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# Rare $B \rightarrow \ell^+ \ell^- \gamma$ decay and new physics effects

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## Abstract

Using the most general, model independent form of the effective Hamiltonian rare decays  $B \rightarrow \ell^+ \ell^- \gamma$  ( $\ell = \mu, \tau$ ) are studied. The sensitivity of the photon energy distribution and branching ratio to the new Wilson coefficients is investigated. © 2001 Elsevier Science B.V. Open access under [CC BY license](#).

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## 1. Introduction

Started to work, the two  $B$ -factories open an excited new era in studying  $B$  meson decays [1,2]. The main research program of these factories is studying CP violation in  $B$  meson system and investigating their decays. From theoretical point of view, interest to the rare decays can be attributed to the fact that they occur at loop level in the Standard Model (SM) and they are very sensitive to the flavor structure of the SM as well as to the new physics beyond the SM. From experimental point of view studying radiative  $B$  meson decays can provide us essential information on the parameters of the SM, such as the elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix, the leptonic decay constants etc., which are yet poorly known.

It is well known that the flavor-changing neutral current process  $B_{s(d)} \rightarrow \ell^+ \ell^-$  has helicity suppression. These decays are proportional to the lepton mass and because of this reason the decay width of these processes are too small to be measured for the light lepton modes. It should be noted that in the SM the branching ratio of the  $\mathcal{B}(B_s \rightarrow e^+ e^-, \mu^+ \mu^-) \simeq 4.2 \times 10^{-14}$  and  $1.8 \times 10^{-9}$ , respectively. Although  $\tau$  channel is free of this suppression, its experimental detection is quite hard due to the low efficiency. It has been observed that the radiative leptonic  $B^+ \rightarrow \ell^+ \nu \gamma$  ( $\ell = e, \mu$ ) decays have larger branching ratio compared to that of the purely leptonic models [3–9]. It was shown in [10,11] that similar situation takes place for the radiative decays  $B_{s(d)} \rightarrow \ell^+ \ell^- \gamma$ . In these decays the contribution of the diagram when photon is radiated from an intermediate charged line, can be neglected, since it is strongly suppressed by a factor  $m_b^2/m_W^2$ . Moreover, the internal Bremsstrahlung (IB) part when photon is emitted from external charged leptons is proportional to lepton mass, which follows from helicity arguments, gives small contribution. For this reason in  $B \rightarrow \ell^+ \ell^- \gamma$  decay the main contribution should come from the diagrams, when photon is emitted from the initial quarks, i.e., structure-dependent part (SD), since they are free of the helicity

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suppression. Therefore, the decay rate of the  $B_{s(d)} \rightarrow \ell^+ \ell^- \gamma$  ( $\ell = e, \mu$ ) might have an enhancement in comparison to the pure leptonic models of  $B_{s(d)} \rightarrow \ell^+ \ell^-$  decay if the SD contributions to the decays are dominant and hence  $B_q \rightarrow \ell^+ \ell^- \gamma$  decay might be sensitive to the new physics effects beyond SM. New physics effects in rare  $B_q$  decays can appear in two different ways; either through new contributions to the Wilson coefficients existing in the SM or through the new operators in the effective Hamiltonian which are absent in the SM. The goal of this work is combining both these approaches to study the sensitivity of the physically measurable quantities, like branching ratio, photon energy distribution, to the new physics effects.

The work is organized as follows. In Section 2, we derive the general expression for the photon energy distribution using the most general form of four-Fermi interaction. In Section 3 we investigate the sensitivity of photon energy distribution and branching ratio to the new Wilson coefficients.

## 2. Matrix element for the $B_q \rightarrow \ell^+ \ell^- \gamma$ decay

In this section we calculate the photon energy distribution and branching ratio for the  $B_q \rightarrow \ell^+ \ell^- \gamma$  decay using the most general model independent form of the effective Hamiltonian. The matrix element for the process  $B \rightarrow \ell^+ \ell^- \gamma$  can be obtained from that of the purely leptonic  $B \rightarrow \ell^+ \ell^-$  decay. The effective  $b \rightarrow q \ell^+ \ell^-$  transition can be written in terms of twelve model independent four-Fermi interactions can be written in the following form [12]:

$$\mathcal{H}_{\text{eff}} = \frac{G\alpha}{\sqrt{2}\pi} V_{iq} V_{ib}^* \left\{ C_{SL} \bar{q} i \sigma_{\mu\nu} \frac{q^\nu}{q^2} L b \bar{\ell} \gamma^\mu \ell + C_{BR} \bar{q} i \sigma_{\mu\nu} \frac{q^\nu}{q^2} R b \bar{\ell} \gamma^\mu \ell + C_{LL}^{\text{tot}} \bar{q} L \gamma_\mu b_L \bar{\ell} L \gamma^\mu \ell_L \right. \\ \left. + C_{LR}^{\text{tot}} \bar{q} L \gamma_\mu b_L \bar{\ell} R \gamma^\mu \ell_R + C_{RL} \bar{q} R \gamma_\mu b_R \bar{\ell} L \gamma^\mu \ell_L + C_{RR} \bar{q} R \gamma_\mu b_R \bar{\ell} R \gamma^\mu \ell_R \right. \\ \left. + C_{LRLR} \bar{q} L b_R \bar{\ell} L \ell_R + C_{RLLR} \bar{q} R b_L \bar{\ell} L \ell_R + C_{LRRL} \bar{q} L b_R \bar{\ell} R \ell_L + C_{RLRL} \bar{q} R b_L \bar{\ell} R \ell_L \right. \\ \left. + C_T \bar{q} \sigma_{\mu\nu} b \bar{\ell} \sigma^{\mu\nu} \ell + i C_{TE} \epsilon^{\mu\nu\alpha\beta} \bar{q} \sigma_{\mu\nu} b \bar{\ell} \sigma_{\alpha\beta} \ell \right\}, \quad (1)$$

where the chiral projection operators  $L$  and  $R$  in (1) are defined as

$$L = \frac{1 - \gamma_5}{2}, \quad R = \frac{1 + \gamma_5}{2},$$

and  $C_X$  are the coefficients of the four-Fermi interactions. It can easily be seen from Eq. (1) that several of all Wilson coefficients do already exist in the SM. The coefficients  $C_{SL}$  and  $C_{BR}$  correspond to  $-2m_s C_7^{\text{eff}}$  and  $-2m_b C_7^{\text{eff}}$  in the SM, respectively. The next four terms in this expression are the vector interactions. The interaction terms containing  $C_{LL}^{\text{tot}}$  and  $C_{LR}^{\text{tot}}$  exist in the SM in the form  $C_9^{\text{eff}} - C_{10}$  and  $C_9^{\text{eff}} + C_{10}$ , respectively. Therefore,  $C_{LL}^{\text{tot}}$  and  $C_{LR}^{\text{tot}}$  describe the contributions coming from the SM and the new physics, whose explicit are

$$C_{LL}^{\text{tot}} = C_9^{\text{eff}} - C_{10} + C_{LL}, \quad C_{LR}^{\text{tot}} = C_9^{\text{eff}} + C_{10} + C_{LR}.$$

The terms with coefficients  $C_{LRLR}$ ,  $C_{RLLR}$ ,  $C_{LRRL}$  and  $C_{RLRL}$  describe the scalar type interactions. The last two terms in Eq. (1) with the coefficients  $C_T$  and  $C_{TE}$  describe the tensor type interactions.

Having presented the general form of the effective Hamiltonian the next problem is calculation of the matrix element of the  $B_q \rightarrow \ell^+ \ell^- \gamma$  decay. This matrix element can be written as the sum of the structure-dependent and internal Bremsstrahlung parts

$$\mathcal{M} = \mathcal{M}_{\text{SD}} + \mathcal{M}_{\text{IB}}. \quad (2)$$

It follows from Eq. (1) that, in order to calculate the matrix element  $\mathcal{M}_{\text{SD}}$  for the structure-dependent part, the following matrix elements are needed

$$\langle \gamma | \bar{s} \gamma_\mu (1 \mp \gamma_5) b | B \rangle, \quad \langle \gamma | \bar{s} \sigma_{\mu\nu} b | B \rangle, \quad \langle \gamma | \bar{s} (1 \mp \gamma_5) b | B \rangle. \quad (3)$$

The first two of the matrix elements in Eq. (3) are defined as [4,10]

$$\langle \gamma(k) | \bar{q} \gamma_\mu (1 \mp \gamma_5) b | B(p_B) \rangle = \frac{e}{m_B^2} \left\{ \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} q^\lambda k^\sigma g(q^2) \pm i [\varepsilon^{*\mu}(kq) - (\varepsilon^* q) k^\mu] f(q^2) \right\}, \quad (4)$$

$$\langle \gamma(k) | \bar{q} \sigma_{\mu\nu} b | B(p_B) \rangle = \frac{e}{m_B^2} \epsilon_{\mu\nu\lambda\sigma} [G \varepsilon^{*\lambda} k^\sigma + H \varepsilon^{*\lambda} q^\sigma + N (\varepsilon^* q) q^\lambda k^\sigma], \quad (5)$$

respectively, where  $\varepsilon_\mu^*$  and  $k_\mu$  are the four vector polarization and four momentum of the photon, respectively,  $q$  is the momentum transfer and  $p_B$  is the momentum of the  $B$  meson. The matrix element  $\langle \gamma(k) | \bar{s} \sigma_{\mu\nu} \gamma_5 b | B(p_B) \rangle$  can be obtained from Eq. (5) using the identity

$$\sigma_{\mu\nu} = -\frac{i}{2} \epsilon_{\mu\nu\alpha\beta} \sigma^{\alpha\beta} \gamma_5.$$

The matrix elements  $\langle \gamma(k) | \bar{s} (1 \mp \gamma_5) b | B(p_B) \rangle$  and  $\langle \gamma | \bar{s} i \sigma_{\mu\nu} q^\nu b | B \rangle$  can be calculated by contracting both sides of the Eqs. (4) and (5) with  $q_\nu$ , respectively. We get then

$$\langle \gamma(k) | \bar{s} (1 \mp \gamma_5) b | B(p_B) \rangle = 0, \quad (6)$$

$$\langle \gamma | \bar{s} i \sigma_{\mu\nu} q^\nu b | B \rangle = \frac{e}{m_B^2} i \epsilon_{\mu\nu\alpha\beta} q^\nu \varepsilon^{\alpha*} k^\beta G. \quad (7)$$

Using Eqs. (5) and (7) the matrix element  $\langle \gamma | \bar{s} i \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b | B \rangle$  can be written in terms of form factors that are calculated in framework of the QCD sum rules [10] as follows

$$\langle \gamma | \bar{s} i \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b | B \rangle = \frac{e}{m_B^2} \left\{ \epsilon_{\mu\alpha\beta\sigma} \varepsilon^{\alpha*} q^\beta k^\sigma g_1(q^2) + i [\varepsilon_\mu^*(qk) - (\varepsilon^* q) k_\mu] f_1(q^2) \right\}. \quad (8)$$

It should be noted that these form factors were calculated in framework of the light-front model in [13]. So, using Eqs. (5), (7) and (8) we can easily express  $G$ ,  $H$  and  $N$  in terms of the form factors  $g_1$  and  $f_1$ . The matrix element which describes the structure-dependent part can be obtained from Eqs. (4)–(8)

$$\begin{aligned} \mathcal{M}_{SD} = & \frac{\alpha G_F}{4\sqrt{2}\pi} V_{tb} V_{tq}^* \frac{e}{m_B^2} \\ & \times \left\{ \bar{\ell} \gamma^\mu (1 - \gamma_5) \ell [A_1 \epsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} q^\alpha k^\beta + i A_2 (\varepsilon_\mu^*(kq) - (\varepsilon^* q) k_\mu)] \right. \\ & + \bar{\ell} \gamma^\mu (1 + \gamma_5) \ell [B_1 \epsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} q^\alpha k^\beta + i B_2 (\varepsilon_\mu^*(kq) - (\varepsilon^* q) k_\mu)] \\ & + i \epsilon_{\mu\nu\alpha\beta} \bar{\ell} \sigma^{\mu\nu} \ell [G \varepsilon^{*\alpha} k^\beta + H \varepsilon^{*\alpha} q^\beta + N (\varepsilon^* q) q^\alpha k^\beta] \\ & \left. + i \bar{\ell} \sigma_{\mu\nu} \ell [G_1 (\varepsilon^{*\mu} k^\nu - \varepsilon^{*\nu} k^\mu) + H_1 (\varepsilon^{*\mu} q^\nu - \varepsilon^{*\nu} q^\mu) + N_1 (\varepsilon^* q) (q^\mu k^\nu - q^\nu k^\mu)] \right\}, \quad (9) \end{aligned}$$

where

$$\begin{aligned} A_1 &= \frac{1}{q^2} (C_{BR} + C_{SL}) g_1 + (C_{LL}^{\text{tot}} + C_{RL}) g, & A_2 &= \frac{1}{q^2} (C_{BR} - C_{SL}) f_1 + (C_{LL}^{\text{tot}} - C_{RL}) f, \\ B_1 &= \frac{1}{q^2} (C_{BR} + C_{SL}) g_1 + (C_{LR}^{\text{tot}} + C_{RR}) g, & B_2 &= \frac{1}{q^2} (C_{BR} - C_{SL}) f_1 + (C_{LR}^{\text{tot}} - C_{RR}) f, \\ G &= 4C_T g_1, & N &= -4C_T \frac{1}{q^2} (f_1 + g_1), & H &= N(qk), \\ G_1 &= -8C_{TE} g_1, & N_1 &= 8C_{TE} \frac{1}{q^2} (f_1 + g_1), & H_1 &= N_1(qk). \end{aligned} \quad (10)$$

For the inner Bremsstrahlung part we get

$$\mathcal{M}_{\text{IB}} = \frac{\alpha G_F}{4\sqrt{2}\pi} V_{tb} V_{tq}^* e f_B i \left\{ F \bar{\ell} \left( \frac{\not{\epsilon}^* \not{p}_B}{2p_1 k} - \frac{\not{p}_B \not{\epsilon}^*}{2p_2 k} \right) \gamma_5 \ell + F_1 \bar{\ell} \left[ \frac{\not{\epsilon}^* \not{p}_B}{2p_1 k} - \frac{\not{p}_B \not{\epsilon}^*}{2p_2 k} + 2m_\ell \left( \frac{1}{2p_1 k} + \frac{1}{2p_2 k} \right) \not{\epsilon}^* \right] \ell \right\}. \quad (11)$$

In obtaining this expression we have used

$$\langle 0 | \bar{s} \gamma_\mu \gamma_5 b | B \rangle = -i f_B p_{B\mu}, \quad \langle 0 | \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) b | B \rangle = 0,$$

and conservation of the vector current. The functions  $F$  and  $F_1$  are defined as follows

$$F = 2m_\ell (C_{LR}^{\text{tot}} - C_{LL}^{\text{tot}} + C_{RL} - C_{RR}) + \frac{m_B^2}{m_b} (C_{LRLR} - C_{RLLR} - C_{LRRL} + C_{RLRL}),$$

$$F_1 = \frac{m_B^2}{m_b} (C_{LRLR} - C_{RLLR} + C_{LRRL} - C_{RLRL}). \quad (12)$$

The double differential decay width of the  $B \rightarrow \ell^+ \ell^- \gamma$  process in the rest frame of the  $B$  meson is found to be

$$\frac{d\Gamma}{dE_\gamma dE_1} = \frac{1}{256\pi^3 m_B} |\mathcal{M}|^2, \quad (13)$$

where  $E_\gamma$  and  $E_1$  are the photon and one of the final lepton energy, respectively. The boundaries of  $E_\gamma$  and  $E_1$  are determined from the following inequalities

$$0 \leq E_\gamma \leq \frac{m_B^2 - 4m_\ell^2}{2m_B}, \quad \frac{m_B - E_\gamma}{2} - \frac{E_\gamma}{2} v \leq E_1 \leq \frac{m_B - E_\gamma}{2} + \frac{E_\gamma}{2} v, \quad (14)$$

where

$$v = \sqrt{1 - \frac{4m_\ell^2}{q^2}},$$

is the lepton velocity.

The  $|\mathcal{M}_{\text{SD}}|^2$  term is infrared free; interference term has an integrable infrared singularity and only  $|\mathcal{M}_{\text{IB}}|^2$  term has infrared singularity due to the emission of soft photon. In the soft photon limit the  $B_q \rightarrow \ell^+ \ell^- \gamma$  decay cannot be distinguished from the pure leptonic  $B_q \rightarrow \ell^+ \ell^-$  decay. For this reason, in order to obtain a finite result the  $B \rightarrow \ell^+ \ell^- \gamma$  and the pure leptonic  $B_q \rightarrow \ell^+ \ell^-$  decay with radiative corrections must be considered together. It was shown explicitly in the second reference of [10] that when both processes are considered together, all infrared singularities coming from the real photon emission and the virtual photon corrections are indeed canceled and the final result is finite. In the present work our point of view is slightly different from the standard description, namely, we consider the  $B_q \rightarrow \ell^+ \ell^- \gamma$  decay as a different process but not as the  $\mathcal{O}(\alpha)$  correction to the  $B \rightarrow \ell^+ \ell^-$  decay. In other words, we consider the photon in the  $B_q \rightarrow \ell^+ \ell^- \gamma$  decay as a hard photon. For this reason, in order to obtain the decay width of the  $B_q \rightarrow \ell^+ \ell^- + (\text{hard photon})$  we must impose a cut on the photon energy, which will correspond to the experimental cut imposed on the minimum energy for detectable photon. We require the photon energy to be larger than 25 MeV, i.e.,  $E_\gamma \geq \delta m_B / 2$ ,  $\delta \geq 0.010$ .

After integrating over lepton energy, we get the following expression for the photon energy distribution

$$\begin{aligned}
 \frac{d\Gamma}{dx} = & - \left| \frac{\alpha G_F}{4\sqrt{2}\pi} V_{tb} V_{tq}^* \right|^2 \frac{\alpha}{(2\pi)^3} \frac{\pi}{4} m_B \\
 & \times \left( x^3 v \left\{ 4m_\ell \operatorname{Re}([A_1 + B_1]G^*) - 4m_B^2 r \operatorname{Re}(A_1 B_1^* + A_2 B_2^*) \right. \right. \\
 & \quad - 4[|H_1|^2(1-x) + \operatorname{Re}(G_1 H_1^*)x] \frac{(1+8r-x)}{x^2} \\
 & \quad - 4[|H|^2(1-x) + \operatorname{Re}(GH^*)x] \frac{(1-4r-x)}{x^2} \\
 & \quad + \frac{1}{3} m_B^2 [2\operatorname{Re}(GN^*) + m_B^2 |N|^2(1-x)](1-4r-x) \\
 & \quad + \frac{1}{3} m_B^2 [2\operatorname{Re}(G_1 N_1^*) + m_B^2 |N_1|^2(1-x)](1+8r-x) \\
 & \quad - \frac{2}{3} m_B^2 (|A_1|^2 + |A_2|^2 + |B_1|^2 + |B_2|^2)(1-r-x) - \frac{4}{3} (|G|^2 + |G_1|^2) \frac{(1+2r-x)}{(1-x)} \\
 & \quad \left. \left. + 2m_\ell \operatorname{Im}([A_2 + B_2][6H_1^*(1-x) + 2G_1^*x - m_B^2 N_1^*x(1-x)]) \frac{1}{x} \right\} \right. \\
 & + 4f_B \left\{ 2v \left[ \operatorname{Re}(FG^*) \frac{1}{(1-x)} - \operatorname{Re}(FH^*) + m_B^2 \operatorname{Re}(FN^*) + m_\ell \operatorname{Re}([A_2 + B_2]F_1^*) \right] x(1-x) \right. \\
 & \quad + \ln \frac{1+v}{1-v} \left[ m_\ell \operatorname{Re}([A_2 + B_2]F_1^*)x(x-4r) + 2\operatorname{Re}(FH^*)[1-x+2r(x-2)] \right. \\
 & \quad \left. \left. - 4rx \operatorname{Re}(FG^*) + m_B^2 \operatorname{Re}(FN^*)x(x-1) - m_\ell \operatorname{Re}([A_1 + B_1]F^*)x^2 \right] \right\} \\
 & + 4f_B^2 \left\{ 2v(|F|^2 + (1-4r)|F_1|^2) \frac{(1-x)}{x} + \ln \frac{1+v}{1-v} \left[ |F|^2 \left( 2 + \frac{4r}{x} - \frac{2}{x} - x \right) \right. \right. \\
 & \quad \left. \left. + |F_1|^2 \left( 2(1-4r) - \frac{2(1-6r+8r^2)}{x} - x \right) \right] \right\} \Bigg\}, \tag{15}
 \end{aligned}$$

where  $x = 2E_\gamma/m_B$  is the dimensionless photon energy,  $r = m_\ell^2/m_B^2$ .

It follows from Eq. (15) that in order to calculate the decay width explicit forms of the form factors  $g$ ,  $f$ ,  $g_1$  and  $f_1$  are needed. These form factors are calculated in framework of light-cone QCD sum rules in [4] and [10], and their  $q^2$  dependences, to a very good accuracy, can be represented in the following dipole forms,

$$\begin{aligned}
 g(q^2) &= \frac{1 \text{ GeV}}{(1 - \frac{q^2}{5.6^2})^2}, & f(q^2) &= \frac{0.8 \text{ GeV}}{(1 - \frac{q^2}{6.5^2})^2}, \\
 g_1(q^2) &= \frac{3.74 \text{ GeV}^2}{(1 - \frac{q^2}{40.5})^2}, & f_1(q^2) &= \frac{0.68 \text{ GeV}^2}{(1 - \frac{q^2}{30})^2}, \tag{16}
 \end{aligned}$$

which we will use in the numerical analysis.

### 3. Numerical analysis and discussion

In this section we will present our numerical analysis. Numerical results are presented only for the  $B_s \rightarrow \ell^+ \ell^- \gamma$  decay. It is clear that in the SU(3) limit the difference between the decay rates is attributed to the CKM matrix elements only, i.e.,

$$\frac{\Gamma(B_d \rightarrow \ell^+ \ell^- \gamma)}{\Gamma(B_s \rightarrow \ell^+ \ell^- \gamma)} \simeq \left| \frac{V_{tb} V_{td}^*}{V_{tb} V_{ts}^*} \right|^2 \simeq \frac{1}{20}.$$

The values of the main input parameters which have been used in the present work are:  $m_b = 4.8$  GeV,  $m_c = 1.35$  GeV,  $m_\tau = 1.78$  GeV,  $|V_{tb} V_{ts}^*| = 0.045$ ,  $\alpha^{-1} = 137$ ,  $G_F = 1.17 \times 10^{-5}$  GeV $^{-2}$ . For the Wilson coefficients  $C_7^{\text{eff}}(m_b)$  and  $C_{10}(m_b)$  we have used the results given in [12,13]. In the leading logarithmic approximation, at the scale  $\mathcal{O}(\mu = m_b)$  they are given as  $C_7^{\text{eff}}(m_b) = -0.315$ ,  $C_{10}(m_b) = 4.6242$ . Although individual Wilson coefficients at  $\mu \sim m_b$  level are all real, the effective Wilson coefficient  $C_9^{\text{eff}}(m_b)$  has a finite phase. The analytic expression of  $C_9^{\text{eff}}(m_b)$  for the  $b \rightarrow s$  transition, in next-to-leading order approximation is given as

$$\begin{aligned} C_9^{\text{eff}}(m_b, \hat{s}) &= C_9(m_b) + 0.124w(\hat{s}) + g(\hat{m}_c, \hat{s})(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \\ &\quad - \frac{1}{2}g(\hat{m}_q, \hat{s})(C_3 + 3C_4) - \frac{1}{2}g(\hat{m}_b, \hat{s})(4C_3 + 4C_4 + 3C_5 + C_6) + \frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6) \end{aligned} \quad (17)$$

where  $m_q = m_q/m_b$ ,  $\hat{s} = q^2/m_b^2$  and the values of the individual Wilson coefficients are listed in Table 1. In Eq. (17)  $w(\hat{s})$  describes one gluon corrections to the matrix element of the operator  $\mathcal{O}_9$  and the function  $g(\hat{m}_q, \hat{s})$  stands for the one loop corrections to the four quark operators  $\mathcal{O}_1$ – $\mathcal{O}_6$  with mass  $m_q$  at the dilepton invariant mass  $s$  [14,15]:

$$\begin{aligned} g(\hat{m}_q, \hat{s}') &= -\frac{8}{9} \ln \hat{m}_q + \frac{8}{27} + \frac{4}{9} y_q - \frac{2}{9} (2 + y_q) \sqrt{|1 - y_q|} \\ &\quad \times \left[ \Theta(1 - y_q) \left( \ln \frac{1 + \sqrt{1 - y_q}}{1 - \sqrt{1 - y_q}} - i\pi \right) + \Theta(y_q - 1) 2 \arctan \frac{1}{\sqrt{y_q - 1}} \right], \end{aligned} \quad (18)$$

where  $y_q = 4\hat{m}_q^2/\hat{s}'$  and  $\hat{s}' = q^2/m_b^2$ . It is well known that the Wilson coefficient  $C_9^{\text{eff}}$  receives also long distance contributions, which have their origin in the real  $c\bar{c}$  intermediate states, i.e.,  $J/\psi$ ,  $\psi'$ , ... (see [16]). In this work we restrict ourselves only to short contributions. Furthermore we assume that all new Wilson coefficients are real and varied in the region  $-4 \leq C_X \leq +4$ .

In Fig. 1 we present the dependence of the integrated branching ratio of the  $B \rightarrow \tau^+ \tau^- \gamma$  decay on the new Wilson coefficients for the cut  $\delta = 0.01$  imposed on the photon energy, without long distance effects. It clearly follows from this figure that as the new Wilson coefficients  $C_T$ ,  $C_{RL}$ ,  $C_{LR}$ ,  $C_{LRLR}$  and  $C_{RLRL}$  increase from  $-4$  to  $+4$  branching ratio decreases. However, this behavior is reversed for the coefficients  $C_{LL}$ ,  $C_{RR}$ ,  $C_{LRRL}$  and  $C_{RLLR}$ , i.e., when these coefficients increase from  $-4$  to  $+4$  branching ratio also increases accordingly. Exception to these cases takes place for the coefficient  $C_{TE}$ . In the region  $-4 \leq C_{TE} \leq 0$  branching ratio decreases and in the region  $0 \leq C_{TE} \leq +4$  it tends to increase.

Table 1

The numerical values of the Wilson coefficients at  $\mu \sim m_b$  scale within the SM

$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$	$C_7^{\text{eff}}$	$C_9$	$C_{10}^{\text{eff}}$
-0.248	1.107	0.011	-0.026	0.007	-0.031	-0.315	4.344	-4.6242

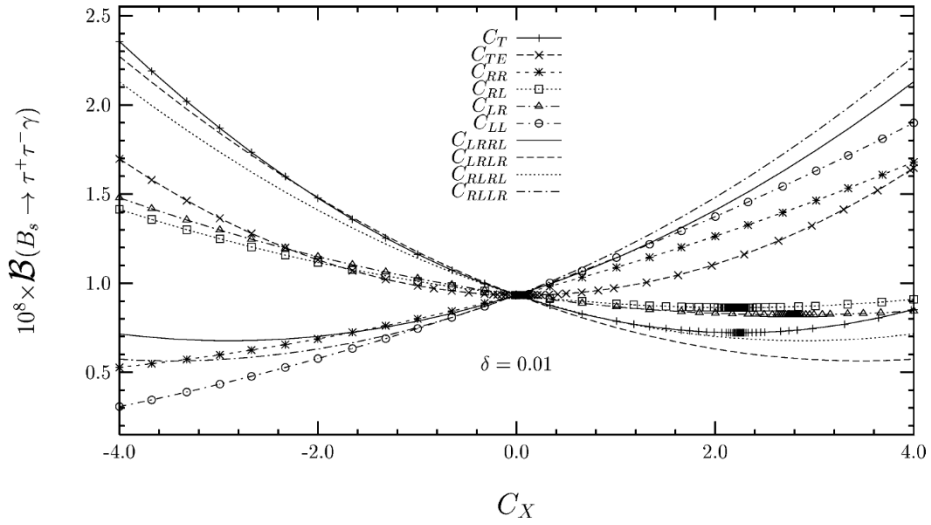


Fig. 1. The dependence of the integrated branching ratio of the  $B_s \rightarrow \tau^+\tau^-\gamma$  decay on the new Wilson coefficients for the cut  $\delta = 0.01$  imposed on the photon energy, only for short distance effects.

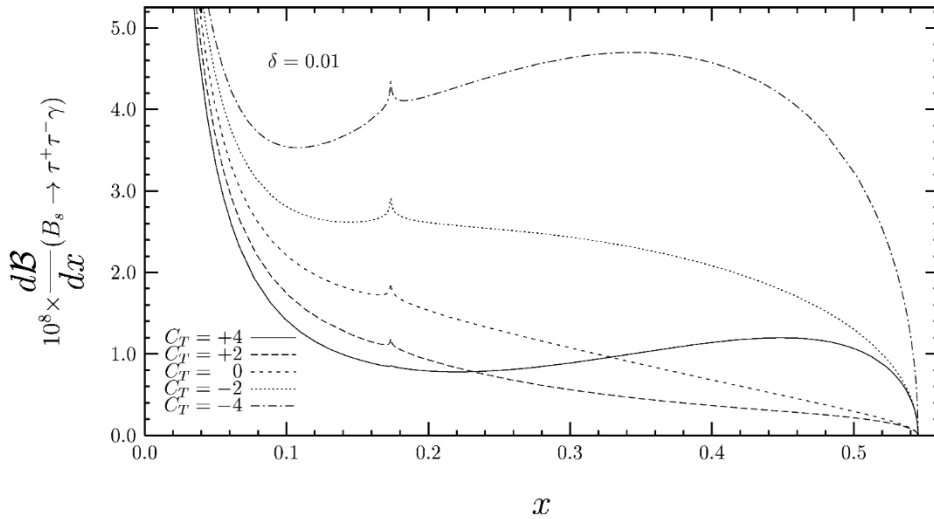


Fig. 2. The dependence of the differential branching ratio for the  $B_s \rightarrow \tau^+\tau^-\gamma$  decay on the dimensionless variable  $x = 2E_\gamma/m_B$  at different values of tensor interaction with coefficient  $C_T$ , without the long distance effects.

For the choice of the photon energy cut  $\delta = 0.02$  all the previous arguments remain valid with only a slight decrease in the value of the branching ratio.

From all present figures we observe that when all Wilson coefficients lie in the range  $-4 \leq C_X \leq -2$ , the branching ratio is more sensitive to the existence of tensor  $C_T$ , scalar  $C_{LRRL}$ ,  $C_{RLRL}$  and vector  $C_{LL}$  type interactions. On the other side, when Wilson coefficients lie in the region  $+2 \leq C_X \leq +4$  the branching ratio is more sensitive to the scalar type interaction with coefficients  $C_{LRRL}$  and  $C_{RLLR}$ .

Photon energy distribution can also give useful information about new physics effects. For this purpose, in Fig. 2 we present the dependence of the differential branching ratio for the  $B \rightarrow \tau^+\tau^-\gamma$  decay on the dimensionless

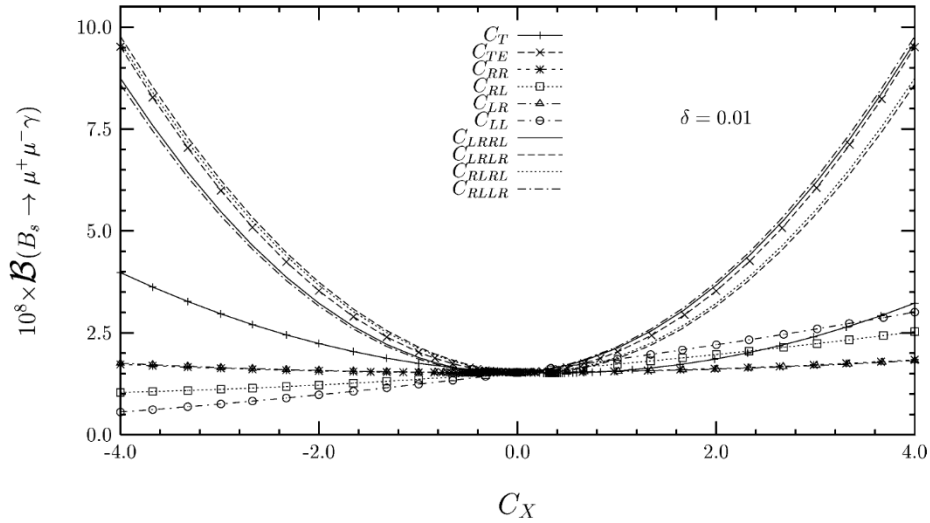


Fig. 3. The same as in Fig. 1, but for the  $B_s \rightarrow \mu^+ \mu^- \gamma$  decay.

variable  $x = 2E_\gamma/m_B$  at different values of tensor interaction with coefficient  $C_T$ . We observe from this figure that when  $C_T < 0$  then the related tensor interaction gives constructive contribution to the SM result, and when  $C_T > 0$  the contribution is destructive. In other words measurement of the differential branching ratio can give essential information about the sign of new Wilson coefficients.

Performing measurement at different photon energies can give information not only about magnitude but also about the sign of the new Wilson coefficient interaction.

Note that the results presented in this work can easily be applied to the  $B_s \rightarrow \mu^+ \mu^- \gamma$  decay. For example, the branching ratio for the  $B_s \rightarrow \mu^+ \mu^- \gamma$  decay at  $\delta = 0.01$ , without the long distance effects at  $C_{TE} = C_T = \pm 4$  is larger about 5 times, compared to that of the SM prediction of the branching ratio for the  $B_s \rightarrow \mu^+ \mu^- \gamma$  decay. Additionally, the dependence of the branching ratio on the new Wilson coefficients is symmetric with respect to the zero point (see Fig. 3). It should be stressed that by studying the Dalitz distribution  $d\Gamma/dE_\gamma dE_1$  at different fixed values of the final lepton (or photon) energies we can get useful information not only about the magnitude of the new Wilson coefficients but also about their sign.

In conclusion, using a general, model independent effective Hamiltonian, the  $B_s \rightarrow \ell^+ \ell^- \gamma$  decay is studied. It has been shown that the branching ratio and photon energy distribution are very sensitive to the existence of new physics beyond SM. We conclude that the radiative  $B_s \rightarrow \ell^+ \ell^- \gamma$  decay can be measured in the  $B$  factories as well as LHC-B experiments, in which  $\approx 2 \times 10^{11}$   $B_s$  mesons are expected to be produced per year.

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