

Lepton flavor violating $Z \rightarrow l_1^+ l_2^-$ decay in the general two Higgs Doublet model

E. O. Iltan * and **I. Turan** †

Physics Department, Middle East Technical University
Ankara, Turkey

Abstract

We calculate lepton flavor violating $Z \rightarrow l^+ l^-$ decay in the framework of the general two Higgs Doublet model. In our calculations we used the constraints for the Yukawa couplings $\bar{\xi}_{N,\tau e}^D$ and $\bar{\xi}_{N,\tau\mu}^D$ coming from the experimental result of muon electric dipole moment and upper limit of the $BR(\mu \rightarrow e\gamma)$. We observe that it is possible to reach the present experimental upper limits for the branching ratios of such Z decays in the model III.

*E-mail address: eiltan@heraklit.physics.metu.edu.tr

†E-mail address: ituran@.metu.edu.tr

1 Introduction

Lepton Flavor Violating (LFV) interactions reached great interest since the related experimental measurements are improved at present. Among them, the lepton flavor changes in Z decays, such as $Z \rightarrow e\mu$, $Z \rightarrow e\tau$ and $Z \rightarrow \mu\tau$ are important for the search of neutrinos, their mixing and possible masses, and the physics beyond the Standard model (SM). With the Giga-Z option of the Tesla project, the production of Z bosons at resonance is expected to increase [1] and this forces to study on such Z decays more precisely.

Since the lepton flavor is conserved in the SM, one needs an extended theory to describe the lepton flavor violating Z decays. One of the possibility is the extension of the SM, so called ν SM, by taking neutrinos massive and permitting the lepton mixing mechanism [2]. In this case the lepton sector is analogous to the quark sector. Considering the branching ratio (BR)

$$BR(Z \rightarrow l_1^\pm l_2^\pm) = \frac{\Gamma(Z \rightarrow \bar{l}_1 l_2 + \bar{l}_2 l_1)}{\Gamma_Z},$$

the first predictions for such Z decays are given in [3, 4]. The best experimental limits obtained at LEP 1 [5] are

$$\begin{aligned} BR(Z \rightarrow e^\pm \mu^\pm) &< 1.7 \times 10^{-6} \text{ [6]}, \\ BR(Z \rightarrow e^\pm \tau^\pm) &< 9.8 \times 10^{-6} \text{ [6, 7]}, \\ BR(Z \rightarrow \mu^\pm \tau^\pm) &< 1.2 \times 10^{-5} \text{ [6, 8]} \end{aligned} \tag{1}$$

and with the improved sensitivities at Giga-Z [9] these numbers could be pulled down to

$$\begin{aligned} BR(Z \rightarrow e^\pm \mu^\pm) &< 2 \times 10^{-9}, \\ BR(Z \rightarrow e^\pm \tau^\pm) &< f \times 6.5 \times 10^{-8}, \\ BR(Z \rightarrow \mu^\pm \tau^\pm) &< f \times 2.2 \times 10^{-8} \end{aligned} \tag{2}$$

with $f = 0.2 - 1.0$. In the framework of ν SM with light neutrinos the theoretical prediction is extremely small and far from these limits [3, 10]

$$\begin{aligned} BR(Z \rightarrow e^\pm \mu^\pm) \sim BR(Z \rightarrow e^\pm \tau^\pm) &\sim 10^{-54}, \\ BR(Z \rightarrow \mu^\pm \tau^\pm) &< 4 \times 10^{-60} \end{aligned} \tag{3}$$

Another possibility to increase the corresponding BR is the extension of the ν SM with one heavy ordinary Dirac neutrino [10]. Heavy neutrinos are expected in string-inspired models [11] and some GUTs [12]. In the ν SM with one heavy neutrino, it is necessary to include a

heavy charged lepton in the theory. In the framework of this model, it is possible to observe their effects from Z decays if the neutrinos with a mass of several hundred GeV exist. Further scenario to increase the BR is the ν SM extended with two heavy right-handed singlet Majorana neutrinos [10]. In this case, it is possible to reach the experimental upper limits in the large neutrino mass region. Lepton flavor Violating Z decays are also studied in the framework of the Zee model [13]. According to this work, among all three lepton flavor violating decay modes of Z , only $Z \rightarrow e\tau$ decay has the largest contribution which is less than the present limits. Other two decays are small to be observed in the next linear colliders.

In our work, we study $Z \rightarrow e^\pm\mu^\pm$, $Z \rightarrow e^\pm\tau^\pm$ and $Z \rightarrow \mu^\pm\tau^\pm$ decays in the model III version of 2HDM, which is the minimal extension of the SM. Since there is no CKM type matrix and therefore no charged FC interaction in the leptonic sector according to our assumption, the source of lepton flavor violating Z decays are the neutral Higgs bosons h^0 and A^0 with the Yukawa couplings which allow tree level flavor changing neutral currents (FCNC). The choice of complex Yukawa couplings brings the possibility of non-zero electric dipole moments (EDM) of leptons and this ensures to restrict the Yukawa couplings using the present experimental limits. Further the lepton flavor violating interaction $\mu \rightarrow e\gamma$ is possible in this model and can be used to predict the constraint for the Yukawa couplings (see [14]). Calculations are done in one loop level and it is shown that the experimental upper limits for Z decays under consideration can be reached by playing with the free parameters of the model III respecting the above restrictions.

The paper is organized as follows: In Section 2, we present the explicit expressions for the Branching ratios of $Z \rightarrow e^-\mu^+$, $Z \rightarrow e^-\tau^+$ and $Z \rightarrow \mu^-\tau^+$ in the framework of the model III. Section 3 is devoted to discussion and our conclusions.

2 $Z \rightarrow l_1^- l_2^+$ decay in the general two Higgs Doublet model.

In the SM and the model I and II version of 2HDM the FCNC at tree level is forbidden. However model III version of 2HDM permits FCNC interactions at tree level and makes flavor violating interactions possible. With the choice of complex Yukawa couplings, the CP violation can also exist and this leads to appearance of non-zero EDM of fermions. The Yukawa interaction for the leptonic sector in the model III is

$$\mathcal{L}_Y = \eta_{ij}^D \bar{l}_{iL} \phi_1 E_{jR} + \xi_{ij}^D \bar{l}_{iL} \phi_2 E_{jR} + h.c. \quad , \quad (4)$$

where i, j are family indices of leptons, L and R denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, ϕ_i for $i = 1, 2$, are the two scalar doublets, l_{iL} and E_{jR} are lepton doublets and singlets respectively.

Here ϕ_1 and ϕ_2 are chosen as

$$\phi_1 = \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2}\chi^+ \\ i\chi^0 \end{pmatrix} \right]; \phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}H^+ \\ H_1 + iH_2 \end{pmatrix}, \quad (5)$$

and the vacuum expectation values are

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}; \langle \phi_2 \rangle = 0. \quad (6)$$

This choice ensures that the SM particles are collected in the first doublet and the ones beyond in the second doublet. The part which produce FCNC at tree level is

$$\mathcal{L}_{Y,FC} = \xi_{ij}^{D\bar{}} \bar{l}_{iL} \phi_2 E_{jR} + h.c.. \quad (7)$$

Here the Yukawa matrices ξ_{ij}^D have in general complex entries. Note that in the following we replace ξ^D with ξ_N^D where "N" denotes the word "neutral" and define $\bar{\xi}_N^D$ which satisfies the equation $\xi_N^D = \sqrt{\frac{4G_F}{\sqrt{2}}} \bar{\xi}_N^D$. In our analysis, we take H_1 and H_2 (see eq. (5)) as the mass eigenstates h^0 and A^0 respectively since no mixing between CP-even neutral Higgs bosons h^0 and the SM one, H^0 , occurs at tree level.

The general effective vertex for the interaction of on-shell Z-boson with a fermionic current is given by

$$\Gamma_\mu = \gamma_\mu (f_V - f_A \gamma_5) + \frac{i}{m_W} (f_M + f_E \gamma_5) \sigma_{\mu\nu} q^\nu \quad (8)$$

where q is the momentum transfer, $q^2 = (p - p')^2$, f_V (f_A) is vector (axial-vector) coupling, f_M (f_E) magnetic (electric) transitions of unlike fermions. Here p ($-p'$) is the four momentum vector of lepton (anti-lepton). The necessary 1-loop diagrams due to neutral Higgs particles are given in Fig. 1.

Taking into account all the masses of internal leptons and external lepton (anti-lepton), the explicit expressions for f_V , f_A , f_M and f_E read as

$$\begin{aligned} f_V &= \frac{g}{64 \pi^2 \cos \theta_W} \int_0^1 dx \frac{1}{m_{l_2^+}^2 - m_{l_1^-}^2} \left\{ c_V (m_{l_2^+} + m_{l_1^-}) \right. \\ &\quad \left((-m_i \eta_i^+ + m_{l_1^-} (-1+x) \eta_i^V) \ln \frac{L_{1,h^0}^{self}}{\mu^2} + (m_i \eta_i^+ - m_{l_2^+} (-1+x) \eta_i^V) \ln \frac{L_{2,h^0}^{self}}{\mu^2} \right. \\ &\quad \left. + (m_i \eta_i^+ + m_{l_1^-} (-1+x) \eta_i^V) \ln \frac{L_{1,A^0}^{self}}{\mu^2} - (m_i \eta_i^+ + m_{l_2^+} (-1+x) \eta_i^V) \ln \frac{L_{2,A^0}^{self}}{\mu^2} \right) \\ &\quad \left. + c_A (m_{l_2^+} - m_{l_1^-}) \right. \\ &\quad \left((-m_i \eta_i^- + m_{l_1^-} (-1+x) \eta_i^A) \ln \frac{L_{1,h^0}^{self}}{\mu^2} + (m_i \eta_i^- + m_{l_2^+} (-1+x) \eta_i^A) \ln \frac{L_{2,h^0}^{self}}{\mu^2} \right. \end{aligned}$$

$$\begin{aligned}
& + \left. \left(m_i \eta_i^- + m_{l_1^-} (-1+x) \eta_i^A \right) \ln \frac{L_{1,A^0}^{self}}{\mu^2} + \left(-m_i \eta_i^- + m_{l_2^+} (-1+x) \eta_i^A \right) \ln \frac{L_{2,A^0}^{self}}{\mu^2} \right\} \\
& - \frac{g}{64 \pi^2 \cos \theta_W} \int_0^1 dx \int_0^{1-x} dy \left\{ m_i^2 (c_A \eta_i^A - c_V \eta_i^V) \left(\frac{1}{L_{A^0}^{ver}} + \frac{1}{L_{h^0}^{ver}} \right) \right. \\
& - (1-x-y) m_i \left(c_A (m_{l_2^+} - m_{l_1^-}) \eta_i^- \left(\frac{1}{L_{h^0}^{ver}} - \frac{1}{L_{A^0}^{ver}} \right) + c_V (m_{l_2^+} + m_{l_1^-}) \eta_i^+ \left(\frac{1}{L_{h^0}^{ver}} + \frac{1}{L_{A^0}^{ver}} \right) \right) \\
& - (c_A \eta_i^A + c_V \eta_i^V) \left(-2 + (q^2 x y + m_{l_1^-} m_{l_2^+} (-1+x+y)^2) \left(\frac{1}{L_{h^0}^{ver}} + \frac{1}{L_{A^0}^{ver}} \right) - \ln \frac{L_{h^0}^{ver}}{\mu^2} \frac{L_{A^0}^{ver}}{\mu^2} \right) \\
& - (m_{l_2^+} + m_{l_1^-}) (1-x-y) \left(\frac{\eta_i^A (x m_{l_1^-} + y m_{l_2^+}) + m_i \eta_i^-}{2 L_{A^0 h^0}^{ver}} + \frac{\eta_i^A (x m_{l_1^-} + y m_{l_2^+}) - m_i \eta_i^-}{2 L_{h^0 A^0}^{ver}} \right) \\
& \left. + \frac{1}{2} \eta_i^A \ln \frac{L_{A^0 h^0}^{ver}}{\mu^2} \frac{L_{h^0 A^0}^{ver}}{\mu^2} \right\}, \\
f_A & = \frac{-g}{64 \pi^2 \cos \theta_W} \int_0^1 dx \frac{1}{m_{l_2^+}^2 - m_{l_1^-}^2} \left\{ c_V (m_{l_2^+} - m_{l_1^-}) \right. \\
& \left(\left(m_i \eta_i^- + m_{l_1^-} (-1+x) \eta_i^A \right) \ln \frac{L_{1,A^0}^{self}}{\mu^2} + \left(-m_i \eta_i^- + m_{l_2^+} (-1+x) \eta_i^A \right) \ln \frac{L_{2,A^0}^{self}}{\mu^2} \right. \\
& + \left. \left(-m_i \eta_i^- + m_{l_1^-} (-1+x) \eta_i^A \right) \ln \frac{L_{1,h^0}^{self}}{\mu^2} + \left(m_i \eta_i^- + m_{l_2^+} (-1+x) \eta_i^A \right) \ln \frac{L_{2,h^0}^{self}}{\mu^2} \right) \\
& + c_A (m_{l_2^+} + m_{l_1^-}) \\
& \left(\left(m_i \eta_i^+ + m_{l_1^-} (-1+x) \eta_i^V \right) \ln \frac{L_{1,A^0}^{self}}{\mu^2} - \left(m_i \eta_i^+ + m_{l_2^+} (-1+x) \eta_i^V \right) \ln \frac{L_{2,A^0}^{self}}{\mu^2} \right. \\
& \left. + \left(-m_i \eta_i^+ + m_{l_1^-} (-1+x) \eta_i^V \right) \ln \frac{L_{1,h^0}^{self}}{\mu^2} + \left(m_i \eta_i^+ - m_{l_2^+} (-1+x) \eta_i^V \right) \frac{\ln L_{2,h^0}^{self}}{\mu^2} \right) \left. \right\} \\
& + \frac{g}{64 \pi^2 \cos \theta_W} \int_0^1 dx \int_0^{1-x} dy \left\{ m_i^2 (c_V \eta_i^A - c_A \eta_i^V) \left(\frac{1}{L_{A^0}^{ver}} + \frac{1}{L_{h^0}^{ver}} \right) \right. \\
& - m_i (1-x-y) \left(c_V (m_{l_2^+} - m_{l_1^-}) \eta_i^- + c_A (m_{l_2^+} + m_{l_1^-}) \eta_i^+ \right) \left(\frac{1}{L_{h^0}^{ver}} - \frac{1}{L_{A^0}^{ver}} \right) \\
& + (c_V \eta_i^A + c_A \eta_i^V) \left(-2 + (q^2 x y - m_{l_1^-} m_{l_2^+} (-1+x+y)^2) \left(\frac{1}{L_{h^0}^{ver}} + \frac{1}{L_{A^0}^{ver}} \right) - \ln \frac{L_{h^0}^{ver}}{\mu^2} \frac{L_{A^0}^{ver}}{\mu^2} \right) \\
& - (m_{l_2^+} - m_{l_1^-}) (1-x-y) \left(\frac{\eta_i^V (x m_{l_1^-} - y m_{l_2^+}) + m_i \eta_i^+}{2 L_{A^0 h^0}^{ver}} + \frac{\eta_i^V (x m_{l_1^-} - y m_{l_2^+}) - m_i \eta_i^+}{2 L_{h^0 A^0}^{ver}} \right) \\
& \left. - \frac{1}{2} \eta_i^V \ln \frac{L_{A^0 h^0}^{ver}}{\mu^2} \frac{L_{h^0 A^0}^{ver}}{\mu^2} \right\}, \\
f_M & = -\frac{g m_W}{64 \pi^2 \cos \theta_W} \int_0^1 dx \int_0^{1-x} dy \left\{ \left((1-x-y) (c_V \eta_i^V + c_A \eta_i^A) (x m_{l_1^-} + y m_{l_2^+}) \right. \right. \\
& \left. \left. + m_i (c_A (x-y) \eta_i^- + c_V \eta_i^+ (x+y)) \right) \frac{1}{L_{h^0}^{ver}} \right.
\end{aligned}$$

$$\begin{aligned}
& + \left((1-x-y) (c_V \eta_i^V + c_A \eta_i^A) (x m_{l_1^-} + y m_{l_2^+}) - m_i (c_A (x-y) \eta_i^- + c_V \eta_i^+ (x+y)) \right) \frac{1}{L_{A^0}^{ver}} \\
& - (1-x-y) \left(\frac{\eta_i^A (x m_{l_1^-} + y m_{l_2^+})}{2} \left(\frac{1}{L_{A^0 h^0}^{ver}} + \frac{1}{L_{h^0 A^0}^{ver}} \right) + \frac{m_i \eta_i^-}{2} \left(\frac{1}{L_{h^0 A^0}^{ver}} - \frac{1}{L_{A^0 h^0}^{ver}} \right) \right) \Bigg\}, \\
f_E & = -\frac{g m_W}{64 \pi^2 \cos \theta_W} \int_0^1 dx \int_0^{1-x} dy \left\{ \left((1-x-y) \left(- (c_V \eta_i^A + c_A \eta_i^V) (x m_{l_1^-} - y m_{l_2^+}) \right) \right. \right. \\
& \left. \left. - m_i (c_A (x-y) \eta_i^+ + c_V \eta_i^- (x+y)) \right) \frac{1}{L_{h^0}^{ver}} \right. \\
& + \left. \left((1-x-y) \left(- (c_V \eta_i^A + c_A \eta_i^V) (x m_{l_1^-} - y m_{l_2^+}) \right) + m_i (c_A (x-y) \eta_i^+ + c_V \eta_i^- (x+y)) \right) \frac{1}{L_{A^0}^{ver}} \right. \\
& \left. + (1-x-y) \left(\frac{\eta_i^V}{2} (m_{l_1^-} x - m_{l_2^+} y) \left(\frac{1}{L_{A^0 h^0}^{ver}} + \frac{1}{L_{h^0 A^0}^{ver}} \right) + \frac{m_i \eta_i^+}{2} \left(\frac{1}{L_{A^0 h^0}^{ver}} - \frac{1}{L_{h^0 A^0}^{ver}} \right) \right) \right\}, \quad (9)
\end{aligned}$$

where

$$\begin{aligned}
L_{1, h^0}^{self} & = m_{h^0}^2 (1-x) + (m_i^2 - m_{l_1^-}^2 (1-x)) x, \\
L_{1, A^0}^{self} & = L_{1, h^0}^{self} (m_{h^0} \rightarrow m_{A^0}), \\
L_{2, h^0}^{self} & = L_{1, h^0}^{self} (m_{l_1^-} \rightarrow m_{l_2^+}), \\
L_{2, A^0}^{self} & = L_{1, A^0}^{self} (m_{l_1^-} \rightarrow m_{l_2^+}), \\
L_{h^0}^{ver} & = m_{h^0}^2 (1-x-y) + m_i^2 (x+y) - q^2 x y, \\
L_{h^0 A^0}^{ver} & = m_{A^0}^2 x + m_i^2 (1-x-y) + (m_{h^0}^2 - q^2 x) y, \\
L_{A^0}^{ver} & = L_{h^0}^{ver} (m_{h^0} \rightarrow m_{A^0}), \\
L_{A^0 h^0}^{ver} & = L_{h^0 A^0}^{ver} (m_{h^0} \rightarrow m_{A^0}), \quad (10)
\end{aligned}$$

and

$$\begin{aligned}
\eta_i^V & = \xi_{N, l_1 i}^D \xi_{N, i l_2}^{D*} + \xi_{N, i l_1}^{D*} \xi_{N, l_2 i}^D, \\
\eta_i^A & = \xi_{N, l_1 i}^D \xi_{N, i l_2}^{D*} - \xi_{N, i l_1}^{D*} \xi_{N, l_2 i}^D, \\
\eta_i^+ & = \xi_{N, i l_1}^{D*} \xi_{N, i l_2}^{D*} + \xi_{N, l_1 i}^D \xi_{N, l_2 i}^D, \\
\eta_i^- & = \xi_{N, i l_1}^{D*} \xi_{N, i l_2}^{D*} - \xi_{N, l_1 i}^D \xi_{N, l_2 i}^D. \quad (11)
\end{aligned}$$

The parameters c_V and c_A are $c_A = -\frac{1}{4}$ and $c_V = \frac{1}{4} - \sin^2 \theta_W$. In eq. (11) the flavor changing couplings $\bar{\xi}_{N, l_j i}^D$ represent the effective interaction between the internal lepton i , ($i = e, \mu, \tau$) and outgoing (incoming) $j = 1$ ($j = 2$) one. Here we take $\bar{\xi}_{N, l_j i}^D$ complex in general and use the parametrization

$$\xi_{N, i l_j}^D = |\xi_{N, i l_j}^D| e^{i\theta_{ij}}, \quad (12)$$

where i, l_j denote the lepton flavors and θ_{ij} are CP violating parameters which are the sources of the lepton EDM.

Now, using the couplings f_V, f_A, f_M and f_E the BR for $Z \rightarrow l_1^- l_2^+$ can be written as

$$BR(Z \rightarrow l_1^- l_2^+) = \frac{1}{48\pi} \frac{m_Z}{\Gamma_Z} \left\{ |f_V|^2 + |f_A|^2 + \frac{1}{2 \cos^2 \theta_W} (|f_M|^2 + |f_E|^2) \right\} \quad (13)$$

where $\alpha_W = \frac{g^2}{4\pi}$ and Γ_Z is the total decay width of Z boson. Note that, in general, the production of sum of charged states is considered with the corresponding BR

$$BR(Z \rightarrow l_1^\pm l_2^\pm) = \frac{\Gamma(Z \rightarrow (\bar{l}_1 l_2 + \bar{l}_2 l_1))}{\Gamma_Z}, \quad (14)$$

and in our numerical analysis we use this branching ratio.

3 Discussion

In the model III, there are number of free parameters such as the masses of charged and neutral Higgs bosons, complex Yukawa couplings. The Yukawa couplings in the lepton sector are $\bar{\xi}_{N,ij}^D, i, j = e, \mu, \tau$ and they should be restricted by present and forthcoming experiments. The first assumption is that $\bar{\xi}_{N,ij}^D, i, j = e, \mu$, are small compared to $\bar{\xi}_{N,\tau i}^D, i = e, \mu, \tau$ since the strength of these couplings are related with the masses of leptons denoted by the indices of them, similar to the Cheng-Sher scenario [15]. Second we assume that $\bar{\xi}_{N,ij}^D$ is symmetric with respect to the indices i and j .

In our work we study on the decays $Z \rightarrow e^\pm \mu^\pm, Z \rightarrow e^\pm \tau^\pm$ and $Z \rightarrow \mu^\pm \tau^\pm$. In the case of $Z \rightarrow e^\pm \mu^\pm$ decay we need the couplings $\bar{\xi}_{N,\mu i}^D$ and $\bar{\xi}_{N,ei}^D$ with $i = e, \mu, \tau$. Here we use the first assumption and neglect the contributions of $\bar{\xi}_{N,ij}^D$ where $i, j = \mu, e$, by taking only the internal τ lepton into account. Now, we should restrict $\bar{\xi}_{N,i\tau}^D, i = e, \mu$. For $\bar{\xi}_{N,\mu\tau}^D$ we use the constraint coming from the experimental limits of μ lepton EDM [16],

$$0.3 \times 10^{-19} e - cm < d_\mu < 7.1 \times 10^{-19} e - cm \quad (15)$$

(see [14] for details).

For the restriction of the coupling $\bar{\xi}_{N,\mu\tau}^D$, the deviation of the anomalous magnetic moment (AMM) of muon over its SM prediction [17] due to the the recent experimental result of muon AMM by g-2 Collaboration [18], can also be used. However, AMM of muon is possible in the model III even for vanishing complex Yukawa couplings. In the LFV $Z \rightarrow l_1 l_2$ decay, the part which depends on the couplings η_i^A and η_i^- (see eq. (11)) is non-vanishing when the complex Yukawa couplings are permitted in the model. Here, we choose EDM of muon since the

restriction is sensitive to complex phases, existing also in the $Z \rightarrow l_1 l_2$ decay. The coupling $\bar{\xi}_{N,e\tau}^D$ is restricted using the experimental upper limit of the BR of the process $\mu \rightarrow e\gamma$ and the above constraint for $\bar{\xi}_{N,\mu\tau}^D$, since $\mu \rightarrow e\gamma$ decay can be used to fix the Yukawa combination $\bar{\xi}_{N,\mu\tau}^D \bar{\xi}_{N,e\tau}^D$ (see [14]). This ensures us to determine the upper and lower limits of the coupling $\bar{\xi}_{N,e\tau}^D$. The maximum value of the $BR(Z \rightarrow \mu e)$ is calculated by taking the combination $\bar{\xi}_{N,\mu\tau}^D \bar{\xi}_{N,e\tau}^D$, which respects the upper bound of $\mu \rightarrow e\gamma$ decay. For the minimum value of $BR(Z \rightarrow \mu e)$, we use the combination $\bar{\xi}_{N,\mu\tau}^D \bar{\xi}_{N,e\tau}^D$ if each coupling is at its minimum value. In fact, this minimum value is artificial. Note that, $\bar{\xi}_{N,e\tau}^D$ can be restricted and the minimum value of $BR(Z \rightarrow \mu e)$ can be obtained by using the experimental result of the EDM of electron [19]. However, we expect that the experimental result of the EDM of electron is not more reliable than the one of the process $\mu \rightarrow e\gamma$.

This analysis shows that $|\bar{\xi}_{N,\mu\tau}^D|, (|\bar{\xi}_{N,e\tau}^D|)$ is at the order of the magnitude of 10^3 (10^{-4}) GeV. Note that these couplings are chosen complex to be able to describe the EDM which is possible in the case of CP violating interactions. Throughout our calculations we use the input values given in Table (1).

Parameter	Value
m_μ	0.106 (GeV)
m_τ	1.78 (GeV)
m_W	80.26 (GeV)
m_Z	91.19 (GeV)
G_F	$1.1663710^{-5}(GeV^{-2})$
Γ_Z	2.490 (GeV)
$\sin \theta_W$	$\sqrt{0.2325}$

Table 1: The values of the input parameters used in the numerical calculations.

Fig. 2 (3) represents $\sin\theta_{\tau e}$ dependence of the maximum (minimum) value of the $BR(Z \rightarrow \mu^\pm e^\pm)$ for $\sin\theta_{\tau\mu} = 0.5$, $m_{h^0} = 70 GeV$ and $m_{A^0} = 80 GeV$. Here the maximum and minimum values are predicted by taking upper and lower limits of μ lepton EDM into account. The maximum (minimum) value of the BR is 7×10^{-11} (10^{-13}) for small values of $\sin\theta_{\tau e}$ and its sensitivity to this parameter is weak. BR decreases at the order of the magnitude 15% for $\sin\theta_{\tau e} \geq 0.5$ and it becomes more sensitive to $\sin\theta_{\tau e}$. $\sin\theta_{\tau\mu}$ dependence of the maximum (minimum) value of the $BR(Z \rightarrow \mu^\pm e^\pm)$ for $\sin\theta_{\tau e} = 0.5$, $m_{h^0} = 70 GeV$ and $m_{A^0} = 80 GeV$ almost the same as $\sin\theta_{\tau e}$ dependence of the BR under consideration.

In Fig. 4 we present m_{A^0} dependence of the minimum value of the $BR(Z \rightarrow \mu^\pm e^\pm)$ for $\sin\theta_{\tau e} = 0.5$, $\sin\theta_{\tau\mu} = 0.5$ and $m_{h^0} = 70 GeV$. The BR is strongly sensitive to m_{A^0} and

decreases with increasing values of m_{A^0} . The same dependence appears for the maximum value of the BR .

In the $Z \rightarrow \tau^\pm e^\pm$ decay, the couplings $\bar{\xi}_{N,\tau i}^D$ and $\bar{\xi}_{N,ei}^D$ with $i = e, \mu, \tau$ play the main role, in the model III. Again the first assumption permits us to neglect the contributions of the couplings $\bar{\xi}_{N,ij}^D$ where $i, j = \mu, e$, by taking only the internal τ lepton into account. For the coupling $\bar{\xi}_{N,e\tau}^D$ the same restriction is used, however for $\bar{\xi}_{N,\tau\tau}^D$ we do not use any constraint.

Fig. 5 (6) shows $\sin\theta_{\tau e}$ and $\sin\theta_{\tau\mu}$ dependence of the maximum (minimum) value of the $BR(Z \rightarrow \tau^\pm e^\pm)$ for $\sin\theta_{\tau\mu} = 0.5$ and $\sin\theta_{\tau e} = 0.5$ respectively. Here the coupling $\bar{\xi}_{N,\tau\tau}^D$ is taken as $\bar{\xi}_{N,\tau\tau}^D = 10^3 \text{ GeV}$. The maximum (minimum) value of the BR for this process is at the order of the magnitude of 10^{-11} (10^{-12}). BR increases with increasing values $\sin\theta_{\tau\mu}$, however this behaviour appears in reverse for $\sin\theta_{\tau e}$ dependence. The sensitivity of $BR(Z \rightarrow \tau^\pm e^\pm)$ to both CP violating parameters, $\sin\theta_{\tau e}$ and $\sin\theta_{\tau\mu}$, is strong.

We present the coupling $\bar{\xi}_{N,\tau\tau}^D$ dependence of $BR(Z \rightarrow \tau^\pm e^\pm)$ in Figs. 7 and 8. These figures shows that the BR enormously increases with increasing values of the coupling $\bar{\xi}_{N,\tau\tau}^D$. Note that we take the coupling $\bar{\xi}_{N,\tau\tau}^D$ real in our calculations.

Finally, we study the $Z \rightarrow \mu^\pm \tau^\pm$ decay by taking τ lepton as an internal one similar to previous analysis. Here the experimental upper limit for $BR(Z \rightarrow \mu^\pm \tau^\pm)$ can be reached for the small values of $\bar{\xi}_{N,\tau\tau}^D$. Further, the theoretical calculations are consistent with this upper limit for the case where the mass differences of neutral Higgs bosons h^0 and A^0 are large, even for large values of the coupling $\bar{\xi}_{N,\tau\tau}^D$.

As a summary, we study the BR 's of the decays $Z \rightarrow e^\pm \mu^\pm$, $Z \rightarrow e^\pm \tau^\pm$ and $Z \rightarrow \mu^\pm \tau^\pm$ and observe that it is possible to reach the present experimental upper limits in the model III, playing with the model parameters in the restriction region. This result is important since the theoretical work in the SM shows that the branching rates are less than 10^{-54} and compared to this number large rates are expected with massive and mixing neutrinos. In our analysis, we predict that the BR for the $Z \rightarrow e^\pm \mu^\pm$ decay can reach to the values at the order of the magnitude 10^{-11} . BR for the process $Z \rightarrow e^\pm \tau^\pm$ depends strongly on the Yukawa coupling $\bar{\xi}_{N,\tau\tau}^D$ and for its large values, $10^3 - 10^4 \text{ GeV}$, it can be in the range $10^{-10} - 10^{-9}$. The process $Z \rightarrow \mu^\pm \tau^\pm$ can have larger BR compared to the previous ones, since the Yukawa couplings entering in the expressions are $\bar{\xi}_{N,\mu\tau}^D$ and $\bar{\xi}_{N,\tau\tau}^D$. Furthermore, BR 's of the processes under consideration are sensitive to the CP-violating parameters since the source of the parts which depend on couplings η_i^A and η_i^- are the non-vanishing complex Yukawa couplings, in the model III.

In future, with the reliable experimental result of upper limits of the BR 's of above processes it would be possible to test models beyond the SM and free parameters of these models

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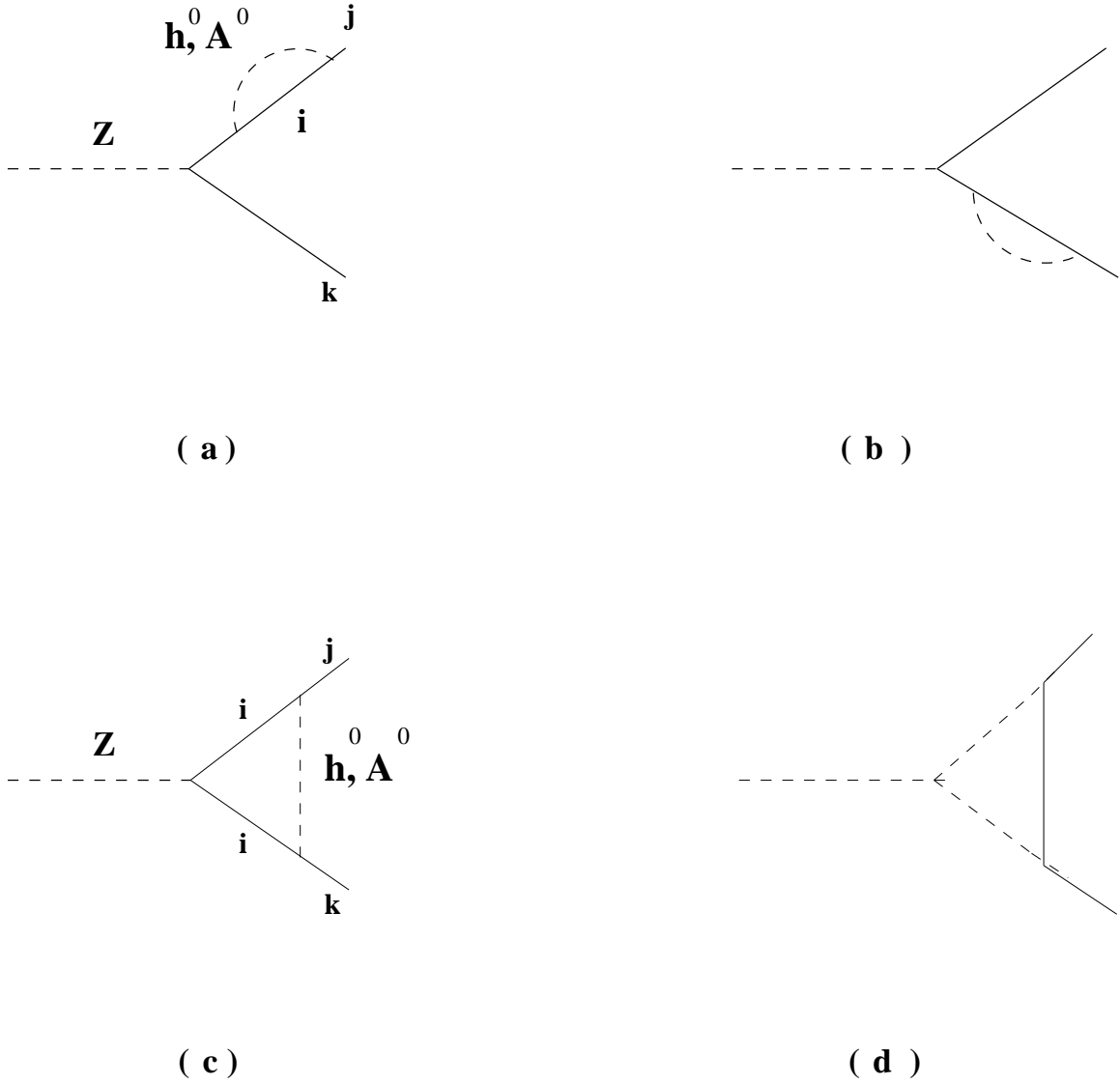


Figure 1: One loop diagrams contribute to $Z \rightarrow k^+ j^-$ decay due to the neutral Higgs bosons h_0 and A_0 in the 2HDM. i represents the internal, j (k) outgoing (incoming) lepton, dashed lines the vector field Z , h_0 and A_0 fields.

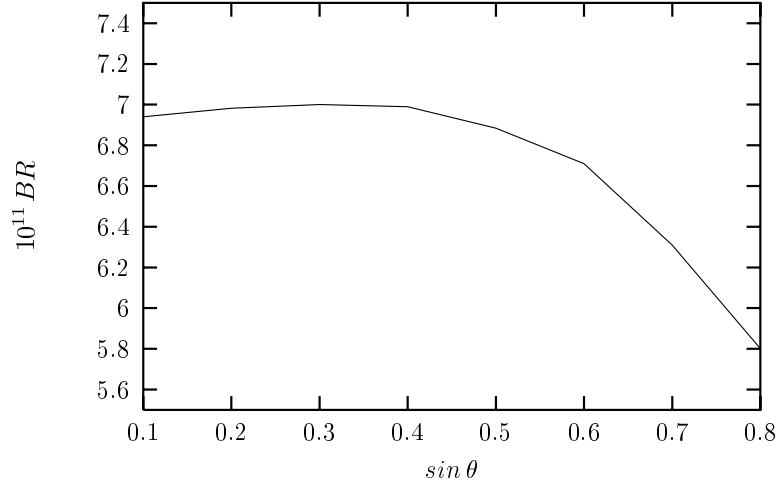


Figure 2: The maximum value of $BR(Z \rightarrow \mu^\pm e^\pm)$ as a function of $\sin\theta_{\tau e}$ for $\sin\theta_{\tau\mu} = 0.5$, $m_{h^0} = 70 \text{ GeV}$ and $m_{A^0} = 80 \text{ GeV}$.

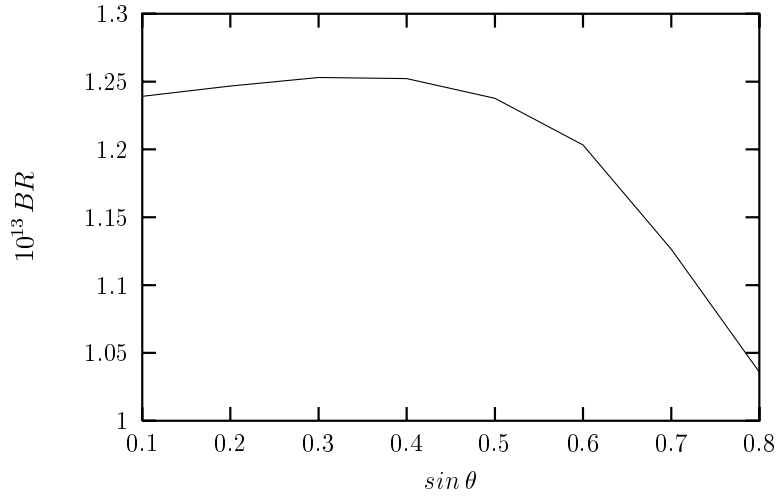


Figure 3: The same as Fig. 2 but for the minimum value of $BR(Z \rightarrow \mu^\pm e^\pm)$.

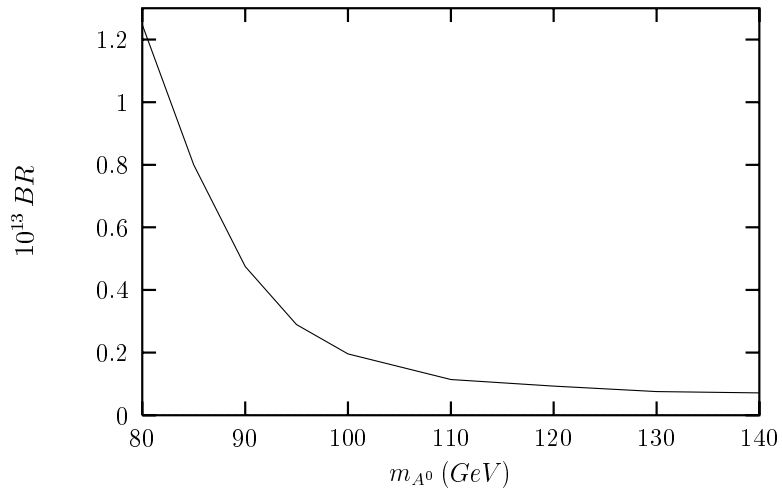


Figure 4: The minimum value of $BR(Z \rightarrow \mu^\pm e^\pm)$ as a function of m_{A^0} for $\sin\theta_{\tau\mu} = 0.5$, $\sin\theta_{\tau e} = 0.5$ and $m_{h^0} = 70 GeV$.

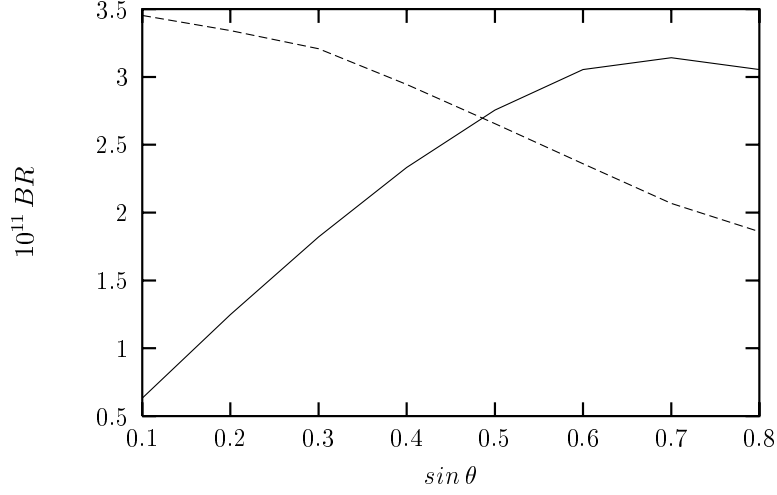


Figure 5: The maximum value of $BR(Z \rightarrow \tau^\pm e^\pm)$ as a function of $\sin\theta$ for $\bar{\xi}_{N,\tau\tau}^D = 10^3 \text{ GeV}$, $m_{h^0} = 70 \text{ GeV}$ and $m_{A^0} = 80 \text{ GeV}$. Here solid line represents the dependence with respect to $\sin\theta_{\tau\mu}$ for $\sin\theta_{\tau e} = 0.5$ and dashed line to $\sin\theta_{\tau e}$ for $\sin\theta_{\tau\mu} = 0.5$.

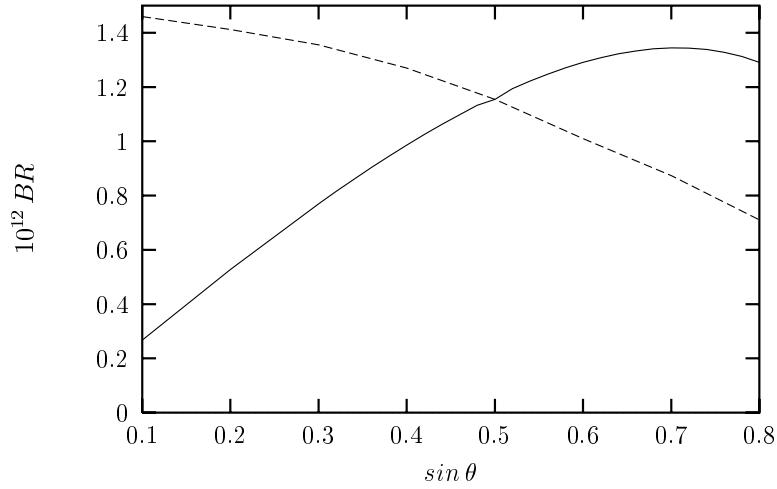


Figure 6: The same as Fig. 5 but for the minimum value of $BR(Z \rightarrow \tau^\pm e^\pm)$.

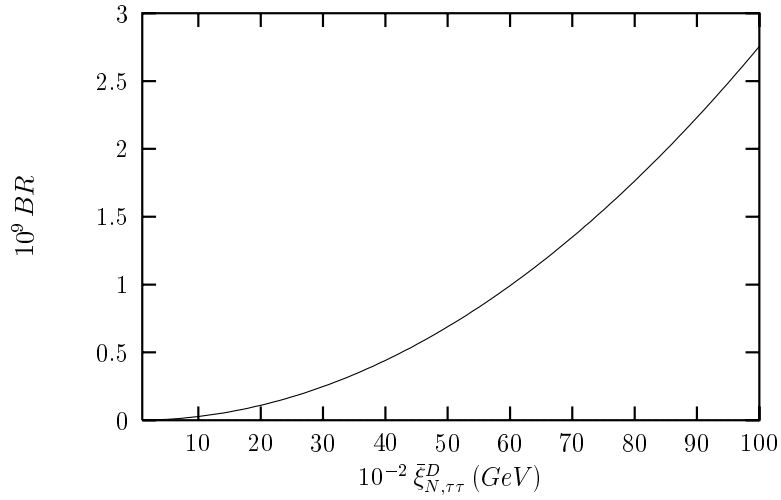


Figure 7: The maximum value of $BR(Z \rightarrow \tau^\pm e^\pm)$ as a function of $\bar{\xi}_{N,\tau\tau}^D$ for $\sin\theta_{\tau\mu} = 0.5$, $\sin\theta_{\tau e} = 0.5$, $m_{h^0} = 70 \text{ GeV}$ and $m_{A^0} = 80 \text{ GeV}$.

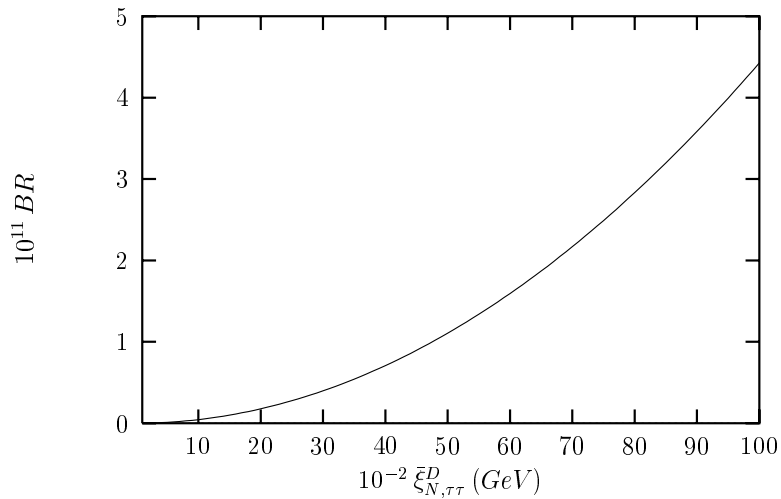


Figure 8: The same as Fig. 7 but for the minimum value of $BR(Z \rightarrow \tau^\pm e^\pm)$.