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Heavy χ_{Q2} tensor mesons in QCD

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ABSTRACT

The masses and decay constants of the ground state heavy χ_{Q2} ($Q = b, c$) tensor mesons are calculated in the framework of the QCD sum rules approach. The obtained results on the masses are in good consistency with the experimental values. Our predictions on the decay constants can be verified in the future experiments.

1. Introduction

During last few years very exiting experimental results are obtained in the charm and beauty meson and baryon spectroscopies [1]. Recent CLEO measurements on the two-photon decay rates of the even-parity, scalar 0^{++}, $\chi_{b(c)0}$ and tensor 2^{++}, $\chi_{b(c)2}$ states ([1,2] and references therein) were motivation to investigate the properties of these mesons and their radiative decays.

In the present work, we calculate the mass and decay constants of the ground state heavy bottomonium, $\chi_{b2}(1P)$ and charmonium, $\chi_{c2}(1P)$ tensor mesons with $I^G(J^{PC}) = 0^+(2^{++})$ in the framework of the QCD sum rules approach. QCD sum rules approach as a non-perturbative approach is one of the most powerful and applicable tools to hadron physics and can play an important role in calculation of the characteristic parameters of the hadrons (for details about this method and some applications see [3,4]). Note that the mass and decay constant of the strange tensor $K_2^*(1430)$ with quantum numbers $I(J^P) = 1/2(2^+)$ have been calculated in [5] in the same framework. These parameters for light unflavored tensor mesons have also been calculated in [6]. The obtained results for the masses and decay constants are used in calculation of the magnetic dipole moments of the light tensor mesons using the QCD sum rules method in [7].

The Letter is organized as follows: in next section, sum rules for the mass and decay constant of the ground state heavy quarkonia, χ_{Q2} tensor mesons are derived in the context of the QCD sum rules method. Section 3 is devoted to the numerical analysis of the mass and decay constants as well as the comparison of the obtained results on the mass with the experimental values.

2. Theoretical framework

In this section, we obtain the sum rules for the mass and decay constant of the heavy $\chi_{Q2}(1P)$ tensor meson in the framework of the QCD sum rules approach. For this aim we consider the following correlation function

$$\Pi_{\mu\nu,\alpha\beta} = i \int d^4x\, e^{iq(x-y)} \langle 0|\mathcal{T}\big[j_{\mu\nu}(x)\bar{j}_{\alpha\beta}(y)\big]|0\rangle, \qquad (1)$$

where, $j_{\mu\nu}$ is the interpolating current of the $\chi_{Q2}(1P)$ tensor meson and $\mathcal{T}$ is the time ordering operator. The explicit form of the current $j_{\mu\nu}$ creating the ground state heavy tensor $\chi_{Q2}(1P)$ state with quantum numbers $I^G(J^{PC}) = 0^+(2^{++})$ from the vacuum can be written in the following form:

$$j_{\mu\nu}(x) = \frac{i}{2}\big[\bar{Q}(x)\gamma_\mu \overleftrightarrow{\mathcal{D}}_\nu(x) Q(x) + \bar{Q}(x)\gamma_\nu \overleftrightarrow{\mathcal{D}}_\mu(x) Q(x)\big], \qquad (2)$$

where Q stands for heavy b or c quark and the $\overleftrightarrow{\mathcal{D}}_\mu(x)$ represents the derivative with respect to four-x acting on left and right, simultaneously. This two-side covariant derivative is defined as:

$$\overleftrightarrow{\mathcal{D}}_\mu(x) = \frac{1}{2}\big[\overrightarrow{\mathcal{D}}_\mu(x) - \overleftarrow{\mathcal{D}}_\mu(x)\big], \qquad (3)$$

where,

$$\overrightarrow{\mathcal{D}}_\mu(x) = \overrightarrow{\partial}_\mu(x) - i\frac{g}{2}\lambda^a A_\mu^a(x),$$
$$\overleftarrow{\mathcal{D}}_\mu(x) = \overleftarrow{\partial}_\mu(x) + i\frac{g}{2}\lambda^a A_\mu^a(x). \qquad (4)$$

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In the above relations, the λ^a are the Gell-Mann matrices and $A^a_\mu(x)$ are the external (vacuum) gluon fields, which can be expressed directly in terms of the gluon field strength tensor using the Fock–Schwinger gauge, $x^\mu A^a_\mu(x) = 0$, as the following way:

$$A^a_\mu(x) = \int_0^1 d\alpha\, \alpha x_\beta G^a_{\beta\mu}(\alpha x) = \frac{1}{2} x_\beta G^a_{\beta\mu}(0) + \frac{1}{3} x_\eta x_\beta \mathcal{D}_\eta G^a_{\beta\mu}(0) + \cdots. \tag{5}$$

Since the current contains derivatives with respect to the space–time so we will consider the two currents at points x and y. After doing calculations and applying the derivatives, we will set $y = 0$.

It is well known that in the QCD sum rules approach, the correlation function in Eq. (1) is calculated in two different ways. The physical or phenomenological part, which is obtained in terms of the hadronic parameters such as mass and decay constant inserting a complete set of the states owing the same quantum numbers as the interpolating current $j_{\mu\nu}$. The theoretical or QCD part, which is calculated in terms of the QCD parameters such as quark masses, quark condensates and quark–gluon coupling constants, etc. The correlation function in this part is calculated in deep Euclidean region, $q^2 \ll 0$, via operator product expansion (OPE). The short distance effects are calculated via the perturbation theory, whereas the long distance contributions, which are non-perturbative effects are parameterized in terms of the quark–quark, gluon–gluon and quark–gluon condensates. The sum rules for the observables (masses and decay constants) of the ground state $\chi_{Q2}(1P)$ meson are obtained equating both representations of the correlation function, isolating the ground state and applying Borel transformation to suppress the contribution of the higher states and continuum through the dispersion relation.

To proceed first we calculate the phenomenological part. Inserting a complete set of intermediate state, $\chi_{Q2}(1P)$ to time ordering product in Eq. (1), and performing integral over x we get:

$$\Pi_{\mu\nu,\alpha\beta} = \frac{\langle 0|j_{\mu\nu}(0)|\chi_{Q2}\rangle\langle\chi_{Q2}|j_{\alpha\beta}(0)|0\rangle}{m^2_{\chi_{Q2}} - q^2} + \cdots, \tag{6}$$

where $\cdots$ denotes the contribution of the higher states and continuum. From the above relation, it is clear that we need to know the matrix element, $\langle 0|j_{\mu\nu}(0)|\chi_{Q2}\rangle$, which can be parameterized in terms of the decay constant, $f_{\chi_{Q2}}$:

$$\langle 0|j_{\mu\nu}(0)|\chi_{Q2}\rangle = f_{\chi_{Q2}} m^3_{\chi_{Q2}} \varepsilon_{\mu\nu}, \tag{7}$$

where $\varepsilon_{\mu\nu}$ is the polarization tensor of χ_{Q2} meson. Using Eq. (7) in Eq. (6), we obtain the following final representation of the correlation function in phenomenological side:

$$\Pi_{\mu\nu,\alpha\beta} = \frac{f^2_{\chi_{Q2}} m^6_{\chi_{Q2}}}{m^2_{\chi_{Q2}} - q^2} \left\{ \frac{1}{2}(g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha}) \right\} + \text{other structures} + \cdots, \tag{8}$$

where, the only structure which contains a contribution of the tensor meson has been kept. In calculations, we have performed summation over the polarization tensor using

$$\varepsilon_{\mu\nu}\varepsilon^*_{\alpha\beta} = \frac{1}{2} T_{\mu\alpha} T_{\nu\beta} + \frac{1}{2} T_{\mu\beta} T_{\nu\alpha} - \frac{1}{3} T_{\mu\nu} T_{\alpha\beta}, \tag{9}$$

where,

$$T_{\mu\nu} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{m^2_{\chi_{Q2}}}. \tag{10}$$

The next step is to calculate the theoretical or QCD side of the correlation function in deep Euclidean region, $q^2 \ll 0$. Using the explicit expression for the tensor current presented in Eq. (2) inside the correlation function shown in Eq. (1) and contracting out all quark pairs using the Wick's theorem, we obtain the following expression for the QCD side:

$$\Pi_{\mu\nu,\alpha\beta} = \frac{i}{4} \int d^4x\, e^{iq(x-y)} \times \left\{ Tr\left[S_Q(y-x)\gamma_\mu \overleftrightarrow{\mathcal{D}}_\nu(x) \overleftrightarrow{\mathcal{D}}_\beta(y) S_Q(x-y)\gamma_\alpha \right] + [\beta \leftrightarrow \alpha] + [\nu \leftrightarrow \mu] + [\beta \leftrightarrow \alpha, \nu \leftrightarrow \mu] \right\}. \tag{11}$$

To proceed we need to know the heavy quark propagator, $S_Q(x-y)$. This propagator has been calculated in [8]:

$$S_Q(x-y) = S_b^{free}(x-y) - ig_s \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} \times \int_0^1 dv \left[\frac{\not k + m_Q}{(m_Q^2 - k^2)^2} G^{\mu\nu}\left[v(x-y)\right]\sigma_{\mu\nu} + \frac{1}{m_Q^2 - k^2} v(x_\mu - y_\mu) G^{\mu\nu}\gamma_\nu \right], \tag{12}$$

where,

$$S_Q^{free}(x-y) = \frac{m_Q^2}{4\pi^2} \frac{K_1(m_Q\sqrt{-(x-y)^2})}{\sqrt{-(x-y)^2}} - i\frac{m_Q^2(\not x - \not y)}{4\pi^2(x-y)^2} K_2\left(m_Q\sqrt{-(x-y)^2}\right), \tag{13}$$

and $K_n(z)$ being the modified Bessel function of the second kind. The next step is to use the expression of the heavy propagators and perform the derivatives with respect to x and y in Eq. (11). After setting $y = 0$, the following final expression for the QCD side of the correlation function in coordinate space is obtained:

$$\Pi_{\mu\nu,\alpha\beta} = \frac{i}{64}\left(\frac{m_Q}{\pi}\right)^4 \times \int d^4x\, e^{iqx}\left\{ [\Gamma_{\mu\nu,\alpha\beta}] + [\beta \leftrightarrow \alpha] + [\nu \leftrightarrow \mu] + [\beta \leftrightarrow \alpha, \nu \leftrightarrow \mu] \right\}, \tag{14}$$

where,

$$\begin{aligned}\Gamma_{\mu\nu,\alpha\beta} = \{ & -2m_Q g_{\alpha\nu} g_{\beta\mu} \mathcal{K}_1 \mathcal{K}_2 \\ & + 2\left(m_Q^2 x_\alpha x_\nu g_{\beta\mu} - g_{\alpha\nu} g_{\beta\mu} + g_{\alpha\mu} g_{\beta\nu} + g_{\alpha\beta} g_{\mu\nu}\right)\mathcal{K}_2^2 \\ & - 2m_Q^2 x_\alpha x_\nu g_{\beta\mu} \mathcal{K}_1 \mathcal{K}_3 \\ & - 2m_Q g_{\alpha\nu}\left(2x_\beta x_\mu - x^2 g_{\beta\mu}\right)\mathcal{K}_2 \mathcal{K}_3 \\ & + 2m_Q^2 x_\alpha x_\nu \left(2x_\beta x_\mu - x^2 g_{\beta\mu}\right)\mathcal{K}_3^2 \\ & - 2m_Q^2 x_\alpha x_\nu \left(2x_\beta x_\mu - x^2 g_{\beta\mu}\right)\mathcal{K}_2 \mathcal{K}_4 \} \\ & + \text{nonperturbative contributions},\end{aligned} \tag{15}$$

and

$$\mathcal{K}_n = \frac{K_n(m_Q\sqrt{-x^2})}{(\sqrt{-x^2})^n}. \tag{16}$$

In the present work, we calculate the contributions of the heavy quark and gluon condensates in nonperturbative part of the correlation function in QCD side. After a simple calculation we obtain for the heavy quark condensate (for the coefficient of the aforementioned structure)

$$-\frac{m_Q^3}{2(q^2-m_Q^2)}\langle \bar{Q} Q\rangle.$$

Using the well-known relation between the heavy quark and the gluon condensates

$$m_Q\langle \bar{Q} Q\rangle = -\frac{1}{12\pi}\left\langle \frac{\alpha_s}{\pi} G^2\right\rangle,$$

these two nonperturbative contributions can be written in terms of gluon condensate contribution. Numerical analysis shows that, taking into account quark condensates decreases gluon condensate contribution about 15%.

Few words about the neglected dimension two operator in the operator product expansion are in order. The term proportional to $1/q^2$ introduced in [9] is a phenomenological parametrization of the higher order contributions to the perturbative series. In other words, this term can appear when considering any types of correlation functions where the perturbative series are not zero. Obviously, this term vanishes when considering the difference of the correlators induced by vector and axial vector currents, VV-AA in the chiral limit, $m_q = 0$ (for more details see [10]). In the present work, we neglect this term because we work to leading order in α_s.

Now, we apply the Fourier transformation to the QCD side of the correlation function to get its expression in momentum space. The next step is to select the structure which gives contribution to the tensor state from both sides of the correlation function, equate the coefficient of the selected structure from both sides and apply the Borel transformation to suppress the contribution of the higher states and continuum. After lengthy calculations, finally we obtain the following sum rules for the decay constant of the heavy tensor quarkonia:

$$\begin{aligned} f_{\chi_Q}^2 e^{-m_{\chi_Q}^2/M^2} = {}& \frac{N_c}{m_{\chi_Q}^6}\int_{4m_Q^2}^{s_0} ds\int_1^\infty du\,\frac{e^{-s/M^2}[s-s(u)]}{16\pi^2 u^6} \\ &\times\big\{-2m_Q^2 u^3 + \big[4M^2 - s - s(u)\big] \\ &-2\big[4M^2 - s - s(u)\big]u \\ &+\big[2m_Q^2 + 4M^2 - s - s(u)\big]u^2\big\} \\ &+ I\big(M^2\big)\left\langle \frac{\alpha_s}{\pi}G^2\right\rangle, \end{aligned} \qquad (17)$$

where,

$$s(u) = m_Q^2\left[u + \frac{1}{1-\frac{1}{u}}\right], \qquad (18)$$

and the explicit expression of the function $I(M^2)$ is quite lengthy and therefore we do not present it.

In the above sum rules, M^2 is the Borel mass parameter, s_0 is the continuum threshold and $N_c = 3$ is the color factor. The mass of the heavy tensor meson is also obtained applying derivative with respect to $-\frac{1}{M^2}$ to the both sides of the sum rules for the decay constant and dividing by itself, i.e.,

$$\begin{aligned} m_{\chi_Q}^2 = {}& \left(\int_{4m_Q^2}^{s_0} ds\int_1^\infty du\,\frac{e^{-s/M^2}[s^2 - s\,s(u)]}{16\pi^2u^6}\right. \\ &\times\big\{-2m_Q^2 u^3 + \big[4M^2 - s - s(u)\big] \\ &-2\big[4M^2 - s - s(u)\big]u + \big[2m_Q^2 + 4M^2 - s - s(u)\big]u^2\big\} \\ &+\int_{4m_Q^2}^{s_0} ds\int_1^\infty du\,\frac{e^{-s/M^2}[s-s(u)]}{16\pi^2u^6}\big\{4M^4(1-u)^2\big\} \\ &\left.-\frac{d}{d(1/M^2)} I\big(M^2\big)\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\right) \\ &\times\left(\int_{4m_Q^2}^{s_0} ds\int_1^\infty du\,\frac{e^{-s/M^2}[s-s(u)]}{16\pi^2u^6}\right. \\ &\times\big\{-2m_Q^2 u^3 + \big[4M^2 - s - s(u)\big] \\ &-2\big[4M^2 - s - s(u)\big]u + \big[2m_Q^2 + 4M^2 - s - s(u)\big]u^2\big\} \\ &\left.+ I\big(M^2\big)\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\right)^{-1}. \end{aligned} \qquad (19)$$

3. Numerical analysis

In this section, we numerically analyze the sum rules for the mass and decay constant of the ground state tensor quarkonia. Some input parameters entering the sum rules are quark masses, $m_b = (4.8 \pm 0.1)$ GeV, $m_c = (1.46 \pm 0.05)$ GeV [4] and gluon condensate, $\langle 0|\frac{1}{\pi}\alpha_s G^2|0\rangle = (0.012 \pm 0.004)$ GeV^4. It should be noted that recent analysis of experimental data leads to the $\langle 0|\frac{1}{\pi}\alpha_s G^2|0\rangle = (0.005\pm0.004)$ GeV^4 for the gluon condensate [11]. For conservative estimation in numerical analysis, we also take into account the value of gluon condensate $\langle 0|\frac{1}{\pi}\alpha_s G^2|0\rangle = (2.16 \pm 0.38)\times 10^{-2}$ GeV^4 which follows from sum rules of $e^+e^- \to I = 1$ hadrons [12] and heavy quarkonia [13–15]. Few words about quark mass are in order. The aforementioned masses are the pole masses for the quarks. Using the four loop results for the vacuum polarization operator in [16], the running masses of the charm and beauty quarks, $m_c(3\text{ GeV}) = (0.986 \pm 0.013)$ GeV and $m_b(m_b) = (4.163 \pm 0.016)$ GeV are obtained. These improved values of the running masses of charm and beauty quarks as well as wide range of gluon condensate are used in numerical calculations. To obtain more reliable results for the mass and decay constant of the heavy tensor meson, we will also take into account a more realistic error coming from the range spanned by the pole and running quark masses as well as the range for the value of the gluon condensate.

From the sum rules for the decay constant and mass it is clear that they contain also two auxiliary parameters, continuum threshold s_0 and Borel mass parameter M^2. The standard criteria in QCD sum rules is that the physical quantities should be independent of these mathematical objects, so we should look for working regions for these parameters at which the masses and decay constants practically remain unchanged. To determine the working region for the Borel mass parameter the procedure is as follows: the lower limit of M^2 is obtained requiring that the higher states and continuum contributions constitute, say, 30% of the total dispersion integral. The upper limit of M^2 is chosen demanding that the sum rules for the decay constants and masses should be convergent, i.e., contribution of the operators with higher dimensions is small. As a result, we choose the regions: 8 GeV$^2 \leqslant M_{\chi_{b2}}^2 \leqslant 20$ GeV^2 and 4 GeV$^2 \leqslant M_{\chi_{c2}}^2 \leqslant 7$ GeV^2 for the Borel mass parameter. The continuum threshold s_0 is not completely arbitrary but it is correlated to the energy of the first exited state with quantum numbers of the interpolating current. Our numerical results are in consistency with this point and show that in the interval $(m_{\chi_{Q2}} + 0.4)^2 \leqslant s_0^{\chi_{Q2}} \leqslant$

Table 1
Values for the masses and decay constants of the tensor mesons χ_{Q2}.

	Present work	Experiment [1]
$m_{\chi_{b2}}$	(9.90 ± 2.48) GeV	(9.91221 ± 0.00057) GeV
$m_{\chi_{c2}}$	(3.47 ± 0.95) GeV	(3.55620 ± 0.00009) GeV
$f_{\chi_{b2}}$	0.0122 ± 0.0072	–
$f_{\chi_{c2}}$	0.0111 ± 0.0062	–

$(m_{\chi_{Q2}} + 0.7)^2$, the results are practically insensitive to the variation of this parameter. Here we would like to make the following remark. It is shown in [17] that the continuum threshold s_0 can depend on the Borel mass parameter. Therefore, the standard criteria, namely, weak dependence of the results on variation of the auxiliary parameters does not provide us realistic errors, and in fact the actual error should be large. Following [17], in the present work we will add also the systematic errors to the numerical values.

Our numerical analysis on the masses and decay constants leads to the results presented in Table 1. The quoted errors in our predictions are due to the variations in the continuum threshold and Borel parameter, uncertainties in quark masses and wide range of the gluon condensates presented at the beginning of this section as well as the systematic errors. The results presented in Table 1 show a good consistency between our predictions and the experimental values [1] on the masses of the ground state heavy, χ_{b2} and, χ_{c2} tensor mesons. Our predictions on the decay constants can be verified in the future experiments.

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