Fourth generation effects in rare exclusive $B \to K^* \ell^+ \ell^-$ decay

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Abstract

Influence of the fourth generation, if ever exists, on the experimentally measurable quantities such as invariant dilepton mass distribution, lepton forward–backward asymmetry, and the ratio Γ_L/Γ_T of the decay widths when K^* meson is longitudinally and transversally polarized, is studied. Using the experimental results on the branching ratios for the $B \to X_s \gamma$ and semileptonic $B \to X_c \ell \bar{\nu}$ decays, the two possible solutions of the 4×4 Cabibbo–Kobayashi–Maskawa factor $V_{t's} V_{t'b}$ are obtained as a function of the t'–quark mass. It is observed that the results for the above–mentioned physical quantities are essentially different from the standard model predictions only for one solution of the CKM factor. In this case the above–mentioned physical quantities can serve as efficient tools in search of the fourth generation. The other solution yields almost identical results with the SM.

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1 Introduction

At present Standard Model (SM) describes very successfully all low energy experimental data. But from theoretical point of view SM is an incomplete theory. This theory contains many unsolved open problems, such as the origin of CP violation, mass spectrum, etc. Another one of the unsolved fundamental problems of SM is the number of generations. There is no any theoretical argument to restrict the SM to three known generations of the fermions. From the LEP result of the invisible partial decay width of Z boson it follows that the mass of the extra generation neutrino N should be larger than 45 GeV [1]. In this connection there comes into mind the following question: If extra generations exist, what effect they would have in low energy physics? This problem was studied in many works (see for example [2]–[8] and the references therein).

Contributions of the new generation to the electroweak radiative corrections were considered in papers [9]–[15]. It was shown in [15] that the existing electroweak data on the Z–boson parameters, the W boson and the top quark masses strongly excluded the existence of the new generations with all fermions heavier than the Z boson mass. However the same data allows few extra generations, if one allows neutral leptons to have masses close to 50 GeV.

The most straightforward and economical generalization of the SM to the four–generation case is similar to the three generations present in SM [16], which we consider in this work. One promising area in experimental search of the fourth generation, via its indirect loop effects, is the rare B meson decays. This year the upgraded B factories at SLAC and KEK will provide us with the first experimental data. It is also well known that in the SM $B \to K^* \ell^+ \ell^-$ decay has "large" branching ratio and it has experimentally clean signature because two leptons are present in the final state. For this reason this decay is one of the most probable candidates to be detected in these machines and in our view it is the right time for an investigation in this direction.

In this work we study the contribution of the fourth generation in the rare $B \to K^* \ell^+ \ell^-$ decay. At the same time this decay is sensitive to the various extension of the SM, because it occurs only at loop level in the SM.

New physics effects can manifest themselves through the Wilson coefficients, whose values can be different from the ones in the SM [17, 18], as well as through the new operators [19]. Note that the inclusive $B \to X_s \gamma$ and $B \to X_s \ell^+ \ell^-$ decays have already been studied with the inclusion of the fourth generation in the SM [6, 20, 21].

The paper is organized as follows. In section 2, we present the necessary theoretical expressions for the $B \to K^* \ell^+ \ell^-$ decay in the SM with four generations, as well as the expressions of the other physical observables such as forward–backward asymmetry and the ratio of the decay widths when K^* meson is polarized longitudinally and transversally. Section 3 is devoted to the numerical analysis and our conclusion.

2 Theoretical results

The matrix element of the $B \to K^* \ell^+ \ell^-$ decay at quark level is described by $b \to s \ell^+ \ell^-$ transition for whom the effective Hamiltonian at $O(\mu)$ scale can be written as

$$\mathcal{H}_{eff} = \frac{4 G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} \mathcal{C}_i(\mu)\, \mathcal{O}_i(\mu) \ , \quad (1)$$

where the full set of the operators $\mathcal{O}_i(\mu)$ and the corresponding expressions for the Wilson coefficients $\mathcal{C}_i(\mu)$ in the SM are given in [22, 23]. As has been noted already, in the model we consider in this work where the fourth generation is introduced in the same way the three generations are introduced in the SM, no new operators appear and clearly the full operator set is exactly the same as in SM. The fourth generation changes only the values of the Wilson coefficients $C_7(\mu)$, $C_9(\mu)$ and $C_{10}(\mu)$, via virtual exchange of the fourth generation up quark t'. The above mentioned Wilson coefficients can be written in the following form

$$\begin{aligned} C_7^{tot}(\mu) &= C_7^{SM}(\mu) + \frac{V_{t'b}^* V_{t's}}{V_{tb}^* V_{ts}} C_7^{new}(\mu) \ , \\ C_9^{tot}(\mu) &= C_9^{SM}(\mu) + \frac{V_{t'b}^* V_{t's}}{V_{tb}^* V_{ts}} C_9^{new}(\mu) \ , \\ C_{10}^{tot}(\mu) &= C_{10}^{SM}(\mu) + \frac{V_{t'b}^* V_{t's}}{V_{tb}^* V_{ts}} C_{10}^{new}(\mu) \ , \quad (2) \end{aligned}$$

where the last terms in these expressions describe the contributions of the t' quark to the Wilson coefficients and $V_{t'b}$ and $V_{t's}$ are the two elements of the 4×4 Cabibbo–Kobayashi–Maskawa (CKM) matrix. In deriving Eq. (2) we factored out the term $V_{tb}^* V_{ts}$ in the effective Hamiltonian given in Eq. (1). The explicit forms of the C_i^{new} can easily be obtained from the corresponding Wilson coefficient expressions in SM by simply substituting $m_t \to m_{t'}$ (see [22, 24]). Neglecting the s quark mass, the above effective Hamiltonian leads to following matrix element for the $b \to s \ell^+ \ell^-$ decay

$$\begin{aligned} \mathcal{M} &= \frac{G \alpha}{2\sqrt{2}\pi} V_{tb} V_{ts}^* \Bigg[C_9^{tot}\, \bar{s} \gamma_\mu (1 - \gamma_5) b\, \bar{\ell} \gamma_\mu \ell + C_{10}^{tot} \bar{s} \gamma_\mu (1 - \gamma_5) b\, \bar{\ell} \gamma_\mu \gamma_5 \ell \\ &- 2 C_7^{tot} \frac{m_b}{q^2} \bar{s} \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b\, \bar{\ell} \gamma_\mu \ell \Bigg] \ , \quad (3) \end{aligned}$$

where $q^2 = (p_1 + p_2)^2$ and p_1 and p_2 are the final leptons four–momenta. The effective coefficient C_9^{tot} of the operator $\mathcal{O}_9 = \bar{s} \gamma_\mu (1 - \gamma_5) b\, \bar{\ell} \gamma_\mu \ell$ can be written in the following form

$$C_9^{tot} = C_9 + Y(s) \ , \quad (4)$$

where $s = q^2/m_B^2$ and the function $Y(s)$ contains the contributions from the one loop matrix element of the four quark operators. A perturbative calculation leads to the result [22, 23],

$$\begin{aligned} Y_{per}(s) &= g(\hat{m}_c, s)(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) - \frac{1}{2} g(1, s)(4C_3 + 4C_4 + 3C_5 + C_6) \\ &- \frac{1}{2} g(0, s)(C_3 + 3C_4) + \frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6) \ , \quad (5) \end{aligned}$$

where $\hat{m}_c = m_c/m_b$. The explicit expressions for $g(\hat{m}_c, s)$, $g(0, s)$, $g(1, s)$ and the values of C_i in the SM can be found in [22, 23].

C_1	C_2	C_3	C_4	C_5	C_6	C_7^{SM}	C_9^{SM}	C_{10}^{SM}
-0.248	1.107	0.011	-0.026	0.007	-0.031	-0.313	4.344	-4.669

Table 1: The numerical values of the Wilson coefficients at $\mu = m_b$ scale within the SM. The corresponding numerical value of C^0 is 0.362.

In addition to the short distance contribution, $Y_{per}(s)$ receives also long distance contributions, which have their origin in the real $c\bar{c}$ intermediate states, i.e., J/ψ, ψ', $\cdots$. The J/ψ family is introduced by the Breit–Wigner distribution for the resonances through the replacement [25]–[27]

$$Y(s) = Y_{per}(s) + \frac{3\pi}{\alpha^2} C^{(0)} \sum_{V_i=\psi_i} \kappa_i \frac{m_{V_i}\Gamma(V_i \to \ell^+\ell^-)}{m_{V_i}^2 - s m_B^2 - i m_{V_i}\Gamma_{V_i}} , \tag{6}$$

where $C^{(0)} = 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6$. The phenomenological parameters κ_i can be fixed from $\mathcal{B}(B \to K^* V_i \to K^* \ell^+\ell^-) = \mathcal{B}(B \to K^* V_i)\,\mathcal{B}(V_i \to \ell^+\ell^-)$, where the data for the right hand side is given in [28]. For the lowest resonances J/ψ nad ψ' we will use $\kappa = 1.65$ and $\kappa = 2.36$, respectively. In our numerical analysis we use the average of J/ψ and ψ' for the higher resonances $\psi^{(i)}$ (see [29]).

It follows from Eq. (3) that in order to calculate the decay width and other physical observables of the exclusive $B \to K^* \ell^+\ell^-$ decay, the matrix elements $\langle K^* \left| \bar{s}\gamma_\mu(1-\gamma_5)b \right| B\rangle$ and $\langle K^* \left| \bar{s} i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b \right|\rangle$ have to be calculated. In other words, the exclusive $B \to K^* \ell^+\ell^-$ decay which is described in terms of the matrix elements of the quark operators given in Eq. (3) over meson states, can be parametrized in terms of form factors. For the vector meson K^* with polarization vector ε_μ the semileptonic form factors of the V–A current is defined as

$$\begin{aligned}
&\langle K^*(p,\varepsilon) \left| \bar{s}\gamma_\mu(1-\gamma_5)b \right| B(p_B)\rangle = \\
&\quad -\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p^\rho q^\sigma \frac{2V(q^2)}{m_B + m_{K^*}} - i\varepsilon_\mu (m_B + m_{K^*}) A_1(q^2) + i(p_B + p_{K^*})_\mu (\varepsilon^* q) \frac{A_2(q^2)}{m_B + m_{K^*}} \\
&\quad + i q_\mu \frac{2m_{K^*}}{q^2} (\varepsilon^* q) \left[A_3(q^2) - A_0(q^2) \right] ,
\end{aligned} \tag{7}$$

where ε is the polarization vector of K^* meson and $q = p_B - p_{K^*}$ is the momentum transfer. Using the equation of motion, the form factor $A_3(q^2)$ can be written in terms of the form factors $A_1(q^2)$ $A_2(q^2)$ as follows

$$A_3 = \frac{m_B + m_{K^*}}{2m_{K^*}} A_1 - \frac{m_B - m_{K^*}}{2m_{K^*}} A_2 . \tag{8}$$

In order to ensure finiteness of (8) at $q^2 = 0$, we demand that $A_3(q^2 = 0) = A_0(q^2 = 0)$. The semileptonic form factors coming from the dipole operator $\sigma_{\mu\nu}q^\nu(1+\gamma_5)b$ are defined as

$$\langle K^*(p,\varepsilon)\,|\bar{s} i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|\,B(p_B)\rangle =
4\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p^\rho q^\sigma T_1(q^2) + 2i\left[\varepsilon^*_\mu(m_B^2-m_{K^*}^2) - (p_B+p_{K^*})_\mu(\varepsilon^* q)\right]T_2(q^2)
+2i(\varepsilon^* q)\left[q_\mu - (p_B+p_{K^*})_\mu \frac{q^2}{m_B^2-m_{K^*}^2}\right]T_3(q^2)\ . \qquad (9)$$

Using the form factors, the matrix element of the $B \to K^*\ell^+\ell^-$ decay takes the following form

$$\mathcal{M} = \frac{G\alpha}{4\sqrt{2}\pi}V_{tb}V_{ts}^*\Bigg\{\Big[(C_9^{tot}-C_{10}^{tot})\bar{\ell}\gamma_\mu(1-\gamma_5)\ell + (C_9^{tot}+C_{10}^{tot})\bar{\ell}\gamma_\mu(1+\gamma_5)\ell\Big]
\times\Bigg[-\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p^\rho_{K^*}q^\sigma\frac{2V(q^2)}{m_B+m_{K^*}} - i\varepsilon^*_\mu(m_B+m_{K^*})A_1(q^2) + i(p_B+p_{K^*})_\mu(\varepsilon^* q)\frac{A_2(q^2)}{m_B+m_{K^*}}
+iq_\mu\frac{2m_{K^*}}{q^2}(\varepsilon^* q)\Big[A_3(q^2)-A_0(q^2)\Big]\Bigg] - 4\frac{C_7^{tot}}{q^2}m_b\Bigg[4\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p^\rho_{K^*}q^\sigma T_1(q^2) + 2i\Big(\varepsilon^*_\mu(m_B^2-m_{K^*}^2)
+(p_B+p_{K^*})_\mu(\varepsilon^* q)\Big)T_2(q^2) + 2i(\varepsilon^* q)\left(q_\mu - (p_B+p_{K^*})_\mu\frac{q^2}{m_B^2-m_{K^*}^2}\right)T_3(q^2)\Bigg]\bar{\ell}\gamma_\mu\ell\Bigg\}\ . \qquad (10)$$

From Eqs. (7), (9) and (10) we observe that in calculating the physical observables at hadronic level, i.e., for the $B \to K^*\ell^+\ell^-$ decay, we face the problem of computing the form factors. This problem is related to the nonperturbative sector of QCD and it can be solved only in framework a nonperturbative approach. In the present work we choose light cone QCD sum rules method predictions for the form factors. In what follows we will use the results of the work [30, 31, 32] in which the form factors are described by a three–parameter fit where the radiative corrections up to leading twist contribution and SU(3)–breaking effects are taken into account. Letting

$$F(q^2) \in \{V(q^2), A_0(q^2), A_1(q^2), A_2(q^2), A_3(q^2), T_1(q^2), T_2(q^2), T_3(q^2)\}\ ,$$

the q^2–dependence of any of these form factors could be parametrized as [30, 31]

$$F(s) = \frac{F(0)}{1 - a_F\, s + b_F\, s^2}\ ,$$

where the parameters $F(0)$, a_F and b_F are listed in Table 3 for each form factor.

Using this matrix element and the helicity amplitude formalism (see for example [33, 34, 35]), we get for the $B \to K^*\ell^+\ell^-$ decay width

	$F(0)$	a_F	b_F
$A_0^{B\to K^*}$	0.47	1.64	0.94
$A_1^{B\to K^*}$	0.35	0.54	−0.02
$A_2^{B\to K^*}$	0.30	1.02	0.08
$V^{B\to K^*}$	0.47	1.50	0.51
$T_1^{B\to K^*}$	0.19	1.53	1.77
$T_2^{B\to K^*}$	0.19	0.36	−0.49
$T_3^{B\to K^*}$	0.13	1.07	0.16

Table 2: The form factors for $B \to K^* \ell^+ \ell^-$ in a three–parameter fit [30].

$$\begin{aligned}\frac{d\Gamma}{dq^2 dx} &= \frac{G^2\alpha^2}{2^{14}\pi^5 m_B} \left|V_{tb}V_{ts}^*\right|^2 v\lambda^{1/2}(1,r,s) \\ &\times\Big\{ \left|\mathcal{M}_+^{+-}\right|^2 + \left|\mathcal{M}_-^{+-}\right|^2 + \left|\mathcal{M}_+^{++}\right|^2 + \left|\mathcal{M}_-^{++}\right|^2 + \left|\mathcal{M}_+^{-+}\right|^2 + \left|\mathcal{M}_-^{-+}\right|^2 + \left|\mathcal{M}_+^{--}\right|^2 \\ &+ \left|\mathcal{M}_-^{--}\right|^2 + \left|\mathcal{M}_0^{++}\right|^2 + \left|\mathcal{M}_0^{+-}\right|^2 + \left|\mathcal{M}_0^{-+}\right|^2 + \left|\mathcal{M}_0^{--}\right|^2 \Big\} , \qquad (11)\end{aligned}$$

where superscripts denote helicities of the leptons and subscripts correspond to the helicity of the vector meson (in our case K^* meson). In Eq. (11)

$$\begin{aligned}&\lambda(1,r,s) = 1 + r^2 + s^2 - 2rs - 2r - 2s \ , \\ &q^2 = (p_B - p_{K^*})^2 \ , \\ &v = \sqrt{1 - 4m_\ell^2/q^2}, \quad \text{(velocity of the lepton), and} \\ &x = \cos\theta, \quad (\theta = \text{angle between } K^* \text{ and } \ell^-), \\ &r = m_{K^*}^2/m_B^2 \ .\end{aligned}$$

The explicit forms of the helicity amplitude $\mathcal{M}_{\lambda_V}^{\lambda_\ell\lambda_\ell}$ are as follows:

$$\begin{aligned}\mathcal{M}_\pm^{++} &= \pm\sqrt{2} m_\ell \sin\theta \left(2C_9^{tot} H_\pm + 4C_7^{tot}\frac{m_b}{q^2}\mathcal{H}_\pm\right) \ , \\ \mathcal{M}_\pm^{+-} &= (-1 \pm \cos\theta)\sqrt{\frac{q^2}{2}}\left\{\left[2\left(C_9^{tot} + vC_{10}^{tot}\right)\right]H_\pm + 4C_7^{tot}\frac{m_b}{q^2}\mathcal{H}_\pm\right\} , \\ \mathcal{M}_\pm^{-+} &= (1 \pm \cos\theta)\sqrt{\frac{q^2}{2}}\left[2\left(C_9^{tot} - vC_{10}^{tot}\right)H_\pm + 4C_7^{tot}\frac{m_b}{q^2}\mathcal{H}_\pm\right] , \\ \mathcal{M}_\pm^{--} &= -\left(\mathcal{M}_\pm^{++}\right) \ ,\end{aligned}$$

$$\mathcal{M}_0^{++} = 2m_\ell \cos\theta \left(2C_9^{tot} H_0 - 4C_7^{tot} \frac{m_b}{q^2} \mathcal{H}_0 \right) + 4m_\ell C_{10}^{tot} H_S^0 \, ,$$

$$\mathcal{M}_0^{+-} = -\sqrt{q^2} \sin\theta \left[2\left(C_9^{tot} + vC_{10}^{tot} \right) H_0 - 4C_7^{tot} \frac{m_b}{q^2} \mathcal{H}_0 \right] \, ,$$

$$\mathcal{M}_0^{-+} = \mathcal{M}_0^{+-}(v \to -v) \, ,$$

$$\mathcal{M}_0^{--} = -2m_\ell \cos\theta \left(2C_9^{tot} H_0 - 4C_7^{tot} \frac{m_b}{q^2} \mathcal{H}_0 \right) + 4m_\ell C_{10}^{tot} H_S^0 \, , \qquad (12)$$

where

$$H_\pm = m_B \left[\pm \lambda^{1/2}(1,r,s) \frac{V(q^2)}{1+\sqrt{r}} + (1+\sqrt{r}) A_1(q^2) \right] \, ,$$

$$H_0 = \frac{m_B}{2\sqrt{rs}} \left[-(1-r-s)(1+\sqrt{r}) A_1(q^2) + \lambda(1,r,s) \frac{A_2(q^2)}{1+\sqrt{r}} \right] \, ,$$

$$H_S^0 = -\frac{m_B \lambda^{1/2}(1,r,s)}{\sqrt{s}} A_0(q^2)$$

$$\mathcal{H}_\pm = 2m_B^2 \left[\pm \lambda^{1/2}(1,r,s) T_1(q^2) + (1-r) T_2(q^2) \right] \, ,$$

$$\mathcal{H}_0 = \frac{m_B^2}{\sqrt{rs}} \left\{ (1-r)(1-r-s) T_2(q^2) - \lambda(1,r,s) \left[T_2(q^2) + \frac{s}{1-r} T_3(q^2) \right] \right\} \, , \qquad (13)$$

In the present paper, we study the dependence of the following measurable physical quantities, such as

(i) Γ_+/Γ_-,

(ii) $\Gamma_L/\Gamma_T = \Gamma_0/(\Gamma_+ + \Gamma_-)$,

(iii) the lepton forward–backward asymmetry

on q^2 and on the t' quark mass for the fixed values of the "new" CKM factor $V_{t's}^* V_{t'b}$. Here the subscripts 0, L and T indicate the helicities of the K^* meson, respectively. From Eq. (11), we can easily obtain the explicit expressions for Γ_+, Γ_- and Γ_0 as

$$\Gamma_\pm = \frac{G^2 \alpha^2}{2^{14} \pi^5 m_B} |V_{tb} V_{ts}^*|^2 \int dq^2 \int dx \, v \lambda^{1/2} \left\{ \left| \mathcal{M}_\pm^{+-} \right|^2 + \left| \mathcal{M}_\pm^{++} \right|^2 + \left| \mathcal{M}_\pm^{-+} \right|^2 + \left| \mathcal{M}_\pm^{--} \right|^2 \right\} \, , \qquad (14)$$

where the upper(lower) subscript in Γ corresponds to $\mathcal{M}_+ (\mathcal{M}_-)$ and

$$\Gamma_0 = \frac{G^2 \alpha^2}{2^{14} \pi^5 m_B} |V_{tb} V_{ts}^*|^2 \int dq^2 \int dx \, v \lambda^{1/2} \left\{ \left| \mathcal{M}_0^{+-} \right|^2 + \left| \mathcal{M}_0^{++} \right|^2 + \left| \mathcal{M}_0^{-+} \right|^2 + \left| \mathcal{M}_0^{--} \right|^2 \right\} \, . \qquad (15)$$

From Eqs. (14) and (15) the expressions for the ratios Γ_+/Γ_- and $\Gamma_L/\Gamma_T = \Gamma_0/(\Gamma_+ + \Gamma_-)$ can easily be obtained. These quantities are measurable from the experiments.

The normalized forward–backward asymmetry A_{FB} is one of the most useful tools in search of new physics beyond SM. Especially the determination of the position of the zero of A_{FB} can predict about new physics [36]. Indeed, existence of the new physics can be confirmed by the shift in the position of the zero of the forward–backward asymmetry. This shift of zero position can be used in looking for new physics. Therefore in the present work the forward–backward asymmetry A_{FB} is considered, which defined in the following way

$$\frac{d}{dq^2} A_{FB}(q^2) = \frac{\displaystyle\int_0^1 dx \frac{d\Gamma}{dq^2 dx} - \int_{-1}^0 dx \frac{d\Gamma}{dq^2 dx}}{\displaystyle\int_0^1 dx \frac{d\Gamma}{dq^2 dx} + \int_{-1}^0 dx \frac{d\Gamma}{dq^2 dx}} \tag{16}$$

To obtain quantitative results we need the value of the fourth generation CKM matrix element $|V_{t's}^* V_{tb}|$. For this aim following [20], we will use the experimental results of the decays $\mathcal{B}(B \to X_s \gamma)$ and $\mathcal{B}(B \to X_c e \bar{\nu}_e)$ to determine the fourth generation CKM factor $V_{t's}^* V_{t'b}$. In order to reduce the uncertainties arising from b–quark mass, we consider the following ratio

$$R = \frac{\mathcal{B}(B \to X_s \gamma)}{\mathcal{B}(B \to X_c e \bar{\nu}_e)} .$$

In leading logarithmic approximation this ratio can be written as

$$R = \frac{\left|V_{ts}^* V_{tb}\right|^2}{\left|V_{cb}\right|^2} \frac{6\alpha \left|C_7^{tot}(m_b)\right|^2}{\pi f(\hat{m}_c) \kappa(\hat{m}_c)} , \tag{17}$$

where the phase factor $f(\hat{m}_c)$ and $\mathcal{O}(\alpha_s)$ QCD correction factor $\kappa(\hat{m}_c)$ [37] of $b \to c \ell \bar{\nu}$ are given by

$$\begin{aligned} f(\hat{m}_c) &= 1 - 8\hat{m}_c^2 + 8\hat{m}_c^6 - \hat{m}_c^8 - 24\hat{m}_c^4 \ln(\hat{m}_c^4) , \\ \kappa(\hat{m}_c) &= 1 - \frac{2\alpha_s(m_b)}{3\pi} \left[\left(\pi^2 - \frac{31}{4} \right) (1 - \hat{m}_c)^2 + \frac{3}{2} \right] . \end{aligned} \tag{18}$$

Solving Eq. (17) for $V_{t's}^* V_{t'b}$ and taking into account Eqs. (2) and (18), we get

$$V_{t's}^* V_{t'b}^{\pm} = \left[\pm \sqrt{\frac{\pi R \left|V_{cb}\right|^2 f(\hat{m}_c) \kappa(\hat{m}_c)}{6\alpha \left|V_{ts}^* V_{tb}\right|^2}} - C_7^{SM}(m_b) \right] \frac{V_{ts}^* V_{tb}}{C_7^{new}(m_b)} . \tag{19}$$

It is observed from Eq. (19) that $V_{t's}^* V_{t'b}$ depends on the t'–quark mass and its values for different choices of the t'–quark mass and for the experimentally measured branching ratio $\mathcal{B}(B \to X_s \gamma) = 3.15 \times 10^{-4}$ [38] (see also [20]), are listed in Table 3 (Here we present only the central value. It should be noted that the LEP result on $(B \to X_s \gamma)$ [39] coincide with the CLEO result within the error limits and for this reason we don't present LEP result.

$m_{t'}$ (GeV)	50	100	150	200	250	300	400
$V_{t's}^* V_{t'b}^{(+)} \times 10^{-2}$	-14.48	-10.01	-8.37	-7.55	-7.07	-6.75	-6.36
$V_{t's}^* V_{t'b}^{(-)} \times 10^{-3}$	3.45	2.39	2.00	1.80	1.69	1.61	1.52

Table 3: The numerical values of $V_{t's}^* V_{t'b}$ for different values of the t'–quark mass. The superscripts $(+)$ and $(-)$ correspond to the respective signs in front of the square root in Eq. (19)

From unitarity condition of the CKM matrix we have

$$V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} + V_{t's}^* V_{t'b} = 0 \ . \tag{20}$$

If the average values of the CKM matrix elements in the SM [28]are used, the sum of the first three terms in Eq. (20) is about 7.6×10^{-2}. Substituting the value of $V_{t's}^* V_{t'b}^{(+)}$ from Table 3, we observe that the sum of the four terms on the left–hand side of Eq. (19) is closer to zero compared to the SM case, since $V_{t's}^* V_{t'b}$ is very close to the sum of the first three terms, but with opposite sign. On the other if we consider $V_{t's}^* V_{t'b}^{(-)}$, whose value is about 10^{-3} and one order of magnitude smaller compared to the previous case. However it should be noted that the data for the CKM is not determined to a very high accuracy, and the error in sum of first three terms in Eq. (19) is about $\pm 0.6 \times 10^{-2}$. It is easy to see then that the value of $V_{t's}^* V_{t'b}^{(-)}$ is within this error range. In summary both $V_{t's}^* V_{t'b}^{(+)}$ and $V_{t's}^* V_{t'b}^{(-)}$ satisfy the unitarity condition (19) of CKM. Moreover, since $\left| V_{t's}^* V_{t'b}^{(-)} \right| \leq 10^{-1} \times \left| V_{t's}^* V_{t'b}^{(+)} \right|$, $V_{t's}^* V_{t'b}^{(-)}$ contribution to the physical quantities should be practically indistinguishable from SM results, and our numerical analysis confirms this expectation.

3 Numerical analysis

Having the explicit expressions for the physically measurable quantities, in this Section we will study the influence of the fourth generation to these quantities. The values of the main input parameters, which appear in the expression for the decay widths Γ_0, Γ_+, Γ_- and A_{FB} are:

$$\begin{aligned} m_b &= 4.8\ GeV, \quad m_c = 1.35\ GeV, \quad m_\tau = 1.78\ GeV, \\ m_\mu &= 0.105\ GeV, \quad m_B = 5.28\ GeV, \quad m_{K^*} = 0.892\ GeV. \end{aligned}$$

The invariant dilepton mass distribution for the $B \to K^* \mu^+ \mu^-$ and $B \to K^* \tau^+ \tau^-$ decays, with and without the long distance effects are presented in Fig. 1 and Fig. 2, respectively, for the choice of $V_{t's}^* V_{t'b}^{(-)}$, and at $m_{t'} = 50\ GeV, 100\ GeV, 150\ GeV$ and $200\ GeV$. From these figures we see that the predictions of the fourth generation model and SM on the differential branching ratio practically coincide in this case. This result can be explained by the fact that $V_{t's}^* V_{t'b}^{(-)}$ is of the order of 10^{-3} and it is one order of smaller than $V_{ts}^* V_{tb} = 0.038$ [28] in the SM. For this reason, the effect of the fourth generation in this case is small. Similar conclusion for the same case can be drawn for the lepton forward–backward asymmetry (see Figs. (3) and (4)). However, when we consider the $V_{t's}^* V_{t'b}^{(+)}$ case, the situation changes essentially and Figs. (5) and (6) depict the invariant mass distribution for the $B \to K^* \mu^+ \mu^-$ and $B \to K^* \tau^+ \tau^-$ decays for this case, respectively. We observe from Fig. (5) that the four generation model predicts lower value compared to SM when $m_{t'}$ is less than m_t. At $m_{t'} = 200\ GeV$, the differential decay rate is enhanced almost twice. The behavior of the differential decay width between J/ψ and ψ' regions differ essentially in the two considered models. Further, we observe from Fig. (6) that, for the $B \to K^* \tau^+ \tau^-$ case at $m_{t'} = 150\ GeV$, four generation model predicts twice as much lower value compared to SM. As $m_{t'}$ increases, departure from SM prediction becomes exaggerated even further.

The dependence of the lepton forward–backward asymmetry A_{FB} on q^2 of the $B \to K^* \mu^+ \mu^-$ and $B \to K^* \tau^+ \tau^-$ decays are depicted in Figs. (7) and (8), for the $V_{t's}^* V_{t'b}^{(+)}$ case. We see from Fig. (7) at $m_{t'} = 50\ GeV$, A_{FB} is positive in the subinterval up to its zero value from origin, while it is negative in the same range at all other choices of $m_{t'}$. Therefore determination of the sign of the lepton forward–backward asymmetry in experiments can yield useful information for establishing new physics. A careful analysis of the same figure suggests that the shift in the position of the A_{FB} for different values of $m_{t'}$ could be an indication of the unambiguous information about the existence of new physics [36].

In Figs. (9) and (10) we investigate the the dependence of the ratios Γ_+/Γ_- and Γ_L/Γ_T on q^2 for both roots of the $V_{t's}^* V_{t'b}$, at different values of the $m_{t'}$ for the $B \to K^* \mu^+ \mu^-$ and $B \to K^* \tau^+ \tau^-$ decays, respectively. As have already been noted, due to the smallness of the $V_{t's}^* V_{t'b}^{(-)}$, these ratios practically display the same behavior as predicted by SM for this choice, confirming our expectation. So, this case can yield no information about new physics. On the other hand, it follows from Fig. (9) that for the $V_{t's}^* V_{t'b}^{(+)}$ case, the essential departure from the SM result is quite obvious in the $B \to K^* \mu^+ \mu^-$ decay for the ratio Γ_+/Γ_-. Moreover, the effect of the fourth generation in the ratio Γ_L/Γ_T is depicted in Fig. (10), whose behavior clearly shows a strong dependence on the value of the $m_{t'}$, especially in the $B \to K^* \tau^+ \tau^-$ decay. Such a dependence can be explained as an indication of the fact that for Γ_L/Γ_T, the terms proportional to the lepton mass can give considerable contribution.

Finally we would like to note that 4×4 CKM matrix contains three CP–violating phases and hence CP violation might be sizeable. We will discuss this problem elsewhere in one of the future works.

To summarize, the exclusive rare $B \to K^* \ell^+ \ell^-$ ($\ell = \mu,\ \tau$) decay has a clean experimental signature and will be measured at the present asymmetric B factories and future hadronic HERA-B, B-TeV and LHC-B machines and is very sensitive to the various extensions of the Standard Model. In the present work this decay is studied in the SM with the four generation model. The two solutions of the fourth generation CKM factor $V_{t's}^* V_{t'b}$ have been used. It is found out that for the choice of the positive root of the factor $V_{t's}^* V_{t'b}^{(+)}$, the measurements of the invariant mass distribution, lepton forward–backward asymmetry and the ratio Γ_+/Γ_- could be quite efficient in establishing the fourth generation in the $B \to K^* \mu^+ \mu^-$ decay, while the measurement of the ratio Γ_L/Γ_T in the $B \to K^* \tau^+ \tau^-$ decay seems to be very informative in searching new physics.

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Figure captions

Fig. 1 The dependence of the invariant dilepton mass distribution for the $B \to K^* \mu^+ \mu^-$ decay on q^2 at different values of $m_{t'}$, with and without the long distance effects, for the choice of $V_{t's}^* V_{t'b}^{(-)}$. In Figs. (1) – (8) the ordering of the lines with respect to different values of $m_{t'}$ are the same, as indicated in Fig. 1.

Fig. 2 Same as Fig. 1 but for the $B \to K^* \tau^+ \tau^-$ decay.

Fig. 3 The dependence of the forward–backward asymmetry for the $B \to K^* \mu^+ \mu^-$ decay on q^2 at different values of $m_{t'}$, with and without the long distance effects, for the choice of $V_{t's}^* V_{t'b}^{(-)}$.

Fig. 4 Same as Fig. 3 but for the $B \to K^* \tau^+ \tau^-$ decay.

Fig. 5 Same as Fig. 1 but for the choice of $V_{t's}^* V_{t'b}^{(+)}$.

Fig. 6 Same as Fig. 2 but for the choice of $V_{t's}^* V_{t'b}^{(+)}$.

Fig. 7 Same as Fig. 3 but for the choice of $V_{t's}^* V_{t'b}^{(+)}$.

Fig. 8 Same as Fig. 4 but for the choice of $V_{t's}^* V_{t'b}^{(+)}$.

Fig. 9 The dependence of the ratio Γ_+/Γ_- on $m_{t'}$ for both roots of the $V_{t's}^* V_{t'b}$, for the $B \to K^* \mu^+ \mu^-$ and $B \to K^* \tau^+ \tau^-$ decays.

Fig. 10 The dependence of the ratio Γ_L/Γ_T on $m_{t'}$ for both roots of the $V_{t's}^* V_{t'b}$, for the $B \to K^* \mu^+ \mu^-$ and $B \to K^* \tau^+ \tau^-$ decays.

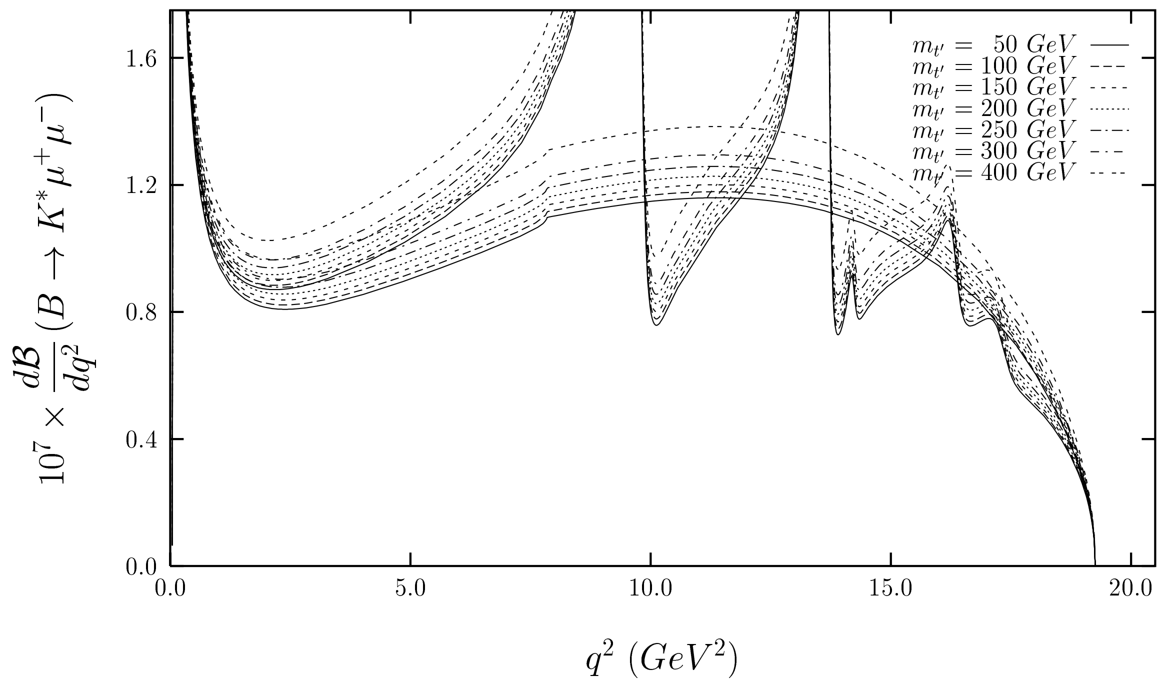
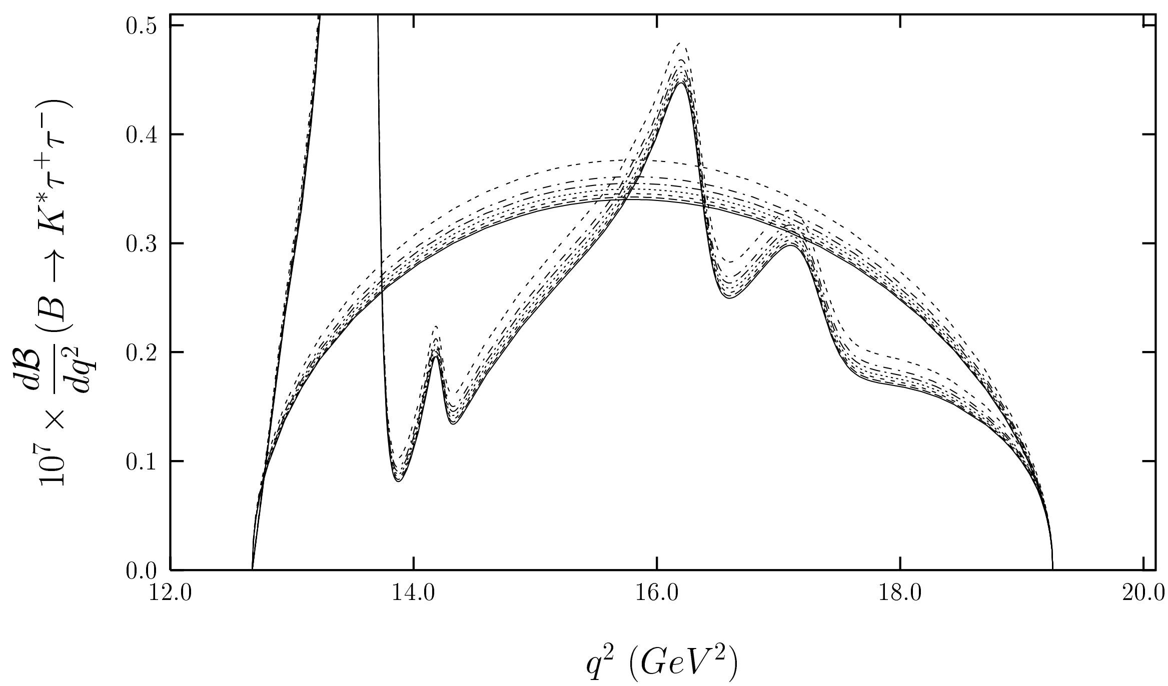


Figure 1:



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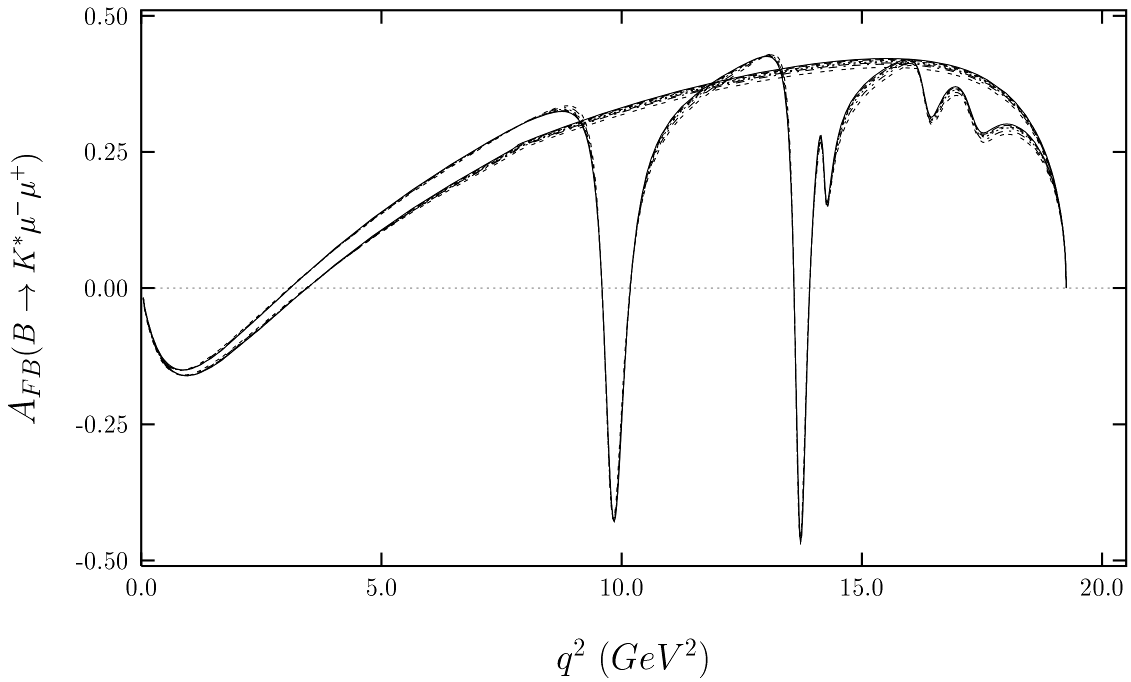

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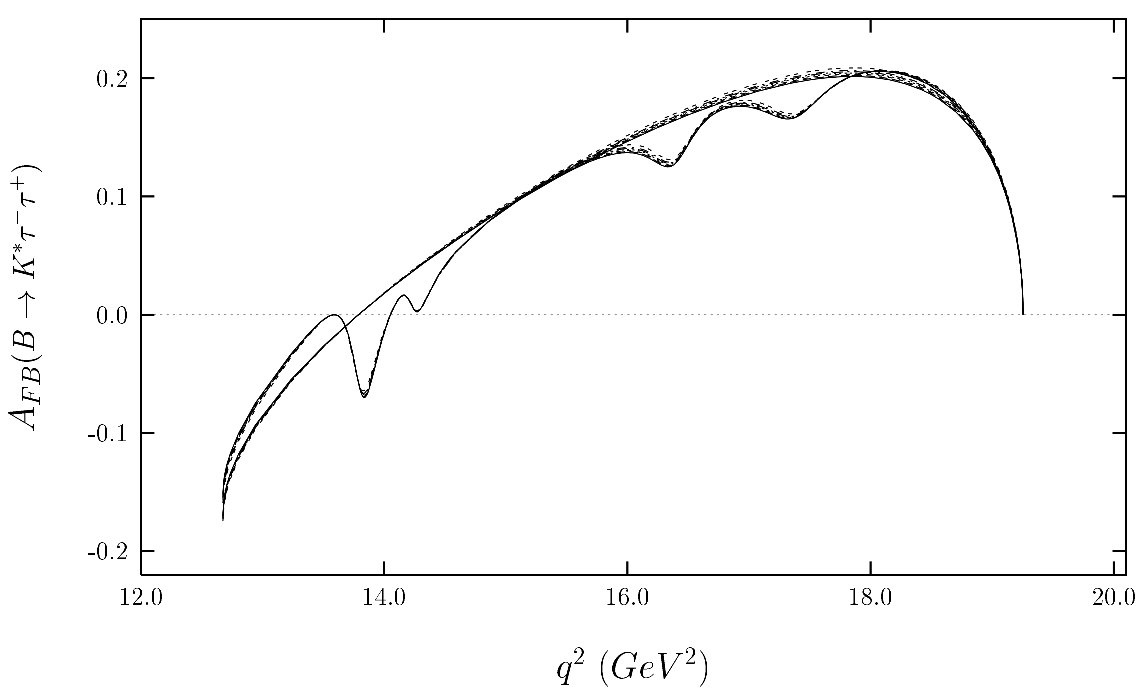

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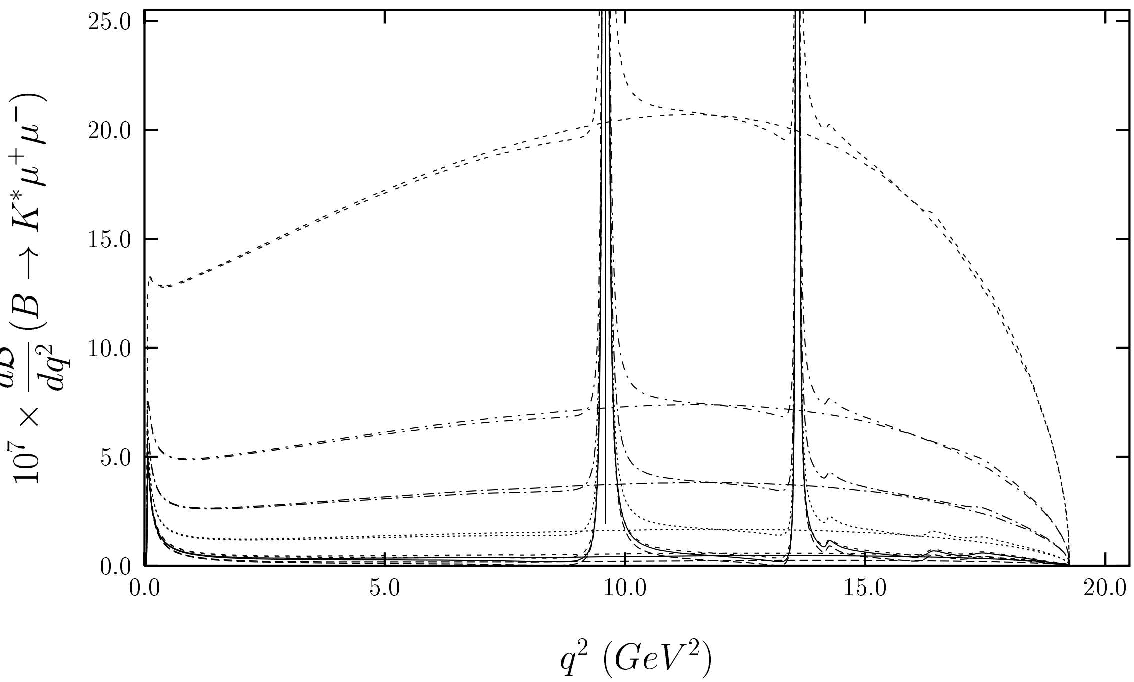

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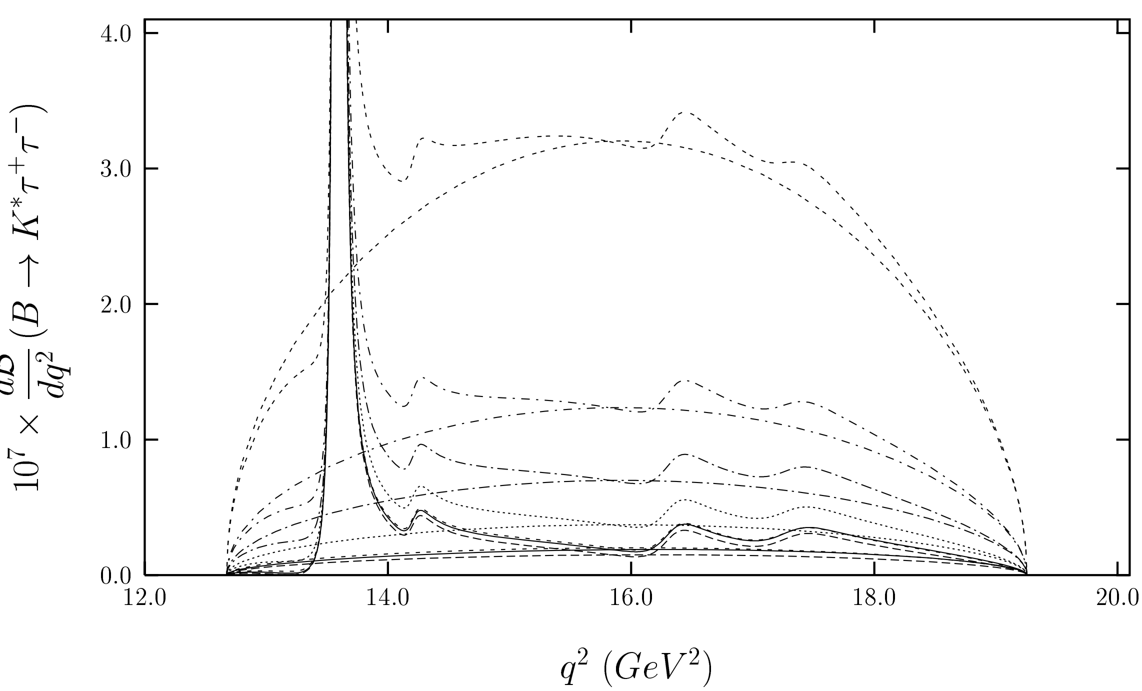

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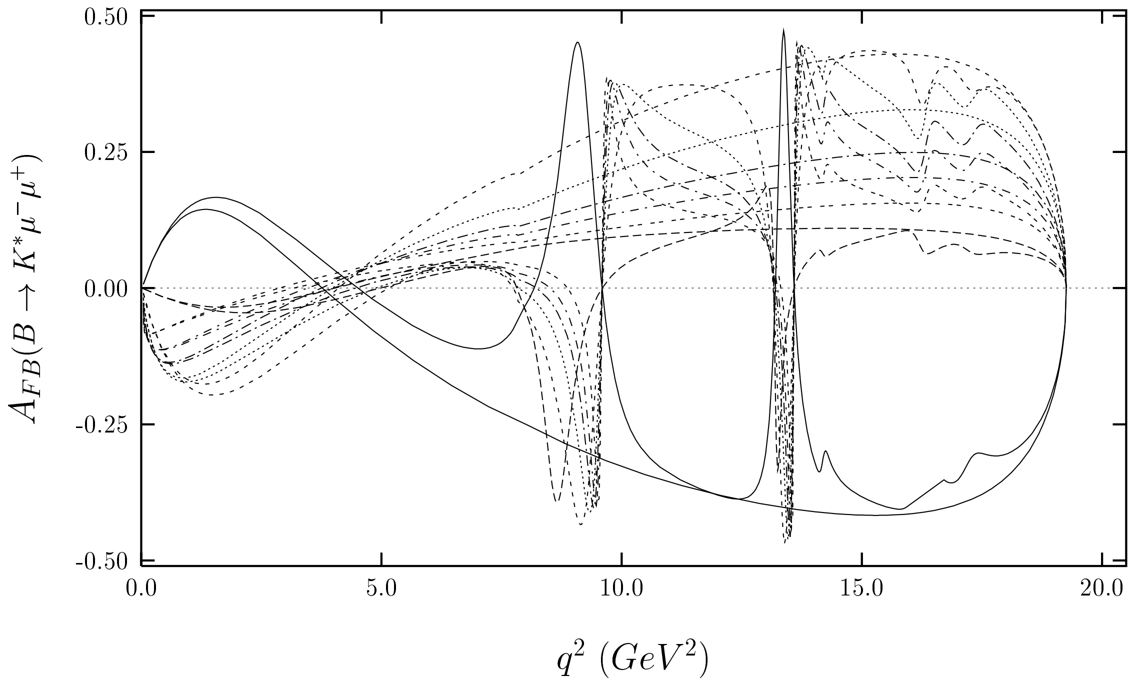

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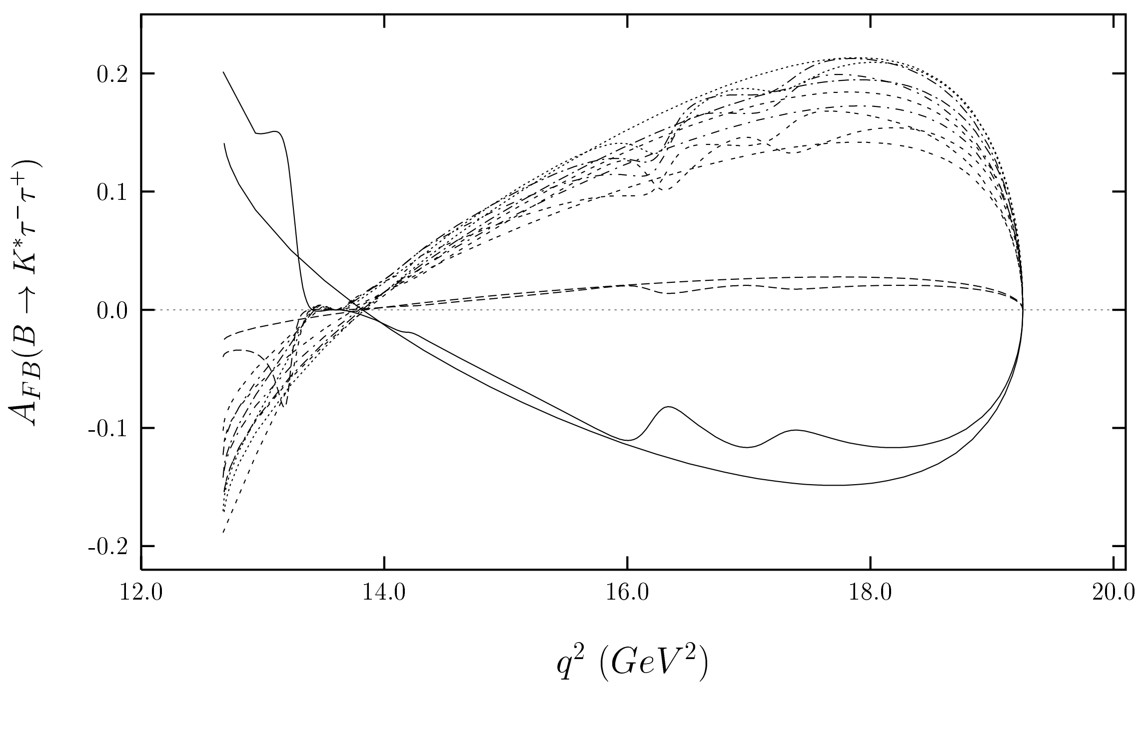

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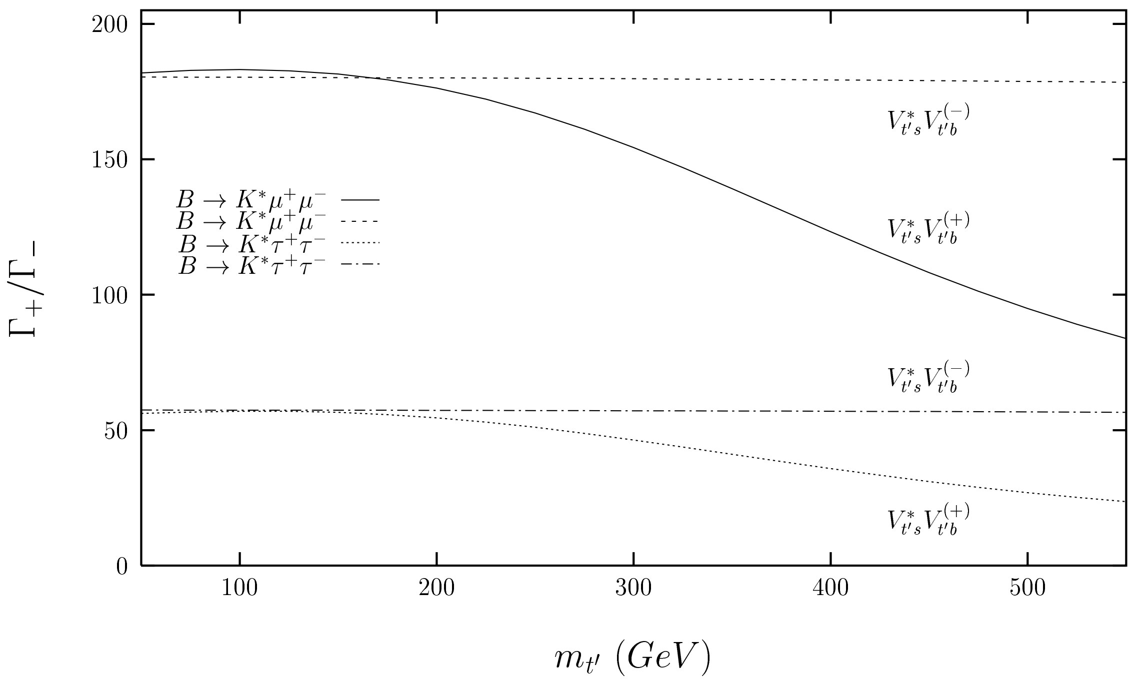

Figure 9:

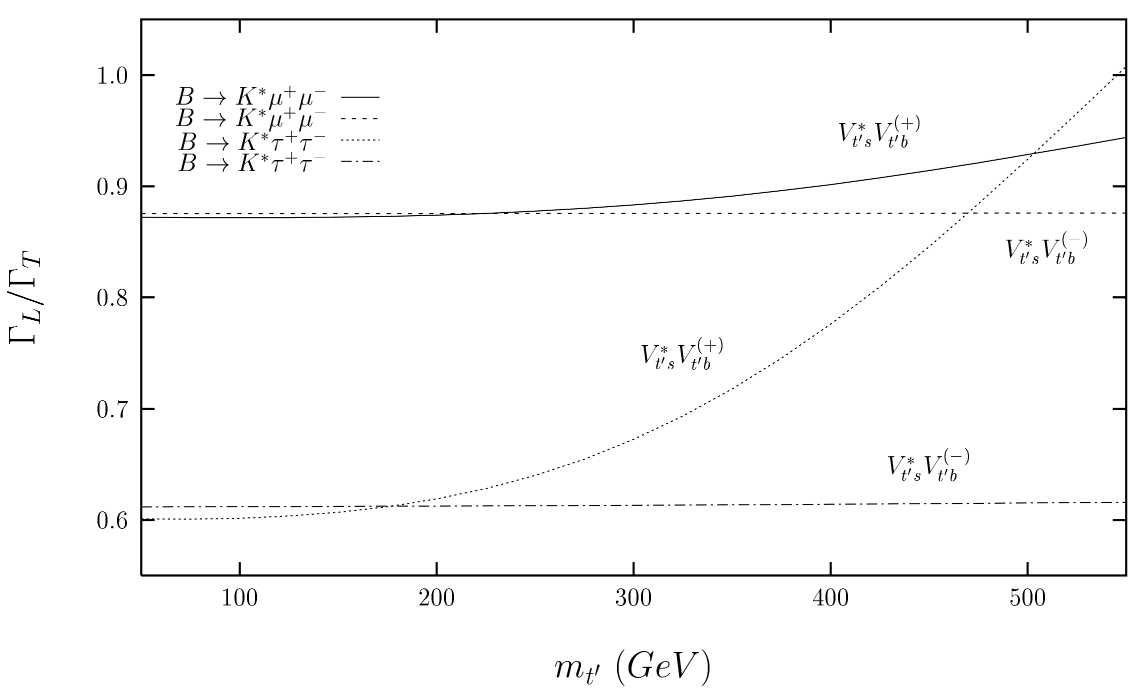

Figure 10: