

$B_q \rightarrow l^+l^-\gamma$ DECAYS IN LIGHT CONE QCD

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Abstract

The radiative dileptonic decays $B_s(B_d) \rightarrow l^+l^-\gamma$ ($l = e, \mu$) are investigated within the Standard Model . The transition formfactors are calculated in the framework of the light cone QCD sum rules method and it is found that the branching ratios are $B(B_s \rightarrow e^+e^-\gamma) = 2.35 \times 10^{-9}$, $B(B_s \rightarrow \mu^+\mu^-\gamma) = 1.9 \times 10^{-9}$, $B(B_d \rightarrow e^+e^-\gamma) = 1.5 \times 10^{-10}$ and $B(B_d \rightarrow \mu^+\mu^-\gamma) = 1.2 \times 10^{-10}$. A comparison of our results with the constituent quark model predictions on the branching ratios is presented.

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1 Introduction

The Flavour Changing Neutral Current (FCNC) processes are the most promising field for testing the Standard Model (SM) predictions at loop level and for establishing new physics beyond that (for a review see [1] and references therein). At the same time the rare decays provide a direct and reliable tool for extracting information about the fundamental parameters of the Standard Model (SM), such as the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V_{td} , V_{ts} and V_{ub} [2].

After the experimental observation of the $b \rightarrow s\gamma$ [3] and $B \rightarrow X_s\gamma$ [4] processes, the interest is focused on the other possible rare B -meson decays, which are expected to be observed at future B -meson factories and fixed target machines. In addition to being used in the determination of the CKM matrix elements, the rare B -meson decays could play an important role in extracting information about some hadronic parameters, such as the leptonic decay constants f_{B_s} and f_{B_d} . Pure leptonic decays $B_s \rightarrow \mu^+\mu^-$ and $B_s \rightarrow e^+e^-$ are not useful for this purpose, since these decays are helicity suppressed and as a result they have branching ratios $B(B_s \rightarrow \mu^+\mu^-) \simeq 1.8 \times 10^{-9}$ and $B(B_s \rightarrow e^+e^-) \simeq 4.2 \times 10^{-14}$ [5]. For B_d meson case the situation becomes worse due to the smaller CKM angle. Although the process $B_s \rightarrow \tau^+\tau^-$, whose branching ratio in the SM is $B(B_s \rightarrow \tau^+\tau^-) = 8 \times 10^{-7}$ [6], is free of helicity suppression, its observability is expected to be compatible with the observability of the $B_s \rightarrow \mu^+\mu^-$ decay only when its efficiency is better than 10^{-2} .

When a photon is emitted in addition to the lepton pair, no helicity suppression exists anymore and larger branching ratios are expected. For that reason, the investigation of the $B_{s(d)} \rightarrow l^+l^-\gamma$ decay becomes interesting. The branching ratios of these processes depend quadratically on the leptonic decay constants of B mesons and hence it could be a possible alternate in determining f_{B_s} and f_{B_d} . In [7], these decays are investigated in the SM using the constituent quark approach and it is shown that the diagrams with a photon radiation from the light quark give the dominant contribution to the decay amplitude which is inversely proportional to the constituent light quark mass. However the concept of the "constituent quark mass" is itself poorly understood. Therefore, any prediction on the branching ratios, in the framework of the above mentioned approach, is strongly model dependent.

In this work, we investigate the $B_{s(d)} \rightarrow l^+l^-\gamma$ processes practically in a model independent way, namely, within the framework of the light cone QCD sum rules method (more

about the method and its applications can be found in a recent review [8]). The paper is organized as follows: In sect.2 we give the relevant effective Hamiltonian for the $b \rightarrow ql^+l^-$ decay. In sect.3 we derive the sum rules for the transition formfactors. Sect.4 is devoted to the numerical analysis of the formfactors, and the calculation of the differential and total widths for the $B_q \rightarrow l^+l^-\gamma$ ($q = s, d$) decays. In this section we also present a comparison of our results with those of [7].

2 Effective Hamiltonian

The most important contribution to $B_q \rightarrow l^+l^-\gamma$ ($l = e, \mu$) stems from the effective Hamiltonian which induces the pure leptonic process $B_q \rightarrow l^+l^-$. The short distance contributions to $b \rightarrow l^+l^-q$ decay, comes from the box, Z-boson and photon mediated diagrams (Fig.1). The QCD corrected quark level amplitude in the SM can be written as [9, 10] :

$$\mathcal{M} = \frac{\alpha G_F}{\sqrt{2}\pi} V_{tb} V_{tq}^* \left[C_9^{eff} (\bar{q}\gamma_\mu P_L b) \bar{l}\gamma_\mu l + C_{10} \bar{q}\gamma_\mu P_L b \bar{l}\gamma_\mu \gamma_5 l - 2 \frac{C_7}{p^2} \bar{q} i \sigma_{\mu\nu} p_\nu (m_b P_R + m_q P_L) b \bar{l}\gamma_\mu l \right] \quad (1)$$

Here $P_{L(R)} = [1 - (+)\gamma_5]/2$, and p is the momentum of the lepton pair. The analytic expressions for all Wilson coefficients can be found in [9, 10]. In further considerations we shall neglect the mass of the light quarks.

As we have already noted, the pure leptonic processes $B_q \rightarrow l^+l^-$ ($l = e, \mu$) are helicity suppressed. If a photon is attached to any of the charged lines in Fig.1, the situation becomes different; helicity suppression is overcome. If a photon is emitted from the final charged lepton lines, it follows from the helicity arguments that the amplitude of such diagrams must be proportional to the lepton mass m_l ($l = e, \mu$). Therefore the contribution of such diagrams are negligible. When a photon is attached to any charged internal line, the contributions of these diagrams will be strongly suppressed by a factor of m_b^2/m_W^2 in the Wilson coefficients, since the resulting operators have dimension 8, which are two orders higher than usual operators in (1). So, we conclude that the main contribution comes from the diagrams in Fig.1 with a photon radiation from the initial quark lines. Thus the corresponding matrix element for the process $B_{s(d)} \rightarrow l^+l^-\gamma$ can be written as

$$\begin{aligned}
\langle \gamma | \mathcal{M} | B \rangle &= \frac{\alpha G_F}{2\sqrt{2}\pi} V_{tb} V_{tq}^* \left\{ C_9^{eff} \bar{l} \gamma_\mu l \langle \gamma(q) | \bar{q} \gamma_\mu (1 - \gamma_5) b | B(p+q) \rangle + \right. \\
&\quad + C_{10}^{eff} \bar{l} \gamma_\mu \gamma_5 l \langle \gamma(q) | \bar{q} \gamma_\mu (1 - \gamma_5) b | B(p+q) \rangle - \\
&\quad \left. - 2C_7 \frac{m_b}{p^2} \bar{l} \gamma_\mu l \langle \gamma(q) | \bar{q} i \sigma_{\mu\alpha} p_\alpha (1 + \gamma_5) b | B(p+q) \rangle \right\} \quad (2)
\end{aligned}$$

These transition amplitudes can be written in terms of two independent, gauge invariant (with respect to the electromagnetic field) structures:

$$\begin{aligned}
\langle \gamma(q) | \bar{q} \gamma_\mu (1 - \gamma_5) b | B(p+q) \rangle &= e \left\{ \epsilon_{\mu\alpha\beta\sigma} e_\alpha^* p_\beta q_\sigma \frac{g(p^2)}{m_B^2} + \right. \\
&\quad \left. + i [e_\mu^*(pq) - (e^* p) q_\mu] \frac{f(p^2)}{m_B^2} \right\}, \\
\langle \gamma(q) | \bar{q} i \sigma_{\mu\alpha} p_\alpha (1 + \gamma_5) b | B(p+q) \rangle &= e \left\{ \epsilon_{\mu\alpha\beta\sigma} e_\alpha^* p_\beta q_\sigma \frac{g_1(p^2)}{m_B^2} + \right. \\
&\quad \left. + i [e_\mu^*(pq) - (e^* p) q_\mu] \frac{f_1(p^2)}{m_B^2} \right\}. \quad (3)
\end{aligned}$$

Here, e_μ and q_μ stand for the polarization vector and the momentum of the photon, p is the momentum of the lepton pair, $g(p^2)$, $g_1(p^2)$, and $f(p^2)$, $f_1(p^2)$ describe the parity conserving and parity violating formfactors. Thus, the main problem is to calculate the formfactors g , g_1 and f , f_1 including their momentum dependence. For this aim we will employ the light cone QCD sum rules method.

Note that the formfactors g and f are calculated in the light cone QCD sum rules in [11]. Therefore we concentrate ourselves to the calculation of formfactors g_1 and f_1 induced by the magneto-dipole interaction.

3 Sum rules for the transition formfactors $f_1(p^2)$ and $g_1(p^2)$

According to the QCD sum rules ideology, in order to calculate the transition formfactors $f_1(p^2)$ and $g_1(p^2)$, it is necessary to write the representation of a suitable correlator function in the hadronic and quark-gluon languages. We start by considering the following correlator function

$$\Pi_\mu(p, q) = i \int d^4x e^{ipx} \langle \gamma(q) | \bar{q}(x) i \sigma_{\mu\alpha} p_\alpha (1 + \gamma_5) b(x) \bar{b}(0) i \gamma_5 q(0) | 0 \rangle \quad (4)$$

This correlator can be calculated in two different ways. On one side we insert to $\Pi_\mu(p, q)$ the hadronic states with B meson quantum numbers. Then we have

$$\begin{aligned}
\Pi_\mu(p, q) &= \frac{m_{B_q}^2 f_{B_q}}{m_b} \frac{1}{m_{B_q}^2 - (p+q)^2} \langle \gamma(q) | \bar{q} i \sigma_{\mu\alpha} p_\alpha (1 + \gamma_5) b | B(p+q) \rangle \\
&= e \frac{m_{B_q}^2 f_{B_q}}{m_b} \frac{1}{m_{B_q}^2 - (p+q)^2} \times \\
&\quad \times \left\{ \epsilon_{\mu\alpha\beta\sigma} e_\alpha^* p_\beta q_\sigma \frac{g_1(p^2)}{m_B^2} + +i \left[e_\mu^*(pq) - (e^*p) q_\mu \right] \frac{f_1(p^2)}{m_B^2} \right\}. \tag{5}
\end{aligned}$$

In deriving eq(5) we used

$$\langle B | \bar{b} i \gamma_5 q | 0 \rangle = \frac{m_{B_q}^2 f_{B_q}}{m_b}$$

On the other hand, the correlation function (4), can be calculated in QCD at large Euclidean momenta $(p+q)^2$. In general, the correlator (4) can be decomposed into the parity conserving and parity violating parts

$$\Pi_\mu(p, q) = \epsilon_{\mu\alpha\beta\lambda} e_\alpha^* p_\beta q_\lambda \Pi_1 + i \left[e_\mu^*(pq) - q_\mu (e^*p) \right] \Pi_2 \tag{6}$$

Equating eqs.(5) and (6) we get sum rules for the formfactors $g_1(p^2)$ and $f_1(p^2)$.

Let us start calculating $\Pi_\mu(p, q)$ from QCD side. The virtuality of the heavy quark in the correlator function under consideration, is large and of order $m_b^2 - (p+q)^2$. Thus, one can use the perturbative expansion of the heavy quark propagator in the external field of slowly varying fluctuations inside the photon. The leading contribution is obtained by using the free heavy quark propagator in eq.(4). Then we have

$$\begin{aligned}
\Pi_\mu(p, q) &= \int \frac{d^4x d^4k}{(2\pi)^4} \frac{e^{i(p-k)x}}{(m_b^2 - k^2)} \langle \gamma | \bar{q}(x) i \sigma_{\mu\alpha} p_\alpha (1 + \gamma_5) (\not{k} + m_b) i \gamma_5 q(0) | 0 \rangle \\
&= - \int \frac{d^4x d^4k}{(2\pi)^4} \frac{e^{i(p-k)x}}{(m_b^2 - k^2)} p_\alpha \left\{ m_b \langle \gamma | \bar{q}(x) \sigma_{\mu\alpha} (1 + \gamma_5) q(0) | 0 \rangle - \right. \\
&\quad \left. - k_\rho \langle \gamma | \bar{q}(x) \sigma_{\mu\alpha} \gamma_\rho (1 - \gamma_5) q(0) | 0 \rangle \right\} \tag{7}
\end{aligned}$$

In this equation a path ordered gauge factor between the quark fields is omitted, since in the Fock-Schwinger gauge $x_\mu A^\mu(x) = 0$, where $A^\mu(x)$ is the external electromagnetic field, it is irrelevant.

The diagrams (a) and (b) in Fig.2 describe only the short distance (perturbative) part of these matrix elements corresponding to the photon emission from the freely propagating

heavy and light quarks. The non-perturbative contributions correspond to the propagation of the light quark in the presence of external electromagnetic field (Fig.2c and 2d).

We consider now the perturbative contributions. For the diagrams (2-a and 2-b) we can write down the double dispersion representation

$$\Pi^{(1,2)} = \int \frac{ds dt \rho_i^{(1,2)}(s, t)}{[s - (p + q)^2](t - p^2)} + \text{subtr. terms.} \quad (8)$$

Here, superscripts 1, and 2 correspond to the contributions of the spectral densities to the structures $\epsilon_{\mu\alpha\beta\lambda} e_\alpha^* p_\beta q_\lambda$ and $e_\mu^*(pq) - q_\mu(e^*p)$ respectively.

For calculating the spectral densities ρ_l and ρ_H we use the method given in [12]. After a rigorous calculation for spectral densities, we have

$$\rho_l^{(1)}(s, t) = -\frac{N_c}{16\pi^2} e e_q s \delta(t - s) \left(1 - \frac{m_b^4}{s^2}\right), \quad (9)$$

$$\rho_H^{(1)}(s, t) = -\frac{2N_c}{16\pi^2} e e_b s \delta(t - s) \left(1 - \frac{m_b^2}{s}\right), \quad (10)$$

$$\begin{aligned} \rho_l^{(2)}(s, t) = & \frac{2N_c}{16\pi^2} e e_q \left\{ \delta(t - s) \left(1 - \frac{m_b^2}{s}\right) \left(-\frac{m_b^2}{2} + \frac{3}{2}s\right) - \right. \\ & \left. -\delta'(t - s) \left(1 - \frac{m_b^2}{s}\right) (s - m_b^2) s \right\}, \quad (11) \end{aligned}$$

$$\begin{aligned} \rho_H^{(2)}(s, t) = & \frac{2N_c}{16\pi^2} e e_b \left\{ \delta(t - s) \left[\left(1 - \frac{m_b^2}{s}\right) \left(\frac{3}{2}s + \frac{1}{2}m_b^2\right) - 2m_b^2 \ln\left(\frac{s}{m_b^2}\right) \right] - \right. \\ & \left. -\delta'(t - s) \left[\left(1 - \frac{m_b^2}{s}\right) (s^2 + s m_b^2) - 2s m_b^2 \ln\left(\frac{s}{m_b^2}\right) \right] \right\}. \quad (12) \end{aligned}$$

In eqs.(9-12) ρ_l and ρ_H corresponds to the interaction of the photon with the light and b quarks, $N_c = 3$ is the color factor, e_q and e_b the electric charge of the light and b quarks and m_b is the mass of the b -quark $\delta'(t - s) = \frac{d}{dt}\delta(t - s)$.

Next consider the non-perturbative contributions. From eq.(7) it follows that the non-perturbative contributions are expressed via the matrix elements of the gauge invariant nonlocal operators, sandwiched in between the vacuum and the photon state. These matrix elements define the following light cone photon wave functions ([10, 13], see also the first reference in [11]):

$$\begin{aligned}
\langle \gamma | \bar{q}(x) \sigma_{\mu\alpha} q(0) | 0 \rangle &= i e e_q \langle \bar{q}q \rangle \int_0^1 du e^{iuqx} \\
&\quad \left\{ (e_\mu q_\alpha - e_\alpha q_\mu) [\chi \phi(u) + x^2 (g_1(u) - g_2(u))] + \right. \\
&\quad \left. + g_2(u) [qx (e_\mu x_\alpha - e_\alpha x_\mu) + ex (x_\mu q_\alpha - x_\alpha q_\mu)] \right\} \quad \text{and} \\
\langle \gamma | \bar{q}(x) \gamma_\mu \gamma_5 q(0) | 0 \rangle &= \frac{1}{4} e \epsilon_{\mu\alpha\beta\lambda} e_\alpha p_\beta x_\lambda f \int_0^1 g_\perp(u) e^{iuqx} . \tag{13}
\end{aligned}$$

Here χ is magnetic susceptibility of the quark condensate, $\phi(u)$, $g_\perp(u)$ are the leading twist $\tau = 2$ photon wave functions, $g_1(u)$ and $g_2(u)$ are the two particle $\tau = 4$ wave functions. Note that for calculating the matrix elements

$$\begin{aligned}
&\langle \gamma(q) | \bar{q}(x) \sigma_{\mu\alpha} \gamma_\rho (1 - \gamma_5) q(0) | 0 \rangle \quad \text{and} \\
&\langle \gamma(q) | \bar{q}(x) \sigma_{\mu\alpha} \gamma_5 q(0) | 0 \rangle
\end{aligned}$$

we use the following identities:

$$\sigma_{\mu\alpha} \gamma_5 = \frac{1}{2} i \epsilon_{\mu\alpha\lambda\rho} \sigma_{\lambda\rho} , \tag{14}$$

$$\sigma_{\mu\alpha} \gamma_\rho = i (\gamma_\mu g_{\alpha\rho} - \gamma_\alpha g_{\mu\rho}) + \epsilon_{\mu\alpha\rho\lambda} \gamma_\lambda \gamma_5 . \tag{15}$$

After lengthy calculations for Π_1 and Π_2 we get the following results, which describe the non-perturbative contributions:

$$\begin{aligned}
\Pi_1 &= m_b e e_q \langle \bar{q}q \rangle \int_0^1 du \left\{ -\frac{\chi \phi(u)}{\Delta} + 8m_b^2 \frac{g_1(u) - g_2(u)}{\Delta^3} - \frac{4(m_b^2 - p^2)}{\Delta^3} g_2 \right\} - \\
&\quad -\frac{e}{4} f \int_0^1 du \left[\frac{1}{\Delta} + \frac{p^2 + m_b^2}{\Delta^2} \right] g_\perp(u) + e e_b m_b \langle \bar{q}q \rangle \frac{1}{(m_b^2 - p^2) [m_b^2 - (p+q)^2]} , \tag{16}
\end{aligned}$$

$$\begin{aligned}
\Pi_2 &= m_b e e_q \langle \bar{q}q \rangle \int_0^1 du \left\{ -\frac{\chi \phi(u)}{\Delta} + 8m_b^2 \frac{g_1(u) - g_2(u)}{\Delta^3} - \frac{4(m_b^2 + p^2)}{\Delta^3} g_2 \right\} - \\
&\quad -\frac{e}{4} f \int_0^1 du \left[\frac{2}{\Delta} + \frac{2upq}{\Delta^3} \right] g_\perp(u) + e e_b m_b \langle \bar{q}q \rangle \frac{1}{(m_b^2 - p^2) [m_b^2 - (p+q)^2]} \tag{17}
\end{aligned}$$

Here $\Delta = m_b^2 - (p + uq)^2$. Last term in eqs.(16) and (17) describes the case when a photon is emitted from the heavy quark (see Fig.2d). Collecting eqs.(9-12) and (16-17) we finally get the following expressions for the invariant functions Π_1 and Π_2 :

$$\begin{aligned}
\Pi_1 &= -\frac{N_c e}{16\pi^2} \int_0^1 du \frac{m_b^2 - p^2 \bar{u}}{u^2 \Delta_1} \left(1 - \frac{m_b^2 u}{m_b^2 - p^2 \bar{u}}\right) \left[e_q \left(1 + \frac{m_b^2 u}{m_b^2 - p^2 \bar{u}}\right) + 2e_b \right] + \\
&+ m_b e e_q \langle \bar{q} q \rangle \int \frac{du}{u} \left\{ -\frac{\chi \phi(u)}{\Delta_1} + 8m_b^2 \frac{g_1(u) - g_2(u)}{u^2 \Delta_1^3} - 4 \frac{(m_b^2 - p^2)}{u^2 \Delta_1^3} g_2 \right\} - \\
&- \frac{e}{4} f \int \frac{du}{u} \left[\frac{1}{\Delta_1} + \frac{p^2 + m_b^2}{u \Delta_1^2} \right] g_\perp(u) + e e_b m_b \langle \bar{q} q \rangle \frac{1}{(m_b^2 - p^2) [m_b^2 - (p+q)^2]} , \tag{18}
\end{aligned}$$

$$\begin{aligned}
\Pi_2 &= \frac{N_c e}{16\pi^2} \int_0^1 \frac{du}{\Delta_1 (m_b^2 - p^2)} \left\{ \left(1 - \frac{u m_b^2}{m_b^2 - p^2 \bar{u}}\right) \left[(e_b + e_q) \frac{m_b^2 - p^2 \bar{u}}{u} \left(\frac{m_b^2 - p^2 \bar{u}}{u} - 3p^2 \right) + \right. \right. \\
&+ \left. (e_q - e_b) m_b^2 \left(\frac{m_b^2 - p^2 \bar{u}}{u} + p^2 \right) \right] + 4e_b m_b^2 p^2 \ln \left(\frac{m_b^2 - p^2 \bar{u}}{m_b^2 u} \right) \left. \right\} + \\
&+ m_b e e_q \langle \bar{q} q \rangle \int_0^1 \frac{du}{u} \left\{ -\frac{\chi \phi(u)}{\Delta_1} + 8m_b^2 \frac{g_1(u) - g_2(u)}{u^2 \Delta_1^3} + 4 \frac{(m_b^2 + p^2)}{u^2 \Delta_1^3} g_2 \right\} - \\
&- \frac{e}{4} f \int_0^1 \frac{du}{u} \left[\frac{2}{\Delta_1} + \frac{2(pq)}{u \Delta_1^3} \right] g_\perp(u) + e e_b m_b \langle \bar{q} q \rangle \frac{1}{(m_b^2 - p^2) [m_b^2 - (p+q)^2]} , \tag{19}
\end{aligned}$$

where $\Delta_1 = \frac{(m_b^2 - p^2 \bar{u})}{u} - (p+q)^2$, $\bar{u} = 1 - u$. In eqs.(18) and (19) we have rewritten the dispersion integral in terms of the variable $u = (m_b^2 - p^2)/(s - p^2)$.

Here we would like to make the following remark. As we noted earlier, the functions $g_1(u)$ and $g_2(u)$ represent twist $\tau = 4$ contributions to the two-particle photon wave function. To this accuracy, in eq.(19) we must take into account other twist $\tau = 4$ photon wave functions (see for example [17]). Using the equation of motion, one can relate them to the three-particle wave functions of twist $\tau = 4$ with an additional gluon from heavy quark [17]. But, these three-particle wave function contributions, in general, are small and we will neglect them in further analysis.

The remaining task is now to match eqs.(18) and (19) with the corresponding hadronic representation (see eq.(5)) and to extract the formfactors $g_1(p^2)$ and $f_1(p^2)$. As usual, invoking duality, we assume that above certain threshold $s_0 = 35 \text{ GeV}^2$ (this value follows from two-point sum rules analysis) the spectral density $\rho(s)$ associated with higher resonances and continuum states coincides with the spectral density from perturbative part.

This procedure is equivalent to writing $(m_b^2 - p^2)/(s_0 - p^2)$ in the lower limit of the integration over u in eqs.(18) and (19) (for more detail see [11, 15]). Finally applying the Borel transformation on the variable $-(p + q)^2 \rightarrow M^2$ to suppress both higher state resonances and higher Fock states in the full photon wave functions, we get the following sum rules for the formfactors:

$$\begin{aligned}
g_1(p^2) &= -\frac{m_b}{f_B} e^{\frac{m_B^2}{M^2}} \left\{ \frac{N_c}{16\pi^2} \int_\delta^1 \frac{du}{u^2} (m_b^2 - p^2\bar{u}) \left(1 - \frac{m_b^2 u}{m_b^2 - p^2\bar{u}} \right) \times \right. \\
&\quad \times \left[e_q \left(1 + \frac{m_b^2 u}{m_b^2 - p^2\bar{u}} \right) + 2e_b \right] e^{-\frac{(m_b^2 - p^2\bar{u})}{uM^2}} + \\
&\quad + m_b \langle \bar{q}q \rangle e_q \int_\delta^1 \frac{du}{u} \left[\chi\phi(u) - 4m_b^2 (g_1 - g_2) \frac{1}{u^2 M^4} - 2\frac{(m_b^2 - p^2)}{u^2 M^4} g_2 \right] e^{-\frac{(m_b^2 - p^2\bar{u})}{uM^2}} + \\
&\quad \left. + \frac{f}{4} \int_\delta^1 du \frac{g_\perp(u)}{u} \left(1 + \frac{p^2 + m_b^2}{uM^2} \right) e^{-\frac{(m_b^2 - p^2\bar{u})}{uM^2}} - e_b m_b \frac{\langle \bar{q}q \rangle}{m_b^2 - p^2} e^{-\frac{m_b^2}{M^2}} \right\}, \\
f_1(p^2) &= \frac{m_b}{f_B} e^{\frac{m_B^2}{M^2}} \left\{ \frac{N_c}{16\pi^2} \int_\delta^1 \frac{du}{m_b^2 - p^2} e^{-\frac{(m_b^2 - p^2\bar{u})}{uM^2}} \left[\left(1 - \frac{m_b^2 u}{m_b^2 - p^2\bar{u}} \right) \times \right. \right. \\
&\quad \times \left((e_q + e_b) \frac{m_b^2 - p^2\bar{u}}{u} \left(\frac{m_b^2 - p^2\bar{u}}{u} - 3p^2 \right) + (e_q - e_b) m_b^2 \left(\frac{m_b^2 - p^2\bar{u}}{u} + p^2 \right) \right) \\
&\quad + 4m_b^2 p^2 \ln \frac{m_b^2 - p^2\bar{u}}{m_b^2 u} \left. \right] + m_b e_q \langle \bar{q}q \rangle \int_\delta^1 \frac{du}{u} \left[-\chi\phi(u) + \frac{4m_b^2}{M^4 u^2} (g_1 - g_2) + \right. \\
&\quad + \frac{2(p^2 + m_b^2)}{u^2 M^4} g_2 \left. \right] e^{-\frac{(m_b^2 - p^2\bar{u})}{uM^2}} + \\
&\quad \left. + \frac{f}{4} \int_\delta^1 \frac{du}{u} g_\perp(u) \left(-1 + \frac{p^2 - m_b^2}{uM^2} \right) e^{-\frac{(m_b^2 - p^2\bar{u})}{uM^2}} + e_b m_b \frac{\langle \bar{q}q \rangle}{m_b^2 - p^2} e^{-\frac{m_b^2}{M^2}} \right\} \quad (20)
\end{aligned}$$

At the end of this section we give the result for the differential decay widths:

$$\begin{aligned}
\frac{d\Gamma}{d\hat{s}} &= \frac{\alpha^3 G^2}{768\pi^5} |V_{tb}V_{tq}^*|^2 m_B^5 \hat{s} (1 - \hat{s})^3 \sqrt{1 - 4\frac{m_l^2}{m_B^2 \hat{s}}} \times \\
&\quad \times \left\{ \frac{1}{m_B^2} [|A|^2 + |B|^2] + \frac{1}{m_B^2} |C_{10}|^2 [f^2(p^2) + g^2(p^2)] \right\}, \quad (21)
\end{aligned}$$

where

$$\begin{aligned}\hat{s} &= p^2/m_B^2, \\ A &= C_9^{eff} g(p^2) - 2C_7 \frac{m_b}{p^2} g_1(p^2), \text{ and} \\ B &= C_9^{eff} f(p^2) - 2C_7 \frac{m_b}{p^2} f_1(p^2).\end{aligned}$$

4 Numerical Analysis

For calculating formfactors $f_1(p^2)$ and $g_1(p^2)$ we use the following input parameters: $m_b = 4.7 \text{ GeV}$, $s_0 \simeq 35 \text{ GeV}^2$, $f_B = 140 \text{ MeV}$ [14, 15], $\phi(u) = 6u(1-u)$ [16, 17]. To the leading twist accuracy we use for $g_\perp(u) = 1$ (see first reference in [11]) and for $g_1(u)$ and $g_2(u)$ the following expressions [13]:

$$g_1(u) = -\frac{1}{8}(1-u)(3-u) \quad (22)$$

$$g_2(u) = -\frac{1}{4}(1-u)^2 \quad (23)$$

The magnetic susceptibility χ was determined in [18], $\chi = -3.4 \text{ GeV}^{-2}$ at the scale $\mu_b \sim \sqrt{m_B^2 - m_b^2}$, $\langle \bar{q}q \rangle = -(0.26 \text{ GeV})^3$. The Borel parameter M^2 has been varied in the region $8 \text{ GeV}^2 < M^2 < 20 \text{ GeV}^2$. Numerical analysis shows that the variation of M^2 in this region, changes the results by less than 8%. The predictions of the sum rules are very stable in this region of the Borel parameter and vary only a few percent with the changes of m_b , s_0 and f_B within the intervals allowed by the two point sum rules for f_B .

The sum rules is reliable in the region $m_b^2 - p^2 \sim \text{a few GeV}^2$, which is smaller than $p^2 = m_b^2$. In order to extent our results to the whole region of p^2 we use some extrapolation formulas. We found that the best agreement is achieved by the dipole type formulas

$$g_1(p^2) = \frac{3.74 \text{ GeV}^2}{(1 - \frac{p^2}{m_1^2})^2}, \quad (24)$$

$$f_1(p^2) = \frac{0.68 \text{ GeV}^2}{(1 - \frac{p^2}{m_2^2})^2}, \quad (25)$$

where $m_1^2 = 40.5 \text{ GeV}^2$ and $m_2^2 = 30 \text{ GeV}^2$. For calculating differential and total decay widths, we need the values of C_9^{eff} , C_7 and C_{10} coefficients and the explicit forms of the

formfactors $g(p^2)$ and $f(p^2)$. These formfactors are calculated in [11]:

$$g(p^2) = \frac{1 \text{ GeV}}{\left(1 - \frac{p^2}{5.6^2}\right)^2}, \quad (26)$$

$$f(p^2) = \frac{0.8 \text{ GeV}}{\left(1 - \frac{p^2}{6.5^2}\right)^2}. \quad (27)$$

The values of the Wilson coefficients C_7 and C_{10} are taken from [9, 10] as

$$C_7 = -0.315, \quad C_{10} = -4.642,$$

and the expression C_9^{eff} for $b \rightarrow s$ transition, in the next-to-leading order approximation is given as (see [19])

$$\begin{aligned} C_9^{eff} = & C_9 + 0.124w(\hat{s}) + g(\hat{m}_c, \hat{s})(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) - \\ & - \frac{1}{2}g(\hat{m}_q, \hat{s})(C_3 + 3C_4) - \frac{1}{2}g(\hat{m}_b, \hat{s})(4C_3 + 4C_4 + 3C_5 + C_6) + \\ & + \frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6), \end{aligned} \quad (28)$$

with

$$\begin{aligned} C_1 = & -0.249, \quad C_2 = 1.108, \quad C_3 = 1.112 \times 10^{-2}, \quad C_4 = -2.569 \times 10^{-2} \\ C_5 = & 7.4 \times 10^{-3}, \quad C_6 = -3.144 \times 10^{-2}, \quad C_9 = 4.227. \end{aligned}$$

The value of C_9^{eff} for $b \rightarrow d$ transition, can be obtained by adding to eq.(28) the term $\lambda_u [g(\hat{m}_c, \hat{s}) - g(\hat{m}_d, \hat{s})] (3C_1 + C_2)$, where

$$\lambda_u = \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}.$$

For obtaining these values we used $\Lambda_{QCD} = 225 \text{ MeV}$, $\sin^2\theta_W = 0.23$, $m_t = 176 \text{ GeV}$, $m_W = 80.2 \text{ GeV}$ and $\hat{m}_q = m_q/m_b$. In the above formula $w(\hat{s})$ represents the one-gluon correction to the matrix element O_9 and explicit expression can be found in [10], while the function $g(\hat{m}_q, \hat{s})$ arises from the one loop contributions of the four quark operators $O_1 - O_6$ (see for example [9, 10]), i.e.

$$\begin{aligned} g(\hat{m}_q, \hat{s}) = & -\frac{8}{9}\ln\hat{m}_q + \frac{8}{27} + \frac{4}{9}y_q - \frac{2}{9}(2 + y_q)\sqrt{11 - y_q} + + \left\{ \Theta(1 - y_q) \times \right. \\ & \left. \times \left(\ln \frac{1 + \sqrt{1 - y_q}}{1 - \sqrt{1 - y_q}} - i\pi \right) + \Theta(y_q - 1) \text{arctg} \frac{1}{\sqrt{y_q - 1}} \right\} \end{aligned} \quad (29)$$

with $y_q = \hat{m}_q^2/\hat{s}'$, and $\hat{s}' = p^2/m_b^2$.

For a more complete analysis of the above decay, one has to take into account the long distance contributions. In the case of the J/ψ family, this is accomplished by introducing a Breit-Wigner formula through the replacement (see [20])

$$g(\hat{m}_c, \hat{s}') \rightarrow g(\hat{m}_c, \hat{s}') - \frac{3\pi}{\alpha^2} \sum_{V=J/\psi, \psi'} \frac{\hat{m}_V Br(V \rightarrow l^+l^-) \hat{\Gamma}_{tot}^V}{\hat{s}' - \hat{m}_V^2 + im_V \hat{\Gamma}_{tot}^V} \quad (30)$$

where $\hat{m}_V = m_V/m_b$, $\hat{\Gamma}_{tot} = \Gamma/m_b$. The masses and decay widths of the corresponding mesons are listed in [21]. In Fig.3 we present the differential decay rate for $B_s \rightarrow \mu^+\mu^-\gamma$ decay (behavior of the differential decay rate for $B_s \rightarrow e^+e^-\gamma$ decay is similar) as a function of \hat{s} , with and without resonance (J/ψ and ψ') contributions. From this figure we see that the contribution from soft photons, corresponding to large \hat{s} region is negligible.

Using the above mentioned values of the parameters and $|V_{tb}V_{ts}^*| = 0.045$, $|V_{tb}V_{td}^*| = 0.01$, $\tau(B_s) = 1.34 \times 10^{-12} s$, $\tau(B_d) = 1.5 \times 10^{-12} s$ [21], for branching ratios we get (without the long distance contributions):

$$\begin{aligned} B(B_s \rightarrow e^+e^-\gamma) &= 2.35 \times 10^{-9} \\ B(B_s \rightarrow \mu^+\mu^-\gamma) &= 1.9 \times 10^{-9} \\ B(B_d \rightarrow e^+e^-\gamma) &= 1.5 \times 10^{-10} \\ B(B_d \rightarrow \mu^+\mu^-\gamma) &= 1.2 \times 10^{-10} \end{aligned} \quad (31)$$

For comparison we present also the constituent model prediction (at $f_B = 140 MeV$, $m_s = 0.57 GeV$, $m_d = 0.35 GeV$) [7]:

$$\begin{aligned} B(B_s \rightarrow e^+e^-\gamma) &= 3 \times 10^{-9} \\ B(B_s \rightarrow \mu^+\mu^-\gamma) &= 2.3 \times 10^{-9} \\ B(B_d \rightarrow e^+e^-\gamma) &= 4 \times 10^{-10} \\ B(B_d \rightarrow \mu^+\mu^-\gamma) &= 3 \times 10^{-10} \end{aligned} \quad (32)$$

We see that the constituent quark model and light cone sum rules method predictions on the branching ratios are very close. Let us compare our results on branching ratios with those of pure leptonic decays. The rates for the pure leptonic decays are (see for example [6, 7])

$$\Gamma(B_q \rightarrow l^+l^-) = \frac{\alpha^2 G_F^2 f_{B_q}^2 m_{B_q} m_l^2}{16\pi^3} |V_{tb}V_{tq}^*|^2 C_{10}^2 \quad (33)$$

If we use the value of $f_{B_s} \simeq f_{B_d} \simeq 140 \text{ MeV}$, for the corresponding Branching ratios we get:

$$\begin{aligned}
B(B_s \rightarrow e^+e^-) &= 3 \times 10^{-14} \\
B(B_s \rightarrow \mu^+\mu^-) &= 1.3 \times 10^{-9} \\
B(B_d \rightarrow e^+e^-) &= 2.1 \times 10^{-15} \\
B(B_d \rightarrow \mu^+\mu^-) &= 9 \times 10^{-11}
\end{aligned} \tag{34}$$

From these values and eq.(30) it follows that, the radiative decays dominate over the pure leptonic decays in the corresponding channels and $B_s \rightarrow e^+e^-\gamma$ decay mode has a larger branching ratio. Few words about the experimental detectability of these processes is in order. In future B -factories and LHC approximately $6 \times 10^{11}(2 \times 10^{11})$ $B_d(B_s)$ mesons are expected per year. Therefore the decays $B_{s(d)} \rightarrow l^+l^-\gamma$ are expected to be quite detectable in these machines.

In conclusion, we have analyzed the rare $B_q \rightarrow l^+l^-\gamma$ decays in SM and obtain the branching ratios for $B_s \rightarrow l^+l^-\gamma$ to be around 2×10^{-9} and around 2×10^{-10} for $B_d \rightarrow l^+l^-\gamma$.

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Figure Captions

1. Feynman diagrams in the Standard Model for $b\bar{q} \rightarrow l^+l^-$
2. Diagrams describing the perturbative and non-perturbative contributions to the correlator function (4).
3. Differential decay rates of $B_s \rightarrow \mu^+\mu^-\gamma$ versus $\hat{s} = p^2/m_B^2$. Here the thick line corresponds to the case without the J/ψ , ψ' and the thin line with the J/ψ , ψ' contributions.

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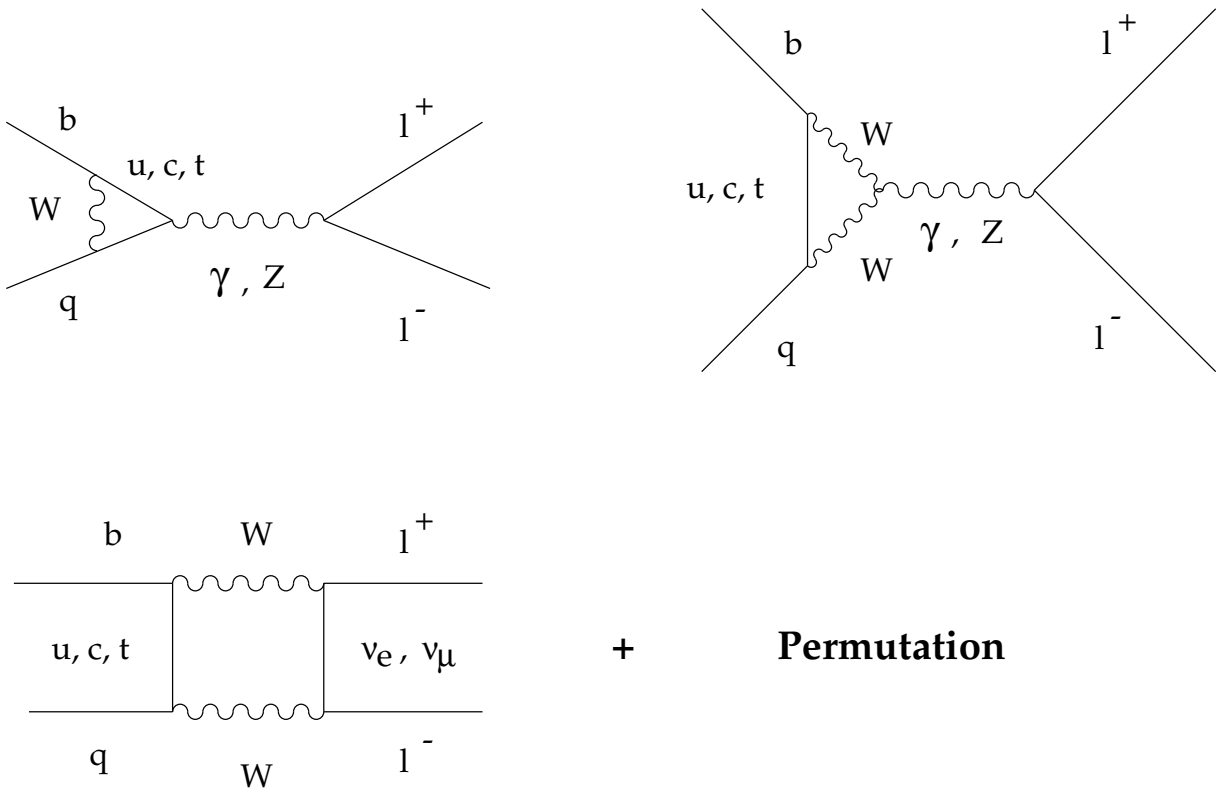


Fig.1

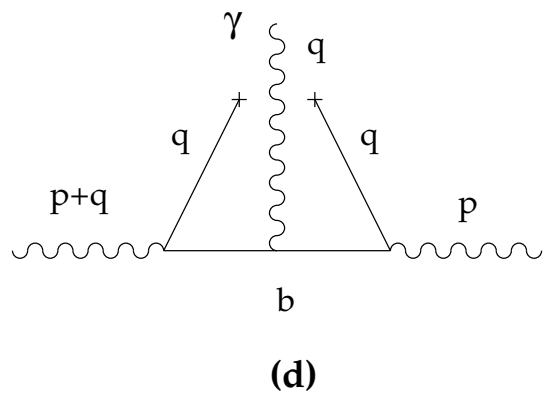
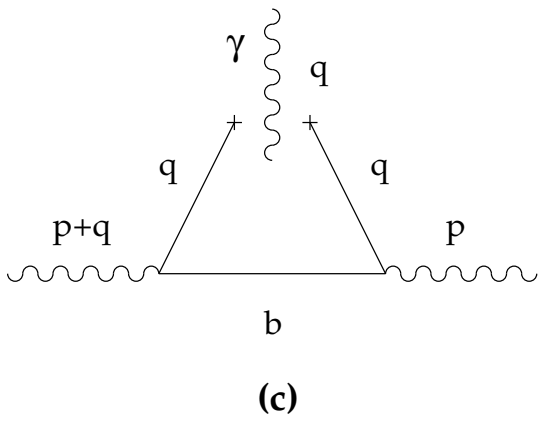
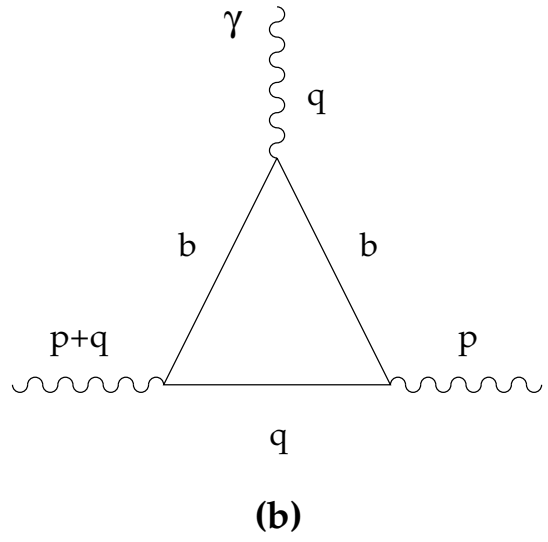
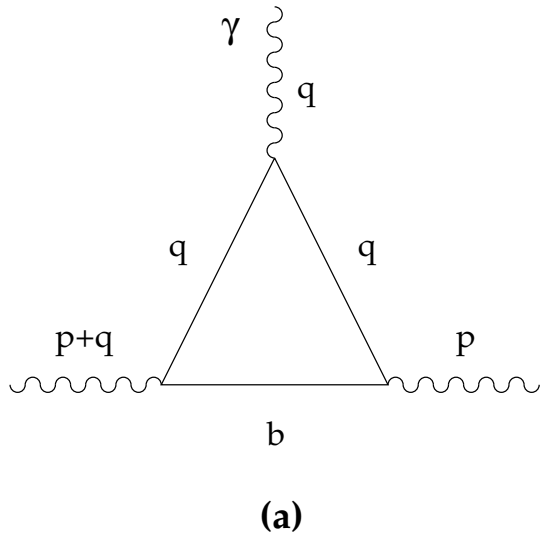


Fig.2

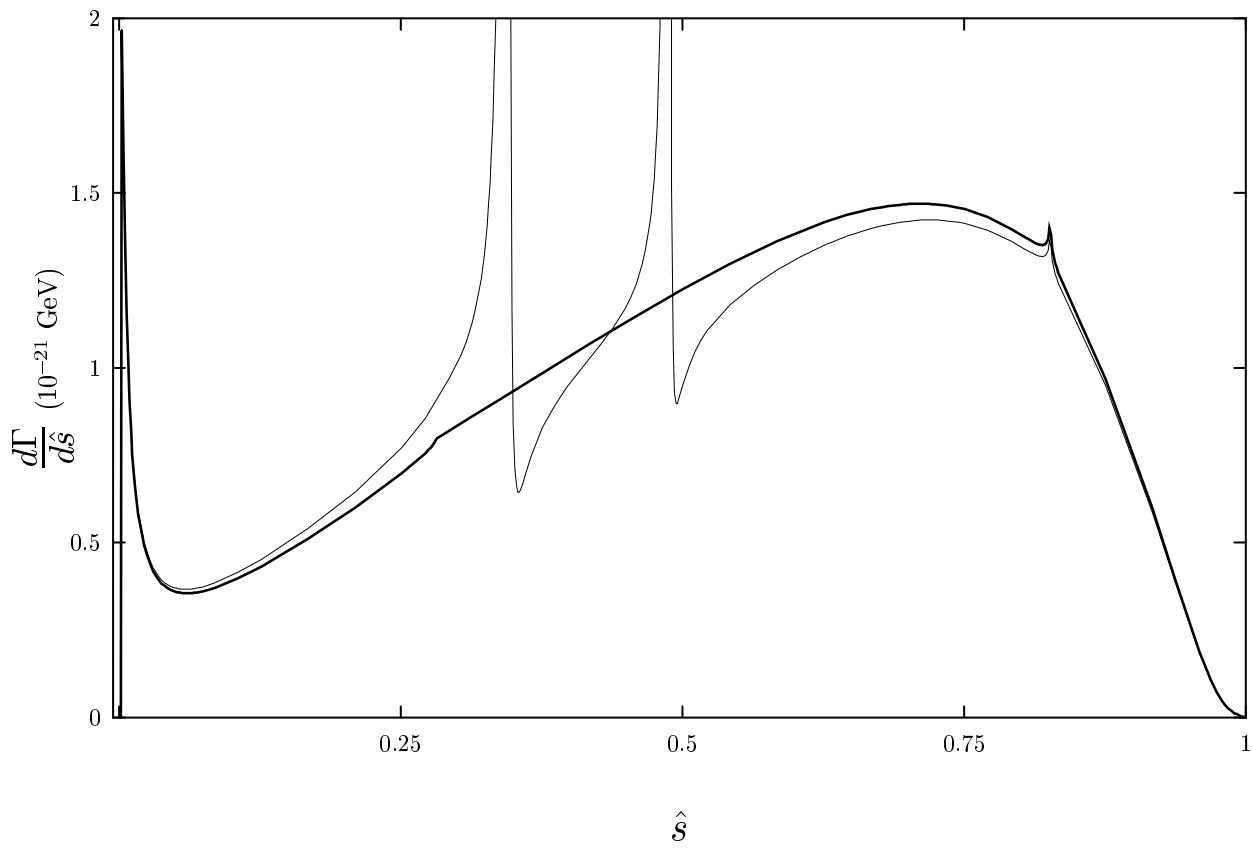


Fig.3