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Determination of the Σ - Λ mixing angle from QCD sum rules

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ABSTRACT: The Σ - Λ mixing angle is calculated in framework of the QCD sum rules by using the general form of the interpolating current. We find that our prediction for the mixing angle is $(1.15 \pm 0.05)^0$. A comparison of our result with the predictions of the quark model, chiral perturbation theory, and lattice QCD approach, and also with the results of the QCD sum rules method existing in literature, is presented.

KEYWORDS: QCD Phenomenology

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1 Introduction

Flavor symmetry plays essential role in classification of the hadrons. The light hadronic states are successfully described by using SU(3) flavor symmetry. In the case this symmetry is exact, hadrons belonging to the same representation of SU(3) flavor group could be degenerate. Experimentally it is known that the hadrons belonging to the same representation have different masses, which leads to SU(3) flavor symmetry breaking. At quark level, this symmetry is broken due to the mass difference of the light u, d and s quarks.

The breaking of the SU(3) flavor symmetry might lead to mixing of hadrons. In other words, the definite flavor eigenstates can mix to form the physically observed states.

Long time ago, it is observed that the lowest lying hadrons Λ and Σ can be represented as the combination of the SU(3) octet, pure isospin $I = 0$ (Λ), and $I = 1$ (Σ^0) baryons in the following form [1],

$$\begin{aligned}\Lambda &= \Lambda^0 \cos \alpha - \Sigma^0 \sin \alpha, \\ \Sigma &= \Lambda^0 \sin \alpha + \Sigma^0 \cos \alpha.\end{aligned}\tag{1.1}$$

The Σ - Λ mixing angle is estimated in the framework of different approaches such as, quark model [1–3], chiral perturbation theory [4], QCD sum rules in vacuum [5, 6], QCD sum rules in isospin asymmetric medium [7], and in lattice QCD [8]. The mixing angle predicted by these models are, $|\alpha| = 1.4 \times 10^{-3}$ [5], $|\alpha| = 7.0 \times 10^{-3}$ [6], $|\alpha| = 1.5 \times 10^{-2}$ [1, 2], $|\alpha| = 1.0 \times 10^{-2}$ [3] and [4], $|\alpha| = 7.0 \times 10^{-3}$ [8] (all results for the mixing angle are given in radians).

We see from these values that different approaches predict different values for the mixing angle. In the present work we determine the Σ - Λ mixing within the QCD sum rules method and compare our result with the predictions of the above-mentioned approaches.

The main difference of our approach in determination of the Σ - Λ mixing angle compared to the QCD sum rules calculation existing in literature is that, in our case the mixing angle is introduced in the interpolating current, rather than in the hadronic part. The other novel property that our approach possesses is the hadronic part of the QCD sum rules is zero in our case, i.e., our approach is free of uncertainties coming from the hadronic part.

In determination of the Σ - Λ mixing angle within the QCD sum rules we follow the method suggested in [9], and for this goal we start by considering the following correlation function,

$$\Pi = i \int d^4x e^{ipx} \langle 0 | T \{ \eta_H(x) \bar{\eta}_H(0) \} | 0 \rangle,\tag{1.2}$$

where \mathbb{T} is the time ordering operator, η_H is the interpolating current, carrying the same quantum numbers as the corresponding hadron. If the bare H_1^0 and H_2^0 states are mixed, the corresponding physical states with definite mass should be the linear combinations of these bare states. In this case, the interpolating currents corresponding to the physical states could be represented as the superposition of the interpolating currents corresponding to the bare states, i.e.,

$$\begin{aligned}\eta_\Lambda &= \sin \alpha \eta_{\Lambda^0} + \cos \alpha \eta_{\Sigma^0}, \\ \eta_\Sigma &= \cos \alpha \eta_{\Lambda^0} - \sin \alpha \eta_{\Sigma^0},\end{aligned}\tag{1.3}$$

where α is the mixing angle between Λ^0 and Σ^0 states. In presence of only two physical states, eq. (1.2) can be written as,

$$\Pi = i \int d^4x e^{ipx} \langle 0 | \mathbb{T} \{ \eta_\Lambda(x) \bar{\eta}_\Sigma \} | 0 \rangle .\tag{1.4}$$

It should be remembered that the general form of the correlator function is,

$$\Pi(p) = \Pi_1(p^2) \not{p} + \Pi_2(p^2) I,$$

and coefficients of the \not{p} and I (unit operator) structures, i.e., $\Pi_1(p^2)$ and $\Pi_2(p^2)$ can both be used in determining the mixing angle.

In order to construct the sum rules for the mixing angle α , the correlation function (1.4) is calculated in terms of hadrons, quarks and gluons. Using the duality ansatz these two representations are matched and the sum rules for the corresponding physical quantity is obtained.

The hadronic representation of the correlation function is obtained by saturating it with the full set of baryons having the same quantum numbers as the corresponding interpolating current. Since η_{H_1} and η_{H_2} can create only the states H_1 and H_2 , correspondingly, the hadronic part of the correlation function is obviously zero if we isolate the contributions of the ground state baryons to the correlation function. It should be noted here that the correlation function also contains contributions coming from the higher states, and therefore in principle the correlation function may not vanish. But contributions of the higher states are taken into account by the quark-hadron duality, i.e., higher state contributions are equal to perturbative contributions starting from some threshold s_0 . Therefore, the physical part of the correlation function becomes zero after continuum subtraction procedure is performed. In other words, coefficients of the structures \not{p} and I should independently be equal to zero.

Using eq. (1.3) in eq. (1.4), one can easily obtain the expression for the mixing angle for both structures,

$$\tan 2\alpha = \frac{2\Pi_{\Sigma\Lambda}^0}{\Pi_{\Sigma\Sigma}^0 - \Pi_{\Lambda\Lambda}^0},\tag{1.5}$$

where Π_{ij}^0 are the correlation functions corresponding to the unmixed states, i.e.,

$$\Pi_{ij}^0 = i \int d^4x e^{ipx} \langle 0 | \mathbb{T} \{ \eta_i^0(x) \bar{\eta}_j^0 \} | 0 \rangle ,\tag{1.6}$$

where $(i, j = \Lambda^0 \text{ or } \Sigma^0)$. So the problem of determination of the mixing angle requires the calculation of the theoretical part of the correlation function, for which the expressions of the interpolating currents are needed.

According to the $SU(3)_f$ classification the interpolating currents for the unmixed Λ^0 and Σ^0 are chosen as [10, 11],

$$\begin{aligned} \eta_{\Lambda^0} &= 2\sqrt{\frac{1}{6}}\varepsilon^{abc}\left\{2(u^{aT}Cd^b)\gamma_5s^c + 2\beta(u^{aT}C\gamma_5d^b)s^c + (u^{aT}Cs^b)\gamma_5d^c + \beta(u^{aT}C\gamma_5s^b)d^c \right. \\ &\quad \left. - (d^{aT}Cs^b)\gamma_5u^c - \beta(d^{aT}C\gamma_5s^b)u^c\right\}, \\ \eta_{\Sigma^0} &= \sqrt{2}\varepsilon^{abc}\left\{(u^{aT}Cs^b)\gamma_5d^c + \beta(u^{aT}C\gamma_5s^b)d^c + (d^{aT}Cs^b)\gamma_5u^c + \beta(d^{aT}C\gamma_5s^b)u^c\right\}, \end{aligned} \quad (1.7)$$

where a, b, c are the color indices, C is the charge conjugation operator, and β is the arbitrary constant with $\beta = -1$ corresponding to the so-called Ioffe current.

Using the operator product expansion at $p^2 \ll 0$, one can easily obtain the expressions for the correlation functions $\Pi_{\Sigma\Sigma}^0 - \Pi_{\Lambda\Lambda}^0$, and $\Pi_{\Sigma\Lambda}^0$ from eq. (1.6) from the QCD side for the \not{p} and I structures. The expressions of these correlation functions are presented in the appendix.

In order to proceed for the numerical calculations we need the values of the input parameters that are given as: $\langle\bar{u}u\rangle(1\text{ GeV}) = (-0.246_{-19}^{+28}\text{ MeV}^3)$ [12], $\langle\bar{s}s\rangle = 0.8\langle\bar{q}q\rangle$, $\langle g^2G^2\rangle = 0.47\text{ GeV}^4$, $m_0^2 = (0.8 \pm 0.2)\text{ GeV}^2$ [13]. For the masses of the light quarks we use their \overline{MS} values given as: $\bar{m}_u(1\text{ GeV}) = 2.3_{-0.5}^{+0.7}\text{ MeV}$, $\bar{m}_d(1\text{ GeV}) = 4.8_{-0.3}^{+0.5}\text{ MeV}$, $\bar{m}_s(1\text{ GeV}) = 95_{-5}^{+5}\text{ MeV}$ [14]. In our numerical calculations we also take into account isospin symmetry breaking effects in the quark condensates, i.e., $\langle\bar{d}d\rangle/\langle\bar{u}u\rangle \simeq (1.0 - 9.0 \times 10^{-3})$ [15]. It should be noted here that the isospin breaking has two different sources: i) The electromagnetic effect due to the electric charge difference between up and down quarks; ii) the mass difference between up and down quarks. It is shown in [16] that in analysis of the Σ - Λ mixing angle, the contributions coming from electromagnetic effects are smaller compared to the ones coming from quark mass differences. For this reason we neglect the electromagnetic effects in our calculations.

It follows from the expressions of the invariant functions that in order to determine the Σ - Λ mixing angle three arbitrary parameters are involved, namely, the continuum threshold s_0 , the Borel mass parameter M^2 , and the parameter β (see the expressions of the interpolating currents); and of course the mixing angle should be independent of them all. As is well known, the continuum threshold is related to the energy of the first excited state. The difference $\sqrt{s_0} - m_{\text{ground}}$, where m_{ground} is the mass of the ground state, is equal to the energy needed to excite the particle to its first energy state. This difference usually changes in the range between 0.3–0.8 GeV. It follows from the analysis of the mass sum rules that in order to reproduce the experimental values of the masses of the Σ and Λ baryons, the continuum threshold s_0 should lie in the range $2.5\text{ GeV}^2 \leq s_0 \leq 3.2\text{ GeV}^2$ [10, 11]. Moreover, the working region of the Borel mass parameter should be such that, the results for the Σ - Λ mixing angle should exhibit good stability with respect to the variation of M^2 at fixed values of s_0 . The upper bound of M^2 is obtained by demanding that the higher states and continuum contributions should be less than 30% of the total result. The

lower bound of M^2 is determined from the condition that the sum of the contributions of the condensate terms should be less than 40% of the contributions coming from the perturbative part. From these conditions the working region of M^2 is determined to be $1.4 \text{ GeV}^2 \leq M^2 \leq 2.2 \text{ GeV}^2$.

In figures 1 and 2, we present the dependence of the mixing angle α on M^2 at the value of the continuum threshold $s_0 = 3.2 \text{ GeV}^2$ and, at several fixed values of the auxiliary parameter β , for the coefficients of the structures \not{p} and I , respectively. We observe from figure 1 that in the range $1.4 \text{ GeV}^2 \leq M^2 \leq 2.2 \text{ GeV}^2$ of the Borel parameter, the mixing angle α exhibits good stability for the values of the auxiliary parameter $\beta = -3; \pm 1$ for the structure \not{p} . As can be traced from figure 2, the mixing angle α seems to be rather stable at all considered values of the auxiliary parameter β for the structure I at the fixed value of the continuum threshold $s_0 = 3.2 \text{ GeV}^2$.

Our final attempt for determination of the mixing angle is to find the region of β where the mixing angle exhibits insensitivity to its variation. For this aim we study the dependence of the mixing angle α on $\cos \theta$ where $\beta = \tan \theta$, at several fixed values of M^2 and at $s_0 = 3.2 \text{ GeV}^2$, and presented them in figures 3 and 4 for the coefficients of the structures \not{p} and I , respectively. In this respect, the results of our numerical analysis depicted in figures 3 and 4 can be summarized as follows:

- For the structure \not{p} , in the above-determined working regions of M^2 and s_0 , the best stability for the mixing angle is achieved when $-1 \leq \cos \theta \leq -0.5$, and the mixing angle is found to have the value $\alpha = (1.15 \pm 0.05)^0 \simeq 2.0 \times 10^{-2}$.
- For the structure I not only there is no stability region for the mixing angle, but also the mixing angle changes its sign. Therefore prediction for the value of the mixing angle from the structure I is not reliable.

Therefore we conclude that, the final result for the mixing angle is $\alpha = (1.15 \pm 0.05)^0 \simeq 2.0 \times 10^{-2}$ (in radians) which is obtained from the \not{p} structure. The error in determination of the mixing angle can be attributed to the uncertainties in the value of the continuum threshold s_0 , the quark condensates, and the scale parameter Λ . The results presented in this work can further be improved by taking $\mathcal{O}(\alpha_s)$ corrections in to account.

Finally, we compare our result on the Σ - Λ mixing with the predictions of the other approaches, as the result of which we observe that our result is very close to the result predicted by the quark model, while it is larger than the predictions of the QCD sum rules method existing in literature [6], and lattice QCD methods. The observed difference between our result and that of the QCD sum rules can mainly be attributed to the input parameters used in the numerical calculations. In our analysis we use the latest and more refined values of the input parameters. The second reason for this difference is that, we use the most general form of the interpolating currents. Finally, we note that our sum rules do not contain phenomenological part, which brings its own uncertainty into sum rules.

As has already been noted, our result is also larger compared to the prediction of the lattice QCD method. A more reliable determination of the Σ - Λ mixing angle requires an

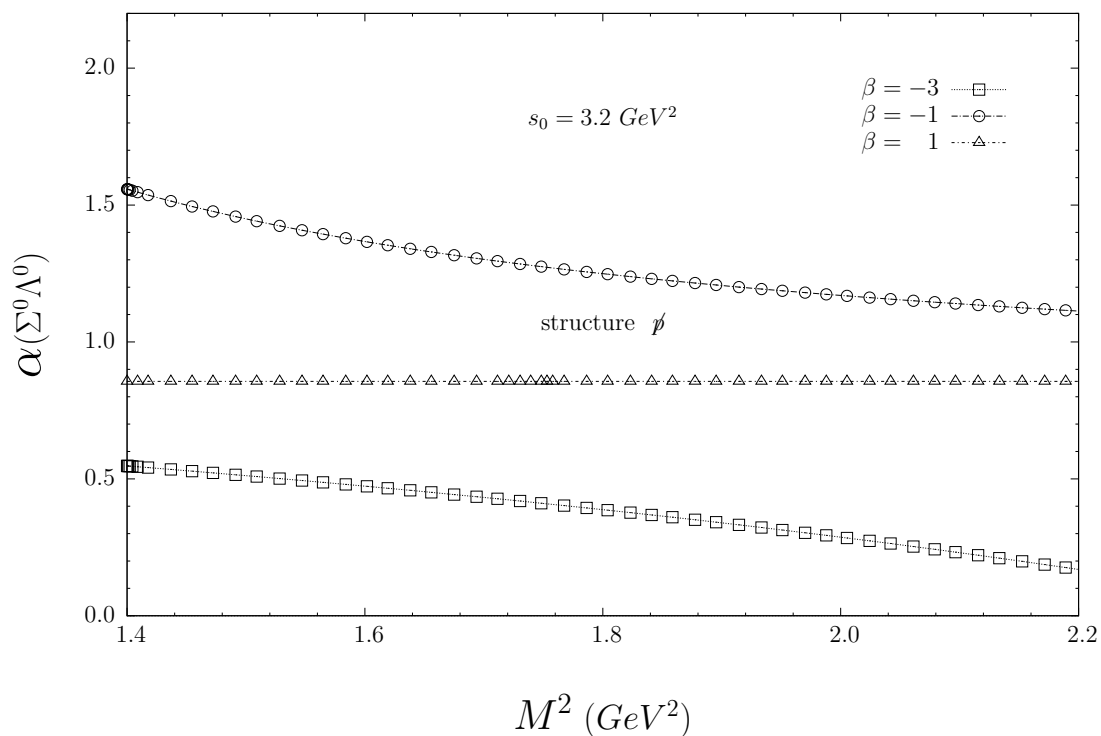


Figure 1. Dependence of the Λ - Σ mixing angle (in degrees) on the Borel mass parameter M^2 at the fixed value of the continuum threshold $s_0 = 3.2 \text{ GeV}^2$, and at several fixed values of the auxiliary parameter β , for the structure \bar{p} .

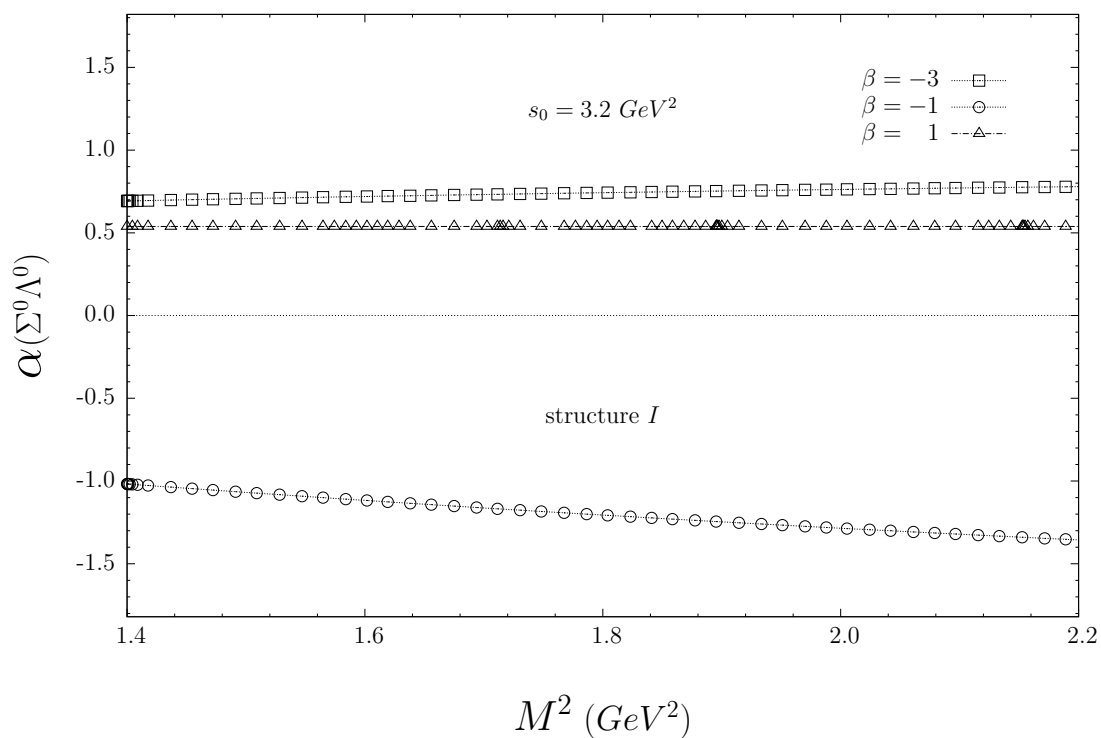


Figure 2. The same as in figure 1, but for the structure I .

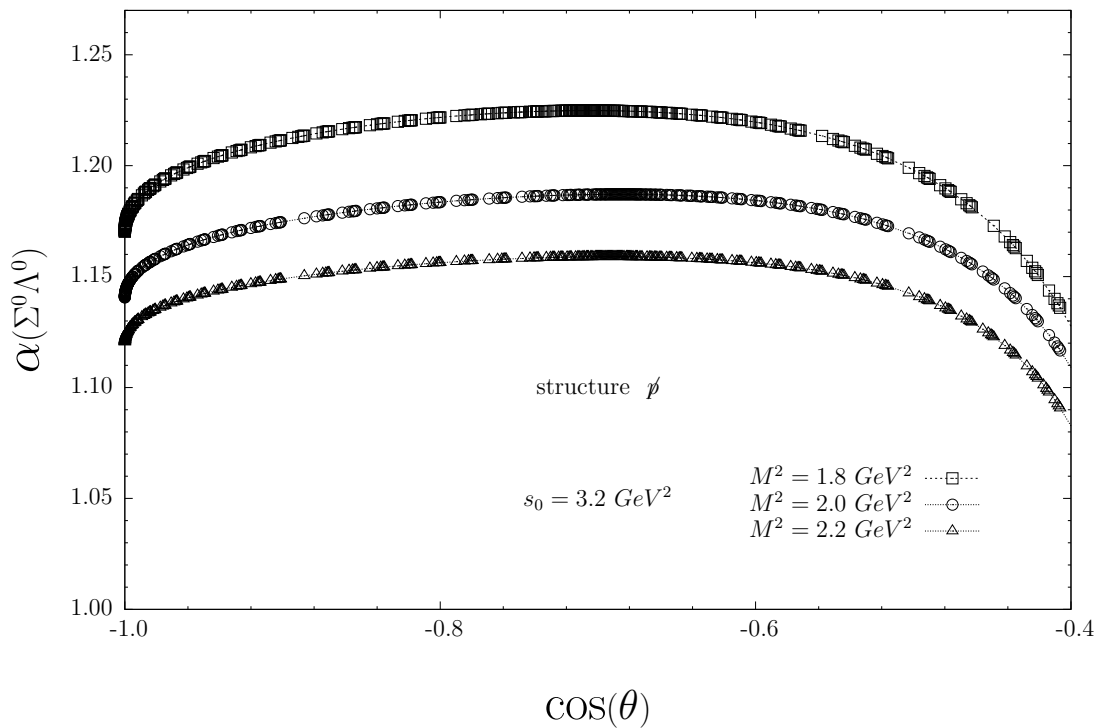


Figure 3. Dependence of the Λ - Σ mixing angle (in degrees) on $\cos\theta$ at the fixed value of the continuum threshold $s_0 = 3.2 \text{ GeV}^2$, and at several fixed values of the Borel mass parameter M^2 , for the structure \bar{p} .

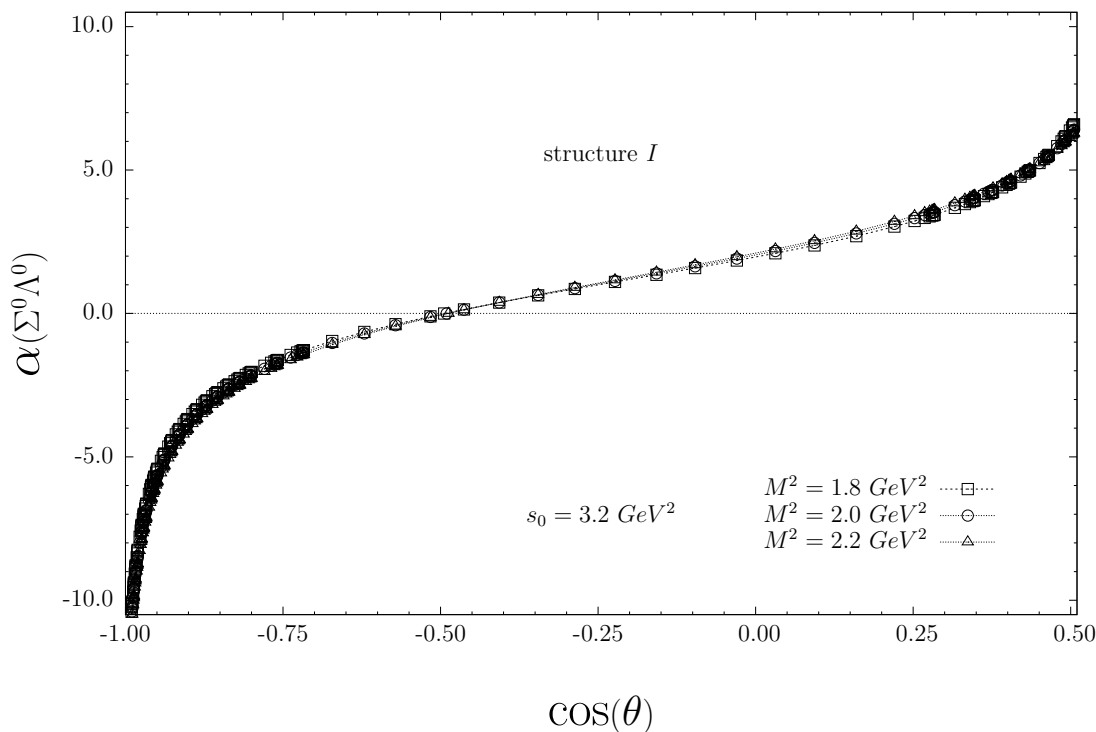


Figure 4. The same as in figure 1, but for the structure I .

equally highly accurate reproduction of the octet baryon mass differences, which has not yet been established.

In conclusion, the mixing angle between the Σ and Λ baryons is estimated within the framework of the QCD sum rules method by using the most general form of the interpolating current. A comparison of our result with the predictions of the quark model, chiral perturbation theory, and also with the result of the QCD sum rules method existing in literature, is presented.

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A Expressions of the invariant functions $\Pi_{\Sigma\Lambda}^0$ and $\Pi_{\Sigma\Sigma}^0 - \Pi_{\Lambda\Lambda}^0$

$\Pi_{\Sigma^0\Lambda^0}^0(u, d, s)$ for the structure \not{p}

$$\begin{aligned}
& e^{m_{\Sigma^0}^2/2M^2} e^{m_{\Lambda^0}^2/2M^2} \Pi_{\Sigma^0\Lambda^0}^0(u, d, s) = \\
& - \frac{1}{3072\sqrt{3}\pi^2 M^4} (1-\beta) \left[(3m_s + 4\beta m_s + m_u + \beta m_u) \langle \bar{d}d \rangle \right. \\
& + (2+3\beta)(m_d - m_u) \langle \bar{s}s \rangle - (m_d + \beta m_d + 3m_s + 4\beta m_s) \langle \bar{u}u \rangle \left. \right] \langle g^2 G^2 \rangle m_0^2 \\
& + \frac{1}{1152\sqrt{3}\pi^2 M^2} \left\{ \left[3(1+\beta+\beta^2)m_d + 2(1-\beta)(2+\beta-3(1+\beta)\gamma_E) m_s \right. \right. \\
& - 3(1-\beta^2)\gamma_E m_u \left. \right] \langle g^2 G^2 \rangle \langle \bar{d}d \rangle + (1-\beta) \left[2(2+\beta) - 3(1+\beta)\gamma_E \right] \langle g^2 G^2 \rangle (m_d - m_u) \langle \bar{s}s \rangle \\
& - 12\pi^2(1-\beta)(7+5\beta)m_0^2 \langle \bar{d}d \rangle \langle \bar{s}s \rangle - \left[(1-\beta)(2(2+\beta)m_s \right. \\
& - 3(1+\beta)\gamma_E(m_d + 2m_s)) \langle g^2 G^2 \rangle + 3(1+\beta+\beta^2) \langle g^2 G^2 \rangle m_u \\
& - 12\pi^2(1-\beta)(7+5\beta)m_0^2 \langle \bar{s}s \rangle \left. \right] \langle \bar{u}u \rangle + 3(1-\beta^2) \langle g^2 G^2 \rangle \left[\langle \bar{d}d \rangle (2m_s + m_u) \right. \\
& + (m_d - m_u) \langle \bar{s}s \rangle - (m_d + 2m_s) \langle \bar{u}u \rangle \left. \right] \ln \frac{M^2}{\Lambda^2} \left. \right\} \\
& + \frac{1}{128\pi^2} \sqrt{3}(1-\beta^2) \left(\gamma_E - \ln \frac{M^2}{\Lambda^2} \right) m_0^2 \left(m_s \langle \bar{d}d \rangle + m_d \langle \bar{s}s \rangle - m_u \langle \bar{s}s \rangle - m_s \langle \bar{u}u \rangle \right) \\
& - \frac{1}{32\sqrt{3}\pi^2} M^2 (2-\beta-\beta^2) \left(m_s \langle \bar{d}d \rangle + m_d \langle \bar{s}s \rangle - m_u \langle \bar{s}s \rangle - m_s \langle \bar{u}u \rangle \right) \\
& - \frac{1}{384\sqrt{3}\pi^2} \left\{ 3 \left[2(1+\beta+\beta^2)m_d - 6m_s + m_u + \beta(m_s + 5\beta m_s - \beta m_u) \right] m_0^2 \langle \bar{d}d \rangle \right. \\
& - 32\pi^2(2-\beta-\beta^2) \left(\langle \bar{d}d \rangle - \langle \bar{u}u \rangle \right) \langle \bar{s}s \rangle \\
& - 3 \left[(7-\beta-6\beta^2)(m_d - m_u) \langle \bar{s}s \rangle + \left((1-\beta)(m_d + \beta m_d - 6m_s - 5\beta m_s) \right. \right. \\
& \left. \left. + 2(1+\beta+\beta^2)m_u \right) \langle \bar{u}u \rangle \right] m_0^2 \left. \right\} .
\end{aligned}$$

$(\Pi_{\Sigma^0 \Sigma^0}^0 - \Pi_{\Lambda^0 \Lambda^0}^0)(u, d, s)$ for the structure \not{p}

$$\begin{aligned}
 e^{m_{\Sigma^0}^2/2M^2} e^{m_{\Lambda^0}^2/2M^2} (\Pi_{\Sigma^0 \Sigma^0}^0 - \Pi_{\Lambda^0 \Lambda^0}^0)(u, d, s) = & \\
 & \frac{1}{4608\pi^2 M^4} (1-\beta) \left\{ [m_s + 2\beta m_s - (5+7\beta)m_u] \langle \bar{d}d \rangle \right. \\
 & + (4+5\beta)(m_d + m_u) \langle \bar{s}s \rangle - [(5+7\beta)m_d - (1+2\beta)m_s] \langle \bar{u}u \rangle \left. \right\} \langle g^2 G^2 \rangle m_0^2 \\
 & - \frac{1}{1728\pi^2 M^2} \left\{ [(1-\beta)(2(2+\beta) - 9(1+\beta)\gamma_E)(m_u + m_d) + 6(1+\beta + \beta^2)m_s] \langle g^2 G^2 \rangle \langle \bar{s}s \rangle \right. \\
 & - \left\{ [3(1+\beta + \beta^2)m_d - (1-\beta)(2(2+\beta)m_s - [4(2+\beta) - 9(1+\beta)\gamma_E]m_u)] \langle g^2 G^2 \rangle \right. \\
 & + 12\pi^2(1-\beta)(7+5\beta)m_0^2 (\langle \bar{s}s \rangle - 2\langle \bar{u}u \rangle) \left. \right\} \langle \bar{d}d \rangle - \left\{ (1-\beta) \left[(4(2+\beta) \right. \right. \\
 & - 9(1+\beta)\gamma_E)m_d - 2(2+\beta)m_s \left. \right] \langle g^2 G^2 \rangle + 3(1+\beta + \beta^2) \langle g^2 G^2 \rangle m_u \\
 & + 12\pi^2(1-\beta)(7+5\beta)m_0^2 \langle \bar{s}s \rangle \left. \right\} \langle \bar{u}u \rangle \\
 & - 9(1-\beta^2) \left[\langle \bar{d}d \rangle m_u - (m_d + m_u) \langle \bar{s}s \rangle + m_d \langle \bar{u}u \rangle \right] \langle g^2 G^2 \rangle \ln \frac{M^2}{\Lambda^2} \left. \right\} \\
 & - \frac{1}{64\pi^2} (1-\beta^2) \left(\gamma_E - \ln \frac{M^2}{\Lambda^2} \right) m_0^2 \left[\langle \bar{d}d \rangle (m_s - 2m_u) + m_u \langle \bar{s}s \rangle + m_d (\langle \bar{s}s \rangle - 2\langle \bar{u}u \rangle) + m_s \langle \bar{u}u \rangle \right] \\
 & + \frac{1}{48\pi^2} M^2 (2-\beta - \beta^2) \left[\langle \bar{d}d \rangle (m_s - 2m_u) + m_u \langle \bar{s}s \rangle + m_d (\langle \bar{s}s \rangle - 2\langle \bar{u}u \rangle) + m_s \langle \bar{u}u \rangle \right] \\
 & - \frac{1}{576\pi^2} \left\{ \left[3(2(1+\beta + \beta^2)m_d + (1-\beta)(8m_s + 7\beta m_s - 13m_u - 11\beta m_u)) m_0^2 \right. \right. \\
 & + 32\pi^2(2-\beta - \beta^2) (\langle \bar{s}s \rangle - 2\langle \bar{u}u \rangle) \left. \right] \langle \bar{d}d \rangle + 32\pi^2(2-\beta - \beta^2) \langle \bar{s}s \rangle \langle \bar{u}u \rangle \\
 & - 3 \left[4(1+\beta + \beta^2)m_s \langle \bar{s}s \rangle - (5-\beta - 4\beta^2)m_u \langle \bar{s}s \rangle - (8-\beta - 7\beta^2)m_s \langle \bar{u}u \rangle \right. \\
 & \left. \left. - 2(1+\beta + \beta^2)m_u \langle \bar{u}u \rangle - (1-\beta)m_d (\langle \bar{s}s \rangle - (13+11\beta)\langle \bar{u}u \rangle) \right] m_0^2 \right\} .
 \end{aligned}$$

$\Pi_{\Sigma^0 \Lambda^0}^0(u, d, s)$ for the structure I

$$\begin{aligned}
 e^{m_{\Sigma^0}^2/2M^2} e^{m_{\Lambda^0}^2/2M^2} \Pi_{\Sigma^0 \Lambda^0}^0(u, d, s) = & \\
 & - \frac{1}{128\sqrt{3}\pi^4} M^6 (2-\beta - \beta^2) (m_d - m_u) \\
 & + \frac{1}{32\sqrt{3}\pi^2} M^4 (2-\beta - \beta^2) (\langle \bar{d}d \rangle - \langle \bar{u}u \rangle) \\
 & - \frac{1}{512\sqrt{3}\pi^4} M^2 (1-\beta)(1+2\beta) \left(\gamma_E - \ln \frac{M^2}{\Lambda^2} \right) (m_d - m_u) \langle g^2 G^2 \rangle \\
 & - \frac{1}{768\sqrt{3}\pi^4} M^2 (1-\beta) \left[(1+5\beta)(m_d - m_u) \langle g^2 G^2 \rangle + 18\pi^2(1+\beta) (\langle \bar{d}d \rangle - \langle \bar{u}u \rangle) m_0^2 \right] \\
 & + \frac{1}{12288\sqrt{3}\pi^4 M^2} \left[(1-\beta)(1+2\beta)(m_d - m_u) \langle g^2 G^2 \rangle^2 \right]
 \end{aligned}$$

$$\begin{aligned}
& + 512\pi^4(1+\beta+\beta^2)\left(\langle\bar{d}d\rangle m_u - m_d\langle\bar{u}u\rangle\right)m_0^2\langle\bar{s}s\rangle] \\
& - \frac{1}{384\sqrt{3}\pi^2}(1-\beta)\left\{\left[(1+2\beta)\langle g^2G^2\rangle + 16\pi^2(2+\beta)m_s\langle\bar{s}s\rangle\right]\langle\bar{u}u\rangle\right. \\
& \left. - \left[(1+2\beta)\langle g^2G^2\rangle + 16\pi^2(2+\beta)\left(m_s\langle\bar{s}s\rangle - (m_d - m_u)\langle\bar{u}u\rangle\right)\langle\bar{d}d\rangle\right]\right\}.
\end{aligned}$$

$(\Pi_{\Sigma^0\Sigma^0}^0 - \Pi_{\Lambda^0\Lambda^0}^0)(u, d, s)$ for the structure I

$$\begin{aligned}
& e^{m_{\Sigma^0}^2/2M^2} e^{m_{\Lambda^0}^2/2M^2} \left(\Pi_{\Sigma^0\Sigma^0}^0 - \Pi_{\Lambda^0\Lambda^0}^0\right)(u, d, s) = \\
& - \frac{1}{192\pi^4} M^6(2-\beta-\beta^2)(m_d - 2m_s + m_u) \\
& + \frac{1}{48\pi^2} M^4(2-\beta-\beta^2)\left(\langle\bar{d}d\rangle - 2\langle\bar{s}s\rangle + \langle\bar{u}u\rangle\right) \\
& - \frac{1}{768\pi^4} M^2(1-\beta)(1+2\beta)\left(\gamma_E - \ln\frac{M^2}{\Lambda^2}\right)(m_d - 2m_s + m_u)\langle g^2G^2\rangle \\
& - \frac{1}{1152\pi^4} M^2(1-\beta)\left[(1+5\beta)(m_d - 2m_s + m_u)\langle g^2G^2\rangle\right. \\
& \left. + 18\pi^2(1+\beta)\left(\langle\bar{d}d\rangle - 2\langle\bar{s}s\rangle + \langle\bar{u}u\rangle\right)m_0^2\right] \\
& + \frac{1}{18432\pi^4 M^2}\left\{(1-\beta)(1+2\beta)(m_d - 2m_s + m_u)\langle g^2G^2\rangle^2\right. \\
& \left. - 512\pi^4(1+\beta+\beta^2)\left(m_u\langle\bar{d}d\rangle\langle\bar{s}s\rangle - 2m_s\langle\bar{d}d\rangle\langle\bar{u}u\rangle + m_d\langle\bar{s}s\rangle\langle\bar{u}u\rangle\right)m_0^2\right\} \\
& + \frac{1}{576\pi^2}(1-\beta)\left\{16\pi^2(2+\beta)(m_s - 2m_u)\langle\bar{s}s\rangle\langle\bar{u}u\rangle - (1+2\beta)\left(2\langle\bar{s}s\rangle - \langle\bar{u}u\rangle\right)\langle g^2G^2\rangle\right. \\
& \left. + \left[(1+2\beta)\langle g^2G^2\rangle - 16\pi^2(2+\beta)\left((2m_d - m_s)\langle\bar{s}s\rangle - (m_d + m_u)\langle\bar{u}u\rangle\right)\right]\langle\bar{d}d\rangle\right\},
\end{aligned}$$

where M^2 is the Borel parameter and Λ is the energy cut off separating perturbative and nonperturbative regimes; and γ_E is the Euler constant.

Note that the scale parameter Λ is calculated in [17, 18] whose value is in the range $0.5 \div 1.0$ GeV.

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