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Exclusive $\Lambda_b \to \Lambda\ell^+\ell^-$ decay beyond standard model

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Abstract

Using the most general, model independent form of the effective Hamiltonian, the exclusive rare baryonic $\Lambda_b \to \Lambda\ell^+\ell^-(\ell = \mu, \tau)$ decay is analyzed. We study the sensitivity of the branching ratio and lepton forward–backward asymmetry to the new Wilson coefficients. It is shown that these physical quantities are quite sensitive to the new Wilson coefficients. Determination of the position of zero value of the forward–backward asymmetry can serve as a useful tool for establishing new physics beyond the standard model, as well as fixing the sign of the new Wilson coefficients.

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1. Introduction

Rare decays, induced by flavor changing neutral current (FCNC) $b \to s(d)$ transitions, provide a potential precision testing ground for the standard model (SM) at loop level. For this reason studying these decays constitutes one of the main research directions of the two operating B-factories BaBar and Belle [1]. Rare decays can give valuable information about the poorly studied aspects of the SM at present, such as the Cabibbo–Kobayashi–Maskawa matrix elements V_{td}, V_{ts}, V_{ub} and the leptonic decay constant. After the CLEO measurement of the radiative $b \to s\gamma$ decay [2], the main interest has been focused on the rare decays induced by the $b \to s\ell^+\ell^-$ transition, which have relatively "large" branching

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ratio in the SM. These decays have been investigated extensively in the SM and its various extensions [3–18].

The theoretical analysis of the inclusive decays is rather easy since they are free of long distance effects, but their experimental detection is quite difficult. For exclusive decays the situation is contrary to the inclusive case; i.e., their experimental investigation is easy, but theoretical analysis is difficult due to the appearance of the form factors. It should be noted that the exclusive $B \to K^*(K)\ell^+\ell^-$ decays, which are described by the $b \to s\ell^+\ell^-$ transition at inclusive level, have been widely studied in literature (see [19–22] and references therein). Another exclusive decay which is described at inclusive level by the $b \to s\ell^+\ell^-$ transition is the baryonic $\Lambda_b \to \Lambda\ell^+\ell^-$ decay. It should be emphasized that, in order to analyze the helicity structure of the effective Hamiltonian for the $b \to s$ transition in the SM and beyond the SM, investigation of the mesonic decays alone is not enough, since the information about the handedness of the quark is lost during the hadronization process. In order to maintain the helicity of the quarks, investigation of the baryonic decays is the only choice. For this reason study of the baryonic decays receive special interest. Note that $\Lambda_b \to \Lambda\ell^+\ell^-$ decay has been studied in context of the SM and two Higgs doublet models in [23] and [24], respectively.

Rare decays are very sensitive to the new physics beyond the SM and, therefore, constitute quite a suitable tool for looking such effects. In general, new physics effects manifest themselves in rare decays either through new contributions to the Wilson coefficients existing in the SM or by introducing new structures to the effective Hamiltonian which are absent in the SM (see, for example, [21,25–27] and the references therein). At this point we would like to remind the reader that, the sensitivity of the physical observables to the new physics effects in the "heavy pseudoscalar meson → light pseudoscalar (vector) meson" transitions such as $B \to K(K^*)\ell^+\ell^-$, are studied systematically in [21,27,28] using the most general form of the effective Hamiltonian.

The intriguing questions that follow next are what happens in the "heavy baryon → light baryon" transition and which physical quantity is most sensitive to the new physics effects. The present work is devoted to look for the answers to these questions.

In this work we present a systematic study of the baryonic $\Lambda_b \to \Lambda\ell^+\ell^-$ decay. The paper is organized as follows. In Section 2, using the most general model independent form of the Hamiltonian, we derive the matrix element, differential decay width and forward–backward asymmetry in terms of the form factors. Section 3 is devoted to the numerical analysis and concluding remarks.

2. Theoretical background

The matrix element of the $\Lambda_b \to \Lambda\ell^+\ell^-$ decay at quark level is described by the $b \to s\ell^+\ell^-$ transition. The decay amplitude for the $b \to s\ell^+\ell^-$ transition, in a general model independent form, can be written in the following way (see [21,25,26])

$$\mathcal{M} = \frac{G\alpha}{\sqrt{2}\pi} V_{tb}V_{ts}^* \times \left\{ C_{SL}\bar{s}i\sigma_{\mu\nu}\frac{q^\nu}{q^2}Lb\bar{\ell}\gamma_\mu\ell + C_{BR}\bar{s}i\sigma_{\mu\nu}\frac{q^\nu}{q^2}Rb\bar{\ell}\gamma_\mu\ell + C_{LL}^{\text{tot}}\bar{s}_L\gamma^\mu b_L\bar{\ell}_L\gamma_\mu\ell_L \right.$$

$$
\begin{aligned}
&+ C_{LR}^{\text{tot}} \bar{s}_L \gamma^\mu b_L \bar{\ell}_R \gamma_\mu \ell_R + C_{RL} \bar{s}_R \gamma^\mu b_R \bar{\ell}_L \gamma_\mu \ell_L + C_{RR} \bar{s}_R \gamma^\mu b_R \bar{\ell}_R \gamma_\mu \ell_R \\
&+ C_{LRLR} \bar{s}_L b_R \bar{\ell}_L \ell_R + C_{RLLR} \bar{s}_R b_L \bar{\ell}_L \ell_R + C_{LRRL} \bar{s}_L b_R \bar{\ell}_R \ell_L \\
&+ C_{RLRL} \bar{s}_R b_L \bar{\ell}_R \ell_L + C_T \bar{s} \sigma^{\mu\nu} b \bar{\ell} \sigma_{\mu\nu} \ell + i C_{TE} \epsilon^{\mu\nu\alpha\beta} \bar{s} \sigma_{\mu\nu} b \bar{\ell} \sigma_{\alpha\beta} \ell \Big\}, \qquad (1)
\end{aligned}
$$

where $L = (1 - \gamma_5)/2$ and $R = (1 + \gamma_5)/2$ are the chiral operators and C_X are the coefficients of the four-Fermi interaction. Part of these Wilson coefficients and structures already exist in the effective Hamiltonian of the $b \to s$ transition in the SM. The first two of the coefficients C_{SL} and C_{BR} are the non-local Fermi interactions which correspond to $-2m_s C_7^{\text{eff}}$ and $-2m_b C_7^{\text{eff}}$ in the SM, respectively. The following four terms describe vector type interactions. Two of these vector interactions containing the coefficients C_{LL}^{tot} and C_{LR}^{tot} do also exist in the SM in the forms $(C_9^{\text{eff}} - C_{10})$ and $(C_9^{\text{eff}} + C_{10})$, respectively. Therefore, C_{LL}^{tot} and C_{LR}^{tot} represent the sum of the combinations from SM and the new physics in the following forms

$$
\begin{aligned}
C_{LL}^{\text{tot}} &= C_9^{\text{eff}} - C_{10} + C_{LL}, \\
C_{LR}^{\text{tot}} &= C_9^{\text{eff}} + C_{10} + C_{LR}. \qquad (2)
\end{aligned}
$$

The terms with C_{LRRL}, C_{LRLR}, C_{RLRL} and C_{RLLR} describe the scalar type interactions. The last two terms in Eq. (1) correspond to the tensor type interactions.

A few words about the Wilson coefficient C_9^{eff} are in order. In the SM, in next-to-leading order at $\mu \approx m_b$ scale, the effective Wilson coefficient C_9^{eff} can be written in the following form:

$$
C_9^{\text{eff}} = C_9(m_b)\left[1 + \frac{\alpha_s(m_b)}{\pi} w(\hat{s})\right] + Y_{\text{SD}}(m_b, \hat{s}) + Y_{\text{LD}}(m_B, \hat{s}),
$$

where $\hat{s} = q^2/m_b^2$ and $w(\hat{s})$ represents the $\mathcal{O}(\alpha_s)$ corrections coming from one gluon exchange in the matrix element of the corresponding operator whose explicit form can be found in [29]. The functions Y_{SD} and Y_{LD} represent the short and long distance contributions of the four-quark operators $\mathcal{O}_i = 1, \ldots, 6$, respectively. The short distance contribution Y_{SD} can be obtained by a perturbative calculation and the result is presented in [14,29].

The long distance part Y_{LD} can be attributed to the real $\bar{c}c$ in the intermediate states, i.e., to the cascade process $B \to K^* \psi_i \to K^* \ell^+ \ell^-$, $(i = 1, \ldots, 6)$. Usually Y_{LD} is parametrized in the form of a phenomenological Breit–Wigner ansatz [8,30], and it is given as

$$
Y_{\text{LD}} = -\frac{3\pi}{\alpha^2} C^{(0)} \sum_{V_i = \psi(1s) \cdots \psi(6s)} \kappa_i \frac{\Gamma(V_i \to \ell^+ \ell^-) m_{V_i}}{q^2 - m_{V_i}^2 + i \Gamma_{V_i} m_{V_i}},
$$

where $C^{(0)} = 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6$. The phenomenological factor κ_i for the lowest two resonances is estimated to be $\kappa_{J/\psi} = 1.65$ and $\kappa_{\psi'} = 1.65$ (see, for example, [23]), and in our numerical calculations we use the average of J/ψ and ψ' for the higher resonances $\psi(3s) \cdots \psi(6s)$.

The amplitude of the exclusive $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay can be obtained by sandwiching the matrix element of the $b \to s \ell^+ \ell^-$ decay between initial and final state baryons. It follows from Eq. (1) that, in order to calculate the amplitude of the $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay the following matrix elements are needed

$$\langle \Lambda | \bar{s} \gamma_\mu (1 \mp \gamma_5) b | \Lambda_b \rangle, \qquad \langle \Lambda | \bar{s} \sigma_{\mu\nu} (1 \mp \gamma_5) b | \Lambda_b \rangle,$$
$$\langle \Lambda | \bar{s} (1 \mp \gamma_5) b | \Lambda_b \rangle. \tag{3}$$

Explicit forms of these matrix elements in terms of the form factors are presented in Appendix A. Using the parametrization of these matrix elements, the matrix form of the $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay can be written as

$$\begin{aligned}
\mathcal{M} = \frac{G\alpha}{4\sqrt{2}\pi} V_{tb} V_{ts}^* \\
\times \Big\{ & \bar{\ell}\gamma^\mu \ell \, \bar{u}_\Lambda \Big[A_1 \gamma_\mu (1+\gamma_5) + B_1 \gamma_\mu (1-\gamma_5) \\
& + i\sigma_{\mu\nu} q^\nu \big[A_2 (1+\gamma_5) + B_2 (1-\gamma_5) \big] \\
& + q_\mu \big[A_3 (1+\gamma_5) + B_3 (1-\gamma_5) \big] \Big] u_{\Lambda_b} \\
& + \bar{\ell}\gamma^\mu \gamma_5 \ell \, \bar{u}_\Lambda \Big[D_1 \gamma_\mu (1+\gamma_5) + E_1 \gamma_\mu (1-\gamma_5) \\
& + i\sigma_{\mu\nu} q^\nu \big[D_2 (1+\gamma_5) + E_2 (1-\gamma_5) \big] \\
& + q_\mu \big[D_3 (1+\gamma_5) + E_3 (1-\gamma_5) \big] \Big] u_{\Lambda_b} + \bar{\ell}\ell \, \bar{u}_\Lambda (N_1 + H_1 \gamma_5) u_{\Lambda_b} \\
& + \bar{\ell}\gamma_5 \ell \, \bar{u}_\Lambda (N_2 + H_2 \gamma_5) u_{\Lambda_b} \\
& + 4C_T \bar{\ell} \sigma^{\mu\nu} \ell \, \bar{u}_\Lambda \big[f_T \sigma_{\mu\nu} - i f_T^V (q_\nu \gamma_\mu - q_\mu \gamma_\nu) - i f_T^S (P_\mu q_\nu - P_\nu q_\mu) \big] u_{\Lambda_b} \\
& + 4C_{TE} \epsilon^{\mu\nu\alpha\beta} \bar{\ell} \sigma_{\alpha\beta} \ell i \bar{u}_\Lambda \big[f_T \sigma_{\mu\nu} - i f_T^V (q_\nu \gamma_\mu - q_\mu \gamma_\nu) \\
& - i f_T^S (P_\mu q_\nu - P_\nu q_\mu) \big] u_{\Lambda_b} \Big\},
\end{aligned} \tag{4}$$

where $P = p_{\Lambda_b} + p_\Lambda$.

Explicit expressions of the functions A_i, B_i, D_i, E_i, H_j and N_j ($i = 1, 2, 3$ and $j = 1, 2$) are given in Appendix A.

Obviously, the $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay introduces a lot of form factors. However, when the heavy quark effective theory (HQET) has been used, the heavy quark symmetry reduces the number of independent form factors to two only (F_1 and F_2), irrelevant with the Dirac structure of the relevant operators [31], and hence we obtain that

$$\big\langle \Lambda(p_\Lambda) \big| \bar{s} \Gamma b \big| \Lambda(p_{\Lambda_b}) \big\rangle = \bar{u}_\Lambda \big[F_1\big(q^2\big) + \not{v} F_2\big(q^2\big) \big] \Gamma u_{\Lambda_b}, \tag{5}$$

where Γ is an arbitrary Dirac structure, $v^\mu = p^\mu_{\Lambda_b}/m_{\Lambda_b}$ is the four-velocity of Λ_b, and $q = p_{\Lambda_b} - p_\Lambda$ is the momentum transfer. Comparing the general form of the form factors with (5), one can easily obtain the following relations among them (see also [23])

$$g_1 = f_1 = f_2^T = g_2^T = F_1 + \sqrt{r}\, F_2,$$
$$g_2 = f_2 = g_3 = f_3 = g_T^V = f_T^V = \frac{F_2}{m_{\Lambda_b}},$$
$$g_T^S = f_T^S = 0, \qquad g_1^T = f_1^T = \frac{F_2}{m_{\Lambda_b}} q^2,$$

$$g_3^T = \frac{F_2}{m_{\Lambda_b}}(m_{\Lambda_b} + m_\Lambda), \qquad f_3^T = -\frac{F_2}{m_{\Lambda_b}}(m_{\Lambda_b} - m_\Lambda), \tag{6}$$

where $r = m_\Lambda^2/m_{\Lambda_b}^2$. These relations will be used in further numerical calculations.

It is a simple matter now to derive the double differential decay rate with respect to the angle between lepton and the dimensionless invariant mass of the dilepton

$$\frac{d^2\Gamma}{ds\,dz} = \frac{G^2\alpha^2 m_{\Lambda_b}}{16384\pi^5}\left|V_{tb}V_{ts}^*\right|^2 v\sqrt{\lambda(1,r,s)}\,\mathcal{T}(s,z), \tag{7}$$

where $s = q^2/m_{\Lambda_b}^2$, $v = \sqrt{1 - 4m_\ell^2/q^2}$ is the lepton velocity and

$$\mathcal{T}(s,z) = \mathcal{T}_0(s) + \mathcal{T}_1(s)z + \mathcal{T}_2(s)z^2. \tag{8}$$

The expressions for $\mathcal{T}_0(s)$, $\mathcal{T}_1(s)$ and $\mathcal{T}_2(s)$ can be found in Appendix B.

In Eqs. (7), (8), $z = \cos\theta$ is the angle between the momenta of ℓ^- and Λ_b in the center of mass frame of dileptons, $\lambda(1,r,s) - 1 + r^2 + s^2 - 2r - 2s - 2rs$ is the triangle function. After integrating over the angle z, the invariant dilepton mass distribution takes the following form

$$\frac{d\Gamma}{ds} = \frac{G^2\alpha^2 m_{\Lambda_b}}{8192\pi^5}\left|V_{tb}V_{ts}^*\right|^2 v\sqrt{\lambda(1,r,s)}\left[\mathcal{T}_0(s) + \frac{1}{3}\mathcal{T}_2(s)\right]. \tag{9}$$

The limit for s is given by

$$\frac{4m_\ell^2}{m_{\Lambda_b}^2} \leqslant s \leqslant \left(1 - \sqrt{r}\right)^2. \tag{10}$$

The lepton forward–backward asymmetry $\mathcal{A}_{\mathrm{FB}}$ is one of the powerful tools in looking for new physics beyond the SM. Determination of the position of the zero value of the $\mathcal{A}_{\mathrm{FB}}$ is very useful for this purpose. New physics effects can shift the position of the zero value of the forward–backward asymmetry. Indeed, it has been shown in [21] that the new physics effects shift the zero value of the forward–backward asymmetry for the $B \to K^*\ell^+\ell^-$ decay. Therefore we will study the sensitivity of the forward–backward asymmetry to the new Wilson coefficients. The normalized forward–backward asymmetry is defined as

$$\mathcal{A}_{\mathrm{FB}} = \frac{\int_0^1 \frac{d\Gamma}{ds\,dz}\,dz - \int_{-1}^0 \frac{d\Gamma}{ds\,dz}\,dz}{\int_0^1 \frac{d\Gamma}{ds\,dz}\,dz + \int_{-1}^0 \frac{d\Gamma}{ds\,dz}\,dz}. \tag{11}$$

It is well known that $\mathcal{A}_{\mathrm{FB}}$ is parity-odd but CP-even quantity, which depends on the chirality of the lepton and quark currents. In order to obtain $z = \cos\theta$ dependence, the differential decay width should contain multiplication of such terms which transform even and odd under parity, respectively.

In the massless lepton case, the zero position of the forward–backward $\mathcal{A}_{\mathrm{FB}}$, similar to the $B \to K^*\ell^+\ell^-$ decay, satisfies the following relation in SM [32]

$$\mathrm{Re}\left[C_9^{\mathrm{eff}}C_{10}^*\right] = -\frac{2m_b(1 - s_0\rho^2)}{s_0[1 - (1-r)\rho^2 - 2\sqrt{r}\,\rho]}\,\mathrm{Re}\left[C_7C_{10}^*\right], \tag{12}$$

where $\rho = F_2/(F_1 + F_2)$ and s_0 is the value of s at which $\mathcal{A}_{\mathrm{FB}}$ vanishes. The effect of introducing a new vector type interaction with coefficient $C_{LL}(C_{LR})$ reduces to redefining C_9^{eff} and C_{10} in the following way:

$$\begin{aligned} &C_9^{\mathrm{eff}} \to C_9^{\mathrm{eff}} + C_{LL}(\to C_9^{\mathrm{eff}} + C_{LR}), \\ &C_{10} \to C_{10} - C_{LL}(\to C_{10} + C_{LR}). \end{aligned}$$

In the presence of other interactions, the change in the form of Eq. (12) can easily be obtained from Eqs. (11) and (B.3). It should be noted that the large energy effective theory and QCD sum rules predict very close results for the ratio $F_2(q^2)/F_1(q^2)$:

$$\frac{F_2(q^2)}{F_1(q^2)} \approx \frac{F_2(0)}{F_1(0)}$$

(see, for example, [33] and the references therein). For this reason, the zero position of the forward–backward $\mathcal{A}_{\mathrm{FB}}$, far from the resonance region, is insensitive to the form factors and depends only on the values of the Wilson coefficients, similar to the B meson decays. Therefore the shift in the zero position of the $\mathcal{A}_{\mathrm{FB}}$ can be attributed to the existence of the new physics.

3. Numerical analysis

In this section we will study the sensitivity of tee branching ratio and lepton forward–backward asymmetry to the new Wilson coefficients. The main input parameters in calculating the above-mentioned quantities are the form factors. Since there exists no exact calculation of the form factors of the $\Lambda_b \to \Lambda$ transition, we will use the form factors derived from QCD sum rules in framework of the heavy quark effective theory, which reduces the number of lots of form factors into two (see, for example, [31]). The q^2 dependence of these form factors can be represented in terms of the three parameters as

$$F(q^2) = \frac{F(0)}{1 - a_F(q^2/m_{\Lambda_b}^2) + b_F(q^2/m_{\Lambda_b}^2)^2},$$

where parameters $F_i(0)$, a and b are listed in Table 1 (see [34]).

The values of other input parameters which appear in the expressions of the branching ratio and forward–backward asymmetry are: $m_b = 4.8$ GeV, $m_{\Lambda_b} = 5.64$ GeV, $m_\Lambda = 1.116$ GeV, $m_c = 1.4$ GeV. Contribution of new physics effects are contained in the new Wilson coefficients (see Eq. (1)). To the leading logarithmic approximation the values of the Wilson coefficients are $C_7^{\mathrm{eff}} = -0.313$, $C_9^{\mathrm{eff}} = 4.344$ and $C_{10}^{\mathrm{eff}} = -4.669$ [14].

Table 1
Transition form factors for the $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay in a three-parameter fit

	$F(0)$	a_F	b_F
F_1	0.462	−0.0182	−0.000176
F_2	−0.077	−0.0685	0.00146

These values of the Wilson coefficients correspond to the short distance contribution. The Wilson coefficient $C_9^{\rm eff}$ receives long distance contributions also coming from the real $\bar{c}c$ intermediate states, i.e., from the J/ψ family. In the present work we take into consideration both short and long distance contributions. In order to estimate the branching ratio and lepton forward-backward asymmetry we need the values of the new Wilson coefficients which describe new physics beyond the SM. In this work we will vary all new Wilson coefficients within the range $-|C_{10}| \leqslant C_X \leqslant |C_{10}|$. The experimental bounds on the branching ratio of the $B \to K^*\mu^+\mu^-$ [35][1] and $B_s \to \mu^+\mu^-$ decays [36] suggests that this is the right order of magnitude range for the vector and scalar Wilson coefficients. We assume that all new Wilson coefficients are real, i.e., we do not introduce any new phase in addition to the one present in the SM.

Let us first study the dependence of the branching ratio for the $\Lambda_b \to \Lambda\ell^+\ell^-$ decay on the new Wilson coefficients. In Figs. 1–4 and 5–8 we present the dependence of the branching ratio for the $\Lambda_b \to \Lambda\mu^+\mu^-$ ($\Lambda_b \to \Lambda\tau^+\tau^-$) decay on C_{LL}, C_{LR}, C_{RR}, C_{RL}, C_{LRLR}, C_T and C_{TE}, respectively. One can easily see from these figures that the branching ratio is strongly dependent on C_{LL} and the tensor interaction coefficients C_T and C_{TE}, while it is weakly dependent on the remaining vector interaction couplings C_{LR}, C_{RR} and C_{RL} and the scalar coupling C_{LRLR}. It should be noted that similar behavior takes place for the other scalar interaction coefficients.

This dependence of the branching ratio on the new Wilson coefficients can be explained in the following way. As an example, let us consider only the terms that come from C_{LL} and C_{LR} in the massless lepton limit. It follows from Eqs. (B.1) and (B.3) that, in this limit the branching ratio defined in Eq. (9) takes the following form:

$$
\begin{aligned}
\frac{d\Gamma}{ds} \sim\, & 32 m_{\Lambda_b}^4 s \\
& \times \Big\{ (1-r+s)\sqrt{r}\,\mathrm{Re}\big[F_1^* F_2\big] \\
& \qquad \times \left[\frac{16 m_b^2}{m_{\Lambda_b}^2 s} |C_7|^2 + \left|C_{LR}^{\rm tot} + C_{LL}^{\rm tot}\right|^2 + \left|C_{LR}^{\rm tot} - C_{LL}^{\rm tot}\right|^2 \right] \\
& \qquad + 4(1-r-s)\frac{m_b}{m_{\Lambda_b}}\,\mathrm{Re}\big[C_7^*\big(C_{LR}^{\rm tot} + C_{LL}^{\rm tot}\big)\big]\left(|F_2|^2 + \frac{1}{s}|F_1|^2\right) \\
& \qquad + 16\sqrt{r}\,\frac{m_b}{m_{\Lambda_b}}\,\mathrm{Re}\big[F_1^* F_2\big]\,\mathrm{Re}\big[C_7^*\big(C_{LR}^{\rm tot} + C_{LL}^{\rm tot}\big)\big] \Big\} \\
& + 8 m_{\Lambda_b}^4 \big[(1-r)^2 - s^2\big] \left[\frac{16 m_b^2}{m_{\Lambda_b}^2} |C_7|^2 \left(|F_2|^2 + \frac{1}{s}|F_1|^2\right) \right.
\end{aligned}
$$

[1] The latest result released by the BaBar Collaboration for the branching ratio of the $B \to K^*\ell^+\ell^-$ decay, is

$$\mathcal{B}\big(B \to K^*\ell^+\ell^-\big) = \big(1.68^{+0.68}_{-0.58} \pm 0.18\big) \times 10^{-6}.$$

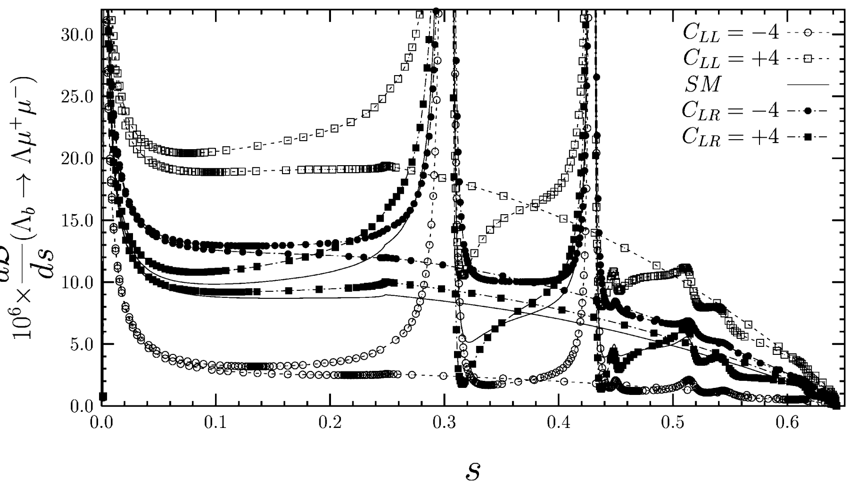

Fig. 1. The dependence of the branching ratio for the $\Lambda_b \to \Lambda\mu^+\mu^-$ decay on the new Wilson coefficients C_{LL} and C_{LR}. In all figures the curves with sharp peaks are the ones in which the long distance contributions are taken into account.

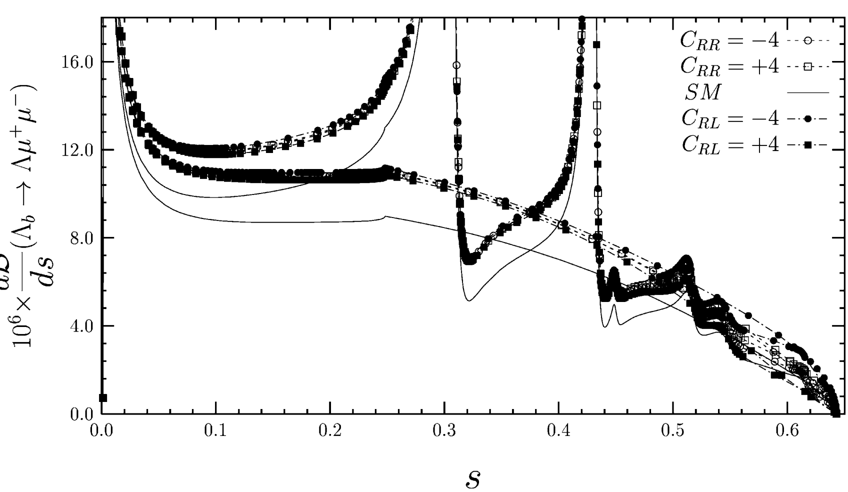

Fig. 2. The same as in Fig. 1, but for the coefficients C_{RR} and C_{RL}.

$$
\begin{aligned}
&+ \left(\left|C_{LR}^{\text{tot}} + C_{LL}^{\text{tot}}\right|^2 + \left|C_{LR}^{\text{tot}} - C_{LL}^{\text{tot}}\right|^2\right)\left(|F_1|^2 + s|F_2|^2\right)\bigg] \\
&+ 8m_{\Lambda_b}^4 \frac{\lambda}{3}\bigg[\frac{16m_b^2}{m_{\Lambda_b}^2}|C_7|^2\left(|F_1|^2 - |F_2|^2\right) \\
&+ \left(\left|C_{LR}^{\text{tot}} + C_{LL}^{\text{tot}}\right|^2 + \left|C_{LR}^{\text{tot}} - C_{LL}^{\text{tot}}\right|^2\right)\left(s|F_2|^2 - |F_1|^2\right)\bigg].
\end{aligned}
$$

We observe from this expression that, if there exist C_{LL} or C_{LR} in addition to SM Wilson

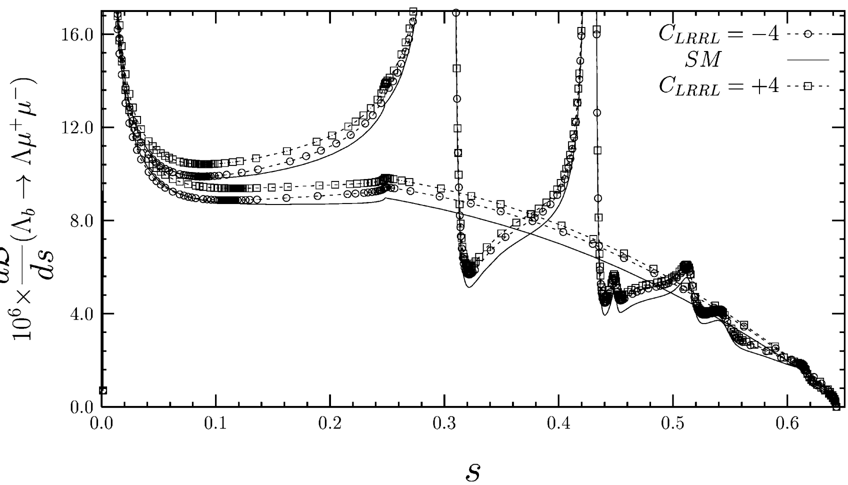

Fig. 3. The same as in Fig. 1, but for the coefficient C_{LRRL}.

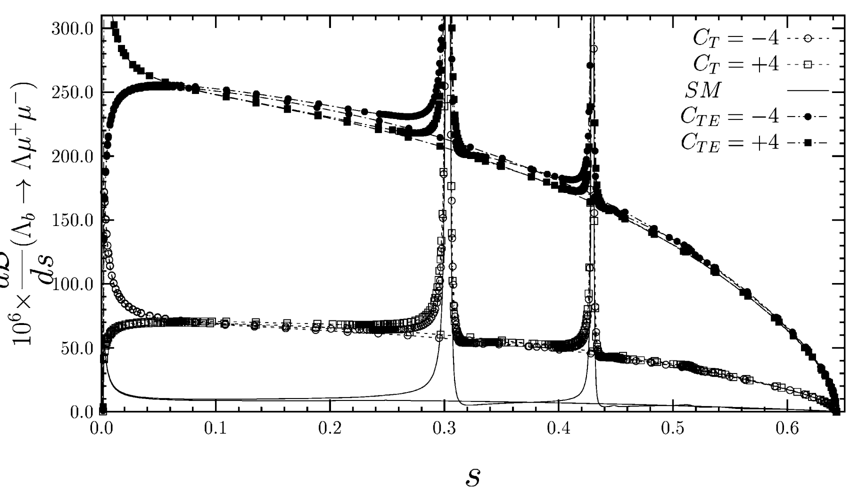

Fig. 4. The same as in Fig. 1, but for the coefficients C_T and C_{TE}, describing the tensor interactions.

coefficients then the leading terms are proportioanal to

$$4\left(\left|C_9^{\text{eff}}\right|^2 + |C_{10}|^2\right) + 2|C_{LL}|^2 + 4\,\text{Re}\left[\left(C_9^{\text{eff}} - C_{10}\right)^* C_{LL}\right], \quad \text{for } C_{LL},$$
$$4\left(\left|C_9^{\text{eff}}\right|^2 + |C_{10}|^2\right) + 2|C_{LR}|^2 + 4\,\text{Re}\left[\left(C_9^{\text{eff}} + C_{10}\right)^* C_{LR}\right], \quad \text{for } C_{LR}.$$

It is well known that in the SM $C_9^{\text{eff}} = 4.344$ (short distance) and $C_{10} = -4.669$, then the terms $\sim \text{Re}[(C_9^{\text{eff}} - C_{10})^* C_{LL}]$ give constructive interference to the SM result, while the terms $\sim \text{Re}[(C_9^{\text{eff}} + C_{10})^* C_{LR}]$ give destructive interference to the SM result, since $C_9^{\text{eff}} + C_{10} < 0$. We can conclude then that the branching ratio is weakly dependent on C_{LR} which is confirmed by the numerical calculations.

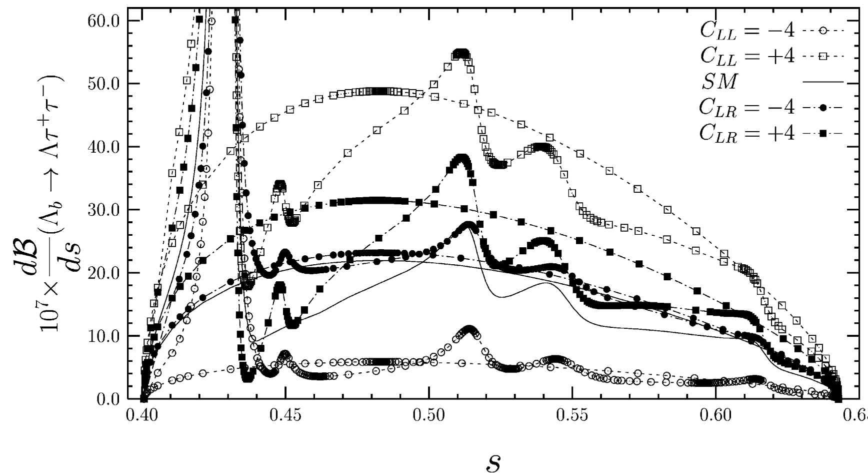

Fig. 5. The dependence of the branching ratio for the $\Lambda_b \to \Lambda\tau^+\tau^-$ decay on the new Wilson coefficients C_{LL} and C_{LR}.

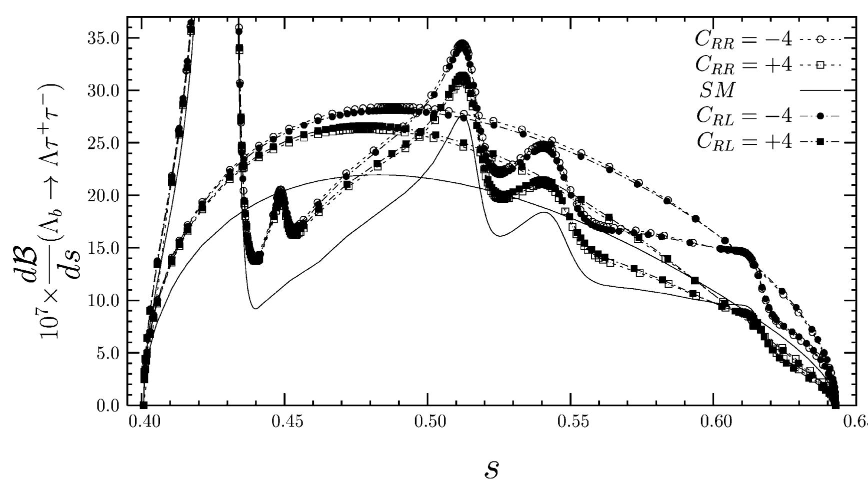

Fig. 6. The same as in Fig. 5, but for the coefficients C_{RR} and C_{RL}.

We observe from Fig. 4 that the branching ratio is strongly dependent on the tensor interaction.

For the $\Lambda_b \to \Lambda\tau^+\tau^-$ decay the situation is analogous to the $\Lambda_b \to \Lambda\mu^+\mu^-$ decay with a slight difference. Contribution coming from different type vector interactions becomes comparable. This fact can be explained by the fact that the terms proportional to $\sim (1 - v^2)$, which are very small in the μ case, contribute more in the τ case.

At this point we would like to point out that, similar dependence on the new Wilson coefficients occurs for the $B \to K^*\ell^+\ell^-$ decay.

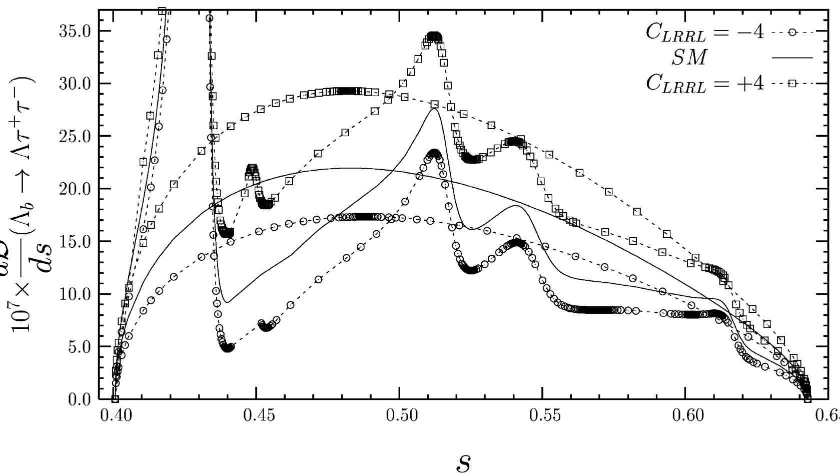

Fig. 7. The same as in Fig. 5, but for the coefficient C_{LRRL}.

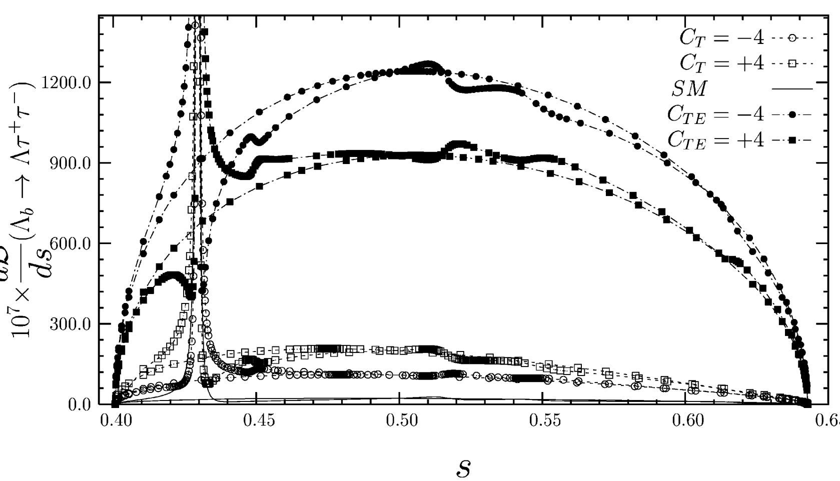

Fig. 8. The same as in Fig. 5, but for the coefficients C_T and C_{TE} describing the tensor interactions.

In Figs. 9–16 we present the dependence of the lepton forward–backward asymmetry on the new Wilson coefficients for the $\Lambda_b \to \Lambda\mu^+\mu^-$ and $\Lambda_b \to \Lambda\tau^+\tau^-$ decays. We observe from Figs. 9–12 that, for the $\Lambda_b \to \Lambda\mu^+\mu^-$ case the lepton forward–backward asymmetry is more sensitive to the coefficients C_{LL} and C_{LR} and weakly depends on rest of the Wilson coefficients. It follows from these figures that when C_{LL} is positive (negative), the zero point of the forward–backward asymmetry is shifted to the left (right) from its corresponding SM value. For all values of the coefficients C_{RR} and C_{RL} the zero position of the forward–backward asymmetry is shifted right and left with respect to its SM value, respectively. From these figures we see that, the new position of $\mathcal{A}_{\mathrm{FB}}$ is far from the resonance region. Moreover, as has already been noted, the zero position of $\mathcal{A}_{\mathrm{FB}}$ is

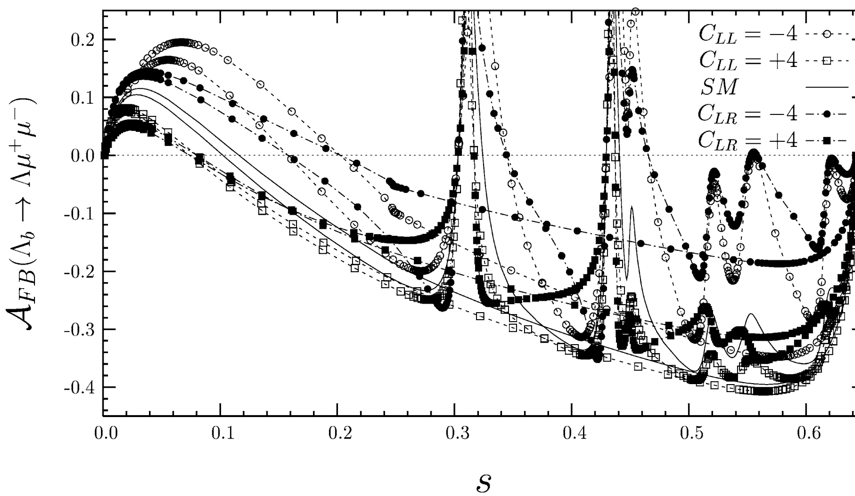

Fig. 9. The dependence of the lepton forward–backward asymmetry for the $\Lambda_b \to \Lambda\mu^+\mu^-$ decay on the new Wilson coefficients C_{LL} and C_{LR}.

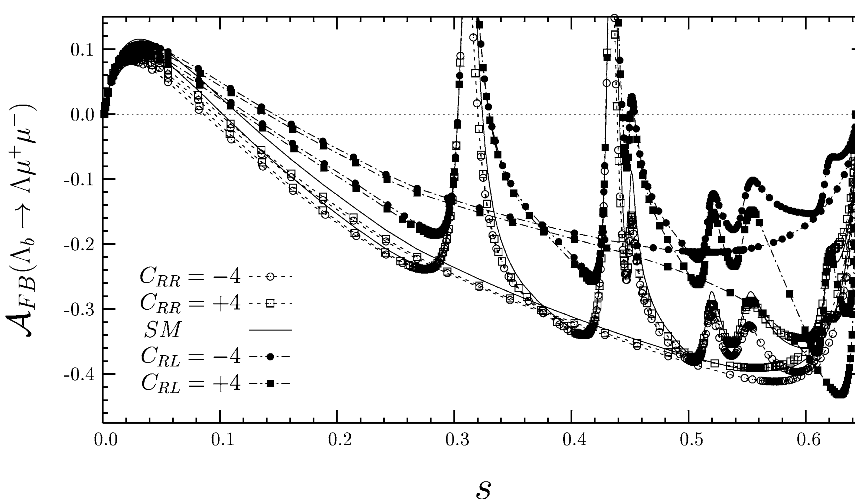

Fig. 10. The same as in Fig. 9, but for the coefficients C_{RR} and C_{RL}.

insensitive to the form factors and depends only on the Wilson coefficients. Therefore, the shift in the zero position of the dilepton forward–backward asymmetry can be attributed to the existence of new physics.

So, in view of all these observations we can say that, determination of the zero point of the forward–backward asymmetry can give us essential information, not only about the existence of new physics, but also about the sign of the new Wilson coefficients.

From Figs. 13–16 we arrive at the following conclusion for the $\Lambda_b \to \Lambda\tau^+\tau^-$ decay. Except tensor interaction coefficients, far from the resonance region, the forward–

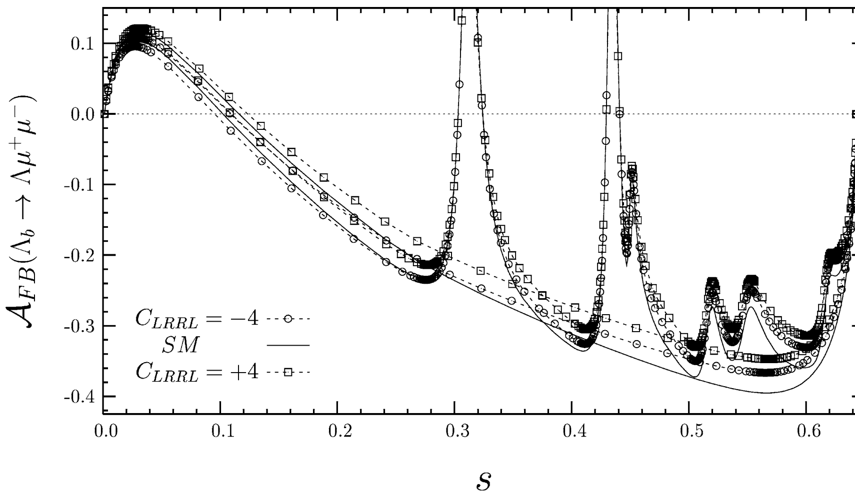

Fig. 11. The same as in Fig. 9, but for the coefficient C_{LRRL}.

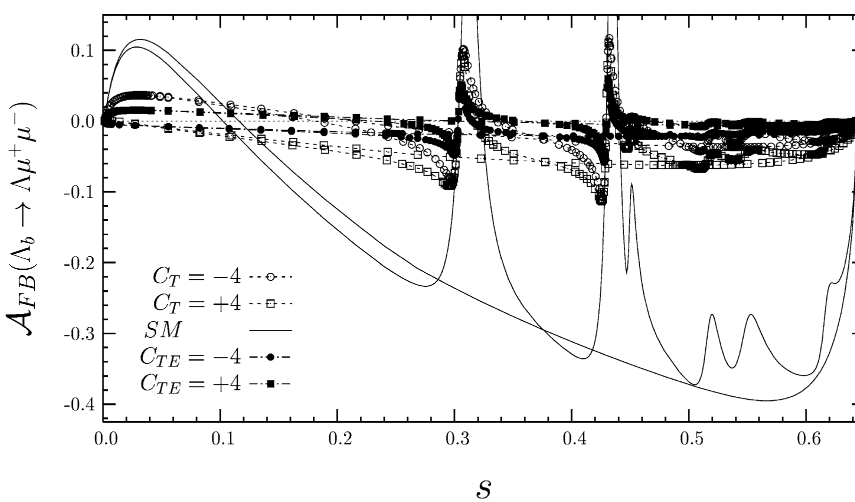

Fig. 12. The same as in Fig. 9, but for the coefficients C_T and C_{TE}, describing the tensor interactions.

backward asymmetry is negative for positive or negative values of the remaining ones. This situation is opposite to the $\Lambda_b \to \Lambda\mu^+\mu^-$ case. The value of the $\mathcal{A}_{\rm FB}$ is more sensitive to the C_{LRRL} and tensor interaction. The sign of the $\mathcal{A}_{\rm FB}$ can give us unambiguous information about the sign of the tensor interaction coefficients.

Obviously, investigation of polarization effects in the $\Lambda_b \to \Lambda\ell^+\ell^-$ decay can provide us new information in addition to the branching ratio and forward–backward asymmetry. We will consider this question in one of our future works.

Finally we would like to discuss briefly the number of expected events. As has already been noted, in the process under consideration, long distance effects can contribute via

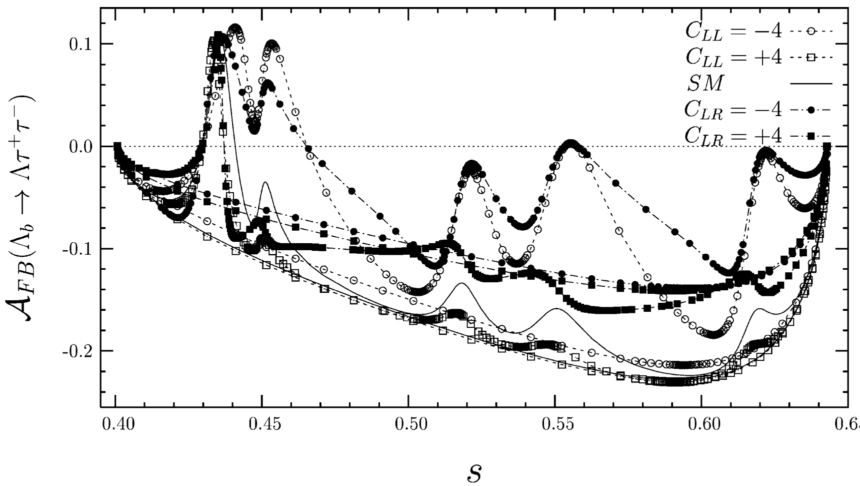

Fig. 13. The dependence of the lepton forward–backward asymmetry for the $\Lambda_b \to \Lambda\tau^+\tau^-$ decay on the new Wilson coefficients C_{LL} and C_{LR}.

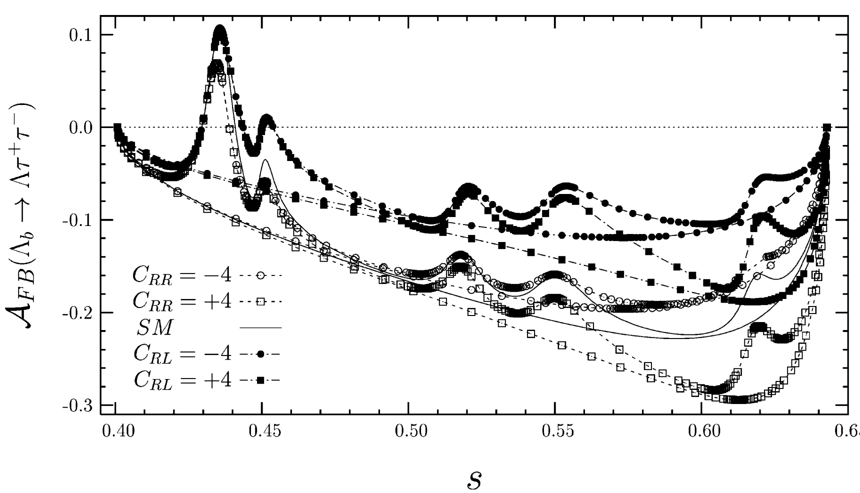

Fig. 14. The same as in Fig. 13, but for the coefficients C_{RR} and C_{RL}.

the real $\bar{c}c$ resonances (see the expression for C_9^{eff}). The dominant contribution to the differential branching ratio comes from the three low lying resonances J/ψ, ψ', ψ'' in the interval 9 $\text{GeV}^2 \leqslant q^2 \leqslant 14.5\ \text{GeV}^2$. In order to minimize the hadronic uncertainties, we will discard this subinterval in estimation of the branching ratio by dividing the q^2 region to low and high dilepton mass intervals

$$\text{(I)} \quad 4m_\ell^2 \leqslant q^2 \leqslant (m_{J/\psi} - 0.02\ \text{GeV})^2 \quad (\text{low } q^2 \text{ region}),$$

$$\text{(II)} \quad (m_{\psi'} + 0.02\ \text{GeV})^2 \leqslant q^2 \leqslant (m_{\Lambda_b} - m_\Lambda)^2 \quad (\text{high } q^2 \text{ region}),$$

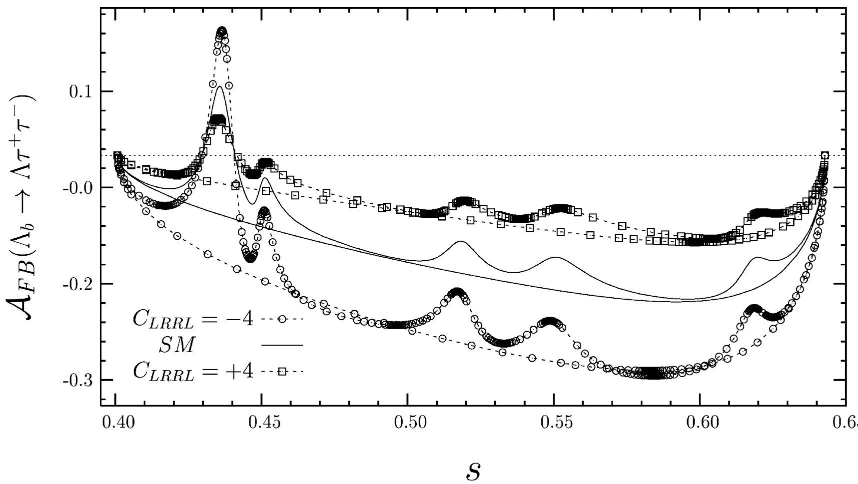

Fig. 15. The same as in Fig. 13, but for the coefficient C_{LRRL}.

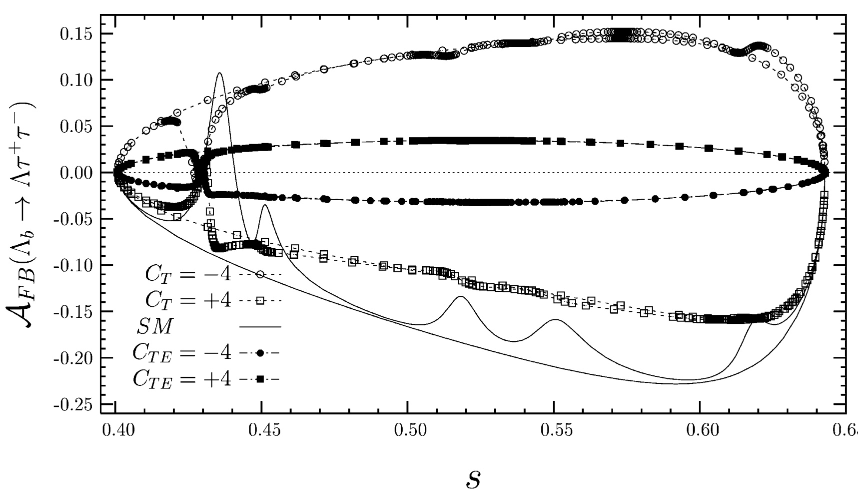

Fig. 16. The same as in Fig. 13, but for the coefficients C_T and C_{TE}, describing the tensor interactions.

where we choose the cutting parameter to be 0.02 GeV. In the SM, the branching ratio for the $\Lambda_b \to \Lambda\ell^+\ell^-$ decay in the above-mentioned kinematical regions is

$$\mathcal{B}(\Lambda_b \to \Lambda\mu^+\mu^-) = \begin{cases} 3.0 \times 10^{-6}, & \text{region (I)}, \\ 0.62 \times 10^{-6}, & \text{region (II)}. \end{cases}$$

Obviously, the $\Lambda_b \to \Lambda\tau^+\tau^-$ decay takes place only in the second kinematical region and our estimation for the branching ratio leads to $\mathcal{B}(\Lambda_b \to \Lambda\tau^+\tau^-) = 1.2 \times 10^{-7}$.

For a comparison of our results, we also present the values of the branching ratio for the $B \to K^* \ell^+ \ell^-$ decays:

$$\mathcal{B}\big(B \to K^* \mu^+ \mu^-\big) = \begin{cases} 1.6 \times 10^{-6}, & 4m_\ell^2 \leqslant q^2 (m_{J/\psi} - 0.02\ \mathrm{GeV})^2, \\ 0.46 \times 10^{-6}, & (m_{\psi'} + 0.02\ \mathrm{GeV})^2 \leqslant q^2 \leqslant (m_B - m_{K^*})^2, \end{cases}$$

$$\mathcal{B}\big(B \to K^* \tau^+ \tau^-\big) = 1.0 \times 10^{-7}, \quad (m_{\psi'} + 0.02\ \mathrm{GeV})^2 \leqslant q^2 \leqslant (m_B - m_{K^*})^2.$$

We observe that the values of the branching ratios for the $B \to K^* \ell^+ \ell^-$ and $\Lambda_b \to \Lambda \ell^+ \ell^-$ decays are close to each other in the corresponding regions.

The number of expected events for the considered decay is estimated to be

$$\mathcal{N} = \mathcal{N}_b f(b \to \Lambda_b) \mathcal{B},$$

where $\mathcal{N}_b$ is the number of $\bar{b}b$ pairs produced per year, f is the fragmentation function of the b quark to Λ_b, which is estimated to have a value about 10%. At LHC-B and BTeV machines, where the double lepton triggering helps high reconstruction efficiencies, $\mathcal{N}_b = 10^{11}$–10^{12} $\bar{b}b$ pairs are expected to be produced per year [37]. Using these values for the branching ratios predicted in the SM, the number of expected events in the above mentioned regions are

$$\mathcal{N}\big(\Lambda_b \to \Lambda \mu^+ \mu^-\big) \approx \begin{cases} 3.0 \times 10^4, & \text{region (I)}, \\ 6 \times 10^3, & \text{region (II)}, \end{cases}$$

and

$$\mathcal{N}\big(\Lambda_b \to \Lambda \tau^+ \tau^-\big) \approx 10^3.$$

We see that although the number of expected events is one order of magnitude less than the corresponding $B \to K^* \ell^+ \ell^-$ decay, it has the potential of being quite detectable in future LHC-B and BTeV machines.

In conclusion, a systematic analysis of the rare $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay is presented. For the form factors describing the $\Lambda_b \to \Lambda$ transition we have used HQET predictions. The sensitivity of the branching ratio and of the lepton forward–backward asymmetry to the new Wilson coefficients is studied systematically. Analysis of the zero position of the lepton forward–backward asymmetry determines not only the magnitude but also the sign of the new Wilson coefficients for the $\Lambda_b \to \Lambda \mu^+ \mu^-$ decay. The sign of the forward–backward asymmetry for the $\Lambda_b \to \Lambda \tau^+ \tau^-$ decay can serve as a useful tool in determining the sign of the Wilson coefficients.

Appendix A. Definition of the form factors

As has already been noted, in describing the $\Lambda_b \to \Lambda$ transition, the following matrix elements

$$\langle \Lambda | \bar{s} \gamma_\mu (1 \mp \gamma_5) b | \Lambda_b \rangle, \qquad \langle \Lambda | \bar{s} \sigma_{\mu\nu} (1 \mp \gamma_5) b | \Lambda_b \rangle, \qquad \langle \Lambda | \bar{s} (1 \mp \gamma_5) b | \Lambda_b \rangle.$$

These matrix elements are generally parametrized in the following way (here we follow [23]

$$\langle \Lambda|\bar{s}\gamma_\mu b|\Lambda_b\rangle = \bar{u}_\Lambda\big[f_1\gamma_\mu + if_2\sigma_{\mu\nu}q^\nu + f_3 q_\mu\big]u_{\Lambda_b}, \tag{A.1}$$

$$\langle \Lambda|\bar{s}\gamma_\mu\gamma_5 b|\Lambda_b\rangle = \bar{u}_\Lambda\big[g_1\gamma_\mu\gamma_5 + ig_2\sigma_{\mu\nu}\gamma_5 q^\nu + g_3 q_\mu\gamma_5\big]u_{\Lambda_b}, \tag{A.2}$$

$$\langle \Lambda|\bar{s}\sigma_{\mu\nu} b|\Lambda_b\rangle = \bar{u}_\Lambda\Big[f_T\sigma_{\mu\nu} - if_T^V\big(\gamma_\mu q^\nu - \gamma_\nu q^\mu\big) - if_T^S\big(P_\mu q^\nu - P_\nu q^\mu\big)\Big]u_{\Lambda_b}, \tag{A.3}$$

$$\langle \Lambda|\bar{s}\sigma_{\mu\nu}\gamma_5 b|\Lambda_b\rangle = \bar{u}_\Lambda\Big[g_T\sigma_{\mu\nu} - ig_T^V\big(\gamma_\mu q^\nu - \gamma_\nu q^\mu\big) - ig_T^S\big(P_\mu q^\nu - P_\nu q^\mu\big)\Big]\gamma_5 u_{\Lambda_b}. \tag{A.4}$$

The form factors of the magnetic dipole operators are defined as

$$\begin{aligned}
&\langle \Lambda|\bar{s}i\sigma_{\mu\nu}q^\nu b|\Lambda_b\rangle = \bar{u}_\Lambda\big[f_1^T\gamma_\mu + if_2^T\sigma_{\mu\nu}q^\nu + f_3^T q_\mu\big]u_{\Lambda_b},\\
&\langle \Lambda|\bar{s}i\sigma_{\mu\nu}\gamma_5 q^\nu b|\Lambda_b\rangle = \bar{u}_\Lambda\big[g_1^T\gamma_\mu\gamma_5 + ig_2^T\sigma_{\mu\nu}\gamma_5 q^\nu + g_3^T q_\mu\gamma_5\big]u_{\Lambda_b}.
\end{aligned} \tag{A.5}$$

Multiplying (A.3) and (A.4) by iq^ν and comparing wit (A.5) and (A.6), respectively, one can easily obtain the following relations

$$\begin{aligned}
&f_2^T = f_T + f_T^S q^2,\\
&f_1^T = \big[f_T^V + f_T^S(m_{\Lambda_b} + m_\Lambda)\big]q^2 = -\frac{q^2}{m_{\Lambda_b} - m_\Lambda}f_3^T,\\
&g_2^T = g_T + g_T^S q^2,\\
&g_1^T = \big[g_T^V - g_T^S(m_{\Lambda_b} - m_\Lambda)\big]q^2 = \frac{q^2}{m_{\Lambda_b} + m_\Lambda}g_3^T.
\end{aligned} \tag{A.6}$$

The matrix element of the scalar (pseudoscalar) operators $\bar{s}b$ and $\bar{s}\gamma_5 b$ can be obtained from (A.1) and (A.2) by multiplying both sides to q^μ and using equation of motion. Neglecting the mass of the strange quark, we get

$$\langle \Lambda|\bar{s}b|\Lambda_b\rangle = \frac{1}{m_b}\bar{u}_\Lambda\big[f_1(m_{\Lambda_b} - m_\Lambda) + f_3 q^2\big]u_{\Lambda_b}, \tag{A.7}$$

$$\langle \Lambda|\bar{s}\gamma_5 b|\Lambda_b\rangle = \frac{1}{m_b}\bar{u}_\Lambda\big[g_1(m_{\Lambda_b} + m_\Lambda)\gamma_5 - g_3 q^2\gamma_5\big]u_{\Lambda_b}. \tag{A.8}$$

Using these definitions of the form factors and effective Hamiltonian in Eq. (1), we get the following forms of the functions A_i, B_i, D_i, E_i, N_j and H_j $(i = 1, 2, 3;\ j = 1, 2)$ entering the matrix element of the $\Lambda_b \to \Lambda\ell^+\ell^-$ decay:

$$\begin{aligned}
A_1 &= \frac{1}{q^2}\big(f_1^T - g_1^T\big)C_{SL} + \frac{1}{q^2}\big(f_1^T + g_1^T\big)C_{BR} + \frac{1}{2}(f_1 - g_1)\big(C_{LL}^{\text{tot}} + C_{LR}^{\text{tot}}\big)\\
&\quad + \frac{1}{2}(f_1 + g_1)(C_{RL} + C_{RR}),\\
A_2 &= A_1(1 \to 2), \qquad A_3 = A_1(1 \to 3),\\
B_1 &= A_1\big(g_1 \to -g_1;\ g_1^T \to -g_1^T\big), \qquad B_2 = B_1(1 \to 2), \qquad B_3 = B_1(1 \to 3),
\end{aligned}$$

$$
\begin{aligned}
&D_1 = \frac{1}{2}(C_{RR} - C_{RL})(f_1 + g_1) + \frac{1}{2}\big(C_{LR}^{\text{tot}} - C_{LL}^{\text{tot}}\big)(f_1 - g_1),\\
&D_2 = D_1(1 \to 2), \qquad D_3 = D_1(1 \to 3),\\
&E_1 = D_1(g_1 \to -g_1), \qquad E_2 = E_1(1 \to 2), \qquad E_3 = E_1(1 \to 3),\\
&N_1 = \frac{1}{m_b}\big(f_1(m_{\Lambda_b} + m_\Lambda) + f_3 q^2\big)(C_{LRLR} + C_{RLLR} + C_{LRRL} + C_{RLRL}),\\
&N_2 = N_1(C_{LRRL} \to -C_{LRRL};\ C_{RLRL} \to -C_{RLRL}),\\
&H_1 = \frac{1}{m_b}\big(g_1(m_{\Lambda_b} + m_\Lambda) - g_3 q^2\big)(C_{LRLR} - C_{RLLR} + C_{LRRL} - C_{RLRL}),\\
&H_2 = H_1\big(C_{LRRL} \to -C_{LRRL}; C_{RLRL} \to -C_{RLRL}\big).
\end{aligned} \tag{A.9}
$$

Appendix B. Double differential rate

The explicit form of the expressions $\mathcal{T}_0(s)$, $\mathcal{T}_1(s)$ and $\mathcal{T}_2(s)$ are as follows:

$$
\begin{aligned}
\mathcal{T}_0(s) = {} & -2048\lambda m_\ell^2 m_{\Lambda_b}^4 |C_T|^2 \operatorname{Re}\big[f_T^* f_T^S\big]\\
& + 384 m_\ell m_{\Lambda_b}^3 \Big\{\big(1+\sqrt{r}\big)\big(1-2\sqrt{r}+r-s\big)\operatorname{Re}\big[(A_1+B_1)^* C_T f_T\big]\\
& + 2\big(1-\sqrt{r}\big)\big(1+2\sqrt{r}+r-s\big)\operatorname{Re}\big[(A_1-B_1)^* C_{TE} f_T\big]\Big\}\\
& + 32 m_\ell^2 m_{\Lambda_b}^4 s(1+r-s)\big(|D_3|^2+|E_3|^2\big)\\
& + 4 m_{\Lambda_b}^4 s\big(1-2\sqrt{r}+r-s\big)\big(4m_\ell \operatorname{Re}\big[(D_3-E_3)^* H_2\big]+|H_2|^2\big)\\
& + 64 m_\ell^2 m_{\Lambda_b}^3 (1-r-s)\operatorname{Re}\big[D_1^* E_3 + D_3 E_1^*\big]\\
& + 256 m_\ell m_{\Lambda_b}^4 \big(1+2\sqrt{r}+r-s\big)\big(2-4\sqrt{r}+2r+s\big)\operatorname{Re}\big[A_2^* C_{TE} f_T\big]\\
& - 128\lambda m_\ell m_{\Lambda_b}^5 \Big\{\big(1+\sqrt{r}\big)\big(\operatorname{Re}\big[(A_1+B_1)^* C_T f_T^S\big]\\
& - 16 m_\ell |C_T|^2 \operatorname{Re}\big[f_T^{S*} f_T^V\big]\big) - m_{\Lambda_b} s \operatorname{Re}\big[(A_2+B_2)^* C_T f_T^S\big]\Big\}\\
& + 64 m_{\Lambda_b}^2 \sqrt{r}\big(6m_\ell^2 - m_{\Lambda_b}^2 s\big)\operatorname{Re}\big[D_1^* E_1\big]\\
& - 128 m_\ell m_{\Lambda_b}^4 \big[(1-r)^2 + \big(1-6\sqrt{r}+r\big)s - 2s^2\big]\operatorname{Re}\big[(A_1+B_1)^* C_T f_T^V\big]\\
& + 64 m_\ell^2 m_{\Lambda_b}^3 \sqrt{r}\Big(2m_{\Lambda_b} s \operatorname{Re}\big[D_3^* E_3\big] + (1-r+s)\operatorname{Re}\big[D_1^* D_3 + E_1^* E_3\big]\Big)\\
& - 128 m_\ell m_{\Lambda_b}^4 \Big\{2\big(1+2\sqrt{r}+r-s\big)\big(2-4\sqrt{r}+2r+s\big)\operatorname{Re}\big[B_2^* C_{TE} f_T\big]\\
& + \big(1-2\sqrt{r}+r-s\big)\big(2+4\sqrt{r}+2r+s\big)\operatorname{Re}\big[B_2^* C_T f_T\big]\Big\}\\
& + 32 m_{\Lambda_b}^2 \big(2m_\ell^2 + m_{\Lambda_b}^2 s\big)\Big\{(1-r+s) m_{\Lambda_b}\sqrt{r}\operatorname{Re}\big[A_1^* A_2 + B_1^* B_2\big]\\
& - m_{\Lambda_b}(1-r-s)\operatorname{Re}\big[A_1^* B_2 + A_2^* B_1\big]\\
& - 2\sqrt{r}\Big(\operatorname{Re}\big[A_1^* B_1\big] + m_{\Lambda_b}^2 s \operatorname{Re}\big[A_2^* B_2\big]\Big)\Big\}\\
& + 8 m_{\Lambda_b}^2 \big\{4m_\ell^2(1+r-s) + m_{\Lambda_b}^2\big[(1-r)^2 - s^2\big]\big\}\big(|A_1|^2+|B_1|^2\big)
\end{aligned}
$$

$$
\begin{aligned}
&+8m_{\Lambda_b}^4\Big\{4m_\ell^2\big[\lambda+(1+r-s)s\big]+m_{\Lambda_b}^2 s\big[(1-r)^2-s^2\big]\Big\}\big(|A_2|^2+|B_2|^2\big)\\
&-8m_{\Lambda_b}^2\Big\{4m_\ell^2(1+r-s)-m_{\Lambda_b}^2\big[(1-r)^2-s^2\big]\Big\}\big(|D_1|^2+|E_1|^2\big)\\
&+512m_{\Lambda_b}^2|f_T|^2\Big\{\big[2m_\ell^2\big(1-6\sqrt{r}+r-s\big)+m_{\Lambda_b}^2\big[\lambda+(1+r-s)s\big]\big]|C_T|^2\\
&+4\big[2m_\ell^2\big(1+6\sqrt{r}+r-s\big)+m_{\Lambda_b}^2\big[\lambda+(1+r-s)s\big]\big]|C_{TE}|^2\Big\}\\
&+8m_{\Lambda_b}^5 s v^2\Big\{-8m_{\Lambda_b}s\sqrt{r}\,\mathrm{Re}\big[D_2^*E_2\big]+4(1-r+s)\sqrt{r}\,\mathrm{Re}\big[D_1^*D_2+E_1^*E_2\big]\\
&-4(1-r-s)\,\mathrm{Re}\big[D_1^*E_2+D_2^*E_1\big]+m_{\Lambda_b}\big[(1-r)^2-s^2\big]\big(|D_2|^2+|E_2|^2\big)\Big\}\\
&+\big(1+2\sqrt{r}+r-s\big)\Big\{1024\lambda m_\ell^2 m_{\Lambda_b}^6|C_T|^2\big|f_T^S\big|^2\\
&+16m_\ell m_{\Lambda_b}^3\big(1-\sqrt{r}\big)\,\mathrm{Re}\big[(D_1+E_1)^*N_2\big]\\
&+4m_{\Lambda_b}^4 s|N_2|^2+4m_{\Lambda_b}^4 s\big(4m_\ell\,\mathrm{Re}\big[(D_3+E_3)^*N_2\big]+v^2|N_1|^2\big)\Big\}\\
&+\big(1-2\sqrt{r}+r-s\big)\Big\{-128m_\ell m_{\Lambda_b}^4\big(2+4\sqrt{r}+2r+s\big)\,\mathrm{Re}\big[A_2^*C_T f_T\big]\\
&+512m_{\Lambda_b}^3\big(1+\sqrt{r}\big)\,\mathrm{Re}\big[f_T^* f_T^V\big]\Big[8m_\ell^2\big(2|C_{TE}|^2-|C_T|^2\big)\\
&-m_{\Lambda_b}^2 s\big(4|C_{TE}|^2+|C_T|^2\big)\Big]-16m_\ell m_{\Lambda_b}^3\big(1+\sqrt{r}\big)\,\mathrm{Re}\big[(D_1-E_1)^*H_2\big]\\
&-24m_{\Lambda_b}^2 s\,\mathrm{Re}\big[(A_2+B_2)^*C_T f_T^V\big]+4m_{\Lambda_b}^4 s v^2|H_1|^2\\
&+256m_{\Lambda_b}^4\big|f_T^V\big|^2\big(\big[m_{\Lambda_b}^2 s^2+4m_\ell^2\big(1+2\sqrt{r}+r+s\big)\big]|C_T|^2\\
&+4m_{\Lambda_b}^2 s^2 v^2|C_{TE}|^2\big)\Big\},
\end{aligned}
\tag{B.1}
$$

$$
\begin{aligned}
\mathcal{T}_1(s)={}&-16m_\ell m_{\Lambda_b}^3 v\sqrt{\lambda}\Big\{\big(1-\sqrt{r}\big)\,\mathrm{Re}\big[(A_1-B_1)^*H_1\big]\\
&-\big(1+\sqrt{r}\big)\,\mathrm{Re}\big[(A_1+B_1)^*N_1\big]\Big\}\\
&-384m_\ell m_{\Lambda_b}^3 v\sqrt{\lambda}\Big\{\big(1+\sqrt{r}\big)\,\mathrm{Re}\big[(D_1-E_1)^*C_T f_T\big]\\
&+2\big(1-\sqrt{r}\big)\,\mathrm{Re}\big[(D_1+E_1)^*C_{TE} f_T\big]\Big\}\\
&-256m_\ell m_{\Lambda_b}^4 v\sqrt{\lambda}(1-r)\big(\mathrm{Re}\big[(D_2-E_2)^*C_T f_T\big]\\
&-2\,\mathrm{Re}\big[(D_2+E_2)^*C_{TE} f_T\big]\big)\\
&+256m_\ell m_{\Lambda_b}^5 v\sqrt{\lambda}\big(1-\sqrt{r}\big)\big(1+2\sqrt{r}+r-s\big)\big(\mathrm{Re}\big[(D_1+E_1)^*C_{TE} f_T^S\big]\big)\\
&+128m_\ell m_{\Lambda_b}^4 s v\sqrt{\lambda}\Big(\mathrm{Re}\big[(C_T-2C_{TE})^*D_3 f_T^*\big]\\
&-\mathrm{Re}\big[(C_T+2C_{TE})^*E_3 f_T^*\big]\Big)-16m_{\Lambda_b}^4 s v\sqrt{\lambda}\Big\{2\,\mathrm{Re}\big[A_1^*D_1\big]\\
&-2\,\mathrm{Re}\big[B_1^*E_1\big]-4\,\mathrm{Re}\big[(N_1+H_2)^*C_T f_T\big]+8\,\mathrm{Re}\big[(N_2+H_1)^*C_{TE} f_T\big]\\
&+m_\ell\,\mathrm{Re}\big[(A_2+B_2)^*N_1\big]\Big\}-16m_{\Lambda_b}^4 s v\sqrt{\lambda}\big(m_\ell\,\mathrm{Re}\big[(A_2-B_2)^*H_1\big]
\end{aligned}
$$

$$
\begin{aligned}
&+2m_{\Lambda_b}\,\mathrm{Re}\big[B_1^* D_2 - B_2^* D_1 + A_2^* E_1 - A_1^* E_2\big]\big) \\
&+256 m_\ell m_{\Lambda_b}^5 s v\sqrt{\lambda}\big(1-\sqrt{r}\big)\,\mathrm{Re}\big[(D_2-E_2)^* C_T f_T^V\big] \\
&+64 m_{\Lambda_b}^5 s v\sqrt{\lambda}\big(1+\sqrt{r}\big)\Big(-\mathrm{Re}\big[N_1^* C_T f_T^V\big] + 2\,\mathrm{Re}\big[N_2^* C_{TE} f_T^V\big] \\
&+4m_\ell\,\mathrm{Re}\big[(D_3+E_3)^* C_{TE} f_T^V\big]\Big) \\
&+32 m_{\Lambda_b}^6 s v\sqrt{\lambda}(1-r)\,\mathrm{Re}\big[A_2^* D_2 - B_2^* E_2\big] \\
&+32 m_{\Lambda_b}^5 s v\sqrt{\lambda}\,\sqrt{r}\,\mathrm{Re}\big[A_2^* D_1 + A_1^* D_2 - B_2^* E_1 - B_1^* E_2\big] \\
&+64 m_{\Lambda_b}^6 s v\sqrt{\lambda}\big(1+2\sqrt{r}+r-s\big)\Big(-\mathrm{Re}\big[N_1^* C_T f_T^S\big] + 2\,\mathrm{Re}\big[N_2^* C_{TE} f_T^S\big] \\
&+4m_\ell\,\mathrm{Re}\big[(D_3+E_3)^* C_{TE} f_T^S\big]\Big) \\
&+256 m_\ell m_{\Lambda_b}^4 v\sqrt{\lambda}\Big\{(1-r)\,\mathrm{Re}\big[(D_1+E_1)^* C_{TE} f_T^V\big] \\
&+s\,\mathrm{Re}\big[(D_1-E_1)^* C_T f_T^V\big]\Big\},
\end{aligned}
\tag{B.2}
$$

$$
\begin{aligned}
\mathcal{T}_2(s) = {}&-8m_{\Lambda_b}^4 v^2\lambda\big(|A_1|^2+|B_1|^2+|D_1|^2+|E_1|^2\big) \\
&-512 m_{\Lambda_b}^4 v^2\lambda\big[\big(4|C_{TE}|^2+|C_T|^2\big)|f_T|^2\big] \\
&+8m_{\Lambda_b}^6 s v^2\lambda\big(|A_2|^2+|B_2|^2+|D_2|^2+|E_2|^2\big) \\
&-256 m_{\Lambda_b}^6 s v^2\lambda\big(4|C_{TE}|^2+|C_T|^2\big)\Big\{2\,\mathrm{Re}\big[f_T^* f_T^S\big] \\
&-\big|f_T^V + m_{\Lambda_b}\big(1+\sqrt{r}\big)f_T^S\big|^2 + m_{\Lambda_b}^2 s\big|f_T^S\big|^2\Big\}.
\end{aligned}
\tag{B.3}
$$

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