Pion–baryon coupling constants in light cone QCD sum rules

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Abstract

We calculate the $\pi\Sigma\Lambda$ and $\pi\Sigma\Sigma$ coupling constants in light cone QCD sum rules for the structure $\sigma_{\alpha\beta}\gamma_5 p^\alpha q^\beta$. A comparison of our results on these coupling constants with prediction of the SU(3) symmetry is presented.

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1 Introduction

Many strong interaction–processes involve meson–baryon coupling constants as the main ingredient. The determination of these fundamental quantities requires information about the physics at large distance. In other words, for a reliable determination of these parameters we need some nonperturbative approach. Among all nonperturbative approaches, QCD sum rules [1] is one of the most powerful method in studying the properties of hadrons. This method is based on the short distance OPE of vacuum–vacuum correlation function in terms of condensates. For the processes involving light mesons π, K or ρ, there is an alternative method to the traditional QCD sum rules, namely, light cone QCD sum rules [2]. In this approach the expansion of the vacuum–meson correlator is performed near the light cone in terms of the meson wave functions. The meson wave functions are defined by the matrix elements of non–local composite operators sandwiched between the meson and vacuum states and classified by their twists, rather than dimensions of the operators, as is the case in the traditional sum rules. Many applications of light–cone QCD sum rules can be found in [3]–[11] and references therein.

In this work we use light cone QCD sum rules approach for determination of the coupling constants of the pion to the lowest states of the baryon octet Σ and Λ, $g_{\pi\Sigma\Lambda}$ and $g_{\pi\Sigma\Sigma}$. Note that these coupling constants were investigated in framework of the QCD sum rules based on pion–to–vacuum matrix element in the leading order of the pion momentum q for the structure $\not{q}\gamma_5$ in [12], where q is the pion momentum. The results of this work are currently under debate in literature (see discussions in [13] and [14]). Moreover in [15] and [16] it was pointed out that there is coupling scheme dependence for the structures γ_5, $\not{q}\gamma_5$, i.e., dependence on the pseudoscalar or pseudovector forms of the effective interaction Lagrangian of pion with hadrons have been used, while the structure $\sigma_{\mu\nu}\gamma_5$ is shown to be independent of any coupling schemes. For this reason, in present work we choose the structure $\sigma_{\mu\nu}\gamma_5 p^\mu q^\nu$, where p and q are the Λ (Σ) and the pion momenta, respectively. It should be noted that the sum rules for the $\sigma_{\mu\nu}\gamma_5 p^\mu q^\nu$ structure was derived in [17] in investigation of $g_{\rho\omega\pi}$ coupling constant.

The paper is organized as follows. In Section 2 we derive sum rules for the pion–baryon coupling constants $g_{\pi\Sigma\Lambda}$ and $g_{\pi\Sigma\Sigma}$ for the structure $\sigma_{\mu\nu}\gamma_5 p^\mu q^\nu$. Section 3 is devoted to the numerical analysis of the sum rules for $g_{\pi\Sigma\Lambda}$ and $g_{\pi\Sigma\Sigma}$ and discussion.

2 Formulation of the pion–baryon sum rule for the $g_{\pi\Sigma\Lambda}$ and $g_{\pi\Sigma\Sigma}$

According to the main philosophy of the QCD sum rules, a quantitative estimation for $g_{\pi\Sigma\Lambda}$ and $g_{\pi\Sigma\Sigma}$ couplings can be obtained by matching the representations of a suitable correlator in terms of hadronic (physical part) and quark–gluon language (theoretical part). For this purpose we consider the following two–point correlator function with pion

$$\Pi(p,q) = \int d^4x\, e^{ipx} \left\langle \pi(q) \left| \mathrm{T} \left[\eta_Y(x) \bar{\eta}_{\Sigma^+}(0) \right] \right| 0 \right\rangle \ , \qquad (1)$$

where p and η_Y are the four–momentum of the hyperon (in our case Λ^0 or Σ^0) and its interpolation current, respectively, η_{Σ^+} is the interpolating current of Σ^+ and q is the pion four–momentum. The interpolating currents for Λ^0, Σ^0 and Σ^+ are [18]

$$
\begin{aligned}
\eta_{Y^0} &= \alpha\,\epsilon_{abc}\left[\left(u_a^T\mathcal{C}\gamma_\mu s_b\right)\gamma_5\gamma^\mu d_c \mp \left(d_a^T\mathcal{C}\gamma_\mu s_b\right)\gamma_5\gamma^\mu u_c\right]~, \\
\eta_{\Sigma^+} &= \epsilon_{abc}\left(u_a^T\mathcal{C}\gamma_\mu u_b\right)\gamma_5\gamma^\mu s_c~, \qquad (2)
\end{aligned}
$$

where $s,~u$ and d are strange, up and down quark fields, the upper(lower) sign corresponds to $\Lambda^0(\Sigma^0)$ and $\alpha=\sqrt{2/3}$ for Λ^0 and $\sqrt{2}$ for Σ^0, respectively, $a,~b,~c$ are the color indices, $\mathcal{C}$ is the charge conjugation operator. Saturating correlator (1) with the Y ($=\Lambda^0$ or Σ^0) and Σ^+ states in the phenomenological part, we get

$$
\Pi = \frac{\langle\pi(q)Y|\Sigma^+\rangle~\langle\Sigma^+(p+q)|\bar\eta_{\Sigma^+}|0\rangle~\langle 0|\bar\eta_Y|Y(p)\rangle}{(p^2-m_Y^2)\,[(p+q)^2-m_{\Sigma^+}^2]} + \text{high. reson.} \qquad (3)
$$

The matrix elements in Eq. (3) are defined in the following way

$$
\begin{aligned}
\langle 0\,|\eta_Y(x)|\,Y(p)\rangle &= \lambda_Y u(p)~, \\
\left\langle\Sigma^+(p+q)\,|\bar\eta_{\Sigma^+}|\,0\right\rangle &= \lambda_\Sigma \bar u(p+q)~, \\
\left\langle\pi(q)Y(p)|\Sigma^+(p+q)\right\rangle &= -g_{Y\Sigma^+\pi^-}~\bar u(p)\gamma_5 u(p+q)~. \qquad (4)
\end{aligned}
$$

Substituting Eq. (4) in Eq. (3), and choosing the structure $i\,\sigma_{\alpha\beta}p^\alpha q^\beta\gamma_5$ for the physical part of Eq. (1), we get

$$
\Pi^{phys} = -\frac{g_{\pi^- Y\Sigma^+}\lambda_Y\lambda_{\Sigma^+}}{(p^2-m_Y^2)\,[(p+q)^2-m_\Sigma^2]} + \text{higher resonances}~. \qquad (5)
$$

Let us now consider the theoretical part of the correlator (1). From this correlator we have (we present only the terms which give contributions to the above–mentioned Lorentz structure)

$$
\begin{aligned}
\Pi &= -\alpha\int d^4x\,e^{ipx}\Big\{-\gamma_5\gamma_\mu\gamma_5\gamma_\varphi\gamma_\rho\,\mathcal{C}\mathcal{S}^T\mathcal{C}^{-1}\gamma_\mu\mathcal{S}^s\gamma_\rho\gamma_5\,\langle\pi\,|\bar u\gamma_5\gamma_\varphi d|\,0\rangle \\
&\quad +\frac{1}{2}\gamma_5\gamma_\mu\sigma_{\alpha\beta}\gamma_\rho\,\mathcal{C}\mathcal{S}^T\mathcal{C}^{-1}\gamma_\mu\mathcal{S}^s\gamma_\rho\gamma_5\,\langle\pi\,|\bar u\sigma_{\alpha\beta}d|\,0\rangle \\
&\quad \pm\Big[-\gamma_5\gamma_\mu\mathcal{S}\gamma_\rho\,\mathcal{C}(\gamma_5\gamma_\varphi)^T\mathcal{C}^{-1}\gamma_\mu\mathcal{S}^s\gamma_\rho\gamma_5\,\langle\pi\,|\bar u\gamma_5\gamma_\varphi d|\,0\rangle \\
&\quad +\frac{1}{2}\gamma_5\gamma_\mu\mathcal{S}\gamma_\rho\,\mathcal{C}\sigma_{\alpha\beta}^T\mathcal{C}^{-1}\gamma_\mu\mathcal{S}^c\gamma_\rho\gamma_5\,\langle\pi\,|\bar u\sigma_{\alpha\beta}d|\,0\rangle\Big]\Big\}~, \qquad (6)
\end{aligned}
$$

where upper(lower) sign corresponds to $\Lambda(\Sigma)$ case and $\alpha=\sqrt{2/3}$ for Λ while $\alpha=\sqrt{2}$ for Σ. Here $\mathcal{S}$ and $\mathcal{S}^s$ are the propagators containing both perturbative and nonperturbative contributions, respectively. Here we present the explicit form of $i\mathcal{S}^s(x)$

$$
\begin{aligned}
i\mathcal{S}^s(x) &= \frac{i}{2\pi^2}\frac{\not{x}}{x^4} - \frac{m_s}{4\pi^2}\frac{1}{x^2} - \frac{1}{12}\langle\bar s s\rangle\left(1-\frac{im_s}{4}\not{x}\right) \\
&\quad -\frac{1}{192}m_0^2\langle\bar s s\rangle\left(1-\frac{im_s}{6}\not{x}\right) - ig_s\frac{1}{16\pi^2}\int_0^1 du\left\{\frac{\not{x}}{x^2}\sigma_{\alpha\beta}G^{\alpha\beta}(ux) - 4i\frac{x_\alpha}{x^2}G^{\alpha\beta}\gamma_\beta\right\}~, \qquad (7)
\end{aligned}
$$

where m_s is the mass of the strange quark and $G_{\alpha\beta}$ is the gluon field strength tensor. The form of $\mathcal{S}$ can be obtained from Eq. (7) by making the replacements $\langle \bar{s}s \rangle \to \langle \bar{q}q \rangle$ and $m_s \to 0$. From Eq. (6) we observe that, in calculation of the correlator function in QCD, the matrix element of the nonlocal operators between the vacuum and pion states are needed. These matrix elements define the two particle pion wave functions and up to twist four they can be written as [6, 7]

$$\begin{aligned}
\left\langle \pi(q) \left| \bar{d}\gamma_\mu\gamma_5 u \right| 0 \right\rangle &= -i f_\pi q_\mu \int_0^1 du\, e^{iuqx} [\varphi_\pi(u) + x^2 g_1(u)] \\
&\quad + f_\pi \left(x_\mu - \frac{x^2 q_\mu}{qx} \right) \int_0^1 du\, e^{iuqx} g_2(u) \ , \\
\left\langle \pi(q) \left| \bar{d}(x)\sigma_{\alpha\beta}\gamma_5 u(0) \right| 0 \right\rangle &= i\left(q_\alpha x_\beta - q_\beta x_\alpha \right) \frac{f_\pi m_\pi^2}{6(m_u+m_d)} \int_0^1 du\, e^{iuqx} \varphi_\sigma(u) \ . \qquad (8)
\end{aligned}$$

Using Eqs. (6), (7) and (8) we get the following result for theoretical part (for the structure $i\sigma_{\alpha\beta}x_\alpha q^\beta \gamma_5$)

$$\begin{aligned}
\Pi^{theor} &= \\
&-\alpha f_\pi \int_0^1 dx\, e^{ipx} \Bigg\{ \left[(1\pm 1)\frac{m_s}{2\pi^4 x^6} + (-\lambda \pm \sigma)\left(\frac{1}{6\pi^2 x^4} + \frac{m_0^2}{96\pi^2 x^2} \right) \right] \int_0^1 du\, e^{iuqx} \varphi_\pi(u) \\
&+ \left[(1\mp 1)\frac{\mu_\pi}{6\pi^4 x^6} + (1\pm 1)\frac{\mu_\pi \langle g^2 G^2 \rangle}{2304\pi^4 x^2} + (1\mp 1)\frac{m_s \mu_\pi \langle \bar{s}s \rangle}{144\pi^2 x^2} \pm \frac{m_s \mu_\pi \sigma}{72\pi^2 x^2} \right] \int_0^1 du\, e^{iuqx} \varphi_\sigma(u) \\
&+ \left[\frac{1}{6\pi^2 x^2}(-\lambda \pm \sigma) + (1\pm 1)\frac{m_s}{2\pi^4 x^4} \right] \int_0^1 du\, e^{iuqx} \left[g_1(u) + G_2(u) \right] \Bigg\} \ , \qquad (9)
\end{aligned}$$

where

$$\begin{aligned}
G_2(u) &= -\int_0^u g_2(v) dv \ , \\
\lambda &= \langle \bar{q}q \rangle - \langle \bar{s}s \rangle \ , \\
\sigma &= \langle \bar{q}q \rangle + \langle \bar{s}s \rangle \ .
\end{aligned}$$

Our next task is to perform integration over x and perform double Borel transformation in Eq. (9) with respect to the variables p^2 and $(p+q)^2$, in order to get an answer for the theoretical part of the sum rules. As an example, let us demonstrate on one of the terms in Eq. (9) how integration over x and double Borel transformation can be carried. Consider the following term

$$\int du \varphi_\pi(u) \int d^4x \frac{e^{i(p+qu)x}}{x^2} i\sigma_{\alpha\beta}x_\alpha q_\beta \ .$$

In performing the x integration, we will make use of the formula [19]

$$\int \frac{d^4x}{(x^2)^n} e^{ipx} = \frac{i(-1)^n 2^{4-2n}\pi^2}{\Gamma(n-1)\Gamma(n)} (p^2)^{n-2} ln(-p^2) + \mathcal{P}_{n-2} \ , \quad (n \geq 2) \ ,$$

where $\mathcal{P}_{n-2}$ is a polynomial of power $n-2$. However this polynomial is inessential, since it vanishes after double Borel transformation. Hence, disregarding this polynomial we have

$$\begin{aligned}
\int du\varphi_\pi(u)\int d^4x\frac{e^{i(p+qu)x}}{x^2}i\sigma_{\alpha\beta}x_\alpha q_\beta &= \int du\varphi_\pi(u)\left(i\frac{\partial}{\partial p_\alpha}\frac{\partial}{\partial p_\rho}\frac{\partial}{\partial p_\rho}\right)\int d^4x\frac{e^{i(p+qu)x}}{x^4}i\sigma_{\alpha\beta}q_\beta \\
&= 8\int du\varphi_\pi(u)\frac{P_\alpha}{P^4}i\sigma_{\alpha\beta}q_\beta ~,
\end{aligned}$$

where $P=p+qu$. The last step in this calculation is performing double Borel transformation over the variables p^2 and $(p+q)^2$ to the expression

$$\int du\varphi_\pi(u)\frac{1}{\left[(p+qu)^2\right]^2} ~.$$

Rewriting $(p+qu)^2 = p^2\bar{u}+u(p+q)^2$ (here the pion mass is neglected) and using the exponential representation for the denominator,

$$\frac{1}{\left[p^2\bar{u}+u(p+q)^2\right]^2} = \int_0^\infty d\alpha\alpha e^{-\alpha\left[p^2\bar{u}+u(p+q)^2\right]} ~,$$

we have

$$\int du\varphi_\pi(u)\frac{1}{\left[(p+qu)^2\right]^2} = \int du\varphi_\pi(u)\int d\alpha\alpha e^{-\alpha\left[p^2\bar{u}+u(p+q)^2\right]} ~.$$

The double Borel transformation over the variable p^2 and $(p+q)^2$ is done with the help of the following general formula

$$\mathcal{B}_{p^2}^{M_1^2}\mathcal{B}_{(p+q)^2}^{M_2^2}\frac{\Gamma(n)}{\left[-\bar{u}p^2-(p+q)^2u\right]^n} = (M^2)^{2-n}\delta(u-u_0) ~,$$

where

$$M^2 = \frac{M_1^2M_2^2}{M_1^2+M_2^2} \qquad \text{and} \qquad u_0 = \frac{M_1^2}{M_1^2+M_2^2} ~,$$

and M_1^2 and M_2^2 are the Borel parameters. Using this expression and performing the integrations over the variables α and u, we finally get

$$\int du\varphi_\pi(u)\int d^4x\frac{e^{i(p+qu)x}}{x^2}i\sigma_{\alpha\beta}x_\alpha q_\beta = 8i\sigma_{\alpha\beta}p_\alpha q_\beta\varphi_\pi(u_0) ~.$$

All other terms can be calculated similarly and for the theoretical part it follows from Eq. (9) that

$$\begin{aligned}
\Pi^{theor} &= \\
&-\alpha f_\pi\Bigg\{\varphi_\pi(u_0)\left[(1\pm1)\frac{m_s}{8\pi^2}M^4f_1(s_0/M^2)-\frac{1}{3}(-\lambda\pm\sigma)M^2f_0(s_0/M^2)+\frac{m_0^2}{12}(-\lambda\pm\sigma)\right] \\
&+\mu_\pi\varphi_\sigma(u_0)\left[(1\mp1)\frac{1}{24\pi^2}M^4f_1(s_0/M^2)+(1\pm1)\frac{8}{2304}\frac{\langle g^2G^2\rangle}{\pi^2}+(1\mp1)\frac{1}{18}m_s\langle\bar{s}s\rangle\pm\frac{1}{9}m_s\sigma\right] \\
&+\left[g_1(u_0)+G_2(u_0)\right]\left[-(1\pm1)\frac{1}{\pi^2}m_sM^2f_0(s_0/M^2)+\frac{4}{3}(-\lambda\pm\sigma)\right]\Bigg\} ~, \qquad (10)
\end{aligned}$$

where the function

$$f_n(s_0/M^2) = 1 - e^{-s_0/M^2} \sum_{k=0}^{n} \frac{(s_0/M^2)^k}{k!} ~,$$

is the factor used to subtract the continuum, which is modeled by the dispersion integral in the region $s_1,~ s_2 \geq s_0,~ s_0$ being the continuum threshold (obviously the continuum thresholds for the Λ and Σ channels are different). Since masses of Λ and Σ are very close to each other, we can choose M_1^2 and M_2^2 to be equal to each other, i.e., $M_1^2 = M_2^2 = 2M^2$, from which it follows that $u_0 = 1/2$.

Performing double Borel transformation over the variables p^2 and $(p+q)^2$ in the physical part (5) and then equating the the obtained result to Eq. (10), we get the sum rules for $g_{\pi\Lambda\Sigma}$ and $g_{\pi\Sigma\Sigma}$ coupling constants

$$g_{Y\Sigma^+\pi^-} \lambda_Y \lambda_{\Sigma^+} = e^{m^2/M^2} \Pi^{theor} ~, \tag{11}$$

where $m \approx m_\Lambda \approx m_\Sigma$.

From Eq. (11) it follows that in determining the strong coupling constants $g_{\pi\Lambda\Sigma}$ and $g_{\pi\Sigma\Sigma}$ the experimentally undetermined residues λ_Λ and λ_Σ need to be eliminated from sum rules. The residues λ_Λ and λ_Σ are determined from corresponding mass sum rules for the λ and Σ hyperons [18, 20] as follows

$$\begin{aligned}
\left|\lambda_\Lambda\right|^2 e^{-m_\Lambda^2/M^2} 32\pi^4 &= M^6 f_2(s_0^\Lambda/M^2) + \frac{2}{3} a m_s (1-3\gamma) M^2 f_0(s_0^\Lambda/M^2) \\
&\quad + b M^2 f_0(s_0^\Lambda/M^2) + \frac{4}{9} a^2 (3+4\gamma) ~, &(12)\\
\left|\lambda_\Sigma\right|^2 e^{-m_\Sigma^2/M^2} 32\pi^4 &= M^6 f_2(s_0^\Sigma/M^2) - 2 a m_s (1+\gamma) M^2 f_0(s_0^\Sigma/M^2) \\
&\quad + b M^2 f_0(s_0^\Sigma/M^2) + \frac{4}{3} a^2 ~, &(13)
\end{aligned}$$

where

$$\begin{aligned}
a &= -2\pi^2 \langle \bar{q} q \rangle ~, \\
b &= \frac{\alpha_s \langle G^2 \rangle}{\pi} \simeq 0.012~GeV^4 ~, \\
\gamma &= \frac{\langle \bar{s} s \rangle}{\langle \bar{q} q \rangle} - 1 \simeq -0.2 ~,
\end{aligned}$$

and the functions $f_0(x),~ f_1(x)$ are presented just after Eq. (10). The ratio of the Eqs. (11), (12) and (13) gives

$$\begin{aligned}
g_{\pi\Lambda\Sigma} &= e^{m^2/M^2} \frac{\Pi^{theor}}{\lambda_\Lambda \lambda_\Sigma} ~, &(14)\\
g_{\pi\Sigma\Sigma} &= e^{m^2/M^2} \frac{\Pi^{theor}}{\lambda_\Sigma^2} ~. &(15)
\end{aligned}$$

The main reason why we consider the above–mentioned ratio rather than the individual sum rules themselves (i.e., first determine λ_Λ and λ_Σ independently from Eqs. (12) and

(13) and substitute their obtained values in Eq. (11)) is that the sum rules obtained from these ratios are more stable as is similar to the baryon mass sum rules case. In addition to that the uncertainties coming from various parameters such as quark condensate, m_0^2, continuum threshold s_0 and Borel parameter, are reduced.

3 Numerical analysis

Now we are ready to perform the numerical analysis. The main nonperturbative input parameters in the sum rules (11) are the pion wave functions. In our calculations we have used the set of wave functions proposed in [6]. The explicit expressions of the wave functions are

$$
\begin{aligned}
\varphi_\pi(u,\mu) &= 6u\bar{u}\left[1 + a_2(\mu)C_2^{3/2}(2u-1) + a_4(\mu)C_4^{3/2}(2u-1)\right] ,\\
\varphi_\sigma(u,\mu) &= 6u\bar{u}\left[1 + C_2\frac{3}{2}\left[5(u-\bar{u})^2-1\right] + C_4\frac{15}{8}\left[21(u-\bar{u})^4 - 14(u-\bar{u})^2+1\right]\right] ,\\
g_1(u,\mu) &= \frac{5}{2}\delta^2(\mu)\bar{u}^2u^2 + \frac{1}{2}\varepsilon(\mu)\delta^2(\mu)\Bigg[u\bar{u}(2+13u\bar{u})\\
&\quad +10u^3\ln u\left(2-3u+\frac{6}{5}u^2\right) + 10\bar{u}^3\ln\bar{u}\left(2-3\bar{u}+\frac{6}{5}\bar{u}^2\right)\Bigg] ,\\
G_2(u,\mu) &= \frac{5}{3}\delta^2(\mu)\bar{u}^2u^2 , \qquad (16)
\end{aligned}
$$

where $\bar{u} = 1-u$, $C_2^{3/2}$ and $C_4^{3/2}$ are the Gegenbauer polynomials defined as

$$
\begin{aligned}
C_2^{3/2}(2u-1) &= \frac{3}{2}\left[5(2u-1)^2+1\right] ,\\
C_4^{3/2}(2u-1) &= \frac{15}{8}\left[21(2u-1)^4 - 14(2u-1)^2+1\right] , \qquad (17)
\end{aligned}
$$

and $a_2(\mu = 0.5~GeV) = 2/3$, $a_4(\mu = 0.5~GeV) = 0.43$. The parameters $\delta^2(\mu = 1~GeV) = 0.2~GeV^2$ [21] and $\epsilon(\mu = 1~GeV) = 0.5$ [6]. Furthermore $f_\pi = 0.132~GeV$, $\mu_\pi(\mu = 1~GeV) = 1.65$, $\langle\bar{s}s\rangle = 0.8\langle\bar{q}q\rangle$ and $\langle\bar{q}q\rangle|_{\mu=1~GeV} = -(0.243)^3~GeV^3$, $s_0 = s_0^\Lambda \simeq s_0^\Sigma = (3.0\pm0.2)~GeV^2$. Moreover all further calculations are performed at $u = u_0 = 1/2$.

Having fixed the input parameters, one must find the range of values of M^2 over which the sum rule is reliable. The lowest possible value of M^2 is determined by the requirement that the terms proportional to the highest inverse power of the Borel parameters stay reasonable small. The upper bound of M^2 is determined by demanding that the continuum contribution is not too large. The interval of M^2 which satisfies both conditions is $1~GeV^2 < M^2 < 2.5~GeV^2$. The analysis of the sum rules (14) and (15) shows that the best stability is achieved in the region of M^2, $1.4~GeV^2 < M^2 < 1.8~GeV^2$. This leads to the following result for the coupling constants

$$
\begin{aligned}
g_{\pi\Lambda\Sigma} &= 5.3 \pm 1.8\\
g_{\pi\Sigma\Sigma} &= 12.5 \pm 4.5 , \qquad (18)
\end{aligned}
$$

in which the errors coming from the quark condensate (which varies in the region $-(0.24\,GeV)^3$ and $-(0.26\ GeV)^3$), m_0^2 parameter (which varies in the region 0.6 GeV^2 and 1.4 GeV^2), variation of the Borel parameter and change of the continuum threshold s_0 are all taken into consideration. Our calculation shows that the main error comes from uncertainties of the quark condensate. The central values of the coupling constants are obtained at $m_0^2 = 0.8\ GeV^2$, $\langle \bar{q} q \rangle = -(0.243\ GeV)^3$ and $s_0 = 3.0\ GeV^2$.

At this point we would like to stress that the above–mentioned strong coupling constant have been analyzed using the individual sum rules themselves by first determining λ_Λ and λ_Σ independently from Eqs. (12) and (13) and substituting their obtained values in Eq. (11). The results predicted by this approach are close to the ones presented in Eq. (18), however our calculations show that the ratio sum rules prediction is more stable and reliable.

Here it should be noted that since the phase of the coupling constants can not be predicted by the sum rules, we take into consideration the magnitudes of these coupling constants to be able to compare them with the predictions of other approaches.

Let us now compare our results on the coupling constants $g_{\pi\Lambda\Sigma}$ and $g_{\pi\Sigma\Sigma}$ with SU(3) symmetry prediction. As is known, SU(3) symmetry predicts

$$\begin{aligned} G_{\pi\Lambda\Sigma} &= \frac{2}{\sqrt{3}}(1-\alpha)\,G_{NN\pi}\ , \\ G_{\pi\Sigma\Sigma} &= 2\alpha\,G_{NN\pi}\ , \end{aligned} \tag{19}$$

where $\alpha = F/(F+D)$ (see for example [22]). Exact SU(3) symmetric analysis of pion–baryon coupling gives $F/D \simeq 0.58$ [22] (exact SU(6) symmetry predicts $F/D = 2/3$). It follows from Eqs. (19) that

$$R = \frac{G_{\pi\Sigma\Sigma}}{G_{\pi\Lambda\Sigma}} = \frac{\sqrt{3}\alpha}{1-\alpha} \approx 1\ , \tag{20}$$

while our analysis yields $R \simeq 2$. It follows from a comparison of these results that SU(3) symmetry is broken significantly. The pion–baryon couplings predicted in [12] are not presented here since we have already noted that the results of this work is currently under debate in literature.

In conclusion we have calculated the strong coupling constants of pion with Λ and Σ hyperons and found out that our results differ significantly from that of the SU(3) symmetry prediction.

References

[1] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, *Nucl. Phys.* **B147** (1979) 385.

[2] V. L. Chernyak and A. R. Zhitnitsky, *JETP Lett.* **25** (1977) 510;
A. V. Efremov and a. V. Radyushkin, *Phys. Lett.* **94B** (1980) 245;
G. P. Lepage and S. J. Brodsky, *Phys. Lett.* **87B** (1979) 359.

[3] V. L. Chernyak and A. R. Zhitnitsky, *Phys. Rep.* **112** (1984) 173.

[4] V. M. Braun, In proc. "Rostock 1997, Progress in heavy quark physics" 105-118, 1997, prep. hep-ph/9801222 (1998).

[5] I. I. Balitskii, V. M. Braun, A. V. Kolesnichenko, *Nucl. Phys.* **B312** (1989) 509.

[6] V. M. Braun, I. E. Filyanov, *Z. Phys.* **C48** (1990) 239.

[7] V. L. Chernyak and A. R. Zhitnitsky, *Nucl. Phys.* **B345** (1990) 137.

[8] V. M. Belyaev, V. M. Braun, A. Khodjamirian and R. Rückl, *Phys. Rev.* **D51** (1995) 6177.

[9] P. Ball, V. M. Braun, *Phys. Rev* **D55** (1997) 5561.

[10] P. Ball, V. M. Braun, *Phys. Rev* **D58** (1998) 094016.

[11] T. M. Aliev, M. Savcı, A. Özpineci, *Phys. Rev.* **D56** (1997) 4260.

[12] M. C. Brise and B. Krippa, *Phys. Rev.* **C54** (1996) 3240.

[13] H. Kim, prep. nucl-th/9903040 (1999).

[14] B. Krippa and M. C. Brise, prep. nucl-th/9904003 (1999)

[15] H. Kim, S. H. Lee, M. Oka, prep. nucl-th/9809004 (1998).

[16] H. Kim, S. H. Lee, M. Oka, prep. nucl-th/9811096 (1998).

[17] V. M. Braun and I. E. Filyanov, *Z. Phys.* **C44** (1989) 157.

[18] B. L. Ioffe, *Nucl. Phys.* **B188** (1981) 317; Errara, *Nucl. Phys.* **B191** (1981) 591.

[19] T. M. Aliev and M. A. Shifman, *Sov. J. Nucl. Phys.* **36** (1982) 89.

[20] L. Reinders, H. Rubinstein and S. Yazaki, *Phys. Rep.* **127C** (1985) 1.

[21] V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, *Nucl. Phys.* **B237** (1984) 525.

[22] A. Reuber, K. Holinde and J. Speth, *Nucl. Phys.* **A570** (1994) 543;
M. M. Nagels *et al.*, *Nucl. Phys.* **B147** (1979) 189;
O. Dumbrajs et. al., *ibid.*, **B216** (1983) 277.