

Radiative $B_c \rightarrow \tau \bar{\nu}_\tau \gamma$ decay

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06531 Ankara, Turkey**Abstract**

The radiative $B_c \rightarrow \tau \bar{\nu}_\tau \gamma$ decay is analyzed in the standard model. The branching ratio of this decay is calculated and the contributions of the Bremstrahlung and structure dependent parts are compared. It is shown that this decay can be detected at LHC.

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1 INTRODUCTION

Recently the CDF collaboration at Fermilab Tevatron in 1.8 TeV $p\bar{p}$ has reported the discovery of B_c meson with ground state mass $m_{B_c} = 6400 \pm 390 \pm 130$ MeV and life time $\tau(B_c) = 0.46^{+0.18}_{-0.16}$ ps [1]. This discovery has stimulated the investigation of the properties of the B_c meson theoretically, as well as experimentally, on a new footing. B_c meson contains two heavy quarks and for this reason the perturbative QCD predictions are more reliable. Therefore the study of the decays of the B_c meson allows to check the QCD predictions more precisely and one can get essential new information about the confinement scale inside hadrons.

The weak decay channels of the B_c meson are richer than the corresponding B_q ($q = u, d, s$). The weak B_c meson decay channels can be divided into three classes: a) The \bar{b} quark decay with the spectator c quark, for example, $B_c \rightarrow J/\psi \ell \bar{\nu}_\ell$; b) the c quark, decay with the spectator \bar{b} quark, for example, $B_c \rightarrow B_s \ell \bar{\nu}_\ell$ and c) the annihilation channels like $B_c \rightarrow \ell \bar{\nu}_\ell (c\bar{s}, u\bar{s})$, where $\ell = e, \mu, \tau$.

From experimental point of view, investigation of the weak decays of the B_c meson gives us the most direct information in determining corresponding elements of the Cabibbo–Kobayashi–Maskawa matrix (CKM), such as V_{cb} etc. A comprehensive analysis of the B_c meson spectroscopy and strong and electromagnetic decays of the excited states is given in [2]. The semileptonic and various nonleptonic decays and the B_c life time have been calculated in many works (see for example [3]–[10] and references listed therein).

The pure leptonic decays of B_c meson are the simplest among all decays and these decays can be useful in determination of the leptonic decay constant f_{B_c} of the B_c meson. The $B_c \rightarrow \ell \bar{\nu}_\ell$ ($\ell = e, \mu$) decay is helicity suppressed and it makes the determination of f_{B_c} of the B_c meson very difficult. Although the $B_c \rightarrow \tau \bar{\nu}_\tau$ channel is free of the helicity suppression, its observation is possible if we have a good efficiency for detection of the τ lepton. In this work we investigate the $B_c \rightarrow \tau \bar{\nu}_\tau \gamma$ decay in frame work of the standard model. In $B_c \rightarrow \tau \bar{\nu}_\tau \gamma$ decay comparable contributions come from diagrams where photon is emitted both from initial quark lines and final τ leptons. These contributions can give very useful information about relative roles of the strong and electroweak interactions. Note that in the $B_c \rightarrow \ell \bar{\nu}_\ell \gamma$ ($\ell = e, \mu$) decay, it follows from helicity arguments that the contribution of the diagram where photon radiated from charged lepton, must be proportional to the lepton mass m_ℓ , and hence it can safely be neglected (see [11, 12]).

In [13, 14] the number of B_c mesons that will be produced in LHC is estimated to be $\sim 2 \times 10^8$. This clearly is an indication of the real possibility of an experimental investigation of the properties of the B_c meson at LHC. This paper is organized as follows. In section 2 we give necessary theoretical formalism for the $B_c \rightarrow \tau \bar{\nu}_\tau \gamma$ decay. Section 3 is devoted to the numerical analysis and the discussion of the results.

2 FORMALISM FOR THE $B_c \rightarrow \tau \bar{\nu}_\tau \gamma$ DECAY

The matrix element for the $B_c \rightarrow \tau \bar{\nu}_\tau \gamma$ decay can be obtained from $B_c \rightarrow \tau \bar{\nu}_\tau$ process if the photon is attached to any charged fermion line. The effective Hamiltonian for the $B_c \rightarrow \tau \bar{\nu}_\tau$

decay is

$$\mathcal{H} = \frac{G}{\sqrt{2}} V_{cb} \bar{c} \gamma_\mu (1 - \gamma_5) b \bar{\tau} \gamma_\mu (1 - \gamma_5) \nu_\tau . \quad (1)$$

Let us start our investigation for the process $B_c \rightarrow \tau \bar{\nu}_\tau \gamma$ by considering the Bremsstrahlung diagram, where photon is radiated from τ lepton. The corresponding matrix element is

$$\mathcal{M}^{Brem.} = i f_{B_c} e \frac{G}{\sqrt{2}} V_{cb} \varepsilon_\alpha \bar{u}(p_1) \left\{ \gamma_\alpha + \frac{m_\tau}{2p_1 q} [2p_{1\alpha} + \gamma_\alpha \not{q}] \right\} (1 - \gamma_5) v(p_2) , \quad (2)$$

where ε_α and q_α are the photon four-vector polarization and momentum, p_1 and p_2 are the momenta of τ lepton and neutrino, respectively, and f_{B_c} is the leptonic decay constant of B_c meson, and defined as

$$\langle 0 | \bar{c} \gamma_\mu \gamma_5 b | B_c \rangle = -i f_{B_c} P_{B\mu} .$$

From Eq. (2) we immediately see that the matrix element $\mathcal{M}^{Brem.}$ is not gauge invariant. In regard to the contribution of the structure dependent part to the $B_c \rightarrow \tau \bar{\nu}_\tau \gamma$ process, i.e., when photon is radiated from initial quark lines, we consider the following correlator:

$$\mathcal{M}_\mu^{SD} = \frac{G}{\sqrt{2}} V_{cb} \varepsilon_\alpha \int d^4 x e^{iqx} \langle 0 | \mathcal{T} \{ \bar{c}(0) \gamma_\mu (1 - \gamma_5) b(0) J_\alpha^{el}(x) \} | B_c \rangle , \quad (3)$$

where J_α^{el} is the electromagnetic current for b or c quarks.

Vector part (\mathcal{M}^V) of this correlator is obviously equal to

$$\mathcal{M}_\mu^V = e \frac{G}{\sqrt{2}} V_{cb} \frac{f_1(p^2)}{m_{B_c}^2} \epsilon_{\mu\alpha\rho\beta} \varepsilon_\alpha p_\rho q_\beta , \quad (4)$$

where $f_1(p^2)$ is the transition form factor, and p is the momentum transfer, $p = p_1 + p_2 = p_1 - q$. In general axial part of the correlator can be written as

$$\mathcal{M}_\mu^A = \frac{G}{\sqrt{2}} V_{cb} \varepsilon_\alpha \{ A_1 g_{\mu\alpha} + A_2 p_\mu p_\alpha + A_3 p_\alpha q_\mu + A_4 p_\mu q_\alpha + A_5 q_\mu q_\alpha \} , \quad (5)$$

where A_i are the form factors. Since $q\varepsilon = 0$, the form factors A_4 and A_5 can be omitted. Obviously, to obtain the matrix element which describes the structure dependent part \mathcal{M}^{SD} we must multiply Eqs. (4) and (5) to the leptonic current. From Eqs. (5) it follows that \mathcal{M}^A part is not gauge invariant as well, while \mathcal{M}^V is gauge invariant itself. Since the total matrix element is supposed to be gauge invariant, we demand that $\mathcal{M}^{Brem.} + \mathcal{M}^A$ must be gauge invariant, i.e.,

$$q_\alpha \{ \mathcal{M}_\alpha^{Brem.} + \mathcal{M}_\alpha^A \} = 0 . \quad (6)$$

This condition allows us to find the relation between different form factors. Indeed from Eqs. (2) and (5) (after multiplying Eq. (5) to the leptonic current), we have

$$i f_{B_c} e \bar{u} (\not{q} + m_\tau) (1 - \gamma_5) v + \bar{u} [A_1 + A_2 (pq)] \not{q} (1 - \gamma_5) v + A_3 (pq) m_\tau \bar{u} (1 - \gamma_5) v = 0 . \quad (7)$$

From this equation we have

$$A_1 + A_2(pq) = -if_{B_c}e, \quad (8)$$

$$A_3(pq) = -if_{B_c}e. \quad (9)$$

If we set $pq = 0$ in Eq. (8), we get $A_1(pq = 0) = -if_{B_c}e$. So the general form of A_1 is as follows:

$$A_1 = -if_{B_c}e + b(pq), \quad (10)$$

where b is a new, unknown form factor. It follows from Eqs. (8) and (10) that

$$A_2 = -b.$$

In the same manner, from Eq. (9) we get

$$A_3 = -\frac{if_{B_c}e}{pq}.$$

Substituting these expressions for A_1 , A_2 and A_3 in the Bremsstrahlung and axial parts of the matrix element, we get

$$\begin{aligned} \mathcal{M}^{Brem.} + \mathcal{M}^A &= \frac{G}{\sqrt{2}} V_{cb}e \left\{ if_{B_c}m_\tau \bar{u}(p_1) \left[\frac{p_1\varepsilon}{p_1q} - \frac{p\varepsilon}{pq} \right] (1 - \gamma_5) v(p_2) \right. \\ &+ if_{B_c} \frac{m_\tau}{2p_1q} \bar{u}(p_1) \not{\varepsilon} \not{q} (1 - \gamma_5) v(p_2) \\ &\left. + i \frac{f_2(p^2)}{m_{B_c}^2} [\varepsilon(pq) - q_\mu(p\varepsilon)] \bar{u}(p_1) \gamma_\mu (1 - \gamma_5) v(p_2) \right\}, \quad (11) \end{aligned}$$

where for convenience we have redefined $b(p^2)$ as

$$b(p^2) = i \frac{f_2(p^2)}{m_{B_c}^2}.$$

The second term in the first line of the Eq. (11), corresponds physically to the contribution of the one B_c intermediate state. It is easy to check that the gauge invariance of the sum of the Bremsstrahlung and axial parts of the matrix element in Eq. (11) is reestablished. Adding the vector part of the correlator to Eq. (11), we finally get for the matrix element of the $B_c \rightarrow \tau \bar{\nu}_\tau \gamma$ process

$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2,$$

where

$$\begin{aligned} \mathcal{M}_1 &= \frac{G}{\sqrt{2}} V_{cb}e \left\{ \frac{f_1(p^2)}{m_{B_c}^2} \epsilon_{\mu\alpha\rho\beta} \varepsilon_\alpha p_\rho q_\beta \right. \\ &\left. + i \frac{f_2(p^2)}{m_{B_c}^2} [\varepsilon_\mu(pq) - q_\mu(p\varepsilon)] \right\} \bar{u}(p_1) \gamma_\mu (1 - \gamma_5) v(p_2), \quad (12) \end{aligned}$$

$$\mathcal{M}_2 = \frac{G}{\sqrt{2}} V_{cb} if_{B_c} m_\tau \bar{u}(p_1) \left\{ \frac{1}{2p_1q} \not{\varepsilon} \not{q} + \left[\frac{p_1\varepsilon}{p_1q} - \frac{p\varepsilon}{pq} \right] \right\} (1 - \gamma_5) v(p_2). \quad (13)$$

As we noted earlier, \mathcal{M}_2 is indeed proportional to the lepton mass, as expected from helicity arguments. If we formally set $m_\tau \rightarrow 0$ in Eqs. (12) and (13), the resulting expression is expected to coincide with the $B_c \rightarrow \ell \bar{\nu} \gamma$ ($\ell = e, \mu$). This decay was investigated in the framework of light cone QCD sum rules and the constituent quark model approach in [11] and [15], respectively.

After lengthy, but straightforward calculation for the squared matrix element, we get

$$|\mathcal{M}|^2 = |\mathcal{M}_1|^2 + 2 \operatorname{Re} [\mathcal{M}_1 \mathcal{M}_2^\dagger] + |\mathcal{M}_2|^2, \quad (14)$$

where

$$\begin{aligned} |\mathcal{M}_1|^2 &= \frac{G^2}{2} |V_{cb}|^2 e^2 16 \left[\frac{|f_1(p^2)|^2}{m_{B_c}^4} + \frac{|f_2(p^2)|^2}{m_{B_c}^4} \right] \\ &\times \left\{ (pp_2)(pq)(p_1q) + (p_2q) \left[(pp_1)(pq) - p^2(p_1q) \right] \right\}, \end{aligned} \quad (15)$$

$$\begin{aligned} 2 \operatorname{Re} [\mathcal{M}_1 \mathcal{M}_2^\dagger] &= \frac{G^2}{2} |V_{cb}|^2 e^2 \left(-16 f_{B_c} m_\tau^2 \right) \frac{1}{(p_1q)(pq)} \\ &\times \left\{ \frac{f_2(p^2)}{m_{B_c}^2} p^2(p_1q)(p_2q) + (pq)^2 \left[\frac{f_2(p^2)}{m_{B_c}^2} (p_1p_2 + p_2q) - \frac{f_1(p^2)}{m_{B_c}^2} (p_2q) \right] \right. \\ &\left. - \frac{f_2(p^2)}{m_{B_c}^2} [(pp_2)(p_1q) + (pp_1)(p_2q)] \right\}, \end{aligned} \quad (16)$$

$$\begin{aligned} |\mathcal{M}_2|^2 &= \frac{G^2}{2} |V_{cb}|^2 e^2 \left(-4 f_{B_c}^2 m_\tau^2 \right) \frac{1}{(p_1q)^2(pq)^2} \\ &\times \left\{ 2p^2(p_1p_2)(p_1q)^2 + (pq)^2 \left[(p_1p_2) (2m_\tau^2 - p_1q) + (p_2q) (m_\tau^2 - 2p_1q) \right] \right. \\ &\left. + (pq)(p_1q) [(pp_2)(p_1q) - (pp_1)(4p_1p_2 + p_2q)] \right\}. \end{aligned} \quad (17)$$

All calculations have been performed in the rest frame of the B_c meson. The dot products of the four-vectors are defined if the photon and neutrino (or electron) energies are specified. The Dalitz boundary for the photon energy E_γ and neutrino energy E_2 is as follows:

$$\begin{aligned} \frac{m_{B_c}^2 - 2m_{B_c}E_\gamma - m_\tau^2}{2m_{B_c}} &\leq E_2 \leq \frac{m_{B_c}^2 - 2m_{B_c}E_\gamma - m_\tau^2}{2(m_{B_c} - 2E_\gamma)}, \\ 0 &\leq E_\gamma \leq \frac{m_{B_c}^2 - m_\tau^2}{2m_{B_c}}. \end{aligned} \quad (18)$$

The expression for the differential decay rate can be written as

$$\frac{d\Gamma}{dE_2 dE_\gamma} = \frac{1}{64\pi^3 m_{B_c}} |\mathcal{M}|^2. \quad (19)$$

The differential ($d\Gamma/dE_\gamma$) and total decay width are singular at the lower limit of the photon energy, and this singularity which is present only in the $|\mathcal{M}_2|^2$ contribution is due to the soft photon emission from charged lepton line. On the other hand, $|\mathcal{M}_1|^2$ and $\text{Re}[\mathcal{M}_1\mathcal{M}_2^\dagger]$ terms are free of this singularity. In this limit the $B_c \rightarrow \tau\bar{\nu}_\tau\gamma$ decay can not distinguished from the $B_c \rightarrow \tau\bar{\nu}_\tau$ decay. In order to obtain a finite result for the decay width, we must consider both decays together. The infrared singularity arising from the $|\mathcal{M}_2|^2$ contribution must be canceled with $O(\alpha)$ virtual correction to the $B_c \rightarrow \tau\bar{\nu}_\tau$ decay. In this work, our consideration is slightly different, namely, the $B_c \rightarrow \tau\bar{\nu}_\tau\gamma$ process is not considered as a $O(\alpha)$ correction to the $B_c \rightarrow \tau\bar{\nu}_\tau$ decay, but rather a separate decay channel with hard photon radiation. Therefore we impose a cut value on the photon energy, which will set an experimental limit on the minimum detectable photon energy. We will consider the case for which the photon energy threshold is larger than 50, MeV i.e., $E_\gamma \geq a m_{B_c}$, where $a \geq 0.01$. Using Eqs. (14)–(19) and imposing the cut as the lower bound of the photon energy, the total decay width takes the following form:

$$\begin{aligned}
\Gamma &= \frac{G^2\alpha m_{B_c}^3}{64\pi^2} |V_{cb}|^2 \left\{ \frac{1}{3} \int_\delta^{1-r} dx \left[|f_1(x)|^2 + |f_2(x)|^2 \right] \frac{1}{(1-x)^2} x^3 (2+r-2x)(1-r-x)^2 \right. \\
&+ 4f_{B_c}^2 \int_\delta^{1-r} dx \frac{r}{x(1-x)} \left[-4 + 8r - 4r^2 + 10x - 14rx + 4r^2x - 9x^2 + 7rx^2 + 3x^3 \right. \\
&+ (1-x)(2 - 2r^2 - 3x + rx + 2x^2) \ln\left(\frac{1-x}{r}\right) \left. \right] \\
&- 4f_{B_c} \int_\delta^{1-r} dx \frac{rx}{1-x} \left[(1-r-x)(f_1(x)x + f_2(x)(1+r-2x)) \right. \\
&\left. \left. - (1-x)(f_1(x)x + f_2(x)(2-r)) \ln\left(\frac{1-x}{r}\right) \right] \right\}, \tag{20}
\end{aligned}$$

where $x = 2E_\gamma/m_{B_c}$ is the dimensionless photon energy, $r = m_\tau^2/m_{B_c}^2$ and $\delta = 2a$.

3 NUMERICAL ANALYSIS

It follows from Eq. (20) that, to be able to calculate the decay width, explicit forms of the form factors f_1 and f_2 are needed. These form factors are calculated in the framework of the light cone QCD sum rules in [11], and it was shown that, to a very good accuracy, their p^2 dependence can be represented in following pole forms:

$$f_1(p^2) = \frac{f_1(0)}{1 - p^2/m_1^2}, \quad f_2(p^2) = \frac{f_2(0)}{1 - p^2/m_2^2}, \tag{21}$$

where

$$\begin{aligned}
f_1(0) &= 0.44 \pm 0.04 \text{ GeV}, & m_1^2 &= 43.1 \text{ GeV}^2, \\
f_2(0) &= 0.21 \pm 0.02 \text{ GeV}, & m_2^2 &= 48.0 \text{ GeV}^2.
\end{aligned}$$

The following set of parameters have been used in the numerical analysis: $f_{B_c} = 0.35 \text{ GeV}$ [16]–[18], $V_{cb} = 0.04$ [19], $\tau(B_c) = 0.46 \times 10^{-12} \text{ s}$ [1]. In Figs. (1) and (2), we present

the photon energy dependence of the branching ratio for two different fixed cut values, i.e., $\delta = 0.016$ and $\delta = 0.032$. The steep increase in the value of the branching ratio at small photon energies, is due to the Bremstrahlung part. Using these values, we have calculated the branching ratios and presented the results in Table 1.

	$\delta = 0.016$	$\delta = 0.032$
Structure dependent part	7.24×10^{-6}	7.24×10^{-6}
Bremstrahlung part	8.60×10^{-5}	6.53×10^{-5}
Interference part	2.17×10^{-6}	2.18×10^{-6}
Total	9.54×10^{-5}	7.47×10^{-5}

Table 1:

From this table we observe that the main contribution to the $B_c \rightarrow \tau \bar{\nu}_\tau \gamma$ decay comes from the Bremstrahlung part and contributions arising from the structure dependent and interference terms are comparable to each other.

Note that the QCD sum rules prediction of the branching ratio of the $B_c \rightarrow \ell \bar{\nu}_\ell \gamma$ ($\ell = e, \mu$) decay is [11]

$$\mathcal{B}(B_c \rightarrow \ell \bar{\nu}_\ell \gamma) \simeq 1.0 \times 10^{-5} . \quad (22)$$

For completeness we present below the predictions of the branching ratios for pure leptonic decays [15]:

$$\begin{aligned} \mathcal{B}(B_c \rightarrow e \bar{\nu}_e) &= 1.25 \times 10^{-9} \\ \mathcal{B}(B_c \rightarrow \mu \bar{\nu}_\mu) &= 5.26 \times 10^{-5} \\ \mathcal{B}(B_c \rightarrow \tau \bar{\nu}_\tau) &= 1.29 \times 10^{-2} . \end{aligned} \quad (23)$$

From a comparison of the results listed in Table 1 and Eq. (22), we observe that the branching ratio of the radiative τ lepton decay is higher than the corresponding $\mathcal{B}(B_c \rightarrow \ell \bar{\nu}_\ell \gamma)$ and the pure light leptonic decays.

Few words about the number of expected events at LHC are in order. As we have noted earlier, approximately $\sim 2 \times 10^8$ B_c mesons will be produced at LHC per year. Using the numerical result for the branching ratio of the $\mathcal{B}(B_c \rightarrow \tau \bar{\nu}_\tau \gamma)$ decay, for the number of expected events at LHC, we get

$$\begin{aligned} N &\simeq 2 \times 10^4 \text{ (for } \delta = 0.016 \text{) and ,} \\ &\simeq 1.5 \times 10^4 \text{ (for } \delta = 0.032 \text{) .} \end{aligned}$$

Even with an efficiency of $\sim 10^{-2}$ in detecting the τ lepton, approximately ~ 200 events are expected to be observed. So, this decay has a good chance to be detected at LHC.

FIGURE CAPTIONS

FIG. 1. The dependence of the branching ratio of the $B_c \rightarrow \tau \bar{\nu}_\tau \gamma$ decay on the dimensionless photon energy. The lower bound of the dimensionless photon energy is taken as $\delta = 0.016$.

FIG. 2. The same as FIG. 1, but at $\delta = 0.032$.

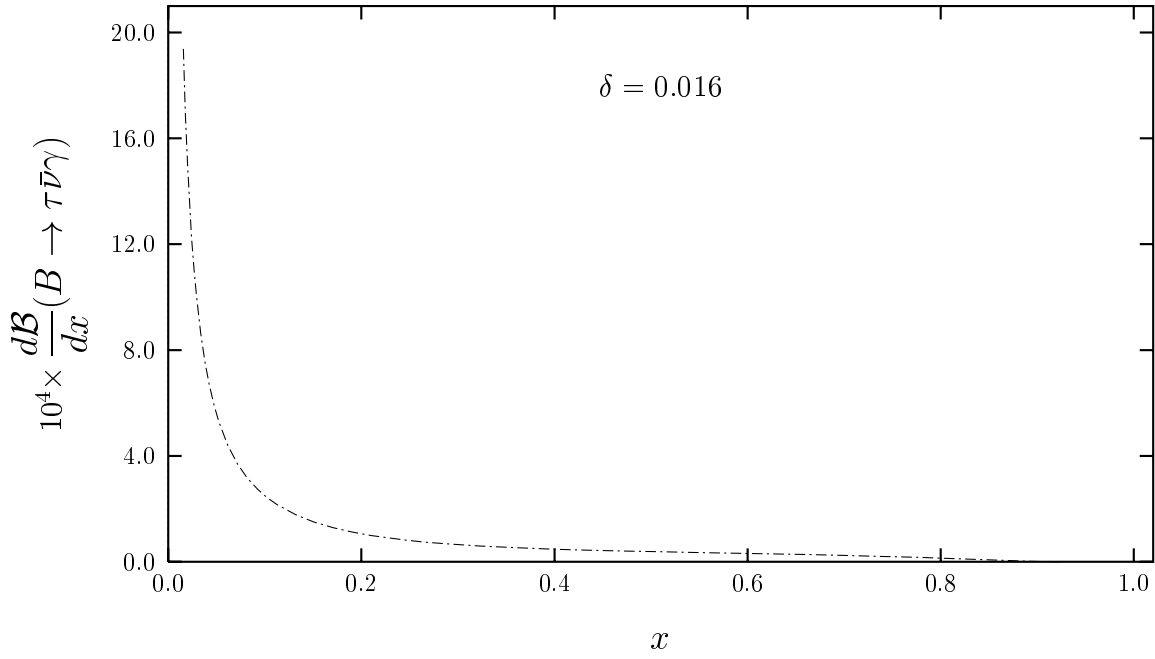


Figure 1:

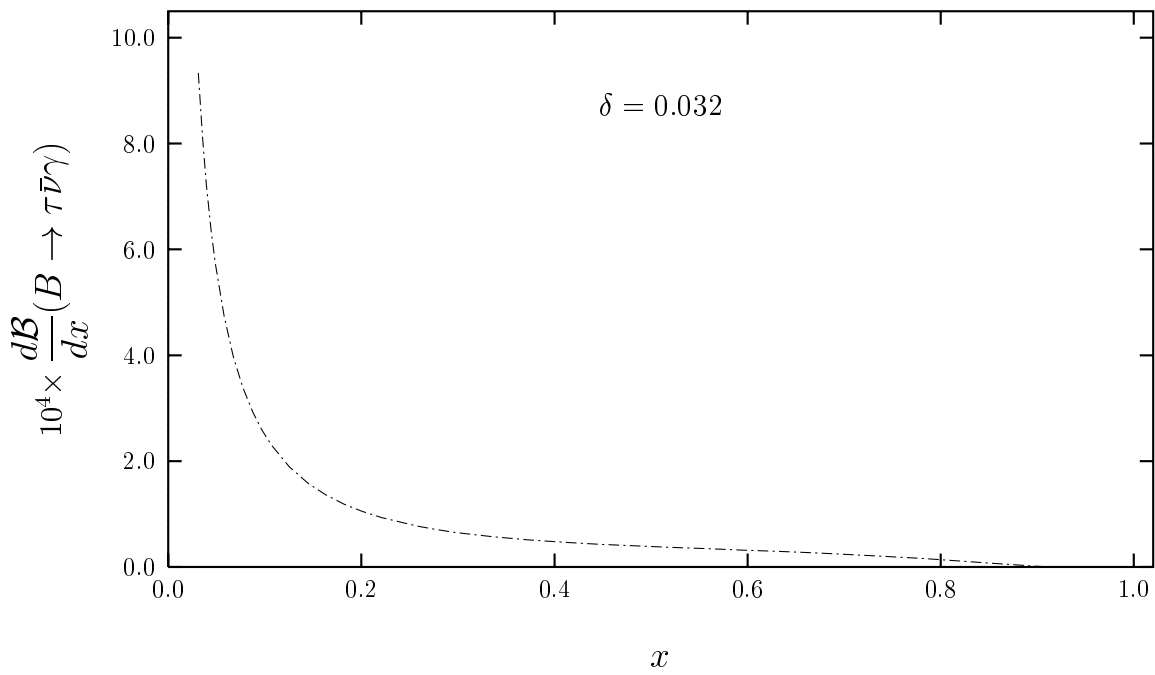


Figure 2:

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