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Fdm solution of mhd flow in a rectangular duct with slipping and partly insulated partly conducting side walls

Sinem Arslan¹ and Münevver Tezer-Sezgin²

^{1,2}Middle East Technical University, Department of Mathematics, Üniversiteler Mahallesi, Dumlupınar Bulvarı No:1 06800 Çankaya Ankara, Turkey

E-mail: ¹arsinem@metu.edu.tr

E-mail: ²munt@metu.edu.tr

Abstract. The magnetohydrodynamic (MHD) flow of an electrically conducting fluid is considered in a long channel (pipe) of rectangular cross-section in which the fluid is driven by a pressure gradient and the flow is steady, laminar, fully-developed. The flow is influenced by an external uniform magnetic field applied perpendicular to the channel-axis. Thus, the velocity field $\vec{V} = (0, 0, V)$ and the magnetic field $\vec{B} = (0, B_0, B)$ have only channel-axis components V and B on the cross-section of the channel which is a rectangular duct. The finite difference method (FDM) is used for solving the governing equations with the boundary conditions which include both the slipping and variably conducting side walls. The well-known characteristics of the MHD flow including the slipping velocity are observed. Thus, the FDM enables one to depict the effects of Hartmann number, conductivity and slip parameters on the behavior of both the velocity of the fluid and the induced magnetic field at a small expense.

1. Introduction

Magnetohydrodynamics is a discipline which is arisen from the main results of fluid mechanics and electrodynamics. It considers the flow of an electrically conducting fluid that is exposed to an external magnetic field and/or an electric current [1]. So, it investigates the influence of these external effects on the behavior of the flow of electrically conducting fluids and on the induced magnetic field. Hartmann [2] who studied the MHD flow between parallel planes introduced the study of magnetohydrodynamics and his results provided an insight for understanding the working principles of MHD flow. There are several studies of MHD flow problem based on the numerical methods such as FDM [3, 4], FEM [5, 6] and BEM [7, 8].

In this paper, the MHD flow of an electrically conducting fluid is considered in a long channel of rectangular cross-section (duct). The numerical solution of the steady, laminar, fully-developed MHD flow equations are investigated by using the finite difference method. The problem solutions are sought when the side walls are both slipping and partly insulated partly conducting whereas the Hartmann walls are no-slip and electrically insulated. The governing equations are solved with the use of FDM by using mixed type boundary conditions on the side walls. The influences of the slipping and the conductivity changes on the velocity and the induced magnetic field are illustrated with equivelocity and the current lines for increasing



values of Hartmann number, slip length and the length of conducting portions, and very well agreement is obtained when the side walls are completely conducting as given in [9].

2. Formulation of the problem

The governing equations of the steady, laminar, fully-developed 2D MHD flow of an incompressible, viscous, electrically conducting fluid in a square duct are given in [10] as

$$\begin{aligned} \nabla^2 V + Ha \frac{\partial B}{\partial y} &= -1 \\ &\text{in } \Omega \\ \nabla^2 B + Ha \frac{\partial V}{\partial y} &= 0 \end{aligned} \quad (1)$$

where $V(x, y)$ and $B(x, y)$ are the velocity and the induced magnetic field, respectively, and Ha is the Hartmann number, $Ha = L_0 B_0 \sqrt{\frac{\sigma}{\rho \nu}}$ where L_0 is the characteristic length, B_0 is the intensity of the external magnetic field, σ , ρ and ν are the electrical conductivity, the density and the kinematic viscosity of the fluid, respectively. $\Omega = \{-1 \leq x \leq 1, -1 \leq y \leq 1\}$ denotes the cross-section of the duct. The external magnetic field B_0 is applied perpendicular to the Hartmann walls $y = \pm 1$ and the boundary conditions are

$$\begin{aligned} V(x, \pm 1) = B(x, \pm 1) = 0, \quad |x| \leq 1 \quad \text{and} \quad \left(V \pm \alpha \frac{\partial V}{\partial x} \right) \Big|_{(\pm 1, y)} = 0, \quad |y| \leq 1 \\ \frac{\partial B}{\partial x} \Big|_{(\pm 1, y)} = 0, \quad |y| \leq l \quad \text{and} \quad B(\pm 1, y) = 0 \quad \text{for} \quad l \leq y \leq 1 \quad \text{or} \quad -1 \leq y \leq -l \end{aligned} \quad (2)$$

where α and l denote the slip and the conducting portion lengths, respectively.

3. Finite difference method application

Discretizing the MHD equations (1) as a whole by central finite differences for both the Laplace operator ∇^2 and the convection operator $\frac{\partial}{\partial y}$, we obtain

$$\begin{aligned} V_{i+1,j} - 4V_{i,j} + V_{i-1,j} + V_{i,j+1} + V_{i,j-1} + \frac{h(Ha)}{2} (B_{i,j+1} - B_{i,j-1}) &= -h^2 \\ B_{i+1,j} - 4B_{i,j} + B_{i-1,j} + B_{i,j+1} + B_{i,j-1} + \frac{h(Ha)}{2} (V_{i,j+1} - V_{i,j-1}) &= 0 \end{aligned} \quad (3)$$

for $i, j = 2, \dots, N$. Here, N is the number of nodes taken on each side and $h = 2/N$ is the step size. Then, the forward finite difference is used on the sides $x = -1$ and $y = -1$ whereas the backward finite difference is used on the sides $x = 1$ and $y = 1$ in order to define boundary values in terms of inner mesh point values. The boundary conditions (2) are approximated in such a way that we use forward and backward FDM when $x = \pm 1$, $i = 1$ or $N + 1$, and $j = 1, \dots, N + 1$, so that $V_{1,j} = \frac{\alpha}{\alpha + h} V_{2,j}$, $V_{N+1,j} = \frac{\alpha}{\alpha + h} V_{N,j}$ and $B_{1,j} = B_{2,j}$, $B_{N+1,j} = B_{N,j}$ and we impose $V_{i,1} = V_{i,N+1} = B_{i,1} = B_{i,N+1} = 0$ when $y = \pm 1$, $j = 1$ or $N + 1$, and $i = 1, \dots, N + 1$. Then, inserting them into the scheme (3), we obtain M unknowns in M equations where $M = 2(N - 1)^2$ for a general N . Then, these equations are written in a matrix-vector system with the coefficient matrix Q of size $M \times M$. Thus, we have the

system $Qx = w$ where $x = [V_{2,2} \ B_{2,2} \ \dots \ V_{2,N} \ B_{2,N} \ \dots \ V_{N,2} \ B_{N,2} \ \dots \ V_{N,N} \ B_{N,N}]^T$ and $w = [-h^2 \ 0 \ -h^2 \ 0 \ \dots \ -h^2 \ 0 \ -h^2 \ 0]^T$ are the unknown and the right hand side vectors of size $M \times 1$, respectively. The coefficient matrix Q of size $M \times M$ is a block diagonal matrix including the Hartmann number Ha , step-size h and the slipping length α in its entries. Finally, the unknown vector x at the discretized points from the solution of the system $Qx = w$ is obtained giving $V(x, y)$ and $B(x, y)$ at the mesh points.

4. Numerical results

The velocity and the induced magnetic field are simulated for increasing values of Ha , α and conducting portion. It is observed that we need to increase the number of nodes N with an increasing Ha since it causes convection dominance in the MHD equations. Figure 1 shows the equivelocity and current lines for increasing values of Ha when $\alpha = 0.1$ and $l = 0.6$. As Ha increases, boundary layers of $O(1/Ha)$ and of $O(1/\sqrt{Ha})$ are developed near the Hartmann (perpendicular) and parallel (side) walls for both V and B which is given in [10]. Near the Hartmann walls the velocity drops sharply within the boundary layers. Furthermore, the slip diminishes with a further increase in Ha because of the formation of the boundary layers.

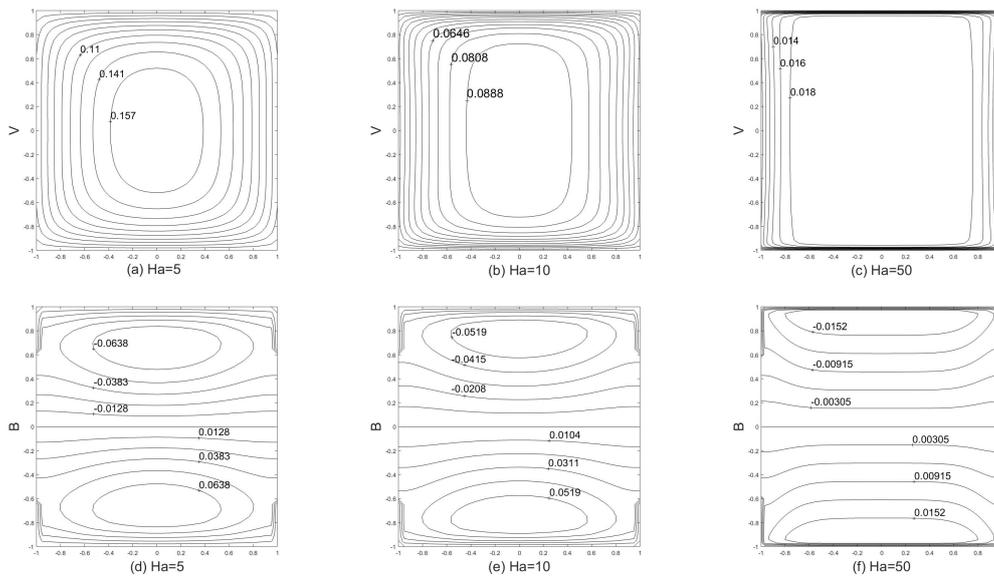


Figure 1. Velocity and the current lines, $\alpha = 0.1, l = 0.6$.

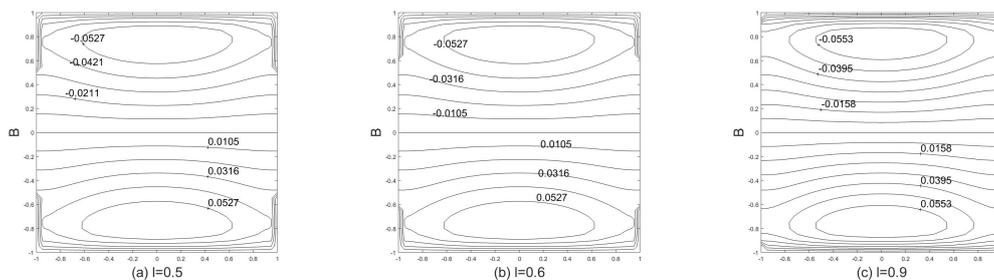


Figure 2. Current lines, $Ha = 10, \alpha = 0.2$.

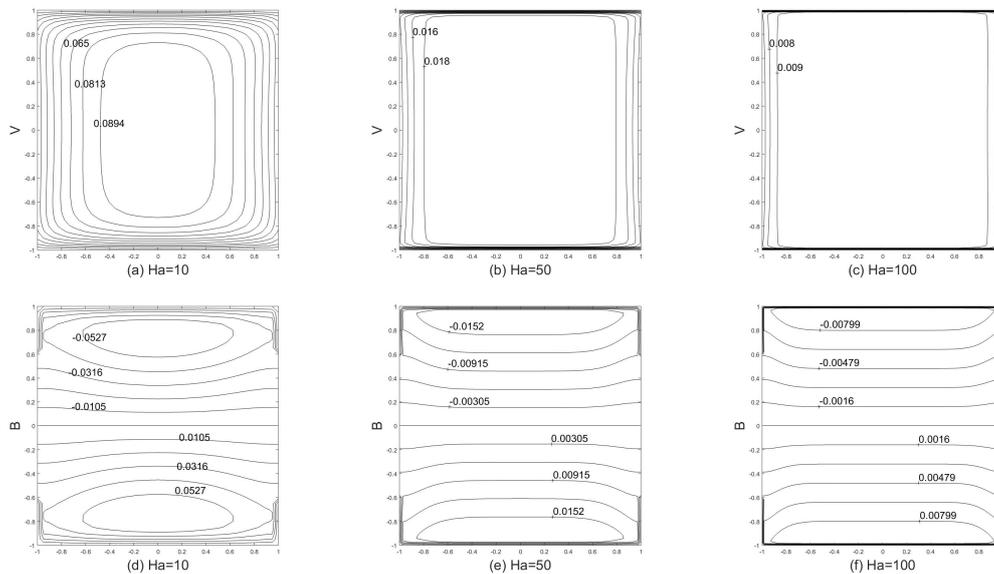


Figure 3. Velocity and the current lines, $\alpha = 0.2$, $l = 0.6$.

The increase in the slip length α causes the increase in the velocity magnitude as shown in Figure 1 and Figure 3 for $Ha = 10$ and $Ha = 50$. Figure 2 depicts the influence of the conducting portions of the side walls on the current lines. As the length of the conducting portions increases, current lines extend between the side walls showing parabolic lines around the vortices close to the Hartmann walls.

5. Conclusion

It is seen that both the velocity and the induced magnetic field magnitudes decrease as Ha increases. Current lines extend between the conducting portions of the side walls and vortices of them are squeezed through the Hartmann walls from the effect of boundary layer formation. As the conducting portion increases, the parabolic boundary layer emanating from the points where the conductivity changes is well observed. The FDM which is simple to implement provides the solution of MHD duct flow with the most general form of wall conditions at a small expense.

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