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Low Complexity Keystone Transform and Radon Fourier Transform Utilizing Chirp-Z Transform

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ABSTRACT Keystone Transform (KT) and Radon Fourier Transform (RFT) are two popular methods proposed to overcome range migration in radars. A major concern in these methods is the computational complexity for real time operations. In this paper, a low complexity implementation of recurrent chirp-z transform (CZT) is offered in order to be employed in fast KT with no loss in performance. Additionally, a novel RFT implementation utilizing recurrent CZT is proposed to take advantage of the fast execution of repeated CZT. A mathematical analysis and simulation results are presented to show the performance and efficiency of the proposed techniques.

INDEX TERMS Keystone transform, Radon Fourier transform, range migration, chirp-z transform.

I. INTRODUCTION

DETECTION of high speed targets with low radar cross section is one of the significant problems and it is taking an increasing attention in the field of radar signal processing. It is well known that increasing signal-to-noise ratio (SNR) by means of long time coherent integration improves radar detection performance. During long coherent processing interval, a high speed target will move across multiple range cells especially for high range resolution radars due to narrower range cells. Therefore, it will be difficult to integrate multiple pulses coherently due to range migration effect and consequently detection performance will degrade.

Numerous target detection algorithms [1]–[24] have been developed in order to eliminate the range migration effect in the last decades. These algorithms can be mainly categorized into two types: noncoherent integration and coherent integration. Radon Transform [1]–[3] , Hough Transform [4]–[6] and the track-before-detect technique [7], [8] are typical noncoherent integration methods. The noncoherent integration methods have poor detection performance under low signal-to-noise ratio (SNR) environment since they do not compensate the phase fluctuation. With regard to the coherent integration detection, there are typically two kinds of methods: non-parametric search and parametric search. The non-parametric search methods include sequence reverse transform (SRT) [9], adjacent cross correlation function (ACCF) [10], [11] and scaled inverse Fourier transform (SCIFT) [12]. These nonparametric search methods have relatively low computational complexity compared to the parametric search methods; however, they demand high SNR input and have degraded detection performance for target echoes with low SNR. As for the parametric search methods employing coherent integration detection, Keystone Transform (KT) [13]–[18] and Radon Fourier Transform (RFT) [19]–[21] are typical algorithms. KT can eliminate range migration by rescaling the slow time axis for each range frequency without considering target's velocity information. Nevertheless, KT needs interpolation operations and an extensive search to determine and eliminate the Doppler ambiguity factor. Zhao et al. [22] proposed to implement KT with lower computational complexity by employing Chirp-Z transform (CZT). However, this implementation still suffers from high complexity because it requires repetitive execution of CZT for each potential Doppler ambiguity factor. RFT can correct range migration by jointly searching along the range and the velocity domains, but its implementation is computationally burdensome due to the two-dimensional searching. Yu et al. [21] proposed a method using CZT for computationally efficient implementation of RFT. Even though KT and RFT have faster implementations utilizing CZT, they can still be improved for real-time processing.

CZT is an efficient algorithm proposed by Rabiner [25] to evaluate z-transform on a spiral contour in the z-plane utilizing Fast Fourier Transform (FFT) operations. In some applications, CZT might be required to be executed repeatedly bringing large computational load as in the fast KT implementation [22]. As we will propose in this paper, RFT can similarly be modified to execute CZT recurrently.

In this work, a fast method is proposed to lower the computational load of performing CZT repeatedly with different starting points and the same angular spacing on the spiral contour. Thus, the proposed technique enables not only range walk compensation methods like KT and RFT to be performed more efficiently but also other applications requiring recurrent CZT executions. We demonstrate by mathematical analysis that the proposed method is equivalent to the existing CZT implementation. Consequently, proposed KT and RFT implementations have lower computational cost without sacrificing coherent integration performance compared to the conventional implementations. Eventually, a simulation result is presented to depict that the proposed method can acquire the same integration performance as the existing algorithms.

The remaining of the paper is organized as follows. In section 2, we establish the signal model of a rectilinearly moving target and briefly summarize the CZT, KT and RFT algorithms. In section 3, the proposed method is described. Computational complexity of the algorithms has been analyzed in section 4. We provide simulation results in section 5. Finally, conclusions are drawn in section 6.

II. SIGNAL MODEL AND BRIEF REVIEW OF CZT, KT AND RFT

A. SIGNAL MODEL AND PROBLEM STATEMENT

Consider a radar that transmits N pulses during the coherent processing interval. The received signal at the n^{th} pulse after down conversion, can be written as

$$s_{Rx}(\tilde{t}, n) \tilde{=} p\big[\tilde{t} - \frac{2(R_0 + vnT_r)}{c}\big]\exp\big[\frac{-j4\pi f_c(R_0 + vnT_r)}{c}\big] \quad (1)$$

where $\tilde{t}$ is fast time, n is pulse number from 0 to N-1, f_c is the carrier frequency, $p(t)$ is an arbitrary envelope signal, c is the light velocity, v denotes the radial velocity of the target receding from the radar, T_r is the pulse repetition interval (PRI), and R_0 is the range location of the target when the first pulse was transmitted, i.e., at $n = 0$. Radial velocity between the radar and the target can be represented as follows

$$v = Fv_{blind} + v_{res}, \quad (2)$$

where Doppler ambiguity factor is $F = round(v/v_{blind})$ [1], blind speed is defined as $v_{blind} = \frac{c}{2f_cT_r}$, v_{res} is the residual velocity with $v_{res} < \frac{v_{blind}}{2}$.

After matched filtering, the signal in the slow time - range frequency domain can be expressed as

$$Y(f, n) = |P(f)|^2\exp\big[-j4\pi(f + f_c)\frac{(R_0 + vnT_r)}{c}\big] \quad (3)$$

[1] $round(.)$ is the function which returns the nearest integer.

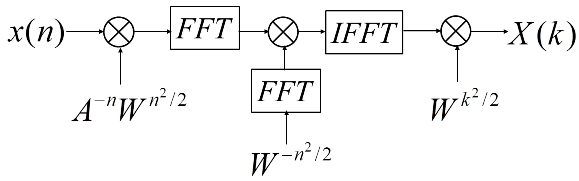

FIGURE 1. Standard implementation of CZT using FFT operations.

where $P(f)$ is the Fourier transform of $p(t)$. The coupling between the range frequency f and slow time index n in the exponential term in (3) causes the range migration problem.

B. REVIEW OF CZT

CZT employs the idea of expressing z-transform on a spiral contour as a discrete convolution and thus FFT operations can be used to compute the z-transform efficiently. K-point CZT of the N points input signal $x(n)$ can be expressed as follows

$$X(k) = \sum_{n=0}^{N-1} x(n)A^{-n}W^{nk} \qquad k = 0, 1, \ldots, K-1 \quad (4)$$

where A is the complex starting point, and W is a complex scalar describing the complex ratio between points on the contour in the z-plane on which the z-transform is computed. The CZT can be implemented as a circular convolution or equivalently by employing FFT operations as shown in Fig. 1.

C. REVIEW OF KT

The traditional KT [16] [17] is usually implemented by sinc interpolation to get rid of range migration as given below.

$$Y_{KT}(f, m) = \sum_{n=0}^{N-1} Y(f, n)\text{sinc}[n\alpha_f - m], \quad m = 0, 1, \ldots, N-1 \quad (5)$$

where $\alpha_f = (f_c + f)/f_c$.

Doppler ambiguity factor F can be corrected by multiplying (5) by the correction term which is defined as

$$C_{KT}(f, n, F) = \exp[-\frac{j2\pi Fn}{\alpha_f}]. \quad (6)$$

Then, KT output in range frequency – slow time domain can be expressed as follows

$$Y_{KT}(f, m, F) = \sum_{n=0}^{N-1} Y_{c,KT}(f, n, F)\text{sinc}\Big[n\alpha_f - m\Big] \quad (7)$$

where

$$Y_{c,KT}(f, n, F) = Y(f, n)C_{KT}(f, n, F). \quad (8)$$

If the ambiguity factor is not known for a given target, it can be estimated by choosing the highest-amplitude coherent integration output after Doppler-ambiguity correction for all possible values of ambiguity factor F.

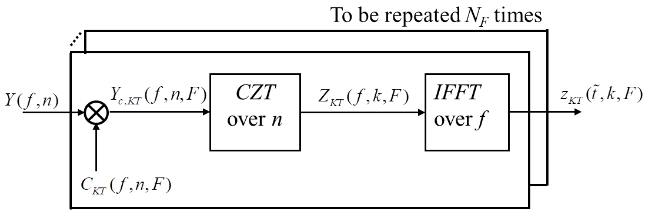

FIGURE 2. Implementation of KT employing repeated CZT operations.

Due to high computation complexity of $sinc$ interpolation, an efficient implementation of KT employing CZT was proposed [22].

In order to perform KT as given in [22], we correct Doppler ambiguity of matched filter output $Y(f,n)$ and take CZT over slow time.

$$Z_{KT}(f,k,F) = \underset{n}{CZT}\Big[Y_{c,KT}(f,n,F)\Big] \tag{9}$$

$$Z_{KT}(f,k,F) = \sum_{n=0}^{N-1} Y_{c,KT}(f,n,F)A^{-n}W_f^{nk} \tag{10}$$

where k is the Doppler frequency index. The parameters of CZT are set as $A = 1$, and $W_f = \exp(-j\frac{2\pi}{N}\alpha_f)$.

Coherent integration output can be obtained by the following expression

$$z_{KT}(\tilde{t},k,F) = \underset{f}{IFFT}\Big[Z_{KT}(f,k,F)\Big]. \tag{11}$$

Fig. 2 shows a block diagram of KT implementation employing N_F times repeated Doppler ambiguity compensation, CZT and coherent integration operations, where N_F is the number of all possible values of F.

D. REVIEW OF RFT

RFT searches jointly along velocity and range domains [19]–[21]. Coherent integration is executed for each possible value of searching velocity $\hat{v}$ as follows.

$$z(\tilde{t},\hat{v}) = \underset{f}{IFFT}\Big[Z(f,\hat{v})\Big] \tag{12}$$

where $Z(f,\hat{v})$ is defined by the following expression

$$Z(f,\hat{v}) = \sum_n Y(f,n)\exp\left[j2\pi(f+f_c)\frac{2\hat{v}}{c}T_r n\right] \tag{13}$$

Expression (12) yields maximum coherent integration amplitude when the searching velocity is equal to the true velocity, i.e. $\hat{v} = v$.

To reduce complexity, it has been proposed to search by using CZT [21] . Velocity can be searched by finding the integer k such that $\hat{v} = k\Delta v$ where Δv is the velocity search resolution. CZT of $Y(f,n)$ can be expressed as

$$Z(f,k) = \underset{n}{CZT}\Big[Y(f,n)\Big] = \sum_n Y(f,n)A^{-n}W_f^{nk}. \tag{14}$$

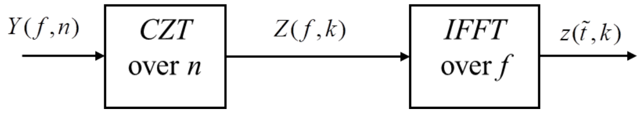

FIGURE 3. RFT detector employing CZT.

Thus, RFT employs CZT for searching velocity by setting CZT parameters as $W_f = \exp\left[j2\pi(f+f_c)\frac{2\Delta v}{c}T_r\right]$ and $A = 1$.

Coherent integration output can be obtained by taking IFFT of CZT output along f:

$$z(\tilde{t},k) = \underset{f}{IFFT}\Big[Z(f,k)\Big] \tag{15}$$

The block diagram corresponding to the RFT detector is depicted in Fig. 3. Coherent integration is performed after executing CZT of the matched filter output.

III. LOW COMPLEXITY KT AND RFT UTILIZING CZT

In the following subsections, we propose to reduce computational load of executing repeated CZT and benefit from it in KT and RFT implementation. For this purpose, KT and RFT implementations need to be expressed as procedures which employ repeated CZT. KT has already been presented in this form as depicted in Fig. 2. On the other hand, existing RFT implementations do not utilize repeated CZT. Consequently, a modification of standard RFT implementation is required so as to take advantage of the fast execution of recurrent CZT. In section III-A, we offer a modified RFT implementation which references the same formulation as KT does in Fig. 2. Afterwards, an efficient method to execute repeated CZT is offered in III-B.

A. NOVEL RFT IMPLEMENTATION EMPLOYING CZT RECURRENTLY

Matched filter output in the slow time - range frequency domain can be reexpressed by substituting (2) in (3):

$$Y(f,n,F) = P(f)^2\exp\left[-j2\pi(f+f_c)\frac{2R_0}{c}\right] \exp\left[-j2\pi(f+f_c)\frac{2v_{res}}{c}T_r n\right]\exp\left[-j2\pi F\alpha_f n\right]. \tag{16}$$

It is clear that Doppler ambiguity can be compensated using following expression

$$Y_{c,RFT}(f,n,F) = Y(f,n,F)C_{RFT}(f,n,F), \tag{17}$$

where Doppler ambiguity compensation factor can be defined as

$$C_{RFT}(f,n,F) = \exp\Big[j2\pi F\alpha_f n\Big]. \tag{18}$$

Then, Doppler ambiguity corrected output can be represented as

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$$Y_{c,RFT}(f,n,F) = P(f)^2 \exp\Big[-j2\pi(f+f_c)\frac{2R_0}{c}\Big] \exp\Big[-j2\pi(f+f_c)\frac{2v_{res}}{c}T_r n\Big]. \quad (19)$$

Instead of searching all possible values of velocity, we can first compensate the ambiguity and search only residual velocity if the Doppler ambiguity factor F is known. Residual velocity index k can be searched by employing CZT:

$$Z_{RFT}(f,k,F) = \underset{n}{CZT}\Big[Y_{c,RFT}(f,n,F)\Big], \quad (20)$$

where CZT parameters are set as given in II-D. Coherent integration of the CZT output given in (20) has maximum amplitude when the searching residual velocity is equal to the true residual velocity, i.e. $v_{res} = k\Delta v$.

When the ambiguity factor is not known, it can be estimated by finding coherent integration outputs for all possible values of the ambiguity factor F, and choosing the one that produces the highest-magnitude peaks. Coherent integration output can be obtained by the following expression

$$z_{RFT}(\tilde{t},k,F) = \underset{f}{IFFT}\Big[Z_{RFT}(f,k,F)\Big], \quad (21)$$

where k is the search velocity index.

In summary, both KT and RFT implementations have been represented by single formulation to utilize recurrent executions of CZT. It can be noticed that expressions (8), (9), and (11) given for KT are equivalent to (17), (20), and (21) given for RFT. Similarly, Fig. 2 can be used to represent the block diagram to implement RFT employing N_F times repeated CZT operations. Doppler ambiguity compensation factors and CZT parameters are the only differences between two implementations.

B. PROPOSED METHOD FOR EFFICIENT IMPLEMENTATION OF RECURRENT CZT

As shown in previous sections, KT and RFT can be implemented by repeating Doppler ambiguity correction and CZT operations respectively for all possible values of F. We can employ two tricks in order to reduce computational complexity of these implementations:

1) To eliminate the multiplication for Doppler ambiguity correction, we can set complex starting point of CZT as a function of f and F. We take $A_f(F) = \exp\big(-j2\pi F\alpha_f\big)$ for RFT and $A_f(F) = \exp\big(j2\pi F/\alpha_f\big)$ for KT.
2) Implementation of CZT can be modified such that repeating CZT for different values of parameter $A_f(F)$ requires less complexity as explained below.

CZT of the matched filter output $Y(f,n)$ can be evaluated for each value of F repeatedly as:

$$Z(f,k,F) = \sum_n Y(f,n)[A_f(F)]^{-n} W_f^{nk}. \quad (22)$$

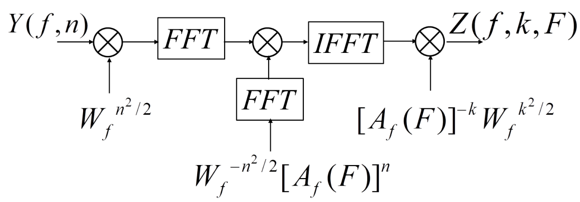

FIGURE 4. Modified implementation of CZT using FFT operations.

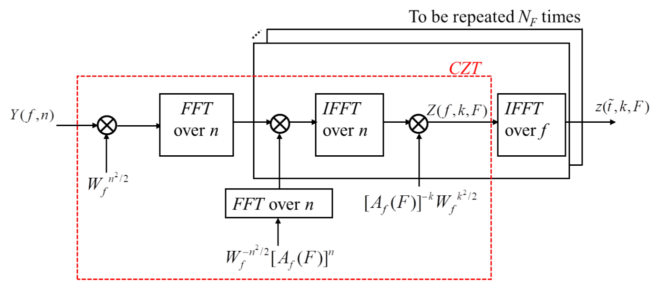

FIGURE 5. Proposed implementation of KT and RFT employing recurrent CZT operations.

For the second trick, we can substitute following expressions in (22):

$$[A_f(F)]^{-n} = [A_f(F)]^{k-n}[A_f(F)]^{-k} \quad (23)$$

$$W_f^{nk} = W_f^{\frac{n^2+k^2-(k-n)^2}{2}}. \quad (24)$$

$$Z(f,k,F) = [A_f(F)]^{-k} W_f^{k^2/2} \sum_n W_f^{n^2/2} x(n) W_f^{-(k-n)^2/2} [A_f(F)]^{k-n} \quad (25)$$

$Z(f,k,F)$ can be represented as a convolution:

$$Z(f,k,F) = [A_f(F)]^{-k} W_f^{k^2/2} \sum_n u_f(n) v_f(k-n,F)$$

$$Z(f,k,F) = [A_f(F)]^{-k} W_f^{k^2/2} \big[u_f(n) * v_f(n,F)\big] \quad (26)$$

where $u_f(n) = W_f^{n^2/2} Y(f,n)$, $v_f(n,F) = W_f^{-n^2/2}[A_f(F)]^n$ and $*$ denotes convolution operation.

Convolution can be performed using FFT and IFFT operations as follows.

$$Z(f,k,F) = [A_f(F)]^{-k} W_f^{k^2/2} \underset{n}{IFFT}\Big[\underset{n}{FFT}[u_f(n)] \underset{n}{FFT}[v_f(n,F)]\Big] \quad (27)$$

CZT can be implemented by employing (27). Block diagram of the modified CZT implementation is depicted in Fig. 4.

After applying aforesaid two maneuvers, a modified implementation of KT and RFT employing recurrent CZT operations is obtained as shown in Fig. 5. Since the prestored values of FFT of $W_f^{-n^2/2}[A_f(F)]^n$ can be used, only one IFFT will be required to be repeated. On the other hand, one FFT and one IFFT must be repeated for each value of F

in KT and RFT implementations employing standard CZT depicted in Fig. 2.

IV. COMPUTATIONAL COMPLEXITY ANALYSIS

Table 1-4 show steps of modified and standard KT and RFT implementation methods and number of complex multiplications required for each step, where M is the number of range bins in PRI, N is the number of pulses to be integrated, N_v is the number of searched velocities, N_F is the total number of ambiguity factors and K is the number of search residual velocities. K is assumed to be an integer such that $N_v = KN_F$. For simplicity of analysis, we define $J = N_v + N$, and $T = K + N$.

TABLE 1. Computational complexity analysis of standard KT implementation utilizing CZT

Step	Processing	Number of complex multiplications
Obtaining $Y(f,n)$	MN-point complex multiplication and N groups of M-point FFT	$MN + \frac{MN}{2} logM$
Ambiguity compensation	N_F groups of MN-point complex multiplication	$N_F MN$
CZT over n	N_F repetition of M groups of N-point CZT of N-point input	$2N_F MNlog2N$
IFFT over f	N_F repetition of N groups of M-point IFFT	$N_F \frac{MN}{2} logM$

TABLE 2. Computational complexity analysis of proposed KT implementation utilizing CZT

Step	Processing	Number of complex multiplications
Obtaining $Y(f,n)$	MN-point complex multiplication and N groups of M-point FFT	$MN + \frac{MN}{2} logM$
CZT over n	N_F repetition of M groups of N-point modified CZT of N-point input	$(N_F + 1)MNlog2N$
IFFT over f	N_F repetition of N groups of M-point IFFT	$N_F \frac{MN}{2} logM$

TABLE 3. Computational complexity analysis of standard RFT implementation utilizing CZT

Step	Processing	Number of complex multiplications
Obtaining $Y(f,n)$	MN-point complex multiplication and N groups of M-point FFT	$MN + \frac{MN}{2} logM$
CZT over n	M groups of N_v -point CZT of N-point input	$MJlogJ$
IFFT over f	N groups of M-point IFFT	$\frac{MN}{2} logM$

Table 1 and Table 3 demonstrate step-by-step computational complexities of standard KT with CZT and standard RFT with CZT, while Table 2 and Table 4 demonstrate step-by-step computational complexities of proposed KT with CZT and proposed RFT with CZT, respectively.

TABLE 4. Computational complexity analysis of proposed RFT implementation utilizing CZT

Step	Processing	Number of complex multiplications
Obtaining $Y(f,n)$	MN-point complex multiplication and N groups of M-point FFT	$MN + \frac{MN}{2} logM$
CZT over n	N_F repetition of M groups of K-point modified CZT of N-point input	$M\frac{(N_F+1)}{2} TlogT$
IFFT over f	N_F repetition of N groups of M-point IFFT	$N_F \frac{MN}{2} logM$

The total number of complex multiplications is given in Table 5 for modified and standard KT and RFT implementations.

TABLE 5. Number of complex multiplications for algorithms

Algorithm	Number of complex multiplications
Standard KT	$MN\left[1 + \frac{(N_F+1)}{2} \log M + N_F(1 + 2\log 2N)\right]$
Proposed KT	$MN\left[1 + \frac{(N_F+1)}{2} \log M + (N_F + 1)log2N\right]$
Standard RFT	$M\left[N + (N \log M + J \log J)\right]$
Proposed RFT	$M\left[N + \frac{(N_F+1)}{2}(N \log M + T \log T)\right]$

The computational complexity advantage of the proposed KT and RFT implementations compared to standard implementations [21] [22] over N_F are given in Fig. 6 where we set $M = 512$, $N = 4$, $N_v = 1024$. The proposed implementation of KT reduces the number of complex multiplications for moderate and high values of N_F by a factor of 1.5. The proposed RFT implementation requires less than half of the number of complex multiplications as required by the traditional one for moderate values of N_F. On the other hand, the proposed RFT implementation is unfavorable for very high values of N_F.

In addition, the effect of N on the computational complexity has been examined. The proposed KT implementation is more advantageous as N increases, whereas the proposed RFT implementation becomes adverse.

It is also observed that the proposed RFT is usually computationally more efficient when N_F is close to K.

The computational complexities of the above four methods as a function of N_v are given in Fig. 7 for $M = 512$, $N = 4$, $N_F = K$. The figure shows that the proposed RFT and KT implementations demand lower computational costs than the conventional ones.

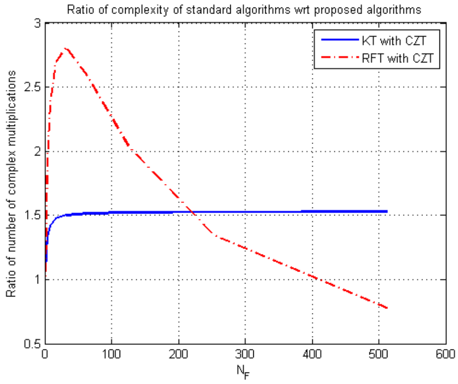

FIGURE 6. Computational complexity advantage of proposed KT and RFT implementations compared to standard implementations over N_F.

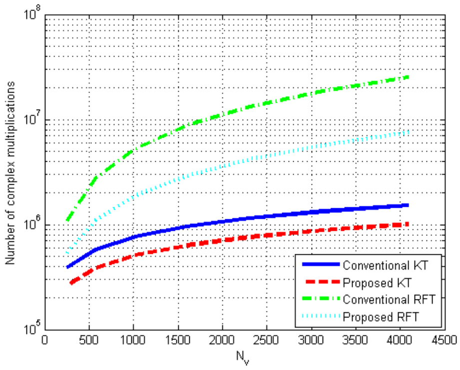

FIGURE 7. Number of complex multiplications required by implementations as a function of N_v when $N_F = K$.

V. SIMULATION RESULTS

An example is presented to show the integration performance of the proposed method is the same as the existing method. Radar parameters are set as follows: Carrier frequency f_c = 3 GHz, bandwidth B = 10 MHz, sampling frequency is 10 MHz, pulse width is 100 μs, pulse repetition interval T_r = 0.5 ms.

Other simulation parameters are given as follows: R_0=60km, v=500m/s, N_v=800, N_F=8, K=100, M=5000, the single-sample SNR after matched filtering is 0dB. Fig. 8 shows coherent integration result of proposed RFT implementations. The maximum differences between normalized coherent integration output results of standard and proposed implementations are 1.08×10^{-12} for RFT and 1.65×10^{-12} for KT. That is, they are identical within the numerical precision tolerances offered by the computers used.

VI. CONCLUSION

In this paper, a computationally efficient implementation was proposed to perform repetitive application of CZT which are characterized by different starting points and with the same angular spacing on the spiral contour. It is shown that the computational complexity of KT can be reduced without any performance degradation utilizing this proposed technique.

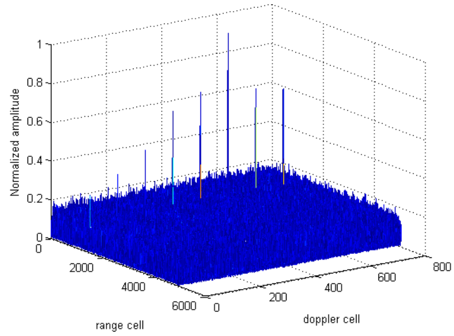

FIGURE 8. Coherent integration result of proposed RFT implementation.

Furthermore, a novel implementation of RFT employing repetitive CZT was proposed. Consequently, it was shown that the novel RFT implementation utilizing the proposed recurrent CZT method demands less computation cost without sacrificing integration performance in certain cases. Finally, a numerical simulation is provided to show that the integration performance is unchanged.

Both KT and RFT implementations proposed in this paper are more favorable when there is Doppler ambiguity. The proposed KT implementation is more advantageous for higher number of pulses to be integrated and higher number of possible ambiguity factors. The proposed RFT implementation is computationally efficient when a low number of pulses are integrated which is commonly encountered for low PRF mode due to limited integration time and the searching ambiguity factor number N_F is not too high.

Future studies will explore further reducing the computational complexity of the proposed algorithm without repeating CZT for Doppler ambiguity search.

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