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Unitary symmetry pattern of the QCD sum rules

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Abstract. Relations between magnetic moments of Σ^0 and Λ baryons are discussed. Physical meaning of the F- and D- couplings in $SU(3)$ is established. The obtained results are generalized to the QCD sum rules which yield unitary pattern with the characteristic F and D structures.

QCD sum rules introduced in [1] are now one of the most reliable methods of analyzing strong interaction. Dozens of works have described couplings of baryons to γ, pseudoscalar and vector mesons, their weak decay constants etc. We have shown that unitary symmetry plays essential role in relating between them various couplings through well-known F- and D-type structures reducing number of independent correlation functions to minimum. We would show here in what way unitary pattern of the QCD sum rules arrives.

Let us begin with the discussion of the baryon octet $1/2^+$ in $SU(3)$.

There are 3 different reductions of the octet along the $SU(2)$.

The 1st one is with Λ being the isosinglet and Σ the isotriplet, isomultiplets formed along the value of Y. The 2nd one is with 11 component $\Sigma^0/\sqrt{2}+\Lambda/\sqrt{6} \equiv -2\Lambda_{us}/\sqrt{6}$ being the U-singlet and n, Ξ^0, $\Sigma^0/2+\sqrt{3}\Lambda/2 \equiv \Sigma_{us}$ form U-triplet, $SU(2)_U$-multiplets having the same charge Q. The 3rd possibility is given by 22-component $-\Sigma^0/\sqrt{2}+\Lambda/\sqrt{6} \equiv -2\Lambda_{ds}/\sqrt{6}$ being the V-singlet and p, Ξ^-, $-\Sigma^0/2+\sqrt{3}\Lambda/2 \equiv \Sigma_{ds}$ form V-triplet, $SU(2)_V$-multiplets having the same $Q-Y$.

For any operator $\mathcal{O}$ the following relations are valid between the matrix elements:

$$2\langle\Sigma_{us}|\mathcal{O}|\Sigma_{us}\rangle + 2\langle\Sigma_{us}|\mathcal{O}|\Sigma_{us}\rangle - \langle\Sigma^0|\mathcal{O}|\Sigma^0\rangle = 3\langle\Lambda|\mathcal{O}|\Lambda\rangle, \tag{1}$$

$$2\langle\Lambda_{us}|\mathcal{O}|\Lambda_{us}\rangle + 2\langle\Lambda_{ds}|\mathcal{O}|\Lambda_{ds}\rangle - \langle\Lambda|\mathcal{O}|\Lambda\rangle = 3\langle\Sigma^0|\mathcal{O}|\Sigma^0\rangle.$$

To show its applicability we consider an example of magnetic moments of octet baryons. In NRQM one has $\mu_\Lambda = \mu_s$ [2]. Now instead of proceeding quark framework calculations we just use relations (1) to obtain μ_{Σ^0} from μ_Λ : $2\mu_{\Lambda_{ds}} + 2\mu_{\Lambda_{us}} - \mu_\Lambda = 2\mu_d + 2\mu_u - \mu_s = 3\mu_{\Sigma^0}$.
Let us now assume that photon (or ρ^0, ω, ϕ) interacts in a different way with two quarks of similar flavour of the Σ-like baryon $B(qq, q')$ and with a single quark q'. As an example let the magnetic moment operator has the form $\hat{e}_q \hat{\omega}_q \hat{\sigma}_z^q$, where new operator $\hat{\omega}_q$ just differs between a single q' quark and a diquark $(q_\uparrow q_\uparrow)$ or $(q_\uparrow q_\downarrow)$ through the matrix elements

$$\begin{aligned} &< q_\uparrow q_\uparrow, q'_\downarrow | \hat{\omega}_q | q_\uparrow q_\uparrow, q'_\downarrow >= w_{\uparrow\uparrow}, \quad < q_\uparrow q_\downarrow, q'_\uparrow | \hat{\omega}_q | q_\uparrow q_\downarrow, q'_\uparrow >= w_{\uparrow\downarrow}, \\ &< q_\uparrow q_\uparrow, q'_\downarrow | \hat{\omega}_{q'} | q_\uparrow q_\uparrow, q'_\downarrow >= v_{\uparrow\uparrow}, \quad < q_\uparrow q_\downarrow, q'_\uparrow | \hat{\omega}_{q'} | q_\uparrow q_\downarrow, q'_\uparrow >= v_{\uparrow\downarrow}. \end{aligned} \tag{2}$$

$$\mu_p = \sum\nolimits_{q=u,d} < p_\uparrow | \quad \hat{e}_q \hat{\omega}_q \hat{\sigma}_z^q \quad | p_\uparrow >=$$

$$= \frac{1}{6} < 2u_1u_1d_2 - u_1d_1u_2 - d_1u_1u_2 | \quad \hat{e}_q \hat{\omega}_q \sigma_z^q \quad | 2u_1u_1d_2 - u_1d_1u_2 - d_1u_1u_2 >=$$

$$= \frac{4}{3} e_u w_{\uparrow\uparrow} - \frac{1}{3} e_d (2v_{\uparrow\uparrow} - v_{\uparrow\downarrow}) \equiv e_u \cdot 2\mu_F + e_d \cdot (\mu_F - \mu_D) = \mu_F + \frac{1}{3}\mu_D$$

with $w_{\uparrow\uparrow} = 3F/2, (2v_{\uparrow\uparrow} - v_{\uparrow\downarrow})/3 = (\mu_D - \mu_F)$. Note that m.e. $w_{\uparrow\downarrow}$ drops out and does not contribute to magnetic moments of Σ-like baryons. But it contributes in axial-vector currents and we should put $w_{\uparrow\downarrow} = D$ to come to SU(3) result. It is worth noting that the assumption $w_{\uparrow\uparrow} = w_{\uparrow\downarrow}$ yields $F/D = 2/3$! The main results are:

- The F coupling is related to the interaction of the γ (or π^0, η, ρ^0, ω, ϕ) with 'diquark' composed of two quarks of (almost) equal flavour and equal spin projections
- The $(F - D)$ is related to the interaction of the γ (or π^0, η, ρ^0, ω, ϕ) with the single quark
- Λ quantities can be obtained through Eq.(1).

Coupling constant of $M = \pi^0$, η, ρ^0, ω, ϕ or photon to Σ^0 containing two quarks u, d in a diquark state and a single quark s would have the form

$$g_{M\Sigma\Sigma} = (g_{Muu} + g_{Mdd})F + g_{Mss}(F - D). \tag{3}$$

In order to obtain coupling to Λ hyperon we use the Eq.(1). Results are in accord with the SU(3) ones. This reasoning can be transferred to the QCD sum rules.
The starting point would be polarization operator (correlator)[1],[3] for hyperons Σ^0 and Λ

$$\Pi^{\Sigma^0,\Lambda} = i \int d^4x e^{ipx} \langle 0 | T\{\eta^{\Sigma^0,\Lambda}(x), \eta^{\Sigma^0,\Lambda}(0)\} | V \rangle, \tag{4}$$

where isovector and isocalar interpolating currents [3] could be chosen as
$\eta^{\Sigma^0} = \frac{\epsilon_{abc}}{2}[u^{aT} C s^b \gamma_5 d^c - d^{aT} C s^b \gamma_5 u^c - (C s^b \gamma_5 \to C \gamma_5 s^b)]$, $\sqrt{3}\eta^\Lambda = \tilde{\eta}^{\Sigma^0 (u \leftrightarrow s)} - \tilde{\eta}^{\Sigma^0 (d \leftrightarrow s)}$

where a, b, c are the color indices, C is charge conjugation matrix, while $V = \rho^0, \omega, \phi, \gamma$.

The idea of the **QCD sum rules**[1],[3] is as follows. Polarization operator is calculated in two different ways: (1) Upon using some phenomenological pole model saturated by baryon poles and resonances plus high energy contributions; (2) Upon performing Wilson operator product expansion (OPE) and calculating quark diagrams with insertions of non-zero vacuum expectation values. Equating (1)=(2) and performing Borel transformation one arrives at desired sum rule. Following the reasoning of the previous examples we construct relations which connect QCD sum rules for Σ^0 hyperon with those for Λ making explicit F,D pattern of them. For this purpose we write U-spin and V-spin quantities for the interpolating currents:

$-2\tilde{\eta}^{\Lambda(d\leftrightarrow s)} = \sqrt{3}\eta^{\Sigma^0} + \eta^{\Lambda}, \quad 2\tilde{\eta}^{\Sigma^0(d\leftrightarrow s)} = \eta^{\Sigma^0} - \sqrt{3}\eta^{\Lambda},$

$2\tilde{\eta}^{\Lambda(u\leftrightarrow s)} = \sqrt{3}\eta^{\Sigma^0} - \eta^{\Lambda}, \quad 2\tilde{\eta}^{\Sigma^0(u\leftrightarrow s)} = \eta^{\Sigma^0} + \sqrt{3}\eta^{\Lambda}.$

With these equations two-point functions of the Eq.(4) for Σ^0 and Λ are related as

$2 \cdot [\tilde{\Pi}^{\Sigma^0(d\leftrightarrow s)} + \tilde{\Pi}^{\Sigma^0(u\leftrightarrow s)}] - \Pi^{\Sigma^0} = 3\Pi^{\Lambda}, \quad 2 \cdot [\tilde{\Pi}^{\Lambda(d\leftrightarrow s)} + \Pi^{\tilde{\Lambda}(u\leftrightarrow s)}] - \Pi^{\Lambda} = 3\Pi^{\Sigma^0}.$

We can write now in full analogy to the Eq.(3)

$\Pi^{\Sigma^0}(u, d, s) = (g_{uuV} + g_{ddV})\Pi_F(u, d, s) + g_{ssV}\Pi_{(F-D)}(u, d, s).$

Couplings with strange mesons are obtained through auxiliary relations (below V is vector nonet)

$\Pi^{\Sigma^0_{ds}\Sigma^-\rho^+}(u, d, s) = -\Pi^{\Sigma^0\Sigma^-\rho^+}(u, d, s) + \sqrt{3}\Pi^{\Lambda\Sigma^-\rho^+}(u, d, s).$

By new $d \leftrightarrow s$ on the left we obtain $\Pi^{\Sigma^0\Xi^- K^{*+}}(u, d, s)$ in terms of Π_F and $\Pi_{(F-D)}$. In this way we arrive at all the correlators $\Pi^{BB'V}$ [4]. We also proved our relations for [5] and [6]. Thus,

• Relations between the magnetic moments of Σ^0 and Λ baryons are written in unitary symmetry and quark model

• Physical meaning of the F- and D- couplings of $SU(3)$ baryon currents is established

• The obtained results are generalized to the QCD sum rules

• QCD sum rules for the strong meson-baryon coupling constants yield unitary pattern with the characteristic F and D structures.

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