the consistency between the data that is collected and the findings of the study (Merriam, 2009). There are several techniques to establish dependability of the study: triangulation, peer examination, investigator's position, and the audit trail (Merriam, 2009, p. 222). The first three techniques are used in the current study to establish both dependability and credibility of the findings (see credibility part). In addition to these techniques, audit trail is also used. It is a detailed explanation of how data was collected, how data was analysed on the basis of specific categories and how conclusions are made. In the current study each process is explained in detail in the method chapter. In addition to detailed explanation, the codes were determined on the basis of negotiation between the second coder and researcher. The second coder was a doctoral student in mathematics education. The researcher and the second coder discussed the codes by examining the transcriptions of the data until they have a consensus on them.

The fourth criteria to establish trustworthiness is confirmability or neutrality. Confirmability is related to the extent to which the findings are not affected by the biases and assumptions of researcher (Lincoln & Guba, 1985). To establish confirmability, the researcher explained objectively her role and biases in the study (see part of 3.10). Triangulation of the data, thick description of data analysis process, and analysis of the data and reviews of the findings by the second coder are among the techniques used to establish confirmability in this study.

3.10. Researcher Background and Role in the Study

Researcher background and role in the study are crucial factors on designing and conducting the study, analysing the data, interpreting and reporting the findings. In this study, the researcher is a doctoral student in Elementary Mathematics Education and has been worked as a research assistant for eight years at Department of Mathematics and Science Education in a public university. During doctoral program, she took courses of both quantitative and qualitative research. She has knowledge of major qualitative research methodologies and has experiences in qualitative research such as writing master thesis with a qualitative method and conducting qualitative studies. She also learnt how to analyse the data on the software of MAXQDA. Regarding visual arts, she took elective courses of three different drawing courses (object drawing, human figure drawing, perspective drawing), two sculpture courses, and a watercolour course in art ateliers during master and doctoral programs. Thus, she had opportunities to observe the nature of art studios, and studio tasks, and the interaction between students and artists/instructors. In addition to these courses, she has learnt basic art movements herself and is interested in drawing in her free time.

In this study, there were several roles of the researcher such as acting as a coach, identifying students' critical actions during the studio works, interviewing students regarding their critical actions, keeping all sources of data, transcribing the audio and video files, analysing them. The researcher acted as a coach who gives demonstrations, provides suggestions and does evaluations with visual art and mathematics teacher to help students develop their artworks. The coach helps students to reflect on his/her performances, weaknesses and strengths during tasks such as creating artwork. Another role of the researcher was that the researcher examined their notes during individual observation and observed what they were doing, how they did it, and what kind of struggles they had, and took notes regarding their critical actions and ambiguous words while students were working on their artworks. The third crucial role of the researcher was that the researcher sometimes controlled the use of one of the cameras during group observation and critique of the artworks. The fourth role of the researcher was that the researcher interviewed some students on the basis of their critical actions after studio works. The last role of the researcher was that researcher investigated each audio and video episode, students' documents and analysed them on the basis of visual-spatial thinking after the study was implemented. Transcriptions of the videos were done by the researcher.

3.11. Limitations of the Study

There are three major limitations in the current study. They are limitations regarding direct observation of students' thinking processes, limitation regarding researcher role and background, and limitation regarding the content of the studio works. The first limitation is that it is very difficult to observe students' way of spatial thinking directly. Students might explain their thinking only by drawing or only by verbal language. Some students might not express their thinking process in an efficient way and can not be able to draw what they thought due to lack of psychomotor abilities even though they perceive and understand shapes and transformations mentally. Thus, this study is limited to the students' expression of ideas by verbal and body language, and their documents such as sketches and their notes regarding artworks. It is not just limited to what students documented, but also limited to researchers' analysis and interpretation of the data. The researcher might not understand what students exactly thought in a specific situation.

The second limitation of the study is the researcher role and background. In the current study, the researcher carried out the implementation of studio works and acted as a coach. This could be regarded as an advantage to experience close interaction with students and understand their observable thinking processes through this interaction. On the other hand, it could be also considered as a disadvantage because the researcher both directed the flow of the studio works, observed students' actions and managed the control of the cameras. It was very difficult to observe students' actions and take notes regarding them. Thus, taking notes regarding students' action during the studio work was very limited. To remedy this problem, the researcher recorded students' action with the use of cameras. In addition to several roles of the researcher in the current study, it is important to note to what extent researcher affect student thinking. When a student had more difficulty than the others and wanted to give up the task, the researcher asked questions step by step and demonstrated some techniques to encourage her/him to continue to the task. While it might affect students' thinking process, this kind of help was important for eliciting students' thinking process in further tasks of the studio work and provided motivation for the student.

The researcher background is also important factor in affecting data collection and analysis processes of the study. The researcher has experiences in mathematics education and is interested in visual art. Even so, she does not have experiences as experts or teachers of visual arts and has not experiences like teachers regarding how to communicate with students better and lead the studio works. To handle this problem, visual art teachers and mathematics teachers of the school were invited to the critique part of the study. They investigated students' artworks and made comments on them.

The third major limitation is the change in the number of students during implementing the study. At the beginning of the study, there were six students that participated to the study. After the second studio work, two students (Esra and Burcu) left the study due to personal reasons. The fact that these two students leave the study may have limited eliciting other students' thinking processes in the further studio works.

The last major limitation is the content of the studio works. The content of the studio work is limited to minimalist artworks that involve the use of basic geometric shapes and basically recognition of shapes in two-dimensional surface and their properties, transformations of them such as scaling and rotating, and representing or drawing them. On the other hand, there are other kinds of spatial abilities such as recognizing and transforming shapes in three-dimensions, recognizing and performing other kinds of transformations such as slicing, folding, and bending, and spatial orientation.

3.12. Ethical Issues

Ethical issues that the researchers should consider are the protection of subjects from harm, the right to privacy, the notion of informed consent, and the the issue of deception (Merriam, 2009, p. 230). First of all, to avoid ethical problems, the permission was taken from the Ethical Committee and Ministry of National Education. They investigated all data collection protocols and the content of studio works after investigation of the documents, they gave permission to conduct the study (see Appendix A and Appendix B).

Then, participants and administrator of the school were informed about the content and purpose of the study and the data collection process such as participating to the pre- and post interviews, interviews after each studio works, participating to the studio works almost two weeks. They were also informed about the use of cameras and voice recorder. After students were informed, the researcher asked them to participate to the study voluntarily. Students who were volunteered to attend to the study were given an informed consent form to be signed by their parents (see Appendix C). All parents signed the form voluntarily. The researcher informed each parents about the data collection process with honesty. It was emphasized that they have rights to withdraw the study whenever they want. Students were also reminded that they did not get a grade on this study and it did not affect their grades in the courses that they took.

Moreover, during data analysis and reporting results of the study the researcher did not explicitly use the names of participants and the name of the school and share with the other people. Rather the researcher described students with pseudonymous names to provide privacy for them. The school name was not reported in the study.

CHAPTER 4

FINDINGS

The purpose of this study is to understand how students make use of visual-spatial thinking processes in the Math-Art Studio Environment. In order to achieve this goal, student' thinking processes during observation of famous artists' artworks, creating artworks, and describing and evaluating their own and friend's artwork were analysed. The overall analysis of students' visual-spatial thinking processes indicated that students made use of four main visual-spatial thinking processes: recognizing geometric shapes, decomposing and composing shapes, patterning, and transforming geometric shapes. These visual-spatial thinking processes are interrelated to each other. It does not involve a hierarchical relationship. Detailed analysis indicated that students reflected the processes of disembedding and embedding shapes, identifying shapes on the basis of their visual appearance (e.g. real-world objects) or their geometric properties, spatial proportional reasoning, identifying scaling and rigid transformations, and identifying congruence between shapes etc.

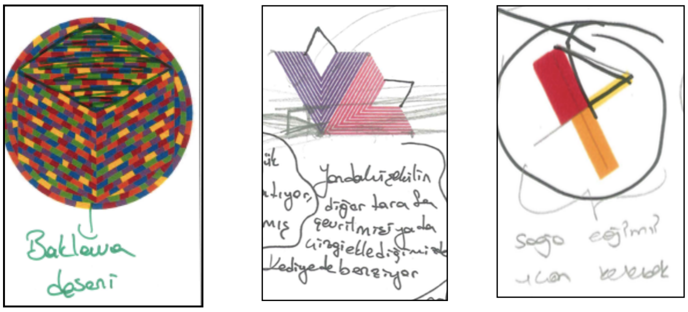
4. 1 Recognizing Geometric Shapes

Recognizing geometric shapes refers to students' identification of two-dimensional shapes and two-dimensional representations of three-dimensional geometric shapes. Analysis of the students' ways of recognizing shapes indicated that they reflected two major ways to identify the geometric shapes. These are identification of geometric shapes as a real-life objects and identification of geometric shapes. Student's identification of geometric shapes was presented in two categories: identification of two-dimensional shapes and two-dimensional representations of three-dimensional shapes. Each category involves identification of shapes through disembedding and embedding, on the basis of their properties, from different orientations & point of views.

4.1.1 Identification of Geometric Shapes as Real-World Objects

Identification of geometric shapes as real-world objects involves associating geometric shapes with real-world objects on the basis of their visual appearance. The analysis of the data indicated that some students (Melek, Fatma, Emre) perceived a geometric shape or combination of geometric shapes as real-world objects especially when they were asked to observe artworks and identify the shapes that they see during individual and group observation. In addition to observation of artworks, some students (Fatma, Emre, Ali) also related geometric shapes with visual images of real-world objects during creating and copying artworks. The evidences of students' such identification are explained respectively.

First of all, some students (Melek, Fatma, Emre) perceived geometrical shapes as real-life objects when they were asked to observe an artwork during individual and group observation in studio work 1 and studio work 2. Findings regarding individual observation indicated two of the students (Fatma and Melek) recognized geometric shapes as real-world objects at some paintings. Three examples of the students' identification of real-life objects were presented in the Figure 13. Students perceived the shape of square as baklava (Figure 13a), the combination of two triangles as a cat head by rotating it mentally and adding missing eyes of cat (Figure 13b), the combination of rotated triangles as a butterfly (Figure 13c) so that the yellow rectangle represents the head of butterfly, and red and orange rectangles represent two wings of butterfly.

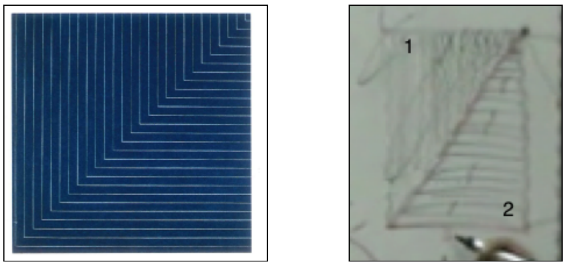


(a) (b) (c)

Figure 13. Students' identification of geometrical shapes a real-world object: (a) baklava (Fatma), (b) a cat head (Fatma), (c) a butterfly (Melek)

In addition to individual observation, students also identified the pattern of line drawings as a path in the artwork of Frank Stella during group observation [studio work 1] and represent it on the board (see Figure 14). In fact, they had not identified it as a path during individual observation. The following quotation explains how they perceive it as road. It seems that they appreciated the role of perspective drawing on perception of depth by decreasing the size of squares. It is important to note that this quotation is a part of discussion on the shapes that students see. Before it, whereas some students claimed that they see a pyramid in the following artwork (Figure 14a), some of them thought it is a combination of squares in which the sizes of squares increase proportionally.

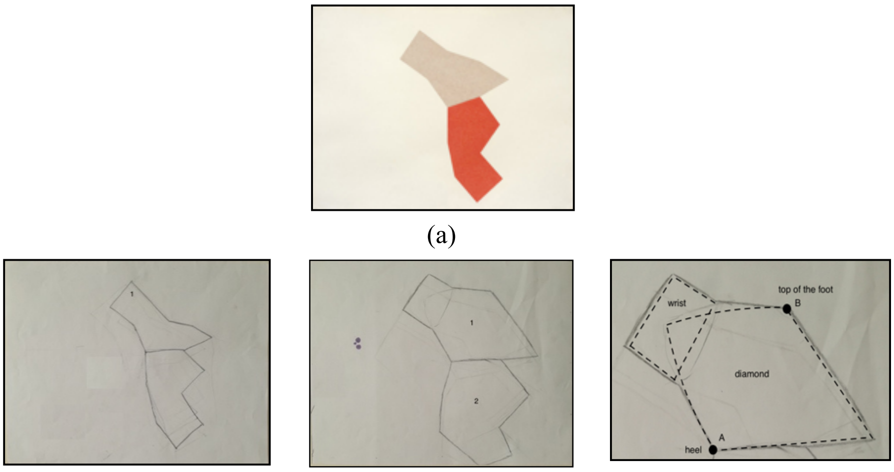
Burcu	Teacher! I see a path getting narrower
Ali	Yea, that's right.
Fatma	Teacher the path is getting smaller and smaller like this.
Ali	Teacher, perspective, approaches to vanishing point!
Burcu	Pencil pencil! [*She wants to draw a path on the board*]
Researcher	Let's draw.
Burcu	Teacher It's like path like this.
Ali	Teacher, It's like vanishing point.
Burcu	Yes, like that, teacher, this is a wall and this is a path [*She points at the region 1 as a wall and region 2 as a path in the Figure 14b*].



(a) (b)

Figure 14. Students' identification of geometric shapes as real-world objects: (a) artwork of Frank Stella, (b) student's identification and representation of the artwork as a path

Another example of students' identification of geometric shapes as real-world objects was observed during copying an artwork [studio work 3] when they were not directly asked to observe what shapes they see. This art work involves two shapes that consist of hidden geometric shapes, presented in Figure 15a. During copying this artwork, students had difficulty in restructuring these shapes in a larger space. To overcome this difficulty, the researcher asked them to find strategies to place each shape. One student (Fatma) perceived the shapes as a shoe (region 1) and moustache of a cartoon character (region 2 in Figure 15c). After she expressed her strategy, another student recognized it as a dragon head by inspiring from her friend strategy (region 1 in Figure 15b). Students' representations of real-world objects were presented in Figure 15b and 15c.



(a)

(b) (c) (d)

Figure 15. Students identification of geometric shapes as real-world object during copying artwork: (a) artwork of Mel Bochner, (b) Ali's identification and representation of a dragon head, (c-d) Fatma's identification and representation of a shoe and moustache of cartoon character

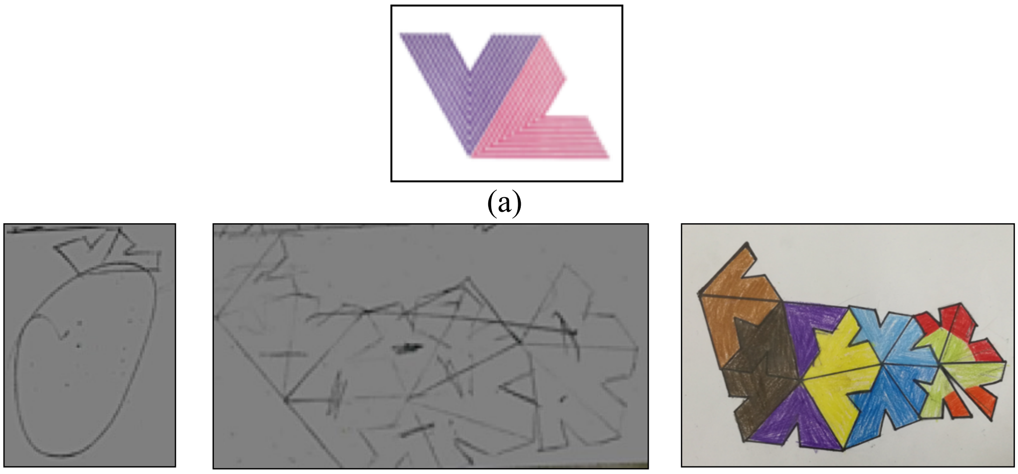
During copying the artwork, for example, Fatma reflected as the following: "*It came to my mind that when I turn it like this [rotates the artwork so that the shoe touches the ground], I thought it is like a shoe.*" during drawing process. The following conversation during retrospective interview also support this claim. It seems that she did not decompose the shapes into smaller geometric shapes with which she is familiar. When she was stuck in drawing, she found a way of representing it as a real-life object to copy it.

Researcher	Can you tell me how did you copy this painting [*shows figure 15a*]?
Fatma	I thought of a cartoon character on that one. When we turn like this [*she turns the original artwork to align the shoe with the ground*], there is a giant's foot; but I could not draw it exactly. When we look like this I see a cartoon character's foot. It looks like sharp drawn cartoon characters. I likened them, that is like a heel [*shows the point at the bottom of the first shape (region 1 in figure 15c)*] or as a moustache, I dreamed of it as a moustache when we turn like this [*shows the region 2 in figure 15c and rotate the artwork 90 degrees to the left*].

Researcher	Yes, it looks like, well, you know, you’ve been erased a lot here and tried to do it again. Where did you had difficulty?
Fatma	Yes, I couldn't adjust the size, I did it before I imagined it as the shoes. I mean, I had a hard time. It was difficult for me before I could not think this way; then I likened it to something and it was easy.
Researcher	What did you think during the drawing? You've done this way so you did it here [*shows the directions of each line segment by using hands*]; so what have you been thinking exactly?
Fatma	Something like that: that's the wrist of that cartoon character [*shows the square area in Figure 15d*] and that’s the foot [*shows the area apart from square area in Figure 15d*], I saw something like a diamond first [*draws a shape like a diamond by adding lines in Figure 15d*]. I've done something a little bit like a diamond by joining these line at the background slightly, then I dreamed like a wrist. Then I imagined the base of that foot, this heel part [*shows point A in Figure 15d*], like the upper part of his foot [*shows the point B*], but there is very little protrusion here [*original painting*], I’ve made more. And I made absolutely straight here [*shows line segment right to point A*], it should have not been flat.

The second finding regarding recognition of shapes as real-world objects was that two students used geometric shapes to model real-life objects. The nature of this thinking is different from the previous thinking process since it involves reverse process in which they are not given an artwork to observe. Rather they create their own two-dimensional representation of real-life objects by making use of geometric shapes. While one student imagined the real-life object firstly and relate it with geometric shapes, the other student perceived real-life object simultaneously with combining the shapes by trial and error.

For example, during creating own artwork in studio 2, Fatma aimed to make a head with hair and then a bird by inspiring from the artwork of Frank Stella. In studio work 2, students were asked to think about the question of what if the artwork of Frank Stella was a beginning of your artwork how they would continue to it (see artwork of Frank Stella in Figure 16a). She deliberately used geometrical shape to create a real-life object. During creating artwork and critiquing process, Fatma reflected her idea as stating: "*Teacher, I drew something different, something like a bird, and that's its tail [points at the last part of the drawing in Figure 16b]*". The stimulated recall interview also reflected on this process. It shows that in the first sketch she imagined to make a head with hairs, presented in Figure 16b. Then she gave up making it and decided to make a bird, presented early sketch of a bird and its final version in Figure 16c and 16d.



(a)

(b) (c) (d)

Figure 16. Fatma's compositon of geometric shape to make a real-world object: (a) artwork of Frank Stella, (b) early representation of a head, (c) early representation of a bird, (d) last representation of a bird.

Researcher: What did you think during drawing them? [*points at her sketches*]

Fatma: I'd supposed we'd draw separately shapes like this [*figure 16a*]. When I learned that we were going to do something different, I thought I would make a circle like this [*shows figure 16b*]and make such a triangular shape like hair, on top of it. Then I gave up because it would be difficult to find the angle; so the bird.

Researcher: So, you thought you couldn't do it? Where did you give up?

Fatma: Yes..I had tried like this circle first [*shows the first composition of triangular shapes in figure 16c*], then I couldn't.

Differently from Fatma's thinking process, Emre imagined the combination of geometric shapes work as a real-life object during he was creating art work. He imagined the combination of the shapes as a clock since it has a circular shape and there are twelve triangular shape. He stated as *"Teacher, mine [the drawing he made] looked like a clock, exactly 12 [counts the numbers on the clock]"* after his first sketch, presented in Figure 17.

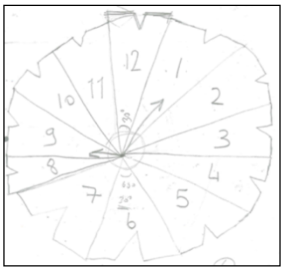


Figure 17. Emre's identification of geometric shapes as real-world object: first representation of a clock

To summary, analysis of students' identification of geometric shapes indicated that students perceived geometrical shapes as real-life objects at different situations such as while they are looking at an artwork, copying an artwork or creating their own artwork. Students reflected two different ways of perceiving geometric shapes as real-life objects: recognizing the geometric shapes as real-life objects when they were already given artworks to observe, recognizing the geometric shapes as real-life objects when they were asked to create their own artwork; rather than they are already given an artwork. It is also worth pointing out that some students recognized the combination of hidden geometric shapes as a real-world object when they did not partition them into familiar geometric shapes during copying of artwork.

4.1.2. Identification of Basic Geometric Shapes

Identification of basic geometric shape refers to recognition of two-dimensional shapes (e.g. square, rectangles, circles, trapezoids, pentagons, hexagons) and two-dimensional representations of three-dimensional shapes (e.g. rectangular and square prism, cube, pyramids). This part presents students' naming of these two- and three-dimensional geometric shapes and identification of them by their properties at different conditions (e.g. change in orientation or perspectives, embedding or hiding them into other shapes). Firstly, students' identification of two-dimensional shapes was explained. Then students' identification of two-dimensional representations of three-dimensional shapes was presented.

4.1.2.1 Identification of Two-Dimensional Shapes

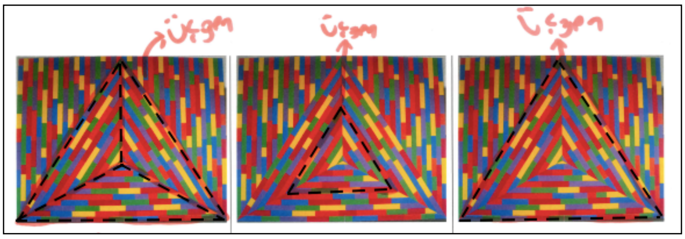
This part involves students' recognition of basic two-dimensional shapes such as triangle, square, rectangle, circle, and parallelogram at all parts of the studio works: demonstration-observation, students-at-work and critique parts. Student identified two-dimensional shapes through disembedding and embedding shapes, on the basis of their geometric properties and from different orientations.

4.1.2.1.1. Identification of Shapes through Disembedding and Embedding Shapes

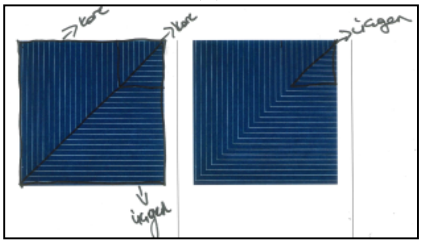
Students identified the shapes that are embedded into other shapes such as triangles, sqaures, rectangles, circles, trapezoid, and parallelogram during individual and group observation and creating artwork. They mostly identifed two-dimensional shapes rather than two-dimensional representations of three-dimensional shapes in reversible artworks that are perceived as both two-and three-dimensional.

For example, one of the students (Ali) disembedded triangles in the artwork. He identified both nested two triangles (see Figure 18a) and three triangles that consist of a triangle. None of the students identified trapezoid shape in the artwork of Sol

LeWitt (see Figure 18a). They appeared to focus on the triangles. In a different artwork (see Figure 18b), most students identified squares embedded into artwork as one of the students stated "*This painting consists of squares*" during group observation in studio work 1. One of the students (Melek) also seemed to identify embedded triangles in the artwork.



(a)

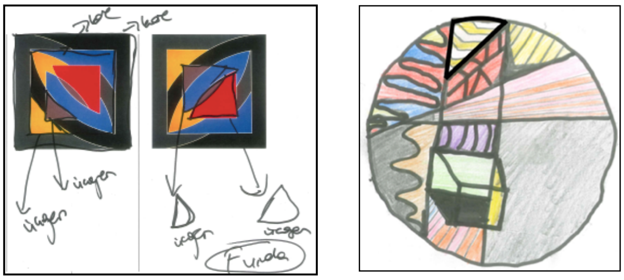


(b)

Figure 18. Students' disembedding two-dimensional geometric shapes in artworks: (a) Ali's identification of triangles [highlighted by researcher with dashed lines], (b) Melek's identification of squares and triangles.

In another artworks, students identified triangles and sqaures that are embedded in the Frank Stella's artwork (see Figure 19a). In fact, the triangle that they showed on the artwork should be a sector. However, all students identified the sector as a triangle since it looks like a triangle.

One of the students (Emre) also identified the sector as a triangle during creating artwork in which students embedded shapes to hide a particular geometric shape as stating *"Teacher, here there is a hidden triangle…umm what kind of triangle is it? both right and its two sides are equal!"*. His artwork is showed in Figure 19b.



(a) (b)

Figure 19. Students' disembedding and embedding geometric shapes: (a) Melek's disembedding of sqaure and triangles, (b) Emre's embedding of a triangle into other shapes.

The following discussion during group observation of the artwork of Frank Stella [studio work 1] indicates that even though students named it as triangle, they admitted it was not a triangle. Even though they admitted it was not a triangle, however, they did not identify it as a sector at first glance. When the researcher draw a sector to the board and asked them what else it could be other than a triangle, one of the students remembered it is a sector.

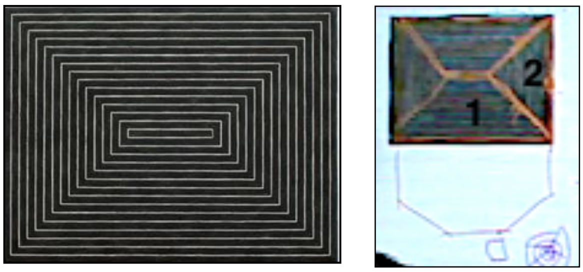
Researcher	So you'd called it a triangle, why is this a triangle [*shows triangles in figure 19a*]?
Melek	It's not exactly a triangle; but it's like a triangle.
Researcher	Can you still call it a triangle?
Melek	I don't know, it's got three sides, but it's not a smooth triangle.
Researcher	Ali and Esra, what do you think this figure looks like? [*draws a sector on the board*]
Esra	A quarter circle! it's not a triangle, I confess I am cunning, I made it up.
Ali	me too. It looks like it, when I see it, I think of the pie. You know, we'd learned in math class.

In summary, students identified basic geometric shapes that are embedded into other shapes ranging from trianles to quadrilaterals such as paraellolograms and rectangles during demonstration and creating artwork parts. Students mostly disembed two-dimensional shapes in artworks that are perceived as both two-dimensional and three-dimensional during individual analysis of artworks.

4.1.2.1.2. Identification of Shapes on the Basis of Properties

Students identified geometric shapes on the basis of their lengths relations (number of length, equal sized lengths), arrangement of parts of shapes (e.g. symmetric), angular relations (acute, obtuse angle). Regarding length relations, students identified geometric shapes by comparing their lengths or counting the number of lengths. For example, during individual observation, one of the students (Fatma) identified triangles as taking note of "equilateral triangle" (see in Figure 20).

Figure 20. Fatma's identification of an equilateral triangle regarding its equal lengths in the artwork of Sol LeWitt



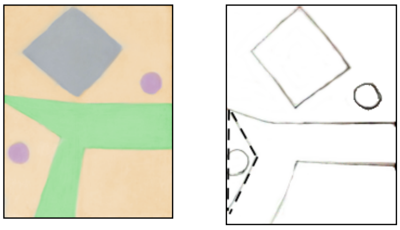
(a) (b)

Figure 21. (a) artwork of Frank Stella, (b) students' identification of geometric shapes in the artwork on the smart board.

During group observation, students similarly determined whether a shape is a triangle or a quadrilateral by comparing the sizes of lengths and determined number of sides. During group observation, they were observing the artwork in Figure 21 again and they realized new geometric shapes. Students realized triangles in addition to rectangles. When the researcher asked them what kind of triangle it is, they made a claim and tried to prove it by measuring the lenghts of triangle with the use of span.

Ali	Teacher, there's a path like before; but, there's a triangle. I've just seen it! [*Shows region 2 in figure 21b*]
Researcher	What kind of triangle is it?
Fatma	Obtuse triangle...
Ali	That part [of the triangle] is missing. [*refers to top of the trapezoid to make a triangle*]
Researcher	What kind of triangle is it? [*shows region 2 in figure 21b*]
Esra	Equilateral triangle
Emre	Triangle, as its name would suggest, should have three sides one-two-three-four, quadrilateral, special quadrilateral; otherwise, it cannot be a square [*counts the number of lengths in the region 1*].
Researcher	so what kind of triangle would it be? [*Shows the region 2*]
Esra	These are equal. This is not equal.
Emre	Isosceles triangle
Researcher	Why not equal?
Esra	Here's a little bit more [*measures two lengths by her span*]

Another student (Emre) determined what is by considering what it is not. To be precise, he determined the number of sides and claimed that it is a quadrilateral, as stating "*quadrilateral, special quadrilateral; otherwise, it cannot be a square*". It seems that he compared quadrilateral with sqaure and decide what it is not. Similary, this student identified a shape by counting number of sides during copying artwork of Agnes Martin in Figure 22a. Half of the students drew that quadrilateral with three sides. For example, after Emre drew the first sketch of the composition, the researcher asked to observe it again. During this process, he realized that he drew the quadrilateral at the left side as a triangle, as stating "*It has been like a triangle, I'll fix it, it should not look like [a triangle].*" during one-to-one conversation. Even though he realized it should not be a triangle, he had difficulty in coordinating the relation between shapes and drew it again with three sides. The drawing of Emre is presented in Figure 22b.

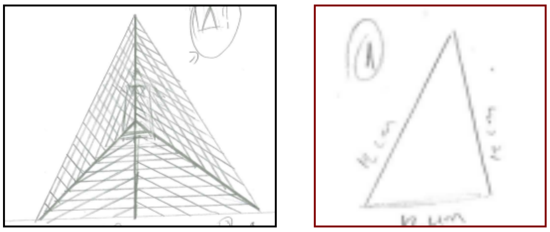


(a) (b)

Figure 22. Students' identification of a quadrilateral during copying artwork: (a) artwork of Agnes Martin, (b) Emre' representation of the triangle instead of quadrilateral in the original artwork [highlighted by researcher with dashed lines]

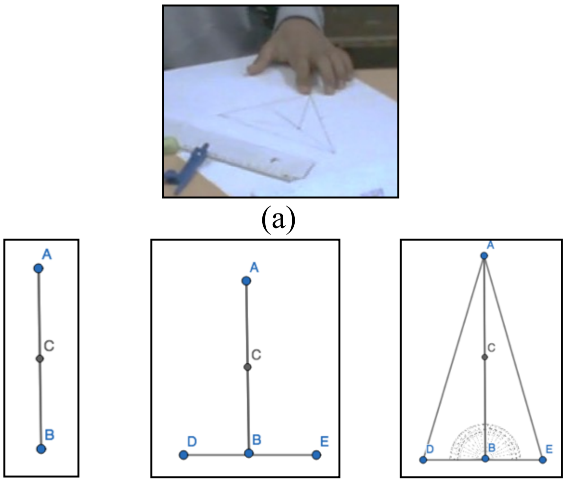
In addition to lenghts relations, students also identified the shapes by considering their symmetric nature with regard to length relations. For example, during creating artwork in studio work 1, Fatma focused on the length relations to draw an equailateral triangle. In fact, in her freehand hand sketches, she did not consider the properties of the triangles (see Figure 23a). When the researcher asked her what kind of triangle it is, she stated it as an equilateral triangle (see Figure 23b).

While she was drawing the last version of artwork in Figure 24a, she took into consideration of metric properties. She identified an equilateral triangle with three equal lenghts. To make equal lenghts, she appeared to recognize the symmetrical property of equilateral triangle while she were drawing final version of artwork. She drew a vertical line with a midpoint in Figure 24b and drew a perpendicular line segment to this line so that its midpoint intersects the the line in Figure 12c. However, she did not reflect on the relation between sides and angles of triangle. During this process, she put the protractor to the midpoint of the horizontal line segment in Figure 24c.



(a) (b)

Figure 23. Fatma’s sketches of equilataeral triangle during creating artwork in studio work 1 (a) early sketch of a triangle (b) identification of properties of the triangle (each side of triangle are 12 cm).



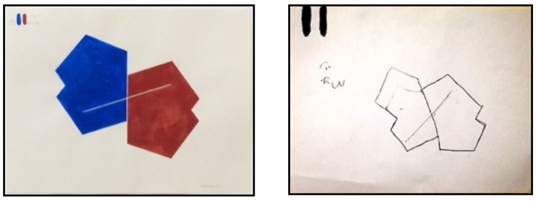
(a)

(b) (c) (d)

Figure 24. Fatma’s identification of properties of an equilateral triangle: (a) sketch of an equilateral traingle in the last version of her artwork, (b-c-d) processes of representatiton of an equilateral triangle with identification of midpoints and symmetrical property

During copying artwork students (Melek and Ali) similarly identified the symmetrical nature of hidden shapes. For example, Ali expressed symmetrical relationship with the claim that mouth shapes should be the same in each geometrical shape in the artwork, presented in Figure 25. In addition, line segments of mouth shape in each geometrical shape should be the same.

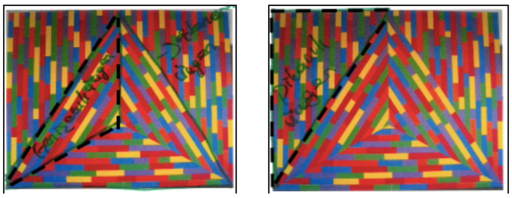
Researcher	So how do you think about Emre's drawing?
Ali	The shape is not much symmetric, for example, here is longer
Researcher	Ali said it isn't symmetrical, could you say what makes something symmetrical?
Ali	Mouths of both of shapes are at equal distance [*at the artwork of Mel Bochner*]. Here these have same lengths; but these are not equal [*points that side lengths are not equal at the shape on the left in the drawing of Emre*]



(a) (b)

Figure 25. (a) Artwork of Mel Bochner, (b) Emre's drawing of the artwork of Mel Bochner

Lastly, student identified shapes by considering their angles. They determined visually to what extent the distance between two line segments is wide. For example, it was only Fatma who identified a shape with the consideration of its angle such as "*obtuse triangle and right triangle*" in the art of Sol LeWitt (see Figure 26).



(a) (b)

Figure 26. Fatma's identification of triangles in terms of their angles in the artwork of Sol LeWitt (a) obtuse triangle (b) right triangle [highlighted by the researcher with dashed lines]

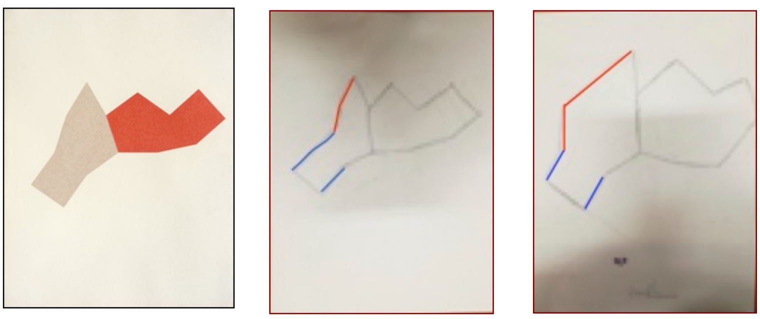
Students also identified the extent to which two line segments are close to each other qualitatively (degree of sharpness or the distance between two line segments) when they did not identified shapes as a particular geometric shape. For example, in the artwork of Mel Bochner (see Figure 27a), Emre noticed the angular relation between two line segments in the first shape during observing and evaluating his drawing and took note of "*the top of orange colored shape was very sharp, it was revised*". In the critiquing part**,** he explained how he revised angles between line segments. In the first sketch he drew a narrower angle, and then he revised his painting though enlarging it. His thinking was elicited in the following conversation:

(a) (b) (c)

Figure 27. Emre's description of his drawing in terms of the change in angular relations (from b to c) on the smart board during critiquing part

Emre	First, I started from the big one that is above [*refers to orange colored shape in Figure 27a*]. I closed my eyes; well I did focus on the original painting, I did not look at my paper, then I have corrected [*inaccurate parts*]
Researcher	Where did you fix?
Emre	[...] there were problems here [*shows the angle in figure 27b*], for example, it was coming from here to there [*describes the direction of line segments in figure 27b*], I changed it like this [*figure 27c*].
Researcher	So what did you change?
Emre	line... angle angle! I've expanded it.

In another artwork [Figure 28a], they identified the angular relations of shapes with the consideration of slope concept even though they did not use the term formally. They compared the steepness of the line segments during copying artworks in studio work 3. When researcher asked Ali and Fatma to compare the first shape (beige colored shape in Figure 28a) in their drawings. During this process, Ali evaluated his friend's drawing and uncovered his spatial strategy that he used: detecting critical points at which the slope of line segment changed and comparing their steepness rather than just considering direction of line segments. [It is presented with red colored line segments in the Figure 28b and 28c]. The following discussion between Ali and Fatma explains this process:



(a) (b) (c)

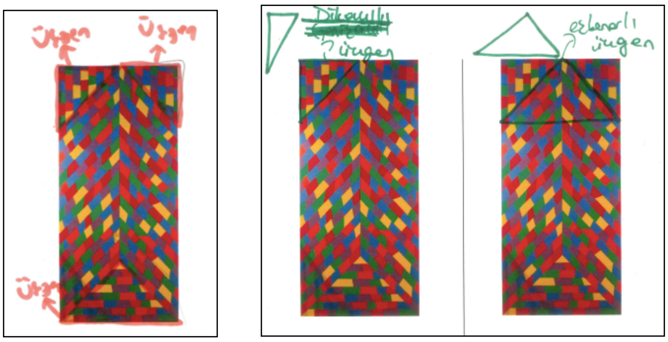
Figure 28. Ali's identification of angular relations between line segments during critiquing his friend's artwork

Researcher	Ali and Fatma, how do you compare your first shapes? [*shows beige colored shape in both students' drawings*]
Ali	[...] look, do you [*Fatma*] see there are angles here [*shows the change in slope of line segments at the beige colored shape*], you made it flat. There is slight angle here. You've been drawn right angle, steeper angle, here [*the angle between two red colored line segments in the Fatma's drawing*] then you continued straight.

In summary, students focused on different properties of shapes to identify them. These properties are length relations (size of lengths, number of sides, parallelism of line segments), angular relations (the amount of distance between two line segments or comparison of steepness of line segments), and symmetrical nature of shapes.

4.1.2.1.3. Identification of Shapes from Different Orientations

Identification of shapes from different orientations refers to recognizing the same geometric shapes even though they are rotated. Analysis of students' identification of shapes from different orientation indicated that students named some triangles as "equilateral triangle" and "right triangle" differently when they are rotated while students identified squares as the same even though it is rotated. First of all, two of the students (Ali and Fatma) named geometric shapes in the artwork of Sol LeWitt differently (Figure 29). Ali, for example, named the same shape as both right triangle and equilateral triangle. The retrospective interview supports this process. It seems that Ali recognized right angle triangle when it was vertical. However, he was not sure when it was rotated.



(a) (b)

Figure 29. (a) Ali's identification of triangles (b) Fatma's identification of triangles in the artwork of Sol LeWitt.

Researcher	You'd written triangles here. What kind of triangles are they? Do they have any similarities or differences?
Ali	So equilateral [*shows the triangle at the bottom in figure 29a*]
Researcher	What makes you say that it's equilateral?
Ali	Just a second. Let me look at this [*turns the paper a little*]. No, this is equilateral. And that is a right triangle [*shows triangles at the corners in figure 29a*]. Such…one second! [*tests it again with eyes*] I think this is equilateral, and this is a right triangle.
Researcher	Why do you think so?
Ali	Because these are very simple, they are at the corner right here, since it is 90 degrees, so you know it's the corner of the rectangle, this is 90-degrees angle.
Researcher	Why did you call that one an equilateral triangle? how did you decide?
Ali	How did I decide to do this, mmm. one second. Can I look like this again? [*turns the paper again*] I made a mistake. We can call it 90 degrees, right? It's 60-60-60. That's why I call it equilateral.

In observation of another artwork of Sol LeWitt (Figure 30) in which perspective drawings of squares are rotated. Students (Fatma and Ali) identified one of them as a sqaure and the other one as a quadrilateral or baklava. Interviews with students after studio work 1 indicated that students, in fact, realized they are same shape.

However, they did not prefer to name them the same at first glance. For example, Fatma indicated as following.

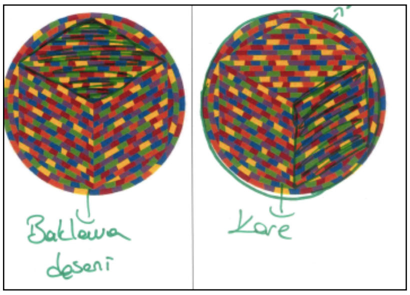
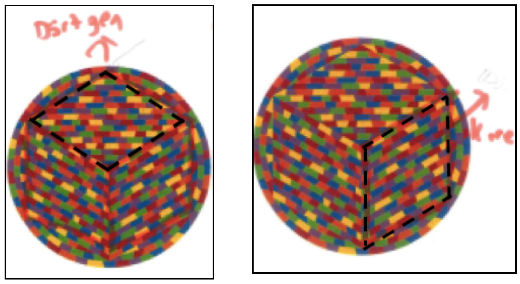


Figure 30. Fatma's identification of squares with different names (baklava and square)

Researcher	How did you identified these shapes?
Fatma	It drew my attention because it has such corners on it [*cube*]. And I realized this diamond shape. For example, here it is diagonal now. If we look from the side, it would look like a normal square; but since it looks like diagonal, I said it is baklava shape.
Researcher	It may be like another geometric shape you know?
Fatma	What else…[*baklava*] looks like a square that is turned.

Similarly, Ali identified the square that stands on its one of the vertices as quadrilateral rather than regular polygon (Figure 31). When the researcher asked him why he named two shapes differently, he realized they are the same. He named them differently since he perceived one of them as bigger than the other one because of perspective drawing. During interview, he imagined the rotate the shape in his hand to match it with square on the top and he identified they are identical.



(a) (b)

Figure 31. Ali’s identification of squares with different names: (a) quadrilateral and (b) square [highlighted by the researcher with dashed lines]

Researcher	Well, you've given the names like "quadrilateral" and "square."
Ali	The same things, I should have been confused there.
Researcher	So why did you think it was a square here [Figure 31b]?
Ali	You know it is normally quadrilateral. Because this place [Figure 31a] is a little bigger I said quadrilateral. I mean, it immediately looked like it to me.
Researcher	You didn't think it would be a square at first glance.
Ali	In fact, the quadrilateral equals square.
Researcher	Then you didn't think it was a square at first looking?
Ali	I didn't exactly think firstly, but when I look at it like this, it remembered me a square [*changes her head to the right as if he stands on the right front of the square face*]. When you turn it like this.
Researcher	Well, what did you see first?
Ali	I saw the cube firstly, then I said “a-ha there is a quadrilateral here” then…[square]
Researcher	What made you say that it is a square?
Ali	There is no difference at all. They have same widths and lengths. Because if I look like this, it looks a little big because of the perspective, and if I keep it like this again, it will look like that [u*ses hand to rotate the shape to match it with square*], so it's the same thing.

Secondly, students identified the squares when they are not embedded into other shapes, even though they are rotated. For example, during copying artwork students copied two artworks: one of them involves four squares that stands on one of their sides (figure 32a); the other one involves square-like shape (deltoid) that stands on one of its vertices and irregular quadrilaterals (figure 32b). Students identfied both of them as a square. However, they focused on different properties of sqaures to draw it them. Students' reconstruction of these artworks were presented in figure 32c and 32d.

For example, in the art work of Robert Mangold (figure 32a) all students drew each square so that lengths of squares are identical and parallel to each other. For example, Fatma took notes regarding what was wrong in her sketch by distinguishing it from rectangle as "*I could not draw the bottom line of the largest square straight. Square under the biggest square looked like a rectangle*". (see following her hand writing in Figure 33).

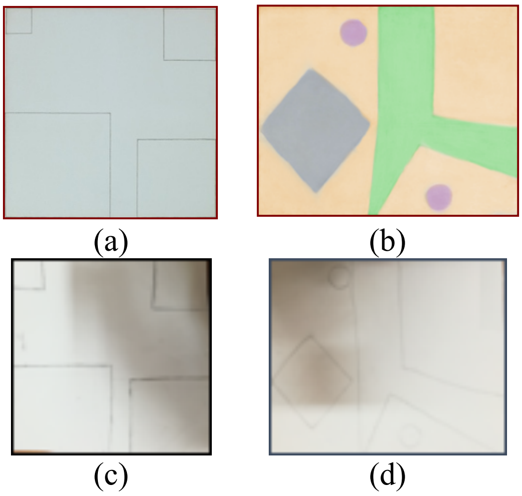


Figure 32. (a) Artwork of Robert Mangold (b) Artwork of Agnes Martin (c) Fatma's reconstruction of the artwork of Robert Mangold (d) Ali's reconstruction of the artwork of Agnes Martin

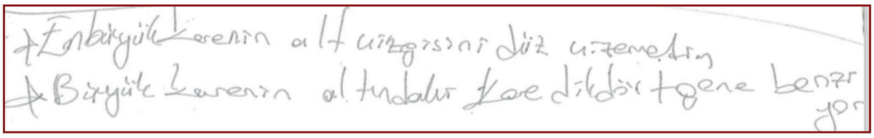
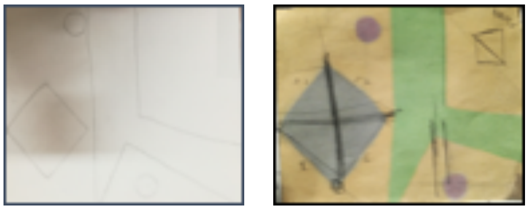


Figure 33. Fatma’s hand writing (critique of her artwork) during copying artwork of Robert Mangold

When the squares are presented as standing on the one of vertices (figure 32b), some students (Ali and Melek) took into consideration of diagonals and alignment of vertices. For example, the following conversation explains this process. It seems that he drew a diagonal to align two vertices of squares and considered symmetrical nature of square.



(a) (b)

Figure 34. (a) Ali’s reconstruction of artwork of Agnes Martin, (b) Melek’s identification of diagonals on the artwork of Agnes Martin.

Ali	Teacher! Mine is over! I'm sure it is good.
Researcher	Well, did you observe anything wrong in the drawing? Let’s write a note Lets observe a little more.
Ali	[*checks the distance between diagonals*]
Ali	This time I've done! I've adjusted them.
Researcher	Which place have you adjusted?
Ali	It was higher than the other [*discusses alignment of two reciprocal corners*] It was not symmetric; I drew a line [diagonal] slightly here [*observes again and revise it again*] I think it’s almost pretty good.

In a similar way, checking process of drawing with a ruler indicated that Melek aligned two vertices through drawing the diagonals and checked the length of the diagonal in two different sized artworks so that the scale regarding their sizes are 1:4. However, these two students did not still check whether two diagonals are equal to each other. Other students only reflected that they tried to make the lengths of square equal. For example, while Emre aligned the horizontal vertices, he did not align the vertical vertices.

In summary, students identified some shapes differently when they are rotated and focused on different properties of shapes when they are rotated. This might be related to several factors such as embedding the shape into other shapes, or the image of prototypical shapes in their mind, or transformations in their size or orientations.

4.1.2.2 Identification of Three-Dimensional Shapes

This part involves students' identification of two-dimensional representations of three-dimensional shapes in the artworks. The analysis of the students' identification of three-dimensional shapes indicated that there are three major findings. The first finding is that they disembedded or embedded three-dimensional shapes. The second finding is that they attempted to identify these shapes on the basis of their properties. The third finding is that they identified two-dimensional representation of three-dimensional shapes from different point of views.

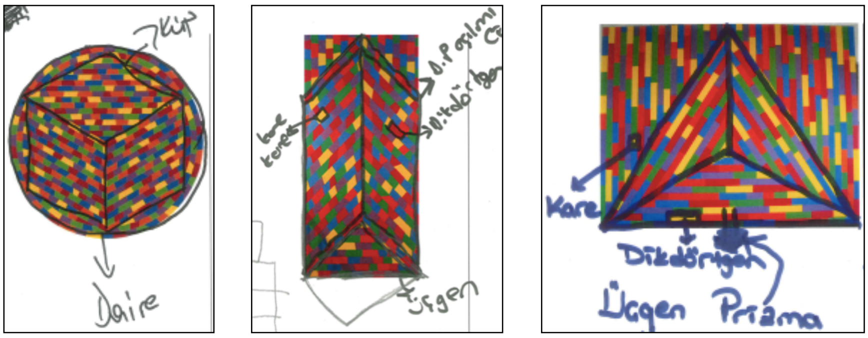
4.1.2.2.1 Identification of Three-Dimensional Shapes through Disembedding and Embedding Shapes

First of all, students identified the two-dimensional representation of three-dimensional shapes embedded in an artwork. In other words, they interpreted the composition of shapes in a different way by perceiving them three-dimensional. This kind of identification involves discrimination of figures/shapes from the ground.

There are five paintings that could be interpreted as both two-dimensional and three-dimensional (see table 10 in the method chapter).

Participants mostly tended to identify two-dimensional shapes rather than two-dimensional representations of three-dimensional shapes during individual observation. Only few participants attempted identify two-dimensional representations of three-dimensional shapes during individual observation of artworks. After individual observation, at some cases such as creating artwork, critiquing, or group observation, students realized two-dimensional representations of three-dimensional shapes.

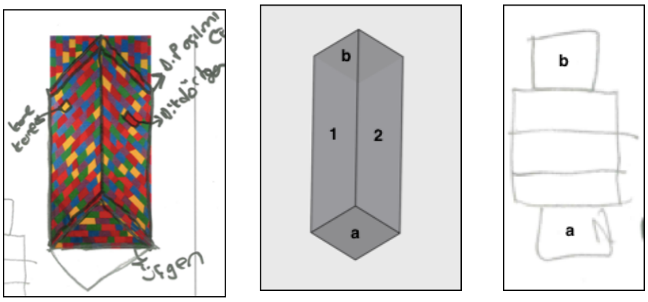
During individual observation of three artworks of Sol LeWitt [studio work 1] (figure 35), all students realized the cube in Figure 35a. Three students identified (Fatma Esra, Emre) three-dimensional shape as rectangular prism and triangular prism in the figure 35b. Two students recognized three-dimensional shape as triangular prism in the Figure 35c.



(a) (b) (c)

Figure 35. Students' identification of two-dimensional representation of three-dimensional shapes in the artworks of Sol LeWitt. (a) a cube (Melek) (b) part of a rectangular prism (Emre) (c) a triangular prism (Esra)

Regarding the artwork in Figure 36a, for example, it seems that Emre discriminated the artwork so that the half of the rectangular prism become a figure on the ground. In other words, it is an evidence of how he pulled out the rectangular prism from the ground. When asked how he imagined it as a rectangular prism, he decomposed it into its parts when it is unfolded (see Figure 36c). The interview after studio work 1 illustated this situation:



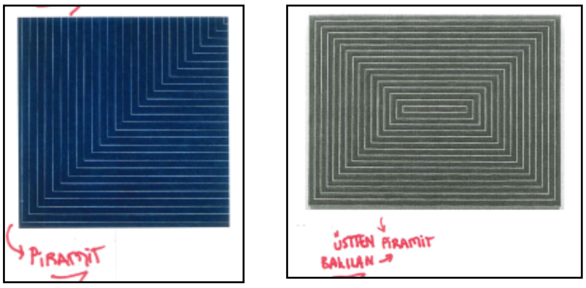
(a) (b) (c)

Figure 36. (a) Emre's identification of rectangular prism (b) representation of rectangular prism by the researcher to illustrate Emre's thinking process (c) Emre's representation of nets of a rectangular prism

Researcher	You said that "RP unfolding" [*"D.P. açılımı" in Figure 36a*], what did you meant?
Emre	Rectangular prism. Something like that [*shows the triangular prism in the figure 36*], not a full rectangle unfolding, it seems like.
Researcher	How did you imagine folding rectangular prism? Could you draw it here?
Emre	It might be like this [*draws nets of rectangular prism (figure 36c)*]. As far as I remember.
Researcher	Okay. How is it exactly looks like [a rectangular prism]?
Emre	It is as folded on both faces [*folds faces of 1and 2 in the figure 36b*]. Two faces closed like this [*show the movement of folding by hand*]
Researcher	Could you match it with its unfolded image? For example, small squares
Emre	When they join together, it will be like that [*square base; shows the face of a in the figure 36b*], and the top square is the

part that is not visible, that's on the back side [*shows the face of b in the figure 36b*]

Regarding the artworks of Frank Stella that are perceived as both two-and three-dimensional (figure 37), it was only Burcu who realized three-dimensional shapes in the painting during individual observation of artworks. She identified them as pyramids. Other students identified the shapes of rectangles, squares and triangles in these paintings.



(a) (b)

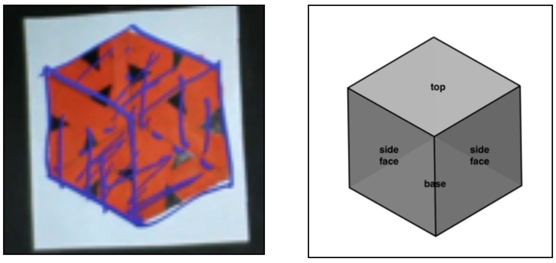
Figure 37. Burcu's identification of pyramids in the artworks of Frank Stella during individual observation of artworks: (a) pyramid, (b) pyramid that is seen from the top view

In the process of group observation, regarding the first artwork students realized new geometrical shapes and forms due to the facts that they observe the same painting the second time or their thinking processes are affected by their friends' thoughts or ideas. Three of the students (Ali, Melek, and Fatma) just realized the painting can be perceived as pyramid after Burcu showed a pyramid in the paintings. However, at first glace, they could not perceive it.

Burcu	I saw something directly without drawing anything! The pyramid has been seen from the sides
Researcher	How did you see it?
Melek	I saw it too! I've just seen.
Ali	Exactly.

Researcher	Could you imagine in your mind? [*asks other students*]
Emre	No, now I imagined it, normally I did not see.

Another example of students' disembedding three-dimensional shapes from the artworks was observed during the critiquing part [studio work 2]. After Ali described and explained how he constructed his artworks by making use of paired triangles and rotating them. During this process, it was only Emre who could realize the shape of cube in the Ali's artwork (figure 38a). He stated the following "*He is not seeing it, but there is a cube here!*" When the researcher asked him to show it, he draw contours of the cube and showed its faces as stating "*Teacher, here is the base and top, here are the side faces, I saw it!*". It was represented by the researcher in Figure 38b.



(a) (b)

Figure 38. (a) Emre's identification of cube in the Ali's artwork on the smart board during critiquing part (b) representation of the cube by the researcher to illustrate Emre's thinking process

Students (Fatma, Burcu, Melek) also attempted to identify three-dimensional shapes through disembedding during creating artwork in studio work 1. Students identified square pyramid that is perceived from the composition of nested square (Fatma), pentagonal pyramid that is perceived from the composition of nested pentagons (Melek), and cube (Burcu) through disembedding. For example, observation of videos indicated that Burcu drew several circles and vertical, horizontal, and oblique lines. After that, she thought a while and observed her artwork. Then she realized the shape of cube and underlined its edged to make it visible. It seems that she picked the cube from the overlapping shapes (see her sketch in Figure 39a). Then she realized there are several cubes that are nested along the dashed line, represented on her artwork by researcher in figure 39b. In fact, it is a rectangular or square prism. The following conversation during students-at-work part how she described what she realized.

(a) (b)

Figure 39. (a) Burcu's disembedding rectangular prisms in her sketch (b) nested retangular prisms represented by the researcher through dashed lines on her final artwork

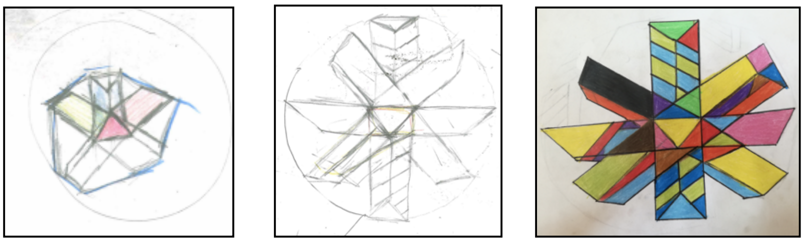
Burcu	Teacher, I gave up drawing "S", I decided on the shape of the cube
Researcher	How did you come up with this idea?
Burcu	I imagined the cube shape
Researcher	Where is it?
Burcu	There are actually more
Researcher	Let's have a look
Burcu	There is a lot going on in this direction.

In addition to identification of the cube, students also identified the shapes of rectangular and square prism when they were asked to identify the shapes in their friend's artwork at the critiquing part. Students appeared to ignore other shapes to pick rectangular/square prism or cube that are embedded into other shapes.

The following quotation presents students' identification of three-dimensional shapes verbally.

Burcu	I tried to hide the shape here.
Researcher	Let's see if you can see what the shape is hidden there.
Burcu	You can get a lot on it [*tells the painting teacher*]
Teacher	I can take out a lot in parallel with the same movement.
	[*Burcu draws the shape that she hided on the smart board*]
Researcher	What shape did Burcu hide?
Emre	Square, Cube
Ali	Rectangle a-hah rectangular prism
Researcher	So, here you said the cube?
Emre	Square prism!
Ali	Rectangular prism!
Fatma	Square prism or rectangular prism!
Emre	Rectangle also.

While these are related to disembedding two-dimensional representations of three-dimensional shapes, one of the students (Melek) also attempted to embed two-dimensional representations of three-dimensional shapes during students-at-work part in the studio work 1 in which students were asked to create own artwork that hide a shape inspiring from the artworks of artists. This process gives clues about how she identified three-dimensional shapes. For example, Melek embedded triangular prisms on the basis of their common face. She identified the shapes of triangular prism and rectangular prisms. She embedded triangular prisms that require coordination between different viewpoints (figure 40a and 40b). She also attempted to embed triangular prism and rectangular prisms (figure 40c). Her representations of triangular prism and rectangular prism were different from each other. She did not draw invisible faces of these prism. She did not represent parallel faces of shapes appropriately as well as number of their faces, which gives clues how she considered the attributes of shapes and their relations. The following discussion of students shows which shapes she identified in her artwork.

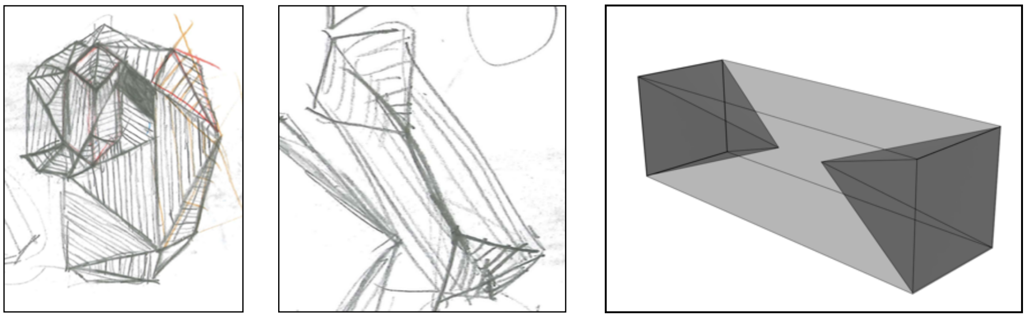


(a) (b) (c)

Figure 40. Melek’s embedding triangular prisms and rectangular prism: (a) triangular prisms in early sketch (b) triangular prisms added in the second sketch (c) triangular and rectangular prisms in the final artwork

Researcher	Melek, could you describe your artwork to your friends?
Melek	The shape I was trying to hide was a rectangle prism, but I couldn't draw it exactly. The rectangular prism is here. [*figure 40c*]
Teacher	Yes, it wasn’t. You would draw faces parallel to each other, the shape you’d drawn here would be parallel.
Researcher	What other shape did you hide?
Melek	I changed the triangle prism from there.
Teacher	You did in the same way. Can you show me again, yeah? I saw the prism.
Melek	But there are a lot of prisms.
Teacher	These are also not parallel, they should be parallel, if you miss that parallelism, and then your geometric shape is distorted.

In her another sketch, she embedded two square pyramids into square prism. In other words, she envisioned to hide square pyramids into square prism. She attempted to place two square pyramids inside of the prism so that they have same bases with square prism as they looked to each other. She attached two shapes on the basis of their bases. She stated “*they are reverse and looking at each other*” [*draws the figure in 41b*], when the researcher asked her what she tried to hide.



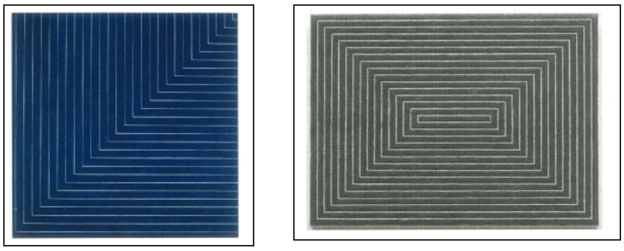
(a) (b) (c)

Figure 41. Melek’s emdedding square pyramids into square prism (a) Sketch of artwork during student-at-work part in studio work 1 (b) emdedding square pyramids into square prism (c) representation of embedding by the researcher to illustrate Melek’s thinking.

In summary, participants mostly tended to disembed two-dimensional shapes rather than two-dimensional representations of three-dimensional shapes at first glance during individual observation. However, after students observe them, they realized them as three-dimensional. Besides students disembedding figures, one of the students also attempted to embed geometric shapes into each other so that they provide a perception of three-dimensional.

4.1.2.2.2 Identification of Three-Dimensional Shapes by Properties

This part involves students’ identification of three-dimensional shapes with the consideration of their geometric properties. The analysis of students’ thinking indicates that they had a confusion regarding naming the three-dimensional shapes and considering their geometric properties. For example, during group observation of the following artworks in studio work 1 (figure 42a), there was a discussion whether the first artwork involves a pyramid or not. While some of the students accept it as a pyramid (Fatma, Burcu, Ali), other three students were not sure whether it is a pyramid. They identified a pyramid on the basis of number of edges of its base. Students conceptualized a pyramid with a base that has three edges or four edges. Since the first artwork involves two faces, they think it is not a pyramid. They perceive it as two-dimensional shape that consists of squares.



(a) (b)

Figure 42. Artworks of Frank Stella

Emre	This is not a complete pyramid. There are two sides of the pyramid and two sides are missing
Researcher	What makes you say that?
Emre	I think this can't be the pyramid anyway. A picture created by growing in certain dimensions only in a certain order.
Melek	There should be three sides of a pyramid. It was a pyramid from the top [*shows figure 42b*], but not this [*shows figure 42a*])
Esra	That's not [*shows figure 42a*]
Melek	These are squares.

Another confusion about geometric properties of pyramids was observed during group observation of artwork of Sol LeWitt during studio work 1. After discussion of the artworks above, the researcher asked students to observe artwork again. When she asked them to think from which perspective they could identify it as a pyramid, students discussed how they see it: from top view or front view. During this discussion, one of the students (Fatma) claimed that it could not be a pyramid. The reason behind her claim was that she conceptualized the pyramid with a base that has four edges.

Figure 43. Artwork of Sol LeWitt

Researcher	Lets' look at this painting again. Is there anything that you've not observed before?
Fatma	Yes, pyramid!
Ali	Exactly, it is.
Researcher	Did you see it at first glance?
Fatma	No teacher
	[...students discuss the perspective of pyramid]
Fatma	But it can't be! it's three-sided, but the pyramid has four sides. pyramid that is made of shrinking squares.

During individual observation of this art work (figure 43), one of the students (Emre) identified a triangular pyramid as a triangular prism. After the studio work 1, researcher asked him how he perceives it as a triangular prism. He identified it as triangular prism since it consists of triangles. It seems that he could be more familiar with the concept of triangular prism than triangular pyramid. He perceived triangular prism since he thought it is a three-dimensional shape.

Researcher	Emre said triangle prism here. Why do you think it's a triangular prism?
Emre	Three-dimensional
Researcher	You saw a three-dimensional shape, so why the prism?
Emre	Triangle is prism. Prism because it is three-dimensional. If you said tthe triangle, it would be two-dimensional.
Researcher	So what makes you say that it is a pyramid?
Emre	For example, there is a rectangular prism, like it
Researcher	OK, what are the properties of rectangular prism?
Emre	Bottom and top faces are equal and parallel to each other
Researcher	So how would it be a triangular prism?
Emre	We see only the front of it. The two faces are equal; the bottom

edge is different.

Researcher	hıımm you said the triangular prism firstly, why did you change it?
Emre	Triangular prism I did not liken it to anything else!

In a similar way, Melek identified pyramids with four side faces in her artwork that she created during studio work 1. She represented a pyramid with two faces and combined two faces of pyramids. When the researcher asked to show one of the pyramids in the artwork, she showed two pyramids that are attached to each other. That is, she represented a pyramid with two faces. Following conversation between the researcher and students explains this process.

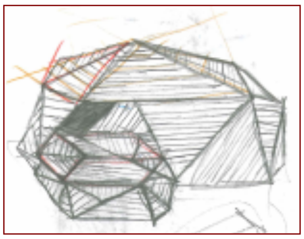


Figure 44. Melek's identification of pyramids in her sketch during studio work 1.

Researcher	There is something that I am confused of. How did you make this pyramid?
Melek	Like this [*figure 45a*]
Researcher	How many side faces does it have in total?
Melek	That's the way it should be [*draws a pyramid with four faces*] four, unfolded like this [*figure 45b*]
Researcher	Something like with a square or a rectangle base
Melek	yes
Researcher	So, think about how two pyramids with four faces are connected to each other in real-life context.
Melek	we can think of it as a continuation of it [*underlines and adds new faces in the sketch with brown color in figure 44*] okay, I'll think about it.

(a) (b) (c)

Figure 45. Melek's representations of pyramids

When the researcher asked her what kind of pyramid it is, she identified it as the pyramid with four side faces. Then, the researcher asked her to think about how a pyramid could be attached to each other in real world if they have four faces. The reason of such a question was to encourage her to think invisible faces. However, she increased the number of visible faces. After this conversation, she sketched a few pyramids with three faces (figure 45c). However, she did not try to combine pyramids whose three faces are seen. She did not still show all invisible faces.

On the other hand, interview after studio work 1 indicated that she extended her identification of pyramids with four side faces to five side faces by combining two pyramids that she had shown before. During interview, she claimed that she drew a pentagonal pyramid in Figure 44. In fact, she had expressed they were two different pyramids with four faces. She justified his thinking by counting the faces through showing the invisible face as "*I thought it as pentagonal pyramid, 1-2-3-4 and one face is at the back*".

To summary, students were not sure about the properties of three-dimensional shapes to identify them. They identified a pyramid on the basis of number of edges of its base or number of side faces. Students conceptualized a pyramid with a base that has three edges or four edges. Their identification depended on the transformation in the visual appearance or perspective of three-dimensional shapes.

4.1.2.2.3 Identification of Three-Dimensional Shapes from Different Viewpoints

This part presents students' imagination of a shape's view and imagination of a shape's view when one changed the view point. Students' recognition of shapes from different perspectives was observed at different phases of studio works such as observing art works, creating their own works, critiquing parts. There are two main findings that are regarded as evidences of students' imagination of a shape from different perspectives: imagining the view of a shape and comparison of different views of a particular shape.

The first finding is related to imagination of the view of a shape. This shape could be drawn at one direction (e.g. only top view or bottom view) or at combination of more than one direction (combination of top view and side view). For example, regarding the view of the shape from only one direction, all students identified the view of the pyramid as top view (bird's eye view) in the artwork in the Figure 46 when the researcher asked them how they see this pyramid during group observation [studio work 1].

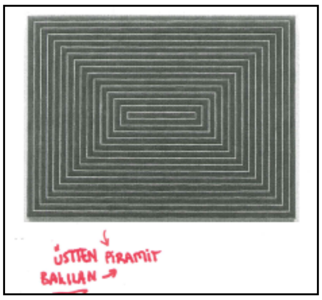


Figure 46. Burcu's identification of shape's view during individual observation

During creating artwork process in studio work 2, Emre realized the shape from the top view. He explained how it is seen from a particular view point both students-at-work and critiquing parts. The following conversation takes place in the critique part.

Teacher	What did you do in your artwork?
Emre	Pyramid
Fatma	12 door pyramid
Emre	There is a 12-door pyramid, if we think it as viewed from the top view, there could be twelve faces. So this is all pyramid seen from the top view.
Teacher	So, it is as if seen from hill.
Emre	Man comes from here [*shows small triangles in his artwork in figure 47*]. There are many entrances [*of the pyramid*] here.

Figure 47. Emre’s representation of a pyramid from the top view

On the other hand, regarding the view of a shape from the combination of different directions such as top view and side view, one of the students explained how a prism could be seen when he was asked to think about its view in the stimulated recall interview (see artwork in Figure 48). Emre focused on only view of the shape: focused on only front view. In fact, it is seen when one stands on the intersection of top and front view and one sees its’ front and sides since the rectangular prism is rotated as to stand on one of the edges. He explained the front view of rectangular prism by pointing at the triangular region as stating “*This is. two parts are combined* [*two triangles forms one face of rectangular prism that he regarded it as front face*]”

Figure 48. Emre's identification of part of a rectangular prism

While these students realized the shapes only from one view point, two students (Ali, Fatma) also perceived reversible views of the shapes. For example, when students were asked to observe the artwork of Sol Lewitt [studio work 1] (Figure 49) and describe how they see the pyramid in the painting, they reflected two views of the pyramid. This pyramid could be perceived from the top view and from the front view. In the following conversation, it seems that Fatma perceived it from two different perspectives at first glance. Ali realized the front view after he imagined the rotation of the pyramid. However, it is not clear how Esra imagined the pyramid and its view.

Figure 49. Artwork of Sol LeWitt

Researcher	[…] and how do we look at this pyramid?
Melek	Top
Fatma	Top
Ali	From the top.
Esra	From the front
Ali	No, it cannot be from the front.
Fatma	It can be seen both from the front and from the top. but when we put the paper and look at the paper from the top; looking at it, it is unfolded in a way.
Researcher	Do you mean it could be seen from the front as well?
Fatma	Yes, teacher.
Esra	Lets' say there is a pyramid here. When we look at from the front and cut the front face, we see inside of the pyramid.; but when we looked at from the top, because we see sharp point of the pyramid, we should have been looked from the front.
Ali	You know, something like this. It's either the pyramid that is seen from the top or the pyramid that stands normal [*flat*]. Then this pyramid shook, and that front side was normally on the ground. Now it [*front face*] is lifted up as if we pulled it up.

In addition to demonstration process, during student-at-work phase, one student (Fatma) described how her artwork is perceived from different perspective. She drew nested squares to make a pyramid. She perceived the square pyramid from top view, front view and bottom view. While she was creating the artwork during studio work 1, she explained how the shape in the artwork could be seen. It was one of the sketches in studio work 1 (figure 50).

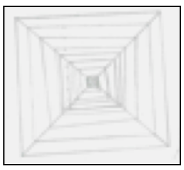
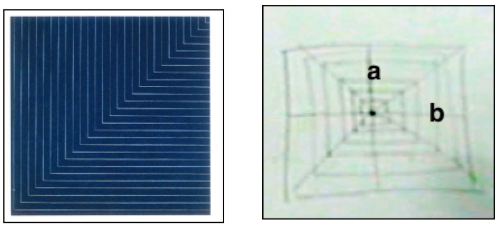


Figure 50. Fatma's representation of a square pyramid from the top or bottom view

Fatma	Teacher, I drew my first thought. Like in a room, I've hidden four shapes in it.
Researcher	What did you hide?

Fatma	There's a square like that. There is a top view of the pyramid. There's a triangle. And what I'm imagining is… we're looking at the dark room through the door. It's getting bigger or less depending on the point of view.
Researcher	So, you mean We'll think of it as the opposite wall.
Fatma	Yes, we are looking towards the opposite wall in the corridor.

The second main finding is related to distinguishing different perspectives of shapes. For example, during group observation process in the studio work 1, one of the students (Burcu) realized a two-dimensional representation of a pyramid. After she realized it, students attempted to envision how the pyramid in the the artwork (figure 51a) can be seen. One of students (Melek) draw how it can be seen from the top view. Another student (Burcu) described how it can be seen by making use of body language.



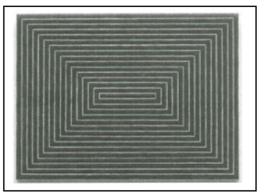
(a) (b)

Figure 51. (a) Artwork of Frank Stella (b) Melek's representation of the pyramid from the top.

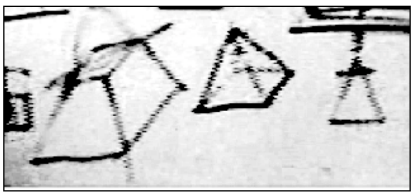
Researcher	What kind of pyramid is it? [*after Burcu says it is a pyramid*]
Fatma	Pyramid seen from side-view
Researcher	How do we look from the side?
Ali	This is my teacher [*shows with body movements*]
Melek	It looks like as if you're looking from the top.
Fatma	As seen from the side and from the top
Researcher	Can you draw on the board Melek? [*draws the shape in the figure 51a on the white board*]
Fatma	For example, we see it when we look at a pyramid from the side and a little bit from the top.
Researcher	What do you think it is possible? [*asks other students*]

Emre	This is already a bird's eye view. It's already a top view [*figure 51b*]
Ali	We stand here and are not seeing the other parts [p*ointing at the a and b parts in figure 51b*]. we see just these parts. You know this is the view from the top (figure 51b). so we come a little bit to the side, teacher. Let me try to draw…I can't draw anyway. It's kind of like a shift to the side [*shows with body movement*]

Another example of such a comparison of different perspectives is observed during group observation of artworks in studio work 1. Students related drawings of a particular shape that were drawn from different perspectives. When students were asked to observe the following artwork again after individual observation and share what they saw, students discussed how a pyramid can be seen from top view. This discussion seems to give clues about how students imagine a shape from different views. For example, Ali explained his thinking process through comparison of different views of this pyramid. To support his ideas, Ali envisioned how cross-section of a pyramid is seen when it is truncated. While one of the students (Ali) perceives it as a rectangle when it is cut from the top, another student (Esra) student perceives it as a parallelogram. The reason behind the difference in their perception, Esra has stick to perspective drawing of a pyramid. Thus, she could not perceive it as a rectangle.



(a)



(b)

Figure 52. (a) Artwork of Frank Stella (b) Ali’s representation of the pyramid in the artwork from a different perspective

Researcher	How do we look at this pyramid *[figure 52a*], Fatma?
Burcu	We look from the top.
Esra	But when we look from the top, we don't see that base [*shows the smallest rectangle in the Figure 52a*] [*Burcu draws a pyramid at this time*]
Researcher	Burcu made a pretty good drawing here.
Fatma	It's like an unfinished pyramid. Looks like it's not half finished, as seen from the top.
Ali	Lets' cut it out of here. This is something like a rectangle. Then let's rotate [*imagines the rotated the pyramid so that it is seen from the top*]
Burcu	No need to rotate, lets' look at it like this [*imagines to change her perspective with body*]
Ali	When we looked at it right there.
Esra	When we cut the shape, don't we see parallelogram? [*figure 52b*] when we throw it away, we see the base of that triangle.
Fatma	but, it is seen as this one [*figure 52a*] when we look from the top
Ali	No, it looks the same [*with figure 52a*] when looking here [*from the top*].

This discussion also indicated that students might think to change the position of the object or change own perspective to perceive how it is seen from a particular point of view. Whereas Ali imagined how it is seen from the top view through mental rotation of the pyramid, Burcu perceived it through imagination of changing her perspective.

To summary, there are two main findings regarding imagination of a shape from different perspectives: imagining the view of a shape and comparison of different perspectives of a particular shape. During imagination of the shapes from the different perspectives, students envisioned to imagine to either change their position or to change the position of the object through rotation.

4.2. Decomposing and Composing Shapes

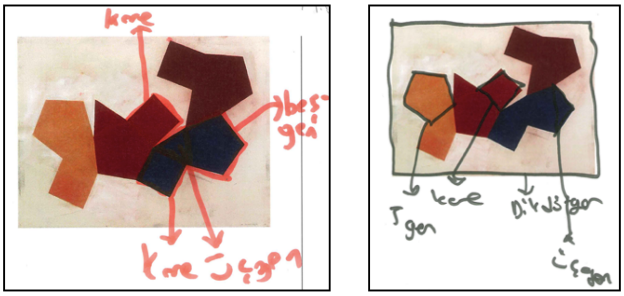
This part presents students' thinking process regarding putting shapes together to produce new shapes (composition of shapes) or taking apart shapes into small shapes (decomposition of shapes) during observation of art work or creating of art works.

4.2.1. Decomposing Shapes (Taking Apart Shapes)

The analysis of the students' thinking processes regarding decomposition of shapes indicated that when they were asked to observe an art work and note what kind of geometric shapes they see, they attempted to decompose colored shapes in the art works into smaller geometric shapes.

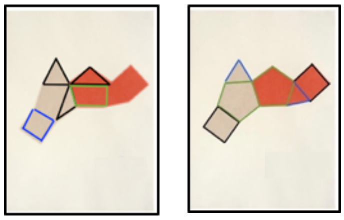
It seems that students partitioned these shapes into triangle, pentagon, square, and trapezoid shapes. Only two of the students (Ali and Melek) partitioned a colored whole shape into familiar regular polygons. The figure 53a and figure 53b shows two ways of student' partition of an unfamiliar shape into geometrical shapes.

This finding was also observed in the critique part of the studio work 3 in which students copy artworks. In the critiquing part, after students explained their artworks and questioned them, the researcher asked them what geometric shape(s) they could see in the painting. Students decomposed the shapes into the shapes of triangle [Emre, Ali], square [Emre, Fatma], and rectangular [Emre, Ali] as stating "*even though it is not exactly rectangle (Emre), not parallel (Ali)*" in the artwork of Mel Bochner (Figure 54a). However, they were not sure about whether it is a rectangle or not. It seems that they coded sharp regions as triangle. They recognized the squares that are hidden in the painting. However, it was not observed that they compose regular polygons of equilateral triangle, pentagon, and square so that there is no a space between them.



(a) (b)

Figure 53. (a) Ali's decomposition of shapes: sqaure, pentagon, triangle (b) Melek's partial decomposition of shapes: pentagon, sqaure, rectangle, triangle



(a) (b)

Figure 54. (a) Students' decomposition of shapes (blue: square, black: triangle, green: rectangle; (b) One of the decompositions of shapes by the researcher

Moreover, there was noteworthy finding is that student who decompose this unfamiliar shape into geometric shapes during individual observation [studio work 1] did not used this thinking process in copying of art work in studio work 3 that is similar with the artwork in studio work 1. It seems that when students were asked to find the shapes, they attempted to decompose the shape.

On the other hand, when students were not asked to find the shapes, they did not consciously decompose the shape in copying the artwork even though they were able to decompose the shapes into smaller shapes. This finding is also supported in copying of another artwork that is composed by two congruent regular pentagons (figure 55).

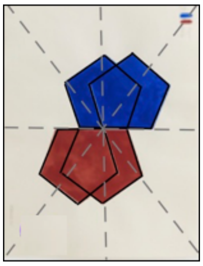
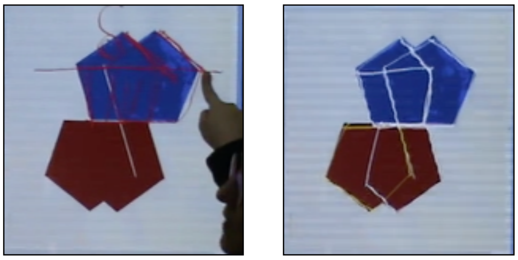


Figure 55. Decomposition of the shapes in the artwork of Mel Bochner by the researcher



(a) (b)

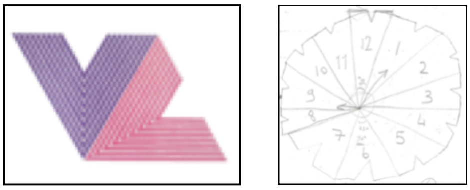
Figure 56. The process of decomposition of shapes by students

During the copying the artwork above [studio work 3] observation of videos indicated that students did not attempt to take apart two congruent colored shapes. Only when students were asked to observe the artwork again and find geometric shapes in these figures after critique the Emre's drawing, only one student (Fatma) noticed overlapped pentagons that are hidden in two colored shapes. Even though Fatma showed one of the pentagons, other students still had difficulty in perceiving pentagons (figure 56a) since they could not draw contours of nested pentagons.

They stated they saw triangle and rectangle. When asked to find the pentagons inside these symmetric shapes, Ali and Emre had difficulty in finding them. Only Fatma could draw outlines of pentagons (figure 56b).

Another finding regarding decomposition of shapes was observed during creating art work in studio work 2. This kind of decomposition was partitioning the shapes into equal parts. Two students attempted to partition the shapes into equal parts. However, it is important to note that students decomposed shapes after composition of shapes.

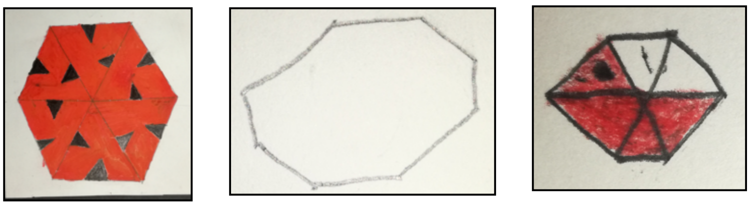
In studio work 2, students were asked to think about the question of 'what if the art work of Frank Stella was a beginning of your art work, how would you continue with it' (artwork in Figure 57a). The researcher asked them to rotate them and compose a new shape. Emre imagined making a circular shape by rotating and combining triangular shapes. During this process, after Emre imagined to make a clock with a circular shape (figure 57b), the researcher asked him how he partitioned the shape into equal parts. However, Emre did not consider the angles of triangles to compose the circle. He partitioned the circle into twelve triangles even though angles of triangles were 60 degrees. It seemed that he did not consider how many triangles with 60 degrees fit into a circle. When the researcher asked him what the round angles is, he stated it is 360 degrees. Then researcher asked further probing question of "*what could angle of a triangle be so that the number of triangle is 12 in the circle*?". Then, he identified 30 degrees for angle of each triangle.



(a) (b)

Figure 57. (a) Artwork of Frank Stella with equilateral triangles; (b) Emre's composite shapes of circle and its decomposition into equal units of triangles

On the other hand, Ali did not imagine making a particular shape and he tried to combine paired triangles by trial and error (final artwork in figure 58a). In other words, he did not deliberately imagine to produce a particular composite shape at first glance. Ali realized the decomposition of the hexagon into six equal triangles after he completed his art work. At first glance, he imagined to rotate the paired equilateral triangles. After composition, he constructed hexagonal shape and observe his artwork (figure 58b). Then he drew a hexagon and partition into equal parts as a very small sketch (figure 58c).



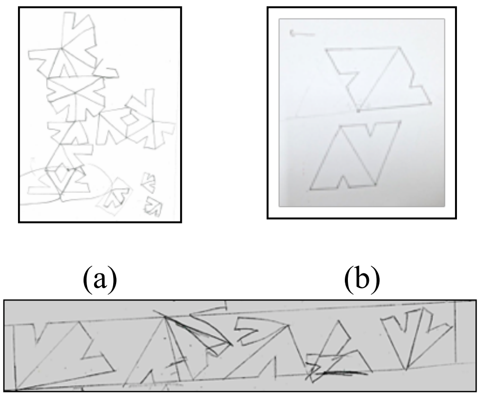
(a) (b) (c)

Figure 58. (a) Ali's final artwork (b) Ali's recognition of composite shape of hexagon (c) Ali's decomposition of the hexagonal shape into six parts

In summary, students attempted to decomposed shapes to identify shapes in art works by finding shapes in the art works that involve hidden and/or overlapped shapes. [studio work 1 and studio work 3]. It seemed that when students were asked to find the shapes, they attempted to decompose the shape. On the other hand, when students were not asked to find the shapes, they did not deliberately decompose the shapes. Another finding was that students also attempted to decomposed shapes by partitioning them into equal parts in the art works during creating art works [studio work 2]. However, students decomposed the shapes after they created their own composite shapes.

4.2.2. Putting Together Shapes

This part involves participants' combinations of shapes to produce new shapes as an independent shape. It was observed that students (Fatma, Melek, Ali, Emre) mostly composed shapes to make an artwork during creating artwork. During studio work 2, students were asked to choose one of the triangular shapes and to imagine what would happen if they continue to this art work and combine them as a whole (see one of the artworks in figure 79a. At first glance, all students had a tendency to draw the shapes without a coherent whole (Figure 59).



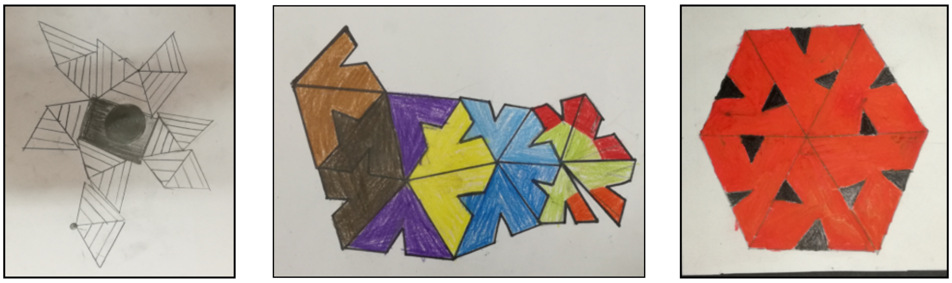
(a) (b) (c)

Figure 59. Students' attempts to compose and rotate shapes: (a) Melek, (b) Ali, (c) Fatma

For example, Fatma drew different version of triangular shapes that were not combined together. It seems that she did not either understand what the researcher asked to do or had difficulty in composing shapes in her first try (see first sketch). When researcher asked to think about how they could be composed together, she imagined to make a real-life object such as a heart, a head, or a bird.

Fatma	Teacher I drew four [*four paired triangles (figure 59c*)]
Researcher	How do you compare these four paired triangles?
Fatma	How so?
Researcher	Now they're all different, I want you to create a picture, not separately
Fatma	With all of them? For example, like a heart shape?
Researcher	You can, for example, or not necessarily have to look like something, it's going to be rotated for a whole, not separate triangles
Fatma	Hı hı
Researcher	Let's take a look at again, draw what imagine you.

After the researcher asked to draw rotated images in a coherent whole, they still had difficulty in drawing rotated images in a coordinated manner. While some of them (Melek, Esra) did not compose the shapes to make a new shape, some of them attempted to combine in a coordinated manner. For example, Melek combined rotated and flipped images of double triangle partially, which did not result in a new shape (Figure 60a). On the other hand, other students made a bird (Figure 60b-Fatma), a hexagon (Figure 60c-Ali), and a clock (Figure 47-Emre).



(a) (b) (c)

Figure 60. Students' attempts to compose shapes: (a) Melek's final artwork (b)

For example, Fatma tried to make a head with hair (Figure 61a) after the conversation above. However, she gave up making a head with hair. Then, she envisioned to make a bird. After studio work 2, stimulated recall interview showed that she thought she had difficulty in combining triangular shapes to make a head.

Researcher	So you did these first [*figure 59c*]. What did you do then?
Fatma	The first thing that comes to my mind is to fill a round with the following triangles and think of doing the hair with triangles next to each other [*figure 61a*], then I give up, the angle to find it would be difficult to find the exact angle. So I decided to make a bird [*figure 61b*]
Researcher	Where did you give up?
Fatma	So I said it drew a circle and I tried, but then I could not… I mean their corners will be round, like hexagon, I tried to do like it [*hexagon*] [*Figure 61b-green colored area*], but I could not. So I turned it to a bird.

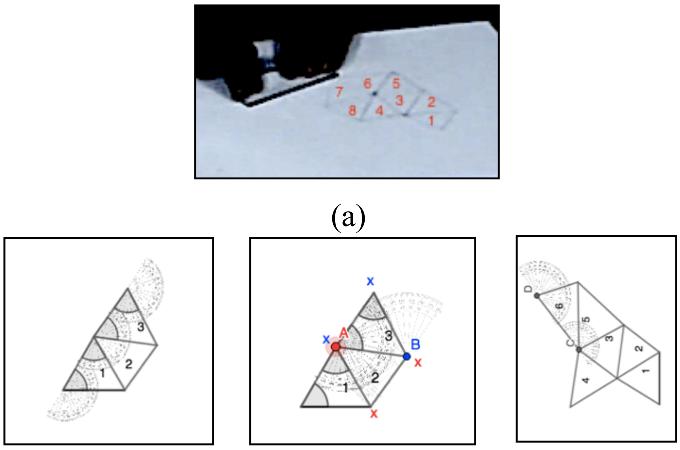
In Figure 61, the green frame refers to the first try to make a head (figure 61b). After drawing this part, it seems that she gave up making it since she was afraid of composing triangular shapes with the consideration of angle. Then, she joined three similar shapes together so that they would be one after another (figure 61c). This was a free-hand sketch in which she did not think properties of triangles (angle, length size) to compose them. In other words, she added each shape randomly.

(a) (b) (c)

Figure 61. Fatma's composition process during studio work 2 (a) first sketch of a head (b-c) sketch of a bird

After she decided to make a bird on the basis of third sketch (figure 61c), researcher asked to think about attributes of triangles and measure them. Thus, she had to think how to compose each shapes in a valid way. Drawing process of the last version of artwork indicates that she had difficulty in composing shapes with the consideration of geometrical properties (figure 62a). Observation of videos showed that she erased what she drew many times during exploration process of exact location of each triangle. While she was drawing, she encountered with a problem that there was a narrow space between two triangles to place a triangle after she drew six triangles. Thus, she made adaptation on two triangles that she had drawn before and changed their lengths and angles visually without consideration properties of equilateral triangle. It seems that when she had difficulty in composing them, she drew through free hand sketching so that it fits in the image on her mind. This process was explained in detail.

She drew the second triangle in the artwork measuring neither the lengths nor angles of the triangle. Then she drew the third triangle next to it. She measured its base angles (figure 62b). Then she made adaptations since the positions of the sides are not correct. After she made revisions, she measured the angle from the point of A, top vertex of the the second triangle. She put signs on the two points referring to 60 degrees (represented red *x* in the figure 62c). She then measured angle from the point of B and she put two signs, represented as blue *x* in the following figure. Then she made revision in the third triangle again. It seems that she checked whether something is wrong and to correct that wrong drawing. During this process, she did not check whether the sides of the triangles are equal to each other. She did not measure two base of angles of the the second triangle.



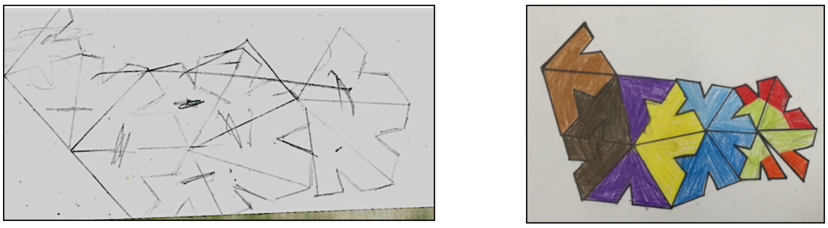
(a)

(b) (c) (d)

Figure 62. Fatma's composition process of triangles on the basis of shapes' properties (a) order of triangles that were drawn during creating last version of artwork (b-c) use of proctractor to measure angles of triangles (d) checking of angles when the distance

She drew the fourth and fifth triangles without measuring lengths and angles (figure 62d). She made free hand sketching. Then she drew the sixth triangle in the same way. Then she attempted to erase it. It seems that she realized that it did not look like others. Before erasing she checked its angles (figure 62d). She then made some revisions. Then she filled the space between fourth and sixth triangles with two triangles (seventh and eight triangles) However, they were smaller than the others. She drew one of sides of the eight triangles so that its angles would be 60 degrees. This time it became smaller than the previous one. The vertex of eight one did not come up to the vertex of fourth triangle. She then deleted two last triangles and she visually divided the space into two parts and drew two triangles without measuring.

She drew other triangles without measuring and drew them smaller than the first eight triangles. It seems that she did not make coordination between triangles that are attached to each other. Thus, the last version of the bird (figure 63b) became different from first sketch (figure 63a). The interview after studio work 3, she explained how she changed the sketch to make a bird. Combining pair of triangular shapes and increasing number of shapes in the last version might lead her to make adaptation on her sketch.



(a) (b)

Figure 63. The change in Fatma's composition of shapes to make a bird (a) early sketch (b) final artwork

Researcher	What did you pay attention to when doing this? [*figure 63a*]
Fatma	I didn't do a lot because it was a draft, I just wanted to get all of them together and, but again some of them has not a pair [t*ries to combine paired triangles like in the artwork of Frank Stella*]
Researcher	And then did you change your any idea?
Fatma	The tail [changed], there were 3 [*triangles*] here; then it became 4, then it was the only a triangle without a pair [*the first triangle in figure 63a*], here is paired [*shows light brown colored triangles in figure 63b*] [*compares the draft and final drawing*]
Researcher	Why did it happen?
Fatma	I did a couple here because it looks more like a bird [*figure 63b*] but I couldn't do the sharpness on it, so this came straight [*sharpness part (above the bird) in the figure 63a became flat in figure 63b*]

This process indicates that she focused on drawing each triangle by combining them. When she had problems in drawing triangles, she faced with the difficulty of filling the space with identical equilateral triangles. To solve this problem, she drew triangles through free hand sketching. It seems that she did not predict what shape is produced when combining equilateral triangles. If she had thought holistically, she would combine them in a coordinated manner by considering compositions of 60 degrees to make a round angle.

In contrast to Fatma, Ali composed shapes by considering angles of each triangles and imagining the rotations of triangles. His composition of shapes (figure 65a) was more mathematically valid compared to that of Fatma even though Ali had difficulty in drawing rotated image of a triangle too. In fact, he could draw the first rotated image of the triangle. He had difficulty in drawing the next step. He used a paper triangle to envision it and then drew a right triangle. Then he realized that he made somehing wrong since the last triangle did not look like the others as stating *"[...]this time equilateral triangle does not occur, right triangle 90 degrees*" (figure 64)

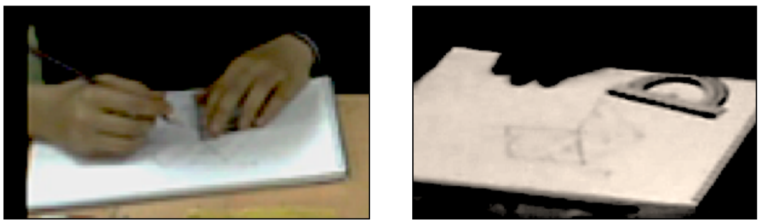
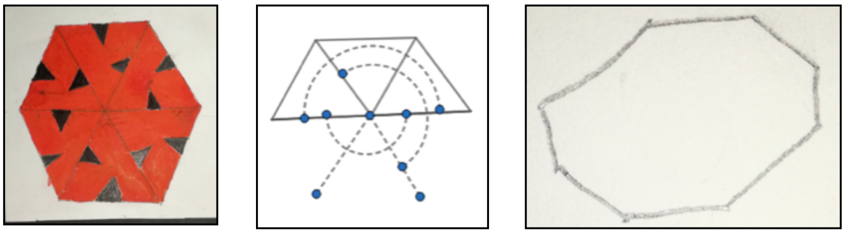


Figure 64. Ali's compostion of triangles by making use of protractor

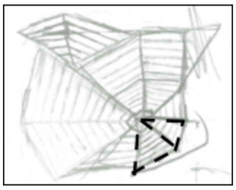


(a) (b) (b)

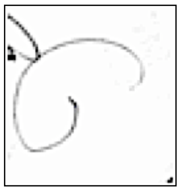
Figure 65. (a) Ali's final artwork through composition of shapes (b) the way of composition with diving protractor into equal parts visually (c) hexagon as a composite shape

When the researcher asked him what the interior angle of each triangle is and how one of the edges is turned to the place of the next edge, he realized that he should identify the angle of rotation as 60 degrees. He divided a protractor into three parts and signed the points corresponding to 60 degrees (figure 65b). He perceived the shape holistically that consisting three triangles and predicted the resulting shape. Then, he could combine the triangles as a whole even though he realized it as a hexagon after he completed it (figure 65c).

Another example of putting together shapes was observed during studio work 1. For example, Melek aimed to compose square pyramids by envisioning making a spiral. Observation of videos indicates that she firstly drew a pyramid with two faces and added pyramids with smaller and bigger size next to it (figure 66a). Then she added new pyramids so that their sizes increase. When we observed her drawing from a distance and make the spiral movement with her finger, we could understand she envisioned to make a spiral (figure 66b). The following conversation after studio work 1 supports how she thought.



(a)



(b)

Figure 66. (a) Melek’s composition of pyramids (b) spiral as a composite shape

Researcher	Could you explain how did you do it? [*figure 66a*]
Melek	I've drawn a lot of pyramids in it...using a lot of pyramids to try to hide a shape [*figure 66b*], that's good
Melek	Here again I tried to draw a square pyramid. I could not rotate from the squares. That’s why I gave up.
Researcher	You tried to draw a square pyramid? Could you show one of them?[*Melek underlines one of them (dashed lines in figure 66)]*

It seems that this kind of composition of two-dimensional representations of three-dimensional shapes was informal. Even though she claimed that she drew square pyramids, she drew them with two faces without envisioning how to put together other faces of square pyramids. She used the rotation consciously. However, she focused only one component of pyramid, its edges. She did not consider how aligned pyramids. It seems that she did not think whether they are vertical pyramids or they are oblique pyramid.

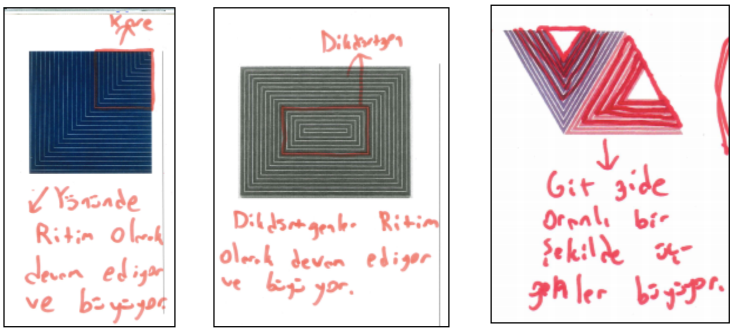
In summary, students’ free hand sketches indicated that they mostly combined the shapes informally and by trial and error. Except one student (Fatma), all students did not anticipate what shape they would create by combining the geometric shapes. In addition, most of them did not focus on the geometric properties of the shapes to composite the shapes into new shapes.

Students' ability to imagine the rotations of shapes and to represent it could be crucial factors that affect students' ability to compose shapes.

4.3 Spatial Patterning

In this study, spatial patterning refers to searching for visual/geometric regularities. It involves identifying the parts of a spatial pattern (segmentation) and combining the parts into a coherent whole (integration). The analysis of the students' ways of spatial patterning indicated that students both segmented the visual patterns into individual units and integrated the units to create a whole with a rule. Each way of patterning is described respectively. These two processes could be interrelated. In this part, particular examples regarding each way of patterning was presented.

First of all, students recognized the patterns that are visually presented by identification of units of the patterns in the artworks. These art works mostly involved growing patterns. Students' identification of patterns by segmenting them into smaller parts was observed during all studio works. For example, during individual observation of artworks in studio work 1 and studio work 2, it was only Ali who realized the growing patterns in two artworks when students were asked to observe what kind of geometric shapes they see and took notes about it. It seems that he identified visual rhythm in the artwork in which the squares as stating *"continues and grows as a rhythm in the [...] direction"* (artwork 1 in Figure 67a) and rectangles as stating "*Rectangles continue as rhythm and growing*" (artwork 2 in Figure 67b) and triangles are repeated with a variation in their sizes as stating "*Triangles are growing in a proportional manner*" (artwork 3 in Figure 67c). In this sense, he segmented out squares and rectangles as units in the patterns.



(a) (b) (c)

Figure 67. Ali's recognition of the patterns through identification of units of the patterns in the artworks.

During group observation, another student (Emre) also explained that the first art work involves a pattern of squares. During this process, students were explaining geometric shapes that they saw in the artworks and discussing whether it could be a pyramid or not. On the basis of this discussion, he stated as "*Teacher, I think this can't be the pyramid. this picture is created by growing squares in certain sizes only within a certain order.*" (artwork in the figure 67a). It seems that he realized the regularity in arrangement of squares whose sizes increase. Students also attempted to identify the rule of this pattern either numerically or visually when the researcher asked them to describe how the sizes of these squares increase. For example, regarding visual description of the rule, Fatma described the rule as stating "*So, leaving a finger space…I think 1 cm 1 cm 1 cm [indicates with finger]*". Emre described it as structuring spatially the arrays of squares as stating "Teacher we already have a square her, but when we do this, there are three squares. So both increased" (figure 75). He attempted to fill the second square with the first and the smallest square. Regarding numerical description of the rule, students could not find a rule on which all of them had a consensus.

In addition to the pattern of squares, in the further process of the discussion, students also identified the patterns of repeating the letter of L as stating "*And also* *there's something like that: It’s getting like an L*" (Emre) and arrangement of repeating vertical and horizontal parallel lines by perceiving it as a road as stating “*Teacher, the way is big and getting smaller*” (Fatma). She segmented out the square into equal parts (one part: wall, other part: road).

Another example of segmenting the pattern into units was observed during studio work 3 in which students copied artworks. During copying artwork of Robert Mangold (figure 68), students did not reflect about the relation between the sizes of all squares. During critiquing the drawing of students, it was observed that one of the students (Melek) described a pattern to solve the problem in her friend’s drawing (adjusting the sizes of squares in a larger space). She explained the pattern by explained its rule numerically. She found this rule by segmenting the biggest square into equal squares.

Figure 68. Artwork of Robert Mangold

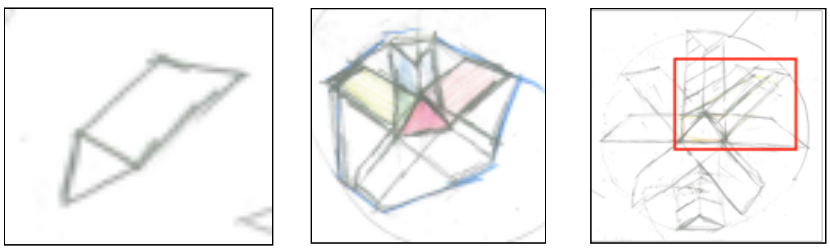
Researcher	Did you have any difficulty in copying this artwork? [figure 57]
Fatma	Teacher, their sizes. I drew the first square here, here are the others.
Researcher	You made it a little smaller. Then, how did you realize you were doing small?
Fatma	I would draw 4 times the size of my teacher. When I drew it that way and it was coming to that part of the paper.
Researcher	So how can we solve this problem? [*Fatma drew smaller than it should be*]
Emre	I think we cannot fix it.
Melek	If we use something like this, this is 1x and this is 2x, which is

	3x 4x…For example, if we take the edge of it as 2, this is 4 times.
Researcher	Did you notice the relationship between them, Melek said something nice.
Emre	It's growing.
Researcher	How?
Emre	2 4 6
Researcher	And what will be the last?
Melek	This will be 4 times bigger: 8.

Interview after studio work 3 indicated how she identified the pattern during copying this art work. She explains how she segmented the biggest square into equal squares. It seems that she identified the pattern in the sizes of square by structuring arrays of squares during encoding the relation between the smallest and biggest squares. The following conversation supports this process.

Researcher	How did you do this artwork? [*figure 68*]
Melek	At first, I knew there would four [*the smallest square*] vertically and four [*the smallest square*] horizontally [*inside the biggest square*]. But, I never thought of the other ones in a similar way.
Researcher	4? You mean 4 in here [*biggest square*]
Melek	There are 4 here and 4 there,16 pieces
Researcher	so this is what you thought?
Melek	I thought but just thought of it, I've said half of this is that [*compares side lengths of two squares*], 2, this is 4, this 6, this 8.
Researcher	Yes, did you think of doing it during copying artwork?
Melek	Of course
Researcher	You tried to do it twice as much, then you tried to do it 3 times, how did you make the distances between them?
Melek	I never thought of the distance between them.

The second finding was that students created visual patterns (repeating and growing patterns) by combining individual units during studio work 1 and studio work 2. They mostly constructed patterns during students-at-work parts in which they created their own art works. First of all, four students created repeating pattern. For example, Melek drew a shape of triangular prism. Observation of videos indicated that she started with the shape of triangular prism (unit of the pattern) (figure 69a). She added another triangular prism by reflecting the first one with the help of a reflection/symmetry line. After drawing the second triangular prism, she drew another triangular prism to the middle of other triangular prisms. It seems that she combined three triangular prisms by linking them on one of their faces. After she drew three prisms, she colored them and observed her drawing. After observation, she extended two symmetric prisms (figure 69b). After extension of prisms, she must have been imagined the rotation of them since she made a gesture of rotation with her hand. This was early draft of her artwork. Then she drew it again by adding other triangular prisms (figure 69c). After she drew them, she again made a gesture of rotation by hand and drew a circle around them to make the sizes of triangular prisms equal. This whole process indicates that she discovered a pattern of triangular prisms in which they are connected on a common face by reflecting them and rotated on the center with the imagination of circle movement.



(a) (b) (c)

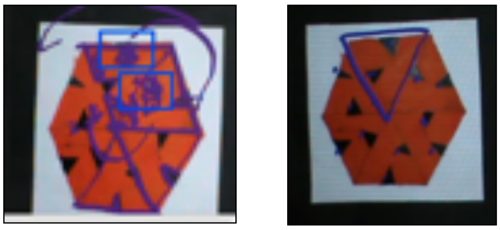
Figure 69. Melek's construction process of a repeating pattern (a) unit of pattern (b) symmetric prisms (b) rotation of unit of pattern

Another example was also observed during studio work 2 in which they were asked to continue to one of the artworks of Frank Stella with imagination of its rotation. Ali chooses the following art work of Frank Stella (Figure 70a). He constructed a hexagon by combining a triangle as a unit with the rule of rotating 60 degrees, which resulted in pattern of rotating nested Z letter. He copied this artwork on his sketch book. Then he imagined its rotation and drew its rotated version (Figure 70b). The researcher asked him what would happen if he combined them together. However, he could not understand how to do it. Then, researcher asked him step by step questions. After these specific questions, he imagined a pattern that involve nested Z letter (figure 70c).

(a) (b) (c)

Figure 70. Ali’s construction of a repeating pattern: (a) Artwork of Frank Stella (b) Copying of the artwork and representing its rotated image (c) Pattern of nested rotated Z letter in the artwork of Frank Stella

Researcher	What would happen if you think it is as the continuation of this [*figure 70b*]. How about if you think of combining rather a separate drawing?
Ali	How?
Researcher	So when you rotate that whole shape, think like you didn't draw it separately, if you turn it 60 degrees, where is its new position?? Each of these was 60 degrees.
Ali	He comes over here, and this comes to the top. A-haaa! [*imagines rotation the first paired triangle in the figure 70b*]
Researcher	For example, you can also turn to the other side [*left*]. It is up to you. Think about it and let’s look at later.



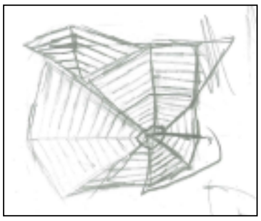
(a) (b)

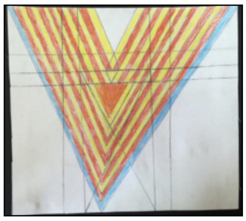
Figure 71. (a) Ali's description of his artwork during critiquing part (b) Teacher's identification of unit of pattern.

After this one-to-one conversation, he deleted the second the shape and imagined the rotation of each triangle. During this process, he added small triangles into the triangles that give clues how he imagined the pattern of nested artworks of Frank Stella. During the critiquing process, he described how he imagined the pattern when he was asked to explain his artwork (Figure 71).

Researcher	Can you tell us what you did Ali?
Ali	At first I started with the general shape that there were no small triangles of blackness [*figure 71a*]. After that I continued this. now that I come here, I turn to that side [*to left in the figure 71a*])
Teacher	How many degrees do you rotate?
Ali	This is 60. That came here when I turned it [*shows the change in position of each side*]. It came here when I turned this. That's how I went, my teacher.
Teacher	You are constantly rotating 60 degrees
Ali	Yes, then after all, like this [*shows the last step in rotation*]
Teacher	Then you have rotated a single triangle, then this is your original shape, you are rotating it constantly [u*nderlines one of the triangles as unit of pattern*]

In addition to repeating patterns, students also created patterns of growing shapes by combining individual units such as a pyramid (Melek), triangle (Ali) during creating their own art work [studio work1]. While Melek constructed a pattern of arranging the pyramid by rotating in a spiral way and increasing their sizes (Figure 72a), Ali constructed a pattern of nested triangles by increasing their sizes (Figure 72b).





(a) (b)

Figure 72. (a) Melek's representation of a growing pattern of pyramids in her artwork (b) Ali's representation of a growing pattern of triangles in his artwork

Observation of videos, for example, indicated that Melek did not aim to create a pattern of pyramid at first glance. She explored connections of pyramids as stating "*Teacher in this way, all of them are pyramids, they are arranged in this way [spiral]*". She started by drawing a pyramid with two faces. Then she added a smaller pyramid to top of that pyramid. Then, she tried to fill the left gap between the smallest pyramid and largest pyramid. She placed shapes that look like pyramids. However, they were not very clearly observable in the in the videos. Then she filled right gap between two pyramids by adjusting the size of these pyramids, which resulted in the pattern of increasing size of pyramids (figure 72a). After drawing it, she observed her artwork from a certain distance. She realized the pyramids has a spiral configuration that includes growing pattern of pyramids (figure 73) even though some parts of the artwork ruin the rule of the pattern. She explained how she thought during process during interview after studio work 1.

Figure 73. Melek's representation of the pattern in alignment of pyramids as a spiral

Researcher	How did you do it?
Melek	I've drawn a lot of pyramids, like a spiral turn, using a lot of pyramids, trying to hide a shape. I tried to hide this shape by making use of a lot of pyramids.
Researcher	This is pretty good [...]

Similarly, Ali drew a pattern with growing triangles (figure 72b). He placed each triangle with identifying equal units between triangles. However, he did not measure during the drawing. He determined the equal space between triangles visually. He tried to make it symmetric by visually determining equal distance to the middle. How he determined equal distance to the center could be explained by which he aligned top point of each triangle. However, it seems that there are some distortions in symmetrical configurations of triangles.

In summary, students both detected spatial patterns by identifying the unit of the pattern and formed patterns by combining individual units. They identified both repeating and growing patterns of shapes. However, their identification of the pattern and rule of the pattern was mostly informal.

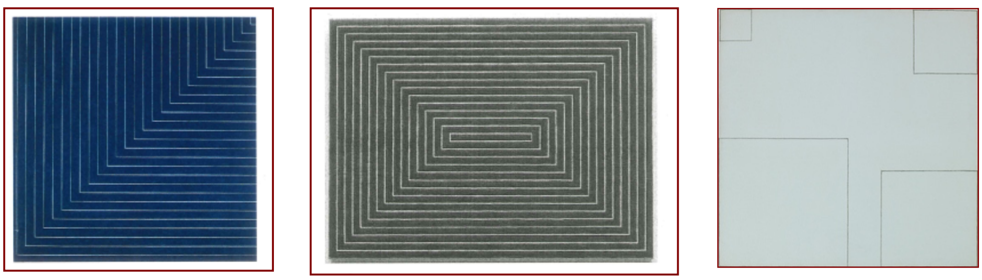
4.4. Transforming Geometric Shapes

This part involves students' identification of transformations of shapes rigidly or non-rigidly. First of all, students' identification of scaling transformations as non-rigid transformations was explained. Then, students' identification of rigid transformations was presented.

4.4.1 Transforming Shapes Non-Rigidly: Scaling Transformations

This part represents students' identification of scaling transformations in artworks through encoding geometrical cues in the artworks. Encoding geometrical cues could be length and angular relations within and between shapes (e.g. proportional relationships), overall geometric arrangement of shapes, identifying geometric shapes and their properties.

The analysis of students' identification of scaling transformations indicated that students attempted to identify the proportional relationships in an artwork when they were give an artwork to observe. They also expressed proportional relationships numerically based on additive and multiplicative comparisons through identifying shapes' properties (e.g. equal lengths of squares and equal distance between two squares in figure 74a) and structuring the shapes into units (e.g. filling the second square with the smallest square in figure 74a; filling the second rectangle with the smallest rectangle in the in figure 74b).



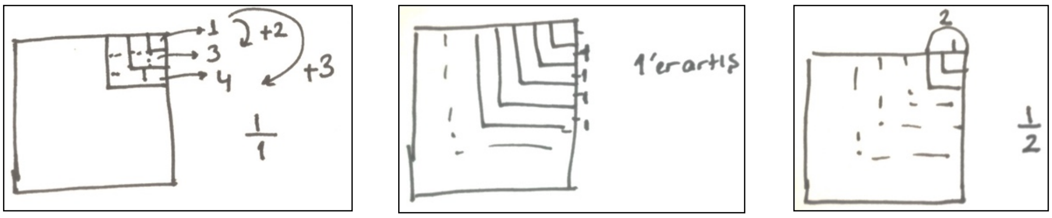
(a) (b) (c)

Figure 74. (a-b) Artworks of Frank Stella (c) Artwork of Robert Mangold

Emre, for example, identified the pattern of squares in the figure 74a above as if their sizes increased proportionally when they were discussing whether it could be perceived as a pyramid or not [group observation in studio work 1]. In order to understand his thinking process, the researcher asked how he thought the sizes of squares increase proportionally. He described the increase in size of squares by thinking about the rate of change in areas of two squares. The conversation about the rate of change in size of squares that Emre started elicited other students' thinking process as well.

Emre	[…] I think this can't be the pyramid anyway. A picture created by growing of squares in certain dimensions only in a certain order.
Researcher	You said a certain size of growth; similar with Ali
Emre	Ratio and proportions, my teacher.
Researcher	You say a certain growth? how do you identify this growth?
Burcu	So leaving a finger space like this [*figure 75a*]
Researcher	Well, how can you identify this relation mathematically?
Esra	It grows at the rate of 1/29.
Researcher	hımm you counted until the end [she counts the number of squares one by one]. What do you think?
Burcu	I think 1 cm 1 cm 1cm [*shows with her fingers*]
Emre	Can I show on the board? I think 1/1
Melek	As if it grows half, half; so 1/2
Researcher	How do you calculate it?
Melek	So how can I say that my teacher… the bigger square twice the smallest one. [*figure 75b*]
Emre	[*draws on the smart board*] There are already one square here [*shows the smallest square*], but there are three squares here [*fill the second square with the smallest one*] I mean, two more. Then when we look here, it's 1-2-3-4 [*counts number of smallest square in the third square*], 3. It goes from 1 to 2, then to 3. [*figure 75c*]

This discussion reveals that students' thinking about the ratio for growth of squares differed from each other. While some of the students (Melek, Fatma, Burcu, Esra) related lengths of the squares, one of the student related areas of the squares (Emre). Emre investigated this relation through additive comparison and explained in the multiplicative structure. He attempted to spatially structure the squares into unit of squares and determined how much increase in the area of the square by identifying difference in the number of units of squares in two squares. He expressed the relation between areas of the squares in terms of rate of change in areas of squares [figure 64c]. In fact, the rate of change was not constant across the different sized squares. It seems that he interpreted it as linear relation in which the sizes of squares should increase proportionally. He had thought the growth of growth in the area of squares are the same and determined it as 1 (see figure 75 illustrated by the researcher). In fact, the ratio between sides of each squares should have been equal to 1.



(a) (b) (c)

Figure 75. Representation of students' thinking regarding the rate of growth in the sizes of squares by the researcher (a) Burcu (b) Melek (c) Emre

Contrary to Emre' thinking about the increase in areas of squares, Burcu thought the lengths of the squares increased by 1 [figure 75a]. She explained it by using her fingers visually and by using numerical expressions such as increase by 1 cm. However, it does not refer to proportional relation between squares. She expressed the difference between lengths of the squares. Unlike Burcu's thinking process, Melek attempted to explain the relation between shapes by multiplicative thinking. She just focused on two steps of the patterns rather than considering all steps and explained the ratio of two corresponding sides of squares as stating "*the smallest square is twice the size of a larger one.*" [figure 75c].

However, it should have been a reverse relationship. It shows the ratio between two sides of squares rather than growth in lengths of sides.

Another finding was that students (Emre and Melek) used similar strategies during analysis other artworks (figure 74b and 74c). Emre divided the square or rectangles into unit of shapes whereas Melek explained the multiplicative relation between sides of squares in the last art wok. For example, Emre spatially structured the rectangles into units of rectangles (figure 76) and counted the number of these unit rectangles to determined the increase in the sizes of rectangles when the researcher asked him to justify his idea. He compared the areas of squares additively and explained it as if it has a multiplicative relation. He stated the rate of change in areas of squares as 1/3.

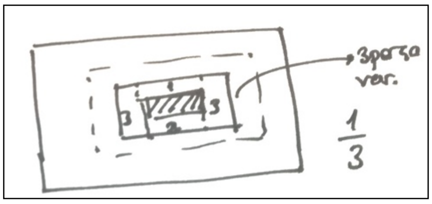
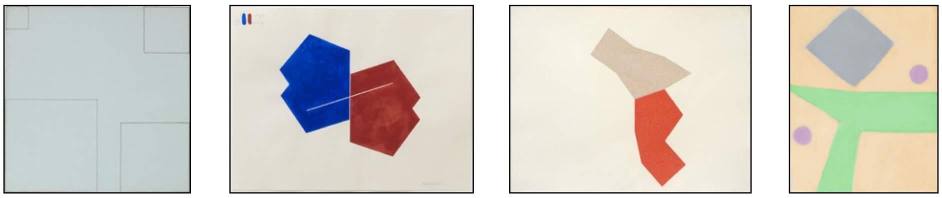


Figure 76. Emre’s spatial structuring unit of rectangles illustrated by researcher

Emre	The ratio here is 1/3
Researcher	Why do you think so?
Emre	Look, there are two of them here, same as the first rectangle.
Researcher	Which one?
Esra	But what about the ones in sides [*questions missing part after filling the unit rectangle into the second rectangle*]
Emre	okay I say that. There are two here, my teacher. ½ rate of increase
Researcher	Where is two of them?
Emre	Here and here, both fit into the rectangle [*area 1 and 2 in the figure 76*] There are two here, then on the sides, when it is combined [*missing parts*], it is equal to 1/3.

Another finding was that students similarly compared lengths of shapes both additively and multiplicatively when they were already given a scaling factor (1:4) to transform the sizes of four artworks during creating artwork part (figure 77). They compared the lengths of shapes multiplicatively or imagined the shape expanding when some students were looking for relations between the original artwork and their sketch in the larger paper and the relations between lengths of shapes in the original artwork. When they did not realize any proportional relationships between lengths of shapes, they considered relative difference between size of lengths.



(a) (b) (c) (d)

Figure 77. Artworks used in the studio work 3 (a) Robert Mangold (b-c) Mel Bochner (d) Agnes Martin

For example, one of the students (Ali) mostly multiplied the lengths, including distance between shapes, in the original painting (figure 77a) by four to draw in larger painting. He multiplied it by four through repeated addition of the shapes in the original painting. The following interviews after studio work 3 explain this situation. It shows that Ali understand each part of the painting become bigger in the larger painting at the same ratio.

Researcher	Could you explain what you thought during copying this artwork?
Ali	I considered the distance between them through my eyes.
Researcher	For example, how did you place the largest rectangle?

Ali	I placed it in this way, I looked the distance between it and a finger like this, if it is four times 1-2-3-4 there is a gap between it. When I look at that distance, it is similar.
Researcher	So you measured with a pen?
Ali	I didn't get it with a pen. I measured with my eye. After that, it was 1-2-3 times solid [*scales the distance*], not exactly 4 times. I said that if we make four times bigger, then there could be so much space…
Researcher	If I understood correctly, you tried to get 4 times this distance?
Ali	Yes, I did so; I drew their sizes four times bigger by imagining on my mind.

On the other hand, there are students who focus on partially on multiplying length of each shape by four. For example, Melek determined the size of the two signs and its distance with the two identical shapes in the artwork with nested and hidden pentagons (figure 77b) by multiplying their sizes whereas she did not focus on multiplying lengths of shapes by four. She reflected about her thinking process during interview after studio work 3.

Researcher	What did you pay attention to when you were making them?
Melek	I adjusted the whole picture on the basis of these [*lines in figure 77b*] when it grows 4 times: it is 1 cm becomes 4 cm or 2 cm is 8 cm.
Researcher	How many centimetres did you think? Like 1cm?
Melek	no, 2 cm. as this part is stretched; the other part [*its corresponding in the larger space*] becomes bigger.
Researcher	Can you tell me about how you get four times?
Melek	So we didn't measure with anything. I imagined on my mind it would be four times.

Another student (Emre) did not perceived that every part of the painting become bigger/larger during scaling transformations during drawing the artwork (figure 66a). When the researcher asked him to observe his drawing again and check what works or does not work, he observed it again and reflected that only sizes of the squares become bigger and the distance between them do not increase by stating "*The space here is bigger, but since the shapes are growing, it [distance between* *shapes] also shrinks.*" He had difficulty in understanding the proportional relation between shapes including spaces between them.

Regarding the relation between lengths of shapes and distance between shapes in an artwork (figure 77a), students mostly identified the relation between equal lengths of a shape. For example, all of them identified the lengths of sides as equal in the artwork with four squares. However, in another artwork (figure 77c), they did not express the equal lengths of the shapes. They only detected this relation in the lengths of square that are hidden in the artwork. In the artwork with symmetrical configuration and hidden pentagons (figure 77b), two students (Ali and Melek) reflected the proportional relation between equal sides. For example, while drawing the second shape, he observed his drawing and tried to make all lengths of the shape equal, stating as "*I can't adjust the proportion. If I make it long, it becomes short, if I make it short, it is long. I couldn't do anything exactly*". During this process, as he revised the sizes of the lengths, he had difficulty in adapting this change to the angle of mouth shape since he aimed to make them equal. In the critiquing part, he expressed the relation between sides of a shape similar to a mouth by evaluating his friend's drawing (figure 25b) as stating "*The mouths of both of them are equal to each other and at equal distance [from the center of the mouth in the figure 77b]*"

When the line segments of shapes are not identical, they mostly used additive comparison. For example, students drew the shapes on the basis of the size differences between shapes by using the words of "more" or "less". For example, Ali expressed how he determined the distances between shapes and the frame. He compared the distance on the right and left. Since the distance between shape and the left side of the frame is bigger in the original painting, he drew it bigger in the larger drawing too, as stating "*for example, there was less space on the left compared to the right (figure 77c)"*. Regarding the size differences between shapes, Melek and Fatma drew lines and imagine shapes between them to determine how much difference there is between two squares in the first artwork in figure 66a. For example, Fatma stated as following:

Researcher	What did you pay attention to when you made it?
Fatma	After drawing the biggest one, I determined the size of the smaller square [*by comparing with the biggest square*], then I drew a straight line [*to compare them*]. I drew its on of the sides after that. There was space into which a square could fill. Then I left a space including a square. Then I drew line again to compare it with the smaller than it. Half of it was here. There was a spaced into which a square or rectangle fit again. I did at this way.

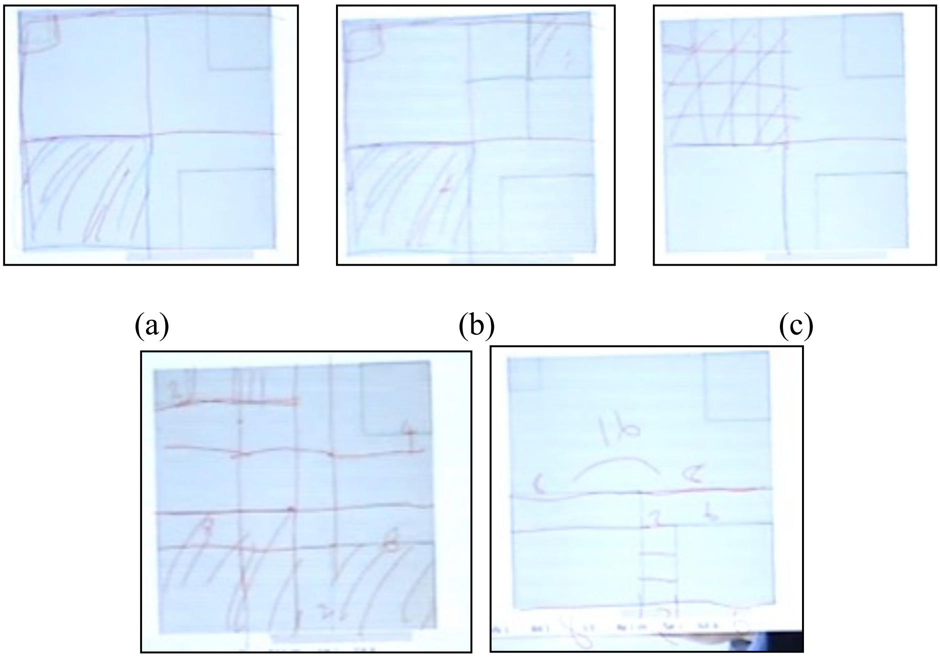
While students encoded the lengths relations between shapes by identifying visual difference between them, two students (Melek and Fatma) reflected also some relations multiplicatively during creating art work and critiquing parts. For example, Fatma detected a proportional relation between two line segments of squares as stating "*Half of it was here [compares sides of the squares]*" during interview after studio work 3. In fact, the ratio between the lengths of these squares is 2:3. She realized another proportional relation between two line segments in last artwork (figure 77d), taking note of "*I drew the half line near the line*".

Melek also reflected the proportional relation between lengths of squares in the first art work (figure 77a). She used similar strategy in identification of proportional relations through realizing the multiplicative relation between sides of squares as in the artwork with nested squares. During critiquing artwork [studio work 3] Melek suggested a strategy that is related with proportional relation between shapes] when one of the students (Fatma) reflected that she had difficulty in coordination of sizes of squares and researcher asked to students how they can solve the problems in the Fatma's drawing. However, she did not reflect about the proportional relation between spaces and squares. She identified the relation between squares by comparing each square with the smallest square.

Researcher	Did you have any difficulty in copying this artwork? [figure 77a]
Fatma	Teacher, their sizes. I drew the first square here, here are the others.
Researcher	You made it a little smaller. Then, how did you realize you

	were doing small?
Fatma	I would draw 4 times the size of my teacher. When I drew it that way and it was coming to that part of the paper.
Researcher	So how can we solve this problem? [*Fatma drew smaller than it should be*]
Emre	I think we cannot fix it.
Melek	If we use something like this, this is 1x and this is 2x, which is 3x 4x…For example, if we take the edge of it as 2, this is 4 times.
Researcher	Did you notice the relationship between them, Melek said something nice.
Emre	It's growing.
Researcher	How?
Emre	2 4 6
Researcher	And what will be the last?
Melek	This will be 4 times bigger: 8.

When the students did not reflect about the relation between squares and the the frame of the artwork with a square shape (part-whole relation), the researcher asked some questions to elicit students' thinking about how they perceive proportional relationship between squares even though they had not noticed during their drawing process. She asked four questions: how the biggest square and the frame related proportionally (figure 78a), how the biggest square and the second sized square is related to each other proportionally (figure 78b), how the biggest square and the smallest square is related to each other proportionally (figure 78c), how the biggest square and the third sized square is related to each other (figure 78d). Students thought the relation between squares in terms of areas rather than their lengths. They divided squares into unit of squares. However, they had difficulty in structuring spatially two squares in case that the ratio of corresponding sides of squares is not an integer. For the first three questions, they proposed a strategy to find proportional relation between shapes by using unit squares. They expressed it though factions of 1/4, ¼, and 1/16 respectively. They investigated proportional relationship between areas of squares in terms of unit square (the smallest square).



(a) (b) (c)

(d)

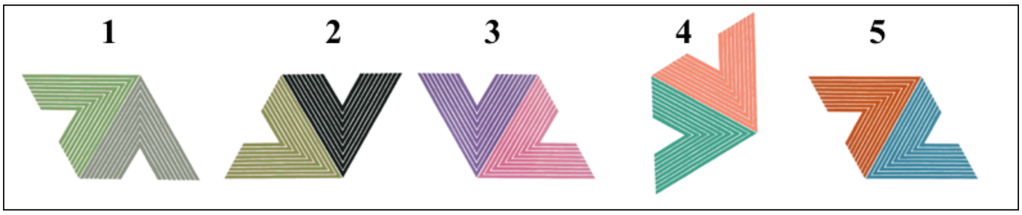
Figure 78. Students' identification of part-whole and part-part relation between squares

Lastly, the researcher asked how the third square is related with the biggest square. They had difficulty in adapting their ideas in the previous tasks. They thought the relationship as the comparison of lengths of the square proportionally as at the beginning as stating "*8 divided by 6*" (Emre) rather than comparing areas of squares. After a while, Emre attempted to express relationship in terms of comparison of areas proportionally. However, he had stuck with how to place the the smallest square within the third square. Then his friend tried to do it. Similarly, he could not fill the gaps with unit of squares (figure 78d). It seems that when the ratio of corresponding sides of squares is not an integer, they had difficulty in structuring spatially two squares and changed their strategy of comparison of areas to comparison of lengths.

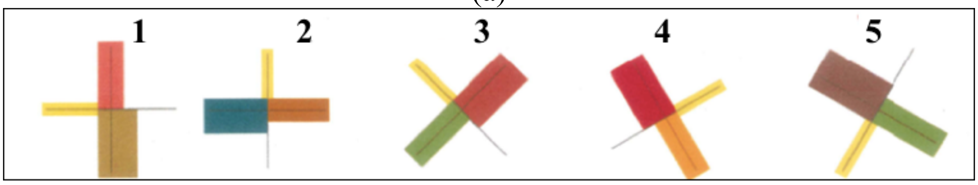
In summary, when students were asked to observe artworks with growing patterns, students attempted to identify the proportional relationships in based on recognizing shapes' properties (four equal lengths of squares) and structuring a shape into units. On the basis of these strategies, they expressed proportional relationships numerically based on additive and multiplicative comparisons. In fact, they mostly tended to identify proportional relations based on additive comparison and expressed it in a multiplicative structure. Another important finding was that when students were given a scaling factor to copy artworks, students similarly compared lengths of shapes both additively and multiplicatively when they were already given a scaling factor (1:4) to transform the sizes of four artworks during creating artwork part. They compared the lengths of shapes multiplicatively or imagined the shape expanding when some students were looking for relations between the original artwork and their sketch in the larger paper and the relations between lengths of shapes in the original artwork. When they did not realize any proportional relationships between lengths of shapes, they considered relative difference between sizes of lengths. When the researcher asked them to think about proportional relations between lengths of squares, then they compared the size of squares multiplicatively even though they did not use it during copying artworks by structuring them into units, similar with the first finding. During this process, they also considered other geometrical cues such as encoding angular relations between line segments, identifying geometric shapes and preserving their properties (see for detailed information in the first part of results), geometrical configuration of shapes, relative positions between shapes. This is not focus of this study.

4.4.2 Transforming Shapes Rigidly: Rotation and Flip

These parts involve students' identification of rigid transformations that preserves objects' shapes and sizes. In this study rotation (turn) and flip (reflection) among the rigid transformations were mostly observed. In this study, the artworks in figure 79 were used to elicit students' identification of rigid transformations. The analysis of students' identification of rigid transformations indicated that there are three main findings: comparison of shapes through mental rotation and flip, identification of congruence between shapes, and identification of angle, center and direction of rotation.



(a)



(b)

Figure 79. (a) Artworks of Frank Stella (b) Artworks of Robert Mangold

4.4.2.1 Comparison of Shapes through Mental Rotation and Flip

It involves students' thinking processes to decide whether shapes are identical or not through mental rotation and flip. The first finding was that some students identified the identical artworks subject to rotation in both asymmetric and symmetric series of artworks whereas some of them could not identify them at first glance (during individual observation). Students' thinking processes were differentiated at different artwork series. Even though students seemed to rotate the artworks to decide whether they are identical or not, it is not easy to claim that they rotated shape. In fact, this process could be more complex than it seems. They also thought flip or combination of rotation and flip.

First of all, students identified similarities and differences between artworks when they were asked to observed the artworks individually in studio work 2. Three of the students (Melek, Ali and Burcu) realized the difference between the first four paintings and the last painting (figure 79a). They decided that regarding the first four paintings, two triangles in each painting have a same direction whereas the last paintings have two triangles with different directions. For example, Burcu took note of "*They all look alike. We can say they are only rotated versions of the first artwork. But, the fifth artwork seemed to be different since its first region looks different from the second region in terms of its direction*" (figure 80).

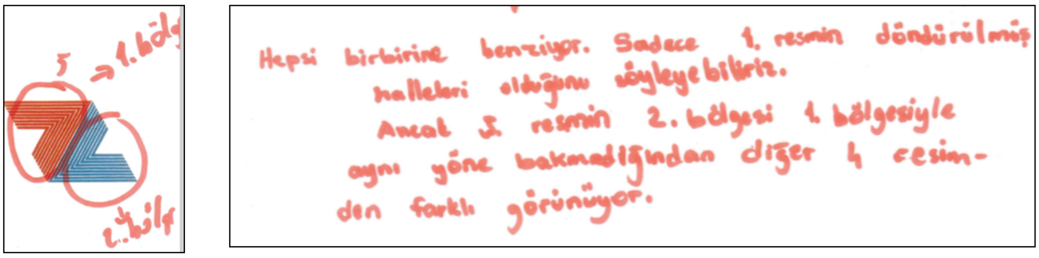


Figure 80. Burcu's identification of differences between shapes

Similarly, during group observation, Melek expressed her ideas on differences between shapes verbally. When researcher asked her what makes you say that. She supported her thinking process as stating "*All of the triangles are looking down and looking in the same direction. One of its triangles [looks at] one direction; the other [looks at] another direction*." (figure 79a). She described the difference between the paintings in terms of differences in direction of two triangles in each painting.

On the other hand, three of the students (Fatma, Esra, and Emre) did not explicitly state the difference between the first four paintings and the last painting (figure 79a). They described the difference on the basis of their different directions. For example, Emre took note of "*They all have the same shape and letters [refers to A and V letters], only the directions and shapes are different. One of them looks down,* *one [look] up, one [looks] right, one [looks] left.*" (figure 81). It seems that he considered only triangles or letters as units and each artwork has same shapes whose directions are different.

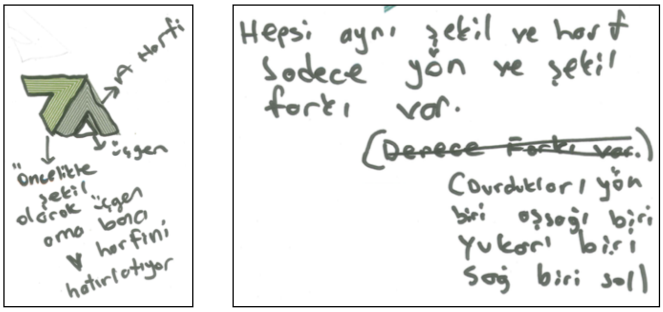
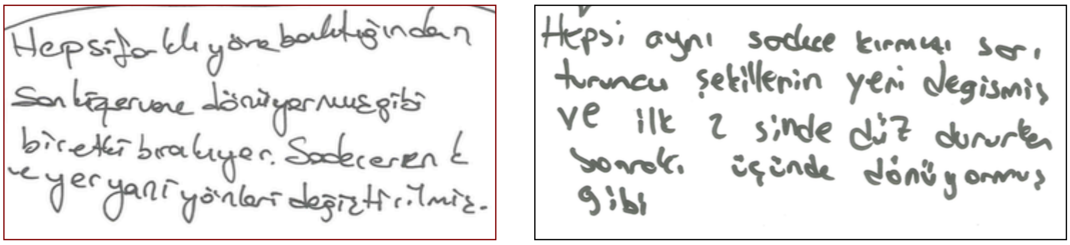


Figure 81. Emre's identification of similarities and differences between artworks

Similarly, in the artwork series created by Robert Mangold, almost all students identified them as rotated shapes. However, they did not realize that all of the artworks are not the same. In fact, the third and fifth paintings were different from each other. It seems that they did not trace where each part of the shape place when they are rotated. For example, Fatma and Emre identified the artworks as identical. The difference between them is the direction of the shapes. While Fatma perceived them as a propeller that has a turn effect as stating "*They all look at different directions, as if the propeller is turning. Only the color and location, i.e. directions, have been changed*" (figure 82a), Emre perceived the rotation of a shape as being oblique rather than being horizontal and vertical as stating "*All are the same, red, yellow and orange shapes just changed their positions and the first two [artworks] stands straight while next three [artworks] as if turning.*" (Figure 82b).



(a) (b)

Figure 82. Fatma’s (a) and Emre’s (b) notes regarding identification of similarities and differences between artworks

It was only Melek who imagined that the third and the fifth paintings are the same as seen her note of “*They become the same when we correct [their positions as vertically]*” (figure 83). It seems that she imagined to make them vertical and compared some specific parts of the artworks to decide whether they are identical or not. She also classified first, second, and the fourth paintings as identical shapes.

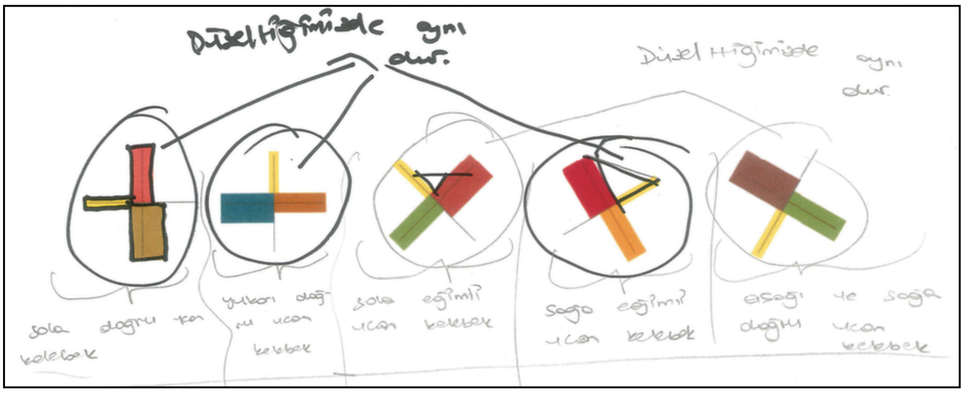
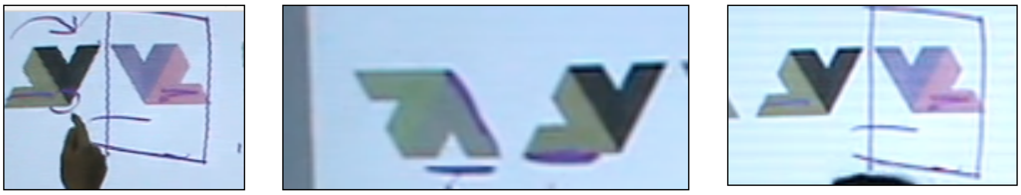


Figure 83. Melek’s identification of similarities and differences between artworks

Secondly, even though students seemed to have rotated the artworks to decide whether they are identical or not during individual observation, students also reflected transformation of flipping (reflection) during group observation and during creating artwork. In other words, they imagined transformations of shapes by either rotating or flipping over, imagining their mirror images as a flip. This finding describes how their thinking on transformation evolved after dividual observation when the researcher asked them to justify their ideas on rotation or changing the orientations of shapes.



(a) (b) (c)

Figure 84. (a) Ali's description of rotation, (b) Burcu's description of rotation, (c) Fatma's description of reflection and Esra's description of flip

Students envisioned either artworks (figure 79a) are flipped over to match it with third painting (Esra), or its rotation (Melek and Burcu), or its mirror images (Fatma) without referring to the word of flip (figure 84). They used the word of *turning* as rotation and flip. They did not relate the word of flip with reflection. It was only Ali who recognized the combinations of different transformations. The following conversation between students explained how Esra and Emre imagine the flip of an object. While Emre imagined the flip as flipping front to back, Esra described it as vertical flip (left goes to right).

Researcher	Now, you've took note of their directions are different. How are they different?
Esra	It's flipped.
Researcher	Which one?
Esra	How could I say ...it is flipped over like this [*uses her hands*]
Researcher	Which one? 2 and 3 [*second and third artworks in figure 79a*]
Esra	Yeah, yeah, yeah, it's turned over, it's flipped
Researcher	Well, what else can be apart from flipping?
Esra	It can not be a rotation. Can't we say that flipping is 90 degrees

rotation?

Emre	No.

Researcher	Do you know any difference between flip and rotation?

Esra	Flipping is turning over by changing its direction, and rotation is just movement of shapes

Emre	You're being ridiculous. Flipping is that bottom comes to top [*uses his hands*]

While these students imagined the flip of artworks to decide whether they are identical or not, another student (Burcu) imagined the rotation by imagining movement of the first painting to the second painting, and movement of the second painting to third painting. She explained the rotation with the gesture of turning hand. To support her reasoning, she mapped the relation between the parts of two artworks as stating "*Teacher, I think it's done like this: [moving her hands as if they are rotating] this part comes to the ground and naturally that object comes to the here*." (Figure 84b)

In addition to transformations of flipping and rotating, a student (Fatma) claimed that the second painting is reflected over vertical line to make it identical with the third painting, like mirror reflection (figure 84c). Despite of different ideas on transformations of artworks, Esra still insisted on the idea of flipping. It seems that Fatma and Esra did not realize that a reflection is a kind of flip over vertical line.

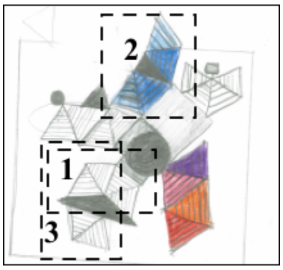
(a) (b) (c)

Figure 85. Students' identification of transformations of shapes

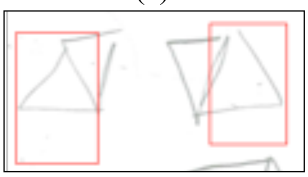
Fatma Can I say something, as if it were a mirror like this, this part of the mirror is in the opposite side?
Esra I think it is flipped
Emre They are obsessed with "flipped". There is no rotation there.
Researcher Fatma says there may be reflection. Do you agree with her?
Emre No. There is rotation there. Green one is rotated.
Fatma When it is reflected, it crosses this side [*figure 85a*]
Researcher Let us listen Emre. You say it's rotated right? [*2nd and 3rd paintings]*
Fatma Teacher it is already reflection in the mirror here, this is coming here [figure 85a]
Emre One minute, teacher. No! it is flipped.
Researcher How does it flip?
Emre Yes, my teacher. I am totally sure, flipped
Melek This is how it looks at this way, if it comes here [*points at the corner*], [*decides according to where the arrows are pointing through rotation in figure 85b*]
Researcher Do you mean they are rotated?
Melek Yes
Researcher What about this one? [*2nd & 3rd paintings*]
Fatma Reflection
Emre Now we think of it as flipped, something like this is happening [*uses hand gestures*]. It [*point of the arrow*] come to the top [*flips the first shape in the figure 85c*], then we turn it come to the bottom. Then, we turn pink comes here and the other one is here [*talks about one flip and two rotations in figure 85c*]
Ali Here both of them are happening, if it is flipped to the side [*vertically to right side*], it becomes like this one [*pink colored shape*]. If we turn like this [*uses hand gestures for rotation*], it becomes like this one again. This part comes to the ground [*matches one side of the first shape with the its corresponding position in the second shape in figure 85c*]

It seems that Emre was hesitant whether it is a rotation or a flip. He thought it was a rotation. Then he changed his idea and thought it was a combined transformation of flip and rotation. After all student reflected about their thinking process, Ali realized that both strategy could be used in explaining the transformation of the second painting during comparison of two artworks. It seems that student have different envisioning process regarding transformations of paintings.

Students' different thinking processes regarding transformations of shapes were also observed during creating artwork in studio work 3. Even though the researcher emphasized to imagine rotations of artworks, they used transformation of flip (reflection). In this studio work, the researcher asked them to choose one of artworks of Frank Stella, think as if it is just beginning of this artwork, and continue to it by rotating it. While Melek and Fatma attempted to imagine both rotation and flips of shapes, Esra insisted on imagining the flip of shapes. Other students (Ali, Emre, Burcu) attempted to imagine rotations of shapes. For example, during one to one conversation between the researcher and Melek, she explained how she used both strategies. She flipped paired triangles [number 2 in the figure 86a] and rotated paired triangles [number 1 in the figure 86a] in her artwork. During flipping, it seems the she flipped each triangle over a vertical line and drew them by changing the place of the second triangle as stating "*now I flipped this triangle, when it is flipped, it comes to here*" [from left to right in figure 86b].



(a)



(b)

Figure 86. Melek's identification of rotation and flip in her artwork during studio work 1

On the other hand, in another part of her artwork, she rotated paired triangle and drew its image even though she did not identify a center of rotation. It was observed when she used her fingers with a rotation movement to image how it is rotated. When the researcher also asked her to identify the transformation in another part of her art work, she identified it as a reflection [number 3 in figure 86a]

In summary, students compared shapes to determine whether they are identical or not through rotation and flip (reflection). During individual observation of artworks, students described shapes as rotated. During group and creating artwork process, their thinking process was elicited how they imagined the rotation of shapes. Students' thinking processes were different from each other. While some students imagined the rotation in plane, some students imagined depth in rotation (flip). A few students also realized the similar results of different transformations or imagined combinations of different transformations to compare geometric shapes to decide whether they are same or different.

4.4.2.2. Identification of Congruence between Shapes

It refers to mapping the relation between shapes and their rotated images. Mapping relation between shapes involves identification of congruent line segments and angles between shapes and their directions. The analysis of students' identification of congruence between shapes indicated that students mapped the relation between identical artworks by considering visual aspects of shapes such as line segments of shapes (Fatma, Melek, Burcu, Ali), especially irregular ones (zigzag), corners (Fatma) and their directions (Melek, Ali, and Burcu). For example, Fatma showed the congruent line segments of an artwork and its rotated image during group observation of artworks as stating "*Teacher this is already its reflection, it comes to here [from left to right]".* She showed the identical line segments by tracing them [figure 87].

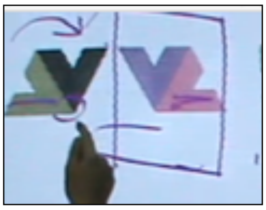


Figure 87. Fatma's matching of line segments in artworks.

Interview after studio work 2 revealed consistent finding regarding how she identified corresponding line segments in rotated image of the artwork while describing its rotation. In addition to line segment, she mapped corners of triangles.

Researcher	You meant rotated and displaced shapes in your notes, what do you mean by displacement?
Fatma	So we have a shape. We firstly lay it to the right, i.e. we lay this side of the shape to the right [*figure 88a*]; then we lay this side to here [*figure 88b*]; then lift it up here [*figure 88c*]; here we lay it and change their places, here while one of the look at this direction, that one looks other direction [*figure 88d*].

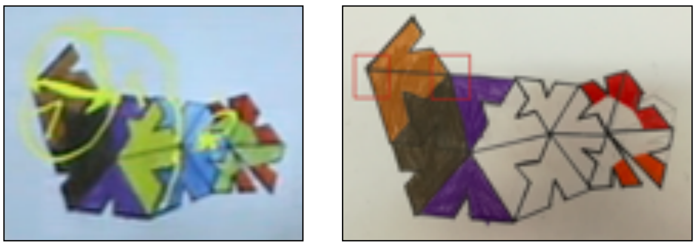
(a) (b) (c) (d) (e)

Figure 88. Fatma's matching of sides and corners of shapes with their rotated images

She mapped the relation between corners as well as edges (figure 88e). When the researcher asked how she rotated the third artwork to make the fourth artwork. She identified the direction of rotation. She matched the corners of whole art works. It seemed that she perceived it as a whole rather than decomposing it into two triangles. When the researcher asked her to explain how the second artwork is turned. She explained its transformation as flipping by using hand gesture. At the same time, she described this transformation with the consideration of rotation.

Researcher	Well, here's how you're bringing it here [*3rd to 4th painting*]
Fatma	So we turn like this [*uses hand by turning it to the right*]. Now I'm holding this corner [*red colored*] and the corner is coming here, this corner comes here [*purple colored*], and other comes here [*blue colored*]

During critiquing process, she reflected how she related identical corners of shapes rather than considering matching line segments of artworks. She identified identical paired triangles. Then the teacher asked her who she rotated one of the paired triangles. She explained that she just considered relating one of their corners and she did not rotate the line segments of each shape, as stating "*I just rotated this triangles [light brown colored pair of triangles]. When I rotated it [to right-down], it came over here, so, I did not pay attention to rotate the edges too. When we rotate the corner, it comes to the top*" (figure 89a and 89b)



(a) (b)

Figure 89. (a-b) Fatma's identification of identical corners in her artwork during critiquing part of studio work 2

In addition to edges and corners, some students (Melek, Ali, and Fatma) matched the directions of triangles based on irregular edges. For example, Ali identified the identical or non-identical artworks on the basis of direction of each triangle (figure

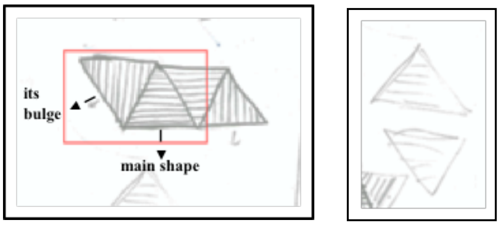
90). He showed the difference between two artworks by indicating the differences in directions of triangles as taking notes of "*The bottom parts of triangles points in the same direction*" (figure 90b) and "*The bottom parts of triangles looking at different directions*" (figure 90c).

(a) (b) (c)

Figure 90. Matching the directions of triangles based on irregular edges (Ali)

During critiquing part, Ali similarly explained how he constructed his artwork by identifying congruent line segments of each triangle by perceiving irregular ones as a knife. He imagined the movement of each segment and he stated the following during critiquing part: "*when we turn it [to the left], it comes to here [black colored small triangles], and this one comes here. It is like a knife and it is turning*" (figure 91).

Figure 91. Ali's identification of congruence on the basis of line segments of the triangles



(a) (b)

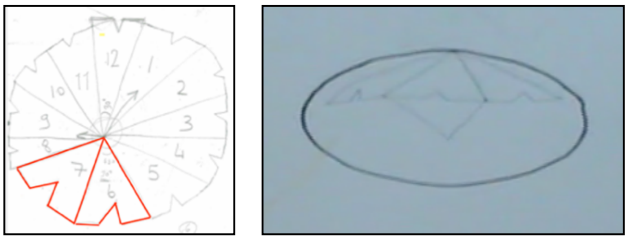
Figure 92. Melek's identification of congruence between a shape and its rotated image

In addition to irregular line segments of shapes, Melek also considered two identical shapes so that one of them is a main shape and the other one is its part (bulge). Even though there was not an irregular part of shape, she imagined one of triangles as an irregular part of paired triangle. It seems that she ignored one of them and considered its rotation in the second order.

Melek	Can we do like this, this is main shape and this is its full rotated shape [*figure 92a*]
Researcher	How do you rotate it?
Melek	I actually see this as the main figure and I see this as a bulge.
Researcher	You only perceive this triangle [*main shape*]. Well, how did you rotate it?
Melek	Actually the bulge is on the left. When I rotated the paper and did it, I added another triangle on the left side [*figure 92b*]
Researcher	So you turn this triangle, how to turn it?
Melek	In the opposite direction [*figure 92b*], we hold the point [*the top vertex of the triangle*], and we rotate [*it around this point*]; I'm holding it from here … this is how it is seen when we rotate, and I thought it [*bulge*] is the left side of the main shape again.

She firstly drew rotated images of the main shape by turning the sketchbook 180 degrees and then added the other triangle next to it. To decide the location of other triangle she identified it stands on the left of the main shape. Then she assumed that its rotated image should also be on the lefts of the rotated image of the main triangle. Then, she combined two double triangles. She also considered the orientations of lines inside the triangles to make them identical.

While they considered identical parts of shapes, one of the students (Emre) seemed to identify congruence in rotation of triangles that he drew on the paper and rotation of triangles in his mind or in his sketch (figure 93a). Even though he did not reflect about mapping particular parts of triangles, he appeared to identify the rotation of triangles holistically because he erased what he drew as rotated images so many times. In each try, he realized that something went wrong. He aligned the bases of each triangle and joined their top vertices at a point on the circle, which he drew on the basis of his friend's suggestion (figure 93b). It seemed that he realized it was not rotated image of the first shape or it was not the same as the image in his mind.



(a) (b)

Figure 93. Emre's representation of rotation of triangles (a) in his sketch and (b) in the final artwork

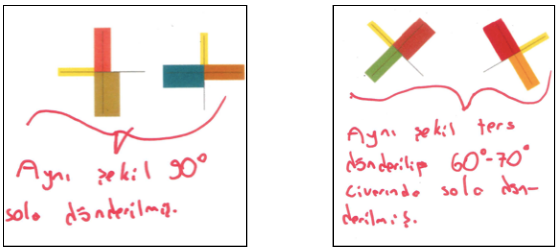
In summary, students mapped the relation between identical artworks by considering visual aspects of shapes such as line segments of shapes (Fatma, Melek, Burcu, Ali), especially irregular ones (zigzag), corners (Fatma) and their directions (Melek, Ali, and Burcu), or holistic visual image of triangle (Emre).

Student focused on different parts of shapes to decide their congruence at different situations such as observing artworks and creating artworks.

4.4.2.3. Identification of Angle, Center, and Direction of Rotation

The analysis of students' identification of angle, center and direction of rotation indicated that students mostly identified the direction of rotation, rather than center of rotation and angle of rotation. When the researcher asked them to think about amount of rotation during group observation of artworks in studio work 2, they mostly identified the amount of rotation by making use of benchmarks such as 45, 90, and 180 degrees without using a measurement tool.

It was only Ali who identified the angles of rotation without a measurement tool during individual observation in which the researcher did not particularly asked to think about the amount of rotation. It seems that he considered the angle between two rectangles (90 degrees) and he used it as a benchmark to estimate the angle of rotation (figure 94a). He envisioned to flip the third painting along the axis of green and brown rectangles and rotated it mentally to compare with the fourth painting. Thus, he realized the combination of transformations in a painting to create the other painting (figure 94b). Since the slope of the line segment involving green and brown rectangles in the third painting is smaller than the line segment involving yellow rectangle in the fourth painting, the amount of rotation should be smaller than 90 degrees.

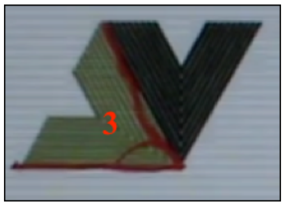


(a) (b)

Figure 94. Ali's identification of angle of rotation by making use of benchmark angle

Another identification of angle of rotation was observed during group observation of artworks in studio work 2 when the researcher asked them to think about how much the green triangle should have been rotated to the location of black triangle. Students estimated it by using benchmark angles such as 45, 90, and 180 degrees (figure 96). However, they had difficulty in showing the angle of rotation. While Burcu show the angle of rotation as stating "*it has been turned up to here*", referring to number 1 in the figure 95a, Melek showed it as stating "*it is turned like this*", referring to number 2 in figure 95a. They seemed to imagine the distance between the starting and point during rotation. When the researcher emphasized the rotation of the first triangle (green triangle) to make it identical with black triangle in terms of their directions, Melek and Emre showed the angle, referring to number 3 in figure 95b, by describing the rotation with hand gesture. The researcher explained what students are asked to thinking about a couple of times since students had difficulty in understanding the task.

(a)



(b)

Figure 95. Students identification of angle of rotation visually

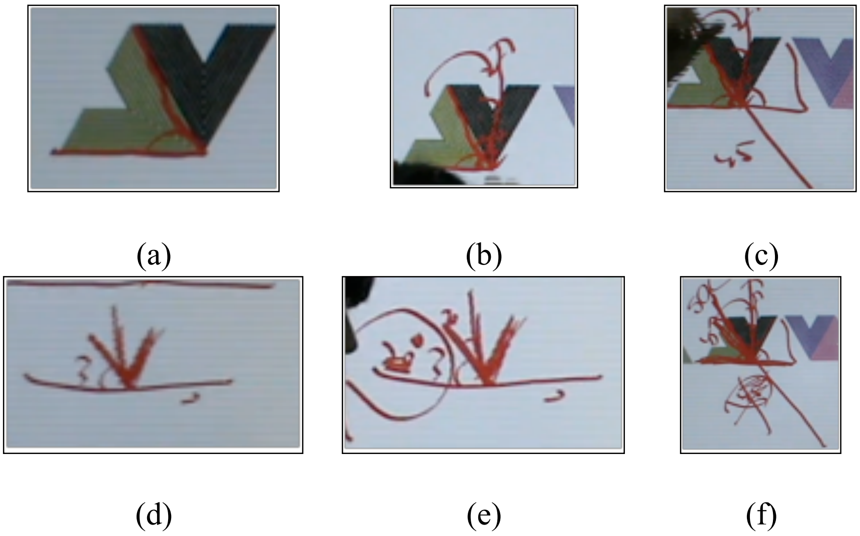


Figure 96. Students' identification of angle of rotation with making use of benchmark angles

Researcher	What do you think how much we are turning the green triangle?
Emre	Look at my teacher, we're going to have to do it [*green triangle*] to make like this [*black triangle*]; so here it turns 45 from here [*ground in the figure 96a*].
Researcher	45 degrees?
Esra	30 I think.
Researcher	Come to the smart board. How much do you think?
Ali	45! 45!
Emre	Teacher, the angle you want is this [*figure 96a*]. One minute, I'll tell you now. To make it congruent with angle of black, it should have been turned to this direction [*shows rotation of black triangle to the right with his head movement*], so, it is as much as the angle of black triangle.
Researcher	We are trying to bring the green to where the black is.
Emre	Exactly the same.
Researcher	How will it be? How many degrees will be?
Emre	One minute, my teacher, isn't it 90? [*figure 96b*] this is 45 [*figure 96c*].

In order to identify the angle of rotation, he made use of 90 degrees as a benchmark and drew three arrays to see the relationship between angles. It seems that he perceived it as 45 degrees since it is in the middle of two arrays. When the researcher asked students to decide whether it is located in the middle of 90 degrees or not, they considered the distances between arrays.

Researcher	Is that line exactly in the middle of 90 degrees?
Esra	Not. This is small and this is big here [*figure 96c*]
Emre	I say 45.
Burcu	Can I say, my teacher, it's going to be 180 degrees, and it is 90 degrees, and what is this? [*refers to question mark in her drawing in figure 96d*]
Researcher	Could you imagine it? [*asks other students*]
Esra	I imagined teacher.
Burcu	I think it's probably about 20 or 25. It is about 70 degrees when we subtract 20 from 90 [*figure 96e*].
Researcher	You said 45 Emre, what do you think?
Emre	Friends, we have already got degrees of 45, and 90 degrees there. To complete it to 90, ok let it be 30 degrees, get 60 or 45 [*subtracts 30 or 45 from 90*], but it can not be 70-80.
Researcher	Well, what do you think it is greater than or less than 45.
Esra	It will be greater than 45. I think if here is 90, its half is there. It's something like 45, since it is its on the right side, it will be tiny little less from 45 [*figure 96e*]
Researcher	Okay, how many degrees do you think?
Esra	50, it would be 50 degrees, I think, my last decision.

Esra claimed that it should be larger than 45 degrees since she realized the area between two arrays is greater than the others. She showed the 45 degrees and compared it with the angle of rotation. Since there is a small difference between, she though it should be 50 degrees. Similarly, Burcu represented angle relations with the consideration of 90 degrees and 180 degrees. She claimed that it should be almost 70 degrees on the basis of her visual representation of angles. However, Emre thought that it can not be as great as 70 degrees since it should be closer to 45 degrees.

This conversation indicated that students identifies the angle of rotation based on the visual images of benchmark angles of 45, 90 and 180 degrees and they compared visual appearances of angles on the basis of these images.

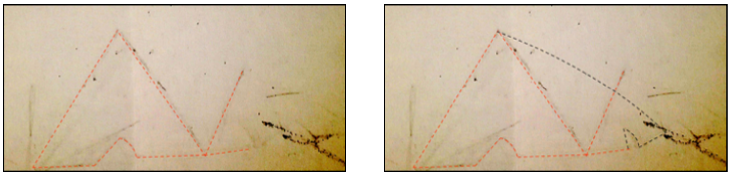
Similar process was observed during creating art work part. However, it was hard for students to identify angle of rotation since they were not given the rotated images of shapes during creating artwork in which they had to draw a shape and it rotated images. Students tended to draw rotated images of the artwork on the basis of changing their directions rather than determining a center of rotation, angle of rotation, and flip line (see Fatma and Esra's sketches in figure 97a). Their sketches were like free hand sketches. For example, Esra constructed her artwork by imagining flip of shapes rather than rotation. During this process, she did not identify any flip line. When the researcher asked her how she flipped the figure, she described the flip by rotating it diagonally by using paper triangle, by stating "*So when flipped, this place has to go up, and I said that one also should go down, but it didn't happen*" It seems she identified the change in the direction of triangles. However, she tried to flip the paper triangle without specifying a line of flip. Rather, she randomly flipped it. She had difficulty in drawing the flipped image of the artwork. Then she got help of her friend. She suggested turning the paper and drawing its flipped image. However, Esra drew the same shape again as stating "*I am turning but, it becomes the same*" (figure 97b) since she did not think about the change in direction of triangles.

(a) (b)

Figure 97. Students' free hand sketches without taking into consideration of angles of rotation and line of flip: (a) Fatma's drawing of rotated images of paired triangle (b) Esra' drawing the same two Z shapes that she claimed they are turned

Similarly, Emre focused on the change in the direction of a triangle when he rotated paper triangle, as stating "*I'm going to change the direction now, teacher, I'm going to bring this down*". In fact, the researcher encouraged them to imagine it without using concrete material and remined they could use in case they had diffciulty. He put paper triangle on the original artwork and matched with one of the triangles. Then, he rotated the paper and translated it to the next of two triangles in the original painting. Then he drew it as a rotated image of the first triangle in the skecthbook. He did not identify the center of rotation. He just changed the direction of the triangle. He did not turn the two triangles as a whole. When the researcher asked him to think about, what would happen if he made a point as constant to rotate it. He explored the rotation of a triangle with the paper with exploring rotation of the triangles around different points. After a few tries, he realized the pattern of rotation that a pyramid perceived from top view is formed by rotation of triangles (figure 98a). He started with drawing of two triangles at first. Then, he continued to drawing by adding one triangle. While drawing the rotated images of the triangles, he rotated the sketchbook and did not use any measurement tool.

(a)



(b)

Figure 98. (a) Emre’s first skecth of a clock (b) second sketch to make final artwork

When the researcher asked him to draw it to the larger paper as a final version of his artwork, he tried to use the protractor. However, during this process, he did not look at back and forth from the sketch (figure 98a) to the larger paper on which he made final artwork (figure 98b). It was very had for him to draw each triangle and their rotated images with the consideration of their angles and sizes. He could not draw it at first try. He asked for his friend help to draw an equilateral triangle and tried to draw a triangle. While drawing rotated image of the first triangle, he held the protractor as he used to draw the first triangle. Uncoordinated drawing of triangles indicated that he did not think about the amount of rotation around a point. After a few try, he became disappointed and did not want to continue it as stating “*How do I find 60 degrees? I didn't get 60 degrees. I could not fit six triangles [into circle] I just could not. I got nervous*.”

The next day, he decided to draw triangle with 30 degrees to fill 12 triangles into a circle, representing a clock. However, it seemed that he still had difficulty in identifying angle of rotation even though he imagined movement of rotation in his first sketch (figure 99a). After the researcher demonstrated how to draw an isosceles triangle with 30 degrees on the circle, he started to draw a a triangle with 30 degrees angle. He realized that the direction of the triangle is wrong (figure 99a). Then he measured the angle clockwise and draw it again (figure 99b). It seems that he wanted to draw the triangle that is placed ath the top of the clock. Then he deleted it too. It seems that it did not match with the visual image in his mind. Then, he changed the size of sides and drew it again (figure 99c.). He must have been realized that it did not look like a symmetrical triangle vertically. However, he did not realize angle of 30 degrees was not angle of rotation in last triangle. At the end, the researche helped him to draw the first two triangles again and continue it to make a clock.

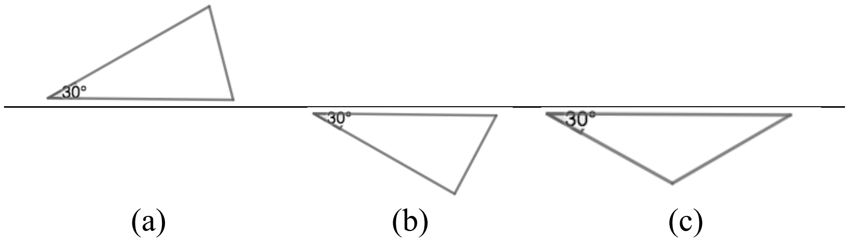


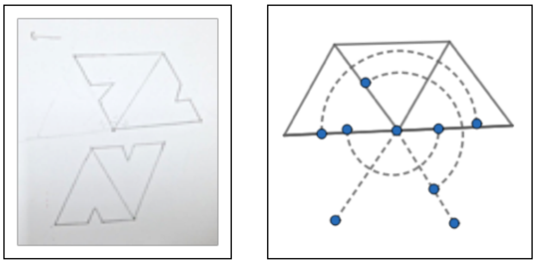
Figure 99. Emre's sketches of one the isosceles triangles in a circle, each base angle is 30 degrees, illustrated by researcher

Even though he was encouraged to identify center and angle of rotation during creating artwork, he appeared to combine shapes to picture a clock without identifying a point o rotation during critiquing part. After the researcher asked him to remember where is the point of rotation that she demonstrated before, he reflected about making the center of rotation as constant and imagination of rotation of a triangle.

Researcher	How many degrees have you rotated around the center?
Emre	30
Teacher	30, 360/12 ohh
Emre	I could not adjust 30 degrees firstly. The teacher helped me. After she demonstrated a few of them, I continued to it.
Teacher	Well, you turned around this point?

Emre	No, they were spontaneously merged together. I drew each shape then they merged there.
Researcher	Where was the point of rotation?
Emre	My rotation point will remain constant, and I turn around it.

On the other hand, one of the students (Ali) determined a center of rotation. He identified the center of rotation and imagined the rotation of each part of triangles, stated as "*I've set this point. Then I rotated it to this direction [to the left*]." However, he did not draw the rotated image of the artwork so that it is attached to the artwork at his first try (figure 100a). In the second sketch, he understood angle of rotation informally by discovering interior angle of equilateral triangles and dividing protractor visually into three equal parts (figure 100b).



(a) (b)

Figure 100. (a) Ali's first sketch of rotation of paired triangles (b) Ali's identification of angle of rotation by dividing protractor visually into three equal parts visually, illustrated by the researcher

In summary, students identified the angle of rotation based on the visual images of benchmark angles of 45, 90 and 180 degrees and they compared visual appearances of angles on the basis of these images when they are already given artworks and their rotated images. When students are creating artworks in which they should imagine each of them and represent their rotated images on the paper, students focused on the change in the direction of triangles rather than identifying angle of rotation and point of rotation or flip line.

4.5. Summary of Findings

The aim of this study was to understand how students make use of visual-spatial thinking processes in an Math-Art Studio Environment that was designed under three main structures of studio: demonstration, students-at-work and critique part. Students made use of four visual-spatial thinking processes mainly: recognizing geometric shapes, decomposing and composing shapes, patterning, transforming shapes (figure 101). Each way of visual-spatial thinking is interrelated to each other. Analysis of artworks and creating artworks are complex process requiring coordination between different visual-spatial thinking processes. Even though they are related to each other, the specific examples regarding each thinking process was presented in this study. Table 13 also presents at what structure of studio environment they were observed. Main findings regarding each way of visual-spatial thinking was summarized in the next part.

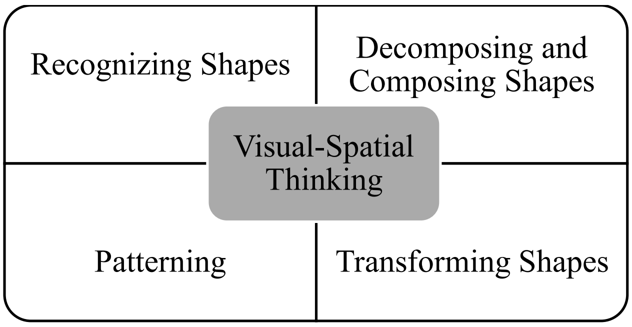


Figure 101. Students' major visual-spatial thinking processes in Math-Art Studio Environment

Table 13. *Students' Visual- Spatial Thinking Processes at Three Structures of Studio Environment*

				Demonstration		Students-at-Work	Critique	
				Individual Observ.	Group Observ.		Describe	Evaluate
Recognizing Shapes	Identifying shapes as real-world objects	Perceiving geometric shapes as real-world objects		x	x	x		
		Combining geometric shapes to make a 2D representation of real-world object				x	x	
	Identifying geometric shapes	Identifying shapes through disembedding & embedding	2D	x	x	x		
			3D	x	x	x	x	
		Identifying shapes by their properties	2D	x	x	x	x	x
			3D		x	x		
		Identifying shapes by changing orientation or viewpoint	2D	x		x		x
			3D	x	x	x	x	
Decomposing and composing shapes		Decomposing shapes		x	x	x		x
		Composing shapes				x	x	x
Patterning		Segmenting the pattern into units		x	x	x		
		Integrating units to make a pattern				x	x	
Transforming geometric shapes		Transforming shapes non-rigidly		x	x	x	x	x
		Comparing shapes through rigid transformations		x	x	x	x	
		Identifying congruence between shapes			x	x	x	x
		Identifying angle, center and direction of rotation			x	x	x	x

4.5.1 Recognizing Geometric Shapes

The analysis of students' processes of recognizing geometric shapes indicated that students made use of two major ways to identify the geometric shapes: Identification of the shapes as a real-life objects and identification of geometric shapes. Students' identification of geometric shapes as real-world objects involved perceiving geometric shapes as real-world object and making a two-dimensional representation of a real-world object with geometric shapes. Students identified geometrical shapes as real-life objects at different situations such as while they are looking at an art work, copying an art work or creating their own art work.

Students' identification of geometric shapes involved three categories: involves identification of shapes through disembedding and embedding, on the basis of their properties, from different orientations and point of views. Students mostly disembedded two-dimensional shapes rather than two-dimensional representation of three-dimensional geometric shapes during individual observation of artworks. Students identified two geometric shapes that are embedded into other shapes ranging from triangles to quadrilaterals during demonstration and creating artwork parts. When students observed artworks again and analysed artworks in a group, they saw new shapes such as two-dimensional representation of three-dimensional geometric shapes in the artworks. Studets also embedded two-and two-dimensional representations of 3D geometric shapes into each other during creating artwork part which gave clues about how they identify geometric shapes.

Students also identified two-dimensional geometric shapes from different orientations (e.g. rotated) or two-dimensional representation of three-dimensional shapes from different viewpoints (changing the perspective). Students identified some shapes differently when they were rotated during individual observation of artworks and focused on different properties of shapes when they are rotated during copying artwork. Students imagined the view and compared different views of a three-dimensional geometric shape that are represented in plane. During this process, students either imagined to change their position or changed the position of the shapes through rotation. Regarding considering shapes' properties, students focused on different properties of two-dimensional shapes to identify them during demonstration, copying artwork and creating artwork, and critiquing parts. These properties are length relations (size of lengths, number of sides, parallelism of line segments), angular relations (the amount of distance between two line segments or comparison of steepness of line segments), and symmetrical nature of shapes. Regarding considering properties of three-dimensional shape, students focused on number of edges of its base or number of side faces to identify shapes. Their identification depended on the transformation in the visual appearance and perspective of three-dimensional shapes.

4.5.2 Decomposing and Composing Shapes

The analysis of students' decomposition and composition of shapes indicated that they attempted to decompose the shape into smaller geometric shapes or into equal parts mostly during demonstration (individual and group observation of artworks) and copying artwork. They did not deliberately decompose the shapes before students were asked to find the shapes in the artwork during copying artwork process. Regarding composition of shapes, students attempted to combine units of shapes to make a shape without a gap mostly during creating artworks. They mostly combined shapes informally and by trial and error. Students' ways of imagination of rotation of shapes and its representation are among the crucial factors to compose shapes. At some cases, students decomposed the shapes after they created their own composite shapes, which indicate two ways of spatial thinking could be interrelated.

4.5.3 Patterning

The analysis of students' patterning indicated that students both identified the patterns in artworks during demonstration and critiquing parts. In addition to identification of patterns that were already represented in artwork, students also created patterns in their own art works. During these process, students identified the unit of pattern and attempted to find the rule of pattern. Also, they combined individual units with a pattern in creating art works such as rotating shapes in a particular way, increasing the sizes of shapes in predictable manner to make another shapes. However, their identification of patterns was mostly informal. They mostly considered additive relations regarding the lengths of shapes during patterning. They created both repeating and growing patterns during creating artworks.

4.5.4 Transforming Geometric Shapes

The analysis of students' transforming geometric shapes indicated that students transformed shapes non-rigidly by changing their size (scaling transformations) and rigidly by preserving their shape and properties other than directions.

Regarding scaling transformations, students encoded length relations on the basis of additive and multiplicative comparisons (proportional) based on recognizing shapes' properties (four equal lengths of squares) and structuring a shape into units when students are asked to analyse an artwork with growing pattern. When students are already given a scaling factor (1:4) in order to copy artworks, students compared lengths of shapes on the basis of additive and multiplicative comparisons. They compared the lengths of shapes multiplicatively or imagined the shape expanding when some students were looking for relations between the original artwork and their sketch in the larger paper and the relations between lengths of shapes in the original artwork. When they did not realize any proportional relationships between lengths of shapes, they considered relative difference between size of lengths. When the researcher asked them to think about proportional relation between lengths, they tried to relate the sizes or areas of shapes multiplicatively through structuring shapes into unit of squares. In addition to length relations, students also encoded other geometrical cues such as angular relations, recognizing shapes and properties, arrangement of shapes.

Regarding rigid transformations, students compared shapes through mental rotation and flip, identified congruence between shapes, and identified angle, center and direction of rotation. First of all, students attempted to determine whether they are same or different. Students mostly compared shapes on the basis of their directions. While some of them rotated shapes in place, some of them imagined to rotate shapes in depth (flip). Two of students also considered combination of different transformations and predicted the same result of different transformations. Student identified congruence between shapes with the consideration of several visual aspects of artworks. Students mapped the relations between a shape and its rotated regarding line segments of shapes (regular or irregular), corners, and directions of the shapes or parts of shapes by decomposing shapes. Regarding identification of angle, direction and center of rotation, students mostly identified the differences in direction of rotated shapes at first glance. When the researcher asked to identify center and angle of rotation, they attempted to identify them. However, they had difficulty in showing angle of rotation visually. When they were given a shape and its rotated image, students made use of benchmark angles of 45, 90 and 180 degrees and compared visual appearances of angles with the consideration of these benchmarks. When students were asked to create rotated image of a shape, they mostly focused on the change in direction of shape rather than angle of rotation. It was very hard for students to construct rotated images of a shape to make a coherent whole.

CHAPTER 5

CONCLUSION, DISCUSSION AND IMPLICATIONS

The aim of the study was to understand how students make use of visual-spatial thinking processes in a Math-Art Studio Environment based on Studio Thinking Framework that involves studio works with geometric-rich content. Findings regarding each process of visual-spatial thinking were discussed in the first part. Then, implications and suggestions for future studies were presented. Implications of the study are discussed under two issues: contribution to literature, contribution to educational settings. The important points to be taken into consideration were presented for future researchers at the end of the chapter.

5.1. Conclusion and Discussion of Findings

In this study, students reflected four major visual-spatial thinking processes in Math-Art Studio Environment in which students were encouraged to analyse artworks individually and in group, creating and copying artworks, and critiquing artworks with geometric shapes. They are recognizing geometric shapes, decomposing and composing shapes, patterning, transforming geometric shapes. In addition to major visual-spatial thinking processes, there are several sub-processes of visual-spatial thinking such as identifying shapes with their properties, relating geometric shapes with real-world objects, dis-embedding and embedding shapes, scaling transformations and proportional reasoning, mental rotation and perspective taking (identifying shapes from different view points). Findings of this study indicated that this environment has a potential to elicit different students' visual-spatial thinking processes that are interrelated to each other. This process is so complex that it requires the use of different thinking processes simultaneously.

5.1.1. Recognizing Geometric Shapes

The findings regarding recognizing geometric shapes indicated that students identified geometric shapes as relating them with real-world objects, identifying their properties, identifying them through disembedding and embedding, and identifying them from different directions or viewpoints. Findings regarding each way of recognition of shapes are discussed respectively.

The first major finding was that students identified geometric shapes as real-world objects. They identified them in two ways: perceiving a geometric shape or composite shapes as a real-world object and using them to make representations of real-world objects. Some students perceived real-world objects, especially when they did not identify hidden geometric shapes in an irregular shape that is formed with combination of more than one regular geometric shape. It seems that they related the image of real-world object in their minds with the irregular shape in the artwork. This finding implies that students might have perceived shapes on the basis of their appearance since students identified a shape so that it looks like a real-life object. The reason such a visual thinking could be that they did not attempt to decompose these irregular shapes into basic geometric shapes. In fact, it is important to note that these students could reflect different levels of thinking at different contexts (Burger & Shaughnessy, 1986) since they also attempted to identify geometric shapes in the artworks. On the other hand, some students also sketched a real-world object (e.g. a head, a bird, a clock) that consists of geometric shapes. This thinking process is important for especially visual artists and designers to understand and imagine basic structure of objects with the consideration of geometric shapes (Goldsmith et. al., 2016). Students’ picture making process could also be effective to encourage students to compose shapes and transform shapes’ orientation, and reflect on them (Clements, Sarama, & DiBiase, 2004).

The second major finding was that all students attempted to identify geometric shapes on the basis of their geometric properties in addition to identification of shapes as real-world objects during individual and group observation of artworks, creating artworks and critiquing parts. Students mostly focused on one aspect of shapes such as length relations (size of lengths and number of sides) to discriminate a shape from another. For example, a student determined a shape cannot be a square, thereby it must another shape with four sides, which is a quadrilateral. It seems that naming process and considering non-examples of a shape are critical factors in identifying geometric shapes (Tsamir et. al., 2008).

Students also attempted to identify properties of two-dimensional geometric shapes at different orientations. They identified triangles and perspective drawings of squares with different names and focused on their different properties when they are presented at different orientations. Students' identification of geometric shapes with different names could be related to prototypes of a shape in their mind (Tsamir, Tirosh, Levenson, 2008; Ubuz & Gökbulut, 2015; Ulusoy & Cakiroglu, 2017). For example, students named right-angle triangles at different directions differently. The reason such a differences could be that students used prototype image of right angle triangle based on its vertical and horizontal relations (Herzkowitz, 1989). Students might also have limited storage of shapes in their mind and identified shapes intuitively on the basis of visual prototype (Tsamir et. al, 2008). As seen in the students' identification of a square as a diamond or baklava in the current study, this prototype could be sometimes a real life representation of a shape (Ubuz & Gökbulut, 2015). Even though students identified a shape with different names at first glance, students realized they are the same during interviews in which students have time to think about attributes of the shape, which indicates students' might have decided on the basis of visual judgment at first glance and then analytic judgment (Tsamir et. al, 2008) and students might have difficulty in transforming shapes mentally to identify whether they are congruent or not. In addition to identification of a shape with different names, students also identified a shape (a square) with the same name even though the artwork involves its rotated image. However, the number of critical attributes to identify a shape changed when it stands on one of the corners. They also considered property of diagonals and symmetry in the shape, which suggest that the number of critical attributes is important to identify a geometric shape (Herzkowitz, 1989).

The third finding regarding recognizing shapes indicated that in addition to identification of two-dimensional shapes, student identified properties of three-dimensional shapes through identifying the number of edges of base or the number of side faces. Students had confusion about whether a two-dimensional representation of a shape is a pyramid or not. They discussed the number of edges of the base or number of faces as three and four, which is consistent with the findings of Ubuz and Gökbulut (2015) in which even primary school teachers defined the base of pyramids with a triangle or a square on the basis of visual prototypes such as Egypt pyramids. In the current study, students' visual-spatial thinking processes were elicited through analysis of artworks with non-prototypical examples of three-dimensional shapes such as two-dimensional representation of a truncated pyramid form top view. On the other hand, some students had confusion between a triangular prism and a triangular pyramid besides considering the number of edges of the base in a pyramid. They might have conceived them as three-dimensional shape with triangular face without identifying their critical attributes, which can be referred to an example of identification of non-critical attributes of shapes (Hershkowitz, 1989).

The fourth major finding regarding recognizing shapes was that student identified two-dimensional representations of three-dimensional shapes from different point of views. Regarding three-dimensional shapes, students identified geometric shapes on the basis of view of the shapes in the artwork (e.g. the pyramid seen from the top), compared different views of a shape, and also sketched different views of a shape. Students recognized geometric shapes on the bases of its visible faces. For example, if they see a two faces of a pyramid, they claimed that it cannot be a pyramid since it involves four faces. This level of thinking refers to the Level 1 (visibility of objects) in which kindergarten students are asked to only decide which objects or parts of objects are seen (van den Heuvel-Panhuizen, Elida, Robitzsch, 2015). Comparing and sketching shapes that are presented from particular point of view in the artwork with another views of the shape that are not given would be related to the level 2 (appearance of objects) in which students are asked to imagine a shape from a different point of view (Michelon & Zack, 2006). It implies that these levels could be interpreted for older students' perspective taking due to the different nature of tasks.

Unlike the previous studies, students in the current study were given one view of a shape in the artwork rather than giving several views of a shape and draw its missing view or whole shape on the basis of given views. Even though they were only given one view of shapes in the artwork, students attempted to imagine different views of a three-dimensional shape and represent it through sketching. During this process, students also envisioned how a cross-section of shapes is seen from different perspectives, as an evidence of relating two-dimensional and three-dimensional views (Newcombe and Shipley, 2015). While a student stuck to perspective drawing of a cross-section in a truncated pyramid, another student could imagine the change in its representation from different perspectives (Cohen & Hegarty, 2014). This might occur due to students' egocentrism (Piaget & Inhelder, 1967). Student who thinks from egocentric frame of reference could not image one object from different point of views; rather they perceive a shape only from their own view and have ideas on the basis of what they see.

In addition to identifying from which point of view shapes are seen, this study provided some evidences regarding how students identify the shapes' view. While a student identified it by imagining rotating of the given shape in the artwork, another student identified it by imagining changing own view perspective. It was inferred on the basis of their use of gestures and verbal explanations. This implies that students could use different mental strategies to identify shapes from different points of view at the small scale, referring to the terms of mental rotation and perspective taking abilities in spatial thinking studies (Hegarty & Waller, 2004).

The last major finding regarding recognizing shapes was related to disembedding and embedding shapes. Students attempted to disembed geometric shapes in the artworks. This kind of disembedding is somewhat different from the previous studies in which students are asked to find simple shapes in a complex configuration in embedded figures tests (Ghent, 1956; Hodgkiss et. al., 2018; Oltman, Raskin, Witkin, 1971; Sarama and Clements, 2009; Witkin, 1950). In the current study, students were not asked to find certain shapes in the artworks. Rather, they are asked to identify what they see in the artworks. Some students disembedded two-dimensional geometric shapes in various ways from the others, which have potential to facilitate realizing geometrical shapes in new ways in the geometric problems (Sarama & Clements, 2009). Supportively, disembedding ability was found as significant predictor of science performance of students from seven to eleven years old (Hodgkiss et.al, 2018). Moreover, students also disembedded two-dimensional representation of three-dimensional shapes in addition to two-dimensional shapes because some artworks involved reversible figures that could be perceived as both two and three-dimensional or perceived from different point of views, which involves flexible transition from one shape to another (Attneave, 1971; Sarama & Clements, 2009). However, it was rarely observed especially during individual analysis of artworks. In the further process of studio works, some of them became to realize them. The reason behind such difficulties could be that shapes shared their contours rather than sharing a point, especially on reversible figures as an example of extreme example of sharing contours (Ghent, 1956). Another reason could be the fact that student had difficulty in identifying critical and non-critical properties of geometric shapes (Hershkowitz, 1989; Tsamir et. al, 2008).

In addition to disembedding, students also attempted to embed geometric shapes to create an artwork differently from the previous studies. There is little research on disembedding and embedding geometric shapes, especially on embedding shapes (Sarama and Clements, 2009). In the current study, a noteworthy finding was observed during a student's attempt (Melek) in embedding two-dimensional representations of three-dimensional shapes at students-at-work part. She tried to embed triangular prisms so that they share one of their faces and rotate along a circular path or embed a square pyramid into a square prism so that their bases are the same. It seemed that she aimed to hide shapes so that they share faces rather than a point, which makes difficulty to identify shapes in embedded figures (Ghent, 1956). However, she had difficulty in representing these embedded shapes. She might not have identified critical properties of three-dimensional shapes or thought visual prototypes of them in her mind (Tsamir et. al, 2008). This could be also due to the fact that she mostly represented three-dimensional shapes on the basis of visible faces rather than imagining invisible faces, which is consistent with the stage 3 (prerealistic) in representation of three-dimensional shapes in the work of Mitchelmore (1978; 1980). At this stage, elementary and middle school students attempt to draw three-dimensional shapes with some distortions in their faces to give depth even though it is not well coordinated and they draw only visible faces.

5.1.2 Decomposing and Composing Shapes

In this study, student decomposed and composed shapes during observation of artworks, copying and creating artworks, which are also observed in the studies which investigated young children's use of pattern blocks to compose a shape (Clements, Wilson, & Sarama, 2004; Wilson, 2002), decomposition of shapes in virtual environment (Spitler, 2009), and middle grade students' composition and decomposition of geometric shapes in paper-pencil test with using pattern blocks, using pencil or scissor, and without using any materials (Alaylı & Türnüklü, 2013; 2014).

The findings of this study regarding decomposition of shapes indicated that students attempted to decompose shapes when they were asked to find geometric shapes in artworks that involve hidden and/or overlapped shapes. Students decomposed shapes even though these invisible shapes are not delineated in the whole shape, which is consistent with the study of Spitler (2009) that found kindergarten students could be able to split the whole such as hexagon into its parts even though they are invisible in the hexagon presened in the virtual environment. In a similar way, in the work of Alaylı and Türnüklü (2013) students could decompose a whole shape into smaller shapes. However, they splitted the whole depending on the shapes in their mind rather than finding certain geometric shapes asked by the researchers. In the current study, students similarly splitted the artworks as to what they imagined inside it on the basis of perceptual cues such as sharpness of corner, lengths sizes. However, differently they splitted the shapes breaking gap. This could be due to the fact that the shapes to be decomposed were not regular geometric shapes with which students are not familiar. In fact, they were compositions of regular polygons. In addition to findings that are consistent with the previous studies, this study also indicated that they attempted to decompose the shape when students were asked to find the shapes. On the other hand, when students were not asked to find the shapes, they did not deliberately decompose the shapes during copying artworks.

The findings of this study regarding decomposing and composing shapes indicated that some students combined geometric shapes without anticipation of new geometric shapes to make a picture of real-world object. This finding is consistent with picture maker level of composition of shapes proposed by the study of Clements, Wilson and Samara (2004) in which young children were asked to used pattern blocks to compose a shape. This finding is consistent with the findings of Alaylı and Türnüklü (2014) in which they found students performed at the first four levels including picture maker level identified by Clements et. al. (2004). While some students reflected picture maker level, some students concatenated shapes to make a particular part of real-world object through rotating shapes. The drawing process elicited that they anticipated the new shapes by exploring the combination of shapes even though they did not deliberatively use the triangles to make a new shape, which is consistent with the level of shape composer (Clements et al., 2004) in which students use three rigid motions with anticipation. On the basis of previous studies, this finding also implies that different aged children could be at the same level of composing shapes depending on the difficulty level of tasks and nature of tasks that they are engaged in.

Another striking finding arising from the study was that students mostly focused on one component of geometric shapes such as side lengths during both decomposition and composition of shapes. This finding is consistent with the study in which middle school students mostly focused on the property of side lengths of geometric shapes (Alaylı & Türnlüklü, 2013). A few students used lengths and angles in a more coordinated manner during creating artwork and combine units or units of units, referred to the level of shape composition defined by Clements et al. (2004). During this process students needed to use of concrete materials such as pattern blocks and paper shapes during this process, which emphasizes the importance of such materials in imagination and exploration of composition of shapes (Clements, 1998).

Moreover, students' drawing composition of shapes indicated that imagining and representing rotations of shapes and considering shapes' properties such as side lengths and angle was crucial factors for composition of shapes especially during creating their artwork, which indicates visual-spatial thinking processes could be interrelated to each other (Sarama & Clements, 2009). In addition, composing and decomposing shapes could be interrelated since a student composed unit of triangles and made artwork with a hexagon even though he did not predict it as a hexagon at first. After he completed it, he realized it is a hexagon and could be divided into equal parts, which provide evidence for interrelated process of composing and decomposing shapes that support each other.

5.1.3. Patterning

The findings of the study regarding patterning indicated that students analysed patterns by segmenting the visual patterns into individual units to identify the rule of pattern and integrated individual units with a regularity by predicting the whole shape. During analysis of artworks, students focused on the local elements of the visual pattern (e.g. considering the first two or three squares in the artwork with nested squares) to identify multiplicative relation between sizes of shapes. Students did not consider the relation between other squares and the whole, which is consisted with findings of previous studies in which young children mostly focused more on local elements (parts of shapes) than the whole. To be able to better patterning, these two processes should be coordinated (Akshoomott & Stiles, 1995; Feeney &Stiles, 1996; Tada & Stiles, 1996; Vinter, Puspitawati, & Witt, 2010).

During creating patterns, students juxtaposed shapes so that one shape is next to to other by preserving their sizes or transforming their sizes repeatedly without predicting the overall pattern or plan at first. After a few juxtaposition of shapes, they realized the whole shape (Akshoomott & Stiles, 1995). Students juxtaposed the shapes through informal use of symmetry, leaving equal distance between them or rotating shapes during creating pattern. Whereas students used informal strategies of patterning during creating artworks, students structured the shapes into smaller shapes and investigated the relations between parts by searching for a rule during group analysis of artworks with patterns. Structuring is important ability for patterning, providing as a foundation for mathematics learning (Lüken, 2012; Mullihan & Mitchelmore, 2009; Sarama and Clements, 2009). Students used both figural and numerical reasoning during analysing artworks with growing patterns. Thus, the use of pictorial growth pattern could be useful for algebraic thinking in early grades (Walkowiak, 2014).

5.1.4. Transforming Shapes

The findings regarding transforming shapes indicated that students identified transformations of shapes in the artworks and transformed shapes in terms of their sizes (scaling) and directions (rotation, flip) during creating artworks.

5.1.4.1. Scaling Transformations

Regarding scaling transformations, students attempted to encode geometrical cues in the artworks and imagined scaling transformations of them during copying shapes in the artworks to a larger space. The geometrical cues that students encoded are mainly encoding length and angular relations, overall arrangement of shapes (symmetric or asymmetric), identifying geometric shapes and their properties. This finding is consistent with the findings of the study of Vasilyeva and Bowers (2006) in which young children transferred objects from one space to a larger space through coding the relations between line segments and/or angles of triangle layout and did not need for scaling transformations. Unlike this study, it was also observed that students used both scaling and encoding geometric cues simultaneously during copying artworks.

The second major finding regarding scaling transformations was that students attempted to identify proportional relationships between shapes. It showed that proportional reasoning is crucial part of scaling transformations (Möhring, Frick, & Newcombe, 2018; Möhring, Newcombe, Levine, & Frick, 2016). However, their proportional reasoning mostly was based on additive thinking by identifying the difference in the number of units in two shapes. They explained in multiplicative structure, which is consistent with the studies on proportional reasoning in which they found students' tendency in additive comparisons and difficulty in understanding multiplicative comparisons of lengths or areas (Sowder et.al., 1998; Lamon,1994). For example, to compare areas of squares, students determined differences in the number of units in each square rather than considering them as composite units to think multiplicatively (Lamon, 1994). It is also noteworthy that to identify proportional relation between areas of squares or rectangles students structured shapes into unit of squares or rectangles in the artwork since they are not given only numerical values and are not allowed to use measurement tool. This finding implied the use of spatial structuring as important skill in measurement of areas and spatial proportional reasoning (Sarama & Clements, 2009).

Third significant finding was that when the researcher asked students to identify proportional relation between shapes, they attempted to use their fraction knowledge by considering areas of parts and whole. However, they did not use this knowledge during copying artwork. In case the researcher asked to find proportional relation part-part and part-whole, they attempted to find it. This implies that students might have had difficulty in applying their knowledge to the context of visual arts and mathematics. This finding supported the claim of Perkins (2013) that he critiqued the current education tends to delay the use of knowledge in their current tasks.

Another significant finding regarding scaling transformation is that during scaling transformations, students mostly compared the lengths of shapes. However, this comparison was limited to comparison of a few shapes, rather than them considering them as parts of overall configuration. For example, most of them did not realize the pattern in sizes of squares in the artwork of Robert Mangold with pattern of squares. It seems that student might have difficulty in predicting the geometric relations between shapes as part of overall configuration or pattern and the relation among shapes simultaneously (Vasilyeva & Bowers, 2006; Uttal, Sandstrom, & Newcombe, 2006).

Other significant finding was that students had more difficulty in copying artwork with asymmetrical shapes in an asymmetrical layout compared to copying the artwork with symmetrical shapes in a symmetric-like layout, which is consistent with the findings of Uttal (1996) in which young children (from 4 to 7 years) and adults were asked to reconstruct the object, presented in a map, in the room. In this study, students had difficulty in encoding geometric information regarding relation between shapes or parts of shapes in asymmetrical shapes or in asymmetrical configurations. Students might have found easier to copy symmetrical shapes or symmetrical configurations since symmetrical configurations has an organized structure or pattern or students might easily have placed each shape or parts of shapes in relation to the its symmetrical counterpart (Uttal, 1996).

5.1.4.2. Rigid Transformations

The findings regarding rigid transformations indicated that students compared rotated or flipped geometrical artworks, identified congruence between them and identified angle, center, and direction of rotation. During imagination of transformation of shapes, students encoded shapes' structure by matching their line segments, corners, directions or holistic visual image of the shapes. Some students focused on the parts of shapes or decomposed composite shapes into smaller shapes and imagined its rotation, which could be related to encoding object structure and might result in high achievement in mental rotation tasks (Xu & Franconeri, 2015). Such thinking process is also consistent the phases of mental rotation that involves identifying visual structure of objects, rotating one of them, and comparing it with another object to identify whether they are identical or not, and responding. However, students differed from each other in each phase (Wright, Thompson, Ganis, Newcombe, & Kosslyn, 2008). During this process, some students used gestures to explain the movement of rotation by rotating their hands, heads, or bodies (dynamic gestures) or just pointing specific parts of shapes (static gestures) (Göksun, Goldin-Meadow, Newcombe, & Shipley, 2013).

Another important finding regarding rigid transformations was that while some students focused on the parts of shapes to imagine rotation, they did not identify angle and center of rotation during individual observation. This finding is aligned with the findings of Harper (2002) in which she investigated pre-service teachers' understanding of rigid transformations. Similarly, in another study on angles, when students are asked to find angles of rotation when they are given rotated artworks, students had difficulty in showing angle of rotation (Mitchelmore & White, 1998). This might be due to lack of understanding dynamic nature of angle during rotation (Foxman & Ruddock, 1984; Sarama & Clements, 2009). Students did not spontaneously think amount of rotation. Rather, they encoded area between two line segments or proximity of two line segments only when they were prompted to think by specific questions. This finding is consistent with Foxman and Ruddock's study with 15 years old in which students could attempted to think amount of rotation only if they were encouraged by questions of the researcher and experiences.

The third major finding regarding transformations was that when students were asked to identify the angle of rotation, they mostly used visual benchmarks of angles such as 45, 90, 180 degrees, which is consistent with the findings of Clements and Burn (2000). They decided angle of a shape by comparing its area with those of benchmark angles visually. This is consistent with the study of Sarama and Clements (2009) that young children associated obliques lines with 45 degrees and vertical and horizontal line with 90 degrees on the basis their 45-90 schemes in their minds. They also suggested to use of them as units of turn to decide amount of turn accompanied with physical rotation such as body rotation and use of concrete materials.

Lastly, student performed different process during observation of artworks that involve rotated images of an artwork and during creating artwork in which they are not given rotated image of the artwork and asked to represent its rotated image. Students had difficulty in drawing rotated images of artworks even though they could discriminate rotated images of artworks during observation of artworks. Some students realized something is wrong in their drawing. However, they did not draw its rotate image correctly. It is a striking finding arising from this study. The reason such a difference could be that drawing shapes is difficult and holistic object recognition is not enough for representing shapes even though it helps to discriminate shapes. Drawing requires students to coordinate what one has drawn already and what one has in mind that is not drawn (Fuson & Murray, 1978).

5.2. Implications

Implications of this study are discussed by describing possible contributions to the literature and educational settings. First of all, how the findings of the study could shed light into the studies on visual arts and mathematics integration, visual-spatial thinking and artful/studio thinking was discussed. Then, what contributions this study could make was discussed for teachers and curriculum developers of mathematics education in school context or out-of school contexts such as summer school programs, art-science centers, and museums.

5.2.1 Implications for Literature

There are three main contributions of this study to the literature. First of all, this study provides an insight into investigation of students' thinking processes in such a visual arts and mathematics environment and provides clues for researchers regarding how to design tasks and environment to make their thinking visible. Studies on the relation of visual arts and mathematics mostly focused on different topics of mathematics such as symmetry (Schaffer, 1997), space filling, similarity and proportions, golden mean, transformations (Kappraff, 1986), tessellation, origami, Islamic pattern, op-art, quilt patterns (Ugurel Okbay, 2013), polyhedra (Hart & Heatfild, 2017; Morgan, Sack, & Knoll, 2010). These studies did not mostly provide detailed information about the nature of tasks and specify learning outcomes of art-based activities and at what conditions or at what type of tasks learning outcomes were observed (Winner, Goldstein, & Vincent-Lancrin, 2013).

On the other hand, some studies (Goldsmith et. al, 2016; Walker et. al, 2011) investigated the transfer from arts education to geometry and investigate correlation relation between visual arts and geometry. These studies provided valuable contributions to the literature. On the other hand, how does transfer occurred still remains questionable. There could be mediating factors that affect this relationship (Winner, Goldstein, & Vincent-Lancrin, 2013). On the basis of visual-spatial thinking regarded as potential overlap between visual arts and geometry (Goldsmith et. al, 2016), this study proposed to investigate what visual-spatial thinking processes arises from the connection of visual arts and geometry by maximizing the probability of transfer with the use of geometric shapes directly in the art context and critical features of art education (mostly studio thinking and artful thinking). Directly integration of two disciplines could be resulted in rich transfer (Perkins & Salomon, 1992). Thus, this study suggested investigating students' thinking processes in detail in such a directly combined context rather investigating it in two different disciplines separately and searching for the relation between them to understand the transfer from art education to geometry education.

In addition to findings of the previous study, this study investigated what visual-spatial thinking processes arises from the connection of arts and geometry and found four main visual-spatial thinking processes that are interrelated to each other by providing detailed information at what condition they were observed. This study could provide some clues for future researchers regarding how to design tasks to develop students' visual-spatial thinking in a studio environment even though it was not the main purpose of the current study.

Secondly, this study adapted some aspects of visual-spatial thinking defined in psychology literature and mathematics education to the contexts of visual arts and mathematics and might enrich the investigation of visual-spatial thinking in various contexts. It is important to note that this study only focused on a particular aspect of visual arts (minimalist art) and mathematics (geometry). There could be different designs for other combinations of visual arts and mathematics. The findings of this study need to be tested and revised in similar and different contexts. Although there are limitations of this study (see method part), this study could have some contributions to the literature. Findings of the study could be related with the typology of spatial thinking proposed by Newcombe and Shipley (2015). Newcombe and Shipley (2015) proposed four main categories for spatial thinking: intrinsic-static, intrinsic-dynamic, extrinsic-static, and extrinsic-dynamic. It is important to note that the identification of the category depends on what we mean by an object that might vary at different scales. The thinking process of identifying geometric shapes as real-world objects, their properties, disembedding and embedding geometric shapes in the artworks could be related with the category of intrinsic-static. Imagining rotations and flips of shapes to decide whether they are identical, imagining the cross-section of a truncated pyramid that is perceived in artwork or during creating artwork could be related with the category of intrinsic-dynamic. During copying artworks, identifying location of a shape in original artwork in a corresponding larger paper with the consideration of its relation with other shapes could be considered as an example of extrinsic-static spatial abilities even though it was examined at small-scale space. The findings of the study provide not only examples regarding each category of spatial thinking but also examples that fall into intersection of categories. For example, identifying geometric properties of shapes that are rotated in the artwork could fall into the category of intrinsic-static and intrinsic-dynamic. Another example is that encoding the proportional relation between parts and whole and using this information to place each shape to the target location with the consideration as well as changing its size (scaling) include intrinsic-static, intrinsic-dynamic and extrinsic-static abilities. Drawing a three-dimensional shape from different points of view requires coordination between intrinsic static (identifying properties of a shape), intrinsic- dynamic (relation 2D and 3D representation of a shape), and extrinsic-dynamic skills (adaptation of the representation of a three-dimensional shape while they are rotating or while changing own perspective at the small scale). This might lead us to think and discuss whether artists are object visualizers and/or spatial visualizers depending on the task they engaged in (Kozhevnikov, Kosslyn, & Shephard, 2005). This study might also have contributions to the studies that aim to develop psychological tests for understanding students' individual differences in visual-spatial thinking in the context of visual arts and mathematics by providing some resources (tasks, artworks, questions).

The findings of the study also share common points with respect to visual-spatial thinking in arts education, called as artistic envisioning, proposed by Goldsmith et. al. (2016) such as mental rotation, flatting the space (2D and 3D relation). However, they differed from each other in terms of nature of task (use of mathematic vs non-mathematical objects). Flattening the space is related to making relation between 2D and 3D in which students draw real-life objects on the two-dimensional page. However, in this study students saw two-dimensional representations of three-dimensional shapes rather than seeing them directly, which also involved relating 2D and 3D shapes. Another connection could be that drawing rotation of a human figure based on envisioning could be related with the drawing a geometric shape from a different point of view based on envisioning mental rotation or perspective taking in the current study. Differently, this study encouraged students to use analytic strategies in addition to visual strategies at different conditions (observation of artworks, creating artworks, and critiquing artworks) apart from just drawing.

Thirdly, studio thinking framework was adapted to the context of of the Math-Art Studio Environment. This study both validated the use of artful/studio thinking dispositions at a different context and revised its some aspects. In the studio thinking framework researchers identified three structures of studio; demonstration, students-at-work, and critiquing. In this study, demonstration part of the studio was revised and described under two main analyses of artworks: individual and group analysis of artworks in which students are asked to observe artworks and find what geometric shapes they see. Use of artworks could be regarded as milestone of such an environment because they provided a base for creating artworks and critiquing of artworks and elicit a variety of thinking skills depending on the nature of artworks. There is a transition part between demonstration and students-at-work parts, called as warm-up activities that were used to engage students in creating artworks since some tasks (e.g. creating artwork though rotation of shapes) was difficult to comprehend. Combined process of reflecting, envisioning, observing, and exploring was mostly observed during students-at-work process. Student reflected their thoughts by mostly explaining their artworks rather than evaluating their friends' artworks during critiquing part. This might imply that students' grade level is not well suited with the critiquing others' art work, which is an expected situation, or the critiquing parts of the study could be revised so that students have an active role in critiquing. Moreover, individual and group observation of artworks seemed to have potential role in encouraging students to use observation skills since it was observed that students extended what geometric shapes they see during group observation of artworks. Group observation of artworks encouraged students to observe again artwork and see the artworks in news ways. Investigation of students' sketches throughout the study might provide a rich understanding of students' thinking over a process. Writing process did not work well since students were not so interested that they explain their ideas and their artworks.

Lastly, this study could provide an insight into the students' thinking processes regarding mathematical concepts. In this study, student have experienced the use and making sense of mathematical ideas in a studio environment. Students' thinking process was observed at the different conditions such as observing artworks individually and in group, creating and copying artworks, and critiquing artworks, which resulted in observation of different levels and types of visual-spatial thinking depending on the nature and content of the tasks. Students had a chance to use knowledge of mathematics. Students not only used previous knowledge, but also had ideas about the concepts that have not been taught yet. For example, even though students were not taught transformational geometry at seventh-grade, they attempted to imagine transformations of shapes, which imply that such an environment could also be used for both understanding students' thinking about concepts that have been already taught and advanced concepts that have not been taught yet.

5.2.2 Implications for Educational Practices

There are two main implications of this study for educational practices. They are investigated under two questions: "How could the teachers benefit from the findings of the study?" and "How do the findings of the study provide implications for curriculum developers and policy makers?"

First of all, this study provided initial pedagogical principles for teachers to implement such studio works in their classrooms or in art-studios or in maker spaces. This studio works could be used for both visual art teachers and mathematics teachers. It informs teachers about how, when and where to use studio thinking. Teacher could understand where to position himself/herself in the Math-Art Studio Environment. In other words, to what extent he/she has active role during studio works and acted as a coach and directed all studio works to experience close interaction with students.

It also provides a guideline regarding how to make student' thinking visible in this environment. Teachers could better understand what is visual-spatial thinking and its use in the context of visual art and mathematics integration and identify individual differences between students' visual-spatial thinking at different tasks.

For example, teacher could use these studio works involving different kind of artworks (e.g. artworks with nested squares vs. artwork with discrete arrangement of square; artworks with a pyramid from top view vs artwork with a pyramid from side and top view) to elicit different thinking processes to understand individual differences between them. Art educators could use these studio works to integrate mathematics into their courses.

The findings of the study might raise teachers' and students' awareness with respect to the fact that visual arts are not only fun for learning mathematics but also, it requires cognitive thinking even though it has been regarded as a non cognitive subject in the previous years (Arnheim, 2015; Efland, 2002). It could also raise awareness regarding the fact that mathematics is not only about logical reasoning but also about spatial reasoning, required for mathematical giftedness according to Krutetskii (as cited in Presmeg, 1986). This implies that teacher should have a responsibility for developing students' spatial abilities and developing their spatial abilities (Bishop, 1980).

Another major implication of study is that the findings provided concrete evidences regarding implementing such studio works to elicit a variety of visual-spatial thinking processes. Thus, this study provided educational materials (studio works) for especially child centers, museums, art and science centers, after school or summer programs. Especially for art and science centers, it provides rich contexts for arts and mathematics integration. Thereby, students who have interest and abilities in mathematics and visual arts get opportunities to develop skills. In addition to providing materials for such educational communities, this study could also shed light on the future studies to revise nature of mathematics and mathematics application courses in Turkey, taking into consideration the nature of visual art courses in the long run (revision of classroom atmosphere in schools as art studios or ateliers; using studio thinking to make students thinking visible). This might create a vision towards mathematics education in which students experience themselves with their hands and minds by using their knowledge of mathematics and students have opportunities for sharing their ideas, feelings, thoughts. This new vision could also provide new opportunities for gaining skills required in the 21st century such as creativity, innovation, imagination, spatial reasoning.

5.3. Recommendations for Future Studies

The findings of this study uncover several issues for future research on arts and mathematics integration. Four critical points arise from this study that needs to be considered by further studies. Each of them explained in the following.

First of all, this study provided some clues regarding students' different thinking levels. For example, while a student imagined and represented transformations of shapes analytically, another student had more difficulty in transformation of shapes. The question of "to what extent students could transform shapes in the artworks or in creating artwork" needed to be considered in the future studies. Researchers could examine these different levels of students' thinking to identify their individual differences in such a Math-Art Studio Environment.

Secondly, the findings of the study indicated that students' use of visual-spatial thinking is a complex process than it seemed to be. They are interrelated to each other. Future studies could examine the dynamic relation between different visual-spatial thinking processes. It is important to investigate to understand underlying mechanism of this complex process and identify in what ways a kind of visual-spatial thinking affect the use of another kind of visual-spatial thinking that is used simultaneously.

Thirdly, this study investigated students' visual-spatial thinking processes in a Math-Art Studio Environment. Several pedagogical principles were determined to elicit students' thinking. This arises the next question of "What characteristics of such an environment could facilitate students' spatial thinking?". In other words, researchers could aim to facilitate students' visual-spatial thinking through revising these principles in a series of studies and identifying the effects of each principle in promoting students' visual-spatial thinking.

Besides, researcher also could investigate students' visual-spatial thinking processes in relation to each studio thinking dispositions and structures of the studio proposed by Hetland et. al. (2013).

Another recommendation of this study could be the use of technology in the context of visual arts and mathematics. There have been some studies on the use of technology in visual arts and mathematics context (Shaffer, 2005; Sinclair, 2006). For last ten years, there has been increase in use of digital media tools. This lead to transformations in the education of visual arts by giving importance on digital artworks (Sheridan, 2011). In this regard, the studio works proposed in this study could be integrated with technology in the future studies.

Lastly, this study focused on the use of artworks with minimalism art movement in the small scale spaces. In the further studies, researcher could focus on different art movements in both small scale and large scale spaces, which are important for eliciting different visual-spatial thinking processes (Newcombe, Uttal, and Sauter, 2013). Moreover, researchers could design studio works including making three-dimensional artworks or making both static and dynamic artworks to elicit different visual-spatial thinking processes.

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APPENDICES

A. APPROVAL OF HUMAN SUBJECTS ETHICS COMMITTEE OF THE UNIVERSITY

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
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05 Mayıs 2017

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Prof. Dr. Erdinç ÇAKIROĞLU ;

Danışmanlığını yaptığınız doktora öğrencisi Mehtap KUŞ' un ***"Sanat Stüdyosunda Matematik ile Oynamak: Sanatsal Düşünme Yaklaşımı ile Öğrencilerin Görsel-Uzamsal Düşünme Yollarının Araştırılması"*** başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülerek gerekli onay **2017-EGT-084** protokol numarası ile **08.05.2017 – 31.12.2017** tarihleri arasında geçerli olmak üzere verilmiştir.

Bilgilerinize saygılarımla sunarım.

Prof. Dr. Ş. Halil TURAN
Başkan V

Prof. Dr. Ayhan SOL
Üye

Prof. Dr. Ayhan Gürbüz DEMİR
Üye

Doç. Dr. Yaşar KONDAKÇI
Üye

Doç. Dr. Zana ÇITAK
Üye

Yrd. Doç. Dr. Pınar KAYGAN
Üye

Yrd. Doç. Dr. Emre SELÇUK
Üye

B. APPROVAL OF ETHICS COMMITTEE OF NATIONAL MINISTRY EDUCATION

T.C.
ANKARA VALİLİĞİ
Milli Eğitim Müdürlüğü

Sayı : 14588481-605.99-E.7798514 29.05.2017
Konu : Araştırma İzni

ORTA DOĞU TEKNİK ÜNİVERSİTESİNE
(Öğrenci İşleri Daire Başkanlığı)

İlgi: a) MEB Yenilik ve Eğitim Teknolojileri Genel Müdürlüğünün 2012/13 nolu Genelgesi.
b) 17/05/2017 tarihli ve 2431 sayılı yazınız.

Fakülteniz Matematik ve Fen Bilimleri Eğitimi Bölümü Doktora programı öğrencisi Mehtap KUŞ'un **"Sanat Stüdyosunda Matematik ile Oynamak: Sanatsal Düşünme Yaklaşımı ile Öğrencilerin Görsel-Uzamsal Düşünme Yollarının Araştırılması"** konulu tez kapsamında uygulama talebi Müdürlüğümüzce uygun görülmüş ve uygulamanın yapılacağı İlçe Milli Eğitim Müdürlüğüne bilgi verilmiştir.

Görüşme formunun (16 sayfa) araştırmacı tarafından uygulama yapılacak sayıda çoğaltılması ve çalışmanın bitiminde bir örneğinin (cd ortamında) Müdürlüğümüz Strateji Geliştirme (1) Şubesine gönderilmesini rica ederim.

Vefa BARDAKCI
Vali a.
Milli Eğitim Müdürü

06-06-2017- 9466

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C. PARENTAL CONSENT FORM

Veli Onay Mektubu

Sayın Veliler,

Bu çalışmanın amacı, stüdyo düşünme aracılığı ile öğrencilerin, zengin matematiksel içeriği olan sanatsal çalışmalarındaki, görsel-uzamsal düşünme süreçlerini incelemektir. Çalışma kapsamında matematik ve görsel sanatların birleşimine yönelik etkinlikler; görsel sanatlar öğretmeni ve araştırmacı tarafından sanat atölyesinde uygulanacaktır. Bu mektubun yollanış amacı çocuğunuzun bu çalışmaya katılmasını onaylayıp onaylamadığınızı belirtmenizdir.

Bu çalışma, Orta Doğu Teknik Üniversitesi Eğitim Fakültesi Matematik ve Fen Bilimleri Eğitimi Bölümü öğretim elemanı Mehtap KUŞ ve öğretim üyesi Prof. Dr. Erdinç ÇAKIROĞLU danışmanlığında yürütülen doktora tezi kapsamında yapılan bir çalışmadır. Çalışma kapsamında resim atölyesinde matematik ve sanatın birleşimine yönelik etkinlikler uygulanacaktır. Çocuklarınızla ses kaydı alınmak üzere birebir görüşmeler yapılacak olup, atölye ortamında yapılan çalışmalar video ile kayıt altına alınacaktır. Çalışmanın hiçbir aşamasında öğrencilerden kimlik belirleyici hiçbir bilgi istenmemektedir. Öğrencilerle yapılan görüşmeler ve video kaydı gizli tutulacak ve sadece bilimsel amaçlar için araştırmacı tarafından değerlendirilecektir. Çalışmaya katılım tamamıyla gönüllülük temelindedir. Görüşme soruları kişisel rahatsızlık verecek herhangi bir ayrıntı içermemektedir. Katılım sırasında, katılımcılar sorulardan ya da herhangi başka bir nedenden ötürü rahatsız hissederlerse çalışmayı yarıda bırakıp çıkmakta serbesttir.

Bu çalışmanın sonucunda ulusal ve uluslararası alanda sanat ve matematik arasındaki ilişkiye yönelik eğitim içeriklerinin araştırılması ve geliştirilmesinden dolayı çalışmaya katılımınız bizim için oldukça önem taşımaktadır.

Çocuğunuzun bu çalışmaya katılmasını istiyorsanız, lütfen aşağıdaki ilgili bölümü doldurunuz. Çalışma hakkında daha fazla bilgi almak için Mehtap KUŞ ile iletişime geçebilirsiniz (e-mail: ozmehtap@metu.edu.tr)

Saygılarımla,

Mehtap KUŞ

Bu araştırmaya çocuğumun gönüllü olarak katılımcı olmasına izin veriyorum. Çalışmayı istediğimiz zaman yarıda kesip bırakabileceğimizi biliyorum ve bu çalışmanın sonuçlarının bilimsel amaçlı olarak kullanılmasını kabul ediyorum.

Anne Adı-Soyadı .... Baba Adı-Soyadı ....
İmza .... İmza:

D. INTERVIEW PROTOCOLS : PRE-IMPLEMENTATION QUESTIONS

Uygulama Öncesi Sorular

[*Bu görüşmenin amacı, uygulama başlamadan önce öğrencilerin görsel sanata yönelik önceki deneyimleri hakkında bilgi edinmek ve görsel sanat ve matematiğe yönelik hisleri, düşünceleri ya da fikirleri hakkında genel bir fikir edinmektir*.]

1- İlk olarak, bu okuldaki görsel sanatlara yönelik deneyimlerinden konuşalım ne dersin?
   - Burada ne gibi çalışmalar yapıyorsun?
   - Yaptığın resimlerden biraz bahseder misin?
   - Bunlardan en çok hangisi hoşuna gitti? Neden hoşuna gitti?
   - En çok hangisinde zorlandın? Neden zorlandığını düşünüyorsun?
   - Görsel sanatlar dersinin gerekli olup olmadığı konusunda ne düşünüyorsun?
   - Görsel sanatlar dersinde eğleniyor musun?
   - Görsel sanat etkinlikleriyle uğraşmaktan hoşlanıyor musun?

2- Görsel sanatlar ifadesi sana göre ne demek?

3- Görsel sanatlar dersi dışında görsel sanatlarla ilgili neler yapardın/yapıyorsun
   - Daha önceden resim kursuna gittin mi?
   - Daha önceki yaptığın resimlerden bahseder misin?
   - Boş vakitlerinde çizimler (karalamalar) ya da resim yapar mısın? Ne tip çizimler bunlar, örneklerle açıklar mısın?
   - Daha önce hiç resim sergisine/resim müzesine gittin mi? Peki gitmekten hoşlanır mıydın/hoşlanıyor musun?
   - Daha önce katılmış olduğun bir resim yarışması var mı? Varsa resmini gösterebilir misin?
   - Farklı sanatçıların çalışmalarını inceleme imkanın oldu mu? Peki bu sanatçıların çalışmaları ilgini ne derece çekiyor?
   - İlerde bir ressam/tasarımcı/resim öğrt olmak ister misin?

4- Bu okuldaki görsel sanatlar dersini nasıl değerlendirirsin?
   - Dersin faydası var mı? Ne tip bir faydası var?
   - Dersten ne tip bir fayda beklediğin ama karşılanmayan bir durum var mı? Neden?

5- Matematik konularını düşündüğünde en çok hangileri senin ilgini çekiyor / seviyorsun / zorlanıyorsun?
   - Matematik dersinde kendini güçlü hissettiğin konular hangileri bahseder misin?
   - Matematik dersinde yaşadığın güçlükler neler; bahseder misin? Örneklerle açıklayabilir misin?

- Matematik öğrenmenin ne derece zevkli olduğunu düşünüyorsun?
- Matematik ile zaman geçirmekten hoşlanıyor musun?
- Geometri konularını mı matematik konularını mı daha çok seviyorsun?
- Okulda daha çok matematik/geometri dersi olsun ister miydin?
- Matematik dersinin gerekli olup olmadığı konusunda ne düşünüyorsun?
- Matematik/Geometri ilgini çekiyor mu?
- Boş zamanlarında matematik ile ilgili uğraşıyor musun? Örneğin, matematik ile ilgili araştırmalar yapıyor musun? Matematiğe yönelik sorular çözüyor musun boş zamanlarda?
- Daha önceden matematik ile ilgili yarışmalara katıldın mı? Katılmak ister miydin?
- İlerde bir matematikçi ya da matematik öğretmeni olmak ister misin?

6- Matematik dersini nasıl değerlendirirsin?
- Dersin faydası var mı? Ne tip bir faydası var?
- Dersten ne tip bir fayda beklediğin ama karşılanmayan bir durum var mı? Neden?
- Matematik dersinin nasıl olmasını isterdin?

7- Matematik ve görsel sanatlar arasında bir ilişki olabileceğini düşünüyor musun? Nasıl bir ilişki olabilir?
- Görsel sanatlar dersinde matematikten yararlandınız mı? Hangi çalışmalarda matematiksel bilgini kullandığını düşünüyorsun?
- Matematik dersinde görsel sanatlardan yararlandınız mı? (Peki teknoloji tasarım dersinde?)

8- Son olarak, bu çalışmada görsel sanatlar ve matematiğin birleşimine yönelik resimler yapacağız. Bu çalışmadan neler bekliyorsun? Nasıl bir ders olacağını hayal ediyorsun?

9- Eklemek istediğin başka bir şey var mı?

E. INTERVIEW PROTOCOLS : POST-IMPLEMENTATION QUESTIONS

Uygulama Sonrası Sorular

[*Bu görüşmenin amacı, uygulamadan sonra öğrencilerin bu çalışmadaki deneyimleri hakkında bilgi edinmek ve bu deneyimler doğrultusunda sanat ve matematik arasındaki ilişkiye yönelik düşüncelerinde ne gibi değişiklikler olduğunu açığa çıkarmaktır.]*

Bu çalışmada 6 tane birbirinden farklı etkinliği tamamladık. Bu etkinliklerle ilgili düşüncelerini merak ediyorum.

1- Bu etkinliklere başlamadan önce bu çalışmayla ilgili kafanda ne canlandırmıştın?
   - Bu uygulamalar canlandırdıklarına ne kadar benziyor?
   - Hangi açılardan benziyor?
   - Hangi açılardan farklı olduğunu düşünüyorsun?

2- Bu çalışmada geçirdiğin süreci nasıl değerlendirirsin?
   - Hoşuna giden şeyler nelerdi?
     - En çok hoşuna giden/olumlu şeyler nelerdi?
   - Hoşuna gitmeyen şeyler nelerdi?
     - En sıkıcı/olumsuz şeyler nelerdi?
   - Hangi etkinlikleri çok kolay yaptın? Neden kolay yapabildiğini düşünüyorsun?
   - Hangi etkinlikler daha çok zorlandın? Neden zorlanmış olabilirsin?

3- Bu süreçte neler öğrendiğini düşünüyorsun? Örneklerle açıklayabilir misin?
   - Bu çalışma sana ne gibi yeni fikirler verdi?
   - Bu çalışmadan sonra sanat ve matematiğin nasıl ilişkili olduğunu düşünüyorsun? Neden böyle düşünüyorsun?
   - Biz bu çalışmaya başladığımız zaman, sizlerin matematik ve sanat arasındaki ilişki ile ilgili ilk fikirleriniz vardı. Sizden bir kaç cümle ile daha önceden yapmış olduğumuz bütün etkinlikleri düşünerek bu etkinlikler ile ilgili ne düşündüğünüzü yazmanızı istiyorum. Hatırlamak için bir dakikanızı ayırın ve cevabınızı aşağıdaki *Önceden şöyle düşünürdüm*...kısmına yazınız. Şimdi ise, biz bu konuyu çalıştıktan sonra fikirlerinizde neler değiştiğini düşünmenizi istiyorum. Birkaç cümle içinde ne düşündüğünüzü *Önceden şöyle düşünürdüm*...kısmına yazınız.

     Önceden şöyle düşünürdüm...
     Şimdi böyle düşünüyorum...

4- Son olarak, biz bu çalışmayı bu şekilde uyguladık. Bütün etkinliklerimizden sonra sence bu ders nasıl olmalıydı? Ya da nasıl olmamalıydı?
   - Eklemek istediğin başka bir şey var mı?

- Sormamı beklediğin, ama sormadığım “keşke sorsaydı anlatacak çok şeyim vardı” dediğin bir soru var mı?
- Sorsam cevaplar mısın?

F. INTERVIEW PROTOCOLS : DURING-IMPLEMENTATION QUESTIONS

Araştırma Sırasında Uyarılmış Hatırlama Görüşme Soruları

[*Bu görüşmelerin amacı, uygulama tamamladıktan sonra öğrencilerin her bir etkinlikte belirli kritik noktalarda görsel-uzamsal düşünme süreçlerini derinlemesine araştırmaktır.*]

Genel sorular:

1- Bu derste öğrendiklerin ya da deneyimlerini düşünmeni istiyorum. Bunları aşağıdaki ölçeğe basitten karmaşığa doğru nasıl yerleştirirsin? (1-10 arası ölçeklendirilmiş olacak)

Basit ———————————————————— Karmaşık

2- Nelerde zorlandığını düşünüyorsun? Örneklerle açıklar mısın?
   a) Zorlanıp daha sonra sonuçlandırdığın çalışmalar var mı?
   b) Baştan zorlanmayıp sonra yapamadığın çalışmalar var mı?

3- Bu ders sonrası aklına takılan sorular neler?

4- Bu derste hoşuna giden şeyler nelerdi? Örneklerle açıklar mısın?

5- Bu dersle ilgili ne tip yeni düşünceler geliştirdiğini düşünüyorsun?

6- Biz bu dersi bu şekilde yaptık ama sence bu ders nasıl olmalı ya da olmalıydı?

Spesifik Sorular:

7- Stüdyo çalışmalarına yönelik spesifik sorular:

Tablo Görüşne Soruları

	Görüşme Soruları
Stüdyo Çalışması 1	İlk olarak nasıl başladın?
	Neden bu şekilleri seçtiğinden bahseder misin?
	Aklına ilk hangi fikirler geldi? Nasıl devam ettin çalışmana? Bu süreçte değişiklikler oldu mu? (Varsa) bu değişikliklerden bahseder misin?
	Bunu yapmaktan vazgeçmişsin neden vazgeçtin?
	Bu şekilleri yerleştirirken neler düşündün?
	Ne tür zorluklar yaşadığından biraz bahseder misin?
	Hangi şekli gizlemek istedin? Neden bu şekli seçtin?
	Bu şekli gizleyebilmek için neler yaptığından bahseder misin?
	Neden öyle düşündün?
	Neden bu renkleri kullanmayı tercih ettin? Bu renklerin nasıl bir etkisi olabilir bu şekli gizlemede?
	Hangi şekillerden yararlandın?

Tablo (Devamı)

Stüdyo Çalışması 2	Bu resme devam etmek için aklına ilk hangi fikirler geldi? Neden bunu yapmaktan vazgeçtin? Bundan sonraki şekli yapmaya nasıl başladın? Bu şeklin (2V) yönelimi nasıl değişiyor? Sence neden böyle değişiyor olabilir? Bir başka kişi senin yapmış olduğun çalışmanın aynısını yapmak isteseydi, nasıl yapmasını söylerdin?
Stüdyo Çalışması 3	4 tane resmimiz var. Bu resimlere yönelik aşağıdaki sorular: İlk hangi şekli yerleştirmeye çalıştın? Ondan sonra hangi şekli yerleştirdin? Nasıl yerleştirdin? Neye göre böyle yerleştirdin? Hangi şeklin doğru yerde olup olmadığından emin olamadın? En çok hangi resmin büyük halini yapmakta zorlandın? Neden? En çok hangi resmin büyük haline yapmak kolay oldu? Neden? Burada hangi şekilleri görüyorsun (özellikle Mel Bochner'in çalışmaları için)? Yanlış yaptığını düşünüp sonra silip düzeltmeye çalıştığın yerler oldu mu? Başka birisi bu resimlerin büyük halini yapacak olsaydı nasıl yapmasını önerirdin?

G. OBSERVATION PROTOCOL

GÖZLEM FORMU

Amaç
Bu gözlemin amacı, stüdyo düşünme tabanlı matematik-sanat stüdyo ortamı bağlamında, öğrencilerin görsel-uzamsal düşünme yollarını incelemektir.
Veri Toplama
Video kayıt cihazı kullanılarak öğrencilerin çalışma süreci gözlenmiştir. Ortamı, etkinlikleri ve süreci anlatacak şekilde betimsel notlar tutulmuştur. Yorumlar, betimsel notlardan ayrı olarak not alınmıştır.
Gözlem Soruları
Aşağıdaki sorular gözlem yaparken genel olarak kılavuz olarak kullanılmıştır.

1) Öğrenciler verilen göreve / etkinliğe nasıl başlıyor?
2) Öğrenciler verilen görevlerde nasıl çalışıyor?
3) Hangi durumlarda öğrenciler sanat yapma sürecinde değişiklikler yapıyor?
4) Hangi durumlarda öğrenciler zorluk yaşıyor?
5) Öğrenciler çalışma süreçlerinde ne tip yeni olası durumları deniyor?
6) Öğrenciler öğretmene ya da araştırmacıya ne tip sorular yöneltiyor?

Gözlem Boyutları
Atölye ortamını ve öğrencilerin düşünme sürecini tanımlamak için aşağıdaki üç önemli nokta hakkında gözlem yapılacaktır.

1) <u>Bağlam:</u> Fiziksel düzen hakkında bilgi (oturma planının, masaların, diğer nesnelerin konumlarının çizimi), atölyedeki materyaller hakkında bilgi, her bir stüdyo çalışamasının ne kadar sürdüğüne dair bilgi)
2) <u>Öğrencilerin stüdyonun farklı aşamalarındaki öğrencilerin düşünme süreçleri</u> Gösterim, Öğrenciler İş Başında ve Eleştiri aşamalarında öğrenciler görsel-uzamsal düşünme süreçlerini nasıl kullanıyor?
3) <u>Stüdyo düşünme yaklaşımına ilişkin olarak öğrencilerin düşünme süreçleri:</u> Bu bölüm, stüdyo düşünme bağlamında öğrencilerin derinlemesine düşünme süreçlerini incelenmesini içermektedir. Gözlem yaparken aşağıdaki sorulardan yararlanılacaktır.
    - *Gözlem*, *araştırma*, *zihinden canlandırma*, *araştırma yapma, zanaat (teknik geliştirme), sanat dünyasını anlama, fikirlerini başkalarına aktarma veya başkalarının çalışmalarını değerlendirme sırasında* öğrenciler ne tip görsel-uzamsal düşünme yollarından yararlanıyor?
    - Çalışmaya devam etme azim durumları nasıl değişiyor?
    - Etkinlik boyunca düşünme yolları nasıl değişiyor?
    - Ne tip geometrik ya da matematiksel özellikler üzerinde odaklanıyorlar?

Ne tip durumlarda zorluk yaşıyorlar?

H. SOME OF THE MINIMALIST ARTWORKS USED IN THE CURRENT STUDY

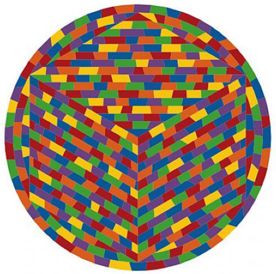


Figure 102

Figure 103

Figure 104

Figure 105

Figure 102. *LeWitt, S., Cube Circle 4*. Retrieved from https://joeleriksson.com/progetto-polymath-gyre-e-gimble.html
Figure 103. LeWitt, S. *Wall Drawing #1113*. Retrived from http://www.cavetocanvas.com/post/20818210226/sol-lewitt-wall-drawing-1113-on-a-wall-a
Figure 104. Bochner, M. (1973/1976). *Four Shapes*. Retrieved from https://www.wikiart.org/en/mel-bochner/four-shapes-1976
Figure 105. Bochner, M. (1976). *Two Shapes*. Retrieved from http://www.marcselwynfineart.com/exhibitions/mel-bochner-2

H. SOME OF THE MINIMALIST ARTWORKS USED IN THE CURRENT STUDY

Figure 106

Figure 107

Figure 108

Figure 109

Figure 106. Stella, F. (1961). *Hampton Roads.* Retrieved from https://www.wikiart.org/en/frank-stella/hampton-roads-1961
Figure 107. Stella, F. (1967). *Tomlinson Court Park from Black Series*. Retrived from https://www.wikiart.org/en/frank-stella/tomlinson-court-park-1959
Figure 108. Stella, F. (1967). *Ifata II, V Series*. Retrieved from https://www.wikiart.org/en/frank-stella/ifafa-ii-1967
Figure 109. Stella, F. *River of Ponds*. Retrieved from https://humphries346.wordpress.com/2015/08/24/post-painterley-abstraction-the-art-of-kenneh-noland-1924-2010-jules-olitski-1922-2007-frank-stella-1936-present-and-louis-morris-1912-1962/

H. SOME OF THE MINIMALIST ARTWORKS USED IN THE CURRENT STUDY

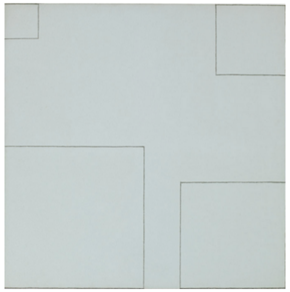


Figure 110


Figure 111

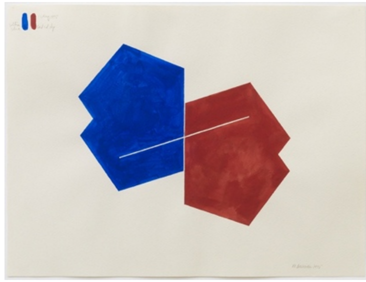


Figure 112

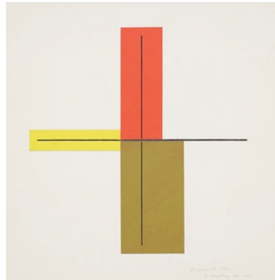


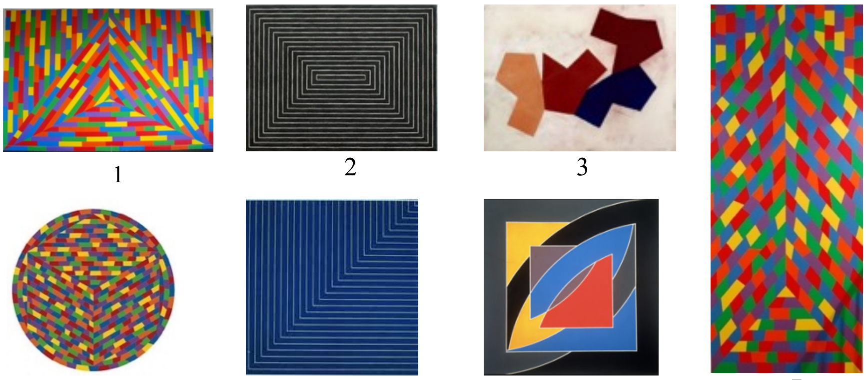
Figure 113

Figure 110. Mangold, R. (1974). *Four Sqaures Witihin a Square*. Retrieved from https://art21.org/gallery/robert-mangold-artwork-survey-1970s/
Figure 111. Martin, A. (1957). *Harbor Number 1*. Retrieved from https://www.moma.org/collection/works/79797
Figure 112. Bocher, M. (1975). *First Fulcrum*. Retrieved from http://mentaltimetraveller.tumblr.com/post/118383227307/mel-bochner-first-fulcrum-study-1975
Figure 113. Mangold, R. *Three Color Series + Series*. Retrieved from https://www.jklworldwide.com/robert-mango

I. STUDIO WORKS

STÜDYO ÇALIŞMASI 1: ŞEKİL SAKLAMBAÇ

Stüdyo Çalışması Planı
Aşama 1: Gösterim (Demonstration) Bu bölümde, öğretmen ısınma etkinliği ile başlar. Sol LeWitt ve çalışmaları hakkında kısa bilgi verdikten sonra öğrencilere Sol LeWitt'in çalışmaları ile ilgili bir video izletir. *Bağlantı:* *https://vimeo.com/ondemand/lewitt/136656840* *[Ö: "Sol LeWitt popüler Amerikalı sanatçılardan biridir. Başta duvar resimleri olmak üzere, çeşitli heykel ve çizimler yapmıştır. Minimalist sanat akımında önemli çalışmalar yapmıştır.* *Ö: Minimalist sanat akımını daha önceden duyan var mı? Minimalist sanat akımında, sanatçılar, sadeliğe ve yalınlığa önem vermişlerdir ve çoğunlukla geometrik şekillerden ve formlardan yararlanmışlardır.* *Ö: Şimdi Sol LeWitt'in bazı çalışmalarını içeren bir video izleyeceğiz.]* Bu sırada öğretmen öğrencilere videoyu izlerken ne gördüklerine dair not almalarını ister. *[O: Şimdi sizden istediğim şey, videoda ne gördüğünüzü yazmanız. Anahtar kelimelerle not alabilirsiniz]* Videoyu izledikten sonra, öğretmen aşağıdaki soruları öğrencilere yöneltir. *[Ö: Bu videoda dikkatinizi çeken şeyler neler?* *Daha önceden görmediğiniz ne var?* *Sol LeWitt'in sanat çalışmaları hakkında aklınıza ne gibi sorular geldi?* *İlginizi çeken şeyler oldu mu?* *Nasıl yapıyorlar? Neler yapıyorlardı?* *Bu resimlerin ortak özellikleri neler olabilir?* *Renkler nasıl kullanılıyor?]* Daha sonra öğretmen, aşağıdaki Sol LeWitt, Frank Stella, Mel Bochner gibi sanatçıların sanat çalışmalarını öğrencilere dağıtır. Öğretmen, öğrencilerden bireysel olarak ne tip geometrik şekiller gözlemlediklerini kağıtlara not almalarını ister.



1 2 3 4 5 6 7

Gözlem yapmak üzere öğrencilere verilen sanatçıların eserleri

Daha sonra öğrencilere verilen kağıtlar toplanır. Öğrencileri cevaplarının çeşitliğine göre resimlerden biri seçilir ve ayrıntılı olarak hep birlikte incelenir. Her bir öğrenciden keşfetmiş olduğu şekli akıllı tahta üzerinde göstermeleri istenir ve aşağıdaki sorular yöneltilir. Örneğin 1. resmi seçtiğimizi varsayalım.

[O: Şimdi hep birlikte Sol Lewitt'in bir resmini daha ayrıntılı olarak inceleyelim.
Ö: Siz de arkadaşınızın gördüğünü hayal edebiliyor musunuz? Arkadaşınıza siz de katılıyor musunuz?
O: Başka ne görüyorsunuz?
O: O hangi şekil olabilir? Neden öyle olduğunu düşünüyorsun?
Ö: Bu resimde kaç tane üçgen görüyorsunuz?
Hangi üçgenler birbirine benziyor hangileri daha farklı? Neden benzer neden farklı olduğunu düşünüyorsun?
Ö: Bu üçgenlerin boyutları nasıl değişiyor? İsterseniz cetvel de kullanabilirsiniz
Ö: Bu üçgenlerin birbirlerine nasıl benzer özellikleri neler?]

Aşama 2: Öğrenciler İş Başında (Students-at-Work)

Bu bölümde, öğretmen öğrencilerin yapacakları işi tanıtıyor. Her öğrenciden, başkaları tarafından görülmesi zor olacak şekilde resimlerde bir şekil yerleştirerek ya da hem iki boyutlu hem üç boyutlu olarak algılanabilecek bir resim çalışması ortaya koymaları istenir. Bu süreçte farklı tipte geometrik şekillerden ya da formlardan (iki boyutlu ve üç boyutlu) yararlanabileceklerinden bahsedilir. Öğretmen, öğrencileri farklı boyut ve yönelimlere sahip geometrik şekilleri denemeye, birleştirmeye ve geometrik şekilleri keşfetmeye teşvik eder. Bu süreç boyunca, öğretmen öğrencilerden eskiz defterlerine not tutmalarını ve çizim yapmalarını ister.

- İlk başta hayal etmeleri sonra çizimler yapmaları istenir. Ara ara kendi çalışmalarını gözlemler problemleri gözler.
- Öğretmen öğrencilerden eskiz defterlerine ilk olarak ne yapmayı planladıklarını (aklına gelenleri) yazıp çizmelerini ister. Sürekli çizim yapmalarını vurgular. Çalışmaları sırasında yaptıkları değişiklikleri ya da ölçüm yapıyorlarsa ölçüm diğerlerini not alabileceklerini hatırlatır.
- Öğretmen öğrencilerin şekil çiziminde ne derece o şekillerin geometrik özelliklerini düşündüklerini ve ne tip zorluklar yaşadığını bire-bire görüşmelerle

not alır ve ne yapmak istiyorlar? nasıl yapıyorlar gibi sorulara da yanıt arar.

- Öğrenciler bir kaç çizimden sonra, öğrencilerden aralarından birine karar vererek daha büyük bir kağıda çizerek ve boyayarak resimlerini tamamlamaları istenir.

Öğretmen, öğrenciler iş başındayken öğrencilerle birebir konuşmalar yapar.
[Ö: "Yaptıklarınıza ve bu çalışmanın izleyiciye ne anlattığına bir göz atalım."
Ö: "İşte size yardımcı olabilecek bir araç"
Ö: "Neler yapıyorsun biraz bahseder misin?"
Ö: "Neden bu şekilde yaptığından bahseder misin? Prizma olması için nasıl olması gerekir?
Ö: Burada ne yapmayı planladın? , Hangi şekilleri gizledin? Nelere dikkat ettin şekilleri yerleştirirken?]

Öğrenciler çalışırken, öğretmen hem bire bir öğrencilere yönelik hem de tüm sınıfa çalışmaların teşvik edici, cesaret veri konuşmalar yapar.

[Ö: "Pes etme. İyi bir iş çıkardın "
Ö: "Yapabileceğin birçok iş var, ama iyi bir başlangıç yapmışsın. Gerçekten odak noktasını yakalamışsın. "]
Ö: Yaptığın şeyleri beğendim, sadece bazen farklı şeyler yapmanı istiyorum, böylece biraz daha farklı yollar görebilir ve öğrenebilirsin.]

Aşama 3: Eleştiri (The Critique)

Öğrenciler resimlerini tamamladıktan sonra öğretmen tüm çizimleri duvara yerleştirir ya da akıllı tahtada yansıtır. Öğretmen, zaman kısıtlamasına bağlı olarak ya bütün öğrencilerden kendi çalışmalarını ya da sadece bir kaç öğrenciden çalışmalarını açıklamalarını ister. Bu sırada öğrencilerden arkadaşlarını gözlemlemelerini ve onların neler gördüğünü açıklamalarını ister.

[Ö: Burada hangi şekilleri saklamayı düşündün?
Ö: Ne tür zorluklar yaşadığından biraz bahseder misin?
Ö: İlk olarak nasıl başladın? Aklına ilk hangi fikirler geldi? Nasıl devam ettin çalışmana? Bu süreçte değişiklikler oldu mu? (Varsa) bu değişikliklerden bahseder misin?
Ö: Bu şeklin çizimi hakkında ne düşünüyorsunuz? Bir prizma hangi özelliklere sahip olmalı?
Ö: Sizce senin çalışmanı arkadaşlarından farklı kılan özellikler neler olabilir?
Ö: Şekilleri yerleştirirken hangi araçlardan yararlandın nasıl yerleştirdin her bir şekli?

STÜDYO ÇALIŞMASI 2: RESMİ TAMAMLA

Stüdyo Çalışması Planı

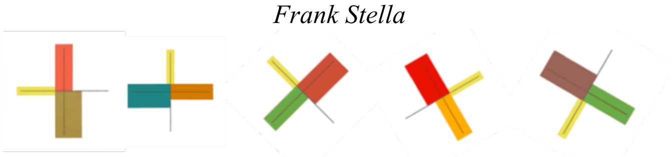
Aşama 1: Gösterim (Demonstration)

Aşama 1: Gösterim (Demonstration)

Bu bölümde, öğretmen Frank Stella ve Robert Mangold'un sanat çalışmalarını öğrencilere tanıtır. Bu sanatçıların daha çok geometrik şekiller kullandıklarından bahsedilir. Öğrencilere gerektiğinde üzerine çizim yapabilmeleri için bu resimlerin birer örneğini dağıtır. Öğretmen öğrencilerden bu resimlerde ne tip şekiller gördüklerini ve bu serilerdeki resimler arasında ne gibi benzerlik ya da farklılıklar gördüklerini not almalarını ister. Bu sorulara bireysel olarak yönelik verilen kağıtlara çizimler yapmaları ya da not almaları istenir.

[Ö: Bugün yeni bir sanatçının çalışmalarını inceleyeceğiz. Bu resimler Frank Stella ve Robert Mangold tarafından bazı geometrik şekiller kullanılarak yapılmıştır.

Frank Stella



Robert Mangold

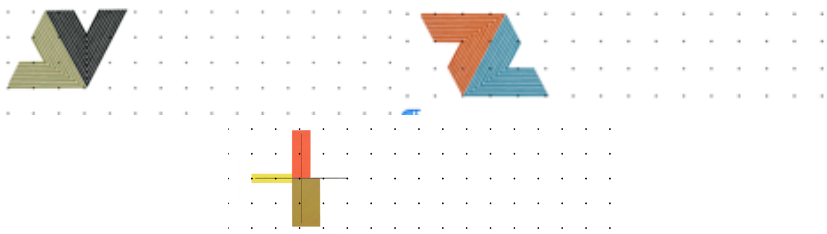
Gözlem yapmak üzere öğrencilere verilen sanatçıların eserleri

Daha sonra öğrencilerden gözlem notları toplanır. Öğretmen Frank Stella'nın V serilerini akıllı tahtada gösterir ve öğrencilere ne tip geometrik şekiller gördüklerini arkadaşlarıyla paylaşmaları istenir. Öğrencilerden gelen cevaplara göre öğretmen daha spesifik sorular yöneltir öğrencilere.

[Ö: Şimdi hep birlikte Frank Stella'nın V serilerini ayrıntılı olarak inceleyelim. Ne tip geometrik şekiller fark ettiniz bu resimlerde? Bu resimleri gördüğünüzde aklınıza ilk neler geldi?
O: Başka ne görüyorsunuz?
O: O hangi şekil olabilir
Ö: Bu sanat çalışmalarının en küçük parçası ne olabilir?
Ö: Siz de arkadaşınızın gördüğünü hayal edebiliyor musunuz? Arkadaşınıza siz de katılıyor musunuz?
O: Başka ne görüyorsunuz?
O: O hangi şekil olabilir? Neden öyle olduğunu düşünüyorsun?
[Ö: Sanatçı bu çalışmayı nasıl yapmış olabilir? Ne düşünüyorsunuz?
Eğer sanatçı bu sanatsal çalışmaları iki parçalı (2 V) olarak ele alırsak, sanatçı birinci parçadan sonra ikinci parçayı nasıl yapmış olabilir?

Birinci ve ikinci parça arasında nasıl bir ilişki olabilir? (Burada öğrencinin rotasyon, simetri alma gibi yöntemleri düşünmesi bekleniyor)
Birbirlerinin döndürülmüş halleriyse, nasıl bir döndürme gerçekleşmiş olabilir, kaç derece döndürülmüş olabilir?
Bu resimler arasında nasıl bir bağlantı var?
Bu resimlerin birbirleriyle benzer özellikleri neler olabilir?
Peki ne gibi farklılıklar var aralarında?
Neden en sondaki resim diğerlerinden farklı? Zihninde nasıl canlandırıyorsun? Açıklayabilir misin?
Ö: Neden böyle olduğunu düşünüyorsun?]

Öğrenciler döndürmede zorluk çekerlerse aşağıdaki noktalı kağıtlara çizim yapabilirler (aşağıda noktalı kağıdın bir örneği gösterilmiştir). Ayrıca somut materyaller (karton parçası) verilebilir rotasyonu uygulamaları için. Fakat ilk olarak zihinde canlandırmaları istenir. Bu sırada öğretmen öğrencilerin kendi düşüncelerini açıklamaları için akıl yürütme düşünme rutinlerinden "What makes you say that / Neden böyle düşünüyorsun?" i kullanır. Zaman kısıtlamasına bağlı olarak aşağıda gösterilen resimlerden sadece bir tanesi seçilebilir.



Döndürmeyi keşfetmek üzere öğrencilere verilen noktalı kağıt

Aşama 3: Öğrenciler İş Başında (Students-at-work)

Bu gözlem sürecini takiben, öğretmen başlangıç / orta / son isimli düşünme rutininden yararlanarak öğrencileri bu derste yapacakları çalışmaya yönlendirir. Bu aşamada, öğrenciler kendilerine anlatılan görevi yapmaya çalışır. Bu resimlerden birini seçmeleri istenir. Öğretmen "*bu sanatsal çalışma başka bir sanatsal çalışmanın sadece bir başlangıcı olsaydı nasıl devam ederlerdi*" sorusunu yönelterek öğrenciler kendi deneyimleri doğrultusunda bir sanatsal ürüne ulaşmaları beklenir.

Ö: Eğer bu sanat çalışması bütün bir sanat çalışmasının sadece bir başlangıcı olsaydı, bir sonraki adımı ne olurdu?

Öğretmen öğrencilerden eskiz defterlerine ilk olarak ne yapmayı planladıklarını (aklına gelenleri) yazıp çizmelerini ister. Birden fazla çizim yapmalarını vurgular. İlk başta taslak çizimler çizerek ne yapabileceklerini hayal etmeleri önerilir. Daha sonraki süreçlerde eğer öğrenci şekilleri geometri bilgileri kullanmadan çizme eğilimde olursa öğretmen şekillerin büyüklük ve açılarına yönelik daha belirli sorular yöneltir.

[Ö: İlk olarak zihninizde canlandırın sonra eskiz defterinize çizimler yapın. Nasıl olduğunu görün.
Onu döndürdüğünde nasıl bir şekil ortaya çıkacağını tahmin edin
Nasıl döndürüldü her bir şekil. Hangi açılarda hangi yönde üçgenlerin büyüklükleri neler?
Hangi noktadan döndürüldü
Aynı birimi sürekli döndürürsen nasıl bir şekil hayal ediyorsun? Başka şekilde döndürseydin nasıl olurdu peki?

Bu şekil döndürüldüğünde nasıl olur? Büyüklüğü değişir mi?]

Problemli durumları fark etmeleri için öğrencilerden çalışmalarını ara ara kontrol etmeleri gözlemlemeleri istenir. Öğretmen birebir bir görüşmelerde ne yapmayı planladıklarını nasıl yaptıklarını saptar. Bu sırada neresi olmuyor, niçin olmadı, neden vazgeçtin, tekrar kontrol edebilir misin gibi sorular yöneltir. Hata yapmaktan çekinmemeleri vurgulanır. Öğrenciler zorluk yaşamaları durumunda bir kağıt parçasının nasıl döndüğünü test edilmesi gibi somut materyallerden yararlanabilirler. Bu şekilde deneme yanılma yoluyla bir şeklin nasıl döndürülebileceğini hayal etmeleri sağlanabilir. Öğrenciler döndürmek ile ilgili farklı olası durumları taslak olarak çizmeye teşvik edilir (aşağıda bazı olası durumlar belirtilmiştir.) Burada öğrenciler döndürme üzerine odaklanmayabilirler. Bu durumda takla atma (flip) ve yansıtma gibi dönüşüm geometrisinden de yararlananlar olabilir.

Kullanılabilecek olası durumlar:
Büyük üçgen içinden farklı büyüklükte üçgen kesme
Öteleme-Dönme-Yansıma tekniklerinin farklı kombinasyonlarını kullanma
Farklı tipte bir üçgen kullanma (dik üçgen)
Üçgeni farklı bir kenardan kesme yada üçgenin tepeden kesilmesi
İkili, üçlü, dörtlü, beşli ya da altılı üçgen dizilimleri

Bu sırada çalışmalarının açıklamasını yazmaları istenir. Nasıl döndürdüklerini açı ve döndürme yönlerini de belirterek açıklamaları istenir. Bu şekilde öğrenciler geometri bilgilerini kullanmaya teşvik edilir. Öğrenciler taslak çizimler yaptıktan sonra içlerinden birini seçerek büyük resim kağıdına yapmak istedikleri çizimi aktarmaları istenir.

Aşama 4: Eleştiri (The Critique)

Öğretmen, eleştiri bölümüne öğrencilerden hem kendi çalışmalarını hem de arkadaşlarının çizimlerini incelemelerini isteyerek başlar. Öğrencilerden ne tip stratejiler kullandıklarından, nasıl yaptıklarından bahsetmelerini ister. Öğrenciler kullandıkları birim şekilden ve birim şekli döndürerek nasıl bir şekil oluşturduklarından bahseder. Öğretmen bu döndürmeyi nasıl yaptıklarına dair sorular yöneltir.

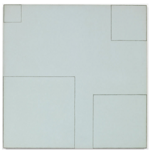
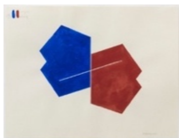
[Ö: Hangi şekilden başladın döndürmeye? Aklına ilk hangi fikirler geldi? Nasıl devam ettin çalışmana? Bu süreçte değişiklikler oldu mu? (Varsa) bu değişikliklerden bahseder misin?
Ö: Nasıl bir şekil oluşturdun?
Ö:Ne tür zorluklar yaşadığından biraz bahseder misin?
Ö:Burada döndürme yaparken nelere dikkat ettin?
Ö:Başka türlü nasıl döndürebilirdin?
Ö:Döndürülen şekilleri birbirine benzerliği konusunda ne düşünüyorsun?]

STÜDYO ÇALIŞMASI 3: SÜNDÜR

Stüdyo Çalışması Planı

Aşama 1: Gösterim (Demonstration)

Bu bölümde, öğretmen yine Minimalist sanatçıların resimlerini gösterir. Öğrencilerden Agnes Martin, Mel Bochner, Robert Mangold tarafından yapılan aşağıda gösterilen resimleri büyük ölçekte tekrar yapmaları istenir. Bu resimler, şekilllerin dikdörtgensel kanvasta yerleştirme düzenine (sıralanmış vs. dağınık) ve şekillerin doğasına (birleşik vs. ayrık geometrik şekiller) göre gruplandırılmıştır. Öğrenciler 1:4 ölçekte seçtikleri resmi yeniden oluşturacaklardır. Diğer bir ifadeyle, öğrenciler küçük ölçekli sanat çalışmasını 4 katı büyüklüğünde büyük ölçekli bir resim haline dönüştüreceklerdir. Bu resimlerin çıktıları alınarak öğrencilere dağıtılır.

	Şekiller	
	Ayrık	Birleşik(Gizli
Sıralı	 Robert Mangold	 Mel Bochner
Dağınık	 Agnes Martin	 Mel Bochner

Büyük ölçekli resim kağıdına yeniden çizilmek üzere öğrencilere verilen sanatçıların eserleri

[Ö: Şimdi hep birlikte bu sanatçıların resimlerini yeniden yapacağız.]

Öğretmen karmaşıklık derecesi ölçeğini öğrencilere dağıtır. Öğrencilerden 1-10’ kadar olan ölçeğe her bir resmi yerleştirmeleri istedir. Bu ölçek öğrencilerin resimleri daha büyük bir resim kağıdına aktarımanın ne derece zorluk olduğuna dair algılamalarını gösterir.

[Ö: Burada dört tane resmimiz var. Bunların hangisini yaparken daha çok zorlanırdınız? Bu ölçeğe basitten karmaşığa olacak şekilde nasıl yerleştiridiniz resimleri?
Ö: Neden oraya yerleştirdiğini açıklar mısın? Açıklarken kendini rahat hissedebilirsin. Aynı resmi birden fazla yere de yerleştirebilirsin, bazı açılardan kolay olurken bazı açılardan da zor olabilir bu resmi yapmak.
Ö: Pekii neden bu resmi seçtin? Neden bu resmi yapmanın zor olduğunu düşünüyorsun?
Ö: Resimleri yapmaya başlamadan önce bu resimlerle ilgili aklınıza gelen ilk sorular neler?]

Aşama 2: Öğrenciler İş Başında (Students-at-work)

Bu aşamada öğrenciler artık kendileri çalışmaya başlıyor. Öğretmen öğrencilere süreç içinde öğrencilerin çalışmaya nasıl başladıklarını nasıl devam ettiklerini ve hangi noktalarda sıkıntı yaşadıklarına dair gözlemler yapar. Bu sırada öğrenciler ne tip teknikler kullanıyor gözlemin odak noktasını içerir. Öğretmen öğrencilerle birebir görüşmelerle düşünme süreçlerini araştırır. Öğretmen öğrencilerden bu resimlerdeki şekilleri daha büyük bir resim kağıdına yerleştirmek üzere kolay ve doğru bir şekilde yerleştirmek için çeşitli çözümler ya da yöntemler araştırmalarını ister. Bu süreçte ara ara ihtiyaç duydukça eskiz defterlerine çizim yapmalarını hatırlatır. Bu sırada öğretmen aşağıdaki soruları birebir görüşmeler sırasında öğrencilere yöneltir.Öğrencilerden bu resimdeki parçaları ve parçalar arasındaki ilişkileri ortaya çıkarmaları istenir. Öğrenci daha büyük ölçekli bir tabloya parçaları uygun bir şekilde yerleştirmede zorluk çekerlerse, sürekli denemelerini ve parçalar arasında nasıl bir ilişki olduğunu düşünmelerini önerir ve sratejilerini değiştirdiklerinde nasıl bir değişiklik olacağını keşfetmeye teşvik eder. Bu sırada öğretmen her bir resmi çizerken sürekli gözlemlemeyip kontrol etmelerini ister. Bu şekilde hatalarını ararken ne gibi noktalara odaklandıklarını keşfeder öğretmen. Bu hatalı durumları eskiz defterlerine not olmaları ve çizimlerini yeniden düzenlemeleri istenir. Bu süreç bir şeklin yerinin olabildiğince doğru belirlenmek üzere araştırılmasını içerir.

- Öğretmen her bir resmi ve o resmin 4 katı büyüklüğündeki resim kağıdını sırayla dağıtır. Yani bir resim bittikten sonra diğerine geçilir. Zaman kısıtlamasına bağlı olarak her bir resmi tamamlamak için belirli bir süre verilir (örn. 5dk.) ve öğrencilerden resimleri boyamaları istenmeyebilir.
- Öğretmen ilk olarak öğrencilerden taslak bir çizim yapmalarını iste. İlk aşamada herhangi bir nesne (cetvel, kalem, kalem kutusu) kullanmadan çizim yapmaları istenir. Bu sırada öğretmen şekilleri nasıl yerleştirdiklerine dair öğrencilerle birebir görüşmeler yapar. Öğretmen öğrencilere sürekli çizimlerini gözlemleyip hatalı yerleri not alıp bu doğrultuda çizimlerini yeniden düzenlemeleri istenir.

[Ö: Sizden çalışmaya başlamadan önce resim kağıtlarınıza bu resimdeki herbir şekli yerleştirmenizi istiyorum.]
Ö: Bu şekilleri büyük bir resim kağıdına nasıl yerleştirdin?
Ö: Örneğin .... şeklini .....şeklinin yanına nasıl yerleştirdiğinden bahseder misin?
Ö: Bu şekli neden biraz daha aşağıya yerleştirdin?]
Ö: Bu şekli biraz daha aşağı ya da yukarı yerleştirseydin nasıl olurdun?
Ö: Bu şekli biraz daha yatık yapsaydın nasıl olurdu?
Ö: Çalışmanı tekrar gözlemler misin? Herhangi bir problemli durum görüyor musunuz?]

Öğrenciler çizimleri tamamladıktan sonra resimlerden birini seçilir ve öğrencilerden bu resmi çizerken ne tip stratejiler kullandıklarına dair ve hangi sırayla şekilleri çizdiklerine dair not almaları istenir. Bir başkası bu çizimleri yapacak olsaydı nasıl yaptıklarını açıklamaları istenir.

Aşama 3: Eleştiri 1 (The Critique)

Öğrenciler çizimlerini tamamladıktan sonra çizimleri akıllı tahtada gösterilir ya da duvara asılır. Her bir öğrenciden çizimlerini anlatmaları istenir. Aşağıdaki sorular öğrencilere yöneltilir.

> *[Ö: İlk olarak nasıl başladın? Aklına ilk hangi fikirler geldi? Nasıl devam ettin çalışmana? Bu süreçte değişiklikler oldu mu?*
> *Ö:Nerede zorluk çektiler (en kolay en zor) Nasıl çözdü bu problemi ya da çözülebilir?)*
> *Ö: Öğrenciler ne tip strateji kullandıkları söyler. Nasıl yerleştirdiklerini ne tip kriterler düşündüler?*
> *Ö: Çizimleriniz arasında ne gibi farklılıklar var? Neden öyle olabilir?*
> *Ö: Arkadaşlarınızın ya da kendi çizimlerinizde nereler düzeltilebilir?]*

Eleştiri 2 (The Critique)

Öğrenciler birbirlerinin çizimlerini inceleyip stratejilerini öğrendikten sonra, resimlerden biri seçilerek cetvel yardımıyla ne kadar doğru bir çizim yapıp yapmadıklarını test etmeleri istenir. Bu çalışmada Agnes Martinin sanat eseri seçildi. Öğrencilerden ölçüm yaparak çizimlerinin üzerine not almaları istenir. Bu şekilde öğrenciler ölçüm yaparken hangi noktalara dikkat ediyor fark edilebilir. Öğrencilerden cetvelle ölçüm yaparken nelere dikkat ettiklerini açıklamaları ve çizimlerindeki sıkıntıları anlatmaları istenir.

> *[Ö: Nereleri ölçtün?*
> *Ö: Senin yaptığın çizimle karşılaştırdığında ne gibi farklılıklar var?*
> *Ö: Dairenin yerini nasıl belirledin?]*

J. CURRICULUM VITAE

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EDUCATION

Degree	Institution	Year
Doctor of Philosophy	Elementary Mathematics Education, Middle East Technical University, Ankara-TURKEY	2019
Master of Science	Elementary Science and Mathematics Education, Middle East Technical University, Ankara-TURKEY	2013
Bachelor of Science	Elementary Mathematics Education, Hacettepe University, Ankara-TURKEY	2009

WORK EXPERIENCE

Year	Place	Enrollment
2010-…..	Department of Mathematics and Science Education, Middle East Technical University, Ankara-TURKEY	Research Assistant
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FOREIGN LANGUAGES

English (Advanced) and German (Beginner)

RESEARCH INTERESTS

Mathematics and Visual Arts, Statistics in Mathematics Education, Critical Thinking

HOBBIES

Drawing, Sculpture, Trekking

K. TURKISH SUMMARY/TÜRKÇE ÖZET

SANAT STÜDYOSUNDA MATEMATİK İLE OYNAMAK: STÜDYO DÜŞÜNME TABANLI ORTAM BAĞLAMINDA ÖĞRENCİLERİN GÖRSEL-UZAMSAL DÜŞÜNME SÜREÇLERİ

Matematik eğitimindeki yeni yaklaşımlar, matematiksel bilginin günlük yaşam durumlarına uygulanmasını vurgulamaktadır. Matematik derslerinde öğrenilen bilgilerin özellikle mühendislik, fen ve mimarlık gibi mesleklerde uygulanabilmesi önem kazanmıştır (Quinn ve Bell, 2013). Ancak, günümüzdeki eğitim anlayışının bilginin uygulanmasını öteleme eğiliminde olduğu tartışılmaktadır. Oysa, öğrenciler geleceği de öngörerek bilgilerini mevcut girişim ve etkinliklerinde kullanabilir (Perkins, 2013). Başka bir deyişle, öğrencilerin bilgiyi uygulayarak öğrenmesi gelecekteki kapasitelerini olumlu yönde etkileyecektir (Papert ve Harrel, 1991).

Papert'in çalışmalarına dayanan inşacılık (constructionism) kuramı bu yaklaşım üzerine önerilmiş bir eğitim kuramıdır. İnşacılık eğitim kuramı, öğrencilerin kişisel olarak anlamlı ürünler veya projeler yaparak öğrendikleri süreçlere önem verir (Paper ve Harrel, 1991). Bu kuram, öğrenme ortamı tasarlamak ve bu ortamda öğrencilerin anlam inşa etme süreçlerini yorumlamak için bir mercek olarak kullanılabilir. Bu ortam, öğrencilerin el ve beyin koordinasyonunu geliştirerek öğrencileri düşünmeye, öğrenmeye ve çeşitli materyaller kullanarak kendi duygu ve düşüncelerini ifade etmeye teşvik eder. Bu bağlamda, inşacılık kuramı öğrencilere bilgilerini uygulayabilme fırsatı yaratarak matematik eğitimine yeni bir vizyon sağlayabilir (Papert ve Harrel, 1991).

Matematik eğitimine yeni bir vizyon sağlamanın yanında, bu kuramın sanat eğitimi ile oldukça uyumlu bağlamlardan biri olduğu görülmektedir (Papert ve Harrel, 1991). Sanat stüdyosunda öğrenciler, kendi sanat çalışmalarını ortaya koymak üzere çalışırlar. Görsel sanatlar eğitimi inşacılık kuramı çerçevesinde matematik eğitimi için önemli ve zengin bir bağlam oluşturabilir. Görsel sanatlar matematik eğitiminde öğrencileri motive etmek ve öğrencileri matematik öğrenmeye teşvik etmek için bir bağlam olarak kullanılabileceği gibi, görsel sanatlar ve matematik eğitimi ile birlikte yeni olası disiplinler arası etkileşimlerin (ör. STEAM (Fen, Teknoloji, Mühendislik, Sanat, Matematik)) önemli bir parçası olarak görülebilir (Goldsmith, Hetland, Hoyle ve Winner, 2016).

Görsel sanatların matematikle veya diğer öğrenme alanlarıyla nasıl etkileşimde olacağı araştırmacılar tarafından tartışılmıştır. Görsel sanatların matematiğe entegrasyonu veya görsel sanatlarda öğrenmenin matematiğe aktarılması üzerine birçok çalışma yapılmıştır. Çalışmaların bazıları (Hanson, 2002; James, 2011; Marino, 2008) deneysel metot kullanarak görsel sanatların öğrencilerin matematikteki performansları üzerindeki olumlu etkilerini göstermiştir. Diğer bir yandan, bazı araştırmalar (Ben-Chetrit, 2010; Walker, Winner, Hetland, Simmons ve Goldsmith, 2011.) yarı deneysel metot veya korelasyon analizi kullanarak görsel sanatlar eğitimi almış öğrenciler ile almamış öğrenciler arasında matematik performansları açısından fark olup olmadığını incelemişlerdir ancak tutarlı sonuçlara ulaşamadıkları gözlenmiştir. Deneysel çalışmaların birçoğu kontrol grubu içermemekle birlikte öğrenciler bu gruplara rastgele (yansız) olarak atanmamıştır. Bu yöntemsel problemlere ek olarak, bu çalışmaların çoğu, sanat tabanlı etkinliklerin nasıl tasarlandığı, hangi durumlarda belirli sonuçlara ulaştıklarına dair yeterince bilgi sağlamamaktadır (Winner, Goldstein ve Vincent-Lancrini, 2013). Bu problem, bu çalışmanın yürütülmesindeki en önemli etmen olarak görülmektedir.

Sanatın başka alanlara entegre edilmesi üzerinde yapılan çalışmaların çoğunun yeterli bilgi sunmaması, bu konuda bir teorik çerçeveye dayanarak güçlü bulgular edinmemizi sağlayan çalışmalara ihtiyaç doğurmaktadır. Bu ihtiyaç, sanatın diğer alanlardaki etkisi üzerine olan OECD (Ekonomik Kalkınma ve İşbirliği Örgütü) raporunda Winner ve çalışma arkadaşları (2013) tarafından da "Sanatın diğer alanlardaki dönüştürücü etkilerine ilişkin iddialar, kanıtları aşıyor. Bu durum iddiaların yanlış olduğu anlamına gelmez. Ancak, henüz doğru oldukları kanıtlanmamıştır." (s. 41) ifadesi ile de belirtilmiştir. Bu bağlamda son zamanlarda yapılan çalışmalardan birinde, Goldsmith, Hetland, Hoyle ve Winner (2016), geometrik akıl yürütme, uzamsal düşünme ve sanatsal zihinde yaratma (artistic envisioning) arasındaki ilişkiye yönelik kanıtlar ortaya koymuştur. Görsel-uzamsal düşünmenin, görsel sanatlar ve matematiğin ortak noktası olarak ele alınmasını önermişlerdir. Görsel-uzamsal düşünme sadece görsel sanatlar ve matematiğin ortak noktası olarak görülmemekle birlikte, STEM (Fen, Teknoloji, Mühendislik, Matematik) ve sanat/mimarlık gibi alanların da kesişimi olarak görülmektedir (Newcombe, 2010; 2013).

Öğrencilerin görsel-uzamsal düşünme süreçlerini sanat çalışması yapılan ortamlarda inceleyebilmek için, düşünme süreçlerinin bu ortamlarda nasıl görünür hale getirilebileceğini düşünmek oldukça önemlidir. Bu çalışmada, Matematik-Sanat Stüdyosu Ortamı araştırmacı tarafından bu amaca ulaşmak için tasarlanmıştır. Matematik-Sanat Stüdyosu Ortamı, bu çalışmada, araştırmacının öğrencilere (1) minimalizm akımı sanat eserlerini tanıttığı, öğrencilerden onları gözlemlemelerini, (2) kendi sanat çalışmalarını yaratmalarını ve (3) kendi ve arkadaşlarının sanat çalışmalarını eleştirmelerini istediği bir ortam olarak tanımlanmıştır. Bu ortam, geometrik şekillerin doğrudan kullanıldığı minimalizm akımı eserlerinden yararlanarak Stüdyo Düşünme Çerçevesi (Studio Thinking Framework) (Hetland vd., 2013) ve görsel-uzamsal düşünme üzerine yapılan çalışmalar temel alınarak tasarlanmıştır. Stüdyo Düşünme Çerçevesi, görsel sanat atölyelerinde öğrenmenin ve öğretmenin doğasını tanımlar ve sanatın diğer öğrenme alanlarıyla entegrasyonu üzerine çalışmalar tasarlamak için bir araç olarak kullanılabilir (Sheridan, 2011). Stüdyo Düşünme Çerçevesi, sanat stüdyosunda öğretilen bazı düşünme eğilimlerini (örn. gözlem yapma, zihinde canlandırma, araştırma) tanımlamakla birlikte, bu stüdyonun yapısını oluşturan üç temel bölümü tanımlar; (1) gösterim (sanat eserlerini tanıtma, teknik gösterme), (2) öğrenciler iş başında (öğrencilerin kendi başlarına sanat çalışması yapması) ve (3) eleştiri (kendi ya da başkalarının sanat çalışmasını açıklama ve eleştirme). Bu çerçeve, sanat stüdyolarının doğasını kapsamlı bir şekilde tanımladığından, Matematik-Sanat Stüdyo Ortamı için bir temel oluşturmak için kullanılmıştır. Matematik-Sanat Stüdyo Ortamı ise, bu çalışmada görsel sanatlar ve matematik bağlamında öğrencilerin görsel-uzamsal düşünme süreçlerini araştırmada bir araç olarak kullanılmıştır. Bu bağlamda, bu araştırma öğrencilerin görsel-uzamsal düşünme süreçlerini Matematik-Sanat Stüdyo Ortamı gibi stüdyo düşünme tabanlı bir ortamda araştırmayı amaçlamıştır. Çalışmanın amacı doğrultusunda, çalışmanın araştırma sorusu şöyledir:

•Stüdyo Düşünme yoluyla öğrencilerin zengin geometrik sanat çalışmaları yaptıkları bir Matematik-Sanat Stüdyo Ortamında 7. sınıf öğrencilerinin görsel-uzamsal düşünme süreçleri nasıldır?

1.1. Çalışmadaki Önemli Terimler

Matematik-Sanat Stüdyo Ortamı, bu çalışmada öğrencilerin görsel-uzamsal düşünme süreçlerinin incelenebilmesi için araştırmacı tarafından hazırlanmıştır. Stüdyo Düşünme Çerçevesi (Hetland vd., 2013) tabanında geliştirilen bir ortamdır. Bu çalışmada, bu ortam stüdyo çalışmaları (geometrik içerikli minimalizm sanat eserlerinin kullanılarak görsel-uzamsal düşünmeyi gerektirecek etkinlikler) ve bu stüdyo çalışmalarının stüdyo düşünme aracılığıyla uygulanması, öğrencilerin bu ortama gösterdiği reaksiyon, öğretmenin/araştırmacının rolü, ve fiziksel ortamın yapısı gibi elemanların organik birleşimini içeren bir ekoloji olarak tanımlamıştır.

Stüdyo Düşünme Çerçevesi sanat stüdyosunda öğretilen bazı düşünme eğilimlerini (örn. gözlem yapma, zihinde canlandırma, araştırma) tanımlamakla birlikte, bu stüdyonun yapısını oluşturan üç temel bölüm (1) gösterim (sanat eserlerini tanıtma, teknik gösterme), (2) öğrenciler iş başında (öğrencilerin kendi başlarına sanat çalışması yapması), (3) eleştiri (kendi ya da başkalarının sanat çalışmasını açıklama ve eleştirme) tanımlar. Alanyazın bölümünde daha detaylı ele alınmıştır.

Görsel-Uzamsal Düşünme uzayda nesnelerin şekilleri ve dizilimleri hakkında ve nesnelerin manipülasyonu, nesnelerin hareketi gibi uzamsal süreçler hakkında düşünme olarak tanımlanır (Hegarty, 2010, p.266). Görsel-uzamsal düşünmenin farklı tipte göstergeleri tanımlanmıştır (Newcombe ve Shipley, 2015). Bu çalışmada doğrudan Newcombe ve Shipley'in çalışmasından yararlanılmasa da, bu çalışma öğrencilerin düşünme süreçlerini araştırmak için önemli bir temel oluşturur. Newcombe ve Shipley görsel-uzamsal düşünmeye yönelik 4 kategori belirlemiştir: içsel, dışsal, durgun ve dinamik. İçsel özellikler, nesnelerin şekilleri, düzenleri, boyutları, yönelimleri (statik) ve nesnelerin bu özelliklerinin dönüştürülmesi (dinamik) ile ilgilidir. Diğer yandan, dışsal özellikler nesnelerin aralarındaki ilişki ve nesne ve bir referans çerçevesi (bakış açısı) arasındaki ilişki (statik) ve bu ilişkilerin değişimi (dinamik) ile ilgilidir. Her bir kategori ile ilgili örnek durumlara alanyazında yer verilmiştir.

Sanat Stüdyosu öğrencilerin belirli bir süre boyunca projeleri üzerinde çalıştıkları fiziksel bir çevreye olarak tanımlanmaktadır (Gandini, Hill, Cadwell, ve Schwall, 2005; Shaffer, 2005). Bu fiziksel ortamda çeşitli materyaller (kağıt, makas, karton, açıölçer vb.), öğrencilerin materyallerini muhafaza etmelerini sağlayan dolap, sanat eserlerinin arkadaşlarıyla paylaşmalarını ve sanat eserlerini gözlemlemelerini kolaylaştıran bir akıllı tahta bulunmaktadır. Stüdyonun düzeni stüdyo çalışmasına göre yer değiştirebilir. Öğrencilerin istedikleri zaman ara verebilecekleri bir ortamdır.

1.2. Çalışmanın Önemi

Bu çalışmanın önemi, çalışmanın bulgularının alanyazına ve eğitim ortamlarına katkıları açısından ele alınacaktır. İlk olarak bu çalışmanın alanyazına katkıları düşünüldüğünde, alanyazında sanatın diğer alanlara entegre edilmesine yönelik tartışmalara, görsel sanatlar ve matematiğin etkileşiminden doğan düşünme süreçlerini açığa çıkararak bir katkı sağlanmaktadır. Çalışmanın bulguları, görsel sanatlar ve matematik nasıl entegre edilebilir veya bu entegrasyon öğrencilerin ne tip düşüncelerini açığa çıkarır gibi tartışmalara başlangıç niteliğinde açıklık getireceği düşünülmektedir. Alanyazına diğer bir katkısı ise, bilişsel bilimler ve psikoloji alanlarında tanımlanan görsel-uzamsal düşünmenin ve sanat eğitiminde önerilen stüdyo düşünme çerçevesinin görsel sanatlar ve matematik bağlamına adapte edilmesidir. Bu sayede, bu teorik çalışmalar matematik ve sanat gibi farklı bağlamlarda araştırılarak zenginleştirilebilir. Aynı zamanda, stüdyo düşünme çerçevesinin matematik eğitimi bağlamına aktarılmasının matematik eğitimi alanına önemli derecede katkı sağlayacağı düşünülmektedir. Diğer bir yandan, görsel-uzamsal düşünmenin görsel sanatlar ve matematik bağlamında incelenmesinin, STEAM (Fen, Teknoloji, Mühendislik, Sanat, Matematik) çalışmalarının tasarımı için de bir temel oluşturacağı öngörülmektedir.

Çalışmanın bulgularının eğitim ortamlarına sağlayacağı katkı şöyledir. Eğitim ortamlarının önemli bir parçası olan öğrenciler açısından çalışmanın olası katkıları incelendiğinde, farklı tipte düşünen öğrenciler, sözel olarak, yazarak, çizim yaparak, ya da vücut dillerini kullanarak farklı yollarla kendilerini ifade etme olanağı bulabilir. Özellikle hem görsel sanatlar hem de matematiğe ilgili duyan öğrenciler kendilerini bu matematik-sanat stüdyosunda keşfetme imkanı bulabilir. Diğer bir yandan, bu çalışma, öğrencilerde matematiğin sadece hesaplamalardan ibaret olmadığına aynı zamandan uzamsal ilişkileri analiz etmeyi de içerdiğine yönelik farkındalık oluşturabilir. Öğretmenler açısından olası katkısı incelendiğinde, bu çalışma öğretmenler için stüdyo düşünmenin ne zaman, hangi durumda, nasıl uygulanacağına yönelik bir kılavuz niteliği taşımaktadır. Aynı zamanda öğretmenlerin kendi performanslarını ve bu ortamdaki rollerini anlama fırsatı da sağlayacağı öngörülmektedir. Ayrıca, öğretmenlere, bu tip stüdyo düşünme tabanlı ortamlarda öğrencilerin görsel-uzamsal düşünme süreçlerinin farkına varması ve yorumlayabilmesi için öğrencilerin düşünme süreçlerine ilişkin somut örnekler sunar. Son olarak, bu çalışmanın önemi eğitim materyali geliştirenler açısından incelendiğinde ise, bu çalışmada tasarlanan stüdyo çalışmalarının okulda, bilim ve sanat merkezi ve müzeler gibi okul dışı ortamlarda kullanılabilecek materyal niteliği taşımaktadır. Örneğin, bu materyaller bilim sanat merkezlerinde hem matematiğe hem de görsel sanatlara yetenekli öğrencilerin ihtiyaçlarına yönelik program geliştirilmesinde kullanılabilir. Aynı zamanda, bu çalışma teoriden uygulamaya geçişin bir örneği olarak, bu materyallerin bu ortamlarda nasıl kullanılabileceği hakkında da zengin örnekler içermektedir.

2. ALANYAZIN TARAMASI

2.1. Görsel Sanatlar ve Matematik

Matematik ve görsel sanatlar entegrasyonu üzerine yapılan çalışmaların odaklandıkları konular şöyledir: Eğitim etkinliklerinin tasarlanması (Frantz, Crannell, Maki ve Hodgson, 2006; Hart ve Heathfield, 2017; Jarvis ve Adams, 2007; Kappraff, 1986; O‘ Dell, 2014; Wilcock, 2014), sanat tabanlı eğitimin matematik performansına etkisi (Ben-Chetrit, 2010; Hanson, 2002; James, 2011; Marino, 2008), matematiğe yönelik tutum (Healy, 2004; Marino, 2008), estetik ve matematiksel problem çözme (Sinclair, 2006), geometri ve görsel sanatlar arasındaki ilişki (Goldsmith vd., 2016; Walker vd., 2011).

Nicel araştırma yöntemi kullanan deneysel çalışmalardan bazıları (Hanson, 2002; James, 2011; Marino, 2008) sanat tabanlı öğrenme ortamının matematik ve geometri performansına olumlu etkisini ortaya koyarken, başka bir çalışmada (Ben-Chetrit, 2010) aslında görsel sanatların geometri performansında anlamlı olarak bir etki yaratmadığı bulunmuştur. Nicel araştırmaların yanında, nitel analiz yöntemiyle (Shaffer, 1997) ve korelasyon analizi yoluyla yürütülen çalışmalar da (Goldsmith vd., 2016; Walker vd., 2011) bulunmaktadır. Bu çalışmalar alanyazına önemli katkılar sağlamalarına rağmen sanat eğitiminden matematik eğitimine transferin nasıl gerçekleştiği hala sorgulanmaktadır. Buna ek olarak, hala sanat ve matematik eğitiminin birleşimine yönelik bir teorik çerçeve bağlamında nasıl bir ortam yaratılabileceğine dair çalışma yok denecek kadar azdır.

Bunların yanı sıra, son zamanlarda STEAM üzerine pek çok çalışma yürütülmektedir. Bu çalışmaların bir kısmı yaratıcı ve yenilikçi beceriler kazandırmak üzere sanatı bir katalizör olarak ele alırken (Clapper ve Lafratte, 2015, Connor vd., 2015, Ghanbari, 2015, Land, 2013, Madden vd., 2013), bir diğer kısmı ise, katalizör etkisinin yanında sanatın kendi başına bir disiplin olduğunu vurgulamıştır (Guyotte, Sochacka, Constantino, Walther, ve Kellam, 2014, Quigley ve Herro, 2016, Sochacka, Guyotte ve Walther, 2016). Bu çalışmalar çoğunlukla problem tabanlı, stüdyo tabanlı öğrenme, araştırma tabanlı öğrenme ve proje tabanlı öğrenme süreçlerinin önemini vurgulamışlardır. Fakat, bu çalışmalarda bahsedilen öğrenme süreçlerinin nasıl kullanıldığı ve bu çalışmaların nasıl yürütüldüğü konusunda verilen bilgi oldukça azdır.

Ulusal alanyazında ise, görsel sanatlar ve matematiğin birleşimine yönelik oldukça az çalışma bulunmaktadır (Alyeşil Kabakçı ve Demirkapı, 2016; Erdogan-Okbay, 2013; Ugurel, Tuncer ve Toprak, 2012). Ugurel ve çalışma arkadaşları (2012) görsel sanatlar ve matematiği birleştirme üzere öğretmen adaylarının ders tasarlamalarını incelemiştir. Diğer bir çalışmada Erdogan-Okbay (2013) sanat tabanlı matematik etkinliklerinin öğrencilerin matematiğe yönelik motivasyonlarına etkisini araştırmıştır. En son olarak, Alyeşil ve çalışma arkadaşları (2016) öğrencilerin görsel-uzamsal düşünmelerini geliştirmek üzere sanat ve matematiği entegre eden bir ders tasarlamıştır.

Özetle, ulusal ve uluslararası alanyazında görsel sanatlar ve matematiğin birleşimine yönelik deneysel çalışmalardan nitel çalışmalara kadar çeşitli çalışmalar yapılmıştır. Görsel sanatlar ve matematiğin birleşimini ele alan bu çalışmalarda etkinliklerin, derslerin içeriklerinin, ve bu etkinliklerin ve içeriklerin öğrenci performansını nasıl etkilediğine dair yeterince bilgi sunulmamıştır. Bu bağlamda, Winner ve çalışma arkadaşları (2013) teorik çerçeveye dayanan çalışmalara ihtiyaç duyulduğunu önemle vurgulamıştır.

2.2. Teorik Çerçeve

2.2.1. Stüdyo Düşünme ve Sanatsal Düşünmenin Tanımlanması

Bu çalışmada matematik-sanat stüdyo ortamını oluşturmak üzere başta Stüdyo Düşünme Çerçevesi olmak üzere, Harvard Üniversitesi’ne ait Project Zero kapsamında geliştirilen Stüdyo Düşünme (Hetland vd., 2013) ve Sanatsal Düşünme

Çerçevelerinden (Tishman ve Palmer, 2006) yararlanılmıştır.

Stüdyo Düşünme Çerçevesi görsel sanatlar dersinin gözlemlenmesi sonucunda belirlenmiştir. Stüdyo düşünme, görsel sanatlar dersinde öğretmenlerin öğrencilere kazandırmayı planladığı düşünme eğilimleri olarak tanımlanmıştır. 8 temel düşünme eğilimi belirlenmişlerdir. Bu düşünme eğilimleri; Zanaat geliştirme (Develop Craft), Uğraşma ve sürdürme (Engage and Persist), Gözlem yapma (Observe), Zihinden Canlandırma (Envision), Dışa Vurma (Express), Yansıtıcı Düşünme (Reflect), Esneme ve Araştırma (Stretch and Explore), Sanat Dünyasını Anlama (Understand Art World) şeklindedir. Bu düşünme eğilimlerinin yanı sıra, projede stüdyo ortamının üç temel bölümü belirlenmiştir. Bu bölümler, Gösterim (Demonstration), Öğrenciler İş Başında (Students-at-Work), ve Eleştiri (Critique) şeklindedir. Bu üç temel yapı öğretmenin üç temel ana yolla öğrencilerle iletişim kurmasına imkan sağlamıştır.

Sanatsal düşünme çerçevesi ise, Harvard Üniversitesi Project Zero tarafından öğretmenler tarafından kullanılmak üzere geliştirilmiş olan bir programdır. Bu program sanat yapmaktan ziyade, sanat eserlerinin kullanarak sanatın eğitim ortamında onanmasına odaklanır. Bu program çerçevesinde 6 temel düşünme eğilimi tanımlanmıştır: Karşılaştırma ve Bağlantı Kurma (Comparing and Connecting), Sorgulama ve Araştırma (Questioning and Investigating), Akıl Yürütme (Reasoning), Karmaşıklığı Bulma (Finding Complexity), Gözlem ve Tasvir Etme (Observing and Describing), Bakış Açılarını Araştırma (Exploring Viewpoints).

Bu çalışmalar, sanat yoluyla öğrenme için düşünme eğilimleri tabanında bir yaklaşım önermekte olup, çeşitli öğrenme alanlarında görsel sanatlar ile nasıl ders içerikleri tasarlanabileceği konusunda aydınlatıcı bilgi sunmaktadır. Öğrenme alanlarından biri olan matematik eğitiminde, bu yaklaşımların, sanat ve matematiğin entegrasyonu bağlamına uyarlanabileceğini söylemek mümkündür.

Bu çalışmada, stüdyo ortamında öğrencilerin düşünme süreçlerini ortaya çıkarmak üzere özellikle Stüdyo Düşünme kullanılarak Matematik ve Sanat Stüdyosu Ortamı tasarlanmıştır.

2.2.2. Görsel-Uzamsal Düşünmenin Tanımlanması

Görsel-uzamsal düşünme, bilişsel bilimler, psikoloji, sanat eğitimi ve matematik eğitimi gibi farklı bağlamlarda ele alınmıştır. Geçmişten günümüze kadar, özellikle bilişsel bilimler ve psikoloji alanlarında görsel-uzamsal düşünmenin birçok tanımı söz konusudur (Carroll, 1993; Linn ve Petersen, 1985; Lohman, 1979; McGee, 1979). İlk yıllarda araştırmacılar uzamsal yetenek üzerine birçok test geliştirmişlerdir. Ancak günümüzde bu testlerin niteliği (ör. dinamik özelliklerin durgun bir ortamda incelenmesi) ve içerdiği soruların doğası (ör. küçük ölçekte soruların yanıtlanması) sorgulanarak aslında tutarlı sonuçlar vermediği tartışılmıştır (Hegarty ve Waller, 2005).

Son zamanlarda görsel-uzamsal düşünmenin yeni tanımları da önerilmiştir (Newcombe ve Shipley, 2013; Tversky, 2005). Yeni çalışmalardan biri olan Newcombe ve Shipley (2015) uzamsal düşünmenin farklı düşünme süreçleri içerdiğini açıklamakla birlikte görsel-uzamsal düşünmenin tanımlanmasına yönelik yeni bir sınıflama önermiştir. Bu sınıflama 4 temel kategori içermektedir: içsel ve dışsal özellikler, durgun ve dinamik özellikler. İçsel özellikler, nesnelerin şekilleri, düzenleri, boyutları, yönelimleri (statik) ve nesnelerin bu özelliklerinin dönüştürülmesi (dinamik) ile ilgilidir. Diğer yandan, dışsal özellikler nesnelerin aralarındaki ilişki ve nesne ve bir referans çerçevesi (bakış açısı) arasındaki ilişki (statik) ve bu ilişkilerin değişimi (dinamik) ile ilgilidir. Tablo 1’de her bir kategoriye yönelik örneklere yer verilmiştir.

Tablo 1. *Görsel-uzamsal düşünmenin sınıflanması (Newcombe & Shipley, 2015)*

	İçsel	Dışsal
Durgun	Nesnelerin görsel özelliklerinin belirlenmesi, karmaşık bir yapı içinde gizli figürleri fark etme	Bir nesnenin konumunu diğerine göre ya da belirli bir referans çerçevesine göre belirlemek
Dinamik	Nesnelerin özelliklerini döndürme, katlama, kıvırma, ve dilimleme yoluyla dönüştürme; Üç boyutlu bir nesneyi iki boyutlu düzlem üzerinde temsil etme	Perspektif alma, navigasyon (bir ortamı farklı bakış açılarında temsil etmek, farklı bakış açıları arasında ilişki kurma)

Özetle, birçok araştırmacı görsel-uzamsal düşünmeyi tanımlamıştır. Son zamanlarda yapılan güncel çalışmalardan birinde Newcombe ve Shipley (2015) görsel-uzamsal düşünmeye yönelik kapsamlı bir sınıflama önermiştir. Bu çalışmada başta Newcombe ve Shipley'in çalışması olmak üzere, alanyazında görsel uzamsal düşünmenin tanımlanmasına yönelik çalışmalar, öğrencilerin görsel-uzamsal düşünme süreçlerini incelemek için bir temel oluşturmaktadır. Bunun yanı sıra, matematik eğitiminde görsel-uzamsal düşünmenin nasıl ele alındığına dair alanyazında inceleme yapılmıştır. Bir sonraki bölümde, görsel-uzamsal düşünme ve matematik ile ilgili yapılan çalışmalara yer verilmiştir.

2.2.2.1 Matematik Eğitiminde Görsel-Uzamsal Düşünme

Araştırmacılar, geçmişten günümüze görsel-uzamsal düşünme ve matematik arasındaki ilişkiyi, birbirleriyle nasıl ilişkili olduklarını ve matematik eğitiminde uzamsal düşünmenin nasıl geliştirilebileceğini tartışmışlardır. Görsel-uzamsal düşünmenin matematikle yakından ilgili olduğu konusunda bir fikir birliğine sahip olduklarını söylemek mümkündür. Ancak bu aralarındaki ilişkinin doğrudan belirgin olmayabileceğini ileri süren çalışmalar da bulunmaktadır (Clements, 1998; Hawes vd., 2015).

Bazı araştırmacılar, bu ilişkiyi nicel yöntemler yoluyla araştırarak, uzamsal-düşünme testlerini ve matematiksel/geometrik düşünme testlerini kullanmışlardır (Goldsmith vd., 2016; Pittalis ve Christou, 2010). Bunların yanı sıra, uzamsal-düşünmeyle ilgili bazı kavramlar, matematiksel veya geometrik düşünmeyle ilişkilendirilerek incelenmiştir. Bu kavramlar şunlardır: Uzamsal ölçeklendirme (Möhring, Frick, ve Newcombe, 2018; Vasilyeva ve Bowers, 2006), kesit alma (Cohen ve Hegarty, 2012); zihinden döndürme (Bruce ve Hawes, 2015); şekillerin ve örüntülerin tanınması (Craine, 1994; Gal ve Linchevski, 2010; Mulligan ve Mitchelmore, 2009; Pittalis ve Christou, 2013), şekilleri parçalarına ayırma ve şekilleri birleştirme (Clements, Wilson ve Sarama, 2004; Spitler, 2009), şekillerin gömülümü veya ortaya çıkarılması (Sarama ve Clements, 2009; Liu ve Toussaint, 2011); geometrik şekillerin çizimi (Mitchelmore, 1978, 1980; Olkun, 2003; Pittalis ve Christou, 2013).

Sonuç olarak, yukarıda belirtilen çalışmalar matematik eğitimi ve psikoloji alanlarında yürütülmüş olup, görsel-uzamsal düşünmenin matematik bağlamında incelenmesine yönelik örneklerdir. Fakat görsel-uzamsal düşünmenin görsel sanatlar ve matematik birleşimi bağlamında bütünsel olarak nasıl ele alınabileceğine dair çalışma yok denecek kadar azdır. Bu çalışmada, yukarıda belirtilen görsel-uzamsal düşünmenin sınıflanması ve görsel-uzamsal düşünme ile ilgili yukarıda belirtilen kavramlar görsel sanatlar ve matematiğin birleşimi bağlamına adapte edilmiştir.

2.3. Çalışmanın Alanyazındaki Yeri

Geçmişten günümüze sanatın diğer öğrenme alanlarına entegrasyonu ya da sanatta öğrenilenlerin başka bir öğrenme ortamına aktarılması, araştırmacılar tarafından yoğun ilgi görülmüştür. Ancak, alanyazın incelendiğinde çalışmaların tutarlı sonuçlar ortaya koymadığı ve araştırmalarda çalışmanın nasıl tasarlandığına, sonuçların nasıl elde edildiğine dair bilgilerin oldukça yetersiz olduğu tartışılmıştır. Bu bağlamda, yapılan çalışmalarda teorik bir arka planının eksikliğine vurgu yapılarak, etkinliklerin nasıl tasarlandığı, sonuçlara nasıl ulaşıldığı gibi bilgilerin detaylı olarak verildiği çalışmalara ihtiyaç duyulduğu belirtilmiştir (Winner vd., 2013).

Görsel sanatlar ve matematiğin nasıl birleştirebileceğine dair yapılan teorik çalışmalardan birinde, bu iki alanın ortak bilişsel süreçlerinin belirlenmesi gerektiği belirtilmiştir (Bickley-Green, 1995). Son zamanlarda, sanatta öğrenmenin, matematik öğrenimine aktarılmasına yönelik yapılan bir çalışmada, görsel-uzamsal düşünmenin görsel sanatlar ve matematiğin ortak bir noktası olabileceği önerilmiştir (Goldsmith vd., 2016). Bu çalışmada, öğrenmenin dolaylı olarak bir öğrenme alanından farklı bir öğrenme alanına aktarımının incelenmesinden ziyade, doğrudan bu iki alanın birleştirilerek aralarındaki etkileşimin incelenmesini önermektedir. Çünkü öğrenmenin bir alandan başka bir alana dolaylı olarak aktarılması birçok değişkenle açıklanabilir ve bu aktarımın nasıl gerçekleştiği net olarak belirlenemeyebilir (Goldsmith vd., 2016). Bu bağlamda, bu çalışmada bir Matematik-Sanat Stüdyosu Ortamı tasarlanarak bu ortamda görsel sanatlar ve matematiğin ortak bilişsel süreci olarak belirlenen öğrencilerin görsel-uzamsal süreçleri araştırılmıştır.

3. YÖNTEM

3.1. Araştırmanın Deseni

Bu çalışmada, Matematik-Sanat Stüdyosu Ortamında öğrencilerin görsel uzamsal düşünme süreçlerini incelemek için, nitel araştırma yöntemlerinden birisi olan araçsal durum çalışmasından (instrumental case study) yararlanılmıştır (Stake, 2005). Bu çalışmada durum, Matematik-Sanat Stüdyosu Ortamı olarak belirlenmiş olup, “araçsal durum çalışmaları” nın doğası gereği ikincil bir rol oynamaktadır (Grandy, 2010). Diğer bir deyişle, bu çalışmada amaç, Matematik-Sanat Stüdyosu Ortamını anlamaktan ziyade, bu ortamda öğrencilerin görsel-uzamsal düşünme süreçlerini incelemektir.

3.2. Matematik-Sanat Stüdyosu Ortamının Katılımcıları

Bir devlet okulunda okumakta olan altı 7. sınıf öğrencileri Matematik-Sanat Stüdyosu Ortamının katılımcılarını oluşturmaktadır. Katılımcılar, görsel sanatlar ve matematik öğretmenlerinin görüşleri doğrultusunda matematik/görsel sanatlara ilgilerine, bu derslerdeki performanslarına ve bu derslerde yaratıcı düşünme yaklaşımlarına göre belirlenmiştir. Ebeveynlerinin onayları doğrultusunda altı öğrenci çalışmaya gönüllü olarak katılmıştır. Öğretmenler, üç öğrenciyi (Fatma, Melek, ve Burcu) görsel sanat eğitiminde diğerlerine göre daha ilgili ve başarılı olarak, diğer üç öğrenciyi (Emre, Ali ve Esra) matematiğe daha ilgili ve başarılı olarak belirlemiştir. Farklı olarak Burcu hem matematik hem de görsel sanatlara ilgili ve başarılı olarak görülmüştür.

3.3. Araştırmanın Genel Süreci

Bu çalışmada görsel sanatlar ve matematik bağlamında, öğrencilerin görsel-uzamsal düşünme süreçlerini inceleyebilmek için ilk atılan adım, Matematik-Sanat Stüdyo Ortamının tasarlanması olmuştur. Alanyazın taraması ve uzmanların (güzel sanatlar bölümünden iki öğretim üyesi) görüşleri çerçevesinde, bu ortamın temel özellikleri belirlenmiştir. Daha sonra araştırmacı tarafından farklı görsel-uzamsal düşünme süreçlerine yönelik stüdyo çalışmaları (stüdyoda uygulanan etkinlikler) geliştirilmiştir. Stüdyo çalışmalarının ilk taslakları bir resim öğretmeni tarafından incelenerek bazı düzenlemeler (ör. stüdyo çalışmalarının zorluk derecesi) yapılmıştır. Daha sonrasında, pilot çalışma Ankara'da bulunan bilim sanat merkezlerinden birinde uygulanmıştır. Pilot çalışma ve uzamsal düşünme üzerinde çalışan bir öğretim üyesinin değerlendirmesi sonucunda, stüdyo çalışmaları yeniden incelenerek ana çalışmada uygulanmak üzere çalışmalara son hali verilmiştir. Bir projenin parçası olarak bu çalışmada, toplamda altı stüdyo çalışması geliştirilmiş olup, bunların ilk üçü derinlemesine araştırılmıştır. Bu stüdyo çalışmaları iki boyutlu sanat eserine yönelik olup, diğerleri üç boyutlu sanat eserlerine yöneliktir. Bu çalışmada derinlemesine ve tutarlı olarak düşünme süreçlerini analiz edebilmek için öğrencilerin düşünme süreçlerini etkilemeyecek şekilde ilk üç stüdyo çalışması incelenmiştir. Bu stüdyo çalışmaları Tablo 2'de kısaca açıklanmıştır.

Tablo 2. *Stüdyo Çalışmaları ve Açıklamaları*

Stüdyo Çalışmaları	Stüdyo Çalışmanın Açıklaması
Stüdyo Çalışması 1	Farklı özelliklere sahip (örn. iç içe geçmiş şekiller içeren) minimal sanat eserlerinin gözlemlenmesi, şekilleri birbiri içine gizleyerek sanat çalışması oluşturma, sanat çalışmalarını arkadaşlarına açıklama ve eleştiriler yapma
Stüdyo Çalışması 2	Döndürülmüş simetrik ve asimetrik minimal sanat eseri serilerinin gözlemlenmesi, bu sanat eserlerinden bir tanesi seçilerek, başka bir sanat eserinin sadece bir başlangıcı gibi düşünülerek, döndürme yoluyla yeni bir sanat çalışması oluşturma, sanat çalışmalarını arkadaşlarına açıklama ve eleştiriler yapma
Stüdyo Çalışması 3	Farklı özelliklere sahip (simetrik/asimetrik, gizli şekiller içeren kompozisyon) minimal sanat eserlerini 4 katı büyüklükte bir kağıda yeniden çizme, yapılan çizimleri arkadaşlarına açıklama ve eleştiriler yapma

3.3.1 Ana Çalışma

Ana çalışma, Ankara'da bulunan bir devlet okulunun görsel sanatlar atölyesinde uygulanmıştır. Bu atölyede, öğretmen ve öğrenciler için masa ve sandalyeler, öğrencilerin eşyalarını muhafaza edebilmeleri için iki tane eşya dolabı ve bir akıllı tahta yer almaktadır. Her bir öğrencinin çalışma sürecini kayıt alan dört adet kamera yerleştirilmiştir. Buna ek olarak, iki adet ses kayıt cihazı da öğrencilerin çalıştıkları masaların üzerine yerleştirilmiştir. Çalışma için gerekli materyaller araştırmacı tarafından sağlanmıştır. Bu materyallerden bazıları şunlardır: Çizim kalemi, çizim defteri, yapıştırıcı, farklı tipte ve renkte kağıt ve kartonlar, kuru ve pastel boya, cetvel, gönye, açıölçer. Araştırmacı aynı zamanda öğrencilerin projelerine yönelik araştırma yapmaları için bir bilgisayar temin etmiştir. Çalışma sırasında öğrencileri motive etmek için klasik müzik dinlenilmiştir. Bu stüdyoda, öğrenciler ihtiyaç duydukları zaman ara vermişlerdir. Akıllı tahta sanatçıların eserlerini ve öğrencilerin kendi çalışmalarını yansıtmak üzere kullanılmıştır. Böylece öğrenciler bu sanat çalışmaları üzerinde daha kolay gözlem yapma ve düşüncelerini ifade edebilme olanağı bulmuşlardır.

3.4. Veri Toplama Süreci

Öğrencilerin her bir stüdyo çalışmasına katılması beklenmiştir. Ancak, öğrenciler istedikleri zaman çalışmadan ayrılma haklarına sahip olup çalışmaya katılım zorunlu olmamıştır. 4 öğrenci bütün stüdyo çalışmalarına katılmış olup, 2 öğrenci ikinci stüdyo çalışmasından sonra çalışmadan kişisel nedenlerden dolayı çalışmadan ayrılmıştır. Stüdyo çalışmalarının öğrencilerin performanslarına bağlı olarak iki hafta içinde tamamlanması planlamıştır. Öğrenciler stüdyo çalışmalarına her sabah yaklaşık 3-4 saat katılmışlardır. Öğrenciler öğleden sonra aynı okulda eğitim görmekteydiler. Çalışma bu şekilde sekiz gün boyunca devam etmiştir. Altı gün sonrasında bir gün ara verilmiştir. Daha sonra tekrar devam edilmiştir. Bu şekilde yoğun bir programın olmasının sebebi, öğrencilerin bir gün önceki yaptıklarını unutma ve çalışmaya yeterince dahil olmama ihtimalini önlemektir. Genellikle eleştiri bölümleri ertesi gün yapılmıştır. Her bir stüdyo çalışmasından sonra, her öğrenciyle uyarılmış hatırlama görüşmeleri (stimulated recall interview) yapılmıştır (De Smet, Van Keer, De Wever, ve Valcke, 2010). Uyarılmış hatırlama görüşmelerine ek olarak, uygulamadan önce ve sonra görsel sanatlar ve matematikte önceki deneyimleri ve bu çalışmadaki deneyimlerini öğrenmek üzere görüşmeler yapılmıştır (bknz. Tablo 3).

Çalışma sırasında araştırmacı bir koç görevi üstlenerek öğrencilere ihtiyaç duydukları zaman yardımcı olmuştur ve öğrencilerle doğrudan iletişim kurmak için tüm stüdyo çalışmalarını yönetmiştir. Sadece eleştiri bölümlerine çalışmanın yürütüldüğü okuldaki matematik ve resim öğretmenleri davet edilmiştir. Bu bölümde öğrencilerin çalışmalarını inceleme ve eleştirme uzmanlık gerektirdiği için matematik ve resim öğretmenlerinden öğrencilerin çalışmalarına yönelik görüşlerini belirtmeleri istenmiştir.

3.5. Veri Toplama ve Analizi

Bu çalışmanın veri toplama araçlarını, katılımcılarla yapılan görüşmeler, video kayıtlarının gözlemi, ve öğrencilerin eskiz, yazılı not ve sanat çalışmaları gibi dokümanları oluşturmaktadır. Katılımcılarla üç farklı görüşme gerçekleştirilmiştir: Uygulama öncesi görüşmeler, uygulama sırası görüşmeler (uyarılmış hatırlama görüşmeleri) ve uygulama sonrası görüşmeler. Her bir veri toplama aracı ve kullanım amacı Tablo 3'te gösterilmiştir. Uygulama öncesi görüşmeler ses kayıtlı olup, diğer görüşmeler hem sesli hem de görüntülü olarak kaydedilmiştir. Çalışma sırasında yapılanları tekrar gözlemlemek üzere bütün stüdyo çalışmaları sesli ve görüntülü olarak kaydedilmiştir.

Tablo 3. *Veri toplama araçları ve amaçları*

Veri Toplama Araçları	**Veri Toplama Aracının Amacı**
Görüşmeler	Öğrencilerin özelliklerini tanımlamak ve diğer veri kaynaklarını (doküman, gözlem) desteklemek
Uygulama Öncesi Görüşmeler	Öğrencilerin görsel sanat ve matematik hakkındaki görüşlerini ve bu alanlardaki geçmiş deneyimlerini öğrenmek
Uygulama Sırası Görüşmeler (Uyarılmış Hatırlama Görüşmeleri)	Öğrencilerin düşünme süreçlerini, geçmiş deneyimlerini hatırlamalarını isteyerek incelemek ve neden yaptıklarını açıklamak.
Uygulama Sonrası Görüşmeler	Öğrenciler bu çalışmadaki deneyimlerini ve bu çalışmaya yönelik fikirlerini öğrenmek
Gözlem	Görsel-uzamsal düşünme ile ilgili öğrencilerin kritik eylemlerini not etmek (sözlü ifadeler, jestler, eylemlerin sırası, araştırmacı ve öğrenciler arasındaki iletişim)
Dokümanlar (yazılı notlar, çizimler, sanat çalışmaları)	Araştırmanın diğer veri kaynaklarını destekleyerek öğrencilerin görsel düşünme süreçlerini öğrenmek.

Öğrencilerle yapılan bütün sesli ve görüntülü kayıtlar ve öğrencilerin dokümanları MAXQDA yazılımına aktarılmıştır. Açık kodlama tekniği ile veri analiz edilerek yorumlanmıştır. Veri analizi sırasında görsel-uzamsal düşünme alanında yapılan çalışmalar incelenmiştir. Veri analizinde temel olarak Newcombe ve Shipley (2015) ve Sarama ve Clements'in (2009) çalışmaları incelenmiştir. Verilerin analizi sürecinde araştırmacı öğrencilerin ifadelerine ve dokümanlarına çeşitli kodlar atayarak ve bu kodlar gözden geçirilerek bir kod kitapçığı oluşturulmuştur. Bu kod kitapçığı kodların tanımlarını ve her bir kodun alt kodlarını da içermektedir. Güvenirliği artırmak üzere kodlar matematik eğitiminde uzman bir kişi ile incelenerek tamamen uzlaşıncaya kadar tartışılmıştır ve kodlara son hali verilmiştir. Belirlenen kodlar aynı zamanda çalışmanın bulgularını oluşturmaktadır. Bir sonraki bölümde çalışmanın bulguları detaylı olarak incelenebilir.

4. BULGULAR

Bu çalışmada öğrencilerin görsel-uzamsal düşünme süreçlerinin analizi, öğrencilerin 4 temel görsel-uzamsal düşünme sürecinden yararlandıklarını ortaya çıkarmıştır: Geometrik şekilleri tanıma, şekilleri parçalarına ayırma ve birleştirme, örüntüleme ve şekilleri dönüştürme. Bu çalışma bu düşünme süreçlerinin birbiriyle bağlantılı olduğunu ve her bir düşünme sürecinin diğerleriyle koordineli bir şekilde kullanılması gerektiğini ortaya çıkarmıştır. Ayrıca, Stüdyo düşünmesine dayanan bu Matematik-Sanat Stüdyosu Ortamının öğrencilerin farklı görsel-uzamsal düşünme yollarını ortaya çıkarma potansiyeline de sahip olduğunu göstermiştir. Her bir görsel-uzamsal düşünme sürecine yönelik bulgular aşağıda belirtilmiştir.

4.1. Şekilleri Tanıma

Şekil tanıma süreçlerinin analizi sonucunda, öğrencilerin şekilleri gerçek yaşam objelerine benzettikleri ya da şekilleri geometrik şekil olarak belirledikleri bulunmuştur.

Öğrenciler sanat çalışmalarını gözlemlerken geometrik şekilleri günlük yaşam objelerine benzetmişlerdir. Şekilleri gerçek yaşam objelerine benzetirken görsel benzerliklerini dikkate almıştır. Örneğin, Fatma geometrik şekillerden birini kediye benzetirken, Melek uçan bir kelebeğe benzetmiştir. Bunun yanı sıra, kendileri sanat çalışması oluştururken günlük yaşam nesnelerini modellemek üzere geometrik şekillerden yararlanmışlardır. Örneğin, öğrencilerden Fatma stüdyo çalışması 2 sırasında kuş yapmak üzere geometrik şekilleri bir araya getirirken, Emre saat yapmak üzere geometrik şekillerden yararlanmıştır.

Öğrencilerin geometrik şekilleri belirleme süreci analiz edildiğinde, öğrencilerin şekilleri geometrik özellikleri doğrultusunda, gömme veya ortaya çıkarma yoluyla (embedding/disembedding) ve şekilleri farklı yön/perspektiflerden belirledikleri bulunmuştur. İlk olarak, öğrenciler şekilleri tanıma süreçlerinde şekillerin özelliklerini dikkate almışlardır. Sanat eserlerini gözlemlerken, kendileri sanat çalışması ortaya çıkarırken, veya sanat eserlerini kopyalarken geometrik şekillerin özelliklerini ayırt etmişlerdir. Bu özellikler, kenar ilişkileri (uzunlukların büyüklüğü, kenar sayısı, kenarların paralelliği), açısal ilişkiler (iki kenar arasındaki mesafe, kenarların diklik derecesinin karşılaştırılması) ve şekillerin simetrik olma özellikleridir. Üç boyutlu şekillerin temsillerini tanıma süreçlerinde ise, tabanın kenar sayısına ya da yanal yüzeylerin sayısına odaklandıkları belirlenmiştir.

Şekilleri gömme veya ortaya çıkarmaya ilişkin olarak, öğrenciler çoğunlukla iki boyutlu geometrik şekilleri sanat çalışmalarının içinden çekip ortaya çıkarmıştır. Üç boyutlu geometrik şekillerin iki boyutlu düzlemdeki temsillerini ilk bakışta ortaya çıkaramamışlardır. Öğrencilerden, sanat çalışmasını arkadaşlarıyla birlikte gözlemeleri istendiğinde, öğrenciler sanat eserlerindeki üç boyutlu geometrik şekillerin iki boyutlu temsillerini fark etmeye başlamışlardır. Ayrıca, sanat çalışması ortaya koyarken öğrenciler şekilleri ortaya çıkarmanın yanı sıra şekilleri birbiri içine gömerek geometrik şekilleri belirlemişlerdir.

Öğrenciler ayrıca iki boyutlu şekilleri farklı yönlerden tanımaya çabalamışlardır. Ancak, bazı durumlarda aynı şeklin farklı yönlerdeki hallerini farklı isimlendirmişlerdir. Öğrenciler ayrıca iki boyutlu temsilleri verilen üç boyutlu geometrik şekillerin farklı bakış açılarından görünüşlerini de hayal etmişlerdir. Bu süreçte öğrencilerin zihinlerinde ya şeklin dönüşünü ya da şeklin etrafında kendi dönüşlerini canlandırabildikleri tespit edilmiştir.

4.2. Şekilleri Oluşturma ve Parçalarına Ayırma

Öğrencilerin şekilleri oluşturma ve parçalarına ayırma süreçleri incelendiğinde, öğrencilerin geometrik şekilleri daha küçük geometrik şekillere böldükleri ortaya çıkmıştır. Öğrenciler ilk başta farkında olarak şekilleri daha küçük parçalara bölmemişlerdir. Araştırmacı, bu sanat eserlerinde hangi şekillerin olduğunu sorduğunda, bu şekilleri parçalarına ayırmaya çalışmışlardır. Öğrenciler şekilleri parçalarına ayırmanın yanı sıra, şekilleri birleştirerek yeni bir şekil oluşturmuşlardır. Çoğunlukla sanat çalışması sırasında ortaya çıkan bu süreçte, öğrenciler birim şekilleri deneme yanılma yoluyla bir araya getirmişlerdir. Öğrencilerin şekillerin döndürülmesini hayal etmelerinin, bu şekillerin birleştirilmesinde önemli rol oynadığı ortaya çıkmıştır.

4.3. Örüntüleme

Öğrencilerin örüntüleme süreçlerinin analiz edilmesi sonucunda, öğrencilerin örüntüyü küçük parçalara ayırarak tanıdıkları veya parçaları birleştirerek örüntü oluşturdukları bulunmuştur. Örüntüyü küçük parçalara ayırarak tanıma süreçleri gösterim ve eleştiri aşamalarında ortaya çıkmıştır. Sanat çalışmalarındaki örüntüleri tanımanın yanı sıra, kendi sanat çalışmalarını yaparken örüntülemeden yararlanmışlardır. Bu süreç sırasında, öğrenciler örüntünün birimini belirleyip belirli bir kural çerçevesinde tekrar eden veya büyüyen örüntü oluşturmuşlardır. Öğrenciler birimleri belirli bir şekilde döndürerek ya da birimlerin büyüklüklerini tahmin edilebilir şekilde artırarak örüntüyü oluşturmuşlardır. Şekillerin kenar uzunlukları ya da alanları arasındaki orantısal ilişkileri çoğunlukla toplamsal akıl yürütme ile analiz etmeye çalışmışlardır.

4.4. Şekilleri Dönüştürme

Şekilleri dönüştürmeye yönelik düşünme süreçlerinin analizi sonucunda, öğrencilerin şekillerin boyutlarını değiştirerek (ölçeklendirme) ya da şekillerin geometrik özelliklerini ve büyüklüklerini koruyacak şekilde yönlerini değiştirerek (rigid dönüşüm) dönüşüm yaptıkları ortaya çıkmıştır. Ölçeklendirmeye ilişkin olarak, şekillerin kenar uzunluklarını veya alanlarını toplamsal ve çarpımsal olarak (orantısal) karşılaştırarak ve bir şekli birimler halinde yapılandırarak akıl yürütmüşlerdir. Şekillerin kenar uzunlukları arasındaki orantısal ilişkiyi fark edemediklerinde ise kenar uzunluları arasındaki göreceli farka bakmışlardır.

Rigid dönüşümlere ilişkin olarak ise, öğrenciler şekillerin birbirine benzeyip benzemediğini anlamak için şekillerin ya dönmesini ya da takla atmasını zihinlerinde canlandırmışlardır. Bunun yanı sıra, bazı öğrenciler şekiller arasındaki benzerliği, şekillerin benzer özelliklerini (eş kenarlar, köşeler, şekillerin işaret ettiği yön) eşleştirerek belirlemişlerdir. Öğrencilerin şekilleri dönüştürmelerine yönelik bulgulardan bir diğeri ise, öğrencilerin ilk bakışta şekillerin yönlerini dikkate alarak karşılaştırma yapmış olduklarıdır. Öğrencilerden dönme açısı ve dönme merkezini belirlemeleri istendiğinde, görsel olarak belirlemeye çalışmışlardır. Açı ölçüsü belirlerken 45, 90, ve 180 gibi referans açılar kullanarak açıların görüntülerini karşılaştırmışlardır. Ancak, öğrenciler kendi sanat çalışmalarını yaparken şekillerin dönüşümlerini çizmede oldukça zorlanmışlardır.

5. TARTIŞMA

Bu çalışmada, öğrencilerin Matematik-Sanat Stüdyosu Ortamında sanat çalışmalarını eleştirirken, sanat çalışması oluştururken ve sanat çalışmalarını gözlemlerken dört temel görsel uzamsal düşünme sürecinden yararlandıkları gözlenmiştir. Bu düşünme süreçlerinin birbirleriyle ilişkili olacak şekilde karmaşık oldukları tespit edilmiştir.

Ayrıca bu çalışma, tasarlanan Matematik-Sanat Stüdyosu Ortamının öğrencilerin farklı düşünme süreçlerini açığa çıkarma potansiyeline sahip olduğunu ortaya çıkarmıştır.

5.1 Geometrik Şekilleri Tanıma

Öğrenciler sanat çalışmalarındaki bazı geometrik şekilleri ya da geometrik şekillerin birleşimini gerçek yaşam objelerine benzettiler. Öğrenciler geometrik şekilleri, özellikle bildikleri bir geometrik şekle benzetemediklerinde, zihinlerindeki günlük yaşam nesneleriyle ilişkilendirmiş olabilirler. Diğer yandan, öğrenciler geometrik şekillerden yararlanarak günlük yaşam objelerinin temsillerinin resimlerini oluşturmuşlardır. Bu düşünme sürecinin özellikle sanatçıların ve tasarımcıların objelerin temel yapılarını anlamaları ve hayal etmeleri için önemli olduğu düşünülmektedir (Goldsmith vd., 2016).

Öğrencilerin geometrik şekilleri özelliklerini düşünerek tanımaya çalışmaları sırasında öğrenciler bazı iki boyutlu geometrik şekilleri (dik üçgen, kare) farklı durumlarda (perspektif çizim ve döndürme) farklı isimlendirdikleri görülmüştür. Bu durum, öğrencilerin zihinlerindeki bu geometrik şekillerin prototiplerini düşünerek şekilleri tanıdıklarını gösteriyor olabilir (Tsamir, Tirosh ve Levenson, 2008; Ubuz ve Gökbulut, 2015; Ulusoy ve Cakiroglu, 2017). Bunun yanı sıra, öğrenciler zihinlerinde şekillere yönelik sınırlı bir depolama hafızasına sahip olabilirler ve gördükleri şekilleri görsel olarak prototiplere benzetiyor olabilirler (Tsamir vd., 2008). Diğer bir bulgu ise, öğrencilerin iki boyutlu düzlem üzerinde temsil edilen üç boyutlu geometrik şekillerin özelliklerini belirlemeleridir. Öğrenciler bu üç boyutlu geometrik şekilleri belirlerken üç boyutlu cismin tabanının kenar sayısına ya da yanal yüzeylerin sayısına dikkat etmişlerdir. İki boyutlu şekillerde olduğu gibi, üç boyutlu cisimleri de belirli bir prototip çerçevesinde ayırt etmeye çalışmışlardır. Örneğin, öğrencilerin piramitin 4 yanal yüzeye sahip olması gerektiğini düşünmeleri Ubuz ve Gökbulut'un (2015) çalışmalarından öğretmenlerin piramitleri sadece mısır piramitlerini düşünerek belirlemeleriyle tutarlılık göstermektedir. Diğer yandan, öğrenciler sanat eserlerinde üçgen prizma ve üçgen piramiti birebirinden ayırt edememişlerdir. Bu durum öğrencilerin şekilleri sadece üç boyutlu olup olmadığına göre algıladıklarına ve bir cismi tanımlayan kritik özelliklerinin farkında olarak şekilleri ayırt etmediklerine örnek olarak gösterilebilir (Hershkowitz, 1989).

Son olarak, öğrencilerin şekilleri gömme ve ortaya çıkarma görsel-uzamsal sürecine ilişkin olarak, öğrencilerin sanat eserlerinde hem iki boyutlu hem de üç boyutlu cisimlerin iki boyutlu temsillerini ortaya çıkardığı ortaya koyulmuştur. Öğrenciler üç boyutlu cisimlerin iki boyutlu temsillerini ilk bakışta fark edememişlerdir. Ancak arkadaşlarıyla birlikte sanat eserlerini yeniden gözlemeye başladıklarında fark etmeye başlamışlardır. Bunun sebebi, öğrencilerin üç boyutlu cisimlerin kritik ve kritik olmayan özelliklerini belirlemede zorluk çekiyor olmaları olabilir (Hershkowitz, 1989; Tsamir vd., 2008) ya da bir şeklin hem iki boyutlu hem de üç boyutlu olarak algılanıyor olması olabilir (Attneave, 1971). Ancak bu çalışmadaki öğrencilerin şekilleri ortaya çıkarma süreçleri daha önce geliştirilmiş olan gizli figürler testlerinden farklıdır (Ghent, 1956; Hodgkiss vd., 2018; Oltman, Raskin, Witkin, 1971; Witkin, 1950). Bu çalışmada öğrencilerden karmaşık bir figür içinden belirli basit bir figürü ortaya çıkarmaları istenmemiştir. Aynı zamanda, şekilleri ortaya çıkarma süreçlerinden farklı olarak, öğrenciler şekilleri birbiri içine gömerken de şekilleri ayırt etmişlerdir. Bu durum çoğunlukla öğrenciler kendi sanat çalışmalarını ortaya koyarken açığa çıkmıştır. Bu süreçte şekillerin ortak yüzeylerinin dikkate alındığı ve sadece görünen yüzeylerinin çizilmeye çalışıldığı gözlenmiştir (Mithelmore, 1978; 1980). Şekillerin gömülmesi, önemli bir görsel-uzamsal düşünme süreci olarak görülmesine rağmen, bu alanda yeterince çalışma bulunmamaktadır (Sarama ve Clements, 2009).

5.2. Şekilleri Oluşturma ve Parçalarına Ayırma

5.3. Örüntüleme

Örüntülemeye ilişkin olarak, öğrencilerin görsel örüntünün küçük parçalarına odaklandıkları gözlenmiştir. Bu küçük parçaların bütünle ilişkisini örüntünün kuralını belirlemede kullanmamışlardır. Bu bulgu daha önce yapılmış olan uzamsal örüntü analizi çalışmalarında küçük çocukların daha çok örüntünün küçük parçalarına odaklandığı bulgusuyla örtüşmektedir. Bu çalışmalarda, örüntüleme becerisi için, küçük parçalar arası ilişkinin ve küçük parçaların bütünle ilişkisinin koordineli olarak düşünülmesi gerektiği vurgulanmıştır (Akshoomott ve Stiles, 1995; Feeney ve Stiles, 1996; Tada ve Stiles, 1996; Vinter, Puspitawati, ve Witt, 2010).

Örüntü oluşturma sırasında ise, öğrenciler ilk başta örüntünün kuralını tahmin etmeden, şekilleri ya boyutlarını koruyarak ya da boyutları dönüştürerek bir araya getirdiler. Birkaç birleşimden sonra bütün şekli tahmin etme durumları önceki çalışmalardaki bulguları da desteklemektedir (Akshoomott ve Stiles, 1995). Öğrenciler örüntüleri informel stratejiler kullanarak (simetri, şekiller arasında eşit mesafe bırakma, şekilleri döndürme) bir araya getirdiler. Öğrenciler sanat çalışması sırasında informel stratejiler kullanmalarına rağmen, ünlü sanatçıların sanat çalışmalarını gözlemlerken bir şekli birim parçalara yapılandırdılar ve bu parçalar arasındaki orantısal kuralı araştırdılar. Öğrencilerin görsel bir örüntüyü, birim parçalarla yapılandırması matematik eğitiminde önemli bir beceri olarak görülmektedir (Lüken, 2012; Mullihan ve Mitchelmore, 2009; Sarama ve Clements, 2009).

5.4. Şekilleri Dönüştürme

5.4.1. Ölçeklendirme

Ölçeklendirme süreçlerinde, öğrencilerin sanat çalışmalarındaki geometrik ipuçlarını analiz ettikleri ve şekillerin büyüklük dönüşümlerini zihinden yaptıkları gözlenmiştir. Öğrencilerin analiz ettikleri geometrik ipuçları, uzunluk ve açı ilişkileri, şekilleri dizilimi (simetrik & asimetrik), geometrik şekillerin geometrik özellikleridir. Bu geometrik ipuçları Vasilyeva ve Bowers'in (2006) okul öncesi çocuklarla yapmış olduğu çalışmayla tutarlılık göstermiştir. Bu çalışmada da çocuklar, üçgensel zeminde yer alan nesneleri bir yerden aynı oranda büyütülmüş başka bir yere taşırken, üçgenin açıları ve kenarları arasındaki ilişkileri kodlamışlardır.

Geometrik ipuçlarının yanı sıra, bu çalışmada öğrenciler orantısal ilişkileri de belirlemeye çalışmışlardır. Ölçeklendirmede, orantısal ilişkilerin kodlanmasının önemli olduğu bu çalışmada da ortaya koyulmuştur (Möhring, Frick, ve Newcombe, 2018; Möhring, Newcombe, Levine, ve Frick, 2016). Fakat orantısal ilişkileri, çarpımsal akıl yürütmeden ziyade toplamsal akıl yürütme yoluyla kodlamaya çalışmışlardır. Bu durum, diğer çalışmalarda olduğu gibi öğrencilerin şekillerin uzunluk ve alanları arasındaki çarpımsal ilişkileri anlamada zorlandıklarını göstermiştir (Sowder vd., 1998; Lamon, 1994). Bu süreçte öğrenciler karelerin alanları arasındaki farkı belirlemek için kareleri birim karelere bölerek uzamsal olarak yapılandırmaya (spatial structuring) çalışmışlardır. Böylece uzamsal yapılandırma becerisinin alan ölçme ve uzamsal orantısal düşünmede önemli rol oynadığı ortaya çıkmaktadır (Sarama ve Clements, 2009).

Diğer bir bulgu ise, öğrenciler asimetrik dizilimli şekilleri içeren sanat çalışmalarını 1:4 oranında daha büyük bir kağıda kopyaladıklarında simetrik dizilime sahip olan sanat çalışmasına göre daha çok zorlanmışlardır. Buna göre öğrencilerin simetrik şekilleri veya şekillerin simetrik dizilimlerini kopyalamada geometrik ilişkileri daha kolay kavradıklarını söylemek mümkündür. Bu bulgu, Uttal'ın (1996) okul öncesi çocuklarla ve yetişkinlerle yürütmüş olduğu çalışmasıyla tutarlılık göstermektedir.

5.4.2. Rigid Dönüşüm

Öğrenciler rigid dönüşümlerle ilgili olarak geometrik sanat çalışmalarını döndürmeyi veya takla attırmayı zihinlerinde canlandırarak karşılaştırmışlardır. Şekillerin dönüşümlerini hayal etme sırasında, öğrencilerin sanat eserlerini, sanat eserlerinde yer alan şekillerin doğru parçalarını, köşelerini, işaret ettikleri yönü veya şeklin bütünsel benzerliğini eşleştirerek analiz ettiği bulunmuştur. Böyle bir süreç, Wright, Thompson, Ganis, Newcombe ve Kosslyn (2008) tarafından zihinden döndürmenin aşamaları olarak belirtilen, objelerin görsel yapısını belirleme, objelerden birini döndürme, benzer olup olmadığını diğer objeyle karşılaştırma ve cevap verme aşamaları ile benzerlik göstermektedir. Bu süreçte, bazı öğrenciler ellerini, kafalarını veya vücutlarını döndürerek (dinamik hareketler) veya sadece şekillerin belirli kısımlarını parmaklarıyla işaretleyerek (statik hareketler) (Göksun, Goldin-Meadow, Newcombe ve Shipley, 2013) şekilleri döndürmeyi hayal ettikleri gözlenmiştir.

Öğrenciler sanat eserlerini bireysel olarak gözlemlerken, şekillerin parçalarının dönüşünü hayal etmelerine rağmen, döndürme yaparken dönme merkezi ya da açısını belirlememişlerdir. Bu durum, öğretmen adaylarının dönüşümleri nasıl algıladığına dair olan çalışma (Harper, 2002) ve öğrencilerin açıları belirlemede zorlukları üzerine olan çalışma (Mitchemore ve White, 1998) ile tutarlılık göstermektedir. Öğrencilerin açıyı belirlemede zorluk yaşamalarının sebebi dönme sırasında açının dinamik doğasını kavrayamamaları olabilir (Foxman ve Ruddock, 1984; Sarama ve Clements, 2009). Ancak araştırmacı tarafından yönlendirici sorular yöneltildiğinde dönme açısının miktarı üzerine düşünmüşlerdir. Bu bulgu, Foxman ve Ruddock'ın çalışmasındaki 15 yaşındaki öğrencilerin düşünme süreciyle benzerlik göstermektedir.

Rigid dönüşüme yönelik diğer bir bulgu ise, öğrenciler dönme açısını belirleme süreçlerinde özellikle sanat çalışmalarını gözlemlerken, 45, 90, 180 gibi referans açılardan yararlandılar. Bu bulgu, Sarama ve Clements'in (2009) çalışmasıyla benzeşmektedir. Okul öncesi çocuklarla ilgili bu çalışmada, zihinlerdeki 45-90 şemaları tabanında açıları belirlemeye çalıştıkları ortaya koyulmuştur.

Son olarak, öğrenciler sanat çalışmalarının gözlemi sırasında şekillerin döndürülmüş hallerini belirlemişlerdir. Ancak, bir şekil ve onun döndürülmüş halini eşleştirebildiği halde öğrenciler, sanat eseri oluşturma sürecinde bir şeklin döndürülmüş halini çizmekte oldukça zorlanmışlardır. Öğrenciler çizimlerinde herhangi bir sıkıntının var olduğunu anlasalar bile çizimlerindeki sıkıntıları çözmekte sıkıntı yaşamışlardır. Bu durumun sebebi öğrencilerin çizim yapmakta zorluk çekiyor olmaları olabilir ya da bir nesnenin analitikten ziyade bütünsel olarak tanınması bir şekli çizmeye yetmeyebilir. Çizim yapma süreci öğrencinin kağıda çizdiği ile, zihninde olan hala çizmediği durum arasında koordineli bir işbirliği gerektiriyor olabilir (Fuson ve Murray, 1978).

Sonuç olarak, bu çalışmadan elde edilen bulguların, görsel sanatlar ve matematiğin birleşimini içeren gelecekteki çalışmalara ışık tutacağı öngörülmektedir. Bu çalışma ile matematik ve görsel-sanatların birleştiği bir ortamda öğrencilerin düşünme süreçlerinin somut örnekleri sunulmuştur. Buna ek olarak, çalışmanın nasıl tasarlandığı ve bulgulara nasıl ulaşıldığı konusunda detaylı bilgiler sunulmuştur. Böylece, bu çalışmanın araştırmacılara ya aynı bağlamda çalışmayı tekrar etme veya farklı bağlamlara adapte etme imkanı sağlayacağı öngörülmektedir.

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