

A CASE STUDY IN OFF-LINE QUALITY CONTROL: CHARACTERIZATION
AND OPTIMIZATION OF BATCH DYEING PROCESS DESIGN

by

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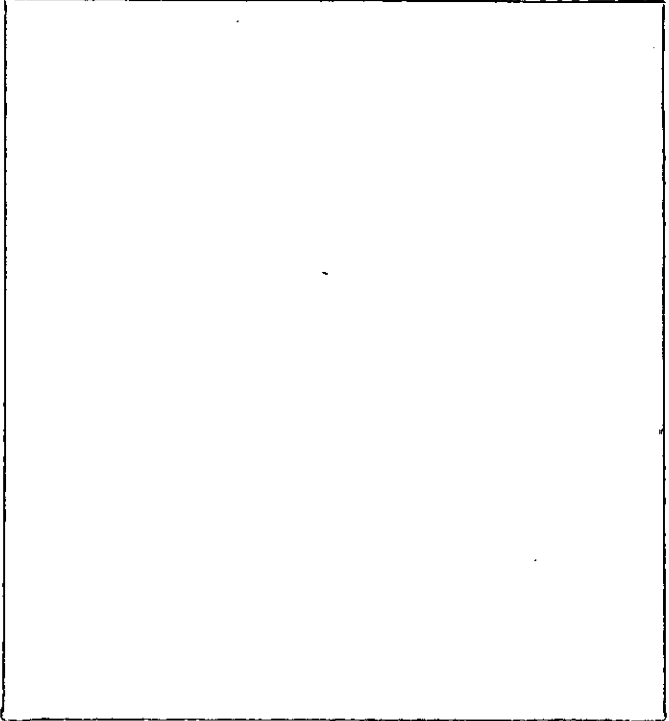
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**A Case Study in Off-Line Quality Control:
Characterization and Optimization of
Batch Dyeing Process Design**

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A method is provided and demonstrated for robust design of the batch dyeing process. This method is used to identify optimal batch dyeing process parameter settings which produce target color with the least color variation within and among dyed fabric pieces. The robust design problem is defined in terms of the design objectives, control factors and noise factors. Performance measures are presented to evaluate mean and dispersion characteristics of the dyeing output. Design and conduct of experiments are discussed for developing empirical models of the performance measures, and these models are developed for the study case. The robust design problem is formulated and solved as a nonlinear programming problem. Confirmation of results and iterative use of the proposed design method are discussed.

Introduction

This paper describes and illustrates an off-line quality control approach developed for improving the quality of batch dyeing process. This approach is used to identify settings of controllable batch dyeing process parameters which minimize adverse effects of manufacturing variations on dyeing performance.

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Customers commonly expect a textile dyer to produce a specific color uniformly on some fabric in repetitive batch dyeings. Batch dyeing performance is affected by several factors which are impossible or expensive to control. These factors cause color variation within and among batches of a fabric and off-target color production. It is common practice in the textile industry to attempt control of variation in dyeing performance by using expensive and sophisticated control systems and by tightening product and process tolerances. However, there might be less need for such a control mechanism, if the process parameter settings were selected so as to minimize the sensitivity of dyeing performance to the manufacturing variations.

In this paper, the batch dyeing process is characterized in terms of the factors affecting its performance and the degree of control over them by the process design engineer. Multiple measures are defined to evaluate the dyeing performance.

The proposed design method is described and illustrated on a case problem. This method requires designing and conducting experiments to support empirical modeling of relations between performance measures and controllable process parameters. Then, these models are used in formulating and solving the parameter design problem as a mathematical programming problem. This approach allows a systematic way of optimizing the multiple design objectives involved.

The Batch Dyeing Process Design Problem

Batch dyeing is a process of applying color to a group of fabric pieces. It is an intermediate step in overall dyehouse operations (see Figure 1). Before dyeing, the fabric is washed and bleached. After dyeing, the fabric is treated chemically or mechanically to improve its appearance and physical properties. Although dyeing performance is affected by performance of the preceding operations and storage conditions between the operations, dyeing is the major step in determining the final color of the fabric. Finishing processes also may cause

unintentional color change or uneven color appearance.

Insert Figure 1 here

A dyed fabric piece is characterized by the closeness of the produced color to the target color (standard) and by the uniformity of the produced color over the piece. The closeness of produced color among different fabric pieces is another important output characteristic.

In conventional color specification, color has three dimensions: In CIELAB color space, these dimensions are identified as L^* , a^* and b^* (see AATCC (1989)). L^* is the lightness of color. a^* and b^* are the other two dimensions, functions of which define hue (H) and chroma (C). The CIELAB color differences, ΔE_{ab} , which can be calculated from the differences in L^* , a^* and b^* between pieces of fabric do not always correlate with visual assessments (see AATCC (1989)). However, the CMC(2:1) color difference, $\Delta E_{cmc(2:1)}$, correlates well with visual assessments of acceptability in commercial color match decisions (AATCC (1989)):

$$\Delta E_{cmc(2:1)} = [(\Delta L^*/2S_L)^2 + (\Delta C^*/S_C)^2 + (\Delta H^*/S_H)^2]^{0.5}.$$

According to this formula, there exists such an ellipsoid around a target color point, STD , that any other color point within this ellipsoid is acceptable as a commercial color match (see Figure 2). This acceptability tolerance depends on the fabric, color and end use.

Insert Figure 2 here

Dyeing is a complex process with many factors affecting its outcome. Köksal and Smith (1990), and Köksal, Smith and Smith (1992) study factors which may cause color variation and off-color dyeings. Some of these factors can be adjusted by the designer to alter the performance of the process. They are referred to as control factors. Many others are difficult or impossible to control causing random changes in the dyeing outcome. These latter are called noise factors.

The objectives involved in robust design of batch dyeing process are to find settings of control factors, which:

1. Minimize color difference of any point on a dyed fabric from the target color,
2. Maximize color uniformity (levelness) of a dyed fabric, and
3. Minimize variation of color patterns produced from one fabric piece to another.

Process Performance Evaluation

Dyeing performance is determined by three important characteristics of the product:

1. The degree of color match between a dyed fabric piece and the color standard,
2. The degree of color uniformity within a dyed fabric piece,
3. The degree of color pattern repeatability among dyed fabric pieces.

The performance of the batch dyeing operation is considered to be improved, if it achieves a higher degree of color match, uniformity, and repeatability over the dyed fabric pieces. Köksal (1992) develops objective measures to determine the dyeing performance. In the following, these measures are presented:

Color Match:

Let $D_{t,rq-s}$ be the CMC(2:1) color difference between the color standard s and a randomly selected point q on a fabric piece r randomly selected from the fabric pieces dyed under the same process parameter settings t . For a given fabric piece i , a measure of color match between the fabric piece and the color standard s is defined as the expected squared color difference of the piece from the standard, $E(D_{t,iq-s}^2)$. This is a combined measure of the mean and the variance of $D_{t,iq-s}$, since $E(D_{t,iq-s}^2) = [E(D_{t,iq-s})]^2 + V(D_{t,iq-s})$. It is necessary to

consider both the mean and the variance because of nonuniform color appearances (see Köksal (1992)). If n measurements are made on fabric i , an unbiased estimator of $E(D_{t,iq-s}^2)$ is the sample mean:

$$Y_{ti} = \frac{1}{n} \sum_{j=1}^n D_{t,ij-s}^2$$

For a randomly selected piece of fabric, Y_t is the expected squared color difference of the piece from the standard. If N fabric pieces are sampled from those dyed under the same process parameter settings t , then the mean $E(Y_t)$, and the variance $V(Y_t)$ can be estimated, respectively, as follows:

$$\bar{Y}_t = \frac{1}{N} \sum_{i=1}^N Y_{ti}$$

$$s_{Y_t}^2 = \frac{1}{N-1} \sum_{i=1}^N (Y_{ti} - \bar{Y}_t)^2$$

In selecting process parameter settings, a minimal value of \bar{Y}_t should be sought after to minimize the color difference of a fabric piece from the target. Similarly, minimizing the $s_{Y_t}^2$ value helps consistent production of this color difference among the fabric pieces.

Color Uniformity:

Dyeing processes do not always produce uniform color on a fabric. Even if all points of a dyed fabric are within acceptable CMC(2:1) color difference units from the color standard, the overall color appearance may not be uniform (see Köksal (1992)).

Let $D_{t,rq-u}$ be the CMC(2:1) color difference between two randomly selected points q and u on a fabric piece r selected randomly from the fabric pieces dyed under the same process parameter settings t . For a given fabric piece i , a measure of color uniformity is defined as the expected squared color difference between any two points of the fabric, $E(D_{t,iq-u}^2)$. If n measurements are made on fabric piece i , then an unbiased estimator of $E(D_{t,iq-u}^2)$ is:

$$Z_{ti} = \frac{1}{C(n:2)} \sum_{j=1}^{n-1} \sum_{k=j+1}^n D_{t,ij-k}^2$$

If N fabric pieces are sampled from those dyed under the same process settings t , then the mean $E(Z_t)$, and the variance $V(Z_t)$ can be estimated, respectively, as follows:

$$\bar{Z}_t = \frac{1}{N} \sum_{i=1}^N Z_{ti}$$

$$s_{Z_t}^2 = \frac{1}{N-1} \sum_{i=1}^N (Z_{ti} - \bar{Z}_t)^2$$

Both of these measures should be minimized to have consistently uniform color appearance on each and every one of the fabric pieces.

Color Pattern Repeatability:

Producing an acceptable color match and uniformity for each and every one of the dyed fabric pieces is not sufficient for an acceptable color match among the fabric pieces (see Köksal (1992)). In this work, repeatability is defined as color pattern match between any two pieces of fabric dyed at different times, but under the same process parameter settings. It is important to note that in comparing color patterns of a pair of fabric pieces, a point on one piece should be compared to the equivalent of that point on the other piece determined by the actual loading positions of the fabric pieces in the dye solution.

Let $D_{t,rq-wq}$ be the CMC(2:1) color difference between a randomly selected point q on a randomly selected fabric piece r and the corresponding point q on a randomly selected (without replacement) fabric piece w , from the fabric pieces dyed under the same process parameter settings t . For a given (i, j) pair, a measure of color pattern match between the fabric pieces is defined as the expected squared color difference between the comparable points of the fabric pieces, $E(D_{t,iq-jq}^2)$. If n measurements are made on each fabric piece, then an unbiased estimator of $E(D_{t,iq-jq}^2)$ is:

$$P_{ij} = \frac{1}{n} \sum_{l=1}^n D_{t,il-jl}^2$$

If N fabric pieces are sampled from those dyed under the same process parameter settings t , then $C(N:2)$ distinct fabric pairs can be found. Clearly, the P_{ij} values corresponding to

these $C(N:2)$ pairs are not mutually independent realizations of the random variable P_t . In this case, the sample variance of P_{tij} 's is not an unbiased estimator of the variance of P_t , $V(P_t)$. Therefore, for small N , it may be desirable to estimate $E(P_t^2)$ instead of estimating the mean $E(P_t)$, and the variance $V(P_t)$, separately. Notice that $E(P_t^2) = [E(P_t)]^2 + V(P_t)$. If a quadratic loss function is assumed, $E(P_t^2)$ is proportional to the expected loss due to poor repeatability. An unbiased estimator of $E(P_t^2)$ is the arithmetic mean of P_{tij}^2 values:

$$\overline{P_t^2} = \frac{1}{C(N:2)} \sum_{(i,j), i < j} P_{tij}^2$$

A minimal value of $E(P_t^2)$ is desired to ensure good repeatability among the color patterns produced on the fabric pieces.

Robust Design Method

The best settings of control factors will be found by modeling and examining the relationships between the control factors and performance measures of the process. These models can be developed either directly by replicating experiments according to a special design ("loss model" or "product array" approach), or indirectly by first modeling the process response and then approximating the performance measures using the response model ("response model" or "combined array" approach) (see Shoemaker, Tsui and Wu (1989)). Based on the performance measure models, optimal process parameter settings can be found either by following Taguchi's two-step approach (see Taguchi (1986), Phadke (1989), Leon, Shoemaker and Kacker (1987)) or by formulating and solving the robust design problem as a nonlinear programming problem (see Fathi (1991), Vining and Myers (1990), Mesenbrink, Lu, McKenzie and Taheri (1992)).

The method proposed for robust design of batch dyeing is outlined in Figure 3. It requires designing and conducting experiments to support empirical modeling of relations between performance measures and controllable process parameters. Then, these models are used in formulating and solving the robust design problem as a mathematical programming problem.

Insert Figure 3 here

It is suggested that the performance measures be modeled directly by using the product array approach. The empirical modeling cost of this approach is less than that of the combined array approach. Following the former approach, it is necessary to develop a total of six empirical models, each corresponding to one of the performance measures described above. However, the latter approach requires that, first, color values, $L^*(i)$, $a^*(i)$ and $b^*(i)$, at each location i of the fabric be modeled, and then the performance measures be approximated from these models using information on the distribution of noise factors. If there are ten measurement points on a piece of fabric (front and back), then it is necessary to have thirty empirical models. Moreover, it is very difficult to estimate the true distribution of noise factors in a dyeing environment. Therefore, the product array approach is more appropriate in developing a robust design of batch dyeing.

The robust design problem can be formulated as a nonlinear programming problem. This approach, as opposed to the Taguchi method, enables better handling of multiple design objectives and constraints. It is also true that, if another customer wants the same color with different expectations concerning color uniformity and/or repeatability, then the optimal parameter settings can be found simply by changing the formulation of the NLP problem accordingly, and solving the revised problem.

Experimental Design

The robust design method is demonstrated on a specific case where a customer asks a dyer to produce a specific color on a certain amount of a fabric. Before dyeing process design, the target color is produced on the selected fabric, in the dye lab, by experimenting with the parameter settings suggested by color match computer programs. Once the customer approves the target color, the L^* , a^* and b^* color dimensions of it become the target in

color comparisons. Types of dyes, chemicals, fabric and equipment to be used in the process are selected. The steps of the dyeing process are identified. These selections are shown in Table 1. Now, it is desired to identify controllable or uncontrollable design factors, region of experimentation and test levels of these factors.

Insert Table 1 here

Dyeing is a complex process influenced by several factors (see Köksal et al. (1990,1992)). For the case problem, the most important factors which are controllable by the design engineer (control factors) are identified by consulting with experts and using earlier work (see Sumner (1976), Köksal and Smith (1991)). They are listed in Table 2 as (liquor) ratio L of volume of dye solution (dyebath) to weight of fabric, amount of dye (based on weight of fabric) D , salt concentration S , alkali concentration A , dyeing temperature T , time before alkali add M , and agitation rate G (measured by twister movements per minute).

Insert Table 2 here

Many factors affecting the dyeing outcome are difficult, expensive or impossible to control precisely. These include variations in the amount of dye, volume of dyebath, amount of fabric, and characteristics of dyes, chemicals and water. The most important of these noise factors are identified as variation in weight of dye, W_d ; variation in volume of dyebath, W_v ; and variation in weight of fabric, W_f (see Table 3). Desired levels of these factors can be simulated in a controlled dye lab environment.

Insert Table 3 here

The experiments are to be performed in a dye laboratory using a dyeing apparatus which simulates the dyehouse dyeing. This apparatus contains glass tubes in which dyebath and

a small piece of fabric is placed spiral wound around a twister (see Figure 4). Agitation is provided by the movements of the twister. At each experiment with certain settings of control and noise factors, a fabric piece of size approximately 6.5" × 8.5" is to be dyed.

Insert Figure 4 here

The L^* , a^* and b^* color measurements are to be made (after dyeing) at each corner and in the middle of the fabric piece, front and back. For the particular fabric of the study case, front and back measurements are not treated separately since there is no texture difference between the front and the back.

As a result, ten sets of L^* , a^* and b^* values are obtained for each experimental run i under each setting t of the control factors (see Figure 5). These values can be summarized into two statistics; one measuring the degree of color match, Y_{ti} , and the other showing the degree of color uniformity, Z_{ti} , as explained before, respectively. If N pieces are sampled from those dyed under the same setting t of the control factors, then $C(N : 2)$ values are obtained for the color repeatability measure P_t . Furthermore, for each control setting t , the mean of the measures Y_t , Z_t and P_t are estimated as the sample mean \bar{Y}_t , \bar{Z}_t and \bar{P}_t , respectively, and similarly the variances of them are estimated as the sample variances $s_{Y_t}^2$, $s_{Z_t}^2$ and $s_{P_t}^2$, respectively. It is also possible to combine \bar{P}_t and $s_{P_t}^2$ into \bar{P}_t^2 , to estimate the associated quality loss due to poor color pattern repeatability. (see Figure 5)

Insert Figure 5 here

The region of experimentation is determined in such a way that the center of the region is located close to the design point (i.e. set of process parameter settings) for producing the color standard. Note that this design point is not necessarily robust to manufacturing variations. The ranges of control factor settings to be tested are found based on expert knowledge and practical limitations of laboratory testing.

The noise factors W_d , W_v and W_f are assumed to have symmetrical distributions with mean 0 and standard deviations 6%, 8% and 2%, respectively, based on earlier studies of variation (see Sumner (1976)).

The experiments necessary for this work are chosen to be performed according to a set-up consisting of two parts as explained before: Control factors are varied according to a design (control array or inner array), and for each row of this design, the noise factors are varied according to another design (noise array or outer array).

Before designing the control array, it is important to postulate a model for the performance measures. It is assumed that the relationships between the control factors L, D, S, A, T, M, G and the performance measures $E(Y_t)$, $V(Y_t)$, $E(Z_t)$, $V(Z_t)$ and $E(P_t^2)$ can be approximated by second-order polynomials.

Dyeing experiments are time consuming and costly. In order to keep the size of the experiments to its minimum, a D-optimal design is constructed for the control array. This design allows estimation of the coefficients of the significant main effects, interactions and quadratic effects. These terms are suggested by expert knowledge considering all relationships between the control factors and the performance measures. The levels selected for the control factors are shown in Table 2. The control array design is constructed by using the DETMAX algorithm (SAS (1989)) as shown in Figure 6. The D-efficiency of this design is 42.39.

Insert Figure 6 here

The noise array is independent of the control array. The noise array design is also desired to have a minimal size to keep the total number of experiments low. An orthogonal array design is selected for the noise array with which two levels of the noise factors can be tested with four experimental runs (see Figure 6). These noise factor levels also shown in Table 3 are selected based on Taguchi's suggestion (Taguchi (1978)) as (mean \pm standard deviation) of the corresponding noise factor.

The experiments are to be conducted in a well-controlled dye laboratory environment. In experimentation, it is important to create exactly the same levels of control and noise factors as are directed by the design, and not to introduce any other variation to the process. This makes it possible to approximate closely the real relationships between the control factors and the performance measures.

Conduct of Experiments

The experiments and color measurements for the particular case problem were performed, but not in all respects as planned. The major problems encountered can be listed as inconsistent color measurement practices and fabric installation procedure, and the use of commercial quality dyestuff (varying dye strength). These inconsistencies were reduced significantly by updating the data through statistical analysis, and the effects of the use of commercial quality dyestuff use were considered negligible. Due to a misunderstanding, the laboratory technician performed a total of sixty more experiments at additional noise settings $(W_v, W_f, W_d) = (+0.08, +0.02, -0.06)$ and $(-0.08, +0.02, +0.06)$.

The color data obtained (in terms of L^* , a^* , b^*) for the aforementioned ten points on each dyed fabric were first converted into performance data Y_{ti} , Z_{ti} , and P_{tij} , and then the performance statistics \bar{Y}_t , \bar{Z}_t , $s_{Y_t}^2$, $s_{Z_t}^2$, and \bar{P}_t^2 were calculated. The values obtained for the performance statistics are shown in Table 4. Also shown in Table 4 are \bar{Y}_t and \bar{Z}_t values obtained at the process parameter settings $t = (T, G, L, D, S, A, M) = (175, 30, 30, 1.0, 40, 2.4, 35)$ of the color standard (C.S.) under a null noise array $(W_v, W_f, W_d) = (0, 0, 0)$. These values were used in developing empirical models of the performance measures $E(Y_t)$ and $E(Z_t)$, respectively.

Insert Table 4 here

Empirical Modeling

Performance data were analyzed and empirical models of the performance measures were developed using the method of least squares. First, a model was selected through stepwise regression, and weighted least squares method was utilized to estimate model parameters. Then, the residuals obtained from the selected fitted model were checked for validity of the assumptions of constant variance, normal error distribution, and independent errors. If the residuals were not distributed as assumed, then the performance data were transformed to an appropriate metric, and the same model building procedure was applied to the transformed data. If the residuals of the resulting model still violated the assumptions, then the transformed data were assigned appropriate weights and modeled again. These weights were based on a model of the sample variances of the data.

In model selection, polynomials of order four and less were considered. Since the improvement in higher order polynomial fits was insignificant, second order fits were selected. Model selection was redone after data transformation and weighting.

Applying this modeling procedure to the color match data, \bar{Y}_t , $t = 1, \dots, 31$, the following model was obtained:

$$\begin{aligned} \log \hat{Y} = & -3.769891 + 0.717084S + 0.838628D + 0.757508D^2 + \\ & 0.006908M + 0.969541M^2 + 0.017368L + 1.478919L^2 - \\ & 0.774098T - 0.433297T^2 + 1.274380LT - \\ & 0.461914G + 2.580223G^2 + 0.753274AT + \\ & 1.018591LM - 1.69807DM + 0.437178DA - \\ & 0.118293A + 1.283416A^2 + 0.898292TM - \\ & 0.680415AG - 0.576239LG - 0.285780LD - \\ & 0.455110LA + 0.256722GM - 0.161762DS + \\ & 0.115107DT - 0.099368SG \end{aligned} \quad (1)$$

In this model and the others, the parameters L, D, S, A, T, M, G denote coded levels having maximum and minimum values of +1 and -1, respectively. Table 5 summarizes the corresponding analysis of variance. A Box-Cox power transformation was applied to the color

match data, since the residuals of the best ordinary least squares (OLS) fit obtained did not have a constant variance and normal distribution. The best power transformation was found to be the log transformation. The OLS fit obtained for the transformed data is better in satisfying the error distribution assumptions, but still the residuals do not appear to be constant when plotted against the model predictions. Therefore, the above model was fitted to the transformed data through weighted least squares (WLS). The data weights are the reciprocal of the predicted variance of the transformed data from an OLS fit of the sample variances. The residuals of this WLS fit validate the error assumptions (see Köksal (1992)), and the resulting F value (156.607) and R^2 (0.9644) are satisfactory in terms of adequacy of the fit. It should be observed from Equation (1) that the model contains many variables. One decision criterion in model selection is to include as many significant terms as possible in the model, and yet have enough degrees of freedom for the error. This is important to increase prediction accuracy of the models in spite of decreased model simplicity, since these models are to be directly used in optimization.

Insert Table 5 here

The model for color uniformity data, Z_t , $t = 1, \dots, 31$, was also obtained after a data transformation and by WLS:

$$\begin{aligned} \hat{Z}' = & -4.350548 - 1.433603 + 0.781364S + 0.022431A + 0.777442AG - \\ & 0.623818LS + 0.155238SA + 1.600393A^2 - \\ & 1.273225S^2 - 0.663185M - 0.298451L + \\ & 0.549965L^2 - 0.520520DM - 0.213697ST - \\ & 0.057746G - 0.345117GT - 0.831621SM + \\ & 0.354170AT + 0.288962DS + 0.856133G^2 - \\ & 0.276246AM - 0.186437GM + 0.096044D. \end{aligned} \quad (2)$$

The residual plots obtained from an OLS fit to the Z_t data indicate that the error assumptions are violated, and the corresponding F value (0.624) is not significant and the R^2 (0.10) is very small. The best Box-Cox transformation for the Z_t data was obtained as $Z'_t =$

$(Z_t^{-0.5} - 1)/(-0.5)$. The OLS model obtained for the transformed data, Z'_t , satisfies the error distribution assumptions. The F value (1.233) is still insignificant and the R^2 (0.1885) is still small. It also becomes apparent in this metric that the residuals are bigger for the data collected from the first thirty one experiments. Since these experiments were done by using a different fabric installation procedure, a blocking variable, O , was added to the model which was defined as -1 for the first thirty one experiments, and as 1 for the rest. Then, a function was fitted to the transformed data by WLS. The weights are again reciprocals of the predicted variances. The analysis of variance table of this fit is shown in Table 6. Residuals satisfy error distribution assumptions (see Köksal (1992)), and the F value (11.197) is significant and R^2 (0.60) is high. Therefore, the resulting Equation (2) was selected for the mean transformed color uniformity measure, Z' . In optimization modeling, this model is to be used with $O = -1$, since the first thirty one experiments did not follow the correct fabric installation procedure.

Insert Table 6 here

The function fitted for the relationship between the variance of the color match measure, $V(Y)$, and the control factors is:

$$\begin{aligned}
 \log \hat{V}(Y) = & 4.447461 + 1.166841S - 0.455444T + 0.863284LS + \\
 & 0.075294D - 0.145547DM - 0.227304L - 0.097189LG + \\
 & 0.260337G + 0.081441A - 1.434977A^2 + \\
 & 0.35108ST + 0.108133DS - 0.730415L^2 - \\
 & 0.106989LT + 0.818556D^2 - 0.802767S^2 - \\
 & 0.401522LA + 0.340872SA - 0.219010G^2 + \\
 & 0.179061M + 0.160714DA - 0.272873T^2 + \\
 & 0.108428M^2 + 0.031408DG - 0.01624SG
 \end{aligned} \tag{3}$$

This model was obtained by applying OLS directly to the logged sample variances $s_{Y_t}^2$, $t = 1, \dots, 30$, since the log transformation stabilizes the variance of chi-squared distributed

data such as sample variances (see Box and Draper (1987)). The residual plots indicate that the model assumptions are satisfied (see Köksal (1992)). The F value (2020.974) is highly significant, and R^2 (0.9999) is high with 4 degrees of freedom for error. The corresponding analysis of variance is shown in Table 7.

Insert Table 7 here

Similarly, the following model was obtained for the variance of the color uniformity measure, $V(Z)$:

$$\begin{aligned}
 \log \hat{V}(Z) = & -0.585206 + 0.73295S + 0.910038T - 1.24336T^2 - \\
 & 1.126104L + 0.838461LG + 1.835292GT + \\
 & 1.959101SA + 1.445285LS - 0.411525A - \\
 & 1.450737A^2 - 0.879793LD - 0.003636M - \\
 & 1.176581GM + 0.680189AG + 0.89319M^2 - \\
 & 0.196459DT - 0.050295D - 0.893826D^2 + \\
 & 0.31491LA + 0.309476DS - 0.344704SG + \\
 & 0.513599DG - 0.579606LT + 0.315453LM + \\
 & 0.455484L^2 - 0.211122TM
 \end{aligned} \tag{4}$$

The corresponding analysis of variance is summarized in Table 8. The F value (199.383) is highly significant, and the R^2 (0.9994) is high with 3 degrees of freedom for error. The residuals also are distributed according to the model assumptions (see Köksal (1992)).

Insert Table 8 here

The functional relationship between the expected value of P squared, $E(P^2)$, and the control factors was estimated by fitting a function to the $\overline{P_t^2}$ data, $t = 1, \dots, 30$. The log transformation was also found to be necessary for the $\overline{P_t^2}$ data. The OLS fit to the transformed data is:

$$\begin{aligned}
\log \hat{P}^2 = & 2.816164 + 0.286856S - 0.07956L - 0.115732LT + \\
& 0.115264M - 0.57959GM - 0.791528LG - \\
& 0.73301DM + 0.294041G - 0.456152T + 0.066769GT + \\
& 0.287497TM + 0.603226LS - 0.510595LD + \\
& 0.633413SA + 0.007322D - 0.876895D^2 + \\
& 0.189137A - 0.302681DS + 0.375822G^2 - \\
& 0.562211T^2 - 0.275001LM - 0.12937SG + \\
& 0.303526S^2 - 0.107998ST.
\end{aligned} \tag{5}$$

Analysis of variance is summarized in Table 9. This model is justified, since the residuals obey the model assumptions (see Köksal (1992)), the F value (76.286) is highly significant and R^2 (0.9973) is high.

Insert Table 9 here

Optimization Modeling

Earlier in this article, three distinct objectives were defined with regard to the analysis of the batch dyeing process. Briefly stated, these objectives are: 1) Minimize color difference from target, 2) Maximize color uniformity within a piece of fabric, and 3) Minimize color variation between different pieces of fabric. In terms of measures Y , Z and P , these objectives can be stated as:

1. Minimize $[E(Y)]^2 + V(Y)$
2. Minimize $[E(Z)]^2 + V(Z)$
3. Minimize $E(P^2)$

These functions are based on the assumption of quadratic loss functions for Y , Z , and P , respectively. For instance, in the case of measure Y , if it is assumed that the quality loss

increases quadratically as Y increases from zero (notice that the ideal value for Y is zero), then it follows that the term $k_1[(E(Y))^2 + V(Y)]$ is the expected loss due to poor color match. In this context, k_1 is the amount of quality loss at $Y = 1$. Similar interpretations apply to color uniformity, measured by Z , and color repeatability, measured by P .

Other formulations of these objectives are possible. One such formulation is discussed in Köksal (1992).

The empirical models of the previous section (or appropriate transformations of these models) can be used to carry out the analysis.

Several approaches are typically used in modeling multiple objective optimization problems. These include goal programming, priority ordering, and the weighted average approach, among others. Here, the weighted average approach is described, and the reader is referred to Köksal (1992) for a discussion of the other two approaches.

The weighted average approach is based on the following model:

$$\begin{aligned} \text{Minimize} \quad & k_1[(E(Y))^2 + V(Y)] + k_2[(E(Z))^2 + V(Z)] + k_3E(P^2) \\ \text{subject to} \quad & -1 \leq L, D, S, A, G, T, M \leq 1 \end{aligned} \tag{6}$$

where k_1, k_2, k_3 are the corresponding weights.

We analyzed Model (6) with several sets of values for k_1, k_2 and k_3 . Based on the results of our analysis, we recommend $k_1 = k_2 = k_3 = 1$. The corresponding optimal solution of Model (6) is shown in Table 10. This solution was obtained by using the NLP software EXPLORE (see Gottfried and Becker (1973)) with fifty randomly selected starting points.

Insert Table 10 here

In order to evaluate the quality level achieved at this solution, two additional sets of measures were defined. The first set is an interpretation of the performance measures $E(Y), E(Z)$,

and $E(P^2)$ in terms of the CMC(2:1) color difference units. Values of these measures are simply the square roots of the corresponding performance measures. To illustrate, it was observed that $E(Y)$ could be expressed as

$$E(Y) = E(D_{rq-s}^2) = [E(D_{rq-s})]^2 + V(D_{rq-s}).$$

Recall that the term $E(D_{rq-s})$ measures the difference between the color of a randomly selected point and the target color. This term has its maximum value, for a given $E(Y)$, when $V(D_{rq-s})$ is zero. Then, the square root of $E(Y)$ is the maximum value of $E(D_{rq-s})$, which is shown in the parenthesis below the value of $E(Y)$ in Table 10. This square root value can be compared with 0.50 which is the maximum commercially acceptable color difference for this particular fabric. The values in parentheses below $E(Z)$ and $E(P)_{max}$ of Table 10 can be interpreted in a similar manner.

The second set of measures consists of two numbers which are referred to as α_Y and α_Z . For a given set of values for $E(Y)$, $V(Z)$, $E(Z)$, and $V(Z)$, these numbers were defined as:

$$\alpha_Y = [(0.50)^2 - E(Y)] / \sqrt{V(Y)}$$

$$\alpha_Z = [(0.50)^2 - E(Z)] / \sqrt{V(Z)}.$$

α_Y is the distance between 0.50^2 and $E(Y)$, in terms of standard deviation of Y . Similarly, α_Z is the distance between 0.50^2 and $E(Z)$, in terms of standard deviation of Z . Naturally, large values of α_Y and α_Z imply better quality.

It can be predicted from the values of the above measures that, at the optimal solution, high and consistent color uniformity can be achieved ($\sqrt{E(Z)} = 0.28 < 0.50$, $\alpha_Z = 9.35$). On the average, color patterns may slightly differ among the pieces dyed ($\sqrt{[E(P)]_{max}} = 0.80 > 0.50$). On the average, a good color match can be achieved ($\sqrt{E(Y)} = 0.16 < 0.50$). However, the proportion of fabric pieces with commercially acceptable color match may not be as high as one would like to achieve ($\alpha_Y = 0.69$).

Confirmation of Results

The process parameter settings found optimal as explained above were tested in the laboratory under the noise array of previous experimentation to confirm the predicted results. Laboratory performance of these parameter settings was determined by estimating $E(Y)$, $E(Z)$, $V(Z)$, $V(Z)$ and $E(P^2)$ from the data collected. These estimates are shown in Table 11 together with the model predictions and 95 % confidence limits on these predictions.

Insert Table 11 here

Comparison of the experimental results with the model predictions shows that the tested parameter settings yield values close to predicted values of the color uniformity measures, $E(Z)$ and $V(Z)$. The value estimated from the laboratory experiments for the loss due to poor color pattern repeatability, $E(P^2)$, is significantly less than the corresponding model prediction and the lower 95 % confidence limit. However, the values obtained from the experimental results for the color match measures, $E(Y)$ and $V(Y)$, turn out to be larger than the upper 95 % confidence limits of the prediction. This implies that at the recommended process parameter settings, uniform and repeatable fabric pieces can be dyed, but the color produced may not be acceptable.

Discussion

The experimental results do not confirm the predicted performance at the recommended parameter settings with respect to the values of $E(Y)$, $V(Y)$ and $E(P^2)$. In this case, the proposed design strategy (see Figure 3) requires that the sources of discrepancies be investigated, and that the process design be fine tuned by repeating the steps of the robust design method according to this feedback.

An analysis of the possible sources of discrepancies for the case problem is shown in Figure 7. These discrepancies might result from design of experiments, data collection process, and/or empirical modeling. It is our belief that the most likely causes are related to the data collection process including experimental and measurement errors (such as use of commercial dyestuff, and measurements on unconditioned fabric).

Insert Figure 7 here

The optimization modeling process is also revisited to investigate if another locally optimal solution obtained could actually perform better. As mentioned before, different optimization models of the robust design problem were developed. A number of solutions obtained from these models were very similar. The recommended solution was only slightly better than the others in terms of the performance measure values. One such similar solution worths mentioning. This solution was obtained from a goal programming formulation:

$$\begin{aligned}
 &\text{Minimize} && [E(Y)]^2 + V(Y) \\
 &\text{subject to} && \\
 &&& [E(Z)]^2 + V(Z) \leq 0.25 \\
 &&& E(P^2) \leq 1.00 \\
 &&& -1 \leq L, D, S, A, G, T, M \leq 1 \qquad (7)
 \end{aligned}$$

This model is aimed at obtaining the best color match with the values of $[E(Z)]^2 + V(Z)$ and $E(P^2)$ restricted to relatively large upper bounds. The solution obtained from Model (7) had a higher amount of dye D value (0.97), and a higher agitation rate G value (35) than the recommended solution. At these parameter values, the models predicted that $\sqrt{E(Y)} = 0.23$, $V(Y) = 6.505 \times 10^{-2}$, $\sqrt{E(Z)} = 0.29$, $V(Z) = 6.270 \times 10^{-4}$ and $\sqrt{E(P)_{max}} = 1.07$. This solution was not selected as the optimal solution, since the predicted maximum color difference between a pair of fabric pieces was high. Now, as a result of the confirmation

experiments, we suspect that the color pattern repeatability model (5) overestimates its actual value. As such, we believe that the Model (7) solution might indeed be better than the recommended solution. This belief is also confirmed by experts in the field based on the argument that the amount of dye suggested by the Model (7) solution is much closer to the amount of dye used in producing the target color than the corresponding value of the recommended solution.

The possibility that the Model (7) solution could produce better results does not answer the question why prediction accuracy of the color match and repeatability models was poor at the recommended solution. Therefore, to fine tune the results obtained here, we suggest that a second set of experiments be designed and constructed to collect more data around the recommended parameter settings, and that the empirical models and the optimal solution be updated accordingly.

Conclusion

In batch dyeing, the main objective is to produce the target color with the least color variation within and among dyed fabric pieces. The method provided in this paper satisfies the need for a systematic way of finding batch dyeing process parameter settings which minimize sensitivity of dyeing performance to manufacturing variations.

Objective quantitative evaluation of dyeing performance is made possible by the measures developed in this work.

Design of experiments and empirical modeling of the performance measures are discussed and demonstrated for the study case.

Formulation and solution of the robust design problem as a nonlinear programming problem enables better handling of the multiple objectives involved. The formulation presented for the study case is based on quality losses.

Confirmation of predicted results requires adequate modeling of the relationships between the performance measures and control factors. Therefore, it is important to prevent deviations from the designed experiment and experimental guidelines. The study case is an example of unconfirmed results at the first iteration of the method due to deviations from the experimental guidelines. It is suggested that a second iteration be performed to design additional experiments and collect more data to update the empirical models.

The proposed design approach can also be used to design other chemical batch processes such as industrial painting or plating. It is especially recommended for problems with many design objectives. For each case, of course, it is necessary to formulate appropriate performance measures.

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References

- AATCC. (1989) "CMC: Calculation of Small Color Differences for Acceptability," *Textile Chemist and Colorist* 21, 11, pp. 18-21.
- Box, G.E.P., and Draper, N.R. (1987). *Empirical Model-Building and Response Surfaces*, New York, John Wiley & Sons.
- Fathi, Y. (1991). "A Nonlinear Programming Approach to the Parameter Design Problem," *European Journal of Operational Research* 53, 3, pp.371-381.
- Gottfried, B.S. and Becker, J.R. (1973). *EXPLORE: A Computer Code for Solving Nonlinear Continuous Optimization Problems*, Technical report no. 73-6, Department of Operations Research, University of Michigan.
- Köksal, G., and Smith, W.A., Jr. (1990). *System Analysis for Dyehouse Quality Control*, Technical report no. 90-3, Dept. of IE, NCSU, Raleigh, NC 27695.
- Köksal, G., and Smith, W.A., Jr. (1991). *Quality Function Deployment: A Multi-stage Application Perspective and a Case Study*, Technical report no. 91-6, Dept. of IE, NCSU, Raleigh, NC 27695.
- Köksal, G., Smith, W.A., Jr., and Smith, C.B. (1992). "A 'System' Analysis of Textile Operations - A Modern Approach For Meeting Customer Requirements," *Textile Chemist and Colorist* 24, pp. 30-35.
- Köksal (1992). *Robust Design of Batch Dyeing Process*, Ph.D. dissertation, Dept. of IE, North Carolina State University, Raleigh, NC 27695.
- Leon, R.V.; Shoemaker, A.C, and Kacker, R.N. (1987). "Performance Measures Independent of Adjustment, An Explanation and Extension of Taguchi's Signal-to-Noise Ratios" (with discussion), *Technometrics* 29, 3, pp.253-285.

Mesenbrink, P., Lu, J.C., McKenzie, R., and Taheri, J. (1992), "Characterization and Optimization of a Wave Soldering Process", in revision for *Journal of the American Statistical Association*.

Phadke, M.S. (1989). *Quality Engineering Using Robust Design*, Prentice-Hall.

SAS Institute Inc. (1989). *SAS/QC Software: Reference*, Version 6, First Edition, Cary, NC: SAS Institute Inc.

Shoemaker, A.C., Tsui, K.L., and Wu, C.F.J. (1989). *Economical Experimentation Methods for Robust Parameter Design*, IIQP Research Report No. RR-89-04, The Inst. for Improvement in Quality and Productivity, Univ. of Waterloo, Waterloo, Ontario, Canada, N2L 3G1

Sumner, H.H. (1976). "Random Errors in Dyeing - The Relative Importance of Dyehouse Variables in the Reproduction of Dyeings," *Journal of the Society of Dyers and Colourists*, pp. 84-99.

Taguchi, G. (1978). "Performance Analysis Design," *International Journal of Production Research* 16, pp. 521-530.

Taguchi, G. (1986). *Introduction to Quality Engineering*, Asian Productivity Organization.

Vining, G.G. and Myers, R.H. (1990). "Combining Taguchi and Response Surface Philosophies: A Dual Response Approach," *Journal of Quality Technology* 22, 1, pp.38-45.

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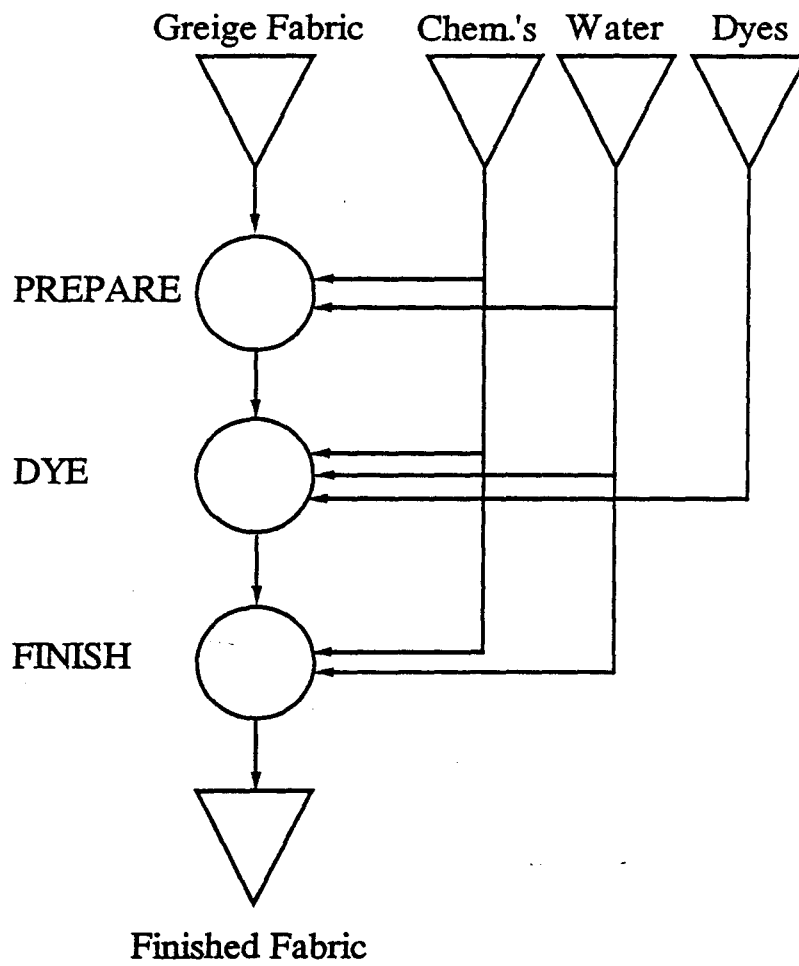


Figure 1: Dyehouse operations flowchart

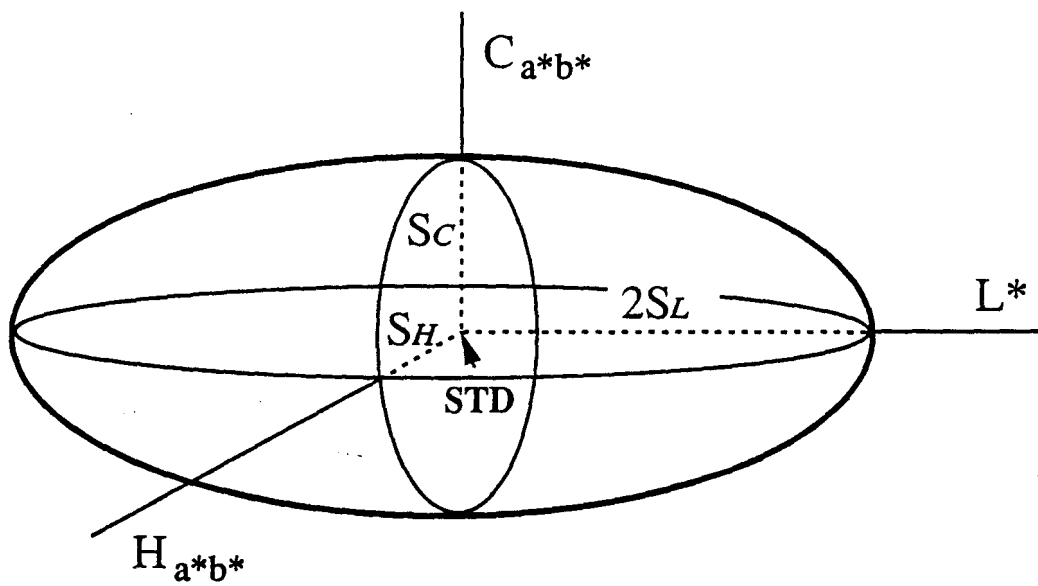


Figure 2: CMC(2:1) unit acceptance volume

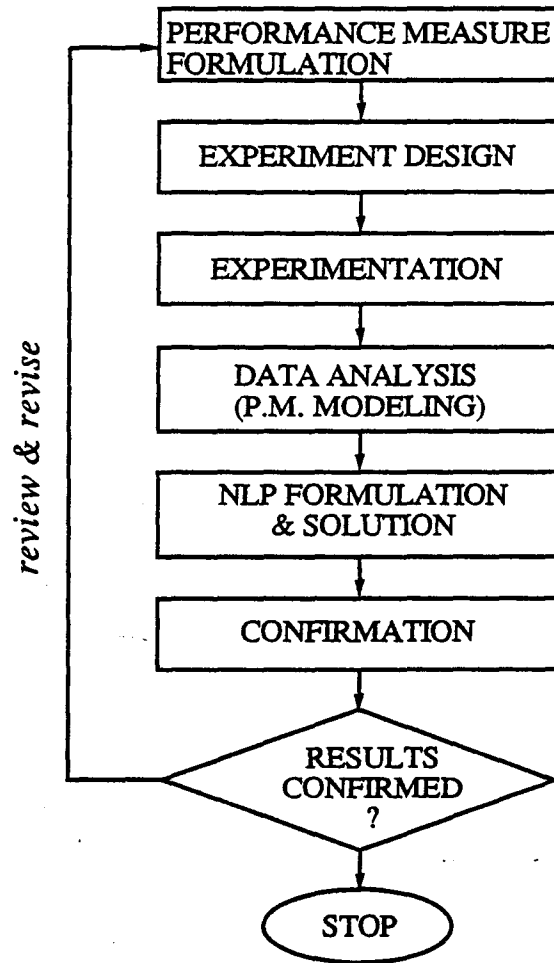


Figure 3: Robust design method

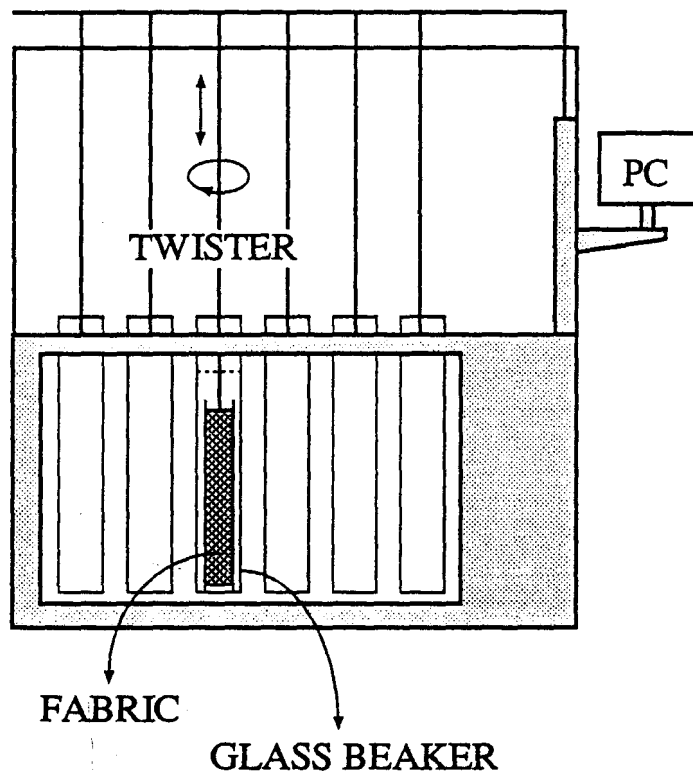


Figure 4: Ahiba-Textomat dyeing machine

Figure 5: Robust design data and performance measures obtained

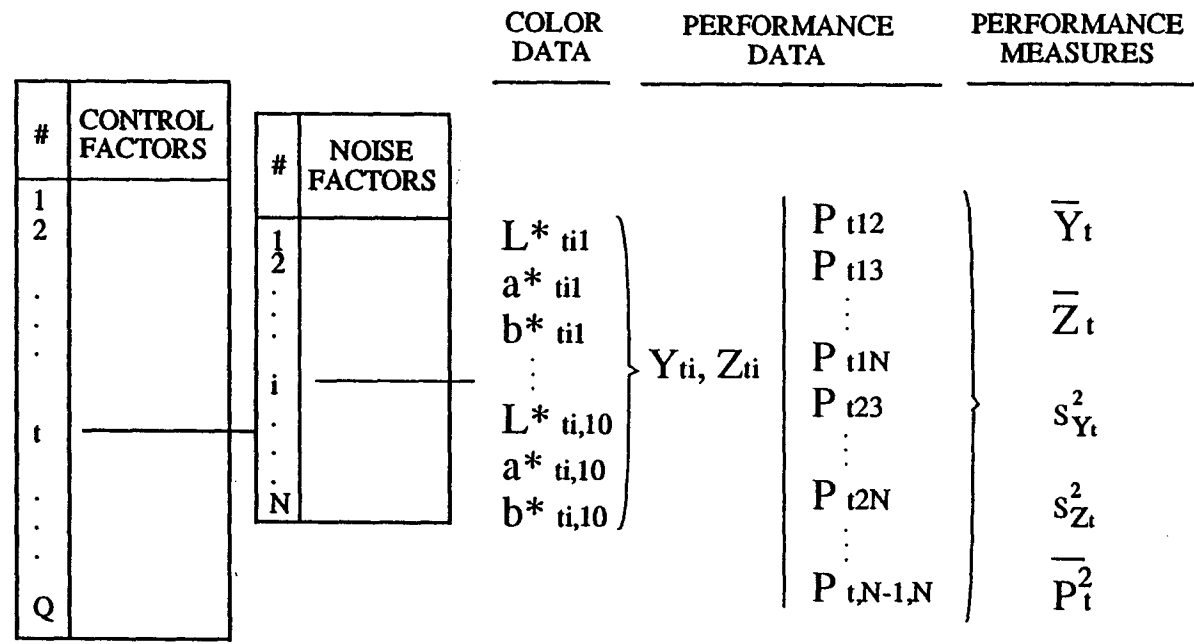


Figure 6: The experimental design for robust design of batch dyeing process

Point number	Control factor							Point number	Noise factor		
	T	G	L	D	S	A	M		W _v	W _f	W _d
1	165	15	25	0.9	60	2.4	35	1	+0.08	+0.02	+0.06
2	165	15	30	0.9	40	3.6	50	2	+0.08	-0.02	-0.06
3	165	15	30	1.0	60	2.4	20	3	-0.08	+0.02	-0.06
4	165	15	35	0.9	20	1.2	20	4	-0.08	-0.02	+0.06
5	165	15	35	1.1	20	3.6	50				
6	165	15	35	1.1	60	1.2	50				
7	165	30	25	1.1	40	1.2	35				
8	165	30	35	1.1	60	3.6	20				
9	165	60	25	0.9	20	3.6	20				
10	165	60	25	1.1	60	3.6	50				
11	165	60	30	1.0	20	1.2	50				
12	165	60	35	0.9	60	1.2	20				
13	165	60	35	1.1	20	2.4	20				
14	175	15	30	1.1	20	1.2	35				
15	175	30	25	1.0	20	3.6	20				
16	175	30	35	0.9	20	2.4	50				
17	175	60	25	0.9	40	1.2	20				
18	175	60	35	1.0	40	3.6	50				
19	185	15	25	0.9	20	3.6	50				
20	185	15	25	1.1	20	1.2	20				
21	185	15	25	1.1	60	3.6	50				
22	185	15	30	0.9	60	1.2	50				
23	185	15	35	0.9	60	3.6	20				
24	185	15	35	1.0	40	2.4	35				
25	185	30	30	1.1	20	3.6	20				
26	185	30	35	1.1	20	1.2	50				
27	185	60	25	0.9	60	3.6	50				
28	185	60	25	1.1	20	3.6	50				
29	185	60	35	0.9	20	1.2	35				
30	185	60	35	1.1	60	1.2	20				

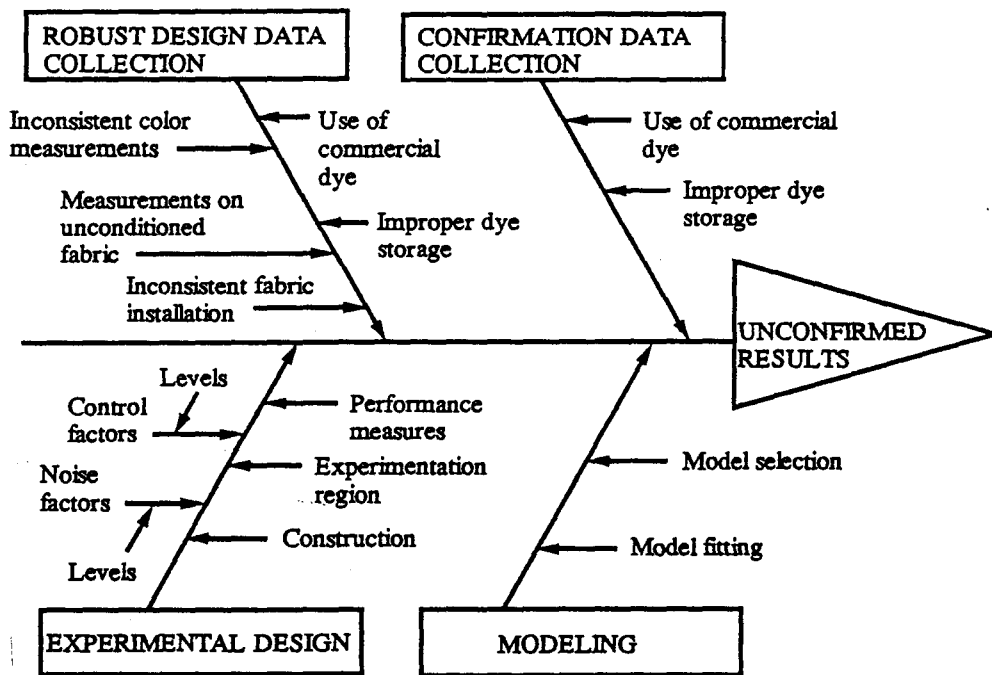


Figure 7: Possible sources of discrepancies

Table 1: The case problem

DYE	Cibacron BR Red 4G-E
CHEMICALS	Irgasol CO-NF, Reserve Salt Flake, sodium chloride, soda ash, caustic soda, Silvatol AS Conc
FABRIC	100% cotton left hand twill, commercially prepared
DYEING MACHINE	Ahiba-Textomat
DYEING PROCEDURE	Batch Exhaust Dyeing

Table 2: Important control factors and their levels

Control Factors	Levels		
	(-1)	(0)	(+1)
L: Liquor ratio [mL/g]	25	30	35
D: Amount of dye [% owf]	0.9	1.0	1.1
S: Salt concentration [g/L]	20	40	60
A: Alkali concentration [g/L]	1.2	2.4	3.6
T: Dyeing temperature [°F]	165	175	185
M: Time before alkali add [min]	20	35	50
G: Agitation rate [movements/min]	15	30	60

Table 3: Important noise factors and their levels

Noise Factors	Levels	
	$(-\sigma)$	$(+\sigma)$
W_d : Variation in dye weight (%)	-0.06	+0.06
W_v : Variation in dyebath volume (%)	-0.08	+0.08
W_f : Variation in fabric weight (%)	-0.02	+0.02

Table 4: Performance statistics calculated from the experimental data

Control Setting t	\bar{Y}_t	\bar{Z}_t	$s_{Y_t}^2$	$s_{Z_t}^2$	\bar{P}_t^2
1	2.479	0.279	4.049	0.168	13.506
2	10.956	0.400	14.854	0.227	5.967
3	16.678	0.386	80.456	0.228	92.842
4	9.234	0.400	16.615	0.317	9.553
5	5.371	0.343	7.086	0.271	7.693
6	3.351	0.244	3.808	0.194	8.364
7	4.055	0.239	6.534	0.185	11.728
8	9.666	0.346	35.306	0.214	43.493
9	3.337	0.317	6.436	0.368	27.526
10	2.496	0.272	0.710	0.261	2.548
11	21.505	0.394	38.587	0.445	3.402
12	12.918	0.317	25.804	0.331	3.560
13	8.108	0.250	47.305	0.171	26.358
14	14.480	0.466	62.544	0.722	24.733
15	4.649	0.177	2.579	0.039	5.859
16	3.254	0.059	0.879	0.000	2.217
17	19.907	0.318	43.599	0.281	6.903
18	2.993	0.086	1.776	0.001	2.327
19	2.367	0.108	0.528	0.003	1.227
20	3.652	0.202	3.076	0.033	5.122
21	3.378	1.069	1.450	5.680	5.649
22	15.460	0.521	26.395	1.067	6.778
23	14.201	0.260	29.391	0.083	15.569
24	7.084	0.248	15.598	0.078	18.269
25	5.250	0.143	9.600	0.008	11.943
26	5.903	0.106	12.989	0.001	9.890
27	14.913	0.201	55.755	0.012	19.720
28	4.085	0.127	8.684	0.009	21.515
29	10.480	0.160	39.596	0.004	22.895
30	4.386	0.106	8.583	0.004	14.136
C.S.	0.036	0.069			

Table 5: Analysis of variance for log Y

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	27	4191.93741	155.25694	156.607	0.0001
Error	156	154.65493	0.99138		
C Total	183	4346.59234			
Root MSE		0.99568	R-square	0.9644	
Dep Mean		0.81132	Adj R-sq	0.9583	
C.V.		122.72328			

Table 6: Analysis of variance for Z'

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	22	142.03562	6.45616	11.197	0.0001
Error	161	92.83252	0.57660		
C Total	183	234.86814			
Root MSE		0.75934	R-square	0.6047	
Dep Mean		-4.00201	Adj R-sq	0.5507	
C.V.		-18.97399			

Table 7: Analysis of variance for $\log s_y^2$

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	25	58.01006	2.32040	2020.974	0.0001
Error	4	0.00459	0.00115		
C Total	29	58.01465			
Root MSE		0.03388	R-square	0.9999	
Dep Mean		2.29033	Adj R-sq	0.9994	
C.V.		1.47946			

Table 8: Analysis of variance for $\log s^2_Z$

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	26	165.23102	6.35504	199.383	0.0005
Error	3	0.09562	0.03187		
C Total	29	165.32664			
Root MSE		0.17853	R-square	0.9994	
Dep Mean		-2.75805	Adj R-sq	0.9944	
C.V.		-6.47311			

Table 9: Analysis of variance for $\log \overline{P^2}$

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	24	26.96182	1.12341	76.286	0.0001
Error	5	0.07363	0.01473		
C Total	29	27.03545			
Root MSE		0.12135	R-square	0.9973	
Dep Mean		2.26256	Adj R-sq	0.9842	
C.V.		5.36348			

Table 10: Locally optimal solution obtained for Model (6)

Performance Measure	Model Prediction	Control Factor	Optimal Settings
$E(Y)$	2.447×10^{-2} (0.16)	L	35
$V(Y)$	1.081×10^{-1}	D	0.90
α_Y	0.69		
$[E(Y)]^2 + V(Y)$	1.087×10^{-1}	S	20.00
$E(Z)$	7.632×10^{-2} (0.28)	A	3.60
$V(Z)$	3.448×10^{-4}		
α_Z	9.35	G	30
$[E(Z)]^2 + V(Z)$	6.169×10^{-3}		
$E(P^2)$	4.032×10^{-1}	T	185
$[E(P)]_{max}$	6.350×10^{-1} (0.80)	M	20

Table 11: Confirmation experiment results and model predictions

Performance Measure	Model Prediction	Lower 95% Conf. limit	Upper 95% Conf. limit	Experiment results
$E(Y)$	2.447×10^{-2} (0.16)	9.911×10^{-3} (0.10)	6.043×10^{-2} (0.25)	6.045 (2.46)
$V(Y)$	1.081×10^{-1}	8.347×10^{-2}	1.400×10^{-1}	2.600×10^{-1}
$E(Z)$	7.632×10^{-2} (0.28)	5.311×10^{-2} (0.23)	1.189×10^{-1} (0.35)	6.531×10^{-2} (0.26)
$V(Z)$	3.448×10^{-4}	1.338×10^{-4}	8.886×10^{-4}	5.633×10^{-4}
$E(P^2)$	4.032×10^{-1}	2.586×10^{-1}	6.286×10^{-1}	9.369×10^{-3}
$[E(P)]_{max}$	6.350×10^{-1} (0.80)	5.085×10^{-1} (0.71)	7.929×10^{-1} (0.89)	9.679×10^{-2} (0.31)

