Radiative Decays of the Heavy Flavored Baryons in Light Cone QCD Sum Rules

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Abstract

The transition magnetic dipole and electric quadrupole moments of the radiative decays of the sextet heavy flavored spin $\frac{3}{2}$ to the heavy spin $\frac{1}{2}$ baryons are calculated within the light cone QCD sum rules approach. Using the obtained results, the decay rate for these transitions are also computed and compared with the existing predictions of the other approaches.

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1 Introduction

In the last years, considerable experimental progress has been made in the identification of new bottom baryon states. The CDF Collaboration [1] reported the first observation of the Σ_b^{\pm} and $\Sigma_b^{*\pm}$. After this discovery, the DO [2] and CDF [3] Collaborations announced the observation of the Ξ_{b^-} state (for a review see [4]). The BaBar Collaboration discovered the Ω_c^* state [5] and observed the Ω_c state in B decays [6]. Lastly, the BaBar and BELLE Collaborations observed Ξ_c state [7].

Heavy baryons containing c and b quarks have been subject of the intensive theoretical studies (see [8] and references therein). A theoretical study of experimental results can give essential information about the structure of these baryons. In this sense, study of the electromagnetic properties of the heavy baryons receives special attention. One of the main static electromagnetic quantities of the baryons is their magnetic moments. The magnetic moments of the heavy baryons have been discussed within different approaches in the literature (see [9] and references therein).

In the present work, we investigate the electromagnetic decays of the ground state heavy baryons containing single heavy quark with total angular momentum $J = \frac{3}{2}$ to the heavy baryons with $J = \frac{1}{2}$ in the framework of the light cone QCD sum rules method. Note that, some of the considered decays have been studied within heavy hadron chiral perturbation theory [10], heavy quark and chiral symmetries [11, 12], in the relativistic quark model [13] and light cone QCD sum rules at leading order in HQET in [14]. Here, we also emphasize that the radiative decays of the light decuplet baryons to the octet baryons have been studied in the framework of the light cone QCD sum rules in [15].

The work is organized as follows. In section 2, the light cone QCD sum

rules for the form factors describing electromagnetic transition of the heavy $J = \frac{3}{2}$, to the heavy baryons with $J = \frac{1}{2}$ are obtained. Section 3 encompasses the numerical analysis of the transition magnetic dipole and electric quadrupole moments as well as the radiative decay rates. A comparison of our results for the total decay width with the existing predictions of the other approaches is also presented in section 3.

2 Light cone QCD sum rules for the electromagnetic form factors of the heavy flavored baryons

We start this section with a few remarks about the classification of the heavy baryons. Heavy baryons with single heavy quark belong to either SU(3) antisymmetric $\bar{3}_F$ or symmetric 6_F flavor representations. For the baryons containing single heavy quark, in the $m_Q \to \infty$ limit, the angular momentum of the light quarks is a good quantum number. The spin of the light diquark is either S = 1 for 6_F or S = 0 for $\bar{3}_F$. The ground state will have angular momentum l = 0. Therefore, the spin of the ground state is 1/2 for $\bar{3}_F$ representing the Λ_Q and Ξ_Q baryons, while it can be both 3/2 or 1/2 for 6_F , corresponding to Σ_Q , Σ_Q^* , Ξ_Q' , Ξ_Q^* , Ω_Q and Ω_Q^* states, where * indicates spin 3/2 states.

After this remark, let us calculate the electromagnetic transition form factors of the heavy baryons. For this aim, consider the following correlation function, which is the main tool of the light cone QCD sum rules:

$$T_{\mu}(p,q) = i \int d^4x e^{ipx} \langle 0 \mid T\{\eta(x)\bar{\eta}_{\mu}(0)\} \mid 0 \rangle_{\gamma}, \qquad (1)$$

where η and η_{μ} are the generic interpolating quark currents of the heavy flavored baryons with J = 1/2 and 3/2, respectively and γ means the external electromagnetic field. In this work we will discuss the following electromagnetic transitions

$$\begin{split} \Sigma_Q^* &\to \Sigma_Q \gamma, \\ \Xi_Q^* &\to \Xi_Q \gamma, \\ \Sigma_Q^* &\to \Lambda_Q \gamma, \end{split}$$
(2)

where Q = c or b quark.

In QCD sum rules approach, this correlation function is calculated in two different ways: from one side, it is calculated in terms of the quarks and gluons interacting in QCD vacuum. In the phenomenological side, on the other hand, it is saturated by a tower of hadrons with the same quantum numbers as the interpolating currents. The physical quantities are determined by matching these two different representations of the correlation function.

The hadronic representation of the correlation function can be obtained inserting the complete set of states with the same quantum numbers as the interpolating currents.

$$T_{\mu}(p,q) = \frac{\langle 0 \mid \eta \mid 2(p) \rangle}{p^2 - m_2^2} \langle 2(p) \mid 1(p+q) \rangle_{\gamma} \frac{\langle 1(p+q) \mid \bar{\eta}_{\mu} \mid 0 \rangle}{(p+q)^2 - m_1^2} + \dots, \quad (3)$$

where $\langle 1(p+q)|$ and $\langle 2(p)|$ denote heavy spin 3/2 and 1/2 states and m_1 and m_2 represent their masses, respectively and q is the photon momentum. In the above equation, the dots correspond to the contributions of the higher states and continuum. For the calculation of the phenomenological part, it follows from Eq. (3) that we need to know the matrix elements of the interpolating currents between the vacuum and baryon states. They are defined as

$$\langle 1(p+q,s) \mid \bar{\eta}_{\mu}(0) \mid 0 \rangle = \lambda_{1} \bar{u}_{\mu}(p+q,s), \langle 0 \mid \eta(0) \mid 2(p,s') \rangle = \lambda_{2} u(p,s'),$$
 (4)

where λ_1 and λ_2 are the residues of the heavy baryons, $u_{\mu}(p, s)$ is the Rarita-Schwinger spinor and s and s' are the polarizations of the spin 3/2 and 1/2 states, respectively. The electromagnetic vertex $\langle 2(p) | 1(p+q) \rangle_{\gamma}$ of the spin 3/2 to spin 1/2 transition is parameterized in terms of the three form factors in the following way [16, 17]

$$\langle 2(p) \mid 1(p+q) \rangle_{\gamma} = e \bar{u}(p,s') \left\{ G_1(q_{\mu} \not\in -\varepsilon_{\mu} \notq) + G_2[(\mathcal{P}\varepsilon)q_{\mu} - (\mathcal{P}q)\varepsilon_{\mu}]\gamma_5 \right. \\ \left. + G_3[(q\varepsilon)q_{\mu} - q^2\varepsilon_{\mu}]\gamma_5 \right\} u_{\mu}(p+q),$$

$$(5)$$

where $\mathcal{P} = \frac{p+(p+q)}{2}$ and ε_{μ} is the photon polarization vector. Since for the considered decays the photon is real, the terms proportional to G_3 are exactly zero, and for analysis of these decays, we need to know the values of the form factors $G_1(q^2)$ and $G_2(q^2)$ only at $q^2 = 0$. From the experimental point of view, more convenient form factors are magnetic dipole G_M , electric quadrupole G_E and Coulomb quadrupole G_C which are linear combinations of the form factors G_1 and G_2 (see [15]). At $q^2 = 0$, these relations are

$$G_M = \left[(3m_1 + m_2) \frac{G_1}{m_1} + (m_1 - m_2) G_2 \right] \frac{m_2}{3},$$

$$G_E = (m_1 - m_2) \left[\frac{G_1}{m_1} + G_2 \right] \frac{m_2}{3}.$$
(6)

In order to obtain the explicit expressions of the correlation function from the phenomenological side, we also perform summation over spins of the spin 3/2 particles using

$$\sum_{s} u_{\mu}(p,s)\bar{u}_{\nu}(p,s) = \frac{(\not\!\!\!p+m)}{2m} \{-g_{\mu\nu} + \frac{1}{3}\gamma_{\mu}\gamma_{\nu} - \frac{2p_{\mu}p_{\nu}}{3m^2} - \frac{p_{\mu}\gamma_{\nu} - p_{\nu}\gamma_{\mu}}{3m}\}.$$
 (7)

In principle, the phenomenological part of the correlator can be obtained with the help of the Eqs. (3-7). As noted in [9, 15], at this point two problems appear: 1) all Lorentz structures are not independent, b) not only spin 3/2, but spin 1/2 states also give contributions to the correlation function. In other words the matrix element of the current η_{μ} between vacuum and spin 1/2 states is nonzero. In general, this matrix element can be written in the following way

$$\langle 0 \mid \eta_{\mu}(0) \mid B(p, s = 1/2) \rangle = (Ap_{\mu} + B\gamma_{\mu})u(p, s = 1/2).$$
(8)

Imposing the condition $\gamma_{\mu}\eta^{\mu} = 0$, one can immediately obtain that $B = -\frac{A}{4}m$.

In order to remove the contribution of the spin 1/2 states and deal with only independent structures in the correlation function, we will follow [9, 15] and remove those contributions by ordering the Dirac matrices in a specific way. For this aim, we choose the ordering for Dirac matrices as $\notin \not{\mu} \not{p}\gamma_{\mu}$. Using this ordering for the correlator, we get

$$\begin{split} T_{\mu} &= e\lambda_{1}\lambda_{2}\frac{1}{p^{2}-m_{2}^{2}}\frac{1}{(p+q)^{2}-m_{1}^{2}} \begin{bmatrix} \\ & [\varepsilon_{\mu}(pq)-(\varepsilon p)q_{\mu}] \left\{-2G_{1}m_{1}-G_{2}m_{1}m_{2}+G_{2}(p+q)^{2} \\ & + [2G_{1}-G_{2}(m_{1}-m_{2})] \not p+m_{2}G_{2} \not q-G_{2} \not q \not p \right\} \gamma_{5} \\ & + [q_{\mu} \not q - \varepsilon_{\mu} \not q] \left\{G_{1}(p^{2}+m_{1}m_{2}) - G_{1}(m_{1}+m_{2}) \not p \right\} \gamma_{5} \\ & + 2G_{1} \left[\not q(pq) - \not q(\varepsilon p)\right] q_{\mu}\gamma_{5} \\ & -G_{1} \not q \not q \{m_{2}+\not p\} q_{\mu}\gamma_{5} \\ & \text{other structures with } \gamma_{\mu} \text{ at the end or which are proportional to } (p+q)_{\mu} \end{split}$$

For determination of the form factors G_1 and G_2 , we need two invariant structures. We obtained that among all structures, the best convergence comes from the structures $\not\in \not\!\!\!/\gamma_5 q_\mu$ and $\not\not\equiv \not\!\!/\gamma_5 (\varepsilon p) q_\mu$ for G_1 and G_2 , respectively. To (9)

get the sum rules expression for the form factors G_1 and G_2 , we will choose the same structures also from the QCD side and match the corresponding coefficients. We also would like to note that, the correlation function receives contributions from contact terms. But, the contact terms do not give contributions to the chosen structures (for a detailed discussion see e. g. [15, 18]).

On QCD side, the correlation function can be calculated using the operator product expansion. For this aim, we need the explicit expressions of the interpolating currents of the heavy baryons with the angular momentums J = 3/2 and J = 1/2. The interpolating currents for the spin J = 3/2baryons are written in such a way that the light quarks should enter the expression of currents in symmetric way and the condition $\gamma^{\mu}\eta_{\mu} = 0$ should be satisfied. The general form of the currents for spin J = 3/2 baryons satisfying both aforementioned conditions can be written as [9]

$$\eta_{\mu} = A\epsilon_{abc} \left\{ (q_1^{aT} C \gamma_{\mu} q_2^b) Q^c + (q_2^{aT} C \gamma_{\mu} Q^b) q_1^c + (Q^{aT} C \gamma_{\mu} q_1^b) q_2^c \right\},$$
(10)

where C is the charge conjugation operator and a, b and c are color indices. The value of normalization factor A and quark fields q_1 and q_2 for corresponding heavy baryons is given in Table 1 (see [9]).

The general form of the interpolating currents for the heavy spin 1/2 baryons can be written in the following form:

$$\eta_{\Sigma_{Q}} = -\frac{1}{\sqrt{2}} \epsilon_{abc} \left\{ (q_{1}^{aT} C Q^{b}) \gamma_{5} q_{2}^{c} + \beta (q_{1}^{aT} C \gamma_{5} Q^{b}) q_{2}^{c} - [(Q^{aT} C q_{2}^{b}) \gamma_{5} q_{1}^{c} + \beta (Q^{aT} C \gamma_{5} q_{2}^{b}) q_{1}^{c}] \right\},$$

$$\eta_{\Xi_{Q},\Lambda_{Q}} = \frac{1}{\sqrt{6}} \epsilon_{abc} \left\{ 2(q_{1}^{aT} C q_{2}^{b}) \gamma_{5} Q^{c} + \beta (q_{1}^{aT} C \gamma_{5} q_{2}^{b}) Q^{c} + (q_{1}^{aT} C Q^{b}) \gamma_{5} q_{2}^{c} + \beta (q_{1}^{aT} C \gamma_{5} Q^{b}) q_{2}^{c} + (Q^{aT} C q_{2}^{b}) \gamma_{5} q_{1}^{c} + \beta (Q^{aT} C \gamma_{5} q_{2}^{b}) q_{1}^{c} \right\},$$

$$(11)$$

| Heavy spin $\frac{3}{2}$ baryons | А | q_1 | q_2 |
|----------------------------------|--------------|--------------|--------------|
| $\Sigma_{b(c)}^{*+(++)}$ | $1/\sqrt{3}$ | u | u |
| $\Sigma_{b(c)}^{*0(+)}$ | $\sqrt{2/3}$ | u | d |
| $\Sigma_{b(c)}^{*-(0)}$ | $1/\sqrt{3}$ | d | d |
| $\Xi^{*0(+)}_{b(c)}$ | $\sqrt{2/3}$ | \mathbf{S} | u |
| $\Xi^{*-(0)}_{b(c)}$ | $\sqrt{2/3}$ | \mathbf{S} | d |
| $\Omega_{b(c)}^{*-(0)}$ | $1/\sqrt{3}$ | \mathbf{S} | \mathbf{S} |

Table 1: The value of normalization factor A and quark fields q_1 and q_2 for the corresponding heavy spin 3/2 baryons.

where β is an arbitrary parameter and $\beta = -1$ corresponds to the Ioffe current. The quark fields q_1 and q_2 for the corresponding heavy spin 1/2 baryons are as presented in Table 2.

| Heavy spin $\frac{1}{2}$ baryons | q_1 | q_2 |
|----------------------------------|-------|--------------|
| $\Sigma_{b(c)}^{+(++)}$ | u | u |
| $\Sigma_{b(c)}^{0(+)}$ | u | d |
| $\Sigma_{b(c)}^{-(0)}$ | d | d |
| $\Xi^{0(+)}_{b(c)}$ | u | \mathbf{S} |
| $\Xi_{b(c)}^{-(0)}$ | d | \mathbf{S} |
| $\Lambda^{0(+)}_{b(c)}$ | u | d |

Table 2: The quark fields q_1 and q_2 for the corresponding heavy spin 1/2 baryons.

After performing all contractions of the quark fields in Eq. (1), we get the following expression for the correlation functions responsible for the $\Sigma_b^{*0} \to \Sigma_b^0 \gamma$ and $\Xi_b^{*0} \to \Xi_b^0 \gamma$ transitions in terms of the light and heavy quark

propagators

$$T_{\mu}^{\Sigma_{b}^{*0}\to\Sigma_{b}^{0}} = \frac{i}{\sqrt{3}} \epsilon_{abc} \epsilon_{a'b'c'} \int d^{4}x e^{ipx} \langle 0[\gamma(q)] | \{-\gamma_{5}S_{d}^{c'c}Tr[S_{b}^{b'b}\gamma_{\mu}S_{u}^{\prime a'a}] \\ + \gamma_{5}S_{u}^{c'c}Tr[S_{d}^{b'b}\gamma_{\mu}S_{b}^{\prime a'a}] + \beta Tr[\gamma_{5}S_{u}^{aa'}\gamma_{\mu}S_{b}^{\prime bb'}]S_{d}^{cc'} - \beta Tr[\gamma_{5}S_{b}^{aa'}\gamma_{\mu}S_{d}^{\prime bb'}]S_{u}^{cc'} \\ - \gamma_{5}S_{d}^{c'a}\gamma_{\mu}S_{u}^{\prime a'b}S_{b}^{b'c} + \gamma_{5}S_{d}^{c'b}\gamma_{\mu}S_{u}^{\prime b'a}S_{u}^{a'c} - \gamma_{5}S_{u}^{c'b}\gamma_{\mu}S_{d}^{\prime b'a}S_{b}^{a'c} \\ + \gamma_{5}S_{u}^{c'a}\gamma_{\mu}S_{b}^{\prime a'b}S_{d}^{b'c} - \beta S_{d}^{ca'}\gamma_{\mu}S_{u}^{\prime ab'}\gamma_{5}S_{b}^{bc'} + \beta S_{d}^{c'b}\gamma_{\mu}S_{b}^{\prime b'a}\gamma_{5}S_{u}^{a'c} \\ - \beta S_{u}^{c'b}\gamma_{\mu}S_{d}^{\prime b'a}\gamma_{5}S_{b}^{a'c} + \beta S_{u}^{c'a}\gamma_{\mu}S_{b}^{\prime a'b}\gamma_{5}S_{d}^{b'c} | 0 \rangle,$$
(12)

$$T_{\mu}^{\Xi_{b}^{s_{0}\to\Xi_{b}^{0}}} = \frac{i}{3} \epsilon_{abc} \epsilon_{a'b'c'} \int d^{4}x e^{ipx} \langle 0[\gamma(q)] | \{\gamma_{5} S_{s}^{c'c} Tr[S_{b}^{b'b} \gamma_{\mu} S_{u}^{'a'a}] \\ - 2\gamma_{5} S_{b}^{c'c} Tr[S_{s}^{b'a} \gamma_{\mu} S_{u}^{'a'b}] + 2\beta Tr[\gamma_{5} S_{u}^{a'b} \gamma_{\mu} S_{b}^{'b'a}] S_{b}^{c'c} - \beta Tr[\gamma_{5} S_{u}^{a'a} \gamma_{\mu} S_{b}^{'b'b}] S_{s}^{c'c} \\ + \gamma_{5} S_{u}^{c'c} Tr[S_{s}^{b'b} \gamma_{\mu} S_{b}^{'a'a}] - \beta Tr[\gamma_{5} S_{b}^{a'a} \gamma_{\mu} S_{s}^{'b'b}] S_{u}^{c'c} - \gamma_{5} S_{u}^{c'b} \gamma_{\mu} S_{s}^{'b'a} S_{b}^{a'c} \\ + 2\gamma_{5} S_{b}^{c'a} \gamma_{\mu} S_{b}^{'b'b} S_{u}^{a'c} - 2\gamma_{5} S_{b}^{c'b} \gamma_{\mu} S_{u}^{'a'a} S_{s}^{b'c} + \gamma_{5} S_{s}^{c'a} \gamma_{\mu} S_{u}^{'a'b} S_{b}^{b'c} \\ - \gamma_{5} S_{s}^{c'b} \gamma_{\mu} S_{b}^{'b'a} S_{u}^{a'c} + 2\beta S_{b}^{c'a} \gamma_{\mu} S_{s}^{'b'b} \gamma_{5} S_{u}^{a'c} - 2\beta S_{b}^{c'b} \gamma_{\mu} S_{u}^{'a'a} \gamma_{5} S_{s}^{b'c} \\ + \beta S_{s}^{c'a} \gamma_{\mu} S_{u}^{'a'b} \gamma_{5} S_{b}^{b'c} - \beta S_{s}^{c'b} \gamma_{\mu} S_{b}^{'b'a} \gamma_{5} S_{u}^{a'c} + \gamma_{5} S_{u}^{c'a} \gamma_{\mu} S_{b}^{'a'b} S_{s}^{b'c} \\ - \beta S_{u}^{c'b} \gamma_{\mu} S_{u}^{'b'a} \gamma_{5} S_{b}^{b'c} - \beta S_{s}^{c'a} \gamma_{\mu} S_{b}^{'b'a} \gamma_{5} S_{u}^{s'c} \beta | 0 \rangle,$$

$$(13)$$

where $S' = CS^T C$.

The correlation functions for all other possible transitions can be obtained by the following replacements

$$\begin{split} T^{\Sigma^{*-}_b \to \Sigma^-_b}_\mu &= ~ T^{\Sigma^{*0}_b \to \Sigma^0_b}_b(u \to d), \\ T^{\Sigma^{*+}_b \to \Sigma^+_b}_\mu &= ~ T^{\Sigma^{*0}_b \to \Sigma^0_b}_b(d \to u), \\ T^{\Sigma^{*+}_c \to \Sigma^+_c}_\mu &= ~ T^{\Sigma^{*0}_b \to \Sigma^0_b}_b(b \to c), \\ T^{\Sigma^{*0}_c \to \Sigma^0_c}_\mu &= ~ T^{\Sigma^{*-}_b \to \Sigma^-_b}_b(b \to c), \\ T^{\Sigma^{*++}_c \to \Sigma^{++}}_\mu &= ~ T^{\Sigma^{*++}_b \to \Sigma^+_b}_b(b \to c), \\ T^{\Xi^{*-}_c \to \Xi^-_b}_\mu &= ~ T^{\Xi^{*0}_b \to \Xi^0_b}_b(u \to d), \\ T^{\Xi^{*0}_c \to \Xi^0_c}_\mu &= ~ T^{\Xi^{*-}_b \to \Xi^-_b}_b(b \to c), \end{split}$$

$$T^{\Xi_c^{*+} \to \Xi_c^+}_{\mu} = T^{\Xi_b^{*0} \to \Xi_b^0}_{\mu}(b \to c),
 T^{\Sigma_b^{*0} \to \Lambda_b^0}_{\mu} = T^{\Xi_b^{*0} \to \Xi_b^0}_{\mu}(s \to d),
 T^{\Sigma_c^{*+} \to \Lambda_c^+}_{\mu} = T^{\Sigma_b^{*0} \to \Lambda_b^0}_{\mu}(b \to c).$$
 (14)

The correlators in Eqs. (12, 13) get three different contributions: 1) Perturbative contributions, 2) Mixed contributions, i.e., the photon is radiated from short distance and at least one of the quarks forms a condensate. 3) Non-perturbative contributions, i.e., when photon is radiated at long distances. This contribution is described by the matrix element $\langle \gamma(q) \mid \bar{q}(x_1)\Gamma q(x_2) \mid 0 \rangle$ which is parameterized in terms of photon distribution amplitudes with definite twists.

The results of the contributions when the photon interacts with the quarks perturbatively is obtained by replacing the propagator of the quark that emits the photon by

$$S^{ab}_{\alpha\beta} \Rightarrow \left\{ \int d^4 y S^{free}(x-y) \ \mathcal{A}S^{free}(y) \right\}^{ab}_{\alpha\beta}.$$
 (15)

The free light and heavy quark propagators are defined as:

$$S_{q}^{free} = \frac{i \not z}{2\pi^{2}x^{4}} - \frac{m_{q}}{4\pi^{2}x^{2}},$$

$$S_{Q}^{free} = \frac{m_{Q}^{2}}{4\pi^{2}} \frac{K_{1}(m_{Q}\sqrt{-x^{2}})}{\sqrt{-x^{2}}} - i \frac{m_{Q}^{2} \not z}{4\pi^{2}x^{2}} K_{2}(m_{Q}\sqrt{-x^{2}}),$$
(16)

where K_i are Bessel functions. The non-perturbative contributions to the correlation function can be easily obtained from Eq. (12) replacing one of the light quark propagators that emits a photon by

$$S^{ab}_{\alpha\beta} \to -\frac{1}{4}\bar{q}^a \Gamma_j q^b (\Gamma_j)_{\alpha\beta} , \qquad (17)$$

where Γ is the full set of Dirac matrices $\Gamma_j = \left\{1, \gamma_5, \gamma_\alpha, i\gamma_5\gamma_\alpha, \sigma_{\alpha\beta}/\sqrt{2}\right\}$ and sum over index *j* is implied. Remaining quark propagators are full propagators involving the perturbative as well as the non-perturbative contributions.

The light cone expansion of the light and heavy quark propagators in the presence of an external field is done in [19]. The operators $\bar{q}Gq$, $\bar{q}GGq$ and $\bar{q}q\bar{q}q$, where G is the gluon field strength tensor give contributions to the propagators and in [20], it was shown that terms with two gluons as well as four quarks operators give negligible small contributions, so we neglect them. Taking into account this fact, the expressions for the heavy and light quark propagators are written as:

$$S_{Q}(x) = S_{Q}^{free}(x) - ig_{s} \int \frac{d^{4}k}{(2\pi)^{4}} e^{-ikx} \int_{0}^{1} dv \left[\frac{\not k + m_{Q}}{(m_{Q}^{2} - k^{2})^{2}} G^{\mu\nu}(vx) \sigma_{\mu\nu} \right. \\ \left. + \frac{1}{m_{Q}^{2} - k^{2}} vx_{\mu} G^{\mu\nu} \gamma_{\nu} \right],$$

$$S_{q}(x) = S_{q}^{free}(x) - \frac{m_{q}}{4\pi^{2}x^{2}} - \frac{\langle \bar{q}q \rangle}{12} \left(1 - i\frac{m_{q}}{4} \not x \right) - \frac{x^{2}}{192} m_{0}^{2} \langle \bar{q}q \rangle \left(1 - i\frac{m_{q}}{6} \not x \right) \\ \left. - ig_{s} \int_{0}^{1} du \left[\frac{\not x}{16\pi^{2}x^{2}} G_{\mu\nu}(ux) \sigma_{\mu\nu} - ux^{\mu} G_{\mu\nu}(ux) \gamma^{\nu} \frac{i}{4\pi^{2}x^{2}} \right. \\ \left. - i\frac{m_{q}}{32\pi^{2}} G_{\mu\nu} \sigma^{\mu\nu} \left(\ln \left(\frac{-x^{2}\Lambda^{2}}{4} \right) + 2\gamma_{E} \right) \right],$$
(18)

where Λ is the scale parameter and we choose it at factorization scale i.e., $\Lambda = (0.5 \ GeV - 1 \ GeV)$ (see [9, 21]).

As we already noted, for the calculation of the non-perturbative contributions, the matrix elements $\langle \gamma(q) | \bar{q}\Gamma_i q | 0 \rangle$ are needed. These matrix elements are calculated in terms of the photon distribution amplitudes (DA's) as follows [22].

$$\langle \gamma(q) | \bar{q}(x) \sigma_{\mu\nu} q(0) | 0 \rangle = -i e_q \bar{q} q(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int_0^1 du e^{i\bar{u}qx} \left(\chi \varphi_\gamma(u) + \frac{x^2}{16} \mathbb{A}(u) \right)$$
$$-\frac{i}{2(qx)} e_q \langle \bar{q}q \rangle \left[x_\nu \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) - x_\mu \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \right] \int_0^1 du e^{i\bar{u}qx} h_\gamma(u)$$

$$\begin{split} &\langle \gamma(q) | \bar{q}(x) \gamma_{\mu} q(0) | 0 \rangle = e_q f_{3\gamma} \left(\varepsilon_{\mu} - q_{\mu} \frac{\varepsilon x}{qx} \right) \int_0^1 du e^{i\bar{u}qx} \psi^v(u) \\ &\langle \gamma(q) | \bar{q}(x) \gamma_{\mu} \gamma_5 q(0) | 0 \rangle = -\frac{1}{4} e_q f_{3\gamma} \epsilon_{\mu\nu\alpha\beta} \varepsilon^\nu q^\alpha x^\beta \int_0^1 du e^{i\bar{u}qx} \psi^a(u) \\ &\langle \gamma(q) | \bar{q}(x) g_s G_{\mu\nu}(vx) q(0) | 0 \rangle = -ie_q \langle \bar{q}q \rangle \left(\varepsilon_{\mu} q_{\nu} - \varepsilon_{\nu} q_{\mu} \right) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{S}(\alpha_i) \\ &\langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu}(vx) \gamma_{\alpha} \gamma_5 q(0) | 0 \rangle = e_q \langle \bar{q}q \rangle \left(\varepsilon_{\mu} q_{\nu} - \varepsilon_{\nu} q_{\mu} \right) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{S}(\alpha_i) \\ &\langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu}(vx) \gamma_{\alpha} \gamma_5 q(0) | 0 \rangle = e_q f_{3\gamma} q_\alpha \left(\varepsilon_{\mu} q_{\nu} - \varepsilon_{\nu} q_{\mu} \right) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{A}(\alpha_i) \\ &\langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu}(vx) i\gamma_{\alpha} q(0) | 0 \rangle = e_q f_{3\gamma} q_\alpha \left(\varepsilon_{\mu} q_{\nu} - \varepsilon_{\nu} q_{\mu} \right) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{V}(\alpha_i) \\ &\langle \gamma(q) | \bar{q}(x) \sigma_{\alpha\beta} g_s G_{\mu\nu}(vx) q(0) | 0 \rangle = e_q \langle \bar{q}q \rangle \left\{ \left[\left(\varepsilon_{\mu} - q_{\mu} \frac{\varepsilon x}{qx} \right) \left(g_{\alpha\nu} - \frac{1}{qx} (q_{\alpha} x_{\mu} + q_{\nu} x_{\alpha} \right) \right) q_{\alpha} \right. \\ &- \left(\varepsilon_{\mu} - q_{\mu} \frac{\varepsilon x}{qx} \right) \left(g_{\beta\mu} - \frac{1}{qx} (q_{\beta} x_{\mu} + q_{\nu} x_{\beta}) \right) q_{\alpha} \\ &- \left(\varepsilon_{\nu} - q_{\nu} \frac{\varepsilon x}{qx} \right) \left(g_{\beta\mu} - \frac{1}{qx} (q_{\beta} x_{\mu} + q_{\mu} x_{\beta}) \right) q_{\mu} \\ &- \left(\varepsilon_{\alpha} - q_{\alpha} \frac{\varepsilon x}{qx} \right) \left(g_{\mu\beta} - \frac{1}{qx} (q_{\mu} x_{\beta} + q_{\beta} x_{\mu}) \right) q_{\mu} \\ &- \left(\varepsilon_{\alpha} - q_{\alpha} \frac{\varepsilon x}{qx} \right) \left(g_{\mu\alpha} - \frac{1}{qx} (q_{\mu} x_{\beta} + q_{\beta} x_{\mu}) \right) q_{\nu} \\ &+ \left(\varepsilon_{\beta} - q_{\beta} \frac{\varepsilon x}{qx} \right) \left(g_{\mu\alpha} - \frac{1}{qx} (q_{\mu} x_{\alpha} + q_{\alpha} x_{\mu}) \right) q_{\mu} \\ &- \left(\varepsilon_{\beta} - q_{\beta} \frac{\varepsilon x}{qx} \right) \left(g_{\mu\alpha} - \frac{1}{qx} (q_{\mu} x_{\alpha} + q_{\alpha} x_{\mu}) \right) q_{\mu} \\ &+ \left(\varepsilon_{\beta} - q_{\beta} \frac{\varepsilon x}{qx} \right) \left(g_{\mu\alpha} - \frac{1}{qx} (q_{\mu} x_{\alpha} + q_{\alpha} x_{\mu}) \right) q_{\mu} \\ &+ \left(\varepsilon_{\beta} - q_{\beta} \frac{\varepsilon x}{qx} \right) \left(g_{\mu\alpha} - \frac{1}{qx} (q_{\mu} x_{\alpha} + q_{\alpha} x_{\mu}) \right) q_{\mu} \\ &+ \left(\varepsilon_{\alpha} - q_{\alpha} \frac{\varepsilon x}{qx} \right) \left(g_{\mu\alpha} - \frac{1}{qx} (q_{\mu} x_{\alpha} + q_{\alpha} x_{\mu}) \right) q_{\mu} \\ &+ \left(\varepsilon_{\alpha} - q_{\beta} \frac{\varepsilon x}{qx} \right) \left(g_{\alpha\mu} - \frac{1}{qx} (q_{\mu} x_{\alpha} + q_{\alpha} x_{\mu}) \right) q_{\mu} \\ &+ \left(\varepsilon_{\alpha} - q_{\beta} \frac{\varepsilon x}{qx} \right) \left(g_{\alpha\mu} - \varepsilon_{\alpha} - \varepsilon_{\alpha} q_{\alpha} \varepsilon^{\alpha} \right) \int \mathcal{D}\alpha_i e^{i(\alpha q + v\alpha q_{\beta})qx} \mathcal{T}_{\alpha$$

where $\varphi_{\gamma}(u)$ is the leading twist 2, $\psi^{v}(u)$, $\psi^{a}(u)$, \mathcal{A} and \mathcal{V} are the twist 3 and $h_{\gamma}(u)$, \mathbb{A} , \mathcal{T}_{i} (i = 1, 2, 3, 4) are the twist 4 photon DA's, respectively and χ

is the magnetic susceptibility of the quarks. The photon DA's is calculated in [22]. The measure $\mathcal{D}\alpha_i$ is defined as

$$\int \mathcal{D}\alpha_i = \int_0^1 d\alpha_{\bar{q}} \int_0^1 d\alpha_q \int_0^1 d\alpha_g \delta(1 - \alpha_{\bar{q}} - \alpha_q - \alpha_g).$$
(20)

The coefficient of any structure in the expression of the correlation function can be written in the form

$$T(q_1, q_2, Q) = e_{q_1} T_1(q_1, q_2, Q) + e_{q_2} T_1'(q_1, q_2, Q) + e_Q T_2(q_1, q_2, Q),$$
(21)

where T_1, T'_1 and T_2 in the right side correspond to the radiation of the photon from the quarks q_1, q_2 and Q, respectively. Starting from the expressions of the interpolating currents, one can easily obtain that the functions T_1 and T'_1 differ only by $q_1 \leftrightarrow q_2$ exchange for $\Sigma_Q^* \to \Sigma_Q$ transition. Using Eq. (14), in SU(2) symmetry limit, we obtain

$$T^{\Sigma_{b(c)}^{*+(++)} \to \Sigma_{b(c)}^{+(++)}} + T^{\Sigma_{b(c)}^{*-(0)} \to \Sigma_{b(c)}^{-(0)}} = 2T^{\Sigma_{b(c)}^{*0(+)} \to \Sigma_{b(c)}^{0(+)}}.$$
(22)

Therefore, in the numerical analysis section, we have not presented the numerical results for $T^{\Sigma_{b(c)}^{*-(0)} \to \Sigma_{b(c)}^{-(0)}}$.

Now, using the above equations, one can get the the correlation function from the QCD side. Separating the coefficients of the structures $\not \in \not p\gamma_5 q_\mu$ and $\not \ll \not p\gamma_5(\varepsilon p)q_\mu$ respectively for the form factors G_1 and G_2 from both QCD and phenomenological representations and equating them, we get sum rules for the form factors G_1 and G_2 . To suppress the contributions of the higher states and continuum, Borel transformations with respect to the variables p^2 and $(p+q)^2$ are applied. The explicit forms of the sum rules for the form factors G_1 and G_2 can be as follows. For $\Sigma_b^{*0} \to \Sigma_b^0$, we obtain

$$G_{1} = -\frac{1}{\lambda_{1}\lambda_{2}(m_{1}+m_{2})}e^{\frac{m_{1}^{2}}{M_{1}^{2}}}e^{\frac{m_{2}^{2}}{M_{2}^{2}}}\left[e_{q_{1}}\Pi_{1}+e_{q_{2}}\Pi_{1}(q_{1}\leftrightarrow q_{2})+e_{b}\Pi_{1}'\right]$$

$$G_{2} = \frac{1}{\lambda_{1}\lambda_{2}}e^{\frac{m_{1}^{2}}{M_{1}^{2}}}e^{\frac{m_{2}^{2}}{M_{2}^{2}}}\left[e_{q_{1}}\Pi_{2}+e_{q_{2}}\Pi_{2}(q_{1}\leftrightarrow q_{2})+e_{b}\Pi_{2}'\right],$$
(23)

where $q_1 = u$, $q_2 = d$. The functions $\prod_i [\prod'_i]$ can be written as:

$$\Pi_{i}[\Pi_{i}'] = \int_{m_{Q}^{2}}^{s_{0}} e^{\frac{-s}{M^{2}}} \rho_{i}(s) [\rho_{i}'(s)] ds + e^{\frac{-m_{Q}^{2}}{M^{2}}} \Gamma_{i}[\Gamma_{i}'], \qquad (24)$$

where,

$$\begin{split} \sqrt{3}\rho_{1}(s) &= \langle q_{1}q_{1}\rangle\langle q_{2}q_{2}\rangle \left[-\frac{\beta}{3}\chi\varphi_{\gamma}(u_{0}) \right] \\ &+ m_{0}^{2}\langle q_{2}q_{2}\rangle \left[-\frac{(1+\beta)}{144m_{Q}\pi^{2}}(1+\psi_{11}) \right] \\ &+ \langle q_{2}q_{2}\rangle \left[\frac{\beta m_{Q}}{4\pi^{2}}(\psi_{10}-\psi_{21}) \right] \\ &+ \langle q_{1}q_{1}\rangle \frac{m_{Q}}{96\pi^{2}} \left[-3(1+\beta)(\psi_{10}-\psi_{21})\mathbb{A}(u_{0}) \\ &+ 2\left(-2(\psi_{10}-\psi_{21})\{(1+5\beta)\eta_{1}-\eta_{2}-4\eta_{3}-\eta_{4}+6\eta_{5}\right. \\ &+ 2\eta_{6}+3\eta_{7}-2\eta_{8}-\beta(\eta_{2}+\eta_{4}-2\eta_{5}-2\eta_{6}-7\eta_{7}+2\eta_{8}) \} \\ &- 3(3+\beta)(-1+\psi_{02}+2\psi_{10}-2\psi_{21})(u_{0}-1)\zeta_{1}+2\{(1+3\beta)\eta_{1} \\ &- (3+\beta)(\eta_{2}-\eta_{4})+(1+3\beta)\eta_{7}\}ln(\frac{s}{m_{Q}^{2}}) \\ &+ 3(1+\beta)m_{Q}^{2}\chi\{2\psi_{10}-\psi_{20}+\psi_{31}-2ln(\frac{s}{m_{Q}^{2}})\}\varphi_{\gamma}(u_{0}) \right) \right] + \\ &+ \frac{m_{Q}^{4}(\beta-1)}{512\pi^{4}} \left[4\psi_{30}-5\psi_{42}-4\psi_{52}+(4\psi_{30}-8\psi_{41}+5\psi_{42}+4\psi_{52})u_{0} \\ &+ 6\{2\psi_{10}-\psi_{20}-2ln(\frac{s}{m_{Q}^{2}})\}(1+u_{0}) \right] \\ &+ \frac{f_{3\gamma}m_{Q}^{2}(\beta-1)}{192\pi^{2}} \left[8\{-(\psi_{20}-\psi_{31})(\eta_{1}'-\eta_{2}')+[\psi_{10}-ln(\frac{s}{m_{Q}^{2}})](\eta_{3}'+\eta_{1}'-\eta_{2}')\} \\ &+ 6(\psi_{20}-\psi_{31})\psi^{a}(u_{0})+(3\psi_{31}+4\psi_{32}+3\psi_{42})(u_{0}-1)[4\psi^{v}(u_{0})-\frac{d\psi^{a}(u_{0})}{du_{0}}] \right], \end{split}$$

$$\sqrt{3}\rho_{1}'(s) = m_{0}^{2}(\langle q_{1}q_{1}\rangle + \langle q_{2}q_{2}\rangle) \left[\frac{5(1+3\beta)(1+\psi_{11})}{144\pi^{2}m_{Q}}\right]$$

$$+ (\langle q_{1}q_{1} \rangle + \langle q_{2}q_{2} \rangle) \frac{m_{Q}(1+3\beta)}{8\pi^{2}} \left[\psi_{10} - \psi_{21} - ln(\frac{s}{m_{Q}^{2}}) \right] + \frac{m_{Q}^{4}(1-\beta)}{768\pi^{4}} \left[2(-3\psi_{20} - 12\psi_{30} + 18\psi_{31} + 8\psi_{32} + 18\psi_{41}) - 3(\psi_{42} + 4\psi_{52}) \right] + \left\{ 2(3\psi_{20} + 6\psi_{30} - 18\psi_{31} - 8\psi_{32}) + 3(-8\psi_{41} + \psi_{42} + 4\psi_{52}) \right\} u_{0} - 12\psi_{10}(7+5u_{0}) - 72(\psi_{10} + u_{0})ln(\frac{m_{Q}^{2}}{s}) + 12\left\{ 7 + 2\psi_{10}(-1+u_{0}) - u_{0} \right\} ln(\frac{s}{m_{Q}^{2}}) \right],$$
(25)

$$\begin{split} \sqrt{3}\rho_{2}(s) &= \langle q_{1}q_{1}\rangle \frac{1}{12m_{Q}\pi^{2}} \left[3(3+\beta)(-1+\psi_{03}+2\psi_{12}+\psi_{22})(u_{0}-1)\zeta_{2} \right. \\ &+ 2\left(-2(1+\beta)+(\beta-1)\psi_{11}+2(1+3\beta)(\psi_{12}+\psi_{22}) \right. \\ &- \frac{3+\beta}{2}(-1+2\psi_{12}+\psi_{22})(\zeta_{9}-\zeta_{5}) \right) \right] + \\ &+ \frac{m_{Q}^{2}(\beta-1)}{128\pi^{4}} \left[(12\psi_{31}+19\psi_{32}+\psi_{33}+12\psi_{42})(u_{0}-1)u_{0} \right] \\ &+ \frac{f_{3\gamma}(\beta-1)}{96\pi^{2}} \left[4\{(1-\psi_{02})\eta_{9}+(-3+3\psi_{02}+2\psi_{10}-2\psi_{21})\eta_{11} \right. \\ &- (-1+\psi_{02}+2\psi_{10}-2\psi_{21})\eta_{10}+(-1+3\psi_{02}-2\psi_{03})(u_{0}-1)\zeta_{11} \right\} \\ &- (-1+3\psi_{02}-2\psi_{03})(u_{0}-1)\psi^{a}(u_{0}) \right], \end{split}$$

$$\sqrt{3}\rho_{2}'(s) = \frac{m_{Q}^{2}(\beta-1)}{192\pi^{4}} \left[(-1+u_{0})u_{0} \{12\psi_{10}-6\psi_{20} + 24\psi_{31}+27\psi_{32}+\psi_{33}+18\psi_{42}-12ln(\frac{s}{m_{Q}^{2}})\} \right], \quad (26)$$

$$\sqrt{3}\Gamma_{1} = m_{0}^{2} < q_{1}q_{1} > < q_{2}q_{2} > \left[\frac{\beta m_{Q}^{4}\mathbb{A}(u_{0})}{48M^{6}} + \frac{m_{Q}^{2}}{432M^{4}}\{12[\eta_{2} - 2\eta_{3} - \eta_{4} + 2\eta_{5} + \beta(2\eta_{1} + \eta_{2} - 2\eta_{3} + \eta_{4} - 2\eta_{6} + 2\eta_{8}) + 3(u_{0} - 1)\zeta_{1}] - \beta\mathbb{A}(u_{0})\}$$

$$+ \frac{5\beta}{54}\chi\varphi_{\gamma}(u_{0}) + \frac{1}{108M^{2}}\left\{(1-u_{0})(\beta-5)\zeta_{1} + 9\beta m_{Q}^{2}\chi\varphi_{\gamma}(u_{0})\right\} \\ + < q_{1}q_{1} > < q_{2}q_{2} > \left[\frac{\beta m_{Q}^{2}\mathbb{A}(u_{0})}{12M^{2}} + \frac{1}{36}\left\{-4[\eta_{2}-2\eta_{3}\right] \\ - \eta_{4} + 2\eta_{5} + \beta(2\eta_{1}+\eta_{2}-2\eta_{3}+\eta_{4}-2\eta_{6}+2\eta_{8}) + 3(u_{0}-1)\zeta_{1}\right] + 3\beta\mathbb{A}(u_{0})\right\} \\ + m_{0}^{2} < q_{2}q_{2} > \left[\frac{-\beta m_{Q}}{16\pi^{2}} + \left\{\frac{\beta m_{Q}^{3}}{24M^{4}} + \frac{(1+\beta)m_{Q}}{216M^{2}}\right\}f_{3\gamma}\psi^{a}(u_{0})\right] \\ + < q_{2}q_{2} > \left[\frac{-\beta}{6}f_{3\gamma}m_{Q}\psi^{a}(u_{0})\right],$$

$$(27)$$

$$\sqrt{3}\Gamma_1' = m_0^2 < q_1 q_1 > < q_2 q_2 > \left[\frac{-\beta m_Q^2}{3M^4}\right] + < q_1 q_1 > < q_2 q_2 > \left[\frac{2\beta}{3}\right],$$

$$\sqrt{3}\Gamma_{2} = m_{0}^{2} < q_{1}q_{1} > < q_{2}q_{2} > \left[-\frac{1}{54M^{4}}(u_{0}-1)(\beta-5)\zeta_{2} + \frac{m_{Q}^{2}}{18M^{6}} \{3(u_{0}-1)\zeta_{2} - \beta(\zeta_{5}-\zeta_{9})\} \right] + < q_{1}q_{1} > < q_{2}q_{2} > \left[\frac{2}{9M^{2}} \{-3(u_{0}-1)\zeta_{2} + \beta(\zeta_{5}-\zeta_{9})\} \right],$$
(28)

$$\Gamma_2' = 0. \tag{29}$$

For $\Xi_b^{*0} \to \Xi_b^0$, we have

$$G_{1} = -\frac{1}{\lambda_{1}\lambda_{2}(m_{1}+m_{2})}e^{\frac{m_{1}^{2}}{M_{1}^{2}}}e^{\frac{m_{2}^{2}}{M_{2}^{2}}}\left[e_{q_{1}}\Theta_{1}-e_{q_{2}}\Theta_{1}(q_{1}\leftrightarrow q_{2})+e_{b}\Theta_{1}'\right]$$

$$G_{2} = \frac{1}{\lambda_{1}\lambda_{2}}e^{\frac{m_{1}^{2}}{M_{1}^{2}}}e^{\frac{m_{2}^{2}}{M_{2}^{2}}}\left[e_{q_{1}}\Theta_{2}-e_{q_{2}}\Theta_{2}(q_{1}\leftrightarrow q_{2})+e_{b}\Theta_{2}'\right],$$
(30)

where $q_1 = u$, $q_2 = s$. The functions $\Theta_i[\Theta'_i]$ are also defined as:

$$\Theta_{i}[\Theta_{i}'] = \int_{m_{Q}^{2}}^{s_{0}} e^{\frac{-s}{M^{2}}} \varrho_{i}(s)[\varrho_{i}'(s)]ds + e^{\frac{-m_{Q}^{2}}{M^{2}}} \Delta_{i}[\Delta_{i}'], \qquad (31)$$

and

$$\begin{split} \varrho_{1}(s) &= \langle q_{1}q_{1} \rangle \langle q_{2}q_{2} \rangle \left[\frac{1+2\beta}{9} \chi \varphi_{\gamma}(u_{0}) \right] \\ &+ m_{0}^{2} \langle q_{2}q_{2} \rangle \left[\frac{(1+\beta)}{144m_{Q}\pi^{2}} (1+\psi_{11}) \right] \\ &+ \langle q_{2}q_{2} \rangle \left[\frac{\beta m_{Q}}{4\pi^{2}} (\psi_{21} - \psi_{10}) \right] \\ &+ \langle q_{1}q_{1} \rangle \frac{m_{Q}}{288\pi^{2}} \left[3(1+5\beta)(\psi_{10} - \psi_{21}) \mathbb{A}(u_{0}) \right. \\ &+ 2 \left(-2(\psi_{10} - \psi_{21}) \{(-1+7\beta)\eta_{1} + \eta_{2} - 4\eta_{3} + \eta_{4} + 2\eta_{5} \right. \\ &- 2\eta_{6} - 3\eta_{7} + 2\eta_{8} + \beta(5\eta_{2} - 8\eta_{3} + 5\eta_{4} - 2\eta_{5} - 10\eta_{6} - 3\eta_{7} + 10\eta_{8}) \} \\ &+ 3(-1+\beta)(-1+\psi_{02} + 2\psi_{10} - 2\psi_{21})(u_{0} - 1)\zeta_{1} \\ &+ 2(-1+\beta)(\eta_{1} + \eta_{2} - \eta_{4} + \eta_{7})ln(\frac{s}{m_{Q}^{2}}) \\ &- 3(1+5\beta)m_{Q}^{2}\chi\{2\psi_{10} - \psi_{20} + \psi_{31} - 2ln(\frac{s}{m_{Q}^{2}})\}\varphi_{\gamma}(u_{0}) \right) \right] + \\ &+ \frac{m_{Q}^{4}(\beta-1)}{512\pi^{4}} \left[6\psi_{20} - 4\psi_{30} + 5\psi_{42} + 4\psi_{52} + (6\psi_{20} - 4\psi_{30} \\ &+ 8\psi_{41} - 5\psi_{42} - 4\psi_{52})u_{0} - 12\{\psi_{10} - ln(\frac{s}{m_{Q}^{2}})\}(1+u_{0}) \right] \\ &- \frac{f_{3\gamma}m_{Q}^{2}(\beta-1)}{576\pi^{2}} \left[8\{(\psi_{20} - \psi_{31})(\eta_{1}' - \eta_{2}') + [\psi_{10} - ln(\frac{s}{m_{Q}^{2}})](3\eta_{3}' - \eta_{1}' + \eta_{2}')\} \\ &+ 18(\psi_{20} - \psi_{31})\psi^{a}(u_{0}) + 3(3\psi_{31} + 4\psi_{32} + 3\psi_{42})(u_{0} - 1)[4\psi^{v}(u_{0}) - \frac{d\psi^{a}(u_{0})}{du_{0}}] \right], \end{split}$$
(32)

$$\begin{aligned} \varrho_1'(s) &= m_0^2(\langle q_1 q_1 \rangle - \langle q_2 q_2 \rangle) \left[\frac{5(\beta - 1)(1 + \psi_{11})}{432\pi^2 m_Q} \right] \\ &+ (\langle q_1 q_1 \rangle - \langle q_2 q_2 \rangle) \frac{m_Q(\beta - 1)}{24\pi^2} \left[\psi_{10} - \psi_{21} - \ln(\frac{s}{m_Q^2}) \right] \end{aligned}$$

+
$$\frac{m_Q^3 m_{q_2}(\beta - 1)}{128\pi^4} \left[6\psi_{10} - \psi_{20} + \psi_{31} - 2\{3 + \psi_{10}\}ln(\frac{s}{m_Q^2}) \right],$$
(33)

$$\begin{split} \varrho_2(s) &= \langle q_1 q_1 \rangle \frac{1-\beta}{36m_Q \pi^2} \left[3(-1+\psi_{03}+2\psi_{12}+\psi_{22})(u_0-1)\zeta_2 \\ &+ 2\psi_{11}(2\zeta_3+\zeta_5-2\zeta_7-\zeta_9)+2\{\frac{1}{2}(\zeta_9-\zeta_5) \\ &+ (2\psi_{12}+\psi_{22})[2\zeta_3+\frac{3}{2}(\zeta_5-\zeta_9)-2\zeta_7]\} \right] + \\ &- \frac{m_Q^2(\beta-1)}{128\pi^4} \left[(12\psi_{31}+19\psi_{32}+\psi_{33}+12\psi_{42})(u_0-1)u_0 \right] \\ &- \frac{f_{3\gamma}(\beta-1)}{288\pi^2} \left[4\{(-1+\psi_{02}+4\psi_{10}-4\psi_{21})\eta_9-3(-1+\psi_{02}+2\psi_{10}-2\psi_{21})\eta_{11} \\ &+ (-1+\psi_{02}-2\psi_{10}+2\psi_{21})\eta_{10}+3(-1+3\psi_{02}-2\psi_{03})(u_0-1)\zeta_{11} \} \\ &- 3(-1+3\psi_{02}-2\psi_{03})(u_0-1)\psi^a(u_0) \right], \end{split}$$

$$\varrho_2'(s) = 0,$$
(34)

$$\begin{split} \Delta_1 &= m_0^2 < q_1 q_1 > < q_2 q_2 > \left[\frac{(1+2\beta)m_Q^4 \mathbb{A}(u_0)}{144M^6} + \frac{m_Q^2}{1296M^4} \{ 12[(1-\beta)(\eta_2 - 2\eta_3) \\ &- 3(1+\beta)\eta_4 + 2(2+\beta)\eta_5 + 2(1+2\beta)(\eta_6 + \eta_7 - \eta_8) + 3(2+\beta)(u_0 - 1)\zeta_1] \\ &+ (1+2\beta)\mathbb{A}(u_0) \} - \frac{5(1+2\beta)}{162}\chi\varphi_\gamma(u_0) \\ &+ \frac{1}{108M^2} \{ (-1+u_0)(5+3\beta)\zeta_1 - 3(1+2\beta)m_Q^2\chi\varphi_\gamma(u_0) \} \right] \\ &+ < q_1 q_1 > < q_2 q_2 > \left[-\frac{(1+2\beta)m_Q^2\mathbb{A}(u_0)}{36M^2} + \frac{1}{108} \{ 4[(\beta-1)\eta_2 + 2\eta_3 + 3\eta_4 - 2(2\eta_5 + \eta_6 + \eta_7 - \eta_8 - 3\zeta_1) - 6u_0\zeta_1 \\ &- \beta(2\eta_3 - 3\eta_4 + 2\eta_5 + 4(\eta_6 + \eta_7 - \eta_8) + 3(u_0 - 1)\zeta_1)] - 3(1+2\beta)\mathbb{A}(u_0) \} \right] \\ &+ m_0^2 < q_2 q_2 > \left[\frac{\beta m_Q}{16\pi^2} - \{ \frac{\beta m_Q^3}{24M^4} + \frac{(1+\beta)m_Q}{216M^2} \} f_{3\gamma}\psi^a(u_0) \right] \end{split}$$

$$+ \quad < q_2 q_2 > \left[\frac{\beta}{6} f_{3\gamma} m_Q \psi^a(u_0)\right], \tag{35}$$

$$\begin{split} \Delta_1' &= m_0^2 < q_1 q_1 > < q_2 q_2 > \frac{5(\beta - 1)}{432} \left[\frac{m_Q^3 m_{q_2}}{M^6} - \frac{m_Q m_{q_2}}{M^4} \right] \\ &- < q_1 q_1 > < q_2 q_2 > \left[\frac{(\beta - 1) m_Q m_{q_2}}{36M^2} \right], \end{split}$$

$$\Delta_{2} = m_{0}^{2} < q_{1}q_{1} > < q_{2}q_{2} > \left[\frac{1}{54M^{4}}(u_{0}-1)(3\beta+5)\zeta_{2} + \frac{m_{Q}^{2}}{54M^{6}}\{3(2+\beta)(u_{0}-1)\zeta_{2} - (1+2\beta)(\zeta_{5}-\zeta_{9})\}\right] + < q_{1}q_{1} > < q_{2}q_{2} > \left[\frac{-2}{27M^{2}}\{3(2+\beta)(u_{0}-1)\zeta_{2} - (1+2\beta)(\zeta_{5}-\zeta_{9})\}\right],$$
(36)

$$\Delta_2' = 0. \tag{37}$$

Note that, in the above equations, the terms proportional to m_s are omitted because of their lengthy expressions, but they have been taken into account in numerical calculations. The contributions of the terms $\sim < G^2 >$ are also calculated, but their numerical values are very small and therefore for customary in the expressions these terms are also omitted. The functions entering Eqs. (25-37) are given as

$$\eta_{i} = \int \mathcal{D}\alpha_{i} \int_{0}^{1} dv f_{i}(\alpha_{i}) \delta(\alpha_{q} + v\alpha_{g} - u_{0}),$$

$$\eta_{i}' = \int \mathcal{D}\alpha_{i} \int_{0}^{1} dv g_{i}(\alpha_{i}) \delta'(\alpha_{q} + v\alpha_{g} - u_{0}),$$

$$\zeta_{i} = \int_{u_{0}}^{1} h_{i}(u) du \quad (i = 1, 2, 11),$$

$$\zeta_{i} = \int \mathcal{D}\alpha_{i} \int_{0}^{1} d\bar{v} h_{i}(\alpha_{i}) \theta(\alpha_{q} + v\alpha_{g} - u_{0}) \quad (i = 3 - 10),$$

$$\psi_{nm} = \frac{(s - m_Q^2)^n}{s^m (m_Q^2)^{n-m}},$$
(38)

and $f_1(\alpha_i) = S(\alpha_i), f_2(\alpha_i) = \tilde{S}(\alpha_i), f_3(\alpha_i) = v\tilde{S}(\alpha_i), f_4(\alpha_i) = h_5(\alpha_i) = \mathcal{T}_2(\alpha_i), f_5(\alpha_i) = h_6(\alpha_i) = v\mathcal{T}_2(\alpha_i), f_6(\alpha_i) = h_8(\alpha_i) = v\mathcal{T}_3(\alpha_i), f_7(\alpha_i) = h_9(\alpha_i) = \mathcal{T}_4(\alpha_i), f_8(\alpha_i) = h_{10}(\alpha_i) = v\mathcal{T}_4(\alpha_i), f_9(\alpha_i) = \mathcal{A}(\alpha_i), f_{10}(\alpha_i) = g_3(\alpha_i) = v\mathcal{A}(\alpha_i), f_{11}(\alpha_i) = g_2(\alpha_i) = v\mathcal{V}(\alpha_i), g_1(\alpha_i) = \mathcal{V}(\alpha_i), h_1(u) = h_{\gamma}(u), h_2(u) = (u-u_0)h_{\gamma}(u), h_3(\alpha_i) = \mathcal{T}_1(\alpha_i), h_4(\alpha_i) = v\mathcal{T}_1(\alpha_i), h_7(\alpha_i) = \mathcal{T}_3(\alpha_i) \text{ and } h_{11}(u) = \psi^v(u) \text{ are functions in terms of the photon distribution amplitudes.}$ Note that in the above equations, the Borel parameter M^2 is defined as $M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}$ and $u_0 = \frac{M_1^2}{M_1^2 + M_2^2}$. Since the masses of the initial and final baryons are very close to each other, we set $M_1^2 = M_2^2 = 2M^2$ and $u_0 = 1/2$.

From Eqs. (23) and (30) it is clear that for the calculation of the transition magnetic dipole and electric quadrupole moments, the expressions for the residues λ_1 and λ_2 are needed. These residues are determined from two point sum rules. For the chosen interpolating currents, we get the following results for the residues λ_1 and λ_2 (see also [9, 23]):

$$\lambda_1^2 e^{\frac{-m_1^2}{M^2}} = A^2 \left[\Pi' + \Pi'(q_1 \longleftrightarrow q_2) \right], \tag{39}$$

where

$$\Pi' = \int_{m_Q^2}^{s_0} ds \ e^{-s/M^2} \left\{ m_0^2 < q_1 q_1 > \left[\frac{(m_{q_1} - 6m_Q)(\psi_{22} + 2\psi_{12} - 1)}{192m_Q^2 \pi^2} \right] - < q_1 q_1 > \left[\frac{2(\psi_{02} + 2\psi_{10} - 2\psi_{21} - 1)m_Q + (\psi_{02} - 1)(3m_{q_1} - 2m_{q_2})}{32\pi^2} \right] - \frac{m_Q^3}{512\pi^4} \left[-8(3\psi_{31} + 2\psi_{32})m_{q_1} + 3\{(2\psi_{30} - 4\psi_{41} + \frac{5}{2}\psi_{42} + 2\psi_{52})m_Q - 4\psi_{42}m_{q_1}\} + 6(2\psi_{10} - \psi_{20})(\frac{3}{2}m_Q - 2m_{q_1}) - 6(4m_{q_1} - 3m_Q)ln(\frac{m_Q^2}{s}) \right] \right\}$$

$$+ e^{-m_Q^2/M^2} \left\{ m_0^2 < q_1 q_1 > < q_2 q_2 > \left[\frac{-5m_Q^3 m_{q_1}}{144M^6} + \frac{m_Q(m_Q + 5m_{q_1})}{24M^4} - \frac{5}{24M^2} \right] + < q_1 q_1 > < q_2 q_2 > \left[\frac{1}{12} - \frac{m_Q m_{q_1}}{12M^2} \right] + m_0^2 < q_1 q_1 > \left[\frac{6m_{q_2} - 7m_{q_1}}{192\pi^2} \right] \right\}, \quad (40)$$

$$-\lambda_{2\Sigma_Q(\Xi_Q)}^2 e^{-m_{\Sigma_Q(\Xi_Q)}^2/M^2} = \int_{m_Q^2}^{s_0} e^{\frac{-s}{M^2}} \sigma_{\Sigma_Q(\Xi_Q)}(s) ds + e^{\frac{-m_Q^2}{M^2}} \omega_{\Sigma_Q(\Xi_Q)}, (41)$$

with

$$\sigma_{\Sigma_Q}(s) = (\langle \overline{d}d \rangle + \langle \overline{u}u \rangle) \frac{(\beta^2 - 1)}{64\pi^2} \left\{ \frac{m_0^2}{4m_Q} (6\psi_{00} - 13\psi_{02} - 6\psi_{11}) + 3m_Q (2\psi_{10} - \psi_{11} - \psi_{12} + 2\psi_{21}) \right\} + \frac{m_Q^4}{2048\pi^4} [5 + \beta(2 + 5\beta)] [12\psi_{10} - 6\psi_{20} + 2\psi_{30} - 4\psi_{41} + \psi_{42} - 12ln(\frac{s}{m_Q^2})],$$

$$(42)$$

$$\sigma_{\Xi_Q}(s) = (\langle \overline{s}s \rangle + \langle \overline{u}u \rangle) \frac{(\beta - 1)}{192\pi^2} \left\{ \frac{m_0^2}{4m_Q} [6(1 + \beta)\psi_{00} - (7 + 11\beta)\psi_{02} - 6(1 + \beta)\psi_{11}] + (1 + 5\beta)m_Q(2\psi_{10} - \psi_{11} - \psi_{12} + 2\psi_{21}) \right\} + \frac{m_Q^4}{2048\pi^4} [5 + \beta(2 + 5\beta)] [12\psi_{10} - 6\psi_{20} + 2\psi_{30} - 4\psi_{41} + \psi_{42} - 12ln(\frac{s}{m_Q^2})],$$

$$(43)$$

$$\begin{aligned}
\omega_{\Sigma_Q} &= \frac{(\beta - 1)^2}{24} < \overline{d}d > < \overline{u}u > \left[\frac{m_Q^2 m_0^2}{2M^4} + \frac{m_0^2}{4M^2} - 1\right], \\
\omega_{\Xi_Q} &= \frac{(\beta - 1)}{72} < \overline{s}s > < \overline{u}u > \left[\frac{m_Q^2 m_0^2}{2M^4} (13 + 11\beta) + \frac{m_0^2}{4M^2} (25 + 23\beta) - (13 + 11\beta)\right],
\end{aligned}$$
(44)

$$\lambda_{2\Lambda_Q} = \lambda_{2\Xi_Q}(s \to d). \tag{45}$$

In the expression for $\lambda_{2\Xi_Q}$ also the terms proportional to the strange quark mass are taken into account in our numerical calculations, but omitted in the above formulas.

3 Numerical analysis

This section is devoted to the numerical analysis for the magnetic dipole G_M and electric quadrupole G_E as well as the calculation of the decay rates for the considered radiative transitions. The input parameters used in the analysis of the sum rules are taken to be: $\langle \bar{u}u \rangle (1 \ GeV) = \langle \bar{d}d \rangle (1 \ GeV) =$ $-(0.243)^3 \ GeV^3$, $\beta(1 \ GeV) = 0.8 \langle \bar{u}u \rangle (1 \ GeV)$, $m_0^2(1 \ GeV) = (0.8 \pm 0.2) \ GeV^2$ [24], $\Lambda = (0.5 - 1) \ GeV$ and $f_{3\gamma} = -0.0039 \ GeV^2$ [22]. The value of the magnetic susceptibility was obtained in different papers as $\chi(1 \ GeV) = -3.15 \pm$ $0.3 \ GeV^{-2}$ [22], $\chi(1 \ GeV) = -(2.85 \pm 0.5) \ GeV^{-2}$ [25] and $\chi(1 \ GeV) =$ $-4.4 \ GeV^{-2}$ [26]. From sum rules for the magnetic dipole G_M and electric quadrupole G_E , it is clear that we also need to know the explicit form of the photon DA's [22]:

$$\begin{split} \varphi_{\gamma}(u) &= 6u\bar{u}\left(1+\varphi_{2}(\mu)C_{2}^{\frac{3}{2}}(u-\bar{u})\right), \\ \psi^{v}(u) &= 3\left(3(2u-1)^{2}-1\right) + \frac{3}{64}\left(15w_{\gamma}^{V}-5w_{\gamma}^{A}\right)\left(3-30(2u-1)^{2}+35(2u-1)^{4}\right), \\ \psi^{a}(u) &= \left(1-(2u-1)^{2}\right)\left(5(2u-1)^{2}-1\right)\frac{5}{2}\left(1+\frac{9}{16}w_{\gamma}^{V}-\frac{3}{16}w_{\gamma}^{A}\right), \\ \mathcal{A}(\alpha_{i}) &= 360\alpha_{q}\alpha_{\bar{q}}\alpha_{g}^{2}\left(1+w_{\gamma}^{A}\frac{1}{2}(7\alpha_{g}-3)\right), \\ \mathcal{V}(\alpha_{i}) &= 540w_{\gamma}^{V}(\alpha_{q}-\alpha_{\bar{q}})\alpha_{q}\alpha_{\bar{q}}\alpha_{g}^{2}, \\ h_{\gamma}(u) &= -10\left(1+2\kappa^{+}\right)C_{2}^{\frac{1}{2}}(u-\bar{u}), \\ \mathbb{A}(u) &= 40u^{2}\bar{u}^{2}\left(3\kappa-\kappa^{+}+1\right) \\ &+8(\zeta_{2}^{+}-3\zeta_{2})\left[u\bar{u}(2+13u\bar{u})\right] \end{split}$$

$$+ 2u^{3}(10 - 15u + 6u^{2})\ln(u) + 2\bar{u}^{3}(10 - 15\bar{u} + 6\bar{u}^{2})\ln(\bar{u})],$$

$$\mathcal{T}_{1}(\alpha_{i}) = -120(3\zeta_{2} + \zeta_{2}^{+})(\alpha_{\bar{q}} - \alpha_{q})\alpha_{\bar{q}}\alpha_{q}\alpha_{g},$$

$$\mathcal{T}_{2}(\alpha_{i}) = 30\alpha_{g}^{2}(\alpha_{\bar{q}} - \alpha_{q})\left((\kappa - \kappa^{+}) + (\zeta_{1} - \zeta_{1}^{+})(1 - 2\alpha_{g}) + \zeta_{2}(3 - 4\alpha_{g})\right),$$

$$\mathcal{T}_{3}(\alpha_{i}) = -120(3\zeta_{2} - \zeta_{2}^{+})(\alpha_{\bar{q}} - \alpha_{q})\alpha_{\bar{q}}\alpha_{q}\alpha_{g},$$

$$\mathcal{T}_{4}(\alpha_{i}) = 30\alpha_{g}^{2}(\alpha_{\bar{q}} - \alpha_{q})\left((\kappa + \kappa^{+}) + (\zeta_{1} + \zeta_{1}^{+})(1 - 2\alpha_{g}) + \zeta_{2}(3 - 4\alpha_{g})\right),$$

$$\mathcal{S}(\alpha_{i}) = 30\alpha_{g}^{2}\{(\kappa + \kappa^{+})(1 - \alpha_{g}) + (\zeta_{1} + \zeta_{1}^{+})(1 - \alpha_{g})(1 - 2\alpha_{g}) + \zeta_{2}[3(\alpha_{\bar{q}} - \alpha_{q})^{2} - \alpha_{g}(1 - \alpha_{g})]\},$$

$$\tilde{\mathcal{S}}(\alpha_{i}) = -30\alpha_{g}^{2}\{(\kappa - \kappa^{+})(1 - \alpha_{g}) + (\zeta_{1} - \zeta_{1}^{+})(1 - \alpha_{g})(1 - 2\alpha_{g}) + \zeta_{2}[3(\alpha_{\bar{q}} - \alpha_{q})^{2} - \alpha_{g}(1 - \alpha_{g})]\}.$$
(46)

The constants entering the above DA's are obtained as [22] $\varphi_2(1 \ GeV) = 0$, $w_{\gamma}^V = 3.8 \pm 1.8$, $w_{\gamma}^A = -2.1 \pm 1.0$, $\kappa = 0.2$, $\kappa^+ = 0$, $\zeta_1 = 0.4$, $\zeta_2 = 0.3$, $\zeta_1^+ = 0$ and $\zeta_2^+ = 0$.

The sum rules for the electromagnetic form factors also contain three auxiliary parameters: the Borel mass parameter M^2 , the continuum threshold s_0 and the arbitrary parameter β entering the expression of the interpolating currents of the heavy spin 1/2 baryons. The measurable physical quantities, i.e. the magnetic dipole and electric quadrupole moments, should be independent of them. Therefore, we look for regions for these auxiliary parameters such that in these regions the G_M and G_E are practically independent of them. The working region for M^2 are found requiring that not only the contributions of the higher states and continuum should be less than the ground state contribution, but the highest power of $1/M^2$ be less than say $30^0/_0$ of the highest power of M^2 . These two conditions are both satisfied in the region 15 $GeV^2 \leq M^2 \leq 30 \ GeV^2$ and 6 $GeV^2 \leq M^2 \leq 12 \ GeV^2$ for baryons containing b and c-quark, respectively. The working regions for the continuum threshold s_0 and the general parameter β are obtained considering the fact that the results of the physical quantities are approximately unchanged. We also would like to note that in Figs. 1-40, the absolute values are plotted since it is not possible to calculate the sign of the residue from the mass sum-rules. Hence, it is not possible to predict the signs of the G_M or G_E . The relative sign of the G_M and G_E can only be predicted using the QCD sum rules. The dependence of the magnetic dipole moment G_M and the electric quadropole moment G_E on $\cos\theta$, where $\beta = tan\theta$ at two fixed values of the continuum threshold s_0 and Borel mass square M^2 are depicted in Figs. 1-20. All presented figures have two following common behavior: a) They becomes very large near to the end points, i.e, $\cos\theta = \pm 1$ and they have zero points at some finite values of $\cos\theta$. Note that similar behavior was obtained in analysis of the radiative decays of the light decuplet to octet baryons (see |15|). Explanation of these properties is as follows. From the explicit forms of the interpolating currents, one can see that the correlation, hence, $\lambda_1 \lambda_2(\beta) G_{M(E)}$ as well as $\lambda_2(\beta)$ is a linear function of the β . In general, zeros of these quantities do not coincide due to the fact that the OPE is truncated, i.e., the calculations are not exact. These points and any region between them are not suitable regions for β and hence the suitable regions for β should be far from these regions. It should be noted that in many cases, the Ioffe current which corresponds to $\cos\theta \simeq -0.71$, is out of the working region of β .

We also show the dependence of the magnetic dipole moment G_M and the electric quadropole moment G_E on M^2 at two fixed values of the continuum threshold s_0 and three values of the arbitrary parameter β in Figs. 21-40. From all these figures, it follows that the sum rules for G_M and G_E exhibit very good stability with respect to the M^2 in the working region. From these figures, we also see that the results depend on s_0 weakly. We should also stress that our results practically do not change considering three values of χ which we presented at the beginning of this section. Our final results on the absolute values of the magnetic dipole and electric quadrupole moments of the considered radiative transitions are presented in Table 3. The quoted errors in Table 3 are due to the uncertainties in m_0^2 , variation of s_0 , β and M^2 as well as errors in the determination of the input parameters. Here, we would like to make the following remark. From Eq. (6), it follows that the G_E is proportional to the mass difference $m_1 - m_2$ and these masses are close to each other. Therefore, reliability of predictions for G_E are questionable and one can consider them as order of magnitude. For this reason, we consider only the central values for G_E in Table 3.

| | $ G_M $ | $ G_E $ |
|---|-----------------|---------|
| $\Sigma_b^{*0} \to \Sigma_b^0 \gamma$ | 1.0 ± 0.4 | 0.005 |
| $\Sigma_b^{*-} \to \Sigma_b^- \gamma$ | 2.1 ± 0.7 | 0.016 |
| $\Sigma_b^{*+} \to \Sigma_b^+ \gamma$ | 4.2 ± 1.4 | 0.026 |
| $\Sigma_c^{*+} \to \Sigma_c^+ \gamma$ | 1.2 ± 0.2 | 0.014 |
| $\Sigma_c^{*0} \to \Sigma_c^0 \gamma$ | 0.5 ± 0.1 | 0.003 |
| $\Sigma_c^{*++} \to \Sigma_c^{++} \gamma$ | 2.8 ± 0.8 | 0.030 |
| $\Xi_b^{*0} \to \Xi_b^0 \gamma$ | 8.5 ± 3.0 | 0.085 |
| $\Xi_b^{*-} \to \Xi_b^- \gamma$ | 0.9 ± 0.3 | 0.011 |
| $\Xi_c^{*+} \to \Xi_c^+ \gamma$ | 4.0 ± 1.5 | 0.075 |
| $\Xi_c^{*0} \to \Xi_c^0 \gamma$ | 0.45 ± 0.15 | 0.007 |
| $\Sigma_b^{*0} \to \Lambda_b^0 \gamma$ | 7.3 ± 2.8 | 0.075 |
| $\Sigma_c^{*+} \to \Lambda_c^+ \gamma$ | 3.8 ± 1.0 | 0.060 |

Table 3: The results for the magnetic dipole moment $|G_M|$ and electric quadrupole moment $|G_E|$ for the corresponding radiative decays in units of their natural magneton.

At the end of this section, we would like to calculate the decay rate of the considered radiative transitions in terms of the multipole moments G_E and G_M :

$$\Gamma = 3 \frac{\alpha}{32} \frac{(m_1^2 - m_2^2)^3}{m_1^3 m_2^2} \left(G_M^2 + 3 G_E^2 \right)$$
(47)

The results for the decay rates are given in Table 4. In comparison, we also present the predictions of the constituent quark model in $SU(2N_f) \otimes O(3)$ symmetry [12], the relativistic three-quark model [13] and heavy quark effective theory (HQET) [14] in this Table.

| | Γ (present work) | Γ [12] | Γ [13] | Γ [14] |
|---|-------------------------|---------------|----------------|---------------|
| $\Sigma_b^{*0} \to \Sigma_b^0 \gamma$ | 0.028 ± 0.016 | 0.15 | - | 0.08 |
| $\Sigma_b^{*-} \to \Sigma_b^- \gamma$ | 0.11 ± 0.06 | - | - | 0.32 |
| $\Sigma_b^{*+} \to \Sigma_b^+ \gamma$ | 0.46 ± 0.22 | - | - | 1.26 |
| $\Sigma_c^{*+} \to \Sigma_c^+ \gamma$ | 0.40 ± 0.16 | 0.22 | 0.14 ± 0.004 | - |
| $\Sigma_c^{*0} \to \Sigma_c^0 \gamma$ | 0.08 ± 0.03 | - | - | - |
| $\Sigma_c^{*++} \to \Sigma_c^{++} \gamma$ | 2.65 ± 1.20 | - | - | - |
| $\Xi_b^{*0} \to \Xi_b^0 \gamma$ | 135 ± 65 | - | - | - |
| $\Xi_b^{*-} \to \Xi_b^- \gamma$ | 1.50 ± 0.75 | - | - | - |
| $\Xi_c^{*+} \to \Xi_c^+ \gamma$ | 52 ± 25 | - | 54 ± 3 | - |
| $\Xi_c^{*0} \to \Xi_c^0 \gamma$ | 0.66 ± 0.32 | - | 0.68 ± 0.04 | - |
| $\Sigma_b^{*0} \to \Lambda_b^0 \gamma$ | 114 ± 45 | 251 | - | 344 |
| $\Sigma_c^{*+} \to \Lambda_c^+ \gamma$ | 130 ± 45 | 233 | 151 ± 4 | - |

Table 4: The results for the decay rates of the corresponding radiative transitions in KeV.

In summary, the transition magnetic dipole moment $G_M(q^2 = 0)$ and electric quadrupole moment $G_E(q^2 = 0)$ of the radiative decays of the sextet heavy flavored spin $\frac{3}{2}$ to the heavy spin $\frac{1}{2}$ baryons were calculated within the light cone QCD sum rules approach. Using the obtained results for the electromagnetic moments G_M and G_E , the decay rate for these transitions were also calculated. The comparison of our results on these decay rates with the predictions of the other approaches is also presented.

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Figure 1: The dependence of the magnetic dipole form factor G_M for $\Sigma_b^{*0} \rightarrow \Sigma_b^0$ on $\cos\theta$ for two fixed values of the continuum threshold s_0 . Bare lines and lines with the circles correspond to the $M^2 = 20 \ GeV^2$ and $M^2 = 25 \ GeV^2$, respectively.



Figure 2: The same as Fig. 1, but for $\Sigma_b^{*+} \to \Sigma_b^+$.



Figure 3: The same as Fig. 1, but for $\Sigma_c^{*+} \to \Sigma_c^+$ and $M^2 = 6 \ GeV^2$ and $M^2 = 9 \ GeV^2$.



Figure 4: The same as Fig. 3, but for $\Sigma_c^{*++} \to \Sigma_c^{++}$.



Figure 5: The same as Fig. 1, but for $\Sigma_b^{*0} \to \Lambda_b^0$.



Figure 6: The same as Fig. 3, but for $\Sigma_c^{*+} \to \Lambda_c^+$.



Figure 7: The same as Fig. 1, but for $\Xi_b^{*0} \to \Xi_b^0$.



Figure 8: The same as Fig. 1, but for $\Xi_b^{*-} \to \Xi_b^-$.



Figure 9: The same as Fig. 3, but for $\Xi_c^{*0} \to \Xi_c^0$.



Figure 10: The same as Fig. 3, but for $\Xi_c^{*+} \to \Xi_c^+$.



Figure 11: The dependence of the electric dipole form factor G_E for $\Sigma_b^{*0} \to \Sigma_b^0$ on $\cos\theta$ for two fixed values of the continuum threshold s_0 . Bare lines and lines with the circles correspond to the $M^2 = 20 \ GeV^2$ and $M^2 = 25 \ GeV^2$, respectively.



Figure 12: The same as Fig. 11, but for $\Sigma_b^{*+} \to \Sigma_b^+$.



Figure 13: The same as Fig. 11, but for $\Sigma_c^{*+} \to \Sigma_c^+$ and $M^2 = 6 \ GeV^2$ and $M^2 = 9 \ GeV^2$.



Figure 14: The same as Fig. 13, but for $\Sigma_c^{*++} \to \Sigma_c^{++}$.



Figure 15: The same as Fig. 11, but for $\Sigma_b^{*0} \to \Lambda_b^0$.



Figure 16: The same as Fig. 13, but for $\Sigma_c^{*+} \to \Lambda_c^+$.



Figure 17: The same as Fig. 11, but for $\Xi_b^{*0} \to \Xi_b^0$.

Figure 18: The same as Fig. 11, but for $\Xi_b^{*-} \to \Xi_b^-$.

Figure 19: The same as Fig. 13, but for $\Xi_c^{*0} \to \Xi_c^0$.

Figure 20: The same as Fig. 13, but for $\Xi_c^{*+} \to \Xi_c^+$.

Figure 22: The same as Fig. 21, but for $\Sigma_b^{*+} \to \Sigma_b^+$.

Figure 23: The same as Fig. 21, but for $\Sigma_c^{*+} \to \Sigma_c^+$ and $M^2 = 6 \ GeV^2$ and $M^2 = 9 \ GeV^2$.

Figure 24: The same as Fig. 23, but for $\Sigma_c^{*++} \to \Sigma_c^{++}$.

Figure 25: The same as Fig. 21, but for $\Sigma_b^{*0} \to \Lambda_b^0$.

Figure 26: The same as Fig. 23, but for $\Sigma_c^{*+} \to \Lambda_c^+$.

Figure 27: The same as Fig. 21, but for $\Xi_b^{*0} \to \Xi_b^0$.

Figure 28: The same as Fig. 21, but for $\Xi_b^{*-} \to \Xi_b^-$.

Figure 29: The same as Fig. 23, but for $\Xi_c^{*0} \to \Xi_c^0$.

Figure 30: The same as Fig. 23, but for $\Xi_c^{*+} \to \Xi_c^+$.

Figure 31: The dependence of the electric dipole form factor G_E for $\Sigma_b^{*0} \rightarrow \Sigma_b^0$ on the Borel mass parameter M^2 for two fixed values of the continuum threshold s_0 . Bare lines, lines with the circles and lines with the dimonds correspond to the Ioffe currents ($\beta = -1$), $\beta = 5$ and $\beta = \infty$, respectively.

Figure 32: The same as Fig. 31, but for $\Sigma_b^{*+} \to \Sigma_b^+$.

Figure 33: The same as Fig. 31, but for $\Sigma_c^{*+} \to \Sigma_c^+$ and $M^2 = 6 \ GeV^2$ and $M^2 = 9 \ GeV^2$.

Figure 34: The same as Fig. 33, but for $\Sigma_c^{*++} \to \Sigma_c^{++}$.

Figure 35: The same as Fig. 31, but for $\Sigma_b^{*0} \to \Lambda_b^0$.

Figure 36: The same as Fig. 33, but for $\Sigma_c^{*+} \to \Lambda_c^+$.

Figure 37: The same as Fig. 31, but for $\Xi_b^{*0} \to \Xi_b^0$.

Figure 38: The same as Fig. 31, but for $\Xi_b^{*-} \to \Xi_b^-$.

Figure 39: The same as Fig. 33, but for $\Xi_c^{*0} \to \Xi_c^0$.

Figure 40: The same as Fig. 33, but for $\Xi_c^{*+} \to \Xi_c^+$.