RARE $B \to K^* \ell^+ \ell^-$ DECAY, TWO HIGGS DOUBLET MODEL, AND LIGHT CONE QCD SUM RULES

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Abstract

The decay width, forward-backward asymmetry and lepton longitudinal and transversal polarization for the exclusive $B \to K^* \ell^+ \ell^-$ decay in a two Higgs doublet model are computed. It is shown that all these quantities are very effective tools for establishing new physics.

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1 Introduction

Experimental discovery of the inclusive and exclusive $B \to X_s\gamma$ and $B \to K^*\gamma$ [1] decays stimulated the study of rare B meson decays in a new manner. These decays take place via flavor-changing neutral current (FCNC) $b \to s$ transitions which are absent in the Standard Model (SM) at tree level and appear only at the loop level. Therefore the study of these decays can provide sensitive tests for investigating the structure of the SM at the loop level, searching for new physics beyond the SM [2].

Currently, the main interest is focused on the rare B meson decays for which the SM predicts large branching ratios and that can be potentially measurable in the near future. The rare $B \to K^*\ell^+\ell^-(\ell = e,\ \mu,\ \tau)$ decays are such decays. For these decays the experimental situation is very promising [3] with e^+e^- and hadron colliders focusing only on the observation of exclusive modes with $l = e,\ \mu$ and τ as the final states. At the quark level, the process $B \to K^*\ell^+\ell^-$ is described by the $b \to sl^+l^-$ transition. In literature [4-17] this transition has been investigated extensively in both SM and two Higgs doublet model (2HDM). It is well known that in the 2HDM, the up type quarks acquire their masses from Yukawa couplings to the Higgs doublet H_2 (with the vacuum expectation value v_2) and down type quarks and leptons acquire their masses from Yukawa couplings to the other Higgs doublet H_1 (with the expectation value v_1). In 2HDM there exist five physical Higgs fields: neutral scalar H^0, h^0, neutral pseudoscalar A^0 and charged Higgs bosons $H^\pm$. Such a model occurs as a natural feature of the supersymmetric models [18]. In these models the interaction vertex of the Higgs boson and fermions depends on the ratio $tan\,\beta = \frac{v_2}{v_1}$ which is a free parameter in the model. The constraints on $tan\,\beta$ are usually obtained from $B - \bar{B},\ K - \bar{K}$ mixing, $b \to s\gamma$ decay width, semileptonic decay $b \to c\tau\bar{\nu}_\tau$ and is given by [19, 20]:

$$0.7 \le tan\,\beta \le 0.6\left(\frac{m_{H^+}}{1\ GeV}\right) \tag{1}$$

(the lower bound $m_{H^+} \ge 200\ GeV$ is obtained in [20]).

In 2HDM the charged and neutral Higgs interactions with fermions induce new contributions to the FCNC processes. For $b \to s\ell^+\ell^-$ $(\ell = e,\ \mu)$ decay the contributions from neutral Higgs boson exchange are usually neglected, due to the fact that the interaction of the neutral Higgs and leptons is proportional to the lepton mass. But for the $b \to s\tau^+\tau^-$ decay the mass of the τ lepton is not too small compared to the mass of the b-quark, therefore the neutral Higgs boson exchange diagrams can give considerable contributions to the process. Recently it has been emphasized by Hewett [21] that τ lepton longitudinal polarization is an important observable and may be accessible in the $B \to X_s\tau^+\tau^-$ decay. In [22] it is shown that in addition to longitudinal component P_L of polarization of the τ lepton two other orthogonal components P_T, which lies in the decay plane, and P_N which is perpendicular to the decay plane, are also very significant for the $\tau^+\ \tau^-$ channel, since they are proportional to the mass of the τ lepton. The polarization components $P_L,\ P_T$ and P_N involve different combinations of the Wilson coefficients $C_7,\ C_9^{eff}$ and C_{10} (see below) and hence contain independent information. Therefore, polarization effects can play a central role for the investigation of the structure of the SM and for establishing new physics beyond it.

In calculating the branching ratios and other observables at hadronic level, e.g., for $B \to K^* \ell^+ \ell^-$decay, we face the problem of computing the matrix element of the effective Hamiltonian responsible for this decay, between B and K^* states. This problem is related to the non-perturbative sector of QCD and it can be solved only by means of a non-perturbative approach.

These matrix elements have been investigated in the framework of different approaches such as chiral theory [23], three point QCD sum rules method [24], relativistic quark model by the light-front formalism [25], effective heavy quark theory [26] and light cone QCD sum rules [27]. The aim of the present work is to calculate these matrix elements using the light cone QCD sum rules formalism in the framework of the 2HDM, taking into account the newly appearing operators due to the neutral Higgs exchange diagrams, and to study the forward-backward asymmetry and final lepton polarization for the exclusive $B \to K^* \ell^+ \ell^-$decay. Taking into account the additional neutral Higgs boson exchange diagrams, the effective Hamiltonian is calculated in [28] as

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left\{ \sum_{i=1}^{10} C_i(\mu) O_i(\mu) + \sum_{i=1}^{10} C_{Q_i}(\mu) Q_i(\mu) \right\} , \tag{2}$$

where the first set of operators in the curly brackets describe the effective Hamiltonian responsible for the $b \to s l^+ l^-$ decay in the SM. The value of the corresponding Wilson coefficients includes the contribution of the diagrams with $H^\pm$ running in the loop (see [7]), i.e.

$$\begin{aligned}
C_7(M_W) &= C_7^{SM}(M_W) + C_7^{H^-}(M_W) , \\
C_9(M_W) &= C_9^{SM}(M_W) + C_9^{H^-}(M_W) , \\
C_{10}(M_W) &= C_{10}^{SM}(M_W) + C_{10}^{H^-}(M_W) ,
\end{aligned}$$

where

$$\begin{aligned}
C_7^{H^-}(M_W) &= -\frac{y}{2}\left[\frac{5y-3}{6(y-1)^2} - \frac{3y-2}{3(y-1)^3} ln\, y\right] - \\
&- \frac{1}{6} ctg^2\beta\, y \left[\frac{8y^3+5y-12}{(y-1)^3} - \frac{3y^2-2y}{2(y-1)^4} ln\, y\right] , \\
C_9^{H^-}(M_W) &= \frac{1-4sin^2\theta_W}{sin^2\theta_W}\left[ctg^2\beta \frac{xy}{8}\left(\frac{1}{y-1} - \frac{1}{(y-1)^2} ln\, y\right)\right] - \\
&- ctg^2\beta\, y \left[\frac{47y^2-79y+108}{108(y-1)^3} - \frac{3y^3-6y+4}{18(y-1)^4} ln\, y\right] , \\
C_{10}^{H^-}(M_W) &= \frac{1}{sin^2\theta_W} ctg^2\beta \frac{xy}{8}\left[-\frac{1}{y-1} + \frac{1}{(y-1)^2} ln\, y\right] .
\end{aligned}$$

The explicit forms of $C_7^{SM}(M_W)$, $C_9^{SM}(M_W)$, and $C_{10}^{SM}(M_W)$ and the corresponding operators can be found in [7].

The second set of operators in the brackets, whose explicit forms are presented in [28], come from the exchange of the neutral Higgs bosons. The corresponding Wilson coefficients are:

$$\begin{aligned}
C_{Q_1}(m_W) &= \frac{m_b m_\ell}{m_{h^0}^2} tan^2\beta \frac{1}{sin^2\theta_W}\frac{x}{4}\left\{ \left(sin^2\alpha + h\, cos^2\alpha\right) f_1(x,y) + \right. \\
&+ \left[\frac{m_{h^0}^2}{m_W^2} + \left(sin^2\alpha + h\, cos^2\alpha\right)(1-z)\right] f_2(x,y) + \\
&+ \left. \frac{sin^2 2\alpha}{2m_{H^\pm}^2}\left[m_{h^0}^2 - \frac{(m_{h^0}^2 + m_{H^0}^2)^2}{2m_{H^0}^2}\right] f_3(y)\right\} , \qquad (3) \\
C_{Q_2}(m_W) &= \frac{m_b m_\ell}{m_{H^\pm}^2} tan^2\beta \left\{ f_1(x,y) + \left[1 + \frac{m_{H^\pm}^2 - m_{A^0}^2}{m_W^2}\right] f_2(x,y)\right\} , \qquad (4) \\
C_{Q_3}(m_W) &= \frac{m_b e^2}{m_\ell g^2}\left[C_{Q_1}(m_W) + C_{Q_2}(m_W)\right] , \qquad (5) \\
C_{Q_4}(m_W) &= \frac{m_b e^2}{m_\ell g^2}\left[C_{Q_1}(m_W) - C_{Q_2}(m_W)\right] , \qquad (6) \\
C_{Q_i}(m_W) &= 0 \qquad i = 5, \ldots, 10 , \qquad (7)
\end{aligned}$$

where

$$x = \frac{m_t^2}{m_W^2} , \quad y = \frac{m_t^2}{m_{H^\pm}^2} , \quad z = \frac{x}{y} , \quad h = \frac{m_{h^0}^2}{m_{H^0}^2} ,$$

$$f_1(x,y) = \frac{x\, lnx}{x-1} - \frac{y\, lny}{y-1} , \quad f_2(x,y) = \frac{x\, lny}{(z-x)(x-1)} + \frac{lnz}{(z-1)(x-1)} ,$$

$$f_3(y) = \frac{1 - y + y\, lny}{(y-1)^2} .$$

The QCD correction to the Wilson coefficients $C_i(m_W)$ and $C_{Q_i}(m_W)$ can be calculated using the renormalization group equations. In [28] it was shown that the operators O_9 and O_{10} do not mix with Q_i $(i = 1, \ldots, 10)$, so that the Wilson coefficients C_9 and C_{10} remain unchanged and their values are the same as in the SM. Their explicit forms can be found in [28], where it is also shown that O_7 can mix with Q_i. But additional terms due to this mixing can safely be neglected since the corrections to the SM value of C_7 arising from these terms are less than 5% when $tan\,\beta \leq 50$.

Moreover the operators O_i $(i = 1, \ldots, 10)$ and Q_i $(i = 3, \ldots, 10)$ do not mix with Q_1 and Q_2 and also there is no mixing between Q_1 and Q_2. For this reason the evolutions of the coefficients C_{Q_1} and C_{Q_2} are controlled by the anomalous dimensions of Q_1 and Q_2 respectively:

$$C_{Q_i}(m_b) = \eta^{-\gamma_Q/\beta_0} C_{Q_i}(m_W) , \quad i = 1,\ 2,$$

where $\gamma_Q = -4$ is the anomalous dimension of the operator $\bar{s}_L b_R$ [29].

Neglecting the strange quark mass, the matrix element for $b \to s\ell^+\ell^-$ decay is [28]:

$$\begin{aligned}
\mathcal{M} &= \frac{G_F\alpha}{2\sqrt{2}\pi} V_{tb}V_{ts}^* \left\{ C_9^{eff}\bar{s}\gamma_\mu(1-\gamma_5)b\,\bar{\ell}\gamma^\mu\ell + C_{10}\bar{s}\gamma_\mu(1-\gamma_5)b\,\bar{\ell}\gamma^\mu\gamma_5\ell - \right. \\
&- \left. 2C_7\frac{m_b}{q^2}\bar{s}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b\,\bar{\ell}\gamma^\mu\ell + C_{Q_1}\bar{s}(1+\gamma_5)b\bar{\ell}\ell + C_{Q_2}\bar{s}(1+\gamma_5)b\bar{\ell}\gamma_5\ell \right\} , \qquad (8)
\end{aligned}$$

where q^2 is the invariant dileptonic mass. Here the coefficient $C_9^{eff}(\mu, q^2) \equiv C_9(\mu) + Y(\mu,\ p^2)$, where the function Y contains the contributions from the one loop matrix element of the four-quark operators [7, 30, 31]. It is clear that the last two terms in eq.(8) can give considerable contribution only for the $B \to K^* \tau^+ \tau^-$decay mode, since C_{Q_1} and C_{Q_2} are proportional to the lepton mass m_ℓ. In addition to the short distance contributions, it is possible to take into account the long distance effects associated with real $c\bar{c}$ in the intermediate states, i.e., the cascade process $B \to K^* J/\psi(\psi') \to K^* \ell^+ \ell^-$. These contributions are taken into account by introducing a Breit-Wigner form of the resonance propagator and this procedure leads to an additional contribution to C_9^{eff} of the form [32]

$$- \frac{3\pi}{\alpha^2} \sum_{V=J/\psi,\ \psi',\ldots} \frac{m_V \Gamma(V \to \ell^+ \ell^-)}{(q^2 - m_V^2) - i m_V \Gamma_V} \ .$$

From eq.(8) it follows that, in order to calculate the branching ratio for the exclusive $B \to K^* \tau^+ \tau^-$decay, the matrix elements $\langle K^* \left| \bar{s} \gamma_\mu (1-\gamma_5) b \right| B \rangle$, $\langle K^* \left| \bar{s} i \sigma_{\mu\nu} q^\nu (1+\gamma_5) b \right| B \rangle$, and $\langle K^* \left| \bar{s} (1+\gamma_5) b \right| B \rangle$ have to be calculated. These matrix elements can be written in terms of the form factors in the following way:

$$\begin{aligned}
&\langle K^*(p,\epsilon) \left| \bar{s} \gamma_\mu (1-\gamma_5) b \right| B(p+q) \rangle = \\
&- \ \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} p^\rho q^\sigma \frac{2V(q^2)}{m_B + m_{K^*}} - i \epsilon_\mu^* (m_B + m_{K^*}) A_1(q^2) + i(2p+q)_\mu (\epsilon^* q) \frac{A_2(q^2)}{m_B + m_{K^*}} + \\
&+ \ i q_\mu (\epsilon^* q) \frac{2 m_{K^*}}{q^2} \left[A_3(q^2) - A_0(q^2) \right] \ ,
\end{aligned} \tag{9}$$

$$\begin{aligned}
&\langle K^*(p,\epsilon) \left| \bar{s} i \sigma_{\mu\nu} q^\nu (1+\gamma_5) b \right| B(p+q) \rangle = \\
&\quad 4 \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} p^\rho q^\sigma T_1(q^2) + 2i \left[\epsilon_\mu^* (m_B^2 - m_{K^*}^2) - (2p+q)_\mu (\epsilon^* q) \right] T_2(q^2) + \\
&+ \ 2i(\epsilon^* q) \left[q_\mu - (2p+q)_\mu \frac{q^2}{m_B^2 - m_{K^*}^2} \right] T_3(q^2) \ ,
\end{aligned} \tag{10}$$

where ϵ is the polarization vector of K^* meson.

To calculate the matrix element $\langle K^* | \bar{s}(1+\gamma_5) b | B \rangle$ we multiply both sides of eq.(9) by q_μ and use the equation of motion. Neglecting the mass of the strange quark, we get:

$$\begin{aligned}
\langle K^*(p,\epsilon) \left| \bar{s}(1+\gamma_5) b \right| B(p+q) \rangle &= \frac{1}{m_b} \Big\{ - i(\epsilon^* q)(m_B + m_{K^*}) A_1(q^2) + \\
&+ \ i(\epsilon^* q)(m_B - m_{K^*}) A_2(q^2) + \\
&+ \ i(\epsilon^* q) 2 m_{K^*} \left[A_3(q^2) - A_0(q^2) \right] \Big\} \ .
\end{aligned} \tag{11}$$

Using the equation of motion, the formfactor $A_3(q^2)$ can be written as a linear combination of the formfactors A_1 and A_2 (see [24]):

$$A_3(q^2) = \frac{m_B + m_{K^*}}{2 m_{K^*}} A_1(q^2) - \frac{m_B - m_{K^*}}{2 m_{K^*}} A_2(q^2) \ . \tag{12}$$

Making use of this relation we obtain:

$$\langle K^*(p,\epsilon)\left|\bar{s}(1+\gamma_5)b\right|B(p+q)\rangle = \frac{2m_{K^*}}{m_b}\left\{-i(\epsilon^* q)A_0(q^2)\right\} \ . \tag{13}$$

From eqs.(8), (9), (10) and (13) we obtain for the matrix element of the $B \to K^* \ell^+ \ell^-$decay:

$$\begin{aligned}
\mathcal{M} &= \frac{G\alpha}{2\sqrt{2}\pi} V_{tb}V_{ts}^* \Big\{ \bar{\ell}\gamma^\mu \ell \left[2A\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^\rho q^\sigma + iB_1 \epsilon_\mu^* - iB_2(\epsilon^* q)(2p+q)_\mu - iB_3(\epsilon^* q)q_\mu \right] + \\
&+ \bar{\ell}\gamma^\mu\gamma_5 \ell \left[2C\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^\rho q^\sigma + iD_1 \epsilon_\mu^* - iD_2(\epsilon^* q)(2p+q)_\mu - iD_3(\epsilon^* q)q_\mu \right] + \\
&+ iF_1(\epsilon^* q)\bar{\ell}\ell + iF_2(\epsilon^* q)\bar{\ell}\gamma_5\ell \Big\} \ ,
\end{aligned} \tag{14}$$

where

$$\begin{aligned}
A &= C_9^{eff}\frac{V}{m_B+m_{K^*}} + 4C_7\frac{m_b}{q^2}T_1 \ , \\
B_1 &= C_9^{eff}(m_B+m_{K^*})A_1 + 4C_7\frac{m_b}{q^2}(m_B^2-m_{K^*}^2)T_2 \ , \\
B_2 &= C_9^{eff}\frac{A_2}{m_B+m_{K^*}} + 4C_7\frac{m_b}{q^2}\left(T_2 + \frac{q^2}{m_B^2-m_{K^*}^2}T_3\right) \ , \\
B_3 &= -C_9^{eff}\frac{2m_{K^*}}{q^2}(A_3-A_0) + 4C_7\frac{m_b}{q^2}T_3 \ , \\
C &= C_{10}\frac{V}{m_B+m_{K^*}} \ , \\
D_1 &= C_{10}(m_B+m_{K^*})A_1 \ , \\
D_2 &= C_{10}\frac{A_2}{m_B+m_{K^*}} \ , \\
D_3 &= C_{10}\frac{2m_{K^*}}{q^2}(A_3-A_0) \ , \\
F_1 &= C_{Q_1}\frac{2m_{K^*}}{m_b}A_0 \ , \\
F_2 &= C_{Q_2}\frac{2m_{K^*}}{m_b}A_0 \ .
\end{aligned} \tag{15}$$

The formfactors $V,\ A_1,\ A_2,\ A_0,\ T_1,\ T_2$ and T_3 are calculated in the framework of light-cone QCD sum rules in [27] and their q^2 dependence, to a good accuracy, can be represented in the following pole form:

$$
\begin{aligned}
V(q^2) &= \frac{0.55}{\left(1-\frac{q^2}{30}\right)^2}\,, \quad A_0(q^2) = \frac{0.26}{\left(1-\frac{q^2}{45}\right)^2}\,,\\
A_1(q^2) &= \frac{0.36}{\left(1-\frac{q^2}{64}\right)^2}\,, \quad A_2(q^2) = \frac{0.40}{\left(1-\frac{q^2}{33}\right)^2}\,,\\
T_1(q^2) &= \frac{0.18}{\left(1-\frac{q^2}{31}\right)^2}\,, \quad T_2(q^2) = \frac{0.18}{\left(1-\frac{q^2}{64.4}\right)^2}\,,\\
T_3(q^2) &= \frac{0.14}{\left(1-\frac{q^2}{32.5}\right)^2}\,,
\end{aligned}
\tag{16}
$$

which we will use in further numerical analysis (for A_3 we use eq.(12)).

Using eq.(14) and performing summation over final lepton polarization, we get for the double differential decay rate:

$$
\begin{aligned}
\frac{d\Gamma}{dq^2dz} &= \frac{G^2\alpha^2\left|V_{tb}V_{ts}^*\right|^2\lambda^{1/2}v}{2^{12}\pi^5 m_B}\Bigg\{2\lambda m_B^4\Bigg[4m_\ell^2\left(|A|^2-|C|^2\right)+m_B^2 s(1+v^2z^2)\left(|A|^2+|C|^2\right)\Bigg]+\\
&+ \frac{\lambda m_B^4}{2r}\Bigg[\lambda m_B^2(1-v^2z^2)\left(|B_2|^2+|D_2|^2\right)+4m_\ell^2\left|D_2\right|^2(2+2r-s)\Bigg]+\\
&+ \frac{1}{2r}\Bigg[m_B^2\left\{\lambda(1-v^2z^2)+8rs\right\}\left(|B_1|^2+|D_1|^2\right)+16m_\ell^2 r\left(|B_1|^2-2\left|D_1\right|^2\right)-\\
&- 2\lambda m_B^4(1-r-s)(1-v^2z^2)\left\{Re\left(B_1B_2^*\right)+Re\left(D_1D_2^*\right)\right\}-16\lambda m_B^2 m_\ell^2 Re\left(D_1D_2^*\right)\Bigg]+\\
&+ \frac{2\lambda m_B^2 m_\ell^2}{r}\Bigg[s m_B^2\left|D_3\right|^2+2Re\left(D_1D_3^*\right)-2(1-r)m_B^2 Re\left(D_2D_3^*\right)\Bigg]+\\
&+ \frac{2\lambda m_B^6 s}{m_b^2}\left(\left|F_1\right|^2 v^2+\left|F_2\right|^2\right)+\\
&+ \frac{4\lambda m_B^3 m_\ell}{m_b\sqrt{r}}\Bigg[Re\left(F_2D_1^*\right)-(1-r)m_B^2 Re\left(F_2D_2^*\right)+m_B^2 s Re\left(F_2D_3^*\right)\Bigg]-\\
&- m_B^2\lambda^{1/2}z\Bigg[4m_B\frac{m_\ell v}{m_b\sqrt{r}}\left\{m_B^2 Re\left(B_2F_1^*\right)-(1-r-s)Re\left(B_1F_1^*\right)\right\}+\\
&+ 8m_B^2 s\left\{vRe\left(B_1C^*\right)+Re\left(AD_1^*\right)\right\}\Bigg]\Bigg\}\,,
\end{aligned}
\tag{17}
$$

where $z=cos\theta$, θ is the angle between the momentum of the ℓ lepton and that of the B meson in the center of mass frame of the lepton pair, $\lambda=1+r^2+s^2-2r-2s-2rs$, $r=\frac{m_{K^*}^2}{m_B^2}$, $s=\frac{q^2}{m_B^2}$ and $v=\sqrt{1-\frac{4m_\ell^2}{q^2}}$, m_ℓ are the lepton velocity and mass respectively.

If we put $F_1 = 0$ and $F_2 = 0$ (which corresponds to the case where the contributions of the neutral Higgs boson exchange diagrams are neglected) we get the results of [27]. As we noted earlier, the forward-backward asymmetry and final lepton polarization asymmetry are very useful tools for extracting more precise information on the Wilson coefficients $C_7,\ C_9^{eff}$ and C_{10} and also for searching new physics. Therefore in this work we shall study these quantities as well. The forward-backward asymmetry A_{FB} is defined in the following way:

$$A_{FB}(q^2) = \frac{\displaystyle\int_0^1 dz \frac{d\Gamma}{dq^2 dz} - \int_{-1}^0 dz \frac{d\Gamma}{dq^2 dz}}{\displaystyle\int_0^1 dz \frac{d\Gamma}{dq^2 dz} + \int_{-1}^0 dz \frac{d\Gamma}{dq^2 dz}} \ .$$

Let us now discuss the final lepton polarization. We define the following three orthogonal unit vectors:

$$\begin{aligned}
\vec{e}_L &= \frac{\vec{p}_1}{|\vec{p}_1|} \ , \\
\vec{e}_T &= \frac{\vec{p}_{K^*} \times \vec{p}_1}{|\vec{p}_{K^*} \times \vec{p}_1|} \ , \\
\vec{e}_N &= \vec{e}_T \times \vec{e}_L \ ,
\end{aligned}$$

where $\vec{p}_1$ and $\vec{p}_{K^*}$ are the three momenta of the ℓ^- and the K^* meson, respectively, in the center of mass of the $\ell^+\ \ell^-$ system. The differential decay rate for any given spin direction $\vec{n}$ of the ℓ^- lepton, where $\vec{n}$ is a unit vector in the ℓ^- lepton rest frame, can be written as

$$\frac{d\Gamma(\vec{n})}{dq^2} = \frac{1}{2} \left(\frac{d\Gamma}{dq^2} \right)_0 \Big[1 + (P_L\, \vec{e}_L + P_N\, \vec{e}_N + P_T\, \vec{e}_T) \cdot \vec{n} \Big] \ , \qquad (18)$$

where the subscript "0" corresponds to the unpolarized case and it can be obtained from eq. (17) by integration over z. From definition of $\vec{e}_i$ it is obvious that P_T lies in the decay plane whose orientation is organized by the vectors $\vec{p}_1$ and $\vec{p}_{K^*}$, and P_N is perpendicular to this plane.

The polarization components $P_i\ (i = L,\ T,\ N)$ are defined as:

$$P_i(q^2) = \frac{\displaystyle\frac{d\Gamma}{dq^2}(\vec{n} = \vec{e}_i) - \frac{d\Gamma}{dq^2}(\vec{n} = -\vec{e}_i)}{\displaystyle\frac{d\Gamma}{dq^2}(\vec{n} = \vec{e}_i) + \frac{d\Gamma}{dq^2}(\vec{n} = -\vec{e}_i)} \ . \qquad (19)$$

After standard calculations for $P_i\, (i = L,\ T)$ we get

$$\begin{aligned}
P_L &= \frac{v}{\Delta} \Bigg\{ \frac{32}{3} \lambda m_B^6 Re\,(AC^*) + \frac{4}{3} \frac{\lambda^2 m_B^6}{r} Re\,(B_2 D_2^*) + 4 \frac{m_B^2}{r} \left[\frac{\lambda}{3} + 4rs \right] Re\,(B_1 D_1^*) - \\
&- \frac{4}{3} \frac{\lambda m_B^4}{r} (1 - r - s) \left[Re\,(B_2 D_1^*) + Re\,(B_1 D_2^*) \right] - 8 \frac{\lambda m_B^6 s}{m_b^2} Re\,(F_1 F_2^*) - \\
&- 8 \frac{\lambda m_B^3 m_\ell}{m_b \sqrt{r}} \Big[Re\,(F_1 D_1^*) - (1 - r)\, Re\,(F_1 D_2^*) + m_B^2 s Re\,(F_1 D_3^*) \Big] \Bigg\} \ , \qquad (20)
\end{aligned}$$

$$
\begin{aligned}
P_T &= \frac{\sqrt{\lambda}\pi}{\Delta}\Bigg\{\frac{\lambda m_B^5 m_\ell\,(1-r)}{r\sqrt{s}} Re\,(B_2 D_2^*) + 8 m_B^3 m_\ell \sqrt{s} Re\,(A B_1^*) - \lambda m_B^5 m_\ell \frac{\sqrt{s}}{r} Re\,(B_2 D_3^*) - \\
&- \lambda m_B^6 \frac{\sqrt{s}}{m_b\sqrt{r}} Re\,(B_2 F_2^*) + m_B^3 m_\ell\,(1-r-s)\,\frac{\sqrt{s}}{r} Re\,(B_1 D_3^*) + \\
&+ m_B^4\,(1-r-s)\,\frac{\sqrt{s}}{m_b\sqrt{r}} Re\,(B_1 F_2^*) - m_B^3 m_\ell\,(1-r-s)\,\frac{(1-r)}{r\sqrt{s}} Re\,(B_1 D_2^*) + \\
&+ m_B m_\ell \frac{(1-r-s)}{r\sqrt{s}} Re\,(B_1 D_1^*) - \lambda m_B^6 \sqrt{s} \frac{v^2}{m_b\sqrt{r}} Re\,(D_2 F_1^*) + \\
&+ m_B^4 \sqrt{s}\,(1-r-s)\,\frac{v^2}{m_b\sqrt{r}} Re\,(D_1 F_1^*) - \lambda m_B^3 \frac{m_\ell}{r\sqrt{s}} Re\,(B_2 D_1^*)\Bigg\}\ . \qquad (21)
\end{aligned}
$$

The denominator Δ in eqs.(20) and (21) can be obtained from eq.(17) by integration over z of the terms within the curly bracket. We also calculate the P_N component of the lepton polarization, but the numerical results show that its value is quite small and because of that we do not present its explicit form here.

2 Numerical Analysis

The values of the main input parameters, which appear in the expression for the decay width are: $m_b = 4.8\ GeV,\ m_c = 1.35\ GeV,\ m_\tau = 1.78\ GeV,\ m_\mu = 0.105\ GeV,\ \Lambda_{QCD} = 225\ MeV,\ m_B = 5.28\ GeV$, and $m_{K^*} = 0.892\ GeV$. We use the pole form of the formfactors given in eq.(16). For B meson lifetime we take $\tau(B_d) = 1.56 \times 10^{-12}\ s$ [33]. The values of the Wilson coefficients $C_7^{SM}(m_b)$ and $C_{10}^{SM}(m_b)$ to the leading logarithmic approximation are [34, 35]:

$$
C_7 = -0.315\ , \quad C_{10} = -4.642\ .
$$

The expression C_9^{eff} for the $b \to s$ transition in the next to leading order approximation is given as (see for example [34])

$$
\begin{aligned}
& C_9^{eff}(m_b) = \\
& \quad C_9^{SM}(m_b) + C_9^{H^-}(m_b) + 0.124 w(\hat{s}) + g(\hat{m}_c,\hat{s})\,(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) - \\
&- \frac{1}{2} g(\hat{m}_q,\hat{s})\,(C_3 + 3C_4) - \frac{1}{2} g(\hat{m}_b,\hat{s})\,(4C_3 + 4C_4 + 3C_5 + C_6) + \\
&+ \frac{2}{9}\,(3C_3 + C_4 + 3C_5 + C_6)\ , \qquad (22)
\end{aligned}
$$

with

$$
C_1 = -0.249\ , \quad C_2 = 1.108\ , \quad C_3 = 1.112 \times 10^{-2}\ , \quad C_4 = -2.569 \times 10^{-2}\ ,
$$

$$
C_5 = 7.4 \times 10^{-3}\ , \quad C_6 = -3.144 \times 10^{-2}\ , \quad C_9^{SM}(m_b) = 4.227,
$$

where $\hat{m}_q = \dfrac{m_q}{m_b},\ \hat{s} = \dfrac{q^2}{m_b^2}$.

In the above expression, $w(\hat{s})$ represents the one gluon correction to the matrix element O_9 and its explicit form can be found in [13], while the function $g(\hat{m_q}, \hat{s})$ arises from the one loop contributions of the four quark operators $O_1 - O_6$ (see for example [34, 35]), i.e.

$$\begin{aligned} g(\hat{m_q}, \hat{s'}) &= -\frac{8}{9} ln\, \hat{m_q} + \frac{8}{27} + \frac{4}{9} y_q - \frac{2}{9}(2+y_q)\sqrt{11-y_q} + \\ &+ \left\{ \theta(1-y_q)\left(ln \frac{1+\sqrt{1-y_q}}{1-\sqrt{1-y_q}} - i\pi \right) + \theta(y_q-1) arctan \frac{1}{\sqrt{y_q-1}} \right\}, \end{aligned} \quad (23)$$

where $y_q = \frac{\hat{m_q}}{\hat{s'}}$, and $\hat{s'} = \frac{4q^2}{m_b^2}$

In all numerical calculations we use the following values for the masses of the Higgs particles: $m_{H^\pm} = 200\, GeV$, $m_{h^0} = 80\, GeV$, $m_{H^0} = 150\, GeV$, and $m_{A^0} = 100\, GeV$. We also take $sin\alpha = \frac{\sqrt{2}}{2}$ and for $tan\, \beta$ we choose the following set of values: $tan\, \beta = 1$, $tan\, \beta = 30$ and $tan\, \beta = 50$.

In Fig.1 we present the q^2 dependence of the differential branching ratio for $B_d \to K^* \tau^+ \tau^-$ decay with and without the long distance effects. It follows from this figure that the differential branching ratio is sensitive to the value of $tan\, \beta$ if the ψ' mass region is excluded. For example at $tan\, \beta = 50$ the differential branching ratio is approximately 2 times larger than the one at $tan\, \beta = 1$.

In Fig.2 we plot the dependence of the forward-backward asymmetry A_{FB} on q^2 with and without the long-distance effects at different values of $tan\, \beta$. From this figure we see that A_{FB} is positive for all values of q^2 except in the ψ' resonance region and it is sensitive to the value of $tan\, \beta$.

In Fig.3 we depicted the q^2 dependence of the longitudinal polarization of the final lepton P_L with and without the long distance effects at different values of $tan\, \beta$. As we see from this figure if we exclude the resonance mass region of ψ', P_L is negative for all values of q^2 at $tan\, \beta = 1$ as well as at $tan\, \beta = 30$ and $tan\, \beta = 50$.

In Fig.4 we present the q^2 dependence of the transversal polarization P_T of the τ lepton which lies in the decay plane, without long distance effects at $tan\, \beta = 1$, $tan\, \beta = 30$ and at $tan\, \beta = 50$. From this figure it follows that at $tan\, \beta = 1$ P_T is positive at all values of q^2. For $tan\, \beta = 30$ and $tan\, \beta = 50$, P_T is positive near the threshold region but far from the threshold it becomes negative. Therefore the determination of the sign of P_T in future experiments is a very important issue and can provide a direct information for the establishment of new physics.

In Fig.5 we present the differential branching ratio versus q^2 for $B \to K^* \mu^+ \mu^-$. From this figure and Fig.1 it follows that the differential branching ratio for the $B \to K^* \mu^+ \mu^-$decay is approximately one order of magnitude greater then that of the $B \to K^* \tau^+ \tau^-$decay. Note that for $tan\, \beta = 30$ and $tan\, \beta = 50$, the results for the differential branching ratio are practically the same.

In Fig.6, we depict the dependence of $A_{FB}(B \to K^* \mu^+ \mu^-)$ on q^2 at $tan\, \beta = 1$ and $tan\, \beta = 50$. From a comparison of this figure with Fig.2 we observe that the behaviour of A_{FB} for the $B \to K^* \mu^+ \mu^-$decay is totaly different from that for the $B \to K^* \tau^+ \tau^-$decay. In the $B \to K^* \tau^+ \tau^-$case A_{FB} is positive for all values of q^2 (without the long distance effects), while it changes its sign here in the $B \to K^* \mu^+ \mu^-$case. This indicates that for the $B \to$

$K^*\tau^+\tau^-$case, the neutral Higgs boson exchange diagrams give considerable contribution. It follows from these two figures then that the determination of the sign of A_{FB} in different kinematical regions is also a very important tool for establishing new physics.

The behaviour of the longitudinal polarization with changing q^2 is presented in Fig.7. In $B \to K^*\mu^+\mu^-$decay, P_L is firstly positive and then it changes sign (without the long distance effects), which is absolutely different from that of the $B \to K^*\tau^+\tau^-$case, for which P_L is negative for all q^2.

Since, as we have already noted, A_{FB}, P_L, and P_T contain independent information, their investigation in experiments will be a very efficient tool in establishing new physics.

At the end of this section we present the values of the branching ratios for the $B_d \to K^*\tau^+\tau^-$ decay. After integrating over q^2 we get for the branching ratios for the $B_d \to K^*\tau^+\tau^-$ decay, without the long distance contributions,

$$B(B_d \to K^*\tau^+\tau^-) = \begin{cases} 0.86 \times 10^{-7} & (\tan\beta = 1) \\ \\ 0.90 \times 10^{-7} & (\tan\beta = 30) \\ \\ 1.11 \times 10^{-7} & (\tan\beta = 50)\ . \end{cases} \tag{24}$$

The ratio of the exclusive and inclusive channels is defined as

$$R = \frac{B(B_d \to K^*\tau^+\tau^-)}{B(b \to s\tau^+\tau^-)}\ .$$

In the SM this ratio is given as $R = 0.270 \pm 0.007$ when $B(b \to s\tau^+\tau^-) = (2.6 \pm 0.5) \times 10^{-7}$ [35]. In our case we have

$$R = \begin{cases} 0.33 & (\tan\ \beta = 1) \\ \\ 0.35 & (\tan\beta = 30) \\ \\ 0.43 & (\tan\beta = 50)\ , \end{cases} \tag{25}$$

where we use the SM value for the inclusive $B(b \to s\tau^+\tau^-)$.

In conclusion, we calculate the rare $B \to K^*\ell^+\ell^-$ decay in 2HDM. It is observed that A_{FB}, P_L and the transversal polarization P_T of the charged lepton are very sensitive to the value of $tan\,\beta$. Therefore, in search of new physics, their experimental investigation can serve as a crucial test.

Figure Captions

In Fig.1, Fig.2 and Fig.3, lines 1, 3 and 4 correspond to the short distance contributions for the values of $tan\,\beta = 50$, $tan\,\beta = 30$ and $tan\,\beta = 1$, respectively; while lines 2, 5 and 6 correspond to the sum of the short and long distance contributions for the values of $tan\,\beta = 50$, $tan\,\beta = 30$ and $tan\,\beta = 1$, respectively.

1. Invariant mass squared (q^2) distribution of the τ lepton pair for the decay $B \to K^* \tau^+ \tau^-$.

2. The dependence of the forward-backward asymmetry A_{FB} on q^2 for the decay $B \to K^* \tau^+ \tau^-$.

3. The dependence of the longitudinal polarization P_L on q^2 for the $B \to K^* \tau^+ \tau^-$.

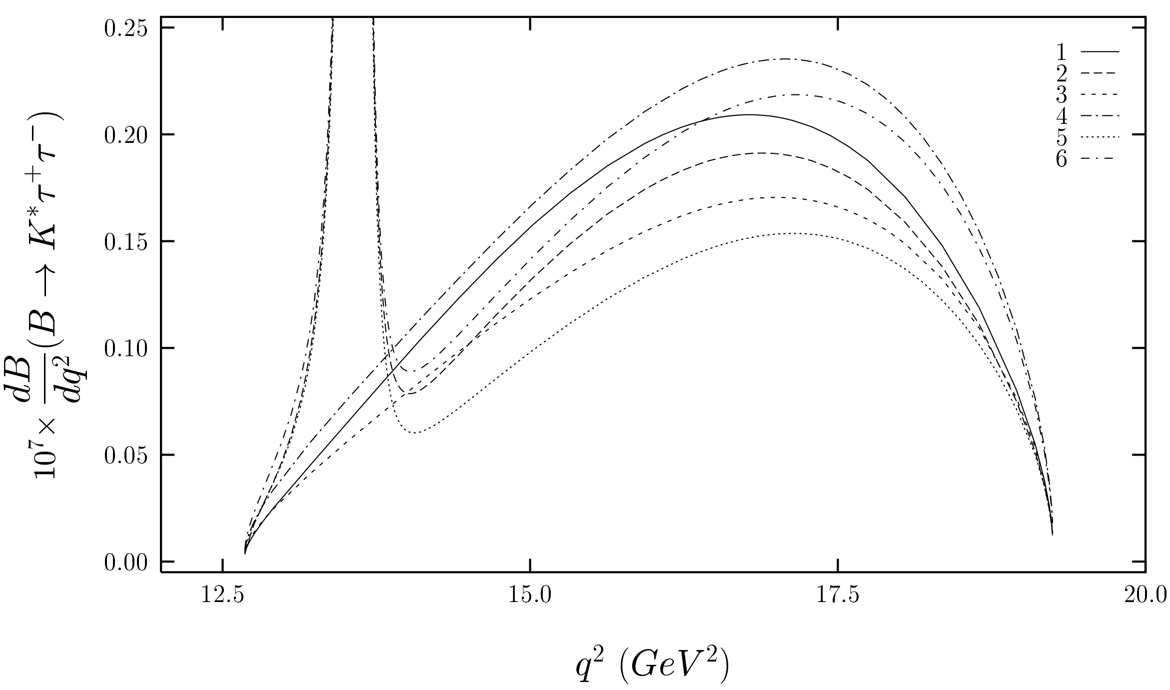
4. The same as in Fig.3 but for the transversal polarization P_T.

5. The same as in Fig.1, but for the $B \to K^* \mu^+ \mu^-$ decay.

In Fig.6 and Fig.7, lines 1 and 3 correspond to the short distance contributions for the values of $tan\,\beta = 50$ and $tan\,\beta = 1$, respectively; while lines 2 and 4 correspond to the sum of the short and long distance contributions for the values of $tan\,\beta = 50$ and $tan\,\beta = 1$, respectively.

6. The same as in Fig.2, but for the $B \to K^* \mu^+ \mu^-$ decay.

7. The same as in Fig.3, but for the $B \to K^* \mu^+ \mu^-$ decay.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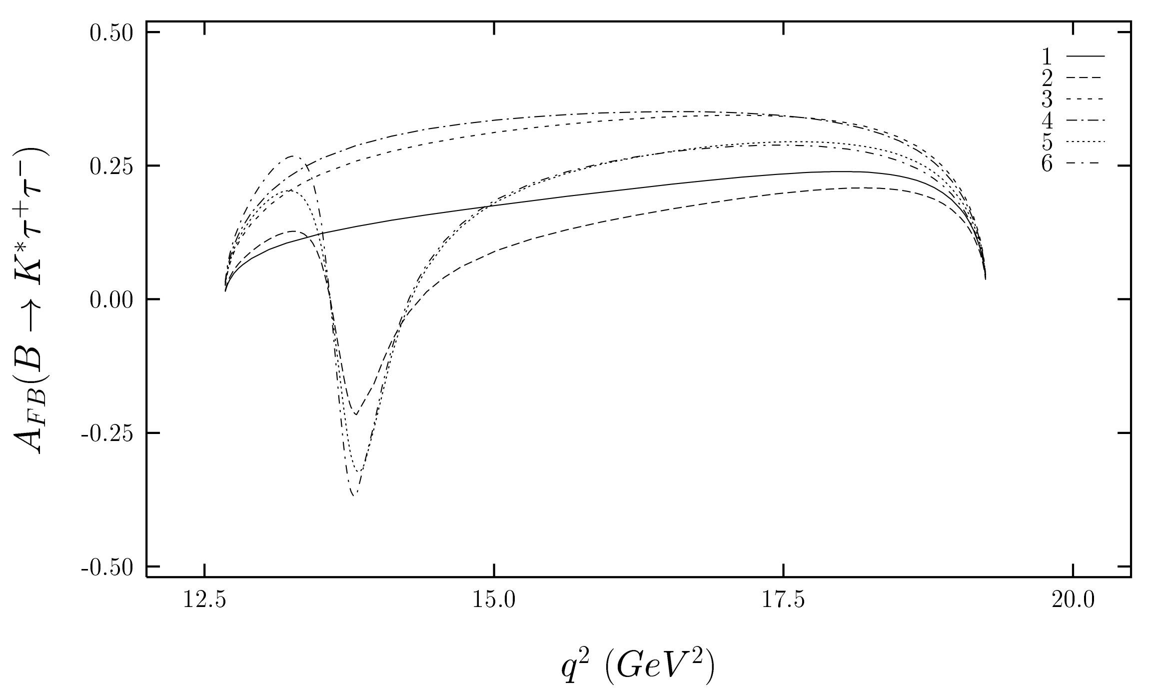
0.20
0.15
0.10
0.05
0.00
12.5
15.0
17.5
20.0
1
2
3
4
5
6
$10^7 \times \frac{dB}{dq^2}(B \to K^* \tau^+ \tau^-)$
$q^2\ (GeV^2)$

Fig.1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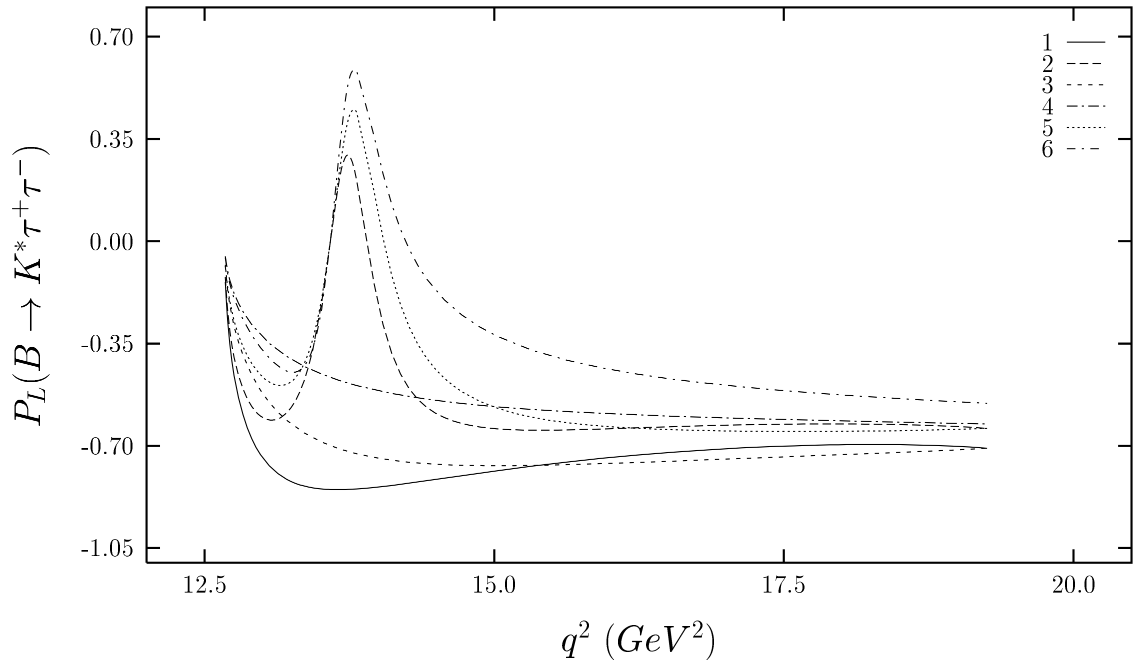
0.25
0.00
-0.25
-0.50
12.5
15.0
17.5
20.0
1
2
3
4
5
6
$A_{FB}(B \to K^* \tau^+ \tau^-)$
$q^2\ (GeV^2)$

Fig.2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0.00
-0.35
-0.70
-1.05
12.5
15.0
17.5
20.0
1
2
3
4
5
6
$P_L(B \to K^* \tau^+ \tau^-)$
$q^2\ (GeV^2)$

Fig.3

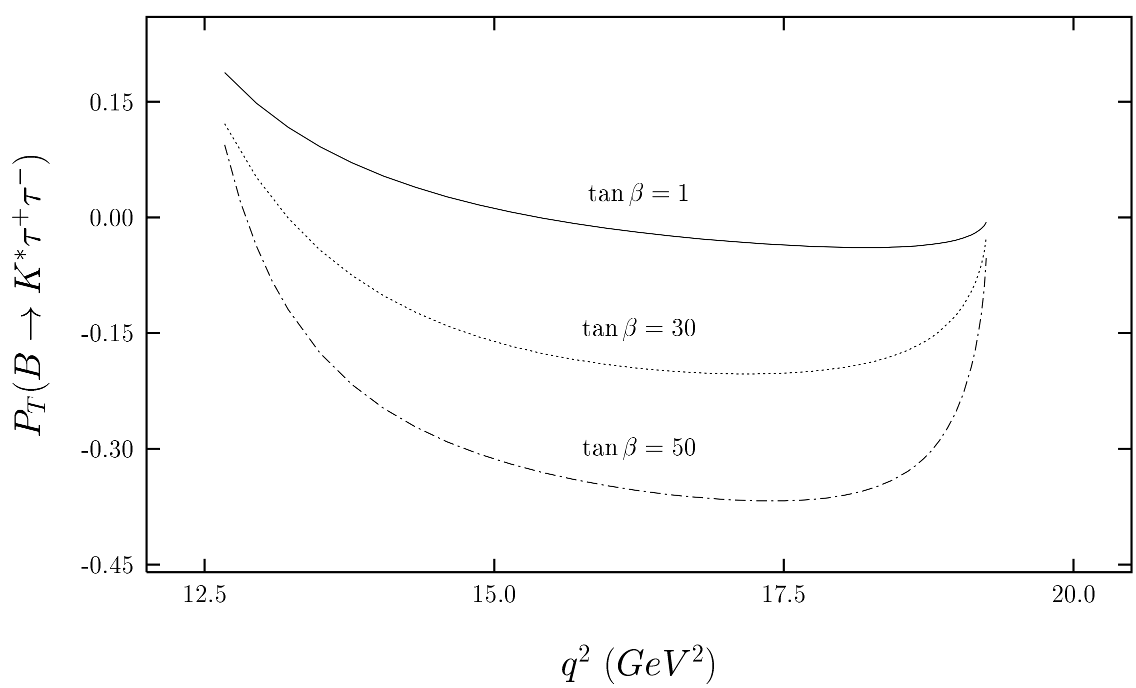
0.15
0.00
-0.15
-0.30
-0.45
12.5
15.0
17.5
20.0
$\tan\beta = 1$
$\tan\beta = 30$
$\tan\beta = 50$
$P_T(B \to K^* \tau^+ \tau^-)$
$q^2\ (GeV^2)$

Fig.4

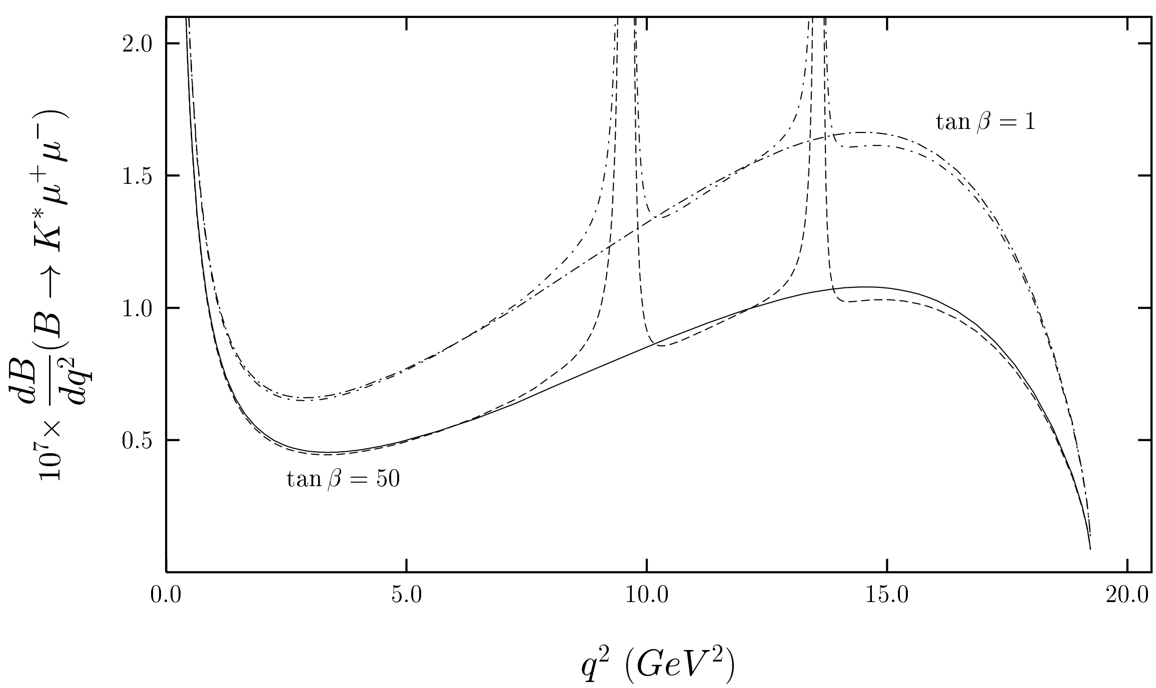
$\tan\beta = 1$
$\tan\beta = 50$
$10^7 \times \frac{dB}{dq^2}(B \to K^* \mu^+ \mu^-)$
$q^2\ (GeV^2)$

Fig.5

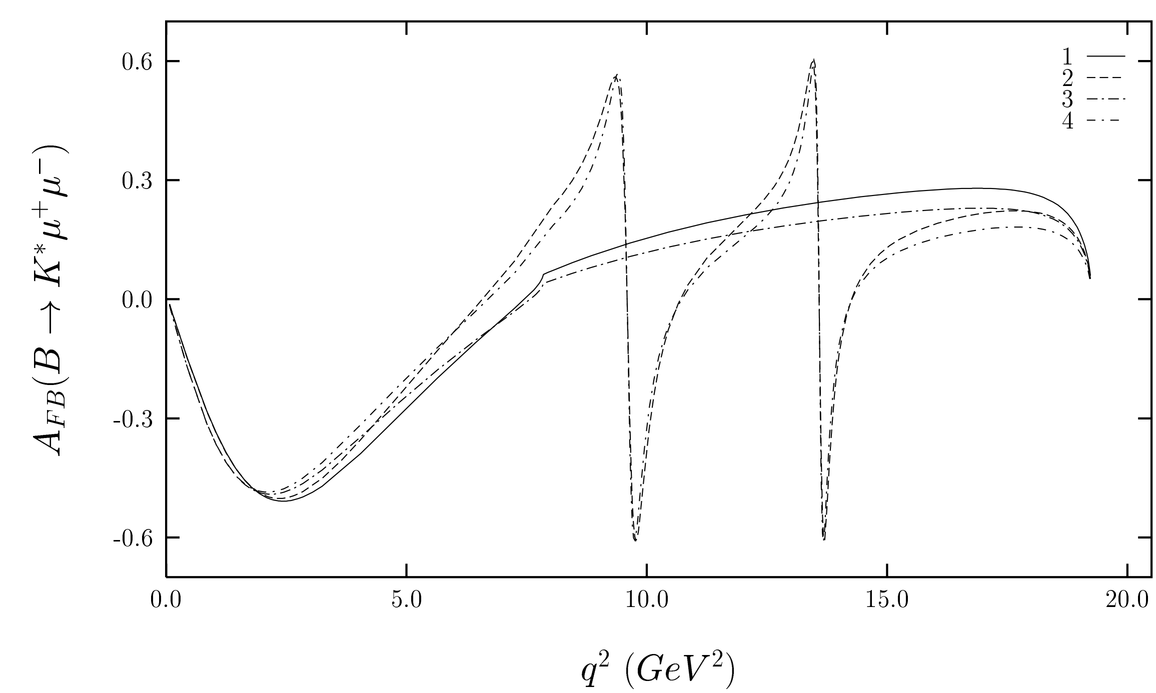
1
2
3
4
$A_{FB}(B \to K^* \mu^+ \mu^-)$
$q^2\ (GeV^2)$

Fig.6

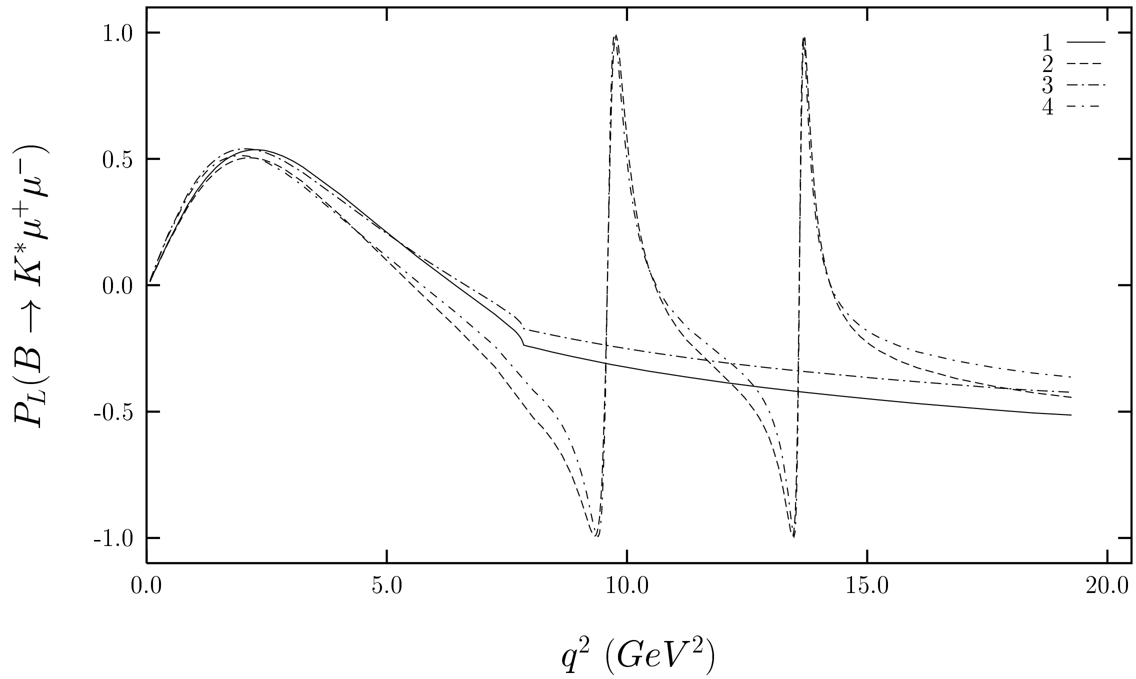

Fig.7

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