DETERMINING AND OVERCOMING PRESERVICE ELEMENTARY TEACHERS' MISCONCEPTIONS IN INTERPRETING AND APPLYING DECIMALS

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ABSTRACT

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The main purpose of the study, first was to determine preservice elementary teachers' misconceptions in interpreting and applying decimals and then to explore and analyse the effects of the Conceptual Change Instruction (CCI) in overcoming the misconceptions. For this purpose the study was conducted in two stages.

The subjects for Stage-1 consisted of 72 first year preservice elementary teachers and the subjects of the Stage-2 consisted of 49 (a subset of the previous 72 preservice elementary teachers) first year preservice elementary teachers as experimental (N=24) and control groups (N=25) at Atatürk Teachers Training College in the Turkish Republic of Northern Cyprus.

At the beginning, *Concept, Problems, and Writing Division and Multiplication Word Problems Tests* were administered to the subjects (N=72). After scoring the first three instruments representative preservice elementary teachers from the three groups were chosen for the interview procedures.

After the first intensive determining process, the findings indicated that some of the preservice elementary teachers had several misconceptions in both interpreting and applying decimals.

Later experimental group studied decimals with CCI and on the other hand control group studied decimals with traditional method. In Stage-2, in order to test the effectiveness of *Conceptual Change Instruction* the Repeated Measures Analysis of Variance procedures were mainly used. The results of Stage-2 revealed that the CCI was effective in overcoming misconceptions of the subjects in interpreting and applying decimals. The experimental group subjects were especially better on the tasks that conceptual understanding was needed than control group subjects.

Keywords: Misconception, Decimal Number, Conceptual Change Instruction, Conceptual Understanding

ÖZ

İLKOKUL ÖĞRETMEN ADAYLARININ ONDALIK SAYILARI YORUMLARKEN VE UYGULARKEN SAHİP OLDUKLARI KAVRAM YANILGILARINI BELİRLEME VE ORTADAN KALDIRMA

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Bu çalışmanın amacı ilk olarak ilkokul öğretmen adaylarının ondalık sayıları yorumlarken ve uygularken sahip oldukları kavram yanılgılarını belirlemek ve daha sonra da bu kavram yanılgılarını ortadan kaldırmada Kavramsal Değişme Öğretiminin etkilerini incelemek ve analiz etmektir. Bu sebeple çalışma iki aşamada ele alınmıştır.

Araştırmanın birinci savhasında 72, ikinci savhasında ise, birinci savhadaki deneklerin bir alt grubu olan ve kendi içerisinde kontrol (N=25) ve deney (N=24) grubu olarak bölünmüş KKTC'deki Atatürk Öğretmen Koleji'nde öğrenim gören 1.sınıf 49 denek kullanılmıştır.

Başlangıçta, tüm deneklere (N=72) *Kavram*, *Problemler* ve *Bölme ve Çarpma Problemleri Yazma* adlı üç ölçek uygulanmıştır. Bu ölçekleri değerlendirdikten sonra, görüşmelerde kullanmak üzere temsili öğretmen adayları seçilmiştir.

Yoğun belirleme süreci sonrası bulgular bazı öğretmen adaylarının ondalık sayıları hem yorumlamada hem de kullanmada birçok kavram yanılgısına sahip olduklarını göstermiştir

Daha sonra deney grubu ondalık sayıları Kavramsal Değişim Öğretimi'ni, öte yandan kontrol grubu ise geleneksel metotları kullanarak işlemişlerdir. Araştırmanın ikinci aşamasında Kavramsal Değişim Öğretimi'nin etkin olup olmadığını ortaya çıkarmak amacıyla verilerin analizinde tekrarlanmış ölçüm tekniği kullanılmıştır. İkinci aşamada elde edilen sonuçlar Kavramsal Değişim Öğretimi'nin deneklerin ondalık sayıları yorumlamada ve uygulamada sahip oldukları kavram yanılgılarından arındırma açısından etkili olduğunu göstermiştir. Kontrol gruptaki deneklere kıyasla deney grubundaki deneklerin özellikle kavramsal anlamanın gerekli olduğu durumlarda daha başarılı oldukları gözlenmiştir.

to Petek and Doğa Cankoy

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Osman Cankoy

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ACT2.1 : Achievement Related to Decimals as Points on Shaded Areas With Subunit Based on Ten

ACT2.2 : Achievement Related to Decimals as Points on Shaded Areas With Subunit Not Based on Ten

ACT3.1 : Achievement Related to Decimals Involving Unit Measures Subunit Based on Ten

ACT3.2 : Achievement Related to decimals Involving Unit Measures Subunit Not Based on Ten

ACT4 : Achievement Related to Denseness of Decimals

ACT5 : Achievement Related to Comparison of Decimals

ACT6 : Achievement Related to Multiplication and Division Involving Decimals

APT : Achievement Related to Choosing the Appropriate Operation for Word Problems

APT1.1 : Achievement Related to Choosing the Appropriate Operation for Multiplication Word Problems

APT1.2 : Achievement Related to Choosing the Appropriate Operation for Multiplication Word Problems Suitable for Direct Proportion

APT1.3 : Achievement Related to Choosing the Appropriate Operation for Multiplication Word Problems Not Suitable for Direct Proportion

APT2.1 : Achievement Related to Choosing the Appropriate Operation for Division Word Problems

APT2.3 : Achievement Related to Choosing the Appropriate Operation for Division Word Problems Not Suitable for Direct Proportion

APT2.4 : Achievement Related to Choosing the Appropriate Operation for Division Word Problems in which the Divisor is Greater than the Dividend

ARDC : Achievement Related to Decimal Concept

ARWWP : Achievement Related to Writing Word Problems for Division and Multiplication Expressions Involving Decimals

ARWWP-DGTD : Achievement Related to Writing Word Problems for Division Expressions in which Divisor is Greater than the Dividend

ARWWP-DIV : Achievement Related to Writing Word Problems for Division Expressions Involving Decimals

ARWWP-DLTD : Achievement Related to Writing Word Problems for Division Expressions in which Divisor is less than the Dividend

ARWWP-MULT : Achievement Related to Writing Word Problems for Multiplication Expressions Involving Decimals

CCI : Conceptual Change Instruction

CG : Control Group

CT : Concept Test

CT1.1 : Sub-Scale of Concept Test in which Decimals are Treated as Points on Number Lines with Subunit Based On Ten

CT1.2 : Sub-Scale of Concept Test in which Decimals are Treated as Points on Number Lines with Subunit Not Based On Ten

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CT4 : Sub-Scale of Concept Test Related to Denseness of Decimals

CT5 : Sub-Scale of Concept Test Related to Comparison of Decimals

CT6 : Sub-Scale of Concept Test Related to Multiplication and Division Involving Decimals

EG : Experimental Group

PT : Problems Test

PT1.1 : Sub-Scale of Problems Test Related to Choosing the Appropriate Operation for Multiplication Word Problems Involving Decimals

PT1.2 : Sub-scale of Problems Test Related to Choosing the Appropriate Operation for Multiplication Word Problems Suitable for Direct Proportion Involving Decimals

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PT2.3 : Sub-scale of Problems Test Related to Choosing the Appropriate Operation for Division Word Problems Not Suitable for Direct Proportion

PT2.4 : Sub-scale of Problems Test Related to Choosing the Appropriate Operation for Division Word Problems in which the Divisor is Greater than the Dividend

WWPT : Writing Division and Multiplication Word Problems Test

WWPT-DGTD : Sub-scale of Writing Division and Multiplication Word Problems Test Related to Writing Word Problems for Division Expressions in which Divisor is Greater than the Dividend

WWPT-DIV : Sub-scale of Writing Division and Multiplication Word Problems Test Related to Writing Word Problems for Division Expressions Involving Decimals

WWPT-DLTD : Sub-scale of Writing Division and Multiplication Word Problems Test Related to Writing Word Problems for Division Expressions in which Divisor is Less than the Dividend

WWPT-MULT : Sub-scale of Writing Division and Multiplication Word Problems Test Related to Writing Word Problems for Multiplication Expressions

CHAPTER I

INTRODUCTION

1.1 Problem Statement

1.1.1 Rationale

There is a great deal of agreement that learning rational number concepts remains a serious obstacle in the mathematical development of children. This consensus is manifested in the similarity of the opening remarks of a number of recent papers on the topic of children's construction of rational number knowledge (Bigelow, Davis, & Hunting, 1989; Freudenthal, 1983; Kieren, 1988). Also there is no clear agreement about how to facilitate learning of rational number concepts. Numerous questions about how to facilitate children's construction of rational number knowledge remain unanswered even if clearly formulated. For one thing we must find out what types of experiences children need in order to develop their rational number knowledge.

Kieren (1976) argues that exposure to numerous rational number constructs is necessary to gain a full understanding of rational number. His list of rational number constructs consists of fractions; decimal fractions; equivalence classes of fractions; numbers of the form p / q, where p and q are integers and $q \neq 0$; multiplicative operators; and elements of an infinite quotion field.

Decimal numbers have been used in mathematics since the fifteenth century A.D. Three mathematicians, Francois Viete, Simon Stevin, and John Napier, introduced and popularized the use of decimals. The whole numbers are used to make crude measurements to the nearest unit. But when we begin to construct houses, engage in commercial trade , assess taxes on land and other possessions, the whole numbers are no longer adequate. When we attempt to measure with precision the properties of an object such as length, area, volume, weight, capacity, and temperature, subdividing of units of measure becomes necessary. To express measures in terms of these subunits, numbers smaller than the whole numbers are needed , such as decimals. Weights, measures, and money systems have been constructed on a decimal base. It is essential to understand decimals in order to function in society. We use these subunits in engineering , the natural sciences, business, and finance. For this reason heavy stress is laid on decimals in schools (Thipkong, 1988, p.1).

Today, in addition to our decimal monetary system, the increasing use of the metric system and the growing number of calculator and computers make decimals much more important.

Students have to learn how to manipulate not only the whole numbers , but decimals as well. It is important for students to know and understand decimal concepts and operations to help them in their daily lives.

In the light of the above paragraphs it is possible to say that two fundamental topics in the elementary school mathematics curriculum are whole numbers and fractions. A great percentage of time is devoted to developing students' skill in working with these two types of numbers. Whole numbers and fractions are taught separately as different symbol systems requiring the application of different sets of rules. The construction of decimal numbers may be viewed as a simple extension of whole number and fraction concepts. But the integration of these two ideas to form a mature notion of decimals represents a major intellectual advance. The decimal system provides new ways of representing quantitative situations and encourages new insights into the properties of number systems themselves.

If students enter instruction on decimals without a full understanding of the whole number system , if their cognitive capacities are exceeded by the new integration demands , or if instruction fails to present decimals in appropriate contexts, students may acquire only a partial understanding of decimals. It is reasonable to assume that the topic of decimals would present difficulties for many students. Previous research (Bell, Swan, & Taylor, 1981; Hart, 1981; Hiebert, & Wearne, 1985) suggests that many students do, in fact , experience great difficulty in learning about decimals.

It is apparent that one of the most important aspects in educating elementary school children is to have highly qualified elementary school teachers who have a very good understanding of and teaching of mathematics.

Recent research has demonstrated that teachers' personal understanding of the subject matter they teach exerts a powerful influence on their instructional practice (Grossman, 1989; Shulman, 1986; Wilson & Wineburg,1988). It has also been demonstrated, however, that many novice teachers have limited conceptual understanding of the content they are preparing to teach. Some researchers attribute such findings to deficiencies in teacher candidates (Vance and Schlechty, 1982). In contrast Stoddart, Connel, Stofflett and Peck (1993) argue that many novice teachers are weakly prepared in content because of deficiencies in the pedagogy practiced in subject matter courses.

It is important to know preservice teachers' weaknesses in order to help them become better in their subject matter in preparation for teaching students since today's preservice teachers are tomorrow's teachers.

Previously, Işeri (1997) conducted a diagnostic study on middle school students' knowledge about decimals and we wondered what would happen if it were applied on preservice elementary teachers in the Turkish Republic of Northern Cyprus. After applying the test we observed many surprising results. The results mainly showed that the preservice elementary teachers had low understandings in decimals and accordingly this gave us motivation and showed us that there was a need to conduct a similar study on preservice elementary teachers.

1.1.2 Background

Several studies have pointed to common misconceptions experienced by students. Results from the mathematics assessment of the National Assessment of Educational Progress (Carpenter et al., 1981) indicated that 9-year olds have little familiarity with decimals and about 50 percent of 13-year-olds lack even basic understanding of decimals. Several items asked students to order decimals by recognizing place value. Although most 13-year-olds realized that a number greater than one is larger than a number less than one, they had substantial difficulty ordering two decimals less than one.

Bell, Swan and Taylor (1981) reported 12 to -16-year-old students' misinterpretation of decimals involving units of measure. For example an average speed of 11.9 miles per hour was read as 11 miles 9 minutes per hour ; pork chops weighting 1.07 pounds was read as 1 pound 7 ounces; and 0.45 hours reported for the marathon winner was read as 45 minutes. In 1982 Bell studied 11-year old British students' choice of an operation for word problems involving decimals. He found that some students misinterpreted decimal numerals ; for example, 0.8 as one-eighth and 9.3 pounds as 9 pounds 3 ounces. Mangan (1986) found that students tend to be confused in converting units involving decimals. From his interview of secondary students, he reported that students interpreted 0.85 hours to be 1 hour and 25 minutes and 0.75 hours to be 1 hour and 15 minutes.

Students may have misconceptions about decimals because the concept of decimals has not been well developed. Students may lack intuitive sense of the size of decimal numbers and cannot relate decimals to every day contexts where the units are not organized by tens.

Many researchers found that a substantial number of preservice teachers and children had difficulty selecting the correct operation to solve multiplication and division word problems involving positive decimal factors, especially, those less than one (Greaber, Tirosh, and Glover, 1989; Bell et al., 1984; Fischbein et al., 1985; İşeri, 1997).

In this present study we also focused on the primitive implicit models of Fischbein et al (1985), which are mainly supposed to be effective in the choice of operations for word problems. They hypothesized that the primitive model associated with multiplication is repeated addition. According to this model a (whole) number of collections of the same size are put together. Multiplication is not seen as commutative in this model. One factor (the number of equivalent collections) is treated as the operator and the other (the magnitude of each collection) as an operand. When this concept of multiplication prevails, the operator "must" be a whole number, and, consequently, the product "must" be greater than the operand. In the domain of whole numbers, where instruction usually begins, possession of the primitive multiplication model can be a source of the belief that "multiplication always makes bigger."

Fischbein et al. (1985) also describe two primitive models for division, a partitive model and a measurement model. In using the primitive partitive model of division, an object or collection of objects is divided into a given whole number of equal parts or subcollections. In using the primitive measurement model, one seeks to determine how many times a given quantity is contained in a larger quantity. Earlier work (Greaber, Tirosh, and Glover, 1986), suggests that American, preservice elementary teachers tend to think of division predominantly in partitive terms. This primitive model, by its behavioral nature, imposes constraints on the operation of division. Two of these constraints are: the divisor must be a whole number and the quotient must be less than the dividend. These constraints can be the source of the belief that "division always makes smaller."

Fischbein et al. (1985) claimed that the "models become so deeply rooted in the learner's mind that they continue to exert an unconscious control over mental behavior even after the learner has acquired formal mathematical notions that are solid and correct." Although a number of researchers from different countries (Bell, 1982; Hart, 1981; Sowder, 1986) have reported that children often explicitly express misbeliefs such as "multiplication always makes larger" and "division always makes smaller," less has been written about whether or not these two beliefs are explicitly held by adults or about adults' sources of support for the beliefs.

Aksu (1997) conducted a study on (n=155) six-grade students and tried to analyze their performance in dealing with fractions. She noticed students' low performances in solving problems.

Previous research (Bell, 1984, Tirosh and Graeber, 1991; Simon, 1993) also revealed that students and preservice teachers had several short-comings in writing or posing problems.

It is well recognized that problem posing is an important component of the mathematics curriculum and , indeed, lies at the heart of mathematical activity (c.f. English, 1998, p.83).

The inclusion of activities in which students generate their own problems in addition to solving preformulated examples, has been strongly recommended by the National Council of Teachers of Mathematics (NCTM, 1989). Such activities have the added benefit of providing insight into children's and preservice teachers' understanding of important mathematical concepts as well as into the nature of their school mathematics activities (Simon, 1993).

Despite its significance in the curriculum, problem writing has not received the attention it warrants from the mathematics education community. We know little about children's and preservice teachers' ability to create their own problems in both numerical and nonnumerical contexts or about the extent to which these abilities are linked to competence in problem solving. We also have insufficient information on how preservice teachers respond to programs designed to develop their problem-posing skills. Research on these issues is particularly warranted.

Main problems that have been reported up to this point can be summarized under the following headings:

a) Understanding of decimal place value,
b) Denseness of decimals,
c) Ordering of decimals,
d) Multiplication and division by numbers smaller than one,
e) Division of a smaller by a bigger one,

f) Dividing by a non-integer,
g) Selection of the correct operation for word problems.

Up to this point we tried to give information about the overall weak conceptual understanding and problems of children and preservice teachers in mathematics. Despite the evidence that a large portion of children and preservice teachers hold serious misconceptions in interpreting and applying decimals, mathematics educators and researchers have apparently paid little attention to develop strategies to prevent or overcome such misconceptions. The following paragraphs give ideas about how to prevent or overcome such misconceptions.

Current reform movements in mathematics education emphasize problem solving and conceptual understanding as outcomes of instruction. A logically necessary condition for instruction that achieves such outcomes is teachers with conceptions of fundamental operations that are relatively rich and misconception free.

Current mathematics education reform efforts, which include a greater emphasis on conceptual understanding (National Council of Teachers of Mathematics, 1989), have focused attention on the adequacy of teachers' mathematical knowledge to provide instruction as envisioned. However many questions remain to be answered. What is adequate mathematical knowledge ? How might it be assessed ? What is the nature of practicing and prospective teachers' mathematical knowledge today ? A research base with respect to prospective teachers' knowledge is essential if we are to develop instructional interventions that will help prospective teachers extend and modify their knowledge.

Mathematical knowledge is defined as knowledge both of and about mathematics (Ball, 1991). Ball defines knowledge of mathematics as conceptual and procedural knowledge about mathematics as "understandings about the nature of the discipline."

According to Hiebert and Lefevre, Procedural knowledge consists of "the formal language, or symbol representation system, of mathematics" and "the algorithms, or rules, for completing mathematical tasks". Conceptual knowledge "can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information" (c.f. Simon, 1993).

Cognitive researchers have demonstrated that traditional didactic approaches to instruction are ineffective in developing learners' conceptual understanding . Teachers are also learners and their understanding of content and pedagogy is powerfully influenced by their own experiences as students. Most teachers learned their content through the same ineffective methods reformers are seeking to replace. Teachers who do not understand the content conceptually are unlikely to be able to teach it conceptually (Stoddart, Connel, Stofflett, Peck, 1993).

In one of his articles (1985) Bell states: "three steps can be identified in the evolution of theories of teaching. The earliest theories assumed that new material needed only to be received and stored in memory. Later , the importance was recognized of making links with the learner's existing knowledge . Theories based on Piagetian psychology go further than this . They take into account the active disposition of the learner to make sense of what he finds or is offered, leading to the assimilation of new material into his existing framework of knowledge, sometimes involving distortion of the new material and hence false conceptions , while at other times the old framework has to be stretched , or abandoned in favor of a new one which holds together the old knowledge and the new. The extensive research in recent years on pupils' understanding has uncovered numerous examples of this, many of them quite striking and unexpected. Teaching methods based on this theory require the prior identification of pupils' conceptions , and the provision of the problems which give rise to the cognitive conflict when inadequate concepts are used. We call such a method Diagnostic Teaching and have experimented with it over the last few years."

Mick and Sinicrope (1989) state: "in Piagetian terms a student needs to modify his present whole-number conception of multiplication to accommodate the new idea of multiplication resulting in smaller numbers. Teachers need to understand these psychological and mathematical difficulties before students can be guided successfully through the necessary accommodations."

Posner, Strike, Hewson and Gertzog (1990) believe that there are analogous patterns of conceptual change in learning. They noticed that sometimes students used existing concepts to deal with new phenomena. They call this variant of the first phase of conceptual change as assimilation. Later they stated that often, however, the students' current concepts were inadequate to allow them to grasp some new phenomenon successfully. Then the student must replace or reorganize his central concepts. They call this more radical form of conceptual change as accommodation.

Posner et al (1990) state conditions which are common to most cases of accommodation as follows:

1) There must be dissatisfaction with the existing conceptions.
2) A new conception must be intelligible.
3) A new conception must appear initially plausible
4) A new concept should suggest the possibility of a fruitful research program."

Dreyfus et al (1990) state: " by means of a Socratic Dialogue the students analyze their views and are led to a confrontation between different perspectives until flabbergasted students are ready to seriously reconsider the validity of the original assumptions."

West and Pines (1984) have called the three main stages of conceptual change as "awareness, disequilibrium, and reformulating."

Onslow (1990) states: " students dealing with numbers between zero and one need to interpret multiplication as a scalar factor, not merely as a process of repeated addition, if they are to overcome the misconception multiplication always makes bigger. Conflict situations have to be devised to provide an imbalance in cognitive structure. Children may then perceive a need to broaden their conceptual framework to more inclusive ones that do not provoke contradictions."

Niaz (1995) states that any conceptual change would have to go through the following sequence: (a) students must become cognizant of the conflicting data; (b) based on this experience, students must discard their existing theoretical framework ; (c) a better theory must be constructed that explains the data."

Graeber and Tirosh (1990) are prescribing some instructional strategies which are in line with a conceptual change model as follow:

"Whenever the skills for multiplication and division by decimals are taught, it seems that the instruction should be designed to assure that understandings of the operation with whole numbers and decimal notation are accessible to students , so that understanding can be brought to the decimal tasks. Possible instructional activities for achieving these goals are:

1. Introduce students in the early grades to the process of estimating whether the answer to a division word problem is greater than , less than, or equal to one.

2. Provide opportunity for students to compare and contrast their varied definitions, models, and understandings of multiplication and division.

3. Relate decimal notation to concrete embodiments and to currency notation. Some students textbooks treat decimal notation in monetary amounts as a topic totally separate from the decimal numeration system. This seems to be totally inappropriate.

4. Use decimal notation and common fraction notation to perform the same calculation.Compare results.

5. Use the area model for multiplication with whole numbers , prior to using it to illustrate the multiplication of decimals greater than and less than one.

6. Introduce multiplication involving decimals less than one in a word problem setting with a whole number multiplier.

7. Explore the extension of the commutative property to indicated products involving a decimal and a whole number."

In line with the above perspectives we aimed to use a conceptual change instruction, in overcoming possible misconceptions, which is based on constructivist approaches to mathematics teaching. Stoddart et al (1993) state: "constructivist research argues that learners actively construct mathematical understanding with concrete materials and an ongoing process of Socratic questions and discussion designed to challenge preconceptions and replace them with mathematically accurate conceptions."

The teaching methodology involves the following steps:

1) *Introductory task* - students are initially confronted with a relatively difficult problem containing a rich exploratory situation which contains a conceptual obstacle. The students write down their individual responses.

2) The individual responses are *discussed in small groups or in pairs*, working toward consensus. results are again recorded.

3) *Class Discussion* - each group leader or spokesperson presents the group's opinions to the rest of the class, one at a time. This helps ensure that if the whole group has accepted an erroneous conclusion it can be exposed and countered. Wrong responses can be challenged by other groups or the teacher. The teacher acts in a way as to make the situation unthreatening, while at the same time not providing any positive or negative feedback. The teacher also acts as facilitator while at the same time providing further provocation or conflicting ideas where necessary in order to ensure the exposure of all misconceptions.

4) *Reflective Class Discussion* - students can discuss how errors are made and which misconceptions they are likely to be based on. The teacher can sum-up the ideas presented although this is not necessary.

5) *Consolidation* - students are presented with further questions. They solve problems using concrete materials and use them to develop new problems and problem representations. Students develop abstract representations using pictorials to help them create mental images.

Great emphasis is put on problem posing exercises.

In summary, the overall picture led us first to determine and then overcome the preservice elementary teachers' misconceptions in interpreting and applying decimals.

1.2 Purpose of the Study

As it is understood from the problem statement, the study had to be conducted in two stages. In the first stage it was aimed to determine possible misconceptions of preservice elementary teachers in interpreting and applying decimals, which could give a chance to observe the understandings and performances of preservice elementary teachers in different contexts. The first stage at the same time would give a good opportunity to design the corrective teaching strategies in a more meaningful and effective way. At the second stage after applying the corrective treatments it would be possible to observe the effect of the various aspects of teaching on overcoming the misconceptions. In this way the preservice elementary teachers will also get the chance of helping their students to overcome their possible misconceptions about decimals and possibly in other topics, in their future experiences.

This overall purpose can be stated more scientifically with possible research questions as follows:

Main purpose of Stage-1 was to determine preservice elementary teachers' misconceptions in interpreting and applying decimals which can be enriched by the following sub-purposes:

i) To investigate preservice elementary teachers' misconceptions and their interpretation of decimal concepts in working with decimal notation, subunits based on ten and not based on ten , comparison of decimals, denseness of decimals, unit measures involving decimals and operations with decimals.

ii) To investigate and analyze preservice elementary teachers' misconceptions and processes used in solving word problems in multiplication and division involving decimals.

iii) To describe the thinking strategies or embodiments used by preservice elementary teachers when writing word problems for mathematical expressions involving decimals.

Main Problem of Stage - 1

What are the misconceptions (if any) of preservice elementary teachers in interpreting and applying decimals ?

Sub-Problems of Stage - 1

What are the misconceptions (if any) of preservice elementary teachers about decimal notation, unit measures involving decimals, denseness of decimals and operations (multiplication and division) involving decimals ?

What are the misconceptions (if any) of preservice elementary teachers and processes used in solving (choosing the appropriate operation) word problems involving decimals?

What thinking strategies or embodiments are used by preservice elementary teachers when writing word problems for mathematical expressions ?

The purpose of Study-2 was to explore and analyse the effects of the Conceptual Change Instruction on overcoming the misconceptions of preservice elementary teachers in interpreting and applying decimals.

Main Problem of Stage - 2

Is the Conceptual Change Instruction effective in overcoming preservice elementary teachers' misconceptions in interpreting and applying decimals ?

Sub-Problems of Stage - 2

Is there any significant difference between experimental and control group subjects in terms of achievement related to decimal notation, comparing of decimals, unit measures involving decimals, denseness of decimals, and operations with decimals ?

Is there any significant difference between experimental and control group subjects in terms of achievement related to choosing the appropriate operation for word problems involving decimals ?

Is there any significant difference between experimental and control group subjects in terms of achievement related to writing word problems for mathematical expressions of experimental group subjects ?

1.3 Significance of the Study

When we go through the background of the study and some related literature , although it is not at a great extent there are several studies which aimed to determine or diagnose preservice elementary teachers' misconceptions in decimals, but very few have some suggestions or attempts in overcoming those misconceptions. In this respect the present study is very important in contributing to the related field. Especially when we go over the recent literature we observe a demand for producing and applying various instructional strategies in overcoming misconceptions. This strengthens the significance of this present study. Demand and importance of producing and applying various instructional strategies in overcoming misconceptions are presented in the following paragraphs.

Owens (1993) stated that research was needed on the relative ease or difficulty of understanding multiplication and division following instruction that had developed a conceptual understanding of basic concepts. Aksu (1997) stated that in developing mathematics curricula and in teaching mathematics, educators should give special attention to the development of concepts in mathematics and the development of problem solving abilities. İşeri (1997) states: "Teaching experiments should be conducted to find suitable ways of recovering students from their misconceptions."

Repeating a lesson or making it clearer will not help students who base their reasoning on strongly held misconceptions (Champagne, Klopfer & Gunstone, 1982; McDermott, 1984; Resnick, 1983). In fact, students who overcome a misconception after ordinary instruction often return to it only a short time later.

Simply lecturing to students on a particular topic will not help most students give up their misconceptions. Since students actively construct knowledge, teachers must actively help them dismantle their misconceptions. Teachers must also help students reconstruct conceptions capable of guiding their learning in the future.

Lochead & Mestre (1988) describe an affective inductive technique for these purposes. The technique induces conflict by drawing out the contradictions in students' misconceptions. Lochead & Mestre (1988) later emphasized the importance of active classroom discussions and state: "active classroom discussion, with the teacher serving as guide, helps students air their misconceptions and, together, truly overcome them."

1.4 Definition of Terms

Decimal Number: A decimal number is a kind of rational number. Decimal means pertaining to ten and comes from the latin word "decima" which means "tithe" or a "tenth" part. *2.4* is an example for a decimal number.

Misconception: Whenever the conception held by someone contradicts its counterpart, the concept, we will refer to it, is a misconception. A misconception is an underlying belief which governs a mistake or error.

Conceptual Change Instruction (CCI): In this present study we define CCI as an instructional approach in overcoming misconceptions in which the preservice teachers are first confronted with a conflicting situation following a disequilibrium and then reformulating their existing conceptions through intensive group/class discussions and manipulative activities.

Partitive Division: Which might also be termed “sharing division”. An object or collection of objects is divided into a number of equal fragments or subcollections.

Quotitive division: Which might also be termed “measurement division”. One seeks to determine how many times a given quantity is contained in a larger quantity.

Multiplication as Repeated Addition: A type of multiplication where the number of elements in each disjoint set is the same.

Multiplication as Cartesian Product: A Cartesian product is found by matching each element from one set with each element from another set to make a set of ordered pairs.

Primitive Multiplication Model: The primitive model associated with multiplication is repeated addition. According to this model a (whole) number of collections of the same size are put together. Multiplication is not seen as commutative in this model. One factor (the number of equivalent collections) is treated as the operator and the other (the magnitude of each collection) as an operand. When this concept of multiplication prevails, the operator “must” be a whole number , and, consequently, the product “must” be greater than the operand. In the domain of whole numbers, where instruction usually begins, possession of the primitive multiplication model can be a source of the belief that “multiplication always makes bigger.”

Primitive Division Models: There are two primitive embodiments for division, a *partitive* model and a *measurement* model. In using the primitive partitive model of division, an object or collection of objects is divided into a given whole number of equal parts or subcollections. In using the primitive measurement model, one seeks to determine how many times a given quantity is contained in a larger quantity. This primitive model, by its behavioral nature, imposes constraints on the operation of division. Two of these constraints are: the divisor must be a whole number and the quotient must be less than the dividend. These constraints can be the source of the belief that "division always makes smaller."

Experimental Group (EG): Experimental group refers to the group which continued to study decimals using the Conceptual Change Instructional Approach.

Control Group (CG): Control group refers to the group which continued to study decimals with the traditional method, the full time allotted for instruction of the decimals unit. The preservice elementary teachers were not first confronted with a conflicting situation following a disequilibrium or reformulating their existing conceptions through intensive group/class discussions and manipulative activities. The preservice elementary teachers were only informed about the overall misconceptions related to decimals.

Preservice Elementary Teachers: Refers to the 1st year students of Atatürk Teachers' Training Collage in the Turkish Republic of Northern Cyprus.

CHAPTER II

REVIEW OF LITERATURE

This chapter is devoted to the presentation of the related literature about the following issues; the nature of misconceptions, misconceptions related to the decimals in general, misconceptions in solving / writing word problems involving decimals, possible reasons or sources of misconceptions related to decimals, suggestions or instructional approaches which can be considered in overcoming misconceptions , and research studies which aimed to determine and/or overcome misconceptions.

2.1 Nature and Definitions of Misconceptions

Fisher (1985) contends that misconceptions serve the needs of the persons who hold them and that erroneous ideas may come from strong word associations, confusion, or lack of knowledge. According to Fisher, some alternative conceptions, judged to be erroneous ideas or misconceptions, have the following characteristics in common:

1. They are at variance with conceptions held by experts in the field.

2. A single misconception, or a small number of misconceptions, tend to be shared by many individuals.

3. Misconceptions sometimes involve alternative belief systems comprised of logically linked sets of propositions that are used by students in systematic ways. at least by traditional methods.

4. Many misconceptions are highly resistant to change or alternation, at least by traditional methods.

5. Some misconceptions have historical precedence: that is, some erroneous ideas put forth by students today mirror ideas espoused by early leaders in the field.

6. Misconceptions may arise as the result of: a) the neurological "hardware" or genetic programming; b) certain experiences that are commonly shared by many individuals; or c) instruction in school or other settings.

Students do not come to school as "blank slates" (Resnick, 1983). Instead, they come with theories constructed from their everyday experiences. They have actively constructed these theories, an activity crucial to all successful learning. Some of the theories that students use to make sense of the world are, however, incomplete half-truths (Mestre, 1987). They are *misconceptions*.

Misconceptions are problems for two reasons. First, they interfere learning when students use them to interpret new experiences, Second, students are emotionally and intellectually attached to their misconceptions, because they have actively constructed them. Hence, students give up their misconceptions, which can have a harmful effect on learning, only with great reluctance. What do these findings mean ? They show teachers that their students almost always come to class with complex ideas about the subject matter at hand (Mestre, 1989).

Frequently, when science and mathematics are taught to elementary school pupils, it is taught as if the children had had no prior experiences relative to the topic being studied. Misconception research findings indicate that this is not a valid assumption. Children come to school already holding beliefs about how things happen, and have expectations based on past experiences, which enable them to predict future events. They also posses clear meanings for words which are used both in everyday language and in more specialized sciences (Blosser, 1987).

A child's view and understanding of the word meanings are incorporated into conceptual structures which provide a sensible and coherent understanding of the world from the child's point of view (Osborne and Gilbert, 1980).

Children hold ideas that were developed by the teachers and/or the textbook. It is possible that children develop parallel but mutually inconsistent explanations of scientific concepts, one for use in school and one for use in *real world* (Trowbridge and Mintzes, 1985).

Two different words are used to denote building of mathematics, or any other science (Sfard, 1991). The word concepts refers to a mathematical idea in its official form. On the other hand, internal representations of a concept will be referred to as conception. While a concept is within the formal universe of ideal knowledge, a conception takes place within the subjective universe of human knowing.

In 1940, Hancock defined misconception as any unfolded belief that does not embody the element of fear, good luck, faith, or supernatural intervention. Hancock considered misconceptions to arise from faulty reasoning (c.f. Blosser, 1987).

Driver and Eastly (1978) state: "those who use the term *misconception* indicate an obvious connotation of a wrong or incorrectly assimilated formal model or theory."

2.2 Several Definitions and Descriptions Related to Cognitive Conflict and Conceptual Change

The main idea is that in order to learn a new concept, pupils must be actively involved in a process of reshaping and restructuring of their knowledge. The starting point of the process of conceptual change is the students' "naive knowledge" which although often imprecise poorly differentiated and different from the intended scientific knowledge, "has served the student successfully" (Champagne et al., 1983).

According to Postner, Strike, Hewson, and Gertzog (1982), the phase of conflict, of dissatisfaction with existing concepts is central to the process of conceptual change : only at this stage will students realize that they must "replace or reorganize" their "central concepts" because they are "inadequate to allow him to grasp some new phenomenon, such process must be, from the point of view of the students, intelligible plausible, and fruitful.

A cognitive conflict can be produced by various situations: (a) surprise produced by a result which contradicts a subject's expectations, resulting in the generation of perturbations; (b) experience of puzzlement, a feeling of uneasiness, a more or less conscious conflict, or a simple intellectual curiosity; (c) experiencing a cognitive gap, as if the person involved were vaguely aware that something within his knowledge structure was missing; (d) disequilibria - that is, questions or felt lacunae that arise when the subject attempts to apply his schema to a given situation. Within the constructivist framework the development of conflicts or contradictions is essential to facilitate conceptual change and has been summarized by saying that cognitive change and learning take place when a scheme, instead of producing the expected result, leads to perturbation and perturbation in turn, leads to accommodation that establishes a new equilibrium (Festinger, 1957; Piaget, 1980; Vygotsky, 1978).

From a philosophy of science perceptive, any conceptual change would have to go through the following sequence: (a) students must become cognizant of the conflicting (anomalous) data; (b) based on his experience, students must discard their existing theoretical framework (core belief); (c) a better theory must be constructed that explains the data (Niaz, 1995).

2.3 Misconceptions Related to Decimals in General

After exploring the works and research studies conducted by many educators and researchers it is possible to say that many of the children and some of the preservice elementary teachers have serious short-comings in understading decimals. Overall misconceptions of children and preservice elementary teachers are presented in the following paragraphs.

2.3.1 The Meaning and Representation of Decimals

It appears that when children are unsure of the meaning of a decimal , then they try to change the unfamiliar into the familiar by treating the number according to what it looks like. Many seem to ignore the decimal point altogether, or treat the number as if it were two separate natural numbers with a mere dot in between. Thus 5.67 may be interpreted as "five hundred and sixty two" . Others appear to confuse the decimal point with other separators such as the dot in 3.56 pm, or the comma in the co-ordinate pair (6,2) (Bell et al, 1985, Bell, 1982, Hart, 1981).

In their struggle to find meaning with a decimal place value, students display a variety of difficulties. We have heard students saying "tens" for "tenths" and "hundreds" for "hundredths" (Owens, 1990, Resnick, 1989).

According to Resnick et. al (1989) the conceptual similarities with whole numbers become apparent when decimal fractions are thought before fractions. This time students would ignore the decimal point or treat it as a separator. They refer this as the *whole number rule*.

2.3.2 Comparing and Ordering Decimals

Many children have great difficulty when asked to compare the relative sizes of two or more decimals which have different numbers of decimal places (Bell et al, 1985, Bell, 1982).

Children make few mistakes when comparing two decimals with different whole number parts, such as 12.7 and 15.56 (Sackur-Grisvard & Leonard, 1985), but they have difficulty comparing decimal fractions if there are different numbers of digits to the right of the decimal point and equal whole number parts (Hiebert & Wearne, 1985). Students' errors such as 0.195 is greater than 0.2 are presumably a result of ignoring the decimal point and treating these as whole numbers. Sackur-Grisvard and Leonard (1985) call this rule the "whole number rule".

The next most frequently encountered rule, leading to error, called "fraction rule" (Sackur-Grisvard and Leonard, 1985), is to select as smaller the number with more digits in the decimal part. For example, choosing 13.564 as being smaller than 13.21.

A third rule is called "zero rule" : select as smaller the decimal that has a zero immediately after the decimal point, and otherwise choose as larger the number with more digits to the right of the decimal (Sackur-Grisvard and Leonard, 1985).

2.3.3 Zero as a Place Holder

One of the most frequently observed difficulties of students and preservice elementary teachers is in using zero as a place holder.

Brown (1981) asked the question "Ring the BIGGER number: 4.06 or 4.5" to the students between ages 12 and 15. Some of the student chose 4.06 as the bigger one, not considering zero as a place holder.

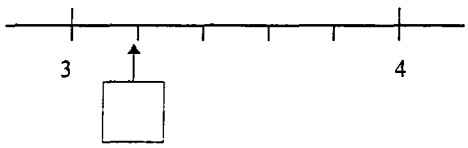
Thipkong and Davis (1991) conducted a research on preservice elementary teachers. They found that 11% of their subjects marked 1.05 as 1.5 ignoring the zero in between and interpreting the decimal point as it was not there.

Bell (1982) reports 79% of 11 year students interpreting 0.1 as less than 0.07, probably because 1 is less than 7.

2.3.4 Interpreting Decimals on Number Lines (Scale Reading) and Area Models

Scale reading discriminates very clearly between pupils who have a deep , genuine understanding of decimals and those who do not (Bell, 1982).

Students, generally concentrate on the labeled calibration immediately to the left of required reading, and then count along a scale, ignoring the value of other marked calibrations and ignoring the size of each interval (Brown 1981). The following is an example from CSMS study:



Common response to the place indicated by the arrow, in terms of a decimal was 3.1, by middle school students.

A preliminary investigation showed that 8th and 10th year students had considerable difficulties in understanding how fractions were to be presented as points on number lines. When number lines are truncated at 0 and 1, students found it fairly easy to locate fractions. However, when the line segments started or ended at different values students seemed to be unsure about the interval on the number line that should serve as a reference. A follow-up study of students and teachers showed that teachers also had doubts about the reference object for fractions on a number line (Carraher, 1993).

Thipkong and Davis (1991) report that 42% of the preservice elementary teachers in their study interpreted 1.4 on a number line as one and four subunits when subunits were not based on ten.

Bell (1982) reports that many students can not read a scale when the value of intermediate markings has to be calculated by proportion.

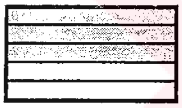
Regions are used in most textbooks as models for common and decimal fractions, but the connections between the regions and the numbers are not always retained (Owens & Super, 1993, p. 142). Hiebert and Wearne (1986) report that students were asked to write decimal fractions to represent the shaded part of a region. They summarize:

"One region was divided into 10 equal parts with three shaded; a second was divided into 100 equal parts with four shaded; and a third was divided into five equal parts with one shaded. Not more that half the students, even in grade nine, responded correctly to any of these items. The most frequent errors on the three tasks in all grades were 3.10, 4.100 and 1.5 respectively. The frequency of these errors declines from about 50% in grade five to about 15% in grade nine" (p. 209).

2.3.5 The Ten-ness of a Decimal Number

There are some children and preservice teachers who have little or no notion that decimals are essentially denary (Brown, 1981).

Bell et. al (1985) conducted a study on middle school students and reported that many of the students were influenced by the base 10 numeration system. For example the responses of many students to the decimal representation of the shaded region was as follows:



"the shaded area is 0.3 square units"

A counter example was given by Thipkong and Davis (1991). They found that 28 of 65 preservice elementary teachers interpreted 1.2 yards as one yard and two feet.

2.3.6 The Denseness of Decimals

Decimals as representation of rational numbers has a very distinct and different property compared to the whole numbers. Within any range of decimal numbers there exist infinitely many others. This situation named the "denseness of decimals" is very hard for children to imagine (İşeri, 1997)

Very few children at any age appreciate that there exist an infinite number of decimals which lie between any two given numbers (Bell et al, 1985, Bell, 1982, Hart, 1981).

In the CSMS study to the question "How many different numbers could you write down which lie between 0.41 and 0.42 ?" there were a variety of answers. For example only 7% of 12 years responded as "infinitely many" and 5% as "lots, hundreds". Twenty - two percent of the answers were "8, 9 or 10" and 9% as "0".

2.4 Multiplication and Division Operations Involving Decimals

Because of the extensive work with whole numbers in the early years of schooling students get used to multipliers and divisors greater than one. The resulting overgeneralization that "multiplication always makes bigger" and "division always makes smaller" are being reported as most deeply structured misconceptions (Bell et al, 1985, Bell, 1982, Hart, 1981, Fischbein et al., 1985, Greaber and Tirosh, 1989, 1990, İşeri, 1997).

Greaber and Tirosh (1989) conducted a study in the United States which was designed to assess the extent to which the beliefs, "multiplication always makes bigger" and "division always makes smaller", are explicitly held by preservice elementary teachers. The preservice teachers were asked to respond to the following six statements related to the misbelief "multiplication always makes bigger" and "division always makes smaller".

A. In a multiplication problem, the product is greater than either factor.
B. The product of 0.45 is less than 90.
C. In a division problem, the quotient must be less than the dividend.
D. In a division problem, the divisor must be a whole number.
E. The quotient for the problem 60 / 0.65 is greater than 60.
F. The quotient for the problem 70 ÷ 1/2 is less than 70.

Eighty-seven percent of the 130 preservice teachers who responded to both of the multiplication statements related to the misbelief "multiplication always makes bigger" responded correctly to both; only 3% of them responded incorrectly to both of the statements.

Four statements related to the misbelief "division always makes smaller" were included on the paper and pencil instrument. Of the 129 preservice teachers who responded to all four of these statements, 28% responded correctly to all four of the statements and 3% responded incorrectly to all four. The majority of the preservice teachers responded incorrectly to statement C (In a division problem, the quotient must be less than the dividend), the statement that most closely parallels this misbelief.

Greaber and Tirosh (1990, p.582) in exploring the interpretations of 4th and 5th graders about multiplication and division involving decimals stated the misleading conceptions of those students as follows:

"The study confirms earlier studies of students' lack of understanding of decimals, their lack of linkages between decimal and fractional knowledge, and their difficulty in seeing a/b as a statement of division. Two other observations about U.S. students are noteworthy. Almost 25% of the U. S. students could provide no definition of division other than that of the inverse of multiplication. While this definition may support learning of the basics facts of division, it does not seem particularly helpful in characterizing instances in which division is used in problem solving.

Tirosh and Graeber (1990) after exploring the beliefs of 21 preservice elementary teachers stated of preservice teachers' conceptions about division as follows:

"1. twelve of 21 gave only a partitive interpretation of division. The following quotation is typical of these responses:' division means sharing , whereby you distribute a certain amount of things among people to see how many each person gets'. 2. Three subjects described division as the inverse operation of multiplication."

In 1990 Tirosh and Greaber investigated cognitive conflict as a means of probing the misconception held by many preservice elementary teachers that in a division problem the quotient must be less than the dividend. In exploring the meaning (conception) of division Tirosh and Greaber found the following four categories:

Twelve of the 21 gave only a partitive interpretation of division. The following quotation is typical of these responses: "Division means sharing, whereby you distribute a certain amount of things among people to see how many each person gets"

Three subjects gave both a partitive and measurement interpretation.

Three subjects described division as the inverse operation of multiplication. The following response was typical: "The opposite of multiplication, I associate it immediately to multiplication."

Three subjects gave responses that did not fall in any of these categories. One subject was unable to express what division meant, and two verbalized only descriptions of the written algorithm. For example, "I see numbers , the little house, the divisor, the dividend and the quotient."

15 subjects argued that the quotient is always less than the dividend and gave one of the following justifications:

Division is sharing. When you share things, each one gets less than the whole amount. Therefore the quotient is less than the dividend.

There are no such examples in which the quotient is greater than the dividend.

Division is the inverse of multiplication. Since multiplication always makes bigger, division always makes smaller.

Arguments based on algorithmic procedures. For example, one of the interviewees argued that in the case of a decimal divisor, "you have to change the divisor to a whole number, add zeros to the dividend, and then , ultimately, the quotient is less than the dividend."

Ball (1990) investigated prospective teachers' (elementary and secondary) knowledge in three applications of division - division by fractions, division by zero, and algebraic equations. She found their knowledge of division to be fragmented and largely procedural. Their explanations did not tend to be connected to an underlying concept of division.

In 1993 Simon investigated prospective teachers' knowledge of division through an open response written instrument and through individual interviews. Problems were designed to focus on two aspects of understanding division: connectedness within and between procedural and conceptual knowledge and knowledge of units. Results indicated that the prospective teachers' conceptual knowledge was weak in a number of areas including the conceptual underpinnings of familiar algorithms, the relationship between partitive and quotitive division, the relationship between symbolic division and real-world problems, and identification of the units of quantities encountered in division computations.

The prospective elementary teachers in this study exhibited serious shortcomings in their understanding of division as a model of situations. They seemed to have appropriate knowledge of the symbols and algorithms associated with division, but many important connections seemed to be missing, leaving a very sparse "web of knowledge".

2.5 Misconceptions in Solving / Writing Word Problems [involving decimals]

In the following paragraphs both students' and preservice elementary teachers' misconceptions in solving or writing word problems [involving decimals] are explored, which are mainly enriched by related research studies:

Previous research suggests that teachers' understanding of fractions which indirectly relates to decimals is limited and replete with misunderstandings about fraction concepts and procedures (Post, Harel, Behr, & Lesh, 1991; Ball 1990; Leinhardt & Smith, 1985). Only a minority could solve the given fraction problems and adequately explain their solutions.

Thipkong and Davis (1991, p.93) state: " preservice teachers who have problems interpreting decimals may also have problems in solving word problems involving decimals."

Brown (1981) stated that 11-to-16 year old British students responded that "multiplication always makes bigger and division always makes smaller." When students are asked to choose the appropriate operation for verbal problems , errors often arose from misconceptions such as multiplication always makes bigger, division always makes smaller, and division must be of a larger number by a smaller number.

Bell (1982, pp.7-8) is one of the researchers who investigated several misconceptions of pupils in solving word problems involving decimals. He classified those misconceptions under the heading of "Choice of Operation in Problems With Decimal Numbers" as follows:

1. Concrete Approach

This is often revealed by pupils' misreading the problem ; for example "an average speed of 11.9 miles per hour" was read as " 11 miles 9 minutes per hour "; and " chops weighting 1.07 lb " was read as " 1 lb 7 ounces ". Similarly, a calculator result to a question about the time then by a marathon winner, 0.453852, was reported as 0.45 hours or 45 minutes. A different but also common misconception was ".... 0.8 ... that's about an eighth ". These are all examples of assimilation of a relatively unfamiliar decimal notation to the more familiar and or concrete units of measure, fractions.

2. Over-generalization

This misconception was very common. This example illustrates how it led to mistaken choice of operation. A question asking, if petrol was 1.17 dollar per gallon what would be the cost of filling a tank containing 8.6 gallons, was answered correctly immediately as 1.17×8.6. The following question asking for the cost , now at 1.20 dollar a gallon , of filling a 0.22 gallon can, was said by the pupils to be different, because you've got a lesser amount. It's under the 1.20 dollar, so obviously it's 1.20/0.22 or something like that. Faced next with the same question with easier numbers - 2 dollar a gallon , and a 5-gallon can to be filled - they immediately said this needed multiplication, 2×5.

In this case , the fact that the cost of 0.22 gallons clearly needed to be smaller than the cost of a gallon led them to think that division was necessary. This is an example of an untaught but sensible awareness relating to whole numbers being over-generalized into the field of fractional numbers.

3. Detachment

There were two misconceptions here, first an assumption, no doubt derived from extensive earlier experience, that the smaller number must always be divided into the larger, and secondly, a mistake about the direction of the $a \div b$ symbolism, which was often confused with the more familiar, and read as " how many *a*'s go into *b* ? " . This is another example of untaught but sensible conclusions being detached from their original meaning and mistakenly extended beyond their sphere of validity.

4. Distraction

" John had 4.5 cwt of coal delivered , which was 1.5 cwt more than Fred, How much did Fred receive ? "

In problems of this type, the word 'more' acts as a distracter, leading pupils to add rather than subtract; and similarly for the word 'times'. This is an assimilation of a more complex problem structure to a simpler one; and is consonant with the notation that reading consists of scanning the sentence until meaning is extracted .(it also highlights the danger of that kind of teaching which encourages the search for cue words rather than the extraction of full meaning . We often have found ourselves fighting our own tendency to 'help' pupils by offering similar 'algorithmic' ways of solving problems without understanding them).

To summarize we have distinguished four main types of misconceptions exhibited by pupils in solving problems. These are related to (i) the necessity for reference to a concrete approach, (ii) the detachment of symbols or models from their meanings, (iii) the over-generalization of rules beyond their domain of truth, and (iv) the distraction caused by cue words or perceptual features.

Several researchers contend that students have primitive partitive and quotitive models of division that they describe as follows (Simon, 1993; Fischbein, Deri, Nello, and Marino, 1985):

Partitive division. In the first model, which might also be termed sharing division, an object or collection of objects is divided into a number of equal fragments or subcollections. The dividend must be larger than the divisor; the divisor must be a whole number; the quotient must be smaller than the dividend.

Qoutitive division. In the second model, which might also be termed measurement division, one seeks to determine how many times a given quantity is contained in a larger quantity. In this case, the only constraint is that the dividend must be larger than the divisor. If the quotient is a whole number, the model can be seen as a repeated subtraction.

In a study fifteen fourth graders and fifteen fifth graders were asked how they would solve the word problem " Five pounds of trial mix was shared equally by fifteen friends. How many pounds of trial mix did each friend get ? " the first response of twenty-four students was "15 ÷ 5 " or " 3 " (Graeber and Tirosh, 1988). This common error is not limited to young students. Forty-two percent of a sample of sixty-five preservice elementary school teachers wrote the response " 15 ÷ 5 " for the same problem.

In a study childrens in grades 4-6 in a local school were asked to complete, in a paper and pencil format and in a group setting, the following computations (O'Brien and Casey, 1983).

1) 6 x 3 =	4) 13 x 16 =
2) 16 x 3 =	5) 3 x 60 =
3) 60 x 1 =	

Then the students were asked to write a story problem for 6 x 3 =

The context of the story was repeated addition.

In the second part of the study children's stories for 6 × 3 were categorized not in terms of "multiplicativeness" of their content, but in terms of their logic and in terms of the realism of the information they contained.

The categories were as follows:

A. Didn't pose a question: made a statement or left question unasked.
B. Incomplete logical structure: left out essential information.
C. Added extraneous information or extraneous computation.
D. Nonsensical or impossible arithmetic operation.
E. Unrealistic data.
F. Nonsensical question.
G. Child's written language makes classification impossible.

Peterson (1991) in exploring pupils mathematical performance in grades 3 and 6 observed the following shortcomings of students in solving word problems:

They used *1 hour = 100 minutes*.

They could not multiply by zero.

Tirosh and Greaber (1989, pp. 83-84) in assessing the preservice elementary teachers' explicit beliefs about multiplication and division stated: "When the operator in a word problem was a decimal less than 1, about 50% of the preservice teachers responded with a division expression. However, when the operator was a whole number, 90-95% of the preservice teachers wrote correct expressions. This high rate of correct response for whole number operators held for both whole number and decimal operands. The influence of the misbelief was also evident in interviews."

The following excerpt (from the same study) is from an interview in which the student was attempting to solve the problem. " The price of one bolt of silk fabric is 12,000 dollar. What is the cost of 0.55 of the bolt ? "

Student:....You want to find out what is the price of just this portion of the bolt. So you have to divide 0.55 into the amount to get the portion.

Interviewer: Can you explain it again ?

Student: OK. This (points to 12,000 dollar) is the price of the bolt of fabric. And you want to know the price of this part, a part, of the bolt. so you are going to divide 0.55 into 12,000 to find out what that part is."

This excerpt is typical of the explanations offered by preservice teachers acting with the related beliefs that "division always makes smaller, and multiplication always makes bigger."

In the above study, about 45% of the preservice teachers wrote multiplication expressions for the division word problems with decimal divisors less than one . A word problem of this type was , " Girls club cookies are packed 0.65 pounds to a box. How many boxes can be filled with 5 pounds of cookies ? " Eighteen of the 40 preservice teachers who responded to this problem did so incorrectly: 14 of those 18 wrote 0.65×5 or 5×0.65.

In another study conducted by Tirosh & Greaber (1990), it was observed that some preservice teachers' misconceptions about division were typically held explicitly, whereas others were held only implicitly. For example , the majority of preservice teachers agreed with the explicit statement "in a division problem, the quotient must be less then the dividend," and their attempts to solve word problems were consistent with their conception. Although the majority of preservice teachers disagreed with the statement "in a division problem, the divisor must be a whole number," many of them attempted to answer word problems as if they believed that the divisor must be a whole number.

In investigating fourth and fifth graders performance in multiplication and division word problems Greaber and Tirosh (1990) stated the following:

"Students given a word problem involving multiplication of a whole number by a decimal less than one did more poorly on it than did on a similar world problem where the role of the factors was reversed. This is generally interpreted as showing the influence of the primitive model of multiplication and accompanying belief ' multiplication always makes bigger.' The definitions the students produced for multiplication and the multiplication word problems they constructed also suggest that the vast majority of them view multiplication as repeated addition.

Resistance to writing an expression that involves division by a larger number seems clear. The majority of initial responses to the word problem with solution 5 ÷ 15 was 15 ÷ 5. The students could reason out an answer, but they rejected the notation $15\overline{)5}$, and even when students were shown the statement 5/15 = 1/3 (which they recognized as correct), they were unable to see what connection this had to the problem. For these students the belief, ' you can't divide by a larger number' seemed to be more a manifestation of an inability to symbolize this operation than it does of inability to conceptualize the operation." In reviewing students' responses to the different division tasks, it was apparent that the model of division evoked varied with the context. For example , some students who used only a measurement model when asked to define division, wrote only partitive problems. The only popular model for illustrating multiplication of two rational less than one was the area model.

In 1978 Vest tried to investigate the disposition of preservice elementary teachers related to measurement and partition division and he observed that the majority of the preservice elementary teachers studied preferred to use partition word problems, 67.8 per cent supplying partition examples on a test and 69.7 per cent using partition to introduce measurement to on another.

Thipkong and Davis (1991) stated that preservice teachers have a lot more success with word problems involving decimals greater than one than decimals *less than one*.

Bell and Onslow (1987) in exploring students multiplicative structures observed that some pupils have a weak grasp of the numerator and denominator roles of the two quantities in a rate, which leads to an error consisting of a reversal of the quantities in the rate, for example, treating miles per hour as if it were hours per mile.

Bell, Greer, Grimison and Mangan (1989) investigating children's performance on multiplicative word problems pointed that an unexpected but striking difference appeared between rate-partition and rate-quotition questions. When errors were made , the proportion that were reversals, rather than choices of multiplication, was high for the partition types, low for the quotitions whenever the numbers allow it , including choosing multiple-groups problems rather than repeated-measures problems; an exception was price, which was frequently used. There was also a universal preference for partition rather than quotition stories.

Type of number used had a large effect on success, but the type of number used as multiplicand generally did not. However , for the 10-year-olds, the type of number used as multiplicand also had an effect.

There were sharp increases in difficulty when the multiplier changed from an integer to a decimal number, and to a decimal number less than one, demonstrating the sensitivity of children and adolescents to these structural aspects of multiplicative problems.

In 1984 Bell, Fischbein, and Greer conducted a research on choice of operation in verbal problems. 12 and 13- year olds were tested with two types of tasks to test their understanding of applications of the multiplication and division of positive numbers.

They observed several misconceptions as follows:

1. Multiplication involving decimals less than 1 was a source of difficulty.
2. Division by a larger number consistently led to reversals in both the problems and the stories.
3. For the problems, division by decimals less than 1 proved difficulty, and in most cases led to a large number of multiplication responses.

2.6 Possible Reasons or Sources of Misconceptions [Related to Decimals]

In the following paragraphs the reasons or sources of both students' and preservice elementary teachers' misconceptions mainly in interpretation and application of decimals are discussed:

2.6.1 Possible Reasons or Sources of Misconceptions Related to Decimals in General

Errors in mathematics have been extensively studied in recent years. Researchers have noted some instability in the errors they have studied. Some errors however , have been found to be stable, and theories have been formulated to explain them. These stable errors are commonly referred to as bugs. The theoretical model for bugs relies on repair theory, which states that errors occur when a student is faced with a difficult or unfamiliar feature of a task that leads the student to an impasse. This impasse is resolved by modifying a known procedure and incorrectly applying it to a task. Bugs, in the context of repair theory, are seen as errors that occur at the production stage. Although readers may be familiar with the term bugs, our study refers to errors as mal-rules. Mal-rules in mathematics are violations of legal mathematical rules. There may be many different etiologies for mal-rules, but Sleeman favors a misgeneralization theory to explain an important subset of them. Sleeman's misgeneralization theory suggests that some errors result when a student infers "several rules which are consistent with the example , and not just the correct rule. The grounded work for these errors occurs during encoding, at the stage at which the student is developing hypotheses (cf. Blando et. al. 1989).

Fuson correctly points out that existing textbook treatments of place-value do not adequately help children construct a multiunit concept . She argues that the features of current textbooks overlook how children think and proposes five new features that should govern place-value instruction. Fuson concludes that instruction of multiunit concepts and multidigit addition and subtraction should be integrated and postponed until second grade. Instruction preceding this, including reading and writing two digit numerals and single-digit sums to 18 and their subtraction complements, should be based on children's unitary concepts (cf. Baroody, 1990, p281).

Sfard (1991) states the importance of forming a concept both operationally and structurally as: "The development of a skill is closely tied to understanding the concept underlying the skill. In the light of this claim it should not surprise us that ever so often, students appear to be learning many mathematical skills at a rote manipulation level and do not understand the concepts underlying the computation. For instance, pupils can be quite successful in computations involving fractions in spite of being unable to treat fractions as numbers."

It is well documented that many students have difficulty working with decimal numbers (Carpenter, Corbitt, Kepner, 1981 Hiebert and Wearne, 1986) Frequently, these students appear to lack some essential conceptual knowledge and have memorized procedural rules that they apply inappropriately (Bell, Swan, and Taylor, 1981 Fischbein, Deri, Nello, and Marino, 1985). In many cases, the rules are tied only to the surface features of tasks rather than to any underlying conceptual rationale.

Wearne and James (1989) stated the difficulties and sources of those difficulties of students in working with decimal numbers as follows:

"Many of the students difficulties can be traced to an incomplete or nonexistent understanding of the written symbols. Without quantitative meanings for decimal numbers, students have little choice but to memorize rules that prescribe how to manipulate the symbols. Lack of meaning for the symbols makes it difficult for students to monitor their own performance or to extend their learned procedures to a new situation."

The major portion of the instruction in elementary school mathematics is devoted to assisting students in becoming proficient with various symbol systems. Students begin their study of symbol systems with whole numbers, then move to fractions, and then to decimals. To many students receiving conventional instruction, each system has its own rules that must be memorized. This kind of learning leads to an overreliance on syntactic features of the written symbol systems and an over-reliance on recalling and applying memorized rules. Often the rules and symbols remain unconnected to the quantities and actions that they represent. The consequence is that even when the rules are practiced frequently, they become flawed and often are applied to problems inappropriately (Brown and VanLehn, 1982 Hiebert and Wearne, 1985).

Larson (1980) in discussing the locating of proper fractions and decimals on number lines stated: "An important difference between a part-whole model and a number line model is that in the number line model the students need also to attend the scaling. Hence a number line model implies a length greater than one. The students can disregard the scaling and respond correctly, as long as they begin counting at the left-at zero. They can still use the rule, count the number of parts in all, in this case equivalent segments, for the denominator and count the number of equivalent segments from zero to the marked point for the numerator. When the line is of length two, this rule does not work."

Using a task in which a child is asked to order decimal fraction numbers , Sackur-Grisvard and Leonard (1985) found that in fourth and fifth grade French classes about half of the children tested used a systematic but incorrect rule to decide which number is greater . There were three different incorrect rules, each used when the numbers to be ordered had the same whole number digit (e.g., 3.214 and 3.8). According to Sackur-Grisvard and Leonard's Rule 1, The number with more decimal places is the larger one; for example , 3.214 is greater than 3.8 because 3.214 has more digits in the decimal part and because 214 as a whole number is larger than 8. Sackur-Grisvard and Leonard suggest that classroom instruction may support Rule 1 by giving students practice mainly in comparing decimals with the same number of digits, in which case treating decimals as whole numbers always works. Sackur-Grisvard and Leonard found that Rule 1 was common ; it was used by 40% of their fourth graders and by about 25% of their fifth graders. Sackur-Grisvard and Leonard's Rule 2 specifies that the number with fewer decimal places is the larger. Thus, given the pair 1.35 and 1.2, Rule 2 chooses 1.2 as greater. Rule 2 was the least common in their sample. Grossman (1989), however, has suggested that a similar rule (choosing the longest decimal number as the smallest) was commonly applied by large numbers of entering U.S. college students. Sackur-Grisvard and Leonard's Rule 3 gives a correct judgment when one or more zeros are immediately to the right of the decimal point in one of the numbers, and otherwise chooses as larger the number with more digits to the right of the decimal point. Thus, given three numbers to order (e.g., 3.214, 3.09, 3.8), Rule 3 correctly chooses the number with the zero as the smallest, but than uses Rule 1 to order the remaining pair: i.e., 3.09, 3.8, 3.214. Rule 3 was used by about 8% of Sackur-Grisvard and Leonard's fourth graders and by 14% of their fifth graders.

Resnick, Nesher, Leonard, Magone, Omanson and Peled (1989) in examining conceptual bases of arithmetic errors drew the following conclusions in line with the findings of Sackur-Grisvard and Leonard:

"We turn now to the cognitive sources of the errorful rules. We have two kinds of data to draw on in making our inferences: the children's verbalizations as they worked on the comparison tasks, and the patterns of answers for the items that more directly examined place value and fractional knowledge. *Whole number rule.* We had hypothesized that the whole number rule results from children's attempts to apply their knowledge about whole numbers to the new kind of numbers they are learning without integrating information about the fractional values. This was confirmed by typical verbalizations of the whole-number-rule children as they respond to comparison items. For example, here are explanations by two Israeli children:

Interviewee: 0.5 < 0.25, "because 25 is bigger."

4.7 < 4.08, "because the zero does not matter and 8 is bigger than 7."

When comparing 2.35 and 2.305 and 2.035, whole number children often referred to a number's decimal portion as a whole number, saying that "three hundred and five" or "three hundred fifty" was bigger than "thirty-five."

Whole number children also showed confusion about the zero's placeholder function. One child, for example , equated 2.35 and 2.035 because the zero is "just a place marker and that would still be thirty five."

Fraction rule. We hypothesized that the fraction rule results from children's efforts to integrate knowledge about fractional parts and ordinary fraction notation with their place value knowledge. In particular, we expected them to know that if a number is divided into more parts, the parts are smaller. We expected however , that fraction rule children might have some difficulty figuring out whether the digits stated explicitly in the decimal form corresponding to the numerator or the denominator of an ordinary fraction."

Resnick, Nesher, Leonard, Magone, Omanson and Peled (1989) briefly states: "*Whole number* errors derive from children's applying rules for interpreting multidigit integers. *Fraction* errors derive from children's efforts to interpret decimals as fractions."

2.6.2 Possible Reasons or Sources of Misconceptions Related to Multiplicaiton and Division Operations [Involving Decimals]

Ball (1990) conducted a research study on prospective elementary and secondary school teachers' understanding of division and she states:" Division with fractions is rarely taught conceptually in school; most of the prospective teachers probably learned to divide with fractions without necessarily thinking about what the problems meant. Indeed, most of them could carry out the procedure to produce the correct answer. The teacher candidates' understandings of division and of their ideas about what is entailed in explaining or justifying something mathematically fit with evidence from other parts of the interview that the prospective teachers' substantive understanding of mathematics tended to be both rule-bound and compartmentalized."

Silver's work demonstrated the importance of connectedness between conceptual and procedural knowledge of division, a connectedness often lacking in the middle school children that he studied. He and Kaput suggest that the lack of connectedness between conceptual and procedural knowledge of division is promoted by the mismatch between students' early partitive models of division and the quotitive approach used to teach division algorithms. This lack of connection is seen particularly in students' inability to interpret remainders (cf. Simon, 1993, pp.233-234).

In a research study Tirosh and Greaber (1990) aimed to explore preservice teachers' thinking about division by evoking cognitive conflict. Throughout the interviews they found the following results:

During the course of the interviews, 14 of the subjects identified the following causes of their misconceptions:

- thought only of whole numbers
- "assumed that with decimals it [division] works in the same way as with natural numbers"
- found the decimals confusing and misleading
- conclusions from the standard algorithm

The following excerpt from O'Brien and Casey (1983) may be an example of unidimensionality of learning which may be a source of misleading beliefs about multiplication:

"In the American elementary school mathematics curriculum, multiplication involving number and place value is regarded as basic. Logical multiplication is virtually ignored. What seems to have happened in these studies, is that the children who failed the task had only a knowledge of number/place value multiplication and none of logical multiplication."

The belief you can not divide by a larger number seemed more a matter of an inability to symbolize the operation than an inability to conceive of the operation (Greaber and Baker, 1992).

It is certainly true that division is first introduced the domain of whole numbers with the whole-number divisors that are factors of the dividends. Thus of initial experiences are those that shape dominant frames or cognitions, the logically acceptable sequence may be seen as a source of the misbelief. nevertheless one would help students break away from this naive idea (Greaber and Baker, 1992).

Although students in fourth to sixth grade typically are introduced to a great deal of information about fractions and learn to operate with fractions, they do not see fractions as indicated division. Students in fourth and fifth grade may not know the long division algorithm for problems such as, $15\overline{)5}$ but potentially they could arrive at an answer to such problems using the knowledge that 5/15 = 3. It appears that the potential connection between information about fractions and information about division is either not made or not used (Greaber and Baker, 1992).

Wiebe (1983) states some reasons for difficulty in performing arithmetical tasks as follows:

"The traditional method for teaching arithmetic is to relate manipulative experiences of the child to written symbols at only the very beginning stages of arithmetic, then to base further instruction on previous symbolic experiences. The problem with this strategy is that children are not able to think in purely abstract, symbolic terms. Thus, relationships and meanings we attempted to develop are understood only by the brightest few, and for the rest arithmetic becomes a process of memorizing algorithms. Eventually for certain children and adults, understanding may come after further cognitive development and after prolonged use of the memorized algorithms."

In 1992 Stefanich and Rokusek conducted a research study to analyze errors made by students in fourth grade in their use of division algorithm and they stated several causes of errors as they perceived as follows:

1. The children do not have a good concept of place value. They do not realize that answers are unreasonable because they can not picture a frame of reference.
2. Children simply do not align their numbers correctly because of inconsistent number size.
3. A lack of knowledge of basic facts. Children have not developed immediate response to basic facts, and, therefore, incorrectly guess or lose interest in the problem and take a short cut. When students are hindered by this lack of success, they develop an easier procedure quickly.

2.6.3 Possible Reasons or Sources of Misconceptions in Solving / Writing Word Problems [involving decimals]

Charles and Lester (cf. Hart, 1984, p.167) stated some factors that affect problem solving performance as follows:

Lack of an experiential framework. When previous experience produced no connection for a student to draw upon in understanding the problem, the student frequently invented definitions or even algorithms for the situation. These invented approaches often seemed cued by some word or set of words in the problem associated with a previously-acquired mathematical concept.

Impositions of unrequired restrictions. Frequently, students would alter the goals of the problem statement by imposing restrictions that were not overtly stated, but that they saw as implicit in the problem. For example, some students assumed that boards could not be cut , or that at least one of each length of board would have to be used. Others tried to impose unnecessary economic constraints.

Lack of individual monitoring or regulating of cognitive activity. At no time did students overtly ask questions such as: " Do I understand this problem ? What am I doing this for ? This doesn't make sense. How will this help me ? "

Unproductive beliefs. The beliefs the students imposed on the problem situation frequently influenced their problem solving performance and may have influenced the other three factors just discussed. For example , one student was confused because she expected that each new sheet of paper would contain a new, unrelated problem. Such confusion might have resulted from a belief formed by the over-generalization of limited classroom experiences. Similarly, the student who assumed that John would not want to buy boards of only one length may have been prompted by the belief that in mathematics word problems, one must use all the numbers.

Thipkong (1988, p.26) summarized the possible reasons of misconceptions in solving word problems as follows:

1. In solving multiplication problems, they are used to solving problems when both multiplier and multiplicand are positive integers, and therefore, the product is always bigger than either the multiplier or multiplicand. Students then conclude that when they solve decimal problems, they will get similar results.

2. The multiplication problems that involve decimals in both the multiplier and multiplicand do not fit students' primitive models of multiplication which are based on repeated addition when the multiplier usually is an integer.

3. In solving division problems, the bigger numbers must always be divided by the smaller numbers.

4. Students generally have had little or no experience with physical and pictorial multiplication and division that involve decimals; for example, 0.23 x 0.58 and 1.76 ÷ 0.38 appear to have no concrete meaning for students.

5. Students also tend to have difficulties when they solve word problems in which they can not analyze a problem situation.

Fischbein et al. (1985) hypothesized that the primitive model associated with multiplication is repeated addition. According to this model a (whole) number of collections of the same size are put together. Multiplication is not seen as commutative in this model. One factor (the number of equivalent collections) is treated as the operator and the other (the magnitude of each collection) as an operand. When this concept of multiplication prevails, the operator "must" be a whole number , and, consequently, the product "must" be greater than the operand. In the domain of whole numbers, where instruction usually begins, possession of the primitive multiplication model can be a source of the belief that "multiplication always makes bigger."

Fischbein et al. (1985) also describe two primitive models for division, a partitive model and a measurement model. In using the primitive partitive model of division, an object or collection of objects is divided into a given whole number of equal parts or subcollections. In using the primitive measurement model, one seeks to determine how many times a given quantity is contained in a larger quantity. Earlier work (Greaber, Tirosh, and Glover, 1986), suggests that American, preservice elementary teachers tend to think of division predominantly in partitive terms. This primitive model, by its behavioral nature, imposes constraints on the operation of division. Two of these constraints are: the divisor must be a whole number and the quotient must be less than the dividend. These constraints can be the source of the belief that "division always makes smaller."

Fischbein et al. (1985) claimed that the "models become so deeply rooted in the learner's mind that they continue to exert an unconscious control over mental behavior even after the learner has acquired formal mathematical notions that are solid and correct." Although a number of researchers from different countries (Bell, 1982; Hart, 1981) have reported that children often explicitly express misbeliefs such as "multiplication always makes larger" and "division always makes smaller," less has been written about whether or not these two beliefs are explicitly held by adults or about adults' sources of support for the beliefs.

If one accepts multiplication and division as inverse operations, the two statements "multiplication always makes larger" and "division always makes smaller" are logically equivalent. The preservice teachers' written justifications, their comments during interviews, and their performance in writing expressions to solve word problems suggest that these beliefs are strongly tied - perhaps to the extent of being one belief. Forty-five percent of the preservice teachers successfully refuted the statement of misbelief about multiplication but did not refute the misbelief about division.

Another possible explanation may be found by considering only procedural knowledge about the operations. In performing the standard multiplication algorithm with one or more decimals, the form of the two factors remain constant and in the final step of the algorithm the decimal point is placed in the answer.

If the preservice teachers' knowledge is limited to or dominated by procedural knowledge, they are more apt to recall the form of the completed multiplication algorithm with a product less than a factor, then to recall the form of a completed division algorithm with a quotient larger than the dividend.

Fischbein, Deri, Nello, and Marino (1985) argued that two primitive models of division (the partitive and quotative) could be the source of misconceptions that explain children's and adolescents' errors in solving multiplication and division word problems with decimals. According to the primitive model of division, an object or collection of objects is divided into a given (whole) number of parts or subcollections. Fischbein et al. (1985) claim that this primitive model has three constraints: (1) The divisor must be less than the dividend, (2) the divisor must be a whole number, and (3) the quotient must be less than the dividend. In applying the primitive measurement model one seeks to determine how many times a given quantity is contained in a larger quantity. The only constraint implied by this model is that the dividend must be larger than the divisor.

Fischbein et al. (1985) suggested two sources for the misconception that the dividend must be less than the quotient. The first is that the misconception is the result of the primitive way of thinking about division as sharing, the partitive model (second) Fischbein et al. (1985) argued that formal schooling first introduces division in the domain of whole numbers and only later is division by rationals, specifically rationals less than one, introduced.

Brown and VanLehn (1982) indicate that a student's errors in solving a problem is frequently the result of the student's invented repair of a known rule or procedure. The repaired rule is invented and applied in order to circumvent a cognitive conflict resulting from differences between the student's existing knowledge and the constraints of the problem space. One can look at the invention of this rule as an attempt of the student to keep his/her knowledge consistent with the constraints stated in the problem or occurred during the solution process.

Farrel (1992) states that students who invent misconceptions seemingly from nowhere, might simply be reacting to the gap between a mathematical concept and meaning. How students learn therefore, depends both on the nature of mathematics and on the intellectual development of the students.

Vergnaud (1983) has developed a theory of the epistomological obstacles that children encounter in learning multiplicative structures and states:

"In addition to the factors that have received systematic study , tentative, ad hoc explanations of pupils' difficulties in solving arithmetic word problems have been offered from time to time . For instance decimals are technically more difficult to handle than whole numbers ; verbal cues may bias the solution process; a concrete context may facilitate finding a solution ; pupils remain bound to the particular meaning originally attached to an operation and most adolescents do not reach the stage of formal operations.

Our thesis is that the concept of an *intervening intuitive model* may explain in a coherent fashion most of the common difficulties children encounter when attempting to solve a problem requiring a single operation . The main exception is the difficulty caused by an unfamiliar text (terms used, situations referred to , etc.), which by itself may lead to considerable confusion and error. The tacit models assumed by us continue to affect the solution process even in adults and, that these models act in a rather unconscious way. We support our claim with the following observations. We have had many opportunities to ask university students, teachers, and researchers to solve some of the typically "difficult" problems that appear in our tests. Most of these adults were not able to indicate directly the operation that would solve the problem. They had to resort to analogies and proportion strategies, and they could report the indirect procedure they had used. Furthermore, all of these adults were surprised to learn that the difficulty derives from the impact of a certain intuitive model. They certainly were not aware of the still-active role that these old, primitive interpretations played in their thinking."

A research conducted by Tirosh and Greaber (1991) aimed to explore preservice elementary teachers performance in division word problems involving decimals. Their findings can be summarized as follows:

1. Preservice teachers' statements during the interviews indicated that the constraints of the primitive partitive division model dominated their thinking even when they solved measurement type problems.

2. For the task of writing expressions for a given word problem , the preservice teachers' success is more affected by conformity to the misconceptions than by the problem type.

3. For the task of writing word problems for given expressions, the preservice teachers' success seemed more affected by the compatibility of the expressions with the partitive interpretation of division than by their adherence to the constraints imposed by the primitive models.

Greaber and Baker (1988) reviewed the third to eighth grade mathematics textbooks of three recent popular series to see if the preservice teachers' recollections were reflected in today's textbooks.To what extend are the examples and word problems dominated by instances in which the divisor is less than the dividend ? In each of the series reviewed less than 14 percent of all the division computation exercises at any grade level included a divisor greater than the dividend . Word problems with the divisor greater than the dividend appeared in grades 4-8 in the series ; however the total number of such word problems in each of the three series was so small (79, 20, 60) that at any given grade level , it is likely a student might never be assigned such a problem. Thus students' early impressions are not frequently challenged in the upper elementary or middle school grades.

2.7 Suggestions or Instructional Approaches which can be Considered in Overcoming Misconceptions [related to decimals]

In the following paragraphs several approaches are discussed which can form a base for the ones who wants to correct students' misconceptions:

First of all we can go over the diagnostic approach.

The aim of diagnostic teaching has been to develop a way of teaching which contributes clearly to long term learning and which promotes transfer. The key aspects of this method are the identification and exposure of pupils' misconceptions and their resolutions through "conflict situations".

Diagnostic teaching in mathematics provides a tool for more effectively directing student remedation as a result of specific error identification. Diagnostic teaching looks at the errors children make and subsequently structures the learning experiences so the errors will be eliminated.

The notion of cognitive conflict derived from Genevan training studies and the importance of feedback of correctness from Gelman's conservation training studies. The value of intensity of experience was highlighted by a Gagne type training study by Trembath and White 1979 in which learning with a stronger mastery criterion took 25% more time but produced 50% more learning (ç.f. Bell, 1994)

The key features of the *diagnostic teaching* methodology are (Bell, 1994) :

- initial presentation of the target tasks, which are those intended for pupils to be able to tackle at the end of the teaching sequence
- choice of tasks to cover the key concepts and likely misconceptions
- choice of sufficiently hard critical tasks to provide cognitive conflict

- provision of some form of feedback of correctness
- intensive discussion aimed at resolving the conflict and forming a newly integrated knowledge structure
- making the key principles explicit, in general terms in the course of this discussion
- further problems, with feedback, to consolidate the insights gained
- flexibility of task, to ensure an appropriate level of challenge for students having varying initial levels of understanding of the concept
- return to the same conceptual points on further occasions, including using different context, until it is clear that the understanding is permanent and transferable

The following suggestions for overcoming misconceptions related to decimals in general are set by Greaber and Tirosh (1990):

"Overcoming or monitoring the misconceptions appears to require acquisition of less primitive conceptual models of the operations which make possible a better linkage between concept and procedure. Preservice teachers ought to be proficient at selecting the correct operation in solving relatively simple word problems. One strategy that first makes students aware of their misconceptions and shows some promise is the conflict teaching method. Preservice teachers might be asked to respond to word problems and then to analyze their errors noting the extent to which the errors reflect a belief that multiplication makes larger. Conflict with the belief "division always make smaller" might be reached by using examples where multiplication results in a product smaller than a factor (e.g., $6 \times 1/2$) to generate the corresponding division sentence $3 \div 1/2 = 6$. Another area of concern is the fact that a substantial number of preservice teachers apparently have only a procedural understanding of division by a decimal less than one. A possible instructional strategy is to review the models for division and require students to estimate or obtain answers without the use of an algorithm. For example , preservice teachers can be asked to provide solutions to "compatible number" problems such as $0.25\overline{)2}$ without using the written algorithm. Such solutions and estimates should be required both before and after the standard algorithm. Since a procedural understanding of division algorithm seems to support the logic of the primitive partitive model (e.g., you can't divide by a decimal) and perhaps strengthen that model, instruction of preservice elementary teachers probably need to stress the measurement model. It seems essential that before students deal with such computations they should (1) exhibit a good understanding of the meaning of decimal notation, (2) be able to interpret division phrases using both the partitive and measurement model, (3) understand that *a/b* notation can signal division, and (4) be facile in translating between common fractions and decimal fractions."

Past studies have shown that misconceptions are not easily monitored or changed. It is essential that instruction for preservice teachers include (Tirosh and Greaber, 1991, p.162):

1) Word problem solving sessions in which the common misconceptions about division are made explicit in discussions concerning reasons for writing a specific expression to solve a problem.

2) Explicit discussion of the primitive models and corresponding constraints and misconceptions , Driver (1987) and Semadeni (1984) have suggested that part of instruction must include a time to discuss a misconception , why it is held and what the correct conception is. Thus, preservice teachers' attention should be focused on those instances in which calculations produce results contrary to the common misconceptions.

3) Wider use of the measurement model. The measurement model is one simple way of giving meaning to expressions with divisors less than one. The similarities and the differences between division with whole numbers and division with non-negative decimals less than one should be explored, made explicit, and applied.

In one of their articles Greaber and Baker (1992) compare the suggestions in overcoming lower and upper graders' misconceptions related to decimals as follows:

"For lower grades (4^{th} , 5^{th}, ...):

1) Offer experiences with division that require the students to decide whether the answer to a division word problem will be less than or greater than one.

2) Introduce and use common-fraction form (for example, 3/4) as indicating division.

3) Continue to link the "fraction as indicated division" notation to other understandings students have about fractions.

4) Encourage students to check their calculated answers to word problems with their estimates and with the data given in the problem.

Later grades:

1) Pose a word problem that will evoke incorrect answers of the "big into little" type.

2) Help students learn to identify the divisor and dividend in a variety of word-problem settings.

3) Encourage students always to reflect on the reasonableness of their answer."

As Fischbein et al. (1985), Thorndike (1921) and others have suggested, it is probably not realistic to consider prevention of the beliefs multiplication makes bigger and division makes smaller. They seem to be a natural outcome of years of work with whole numbers. The middle school teacher will undoubtedly always be in the position of needing to help students control the influence of these beliefs. The conflict teaching approach described by Swan (1983) and the contrast of operations in the whole number and rational domains suggested by Semadeni (1984) are likely candidates for approaches that should be tested. Teachers must also consider Fischbein's (1987) concerns about helping each student cope with the idea that "while helping absolutely convinced about the truth of an idea," such as division makes bigger, "he was in fact wrong". The suggestions above can easily be incorporated into such treatments.

Swan (1983) has reported on a conflict teaching approach to involve students in discussion of , and reflection on their errors and misconceptions. The discussion and reflection are intended to bring students to the realization that their conceptions are inadequate and in need of modification. Appropriate conceptions can then be explored and used. The results suggest that when the conflict approach is carefully applied, preservice teachers may form a more accurate conception about the relative size of the quotient and the dividend and improve their performance in writing expressions for multiplication and division word problems.

Onslow (1990) stated the importance of conflict discussion under the heading "*A Rationale for Conflict Discussion*" as follows:

"Discussion provide an opportunity fro students to say what they mean and mean what they say. Cognitive conflict, a term coined by Inhelder, Sinclair, and Bovet (1974) describing a situation which appears contradictory to a child's logical structure, provides the focus for discussion. Such discussions expose children's errors and encourage students to face their mistake in a positive manner and make constructive use of them. To be effective, the climate in the classroom must be one of mutual respect. Students have to value the opinions of others even though they might disagree with them. Under such conditions. it is possible to establish an atmosphere of cooperative learning. Once students feel confident about expressing their ideas, whether correct or incorrect, they can develop a better appreciation of their thoughts and contributions. Students dealing with numbers between zero and one need to interpret multiplication as a scalar factor, not merely as a process of repeated addition, if they are to overcome the misconception multiplication always makes bigger. Conflict situations have to be devised to provide an imbalance in cognitive structure. Children may then perceive a need to broaden their conceptual frameworks to more inclusive ones that do not provoke contradiction."

Meyers (1924) noted it is better for teachers to spend their time analyzing the written work of children and planning corrective instruction rather than using what time they have for scoring.

Ashlock (1972) stated that the diagnosis of errors in arithmetic is an essential part of evaluation in the mathematics program, and any such diagnosis must be accompanied by remedial or corrective instruction.

Rokusek (1992) suggests a process of proceduralizing a new skill as follows:

1. Identify the sequence of steps necessary to perform the task.

2. Write the steps down.

3. Before performing the task, read over the procedure you have written and mentally picture yourself performing the procedure.

4. Occasionally revise your procedure, adding or deleting steps to make it better.

5. When the procedure becomes automatic, disregard your description.

Rokusek (1992) also suggests the following in helping students to address systematic errors:

1. Make sure the children have a step-by-step procedure to follow when going through the division algorithm.

2. Use manipulative whenever possible to help students create a visual image of the process. This will help them bring meaning to the algorithm.

3. Train the children to make an estimate of their answer first. This will help children become aware of reasonable answers.

4. Involve the three processes with cooperative learning group interactions.

5. As a teacher find the trouble spot. This is the place where the child's errors increase as they proceed through the skill levels. If it doesn't happen until level 4, then take advantage of their previous knowledge and build from here. In one-to-one or error group situation, start from where the error starts.

Behr and Harel (1990) state "it is important to recognize that cognitive conflict, or inconsistency, is not a negative aspect of learning. It is likely that no learning takes place unless some degree of conflict exists. Broadly speaking, cognitive conflict can be associated with a Piagetian construct of disequilibrium. For educational practice the issue is on the questions of: (a) how to create situations which establish an appropriate level of cognitive conflict for individual learners and (b) how to guide students to conflict resolutions through the construction of knowledge structures which are consistent with domain principles."

Later Behr and Harel (1990) stated that they believed, however, that in order to answer those questions for educational practice, research in mathematical cognition needed continued emphasis on:

1. Gaining insight into children's knowledge structure in various mathematical domains,
2. Analyses of mathematical domains from both logical and cognitive perspectives,.
3. Identification of cognitive structures which seem to be necessary for expertise in these mathematical domains,
4. The development of experiences, based on the knowledge of the above, that cause cognitive conflict and lead toward conflict resolution,
5. Explanation of how to (a) inhibit the learners' invention or construction of "buggy" strategies and (b) facilitate the learners' invention or construction of domain consistent strategies.

Farrell (1992) suggests: "To study the error patterns or basic misconceptions of students, teachers may need to incorporate feedback-gathering strategies that yield explanatory data. Teachers might create opportunities to have students talk with one another about their approaches to a set of problems, write how they would explain a new concept to a younger sibling or an absent friend, or, using a calculator or computer, test their invented algorithm on a set of examples."

In order to break the cycle of teachers with weak conceptual backgrounds providing conceptually impoverished instruction , preservice mathematics courses will need to prepare prospective teachers more adequately. There is a need for a mathematical education considerably different from what is currently available on most collage campuses. Mathematics course work should provide prospective teachers the opportunity to understand the concepts underlying the mathematics that they will teach and the relationships of these concepts to the algorithms that they previously mastered. This would involve making the development of dense webs of understandings (e.g. multiplicative structures) a higher priority than vertical content coverage (e.g. passing courses through calculus) (Simon, 1993, pp. 252-253).

Sophisticated mathematical concepts such as rational numbers equivalence develops slowly over time. Teachers need to provide many opportunities to explore and reflect on the following ideas: (a) rational numbers have many names or clones, (b) renaming a number does not change its properties, and (c) the best name for a number depends on the situation. Having students talk and write about how they create or recognize equivalent fractions and decimals and how the idea of equivalence is applied in the solving of various problems can strengthen their understanding and provide valuable information for the teacher (Vance, 1992, p.266).

Posner, Strike, Hewson and Gertzog (1982)state: "We believe there are analogous patterns of conceptual change in learning. Sometimes students use existing concepts to deal with new phenomena. This variant of the first phase of conceptual change we call assimilation. Often , however, the students' current concepts are inadequate to allow him to grasp some new phenomenon successfully. Then the student must replace or reorganize his central concepts. This more radical form of conceptual change we call accommodation."

The following four are common to most cases of accommodation (Posner et al., 1982:

1) There must be dissatisfaction with existing conceptions.

2) A new concept must be intelligible

3) A new conception must appear initially plausible.

4) A new concept should suggest the possibility of a fruitful research program.

If we aim to produce rationally based conceptual change in students, the content of mathematics and science courses should be such that it renders scientific theory intelligible, plausible, and fruitful. In order to give expressions to this general requirement, the following conditions appear to be necessary:

1. More emphasis should be given to assimilation and accommodation by students of that content than to content "coverage".

2. "Retrospective anomalies" should be included, particularly if historically valid anomalies are difficult to comprehend, or, as with the special theory, were not responsible for driving the conceptual change in the first place.

3. Sufficient observational theory should be taught for students to understand the anomalies employed.

4. Any available metaphors, models, and analogies should be used to make a new conception more intelligible and plausible (Posner et al., 1982)

For teaching aimed at accommodation the following approaches can be used:

1. Develop lectures, demonstrations, problems, and labs which can be used to create cognitive conflicts in students. Among other things, one might consider what types of homework problems would create the kind of cognitive conflict necessary as preparation for an accommodation (Stavy & Berkovitz, 1980).

2. Organize instruction so that teachers can spend a substantial portion of their time in diagnosing errors in student thinking and identifying defensive moves used by students to resist accommodation (Posner et al., 1982).

3. Develop the kinds of strategies which teachers could include in their repertoire to deal with student errors and moves that interfere with accommodation (Posner et al., 1982).

4. Help students make sense of the content by representing content in multiple modes (e.g. verbal, mathematical, concrete-practical, pictorial), and by helping students translate from one mode of representation to another (Clement, 1977).

5. Develop evaluation techniques to help the teacher track the process of conceptual change in students (Postner & Gertzog, 1982).

In these roles the teacher becomes:

1. An adversary in the sense of a Socratic tutor. In this role the teacher confronts the student with the problem arising from their attempts to assimilate new conceptions (Posner et al., 1982).

2. A model of scientific thinking. Aspects of such a model might include a ruthless pursuit of parsimony among beliefs, a skepticism for excessive "ad hoc-ness" in theories and a critical appreciation of whether discrepancies between results may be in "reasonable agreement" with theory (Posner et al., 1982).

The main idea is that in order to learn a new concept, pupils must be actively involved in a process of reshaping and restructuring of their knowledge. The starting point of the process of conceptual change is the students' "naive knowledge" which although often imprecise poorly differentiated and different from the intended scientific knowledge, "has served the student successfully" (Champagne et al., 1983).

According to Postner, Strike, Hewson, and Gertzog (1982), the phase of conflict, of dissatisfaction with existing concepts is central to the process of conceptual change : only at this stage will students realize that they must "replace or reorganize" their "central concepts" because they are "inadequate to allow him to grasp some new phenomenon such process must be, from the point of view of the students, intelligible plausible, and fruitful.

In the words of the National Council of Teachers of Mathematics (NCTM):

"A conceptual approach enables children to acquire clear and stable concepts by constructing meaning in the context of physical situations and allows mathematical abstractions to emerge from empirical experience. A strong conceptual framework also provides anchoring for skill acquisition. Skills can be acquired in ways that make sense to children and in ways that result in more effective learning. A strong emphasis on mathematical concepts and understanding also support the development of problem solving" (1989, p.17).

"When a student holds a misconception, presenting a principle with supporting examples to show the range of application of the principle may be ineffective. Rather, it appears that examples are more effective when they help students draw on and analogically extend existing valid physical intuitions in constructing a new conceptual model of a target situation. To help students in this constructive effort, first the examples used must be understandable and believable to the students, not simply to the teacher or textbook author. Second, even when an example is compelling to the student, it may not be seen as analogous to target problems drawing out a misconception. In this case, analogy relations may need to be explicitly developed. Third, qualitative, visualisable models may need to be developed which give mechanistic explanations for phenomena" (Brown, 1992, p.17).

If children are actively involved in the situation and are encouraged to decide upon the merits of different strategies among themselves, there is more likelihood that they will retain the concepts being presented (Piaget, 1970).

Recent research in science education shows the importance of cognitive conflict as a teaching strategy (D'Ambrosio & Campos, 1992; Dreyfus, Jungwirth & eliovitch, 1990).

The importance of cognitive conflict in human development has been recognized by psychologists (Festinger, 1957; Piaget, 1980; Vygotsky, 1978). A cognitive conflict can be produced by various situations: (a) surprise produced by a result which contradicts a subject's expectations, resulting in the generation of perturbations; (b) experience of puzzlement, a feeling of uneasiness, a more or less conscious conflict, or a simple intellectual curiosity; (c) experiencing a cognitive gap, as if the person involved were vaguely aware that something within his knowledge structure was missing; (d) disequilibria - that is, questions or felt lacunae that arise when the subject attempts to apply his schema to a given situation. Within the constructivist framework the development of conflicts or contradictions is essential to facilitate conceptual change and has been summarized by saying that cognitive change and learning take place when a scheme, instead of producing the expected result, leads to perturbation and perturbation in turn, leads to accommodation that establishes a new equilibrium.

A teaching strategy based on cognitive conflict, should take into account on the following considerations (Niaz, 1995, pp.969-961):

"1) Cognitive conflict must be based on problem-solving strategies that students find relatively convincing. This is based partially on Mischel's (1971) recommendation that "The cognitive conflicts which the child himself endengers in trying to cope with his world, are what motivates his cognitive development; they are his motives for reconstructing his system of cognitive schemas.

2) Teaching strategies used for introducing cognitive conflicts must be based on data that may be contrary to the expectations of at least some students.

3) After generating a cognitive conflict, it is essential that the students be provided with an experience that could facilitate the resolution of the conflict. This part of the strategy is essential for the Piagetian dialectic concept of bipolar unity to operate. According to Bidell (1988), "the concept of bipolar unity is especially important to the Piagetian corpus because it explicitly formulates the role of dialectical contradiction that is central part of the work of Piaget and Inhelder". According to the bipolar conceptualization, conflict generation would correspond to the Piagetian concept of assimilation, whereas conflict resolution would correspond to accommodation.

From a philosophy of science perceptive, any conceptual change would have to go through the following sequence: (a) students must become cognizant of the conflicting (anomalous) data; (b) based on his experience, students must discard their existing theoretical framework (core belief); (c) a better theory must be constructed that explains the data."

Using readiness, concrete, and semiconcrete experiences would help children develop conceptual understanding of mathematics. By joining the three levels with Bruner's (1963) enactive and iconic ways of knowing , three levels help children know mathematics kinesthetically (enactive) and through mental imagery (iconic). Later, when learners have abstract or symbolic experiences (when there are no manipulatives or diagrams available), they can call on their enactive and iconic knowledge to make sense of the symbols (Eisenhart et al., 1993).

Vest (1985) suggests the following approach in explaining a division algorithm:

"For all levels of students, manipulation of concrete objects serves to communicate division concepts in a way that abstract symbols do not match. When studying complex calculations, students can advance from actual manipulations of concrete objects to carefully described and imagined manipulations aided by careful record keeping. When students think in terms of mental images of concrete manipulations, they gain confidence in their ability to understand mathematics and to solve problems. For example, it is important that students learn to think of $12 \div 4 = x$ by imagining separating a set of 12 objects into sets of 4 each, making 3 sets."

Students must at the beginning, construct meanings for the written symbols and then use the meanings to develop procedures for operating with symbols (Wearne and Hiebert, 1989, p.512).

2.8 Previous Studies in Determining [Mis]conceptions and/or Efforts in Overcoming Misconceptions

In 1982 Bell et. al conducted a study on a group of students (N=18) aged 12-16. They aimed to increase the understanding of students in choice of operation, place value, operations, and unit measures using a diagnostic teaching approach. The students' understandings in each of the dimensions were measured in three occasions; before the treatment, just after the treatment and two weeks after the treatment. There were maintained improvements in understanding of decimal place value and more modest increases elsewhere. The declines at the delayed post-test stage were associated with concepts on which little teaching was given and where the emphasis was on memorising.

In 1983 Swan conducted a study on 42 middle school students. The researcher aimed to observe how conflict discussions can help in overcoming some misconceptions related to the meaning of decimals. For this purpose the subjects were divided into two groups as conflict (N=22) and positive-only (N=25). In the conflict group student were involved in discussions whereas in the positive only group more traditional method were used. The results showed significant differences between the two groups favoring the conflict group.

In 1986 Tirosh conducted a study on 59 college students enrolled in one of the mathematics content or method courses for early elementary education majors in the spring quarter at the University of Georgia. The aim was to diagnose and correct preservice teachers' misconceptions about the operation of division using a diagnostic computer program. The results of the study showed that the developed diagnostic computer program was effective in identifying students who held the misbelief that the divisor must be less than the dividend. The program was also effective in helping students become aware of their tendency to reverse the role of divisor and dividend.

In 1992 Perso conducted a research on middle school students aged 13-16. The aim was to overcome the misconceptions of students related to algebra. For this purpose diagnostic (conflict) teaching materials were developed and used. The results showed that the treatment with each of the four groups was significantly successful, both in the short term and in the long term.

Aksu (1997) conducted a study on 155 sixth grade students. The main purpose of the study was to observe the differences in student performance when fractions were presented in the three contexts of a) understanding the meaning of fractions, b) computations with fractions, and c) solving word problems involving fractions. The results showed that there were no differences in performance on four operations when fractions were presented in computations. On the other hand it was reported that addition problems were the simplest to perform whereas multiplication problems were the most difficult in word problems.

Eryılmaz (1992) administered a diagnostic test to 946 students in 1990-1991 and 401 students in 1991-1992 academic year. The purpose of the study was to find the factors affecting students' misconceptions and achievement in introductory mechanics course at university level. The research has revealed that most of the freshmen students at METU, as well as inservice physics teachers had many misconceptions in introductory mechanics. Although the conventional instruction had some positive effects on the students' misconceptions in introductory mechanics, it was far from being sufficient in removing certain misconceptions which were persistent and highly resistant to change.

Eryılmaz (1996) conducted a study which aimed to find the effect of three instructional methods (conceptual assignments, conceptual change discussion method, and Computer Assisted Instruction (CAI) program emphasising cognitive conflict) on students' misconceptions about force and motion. The study was conducted with 6 physics teachers, 18 classes, a total of 396 high school students. Weak evidence was provided by the study that CAI program, the conceptual assignments, and the treatments interactions effects were not an effective means of reducing the number of misconceptions students held and significantly improving students' physics achievement in force and motion.

Başer (1997) conducted a study which aimed to explore the effectiveness of refutational text over science text on remediation of students' misconceptions related to heat and temperature at 7th grade level (N=72), when both teaching methods were used as a supplement to the regular classroom instruction. The results of the study indicated that students who were taught by refutational texts as a supplement to regular classroom instruction produced significantly greater achievement on heat and temperature concepts achievement than students who were taught by science texts as a complement to regular classroom instruction.

In sum it is possible to say that although there are many efforts in diagnosing or determining misconceptions, still we need to focus heavily on correcting or overcoming misconceptions in decimals and in many other fields.

CHAPTER III

3. METHOD

This chapter is devoted to the presentation of the problem, hypothesis and the design of the study. The purpose of the present study was to determine and overcome preservice elementary teachers' misconceptions in interpreting and applying decimals. Because of the dual nature of the present study it was conducted in two stages as mentioned in the following paragraphs.

3.1 Pilot Study

A pilot study was conducted in March 1996, to investigate the background knowledge of preservice teachers at Atatürk Öğretmen Koleji related to decimals and observe their misconceptions (if any) and decide if there was a need for a further research. The test items were developed previously by Aykut Inan Işeri (1997). Two forms of the test were used to find out the performance of preservice teachers in "basic decimal concepts" and "choosing the appropriate operation for word problems involving decimals".

The instrument was administered on 49, second year, preservice elementary teachers at Atatürk Öğretmen Koleji ,who took two mathematics content courses.

This Pilot study showed that most of the preservice elementary teachers had difficulties with the concept of decimals , units, and multiplication word problems when both multiplicand and multiplier were decimals. In division word problems it seemed that most of the preservice teachers hold the misconception that “divisor should always be less than the dividend” and “the divisor should be a whole number”.

It was obvious that preservice teachers had some problems related to decimals in general and it has been decided to conduct a study in this field. It was also aimed to develop a test form in which preservice teachers can show their word problem writing performance.

3.2 Overall Design of the Study

As it was mentioned before the present study needed to be conducted in two stages and the following figure summarizes the overall flow of the present research.

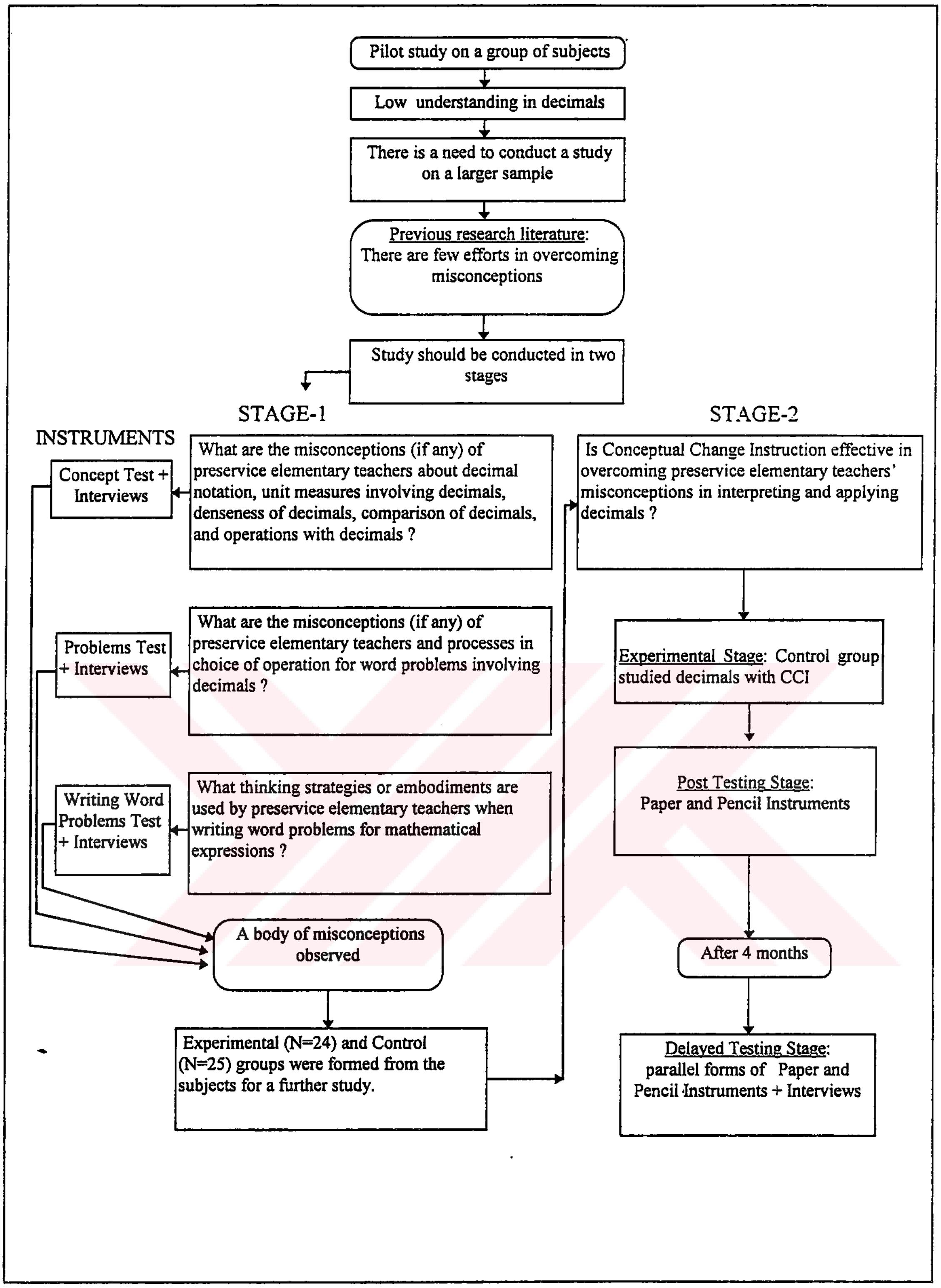


Figure 3.1: Overall Flow of the Study.

3.2.1 Purpose of Stage - 1

The main purpose of Stage - 1 was to determine preservice elementary teachers' misconceptions in interpreting and applying decimals which can be enriched by the following sub-purposes:

i) To investigate preservice elementary teachers' misconceptions and their interpretation of decimal concepts in working with decimal notation, subunits based on ten and not based on ten , comparison of decimals, denseness of decimals, unit measures involving decimals and operations with decimals.

ii) To investigate and analyze preservice elementary teachers' misconceptions and processes used in solving word problems in multiplication and division involving decimals.

iii) To describe the thinking strategies or embodiments used by preservice elementary teachers when writing word problems for mathematical expressions involving decimals.

3.2.2 Purpose of Stage - 2

The purpose of Study - 2 was to explore and analyse the effects of the Conceptual Change Instruction on overcoming the misconceptions of preservice elementary teachers in interpreting and applying decimals.

3.2.3 Problems of Stage - 1

3.2.3.1 Main Problem of Stage - 1

What are the misconceptions (if any) of preservice elementary teachers in interpreting and applying decimals ?

3.2.3.2 Problem 1 of Stage - 1

What are the misconceptions (if any) of preservice elementary teachers about decimal notation, unit measures involving decimals, denseness of decimals, comparison of decimals and operations (multiplication and division) involving decimals ?

3.2.3.3 Sub-problems of Problem 1

i) How do preservice elementary teachers interpret decimals as points on number lines with subunits based on ten and subunits not based on ten ?

ii) How do preservice elementary teachers interpret decimals on shaded areas with subunits based on ten and subunits not based on ten ?

iii) How do preservice elementary teachers compare decimals ?

iv) How do preservice elementary teachers interpret the denseness of decimals ?

v) How do preservice elementary teachers interpret decimals involving unit measures , subunits based on ten and subunits not based on ten ?

vi) How do preservice elementary teachers interpret multiplication and division involving decimals ?

3.2.3.4 Problem 2 of Stage - 1

What are the misconceptions (if any) of preservice elementary teachers and processes used in solving (choosing the appropriate operation) word problems involving decimals?

3.2.3.5 Sub-Problems of Problem 2:

i) What are the misconceptions (if any) of preservice elementary teachers in solving division word problems involving decimals ?

ii) What are the processes used by preservice elementary teachers in solving division word problems involving decimals ?

iii) What are the misconceptions (if any) of preservice elementary teachers in solving multiplication word problems involving decimals ?

iv) What are the processes used by preservice elementary teachers in solving multiplication word problems involving decimals ?

3.2.3.6 Problem 3 of Stage - 1

What thinking strategies or embodiments are used by preservice elementary teachers when writing word problems for mathematical expressions ?

3.2.3.7 Sub-Problems of Problem - 3

i) What thinking strategies or embodiments are used by preservice elementary teachers when writing word problems for multiplicative expressions ?

ii) What thinking strategies or embodiments are used by preservice elementary teachers when writing word problems for division expressions ?

3.2.4 Problems of Stage - 2

3.2.4.1 Main Problem of Stage - 2

Is the Conceptual Change Instruction (CCI) effective in overcoming preservice elementary teachers' misconceptions in interpreting and applying decimals ?

3.2.4.2 Problem 1 of Stage - 2

Is there a significant difference between experimental and control group subjects in terms of achievement related to decimal notation, comparing of decimals, unit measures involving decimals, denseness of decimals, and operations with decimals ?

3.2.4.3 Problem 2 of Stage - 2

Is there a significant difference between experimental and control group subjects in terms of achievement related to choosing the appropriate operation for word problems involving decimals ?

3.2.4.4 Problem 3 of Stage - 2

Is there a significant difference between experimental and control group subjects in terms of achievement related to writing word problems for mathematical expressions ?

3.2.4.5 Hypotheses of Stage - 2

Hypothesis 1: There is no significant difference between mean post CT scores of subjects in the experimental and control groups.

Hypothesis 2: There is no significant difference between mean delayed CT scores of subjects in the experimental and control groups.

Hypothesis 3: There is no significant difference between mean post PT scores of subjects in the experimental and control groups.

Hypothesis 4: There is no significant difference between mean delayed PT scores of subjects in the experimental and control groups.

Hypothesis 5: There is no significant difference between mean post WWPT scores of subjects in the experimental and control groups.

Hypothesis 6: There is no significant difference between mean delayed WWPT scores of subjects in the experimental and control groups.

Hypothesis 7: There is no significant difference between mean post CT1.1 scores of subjects in the experimental and control groups.

Hypothesis 8: There is no significant difference between mean delayed CT1.1 scores of subjects in the experimental and control groups.

Hypothesis 9: There is no significant difference between mean post CT1.2 scores of subjects in the experimental and control groups.

Hypothesis 10: There is no significant difference between mean delayed CT1.2 scores of subjects in the experimental and control groups.

Hypothesis 11: There is no significant difference between mean post CT2.1 scores of subjects in the experimental and control groups.

Hypothesis 12: There is no significant difference between mean delayed CT2.1 scores of subjects in the experimental and control groups.

Hypothesis 13: There is no significant difference between mean post CT2.2 scores of subjects in the experimental and control groups.

Hypothesis 14: There is no significant difference between mean delayed CT2.2 scores of subjects in the experimental and control groups.

Hypothesis 15: There is no significant difference between mean post CT3.1 scores of subjects in the experimental and control groups.

Hypothesis 16: There is no significant difference between mean delayed CT3.1 scores of subjects in the experimental and control groups.

Hypothesis 17: There is no significant difference between mean post CT3.2 scores of subjects in the experimental and control groups.

Hypothesis 18: There is no significant difference between mean delayed CT3.2 scores of subjects in the experimental and control groups.

Hypothesis 19: There is no significant difference between mean post CT4 scores of subjects in the experimental and control groups.

Hypothesis 20: There is no significant difference between mean delayed CT4 scores of subjects in the experimental and control groups.

Hypothesis 21: There is no significant difference between mean post CT5 scores of subjects in the experimental and control groups.

Hypothesis 22: There is no significant difference between mean delayed CT5 scores of subjects in the experimental and control groups.

Hypothesis 23: There is no significant difference between mean post CT6 scores of subjects in the experimental and control groups.

Hypothesis 24: There is no significant difference between mean delayed CT6 scores of subjects in the experimental and control groups.

Hypothesis 25: There is no significant difference between mean post PT1.1 scores of subjects in the experimental and control groups.

Hypothesis 26: There is no significant difference between mean delayed PT1.1 scores of subjects in the experimental and control groups.

Hypothesis 27: There is no significant difference between mean post PT1.2 scores of subjects in the experimental and control groups.

Hypothesis 28: There is no significant difference between mean delayed PT1.2 scores of subjects in the experimental and control groups.

Hypothesis 29: There is no significant difference between mean post PT1.3 scores of subjects in the experimental and control groups.

Hypothesis 30: There is no significant difference between mean delayed PT1.3 scores of subjects in the experimental and control groups.

Hypothesis 31: There is no significant difference between mean post PT2.1 scores of subjects in the experimental and control groups.

Hypothesis 32: There is no significant difference between mean delayed PT2.1 scores of subjects in the experimental and control groups.

Hypothesis 33: There is no significant difference between mean post PT2.2 scores of subjects in the experimental and control groups.

Hypothesis 34: There is no significant difference between mean delayed PT2.2 scores of subjects in the experimental and control groups.

Hypothesis 35: There is no significant difference between mean post PT2.3 scores of subjects in the experimental and control groups.

Hypothesis 36: There is no significant difference between mean delayed PT2.3 scores of subjects in the experimental and control groups.

Hypothesis 37: There is no significant difference between mean post PT2.4 scores of subjects in the experimental and control groups.

Hypothesis 38: There is no significant difference between mean delayed PT2.4 scores of subjects in the experimental and control groups.

Hypothesis 39: There is no significant difference between mean post WWPT-MULT scores of subjects in the experimental and control groups.

Hypothesis 40: There is no significant difference between mean delayed WWPT-MULT scores of subjects in the experimental and control groups.

Hypothesis 41: There is no significant difference between mean post WWPT-DIV scores of subjects in the experimental and control groups.

Hypothesis 42: There is no significant difference between mean delayed WWPT-DIV scores of subjects in the experimental and control groups.

Hypothesis 43: There is no significant difference between mean post WWPT-DLTD scores of subjects in the experimental and control groups.

Hypothesis 44: There is no significant difference between mean delayed WWPT-DLTD scores of subjects in the experimental and control groups.

Hypothesis 45: There is no significant difference between mean post WWPT-DGTD scores of subjects in the experimental and control groups.

Hypothesis 46: There is no significant difference between mean delayed WWPT-DGTD scores of subjects in the experimental and control groups.

3.2.5 Design of the Study

In this section; the subjects, the instruments, the procedure, the data analysis, the assumptions and the limitations of the study will be presented.

3.2.5.1 Subjects

The subjects for Stage - 1 consisted of 72 first year preservice elementary teachers who enrolled in a mathematics content course in the spring semester of 1997 at the *Atatürk Teachers College*, in the Turkish Republic of Northern Cyprus (TRNC). Fifty-seven (57) were females and fifteen (15) were males. In TRNC students who come to the high school level have two alternatives to choose from; they can either choose the *arts* department where the mathematical courses are in an intermediate level or choose the *science* department where the mathematical courses are more advanced compared to the arts department. In this respect for mathematical background in high school we can say that thirty-seven (37) of the subjects were *science* and thirty-five (35) Were *arts* students.

The following table gives detailed information about the mathematical background of the subjects at high school:

Table 3.1: Distribution of Subjects for Stage - 1

FEMALES		MALES	
Arts	*Science*	*Arts*	*Science*
26	31	9	6
N=57		N=15	
TOTAL = 72			

After the first administration of Concept (see Appendix-A) and Problems Tests (see Appendix-B) the preservice elementary teachers were named as Low (preservice elementary teachers below 40^{th} percentile), Medium (preservice elementary teachers between 40^{th} and 80^{th} percentile), and High scorers (preservice elementary teachers above 80^{th} percentile) according to their Concept test and Problems Test scores. The distribution of preservice elementary teachers is shown in table 3.2:

Table 3.2: Distribution of Subjects as Scorers

			Concept Scores			
			Low	*Medium*	*High*	
			3 - 21	*22 - 31*	*32 - 44*	*Total*
Problem Scores	*Low*	*0 - 19*	**13**	**6**	**3**	**22**
	Medium	*20 - 22*	**7**	**8**	**7**	**22**
	High	*23 - 26*	**5**	**9**	**14**	**28**
		Total	**25**	**23**	**24**	**72**

- *The numbers in bold letters show the number of preservice teachers in each cell.*

Later a group of preservice elementary teachers (N=25) were randomly chosen to form a group to be interviewed. Table 3.3 shows the distribution of preservice elementary teachers (interviewees) in each cell.

Table 3.3: Distribution of Preservice Elementary Teachers as Interviewees.

			Concept Scores			
			Low	*Medium*	*High*	
			3 - 21	*22 - 31*	*32 - 43*	*Total*
Problem Scores	*Low*	*0 - 19*	**5**	**5**	**-**	**10**
	Medium	*20 - 22*	**3**	**2**	**3**	**8**
	High	*23 - 26*	**1**	**4**	**2**	**7**
		Total	**9**	**11**	**5**	**25**

- *The numbers in bold letters show the number of preservice teachers in each cell.*

The subjects of the Stage - 2 consisted of 49 (a subset of the previous 72 preservice elementary teachers) first year preservice elementary teachers at Atatürk Teachers Training College in the Turkish Republic of Northern Cyprus. The subjects Were composed of two classes and the classes were randomly assigned as control (N=25) and experimental groups (N=24). Table 3.4 shows the distribution of subjects according to gender.

Table 3.4: Distribution of Subjects for Stage - 2

	Male (#)	Female (#)	Total
Control Group	6	19	25
Experimental Group	6	18	24
Total	12	37	49

In order to see if the CCI was effective in overcoming the preservice elementary teachers' misconceptions and follow the progression of those preservice teachers, a subset of the interviewees (previously interviewed) of the Stage-1 who fell into control and experimental groups were chosen to be followed. Table 3.5 shows the distribution of those subjects.

Table 3.5: Distribution of Preservice Elementary Teachers as Scorers and Interviewees for Stage -2

			Concept Scores		
			Low	*Medium*	*High*
			3 - 21	***22 - 31***	***32 - 43***
Problem Scores	***Low***	***0 - 19***	1(*2*)	1(*1*)	-
	Medium	***20 - 22***	1(*1*)	1(*1*)	2(*0*)
	High	***23 - 26***	1(*0*)	1(*2*)	0(*0*)

- *Italic numbers in the brackets represent the number of interviewees (N=7) in the control group and the other numbers represent the number of interviewees (N=8) in the experimental group.*
- *Bold numbers stand for score ranges.*

3.2.5.2 Variables

The independent variable of this study is the Conceptual Change Instruction.

The dependent variables are the preservice elementary teachers' achievement related to decimals which can be divided into 3 main and 18 sub-dimensions as follows:

Achievement related to:

1) Decimal Concepts

1.1) Decimals as points on number lines with subunits based on ten.

1.2) Decimals as points on number lines with subunits not based on ten.

1.3) Decimals on shaded areas with subunits based on ten.

1.4) Decimals on shaded areas with subunits not based on ten.

1.5) Denseness of decimals.

1.6) Comparison of decimals.

1.7) Multiplication and Division involving decimals.

2) Choosing the appropriate operation for word problems

2.1) Choosing the appropriate operation for multiplication word problems.

2.2) Choosing the appropriate operation for multiplication word problems suitable for direct proportion.

2.3) Choosing the appropriate operation for multiplication word problems not suitable for direct proportion.

2.4) Choosing the appropriate operation for division word problems.

2.5) Choosing the appropriate operation for division word problems suitable for direct proportion.

2.6) Choosing the appropriate operation for division word problems not suitable for direct proportion.

2.7) Choosing the appropriate operation for division word problems in which the divisor is greater than the dividend.

3) Writing word problems for division and multiplication expressions involving decimals.

3.1) Writing word problems for multiplication expressions involving decimals.

3.2) Writing word problems for division expressions involving decimals.

3.3) Writing word problems for division expressions involving decimals in which the divisor is less than the dividend.

3.4) Writing word problems for division expressions involving decimals in which the divisor is greater than the dividend.

3.2.5.3 Instruments

Three paper- and- pencil and a semi-structured interview schedule were used in this study. The instruments were developed / adapted by the researchers. The three paper-and -pencil instruments were used in the forms of pretest (see Appendices-A,B,C), posttest and delayed test (see Appendices-D, E, F), whereas the interview schedule (see Appendices-G and H) was used after the application of pretests and delayed tests.

3.2.5.3.1 Concept Test (CT):

The Concept Test (see Appendix - A) included 44 - items. The test was used in order to investigate the understandings of preservice elementary teachers about decimals in general which can be categorised as follows:

1. marking a point on a number line
2. writing the decimal for the point that the arrow indicates on a number line
3. marking and shading of area models
4. comparing decimals
5. unit measures involving decimals
6. denseness of decimals
7. operations with decimals

For scoring, each problem on the Concept test was scored one (1) for the correct and zero (0) for the wrong answer or no response.

As it was stated in Table 3.6, it has been tried to develop / adapt items for the Concept test in which the preservice teachers could show their understanding related to decimals in a wide range of context, such as marking a point on a number line, writing the decimal for the point that the arrow indicates on a number line, marking and shading of area models, comparing decimals, unit measures involving decimals, denseness of decimals, and operations with decimals. In addition to the variability of the items in terms of context, we also tried to investigate how preservice elementary teachers' performance change in dealing with the decimals in cases where subunits were and were not based on ten. In line with that, we put some items to the Concept test that can show the distinctive characteristics of subunits based on ten and subunits not based on ten. A similar effort was previously done by Thipkong and Davis (1991).

In the concept test we mainly developed / adapted items in multiplication and division that were built either on the so called Primitive Models of Fischbein (1985) or vice versa. Distribution of items of the Concept Test is stated in the following table.

Table 3.6: Distribution of Items of the Concept Test Among the Specified Categories.

CATEGORY	ITEMS	NUMBER OF ITEMS
Decimals as points on number lines	1,2,3,5,9,14,15,16,17[1] *4,6,7,8,10,11,12,13,18*[2]	9 (CT1.1[3]) 9 (CT1.2)
Decimals on shaded areas	20,23,25 *19,21,22,24*	3 (CT2.1) 4 (CT2.2)
Decimals involving unit measures	34,36,37 *33,35,38*	3 (CT3.1) 3 (CT3.2)
Denseness of decimals	31,32	2 (CT 4)
Comparing decimals	26,27,28,29,30	5 (CT 5)
Multiplication and division involving decimals	39,40,41,42,43,44	6 (CT 6)

1: Item numbers in normal letters represent the items in which *subunit based on ten* considered.
2: Item numbers in italics represent the items in which *subunit not based on ten* considered.
3: Abbreviations in the brackets show to which sub-category do the items fall.

The Concept Test (which was administered before the treatment) was tested on 52 second year, preservice elementary teachers at Atatürk Öğretmen Koleji. The preservice teachers had a total of 30 minutes for the Concept Test. There were 45 questions in the Concept test at the beginning but through a classical item -analysis technique we decided to ignore item-26 in which we tried to test if the preservice teachers can compare decimals. Most of the preservice teachers at the extreme ends (low scorers (N=13) and high scorers (N=14)) answered correctly item-26. Finally the number of questions in Concept Test was reduced to 44. The reliability for the preservice elementary teachers' scores in the Concept Test (which was administered before the treatment) was computed using Alpha reliability procedure from SPSS Version 6.0 for Windows. The reliability of the Concept test was 0.92. The content validity of the Concept Test was judged by one foreign (English) expert in the field and by two other mathematics educators from METU. They all decided that the items in the test could help to investigate possible misconceptions about decimals. Although, most of the items could challenge possible misconceptions related to decimals, I also hypothesised that the items that were built on *subunits not based on ten* were more powerful in challenging possible misconceptions than the *subunits based on ten* items. In this respect, in order to test this hypothesis we applied a paired samples t-test for the same group of students (N=52) which could be an estimate for the construct validity of the Concept Test. The following table summarises the related results:

Table 3.7: Comparison of the Mean Scores for the Sub-Scales of the Concept test Related to Subunits Based on Ten and Not Based on Ten

Variable	Number of Pairs	Corr	2-tail Sig.	Mean	SD	t-value	df	2-tail Sig.
CT1.2	52	0.867	0.000	5.3462	2.611	-6.45	51	0.000
CT1.1				6.6538	2.936			
CT2.2	52	0.540	0.005	0.9200	1.187	-3.10	51	0.005
CT2.1				1.7100	1.458			
CT3.2	52	0.621	0.000	1.4808	1.448	-3.73	51	0.000
CT3.1				2.0962	1.257			

Since all p-values were less than 0.05 level, we concluded that the performance of the group was better on the items that were related to subunits based on ten, which means that the Sub-scales that built on subunits not based on ten can challenge misconceptions related to decimals better than the Sub-scales that are built on subunits based on ten. These findings, of course, strengthen the construct validity of the Concept test.

Later we changed only the decimal numbers used in the Concept Test items and formed another test to be used in the post and delayed testing periods. In order to test if the final form of the Concept test was parallel or coincident to the pre-form of the same test the pre and final forms of the test were administered on a group of 25 third year preservice elementary teachers at Atatürk Öğretmen Koleji. For testing the parallelism or either the coincidence of the two forms, Hotellings and Roys tests and the Test of Significance for Average, using sequential sums of squares were used which are available in the Repeated Measures Analysis of Variance procedure form SPSS. Parallelism, coincidence and equal scale means are reflected in the following tables:

Table 3.8: Test of Parallellism of Pre and Post Forms of the Concept test

TEST NAME	VALUE	EXEACT F	HYPOTH. DF	ERROR DF	SIG OF F
HOTELLINGS	4.78400	0.66753	43	6	**0.798**[1]

1: p=0.798 indicates that parallelism is tenable at the 0.05 level.

Table 3.9: Test of Coincidence of Pre and Post Forms of the Concept test

Source of Variation	SS	DF	MS	F	Sig. Of F
Within Cells	103.28	48	2.15	-	-
Application	0.000	1	0.000	0.000	**0.965**[2]

2: This indicates that Pre and Post forms of the Concept test are coincident, since p=0.965 is greater than 0.05.

Table 3.10: Test of Equal Scale Means (Roys Test) for Concept test

TEST NAME	VALUE	EXACT F	HYPOTH. DF	ERROR DF	SIG. of F
ROYS	0.17289	0.66753	43	6	**0.798**[3]

3: This test indicates that equal scale means is tenable at the level 0.05 level because p=0.798 is greater than 0.05.

Briefly, by considering the Tables 3.8-10, we concluded that the pre and post forms of the Concept test were parallel, coincident and means of each item were nearly the same for the two applications. The position can also be enriched by the following figure.

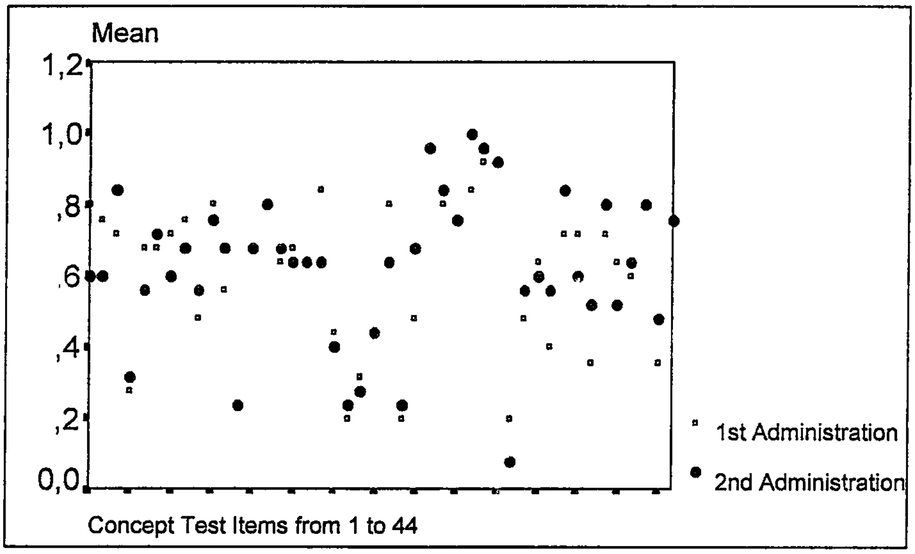


Figure 3.2: The Profiles of Pre and Post Administrations of Concept test

As it is seen from the above figure the profiles of the two applications (*pre and post forms of the Concept test*) are nearly the same.

The Concept test (which was administered after the treatment and at the delayed testing stage) were tested with 52 second year, preservice elementary teachers at Atatürk Öğretmen Koleji. The preservice elementary teachers had a total of 30 minutes for the Concept test (which was administered after the treatment and at the delayed testing stage). The reliability for the preservice elementary teachers' scores in the Post / Delayed form of the Concept test was computed using Alpha reliability procedure from SPSS. The reliability of Post / Delayed form of the Concept Test was 0.90. The validity of the Post / Delayed form of the Concept Test was judged by three maths teachers (who has at least 5 years of teaching experience) from a local high school. They all decided that the items in the test can help to investigate possible misconceptions, who were previously trained about the misconceptions related to decimals.

3.2.5.3.2 Problems Test (PT): Choosing the Appropriate Operation

The Problems Test (see Appendices B and E) included 26 - items. It was used in order to investigate the misconceptions of preservice elementary teachers and processes used in solving (choosing the appropriate operation) word problems involving decimals.

The following table gives detailed information about the distribution of items among the specified sections.

Table 3.11: Distribution of Items of the Problems Test Among the Specified Sections for Stage-1.

		Items
Multiplication problems	Conforming to the primitive model	1,5,10,20 (K=4)
	Not conforming to the primitive model	3,7,9,14,21 (K=5)
Division problems	Partitive conforming to the primitive model	4,11,23 (K=3)
	Partitive, not conforming to the primitive model	6,12,15 (K=3)
	Quotative, conforming to the primitive model	2,13,17,18 (K=4)
	Quotative, not conforming to the primitive model	16,19,24,25 (K=4)
Addition / Subtrac.. problems		8,22,26 (K=3)

•K : Number of Items

Table 3.12: Distribution of Items of the Problems Test Among the Specified Sections for Stage-2.

	ITEMS		# of ITEMS
Multiplication Problems (PT1.1)	1,3,5,7,10,14,20	(PT1.2[1])	7
	9,21	(PT1.3)	2
Division Poblems (PT2.1)	4,6,11,12,13,15,18,19,23	(PT2.2)	9
	2,16,17,24,25	(PT2.3)	5
	4,6,12,15,16,19,24,25	(PT2.4)	8

1: Abbreviations in the brackets show to which sub-category do the items fall.
PT1.2: Items suitable for direct proportion.
PT1.3: Items not suitable for direct proportion.
PT2.2: Items suitable for direct proportion.

PT2.3: Items not suitable for direct proportion.
PT2.4: Items in which the divisor is greater than the dividend.

As it was stated in Table 3.11, we divided the Problems Test into 6 main dimensions, in order to get more information about the items and differences among the specified sections.

For scoring, each problem on the Problems Test was scored one (1) for the correct expression and zero (0) for the incorrect expression or no response.

Some of the items in the Problems Test (which was administered before the treatment) were developed by the researcher and some others were adapted from other researchers works like İşeri (1997), Bell (1982), Thipkong and Davis (1991), Greaber and Tirosh (1989) and Fischbein (1985).

The Problems (which was administered before the treatment) Test was tested with 52 second year, preservice elementary teachers at Atatürk Öğretmen Koleji. The preservice teachers had a total of 35 minutes for the Problems Test. There were 26 questions in the Problems Test at the beginning. Through a classical item -analysis technique I decided not to ignore any item from the test.

Preservice elementary teachers were asked to write an expression that would lead to the solution of each given word problem. Problems to be used in the study include addition, subtraction, multiplication and division. Since the main concern was to explore the performance of preservice elementary teachers in choosing the appropriate operation in solving multiplication and division word problems, the addition and subtraction problems were added to prevent the students from not writing multiplication and division expressions in a procedural manner.

In addition to conformity or non-conformity to the primitive models, the problem types for multiplication and division can also be classified as follows:

DIVISION

1- Partitive
2- Quotative (measurement)

MULTIPLICATION

1- Repeated Addition
2- Cartesian Product or Rate

The reliability for the preservice elementary teachers' scores in the Problems Test (which was administered before the treatment) was computed using Alpha reliability procedure from SPSS. The reliability of the Problems Tests was 0.86. Similar to the Concept Test content validity of the Problems Test was also judged by a foreign (English) expert in the field and by two other mathematics educators from METU. They all decided that the items in the test can help to investigate possible misconceptions about decimals. Similar to the Concept Test in order to test the hypothesis that items which do not confirm to the Primitive Models can better challenge misconceptions related to choosing the appropriate operations for word problems involving decimals (construct validity), I applied a paired samples t-test on the same group (N=52). Results are summarised in the following table.

Table 3.13: Comparison of the Mean Scores for the Sub-Scales of the Problems Test in terms of Conformity or Non-Conformirty to the Primitive Models

Variable	Number of Pairs	Corr	2-tail Sig.	Mean	SD	t-value	df	2-tail Sig.
PT1.1				3.5385	0.828			
	52	0.590	0.000			5.39	51	0.000
PT1.2				2.6346	1.495			
PT2.1				2.7692	0.581			
	52	0.282	0.043			8.59	51	0.000
PT2.2				1.5962	0.975			
PT2.3				3.2115	1.117			
	52	0.352	0.011			7.95	51	0.000
PT2.4				1.5769	1.405			

Since all p-values were less than 0.05 level, we concluded that the performance of the group was better on the items that conform to the primitive models, which means that the sub-scales that did not conform to the primitive models could challenge misconceptions, related to decimal word problems, better than the Sub-scales that conform to the primitive models. These findings, of course, like in the Concept Test, strengthen the construct validity of the Problems Test.

Later we changed only the decimal numbers used in the Problems Test (which was administered before the treatment) and formed another test to be used in the post and delayed testing periods. In order to test if the post form of the Problems Test was parallel to the pre-form of the same test, the pre and post forms of the test were administered on a group of 25 third year preservice elementary teachers at Atatürk Öğretmen Koleji. Similar to the Concept test for testing the parallelism or either the coincidence of the two forms, Hotellings and Roys tests and the Test of Significance for Average, using sequential sums of squares were used which are available in the Repeated Measures Analysis of Variance procedure form SPSS. Parallelism, coincidence and equal scale means are reflected in the following tables:

Table 3.14: Test of Parallelism of Pre and Post Forms of the Problems Test

TEST NAME	VALUE	EXEACT F	HYPOTH. DF	ERROR DF	SIG OF F
HOTELLINGS	1.05946	1.01708	25	24	**0.485**[1]

1: p=0.485 indicates that parallelism is tenable at the 0.05 level.

Table 3.15: Test of Coincidence of Pre and Post Forms of the Problems Test

Source of Variation	SS	DF	MS	F	Sig. Of F
Within Cells	49.23	48	1.03	-	-
Application	0.030	1	0.030	0.03	**0.870**[2]

2: This indicates that Pre and Post forms of the Problems Test are coincident, since p=0.870 is greater than 0.05.

Table 3.16: Test of Equal Scale Means (Roys Test) for Problems Test

TEST NAME	VALUE	EXACT F	HYPOTH. DF	ERROR DF	SIG. of F
ROYS	0.48556	9.73721	25	24	**0.485**[3]

3: This test indicates that equal scale means is tenable at the level 0.05 level because p=0.485 is greater than 0.05.

Briefly, by considering the Tables 3.13-15, we concluded that the pre and post forms of the Problems Test were parallel, coincident and means of each item were nearly the same for the two applications. The position can also be enriched by the following figure.

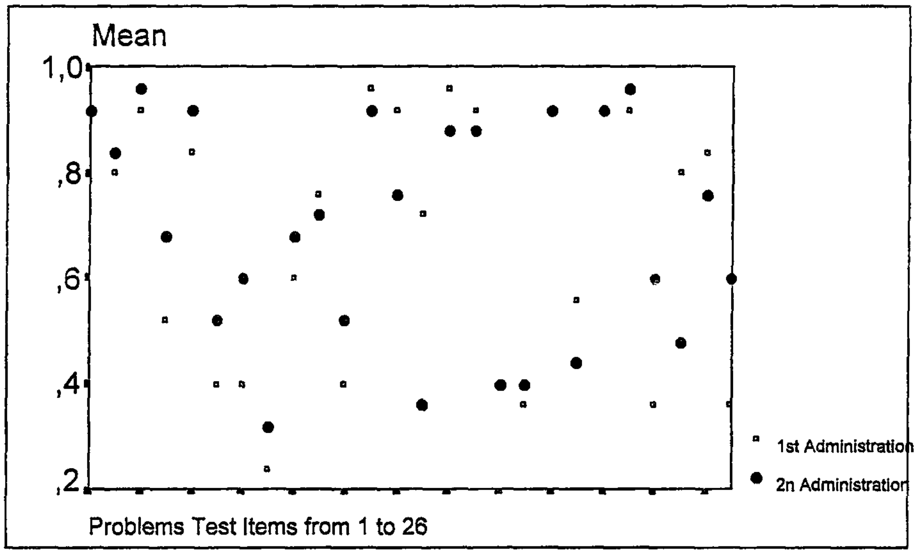


Figure 3.3: The Profiles of Pre and Post Administrations of Problems Test

As it was seen from the above figure the profiles of the two applications (*pre and post forms of the Problems Test*) were nearly the same.

The Post / delayed forms of the Problems Test was tested with 52 second year, preservice elementary teachers at Atatürk Öğretmen Koleji. The reliability for the preservice elementary teachers' scores in the / delayed forms of the Problems Test was computed using Alpha reliability procedure from SPSS. The reliability of Post / delayed forms of the Problems Tests was 0.88. The content validity of the Post / delayed forms of the Concept Test was judged by three expert maths teachers from a local high school. They all decided that the items in the test can help to investigate possible misconceptions about decimals.

3.2.5.3.3 Writing Division and Multiplication Word Problems Test (WWPT):

Writing Division and Multiplication Word Problems Test (see Appendices C and F) included 10 multiplication and division expressions and preservice elementary teachers were asked to write 10 problems for the given expressions. It was used in order to describe the thinking strategies or embodiments used by preservice elementary teachers when writing word problems for mathematical expressions involving decimals.

Writing Division and Multiplication Word Problems Test which aimed to investigate the preservice elementary teachers' performances in writing division and multiplication word problems for multiplication and division expressions before the treatment was referred to *Pre - Writing Division and Multiplication Word Problems Test.*

The following tables give more information about the structure of the Writing Division and Multiplication Word Problems Test (a sample from *Writing Division and Multiplication Word Problems Test* which was administered before the treatment)

Table 3.17: Multiplication Expressions of the Writing Division and Multiplication Word Problems Test (which was administered before the treatment)

Multiplicative Expressions	*Suitable for ...*	*Arithmetic Structure*
5 × 8	Repeated Addition, Cartesian Product, or Rate (conforming to the primitive model)	I x I
5 × 0.68	Repeated Addition, Cartesian Product, or Rate (conforming to the primitive model)	I x d
0.63 × 22	Repeated Addition, Cartesian Product, or Rate (not conforming to the primitive model)	d x I
12.05 × 0.93	Cartesian Product, or Rate (not conforming to the primitive model)	D x d

(I) : an integer, (D) : a decimal greater than 1, (d) : a decimal less than 1

Table 3.18: Division Expressions of the Writing Division and Multiplication Word Problems Test (which was administered before the treatment)

Division Expressions	*Suitable for ...*	*Arithmetic Structure*
24 ÷ 4	Partition, Quotition, or Rate (conforming to the primitive model)	I ÷ I
4 ÷ 24	Fractional Partition, or Rate (not conforming to the primitive model)	I ÷ I
3.86 ÷ 23	Fractional Partition, or Rate (not conforming to the primitive model)	D ÷ I
0.83 ÷ 0.32	Quotition, or Rate (not conforming to the primitive model)	d ÷ d
9.6 ÷ 62.2	Fractional Quotition, or Rate (not conforming to the primitive model)	D ÷ D
0.53 ÷ 1.4	Fractional Quotition, or Rate (not conforming to the primitive model)	d ÷ D

I) : an integer, (D) : a decimal greater than 1, (d) : a decimal less than 1

Writing Division and Multiplication Word Problems Test (which was administered before the treatment) was tested on 20 (a subset of 52) preservice elementary teachers. Through the related literature I collected several possible embodiments that can be used in writing word problems for multiplication and division expressions and then Six, 5-year experienced mathematics teachers from a local high school were asked to evaluate (categorize) the answers of the preservice teachers, according to the categories given in tables 3.19-3.22 , in addition to the researcher in order to find if any other maths teacher could categorize the written problems as in the same way I did. All the teachers were trained for two hours before the evaluation processes. A sample WWPT paper was evaluated by me and 5 other maths teachers. All the written problems were categorized in the same way by me and the others, except some additional embodiments like weak context and alternation of numbers.

Table 3.19: Possible Embodiments For Multiplication Expressions

Possible Embodiment	Definition	Example
Repeated Addition*	This is a model where one can add the same amount repeatedly instead of multiplying like: 5 x 0.65 = 0.65 + 0.65 + 0.65 + 0.65 + 0.65	a typical example for the expression 5 x 0.65: *If the width of a ruler is 0.65 meter then what is the total width of 5 rulers each having the same width ?*
Cartesian Product*	This is a model where one does not attempt to add and find the product by matching each element from one set with each element from another set to make a set of ordered pairs like *area* and *enlargement* problems	*To fit a picture of a dress onto a magazine , the picture has to be reduced to 0.14 of its original size. In the original picture the length of the dress was 2 m. What will its length be in the magazine ?*
Rate*	The idea is the same with the Cartesian product model, but this time I mainly deal with *speed x time* and *amount x money* type of problems.	*One lt of patrol costs 1.33$. How much will it cost to fill up a small tank which can only hold 0.53 lt ?*
Division Embodiment	In this Embodiment preservice elementary teachers wrote some word problems that leads to division instead of multiplication	For expression 0.63 x 22 : *" Ayþe has a rope of 0.63 m. if she wants to divide this into 22 equal parts then how many small ropes will she get ? "*
Addition Embodiment	In this Embodiment preservice elementary teachers wrote some word problems that leads to addition instead of multiplication	For expression 12.05 x 0.93 : *" The length and width of a ruler are 12.05 and 0.93 cm respectively. Then what is the sum of the two dimensions ? "*
Subtraction Embodiment	In this Embodiment preservice elementary teachers wrote some word problems that leads to subtraction instead of multiplication	For expression 0.63 x 22 : *" What I have if I subtract 0.63 of 22 ?"*

* : Embodiments in bold letters stand for Correct Embodiments

Table 3.20: Possible Embodiments For Division Expressions

Possible Embodiment	Definition	Example
Partition Embodiment	In this type an object or collection of objects is divided into a number of equal fragments or subcollections. Can be used whenever the divisor is an integer smaller than the dividend.	*" An electrician has 1400 m of electric cable in stock. He has to rewwere 12 houses. How much cable can he use in each house ? "*
Fractional Partition	In this type, like Partition, an object or collection of objects is divided into a number of equal fragments or subcollections and can be used whenever the divisor is an integer larger than the dividend.	*" 25 men do the football pools together. This Iek they have won 22 $. How much will they receive each ?"*
Quotition	In this type one seeks to determine how many times a given quantity is contained in larger quantity and can be used whenever the divisor is less than the dividend	*" On a sailing boat I need a lot of pieces of rope 0.26 m long. How many can I cut from 0.78 m of rope ?"*
Fractional Quotition	In this type again, like **Quotition,** one seeks to determine how many times a given quantity is contained in larger quantity and can be used whenever the divisor is greater than the dividend.	*" A rabbit is digging its way out of cave. It can dig 3 m in a day. At this rate how many days will it take it to reach a tree 0.914 m away ? "*
Rate.	This type can always be used. I named the division problems as Rate whenever the preservice elementary teachers used context like *speed, price, ratio or proprotion*	*" A rowing crew covers a 3 km course in 7.2 minutes. How far did they row in 1 minute ? "*
Inverse Division	In this Embodiment preservice elementary teachers wrote some word problems that leads to the reversal of divisor and dividend.	For expression 4 ÷ 24 : *" I am packing boxes of cassettes. I fit 4 cassettes in a box. I have 24 cassettes. How many boxes will I fill ? "*
Multiplication	In this Embodiment preservice elementary teachers wrote some word problems that leads to multiplication instead of division	For expression 3.86 ÷ 23 : *" How far does a horse can go in 23 seconds if it can run 3.86 m in a second ?"*
Subtraction	In this Embodiment preservice elementary teachers wrote some word problems that leads to subtraction instead of division	For expression 0.83 ÷ 0.32: *" Total of a cake is 0.83 if take 0.32 of it how much do I have left ?"*

* : Embodiments in bold letters stand for Correct Embodiments

Table 3.21: Common Embodiments For both Division and Multiplication Expressions

Possible Embodiment	Definition	Example
Not a Word Problem	In this Embodiment preservice elementary teachers tried to ask questions where one can only use his/her arithmetical skills to solve.	*" What is the product of 5 and 6 ? "* *" What are the prime factors of 40 ? "*
2-Step Operation	In this embodiment preservice elementary teachers wrote word problems that can only be solved if one performs at least two operations in order to solve a given problem (leads to a wrong expression).	For expression 5 x 8 : *" 5 worker work for 5 days , each day they work for 8 hours. In how many days can they finish the whole work ? "*
Inappropriate Embodiment	in this embodiment preservice elementary teachers wrote some word problems that does not lead to any mathematical expression or the ansIr is explicitly given	For expression 9.6 x 62.2 : *"If I want to make a 9.6 kg rice as a 62.2 box . How much do I need ? "*
No Answer Embodiment	In this embodiment preservice elementary teachers did not attempt to write any word problem.	

- : Embodiment in bold letters stand for Correct Embodiment

Table 3.22 : Embodiments that can be linked to other Embodiments in Writing Division and Multiplication Word Problems

Possible Embodiment	Definition	Example
Iak Context	In this embodiment preservice elementary teachers wrote word problems that Were mathematically correct but not applicable in real life.	*"If I divide 4 marbles among 24 students. How many each will get ? "*
Alternation of Numbers or Adding More Numbers	In this embodiment preservice elementary teachers wrote word problems by either altering or adding more numbers to the original ones .	For expression 0.63 x 22 : *" In order to fill a pool there are 22 water pipes each having a 0.63 lt / 3 minutes filling capacity. How much water do all the pipes pump ?"*

The embodiments which were treated as *Incorrect* scored as zero(0)

The embodiments which were treated as correct were scored by going through the following chart stated in Figure 3.4.

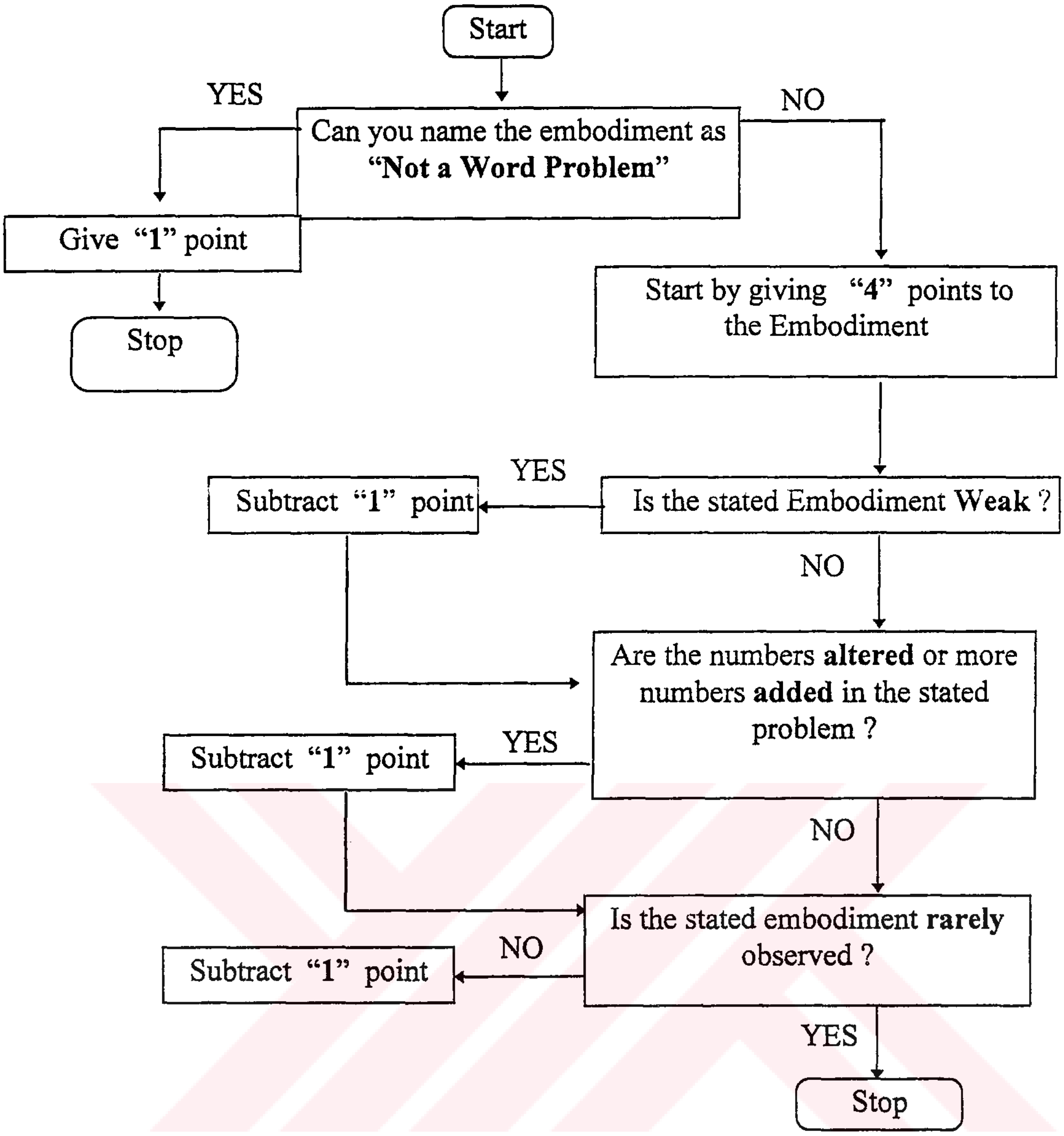


Figure 3.4 : Scoring Chart for Correct Embodiments in Writing Division and Multiplication Word Problems Test

Briefly we scored any word problem by checking the above dimensions one by one and a score for a correct embodiment differed from 1 to 4 accordingly.

The following tables show the responses of the preservice elementary teachers.

Table 3.23: Categorisation of the Embodiments Stated by Preservice Teachers (N=20) for Division Expressions (in Pilot process)

Problem	24 ÷4	4÷24	0.83÷ 0.32	3.36÷23	9.6÷62.2	0.53÷1.4
Correct	18	14	11	13	6	6
Partition	15	0	0	0	0	0
Frac.Partition	0	10(3W)	0	11	0	0
Quotition	1	0	8(5W)	0	0	0
Frac. Quotition	0	1	0	0	3(2W)	2(1W)
Rate	2(1A)	1	2	1	3 W	3 (2W)
Not a word prob.	0	2	1	1	0	1
Multiplication	1	0	0	2	0	0
Subtraction	0	1	2	0	0	0
Inappropriate embodiment	1	2	2	2	4	5
No answer	0	3	5	3	10	9

W: Weak or inappropriate embodiment *A: alternation of numbers or adding more number*

Table 3.24: Categorisation of the Embodiments Stated by Preservice Teachers (N=20) for Multiplication Expressions (in Pilot process).

Problem	5×8	5×0.68	0.63×0.22	12.05×0.93
Correct	18	19	18	10
Repeated addition	14(2W, 1A)	10	13(1A)	1(1W)
Rate	0	9(1W)	5	7(1W)
Cartesian product	3	0	0	1
Not a word problem	1	0	0	0
Inappropriate embodiment	1	1	1	2
Division	1	0	1	0
Addition	0	0	0	1
No answer	0	0	0	7

W: Weak or inappropriate embodiment *A: alternation of numbers or adding more number*

In the preparation of Writing Division and Multiplication Word Problems Test, expert inspection was done. In both of the tables (table 3.23 and table 3.24) it was possible to observe a decline whenever the numbers involved in the given expression were decimals less than 1 and whenever the divisors were greater than the dividend. This fact strengthens the validity of the instrument.

3.2.5.3.4 Interviews

In order to decide whether the observed mistakes of preservice elementary teachers from the three paper and pencil instruments were coming from certain misconceptions or not, we aimed to do interviews with some preservice elementary teachers. For this reason we decided to conduct a pilot study to develop a good interview schedule. In March 1996 we tested an interview schedule (see Appendix - G) on a group of (N=5) second year preservice elementary teachers.

Later we decided to make the interview schedule more specific to the interviewees in addition to more general questions. According to the pre-testing results of the tests, we aimed to ask the interviewees to express their performance in certain problems that they missed in the tests, by giving reasons. We prepared standard forms for each interviewee that showed the problems or questions to be posed and the level of the preservice teacher.

A group of preservice teachers were chosen and interviewed in order to obtain more information about the conceptions they held and the reasoning they used. The preservice teacher was first given problems similar [or same] to those s/he missed on the written instruments and was asked to explain why s/he responded the way s/he had and how s/he could check his/her work.

A typical interview schedule was as follows:

First Part

How can you define decimal numbers ?

What is the basic difference between a decimal number and a whole number ?

What was the most difficult part or question for you in the tests ? Why ?

How can you define multiplication ? Give examples.

How can you define division ? Give examples.

Does decimal involvement in the problems or expressions make the work harder ? Why ?

Did you learn decimals previously in detail when you consider your elementary, middle or high school education ?

Second Part

The preservice teacher first was given problems similar [or same] to those s/he missed on the written instruments and was asked to explain why s/he responded the way s/he had and how s/he could check his/her work.

The most frequently used questions were:

From Concept test: problems 4, 5, 11, 12, 15, 19, 20, 24, 26, 27, 33, 35, 37, 38, 40, 41

From Problems Test: problems 7, 9, 16, 17, 24

From Writing Word Problems Test: spontaneous

Each interviewee was asked approximately 10 questions in a period of 30 -45 minutes.

In the following tables (table 3.25, 3.26, and 3.27) the number of preservice elementary teachers interviewed on each item, after the first application of all the instruments, are given.

Table 3.25: Number of Preservice Elementary Teachers Interviewed on Items of the Concept test (which was administered before the treatment)

Item	Number of Interviews	Item	Number of Interviews
1	1	23	2
2	2	24	10
3	1+(1)	25	0
4	13+(4)	26	14+(4)
5	12+(1)	27	2+(1)
6	0	28	1+(1)
7	1+(1)	29	1+(1)
8	1	30	1+(4)
9	2+(1)	31	5+(15)
10	0	32	5+(17)
11	5+(1)	33	12+(4)
12	0	34	1+(2)
13	2	35	7+(2)
14	1	36	1
15	7	37	5+(2)
16	1	38	2+(9)
17	(1)	39	3+(2)
18	0	40	4+(16)
19	8+(2)	41	14+(1)
20	15	42	2+(1)
21	1	43	8+(5)
22	(1)	44	0

- Numbers in the brackets stand for the number of interviews in which parallel forms of the related items Were asked to the interviewees.

Table 3.26: Number of Preservice Elementary Teachers Interviewed on Items of the Problems Test (which was administered before the treatment)

Item	Number of Interviews	Item	Number of Interviews
1	0	14	0
2	1	15	1
3	2	16	13
4	2	17	4
5	1	18	2
6	2	19	1
7	4	20	2
8	0	21	2
9	8	22	0
10	2	23	3
11	0	24	6
12	8	25	1
13	3	26	0

Table 3.27: Number of Preservice Elementary Teachers Interviewed on Items of the Writing Division and Multiplication Word Problems Test (which was administered before the treatment)

Item	Number of Interviews	Item	Number of Interviews
1	2	6	12
2	8+(2)	7	3
3	6+(1)	8	2+(1)
4	5+(1)	9	4
5	5	10	3+(2)

- Numbers in the brackets stand for the number of interviews in which parallel forms of the related items were asked to the interviewees.

Parallel forms of all the paper and pencil instruments were used after the treatment (Conceptual Change Instruction) as post and delayed tests which had the same content but with different numerical data from the problems that were used in the previous applications.

3.2.6 Procedure

3.2.6.1 Pilot Testing

After conducting a pilot study in March 1996 we decided to develop a Concept, a Problems and a Writing Word Problems tests and we determined the reliabilities of the tests, in November 1996.

The first two instruments were tested with 52 and the third instrument was tested on 20 (a subset of 52) , second year, preservice teachers at Atatürk Öğretmen Koleji. The preservice teachers had a total of 65 minutes for the first two tests and 30 minutes for the third test in another session. There were 45, 26, and 10 questions in each test respectively.

Through a classical item -analysis technique we decided to ignore item-26 in which we tried to test if the preservice teachers can compare decimals, from the Concept test. Most of the preservice teachers at the extreme ends (low scorers (N=13) and high scorers (N=14)) answered correctly item-26.

The problems test (which was administered before the treatment) worked as it was expected.

Pilot testing of the Writing Word Problems Test showed us that preservice elementary teachers had some problems in writing problems, especially when the given expressions involve decimals.

3.2.6.2 Pre - Testing

During the semester break (Spring 1997), N=74 , 1st year preservice elementary teachers were randomly assigned to three groups (administrational activities forced us to form three groups). After that two of the groups were randomly assigned to experimental (N=24) and control groups (N=25), which could enable us to test the effectiveness of the *Conceptual Change Instruction* in overcoming the misconceptions of preservice elementary teachers. At the beginning of the spring semester (March 1997) the three paper-and-pencil instruments (*Concept, Problems, and Writing Division and Multiplication Word Problems Tests*) were administered to 72 first year preservice elementary teachers. Each preservice elementary teacher had 30, 35, and 30 minutes to finish the three instruments respectively. The *Writing Division and Multiplication Word Problems Test* (which was administered before the treatment) was given in another day whereas the first two were given at the same time.

After scoring the first three instruments representative preservice elementary teachers from the three groups were chosen in such a way that we can cover a wide range of performance, for the interview procedures.

Through interview procedures we hoped that it would be possible to obtain more information about the conceptions / misconceptions the preservice elementary teachers hold and the reasoning they used. The preservice teacher first was given problems similar [or same] to those s/he had [missed] on the written instruments and was asked to explain why s/he responded the way s/he had and how s/he could check his/her work.

3.2.6.3 Treatment (Use of CCI) and Post Testing

After the first intensive determining process, according to the results of Stage-1, we determined the most common or typical misconceptions (see Appendix-K) of the interviewees. After that I developed / adapted several instructional strategies and materials (see Appendix-I) and used in the experimental group in which the most common misconceptions are emphasised / challenged. The proposed instructional strategies or materials are based on the intersection of the so called *diagnostic teaching* and *conflict teaching* in a constructivist framework which can be called *Conceptual Change*

Instruction. The intensive group and whole class discussions were the most important dimensions of the proposed strategy.

Constructivist research argues that learners actively construct mathematical understanding with concrete materials and an ongoing process of Socratic questions and discussion designed to challenge preconceptions and replace them with mathematically accurate conceptions.

The teaching methodology that we tried to use involved the following steps:

1) *Introductory task* - students are initially confronted with a relatively difficult problem containing a rich exploratory situation which contains a conceptual obstacle. The students write down their individual responses.

2) *The individual responses are discussed in small groups or in pairs*, working toward consensus. Results are again recorded.

3) *Class Discussion* - each group leader or spokesperson presents the group's opinions to the rest of the class, one at a time. This helps ensure that if a whole group has accepted an erroneous conclusion it can be exposed and countered. Wrong responses can be challenged by other groups or the teacher. The teacher acts in a way as to make the situation unthreatening, while at the same time not providing any positive or negative feedback. The teacher also acts as facilitator while at the same time providing further provocation or conflicting ideas where necessary in order to ensure the exposure of all misconceptions.

4) *Reflective Class Discussion* - students can discuss how errors are made and which misconceptions they are likely to be based on. The teacher can sum-up the ideas presented although this is not necessary.

5) *Consolidation* - students are presented with further questions. They solve problems using pictorial materials and use them to develop new problems and problem

representations. Students develop abstract representations using pictorials to help them create mental images.

After the determining process we observed 32 misconceptions and arranged the related instructional materials in 8 categories as shown in Table 3.28.

Table 3.28: Lessons and modules developed to overcome misconceptions in each category.

Category	Misconceptions Addressed	Number of Lessons and Modules
1. Interpreting Decimals as Points on Number Lines	1, 2, 3, 4, 5, 6, 7, 8, 9	1 lesson = (2 modules)
2. Interpreting Decimals on Area Models	3, 8, 10, 11, 12, 13, 14, 15	1 lesson = (2 modules)
3. Comparing Decimals	11, 16, 17, 18	1 lesson =(2 modules)
4. Denseness of Decimals	3, 19, 20, 21	1 lesson
5. Unit Measures Involving Decimals	22, 23	1 lesson = (2 modules)
6. Multiplication and Division Operations Involving Decimals	24, 25, 26, 27	2 lessons = (2 modules) +1 lesson
7. Choosing the appropriate operation for word problems involving decimals	28, 29, 30, 31, 32	2 lessons = (4 modules)
8. Wiriting Word Problems for Division and Multiplication Expressions	24, 25, 28,29,30,31,32	3 lessons = (4 modules) +1 lesson

The control group continued to study the regular program with the traditional method , full time allotted for instruction of the decimals.

The overcoming procedures took 4 weeks (May 1997) or 12 class hours (each 45 minutes).

At the end of the overcoming stage, in June 1997, parallel forms of concept, problems and writing division and multiplication word problems tests, which were treated as post tests, were used, in order to explore, if there was any change in preservice elementary teachers' performances (interpretation and application of decimals).

3.2.6.4 Delayed Testing

After four (4) months (in October 1997) another set of parallel forms of concept, problems and writing division and multiplication word problems tests were used on the same groups to examine the influences of the treatment over a period of time.

After conducting the three paper and pencil instruments, as delayed tests, a group of preservice elementary teachers (N=7 from CG and N=8 from EG), who were previously interviewed for the Stage - 1 , were again interviewed in order to get more information about their final position, in terms of interpreting and applying decimals and compare the performances and understandings of EG and CG.

3.2.7 Analysis of Data

Data analysis based on both qualitative (asking questions, video-taped interviews) and quantitative (using the Statistical Package for the Social Sciences - SPSS , version 6.0, for Windows) methods.

In Stage - 1, first of all the overall performance of preservice elementary teachers was analyzed using several descriptive statistics like; mean, standard deviation, and frequencies, and percentages. The subjects' thinking processes and overall strategies used in solving the problems were summarized in table formats as student profiles.

In order to decide whether the incorrect responses are simple errors or the result of several misconceptions, great importance was ascribed to interviews. Clinical Method - a technique for analyzing the qualities of thought processes that involves both a question or problem to be solved by subject and some discussion with him or her regarding the processes and reasoning involved (Gorman, 1972) - was used. All the interview sessions were video-taped and transcribed.

In Stage - 2, in order to test the effectiveness of *Conceptual Change Instruction* , I aimed to observe the differences among the mean scores coming from pre, post and delayed testing applications of the three paper and pencil instruments. For this reason I mainly used the Repeated Measures Analysis of Variance procedures. In order to strengthen the findings I added some interview transcriptions, which were done after the delayed testing.

3.2.8 Assumptions of the Study

1. No outside event will occur during the treatment that influence the preservice elementary teachers responses to the tests.
2. All preservice teachers in this study will answer the tests accurately and sincerely.
3. The misconceptions observed on the interviewees can represent the whole sample.
4. Researcher is not biased during the treatment.

3.2.9 Limitations of the Study

1. The scope of the investigation was limited to the preservice elementary teachers at "Atatürk Öğretmen Koleji" and this study was concerned with first year preservice teachers.
2. Evidence for validity of the instruments was only obtained through expert judgment.

CHAPTER IV

RESULTS AND DISCUSSION

In this chapter the results related to Stage-1 and Stage-2 will be presented respectively. The results coming from the three paper and pencil instruments will be enriched by the interview results when it is appropriate. On the other hand the partial and overall results will be discussed at the end of each section related with present research literature.

4.1 Results and Discussion of Stage - 1 (Determining Misconceptions)

Table 4.1 displays the frequency distribution of scores obtained from Concepts Test (which was administered before the treatment). Maximum possible score was 44. These scores ranged from 3 to 43. The mean score was 25.7 with a standard deviation 9.822. This situation can also be seen in histogram stated in figure 4.1.

Table 4.1 Frequency distribution of scores obtained from Pre-Testing of Concept Test

Score	Frequency	Score	Frequency	Score	Frequency	Score	Frequency
3	1	16	5	24	5	32	7
6	1	17	1	25	4	33	3
7	2	18	1	26	1	35	1
9	1	19	1	27	1	36	1
10	2	20	3	28	2	37	2
13	1	21	4	29	1	38	1
14	1	22	1	30	5	39	2
15	1	23	2	31	1	40	3
41	2	42	1	43	1		

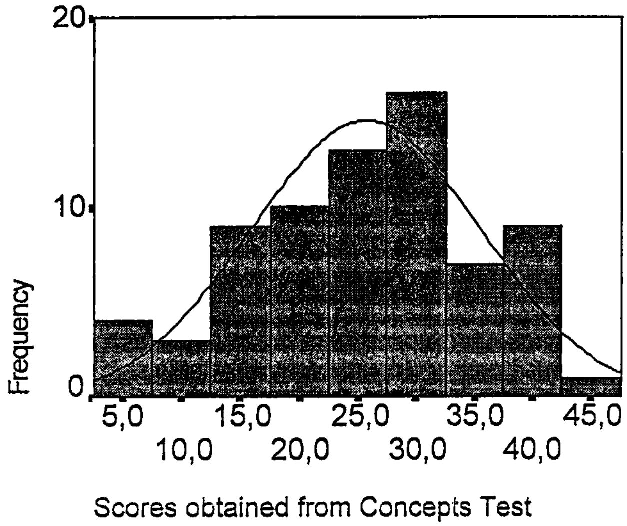


Figure 4.1 Histogram of Concept Scores Coming from Pre-Testing of Concept Test

Table 4.2 displays the frequency distribution of scores obtained from Pre-testing of the Problems Test. Maximum possible score was 23. These scores ranged from 0 to 23. The mean score was 16.4 with a standard deviation 5.341. This situation can also be seen in histogram stated in figure 4.2.

Table 4.2: Frequency distribution of scores obtained from Pre-Testing of the Problems Test

Scores	Frequency
0	1
1	1
4	1
5	3
8	2
10	1
11	2
12	14
13	4
14	5
15	1
16	6
17	11
18	7
19	10
20	2
21	4
22	6
23	2
	Total = 72

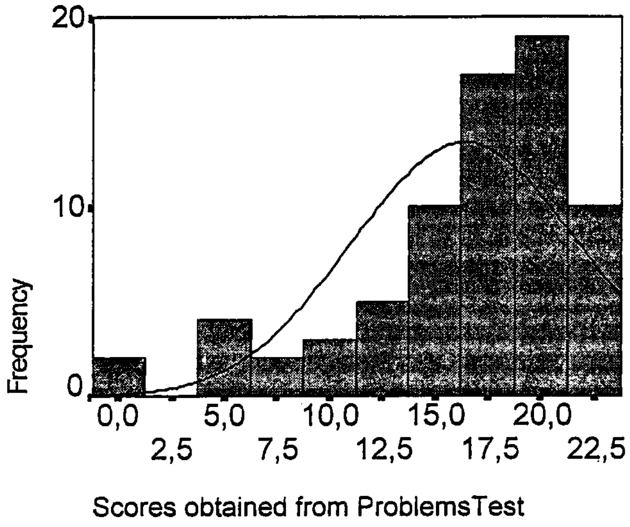


Figure 4.2: Histogram of Problem Scores Coming from the Pre-Testing of Problems Test

Table 4.3 displays the frequency distribution of scores obtained from Pre-testing of Writing Word Problems Test. Maximum possible score was 40. These scores ranged from 1 to 35. The mean score was 20.4 with a standard deviation 7.442. This situation can also be seen in histogram stated in figure 4.3.

Table 4.3: Frequency distribution of scores obtained from Pre-Testing of Writing Word Problems Test

Score	Frequency	Score	Frequency
1	1	20	1
4	1	21	2
7	2	22	5
8	1	23	3
9	3	24	4
10	1	25	1
11	1	26	3
13	1	27	5
15	5	28	4
16	4	29	5
17	5	32	2
18	3	33	1
19	4	35	1

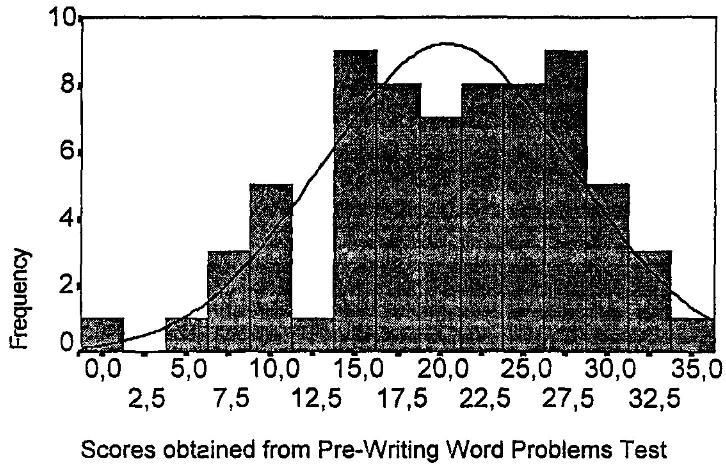


Figure 4.3: Histogram of Writing Word Problems Scores Coming from the Pre-Testing of Writing Word Problems Test

4.1.1 Results Obtained from the Pre-Testing of Concept Test

The Concept Test (see Appendix - A) included 44 - items. The test was used in order to investigate the understandings of preservice elementary teachers about decimals in general which can be categorised as follows:

1. marking a point on a number line (*9 items*)
2. writing the decimal for the point that the arrow indicates on a number line (*9 items*)
3. marking and shading of area models (*7 items*)
4. comparing decimals (*5 items*)
5. unit measures involving decimals (*6 items*)
6. denseness of decimals (*2 items*)
7. operations with decimals (*6 items*)

4.1.1.1 Results Concerning the Interpretations of Preservice Elementary Teachers About Decimals as Points on Number Lines

4.1.1.1.1 Results Concerning the Interpretations of Preservice Elementary Teachers In Marking a Point On a Number line for a Given Decimal

As can be seen in figures 4.4 - 4.12, preservice elementary teachers seemed to do better with subunits based on ten. In this part there were 9 items (items 1, 2, 3, 4, 5, 6, 7, 8, 9 from the Pre-Form of Concept Test) to be answered, so the maximum possible score was 9. Scores ranged from 0 to 9. The mean score was 5.39 with a standard deviation of 2.55. Items 4 and 5 , marking the point 2.4 on a number line with the subunit based on 8 and marking the point 0.6 on a number line with the subunit based on 20, were the most difficult items in this part. The mean percent of the preservice elementary teachers' correct responses in this part was 63.

In the following figures the numbers in thin rectangles ( ) show the number of wrong responses and the numbers in rounded-bold rectangles ( ) show the number of wrong responses where the respondent is just count the number of marks or spaces as a decimal number. The bold rectangles ( ) shows the number of correct responses.

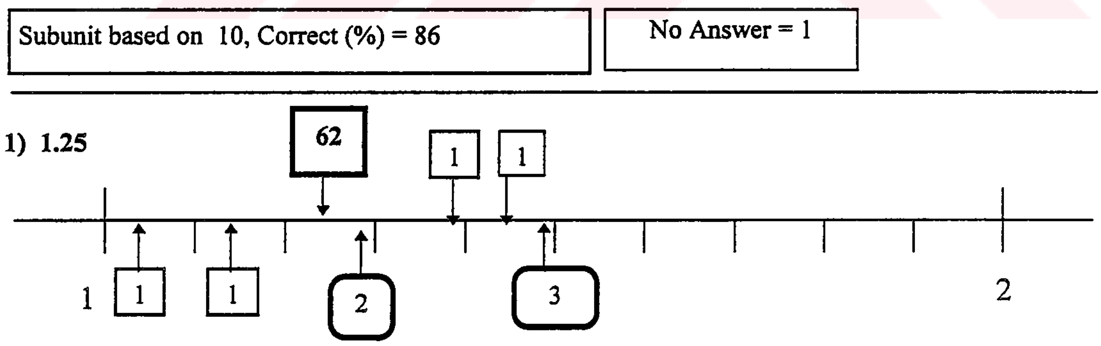


Figure 4.4 : Categories of Responses of Preservice Elementary Teachers as Frequencies on a Number Line for Item 1.

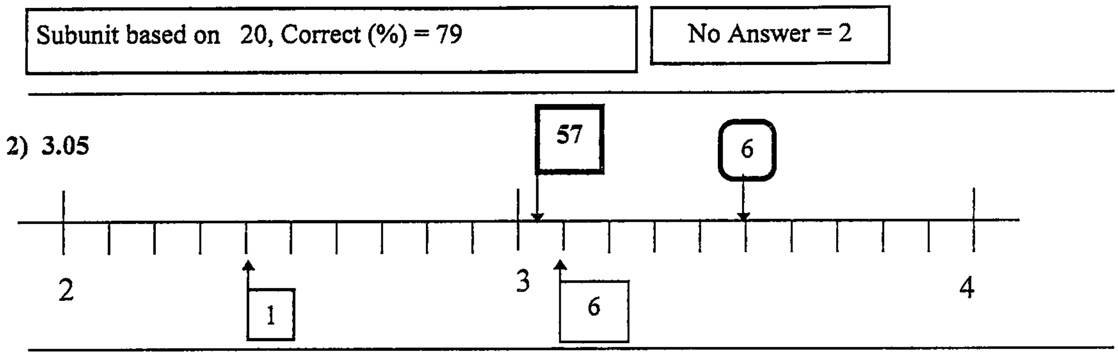


Figure 4.5: Categories of Responses of Preservice Elementary Teachers as Frequencies on a Number Line for Item 2.

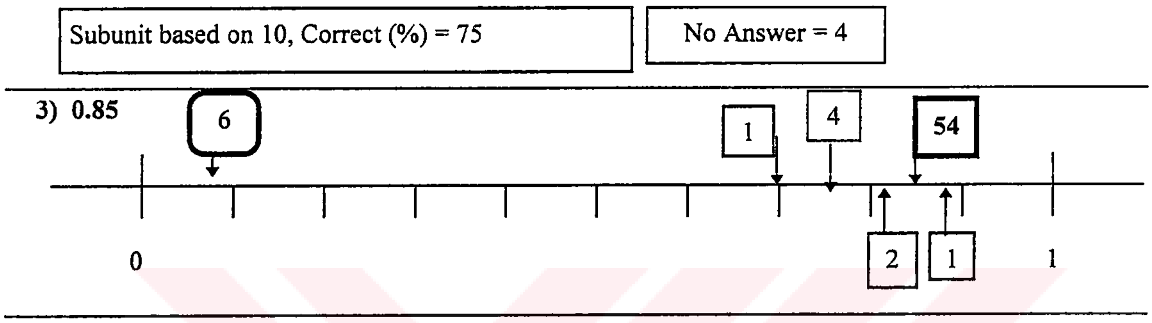


Figure 4.6 Categories of Responses of Preservice Elementary Teachers as Frequencies on a Number Line for Item 3.

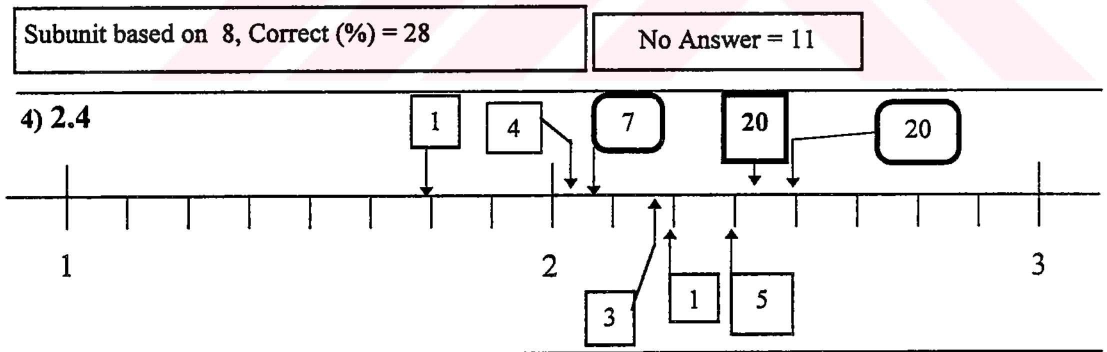


Figure 4.7: Categories of Responses of Preservice Elementary Teachers as Frequencies on a Number Line for Item 4.

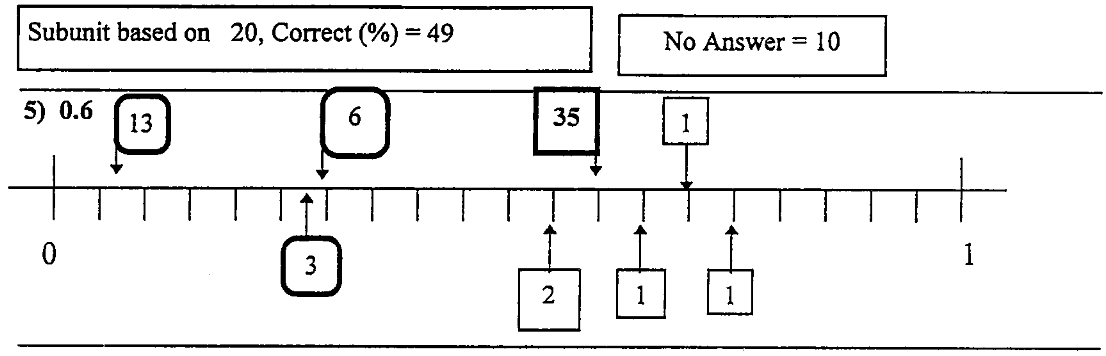


Figure 4.8: Categories of Responses of Preservice Elementary Teachers as Frequencies on a Number Line for Item 5.

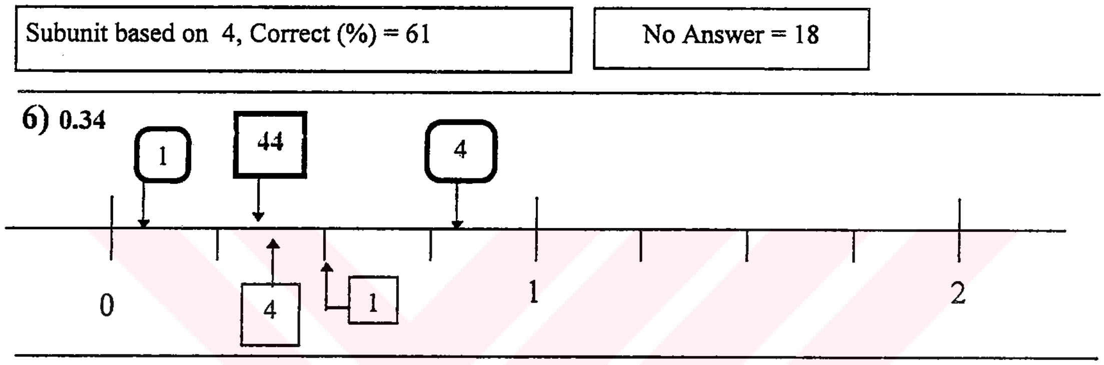


Figure 4.9: Categories of Responses of Preservice Elementary Teachers as Frequencies on a Number Line for Item 6.

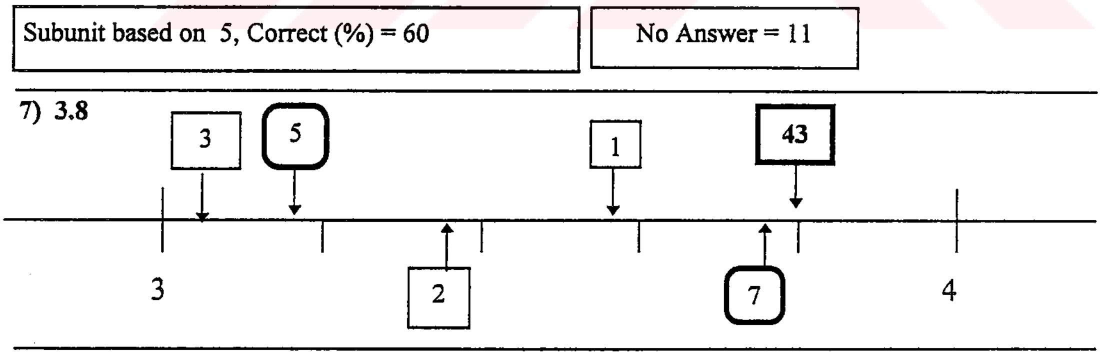


Figure 4.10: Categories of Responses of Preservice Elementary Teachers as Frequencies on a Number Line for Item 7.

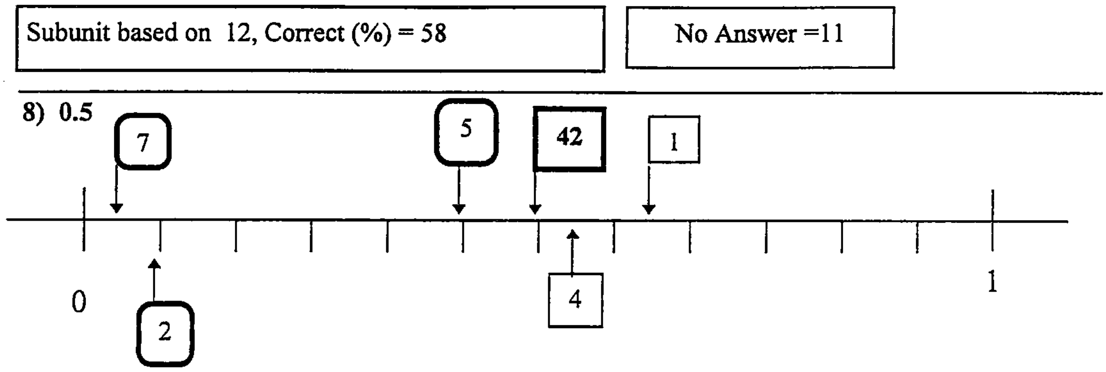


Figure 4.11 Categories of Responses of Preservice Elementary Teachers as Frequencies on a Number Line for Item 8.

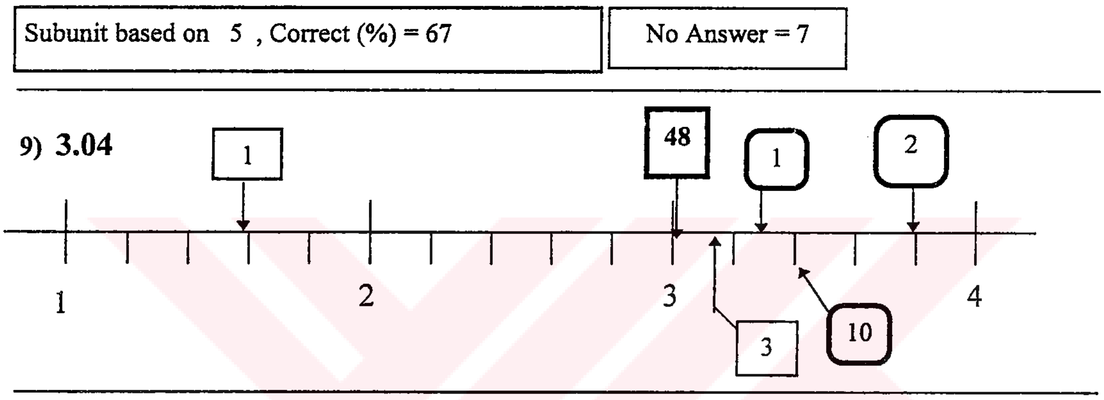


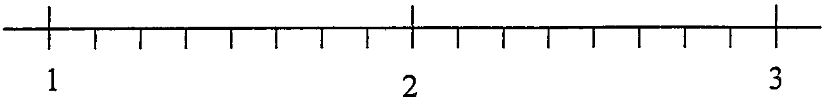
Figure 4.12: Categories of Responses of Preservice Elementary Teachers as Frequencies on a Number Line for Item 9.

In the following paragraphs we present some interview excerpts related to Marking a Point On a Number line for a Given Decimal . The items posed to the preservice elementary teachers in this part are the most challenging ones to whole of the sample or to the individual interviewee.

The following were typical responses of the preservice elementary teachers from the interviews for Item 4 (*13 preservice elementary teachers were interviewed on this item*) :

Item 4: (subunit based on 8)

Mark a point on the given number line to represent the decimal number 2.4



1st Interview Excerpt for Item 4

R[1]: Could you please mark a point on the given number line to represent the decimal number 2.4.
Ç[2]: It is the fourth division.
R[1]: why?
Ç[2]: Because I just count and find.

---

[1] "R": means researcher ,
[2] "... any letter except R": any letter except R represents the first letter of the name of the related interviewee.

As it is seen in the above interview excerpt, the interviewee neglects the calibration of the number line and treats as if it were in subunit -10. In the further steps of the interview the interviewee showed the belief "*decimals are based on subunit 10*".

2nd Interview Excerpt for Item 4

R: Could you please mark a point on the given number line to represent the decimal number 2.4
N: Here there are 8 divisions then it should be at the fourth
R: OK ! why should it be at the fourth ?
N: When we count from the beginning it goes like 2.1, 2.2, 2.3,....2.8 (seemed confused) but 3 comes next which is not possible....(smiling)... do we have to divide each division into two equal parts, where we can get 16 divisions....when it is 16 ...
R: can you explain in detail your way of doing these kind of things ?
N: I try to find by counting the divisions...for example in rulers we have 10 divisions and each division is 0.1 unit. In this problem I think that we have to do many calculations.

In the above interview excerpt, the interviewee first tried to count like the previous one without considering the calibration of the given number line, but suddenly she noticed that it was not possible to exceed 3. Later she tried to increase the calibration of the number line, and this might show that she had a weak conceptualisation of decimals. All over the interview, the most important misbelief of the interviewee was " *in rulers we have 10 divisions each being 0.1 unit*" . This shows that the interviewee has a strong belief about the *denary nature* of decimals.

3rd Interview Excerpt for Item 4

R: Could you please mark a point on the given number line to represent the decimal number 2.4
M: First of all we have 8 divisions ….. (gets nervous)…
R: What did you do for this in the exam ?
M: I divided the 8 divisions into 10.

Up to this point it is possible to say that the interviewee like the other tried to think decimals in subunit 10.

The interview continued as follows:

R: OK do whatever needed.
M: First we can find the value of each division… (it takes time)…
R: if you want you may use a calculator.
M: ..(uses calculator)… 100 over 8.
R: did you say 100 over 8 ?
M: yes! Which is 12.5…. 12.5 + 1 is equal to 13.5…..sorry ! this is not possible !
R: why did you say 100 over 8 ? and why not 10 ?
M: Now I'm thinking about what I did in the exam.

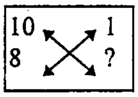
Since the interviewee couldn't solve the problem by subunit 10, this time she tried to find the value of each division by considering that between two numbers there were 100 divisions. This fact may point out the belief that "*decimals are based on subunit 100*".

4th Interview Excerpt for Item 4

R: Could you please mark a point on the given number line to represent the decimal number 2.4
G: it is the fourth division.
R: why is it the forth one ?
G: in fact I am not so sure, but it seems so. Is it 4 of 2 ?

It seems that this interviewee held the same beliefs as the previous ones but interestingly she also held a wrong interpretation of a decimal as a fraction. She interpreted 2.4 as $\frac{4}{2}$.

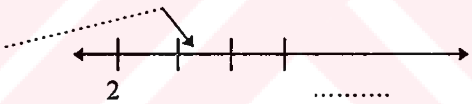
5^{th} Interview Excerpt for Item 4

R: Could you please mark a point on the given number line to represent the decimal number 2.4
S: Normally 2.4 should be at the fourth division but here there is a contradiction.
R: How can you find it exactly ?
S: I use a direct proportion like →


R: Why did you choose 10 ?
S: usually it is divided into 10.

Throughout the interviews many of the interviewees tried to use direct proportion whether it was appropriate or not. This interviewee was a typical example for that. As it is seen from the interview again the decimals are being thought in subunit 10.

6^{th} Interview Excerpt for Item 4

R: Could you please mark a point on the given number line to represent the decimal number 2.4
Ş: It should be somewhere between 2 and the first division.
R: could it be the fourth division ?
Ş: yes.
R: Don't you think that there is a contradiction ? Which choice is correct ?
Ş: The first between 2 and the first division.
R: Do you always mark the decimals between the starting point ant the first division.
Ş: Yes.
R: Think that you have been given 2.6, do you do the same thing ?
Ş: if it were 2.6 the it would be at
I take each division as 5 units.


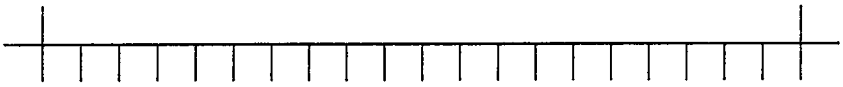
R: what is the reason of taking each division as 5 units ?
Ş: each time I jump 5.
R: OK !

Unlike the previous interviewees this interviewee neglect the given calibration of the number line very strongly and stated that the given decimal could be marked between the starting point and the first division. Later we observed that she mainly considered each division of a number line as 0.5 unit.

The following were typical responses of the preservice elementary teachers from the interviews for Item 5 (*12 preservice elementary teachers were nterviewed on this item*):

Item 5 (subunit based on 20):

Mark a point on the given number line to represent the decimal number 0.6



1st Interview Excerpt for Item 5

R: Could you explain what you do ?
N: First I count the divisions. There are 20 . (thinking) …..
R: OK.
N: it is the 6th division.
R: Why did you say so ?
N: I think that it can also be the third division.

Up to this point, surprisingly, this interviewee stated that it was possible to use different units on the same number line (0.1 or 0.2). Like the others this interviewee also seemed to hold the belief that *"decimals are based on subunit 10"* (rulers are usually divided into 10)

The interview continued as follows:

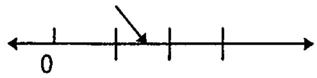
R: Then , what is the value of each division ?
N: Rulers are usually divided into 10. We can take each division as either 0.1 or 0.2 units.
R: Do you say that both can be used ?
N: yes , and I think that in this example we'd better take 0.2.

The difficulties of this interviewee about the calibration of number lines were more highlighted when she said that they had better use 0.2 instead of 0.1. Later in the interview the interviewee stated that *doubling the calibration of a number line could double the value of each division.*

2nd Interview Excerpt for Item 5

R: Could you please mark a point on the given number line to represent the decimal number 0.6.

Ş: It should be somewhere between the 2nd and the 3rd division
R: did you again take each division as 0.5 units ?
Ş: Yes….

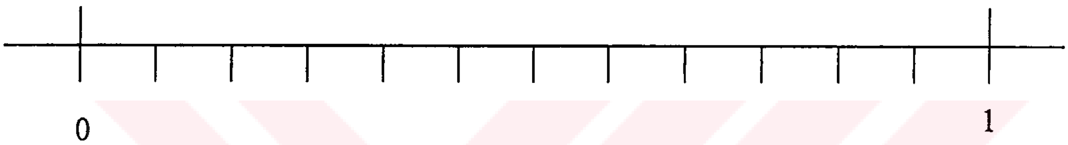


Like many others this interviewee also had some problems about scale reading (calibration of number lines) and showed that she thought that each unit on a number line is 0.5 unit.

The following were typical responses of the preservice elementary teachers from the interviews for Item 8 (2 *preservice elementary teachers were interviewed on this item* :

<u>Item 8 (subunit based on 12)</u>:

Mark a point on the given number line to represent the decimal number 0.5



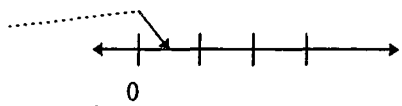
1st Interview Excerpt for Item 8

R: Could you please mark a point on the given number line to represent the decimal number 0.5
H: the first division after 0 is 1 then it will be the fifth one.
R: suppose that there are 100 divisions on the same number line, would it be the same.
H: these divisions ?
R: did you first count the divisions before marking ?
H: no.
R: Briefly I'm trying to say what happens if I divide each division into halves.
H: then it could be the first division.
R: Why did you say one ? Previously you said it could be the fifth.
H: Oh.. yes ,yes it will be the fifth….

It seemed that this interviewee had a very weak understanding of decimals. At the beginning of the interview she completely ignored the zero on the left (zero as a place holder) and considered each division as 1 unit like counting numbers. She also showed that she didn't think the right calibration of the given number line. Later she noticed that 0.5 could be considered as a half but again interpreted it in a wrong manner (calibration problem) and said that it could be marked on the first division, again, by neglecting the calibration of the number line.

2nd Interview Excerpt for Item 8

R: Could you please mark a point on the given number line to represent the decimal number 0.5
U: Oh! I can never do these.
R: OK, try to think now.
U: How can I explain ?
R: How can you read this decimal ?
U: zero point five.
R: where is the place of this decimal in the given number line ?
U: It will be somewhere here
R: why is it so ?



U: I just count as 0.1 , 0.2, 0.3, 0.4 and then 0.5.

This is another interviewee who thinks that "*any decimal on a number line can be marked between the starting point and the first division*" . On the other hand she thinks that from the starting point you count as 0.1, 0.2, 0.3 and etc. She seemed to be influenced by the nature of counting numbers. This also may show the influence of the belief " *decimals are based on subunit 10* ".

4.1.1.1.2 Results Concerning the Interpretations of Preservice Elementary Teachers in Writing the Decimal for the Point or the Place that the Arrow Indicated:

In this part there were 9 items to be answered (items 10, 11, 12, 13, 14, 15, 16, 17, 18, 19 from Pre-Form of Concept Test), so the maximum possible score was 9. Scores ranged from 0 to 9. The mean score was 5.63 with a standard deviation 2.93. The mean percent of the preservice elementary teachers' correct responses in this part was 67. As it is seen in table 4.4, preservice Elementary Teachers seemed to do better on items like 10, 13, 14, and 17 where the subunits were based on ten or derived from familiar decimals like 0.2, 0.25, 0.50 and 0.75.

Items 11, 12, 15 and 16 were the ones in which preservice elementary teachers scored lower and gave interesting responses.

In table 4.4 under the heading of Observation - 2 and Observation - 3 we tried to monitor some strange responses of preservice elementary teachers:

For example for item 10 in which we asked to write the decimal (answer = 2.75) for the point or the place that the arrow indicated where the subunit is 4, three of the preservice elementary teachers considered each subunit mark as 2 tenth and gave 2.6 as the answer.

For item 11 in which we asked to write the decimal (answer = 1.33) for the point or the place that the arrow indicated where the subunit is 3, five of the preservice elementary teachers considered the starting and the end subunit marks in addition to the others so they treated the subunit as it was 4 instead of 3 and gave 1.25 as the answer.

For item 12 in which we asked to write the decimal (answer = 5.375) for the point or the place that the arrow indicated where the subunit is 8, six of the preservice elementary teachers divided the first two subunit marks into halves to make the subunit based on ten and since they counted each subunit mark as one tenth they gave 5.6 as the answer.

For item 15 in which we asked to write the decimal (answer = 2.03) for the point or the place that the arrow indicated where the subunit is 10, three of the preservice elementary teachers considered each subunit mark as 2 tenths and gave 2.6 as the answer.

For item 18 in which we asked to write the decimal (answer =4.5) for the point or the place that the arrow indicated where the subunit is 5, six of the preservice elementary teachers divided 100 by 5 and tried to find the value of each subunit mark and since the arrow indicated the midpoint of two subunit marks they gave 4.45 as the answer.

Table 4.4: Categories of Responses of Preservice Elementary Teachers on Items in Writing the Decimal for the Point or the Place that the Arrow Indicated in Pre-Form of Concepts Test

Item	Decimal	Subunit	Correct Ans (f)	No Answer (f)	Observation - 1 (f)	observation - 2 (f)	Other (f)
10	2.75	4	54	7	6 (2.3)	3 (2.6)	2
11	1.33	3	38	13	11 (1.1)	5 (1.25)	5
12	5.375	8	29	18	7 (5.3)	6 (5.6)	12
13	1.4	5	52	6	8 (1.2)	-	6
14	0.5	20	59	6	6 (0.10)	-	1
15	2.03	10	49	8	9 (2.3)	3 (2.6)	3
16	1.6	20	48	7	12 (1.12)	-	5
17	0.65	10	56	6	4 (0.7)	-	6
18	4.5	5	50	10	2 (4.25)	6 (4.45)	4

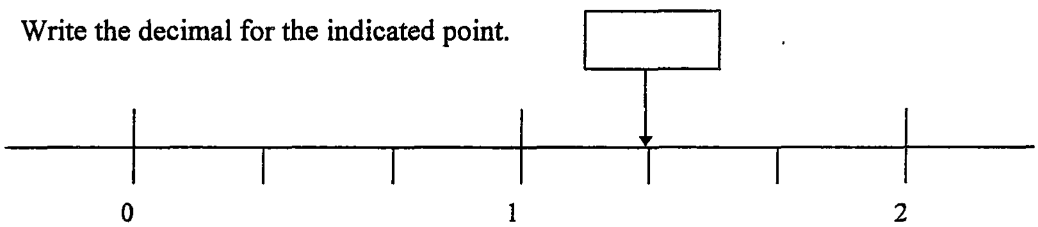
- Observations-1 and 2 stand for the wrong responses.
- The numbers in the dark regions indicates preservice elementary teachers' mistakes by counting each subunit mark as one tenth.
- The numbers in the parenthesis are the typical Categories of Responses of Preservice Elementary teachers.

In the following paragraphs we present some interview excerpts related to Writing the Decimal for the Point or the Place that the Arrow Indicated. The items posed to the preservice elementary teachers in this part are the most challenging ones to whole of the sample or to the individual interviewee.

The following were typical responses of the preservice elementary teachers from the interviews for Item 11 (5 *preservice elementary teachers were interviewed on this item*) :

Item 11 (subunit based on 3):

Write the decimal for the indicated point.



1st Interview Excerpt for Item 11

R: Could you please write the decimal for the point or the place that the arrow indicated.

Ş: is it 1.5 ?
R: Why ?
Ş: Because I consider each division (subunit mark) as 0.5 unit.

As in the previous interviews this interviewee also seemed to be influenced by some familiar units as 0.1, 0.5, 0.25 and etc, so she stated that *each subunit mark could be considered as 0.5 unit.* In some further steps of the interview she also stated that this could be a generalisation.

2nd Interview Excerpt for Item 11

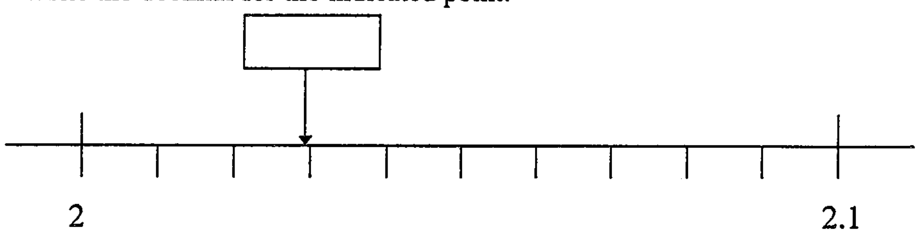
R: Could you please write the decimal for the point or the place that the arrow indicated.
Ç: It is 1.1.
R: Why ?
Ç: Because each subunit mark is 0.1 then it is 1.1.

Although the subunit was based on 3, the interviewee considered each subunit mark as one tenth (0.1) and just by counting the subunit marks he gave 1.1 as the answer.

The following were typical responses of the preservice elementary teachers from the interviews for Item 15 (*7 preservice elementary teachers were interviewed on this item*):

Item 15 (subunit based on 10):

Write the decimal for the indicated point.



1st Interview Excerpt for Item 15

R: Could you please write the decimal for the point or the place that the arrow indicated.
H: The first division is 2 then the place that the arrow indicates will be 5. I take every subunit mark as 1.
R: don't you check the starting and the end points. Is 5 bigger than 2 ?
H: Yes. But I don't know what I'm going to do.

R: thank you.

The interviewee treated decimal numbers as whole numbers and then she counted the subunit marks as the counting numbers each being 1 unit. This empower the belief that "*each division on a number line is 1 unit*".

2^{nd} Interview Excerpt for Item 15

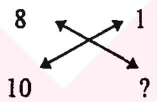
R: Could you please write the decimal for the point or the place that the arrow indicated.
Ç: It will be 2.3.
R: did you again count each subunit mark as one tenth ?
Ç: yes.
R: OK. Thank you.

The above interview excerpt is another example of the belief that "*on a number line between two numbers there are 10 divisions each being 0.1 unit*".

4.1.1.2 Discussion of the Results Concerning the Interpretations of Preservice Elementary Teachers About Decimals as Points on Number Lines

In interpreting decimals as points on numbers lines the mean percent of the preservice elementary teachers' correct responses to the related items was 65. When we go over the test results (figures 4.4 - 4.12 and table 4.4) and interview results it is seen that preservice elementary teachers were, slightly, better on the items which were based on subunit 10. Similar findings were observed by Thipkong and Davis (1991). They stated that in marking a point on each number line for a given decimal preservice elementary teachers seemed to do better with the subunit based on ten. The most difficult items in this part were items 4 and 12, which were both based on subunit 8. Bell (1982) reports that many students can not read a scale when the value of intermediate markings has to be calculated by proportion .

As it is seen in figures 4.4 - 4.12, table 4.4 and from the interviews, most of the preservice elementary teachers counted each subunit mark on the given number lines as 1 tenth and for example interpreted 2.4 on a number line as two and four subunits when subunits were not based on ten (such as eight). In the interviews, ten (40%) of the interviewees explicitly verbalised that on a number line between two numbers there were 10 marks / divisions each being 0.1 unit. This shows that preservice elementary teachers strongly believed that decimals are based on subunit 10. This might be the reason for their better performance on the items based on subunit 10. A similar observation was made by İşeri (1997, p.39) with middle school students. Although there were 5 subunits between 3 and 4, and the number which would represent the place up to the second subunit was asked, students (29 out of 54) just gave the response "3.2" without considering that each subunit represented fifths. Some of the interviewees (3) in interpreting decimals as points on numbers lines tried to find the value of each subunit mark between two numbers just by dividing 1 by 100. This again might show that preservice elementary teachers were familiar with subunit ten or multiples of ten. Some of the preservice elementary teachers counted the number of parts in all for the denominator and counted the numbers of equivalent segments from zero to the marked point for the numerator, when the number line is of length two, which does not work. A group of 3 interviewees used direct proportion in finding the value of each subunit mark which again shows the effect of subunit ten on preservice elementary teachers as follows:



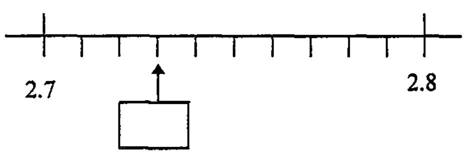
Some of the preservice elementary teachers seemed to have some problems in interpreting decimals as fractions. For example in a typical interview an interviewee interpreted 2.4 as $\frac{4}{2}$. In 1997 one of the respondents of İşeri interpreted 5.8 as $\frac{5}{8}$ and another one as in our research interpreted 5.8 as $\frac{8}{5}$.

Throughout stage-1 of the study we observed that preservice elementary teachers had some problems related to the calibration of number lines. For example 2 of the interviewees neglected the given calibration of number line in items 4 and 5 and treated each subunit mark as 0.5 units. Some stated that different units could be used on the same number line and some others believed that doubling the calibration of a number line doubles the value of each subunit mark and stated that the place of a decimal number could change on a number line whenever the calibration changes. For example 5 of the interviewees totally neglected the calibration of the given number line for item 8 and stated that any decimal on a number line could be marked between the starting point and the first subunit mark. Similar observations about calibration problems of preservice elementary teachers were made by Thipkong and Davis (1991). They stated that the most popular strategies in plotting a point on a number line were dividing the line into half, and then dividing the half into half again and by this way making the given calibration suitable for their thinking. İşeri (1997) similarly stated that in some cases middle school students were totally neglecting the calibration of a given number line.

One of the most interesting observations done on this part was the preservice elementary teachers' whole number mentality. Although it was not possible, two of the interviewees totally neglected the given calibration of the number line for items 8 and 15 and they just counted each subunit mark as 1 unit and stated that in a number line between two consecutive integers each subunit mark could be considered as one unit. In some cases like 0.5 and 0.6 they neglected the zero on the left in order to use the given decimals as integers and treated 0.5 as 5 and 0.6 as 6. Similar observations were made by Thipkong and Davis (1991). They stated that 17 of 65 preservice elementary teachers interpreted 0.4 as 4 when the subunit was based on eight.

Similar erroneous strategies and misconceptions in scale reading were observed in the CSMS research (Brown, 1981) as follows:

Students generally fixed their attention on the labeled calibration immediately to the left of the required reading, and then counted along a scale, ignoring the value of other marked calibrations and ignoring the size of each interval. The following is a question from CSMS study (item 15 from our study was similar to the question given below):



27%, 15%, 11%, and 9% of 12, 13, 14, and 15 years, respectively, appeared to focus all their attention on the 2.7 and then proceeded to count along the scale , thus they responded as : "2.8, 2.9, 2.10, 2.11" or "2.8, 2.9, 3.0, 3.1" or "28, 29, 30, 31" so the 2.8 calibration was ignored.

Briefly we can state that preservice elementary teachers have some erroneous strategies and misconceptions in interpreting and applying decimals as points on number lines. Previous research also suggests that teachers' understanding of decimals and fractions is limited and replete with misunderstandings about decimal and fraction concepts and procedures (Post, Harel, Behr, and Lesh, 1991; Ball, 1990; Leinhart and Smith, 1985).

Preservice elementary teachers' erroneous strategies and misconceptions in interpreting and applying decimals as points on number lines are given in the following paragraphs. The erroneous strategies and misconceptions of preservice elementary teachers mainly were observed during the interviewees and most of them are just the preservice elementary teachers' own words or sentences. In table 4.5 we monitored preservice elementary teachers' erroneous strategies and misconceptions in interpreting and applying decimals as points on number lines from the most frequently observed to the least frequently observed.

Table 4.5 Interviewees' Misconceptions in Interpreting and Applying Decimals as Points on Number Lines

	Misconceptions[1]								
Interviewees	1	2	3	4	5	6	7	8	9
1		*			*		*		
2	*	*			*	*		*	*
3	*	*							
4		*		*					
5									
6									
7	*								
8									
9						*			
10									
11									
12	*								
13	*								
14			*				*	*	*
15	*								
16				*					
17	*	*	*						
18	*		*						
19									
20									
21									
22	*								
23									
24	*								
25									
	40%	20%	12%	8%	8%	8%	8%	8%	8%

(1): Misconceptions:

1. On a number line between two numbers there are 10 divisions each being 0.1 unit
2. Any decimal on a number line can be marked between the starting point and the first subunit mark
3. Decimals are based on subunit 10
4. Each subunit mark on a number line is 0.5 unit
5. Each division on a number line is 1 unit
6. Decimals are based on subunit 100
7. Place of a decimal number changes on a number line when the calibration changes
8. Different subunits can be used on the same number line / model
9. Doubling the calibration of a number line doubles the value of each subunit mark

4.1.1.3 Results Concerning the Interpretations of Preservice Elementary Teachers About Decimals on Shaded Areas

4.1.1.3.1 Results Concerning the Interpretations of Preservice Elementary Teachers In Marking and Shading the Squares for the Given Decimal

In this part there were 3 items (items 19, 20, 21 from Pre-Form of Concept Test) to be answered, so the maximum possible score was 3. Scores ranged from 0 to 3. As it is seen in table 4.6 shading the squares for the given decimal 1.08 on the subunit based on 10 for item 20 was the most difficult one for the preservice elementary teachers in this part. Items 19 and 21 were better responded when compared with item 20 but still they were scored low. The mean score was 1.15 with a standard deviation 1.17. The mean percent of the preservice elementary teachers' correct responses in this part was 46.

Table 4.6: Categories of Responses of Preservice Elementary Teachers on Items in Marking and Shading the Squares for the Given Decimal in Pre-Form of Concept Test

Item	Decimal	Subunit	Correct . Ans	No Answer	Observation -1	Observation -2	Observation -3	*Other*
19	0.6	8	39	12	8 (6-strips)	5 (0.6 of one strip)	3 (3-strips)	5
20	1.08	10	24	14	22 (18 strips)	10 (1+0.8 stps)	-	2
21	2.75	3	37	11	10 (2+0.75 strps)	8 (6+0.75 strps)	2 (whole)	4

- Observations-1 ,2 and 3 stand for the wrong responses.
- The numbers in the dark regions indicates preservice elementary teachers' mistakes by counting each subunit area as one tenth or one hundredth.
- The numbers in the parenthesis are the typical responses of preservice elementary teachers.

In table 4.6 under the heading of Observation - 2 and Observation - 3 we tried to monitor some interesting responses of preservice elementary teachers:

For item 19, in marking and shading the squares / rectangles for the given decimal 0.6 , five (7%) of the preservice elementary teachers neglected the other subunit areas and considered only 0.6 of the first strip. On the other hand three of them treated each subunit region as 2 tenths and shaded only 3 strips.

For item 20, in marking and shading the squares / rectangles for the given decimal 1.08, ten (14%) of the preservice elementary teachers again neglected the other subunit areas and treated each subunit area as 1 unit and shaded 1 + 0.8 regions. They also seemed to ignore zero in 1.08 as a place holder.

For item 21, in marking and shading the squares / rectangles for the given decimal, eight (12%) of the preservice elementary teachers showed interesting responses. This time they didn't neglect the subunit areas but treated the subunit in a wrong manner. They just shaded whole of the first two models and later shaded a portion of only the first area of the third model by pointing out that it showed 0.75 of it. In fact the areas shaded by them could not stand for 2.75.

In the following paragraphs we present some interview excerpts related to marking and shading the squares for the given decimal. The items posed to the preservice elementary teachers in this part are the most challenging ones to the whole of the sample or to the individual interviewee.

The following were typical responses of the preservice elementary teachers from the interviews for Item 19 (8 *preservice elementary teachers were interviewed on this item)* :

Item 19 (subunit based on 8):
Mark and shade the squares/rectangles for the given decimal number.

0.6

1st Interview Excerpt for Item 19

R: Please mark and shade the rectangular / square region(s) for the given decimal.
N: Here we have 8 rectangles.

R: What is the way of solving this ?
N: If there were 6 rectangles each would be 0.1.
R: is it possible to solve ?
N: I don't know, what I should do for this.

In the above interview the interviewee seemed to be influenced mainly by the *subunit based on ten nature of the decimals*. Later in the interview process she couldn't do anything about the problem.

2nd Interview Excerpt for Item 19

R: Please mark and shade the rectangular / square region(s) for the given decimal.

E: I can use a direct proportional procedure like → | In 10 it is 6 |
then "?" is equal to 4.8 which is 0.48. | in 8 ? |
R: How did you decide on 0.48 ?
E: Sorry , it will be 4.8.
R: What is the region that will be shaded ?
E: 4.8 of the total.

This interviewee was slightly better on this item and she stated that she could only solve such problems by using direct proportions.

3rd Interview Excerpt for Item 19

R: Please mark and shade the rectangular / square region(s) for the given decimal.
M: Changing it to a fraction will be easier like $\frac{6}{10} = \frac{3}{5}$ then I should take 3 of 5, the remaining slices are additional there is no need for them.

In the above interview the starting point of the interviewee was not totally wrong but the fractional representation of the given decimal distracted her and gave us the chance of detecting her weaknesses about the calibration of number lines. Her statement *" take 3 of 5, the remaining segments are additional there is no need for them"* was really interesting.

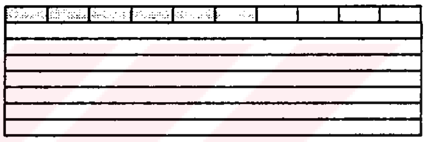
4th Interview Excerpt for Item 19

R: Please mark and shade the rectangular / square region(s) for the given decimal.
M: Since it is 0.6 than we should shade 6 of ten, but in this case we have only 8 rectangles then if we halve the upper two rectangles, we get the chance of shading 6 out of 10.
R: Is this really possible ?
M: Yes, of course.

Unlike the previous one, this time the interviewee tried to increase the calibration of the given number line by only halving the last upper two areas. In addition to the calibration problem, the interviewee also seemed to defend, implicitly, the idea that "*a 1 unit area model can be treated as multiple 1 unit models*"

5th Interview Excerpt for Item 19

R: Please mark and shade the rectangular / square region(s) for the given decimal.
G: I think that we will shade 6 of the 10...well.... Is the given subunit based on 10 ?
R: No, it is 8.
G: Well, I can divide the first rectangle into 10 and shade 6 of them.



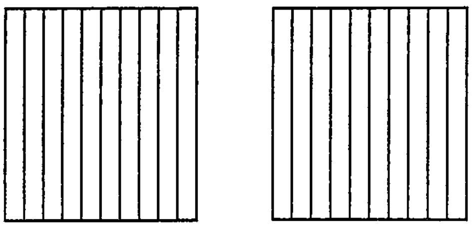
At the beginning of the interview, the interviewee showed that she was influenced about the ten-ness of the decimals. Later like the previous interviewee, she also held the idea that "*a 1 unit area model can be treated as multiple 1 unit models*" by shading only the first area of the given model.

The following are typical responses of the preservice elementary teachers from the interviews for Item 20 (15 *preservice elementary teachers were interviewed on this item*):

<u>Item 20 (subunit based on 10)</u>

Mark and shade the rectangles/squares for the given decimal number (each model is one-unit)

1.08



1st Interview Excerpt for Item 20

R: Please mark and shade the rectangles/squares for the given decimal number.
S: The first model will be totally shaded,... (mmmm !) Am I going to take 8 of the second model ?
R: Please do not ask me such questions.
S: Well , we'll shade 8 of the second model.

The above interviewee seemed to have some problems about *"zero as a place holder"* , because she treated 1.08 as if it were 1.8.

2nd Interview Excerpt for Item 20

R: Please mark and shade the rectangles/squares for the given decimal number.
N: The first one is "1" I will try to find 0.8....(thinking) if we count as 0.1, 0.2, 0.3, ...then we have two rectangles left which should not be shaded.
R: Is there a difference between 1.8 and 1.08 ?
N: Yes.
R: Then how can you read 1.08 ?
N: One and 8 hundredths, ... but when you shade they cover different areas.

At the beginning, the above interviewee seemed to have no problem about *zero as a place holder* but since she stated that although 1.8 and 1.08 were different numbers they could cover the same region on the given models, we can conclude that she still has some problems about zero as a place holder. Like many others, this interviewee also seemed to be influenced by the belief *"decimal numbers are based on subunit ten"*.

3rd Interview Excerpt for Item 20

R: Please mark and shade the rectangles/squares for the given decimal number.
M: The first model can be treated as "1" and we can take 8 of the second model , where each rectangle is 0.1 unit.
R: Is there a difference between 1.8 and 1.08.
M: When we move the decimal point we get 18 and 10.8 which shows that there is a difference.
R: Finally what do you say about shading ?
M: Shade the first model totally and take 8 of the second model.

In the above interview, even if it was totally procedural (moving the decimal point in order to be secure), the interviewee could state that 1.8 and 1.08 were different. It seemed that this benefit couldn't help the interviewee and her *ten-ness belief of decimals* beat her *zero as a place holder* awareness which in turn made her conclude in a wrong manner.

4th Interview Excerpt for Item 20

R: Please mark and shade the rectangles/squares for the given decimal number.

RM: Let us write 1.08 as $\frac{108}{100}$ and then make the required simplifications in which we get $\frac{27}{25}$.

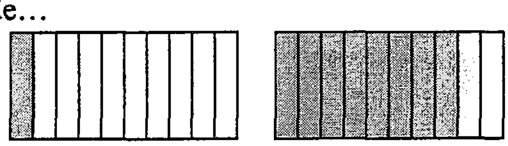
R: Do you mean that we'll get 27 out of 25 ?
RM: Yes.
R: How can you shade this ?
RM: I don't know.

Like many other preservice elementary teachers this interviewee also tried to do the given job by using pure procedural ways without a full understanding. Although, she tried to use her fractional representation knowledge of decimals, it seemed that she had a weak understanding of decimals in terms of fractions. Since she was surprised when she was faced with 27 of 25, it seemed that her fractional numbers knowledge was limited to proper fractions.

5th Interview Excerpt for Item 20

R: Please mark and shade the rectangles/squares for the given decimal number.
U: I can take "1" by shading the first rectangle of the first model ....(thinking)... is there a meaning of that zero.

R: I don't know, you decide...
U: It will be as follows



The above interviewee seemed to have a very limited understanding of decimals. As we stated that each model is one-unit she noticed that there was a need to shade some regions from the two models but this couldn't help her anyway. It seemed that she has *zero as a* *place holder* problem. In addition to that it seemed that she held the belief that *"a 1 unit area model can be treated as multiple 1 unit models"*.

6th Interview Excerpt for Item 20

R: Please mark and shade the rectangles/squares for the given decimal number.
Ü: We shade the whole of the first but I am not sure about the second one.... Ohh. I think that we can shade four of the second model.
R: Why will you shade four of them .
Ü: If it were 8 out of 10 it would be 8 but since it is 8 over 100 then we should take four of them.
R: Are you sure ?
Ü: Yes.

The strategy used by the above interviewee was very interesting. Although she understood the decimal notation, instead of talking about the increasing of the calibration of the given models she preferred to decrease the given calibration in a wrong way.

4.1.1.3.2 Results Concerning the Interpretations of Preservice Elementary Teachers In Writing a Decimal for the Given Shaded Region

In this part there were 4 items (items 22, 23, 24, 25 from Pre-Form of Concept Test) to be answered, so the maximum possible score was 4. Scores ranged from 0 to 4. As it is seen in table 4.7 Preservice elementary teachers seemed to do better with subunit based on ten. The mean score was 1.81 with a standard deviation 1.45. Item 24 was the most difficult problem for preservice elementary teachers in this part. The subunit was based on 6 and there were two models each representing one-unit. The mean percent of the preservice elementary teachers' correct responses in this part was 38.

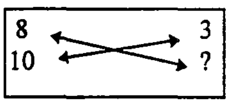
Table 4.7 : Categories of Responses of Preservice Elementary Teachers on Items in Writing a Decimal for the Given Shaded Area in Pre - Form of Concepts Test

Item	Decimal	Subunit	Correct	No	Obs-1	Obs-2	Obs-3	Obs-4	***Other***
22	0.375	8	42	7	7 (0.3)	5 (8.6)	5 (3.75)	-	6
23	0.15	10	53	4	3 (1.5)	2 (6.6)	2 (1.15)	-	3
24	1.67	6	23	8	14 (0.83)	2 ($\frac{10}{12}$)	2 ($\frac{4}{6}$)	1 (1.4)	22
25	1.56	10	32	6	11 (0.78)	2 ($\frac{156}{200}$)	2 ($\frac{56}{100}$)	2(7.8)	17

- Obs-1, 2, 3 and 4 stand for the wrong responses.
- The numbers in the parenthesis are the typical Categories of Responses of Preservice Elementary teachers.

In table 4.7 under the heading of Observaiton - 1, Observation - 2 , Observation - 3 and Observation - 4, we tried to monitor some strange responses of preservice elementary teachers:

For item 22, in writing a decimal for the given shaded region, seven (9%) of the preservice elementary teachers just counted the number of shaded areas and wrote that number as a decimal and gave 0.3 as the answer. Some of the preservice elementary teachers in this group also treated each shaded region as it was one-tenth. The ones who just counted the number of shaded areas as decimals seemed to treat *decimal point as a separator*. Five (7%) of the preservice elementary teachers tried to represent the given shaded area in terms of a fraction ($\frac{8}{3} \approx 2.6$) and gave 2.6 as the answer for item 22. Another group of five (7%) preservice elementary teachers gave 3.75 as the answer after performing the following direct proportion.



Finally for item 22 one of the preservice elementary teachers had really done one of the most interesting procedures. In order to change the given subunt to 10 she even broke some basic rules of arithmetic as follows:

First of all she expressed the given shaded areas as $\frac{3}{8}$ later she tried to enlarge the fractional number by multiplying numerator and denominator by 2

($\frac{3.2}{8.2}$). Although it was impossible she stated that $\frac{3.2}{8.2} = \frac{6}{10} = 0.6$.

For item 23, in writing a decimal for the given shaded region, three (4%) of the preservice elementary teachers treated the given *one unit area model as multiple one unit models* and considered each cell as one-tenth so gave 1.5 as the answer. In addition to that another group of three (4%) preservice elementary teachers completely ignored the given one unit model and treated each cell as 1 unit areas. This time the three preservice elementary teachers handled the *decimal point as a separator* and gave 10.5 as the answer. Two (3%) of the preservice elementary teachers just divide 100 by 15 ($100 \div 15$) and gave 6.6 as the answer for item 23.

For item 24, in writing a decimal for the given shaded region, fourteen (20%) of the preservice elementary teachers treated the given two one-unit area models as a single one unit model so thought the shaded region as $\frac{10}{12}$ and gave 0.83 as the answer. Two of the preservice elementary teachers thought in the same way but they gave $\frac{10}{12}$ as the final answer without turning it into a decimal. Another group of two (3%) preservice elementary teachers didn't consider the first model and tried to express the shaded region, only, on the second model in terms of a fraction as $\frac{4}{6}$. One of the interesting observation for item 24 was the answer 1.4. One of the preservice elementary teachers treated the *decimal point as a separator* and moreover he took each subunit area as 1 unit so just counted the areas and gave 1.4 as the answer.

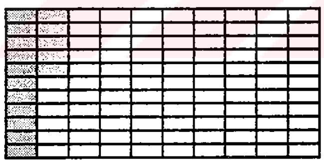
For item 25, in writing a decimal for the given shaded region, eleven (15%) of the preservice elementary teachers treated the given two one-unit area models as a single one unit model so thought the shaded region as $\frac{156}{200}$ and gave 0.78 as the answer. Another two (3%) of the preservice elementary teachers thought the same thing but didn't turn the fraction $\frac{156}{200}$ into a decimal and gave it as the answer. A group of two preservice elementary teachers neglected the first area model and gave $\frac{56}{100}$ as the answer by considering only the second area model. Another group of two (3%) preservice elementary teachers again considered the given two one - unit area models as a whole and also insisted on a subunit system 10 by doing the following procedures:

$$\boxed{\frac{156}{200} = \frac{x}{10} \rightarrow 7.8}$$

The following is a typical interview excerpt for Item 23 (2 *preservice elementary teachers were interviewed on this item*):

Item 23 (subunit based on 100)

Write the decimal for the given shaded area (each model is one-unit)



An Interview Excerpt for Item 23

R: Please write the decimal for the given shaded area
O: It will be 1.05
R: Why ?
O: Each block is 1 unit. We take the whole of the first and half of the second then the shaded region is 1.05.

Although, we stated that the given area model was one unit, as it is seen in the above interview excerpt, the interviewee treated the given model as it consisted of multiple 1 unit models. He stated that each block was 1 unit and then took the whole of the first and half of the second. This contradicts with his subunit idea, because after his explanation 1.5 was the expected answer but he gave 1.05 as the result. He noticed that the given model was based on subunit 100 but his answer points out that he might hold the belief *"different units can be used on the same model"*

The following are typical responses of the preservice elementary teachers from the interviews for Item 24 (10 *preservice elementary teachers were interviewed on this item*):

Item 24 (subunit based on 6):

Write the decimal for the given shaded area.

1st Interview Excerpt for Item 24

R: Please write the decimal for the given shaded area

Ç: Well it will be 10 over 12 which can be shown as $\frac{10}{12}$

R: How can you present the shaded region in terms of a decimal number ?

Ç: Is it 1.2 ?

R: I don't know you will tell us.

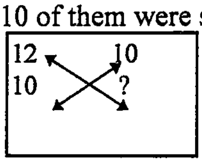
In the above interview, the interviewee considered the given two models as a whole and used his fraction knowledge in order to find the answer. Later he turned the fractional number into a decimal in a wrong way, this may also show the influence of the belief *"decimals are based on subunit 10"*.

2nd Interview Excerpt for Item 24

R: Please write the decimal for the given shaded area

E: 6 + 6 = 12 there are twelve pieces, and 10 of them were shaded.....

we can use direct proportion as



then we get 0.102 and this will be 0.0102

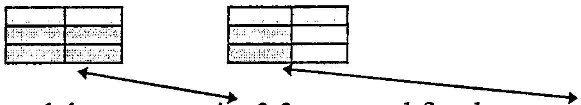
R: Please do not forget that each rectangular model is one unit.

E: (can not do anything...)

Above interviewee seemed to hold the same misbeliefs with the previous interviewee. This time she used a direct proportion in order to find the answer but couldn't manage to find it.

3rd Interview Excerpt for Item 24

R: Please write the decimal for the given shaded area



For the first model we can write 3.3 and for the second 3.1

R: Are you sure about that ?
Ş: Well .... Is it going to be 9.1 ?

The above interview was almost the most interesting one. She just divided each one-unit model into halves and used the halving lines as the decimal points. Moreover she treated the *decimal point as a separator* and counted the subunit areas as numbers to be placed before and after the decimal points. Briefly, it seemed that she held the belief " *a 1 unit area model can be treated as multiple 1 unit models*".

4th Interview Excerpt for Item 24

R: Please write the decimal for the given shaded area
Ü: (thinking)....I don't know how to do it !
R: Is there a representation of this ?
Ü: (thinking)...
R: Can you tell me any way of doing this ?
Ü: There is no representation of this.

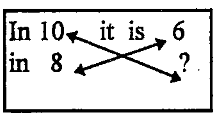
In the above interview the most important observation about the interviewee was his belief that *some area models had no decimal representation.*

4.1.1.4 Discussion of the Results Concerning the Interpretations of Preservice Elementary Teachers About Decimals on Shaded Areas

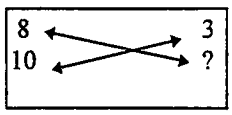
In interpreting decimals on shaded areas the mean percent of the preservice elementary teachers' correct responses to the related items was 42. When we go over the test results (tables 4.5 - 4.6) and interview results, this time it is seen that preservice elementary teachers performance on the items which were based on subunit 10 were almost equal on the items which were not based on subunit ten. In the pre-form of concept test some area models were including one and some others were including two one-unit and this time we observed that preservice elementary teachers were, slightly, better on the items that were including one-unit area models (62% correct) comparing to two-unit models (40% correct).

In this part, the most difficult items for preservice elementary teachers were item 20 (34% correct) and item 24 (32% correct). In both of the items, separate two-unit models were used. In items like 20 and 24 preservice elementary teachers generally treated multiple one unit area models as single one unit models and they counted the number of parts in all for the denominator and counted the numbers of equivalent shaded areas for the numerator. Preservice elementary teachers did not use this strategy for number lines at that much. When we look at the educational system in Northern Cyprus, we observe that number lines are much more emphasised than area models in interpreting and applying decimals. This can be a possible reason for the above case. Especially in item 20 in which we asked the preservice elementary teachers to show 1.08 on a two-unit model each divided into one tenths, most of the preservice elementary teachers ignored the zero in the tenths place. Similar observation was done on five of the interviewees.

As we observed in number lines, preservice elementary teachers again seemed to hold the belief " decimals are based on subunit ten". Preservice elementary teachers proved this idea whenever they used direct proportion in order to find the shaded regions or in shading the regions for the given decimals as follows:



For item 19



For item 22

In some items, like item 19, some of the preservice elementary teachers ignored zero in the given decimal number and treated the decimal point as a separator. For example in item 19 they shaded 6 subunit areas for 0.6 , although it was based on subunit 8. In CSMS research Brown (1981) stated that students had difficulties using zero as a place holder and reports this situation typically in the following question:

Question : Ring the BIGGER number : 4.06 or 4.5

Success rates of 12, 13, 14, and 15 year old were 66%, 72%, 83%, and 80%, respectively.

This misconception is not common only to children. Thipkong and Davis (1991) conducted a research whose subjects were 65 preservice elementary teachers. They had administered a test including reading scales and shading area models. They identified 29 subjects as having low conceptual understanding. More specifically they found that 11% of the preservice elementary teachers marked 1.05 as 1.5. They ignored the zero in between and interpreted the decimal point as it was not there. Bell (1982) reports 79% of 11 year old students interpreting 0.1 less than 0.07, probably because 1 is less than 7.

In some items like item 20, preservice elementary teachers first started by turning the given decimal into a fraction , for example $\frac{108}{100}$ for 1.08. Later when they simplified the fraction they got $\frac{27}{25}$ and concluded that 27 of 25 was impossible. This may show that the preservice elementary teachers are mainly thinking decimals as proper fractions. Similar observations were done by Resnick et al. (1989).

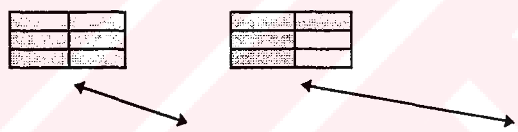
Throughout the interviews especially in discussing the items 19, 20, and 23, some of the interviewees monitored some difficulties similar to calibration problems in number lines. The most frequently observed erroneous strategies and misconceptions were as follows:

"a 1 unit area model can be treated as multiple 1 unit models", "different units can be used on the same model" , "calibration of a model can be increased without changing the size of each subunit area", " multiple one unit area models can be treated as a single 1 unit model", and "calibration of a model can be decreased without changing the size of each subunit area". One of the most interesting observation about the calibration problem was as follows:

An interviewee wrote 0.6 as $\frac{6}{10} = \frac{3}{5}$ and then said that she should have taken 3 of 5 and the remaining slices were additional, there was no need for them.

Another important observation, especially, came out when we discussed item 24. The interviewees showed that they strongly held the belief *"decimal point is a separator"* . The following interview excerpt is an indicative example for that:

R: Please write the decimal for the given shaded area



For the first model we can write 3.3 and for the second 3.1

R: Are you sure about that ?
Ş: Well .... Is it going to be 9.1 ?

We also observed only one interviewee saying *"some area models have no decimal representation"* which was really interesting. This was observed in the following interview:

R: Please write the decimal for the given shaded area
Ü: (thinking)....I don't know how to do it !
R: Is there a representation of this ?
Ü: (thinking)...
R: Can you tell me any way of doing this ?
Ü: There is no representation of this.

Briefly we can state that preservice elementary teachers have some erroneous strategies and misconceptions in interpreting and applying decimals on area models.

Preservice elementary teachers' erroneous strategies and misconceptions in interpreting and applying decimals on area models are given in the following paragraphs. The erroneous strategies and misconceptions of preservice elementary teachers were mainly observed during the interviews and most of them, as in the number lines, are the preservice elementary teachers' own words or sentences. In table 4.8, we monitored preservice elementary teachers' erroneous strategies and misconceptions in interpreting and applying decimals on area models from the most frequently observed to the least frequently observed.

Table 4.8 Interviewees' Misconceptions in Interpreting and Applying Decimals on Area Models

	Misconceptions[1]								
Interviewees	3	10	11	12	8	13	14	6	15
1		*	*						
2	*	*	*						
3	*	*		*					
4	*		*						
5									*
6				*				*	
7									
8									
9		*					*		
10	*		*		*	*			
11									
12		*			*				
13									
14							*	*	
15	*								
16									
17	*	*							
18						*			
19	*		*						
20	*								
21									
22									
23									
24									
25									
	32%	24%	20%	8%	8%	8%	8%	8%	8%

(1): Misconceptions:
3. Decimals are based on subunit 10
6. Decimals are based on subunit 100
8. Different units can be used on the same number line / model
10. A 1 unit area model can be treated as multiple one unit models
11. Zero in a decimal number is not a place holder
12. Multiple one unit area models can be treated as a single 1 unit model
13. Calibration of a model can be decreased without changing the size of each subunit area
14. Calibration of a model can be increased without changing the size of each subunit area
15. Some area models have no decimal representation

4.1.1.5 Results Concerning The Preservice Elementary Teachers' Comparison of Decimals

In this part there were 5 items (items 26, 27, 28, 29, 30 from Pre-Form of Concept Test) to be answered, so the maximum possible score was 5. Scores ranged from 0 to 5. As it is seen in table 4.9, in this part item 26 and 27 were the most difficult problems for the preservice elementary teachers. The mean score was 3.92 with a standard deviation of 1.61. The mean percent of the preservice elementary teachers' correct responses in this part was 82.

Table 4.9: Categories of Responses of Preservice Elementary Teachers to the Items Related to the Comparison of Decimals in Pre-Form of Concept Test

Item	Alternatives			No Answer
26	3.521 (4)	3.6 (13)	3.75 (52)	3
27	15.4 (15)	15.56 (53)	15.327 (3)	1
28	4.09 (0)	4.7 (66)	4.008 (3)	3
29	0.5 (64)	0.36 (3)	-	5
30	0.25 (57)	0.100 (9)	-	6

- *The shaded regions stand for the correct answers and the numbers in the paranthesis are for frequences..*

In some problems in order to prevent recognition some items to be used in the interviews were reformed without distorting the general character.

In the following paragraphs we stated the responses of some interviewees to such problems

Item - 4.3. a (not from the test, 1 *preservice elementary teacher was interviewed on this item*):

Circle the bigger number.
3.52 3.8

A typical interview excerpt for item 4.3.a

R: would you please circle the bigger number.
Ç: 3.52 is bigger then 3.8
R: please give a brief explanation
Ç: 3.52 is closer to 4 .
R: what do you think about 52 and 8 ?
Ç: that is the main reason.

In the above interview excerpt, the interviewee thought that 3.52 was closer to 4 than 3.8 and stated that 52 was bigger that 8 which made him choose 3.52 as the bigger number. It seemed thac the interviewee was mainly under the influence of his whole number knowledge.

Item 4.3.b (not from the test, 4 *preservice elementary teachers were interviewed on this item*):

Circle the bigger number.

4.521 4.6 4.75

A typical interview excerpt for item 4.3.b

R: Please circle the biggest number.
O: 4.6 is the biggest.
R: What is the reason for that ?
O: When the number of digits after the decimal point is getting less and less the number becomes larger.
R: OK. Let's try 0.25 and 0.100
O: Well it is 0.25.
R: why ?
O: because it is closer to an integer.

This time the interviewee seemed to be under the pressure of fractional numbers knowledge (especially proper fractions) because he believed that *"more digits after the decimal point makes a decimal smaller"* . In addition to that he also seemed to hold the belief *" when the number of steps needed to round a decimal to a whole number is limited the number becomes larger".*

Item 4.3.c (not from the test, 4 *preservice elementary teachers were interviewed on this item*):

Circle the bigger number.

0.35 0.200

A Typical Interview Excerpt for Item 4.3.c

R: Please Circle the bigger number.
H: I think that 0.200 is bigger than 0.35.
R: Why ?
H: Because the number 200 after the decimal is much greater than 35.
R: What do you think about 0.52 and 0.006 ?
H: 0.52 is greater.
R: what is the reason ?
H: because 0.0 has no value.
R: Is it to say that we can write 0.006 in a different form ?
H: yes it can also be written as 0.6.

In the above interview excerpt the interviewees' first responses were similar to the previous one, we may say that she thought decimals as whole numbers. In addition to that, she seemed to have some difficulties about *zero as a place holder* in decimals. She stated that 0.006 could also be written as 0.6.

The following are typical responses of the preservice elementary teachers from the interviews for Item 26 (14 *preservice elementary teachers were interviewed on this item*):

Item 26

Circle the biggest number.

3.521 3.6 3.75

1st Interview Excerpt for Item 26

R: Would you please circle the biggest number.
Y: 3.6 is the biggest one.
R: Why ? Would you explain ?
Y: If all the numbers after the decimal point were having two digits like
3.51 3.62 3.42 it would be 3.62

R: Then if the number of the digits after the decimal point ......

Y: ..... increases the number gets smaller.

R: then what do you say about 3.42 and 3.420 ?
Y: 3.42 is greater. In 3.420 , 420 is out of 1000 then we divide it into many parts.

The above interviewee seemed to be influenced mainly by fraction rule, where the decimal number gets smaller if it has many digits after the decimal point. Since she stated that 3.42 was bigger that 3.420, she also showed her weak understanding of zero as a place holder.

The same findings can be observed for the following interviewee.

2nd Interview Excerpt for Item 26

R: Would you please circle the biggest number.
A: 3.6 is the biggest one.
R: Why ?
A: 3.6 is closer to 4.
R: what about 0.25 and 0.100 ?
A: 0.25 is bigger.
R: Are you trying to say that the one which is easier to round is greater ?
A: yes, that's exactly what I'm thinking.

4.1.1.6 Discussion of the Results Concerning The Preservice Elementary Teachers' Comparison of Decimals

In this part although the preservice elementary teachers performance was slightly better when compared with the other dimensions, we observed interesting erroneous strategies and misconceptions in comparing decimals.

Although it was out of our focus we observed that two of the preservice elementary teachers interviewed in this part had problems with *"zero as a place holder"*. In comparing 0.52 and 0.006 one of the interviewees stated that 0.006 could also be written as 0.6 for zero has no value. It is possible to see many similar observation throughout the related literature. From CSMS study Brown (1981) reports that many students selected 0.75 as the answer to the question "Ring The BIGGER of the two numbers: 0.75 0r 0.8". A twelve year old student gave her reason to 0.75 as "this is nothing before and seventy five, this is nothing before and just eight." As it is seen she is just generalising the rules that are correct within whole number context to decimal number context.

Generally students fail to use and interpret zero as a place holder correctly when zero is either before or just after the decimal point. Surprisingly, some of the interviewees failed to interpret zero as a place holder even if it was at the end of the given number (e.g., an interviewee chose 3.420 as the bigger than 3.42, because 420 is bigger than 42). Similar observations were made after the mathematics assessment of the National Assessment of Educational Progress (Carpenter, Corbitt, Kepneri Lindquist, and Reys, 1981). It was indicated that 9 year olds had little familiarity with decimals and about 50% of 13 year olds lacked even basic understanding of decimals. Several items asked students to order decimals by recognising place value. Although most 13 year olds realised that a number greater than one is larger than a number less than one, they had substantial difficulty ordering two decimals less than one. Almost 50% ignored the decimal points and treated the numbers as whole numbers. Most students did not recognised that 0.3 was equivalent to 0.30. Hart (1981) interviewed a student by asking "Is there a difference between 4.90 and 4.9 ?" and the answer was "yes, 4.90 is more."

The most frequently observed misconception of the interviewees in this part was as follows:

For example in comparing 3.521, 3.6, and 3.75, five of the interviewees had chosen 3.521 as the biggest. They were defending themselves by stating that a decimal was greater when the whole number after the decimal point was greater. Similar observations were made by many researchers. Sackur-Grisvard and Leonard (1985, pp.157-174) found that students use several faulty generalisations to develop operational rules that are error prone. The most frequent error (made by 40% of fourth graders and 25% of fifth graders), called *"whole number rule"* is to select as smaller the number whose decimal portion would be a smaller whole number (e.g., choosing 11.7 as smaller than 11.14 because 7 is smaller than 14). The results of decimal concepts in the Second National Assessment (Carpenter et al., 1981) were that 9 year olds demonstrated little familiarity with decimals and treated then as whole numbers, and that 13 and 14 year olds have difficulty with thousandths and smaller decimal numbers. İşeri (1997, p.67) stated that many of the middle school students in his research compared decimal numbers just like whole numbers. Bonotto (1993), in examining fifth-grade students ordering of decimals and fractions stated that many children interpreted decimals as integers or as fractions.

Another important misconception of the interviewees in comparing decimals was to interpret decimal number as fractions in a wrong way. Most of the preservice elementary teachers stated that *"more digits after the decimal made a decimal smaller"*. A typical example is as follows:

In comparing 4.521, 4.6, and 4.75 , three of the interviewees chose 4.6 as the biggest and stated that "when the number of digits after the decimal point was limited the number became larger". This time Sackur-Grisvard and Leonard (1985) called such a misconception as *"fraction rule"* , that is to select as smaller the number with more digits in the decimal part (e.g., choosing 11.723 as smaller than 11.41). Hiebert and Wearne (1986, p.206) report the percentage of students in grades 5, 7, and 9 choosing "0.065 when asked to identify the smallest of 1.006, 0.06, 0.065, and 0.09" increases from 6% in grades 5 to 25% in grade 9. In pointing out the importance of the case, Grossman (1983) states "unfortunately, the fraction error is still made by many students entering college". Resnick et al (1989) gives an explanation to treatment of decimal with least number digits as the biggest as follows: "if students know that thousandths are smaller parts than hundredths and that three digit decimals are read as thousandths whereas two-digit decimals are read as hundredths, they might well infer that longer decimals, because they refer to smaller parts, must have lower values."

In comparing decimals, some of the preservice elementary teachers in this present study used some strategies that were never or rarely observed by previous researchers before. For example two of the interviewees states " *when the number of steps needed to round a decimal to a whole number is limited the number becomes larger.* " This might be treated as a different version of the *fraction rule*.

Briefly we can state that preservice elementary teachers have some erroneous strategies and misconceptions in comparing decimals.

Preservice elementary teachers' erroneous strategies and misconceptions in comparing decimals are given in the following paragraphs. The erroneous strategies and misconceptions of preservice elementary teachers were mainly observed during the interviewees and most of them, as in the number lines and area models are exactly the preservice elementary teachers' own words or sentences. In table 4.10 we monitored preservice elementary teachers' erroneous strategies and misconceptions in comparing decimals from the most frequently observed to the least frequently observed.

Table 4.10 Interviewees' Misconceptions in Comparing Decimals

	Misconceptions[1]			
Interviewees	16	17	18	11
1	*			*
2	*			
3	*			
4				
5				
6				
7		*		
8		*		*
9	*			
10				
11				
12		*	*	
13			*	
14				
15	*			
16				
17				
18				
19				
20				
21				
22				
23				
24				
25				
	20%	12%	8%	8%

(1): Misconceptions:

11. Zero in a decimal number is not a place holder.
16. A decimal is greater when the whole number after the decimal point is greater.
17. More digits after the decimal point makes a decimal smaller.
18. When the number of steps needed to round a decimal to a whole number is limited the number becomes larger.

4.1.1.7 Results Concerning the Interpretations of Preservice Elementary Teachers About the Denseness of Decimals

In this part there were 2 items (items 31 and 32 from Pre-Form of Concept Test) to be answered, so the maximum possible score was 2. Scores ranged from 0 to 2. The mean score was 0.93 with a standard deviation 0.48. The mean percent of the preservice elementary teachers' correct responses in this part was 49.

Although in many questions, throughout the Pre-Form of Concept test, it was possible to observe how preservice elementary teachers interpret the denseness of decimals, we preferred to ask two questions which can directly give us the chance to observe the interpretations of preservice elementary teachers in this respect.

In this part most of the preservice elementary teachers (61) could specify a decimal between 5 and 5.1, but when we asked them to state the number of numbers between 0.56 and 0.57 (or similar) only ten (14%) of 72 preservice elementary teachers could state that there might be infinitely many (see table 4.11 and 4.12). Some of the preservice elementary teachers (who didn't do correctly) stated that there might be 9 or 10 numbers between 0.56 and 0.57. Item 32 (%14 correct) was the hardest item all over the Pre-form of concept Test for the preservice elementary teachers.

Write a number which is between 5 and 5.1. ☐

Figure 4.13 : Item 31 from Pre-form of concept Test

Table 4.11: Categories of Responses of Preservice Elementary Teachers for Item 31 in Pre-Form of Concept Test

Item	Correct	< 5	5.0	5.1 <	No Answer
31	61	1	1	2	7

How many numbers are there between 0.56 and 0.57 ?	☐

Figure 4.14: Item 32 from the Concepts Test

Table 4.12: Categories of Responses of Preservice Elementary Teachers for Item 32 in Concepts Test

Item	Answer	Correct	9	10	6	1	No Answer	Other
32	∞	10	14	5	3	3	31	6

The followings are typical responses of interviewees about the denseness of decimals:

Item 4.4.a (not from the test. 4 *preservice elementary teachers were interviewed on this item*):

Is there any number between 0.63 and 0.64 ?

1st Interview Excerpt for Item 4.4.a

R: Is there any number between 0.63 and 0.64 ?
E: Yes there is.
R: Can you tell me ?
E: 0.631.
R: How many numbers are there in between 0.63 and 0.64 ?
E: There should be 9 numbers.
R: is this the maximum number ?
E: yes it is 9.
R: why ?
E: because 4 comes after 0.639.

Above interviewee seemed to hold the belief that " *there was a limited number of decimals between two consecutive decimals* ". She also seemed to believe that *decimals are based on subunit 10.*

2nd Interview Excerpt for Item 4.4.a

R: Is there any number between 0.63 and 0.64 ?
M: No, there is not.
R: What is the reason for that ?
M: Because it is not possible to write a decimal which has two decimal points like 0.63.1

In the above interview, the interviewee believed that in order to place more numbers, between, the already existing ones you needed to add more decimal points. Later she stated that the situation was impossible and there was no way of finding any number between 0.63 and 0.64. She seemed to have some problems about decimal notation.

3rd Interview Excerpt for Item 4.4.a

R: Is there any number between 0.63 and 0.64 ?
H: yes 0.62.5.
R: would you please read this decimal that you have written ?
H: ……(can not say anything) …..(then) … oh ! sorry I should have said 0. 63 . 5
R: Is it possible to use two decimal points in the same decimal number at the same time ?
H: No, but I can not give any other alternative.
R: OK, how money numbers can you put between 0.63 and 0.64 ?
H: It may be five….

Contrary to the one before, the interviewee in the above interview tried to use two decimal points in the same decimal number in order to do the given task. She also believed that between two decimals there could be a limited number of decimals.

4th Interview Excerpt for Item 4.4.a

R: Is there any number between 0.63 and 0.64 ?
G: Of course there is !
R: Can you give an example ?
G: There can be a half.. but I don't know how to represent this.
R: Then can you tell me how many numbers there are between 0.63 and 0.64 ?
G: There are 9.
R: Why is it so ?
G: Because there are ten divisions between 0.63 and 0.64

The interviewee in the above interview seemed to be influenced by the belief "*on a number line between two numbers there are 10 divisions*" and this might also cause him to conclude that *there was a limited number of decimals between two consecutive decimals.*

Item 4.4.b (not from the test, 2 *preservice elementary teachers were interviewed on this item*):

Is there any number between 0.48 and 0.49 ?

1st Interview Excerpt for Item 4.4.b

R: Is there any number between 0.48 and 0.49 ?
OO: yes , 0.481.
R: how many more can you write ?
OO: we can write at most 8.
R: Does it go like 0.482, 0.483, 0.484 and etc ?
OO: Oh…I'm sorry there is no other.

In the above interview the interviewee first stated that there could be a limited number of decimals between the given two decimals, but suddenly he noticed that the ones that he recommended were bigger then the already existing ones (this was his idea) and said that *there was no any other*.

2nd Interview Excerpt for Item 4.4.b

R: Is there any number between 0.48 and 0.49 ?
D: What can be between 0.48 and 0.49 ?
R: Can we say that 0.49 is the one and only number which follows 0.48 ?
D: Of course there is no other number. Moreover it is not possible to write 0.48 and a half.
R: what do you say about 5 and 5.1 ?
D: Well 5.01 can follow 5, and there should be 10 numbers between them.
R: Why 10 ?
D: Because when you jump 10 steps the number changes.
R: Is it possible to have more than 10 ?
D: No !

Like many others, the above interviewee seemed to hold the belief that " *there is a limited number of decimals between two consecutive decimals* ". She also seemed to believe that *decimals are based on subunit 10*. Since she suddenly changed her previous idea we can briefly say that she has a weak understanding of decimals.

Item 4.4.c (not from the test, 1 *preservice elementary teacher was interviewed on this item*):

Is there any number between 2.53 and 2.54 ?

A typical Interview Excerpt for Item 4.4.c

R: Is there any number between 2.53 and 2.54 ?
F: Yes but no, ... I think that there is a kind of operation of doing these but I don't know.
R: Yes or no, try to be certain.
F: No there is none.
R: What is the reason for that ?
F: Because 2.54 follows 2.53.
R: what is the rate of increase ?
F: one by one.

In the above interview the interviewee seemed to hold the belief that *"there are no numbers between two consecutive decimals"* . Later we observed that she considered the rate of increase as 1. This also may show her weak understanding of place value concepts. She seemed to treat decimals like whole numbers.

4.1.1.8 Discussion of the Results Concerning the Interpretations of Preservice Elementary Teachers About the Denseness of Decimals

In this part the mean percent of the preservice elementary teachers' correct responses was 49. In fact this sudden decrease in the preservice elementary teachers' performance was due to their low performance on item 32. Only 10 (14%) of the preservice elementary teachers did item 32 correctly. In answering item 32 since we did not ask the preservice elementary teachers for any particular procedure of finding the answer , the preservice elementary teachers must have used mainly their conceptual rather than procedural understanding of decimals. Especially during the interviews we observed that many interviewees had a weak conceptual understanding of decimals, so their low performance in item 32 was expected. Previous research also shows that students and preservice elementary teachers' performance on tasks which mainly call for a conceptual understanding is limited (Aksu, 1997; Eisenhart et al., 1993; Stoddart et al., 1993; Fennema and Franke, 1992).

Most of the preservice elementary teachers in the interviews believed that *there was a limited number of decimals between two consecutive decimals.* It seemed that the interviewees who held this misconception also believed that decimals are based on subunit ten, because they generally stated that between two consecutive decimals there were either 10 or 9 decimals (e.g., "when you jump ten steps the number changes"). Similar observations were made in previous research studies. Wearne, Hiebert, and Taber (1991) point out that decimal fractions have a continuous aspect like common fractions and a discrete aspect like whole numbers. They found that the continuous aspect of decimals was especially difficult for students to understand. In a national U.S. sample about one half of grade 7 students could pick the decimal fraction representing the greatest number and about one third could identify a number between two given decimals (Kouba et al., 1988). In the CSMS study Brown (1981) reports examples to this situation. To the question "How many different numbers could you write down which lie between 0.41 and 0.42 ?" there were a variety of answers. Considering only 12 year olds, 7% responded as "infinitely many" (in this present study 14% of the preservice elementary teachers responded as "infinitely many"). İşeri (1997, p.68) stated that many students were not aware that there were infinitely many other decimals between any other two decimals.

Six of the interviewees stated that there was no other number between two decimals. Accordingly their interpretation of the denseness of decimals seemed to be affected by several misconceptions. Their conception was that whole numbers do not have such a property and they are generalising the whole numbers to include decimals. A similar over-generalisation was done, more strongly, by four of the interviewees. They stated that the rate of increase between two decimals was one unit (e.g., "2.54 should follow 2.53")

Three of the interviewees stated that in order to locate another decimal between already stated ones, one needed to put one more decimal point (e.g., 1.4.4 can be between 1.4 and 1.5) and since this was not possible they concluded that there was no number between two decimals. This shows that some of the preservice elementary teachers have problems about decimal notation.

In this part, we mainly observed that the preservice elementary teachers in the present study interpreted decimals as whole numbers so they generally stated that there was no other number between two consecutive decimals or there was a limited number of decimals between two consecutive decimals.

Briefly we can state that preservice elementary teachers have some erroneous strategies and misconceptions in interpreting the denseness of decimals.

Preservice elementary teachers' erroneous strategies and misconceptions in interpreting the denseness of decimals are given in the following paragraphs. The erroneous strategies and misconceptions of preservice elementary teachers were mainly observed during the interviews and most of them as in previous cases are exactly the preservice elementary teachers' own words or sentences. In table 4.13 we monitored preservice elementary teachers' erroneous strategies and misconceptions in interpreting the denseness of decimals from the most frequently observed to the least frequently observed.

Table 4.13 Interviewees' Misconceptions in Interpreting the Denseness of Decimals

	Misconceptions[1]			
Interviewees	19	20	21	3
1	*			
2		*		
3		*		
4		*		
5				
6	*			*
7				
8				
9		*	*	
10	*			
11	*			
12				
13				
14		*	*	
15	*			
16				
17	*			*
18			*	*
19	*			
20				
21		*	*	
22				
23	*			*
24	*			
25	*			
	40%	24%	17%	17%

(1) Misconceptions:

3. Decimals are based on subunit 10.
19. There is a limited number of decimals between two consecutive decimals.
20. There is no number between two consecutive decimals.
21. The difference between two consecutive decimals is one unit.

4.1.1.9 Results Concerning the Interpretations of Preservice Elementary Teachers About Decimals Involving Unit Measures

In this part there were 6 items (items 33, 34, 35, 36, 37, 38 from Pre-Concept Test) to be answered, so the maximum possible score was 6. Scores ranged from 0 to 6. As it is seen in table 4.14, in this part, Preservice elementary teachers seemed to be better on the items that were based on the metric system.. The mean score was 3.39 with a standard deviation 2.24. The mean percent of the preservice elementary teachers' correct responses in this part was 62.

Table 4.14: Categories of Responses of Preservice Elementary Teachers to the Items Related to the Unit Measures Involving Decimals in Pre-Form of Concept Test

Item	Decimal	Subunit	Answer	Correct	No Ans	Obs-1	Obs-2	Other
33	0.45	60	27	41	4	15 (45)		15
34	2.32	1000	2320	49	9	8 (232)	3 (0.00232)	3
35	1.15	60	69	36	5	27 (75)	-	4
36	3.25	100	325	52	13	3 (3250)	-	4
37	6.80	1000	6800	44	18	1 (6.8)	1 (0.0068)	8
38	2.4	12	28.8	42	4	16 (28)	5 (27)	5

- *The expressions in the parenthesis stand for the Responses of Preservice Elementary teachers.*

In this part the most difficult items for the preservice elementary teachers were Items 33, 35, and 38 which were not based on the subunit 10, 100 or 1000.

In this part, 4 (four) contexts, namely time, year, length and weight were used.

4.1.1.9.1 Results Concerning the Interpretations of Preservice Elementary Teachers About Decimals Involving Unit Measures In Relation with the Time Context

As it is seen in table 4.14, the items (33 and 35) which are related to time context were the most difficult items for preservice elementary teachers. The subunit not being based on ten (which is 60) seemed to be the most popular reason.

In table 4.14, under the heading of Observation - 1, Observation - 2, we tried to monitor some strange responses of preservice elementary teachers:

For item 33, in converting 0.45 hours into minutes, fifteen (21%) of the preservice elementary teachers treated the *decimal point as a separator* and interpreted 0.45 hours as 45 minutes. They also seemed to hold the belief that *"the number after the decimal point may represent different units according to the subunit system of a given measure"*

For item 35, in converting 1.15 hours into minutes, this time twenty seven (38%) of the preservice elementary teachers treated the *decimal point as a separator* and interpreted 1.15 hours as 1 hour (60 minutes) + 15 minutes = 75 minutes. As in the previous example, they also seemed to hold the belief that *"the number after the decimal point may represent different units according to the subunit system of a given measure"*. This sudden increase in the number of preservice elementary teachers who calculated 1.15 as 75 minutes may be the effect of "1" before the decimal point, because it may sharpen the belief that the decimal point is a separator.

The following are the typical responses of interviewees related to the unit measures involving decimals which were related to the time context:

Item 33 (12 *preservice elementary teachers were interviewed on this item)*

How many minutes are there in 0.45 hours ?

1st Interview Excerpt for Item 33

R: How many minutes are there in 0.45 hours ?
S: Do you want the operation for that ?
R: You decide.
S: Is it 45 minutes ?
R: Does the number after the decimal point show the amount out of 60 ?
S: yes.

In the above interview, the interviewee seemed to hold the beliefs "*decimal point is a separator*" and "*the number after the decimal point may represent different units according to the subunit system of a given measure*" .

2nd Interview Excerpt for Item 33

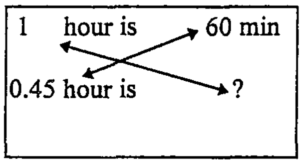
R: How many minutes are there in 0.45 hours ?
A: Well it is 45.
R: Does the number after the decimal point show the amount out of 60 ?
A: yes , of course

Above interviewee, as in the previous one, also treated the decimal points as a separator and interpreted 45 as it was out of 60 and gave 45 minutes as the answer.

3rd Interview Excerpt for Item 33

R: How many minutes are there in 0.45 hours ?

Ak: Let me do a direct proportion like



R: Is this the only way of doing that ?
Ak: I don't know any other way of doing this.

In the above interview, like many others, the interviewee preferred to use a direct proportion in order to be secure about the answer but this may not always show that the respondent has no problems about decimals.

4th Interview Excerpt for Item 33

R: How many minutes are there in 0.45 hours ?
Ş: Do we divide 45 by 60 ?
R: Do whatever you think.
Ş: Will it be 45 times 60 (45 x 60) ?
R: Why ?
Ş:....(thinking) .....
R: Some of your friends said that it would be 45 minutes, what do you say ?
Ş: Yes I agree.

Above interviewee was not fully aware of what she was doing but since she tried to divide 45 by 60 it seemed that she was mainly influenced by the belief "*the number after the decimal point may represent different units according to the subunit system of a given measure*"

Item 35 (*7 preservice elementary teachers were interviewed on this item)*

How many minutes are there in 1.15 hours ?

1st Interview Excerpt for Item 35

R : How many minutes are there in 1.15 hours ?
OO: It will be 75 minutes.
R : Why ?
OO: One hour is 60 minutes. Here the number before the decimal point shows hours and the number after the decimal stands for the minutes.
Then we can write 60 + 15 which gives 75.

In the above interview the interviewee, strongly, held the belief *"the number after the decimal point may represent different units according to the subunit system of a given measure"*

2nd Interview Excerpt for Item 35

R: How many minutes are there in 1.15 hours ?
Ö: It will be 75 minutes.
R: What is the reason for this ?
Ö: One hour is 60 minutes if we add the other 15 minutes on to that we get 75 minutes.
R: How do you read this decimal number ?
Ö: One point fifteen.
R: Any other ?
Ö: One and fifteen hundredth, ….. no no, it should be 15 out of 60.

The interviewee in the above interview held the same beliefs of the previous ones but interestingly she could read the decimal number 1.15 as one and fifteen hundredth.

3rd Interview Excerpt for Item 35

R: How many minutes are there in 1.15 hours ?
H: Sir! Are we going to get 15 as hours or seconds ?
R: Does it change from one person to another ? … Is it personal ?
H: Yes it depends on you.
R: Can you show me an example ?
H: 2 hours and 15 minutes.
R: How can you write this as a decimal ?
H: It is 2.15.
R: Suppose that I say "2 hours and 15 seconds" , do you write the same decimal for this ?
H: Yes.
R: Then can we say that the number after the decimal point shows the amount out of 60 ?
H: Yes.

The above interviewee was the one, out of all the interviews, who most strongly held the belief *"the number after the decimal point may represent different units* *according to the subunit system of a given measure*". One of the most interesting responses of the interviewee was that the unit after the decimal point could change from one person to another.

Item 4.5.1.a (not from the test, 2 *preservice elementary teachers were interviewed on this item*):

How many minutes are there in 0.15 hours ?

A Typical Interview Excerpt for Item 4.5.1.a

R: How many minutes are there in 0.15 hours ?
O: One hour is 60 minutes. 15 out of 60 is one quarter of 60 then it will be 15 minutes.
R: What happens if we say 0.45 hours ?
O: It will be 45 minutes.
R: Is the number 15 after the decimal point out of 60 ?
O: Yes
R: How can you read this decimal (0.15) ?
O: Zero point fifteen…

In addition to the beliefs "*decimal point is a separator*" and "*the number after the decimal point may represent different units according to the subunit system of a given measure*" the above interviewee was also defective in reading the decimals. In order to say zero and fifteen hundredth she read 0.15 as zero point fifteen.

Item 4.5.1.b (not from the test, 2 *preservice elementary teachers were interviewed on this item*):

How many minutes are there in 2.15 hours ?

A typical Interview Excerpt for Item 4.5.1.b

R: How many minutes are there in 2.15 hours ?
Y: One hour is 60 minutes, 2 hours is 120 minutes and finally if we add 15 minutes to that we get 135 minutes.

Above interviewee held the same beliefs of the previous interviewees, we stated up to this point, about unit measures involving decimals.

Item 4.5.1.c (not from the test, 1 *preservice elementary teacher was interviewed on this item*):

How many minutes are there in 0.43 hours ?

A Typical Interview Excerpt for Item 4.5.1.c

R: How many minutes are there in 0.43 hours ?
G: it is 43 minutes.
R: what is your reason ?
G: in order to say it is one hour we need 60 minutes. In this example it is not still full then we can say that it is just 43 minutes.
R: Well, how can you read this decimal ?
G: zero point 43 out of hundred.
R: What is the unit of the number after the decimal point ?
G: It stands for minutes.

In the above interview the interviewee thought the number after the decimal point works as in a simple counter and this of course is supported by the belief that zero is not a place holder. In addition to that the interviewee also held the belief *"the number after the decimal point may represent different units according to the subunit system of a given measure."*

4.1.1.9.2 Results Concerning the Interpretations of Preservice Elementary Teachers About Decimals Involving Unit Measures In Relation with the Year Context

As it is seen in table 4.14, item 38 which is related to year context was one of the most difficult items for preservice elementary teachers. Nearly only half of the subjects (58%) answered this item correctly. Again the subunit not being based on ten (which is 12) seemed to be the most popular reason for the incorrect responses.

In table 4.10 under the heading of Observation - 1, Observation - 2, we tried to monitor some strange responses of preservice elementary teachers:

For item 38, in converting 2.4 years into months, sixteen (22%) of the preservice elementary teachers seemed to follow the same procedures and interpret in the same way as it was done for items 33 and 35. Briefly, they seemed to hold the beliefs *"decimal point is a separator"* and *"the number after the decimal point may represent different units according to the subunit system of a given measure"* . A group of five (7%) preservice elementary teachers followed a very interesting procedure. They considered the number before the decimal point as 2 years and then treated the number 4 , after the decimal point, as it were $\frac{4}{12}$ of a year ($\frac{1}{3}$) which later caused them to conclude that the number 4 after the decimal point stood for 3 months and finally they did the following procedure:

2.4 years = 2 years (24 months) + 3 months = 27 months

The following are the typical responses of interviewees related to the unit measures involving decimals which were related to the year context:

Item 38 (subunit based on 12, 2 *preservice elementary teachers were interviewed on this item*)

How many months are there in 2.4 years ?

1st Interview Excerpt for Item 38

R: How many months are there in 2.4 years ?
S: 2 years is 24 months and then we can add 4 to that which gives 28 months.
R: do you simply add the numbers after the decimal point as months ?
S: yes.

2nd Interview Excerpt for Item 38

R: How many months are there in 2.4 years ?
F: Hmmm. It will be 2 years + 4 months which is 28 months.
R: Do you mean that the number after the decimal point .....

F: ..... Yes the number after the decimal point is 4 out of 12.

In the interviews above the two interviewees again simply showed that they held the beliefs *"decimal point is a separator"* and *"the number after the decimal point may represent different units according to the subunit system of a given measure"*.

Item 4.5.2 (not from the test - subunit based on 12, 3 *preservice elementary teachers were interviewed on this item*):

How many months are there in 3.4 years ?

1st Interview Excerpt for Item 4.5.2

R: How many months are there in 3.4 years ?
Y: A year is 12 months, then 3 years is 36 months and finally if we add 4 we get 40 months.
R: What does the number after the decimal point show us ?
Y: It shows the subunit of the unit that we work on. Subunit of year is month.
R: how can you read 3.4 ?
Y: 3 point 4, out of ten.... But in our example it is 4 out of 12 , because in one year there are 12 months.

In the above interview, although the interviewee knew that 4 was out of ten, she could easily interpret 4 after the decimal point as 4 out of 12. This showed that she, strongly , held the belief that "*the number after the decimal point may represent different units according to the subunit system of a given measure*".

2nd Interview Excerpt for Item 4.5.2

R: How many months are there in 3.4 years ?
H: 3 years is 36 months ... but I didn't understand 4...Does it mean 3 out of 4.
R: It is a decimal number.
H: Probably it is 40 months.
R: Is the number after the decimal point 3 out of 12.
H: yes.

The most interesting observation that came through the above interview was the interpretation of the interviewee of 3.4 as $\frac{3}{4}$. Briefly in the above interview it seemed that the interviewee held the same beliefs of the previous interviewee.

4.1.1.9.3 Results Concerning the Interpretations of Preservice Elementary Teachers About Decimals Involving Unit Measures In Relation With the Length Context

As it is seen in table 4.14, the items (items 34 and 36) which are related to length context were easier for preservice elementary teachers when compared with time and year contexts.

In table 4.14 under the heading of Observation - 1, Observation - 2, we tried to monitor some strange responses of preservice elementary teachers:

For item 34, in converting 2.32 Km into meters, eight (11%) of the preservice elementary teachers considered one km as 100 meter and wrote 2.32 km as 232 meters.

For item 36, in converting 2.35 meters into cm , the preservice elementary teachers were relatively better when compared with the previous example. Three (4%) of the preservice elementary teachers considered a meter as 1000 cm and wrote 2.35 meters as 2350 cm.

The following responses are coming from the interviews that focused on the items related to the length context where the subunit is based on either 100 or 1000:

Item 35 (subunit based on 100, 7 *preservice elementary teachers were interviewed on this item*)

What is 3.25 meter in terms of centimetres ?

A typical Interview Excerpt for Item 35

R: What is 3.25 meter in terms of centimetres ?
OO: it is 325 centimetres.

In the above interview, like many others, it seemed that the interviewee had no problem when the units involved were meters and cm.

Item 4.5.3.a (not from the test, subunit based on 1000, 2 *preservice elementary teachers were interviewed on this item*):

What is 5.26 kilometres in terms of metres ?

A typical Interview Excerpt for Item 4.5.3.a

R: What is 5.26 kilometres in terms of metres ?
Ç: One kilometre is 1000 metres …… I don't know.

Although the units related to length were the easiest ones for the preservice elementary teachers, in the above interview it seemed that the interviewee still had some problems. He couldn't turn 5.26 kilometres into meters.

Item 4.5.3.b (not from the test, subunit based on 100, 1 *preservice elementary teacher was interviewed on this item*):

What is 3.82 km in terms of metres ?

A typical Interview Excerpt for Item 4.5.3.b

R: What is 3.82 km in terms of metres ?
G: It is 3 metres and 82 centimetres…..no.. 3 km and 3000 metres …no 3000 metres and 820 cm.
R: Does the number after the decimal point show cm ?
G: Yes … it will be 3 km 8 meters and 2 cm.

The most interesting observation made on the use and interpretation of units involving decimals was the above. The interviewee interpreted each number in the places as the subunits of the previous ones and read 3.82 as *3 kilometres , 8 meters and 2 cm.* The interviewee also neglected the decimal point.

4.1.1.9.4 Results Concerning the Interpretations of Preservice Elementary Teachers About Decimals Involving Unit Measures In Relation with the Weight Context

In table 4.14 under the heading of Observation - 1, Observation - 2, we tried to monitor some interesting of preservice elementary teachers:

For item 37, in converting 6.80 kg into gr, eighteen (25%) of the preservice elementary teachers didn't write anything. More interestingly one of the preservice elementary teachers seemed to have some problems about zero as a place holder and gave 6.8 as the answer. Another one considered one kg as 1000 gr but went backwards and gave 0.0068 as the answer.

When we look at the results of the concepts test, it is possible to say that some preservice elementary teachers had some problems related to the item 37, which considered the weight context, but the interview results showed that the errors done in this item were not totally due to some misconceptions related to weight context. Only one interviewee showed a lower performance on item 37. This is presented in the following paragraphs.

Item 4.5.4 (subunit based on 1000, 2 *preservice elementary teachers were interviewed on this item*):

What is 2.3 kg in terms of grams ?

An Interview Excerpt for Item 4.5.4

R: What is 2.3 kg in terms of grams ?
U: ….. h mmm ….
R: What is 1 kg in terms of grams ?
U: One point something … I don't know.

Above interviewee showed a faulty performance in converting 2.3 kg into grams.

4.1.1.10 Discussion of the Results Concerning the Interpretations of Preservice Elementary Teachers About Decimals Involving Unit Measures

In this part, first of all it is possible to say that preservice teachers performance on the items which were based on the metric system (mean percent correct, 67) were slightly better than the performance on the items that were based on hour or year contexts (mean percent correct, 55).

Throughout the test and interview results, it was easily observed that some of the preservice elementary teachers treated decimal point as a separator, especially, in converting hours into minutes or years into months. For example 38% of the preservice elementary teachers in converting 1.15 hours into minutes, interpreted 1.15 hours as 1 hour (60 minutes) + 15 minutes = 75 minutes. They also seemed to hold the belief that *"the number after the decimal point may represent different units according to the subunit system of a given measure"*. This sudden increase in the number of preservice elementary teachers who responded to 1.15 hours as 75 minutes may be the effect of "1" before the decimal point, because it may intensify the belief that the decimal point is a separator. Moreover in a typical interview , in converting 2.15 hours into minutes, one of the interviewees gave "2 hours (120 minutes) + 15 minutes = 135 minutes" as the answer and stated that the unit of the number after the decimal point could change from one person to another and for the question "write a decimal number which represents 2 hours and 15 seconds" she gave 2.15 as the answer. Similar observations were made by several researchers. Bell (1981, p.405) in discussing middle school students misinterpretation of decimals states: " *11.9 miles per hour* read as *11 miles, 9 minutes per hour*; *pork chops weighting 1.07 pound* read as *1 pound 7 ounces*; *0.45 hours* reported as *45 minutes;* and *0.8* estimated as *about an eighth.* Greer (1987) stated that some students' conception of decimal numbers is as point separating two whole numbers that can be operated on independently using the rules of whole number arithmetic. Mangan (1986) found that students tend to be confused in converting units involving decimals. From his interview of secondary students, he reported that students interpreted 0.85 hours to be 1 hour and 25 minutes and 0.75 hours to be 1 hour and 15 minutes. İşeri (1997) found that many students have some misconceptions about decimal notation. He stated that some middle school students conceptualised decimal point as a separator between two distinct numbers.The belief of *"the number after the decimal point may represent different units according to the subunit system of a given measure"* was rarely observed in the related literature in such a verbalisation.

It seems that the preservice elementary teachers in this present study also have some problems about the relation between fractions and decimals. For example, in converting 2.4 years into months, 7% of the interviewees considered the number before the decimal point as 2 years and then treated the number 4 , after the decimal point, as it was $\frac{4}{12}$ of a year ($\frac{1}{3}$) which later caused them to conclude that "number 4 after the decimal point stands for 3 months" and finally they did the following procedure:

2.4 years = 2 years (24 months) + 3 months = 27 months

Another interviewee in converting 3.4 years into months treated 3.4 as $\frac{3}{4}$.

Although the conversions related to length context were easier for the preservice elementary teachers in this present study, in converting 3.82 km into metres one of the interviewees interpreted each number in the places as the subunits of the previous ones and read 3.82 as *3 kilometres , 8 meters and 2 cm.* The interviewee also neglected the decimal point and this was one of the most interesting observations done on unit conversions.

Preservice elementary teachers may have misconceptions about unit measures involving decimals because the concept of decimals has not been well developed. Preservice elementary teachers may lack intuitive sense of the size of decimal numbers and cannot relate decimals to everyday contexts where the units are not organised by tens. In this present study for example we observed that some (21%) of the interviewees thought 0.45 hours represented 45 minutes. This problem arose because they did not understand the interpretation of decimals involving different units of measurement. They could not convert from one system to another because of familiarity with working with the base 10 numeration system. Therefore, work done with the operations in the base 10 numeration system may cause problems when students learn to deal with units of 60 such as in minutes or hours.

Briefly we can state that preservice elementary teachers have some erroneous strategies and misconceptions in interpreting decimals involving unit measures.

Preservice elementary teachers' erroneous strategies and misconceptions in interpreting decimals involving unit measures are given in the following paragraphs. The erroneous strategies and misconceptions of preservice elementary teachers were mainly observed during the interviews and most of them, as in previous cases are exactly the preservice elementary teachers' own words or sentences. In table 4.15 we monitored preservice elementary teachers' erroneous strategies and misconceptions in interpreting decimals involving unit measures from the most frequently observed to the least frequently observed.

Table 4.15 Interviewees' Misconceptions in Interpreting decimals involving unit measures.

	Misconceptions[1]	
Interviewees	22	23
1	*	*
2	*	*
3	*	*
4	*	*
5	*	*
6	*	*
7		
8	*	*
9		
10		
11		
12	*	*
13	*	*
14	*	*
15		
16	*	*
17		
18	*	*
19	*	*
20	*	*
21	*	*
22		
23		
24		
25		
	60%	60%

(1) Misconceptions:

22. Decimal point is a separator.
23. The number after the decimal point may represent different units according to the subunit system of a given measure.

4.1.1.11 Results Concerning the Interpretations of Preservice Elementary Teachers About Multiplication and Division Operations Involving Decimals

In this part there were 6 items (items 39, 40, 41, 42, 43, 44 from Pre-Form of Concept Test) to be answered, so the maximum possible score was 6. Scores ranged from 0 to 6. The mean score was 3.36 with a standard deviation of 1.69. The most difficult items for preservice elementary teachers were items 40, 43 and 41 respectively. The mean percent of the preservice elementary teachers' correct responses in this part was 56.

Table 4.16: Preservice Elementary Teachers' Responses to the Items Related Multiplication and Division Operations Involving Decimals

Item	Structure	Arithmetic Structure	Answer	Correct	No Answer	Other
39	48.36 × 0.353	D x d	a	41	6	25 (b)
40	35.67 ÷ 0.478	D ÷ d	b	26	6	40 (a)
41	0.37 × 0.561	d x d	a	34	6	32 (b)
42	35.48 × 5.36	D x D	b	58	7	7 (a)
43	0.236 ÷ 0.617	d ÷ d	b	26	7	39 (a)
44	62.05 ÷ 72.34	D ÷ D	a	57	7	8 (b)

- D : Stands for the decimal numbers which are greater than 1
- d : Stands for the decimals which are less than 1

4.1.1.11.1 Results Concerning the Interpretations of Preservice Elementary Teachers About Multiplication Operations Involving Decimals

4.1.1.11.1.1 Results Concerning the Interpretations of Preservice Elementary Teachers About Multiplication Operations in "d x d" Structure

As it is seen in table 4.16, preservice elementary teachers' performance on multiplication operations where both of the numbers were decimals less than 1, were not as successful (item 41) when compared with other multiplication operations

The following are typical responses of interviewees on Multiplication Operations in "d x d" Structure *(15 preservice elementary teachers were interviewed in this part):*

<u>Item 41</u>

Is the result of 0.37 x 0.561 greater or less than 0.37 ?

1st Interview Excerpt for Item 41

R: Is the result of 0.37 x 0.561 greater or less than 0.37 ? Please, first try to guess.

N: it will be greater than 0.37.
R: Why ?
N: Because this is a multiplication…(thinking)…what a minute ! here we have decimals …hmmm… but no no it doesn't make a difference it is again greater.

In the above interview, the interviewee held the belief that *"multiplication makes larger"*. First she thought that the situation could be different for decimals but then changed her mind.

2nd Interview Excerpt for Item 41

R: Is the result of 0.37 x 0.561 greater or less than 0.37 ? Please, first try to guess.
E: We get a greater result.
R: why ?
E: Because we are multiplying with a greater number.
R: well, suppose you have 0.37 x 0.22 , then what would you say ?
E: The result will be less than 0.37.

In the above interview, if one especially considers the last two lines, one can easily see that the interviewee treated decimals as whole numbers. In other words she *ignored the decimal point.*

3rd Interview Excerpt for Item 41

R: Is the result of 0.37 x 0.561 greater or less than 0.37 ? Please, first try to guess.
Ö: It will be greater.
R: Why ?
Ö: I always think like that…but here the result will have five numerals then it could be less.

The interviewee in the above interview mainly seemed to be influenced by the belief that *"multiplication makes larger"* and the last sentence of the interviewee showed that she also thought that *more digits after the decimal point makes a decimal smaller.*

4th Interview Excerpt for Item 41

R: Is the result of 0.37 x 0.561 greater or less than 0.37 ? Please, first try to guess.
M: The result will be greater than 0.37.
R: why ?

M: Because this is a multiplication. For example when you multiply 6 by 3 you get 18 (6 x 3 = 18).

In the above interview, like many others, the interviewee seemed to hold the belief that *"multiplication makes larger"* but moreover she treated decimals just as *whole numbers*.

5th Interview Excerpt for Item 41

R: Is the result of 0.37 x 0.561 greater or less than 0.37 ? Please, first try to guess.
H: I think that this multiplication can not be done.
R: why ? Is there any inconsistency ?
H: the first one has 3 and the other one has 4 numerals.
R: Are you trying to say that multiplication is impossible ?
H: May be we can multiply but the result will be smaller.
R: why ?
H: Because in 0.561 the zero has no value.
R: then what about 0.37 ?
H: I don't know.

The above interview was one of the most interesting interviews in this part. The interviewee seemed to be very weak in decimals and arithmetic. She stated that the *multiplication could not be done* under the given conditions and this was the most important observation about the interviewee. It was also felt that the interviewee had some problems about *zero as a place holder*.

6th Interview Excerpt for Item 41

R: Is the result of 0.37 x 0.561 greater or less than 0.37 ? Please, first try to guess.
A: There will be a greater result.
R: What is the reason for that ?
A: Because this is a multiplication. On the other hand if we move the decimal points then it can be easily seen that the result is always greater.

The above interviewee mainly thought that *multiplication made larger.* In addition to this she seemed to be under the influence of whole numbers as she tried to move the decimal points.

4.1.1.11.1.2 Results Concerning the Interpretations of Preservice Elementary Teachers About Multiplication Operations in "D x d" Structure

As it is seen in table 4.16 preservice elementary teachers performance was still lower on multiplication operations, where one of the numbers was a decimal number less than 1, when compared with other multiplication operations where the numbers involved were both decimals greater than 1.

The followings are typical responses of interviewees on Multiplication Operations in "D x d" Structure (*5 preservice elementary teachers were interviewed in this part*):

Item 4.6.1.2.a (not from the test)

Is the result of 42.25 x 0.35 greater or less than 42.25 ?

A Typical Interview Excerpt for Item 4.6.1.2.a

R: Is the result of 42.25 x 0.35 greater or less than 42.35 ? Please, first try to guess.
Ç: The result will be greater.
R: Why ?
Ç: Because we are multiplying.
R: Can we say that "multiplication always give greater results" ?
Ç: yes.

In the above interview the interviewee held the belief that *"multiplication makes larger"*.

Item 4.6.1.2.b (not from the test)

Is the result of 42,36 x 0,275 greater or less than 42.36

A Typical Interview Excerpt for Item 4.6.1.2.a

R: Is the result of 42,36 x 0,275 greater or less than 42.36
Y: First I should do the operation.
R: please first try to say something without doing any operation.
Y: Well, is it greater ? .... Let me multiply.
R: Try to guess.

Y: I think that it will be greater than 42.36.

In the above interview, the interviewee couldn't be sure about the result without doing any operation but she seemed to be influenced about the belief that multiplication made larger.

4.1.1.11.1.3 Results Concerning the Interpretations of Preservice Elementary Teachers About Multiplication Operations in "D x D" Structure

As it is seen in table 4.16 preservice elementary teachers performance seemed to be the greatest on item 42 where the numbers involved were both decimals greater than 1.

<u>Item 4.6.1.3 (not from the test)</u>

Is the result of 42,36 x 5.36 greater or less than 42.36

A Typical Interview Excerpt for Item 4.6.1.3

R: Is the result of 42.36 x 5.36 greater or less than 42.36
Oz: The value will be greater.
R: why ?
Oz: Because this is a multiplication.

As in the above interview when we asked preservice elementary teachers questions in "D x D" structure all of them concluded that the result would be larger, but in most of the interviews they stated "multiplication makes larger" as being the main reason for this result.

4.1.1.11.2 Results Concerning the Interpretations of Preservice Elementary Teachers About Division Operations Involving Decimals

4.1.1.11.2.1 Results Concerning the Interpretations of Preservice Elementary Teachers About Division Operations in "d ÷ d" Structure

As it is seen in table 4.16 preservice elementary teachers performance on division operations where both of the numbers were decimals less than 1, were not as good (item 43) when compared to "D ÷ D" structure.

The followings are typical responses of interviewees on Division Operations in "d ÷ d" Structure (*13 preservice elementary teachers were interviewed in this part*):

<u>Item 43</u>

Is the result of 0.236 ÷ 0.617 greater or less than 0.236 ?

1st Interview Excerpt for Item 43

R: Is the result of 0.236 ÷ 0.617 greater or less than 0.236 ?
Me: The result will be smaller.
R: Why ?
Me: Because the divisor is greater than the dividend.

In the above interview the interviewee followed the same procedure as she followed for whole numbers and stated that the result would be smaller

2nd Interview Excerpt for Item 43

R: Is the result of 0.236 ÷ 0.617 greater or less than 0.236 ?
Ü: It will be smaller than 0.236.
R: can you give me a reason for that ?
Ü: Because this is a division operation.

This time the interviewee thought division in a more general way and believed that *division made smaller* without considering the type of the given numbers.

3^{rd} Interview Excerpt for Item 43

R: Is the result of 0.236 ÷ 0.617 greater or less than 0.236 ?
E: It will be greater…No wait… the result will be greater than 0.236 and less than 0.617.

Apart from the other interviews done for this part, in the above interview the interviewee stated that when one divides two decimal numbers in d ÷ d structure , the result would always be between the divisor and the dividend but she could not give any reason for her own rule.

Item 4.6.2.1.a (not from the test)

Is the result of 0.236 ÷ 0.23 greater or less than 0.236 ?

A typical Interview Excerpt for Item 4.6.21.a

R: Is the result of 0.236 ÷ 0.23 greater or less than 0.236 ?
Y: The result will be greater.
R: why ?
Y: In decimal numbers if there are more and more numbers after the decimal point then the number gets smaller.

In the above interview the interviewee first stated that the result of 0.236 ÷ 0.23 would be greater than 0.236 and then stated that in decimal numbers more numbers after the decimal point made the number smaller but this contradicted her first statement.

Item 4.6.2.1.b (not from the test)

Is the result of 0.37 ÷ 0.61 greater or less than 0.37 ?

A typical Interview Excerpt for Item 4.6.21.b

R: Is the result of 0.37 ÷ 0.61 greater or less than 0.37 ?
OO: It will be smaller than 0.37, because we are dividing a smaller number by a greater number.

In the above interview like many others the interviewee treated decimals as whole numbers. In other words he *neglected the decimal point.*

Item 4.6.2.1.c (not from the test)

Is the result of 0.236 ÷ 0.348 greater or less than 0.236 ?

A Typical Interview Excerpt for Item 4.6.21.c

R: Is the result of 0.236 ÷ 0.348 greater or less than 0.236 ?
D: if we move the decimal points we'll get 236 ÷ 348 and since we are dividing a smaller number by a greater then we'll get a smaller result.

In the above interview this time the interviewee moved the decimal points to be more secure before her conclusion but again we can say that she *neglected the decimal point* and took *division makes smaller* as a generalisation.

4.1.1.11.2.2 Results Concerning the Interpretations of Preservice Elementary Teachers About Division Operations in "D ÷ d" Structure

As it is seen in table 4.16 preservice elementary teachers performance on the division operation (item 40) where the dividend is a decimal number greater than 1 also seemed to be low. This may show that the main problem is due to the divisor (less than 1).

The following are typical responses of interviewees on Division Operations in "D ÷ d" Structure (*20 preservice elementary teachers interviewed in this part):*

Item 40

Is the result of 35.67 ÷ 0.478 greater or less than 35.67 ?

1st Interview Excerpt for Item 40

R: Is the result of 35.67 ÷ 0.478 greater or less than 35.67 ?
S: The result will be smaller.
R: would you please do the operation.
S: (…doing the operation ….) it is 74.62
R: is 35.67 greater or less than 74.62 ?
S: …. Mmm ! …result will be greater.

Above interviewee first stated that the result would be smaller but when she performed the operation she noticed that she thought in a wrong way.

2^{nd} Interview Excerpt for Item 40

R: Is the result of $35.67 \div 0.478$ greater or less than 35.67 ?
N: it will be smaller.
R: why ?
N: As you understand from the name of the operation ! It is division which means that you are dividing into small pieces.

Above interviewee believed that *division made smaller* and also she held the *partitive* interpretation of division as she stated that it was divided into small pieces.

3^{rd} Interview Excerpt for Item 40

R: Is the result of $35.67 \div 0.478$ greater or less than 35.67 ?
E: the result will be smaller than 35.67.
R: Why ?
E: because in division operations whether you get involved with decimal numbers or integers the dividend always get smaller. On the other hand in multiplicative structures you get greater results.

In the above interview the interviewee strongly held the beliefs *"division makes smaller"* and *"multiplication makes larger"* no matter what the type of numbers involved.

4^{th} Interview Excerpt for Item 40

R: Is the result of $35.67 \div 0.478$ greater or less than 35.67 ?
M: It will be smaller.
R: What is the reason for that ?
M: In division operations involving decimals first we should move the decimal points.
R: Can't you guess the result ?
M: Certainly, the result will be smaller.
R: What would you say if it were multiplication ?
M: it would be greater.

In the above interview, the interviewee seemed to hold the beliefs *"division makes smaller"* and *"multiplication makes larger"* . In addition to those she also moved the decimal points before the conclusion and forced the numbers to be like whole numbers. This was another way of *ignoring the decimal points*.

5^{th} Interview Excerpt for Item 40

R: Is the result of $35.67 \div 0.478$ greater or less than 35.67 ?
M: it will be smaller.

R: why ?
M: For example when you divide 6 by 3 you get a result which is less than 6.

Above interviewee thought totally in the same way as she thought for whole numbers. Again the decimal points were ignored.

4.1.1.12 Discussion of the Results Concerning the Interpretations of Preservice Elementary Teachers About Multiplication and Division Operations Involving Decimals

When we go over the pre-form of concept test results and the observations made on the interviews (see table 4.16), it is possible to observe that preservice elementary teachers were slightly better on multiplication operations (62%: mean percent correct) compared to division operations (50%: mean percent correct), but when we go into detail, we see that in both of the operations (multiplication and division), preservice elementary teachers were less successful when the decimals involved were less than 1 compared to other structures. Owens (1985) stated that in estimating products, an important condition that affects the performance and understanding was having one factor that was less then one. Later in 1988 Owens reported that the easiest product estimation for students was with factors that have whole number parts. In our research, the results came about because the preservice elementary teachers failed to extent their understandings of whole number operations to fractions and decimals. Therefore they generally conclude "multiplication makes larger" and "division makes smaller". In dealing with operations involving decimals, some of the preservice elementary teachers totally ignored the type of the given numbers and treated the decimals as they would treat whole numbers. For example three of the interviewees stated that multiplication made larger and division made smaller by giving examples from whole numbers (e.g., for question "Is the result of 0.37 x 0.561 greater or less than 0.37 ?" an interviewee states: "*Greater !, Because this is a multiplication. For example when you multiply 6 by 3 you get 18*"). Similar observations have been made by many researchers. Brown (1981) stated that 11 to 16 year old British students responded that "multiplication always makes bigger and division always makes smaller. Greaber and Tirosh (1989) conducted a study in the United States which was designed to assess the extent to which beliefs, "multiplication always makes bigger" and "division always makes smaller", were held by preservice elementary teachers. It is reported that the stated beliefs were held by about 80% of the preservice elementary teachers. In this present study approximately 72% of the preservice elementary teachers held the stated beliefs. Results of the second mathematics assessment (Carpenter, Corbitt, Kepner, Linguist, & Reys, 1981) indicated that division when a decimal was divided by a whole number was as high as 76% for 13 year olds, but dropped to about 37% when the divisor was a decimal. İşeri (1997, p.68) states: "One of the most evident finding about students' conceptions was that they believe that multiplication makes bigger and division makes smaller."

Six of the interviewees moved the decimal points to the right in order to treat the given decimals as whole numbers.

Nine (36%) of the interviewees in estimating the results of the given operations totally ignored the decimal points, which again shows that they can not extent their understandings of whole number operations to fractions and decimals and they insist on treating decimals as whole numbers. Similar observations were made by Brown (1981). The researcher reported that 1/8 of 12-15 year old students read 0.29 as "twenty-nine". Carpenter et al (1981) stated that students' errors such as "0.195 is greater than 0.2" were presumably a result of ignoring the decimal point and treating such numbers as whole numbers.

One of the interviewees stated that in an operation like 0.236 x 0.617 the results would always be between 0.236 and 0.617. Another interesting observation was made on two of the interviewees who stated that multiplication could not be done if the given decimals did not have the same number of digits.

There is evidence to suggest that performance on estimating products and divisions is not related to the performance on other multiplication and division tasks such as calculating operations and solving routine mlutiplicaiton and division word problems. A plausible explanation for this finding is that estimating products and divisions requires conceptual knowledge, while computation of products and divisions requires only procedural knowledge.

Briefly we can state that preservice elementary teachers have some erroneous strategies and misconceptions in interpreting multiplication and division operations involving decimals.

Preservice elementary teachers' erroneous strategies and misconceptions in interpreting multiplication and division operations involving decimals are given in the following paragraphs. The erroneous strategies and misconceptions of preservice elementary teachers were mainly observed during the interviewees and most of them, as in previous cases are exactly the preservice elementary teachers' own words or sentences. In table 4.17, we monitored preservice elementary teachers' erroneous strategies and misconceptions in interpreting multiplication and division operations involving decimals from the most frequently observed to the least frequently observed.

Table 4.17 Interviewees' Misconceptions in Interpreting Division and Multiplication Operations Involving Decimals

	Misconceptions[1]			
Interviewees	24	25	26	27
1	*			*
2	*			
3	*	*		
4	*	*		
5	*	*		
6	*	*	*	*
7	*	*		
8		*	*	
9		*		
10	*	*	*	
11	*		*	
12	*	*	*	
13				
14	*			
15	*	*		
16				
17	*	*	*	
18	*	*		
19	*	*		
20	*			
21	*	*		
22	*	*	*	
23		*	*	
24				
25			*	
	72%	64%	36%	8%

(1) Misconceptions:
24. Division makes smaller.
25. Multiplication makes bigger.
26. Decimal point can be ignored.
27. Two decimals can not be multiplied if they have not the same number of digits.

4.1.2 Results Obtained from the Pre-Testing of Problems Test

The Pre-Form of Problems Test (see Appendices B and E) included 26 - items. It was used in order to investigate the misconceptions of preservice elementary teachers and processes used in solving (choosing the appropriate operation) word problems involving decimals. The Pre- Form of Problems Test consisted of the following dimensions:

1. Multiplication problems

   a) Confirming to the Primitive Model *(4 items)*

   b) Not confirming to the Primitive Model *(5 items)*

2. Division Problems

   a) Partitive, confirming to the Primitive Model *(3 items)*

   b) Partitive, not confirming to the Primitive Model *(3 items)*

   c) Quotitive, confirming to the Primitive Model *(4 items)*

   d) Quotitive, not confirming to the Primitive Model *(4 items)*

3. Addition and Subtraction Problems *(3 items)*

4.1.2.1 Results Concerning the Interpretations of Preservice Elementary Teachers on Items of Multiplication Word Problems Involving Decimals

In this part there were 9 problems (problems 1, 3, 5, 7, 9, 10, 14, 20, 21 from Pre-Form of Problems Test) to be answered. The maximum possible score was 9. Scores ranged from 0 to 9. The mean score was 7.32 with a standard deviation 1.95. The mean percent of the preservice elementary teachers' correct responses in this part was 86.

Table 4.18: Categories of Responses of Preservice Elementary Teachers in Multiplication Word Problems Involving Decimals

Problem	Operation	Type	Confirmity to the primitive models	Correct	No Ans	Division	Subtraction	Addition	Other
1	15 × 2*	Rate	yes	68	1	3	-	-	-
3	0.52 × 0.93*	Rate	no	55	6	7	2	2	-
5	150000 × 3*	Repeated addition	yes	66	2	3	-	-	1
7	0.05 × 85.3*	Rate	no	56	3	11	1	1	-
9	3.2 × 4.6	Rate	no	49	14	3	5	-	1
10	1.5 × 0.75*	Repeated addition	yes	64	3	5	-	-	-
14	0.53 × 1.33*	Rate	no	61	5	4	1	-	1
20	15 × 10.25*	Repeated addition	yes	64	3	5	-	-	-
21	0.23× 4.6	Enlarge ment (Rate)	no	55	11	1	1	4	-

* Stands for the problems that can be solved by using direct proportion.

Table 4.18 Shows that the preservice elementary teachers performance seemed to be high when the numbers involved in a multiplication problem were integers (items 1,5).

The mean percent (for the correct responses) for the problems that were of the rate type was 80 and the mean percent (for the correct responses) for the ones that were repeated addition type was 90. For example, although the numerical structure of problems 9 and 20 are similar the nonparamatric McNemar test ($p < 0.05$) showed that the preservice elementary teachers were better on problem 20.

Table 4.19: Comparison of Preservice Elementary Teachers' Performance on Problem 9 vs Problem 20 in Pre-Form of Problems Test

		Problem 9: Correct	Problem 9: Incorrect
Problem 20	Incorrect	5	9
	Correct	45	15

$p = 0.0414 < 0.05$

The mean percent (for the correct responses) for the problems that confirm to primitive models was 91 and the mean percent (for the correct responses) for the ones that were not confirming to the primitive models was 77.

Generally when the numbers involved get less than 1 the preservice elementary teachers tended to use division instead of multiplication (problems 3 and 7) .

The following are typical responses of and strategies used by preservice elementary teachers (interviewees) in choosing the correct operation for multiplication word problems involving decimals:

Problem 9 *(8 preservice elementary teachers were interviewed on this item):*

An artist uses 4.6 times more red compared to yellow in order to produce a certain colour . If the artist uses 3.2 gr yellow, then how much red should he use to produce that colour ?

1st Interview Excerpt for Problem 9

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
O: (reads aloud) …..(Then reads once more)…. Let's say yellow is X or S then we can say 4.6 times Red is equal to 3.2 (4.6 . R = 3.2) and we'll have

$$\text{Red} = \frac{3.2}{4.6}$$

R: why is it division ?
O: well, I just tried to make an equation.

In the above interview, the subject tried to find the answer in a procedural manner. He first formed an equation but since he did not test his strategy, this caused him to say division.

2nd Interview Excerpt for Problem 9

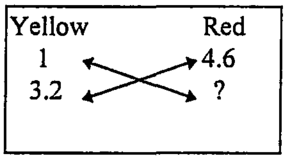
R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.

A: (thinking after reading the problem aloud) ….I usually do direct proportion for such problems.
R: OK, just do it.
A: I can not do this.

In many of the interviews we observed that the interviewees tried to use direct proportion in choosing an appropriate operation. In fact problem 9 was not directly appropriate for a direct proportion and most probably because of this reason, above interviewee couldn't choose any operation for problem 9.

3rd Interview Excerpt for Problem 9

R: Please , first, read the problem aloud and then choose the appropriate operation in which enables you to find the answer of the problem.
G: If yellow is 1 then red is 4.6 then…



R: Do you always use direct proportion ?
G: Yes, even if it is meaningless or inappropriate, I force myself to use it.
R: If I say "don't use direct proportion" that what would you say ?
G: I can not do anything.

Like the previous interviewee the above interviewee also insisted on the use of *direct proportion* in choosing the appropriate operation. This time more strongly, she stated that everytime she tried to use direct proportion even if it was meaningless or inappropriate.

4th Interview Excerpt for Problem 9

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
OO: (reading…)…. (then thinking) …
R: Well, let's write 5 instead of 4.6 and 2 instead of 3.2 and try again. Will it be easier ?
OO: Yes I think it will be easier …but I'm not sure if I can do it or not.

Above interviewee stated that it would be easier after replacing the decimals by whole numbers but in any case he couldn't give the answer.

Up to this point for problem 9, decimal operators seemed to be a source of difficulty but the last interview showed that this was not the only reason. The type (context of the problem) may be another factor.

<u>Problem 7</u> (*4 preservice elementary teachers were interviewed on this item*)

The local government offers 85.3 TL for each ton of disposed water. How much should a family pay for a 0.05 ton of disposed water ?

1st Interview Excerpt for Problem 7

R: Please , first, read the problem aloud and then choose the appropriate operation in which enables you to find the answer of the problem.
U: … I didn't understand ! (reading once more)… Well, we'll subtract.
R: why ?
U: .. NO … since the water is used then we will,…mmm.. it will be addition.
R: Now, if we replace 85.3 by 4 and 0.05 by 2 then what you say ?
U: Will it be 4 x 2 ?
R: Does is become easier when we use integers ?
U: Yes.

In the above interview the interviewee could not do anything with decimal numbers but when we replaced them with integers she managed to choose the appropriate operation.

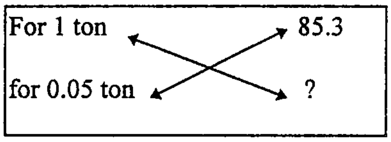
2nd Interview Excerpt for Problem 7

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
H: I think that it will be 85.3 - 0.05, a subtraction.
R: why ?
H: I don't know.

Above interviewee had chosen subtraction as the appropriate operation. In the further steps of the interview we observed that she often tried to detect some cue words in the given problems in order to choose the appropriate operation and for problem 7 she seemed to be affected by the word disposed in saying subtraction.

3rd Interview Excerpt for Problem 7

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
S: we can use a direct proportion like



R: what could you do if you don't use direct proportion ?
S: I'm not sure.

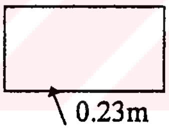
In the above interview like many others the interviewee used direct proportion and automatically found the necessary operation to solve the problem. In reality, this may not always show that she can choose the appropriate operation for word problems.

<u>Problem 21</u> (*2 preservice elementary teachers were interviewed on this item*)

A painting is to be enlarged by a factor of 4.6. If the actual height of the painting is 0.23 meter then what will be the new height ?

A Typical Interview Excerpt for Problem 21

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
D: we can think of a frame like



.....Do we know the ratio between the width and height of the frame ?
R: What are you going to do with width of the painting ?
D: ...by a factor 4.6 ...well... then it will be multiplication.
R: why ?
D: Because we want to make the picture larger.

In the above interview it seemed that the interviewee was under the influence of the belief *"multiplication makes larger"* in choosing the appropriate operation.

<u>Problem 3</u> (*2 preservice elementary teachers were interviewed on this item)*

A runner finished a race in 0.52 hours. If the average speed of the runner is 0.93 km in an hour, then what is the distance covered by the runner ?

A Typical Interview Excerpt for Problem 3

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
M: ..(first read and then tried to do a direct proportion)...
R: Are you trying to use direct proportion ?
M: ...I'm not sure....(it takes too much time)...

In the above interview , although, the interviewee tried to use a direct proportion, in choosing the appropriate operation, she could not carry it out.

<u>Problem 10</u> (*2 preservice elementary teachers were interviewed on this item)*

It is possible to get 0.75 kg of flour from one kg of wheat. How much flour can be produced using 15 kg of wheat ?

A Typical Interview Excerpt for Problem 10

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
U: ....Are we going to add ?
R: What do you mean ? Is 0.75 + 15 what you are trying to say ?
U: mmm! There is no other number. We should add.

The above interviewee seemed to be unsuccessful in choosing the appropriate operation for word problems. In any case she insisted on the use of addition.

4.1.2.2 Discussion of the Results Concerning the Interpretations of Preservice Elementary Teachers on Items of Multiplication Word Problems Involving Decimals

When we go over the results it seems that the type of the multiplication problem affects the performance of the preservice elementary teachers. As it is seen in table 4.18 the preservice elementary teachers were better when the multiplication problem was suitable for a repeated addition model. This may be because what preservice elementary teachers experiencing are extensively with whole numbers. İşeri (1997, p.69) also stated that an important factor affecting the difficulty of verbal problems was the problem type. He reported that it was easier for students to conceptualise repeated addition.

Conformity of the problems to the Primitive Implicit Models (of Fischbein) is another factor which seems to affect the performance of preservice elementary teachers in choosing the appropriate operation for multiplication word problems. According to Fischbein et al (1985) for multiplication the primitive model is repeated addition , which is applicable in simple situations where a number or measure is replicated a number of times. In more complex situations, the mediation of the implicit repeated addition model for multiplication imposes the constraint that the multiplier must be an integer if the operation is to be intuitively perceived as multiplication. Graeber et al (1989) findings were similar to ours. They stated that the violation of the primitive model was a source of difficulty in solving multiplication word problems (p. 97). In 1989 Bell et al also pointed out that the type of the number used as multiplier had a large effect on success (p. 439).

We can say that the preservice elementary teachers were better in the multiplication problems that confirm to the primitive models. In other words the implicit belief "in a multiplication expression the operator should be a whole number" seemed to be held by 20% of the preservice elementary teachers.

Decimals less than 1 seemed to be another important factor which affects the overall success of the preservice elementary teachers. Especially, whenever the operators were decimals less than 1, some of the preservice elementary teachers tended to use division instead of multiplication (look at problems 7 and 3). Many of these may have been due to an intuitive awareness that the answer had to be smaller than the first number, combined with the misconception that to make a number smaller you must divide. Graeber et al (1989, p. 98) reported similar findings. They stated that more than 25% of their subjects (preservice elementary teachers) incorrectly wrote a division expression as appropriate to the solution of the multiplication problems which included decimal operators less than 1. Bell, Swan, and Taylor (1981) have shown that when children were presented with a series of problems with the same content, they might change their minds about the operation needed to solve the problem depending on the specific numerical data that were given. Students could solve a word problem by multiplying when the decimal numbers are larger than one, and then solved the same problem by dividing when one of the numbers was changed to a decimal number less than one, and did not see the contradiction. After conducting a research on middle school students İşeri (1997, p.68) reported that whenever the students were confronted with a situation requiring multiplication or division by a number smaller than one, some tended to change the operation.

In most of the problems preservice elementary teachers used direct proportion procedures in order to choose the operation. Whenever the problem context was not directly suitable for direct proportional procedure , we observed that the preservice elementary teachers performance was lower compared to the others (problems 9 and 21). As in the previous dimensions (interpreting denseness of decimals, unit measure involving decimals and estimating products and etc.) whenever the given task can not be performed by procedural techniques the preservice elementary teachers performance declines. Although the mean percent of the correct responses in estimating products was 62, the mean percent of the correct responses for multiplication word problems was 83. The reason for this sudden increase, of course, does not show that the conceptual understanding of the preservice elementary teachers started rising, because 7 of 9 multiplication word problems were suitable for direct proportional procedures and some of the preservive elementary teachers used these procedures in finding the appropriate operations.

During the interviews for some interviewees who failed to choose any operation for multiplication word problems, we replaced the decimals by whole numbers and repeated the same question. Although there were a group of interviewees who still failed to choose any operation, some of them became successful in choosing an operation. This may show that in some cases mere presence of decimals causes a lower success rate.

Throughout the interviews we observed that some of the interviewees tried to find some cue words which could help them in choosing the appropriate operation but in some cases as in problem 21 "*A painting is to be enlarged by a factor 4.6*" the word *enlarge* distracted some of the preservice elementary teachers and they (5%) had chosen addition instead of multiplication. Bell (1982) in summarising the misconceptions exhibited by pupils in solving problems stated that students could be distracted by cue words or perceptual features and named this as *distraction*.

Briefly, an obvious point to be made is that multiplication problems differ markedly in difficulty. This can partly be attributed to structural and contextual differences.

We can state that preservice elementary teachers are mainly influenced by the primitive multiplication model and hold the belief "in a multiplication expression the operator should be a whole number".

In table 4.20 we monitored the distribution of the interviewees holding the belief "in a multiplication expression the operator should be a whole number".

Table 4.20 Distribution of Interviewees Holding the Belief "in a multiplication expression the operator should be a whole number"

	Misconception No 28
Interviewees	In a Multiplication Expression the Operator Should be a Whole Number".
1	*
2	*
3	*
4	
5	
6	
7	
8	*
9	
10	*
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	
21	
22	
23	
24	
25	
	20%

4.1.2.3 Results Concerning the Interpretations of Preservice Elementary Teachers on Items of Division Word Problems Involving Decimals

In this part there were 14 problems (problems 2, 4, 6, 11, 12, 13, 15, 16, 17, 18, 19, 23, 24, 25) to be answered. The maximum possible score was 14. Scores ranged from 0 to 14. The mean score was 9.28 with a standard deviation 3.56. The mean percent of the preservice elementary teachers' correct responses in this part was 71.

Table 4.21: Categories of Responses of Preservice Elementary Teachers in Division Word Problems Involving Decimals

Problem	Operation	Type	Confirmity to the primitive models	Corr	No Ans	Inverse	Mult	Subtr.	Add..
2	5.25 ÷ 3	Quotition	yes	67	2	-	2	-	1
4	6.25 ÷ 5*	Partition	yes	61	2	4	4	-	1
6	3.25 ÷ 5*	Partition	no	63	2	4	1	1	-
11	13 ÷ 3*	Partition	yes	65	3	1	2	-	1
12	3 ÷ 7.2*	Rate (P)	no	40	6	25	1	-	-
13	13 ÷ 3*	Quotition	yes	62	7	-	3	-	-
15	0.65 ÷ 5*	Fractional	no	60	5	5	1	-	1
16	0.65 ÷ 5	Rate (Q)	no	28	12	18	10	4	-
17	0.98 ÷ 0.245	Quotition	yes	45	14	12	1	-	-
18	3 ÷ 0.174*	Quotition	yes	53	10	5	4	-	-
19	0.5 ÷ 15.3*	Rate (Q)	no	50	10	9	3	-	-
23	5.25 ÷ 3*	Partition	yes	59	8	3	1	1	-
24	0.82÷ 13.5	Rate (Q)	no	24	11	29	3	5	-
25	3.25 ÷ 5	Rate (Q)	no	35	10	10	7	10	-

* Stands for the problems that can be solved by using direct proportion.

As it is seen in table 4.21, preservice elementary teachers' scores in partitive type division problems were higher, compared to quotitive type division problems. The mean percent (for the correct responses) for the problems that were partitive type was 81 and the mean percent (for the correct responses) for the ones that were quotitive type was 63.

Preservice elementary teachers' scores were higher on the problems conforming to the primitive models compared to the problems that were not conforming to the primitive models. The mean percent (for the correct responses) for the problems that were conforming to the primitive models was 82 and the mean percent (for the correct responses) for the ones that were not conforming was 59.

Preservice elementary teachers' performance on problems where the divisor in the needed operation was greater than the dividend was lower compared to the ones which has divisors less than the dividend. The mean percent (for the correct responses) for the problems where the divisor in the needed operation was greater than the dividend was 61 and the mean percent (for the correct responses) for the ones which has divisors less than the dividend was 82. As it is seen in table 4.21 preservice elementary teachers tended to reverse the needed operation whenever the divisors were greater than the dividend (especially in problems 12, 16, 17, 24, and 25).

In some problems (problems 15, 16, 17, 18, 19, 24) the divisors were less than one and on the other hand in some others the divisors were greater than one (problems 2, 4, 6, 11, 12, 13, 23, 25). At this point it seemed that the preservice elementary teachers' performance in problems where the divisors were less than one was slightly lower when compared with the ones in which the divisors were greater than one. The mean percent (for the correct responses) for the problems involving divisors less than one was 61 whereas the mean item percent for the problems involving divisors greater than one was 78.

In some problems like problems 4, 6, 11, 12, 13, 15, 18, 19, and 23 it was suitable to use direct proportional technique to choose the appropriate operation, but in problems like 2, 16, 17, 24, and 25 it was not possible to use a direct proportional technique to choose the appropriate operation. The mean percent for the problems that were suitable to use direct proportional techniques was 79 and the mean percent for the problems that direct proportional techniques were not possible was 55. For example, as it is seen in tables 4.21 and 4.22, although the numerical structure of problems 15 - 16 and problems 6 - 25 are just the same the nonparametric McNemar test ($p< 0.05$) showed that the preservice elementary teachers were better at problems 15 and 6 where the direct proportional approaches were possible.

Table 4.22: Comparison of Preservice Elementary Teachers' Performance on Problem 15 vs Problem 16 in Pre-Form of Problems Test

		Problem16 Correct	Incorrect
Problem 15	Incorrect	3	11
	Correct	28	38

$p = 0.0000 < 0.05$

Table 4.23: Comparison of Preservice Elementary Teachers' Performance on Problem 6 vs Problem 25 in Pre-Form of Problems Test

		Problem 6 Correct	Incorrect
Problem 25	Incorrect	33	7
	Correct	29	2

$p = 0.0000 < 0.05$

The following are typical responses of and strategies used by preservice elementary teachers (interviewees) in choosing the correct operation for division word problems involving decimals:

<u>Problem 16 (</u> *13 preservice elementary teachers were interviewed on this problem*)

A 0.65 kg of peanut is to be put into a box which has a 5 kg capacity, then how much of the box is filled ?

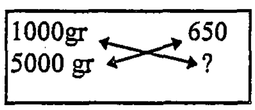
1st Interview Excerpt for Problem 16

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
Oz: In order to find the portion to be filled we should divide 5 by 0.65 ($5 \div 0.65$).
R: why ?
Oz: because we are trying to find how much of the 5 will be filled.

Above interviewee divided 5 by 0.65 instead of dividing 0.65 by 5. In the further steps of the above interview he seemed to hold the belief *"In a quotative division model the dividend should be greater than the divisor"*

2nd Interview Excerpt for Problem 16

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
OO: 0.65 kg is equal to 650 gr.... and 5 kg is equal to 5000 gr. then we can do a direct proportion like
R: what else ?
OO: I don't know.



Above interviewee first converted the given numbers into different units as if he escaped from decimals and tried to use a direct proportion which was, in fact, inappropriate for the given problem. He also seemed to hold the belief *"In a quotative division model the dividend should be greater than the divisor"*. Same interviewees' performance on items 40 and 43 which were related with division operations involving decimals was also low.

3rd Interview Excerpt for Problem 16

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
N: It will be a direct proportion.
R: can you do it ?
N: well, it is 65 out of 100 then we'll say what is it in 5....and it will be $\frac{5x65}{100}$

Above interviewee treated 0.65 as 65% and tried to use a direct proportion but she failed to notice that her answer was meaningless in terms of percentage. Like many others, in this interview we again observed an overuse of direct proportion. The overall performance of the interviewee on items related with division operation in the pre-form of concept test were low. This may show that the interviewee implicitly held some beliefs about operations and the model of operations like "*division makes larger*" and "*In a quotative division model the dividend should be greater than the divisor*".

4th Interview Excerpt for Problem 16

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
H: (it takes too much time for her)...
R: what do you think ?
H: I don't know.
R: Some of your friends tried to use direct proportion for such problems what do you say about this ?
H: I never think like that.
R: why ?
H: I'm very poor in decimals.
R: let's change the 0.65 by 3. What can you do now ? Does it seem to be easier ?
H: It seems easier.
R: Go on.
H: I think that it will be 1 out of 3.
R: why.
H: (....there is no sound....)

In the above interview, it seemed that the interviewee's problems or difficulties were far more than decimals.

5th Interview Excerpt for Problem 16

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.

U: Do we add ? ....no we will multiply.
R: Why ?
U: We have that much of peanut and that capacity of the box and multiplication can give us the answer.

In the above interview the interviewee preferred to use multiplication instead of division. In the further steps of the interview she gave the cue words "how much of" as the reason of choosing multiplication. As it is seen in table 4.21 ten (14%) of the preservice elementary teachers had chosen the same operation (multiplication) for item 16 and their reason might be the same.

6th Interview Excerpt for Problem 16

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
G: I think we'd better draw a figure. Let's take a model and divide it into 5

We can not fill, even one of the strips.
R: well, what will be the operation for this ?
G: We will subtract 0.65 from 1.
R why from 1and not from 5 ?
G: Well, we can also subtract from 5 but it is better to subtract from one-unit instead of 5.

Above interview was one of the most interesting interview done on item 16. Although the interviewee answered only 2 of the items correctly out of 7 which were about marking decimals on area models, promisingly, she started by drawing a figure for the stated case but later she could not interpret the shaded region in a meaningful way. Her contradictory explanations might show that she had not had a fully conceptual understanding on the application and interpretation of decimal numbers.

7th Interview Excerpt for Problem 16

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
M: When we multiply we find the portion of the box that will be filled.
R: How can you define multiplication ?
M: Suppose that we have a group of students and each has 5 pencils, then in order to find the total number of the pencils we do multiplication. This is the short way of addition.

R: then how can you define division ?
M: Well, it is the inverse of multiplication....eeee...it is hard to define it.

In the above interview the interviewee believed that one could find the needed portions of a whole by multiplication. Later he stated that he thought multiplication as a repeated addition process. In the interview it was possible to observe that he was not that good in the use and interpretation of division.

8th Interview Excerpt for Problem 16

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
Ş: Since we should think the whole of the box it will be multiplication.
R: can you use a direct proportion here ?
Ş: no it is hard to use.

In the whole of the above interview excerpt it was possible to observe the influence of the belief *"multiplication makes larger"*. The interviewee was so sure that she did not need to use a direct proportion.

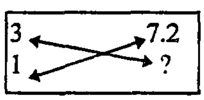
Problem 12 (*8 preservice elementary teachers were interviewed on this problem*)

A rowing team covers 3 km in 7.2 minutes, then how far does the team go in a minute ?

1st Interview Excerpt for Problem 12

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.

U: It is 7.2 in 3 and then what will it be when it is 1



After this direct operation we can say that it will be a division , which is $7.2 \div 3$.

In the above interview, the interviewee set up the direct proportion in a wrong way and ,of course, gave $7.2 \div 3$ as the answer. The Pre-Form of Problems test score of the interviewee (score =1) might show us that she had a low performance in the interpretation and use of decimals in word problems. Later in some further parts of the above interview she seemed to hold the belief *"dividend should be greater than the divisor"*.

2nd Interview Excerpt for Problem 12

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
G: It will be easier if we turn 7.2 minutes into seconds. In this way we do not have to work with decimal points.
R: OK. What is next ?
G: I don't know.

In the above interview the interviewee could not choose the appropriate operation for the given problem but as she tried to change 7.2 into seconds in order to get rid of the decimals, she seemed to hold the belief *"in a partitive division model the divisor should be a whole number"*. Interviewees' low score on the division operations in the pre-form of concept test may strengthen our observation.

3rd Interview Excerpt for Problem 12

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
Ş: It will be an indirect proportion.
R: What do you mean ?
Ş: I'm trying to say that it will be 7.2 ÷ 3.
R: would it be 3 ÷ 7.2 ?
Ş: Well, I think that this is better.
R: why ?
Ş: ...(no sound...)

Above interviewee first stated that it would be an indirect proportion and gave 7.2 ÷ 3 as the answer. Interviewees' score on the operations part (2 out of 6) of the pre-form of concept test and the previous observation seemed to declare that she held the beliefs "*dividend should be greater than the divisor*" and "*in a partitive division model the divisor should be a whole number.*

Problem 24 (6 preservice elementary teachers were interviewed on this *problem*)

Suppose that in order to cover the chairs in your house you need 13.5 meter of a certain kind of fabric. How much of the work can be done by 0.82 meter of that fabric ?

1st Interview Excerpt for Problem 24

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
N: (thinking..) in order to do the whole work we need 13.5 meter and with 0.82 meter of the fabric we can do ....mmm... but we don't know the number of chairs. If we had known the number of chairs then we could have divided 13.5 by the number of chairs and found the amount needed for each chair.... For example if we had 0.41 this could have meant that we could have covered two chairs.
R: Do decimals make the work harder ?
N: yes they confuse me.

When we went over the Pre-Form of Problems test paper of the above interviewee we observed that she generally tried to find the appropriate operation by using direct proportions. Most probably, in order to set the required conditions for the use of a direct proportion for problem 24, the interviewee asked for the number of chairs. The interviewees' effort to divide 13.5 by the number of chairs showed that she might hold the belief *" in a qoutitive division model the dividend should be greater than the divisor"*. The interviewee also stated that decimals made the work harder.

2nd Interview Excerpt for Problem 24

R: Please , first, read the problem aloud and then choose the appropriate operation in which enables you to find the answer of the problem.
A: (thinking. ...)...this is so hard !
R: What do you say if we replace 13.5 by 20 and 0.82 by 5 ?
A: I think it will be easier.
R: What is your answer ?
A: One out of 4 , I mean that $\frac{1}{4}$ of the work can be done.

In the above interview when we replaced the decimals by whole numbers the interviewee could easily give the answer. The interviewee seemed to have problems only about the use of decimals.,

3rd Interview Excerpt for Problem 24

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
Ü: (thinking...)
R: well, this time please do not use direct proportion.
Ü: It gets harder. ......Is it division ?
R: Let's replace 13.5 by 10 and 0.82 by 5, then what do you say ?

Ü: Now it looks better.
R: What will be the operation ?
Ü: ...mmmm...it is $\frac{10}{5}$, in other words we can do the half of the work.

Above interview was one of the most interesting interviews done for problem 24. First of all it was obvious that she had some problems with decimal numbers but later although we replaced the decimals by integers she insisted to divide a greater by a smaller and more interestingly interpret $\frac{10}{5}$ as the half of the work. Briefly it seemed that the interviewee held the belief *"in a qoutitive division model the dividend should be greater than the divisor"*.

4th Interview Excerpt for Problem 24

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
Ç: We will subtract 0.82 from 13.5 (13.5 - 0.82)
R: why ?
Ç: Because we are comparing with respect to the total.

Five of the preservice elementary teachers including the above interviewee had chosen subtraction instead of division for problems 24. Above interviewee treated comparison as subtraction.

5th Interview Excerpt for Problem 24

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
S: ... It will be 13.5 ÷ 0.82.
R: why ?
S: Because we are trying to find the work done with 0.82 meter of fabric.
R: mainly you are saying that it will be a division. Could it be 0.82 ÷ 13.5 ?
S: May be I should use direct proportion.

Above interviewee seemed to hold the belief *"in a qoutitive division model the dividend should be greater than the divisor"* and she did not see any way of finding the exact answer other than direct proportion. This again might show that preservice elementary teachers generally thought in a procedural way.

4.1.2.4 Discussion of the Results Concerning the Interpretations of Preservice Elementary Teachers on Items of Division Word Problems Involving Decimals

When we go over the results, it is seen that preservice elementary teachers were better on partitive type problems than quotitive problems. Throughout the interviews, we felt that preservice elementary teachers were more familiar with partitive type division model. In this present study 70% of the interviewees gave only a partitive interpretation of division. This might be a reason for their better performance on partitive division problems. In 1978, Vest tried to investigate the disposition of preservice elementary teachers related to quotitive and partition division and he observed that the majority of the preservice elementary teachers studied preferred to use partition word problems, 67.8 percent supplying partition examples on a test. Bell et al (1981) conducted a research on 12 and 13 year olds performance on choice of operation in verbal problems and stated that students were better on partitive problems compared to quotitive ones. Tirosh and Graeber (1990, p.102) in exploring preservice elementary teachers' thinking about division stated that twelve of 21 preservice elementary teachers defined division as sharing which recalls the partitive interpretation of division. İşeri (1997, p.69) stated that it was easier for students to conceptualise partitioning.

Another important factor which seemed to affect the preservice elementary teachers performance on the choice of operation for division word problems was the conformity of the given problems to the primitive models. The overall performance of the preservice elementary teachers were better on the problems that were conforming to the primitive models compared to the problems that were not conforming to the primitive models. Tirosh and Graeber (1991, pp. 160-161) in exploring the effect of problems type and common misconceptions (primitive models) on preservice elementary teachers' thinking about division stated that common misconceptions appeared to have an effect on preservice elementary teachers' success in writing expressions for division word problems. They also stated that preservice teachers were more successful with word problems that did not challenge common misconceptions than with word problems that challenged the misconceptions. These are similar to our findings.

The preservice elementary teachers in this present study were more successful on the problems involving divisor less than the dividend than on problems involving divisor greater than the dividend. As it is seen on table 4.20, especially for problems 12 and 24 approximately 40% of the preservice elementary teachers reversed the dividend and the divisor. Many of the interviewees showed the same habit during the interviews. Therefore it is obvious that many of the preservice elementary teachers hold the belief " the divisor must be smaller than the dividend." Similar observations done by many researchers. Greaber and Tirosh (1989, p.99) in researching preservice elementary teachers' misconceptions in solving word problems in multiplication and division stated that the majority of the incorrect responses to problems in which the divisor was larger than the dividend were expressions that reversed the roles of the divisor and the dividend.

In addition to the factors which affect the overall performance on word problems we can also state that preservice elementary teachers in this present study were more successful on the problems which were involving divisors being whole number or decimals greater than one, than problems involving divisors less than one. This was mainly related to the belief that "dividend should be greater than the quotient." Thipkong and Davis (1991) stated that preservice elementary teachers had a lot of success with word problems involving decimals greater than one than decimals less than one. In 1984 Bell, Fischbein, and Greer conducted a research on choice of operation in verbal problems. Twelve and 13 year olds were tested with two types of tasks to test their understanding of applications of the multiplication and division of positive numbers. They observed that for the problems, division by decimals less than 1 proved difficulty, and in most cases led to a large number of multiplication responses. In this present, study although we met some multiplication expressions instead of division expressions, in addition to the test results after exploring the interview results we observed that preservice elementary teachers were mainly influenced by some cue words in the choice of operations. For example approximately 14% of the preservice elementary teachers gave multiplication responses for problems 16 and 25. They seemed to be distracted by the phrase "*how much of* ..." in choosing operations. In middle school grades especially in Northern Cyprus students are very familiar to the questions like "*what is 0.53 of 243 ?*" and in such questions students strongly experienced multiplication. Therefore students overgeneralise the phrase "*how much of* ..." as a representative of the multiplication operation.

As in the multiplication word problems, most of the preservice elementary teachers used direct proportion in order to find the appropriate operation for the division word problems. Again the preservice elementary teachers' fair performance (mean percent of correct responses on division problems is 71) on division problems does not imply to a fair conceptual understanding in interpreting and applying decimals because of the over use of direct proportional procedures. This is really interesting because some educators like Sellke et al (1991) worked on how to improve 7th grade students' problems solving performance by making use of direct proportional process. In Northern Cyprus most of the students starting from grade 3 to 12 follows particular courses to prepare for college and university entrance examinations. In such courses, students usually perform only procedural tasks and try to be equipped with certain tricks in finding the solution without fully understanding the problems. Therefore the preservice elementary teachers in this present study are very familiar with direct proportional procedures.

During some of the interviews whenever the interviewee was faced with a difficulty, we replaced the decimals by whole numbers. In some of the interviews, the interviewees were better after the replacement but some were still having problems in choosing the operations. A typical example from an interview is as follows:

3rd Interview Excerpt for Problem 24

R: Please , first, read the problem aloud and then choose the appropriate operation which enables you to find the answer of the problem.
Ü: (thinking…)
R: well, this time please do not use direct proportion.
Ü: It gets harder. ……Is it division ?
R: Let's replace 13.5 by 10 and 0.82 by 5, then what do you say ?
Ü: Now it looks better.
R: What will be the operation ?
Ü: …mmmm…it is $\frac{10}{5}$, in other words we can do the half of the work.

Above interview was one of the most interesting interviews done for problem 24. First of all it was obvious that she had some problems with decimal numbers but later although we replaced the decimals by integers she insisted to divide a greater by a smaller and more interestingly interpret $\frac{10}{5}$ as the half of the work. Briefly it seemed that the interviewee held the belief *"in a qoutitive division model the dividend should be greater than the divisor"*.

In choosing an appropriate operation for problem 17, which was leading to the expression 0.98 ÷ 0.245, twelve (17%) of the preservice elementary teachers gave 0.245 ÷ 0.98 as the answer. Most probably they thought that 0.245 was greater than 0.98. This was one of the most interesting observations done on this part because it is rarely observed in the related literature.

When we go over the above paragraphs, especially the problems caused by primitive models, divisors less than 1 and reversal and multiplication responses of preservice elementary teachers lead us to conclude that the preservice elementary teachers in this present study have the following misconceptions:

- In a partitive division model the dividend should be greater than the divisor.
- In a partitive division model the divisor should be a whole number.
- In a partitive division model the dividend should be greater than the quotient.
- In a quotitive division model the dividend should be greater than the divisor.

Especially throughout the interviews, we observed that nearly all of the preservice elementary teachers held these misconceptions implicitly. For example although the majority of the interviewees disagree with the statement "in a division problem, the divisor must be a whole number", many of them attempted to answer word problems as if they believed that the divisor must be a whole number.

In table 4.24 we monitored the distribution of the interviewees holding the above misconceptions. This time since we discussed quotition type problems more than partitive type problems then the misconceptions were clustered in that respect.

Table 4.24 Interviewees' Misconceptions in Solving Word Problems in Multiplicaiton and Division

	Misconceptions[1]			
Interviewees	29	30	31	32
1			*	
2				
3	*			*
4		*	*	*
5	*			
6				
7				
8	*			
9	*		*	
10	*			
11				
12	*			
13	*			
14	*	*		
15				
16				*
17	*	*		
18				
19	*			
20				
21				
22				
23	*			
24		*		
25				
	44%	16%	12%	12%

(1): Misconceptions:

29. In a quotitive division model the dividend should be greater than the divisor.
30. In a partitive division model the divisor should be a whole number.
31. In a partitive division model the dividend should be greater than the divisor.
32. In a partitive division model the dividend should be greater than the quotient.

4.1.3 Results Obtained from the Pre-Testing of Writing Division and Multiplicaiton Word Problems Test

Pre-Form of Writing Division and Multiplication Word Problems Test (see Appendices C and F) included 10 multiplication and division expressions and preservice elementary teachers were asked to write 10 problems for the given expressions. It was used in order to describe the thinking strategies or embodiments used by preservice elementary teachers when writing word problems for mathematical expressions involving decimals. The following were the expressions in the Pre-Form of Writing Division and Multiplication Word Problems Test:

Multiplicative Expressions	Division Expressions
5×8	$24 \div 4$
5×0.68	$4 \div 24$
0.63×22	$3.86 \div 23$
12.05×0.93	$0.83 \div 0.32$
	$9.6 \div 62.2$
	$0.53 \div 1.4$

4.1.3.1 Results Concerning the Thinking Strategies or Embodiments Used by Preservice Elementary Teachers When Writing Word Problems for Multiplicative Expressions Involving Decimals

4.1.3.1.1 Results Concerning the Thinking Strategies or Embodiments Used by Preservice Elementary Teachers for Multiplicative Expressions Explored Through the Pre-Form of Writing Division and Multiplication Word Problems Test

4.1.3.1.1.1 Multiplication Stories for the Expression "5 × 8"

In writing multiplication stories for **5 × 8**: As it is seen in table 4.25, 63 (89 %) of 71 attempts were correct.

Table 4.25: Results for Multiplication Stories

Expression	5 × 8	5 × 0.68	0.63 × 22	12.05 × 0.93
Correct	*63*	*59*	*54*	*40*
Rep. Addition	44	32 (2A, 7W)	26 (3W, 1A)	4W
Cartesian prod.	7	3	0	13
Rate	7 (1w)	10 (2W, 1A)	12 (4W)	8 (4W)
Not a Word Prob.	5	14 (2A, 1W)	16 (1A)	15 (1A)
Incorrect	*8*	*12*	*17*	*31*
Inapp. Embodiment	3	8	9 (1A)	12 (1W, 3A)
Division	2	1	2	4
Addition	1	1	1	0
Subtraction	0	0	2	0
No Ans	2	0	3	13
2-step oper.	0	2	0	2

W: Weak or Inappropriate context, A: Alternation of numbers or Adding more numbers

The most popular embodiment was **Repeated Addition** (44 , 62%) - For example:

There are 8 baskets. If there are 5 apples in each basket then what is the total number of apples ?

Five students shared the marbles they had. If each of them took 8 marbles, then find the total number of marbles ?

One of the embodiments that followed repeated addition was **Cartesian Product** (7, 9%). All the preservice elementary teachers used only the area context in writing cartesian product type problems - For example :

Width of a rectangle is 5 cm and length is 8 cm then find the area of the rectangle.

The other embodiment that was used in the same amount as cartesian product was **Rate** (7, 9%). Two of the preservice elementary teachers (out of 7) used money context in writing rate type problems - For example :

Suppose that you can get 5 oranges with 1 TL , then how many oranges can be bought by 8 TL ?

Although the stated problem was mathematically solvable, since it is not possible to buy 5 oranges with 1 TL it is a bit meaningless.

The rest of the preservice elementary teachers (5 out of 7) used only the multiple context - For example:

The pencil of Mine is 5 cm. If the pencil of Cemil is 8 times more than Mines' then what is the length of Cemils' pencil ?

Five of the preservice elementary teachers (7 %) used **Not a Word Problem** embodiment in writing problems - For example:

What do we get if we multiply 5 by 8 ?

What is 5 times 8 ?

Three of the preservice elementary teachers used **Inappropriate Embodiments** where it is not possible to use any mathematical expression to solve the written problem - For example:

In a class there are 5 girls and 8 boys, then what is the addition number of their multiplication ?

One of the most interesting embodiments used by the preservice elementary teachers was **Division** instead of multiplication. Two of the preservice elementary teachers used division embodiment - For example:

Ali has 5 marbles and Ahmet has 40 marbles. Then at what rate should Ali increase his marbles to have the same number as Ali has ?

A grocer sells an apple for 5 TL. If he sells some apples for 40 TL ,then find the number of apples he sold ?

One of the preservice elementary teachers used an **Addition Embodiment** for the given expression - For example:

Mehmet has 5 balls. What will be the total number of balls if you give him 8 more ?

Two of the preservice elementary teachers didn't write any problem for the expression 5 x 8.

4.1.3.1.1.2 Multiplication Stories for the Expression "5 × 0.68"

In writing multiplication stories for **5 × 0.68** : As it is seen in table 4.24, 59 (83%) of 71 attempts were correct.

Like the previous expression the **repeated addition embodiment** (59, 83 %) was still the most popular one. The most frequently used context for this embodiment was length - For example:

We have 5 identical pieces of rope , each of length 0.68. If we put them on the floor one after the other then what will be the total length that can be formed ?

This time a sudden increase was observed in the use of **weak contexts** (9%) in the written problems - For example:

In a class 0.63 pencils were given to 5 successful students. What is the total number of pencils 5 students have ?

In addition to the use of weak contexts two of the preservice teachers added more numbers or altered the original ones in order to write appropriate problems for the given expression - For example:

Hüseyin shared 25 marbles among 17 friends. Each day every single friend is given some more marbles at the same amount of their previous shares. What is the total number of marbles given to all of the friends at the end of 5 days ?

It seems that involvement of a decimal number, in the given expression, increased the use of **Not a Word Problem embodiment** (14, 20 %) - For example:

What do we get if we multiply 0.68 by 5 ?

Two of the preservice elementary teachers also added more numbers in order to write appropriate problems - For example:

Which number should be multiplied by 0.68 in order to get 3.40 ?

Ten of the preservice elementary teachers (14 %) used **Rate Embodiment** for the given expression with an over use of money context (percent follows this) whereas three of the preservice elementary teachers used again weak contexts - For example:

68 % of 5 apples have decayed, then how many of them are decayed ?

0.68 of one apple is 1 TL, then how much of one apple makes 5 TL ?

Only one of the preservice elementary teachers used distance context, but he didn't consider the relation in the units used - For example:

A car can cover 5 km in a second. What is the distance covered in 0.68 minutes by the same car ?

Three (4%) of the preservice elementary teachers used **cartesian product embodiment** with only the area context - For example:

A rectangle's longer side is 5 and its shorter side is 0.68 cm. What will be the area ?

Eight (11%) of the preservice elementary teachers used **Inappropriate Embodiment** - For example:

What would have happened, if a 5 gr 68% alcohol had been fully pure ?

A cake has been sliced into 100 equal parts. Suppose that we do the same to 5 cakes and give 68 slices to a group of people, then what are the number of persons in that group ?

Although we expected the preservice elementary teachers to write problems which can be solved by a single operation we observed that two of them wrote problems that can be solved by using two-step operations - For example:

Ali has 0.68 marbles and his friend gives him 5 times more. What is the total number of marbles Ali has at the end ?

Two of the most interesting embodiments used for the expression "5 x 0.68" were **Division** and **Addition** - Examples are given in the following paragraphs respectively:

We share 0.68 meter rope among 5 students. What is the length of the rope each student can take ?

I bought 5 kg beans from the market and then add 0.68 more. What is the total weight at the final position ? .. it will be 5 x 0.68 = 15.40.

4.1.3.1.1.3 Multiplication Stories for the Expression "0.63 × 22"

In writing multiplication stories for **0.63 × 22**: 54 (76%) of 71 attempts were correct. When compared to the previous one it seemed that the number of **repeated addition** embodiments were, a little bit, decreased (from 32 (45%) to 26 (37%)). The most popular embodiment was still **repeated addition** and the most popular contexts used in the problems were joining, weight, length and money respectively - For example:

Suppose that we want to make a geometric region by joining 22 squares each having a 0.63 cm^2 area. What will be the total area of the region ?

Even the repeated addition embodiment was the easiest one for the preservice elementary teachers. They (3 of them) couldn't help writing weak repeated addition embodiments - For example:

The price of a ball is 0.63 TL. If we want to buy 22 balls for a group of students then what will be the total amount of money we should pay ?

The embodiment used for the expression " 0.63 x 22 " which followed repeated addition embodiment was **Not a Word Problem Embodiment.** Sixteen (23 %) of the preservice elementary teachers used Not a Word Problem Embodiment. Nine of them used the simple multiplication and 7 of them used percent context - For example:

What will be your answer if you multiply 0.63 by 22 ?

What is 63 percent of 22 ?

The last embodiment used by preservice elementary teachers which was treated as correct was **Rate embodiment.** Twelve (17 %) of the preservice elementary teachers used this embodiment. The most popular context for this embodiment was percent. Four of them again used very weak rate embodiments - For example:

63 percent of a class in which there are 22 students are unsuccessful in physics. What is the number of unsuccessful students ?

Nine (13 %) of the preservice elementary teachers used **Inappropriate** embodiments in writing problems for the expression 0.63 x 22. Five of the preservice elementary teachers used weight, two of them used length and the rest used sharing contexts respectively - For example:

From 0.63 kg of rice we can make 22 different kind of meals. What is the amount of rice ?

Weight of an egg is 0.63 gr. Two of a dozen of eggs has broken. What is the total weight of the rest of the eggs ?

Kezban has 63 apples. She shares those apples among 100 students. In every one hour each students takes more apples at the same amount that they previously had. After 8 hours what will be the number of apples each student has ?

One of the most interesting embodiments used by preservice elementary teachers was **Division Embodiment.** Two of the preservice elementary teachers wrote problems that can be solved by dividing the given numbers instead of multiplying - For example:

The value of one book is 0.63. How many books do we have which has a value of 22 ?

What should be the number of students who can share 22 cakes each one having 0.63 of the cakes ?

The second interesting embodiment used by preservice elementary teachers was the **Subtraction Embodiment**. Two of the preservice elementary teachers used this embodiment in writing problems for the expression 0.63 x 22 - For example:

Five less than 22 times of a number is 13.86. What is the number ?

One of the preservice elementary teachers used **Addition Embodiment** - For example:

Suppose that I bought 0.63 kg flour, if I add 22 kg of flour to the previous one then what will be the total weight of flour ?

Three of the preservice elementary teachers didn't write any problem for the expression 0.63 x 22.

4.1.3.1.1.4 Multiplication Stories for the Expression "12.05 × 0.93"

In writing multiplication stories for **12.05 × 0.93 :** As it is seen in table 4.24 , 40 (56 %) of 71 attempts were correct. When compared to the others this seems to be lower. At first sight, it is possible to observe that the *repeated addition* embodiment was the least preferred one when compared to the other correct embodiments (f=4, 6 %). Although it is not possible to write problems in a repeated addition embodiment, for the given expression, four of the preservice teachers attempted to write problems in this embodiment but of course the problems occurred in weak contexts - For example:

In a box there are 0.93 pencils. What is the total number of pencils in 12.05 boxes ?

A baby weights 12.05 kg. What will be the final weight of the baby If he put on weight 0.93 times ?

Most of the preservice elementary teachers (f=15, 21%) used **Not a Word Problem Embodiment** - For example:

What do we get when we multiply 12.05 by 0.93 ?

Cartesian Product Embodiment followed Not a Word Problem Embodiment. Thirteen (18 %) of the preservice elementary teachers used this embodiment in writing problems for the expression 12.05 x 0.93. Twelve of them used *area* and one of them used *force* contexts - For example:

What is the area of a rectangular field which has a length of 12.05 and a width of 0.93 ?

A force is applied to a wooden block which has a mass of 12.05 kg. What is the force applied if the wooden block gains a 0.93 m/s^2 *of acceleration ?*

Eight (11%) of the preservice elementary teachers used **Rate Embodiment** but four of the problems written were weak in terms of the overall context. The most popular contexts used in this embodiment were length , weight and distance - For example:

It is possible to make a skirt using 12.05 m of a certain kind of fabric. What is the amount of fabric needed in order to make 0.93 skirts ?

The weight of a single rice is 12.05 gr. What is the total weight of 0.93 pieces of rice ?

A runner completed a run in 0.95 minutes. He maintained an average speed of 12.05 km per hour. How long did he run ?

Twelve (17 %) of the preservice elementary teachers used **Inappropriate embodiments** in writing problems for the expression 12.05 x 0.93 - For example:

I have some pieces of fabrics in red each being 12.05 cm and some pieces of black fabrics each being 0.93 cm. If I want to make all pieces as one what should I do ?

Some of the preservice elementary teachers who used the Inappropriate embodiment also tried to alter or change the numbers in the given expression - For example:

Hasan collects 241 lemons in 20 days. The number of lemons collected is equal to the number of students in his class. What is 50 % of the class ?

When compared to the other cases we can easily observe that an important number (18 %) of preservice elementary teachers didn't write any problem for the expression 12.05 x 0.93 (see table 4.24).

Again, surprisingly, four of the preservice elementary teachers wrote problems that could only be solved by using **division** instead of multiplication - For example:

How many bottles of Cola, each having a 0.93 lt capacity , is needed to fill up a cup having a 12.05 lt capacity ?

One of the preservice elementary teachers used division with a Not a Word Problem Embodiment as follows:

What do we get if we multiply 12.05 by 0.93 ?

Another preservice elementary teacher used division embodiment and also showed that he treated *decimal point as a separator* as follows:

We divide a stick which is 12 meter and 5 cm into some pieces each having a length of 93 cm. What is the number of small pieces that we can get ?

Two of the preservice elementary teachers wrote problems that can be solved by using at least two operations (**2-step operations**) - For example:

Ahmet is 12.05 cm tall and Alis' height is 0.93 times of Mehmets' height. What is the sum of the heights of the two ?

What will be the total area of a 12.05 m^2 pool If we increase its area by 0.93 ?

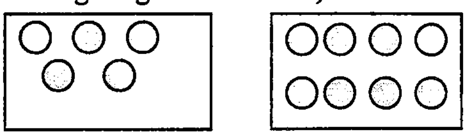
4.1.3.1.2 Results Concerning the Thinking Strategies or Embodiments Used by Preservice Elementary Teachers When Writing Word Problems for Multiplication Expressions Explored Through the Interviews

4.1.3.1.2.1 Multiplication Stories for the Expression "5 × 8"

In writing multiplication stories for 5 × 8 (*5 preservice elementary teachers were interviewed for this expression*) three of the preservice elementary teachers wrote problems that were in the mode of repeated addition. Although the given expression was very basic 2 of them could not write any problem. Some difficulties of the preservice elementary teachers are given in the following paragraphs:

1st Interview Excerpt For Expression 5 x 8

R: Please write a problem about the given expression
H: (thinking)...(drawing a figure as follows)



H: What do we get if we add 5 apples and 8 oranges ?

R: Can you write another one ?
H: (thinking).....no !

2nd Interview Excerpt For Expression 5 x 8

R: Please write a problem about the given expression
U: (Thinking)... In a class 5 of the students speak English, half of the 8 students speak only English and another half of the class don't speak English. Find the number of the rest of the class ?

In the above 2 interviews It was observed that the two preservice elementary teachers were not so capable of writing word problems. It seemed that the difficulties of the preservice elementary teachers were not limited to decimals.

4.1.3.1.2.2 Multiplication Stories for the Expression "5 × 0.68"

In writing multiplication stories for **5 × 0.68** (*8 preservice elementary teachers were interviewed for this expression*) some of the preservice elementary teachers wrote interesting problems and showed their thinking strategies as follows:

1st Interview Excerpt For Expression 5 x 0.68

R: Please write a problem about the given expression
M: Most probably you don't need 5 times more of 0.68 ?
R: Try to write problems out of *not a word problem embodiment.*
M: (thinking)....
R: If I give you 5 x 8, will it be easier ?
M: Well.... In order to make a cake 5 eggs are needed. How many eggs are needed to make 8 cakes ?
R: Do integers make the work easier ?
M: Yes... Let me write another one ∴. We have 5 slices of water melon then what is $\frac{68}{100}$ of the slices ?

In the above interview the interviewee seemed to have difficulties with decimals. She could not help writing problems in the mode of repeated addition and although at the end she tried to write a problem for the original expression it seemed that she was trying to keep away from the decimals as she wrote 0.68 as a fraction ($\frac{68}{100}$). The interviewee could not give correct answers to the items in the pre-form of concept test, which were operations including decimals, then it seemed that the interviewee use decimals in a limited range of context.

2nd Interview Excerpt For Expression 5 x 0.68

R: Please write a problem about the given expression
D: (thinking)... There are 100 apples and 68 students will share the apples. Each student can get 0.68 of the apples. How can we multiply that amount with 5 ?
R: Why don't you consider only 5 and 0.68 ?
D: An apple can not be 0.68 !
R: Do you have to consider only apples ?
D: I don't know !

As it is seen in the above interview the interviewees' problems don't make any sense. Like many others, the interviewee thought narrow ranged embodiment for problems and it seemed that she focused her attention to countable objects (whole numbers) as the source of context for the problems to be written.

4.1.3.1.2.3 Multiplication Stories for the Expression "0.63 × 22"

In writing multiplication stories for **0.63 × 22** (*6 preservice elementary teachers were interviewed for this expression*) 5 of the interviewees used repeated addition embodiment and only one of them used cartesian product embodiment in their problems. Typical thinking strategies of interviewees are monitored in the following paragraphs:

1st Interview Excerpt For Expression 0.63 × 22

R: Please write a problem for the given expression

V: We have 22 , 0.63 meter sticks. What will be the total length of the sticks if we put them one after the other ?
R: Do decimals make the work harder ?
V: No, not that much.

Above interviewee used the commutative property of multiplication and treated 22 as the operator and made the given expression suitable for a repeated addition embodiment. Similar problems written by the interviewee showed that she could apply decimals in a limited range of contexts. All the problems written by the interviewee were repeated addition type. In all the expressions of the same type (decimal x whole number) she interchange the places of operator and operand so she seemed to hold the belief *"in a multiplication expression the operator should be a whole number"*.

2nd Interview Excerpt For Expression 0.63 × 22

R: Please write a problem for the given expression
Ç: (thinking)… We bought 0.63 meter of a certain fabric but later we bought 22 more. What will be the total amount of fabric ?
R: Does 0.63 x 22 give your answer ?
Ç: My problem reflects addition… Yes I found ! ….Width of a fabric is 0.63 and its length is 22 what is its total ?

In the above interview the interviewee seemed to hold strongly the belief *"in a multiplication expression the operator should be a whole number"*. As it is seen at the final line of the interview, although he tried to change his embodiment it was still reflecting addition. Interviewees' low performance on the items related with operations including decimal operators strengthen our conclusion about the belief of the interviewee.

4.1.3.1.2.4 Multiplication Stories for the Expression "12.05 × 0.93"

In writing multiplication stories for **12.05 × 0.93** (*2 preservice elementary teachers were interviewed for this expression*) neither of the two interviewees could write a problem. Thinking strategies of the interviewees are given below:

1st Interview Excerpt For Expression 12.05 × 0.93

R: Please write a problem for the given expression

U: I can not do anything for this !
R: Do decimals make the work harder ?
U: Yes. I can not do anything with decimals.

In addition to the above interview, when we went through pre-form of concept (6 out of 44), Pre-Form of Problems (1 out of 26) and pre-form of writing word problems (7 out of 40) tests scores, it seemed that the above interviewee had some serious difficulties / problems about the use and interpretation of decimals

2nd Interview Excerpt For Expression 12.05×0.93

R: Please write a problem for the given expression
Y: (thinking)…May I ignore the decimal points ?
R: Do you mean to move the decimal points ?
Y: Yes I think it means the same… (thinking)….mmm !
R: Do decimals make the work harder ?
Y: Yes ! It becomes more complicated.

Although the above interviewee was a medium scorer in all three of the tests, she couldn't write a problem for the given expression. From the interview it seemed that she held the beliefs *"in a multiplication expression the operator should be a whole number"* and *"decimal point can be ignored"*.

4.1.3.2 Discussion of the Results Concerning the Thinking Strategies or Embodiments used by Preservice Elementary Teachers When Writing Word Problems for Multiplicative Expressions Involving Decimals

First of all when we look at table 2.25, without going into detail, it is possible to say that decimals make the work harder. For example Graeber, Tirosh, and Glover (1989) states: "Some readers might argue that the mere presence of decimals causes a lower success." The expressions in table 2.24 are listed from the simple to more complex forms and the preservice elementary teachers' correct problem writing performance is changing from 89% (for the simplest expression) to 56% (for the most complex expression). When we go into more detail we can also see that use of *Not a Word*

Problem Embodiment is changing from 4% (for the simplest expression) to 21% (for the most complex expression). Similarly the uses of *Inappropriate* and *No Answer* Embodiments are changing from 4% to 17% and from 3% to 18% respectively. All the above might show that the preservice elementary teachers in this present study have some problems with decimals in writing problems.

Whenever two of the numbers were whole numbers or one of the numbers was a whole number in the given expressions the preservice elementary teachers' most popular embodiment in writing problems was repeated addition. Similar observations were made by Fischbein et al (1985). They stated that decimal operators might cause the overuse of repeated addition embodiment. In a study, children in grades 4-6 in a local school were asked to write a word problem for 6 x 3, the only context used by the students were repeated addition (Brien & Casey, 1983). Bell et al (1984) in exploring 12 and 13 year olds problems writing performance for given expressions stated that for multiplication expressions (e.g., 0.51 x 33) in which one of the numbers involved were decimals, most of the problems written were in repeated addition mode. Preservice elementary teachers used the commutative property of multiplication and changed the roles of operator and operand whenever the operator was a decimal in the given expressions. This may show that they, implicitly, thought that operators should have been whole numbers.

As it is seen in table 2.25 (expression 12.05 x 0.93) when two of the numbers in the given expression were decimals preservice elementary teachers tended to use the *Cartesian Product* embodiment for the given expression (13 out of 71). For expression 12.05 x 0.93, twelve of the 13 cartesian product embodiments were in area context. Similar observation was done by Bell et al (1984). When compared to the previous expressions we can say that an important number of preservice elementary teachers (f=13, 18%) didn't do anything for this expression. This may shows us that not having a whole number somewhat increased the difficulty. For example, in the interviews one of the interviewees tried to ignore the decimal points in 12.05 x 0.93.

In the interviews, some of the interviewees tried to replace a decimal operator by a whole number operand which might show the influence of primitive models on the preservice elementary teachers multiplication stories.

To summarise, repeated addition was the preferred embodiment when one of the numbers involved was a whole number. Although it was not possible, four of the preservice elementary teachers tried to write problems in repeated addition embodiment for the expression 12.05 x 0.93. When one or both of the numbers involved in a multiplication expression were decimals, *not a word problem embodiment* and *inappropriate embodiments* were both used to a great extent. Area and money contexts were used frequently when it was feasible. Most of the problems written were similar to each other. A similar observation was made by Silver et al (1996). They conducted a study on a group of (N=81) middle school and prospective secondary school teachers' mathematical problem posing performance and they stated that a sizeable portion of the posed problems were produced in clusters of related problems. It seems that some extra difficulties occurred for preservice elementary teachers when the numbers involved were less than 1. Previously many other researchers reported the influence of decimals less than 1 (Taylor, 1981; Bell et al, 1984; İşeri, 1997). Whenever both of the numbers were decimals a great number of preservice elementary teachers couldn't write any word problem for the given multiplicative expressions. In such conditions some of the preservice elementary teachers naturally wrote word problems with a Cartesian product embodiment, but this was not that much . Approximately 12% of the problems written for multiplication expressions, which were treated as correct, were in very weak contexts. Therefore we can conclude that the preservice elementary teachers in this present study are relatively poor in writing multiplication problems for expressions involving decimals.

The overall test and interview results showed that some of the preservice elementary teachers in this present study still held the belief "in a multiplication expression the operator should be a whole number." In addition to this the problems written by the preservice elementary teachers declare that they had never had such problem writing experiences. In the interviews all the interviewees stated that they had never tried to write problems for given expressions or posed any kind of problems. This of course affected their overall problem writing performance.

4.1.3.3 Results Concerning the Thinking Strategies or Embodiments Used by Preservice Elementary Teachers When Writing Word Problems for Division Expressions Involving Decimals

4.1.3.3.1 Results Concerning the Thinking Strategies or Embodiments Used by Preservice Elementary Teachers When Writing Word Problems for Division Expressions Explored Through Writing Division and Multiplication Word Problems Test

I stated five types of correct embodiments for division expressions as follows:

1. **Partition** - can be used whenever the divisor is an integer smaller than the dividend.
2. **Fractional partition** - can be used whenever the divisor is an integer larger than the dividend.
3. **Quotition** - can be used whenever the divisor is less than the dividend
4. **Fractional quotition** - can be used whenever the divisor is greater than the dividend.
5. **Rate** - can always be used.
6. **Not a Word Problem** - can always be used.

On this basis six items can be divided into four types as;

Type - 1 (24 ÷ 4) - can be embodied by partition, quotition or rate.

Type - 2 (4 ÷ 24 and 3.86 ÷ 23) - can be embodied by fractional partition, fractional quotition or rate.

Type - 3 (0.83 ÷ 0.32) - can be embodied by quotition or rate.

Type - 4 (9.6 ÷ 62.2 and 0.53 ÷ 1.4) - can be embodied by fractional quotition or rate.

Table 4.26: Results of Division Stories

Problem	24 ÷ 4	4 ÷ 24	0.83 ÷ 0.32	3.86 ÷ 23	9.6 ÷ 62.2	0.53 ÷ 1.4
Correct	*66*	*50*	*48*	*41*	*28*	*29*
Partition	48 (2W, 1A)	0	5 (3W)	3 (1A)	0	0
Fractional Partition	0	32 (12W)	0	26 (12W, 1A)	5 (4W)	2W
Quotition	11	0	19 (3W)	0	0	0
Fractional Quotition	0	0	1	0	4 (1W, 1A)	4 (2W)
Rate	2	10 (4W)	5 (2W)	4 (1W)	6 (2W)	6 (3W)
Not a Word Problem	5	8 (4W)	18 (2A, 1W)	8 (1W)	13 (2A)	17 (2A)
Incorrect	*5*	*21*	*23*	*30*	*43*	*42*
Inappropriate Embodiment	3	11	9 (1A)	16	22 (2A)	19 (1A)
Inverse Division	0	5	0	1	5	1
Multiplication	0	1	2	3	0	3 (1A)
Subtraction	1	2	0	0	1	0
No answer	0	2	12	10	15	19
2-step Oper..	1	0	0	0	0	0

W: Weak or Inappropriate context, A: Alternation of numbers or Adding more numbers

4.1.3.3.1.1 Division Stories for the Expression "24 ÷ 4"

In writing word problems for the expression **24 ÷ 4**: As it is seen in table 4.25, 66 of (93 %) 71 attempts were correct and the most popular embodiment used was partition.

Forty-eight (68 %) of the preservice elementary teachers used **Partition Embodiment** in writing problems for the expression 24 ÷4. All of the preservice elementary teachers used very simple contexts like sharing marbles and balls - For example:

There are 24 marbles and 4 students. How many marbles does each student get ?

One of the preservice elementary teachers beside using partition embodiment added more numbers to the original ones as follows:

I have <u>*48*</u> *balloons and half of them have popped. If there are 4 students , how many balloons does each one get ?*

Although it was quite appropriate only eleven (15 %) of the preservice elementary teachers used **Quotition Embodiment** for the expression 24 ÷4. In this case preservice elementary teachers used "members in a certain group" context - For example:

In a class there are 24 students. For a march, lines consisting of 4 members each are needed. How many lines can be formed ?

One of the preservice elementary teachers used the qoutition embodiment but with a weak unit (couple) as follows:

How many 4 member - couples can be formed in a class which has 24 students ?

Five of the preservice elementary teachers used **Not a Word Problem Embodiment** in writing problems for the expression 24 ÷4 - For example:

What will be the result , if we divide 24 by 4 ?

Only two (3%) of the preservice elementary teachers used **Rate Embodiment** for the given expression. Both of them used a *money* context in their problems - For example:

Four apples costs 1 TL . How much will it cost if we buy 24 apples ?

Three of the preservice elementary teachers used **Inappropriate Embodiments** in writing problems for the expression 24 ÷ 4 - For example:

There are 30 candies to be shared among 4 students. Each student following the previous one can take one more candy additional to the one taken previously. What is the number of candies the first student took ?

What will be the divisor and remainder if we start from 24 and each time subtract 4 in a reversed order ?

Interestingly, one of the preservice elementary teachers used **subtraction** instead of division - For example:

Ahmet has 24 TL and Ali has 4 TL. What is the difference of their money ?

Finally, in writing problems for the expression 24 ÷ 4 , one of the preservice elementary teachers wrote a problem that can only be solved using two operations (**2 - step Operations**) - For example:

Ayşe has 24 marbles and ¼ of her marbles were lost. How many marbles does Ayşe have at the final position ?

4.1.3.3.1.2 Division Stories for the Expression "4 ÷ 24"

In writing word problems for the expression **4 ÷ 24** : As it is seen in table 4.25, 50 (70 %) of 71 attempts were correct. In fact this expression was not suitable for partition embodiments and most of the preservice elementary teachers (32, 45 %) used **fractional partition** embodiment for the given expression. The two contexts used for this expression were simple *sharing* , in which there is an amount of material and we try to find the share of each member in a certain group of people, by dividing a material into equal amounts (*length*) - For example:

A 4 meter wooden stick is to be cut into 24 equal parts. What will be the length of each small part ?

Although the fractional partition was the most popular embodiment for preservice elementary teachers, 12 (38 %) of them used this embodiment in very weak contexts - For example:

There are 4 marbles to be shared among 24 students. Find the share of each student ?

Ten (14 %) of the preservice elementary teachers used **Rate Embodiment** in writing problems for the given expression. Nine of the preservice elementary teachers used rate embodiment in simple *ratio* or *proportion* contexts - For example:

How much of the apples does each one get, if we share 4 apples among 24 students ?

Ali can paint a house in 24 hours. First day Ali works for 4 hours. How much of the work has been done ?

Not a Word Problem Embodiment followed rate embodiment. Eight (11 %) of the preservice elementary teachers used this embodiment in writing problems for the expression $4 \div 24$. Two of the preservice elementary teachers also altered or added more numbers to the original ones, while writing problems - For example:

What do we get if we divide three less of 7 by 24 ?

What do we get if we divide 4 by twice of 12 ?

Another two of the preservice elementary teachers related the given numbers by decimals as follows:

Turn $\frac{4}{24}$ *into a decimal number.*

When we divide 4 by 24 we get a decimal number. What is that number ?

Eleven (16 %) of the preservice elementary teachers used **Inappropriate Embodiment** in writing problems for the expression 4 ÷ 24 - For example:

What do we get if we divide 4 slices of an apple into 24 parts ?

When compared to the previous expression, it seems that more preservice elementary teachers used inappropriate embodiments.

Surprisingly, five (7 %) of the preservice elementary teachers wrote word problems that led to the reversal of the given numbers in the expression 4 ÷ 24. We called this **Inverse Division** Embodiment - For example:

Ayşe brought a birthday cake that is divided into 24 slices. If she has 4 guests then find the number of slices that each guest will take.

Ali has 24 marbles. Later Ahmet, Ayşe, and Leyla come to share the marbles. Find the share of each one ?

One of the preservice elementary teachers wrote a problem that leads to **Multiplication** instead of division as follows:

Four workers do a work in 24 hours, then in how many hours can a worker finish the same job ?

More surprisingly, two of the preservice elementary teachers wrote problems that lead to **subtraction** instead of division as follows:

In a class there are 4 students. If $\frac{4}{24}$ *of the students leave the class , how many students remain in the class ?*

Ahmet gives 1/24 of his 4 chocolate to one of his friends. How much of the chocolate does Ahmet have at this final position ?

The two examples above also lead to **two-step operations.**

Finally, we observed that two of the preservice elementary teachers didn't try to write any problems (**no answer embodiment**) for the given expression.

4.1.3.3.1.3 Division Stories for the Expression "3.86 ÷ 23"

In writing word problems for the expression **3.86 ÷ 23** : As it is seen in table 4.25, 41 (58 %) of 71 attempts were correct.

The numerical structure of this expression sounds like the previous one (4 ÷ 24) , the divisor is greater than the dividend, but this time the dividend is a decimal number. As it is seen in table 4.25, it seems like decimals affect the overall problem writing performance of preservice elementary teachers.

Like the previous one again the most popular embodiment was **fractional partition.** Most of the problems, written, were considering sharing in line with length , and volume. Twenty-six (37 %) of the preservice elementary teachers used this embodiment in writing problems for the expression 3.86 ÷ 23 - For example:

A 3.86 lt bottle is full of water. There are 23 students who need some water. If we divide the water equally among the students, what will be each one's share ?

Again most of the fractional partition problems (46%) written for the expression were in **weak contexts** - For example:

3.86 gr chocolate is to be shared among 23 students. How much will they each receive ?

Eight (11%) of the preservice elementary teachers used **Not a Word Problem Embodiment** for the given expression - For example:

What do we get if we divide 3.86 by 23 ?

When compared with the problems written for the expression 4 ÷ 24, this time we observe a slight decrease in the number of rate type problems. Only 4 (5%) of the preservice elementary teachers used **rate embodiment** in writing word problems for the given expression - For example:

What is 3.86 minutes in terms of hours, if we treat 1 hour as 23 minutes ?

Only one of the four preservice elementary teachers used *speed* but with inappropriate units as follows:

A boy can cover 3.86 km in 23 minutes. What is the speed of the boy ?

Sixteen (23 %) of the preservice elementary teachers used **Inappropriate Embodiments** in writing problems for the expression 3.86 ÷ 23 - For example:

3.86 and 23 are given. What do we get if we turn 3.86 into an integer and divide by 23 ?

In a class there are 23 students. How many of the students in the 3.86 portion of the class know English ?

One of the preservice elementary teachers also used decimal point as a separator in an inappropriate embodiment as follows:

We share a <u>3 meter and 86 cm</u> stick among 23 workers. How many small sticks can be formed to be shared ?

Again when we compare with the problems written for the expression 4 ÷ 24 , this time we observed a sudden increase in the use of **No Answer Embodiment** (from 3 % to 14 %).

Three of the preservice elementary teachers used **Multiplication** instead of division - For example:

A man bought 23 packets of flour each having a 3.86 kg weight. What is the total weight of the flour he bought ?

One of the preservice elementary teachers wrote a problem that can only be solved by reversing the order (**Inverse Division**) of divisor and dividend as follows:

A room costs 3.86. A person has 23. How many rooms can be hired with this ?

4.1.3.3.1.4 Division Stories for the Expression "0.83 ÷ 0.32"

In writing word problems for the expression **0.83 ÷ 0.32** : As it is seen from table 4.25, Sixty-one (68 %) of 71 attempts were correct.

The most popular embodiment used was **quotition.** Twenty (28 %) of the preservice elementary teachers used this embodiment, in writing problems, for the expression 0.83 ÷ 0.32. Most frequently used contexts were just simple quotition divisions, in which one tries to find the number of elements, with the units of length, weight, and volume - For example:

How many 0.32 litre cups are needed to fill up a 0.83 litre bottle ?

In a cup there are some identical marbles which has a total weight of 0.83 gr. If one of the marbles weight 0.32 gr, then how many marbles are there in the cup ?

Although, the contexts used in the previous example was fairly weak, we observed the problems of three preservice elementary teachers which were quite weak in context - For example:

In a market there is a piece of fabric which is 0.83 meter long. A group of people come and each one in the group buys 0.32 meter of that fabric. What is the number of persons in the group that shared the total fabric ?

The embodiment followed the quotition embodiment was **Not a Word Problem Embodiment** . Eighteen (25 %) of the preservice elementary teachers used this embodiment, in writing problems, for the expression 0.83 ÷ 0.32. Seventeen of the preservice elementary teachers used *not a word problem type* like the ones used for previous expressions , but one of the eighteen preservice elementary teachers used the not a word problem embodiment in a very different manner as follows:

$$\begin{array}{r|l} 0.83 & 0.32 \\ -\ 0.64 & \hline \\ & \ 2 \\ \hline 0.19 & \end{array}$$

Although, the given expression 0.83 ÷ 0.32 is not appropriate for **partitive** type problems , five (7%) of the preservice elementary teachers insisted on using this embodiment. Since the given expression was not suitable for partitive embodiments therefore the problems, written by the preservice elementary teachers, occurred in weak contexts - For example:

A teacher wants to share 0.83 balls among 0.32 students. How many will each one receive ?

We have a type of material weighting 0.83 gr. We divide this material into 0.32 parts. What will be the weight of each small part ?

Five (7%) of the preservice elementary teachers used **Rate Embodiment** in writing problems for the expression 0.83 ÷ 0.32. All the problems stated in this embodiment were related to ratio type rate problems which were not that different than *not a word problem types.* Therefore it is not possible to say that the stated contexts were that strong - For example:

In a school 83 percent of the students wear glasses and 32 percent of the students are blond. What is the ratio of the number of students who wear glasses to the number of students who are blond ?

Length of a rectangle is 0.83 cm and the width of the same rectangle is 0.32 cm. What is the ratio of the length to the width ?

Nine (13 %) of the preservice elementary teachers used **Inappropriate Embodiments** in writing problems for the expression 0.83 ÷ 0.32 - For example:

83 percent of a class is willing to go to a picnic. On the other hand 32 percent of the school will not go to a picnic. What is the ratio of the ones that want to go to a picnic to the ones that do not want ?

Ayşe has 0.83 dollies and Emine has 0.32 dollies. What is the share of Fatma ?

One of the preservice elementary teachers in addition to inappropriate embodiment also altered the given numbers in the expression as follows:

There is a set containing 100 marbles. The marbles are needed to be shared among 32 people. What will be the number of persons who will share the marbles ?

Two of the preservice elementary teachers wrote problems that lead to **multiplication** instead of division as follows:

0.32 of a 0.83 gr chocolate will be given to some children. What is that amount ?

What is 0.32 of a rope which is 0.83 long ?

Twelve of (17 %) of the preservice elementary teachers did not attempt to write any problem (**No Answer Embodiment**) for the expression 0.83 ÷ 0.32.

4.1.3.3.1.5 Division Stories for the Expression "9.6 ÷ 62.2"

In writing word problems for the expressions **9.6 ÷ 62.2** : As it is seen in table 4.25, only 28 (39 %) of 71 attempts were correct. When compared with the previous cases it seems that the overall performance of preservice elementary teachers were lower. Although the given expression was leading to fractional quotition and rate problems a limited number of preservice elementary teachers wrote problems at those embodiments.

This time the most popular correct embodiment was **Not a Word Problem Embodiment.** Thirteen of (18 %) the preservice elementary teachers used this embodiment in their problems - For example:

What do we get if we divide 9.6 by 62.2 ?

Two of the preservice elementary teachers also **add more numbers** to the original ones , Although they used not a word problem embodiment - For example:

First add 3.2 to 6.4, then add 11.2 to 51 and find their ratio ?

One of the most appropriate embodiments for the given expression was **Rate Embodiment** but only six of (8 %) the preservice elementary teachers used this embodiment in writing problems for 9.6 ÷ 62.2. The only context used in these rate problems was ratio. No one used speed, money, distance and etc - For example:

We have 62.2 kg of cotton. The first day we sold 9.6 kg of the cotton. How much of the cotton was sold ?

Two of the preservice elementary teachers used rate embodiment type problems but in weak contexts in terms of the units used, which were not suitable in real life - For example:

In order to make a dress we need 62.2 cm of a certain fabric. With a 9.6 cm portion of the fabric how much of the work can be done ?

Although the given expression 9.6 ÷ 62.2 was not appropriate for **fractional partition** type problems, five of (7 %) the preservice elementary teachers insisted on fractional partition problems. Because of the fractional quotition nature of the given expression, all the problems written, occurred in weak contexts - For example:

If we divide a 9.6 gr block into 62.2 equal parts, then what will be the weight of each part ?

The other embodiment which is appropriate for 9.6 ÷ 62.2 was **Fractional Quotition Embodiment** . Only four of (6 %) the preservice elementary teachers used this embodiment in writing problems for the given expression. Like the previous examples all the fractional quotition problems written were in weak contexts, especially in terms of *units* used in the problems - For example:

How many 62.2 ml drops can be obtained from a certain liquid which weights 9.6 cl ?

We have 9.6 tons of sugar to be divided among some boxes. Every small box can get 62.2 kg of sugar. For the purpose how many boxes are needed ?

Twenty - two (31 %) of the preservice elementary teachers used **Inappropriate Embodiments** in writing problems for the expression 9.6 ÷ 62.2.

The problems written in this embodiment were almost the most interesting ones up to this point. Some of the preservice elementary teachers alternated the original numbers in addition to using an inappropriate embodiment as follows:

We want to increase 96 cakes to 622. In order to do this by which number do we have to divide 96 ?

Some of the preservice elementary teachers wrote inappropriate type problems in which the answer is clearly given in the stated problem as follows:

If we divided a 9.6 meter region into 62.2 m equal parts, what will be the number of parts ?

One of the preservice elementary teachers wrote an inappropriate type problem that also considered the *decimal point as a separator* as follows:

Total weight of the foods in a picnic basket is 9.6 kg . The foods are needed to be shared among some children whose total age is 62.2 (62 years and 2 months). Consider the ages of each child in terms of month and find each ones' share ?

Five of (7 %) the preservice elementary teachers wrote problems that led to the inversion of the divisor and the dividend given in the original expression (**Inverse Division Embodiment**) - For example:

In a classroom there are 62.2 desks. There are 9.6 students. How many desks can each student get ?

How many parts, each being 9.6 cm can be obtained from a 62.2 cm long stick ?

Surprisingly, one of the preservice elementary teachers wrote a problem that leads to **subtraction** instead of division as follows:

A material which is 9.6 kg is diminished by a factor 62.2. What will be the final weight of the material ?

Fifteen of (21 %) the preservice elementary teachers did not attempt to write any problem (**No Answer Embodiment**) for the expression 9.6 ÷ 62.2.

4.1.3.3.1.6 Division Stories for the Expression "0.53 ÷ 1.4"

In writing word problems for the expressions **0.53 ÷ 1.4**: As it is seen in table 4.25, only 29 (41 %) of 71 attempts were correct. The overall performance of the preservice elementary teachers were similar to the previous one.

Again, the most popular embodiment for the given expression was **Not a Word Problem Embodiment**. Seventeen of (24 %) the preservice elementary teachers used this embodiment. We think that there is no need to give examples for this embodiment because the idea is always the same but two of the preservice elementary teachers who wrote not a word problem type problems also added more numbers to the original ones or altered the given numbers in the expression as follows:

Two lengths are 0.12 and 0.41 respectively. What do we get if we first add the two lengths and then divide the result by the sum of 0.4 and 1 ?

Like the previous case, **Rate Embodiment** followed *not a word problem embodiment.* Six (8 %) of the preservice elementary teachers used this embodiment in writing problems for the given expression. All the contexts used, were related to *ratio* *approach* in which one can solve the written problem simply by using a direct proportional method. Although, the written problems were very simple in nature, three of the preservice elementary teachers used them in weak contexts where the *units* used were meaningless as follows:

In order to paint the whole of a house, we need 1.4 kg of a certain paint. How much of the work can be done with 0.53 kg of the same paint ?

A metal pipe which weights 1.4 gr is divided into several small part each weighting 0.53 gr. What is the ratio of a single small pipe to the whole pipe ?

Although the given expression was appropriate for **fractional quotition** type problems, only four (6 %) of the preservice elementary teachers used this embodiment in writing problems. Again, like the previous cases all the written problems were including some *weak* dimension in terms of units as follows:

How many boxes, each having a 1.4 gr capacity, are needed to divide a 0.53 kg sugar ?

How many 1.4 mg single rices are included in 0.53 kg rice ?

Two of the preservice elementary teachers forced the written problems to be **fractional partition** type, which is, in fact, impossible - For example:

1.4 children want to share a 0.53 cake. How much does each one get ?

Nineteen (27 %) of the preservice elementary teachers used **Inappropriate Embodiments** in writing problems for the expression 0.53 ÷ 1.4 - For example:

Turn 0.53 and 1.4 into integers and divide them by each other.

A 53 % solution is to be divided into some 14 % cups. How many cups are needed ?

How much 14 % solution is contained in a 53 % solution ?

Again surprisingly, three of the preservice elementary teachers wrote problems that lead to **multiplication** instead of division - For example:

What is the product of 53 mm and 14 cm in terms of dm ?

Three of the preservice elementary teachers wrote problems that led to the inversion of the divisor and the dividend given in the original expression (**Inverse Division Embodiment**) - For example:

A worker can do 0.53 meter of a wall in an hour. In how many hours can the same worker can do 1.4 meter of the wall ?

How many 0.53 are contained in 1.4 ?

Value of a book is 0.53. How many book are there in a shelf which has a value of 1.4 ?

This time seventeen (24 %) of the preservice elementary teachers did not attempt to write any problem (**No Answer Embodiment**) for the expression $0.53 \div 1.4$.

4.1.3.3.2 Results Concerning the Thinking Strategies or Embodiments used by Preservice Elementary Teachers When Writing Word Problems for Division Expressions Involving Decimals Explored through the Interviewes

4.1.3.3.2.1 Division Stories for the Expression "4 ÷ 24"

In writing multiplication stories for **4 ÷ 24** (*12 preservice elementary teachers were interviewed for this expression*) generally preservice elementary teachers wrote problems that could be solved by 24 ÷4 . Some of them noticed their fault and some of them didn't. Some examples of interviewees' thinking strategies are given below:

1st Interview Excerpt for Expression 4 ÷ 24

R: Please write a problem for the given expression.
Ü: We have 24 apples and we want to share them among 4 students. What will be the share of each student ?....(thinking) .... No it will be the reverse ! .... There is a 4 cm long stick. If we divide this into 24 what will be the answer ?

In the above interview first the interviewee wrote a problem whose answer was the reverse of the given expression. Later she tried to correct it but the second problem she wrote was still weak in terms of units and in terms of what was exactly asked. The interviewees' score on the operations part of the pre-form of concept test, including greater divisors was very low. She could not do any operation in the test, so it seemed that she held the belief *"in a division operation, dividend should be greater than the divisor"* which might also affect her problem writing performance.

2nd Interview Excerpt for Expression 4 ÷ 24

R: Please write a problem for the given expression.
O: We want to put some spots on a 4 meter long stick in which the spots are 24 cm away from each other. How many spots are needed ?

Above interviewee noticed that he really wrote a problem that calls for 4 ÷ 24 but since he tried to divide meter by cm in an incomplete manner (because the answer is not 4 ÷ 24) is seemed that he still insisted on dividing a greater by a smaller (meter ÷ cm). The interviewee couldn't give correct answers to division operations where the divisors were greater and he only could choose 4 appropriate operations for the problems in the Pre-Form of Problems test so it seemed that he held the beliefs *"in a division operation dividend should be greater than the divisor"* and *"division makes smaller"*.

4.1.3.3.2.2 Division Stories for the Expression "0.83 ÷ 0.32"

In writing multiplication stories for **0.83 ÷ 0.32** (*5 preservice elementary teachers were interviewed for this expression*) interviewees had many difficulties. Some of them are given in the following paragraphs:

1st Interview Excerpt for Expression 0.83 ÷ 0.32

R: Please write a problem for the given expression.
V: We have two rulers. The length of the first one is 0.83 meter and the length of the second one is 0.32 meter then what is the ratio of the first ruler to the second one ?
R: But this does not sound like a real problem. You just asked the ratio.
V: yes ! but the numbers given make it impossible !
R: OK! How can you define multiplication ?
V: It is the easy way of adding. You multiply two numbers.
R: What about division ?
V: In fact multiplication makes it greater and division makes it smaller.

In the above interview the interviewee seemed to have some difficulties in using decimals in a problem writing situation but more than that we observed explicitly that she held the beliefs *"multiplication makes larger"* and *"division makes smaller"*. Her low performance on the items related to operations including decimals less than one also justifies our observation.

2nd Interview Excerpt for Expression 0.83 ÷ 0.32

R: Please write a problem for the given expression.
Oz: (thinking) ....When decimals enter, the work becomes harder.
R: We are faced with decimals in our daily life, don't we ?
Oz: Yes . For example in weight.
R: So ! what can you write ?

Oz: In a 0.83 kg food packet (oh! It is so complicated) , 0.32 of the food is used, what will be the rest of the food ? .....*oh! this is bad* !

Above interviewee noticed that a problem could be written for the given expression but he couldn't write a problem which was high in quality. Like the previous interviewee, this one also had a low performance on the items related to operations including decimals less than one. This might be the source of his problem writing performance.

4.1.3.3.2.3 Division Stories for the Expression "3.86 ÷ 23"

In writing multiplication stories for **3.86 ÷ 23** (*3 preservice elementary teachers were interviewed for this expression*) two of the interviewees couldn't write any problem and only one of them tried to do something. Two of the interview excerpts are given below:

1st Interview Excerpt for Expression 3.86 ÷ 23

R: Please write a problem for the given expression.
M: (thinking) .... How can we divide 23 cakes equally into 3.86 ?
R: Suppose that you try to solve your problem. Are you sure that the answer will be 3.86 ÷ 23 ?
M: (thinking) ... No ! it will be 23 ÷ 3.86.

In the above interview it seemed that the interviewee held the belief *"in a division operation dividend should be greater than the divisor"*. Her low performance in the Pre-Form of Problems test also justify this conclusion about the interviewee.

2nd Interview Excerpt for Expression 3.86 ÷ 23

R: Please write a problem for the given expression.
D: (thinking)....(it takes a long time)...
R: Do decimals make the work harder ?
D: yes !
R: What is the reason ?
D: Decimals are a little bit harder. In our daily life we usually get involved with whole numbers, decimals are rarely used and their use are limited to certain areas.

R: Think of your previous education.
D: Oh! Yes We were working heavily on whole numbers.

In the above interview the interviewee couldn't write any problem for the given expression. Although she paid attention to the decimal numbers her low performance on previously asked question about operations showed that she also was under the influence of the belief *"in a division operation dividend should be greater than the divisor"*

4.1.3.3.2.4 Division Stories for the Expression "9.6 ÷ 62.2"

In writing multiplication stories for **9.6 ÷ 62.2** (*4 preservice elementary teachers were interviewed for this expression*) none of the interviewees could write any problems but interestingly one of them stated that the given expression had no representation in real life. Three interview excerpts are given below:

1st Interview Excerpt for Expression 9.6 ÷ 62.2

R: Please write a problem for the given expression.
N: (thinking) … The divisor is greater I should think the reverse…(thinking) … A 62.2 lt bowl is full with milk. There are some 9.6 lt bottles. If we divide the water equally among the bottles, what will be each ones' share ?
R: Can't we find more examples for the given expression from real life ?
N: It is so hard to find.
R: In a division operation should the divisor be less then the dividend ?
N: Not always. But when the divisor is greater division becomes harder.

The interviewee in the above interview didn't say explicitly that in a division operation dividend should have been greater than the divisor but she stated that it was better for her. Although she reversed the given expression she wrote a problem that was meaningless.

2nd Interview Excerpt for Expression 9.6 ÷ 62.2

R: Please write a problem for the given expression.
O: (thinking) …. This can not be done the divisor is greater than the dividend.
R: think a little bit more about it.

O: What about the ratio of the two numbers ?
R: Try to write a real problem.
O: (thinking)
R: What would you say if it were 9 ÷ 60 ?
O: There are 9 balls to be shared among 60 students ....*if I replace 60 by 63* ...(thinking)

Above interviewee explicitly showed that he held the belief *"in a division operation dividend should be greater than the divisor"*. Although we replaced the decimals by integers he couldn't write any problem and this might also justify our first conclusion about the interviewee.

3rd Interview Excerpt for Expression 9.6 ÷ 62.2

R: Please write a problem for the given expression.
S: Decimals are difficult !
R: Is choosing appropriate operations for problems easier than this ?
S: yes
R: Have you ever done such things before ?
S: No! never.
R: Do you think that it is useful ?
S: Yes. We will be teachers in the future and it is very important for us.

In the above interview, although, the interviewee was a high scorer she couldn't write any problem for the given expression but at least we got information about the need for working on problem writing.

4.1.3.3.2.5 Division Stories for the Expression "0.53 ÷ 1.4"

In writing multiplication stories for **0.53 ÷ 1.4** , three preservice elementary teachers were interviewed. Two of the interviewees couldn't write any problem and one of the three interviewees tried to write a problem but it was quite weak. A typical interview excerpt is given below:

A Typical Interview Excerpt for Expression 0.54 ÷ 1.4

R: Please write a problem for the given expression.
M: (thinking)... Into how many small parts can a 1.4 gr water melon be divided , each being 0.53 gr ?

R: Is that OK ?
M: I'm not sure.
R: If the numbers in the given expression were integers, do you think that it would be easier to write a problem ?
M: Yes! Decimals make the work harder.

The problem written by the above interviewee was for 1.4 ÷ 0.53 instead of 0.53 ÷1.4. Previously on the items that were related with the division of a smaller by a greater number, the interviewees' performance was low, so it seemed that she held the belief *"in a division operation dividend should be greater than the divisor"*. Since a 1.4 gr water melon is a very small bit, we can say that the units used by the interviewee were not really meaningful and this might show that she had difficulties on decimals.

4.1.3.4 Discussion of the Results Concerning the Thinking Strategies or Embodiments used by Preservice Elementary Teachers When Writing Word Problems for Division Expressions Involving Decimals

As it happened with multiplication expressions, in writing problems for division expressions it is again observed that decimals make the work harder for the preservice elementary teachers. As it is seen in table 2.25 expressions are listed from the simple to more complex forms and the preservice elementary teachers' correct problem writing performance is changing from 93% (for the simplest expression) to 41% (for the most complex expression). When we go into more detail we can also see that use of *Not a Word Problem Embodiment* is changing from 7% (for the simplest expression) to 24% (for the most complex expression). Similarly the uses of *Inappropriate* and *No Answer* Embodiments are changing from 3% to 31% and from 0% to 24% respectively. Similar observations were done by Bell at al (1984) in exploring 12 and 13 year olds problem writing performance for division expressions.

Although it was not suitable, some of the preservice elementary teachers forced the given expressions incorrectly into the forms they are much more familiar with. For example, for expressions 0.86 ÷ 0.32 , 9.6 ÷ 62.2, and 0.53 ÷ 1.4 partition or fractional partition embodiment were not suitable but nearly 7% of the preservice elementary teachers tried to write partition and fractional partition problems which of course occurred in very weak contexts. Bell et al (1984) observed very similar situations for expressions 0.74 ÷ 0.21 and 8.7 ÷ 59.1. In the interviews 80% of the interviewees gave only partition interpretation of division, 10% stated that division was the inverse of multiplication and 10% of them couldn't give any reasonable explanation. This might be the reason of their inclination to use partitive embodiments. This observation is consistent with previous research results (Tirosh & Graeber, 1990 and 1991; Simon, 1993; Ball, 1990; Vest, 1978; Graeber & Tirosh, 1990). Many of the researchers stated that school students and preservice secondary or elementary school teachers are mainly familiar with partitive interpretation of division. in this present study, especially during the interviews, preservice elementary teachers' statements indicated that the constraints of the primitive partitive division model dominated their thinking even when they solved or wrote quotitive type problems.

Whenever the divisor in the given expression was larger than the dividend reversals occurred. Bell et al (1984) reported almost the same for expressions 8.7 ÷ 59.1 and 0.47 ÷ 1.3. Interestingly some of the interviewees didn't reverse the numbers in the expressions but wrote such problems in which again a greater number was divided by a smaller in term of units. A typical example is as follows:

R: Please write a problem for the 4 ÷ 24 .
O: We want to put some spots on a 4 meter long stick in which the spots are 24 cm away from each other. How many spots are needed ?

These observations may show us the influence of primitive models and specify that the preservice elementary teachers think that the divisor must be smaller than the dividend. For example one of the interviewees, in writing a problem for 9.6 ÷ 62.2, explicitly stated that it was not possible to write a problem whenever the divisor was greater (see 2nd Interview Excerpt for Expression 9.6 ÷ 62.2).

Most of the preservice elementary teachers in writing partitive type problems used simple sharing contexts considering their environment like students ,classrooms, and etc. In writing rate type problems we generally observed that the preservice elementary teachers tended to use simple ratio or proportion contexts. Speed or price contexts were used in a limited number.

Especially, during the interviews, it was possible to observe the existence influence of previously observed misconception of the preservice elementary teachers in writing problems. For example some of the preserviece elementary teachers in writing problems for 3.86 ÷ 23 and 9.6 ÷ 62.2 treated *decimal point as a separator* as follows:

We share a <u>3 meter and 86 cm</u> stick among 23 workers. How many small sticks can be formed to be shared ?

Total weight of the foods in a picnic basket is 9.6 kg . The foods are needed to be shared among some children whose total age is 62.2 (<u>62 years and 2 months</u>). Consider the ages of each child in terms of month and find each ones' share ?

In a typical interview, one of the interviewees explicitly stated that *multiplication made larger and division made smaller* (see 1st Interview Excerpt for Expression 0.83 ÷ 0.32).

The overall interview and test results showed that the preservice elementary teachers in this present study were not that good in applying decimals in problem writing situations, because some of the problems written by them occurred in very weak contexts which were not possible in real life. For example 28% of the problems written for division expressions, which were treated as correct, were in very weak contexts.

In summary, the most dominant interpretation of division of the preservice elementary teachers in this present study was the partitive model. The preservice elementary teachers in this present study were more successful in writing problems for expressions with dividends greater than the divisors than they were in writing problems for expressions with dividends less than the divisors. In addition to that decimals made the work a little bit harder for preservice elementary teachers. These findings are consistent with the findings of Graeber et al (1986). This present study showed that many of the preservice elementary teachers were both implicitly and explicitly influenced by the constraints of the primitive division models.

4.1.4 Other Observations Coming Through the Interviews (Stage-1)

4.1.4.1 Meaning of a Decimal and Decimal Notation

Most of the interviewees have some problems about the overall meaning of decimals. Typical responses to the question "How can you define decimal numbers ?" were as follows:

A: Number from 0 to 9.
Ü: Decimals have points.
S: Division of a smaller by a greater.
G: The remainder, after the division of a number by 10.
H: When you write 1.2, then you can draw an apple to the right and two more to the left.
M: Number from 1 to 10... (How can you read 3.8 ?)... 3 out of 8 (3/8).

4.1.4.2 Meaning of Multiplication and Division

The interviewees showed that some of the interviewees didn't understand multiplication and division conceptually. The interviewees mainly preferred repeated addition embodiment of multiplication and held the belief "multiplication makes bigger". Typical responses to the questions "How can you define multiplication ?" and "How can you define division ?" were as follows:

<u>Multiplication</u>

M: It is a kind of addition. We find the factor of a whole.
V: It is a kind of addition, in other words becoming larger.
Ş: In multiplication we get greater results.
Oz: Multiplication makes larger.
D: In multiplication numbers become greater.

<u>Division</u>

M: Division is the inverse of multiplication. It is sharing.
V: Division is becoming smaller.
Ü: Division shows a ratio.
S: We use this in partitioning a whole.
Ş: Dividing a greater by a smaller.
U: Dividing two number to each other.
Y: Partitioning a greater into smaller parts.
D: In division numbers get smaller.

4.1.4.3 The Most Difficult Part in the Tests

The interviewees responses to the question "What was the most difficult part or question for you in the test ?" were as follows (they are listed from the most difficult to the least, according to the frequencies):

1. Problem writing
2. Area models
3. Number lines
4. Questions which are not based on subunit 10

4.1.4.4 Decimal Involvement in the Questions

Most of the interviewees stated that decimal involvement in the given questions made the work harder for them. Typical responses were as follows:

V: I am better in whole numbers.
H: Decimals are very hard for me.
G: I'm not good at decimals.

4.1.5 Overall Discussion of the Results Related to Stage-1

In interpreting and applying decimals as points on number lines, most of the preservice elementary teachers rely heavily on the domain of denary system and whole numbers. Because of this reason they usually counted each subunit mark on a given number line as 1 tenth and for example interpreted 2.4 on a number line as *two and four subunits* when subunits were not based on ten (such as eight). The most difficult items in this part were items 4 and 12, which are both based on subunit 8. Although, we did not ask preservice elementary teachers, especially in the interviews to find the related point in an exact way, nearly all of them tried to apply certain procedures to find the value of each subunit mark. In all the procedures they tried to calculate each subunit by proportion and in the proportions they generally divided 1 unit by 10, which again shows their familiarity to the denary system. Lack of estimation in interpreting decimals on number lines, apparently led most of the preservice elementary teachers to use only procedural methods in plotting a point on a number line. When we again look at the educational system in Northern Cyprus, it is not possible to observe any use of estimation in interpreting decimals or in any other activity. In 1981 Bell reported the importance of estimation in educational activities. In interpreting decimals on number lines many of the preservice elementary teachers had some problems related to the calibration of number lines. For example they believed that the place of a point on a number line could change, when one increased the subunit marks. As in many countries, in Northern Cyprus, in elementary and middle school ages students are not working on locating number on the same number lines with different calibrations. The calibration problems may be due to this fact. Some of the interviewees, on the other hand, totally ignored the given calibration and used only the starting and the first subunit mark in locating the decimals. Some of the interviewees seemed to have a net of misconceptions and in some cases they referred to two or three of their misconceptions in performing certain tasks. For example, two of the interviewees neglected the zeros in 0.5 and 0.6 in order to treat the decimals as whole numbers and considered each subunit mark as 1 unit on the given number lines. This shows again their calibration problems and whole number familiarity. Although we were observing the problems related to decimals, we saw that some the preservice elementary teachers' understanding of fractions was also weak. One of the interviewees interpreted 2.4 as $\frac{4}{2}$. Some of the misconceptions for this reason seemed to grow stronger. The most frequently observed misconceptions in this part were "on a number line between two numbers there are 10 divisions each being 0.1 unit" and "any decimal on a number line can be marked between the starting point and the first subunit mark."

In interpreting and applying decimals as points on shaded areas, generally preservice elementary teachers treated separate two-unit models as single one unit models and counted the number of part in all for the denominator and counted the number of equivalent shaded areas for the numerator. This was another version of the calibration problem applied to area models. This may be due to the overuse of proper fractions in elementary and middle school ages. In this part again the subunit based on ten approach was in charge. For example, although, some of the interviewees could read 1.08 correctly, in shading this portion on a given two-unit model, five of them ignored zero and treated 1.08 as 1.8 to make it suitable for a subunit based on ten system. Preservice elementary teachers also showed "decimals are based on ten" misconception when they used direct proportions. For example in finding the decimal representation of 5 subunit areas on an area model consisting of 8 subunit areas, most of the preservice elementary teachers formed to equations 5 over 8 equal to x over 10 and found x equal 6.25. In this part we again observed the influence of weak fraction knowledge intersecting with the weak calibration understanding as follows:

An interviewee wrote 0.6 as 6/10 = 3/5 and then said that she should have taken 3 of 5 and the remaining 3 subunit areas were additional, there was no need for them.

In interpreting decimals on shaded areas the overall performance of the preservice elementary teachers were lower when compared with the number lines. Giving more emphasis to (in Northern Cyprus) number lines than area models may be a reason for that. The area model often used in order to show proper fractions. On the other hand the number lines are used in the understanding of fractions, decimals, and basic operations. The most frequently observed misconceptions in this part were "decimals are based on subunit ten" , "multiple one unit area models can be treated as a single one unit model", "a unit area model can be treated as multiple one unit models", and "zero in a decimal number is not a place holder."

In comparing decimals, preservice elementary teachers generally treated decimals as whole numbers. For example five of the interviewees in comparing 3.521 , 3.6, and 3.75 had chosen 3.521 as the biggest. They defended themselves by stating that a decimal was greater when the whole number after the decimal point was greater. Although they were saying "*the whole number after the decimal*" it seemed that they were also ignoring the decimal point and , for example, considering 3.521 as 3521. Similar findings observed by many researchers (Sackur-Grisvard and Leonard, 1985; Carpenter et al., 1981; İşeri, 1997). Another faulty procedure, which was called as "*fraction rule*" by Sackur-Grisvard and Leonard (1985) was used by many of the preservice elementary teachers. In comparing 4.521, 4.6, and 4.75, three of the interviewees chose 4.6 as the biggest. As Resnick et al (1989) stated generally the interviewees thought that thousandths were smaller parts than hundredths , then they inferred that longer decimals must have lower values. Another version of this rule, which we haven't met before, observed in this present study as follows:

Two of the interviewees stated that when the number of steps needed to round a decimal to a whole number was limited the number became larger.

In interpreting the denseness of decimals again the preservice elementary teachers rely heavily on the domain of whole numbers and most probably for this reason they stated that there was no number between two consecutive decimals. It seemed that the continuous aspect of decimals was difficult for the preservice elementary teachers. As we mentioned before preservice elementary teachers were referring to a web of misconceptions in dealing with decimals. They seemed to use the misconception "decimals are based on subunit ten", because their answers to the question "How many numbers are there in between 0.41 and 0.42 ?" were "9" or "10". Three of the interviewee in locating a number which is between 1.4 and 1.5, gave 1.4.4 as the answer, in which there was two decimal points. This shows that some of the preservice elementary teachers were really weak in decimals.

In dealing with unit conversions involving decimals, 60% of the preservice elementary teachers seemed to hold the same misconceptions. Fifteen of the interviewees treated decimal point as a separator, especially, in converting hours into minutes and years into months. For example 35 % of the preservice elementary teachers in converting 1.15 hours into minutes, interpreted 1.15 hours as 1 hour (60 minutes) + 15 minutes = 75 minutes. Some of the preservice elementary teachers in this present study also believed that the number after the decimal might represent different units according to the subunit system of a given measure. For example some of the interviewees gave 2.4 as a decimal representation for the questions "write 2 years and 4 months in terms of a decimal number" and "write 2 hours and 4 minutes in terms of a decimal number". Although, most of the preservice elementary teachers were more successful on the unit conversions with the base 10 numeration system (length, weight), more over one of the interviewees in converting 3.82 km into metres, interpreted each number in the places as the subunits of the previous ones and read 3.82 as 3 kilometres, 8 metres, and 2 cm.

As we mentioned in the previous dimensions, in unit conversion some of the preservice elementary teachers seemed to have many problems in relating decimals and fractions. For example in converting 2.4 years into months, 7% of the interviewees considered the number before the decimal point as 2 years and then treated the number 4, after the decimal point, as it was 4/12 of a year (1/3) which later confused them to conclude that "number 4 after the decimal point stands for 3 months" and finally they gave 27 months as the answer.

In summary, some of the preservice elementary teachers could not convert from one system to another because of familiarity with working with base 10 numeration system. Therefore, work done with the operations in the base 10 numeration system may cause problems when students learn to deal with units of 60 and 12 such as in minutes or hours and months.

In interpreting multiplication and division operations, although, the preservice elementary teachers were slightly better on multiplication operations, we observed that the main problem occurred when the decimals involved were less than 1, in both of the operations. Extensive work on the whole numbers, seemed to lead most of the preservice elementary teachers to conclude that "multiplication makes bigger" and "division makes smaller." In this present study approximately 72% of the preservice elementary teachers held those beliefs either explicitly or implicitly. Moreover, the influence of whole number domain caused some preservice elementary teachers to move the decimal points and treat the decimals as they treat whole numbers. Six (24%) of the interviewees moved the decimal points in dealing with multiplication and division operations. On the other hand 36% percent of the interviewees totally ignored the decimal points. This probably, shows that the some of the preservice elementary teachers can not extent their understanding of whole number operations to fractions and decimals and they insist on treating decimals as whole numbers.

In the choice of operation for word problems involving decimals some of the preservice elementary teachers seemed to have many problems. In choosing operations for multiplication word problems, preservice elementary teachers were better on the problems that were suitable for a repeated addition model than cartesian product type problems. This may be due to preservice elementary teachers' extensive experiences on whole numbers. The preservice elementary teachers also seemed to be influenced by the primitive implicit models which imposes the constraint that the multiplier must be an integer. They were better in the multiplication problems that confirmed to the primitive models. Twenty percent of the interviewees seemed to hold the belief " in a multiplication expression the operator should be a whole number." In line with that decimals less than 1 seemed to be another important factor which affects that overall performance of the preservice elementary teachers. Some of the preservice elementary teachers tended to use division instead of multiplication, which may be due to an intuitive awareness that the answer had to be smaller than the first number, combined with the misconception "to make a number smaller you must divide."

Briefly, the preservice elementary teachers seemed to be influenced by the primitive multiplication model and held the belief "in a multiplication expression the operator should be a whole number."

In choosing operations for division word problems, preservice elementary teachers were better on partitive type problems than quotitive problem. Throughout the interviews we observed that 70% of the preservice elementary teachers gave only partitive interpretation of division and this might be a reason for their better performance on partitive division word problems. On the other hand the preservice teachers in this present study were more successful on the problems involving divisors less than the dividend than on problems involving divisor greater than the dividend. For example in problems 12 and 24 (in which divisors were greater than the dividends) , 40% of the preservice elementary teachers reversed the roles of roles of the divisor and the dividend. They seemed to hold the belief "the divisor must be smaller than the dividend.". The preservice elementary teachers also had some problems in the choice of operation for division word problems whenever the divisor was less than 1.

As it was observed in multiplication word problems, in choosing operations for division word problems, we observed that some of the preservice elementary teachers were distracted by some cue words like "*enlarge*" in multiplication word problems and "*how much of*" in division word problems. For example in problem 21 the word "*enlarge*" led some (5%) of the preservice elementary teachers to choose addition instead of multiplication and in problems 16 and 25 the phrase "*how much of*" led some (approximately 14%) of them to choose multiplication instead of division. It seems that these choices are mainly influenced by the belief "multiplication makes larger."

In choosing operations for division and multiplication word problems some of the preservice elementary teachers used direct proportion in order to find the appropriate operations.

During some of the interviews whenever the interviewee faced a difficulty, we replaced the decimals by whole numbers. Some of them were better after the replacements but some were still failing, probably, due to a lack of conceptual understanding of decimals.

In writing word problems for division and multiplication expressions involving decimals, it was possible to observe the influence of all misconceptions that were previously observed. In writing problems for multiplication expressions, repeated addition was the preferred embodiment when one of the numbers involved was a whole number. Although it was not possible, four of the preservice elementary teachers tried to write problems in repeated addition embodiment for the expression 12.05 x 0.93. When one or both of the numbers involved in a multiplication expression were decimals *not a word problem embodiment* and *inappropriate embodiments* were both used to a great extent. Area and money contexts were used frequently when it was feasible. Most of the problems written were similar to each other. A similar observation was done by Silver et al (1996). They conducted a study on a group of (N=81) middle school and prospective secondary school teachers' mathematical problem posing performance and they stated that a sizeable portion of the posed problems were produced in clusters of related problems. It seems that some extra difficulties occurred for preservice elementary teachers when the numbers involved were less than 1. Previously many other researchers reported the influence of decimals less than 1 (Taylor, 1981; Bell et al, 1984; İşeri, 1997). Whenever both of the numbers were decimals a great number of preservice elementary teachers couldn't write any word problem for the given multiplicative expressions. In such conditions some of the preservice elementary teachers naturally wrote word problems with a Cartesian product embodiment, but this was not that much . Approximately 12% of the problems written, which were treated as correct, were in very weak contexts. Therefore we can conclude that the preservice elementary teachers in this present study are not very good in writing multiplication problems for expressions involving decimals.

The over all test and interview results showed that some of the preservice elementary teachers in this present study still held the belief "in a multiplication expression the operator should be a whole number." In addition to this, the problems written by the preservice elementary teachers declare that they had never had such problem writing experiences. In the interviews, all the interviewees stated that they had never tried to write problems for given expressions or simply posed any kind of problems. This of course affected their overall problem writing performance. The most dominant interpretation of division of the preservice elementary teachers in this present was the partitive model. The preservice elementary teachers in this present study were more successful in writing problems for expressions with dividends greater than the divisors than they were in writing problems for expressions with dividends less than the divisors. In addition to that, decimals made the work a little bit harder for preservice elementary teachers. These findings are consistent with the findings of Graeber et al (1986). This present study showed that many of the preservice elementary teachers were both implicitly and explicitly influenced by the constraints of the primitive division models.

Overall distribution of the interviewees holding the observed misconceptions are monitored in Appendix-K.

4.2 Results and Discussion of Stage - 2 (Overcoming Misconceptions)

In this part, we present the results of stage-2 of the study related to the findings about the stated hypothesis. The level of significance for stage-2 of the study was set to be 0.05. The results in this part are also enriched by some observations coming through the interviews.

Although the preservice elementary teachers have been assigned randomly to experimental and control groups, before the beginning of the treatment the CT, PT, and WWPT tests were applied to see the prerequisite knowledge of preservice elementary teachers on the related areas and check if there was any difference between the two groups. Results showed that there was no significant difference between the means of the two groups at any of the three tests (see Appendix -J).

4.2.1 Results Obtained from the Pre, Post, and Delayed - Testing of Concept Test:

In hypothesis-1 of the stage-2 of the study, it was stated that there was no significant difference between mean post CT scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post - Form of Concept Test was administered to the two groups. In hypothesis-2 of the stage-2 of the study, it was stated that there was no significant difference between delayed CT scores of subjects in the experimental and control groups. After the administration of Post -Form of Concept Test (after four months), Post -Form of Concept Test was administered as Delayed Concept Test to the two groups. In this part there were 44 items to be answered by the EG and CG subjects. The following two tables were obtained in the repeated measures analysis of variance on achievement related to decimal concepts.

Table 4.27: Tests of Significance for Group and Achievement Related to Decimal Concepts (ARDC) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	1404.81	94	14.94	-	-
ARDC	1598.52	2	799.26	53.48	0.000
Grp. By ARDC	978.23	2	489.11	32.73	0.000

Table 4.28: Tests of Significance for Group Main Effect on Achievement Related to Decimal Concepts (ARDC), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	8771.99	47	186.64	-	-
Group	4009.39	1	4009.39	21.48	0.000

As it is seen in tables 4.27 and 4.28, main effect for group and main effect for ARDC is significant. Also there is a significant interaction between ARDC and group.

In order to understand the significant differences observed as a result of averaged tests of significance for achievement related to decimals concepts (ARDC), mean scores of experimental and control groups are indicated in the graph below.

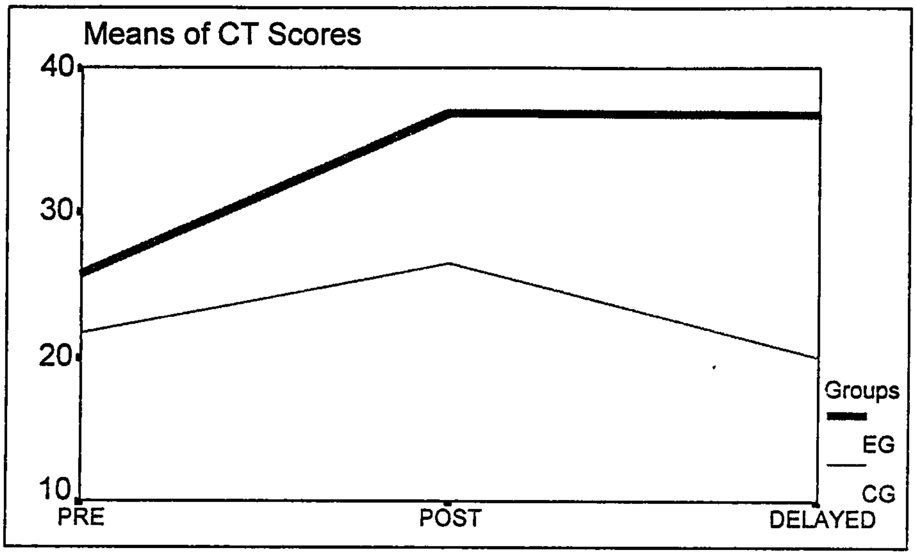


Figure 4.15: Pre-, Post-, and Delayed - CT Mean Scores for Experimental and Control Groups.

As noticed in figure 4.15, the significant difference between the two measures of ARDC might be due to the gradual increase in the experimental group, and the gradual decrease in the control group. Although the increase from post testing to delayed post testing of the experimental group is not that much, individual univariate estimates shows that it is still higher than the control group. In ARDC two of the groups are significantly different from each other. In the light of these findings there is sufficient evidence to reject hypothesis-1 and hypothesis-2. Thus it can be concluded that there are significant differences between mean post CT and mean delayed CT scores of subjects in the EG and CG. Means and standard deviations of experimental and control groups in the three measures of concepts test are given in the following table.

Table 4.29: Means and Standard Deviations of Experimental and Control Groups in the Three Measures of Concepts Test.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	25.79	8.80	36.96	5.97	36.79	7.46
CG	21.64	9.22	26.56	9.86	20.00	8.95

4.2.1.1 Results Obtained from a Sub-dimension of Concept Test (CT1.1) in which Decimals are Treated as Points on Number Lines with Subunit Based On Ten:

In hypothesis-7 of the stage-2 of the study, it was stated that there was no significant difference between mean post CT1.1 scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post - Form of Concept Test was administered to the two groups. In hypothesis-2 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed CT1.1 scores of the subjects in the experimental and control groups. After the administration of Post -Form of Concept Test (after four months), Post -Form of Concept Test was administered as Delayed Concept Test to the two groups. In this part there were 9 items (items 1, 2, 3, 5, 9, 14, 15, 16, 17 from post and delayed-concept tests) to be answered by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to decimals as points on number lines with subunit based on ten.

Table 4.30: Tests of Significance for Group and Achievement Related to Decimals as Points on Number Lines with Subunit Based on Ten (ACT1.1) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	134.25	94	1.43	-	-
ACT1.1	36.16	2	18.08	12.66	0.000
Grp. By ACT1.1	28.81	2	14.41	10.09	0.000

Table 4.31: Tests of Significance for Group Main Effect on Achievement Related to Decimals as Points on Number Lines with Subunit Based on Ten (ACT1.1), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	591.16	47	12.58	-	-
Group	161.74	1	161.74	12.86	0.001

As it is seen in tables 4.30 and 4.31, main effect for group and main effect for ACT1.1 is significant. Also there is a significant interaction between ACT1.1 and group.

In order to understand the significant differences observed as a result of averaged tests of significance for achievement related to decimals as points on number lines with subunit based on ten (ACT1.1), mean scores of experimental and control groups are indicated in the graph below.

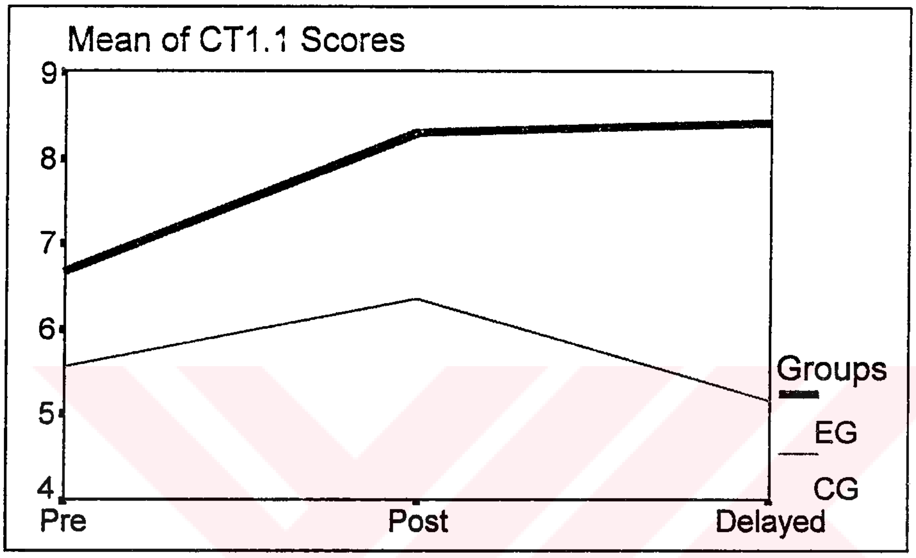


Figure 4.16: Pre-, Post-, and Delayed - CT1.1 Mean Scores for Experimental and Control Groups.

As noticed in the graph, the significant difference among the two measures of ACT1.1 might be due to the gradual increase in the experimental group, and the gradual decrease in the control group, especially from the post to the delayed testing stages. In achievement related to decimals as points on number lines with subunit based on ten, the groups (EG and CG) are significantly different from each other, especially at the final stage. The experimental groups' mean score on CT1.1 is higher than the control groups' mean score on CT1.1. In the light of these findings there is sufficient evidence to reject hypothesis-7 and hypothesis-8. Thus it can be concluded that there are significant differences between mean post CT1.1 and mean delayed CT1.1 scores of subjects in the

EG and CG. Means and standard deviations of experimental and control groups in the three measures of CT1.1 are given in the following table.

Table 4.32: Means and standard deviations of experimental and control groups in the three measures of CT1.1

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	6.67	2.44	8.29	1.12	8.42	0.88
CG	5.56	2.84	6.36	2.71	5.16	2.67

4.2.1.2 Results Obtained from a Sub-dimension of Concept Test (CT1.2) in which Decimals are Treated as Points on Number Lines with Subunit Not Based On Ten:

In hypothesis-9 of the stage-2 of the study, it was stated that there was no significant difference between mean post CT1.2 scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post - Form of Concept Test was administered to the two groups. In hypothesis-10 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed CT1.2 scores of the subjects in the experimental and control groups. After the administration of Post -Form of Concept Test (after four months), Post -Form of Concept Test was administered as Delayed Concept Test to the two groups. In this part there were 9 items (items 4, 6, 7, 8, 10, 11, 12, 13, 18 from post and delayed-concept tests) to be answered by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to decimals as points on number lines with subunit not based on ten.

Table 4.33: Tests of Significance for Group and Achievement Related to Decimals as Points on Number Lines with Subunit Not Based on Ten (ACT1.2) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	147.52	94	1.57	-	-
ACT1.2	124.23	2	62.11	39.58	0.000
Grp. By ACT1.2	66.48	2	33.24	21.18	0.000

Table 4.34: Tests of Significance for Group Main Effect on Achievement Related to Decimals as Points on Number Lines with Subunit Not Based on Ten (ACT1.2), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	701.44	47	14.92	-	-
Group	269.0.1	1	269.01	18.03	0.000

As it is seen in tables 4.33 and 4.34, main effect for group and main effect for ACT1.2 is significant. Also there is a significant interaction between ACT1.2 and group.

In order to understand the significant differences observed as a result of averaged tests of significance for achievement related to decimals as points on number lines with subunit not based on ten (ACT1.2), mean scores of experimental and control groups are indicated in the graph below.

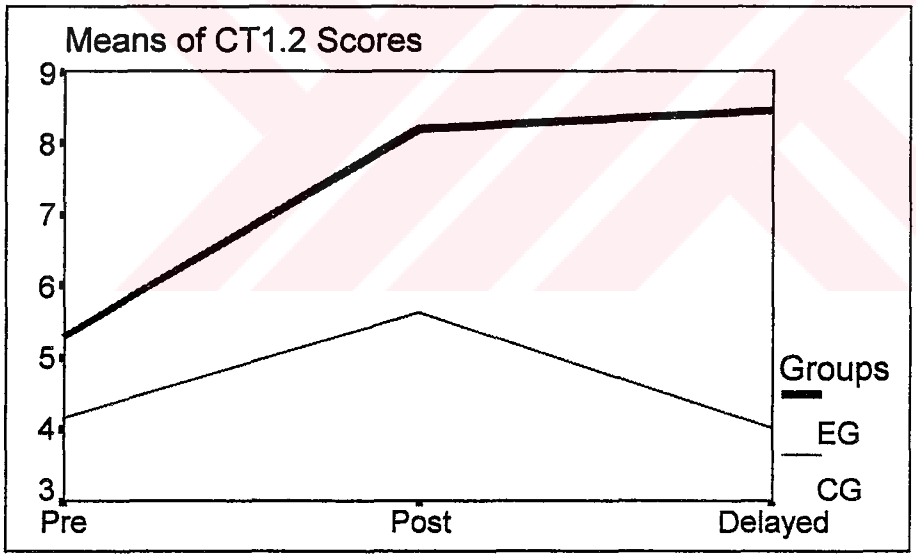


Figure 4.17: Pre-, Post-, and Delayed - CT1.2 Mean Scores for Experimental and Control Groups.

As the graph reveals, the significance of the differences between the groups are evidenced by the gradual increase of the experimental group and the gradual decrease of the control group especially from the post to the delayed post testing stages. Individual univariate estimates and the graph itself shows that although there is an increase in the mean score of control group from pre to post testing, the mean score of the experimental group is still higher. In the light of these findings there is sufficient evidence to reject hypothesis-9 and hypothesis-10. Thus it can be concluded that there are significant differences between mean post CT1.2 and mean delayed CT1.2 scores of subjects in the EG and CG. Means and standard deviations of experimental and control groups in the three measures of CT1.2 are given in the following table.

Table 4.35: Means and standard deviations of experimental and control groups in the three measures of CT1.2.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	5.29	2.71	8.21	1.67	8.46	0.78
CG	4.16	2.97	5.61	2.81	4.04	2.89

4.2.1.3 Results Obtained from a Sub-dimension of Concepts Test (CT2.1) in which Decimals are Treated as Points on Shaded Areas with Subunit Based On Ten:

In hypothesis-11 of the stage-2 of the study, it was stated that there was no significant difference between mean post CT2.1 scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post -Form of Concept Test was administered to the two groups. In hypothesis-12 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed CT2.1 scores of the subjects in the experimental and control groups. After the administration of Post -Form of Concept Test (after four months), Post -Form of Concept Test was administered as Delayed Concept Test to the two groups. In this part there were 3 items (items 20, 23, 25 from post and delayed concept tests) to be answered by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to decimals as points on shaded areas with subunit based on ten.

Table 4.36: Tests of Significance for Group and Achievement Related to Decimals as Points on Shaded Areas With Subunit Based on Ten (ACT2.1) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	43.57	94	0.46	-	-
ACT2.1	9.89	2	4.94	10.67	0.000
Grp. By ACT2.1	6.62	2	3.31	7.14	0.001

Table 4.37: Tests of Significance for Group Main Effect on Achievement Related to Decimals as Points on Shaded Areas with Subunit Based on Ten (ACT2.1), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	95.20	47	2.03	-	-
Group	32.65	1	32.65	16.12	0.000

As it is seen in tables 4.36 and 4.37, main effect for group and main effect for ACT2.1 is significant. Also there is a significant interaction between ACT2.1 and group.

In order to understand the significant differences observed as a result of averaged tests of significance for achievement related to decimals as points on shaded areas with subunit based on ten (ACT2.1), mean scores of experimental and control groups are indicated in the graph below.

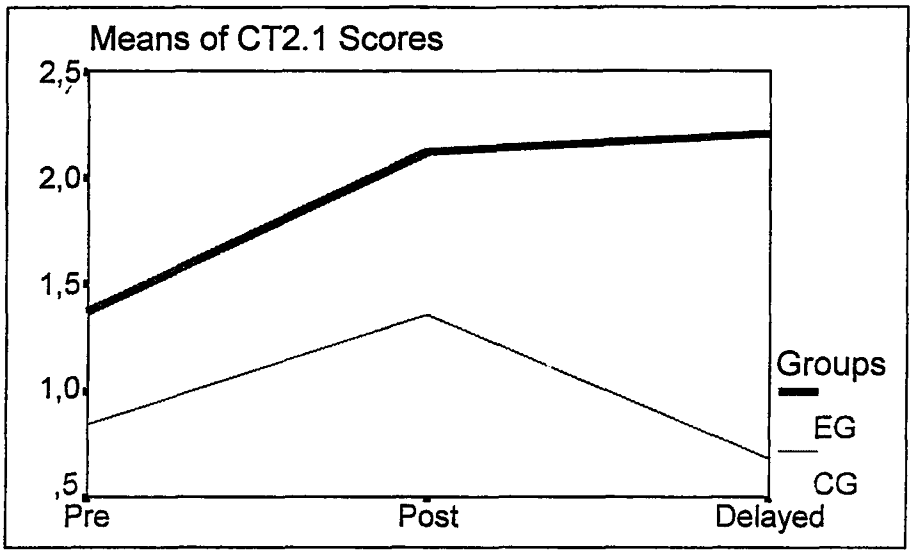


Figure 4.18: Pre-, Post-, and Delayed - CT2.1 Mean Scores for Experimental and Control Groups.

As noticed in the graph, the significant difference between the two measures of achievement related to decimals as points on shaded areas with subunit based on ten might be due to the gradual increase in the experimental and the gradual decrease in the control group. As it is seen in the graph the main difference is due to the sudden decrease in the control group from post to the delayed testing. The experimental group seems to remain nearly the same from post to delayed testing but experimental groups' mean score is still higher than the control groups' mean score. In the light of these findings there is sufficient evidence to reject hypothesis-11 and hypothesis-12. Thus it can be concluded that there are significant differences between mean post CT2.1 and mean delayed CT2.1 scores of subjects in the EG and CG. Means and standard deviations of experimental and control groups in the three measures of CT2.1 are given in the following table.

Table 4.38: Means and standard deviations of experimental and control groups in the three measures of CT2.1.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	1.38	1.14	2.13	0.85	2.21	0.88
CG	0.84	1.03	1.36	1.00	0.68	1.03

4.2.1.4 Results Obtained from a Sub-dimension of Concept Test (CT2.2) in which Decimals are Treated as Points on Shaded Areas with Subunit Not Based On Ten:

In hypothesis-13 of the stage-2 of the study, it was stated that there was no significant difference between mean post CT2.2 scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post -Form of Concept Test was administered to the two groups. In hypothesis-14 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed CT2.2 scores of the subjects in the experimental and control groups. After the administration of Post -Form of Concept Test (after four months), Post -Form of Concept Test was administered as Delayed Concept Test to the two groups. In this part there were 4 items (items 19, 21, 22, 24 from post and delayed concept tests) to be answered by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to decimals as points on shaded areas with subunit not based on ten.

Table 4.39: Tests of Significance for Group and Achievement Related to Decimals as Points on Shaded Areas With Subunit Not Based on Ten (ACT2.2) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	71.15	94	0.76	-	-
ACT2.2	29.98	2	14.99	19.81	0.000
Grp. By ACT2.2	19.45	2	9.73	12.85	0.000

Table 4.40: Tests of Significance for Group Main Effect on Achievement Related to Decimals as Points on Shaded Areas with Subunit Not Based on Ten (ACT2.2), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	161.78	47	3.44	-	-
Group	67.50	1	67.50	19.61	0.000

As it is seen in tables 4.39 and 4.40, main effect for group and main effect for ACT2.2 is significant. Also there is a significant interaction between ACT2.2 and group.

In order to understand the significant differences observed as a result of averaged tests of significance for achievement related to decimals as points on shaded areas with subunit not based on ten (ACT2.2), mean scores of experimental and control groups are indicated in the graph below.

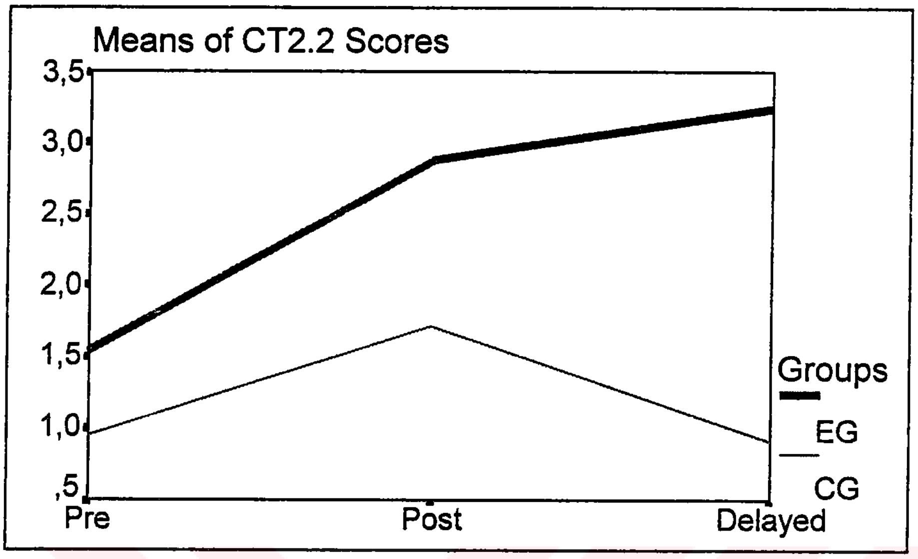


Figure 4.19: Pre-, Post-, and Delayed - CT2.2 Mean Scores for Experimental and Control Groups.

As noticed in the graph, the significant difference between the two measures of ACT2.2 might be due to the gradual increases in the experimental group at post and delayed post testing stages and the gradual decrease in the control group especially from post to delayed post testing. In the light of these findings there is sufficient evidence to reject hypothesis-13 and hypothesis-14. Thus it can be concluded that there are significant differences between mean post CT2.2 and mean delayed CT2.2 scores of subjects in the EG and CG. Means and standard deviations of experimental and control groups in the three measures of CT2.2 are given in the following table.

Table 4.41: Means and standard deviations of experimental and control groups in the three measures of CT2.2.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	1.54	1.44	2.88	1.19	3.25	1.15
CG	0.96	1.24	1.72	1.45	0.92	1.18

4.2.1.5 Results Obtained from a Sub-dimension of Concepts Test (CT3.1) Related to Decimals Involving Unit Measures Subunit Based on Ten:

In hypothesis-15 of the stage-2 of the study, it was stated that there was no significant difference between mean post CT3.1 scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post -Form of Concept Test was administered to the two groups. In hypothesis-16 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed CT3.1 scores of the subjects in the experimental and control groups. After the administration of Post -Form of Concept Test (after four months), Post -Form of Concept Test was administered as Delayed Concept Test to the two groups. In this part there were 3 items (items 34, 36, 37 from post and delayed concept tests) to be answered by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to decimals involving unit measures subunit based on ten (ACT3.1).

Table 4.42: Tests of Significance for Group and Achievement Related to Decimals Involving Unit Measures Subunit Based on Ten(ACT3.1) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	22.64	94	0.24	-	-
ACT3.1	2.40	2	1.20	4.97	0.009
Grp. By ACT3.1	2.40	2	1.20	4.97	0.009

Table 4.43: Tests of Significance for Group Main Effect on Achievement Related to decimals Involving Unit Measures Subunit Based on Ten (ACT3.1), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	194.37	47	4.14	-	-
Group	7.98	1	7.98	1.93	0.171

As it is seen in table 4.42 the significant differences coming from the mean differences between the two measures and the interaction terms. As indicated in table 4.43, the main effect for group is not significant, as well as its interaction with ACT3.1.

In order to understand the significant differences observed as a result of averaged tests of significance for achievement related to decimals involving unit measures subunit based on ten (ACT3.1), mean scores of experimental and control groups are indicated in the graph below.

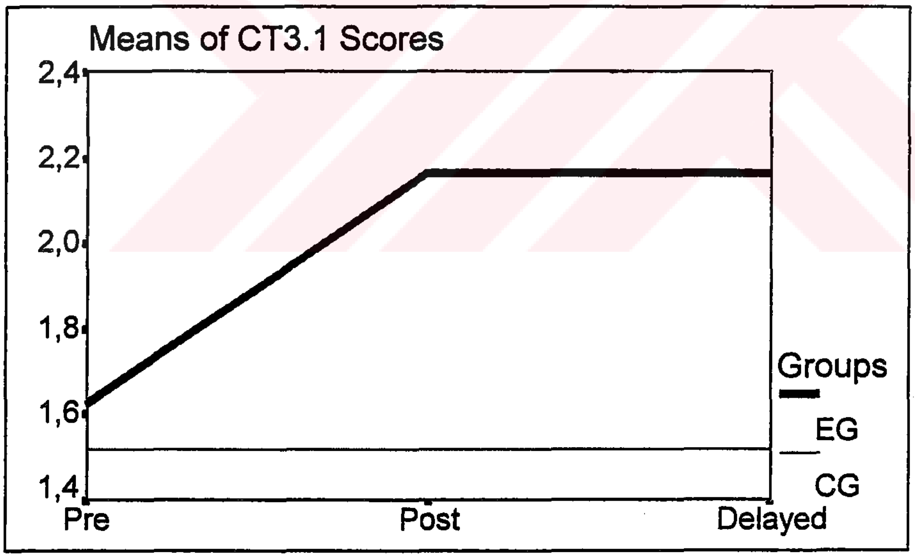


Figure 4.20: Pre-, Post-, and Delayed - CT3.1 Mean Scores for Experimental and Control Groups.

As can be noticed from the graph, the significant difference between the two measures of ARCT3.1 might be due to the gradual increase in the mean score of the experimental group at the post testing stage. As it is seen in the graph control group remains constant through all three measurements. In the light of these findings there is sufficient evidence to reject hypothesis-15 and hypothesis-16. Thus it can be concluded that there are significant differences between mean post CT3.1 and mean delayed CT3.1 scores of subjects in the EG and CG. Means and standard deviations of experimental and control groups in the three measures of CT3.1 are given in the following table.

Table 4.44: Means and standard deviations of experimental and control groups in the three measures of CT3.1.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	1.63	1.21	2.17	1.27	2.17	1.27
CG	1.52	1.23	1.52	1.24	1.52	1.22

4.2.1.6 Results Obtained from a Sub-dimension of Concepts Test (CT3.2) Related to Decimals Involving Unit Measures Subunit Not Based on Ten :

In hypothesis-17 of the stage-2 of the study, it was stated that there was no significant difference between mean post CT3.2 scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post - Form of Concept Test was administered to the two groups. In hypothesis-18 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed CT3.2 scores of the subjects in the experimental and control groups. After the administration of Post -Form of Concept Test (after four months), Post -Form of Concept Test was administered as Delayed Concept Test to the two groups. In this part there were 3 items (items 33, 35, 38 from post and delayed concept tests) to be answered by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to decimals involving unit measures subunit not based on ten (ACT3.2).

Table 4.45: Tests of Significance for Group and Achievement Related to decimals Involving Unit Measures Subunit Not Based on Ten (ACT3.2) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	56.61	94	0.60	-	-
ACT3.2	12.79	2	6.40	10.62	0.000
Grp. By ACT3.2	4.19	2	2.10	3.48	0.035

Table 4.46: Tests of Significance for Group Main Effect on Achievement Related to decimals Involving Unit Measures Subunit Not Based on Ten (ACT3.2), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	140.39	47	2.99	-	-
Group	34.64	1	34.64	11.60	0.001

As it is seen in tables 4.45 and 4.46, main effect for group and main effect for ACT3.2 is significant. Also there is a significant interaction between ACT3.2 and group.

In order to understand the significant differences observed as a result of averaged tests of significance for achievement related to decimals involving unit measures subunit not based on ten (ACT3.2), mean scores of experimental and control groups are indicated in the graph below.

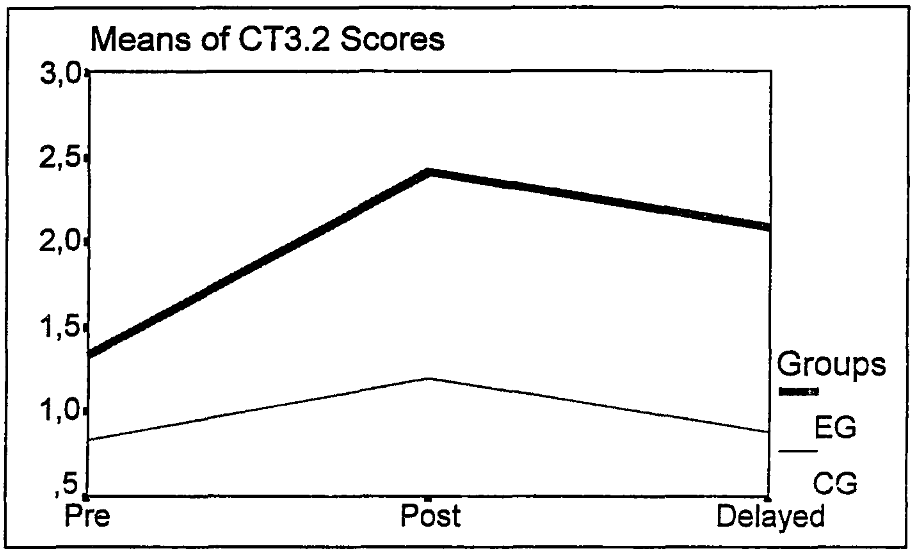


Figure 4.21: Pre-, Post-, and Delayed - CT3.2 Mean Scores for Experimental and Control Groups.

As it is seen in the graph, the significant differences between the groups is evidenced by higher mean scores of experimental group than control groups' mean scores at post testing and delayed post testing stages. Although there is a decrease in the experimental groups' mean score in the delayed post test, it is still better than control groups' mean score. The control group returned to its starting position. In the light of these findings there is sufficient evidence to reject hypothesis-17 and hypothesis-18. Thus it can be concluded that there are significant differences between mean post CT3.2 and mean delayed CT3.2 scores of subjects in the EG and CG. Means and standard deviations of experimental and control groups in the three measures of CT3.2 are given in the following table.

Table 4.47: Means and standard deviations of experimental and control groups in the three measures of CT3.2.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	1.34	1.24	2.42	1.14	2.08	1.18
CG	0.84	1.14	1.20	1.26	0.88	1.13

4.2.1.7 Results Obtained from a Sub-dimension of Concepts Test (CT4) Related to Denseness of Decimals:

In hypothesis-19 of the stage-2 of the study, it was stated that there was no significant difference between mean post CT4 scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post - Form of Concept Test was administered to the two groups. In hypothesis-20 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed CT4 scores of the subjects in the experimental and control groups. After the administration of Post -Form of Concept Test (after four months), Post -Form of Concept Test was administered as Delayed Concept Test to the two groups. In this part there were 2 items (items 31 and 32 from post and delayed concept tests) to be answered by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to denseness of decimals (ACT4).

Table 4.48: Tests of Significance for Group and Achievement Related to Denseness of Decimals (ACT4) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	13.02	94	0.14	-	-
ACT4	5.44	2	2.72	19.63	0.000
Grp. By ACT4	4.38	2	2.19	15.80	0.000

Table 4.49: Tests of Significance for Group Main Effect on Achievement Related to Denseness of Decimals (ACT4), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	26.29	47	0.56	-	-
Group	10.51	1	10.51	18.80	0.000

As it is seen in tables 4.48 and 4.49, main effect for group and main effect for ACT4 is significant. Also there is a significant interaction between ACT4 and group.

In order to understand the significant differences observed as a result of averaged tests of significance for achievement related to denseness of decimals (ACT4), mean scores of experimental and control groups are indicated in the graph below.

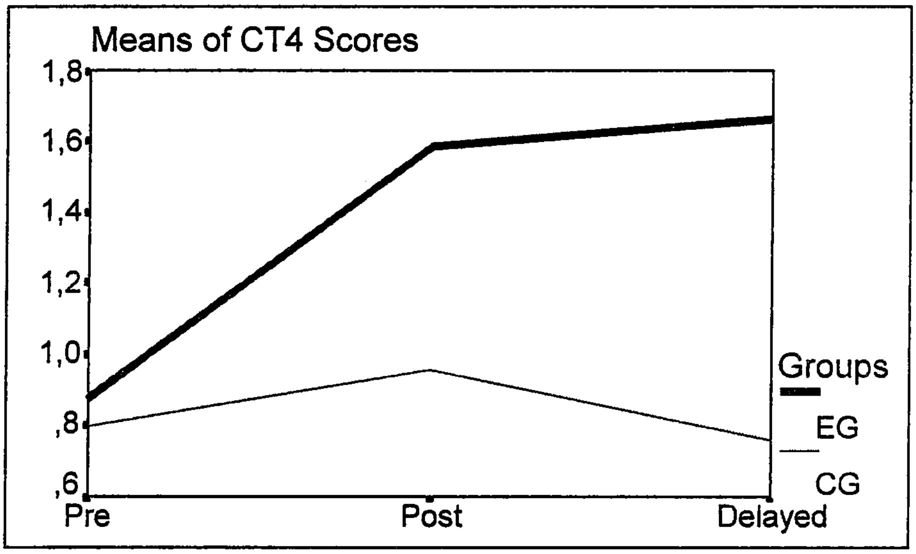


Figure 4.22: Pre-, Post-, and Delayed - CT4 Mean Scores for Experimental and Control Groups.

As noticed in the graph, the significant difference between the two measures of ATC4 might be due to the gradual increases of experimental group at post and delayed post testing stages. Although there is a slight increase for the control group at post testing stage, after that we observe a slight decrease which is lower than its' starting position. In the light of these findings there is sufficient evidence to reject hypothesis-19 and hypothesis-20. Thus it can be concluded that there are significant differences between mean post CT4 and mean delayed CT4 scores of subjects in the EG and CG. Means and standard deviations of experimental and control groups in the three measures of CT4 are given in the following table.

Table 4.50: Means and standard deviations of experimental and control groups in the three measures of CT4.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	0.88	0.34	1.58	0.65	1.67	0.48
CG	0.80	0.58	0.96	0.54	0.76	0.52

4.2.1.8 Results Obtained from a Sub-dimension of Concepts Test (CT5) Related to Comparison of Decimals:

In hypothesis-21 of the stage-2 of the study, it was stated that there was no significant difference between mean post CT5 scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post -Form of Concept Test was administered to the two groups. In hypothesis-22 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed CT5 scores of the subjects in the experimental and control groups. After the administration of Post -Form of Concept Test (after four months), Post -Form of Concept Test was administered as Delayed Concept Test to the two groups. In this part there were 5 items (items 26, 27, 28, 29, 30 from post and delayed concept tests) to be answered by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to comparison of decimals (ACT5).

Table 4.51: Tests of Significance for Group and Achievement Related to Comparison of Decimals (ACT5) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	70.83	94	0.75	-	-
ACT5	7.97	2	3.99	5.29	0.007
Grp. By ACT5	4.73	2	2.37	3.14	0.048

Table 4.52: Tests of Significance for Group Main Effect on Achievement Related to Comparison of Decimals (ACT5), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	228.40	47	4.86	-	-
Group	27.10	1	27.10	5.58	0.022

As it is seen in tables 4.51 and 4.52, main effect for group and main effect for ACT5 is significant. Also there is a significant interaction between ACT5 and group.

In order to understand the significant differences observed as a result of averaged tests of significance for achievement related to comparison of decimals (ACT5), mean scores of experimental and control groups are indicated in the graph below.

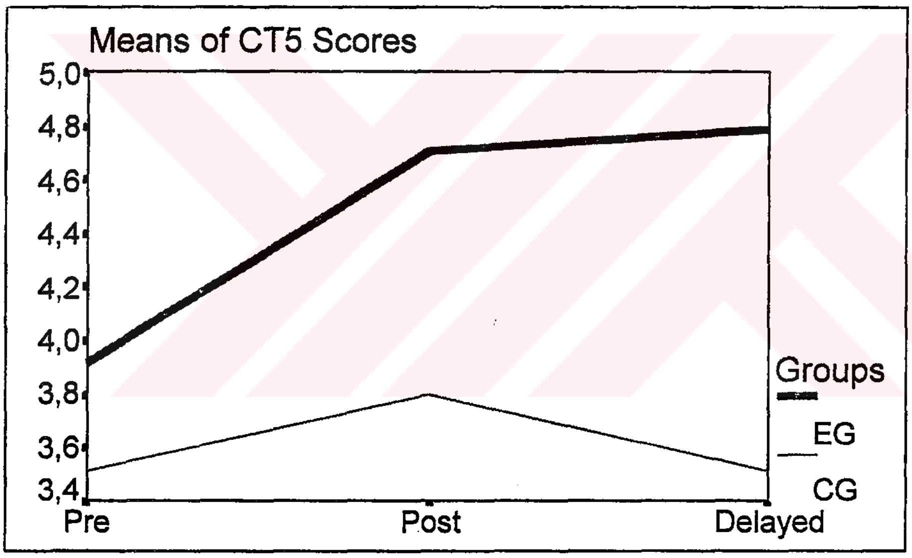


Figure 4.23: Pre-, Post-, and Delayed - CT5 Mean Scores for Experimental and Control Groups.

As noticed in the graph, the significant difference between the two measures of ATC5 might be due to the gradual increases of experimental group at post and delayed post testing stages. Although there is a slight increase for the control group at post testing stage after that we observe a slight decrease at the delayed post testing stage. In the light of these findings there is sufficient evidence to reject hypothesis-21 and hypothesis-22. Thus it can be concluded that there are significant differences between mean post CT5 and mean delayed CT5 scores of subjects in the EG and CG. Means and standard deviations of experimental and control groups in the three measures of CT5 are given in the following table.

Table 4.53: Means and standard deviations of experimental and control groups in the three measures of CT5.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	3.92	1.61	4.70	1.04	4.79	0.51
CG	3.52	1.76	3.80	1.58	3.53	1.75

4.2.1.9 Results Obtained from a Sub-dimension of Concepts Test (CT6) Related to Multiplication and Division Involving Decimals:

In hypothesis-23 of the stage-2 of the study, it was stated that there was no significant difference between mean post CT6 scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post -Form of Concept Test was administered to the two groups. In hypothesis-24 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed CT6 scores of the subjects in the experimental and control groups. After the administration of Post -Form of Concept Test (after four months), Post -Form of Concept Test was administered as Delayed Concept Test to the two groups. In this part there were 6 items (items 39, 40, 41, 42, 43, 44 from post and delayed concept tests) to be answered by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to multiplication and division involving decimals (ACT6).

Table 4.54: Tests of Significance for Group and Achievement Related to Multiplicaiton and Division Involving Decimals (ACT6) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	78.00	94	0.83	-	-
ACT6	13.00	2	6.50	7.83	0.001
Grp. By ACT6	8.26	2	4.13	4.98	0.009

Table 4.55: Tests of Significance for Group Main Effect on Achievement Related to Multiplicaiton and Division Involving Decimals (ACT6), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	209.59	47	4.46	-	-
Group	18.93	1	18.93	4.24	0.045

As it is seen in tables 4.54 and 4.55, main effect for group and main effect for ACT6 is significant. Also there is a significant interaction between ACT6 and group.

In order to understand the significant differences observed as a result of averaged tests of significance for achievement related to multiplication and division involving decimals (ACT6), mean scores of experimental and control groups are indicated in the graph below.

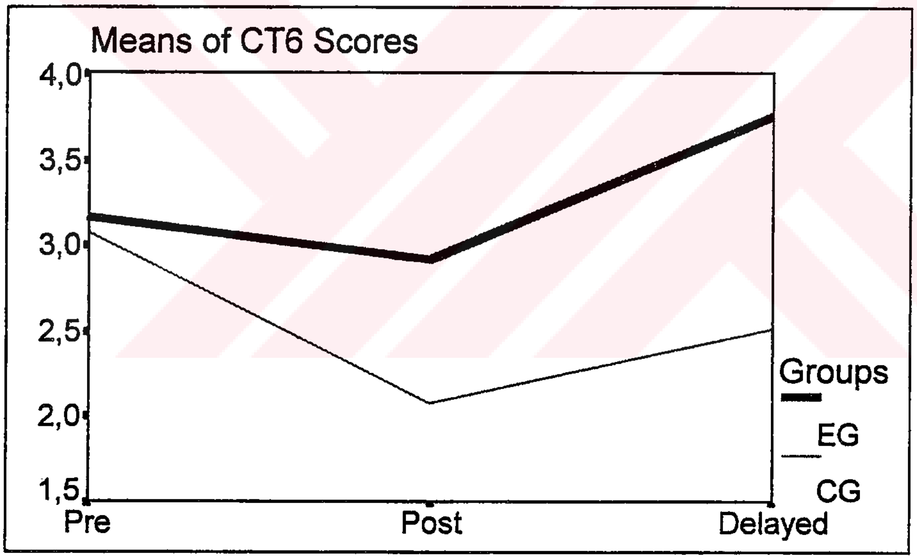


Figure 4.24: Pre-, Post-, and Delayed - CT6 Mean Scores for Experimental and Control Groups.

As it is seen in the graph there is a slight decrease in both of the groups at the post testing stage. When we look at the final positions of the two groups it is clear that the experimental groups' mean score is higher than its' previous positions. On the other hand, although the control groups' mean score shows a slight increase compared to the post testing stage its score is lower than its starting position. Mean scores of the experimental group at post and delayed testing stages are higher than the control groups' mean scores. During the pre-testing although we said the preservice elementary teachers to estimate the operations, they tried to do the calculations therefore a slight decrease occurred in both of the groups from pre to post testing (they did not calculate at post and delayed testing stages). In the light of these findings there is sufficient evidence to reject hypothesis-23 and hypothesis-24. Thus it can be concluded that there are significant differences between mean post CT6 and mean delayed CT6 scores of subjects in the EG and CG. Means and standard deviations of experimental and control groups in the three measures of CT6 are given in the following table.

Table 4.56: Means and standard deviations of experimental and control groups in the three measures of CT6.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	3.17	1.63	2.92	1.32	3.75	1.57
CG	3.08	1.47	2.08	1.22	2.52	1.33

4.2.2 Results Obtained from the Pre, Post, and Delayed - Testing of Problems Test:

In hypothesis-3 of the stage-2 of the study, it was stated that there was no significant difference between mean post PT scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post-Form of Problems Test was administered to the two groups. In hypothesis-4 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed PT scores of the subjects in the experimental and control groups. After the administration of Post-Form of Problems Test (after four months), Post-Form of Problems Test was administered as Delayed Problems Test to the two groups. In this part there were 26 items to be answered by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to choosing the appropriate operation for word problems (APT).

Table 4.57: Tests of Significance for Group and Achievement Related to Choosing the Appropriate Operation for Word Problems (APT) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	1109.22	94	11.80	-	-
APT	117.01	2	58.50	4.96	0.009
Grp. By APT	97.17	2	48.59	4.12	0.019

Table 4.58: Tests of Significance for Group Main Effect on Achievement Related to Choosing the Appropriate Operation for Word Problems (APT), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	1906.26	47	40.56	-	-
Group	115.30	1	115.30	2.84	0.98

Table 4.57 indicates the significant differences coming from the mean differences between the two measures and the interaction terms. As indicated in table 4.58, the main effect for group is not significant, as well as its interaction with APT.

In order to understand the significant differences observed as a result of averaged test of significance for APT, means of PT scores across the two groups were indicated in the graph below.

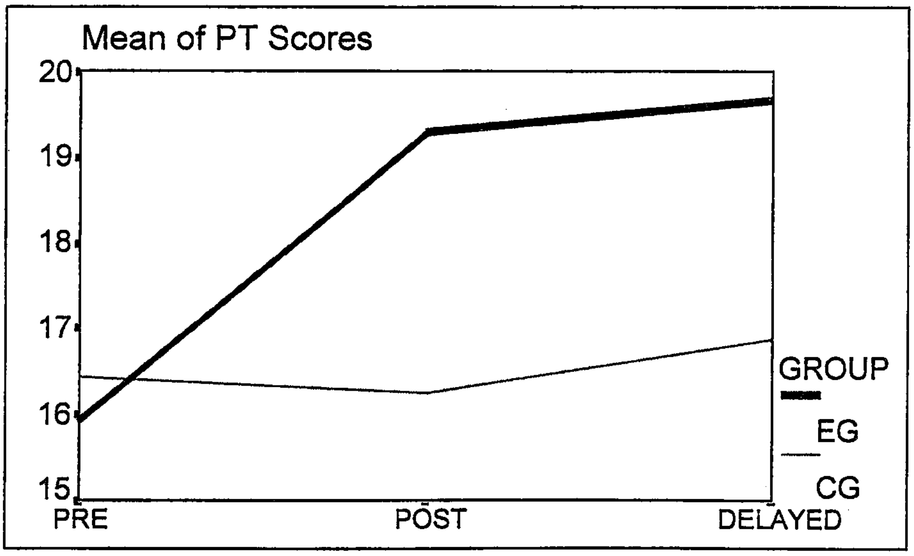


Figure 4.25: Pre-, Post-, and Delayed - PT Mean Scores for Experimental and Control Groups.

As can be noticed from the graph, the significant difference between the two measures of APT might be due to the gradual increase in the mean scores of the experimental group. The significant interaction is also evidenced by the crossing lines observed between the experimental and control groups. As it is seen in the graph, there is a slight increase in the mean score of the control group at the delayed post testing stage which is still lower than the mean score of the experimental group. In the light of these findings there is sufficient evidence to reject hypothesis-3 and hypothesis-4. Thus it can be concluded that there are significant differences between mean post PT and mean delayed PT scores of subjects in the EG and CG. Means and standard deviations of experimental and control groups in the three measures of problems test are given in the following table.

Table 4.59: Means and Standard Deviations of Experimental and Control Groups in the Three Measures of Problems Test.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	15.92	4.84	19.29	3.83	19.67	2.68
CG	16.44	5.30	16.24	5.48	16.88	4.93

4.2.2.1 Results Obtained from a Sub-dimension of Problems Test (PT1.1) Related to Choosing the Appropriate Operation for Multiplication Word Problems Involving Decimals:

In hypothesis-25 of the stage-2 of the study, it was stated that there was no significant difference between mean post PT1.1 scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post-Form of Problems Test was administered to the two groups. In hypothesis-26 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed PT1.1 scores of the subjects in the experimental and control groups. After the administration of Post-Form of Problems Test (after four months), Post-Form of Problems Test was administered as Delayed Problems Test to the two groups. In this part there were 9 items (items 1, 3, 5, 7, 9, 10, 14, 20, 21 from post and delayed problems test) to be answered by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to choosing the appropriate operation for multiplication word problems (APT1.1).

Table 4.60: Tests of Significance for Group and Achievement Related to Choosing the Appropriate Operation for Multiplication Word Problems (APT1.1) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	100.00	94	1.06	-	-
APT1.1	2.67	2	1.33	1.25	0.290
Grp. By APT1.1	0.00	2	0.00	0.00	0.999

Table 4.61: Tests of Significance for Group Main Effect on Achievement Related to Choosing the Appropriate Operation for Multiplication Word Problems (APT1.1), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	331.28	47	7.05	-	-
Group	8.18	1	8.19	1.16	0.287

As it is seen in tables 4.60 and 4.61, main effect for group and main effect for APT1.1 is not significant. Also there is no significant interaction between APT1.1 and group. As a result of averaged test of significance for APT1.1 there is no sufficient evidence to reject hypothesis-25 and hypothesis-26. Thus it is possible to say that there is no difference between the two groups in terms of achievement in any of the measurements. Means and standard deviations of experimental and control groups in the three measures of PT1.1 are given in the following table.

Table 4.62: Means and Standard Deviations of Experimental and Control Groups in the Three Measures of PT1.1.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	7.71	1.27	8.00	0.98	7.71	1.26
CG	7.24	2.23	7.52	2.02	7.24	2.22

4.2.2.2 Results Obtained from a Sub-dimension of Problems Test (PT1.2) Related to Choosing the Appropriate Operation for Multiplication Word Problems Suitable for Direct Proportion Involving Decimals:

In hypothesis-27 of the stage-2 of the study, it was stated that there was no significant difference between mean post PT1.2 scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post-Form of Problems Test was administered to the two groups. In hypothesis-28 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed PT1.2 scores of the subjects in the experimental and control groups. After the administration of Post-Form of Problems Test (after four months), Post-Form of Problems Test was administered as Delayed Problems Test to the two groups. In this part there were 7 items (items 1, 3, 5, 7, 10, 14, 20 from post and delayed problems test) to be answered by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to choosing the appropriate operation for multiplication word problems suitable for direct proportion (APT1.2).

Table 4.63: Tests of Significance for Group and Achievement Related to Choosing the Appropriate Operation for Multiplication Word Problems Suitable for Direct Proportion (APT1.2) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	129.28	94	1.38	-	-
APT1.2	11.39	2	5.69	4.14	0.010
Grp. By APT1.2	6.19	2	3.10	2.25	0.111

Table 4.64: Tests of Significance for Group Main Effect on Achievement Related to Choosing the Appropriate Operation for Multiplication Word Problems Suitable fro Direct Proportion (APT1.2), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	162.44	47	3.46	-	-
Group	8.78	1	8.78	2.54	0.118

As it is seen in tables 4.63 and 4.64, the group main effect and interaction of group and achievement related to choosing the appropriate operation for multiplication word problems suitable for direct proportion (APT1.2) are not significant. As a result of averaged test of significance for APT1.2 there is no sufficient evidence to reject hypothesis-27 and hypothesis-28. Thus it is possible to say that there is no difference between the two groups in terms of achievement in any of the measurements. Means and standard deviations of experimental and control groups in the three measures of PT1.2 are given in the following table.

Table 4.65: Means and Standard Deviations of Experimental and Control Groups in the Three Measures of PT1.2.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	5.62	1.58	6.41	0.88	6.63	0.58
CG	5.68	1.70	5.48	1.94	6.04	1.42

4.2.2.3 Results Obtained from a Sub-dimension of Problems Test (PT1.3) Related to Choosing the Appropriate Operation for Multiplication Word Problems Not Suitable for Direct Proportion Involving Decimals:

In hypothesis-29 of the stage-2 of the study, it was stated that there was no significant difference between mean post PT1.3 scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post-Form of Problems Test was administered to the two groups. In hypothesis-30 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed PT1.3 scores of the subjects in the experimental and control groups. After the administration of Post-Form of Problems Test (after four months), Post-Form of Problems Test was administered as Delayed Problems Test to the two groups. In this part there were 2 items (items 9 and 21 from post and delayed problems test) to be answered by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to choosing the appropriate operation for multiplication word problems not suitable for direct proportion (APT1.3).

Table 4.66: Tests of Significance for Group and Achievement Related to Choosing the Appropriate Operation for Multiplication Word Problems Not Suitable for Direct Proportion (APT1.3) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	29.81	94	0.32	-	-
APT1.3	0.37	2	0.18	0.58	0.562
Grp. By APT1.3	0.48	2	0.24	0.75	0.474

Table 4.67: Tests of Significance for Group Main Effect on Achievement Related to Choosing the Appropriate Operation for Multiplication Word Problems Not Suitable fro Direct Proportion (APT1.3), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	37.57	47	0.80	-	-
Group	0.02	1	0.02	0.02	0.878

As it is seen in tables 4.66 and 4.67, the group main effect and interaction of group and achievement related to choosing the appropriate operation for multiplication word problems not suitable for direct proportion (APT1.3) are not significant. As a result of averaged test of significance for APT1.3 there is no sufficient evidence to reject hypothesis-29 and hypothesis-30. Thus it is possible to say that there is no difference between the two groups in terms of achievement in any of the measurements. Means and standard deviations of experimental and control groups in the three measures of PT1.3 are given in the following table.

Table 4.68: Means and Standard Deviations of Experimental and Control Groups in the Three Measures of PT1.3.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	1.48	0.72	1.46	0.51	1.36	0.65
CG	1.56	0.71	1.32	0.80	1.48	0.71

4.2.2.4 Results Obtained from a Sub-dimension of Problems Test (PT2.1) Related to Choosing the Appropriate Operation for Division Word Problems Involving Decimals:

In hypothesis-31 of the stage-2 of the study, it was stated that there was no significant difference between mean post PT2.1 scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post-Form of Problems Test was administered to the two groups. In hypothesis-32 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed PT2.1 scores of the subjects in the experimental and control groups. After the administration of Post-Form of Problems Test (after four months), Post-Form of Problems Test was administered as Delayed Problems Test to the two groups. In this part there were 14 items (items 2, 4, 6, 11, 12, 13, 15, 16, 17, 18, 19, 23, 24, 25 from post and delayed problems test) to be answered by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to choosing the appropriate operation for division word problems (APT2.1).

Table 4.69: Tests of Significance for Group and Achievement Related to Choosing the Appropriate Operation for Division Word Problems (APT2.1) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	259.23	94	2.76	-	-
APT2.1	8.84	2	4.42	1.60	0.207
Grp. By APT2.1	5.52	2	2.76	1.00	0.371

Table 4.70: Tests of Significance for Group Main Effect on Achievement Related to Choosing the Appropriate Operation for Division Word Problems (APT2.1), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	803.65	47	17.10	-	-
Group	114.29	1	114.29	6.68	0.013

As it is seen in tables 4.68 and 4.69, although the group main effect is significant, since the interaction of group and achievement related to choosing the appropriate operation for multiplication word problems (APT2.1) is not significant it is not possible to say that there is significant difference between the two groups in terms of achievement in any of the measurements. Thus as a result of averaged test of significance for APT2.1 there is no sufficient evidence to reject hypothesis-31 and hypothesis-32. Means and standard deviations of experimental and control groups in the three measures of PT2.1 are given in the following table.

Table 4.71: Means and Standard Deviations of Experimental and Control Groups in the Three Measures of PT2.1.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	10.63	2.08	11.67	1.94	11.00	1.79
CG	9.20	3.50	9.36	3.20	9.44	3.30

4.2.2.5 Results Obtained from a Sub-dimension of Problems Test (PT2.2) Related to Choosing the Appropriate Operation for Division Word Problems Suitable for Direct Proportion:

In hypothesis-33 of the stage-2 of the study, it was stated that there was no significant difference between mean post PT2.2 scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post-Form of Problems Test was administered to the two groups. In hypothesis-34 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed PT2.2 scores of the subjects in the experimental and control groups. After the administration of Post-Form of Problems Test (after four months), Post-Form of Problems Test was administered as Delayed Problems Test to the two groups. In this part there were 9 items (items 4, 6, 11, 12, 13, 15, 18, 19, 23 from post and delayed problems test) to be answered by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to choosing the appropriate operation for division word problems suitable for direct proportion (APT2.2).

Table 4.72: Tests of Significance for Group and Achievement Related to Choosing the Appropriate Operation for Division Word Problems Suitable for Direct Proportion (APT2.2) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	247.99	94	2.64	-	-
APT2.2	16.54	2	8.27	3.14	0.480
Grp. By APT2.2	14.58	2	7.29	2.76	0.068

Table 4.73: Tests of Significance for Group Main Effect on Achievement Related to Choosing the Appropriate Operation for Division Word Problems Suitable for Direct Proportion (APT2.2), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	363.82	47	7.74	-	-
Group	13.08	1	13.03	1.69	0.200

As it is seen in tables 4.72 and 4.73, the group main effect and interaction with group and achievement related to choosing the appropriate operation for division word problems suitable for direct proportion (APT2.2) are not significant. Therefore it is not possible to say that there is significant difference between the two groups in terms of achievement in any of the measurements. Thus as a result of averaged test of significance for APT2.2 there is no sufficient evidence to reject hypothesis-33 and hypothesis-34. Means and standard deviations of experimental and control groups in the three measures of PT2.2 are given in the following table.

Table 4.74: Means and Standard Deviations of Experimental and Control Groups in the Three Measures of PT2.2.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	5.75	2.58	6.83	1.24	7.17	1.24
CG	5.88	2.40	6.32	2.02	5.76	2.52

4.2.2.6 Results Obtained from a Sub-dimension of Problems Test (PT2.3) Related to Choosing the Appropriate Operation for Division Word Problems Not Suitable for Direct Proportion:

In hypothesis-35 of the stage-2 of the study, it was stated that there was no significant difference between mean post PT2.3 scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post-Form of Problems Test was administered to the two groups. In hypothesis-36 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed PT2.3 scores of the subjects in the experimental and control groups. After the administration of Post-Form of Problems Test (after four months), Post-Form of Problems Test was administered as Delayed Problems Test to the two groups. In this part there were 5 items (items 2, 16, 17, 24, 25 from post and delayed forms of problems test) to be answered by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to choosing the appropriate operation for division word problems not suitable for direct proportion (APT2.3).

Table 4.75: Tests of Significance for Group and Achievement Related to Choosing the Appropriate Operation for Division Word Problems Not Suitable for Direct Proportion (APT2.3) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	101.43	94	1.08	-	-
APT2.3	17.63	2	8.82	8.17	0.001
Grp. By APT2.3	12.08	2	6.04	5.60	0.005

Table 4.76: Tests of Significance for Group Main Effect on Achievement Related to Choosing the Appropriate Operation for Division Word Problems Not Suitable for Direct Proportion (APT2.3), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	137.82	47	2.93	-	-
Group	11.94	1	11.94	4.07	0.049

As it is seen in tables 4.75 and 4.76, main effect for group and main effect for APT2.3 is significant. Also there is a significant interaction between APT2.3 and group.

In order to understand the significant differences observed as a result of averaged tests of significance for achievement related to choosing the appropriate operation for division word problems not suitable for direct proportion (APT2.3), mean scores of experimental and control groups are indicated in the graph below.

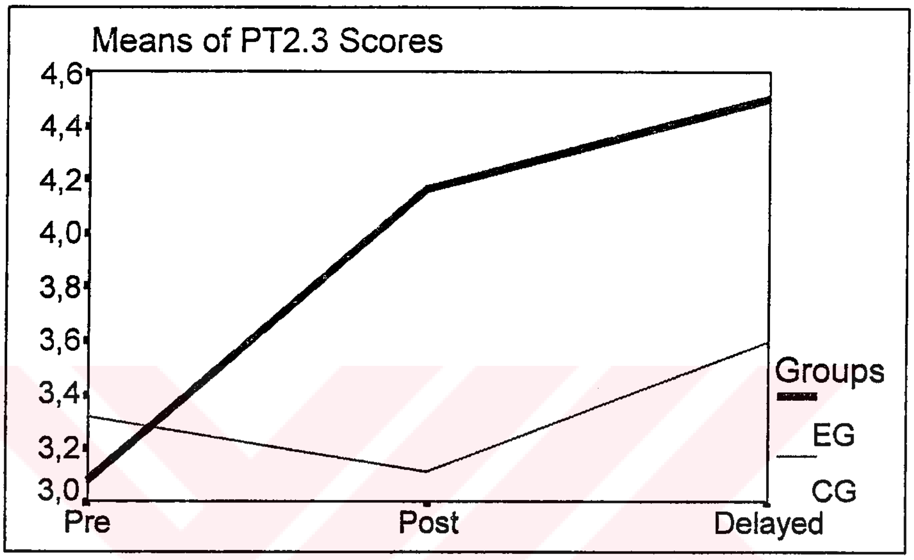


Figure 4.26: Pre-, Post-, and Delayed - PT2.3 Mean Scores for Experimental and Control Groups.

As noticed in the graph, the significant difference between the two measures of APT2.3 might be due to the increases in the control group through pre - testing to delayed testing, and the slight decrease in the control group from pre to post testing. Although there is a slight increase in the control group from post to delayed post testing, the mean score of the experimental group is higher than the mean score of the control group. The graph also reveals a significant interaction, evidenced by the crossing lines of the experimental group and control group. In the light of these findings there is sufficient evidence to reject hypothesis-35 and hypothesis-36. Thus it can be concluded that there are significant differences between mean post PT2.3 and mean delayed PT2.3 scores of subjects in the EG and CG. Means and standard deviations of experimental and control groups in the three measures of PT2.3 are given in the following table.

Table 4.77: Means and Standard Deviations of Experimental and Control Groups in the Three Measures of PT2.3.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	3.08	1.18	4.17	1.20	4.50	1.41
CG	3.32	1.49	3.12	1.56	3.60	1.15

4.2.2.7 Results Obtained from a Sub-dimension of Problems Test (PT2.4) Related to Choosing the Appropriate Operation for Division Word Problems in which the Divisor is Greater than the Dividend:

In hypothesis-37 of the stage-2 of the study, it was stated that there was no significant difference between mean post PT2.4 scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post-Form of Problems Test was administered to the two groups. In hypothesis-38 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed PT2.4 scores of the subjects in the experimental and control groups. After the administration of Post-Form of Problems Test (after four months), Post-Form of Problems Test was administered as Delayed Problems Test to the two groups. In this part there were 8 items (items 4, 6, 12, 15, 16, 19, 24, 25 from post and delayed problems test) to be answered by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to choosing the appropriate operation for division word problems in which the divisor is greater than the dividend (APT2.4).

Table 4.78: Tests of Significance for Group and Achievement Related to Choosing the Appropriate Operation for Division Word Problems in which the Divisor is Greater than the Dividend (APT2.4) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	141.45	94	1.50	-	-
APT2.4	18.84	2	9.42	6.26	0.003
Grp. By APT2.4	22.54	2	11.27	7.49	0.001

Table 4.79: Tests of Significance for Group Main Effect on Achievement Related to Choosing the Appropriate Operation for Division Word Problems in which the Divisor is Greater than the Dividend (APT2.4), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	218.32	47	4.65	-	-
Group	18.14	1	18.14	3.91	0.054

Table 4.78 indicates the significant differences coming from the mean differences between the two measures and the interaction terms. As indicated in table 4.79, the main effect for group is not significant, as well as its interaction with APT2.4.

In order to understand the significant differences observed as a result of averaged tests of significance for achievement related to choosing the appropriate operation for division word problems in which the divisor is greater than the dividend (APT2.4), mean scores of experimental and control groups are indicated in the graph below.

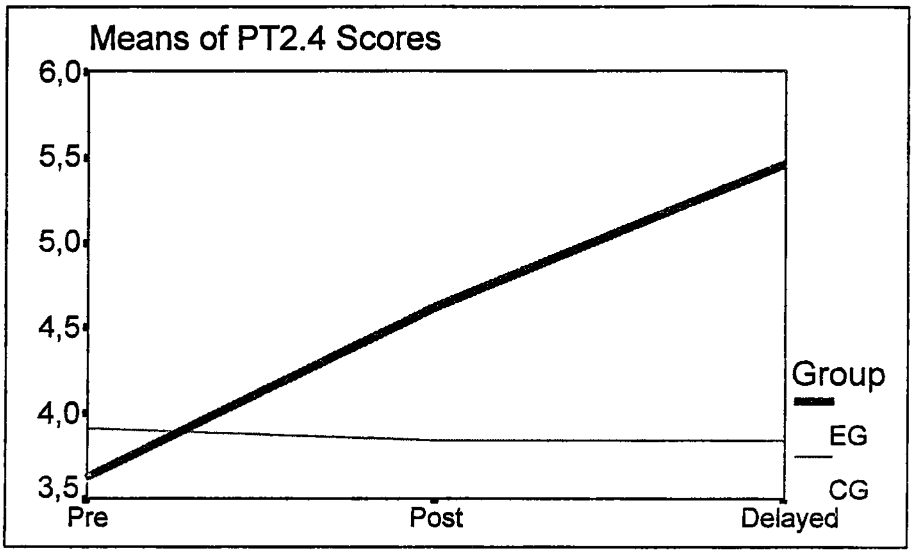


Figure 4.27: Pre-, Post-, and Delayed - PT2.4 Mean Scores for Experimental and Control Groups.

As noticed in the graph, the significant difference between the two measures of APT2.4 might be due to the increases in the experimental group through pre - testing to delayed testing, and the slight decreases in the control group. The graph also reveals a significant interaction, evidenced by the crossing lines of the experimental group and control group. Individual univariate estimates showed that the difference between the groups are mainly due to the differences in mean scores of the two groups at the delayed post testing stage. The control group seems to remain constant at the delayed post testing stage, while the experimental group has been slightly increasing. In the light of these findings there is sufficient evidence to reject hypothesis-37 and hypothesis-38. Thus it can be concluded that there are significant differences between mean post PT2.4 and mean delayed PT2.4 scores of subjects in the EG and CG. Means and standard deviations of experimental and control groups in the three measures of PT2.4 are given in the following table.

Table 4.80: Means and Standard Deviations of Experimental and Control Groups in the Three Measures of PT2.4.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	3.63	1.66	4.62	1.28	5.45	1.18
CG	3.92	1.75	3.84	1.77	3.84	1.79

4.2.3 Results Obtained from the Pre, Post, and Delayed -Testing of Writing Division and Multiplication Word Problems Test (WWPT):

In hypothesis-5 of the stage-2 of the study, it was stated that there was no significant difference between mean post WWPT scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post-Form of Problems Test was administered to the two groups. In hypothesis-6 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed WWPT scores of the subjects in the experimental and control groups. After the administration of Post-Form of Problems Test (after four months), Post-Form of Problems Test was administered as Delayed Problems Test to the two groups. In this part there were 10 expressions for which word problems were expected to be written by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to writing word problems for division and multiplication expressions involving decimals (ARWWP).

Table 4.81: Tests of Significance for Group and Achievement Related to Writing Word Problems for Division and Multiplication Expressions Involving Decimals (ARWWP) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	2875.81	94	30.59	-	-
ARWWP	364.55	2	182.18	5.95	0.004
Grp. By ARWWP	1331.59	2	665.80	21.76	0.000

Table 4.82: Tests of Significance for Group Main Effect on Achievement Related to Writing Word Problems for Division and Multiplication Expressions Involving Decimals (ARWWP), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	4690.37	47	99.80	-	-
Group	4218.43	1	4218.43	42.27	0.000

As the tables 4.80 and 4.82 clearly demonstrate, main effect for group and main effect for ARWWP is significant. Also, there is a significant interaction between group and ARWWP.

In order to understand the significant differences observed as a result of averaged test of significance for ARWWP, means of WWPT scores across the two groups were indicated in the graph below.

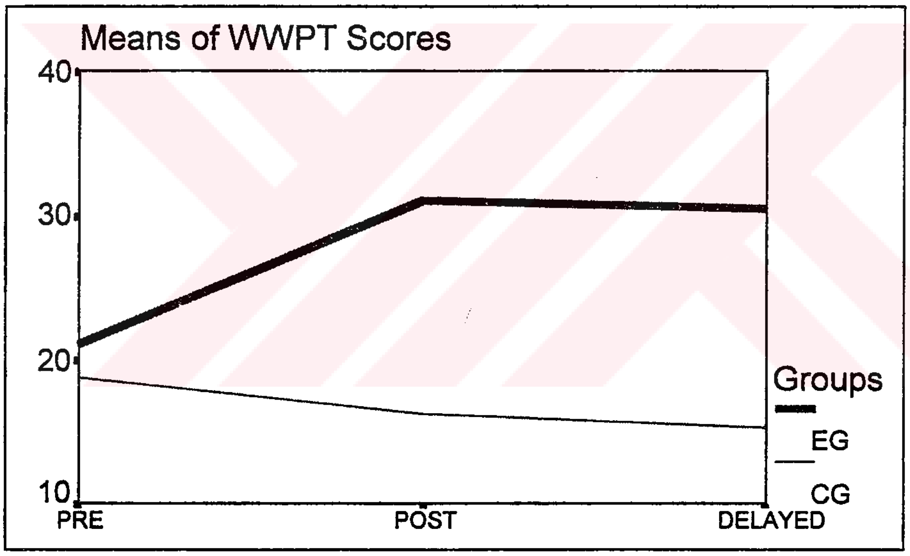


Figure 4.28: Pre-, Post-, and Delayed - WWPT Mean Scores for Experimental and Control Groups.

As revealed in the graph, the significant difference between the two groups is evidenced by the gradual increase of the experimental group and the gradual decrease of the control group, especially from pre to post testing. It is also seen from the graph that both of the groups remains constant from post to delayed testing, but mean score of experimental group is still higher that control groups' mean. In the light of these findings there is sufficient evidence to reject hypothesis-5 and hypothesis-6. Thus it can be concluded that there are significant differences between mean post WWPT and mean delayed WWPT scores of subjects in the EG and CG. Means and standard deviations of experimental and control groups in the three measures of WWPT are given in the following table.

Table 4.83: Means and Standard Deviations of Experimental and Control Groups in the Three Measures of WWPT.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	21.13	6.84	31.04	6.27	30.54	6.19
CG	18.92	7.73	16.28	9.01	15.36	7.43

4.2.3.1 Results Obtained from a Sub-dimension of Writing Division and Multiplication Word Problems Test Related to Writing Word Problems for Multiplication Expressions (WWPT-MULT) Involving Decimals:

In hypothesis-39 of the stage-2 of the study, it was stated that there was no significant difference between mean post WWPT-MULT scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post-Form of Problems Test was administered to the two groups. In hypothesis-40 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed WWPT-MULT scores of the subjects in the experimental and control groups. After the administration of Post-Form of Problems Test (after four months), Post-Form of Problems Test was administered as Delayed Problems Test to the two groups. In this part there were 4 expressions for which word problems were expected to be written by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to writing word problems for multiplication expressions involving decimals (ARWWP-MULT).

Table 4.84: Tests of Significance for Group and Achievement Related to Writing Word Problems for Multiplication Expressions Involving Decimals (ARWWP-MULT) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	691.07	94	7.35	-	-
ARWWP-MULT	100.32	2	50.16	6.82	0.002
Grp. By ARWWP-MULT	191.86	2	95.93	13.05	0.000

Table 4.85: Tests of Significance for Group Main Effect on Achievement Related to Writing Word Problems for Multiplication Expressions Involving Decimals (ARWWP-MULT), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	1140.82	47	24.27	-	-
Group	467.60	1	467.60	19.26	0.000

As the tables 4.84 and 4.85 clearly demonstrate, main effect for group and main effect for ARWWP-MULT is significant. Also, there is a significant interaction between group and ARWWP-MULT.

In order to understand the significant differences observed as a result of averaged test of significance for ARWWP-MULT, means of WWPT-MULT scores across the two groups were indicated in the graph below.

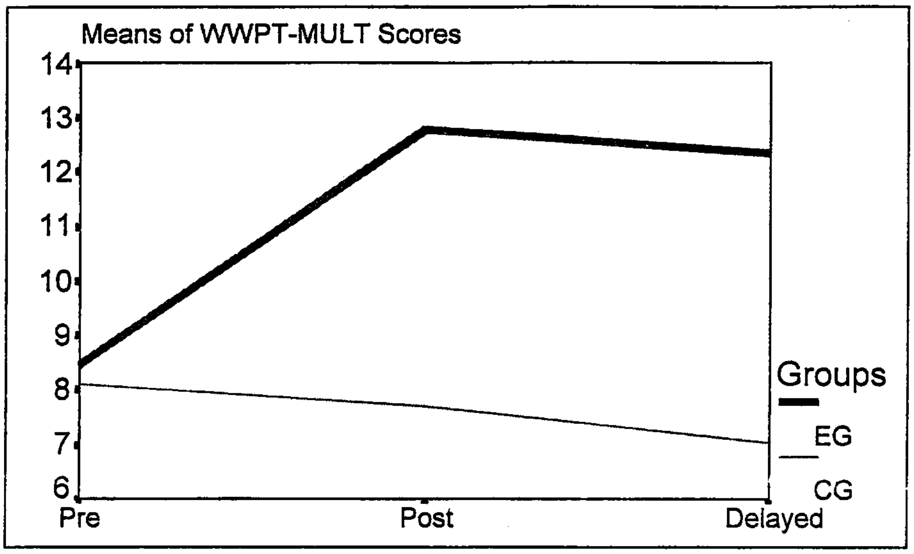


Figure 4.29: Pre-, Post-, and Delayed - WWPT-MULT Mean Scores for Experimental and Control Groups.

As the graph reveals, the significant differences observed between the groups are evidenced by the gradual increase of the experimental group from pre testing to post testing and gradual decrease of the control group from pre to post testing. After the post testing stage both of the groups have slight decreases, but the experimental group is still at a higher point compared to the pre testing. The control group is lower than its starting position at the delayed post testing stage. In the light of these findings there is sufficient evidence to reject hypothesis-39 and hypothesis-40. Thus it can be concluded that there are significant differences between mean post WWPT-MULT and mean delayed WWPT-MULT scores of subjects in the EG and CG. Means and standard deviations of experimental and control groups in the three measures of WWPT-MULT are given in the following table.

Table 4.86: Means and Standard Deviations of Experimental and Control Groups in the Three Measures of WWPT-MULT.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	8.46	3.40	12.79	2.90	12.34	2.71
CG	8.12	3.98	7.72	4.52	7.04	3.71

4.2.3.2 Results Obtained from a Sub-dimension of Writing Division and Multiplication Word Problems Test Related to Writing Word Problems for Division Expressions (WWPT-DIV) Involving Decimals:

In hypothesis-41 of the stage-2 of the study, it was stated that there was no significant difference between mean post WWPT-DIV scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post-Form of Problems Test was administered to the two groups. In hypothesis-42 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed WWPT-DIV scores of the subjects in the experimental and control groups. After the administration of Post-Form of Problems Test (after four months), Post-Form of Problems Test was administered as Delayed Problems Test to the two groups. In this part there were 6 expressions for which word problems were expected to be written by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to writing word problems for division expressions involving decimals (ARWWP-DIV).

Table 4.87: Tests of Significance for Group and Achievement Related to Writing Word Problems for Division Expressions Involving Decimals (ARWWP-DIV) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	1732.70	94	18.43	-	-
ARWWP-DIV	84.21	2	42.10	2.28	0.107
Grp. By ARWWP-DIV	512.62	2	256.31	13.90	0.000

Table 4.88: Tests of Significance for Group Main Effect on Achievement Related to Writing Word Problems for Division Expressions Involving Decimals (ARWWP-DIV), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	1914.69	47	40.74	-	-
Group	1877.09	1	1877.09	46.08	0.000

As the tables 4.87 and 4.88 clearly demonstrate, the group main effect and interaction of group and ARWWP-DIV is significant. As indicated in table 4.87, the main effect for ARWWP-DIV is not significant, as well as its interaction with the group.

In order to understand the significant differences observed as a result of averaged test of significance for ARWWP-DIV, means of WWPT-DIV scores across the two groups were indicated in the graph below.

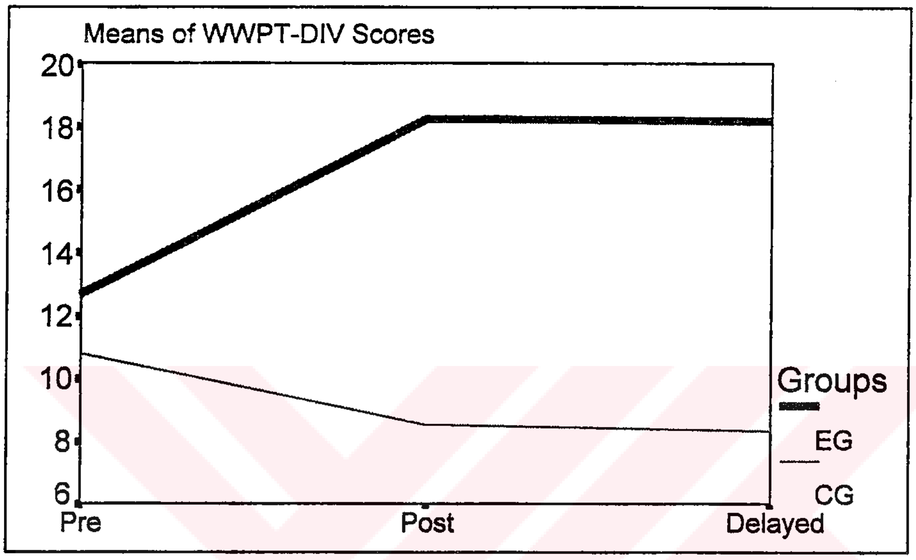


Figure 4.30: Pre-, Post-, and Delayed - WWPT-DIV Mean Scores for Experimental and Control Groups.

As noticed in the graph, the significant difference between the groups is evidenced by the gradual increase in the experimental group especially at the post testing stage and the slight decrease of the control group throughout the post and delayed post testing stages. The position of the experimental group at the delayed post testing is approximately the same when compared to the post testing stage and the overall position of the control group is nearly the same. That is why the main effect for ARWWPT-DIV is not significant but the interaction term shows that the positions of the experimental group at post and delayed post testing stages are higher when compared to the control group. In the light of these findings there is sufficient evidence to reject hypothesis-41 and hypothesis-42. Thus it can be concluded that there are significant differences between mean post

WWPT-DIV and mean delayed WWPT-DIV scores of subjects in the EG and CG. Means and standard deviations of experimental and control groups in the three measures of WWPT-DIV are given in the following table.

Table 4.89: Means and Standard Deviations of Experimental and Control Groups in the Three Measures of WWPT-DIV.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	12.67	5.26	18.25	4.40	18.21	2.92
CG	10.80	5.29	8.56	5.51	8.32	5.04

4.2.3.3 Results Obtained from a Sub-dimension of Writing Division and Multiplication Word Problems Test Related to Writing Word Problems for Division Expressions in which Divisor is Less than the Dividend (WWPT-DLTD):

In hypothesis-43 of the stage-2 of the study, it was stated that there was no significant difference between mean post WWPT-DLTD scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post-Form of Problems Test was administered to the two groups. In hypothesis-44 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed WWPT-DLTD scores of the subjects in the experimental and control groups. After the administration of Post-Form of Problems Test (after four months), Post-Form of Problems Test was administered as Delayed Problems Test to the two groups. In this part there were 2 expressions for which word problems were expected to be written by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to writing word problems for division expressions in which divisor is less than the dividend (ARWWP-DLTD).

Table 4.90: Tests of Significance for Group and Achievement Related to Writing Word Problems for Division Expressions in which Divisor is less than the Dividend (ARWWP-DLTD) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	336.59	94	3.58	-	-
ARWWP-DLTD	3.37	2	1.69	0.47	0.626
Grp. By ARWWP-DLTD	39.56	2	19.78	5.52	0.005

Table 4.91: Tests of Significance for Group Main Effect on Achievement Related to Writing Word Wroblems for Division Expressions in which Divisor is less than the Dividend (ARWWP-DLTD), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	317.28	47	6.75	-	-
Group	105.47	1	105.47	15.62	0.000

As the tables 4.90 and 4.91 clearly demonstrate, the group main effect and interaction of group and ARWWP-DLTD are significant. As indicated in table 4.87, the main effect for ARWWP-DLTD is not significant, as well as its interaction with the group.

In order to understand the significant differences observed as a result of averaged test of significance for ARWWP-DLTD, means of WWPT-DLTD scores across the two groups were indicated in the graph below.

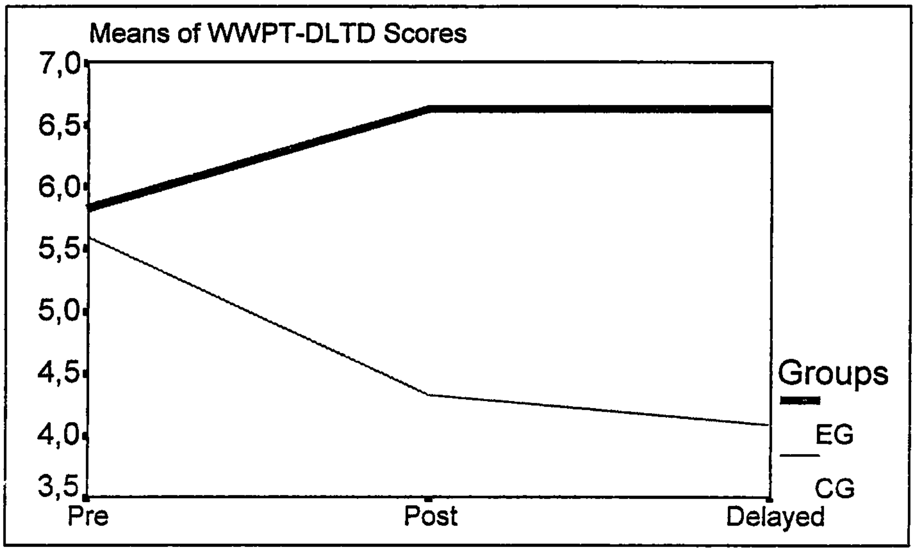


Figure 4.31: Pre-, Post-, and Delayed - WWPT-DLTD Mean Scores for Experimental and Control Groups.

As the graph reveals, the significance of the differences between the groups are evidenced by the gradual decreases of the control group. As we mentioned before main effect for ARWWP-DLTD was not significant, this is evidenced by the graph in which the positions of the experimental group are nearly constant. The interaction terms and the graph reveals that the mean scores of the experimental group are higher than control groups' mean scores at post and delayed post testing stages. In the light of these findings there is sufficient evidence to reject hypothesis-43 and hypothesis-44. Thus it can be concluded that there are significant differences between mean post WWPT-DLTD and mean delayed WWPT-DLTD scores of subjects in the EG and CG. Means and standard deviations of experimental and control groups in the three measures of WWPT-DLTD are given in the following table.

Table 4.92: Means and Standard Deviations of Experimental and Control Groups in the Three Measures of WWPT-DLTD.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	5.84	2.33	6.63	1.44	6.63	1.28
CG	5.60	2.27	4.32	2.29	4.08	2.84

4.2.3.4 Results Obtained from a Sub-dimension of Writing Division and Multiplication Word Problems Test Related to Writing Word Problems for Division Expressions in which Divisor is Greater than the Dividend (WWPT-DGTD):

In hypothesis-45 of the stage-2 of the study, it was stated that there was no significant difference between mean post WWPT-DGTD scores of subjects in the experimental and control groups. After the implementation of the Conceptual Change Instruction, the Post-Form of Problems Test was administered to the two groups. In hypothesis-46 of the stage 2 of the study, it was stated that there was a significant difference between mean delayed WWPT-DGTD scores of the subjects in the experimental and control groups. After the administration of Post-Form of Problems Test (after four months), Post-Form of Problems Test was administered as Delayed Problems Test to the two groups. In this part there were 4 expressions for which word problems were expected to be written by EG and CG. The following two tables were obtained in the repeated measures analysis of variance on achievement related to writing word problems for division expressions in which divisor is greater than the dividend (ARWWP-DGTD).

Table 4.93: Tests of Significance for Group and Achievement Related to Writing Word Problems for Division Expressions in which Divisor is Greater than the Dividend (ARWWP-DGTD) interaction, using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	1171.78	94	12.47	-	-
ARWWP-DGTD	118.56	2	59.28	4.76	0.011
Grp. By ARWWP-DGTD	268.11	2	134.06	10.75	0.000

Table 4.94: Tests of Significance for Group Main Effect on Achievement Related to Writing Word Wroblems for Division Expressions in which Divisor is Greater than the Dividend (ARWWP-DGTD), using unique sums of squares.

Source of Variation	SS	DF	MS	F	Sig. of F
Within cells	1108.13	47	23.58	-	-
Group	1092.67	1	10.92	46.34	0.000

As the tables 4.93 and 4.94 clearly demonstrate, the group main and ARWWP-DGTD main effects and interaction of group and ARWWP-DGTD are all significant.

In order to understand the significant differences observed as a result of averaged test of significance for ARWWP-DGTD, means of WWPT-DGTD scores across the two groups were indicated in the graph below.

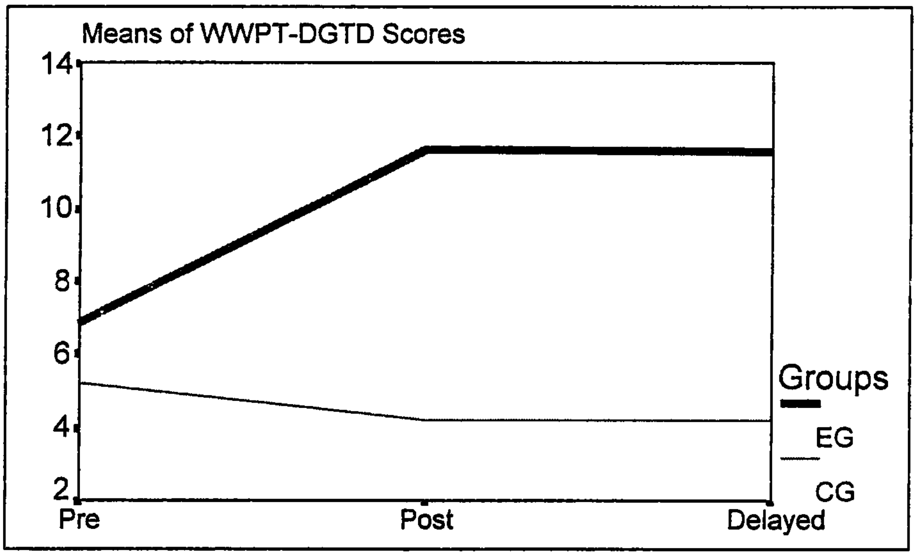


Figure 4.32: Pre-, Post-, and Delayed - WWPT-DGTD Mean Scores for Experimental and Control Groups.

As noticed in the graph, the significant difference between the two groups of ARWWP-DGTD might be due to the slight decrease of the control group at post testing stage and the gradual increase in the experimental group from pre testing to post testing. Although the experimental group remained nearly constant at the delayed posting stage the mean score of the group is still higher than the mean score of the control group. In the light of these findings there is sufficient evidence to reject hypothesis-45 and hypothesis-46. Thus it can be concluded that there are significant differences between mean post WWPT-DGTD and mean delayed WWPT-DGTD scores of subjects in the EG and CG. Means and standard deviations of experimental and control groups in the three measures of WWPT-DGTD are given in the following table.

Table 4.95: Means and Standard Deviations of Experimental and Control Groups in the Three Measures of WWPT-DGTD.

	Pre		Post		Delayed	
Groups	Mean	sd	Mean	sd	Mean	sd
EG	6.84	4.08	11.63	3.93	11.58	4.85
CG	5.20	4.08	4.24	3.68	4.24	3.38

4.2.4 Summary of the Results of Stage-2 Obtained by Statistical Testing

In the light of the findings obtained by statistical testing of the hypothesis, the following conclusions can be stated:

1. There is a significant difference between mean post CT scores of subjects in the experimental and control groups.

2. There is a significant difference between mean delayed CT scores of subjects in the experimental and control groups.

3. There is a significant difference between mean post PT scores of subjects in the experimental and control groups.

4. There is a significant difference between mean delayed PT scores of subjects in the experimental and control groups.

5. There is a significant difference between mean post WWPT scores of subjects in the experimental and control groups.

6. There is a significant difference between mean delayed WWPT scores of subjects in the experimental and control groups.

7. There is a significant difference between mean post CT1.1 scores of subjects in the experimental and control groups.

8. There is a significant difference between mean delayed CT1.1 scores of subjects in the experimental and control groups.

9. There is a significant difference between mean post CT1.2 scores of subjects in the experimental and control groups.

10. There is a significant difference between mean delayed CT1.2 scores of subjects in the experimental and control groups.

11. There is a significant difference between mean post CT2.1 scores of subjects in the experimental and control groups.

12. There is a significant difference between mean delayed CT2.1 scores of subjects in the experimental and control groups.

13. There is a significant difference between mean post CT2.2 scores of subjects in the experimental and control groups.

14. There is a significant difference between mean delayed CT2.2 scores of subjects in the experimental and control groups.

15. There is a significant difference between mean post CT3.1 scores of subjects in the experimental and control groups.

16. There is a significant difference between mean delayed CT3.1 scores of subjects in the experimental and control groups.

17. There is a significant difference between mean post CT3.2 scores of subjects in the experimental and control groups.

18. There is a significant difference between mean delayed CT3.2 scores of subjects in the experimental and control groups.

19. There is a significant difference between mean post CT4 scores of subjects in the experimental and control groups.

20. There is a significant difference between mean delayed CT4 scores of subjects in the experimental and control groups.

21. There is a significant difference between mean post CT5 scores of subjects in the experimental and control groups.

22. There is a significant difference between mean delayed CT5 scores of subjects in the experimental and control groups.

23. There is a significant difference between mean post CT6 scores of subjects in the experimental and control groups.

24. There is a significant difference between mean delayed CT6 scores of subjects in the experimental and control groups.

25. There is no significant difference between mean post PT1.1 scores of subjects in the experimental and control groups.

26. There is no significant difference between mean delayed PT1.1 scores of subjects in the experimental and control groups.

27. There is no significant difference between mean post PT1.2 scores of subjects in the experimental and control groups.

28. There is no significant difference between mean delayed PT1.2 scores of subjects in the experimental and control groups.

29. There is no significant difference between mean post PT1.3 scores of subjects in the experimental and control groups.

30. There is no significant difference between mean delayed PT1.3 scores of subjects in the experimental and control groups.

31. There is no significant difference between mean post PT2.1 scores of subjects in the experimental and control groups.

32. There is no significant difference between mean delayed PT2.1 scores of subjects in the experimental and control groups.

33. There is no significant difference between mean post PT2.2 scores of subjects in the experimental and control groups.

34. There is no significant difference between mean delayed PT2.2 scores of subjects in the experimental and control groups.

35. There is a significant difference between mean post PT2.3 scores of subjects in the experimental and control groups.

36. There is a significant difference between mean delayed PT2.3 scores of subjects in the experimental and control groups.

37. There is a significant difference between mean post PT2.4 scores of subjects in the experimental and control groups.

38. There is a significant difference between mean delayed PT2.4 scores of subjects in the experimental and control groups.

39. There is a significant difference between mean post WWPT-MULT scores of subjects in the experimental and control groups.

40. There is a significant difference between mean delayed WWPT-MULT scores of subjects in the experimental and control groups.

41. There is a significant difference between mean post WWPT-DIV scores of subjects in the experimental and control groups.

42. There is a significant difference between mean delayed WWPT-DIV scores of subjects in the experimental and control groups.

43. There is a significant difference between mean post WWPT-DLTD scores of subjects in the experimental and control groups.

44. There is a significant difference between mean delayed WWPT-DLTD scores of subjects in the experimental and control groups.

45. There is a significant difference between mean post WWPT-DGTD scores of subjects in the experimental and control groups.

46. There is a significant difference between mean delayed WWPT-DGTD scores of subjects in the experimental and control groups.

4.2.5 Interview Results Concerning Stage-2

After conducting the three paper and pencil instruments (in October 1997), as delayed tests, a group of preservice elementary teachers (N=7 from CG and N=8 from EG), who were previously interviewed for the Stage - 1 , were again interviewed in order to get more information about their final position, in terms of interpreting and applying decimals and compare the performances and understandings of EG and CG.

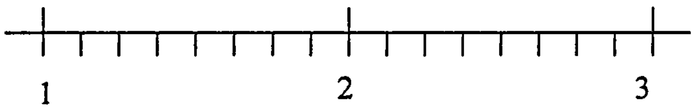
4.2.5.1 Interview Results Concerning Decimal Concepts

4.2.5.1.1 Interview Results Concerning Decimals as Points on Number Lines

In this part six (75%) of the eight interviewees from the experimental group who previously had several misconceptions about interpreting and applying decimals on number lines seemed to overcome all the related misconceptions. On the other hand only one of the interviewees from the control group seemed to overcome her misconceptions in this part, and four (57%) of them seemed to hold their previous misconceptions. Typical interview excerpts are given below (for a better comparison, generally, the interview excerpts for the experimental group were chosen from lower scorers and the interview excerpts for the control group were chosen from higher scorers):

An interview excerpt from the experimental group:

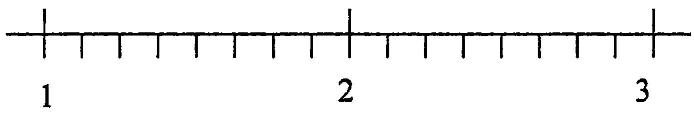
R: Please,mark a point on the given number line to represent the decimal number 2.4



Ç: Well, here we have 8 subunit marks. (thinking)...If there were 4, each would be 25.
R: GO on !
Ç: It will be somewhere just before the fourth subunit mark.

An interview excerpt from the control group:

R: Please,mark a point on the given number line to represent the decimal number 2.4



A: Generally, I just count and mark. Here it will be the fourth subunit mark.
R: Suppose that there were 10 subunit mark. Would it be the fourth, again ?
A: Now, the place of 2.4 will change on the line.

In the following table, the misconceptions in interpreting and applying decimals as points on number lines, corrected or held by the interviewees are presented.

Table 4.96: Misconceptions Overcome or Held by the Interviewees Related to Number Lines.

	After Pre-Test		After Del-Post.Test	
MISCONCEPTIONS	EG	CG	EG	CG
On a number line between two numbers there are 10 divisions each being 0.1 unit	4	3	0	1
Any decimal on a number line can be marked between the starting point and the first subunit mark	1	3	0	2
Decimals are based on subunit 10	1	3	0	1
Each subunit mark on a number line is 0.5 unit	0	1	0	0
Each division on a number line is 1 unit	0	2	0	1
Decimals are based on subunit 100	0	1	0	1
Place of a decimal number changes on a number line when the calibration changes	0	1	0	0
different subunits can be used on the same number line / model	0	1	0	0
Doubling the calibration of a number line doubles the value of each subunit mark	0	1	0	0

4.2.5.1.2 Interview Results Concerning Decimals on Shaded Areas

In this part five (63%) of the eight interviewees from the experimental group who previously had several misconceptions about interpreting and applying decimals on shaded areas seemed to overcome all the related misconceptions. Only one of the interviewees from the experimental group seemed to hold his previous misconceptions. On the other hand, only one of the interviewees from the control group seemed to overcome her misconceptions in this part, and five (71%) of them seemed to hold their previous misconceptions. Typical interview excerpts are given below (for a better comparison, generally, the interview excerpts for the experimental group were chosen from lower scorers and the interview excerpts for the control group were chosen from higher scorers):

An interview excerpt from the experimental group:

R: Please write the decimal for the given shaded area

O: The first one is just "1" and the other one is 4/6. The I can do 1+4/6 and find in terms of a decimal.

An interview excerpt from the control group:

R: Please write the decimal for the given shaded area

M: There are six parts on each. Mmm !. I will take each two parts as one unit. Then the first one will be 3 and the other one will be 3.2.
R: You may also estimate.
M: Let's say each part is one unit.
R: Can anyone else say one unit for 4 of them.
M: Yes.
R: Why ?
M: Because here there is no restriction.

In the following table the misconceptions, in interpreting and applying decimals on shaded areas, corrected or held by the interviewees are presented.

Table 4.97: Misconceptions Overcome or Held by the Interviewees Related to Area Models.

	After pre-Test		After Del.P.T	
MISCONCEPTIONS	EG	CG	EG	CG
Decimals are based on subunit 10	3	2	1	1
Decimals are based on subunit 100	0	1	0	0
Different units can be used on the same number line / model	1	0	0	0
A 1 unit area model can be treated as multiple one unit models	1	3	0	2
Zero in a decimal number is not a place holder	1	2	0	2
Multiple one unit area models can be treated as a single 1 unit model	1	0	1	0
Calibration of a model can be decreased without changing the size of each subunit area	0	0	0	0
Calibration of a model can be increased without changing the size of each subunit area	0	0	0	0
Some area models have no decimal representation	0	0	0	0

4.2.5.1.3 Interview Results Concerning Comparing Decimals

In this part four (67%) of the six interviewees from the experimental group who previously had several misconceptions about comparing decimals seemed to overcome all the related misconceptions. Only one of the interviewees from the experimental group seemed to hold his previous misconceptions. Two of the interviewees from the experimental group couldn't be observed for the comparing of decimals. On the other hand three of the interviewees from the control group seemed to hold their previous misconceptions in this part, and four of them who previously had no problems with the comparison of decimals, kept their positions. Typical interview excerpts are given below (for a better comparison, generally, the interview excerpts for the experimental group were chosen from lower scorers and the interview excerpts for the control group were chosen from higher scorers):

An interview excerpt from the experimental group:

R: Please, circle the biggest number among 3.521 3.6 3.75.
O: It is 3.75.
R: Why ?
O: Because 3.75 is the closest one to 4.

An interview excerpt from the control group:

R: Please, circle the bigger number : 0.75 and 0.7.
A: It will be 0.7.
R: why ?
R: Because 0.7 is closer to 0.8.
R: What about 0.135 and 0.20.
R: I can not say at first sight. First of all we should turn them into fractions.

In the following table the misconceptions in comparing decimals, corrected or held by the interviewees are presented.

Table 4.98: Misconceptions Overcome or Held by the Interviewees Related to the Comparison of Decimals.

MISCONCEPTIONS	After Pre-Test.		After Del.Post Test.	
	EG	CG	EG	CG
Zero in a decimal number is not a place holder.	1	1	0	1
A decimal is greater when the whole number after the decimal point is greater.	1	3	0	2
More digits after the decimal point makes a decimal smaller.	2	1	1	1
When the number of steps needed to round a decimal to a whole number is limited the number becomes larger.	2	1	0	0

4.2.5.1.4 Interview Results Concerning Denseness of Decimals

In this part six (75%) of the eight interviewees from the experimental group who previously had several misconceptions about interpreting and applying the denseness of decimals seemed to overcome all the related misconceptions. Two of the interviewees from the experimental groups who previously did not have any problem about the denseness of decimals kept their position. On the other hand three of the interviewees from the control group seemed to hold their previous misconceptions in this part. Two of them were good and kept their positions and the other two of them could not be observed in this part. Typical interview excerpts are given below (for a better comparison, generally, the interview excerpts for the experimental group were chosen from lower scorers and the interview excerpts for the control group were chosen from higher scorers):

An interview excerpt from the experimental group:

R: Please write a number which is between 0.63 and 0.64.
F: It is 0.63001.
R: Is there a limited number of numbers ?
F: I don't know, there are too many.
R: What do you mean by too many ?
F: Infinitely many !

An interview excerpt from the control group:

R: How many numbers are there in between 0.56 and 0.57 ?
N: There are 9.

R: Why ?
N: If we move the decimal points there are 9 numbers between 560 and 570.

In the following table the misconceptions, in interpreting the denseness of decimals, corrected or held by the interviewees are presented.

Table 4.99: Misconceptions Overcome or Held by the Interviewees Related to Denseness of Decimals.

	After Pre-Test..		After Del-Post.T..	
MISCONCEPTIONS	EG	CG	EG	CG
Decimals are based on subunit 10.	0	1	0	?
There is a limited number of decimals between two consecutive decimals.	4	1	0	1
There is no any number between two consecutive decimals.	2	2	0	1
The difference between two consecutive decimals is one unit.	0	1	0	1

4.2.5.1.5 Interview Results Concerning Unit Measures Involving Decimals

In this part, all (100%) of the eight interviewees from the experimental group who previously had several misconceptions about interpreting and applying unit measures involving decimals seemed to overcome all the related misconceptions. On the other hand four of the interviewees from the control group seemed to hold their previous misconceptions in this part. Two of them seemed to overcome the related misconceptions and one of them was good and kept this position. Typical interview excerpts are given below (for a better comparison, generally, the interview excerpts for the experimental group were chosen from lower scorers and the interview excerpts for the control group were chosen from higher scorers):

An interview excerpt from the experimental group:

R: What is 0.45 hours in terms of minutes ?
Ç: I can easily find by using a direct proportion.
R: Can we say it is 45 minutes.
Ç: No we can not.

An interview excerpt from the control group:

R: What is 0.45 hours in terms of minutes ?
M: One hour is 60 minutes....
R: Can we say it is 45 minutes.
M: Yes of course.

In the following table the misconceptions, in interpreting and applying unit measures involving decimals, corrected or held by the interviewees are presented.

Table 4.100: Misconceptions Overcome or Held by the Interviewees Related to Unit Measures Involving Decimals.

	After Pre-Testing		After Del.Post-Testing	
MISCONCEPTIONS	EG	CG	EG	CG
Decimal point is a seperator	8	6	0	3
The number after the decimal point may reperesent different units according to the subunit system of a given measure	8	6	0	4

4.2.5.1.6 Interview Results Concerning Operations Involving Decimals

In this part, all (100%) of the six interviewees from the experimental group who previously had several misconceptions about operations involving decimals seemed to overcome all the related misconceptions. Two of the eight interviewees was good in operations and they kept their positions at the final stage. On the other hand all (100%) of the six interviewees from the control group seemed to hold their previous misconceptions in this part. One of the seven interviewees, who previously had no problem seemed to keep this position. Typical interview excerpts are given below (for a better comparison, generally, the interview excerpts for the experimental group were chosen from lower scorers and the interview excerpts for the control group were chosen from higher scorers):

An interview excerpt from the experimental group:

R: Is the result of 35.67 ÷ 0.64, greater or less than 35.67 ?
Ç: It is greater.
R: Why ?

Ç: Because the divisor is less than 1.

An interview excerpt from the control group:

R: Is the result of 35.67 ÷ 0.478, greater or less than 35.67 ?
N: First of all, I move the decimal points, then we get 3567 and 478...(thinking)...so the result will be smaller.
R: Do you always turn the given decimal into whole numbers ?
N: Yes, in order to see which one is greater, I do this...mmm... In fact division makes smaller.

In the following table the misconceptions, in interpreting and applying operations involving decimals, corrected or held by the interviewees are presented.

Table 4.101: Misconceptions Overcome or Held by the Interviewees Related to Operations Involving Decimals.

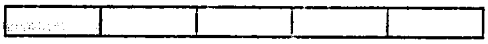
	After Pre-Testing		After Del.Post-Testing	
MISCONCEPTIONS	EG	CG	EG	CG
Division makes smaller.	6	6	0	6
Multiplication makes bigger.	6	6	0	6
Decimal point can be ignored.	2	3	0	3
Two decimals can not be multiplied if they have not the same number of digits.	0	2	0	1

4.2.5.2 Interview Results Concerning the Choice of Operation for Word Problems Involving Decimals

In this part, three (60%) of the five interviewees from the experimental group who previously had several misconceptions about operations involving decimals seemed to overcome all the related misconceptions. Two of the five seemed to keep their positions. One of the interviewees from the experimental group was good at choice of operation and kept this position at the end and another one from this group could not be observed. On the other hand all (100%) of the six interviewees from the control group seemed to hold their previous misconceptions in this part. One of the seven interviewees could not be observed. Typical interview excerpts are given below (for a better comparison, generally, the interview excerpts for the experimental group were chosen from lower scorers and the interview excerpts for the control group were chosen from higher scorers):

An interview excerpt from the experimental group:

R: Please , first, read the problem aloud and then choose the appropriate operation in which enables you to find the answer of the problem (A 0.65 kg of peanut is needed to be put into a box which has a 5 kg capacity, then how much of the box is filled ?)
O: First let's turn them into grams, as 650 and 5000, and let's draw a figure as follows:



O: If each one is 1 kg...
R: You don't have to find the result in an exact form.
O: Oh! Yes it will be 0.65 ÷ 5

An interview excerpt from the control group:

R: Please , first, read the problem aloud and then choose the appropriate operation in which enables you to find the answer of the problem (With 13.5 meter of a certain fabric it is possible to cover the chairs in your house. How much of the work can be done by only 0.82 meter of the same fabric ?)
A: I usually use direct proportion.
R: OK , go on.
A: First I divide 13.5 by 0.83 and then write the answer in terms of a percent.
R: Can we say that in a division expression the divisor is always smaller than the dividend *
A: No.
R: What are you thinking now ?
A: If the number of chairs were given, It would be easier for me.

In the following table the misconceptions, in choice of operations for word problems involving decimals, corrected or held by the interviewees are presented.

Table 4.102: Misconceptions Overcome or Held by the Interviewees Related to Choice of Operations for Word Problems Involving Decimals.

	After Pre-Testing		After Del.Post-Testing	
MISCONCEPTIONS	EG	CG	EG	CG
In a multiplication expression the operator should be a whole number.	3	2	0	2
In a quotitive division model the dividend should be greater than the divisor.	4	3	2	3
In a partitive division model the divisor should be a whole number.	1	2	0	2
In a partitive division model the dividend should be greater than the divisor.	0	1	0	1
In a partitive division model the dividend should be greater than the quotient.	1	1	?	1

4.2.5.3 Interview Results Concerning Writing Word Problems for Multiplication and Division Expressions Involving Decimals

In this part, four (57%) of the seven interviewees from the experimental group who previously had several misconceptions, in writing word problems for multiplication and division expressions involving decimals, seemed to overcome all the related misconceptions. Two of the seven was not good in writing word problems for multiplication and division expressions involving decimals and seemed to keep their positions. One of the interviewees from the experimental group could not be observed. On the other hand all (100%) of the seven interviewees from the control group seemed to hold their previous misconceptions in this part. Typical interview excerpts are given below (for a better comparison, generally, the interview excerpts for the experimental group were chosen from lower scorers and the interview excerpts for the control group were chosen from higher scorers):

An interview excerpt from the experimental group:

R: Please, write a problem for 0.26 ÷ 4
Y: Well, let me think.
R: OK, go on.
Y: In order to make a skirt, we need 4 meter of a certain fabric. How much of the work can be done by only 0.26 meter of the fabric ?

An interview excerpt from the experimental group:

R: Please, write a problem for 0.53 ÷ 1.4
G: It will be meaningless.
R: OK! What does make it difficult for you ?
G: Dividing a smaller by a bigger one is hard for me.
R: OK! Let's change it as 1.4 ÷ 0.53
G: How much of a 1.4 meter fabric can be used each being 0.53 meter ?...(pause)... In decimals I'm not good.

4.2.5.4 Other Interview Results Concerning Stage-2

During the interviews after the treatment we asked the question "How did you find the overall treatment ?" only to the interviewees from the experimental group. Typical responses were as follows:

Y: It was enjoyable, I think that this is better for students. Our previous knowledge about decimals became stronger.

O: On the performing the tasks related to decimals or just marking points on number lines, now I can use estimation.

N: It was great, I really enjoyed it.

In the following table we present the final status of each interviewee from the experimental and control groups, on the eight categories.

Table 4.103: Final Status of Each Interviewee from Experimental and Control Groups.

Interviewees		CAT1	CAT2	CAT3	CAT4	CAT5	CAT6	CAT7	CAT8
EG	OO	↑	↑	↑	↑	↑	↑	↑	↑
	YM	↑	↔	↓	↔	↑	↑	↑	↑
	NK	↑	↑	↑	↑	↑	↑	↑	↑
	FK	↔	↔	⊗	↑	↑	↔	↔	↓
	ÇY	↑	↓	⊗	↑	↑	↑	⊗	⊗
	GA	↔	↑	↑	↔	↑	↑	↓	↓
	OÖ	↑	↑	↑	↑	↑	↑	↓	↓
	SG	↑	↑	⊗	↑	↑	↑	⊗	↑
CG	MV	↔	↓	↔	↔	↓	↔	↓	↓
	AA	↔	↔	↔	↔	↑	↓	↓	↓
	NE	↓	↓	↔	↓	↔	↓	↓	↓
	UÖ	↓	↓	↓	↓	↓	↓	⊗	↓
	AY	↓	↓	↓	↔	↑	↓	↓	↓
	HD	↓	↓	↓	↓	↓	↓	↓	↓
	GS	↑	↑	↔	⊗	↓	↓	↓	↓

CAT1: Decimals as points on number lines.
CAT2: Decimals on shaded areas.
CAT3: Comparing decimals.
CAT4: Denseness of decimals.
CAT5: Unit measures involving decimals.
CAT6: Operations involving decimals.
CAT7: Choice of operation for word problems involving decimals.
CAT8: Writing word problems for multiplication and division operations involving decimals.
↔ : No misconception observed, previously, and the position is kept.
↑ : Previously observed misconceptions have been overcome.
↓ : Previously observed misconceptions have not been overcome.
⊗ : The final position couldn't be observed.

4.2.6 Discussion of the Results of Stage-2

When we go over the results of stage-2, it seems that the Conceptual Change Instruction (CCI) is effective in overcoming preservice elementary teachers' misconceptions in interpreting and applying decimals. Details will be discussed in the following paragraphs.

In the first main dimension of the study which was called as "Decimal Concepts" we tried to see if the CCI was effective on achievement related to 1) decimals as points on number lines, 2) decimals on shaded areas, 3) denseness of decimals , 4) comparing of decimals, 5) unit measures involving decimals, and 6) multiplication and division operations involving decimals. As it is seen in figure 4.15 the mean scores of the experimental group on post and delayed -testing stages related to decimal concepts in general are greater than the mean scores of the control group. The experimental group kept its previous performance (post testing) after four months whereas the performance of the control group declined. In addition to that when we go over table 4.103 it is possible to say that the overall achievement of the experimental group subjects are better than control group subject and they mainly have overcome their previously held misconceptions. None of the interviewees from the control group have overcome their previously held misconceptions related to decimal concepts. Therefore, CCI is effective in overcoming preservice elementary teachers' misconceptions in decimal concepts. In 1982, Bell conducted a study on middle school students to see how conflict teaching can improve their performances and help them to overcome their misconceptions in similar topics like place value, operations, units, and etc. He observed an increase in the overall performance of the students from pre to post and from post to delayed testing measurements. He noted that most of the students had overcome their misconceptions.

In all the sub-dimensions we mentioned above about the decimal concepts, (number lines , area models, comparison, denseness, and etc.) we observed that the experimental group was better than the control group. In interpreting decimals on number lines all of the interviewees from the experimental group had overcome their previously held misconceptions but in the control group only one of the interviewees had overcome her previously held misconceptions. Therefore CCI is more effective in overcoming misconceptions related to interpretation and application of decimals as points on number lines. Similar findings was observed by Bell (1982). He noted that the percentage of students achieving the success criterion in number lines increases from pre to post and from post to delayed post testing as 44%, 56%, and 61% respectively. In area, models only one of the interviewees from the experimental group kept the previously held misconceptions. On the other hand only one interviewee from the control group managed to overcome her previously held misconceptions. Therefore we can conclude that the CCI is also effective in overcoming the preservice elementary teachers' misconceptions in interpreting and applying decimals on area models. The subunit based on ten and not based on ten didn't differ, the experimental group was still better than the control group.

In comparing decimals, as it is seen in figure 4.23, this time the overall performance of the experimental group continued to increase slightly from post to delayed testing. On the other hand, although, the mean score of the control group increased a little bit from pre to post testing, it suddenly fell from post to delayed testing. Again if we look at table 4.103, it is seen that only one of the five interviewees who previously held misconceptions about comparing of decimals still has the related misconceptions. On the other hand, all of the three interviewees from the control group are still holding their misconceptions. Therefore we can conclude that CCI is effective in overcoming the preservice elementary teachers' misconceptions in comparing decimals. Similar observations were made by several researchers. Owens (1981) found that after receiving conceptually oriented instruction, 13 of the 16 students interviewed answered correctly to "which is larger, 2.45 or 2.5 ?." Wearne and Hiebert (1989) reported that after conceptually based instruction on decimal fractions with concrete referents, students in grade 4 were able to indicate equivalence or order.

In interpreting the denseness of decimals, as it is seen in figure 4.22, the overall achievement related to denseness of decimals of the experimental group is still higher than the overall achievement of the control group. Throughout the study, we observed that the performance of the experimental group on the tasks that conceptual understanding was needed, was much more higher than the performance of the control group. Denseness of decimals is a typical example for this case. Again, table 4.103, shows that all the interviewees from the experimental group have overcome their previously held misconceptions but none of the interviewees from the CG have overcome their previously held misconceptions in denseness of decimals. Therefore the CCI is effective in overcoming misconceptions about the denseness of decimals. Bell (1982) reported similar findings.

In dealing with unit measures involving decimals, as it is seen in figures 4.20 and 4.21, the achievement of the EG is still better than the achievement of the CG. This time it is possible to say that, although the EG is better than the CG at all of the occasions, since there is a slight decrease from post to delayed testing, EG itself is better on the unit conversions which are parallel with base 10 numeration system than the conversions which are not parallel with the base 10 numeration system (like subunit based on 60 or 12). This is due to the overuse of base 10 numeration system. When we look at table

4.103, it is observed that all of the eight interviewees from the EG, who previously had some problems, have overcome their misconceptions. On the other hand in the CG, only two of the six interviewees have overcome their misconceptions. Therefore the CCI is effective in overcoming misconceptions in unit conversions involving decimals. Similar findings were observed by many researchers. Bell (1982, p.418) reported that most of the student after conceptually based conflict teaching periods, had overcome their previously held misconceptions related to units involving decimals. Bell, Swan, and Taylor (1981) interviewed, tested, and then applied conflict teachings with 15 year olds of average ability on operations and applications with decimals. They found that the students often misread the decimal; for example, 11.9 miles per hour was read as 11 miles 9 minutes per hour. With the instruction, they found dramatic improvements in understanding of units and decimal place value.

In multiplication and division operations involving decimals, although it is not that sharp for the EG, as it is seen in figure 2.24, the mean scores of both of the groups declined from pre to post testing. In the pre-testing period, although we asked the preservice elementary teachers just to estimate the results of the operations, whenever we checked the test papers we observed that most of them tried to calculate the results. Therefore in this case we can treat the post test scores as the real prerequisite standing of both of the groups. As it is seen again in figure 4.23, the mean score of the EG group is higher than the mean score of the CG. As it is also seen in table 4.103, all of the 8 interviewees from the EG have overcome their previously held misconceptions but none of the 7 interviewees from the CG who previously had problems have overcome their misconceptions in multiplication and division operations involving decimals. Therefore, the CCI is effective in overcoming misconceptions in multiplication and division operations involving decimals. Similar observations were made by many researchers. Tirosh (1986) aimed to diagnose and correct (n=59) some college students' misconceptions about the operation of division, using a diagnostic computer program . He reported that many of the student had overcome their misconceptions about the operation of division. Similar findings were also put forth by Bell (1982). He reported that most of the students, after the conflict teachers sessions, were able to detect that when one multiplies or divides by a decimal less than 1, the results might be different than whole number domain and did not agree with the beliefs "multiplication makes bigger" and division makes smaller." Onslow (1990) used a game based on conflict discussions, on (n=18), 12 and 13 year-old students, to overcome the misconceptions "multiplication makes bigger" and division makes smaller." He reported that the results demonstrated that students participating in discussions were far more successful in overcoming the stated misconceptions than other students who were not involved in the study. In this present study from the test results and interview results it is observed that the number of the subjects, especially from the EG, who previously held the belief "division always makes smaller" and "multiplication always makes bigger" has decreased especially from pre test to post test. Similar observations were made by Tirosh and Graeber (1990, p.99) in evoking cognitive conflict to explore preservice teachers thinking about division. They stated that the total number of errors as well as the number consistent with the misconception "division always makes smaller" declined from pretest to posttest.

In the choice of operation, as it is seen in figure 4.25, the overall performance of the EG is better than the overall performance of the CG, but when we went in detail we observed that in many of the sub-dimensions like: choosing the appropriate operation for multiplication word problems, choosing the appropriate operation for multiplication word problems suitable for direct proportion, choosing the appropriate operation for multiplication word problems not suitable for direct proportion, choosing the appropriate operation for division word problems, and choosing the appropriate operation for division word problems suitable for direct proportion, no significant difference was observed between the EG and CG. This time, all the above sub-dimensions were suitable for some procedures and most of the subjects especially from the CG, used direct proportions in choosing the appropriate operations for the given word problems. Therefore no significant difference was observed in these sub-dimensions. On the other hand, in the last two dimensions which were not suitable for some simple procedures, we observed significant differences between the EG and the CG. In choosing the appropriate operation for division word problems not suitable for direct proportion, as it is seen in figure 4.26, the mean scores of the EG are higher than the mean scores of the CG. Later as it is seen in figure 4.27, in choosing the appropriate operation for division word problems in which the divisor is greater than the dividend, again the mean scores of the EG are higher than the control group. Although, as a result of the statistical testing the main differences between the two groups were due to the differences in the last two sub-dimensions for the choice of operation, this does not guarantee that the preservice elementary teachers have no misconceptions in any of the other sub-dimensions. As it is seen in table 4.103, three of the five interviewees from the EG totally have overcome their previously held misconceptions but none of the interviewees from the CG have overcome their previously held misconceptions related to the choice of operations in word problems. Therefore CCI is effective in overcoming misconceptions related to the choice of operations in word problems. Similar findings were observed by some researchers. Tirosh and Graeber (1990) in evoking cognitive conflict to explore preservice teachers thinking about division, observed that when the conflicting approach was carefully applied, preservice teachers might form more accurate conception about the relative size of the quotient and the dividend and improve their performance in writing expressions for multiplication and division word problems. In this present study, we also observed that the preservice elementary teachers' performance in the choice of operations for word problems continued even after 4 months. Swan (1983) post-tested his subjects 3 moths after the intervention and found that the effects of the "conflict teaching" approach endured over this period. We can briefly say that CCI is also effective in building a conceptual understanding, which in turn empowers the preservice elementary teachers to fight with their misbeliefs or misunderstandings and replace them with more plausible or meaningful concepts. In the literature there are many researchers who gave great emphasis to conceptual understanding (Aksu, 1997; Stoddart et al., 1993; Nancy, 1990).

In writing word problems for division and multiplication expressions involving decimals, as it is seen in figure 4.28, the mean score of the EG increases from pre to post test and then keeps its previous performance even at the delayed testing stage. On the other hand, the mean scores of the of the CG are almost the same at all the three measurements. At the post and delayed testing stages the overall achievements of the EG group are higher than the overall achievements of the CG. As it is also seen in table 4.103, four of the seven interviewees from the EG totally have overcome their previously held misconceptions but none of the interviewees from the CG have overcome their previously held misconceptions related to writing problems for division and multiplication operations involving decimals. Therefore, CCI is effective in overcoming misconceptions related to writing problems for division and multiplication operations involving decimals. As we mentioned before this category had four sub-dimensions as, 1) writing word problems for multiplication expressions involving decimals, 2) writing word problems for division expressions involving decimals, 3) writing word problems for division expressions involving decimals in which the divisor is less than the dividend, and 4) Writing word problems for division expressions involving decimals in which the divisor is greater than the dividend. Although in the 3rd sub-dimension (see figure 4.31) the overall performance of the EG group was nearly the same at the three measurements. This was due to the simple structure of the given expressions (24 ÷4 and 0.83 ÷0.32). In fact, mean scores of the EG at this sub-dimension were still higher than the mean scores of the CG at post and delayed testing stages. In all the other sub-dimensions (see figures 4.29, 4.30, 4.32) it is possible to observe relative increases in the mean scores of the EG, from pre to post, whereas we observe dramatic decreases in the mean scores of the control group from pre to post and from post to delayed testing. Generally in all these dimensions, the performance of the EG remains constant from post to delayed testing stages, which may enlighten the retention effect of CCI.

In writing word problems for division and multiplication expressions involving decimals, as we noticed before, it is possible to see the effect of all kind of previously held misconceptions (e.g., decimal points is a separator, multiplier should be a whole number, divisor should be greater than the dividend, and etc). This dimension seemed to be the most appropriate area to observe the preservice elementary teachers' interpretation and application of decimals, because this was not just a simple problem writing activity; they also showed all their capabilities, misconceptions, and overall conceptualisations. For this reason it is possible to say that high scores in this dimension can guarantee that the preservice elementary teachers increased their conceptual understanding of decimals. The following sentences taken from the related literature are consistent with our statement. Ellerton (1986) and Pegg & Davey (1991) treat problem writing as a way of extenting students' understanding of important mathematical ideas. Hashimoto & Swada (1984); Shimada (1977) and Silver & Cai (1993) consider problem writing as a means of improving students' skills in problem solving. Ellerton (1986) and Krutetskii (1976) consider problem writing as a way of investigating students' difficulties and mathematical abilities. In this present study during the treatment sessions empowering the problem writing activities with conflicting situations and intensive class and group discussions seemed to equip the EG subjects in a better way. Once again we may say that the CCI is not only effective in overcoming misconceptions in writing word problems for division and multiplication expressions involving decimals, but it is also effective in overcoming any kind of misconceptions of preservice teachers and students. The following excerpts from the literature are in line with that idea. In 1971, West stated that diagnostic teaching looked at the errors children made and subsequently structured the learning experiences so the errors would be eliminated. Onslow (1990) stated that discussions provide an opportunity for students to say what they mean and mean what they say. Later he noticed that cognitive conflict, describing a situation which appeared contradictory to a child's logical structure, provided the focus for discussions and such discussions exposed children's errors and encouraged them to face their mistake in a positive manner and made constructive use of them. Behr and Harel (1990) stated that it was likely that no learning took place unless some degree of conflict existed. According to Postner et.al (1982) the phase of conflict of dissatisfaction with existing concepts is central to the process of conceptual change: only at this stage will students realise that they must replace or reorganise their central concepts, because they are inadequate in allowing him to grasp some new phenomenon.

CHAPTER V

CONCLUSIONS AND IMPLICATIONS

The following chapter presents conclusions related to the results that were reported in the previous chapter and their implications.

5.1 Conclusions

The main purpose of the study was to determine preservice elementary teachers' misconceptions in interpreting and applying decimals and then to explore and analize the effects of the Conceptual Change Instruction in overcoming the misconceptions of preservice elementary teachers in interpreting and applying decimals.

As it is understood from the purpose, the study was conducted in two stages. In the first stage of the study, we mainly tried to determine preservice elementary teachers' misconceptions related to place value, operations involving decimals, comparison of decimals, denseness of decimals, unit measures involving decimals, choice of operations for word problems, and writing word problems for division and multiplication expressions involving decimals.

The test and interview results showed that most of the preservice elementary teachers rely heavily on the domain of whole numbers and base 10 numeration system. For example in interpreting and applying decimals as points on number lines, most of the preservice elementary teachers usually counted each subunit mark on the given number line as 1 tenth and for example interpreted 2.4 on a number line as, two and four subunits when subunits were not based on ten. These observations are consistent with previous research results (Tirosh and Graeber, 1990; Thipkong and Davis, 1991; İşeri, 1997). Lack of estimation in interpreting decimals on number lines, apparently, led most of the preservice elementary teachers to use only some algorithmic procedures in plotting a point on a number line which generally led to faulty results. Bell (1982) points out the importance of estimation in interpreting and applying decimals. Reliance on whole number domain and base 10 numeration system caused most of the preservice elementary teachers to ignore the decimal points, zeros on the left of the decimal points and calibration of the given number lines. For example, some of the interviewees stated that any decimal on a number line can be marked between the starting point and the first subunit mark. On the other hand two of the interviewees neglected the zeros in 0.5 and 0.6 in order to treat decimals as whole numbers. Some of the preservice elementary teachers also failed to connect their fractions and decimals knowledge. For example one of the interviewees interpreted 2.4 as 4/2. A similar finding was observed by İşeri (1997). Some of the misconceptions seemed to grow stronger or led to new misconceptions. This idea seems to be valid if we look at concept development from a constructivist point of view. Fisher (1985, p.53) states: "misconceptions sometimes involve alternative belief systems comprised of logically linked sets of proportions that are used by students in systematic ways."

In summary. in interpreting and applying decimals as points on number lines, the preservice elementary teachers, in this present study held the following misconceptions:

- On a number line between two numbers there are 10 divisions each being 0.1 unit
- Any decimal on a number line can be marked between the starting point and the first subunit mark
- Decimals are based on subunit 10
- Each subunit mark on a number line is 0.5 unit
- Each division on a number line is 1 unit
- Decimals are based on subunit 100
- Place of a decimal number changes on a number line when the calibration changes
- Different subunits can be used on the same number line / model
- Doubling the calibration of a number line doubles the value of each subunit mark

In interpreting and applying decimals on shaded areas, generally, preservice elementary teachers treated separate two - 1 unit models as a single one unit model and counted the number of parts in all for the denominator and counted the number of shaded areas for the numerator. This was another version of calibration problem applied to area models. This may be due to the overuse of proper fractions in elementary and middle school ages. In marking or shading the squares/rectangles for the given decimal, the influence of base 10 numeration system was in charge. For example, five of the interviewees in shading 1.08 on a two - 1 unit model, ignored zero and treated 1.08 as 1.8 to make it suitable for a subunit based on ten system. As we mentioned before, mere use of procedures, which do not rely on conceptual bases, or some misconceptions, may cause faulty generalisations. For example an interviewee in shading 0.6 on a 1-unit model, where the subunit was based on eight, wrote 0.6 as 6/10=3/5 and then concluded that 3 subunit areas were additional, by shading 3 of the five subunit areas. This is an over-ignorance of the given calibration. In interpreting decimals on shaded areas, the overall performance of the preservice elementary teachers were lower when compared with the number lines. Giving more emphasis to number lines (especially in Northern Cyprus) than area models may be a reason for this.

In summary, in interpreting and applying decimals as points on area models, the preservice elementary teachers, in this present study held the following misconceptions:

- Decimals are based on subunit 10
- Decimals are based on subunit 100
- Different units can be used on the same number line / model
- A 1 unit area model can be treated as multiple one unit models
- Zero in a decimal number is not a place holder
- Multiple one unit area models can be treated as a single 1 unit model
- Calibration of a model can be decreased without changing the size of each subunit area
- Calibration of a model can be increased without changing the size of each subunit area
- Some area models have no decimal representation

In comparing decimals, as in other cases, preservice elementary teachers, generally, treated decimals as whole numbers. For example in comparing 3.521, 3.6, and 3.75, five of the interviewees chose 3.521, defending themselves by stating that a decimal was greater when the whole number after the decimal point was greater. This finding is consistent with previous research results (Sackur-Grisvard and Leonard, 1985; Carpenter et al., 1981; İşeri, 1997). Some of the preservice elementary teachers, on the other hand, thought that thousandths were smaller parts than hundredths; then they inferred that longer decimals must have lower values. Accordingly, the preservice elementary teachers' comparison of decimals in this present study were affected by these misconceptions and therefore they gave many mistaken answers.

In summary, in comparing decimals, the preservice elementary teachers, in this present study held the following misconceptions:

- Zero in a decimal number is not a place holder.
- A decimal is greater when the whole number after the decimal point is greater.
- More digits after the decimal point makes a decimal smaller.
- When the number of steps needed to round a decimal to a whole number is limited the number becomes larger.

In interpreting the denseness of decimals, preservice elementary teachers again relied heavily on the domain of whole numbers and base 10 numeration system and most probably for this reason they either stated that there was no number between two consecutive decimals or said "there are 9 numbers" or "there are 10 numbers". We can specifically conclude that, the continuous aspect of decimals have not yet developed fully in preservice elementary teachers' minds. In summary, in interpreting the denseness of decimals, the preservice elementary teachers, in this present study held the following misconceptions:

- Decimals are based on subunit 10.
- There is a limited number of decimals between two consecutive decimals.
- There is no any number between two consecutive decimals.
- The difference between two consecutive decimals is one unit.

Nearly, in all the dimensions we observed that the preservice elementary teachers were treating the decimal point as if it were a separator, but in dealing with unit conversions involving decimals we met an overuse of the decimal point as a separator. For example 35% of the preservice elementary teachers in converting 1.15 hours into minutes interpreted 1.15 hours as 1 hour (60 minutes) + 15 minutes = 75 minutes. During the treatments some of the preservice elementary teachers noticed that digital watches might cause such over generalisations. Although many of the preservice elementary teachers were more successful on the unit conversions with the base 10 numeration system, such as length and weight, it seems that they were also affected by the misconception "*decimal point is a separator*" because although it was based on base 10 numeration system, one of the interviewees in converting 3.83 km into metres, read 3.83 as *3 kilometres, 8 metres, and 3 cm*. Briefly, many of the preservice elementary teachers could not convert from one system to another because of familiarity with working with the base 10 numeration system. Therefore, the work done with the operations in the base 10 numeration system may cause problems when students learn to deal with units of 60 and 12 such as minute or hours and months, and lead the students to treat the decimal point as a separator.

In summary, in unit conversion involving decimals, the preservice elementary teachers, in this present study held the following misconceptions:

- Decimal point is a separator.
- The number after the decimal point may represent different units according to the subunit system of a given measure.

Extensive work on the whole numbers domain, seemed to lead most of the preservice elementary teachers to conclude that "multiplication makes bigger" and "division makes smaller". In this present study, approximately 72% of the preservice elementary teachers held those misconceptions either implicitly or explicitly. Form time to time, in order to go in line with their misconceptions some of the preservice elementary teachers either moved the decimal points or totally ignored them in the given operations. Therefore, we can conclude that some of the preservice elementary teachers could not extend their understanding of whole number operations to fractions and decimals.

In summary, in interpreting multiplication and division operations involving decimals, the preservice elementary teachers, in this present study held the following misconceptions:

- Division makes smaller.
- Multiplication makes bigger.
- Decimal point can be ignored.
- Two decimals can not be multiplied if they have not the same number of digits.

In choosing the appropriate operation for multiplication word problems involving decimals, the preservice elementary teachers seemed to be influenced by the primitive implicit models which impose the constraint that the multiplier must be an integer. They were better in the multiplication problems that confirmed to the primitive multiplication model. Twenty percent of the interviewees seemed to hold the belief "in a multiplication expression the operator should be a whole number". Decimals less than one were another source of difficulty for preservice elementary teachers. Some of the preservice elementary teachers for this reason tended to use division instead of multiplication which may have been due to the belief that the answer had to be smaller than the first number, combined with the misconception "to make a number smaller you must divide".

Throughout the interviews we observed that nearly 70% of the interviewees gave only partitive interpretation of division. This might be a reason for their better performance on partitive division word problems. Some of the preservice elementary teachers seemed to hold the belief "divisor must be smaller than the dividend". For example, 40% of the preservice elementary teachers reversed the roles of the divisor and the dividend.

In choosing the appropriate operation for word problems involving decimals, we observed that some of the preservice elementary teachers without fully understanding the given problems searched for some cue words or used direct proportion which many times led them to choose inappropriate operations. In some cases, during the interviews, although we replaced the decimals by whole numbers some were still failing, probably, due to a lack of conceptual understanding in the choice of operations.

In summary, the preservice elementary teachers mainly seemed to be influenced by the primitive division and multiplication models and held the following misconceptions:

- In a multiplication expression the operator should be a whole number.
- In a quotitive division model the dividend should be greater than the divisor.
- In a partitive division model the divisor should be a whole number.
- In a partitive division model the dividend should be greater than the divisor.
- In a partitive division model the dividend should be greater than the quotient.

In writing word problems for multiplication expressions, repeated addition was the most preferred embodiment by the preservice elementary teachers when one of the numbers involved was a whole number in the given expression. This goes parallel with the multiplication problems in the problems test. Therefore, we can conclude that the effect of primitive multiplication model continues for problem writing. The preservice elementary teachers in this present study were more successful in writing problems for expressions with dividends greater than the divisors than they were in writing problems for expressions with dividends less than the divisors. In addition to that the number of written problems which led to reversals shows that some of the preservice elementary teachers were affected by the constraints of the primitive division models. Whenever the structure of the given expressions (also including decimals less than 1) became more complex, the preservice elementary teachers problem writing performance declined. For example, 12% of the problems written for multiplication expressions and 28% of the problems written for division expressions occurred in very weak contexts. Briefly, the present study showed that in problem writing, the preservice elementary teachers were both implicitly or explicitly influenced by the constraints of the primitive models. In problem writing, it was possible to observe many of the previously observed misconceptions. Apart form the mere presence of decimals which in many cases made the work harder, it is obvious that some of the preservice elementary teachers were also weak in problem writing for expressions involving only whole numbers. In the interviews, all of the interviewees stated that they had never tried to write problems for given expressions or posed any kind of problem, so this affected their overall problem writing performance beyond the decimals.

The preservice elementary teachers in this study exhibited serious short-comings in their understanding of decimals and fractions. Most of them seemed to have appropriate knowledge of algorithms associated with decimals, but many important connections seemed to be missing. These findings are consistent with previous research results (Ball, 1990; Simon, 1993). Therefore, many teacher candidates enter elementary education seriously deficient in understanding the mathematical content they will be expected to teach students (Stoddart et al., 1993)

When we go over the table in Appendix-F, it is possible to observe slight increases in the number of misconceptions coming from the dimensions which demand conceptual understanding rather than procedural. As one notices, the dimensions which are related to *denseness of decimals* , *unit measures, operations,* and *word problems* are much more dense in terms of misconceptions observed compared to the dimensions *number lines, area models,* and *comparison of decimals*.

Although most of the preservice elementary teachers could calculate correctly, they had important difficulties with the meaning of operations of decimals indicating a narrow understanding of the concepts underlying the procedural knowledge. Their understanding appeared to comprise remembering the rules for specific cases. Division and multiplication with decimals and fractions is often taught in a procedural manner in schools. Ball (1990) notices similar observations. Most of the preservice elementary teachers probably learned to divide or multiply with decimals without necessarily thinking about what the problems meant. When the preservice elementary teachers tried to write word problems for the given expressions, most of them failed. They seemed to be unable to connect the symbolic computations with real world contexts (Simon, 1993). Aksu (1997) in observing the performance of students in dealing with fractions, stated that the emphasis in mathematics teaching was still primarily on computation and instrumental understanding (p.379).

Weak conceptual understandings of most of the preservice elementary teachers provides an assessment of elementary school, middle school, and high school mathematics. It seems that schools provide students with procedural knowledge which is sparsely connected.

The preservice elementary teachers in this study, dramatically, improved their understanding of decimals after being taught through CCI. Similar findings were reported by Stoddart et al (1993, p.238).

There was maintained improvement in understanding decimal place value and decimal notation, comparison of decimals, denseness of decimals, unit measures, and operations involving decimals which were under the heading of Decimal Concepts. Therefore once more we can say that CCI is effective in overcoming misconceptions related to decimal concepts in general. There are many researchers who defend CCI or teaching approaches similar to CCI in overcoming misconceptions related to decimal concepts. Onslow (1990) stated that students dealing with numbers between zero and one needed to interpret multiplication as a scalar factor, not merely as a process of repeated addition. He later stated that in order to overcome the misconception "multiplication always makes bigger", conflict situations had to be devised. Simon (1993) stated that having students talk and write about how they create or recognise equivalent fractions and decimals and how the idea of equivalence was applied in solving various problems could strengthen their understanding.

In the choice of operations, although the CCI was mainly effective, over use of procedural techniques (e.g., direct proportion) especially by the CG subjects masked their previously observed misconceptions and through statistical testing we found out that the differences between the two groups were mainly due to the differences at two dimensions (choice of operation for division word problems which are not suitable for direct proportion and choice of operation for division word problems in which the divisor is greater than the dividend) which required conceptual understandings. This of course, statistically shows that CCI is effective on building conceptual understanding. In this case we observed that although the written test could mask the misconceptions, the interviews done on the choice of operation showed that the subjects from the CG group still held many of the misconceptions even in the dimensions that did not really required conceptual understanding. Therefore, we can conclude that CCI is effective in overcoming misconceptions related to the choice of operation by improving the preservice elementary teachers' conceptual understandings. Many researchers value CCI and similar approaches in building conceptual understanding and overcoming misconceptions (Başer, 1997; Eisenhart et al., 1993; Stoddart et al., 1993; Eryılmaz, 1992; Bell, 1982). The CCI was effective in helping the preservice elementary teachers become aware of their tendency to reverse the role of divisor and dividend. The EG subjects identified the strategy of estimating answers as the one that was most helpful to them in monitoring their work.

In writing word problems, the EG subject mean scores were much greater than the mean scores of the CG subjects. In this category, it is not enough to say that CCI is effective in overcoming misconceptions related to problem writing because we also checked how the preservice elementary teachers wrote high quality problems and the statistical testing and the interviews revealed that CCI is also effective in improving the preservice elementary teachers' high quality problem writing skills. Although the EG group subjects performance in problem writing was much greater than the CG subjects, the interview results revealed that some of the preservice elementary teachers showed modest increases from pre to post and from post to delayed testing. This reminds us that problem writing is an important topic and there is a need to start from early ages.

Generally, EG subjects maintained improvements or kept their post test performances even at the delayed post testing stage. This shows that CCI is also significantly successful in the long term effect on building conceptual understanding. This is consistent with previous research results. Perso (1992), after using diagnostic conflict teaching to overcome several misconceptions related to algebra of high school students stated that treatment was significantly successful , both in the short term and more importantly in the long term. Many other researchers reported similar results about the retention effect of CCI or similar strategies (Bell, 1982; Stoddart et al, 1993).

Although, the CG subjects were informed about the overall misconceptions related to decimals, it seemed that this was not enough for a better understanding. For example Flavell (1977) stated that awareness of misconceptions was not a sufficient condition for better conceptualisation and improved performance.

In some cases, in order to defend our findings we gave examples from elementary school or middle school students. According to many researchers this did not make a great difference. For example Fischbein and Schnarch (1997) stated that some misconceptions grew even stronger with age.

Within the limits of this study, it can be concluded that CCI has a high probability of overcoming misconceptions in interpreting and applying decimals and building a conceptual understanding of decimals in the short and long term. When we go over the results of Stage-2 in many cases it was possible to observe that the standard deviations of the EG were less than the standard deviations of the CG, this also shows that CCI is also effective in homogenising the groups.

5.2 Implications for Practice

In light of the present findings, it is possible to say that a new approach is required to prepare individuals to teach mathematics in elementary schools. What is needed is an intensified focus on pedagogy. .

Teachers tend to teach as they were taught and expect students to learn as they learned (Lortie, 1975). In teacher education programs, preservice teachers should be helped to develop ideas about conceptual change in learning. Teacher educators must realize that their students have conceptions about teaching and learning that are different from those which the teacher educators hold. Teacher educators should work on changing these students' misconceptions.

Among the most important learning outcomes ***curriculum developers*** should/may address, are the following:

1. A conceptual change view of learning.

2. Knowledge of generic strategies useful in achieving conceptual change.

3. Knowledge of common misconceptions for several important topics and specific strategies for changing them.

4. Skill in selecting and adapting curriculum materials based on common preconceptions held by students.

5. Skill in diagnosing student conceptions and recognizing them from student responses.

6. More math content and method courses should be added to the teacher education programs considering all the misconceptions observed in the related literature.

Several approaches that ***teachers*** may try are as follows:

1. Start with students' ideas and devise teaching strategies to take some account of them.

2. Provide more structured opportunities for students to talk through ideas at length, both in small group and whole class discussions.

3. Begin with known and familiar examples.

4. Explanations of any links between new information and prior knowledge should be made in a variety of ways so that learners are presented with visual, verbal and/or a diagrammatic format of the principles to be taught.

5. Whenever new concepts or definitions are to be introduced, teachers should provide significant numbers of examples and non-examples.

Several approaches that teachers and teacher educators may try in teaching decimals are as follows:

1. Build a strong understanding of decimal concepts before proceeding to computation and application.
2. Encourage students to estimate before doing computation and applications with decimals especially at operations and problem settings.
3. Use both decimal notation and common fraction notation to perform the same calculation.
4. Instruct preservice teachers and students to be familiar also with the quotitive division model.
5. Relate decimal notation to concrete embodiments and to currency notation.
6. Encourage students to write, starting from structured to totally free type of problems and discuss with their friends and with the whole class.

5.3 Implications for Research

1. Subjects of this study were preservice elementary teachers. It is also important to conduct research on smaller grades and compare the results.

2. Subjects of this study was small (at stage-1: 72, at stage-2:49), so a similar study with a larger sample would enable us to conduct factor analysis and path analysis in order to gain more information.

3. This study showed that preservice elementary teachers also had some problems related to fractions. So it is important to conduct similar studies about fractions.

4. Follow up studies can be done in order to observe how preservice elementary teachers can help their students to overcome their misconceptions.

5. Similar studies should be conducted with other content areas and students intuitive misconceptions should be discussed and corrected.

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APPENDICES

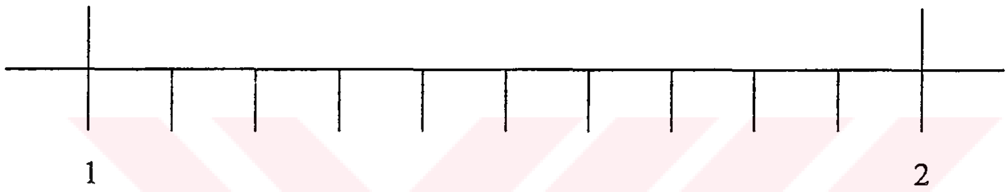
Appendıx - A

Concepts Test (For Pre-Testing)

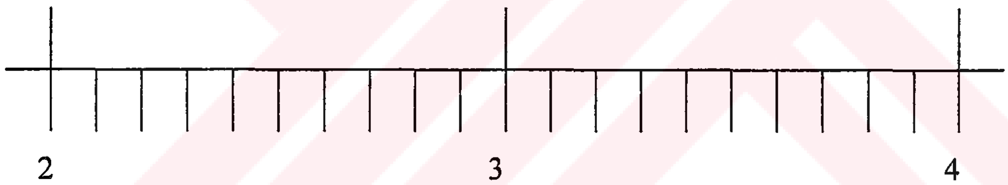
İsim / Soyadı:		Sınıf:	
Cinsiyet :		Tel.No :	
Tarih:			

I. Aşağıdaki ondalık sayıların verilen sayı doğruları üzerindeki yerlerini uygun yere nokta (.) işareti koyarak belirtiniz.

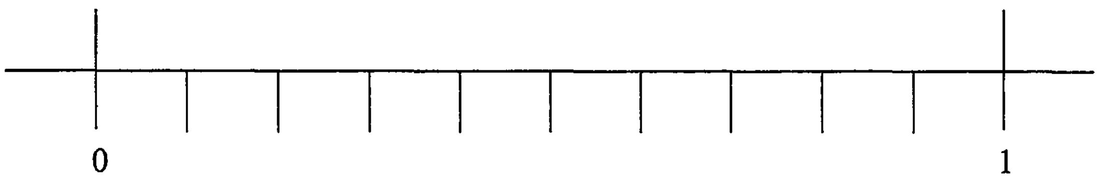
1) 1.25



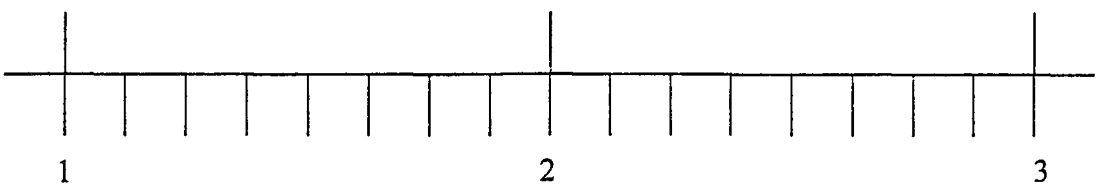
2) 3.05



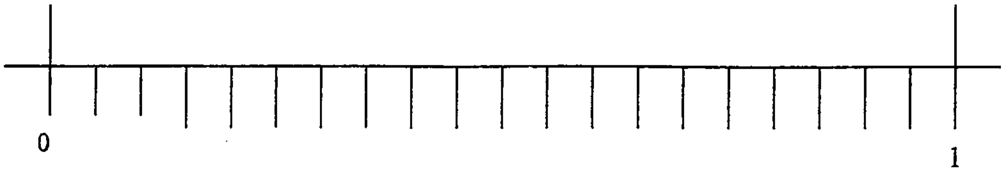
3) 0.85



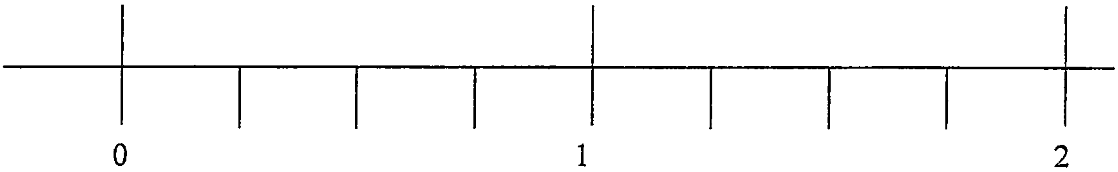
4) 2.4



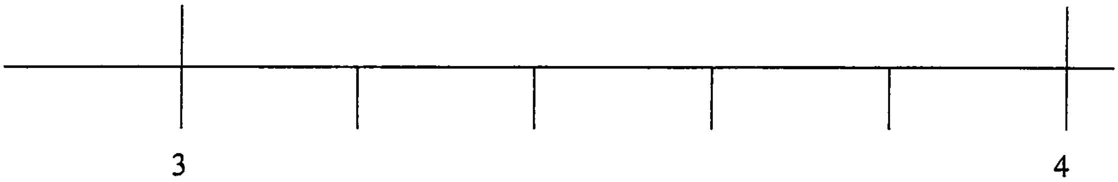
5) 0.6



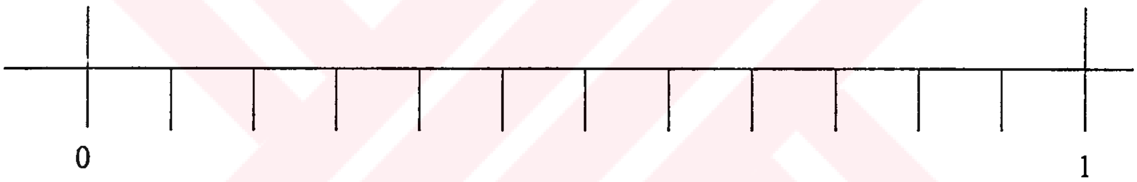
6) 0.34



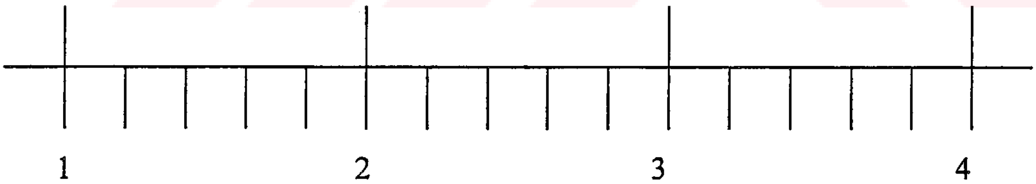
7) 3.8



8) 0.5

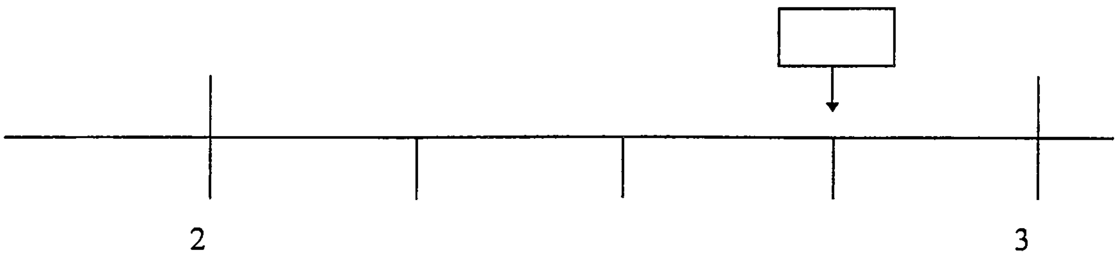


9) 3.04

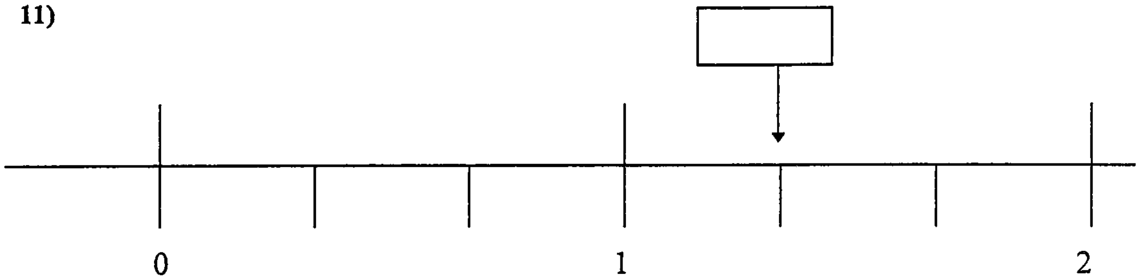


II. Aşağıdaki sayı doğruları üzerinde, okla gösterilen yerlerdeki sayıları verilen boş kutular içerisine ondalık sayı olarak yazınız.

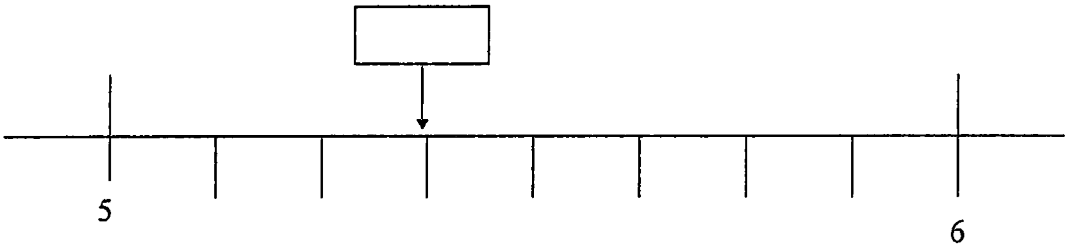
10)



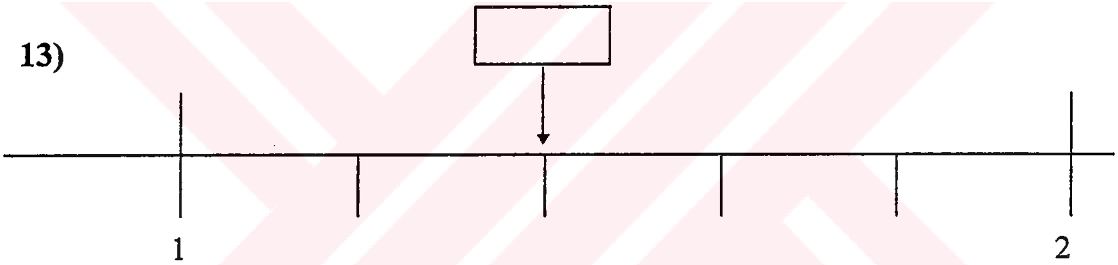
11)



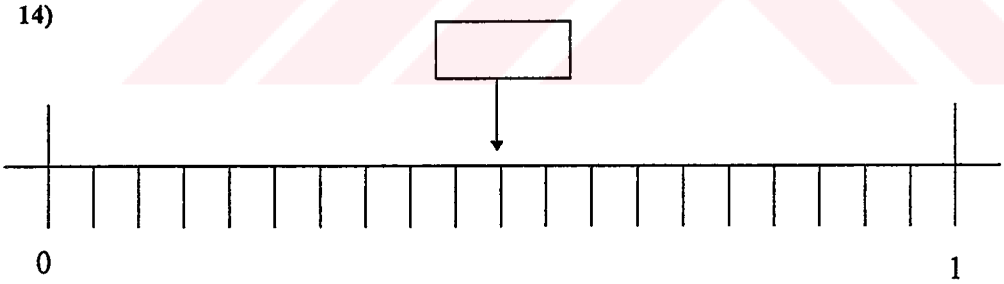
12)



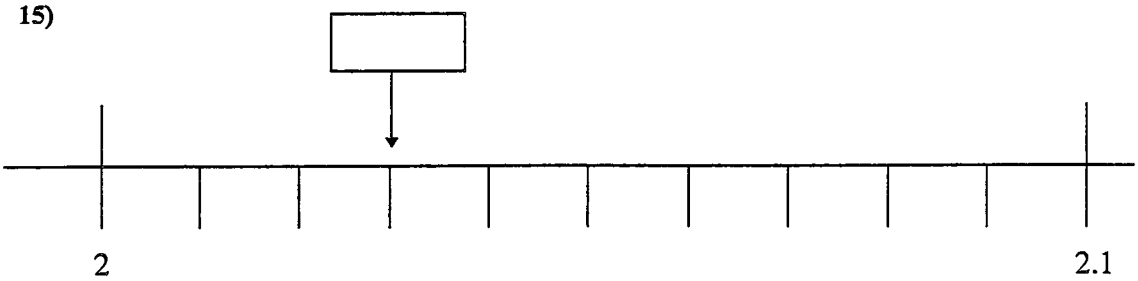
13)



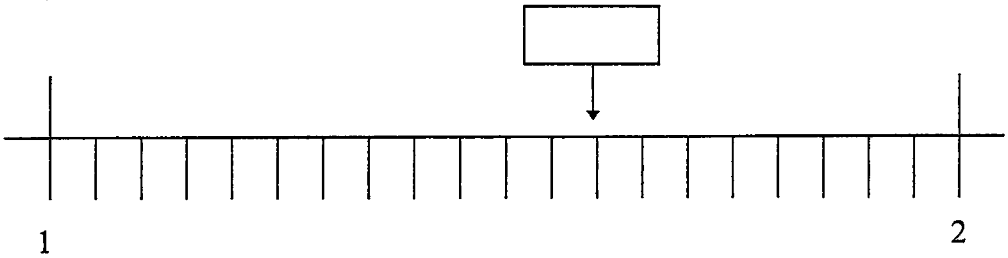
14)



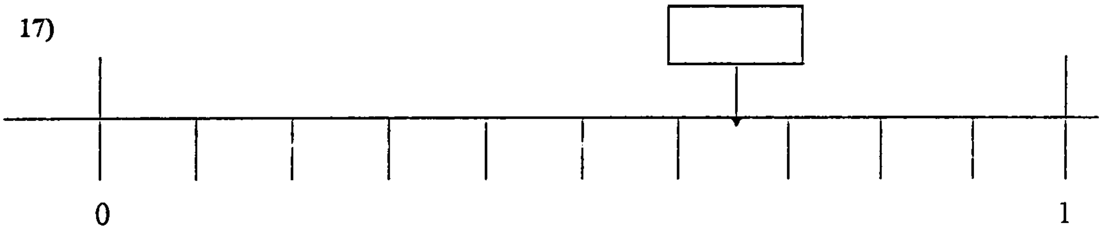
15)



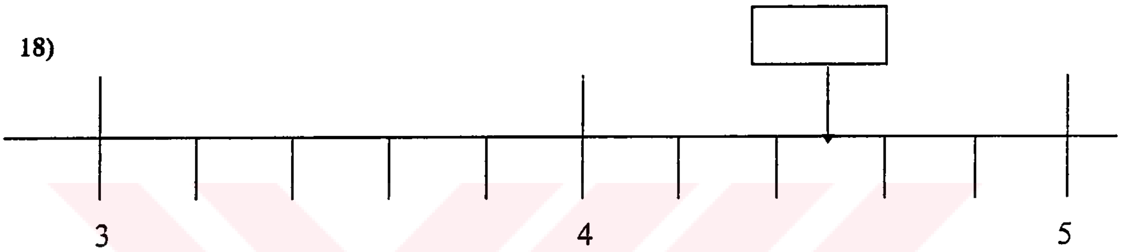
16)



17)



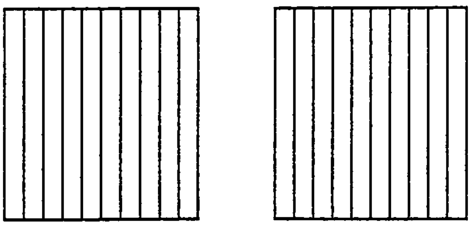
18)



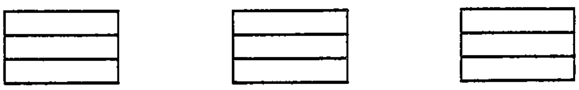
III. Aşağıdaki ondalık sayıları verilen karesel / dikdörtgensel modeller üzerinde tarama yaparak gösteriniz.

19) 0.6

20) 1.08

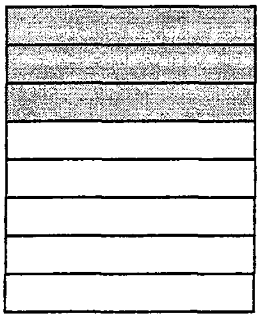


21) 2.75



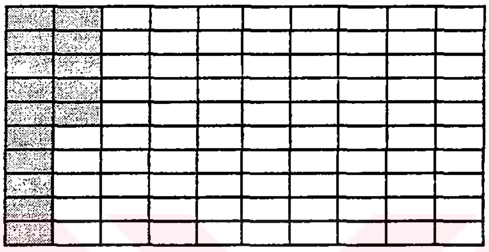
IV. Aşağıda taranmış olarak verilen bölgelerin ondalık sayı olarak karşılıklarını yazınız.
(Her model 1-birimdir.)

22)



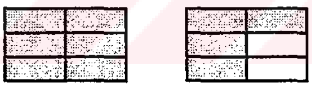
Cevap =

23)



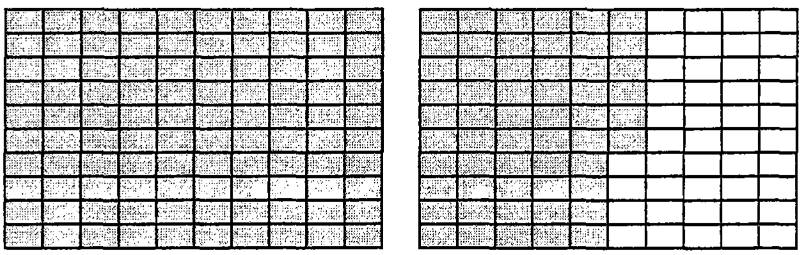
Cevap =

24)



Cevap =

25)



Cevap =

V. Aşağıda verilen sayı gruplarından en büyük sayıyı yuvarlak içerisine alınız.

26) 3.521 3.6 3.75

27) 15.4 15.56 15.327

28) 4.09 4.7 4.008

29) 0.5 0.36

30) 0.25 0.100

VI. Aşağıdaki sorular için uygun olan cevabları, her soru için verilen boş kutuların içine yazınız.

31) 5' den büyük fakat 5.1' den küçük bir sayı yazınız.

32) 0.56 ile 0.57 arasında olan kaç değişik sayı yazılabilir ?

33) 0.45 saat kaç dakikadır ?

34) 2.32 km kaç metredir ?

35) 1.15 saat kaç dakikadır ?

36) 3.25 metre kaç cm'dir ?

37) 6.80 kg kaç gr'dır ?

38) 2.4 yıl kaç aydır ?

VII. Aşağıdaki işlemlerle ilgili doğru seçeneği yuvarlak içerisine alınız. Bunu yaparken işlemlerin sonucunu sadece tahmin ediniz; uzun hesaplamalar yapmayınız.

39) 48.36 × 0.353

a) 48.36' dan küçük

b) 48.36' dan büyük

40) 35.67 ÷ 0.478

a) 35.67' den küçük

b) 35.67' den büyük

41) 0.37 × 0.561

a) 0.37' den küçük

b) 0.37' den büyük

42) 35.48 × 5.36

a) 35.48' den küçük

b) 35.48'den büyük

43) 0.236 ÷ 0.617

a) 0.236'dan küçük

b) 0.236'dan büyük

44) 62.05 ÷ 72.34

a) 62.05'den küçük

b) 62.05'den büyük

Appendix - B

Problems Test (For Pre-Testing)

II. BÖLÜM - Aşağıdaki problemleri çözmek için sadece gerekli olan işlemi yazınız, hesaplama yoluna gitmeyiniz.

Örnek: 8 kardeş ortaklaşa 24 kilogram çikolata satın alıyor. Her birinin payına ne kadar çikolata düşer ?   *İşlem : 24 ÷ 8*

1) Bir araba dakikada 2 km yol alabiliyor. Bu arabanın hızının sabit kaldığı düşünülürse, 15 dakikada ne kadar yol gider ?

İşlem = [ ]

2) Bir doğum günü partisi için 5.25 litrelik kokteyl içecek hazırlanmıştır. 5.25 litrelik bu içecek 3 litrelik kaç adet kaba boşaltılabilir ?

İşlem = [ ]

3) Bir koşucu bir yarışı 0.52 saatte tamamlamıştır. Koşucunun ortalama hızı saatte 0.93 km ise, koşulan mesafe kaç km' dir ?

İşlem = [ ]

4) Aynı büyüklükteki 5 adet şişe toplam 6.25 litrelik bir kapasiteye sahiptir. Her şişenin kapasitesi ne kadardır ?

İşlem = [ ]

5) Kilosu 150000 TL olan, 3 kilo portakal almak için kaç paraya ihtiyaç vardır ?

İşlem = [ ]

6) Aynı büyüklükte beş adet ceket dikmek için 3.25 metre kumaşa ihtiyaç duyulmaktadır. Yalnızca bir adet ceket dikmek için ne kadar kumaşa ihtiyaç vardır ?

İşlem = [ ]

7) Belediye kullanılan her ton su için 85.3 TL atık su parası almaktadır. 0.05 tonluk su harcayan bir aile ne kadar atık su parası verir ?

İşlem = [ ]

8) Ali'nin , Ahmet'in içeceğinden 1.8 litre daha fazla olma koşuluyla 6.3 litrelik içeceği vardır. Buna göre Ahmet'in içeceği kaç litredir ?

İşlem = [ ]

9) Bir ressam belirli bir rengi elde etmek için, sarı renge oranla 4.6 kat daha fazla kırmızı renk kullanmaktadır. Bu rengi elde etmek için 3.2 gr' lık sarı renk kullanılmışsa, ne kadar kırmızı renk kullanılmıştır ?

İşlem = [ ]

10) Bir kilogram buğday öğütüldüğü zaman 0.75 kg' lık un elde ediliyor. 15 kilogram buğday kullanarak ne kadar un elde edilebilir ?

İşlem = [ ]

11) Aynı büyüklükteki üç adet kitap 13 kilo ağırlığındadır. Her kitabın ağırlığı ne kadardır ?

İşlem = [ ]

12) Bir kürek takımı 3 km' lik bir yolu 7.2 dakikada alabilmektedir. Bu kürek takımı 1 dakikada ne kadar yol almaktadır ?

İşlem = [ ]

13) Bir boyacı, bir otel odasını boyamak için, aynı büyüklükteki, üç kutu boyaya ihtiyaç duymaktadır. Aynı büyüklükte 13 kutu boya ile kaç tane otel odası boyanabilir ?

İşlem = [ ]

14) Amerikada benzinin galonu 1.33 dolardan satılmaktadır. 0.53 galon kapasitesinde olan bir depoyu benzinle doldurmak için kaç dolara ihtiyaç vardır ?

İşlem = [ ]

15) Aynı büyüklükteki beş parfüm şişesi toplam 0.65 litrelik bir kapasiteye sahiptir. Bu şişelerden bir tanesi ne kadarlık parfüm kapasitesine sahiptir ?

İşlem = [ ]

16) 0.65 kilogramlık bir miktar fıstığı kapasitesi 5 kg olan bir kutuya koyduğunuzu düşünün. Böyle bir durumda bu kutunun kaçta kaçı dolmuş olur ?

İşlem = [ ]

17) Aynı boyuttaki bir miktar kutuyu sarmak için 0.245 metre uzunlugunda iplere ihtiyaç duyulmaktadır. 0.98 metre uzunlugundaki bir halat kesilerek , ihtiyaç duyulan iplerden kaç tane elde edilebilir ?

İşlem = [ ]

18) Bir mahkum cezaevinden kaçmak için bir tünel kazmaya başlıyor. Birinci günün sonunda sadece 0.174 km kazabiliyor. Bu hızla 3 km ötedeki ormana ulaşması kaç gününü alır ?

İşlem = [ ]

19) 15.3 Avustralya şilini ile 1 adet Amerikan doları alınabilir. 0.5 Avustralya şilini ile ne kadar dolar alınabilir ?

İşlem = [ ]

20) Bir parça çikolata 10.25 gram ağırlığındaysa, aynı çikolatanın 15 parçası kaç gram yapar ?

İşlem =

21) Gerçek boyu 0.23 metre olan bir resim, 4.6 oranında büyültülmek isteniyor. Buna göre resmin yeni boyu ne olur ?

İşlem =

22) Yeni doğan bir çocuğun boyu 32.4 cm' dir. Bir ay sonra boyu 3.5 cm kadar uzuyor. Çocuğun yeni boyu ne olur ?

İşlem =

23) Aynı büyüklükteki 3 kutuyu sarmak için 5.25 metre ipe ihtiyaç vardır. Bu kutulardan sadece bir tanesini sarmak için ne kadar ip gerekir ?

İşlem =

24) Evinizdeki koltukları döşemek için 13.5 metrelik kumaşa ihtiyacınız vardır. Sadece 0.82 metrelik kumaşla işin ne kadarlık kısmı yapılabilir ?

İşlem =

25) Odanızdaki bir duvarı sıvamak için 3.25 litrelik bir karışım hazırladınız. Bu karışımı toplam kapasitesi 5 litre olan bir kaba boşaltırsanız, kabın kaçta kaçı dolmuş olur ?

İşlem =

26) Yakıt deposunda 3.8 litre benzin bulunan bir arabaya 5.3 litre daha benzin eklenirse, arabada ne kadar benzin olur ?

İşlem =

Appendix - C

Writing Word Problems Test (For Pre-Testing)

III. BÖLÜM

Aşağıda verilen işlemler için, en uygun problemi verilen sayıları da göz önüne alarak yazınız.

1) $24 \div 4$

2) 5×0.68

3) 0.63×22

4) $0.83 \div 0.32$

5) 5×8

6) $4 \div 24$

7) $3.86 \div 23$

8) 12.05×0.93

9) $9.6 \div 62.2$

10) $0.53 \div 1.4$

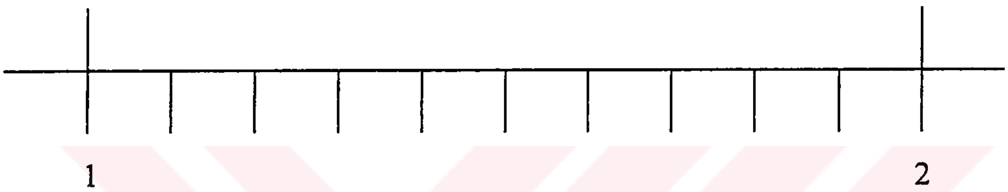
Appendix - D

Concepts Test (For Post-Testing)

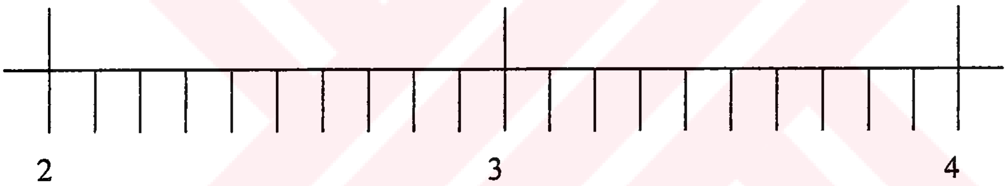
İsim / Soyadı:		Sınıf:	
Cinsiyet :		Tel.No :	
Tarih:			

I. Aşağıdaki ondalık sayıların verilen sayı doğruları üzerindeki yerlerini uygun yere nokta (.) işareti koyarak belirtiniz.

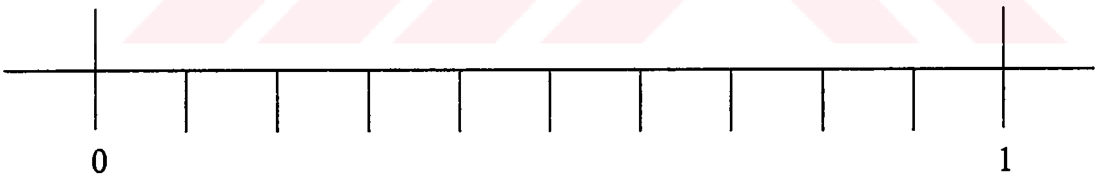
1) 1.35



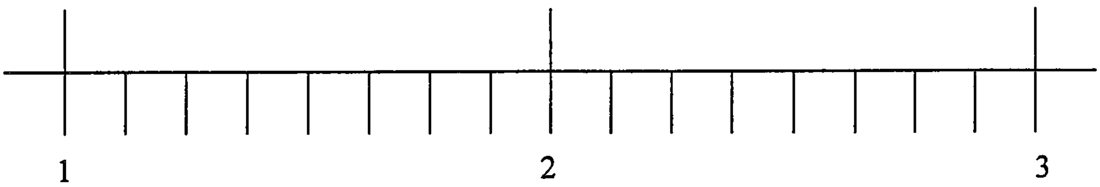
2) 2.



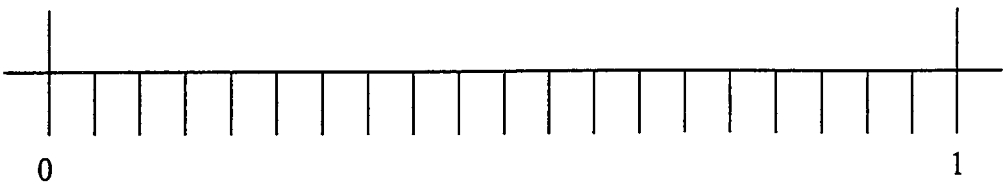
3) 0.65



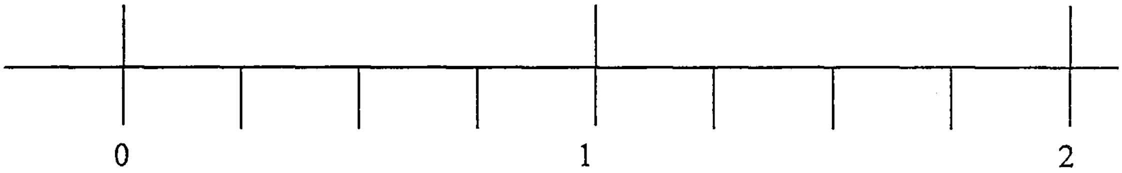
4) 2.3



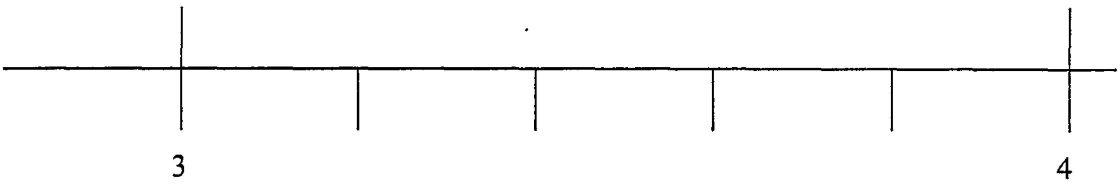
5) ...



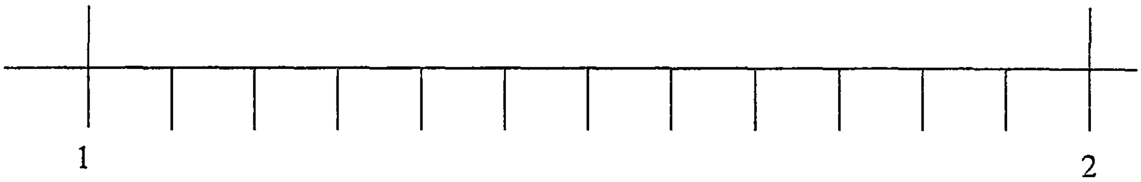
6) 0.24



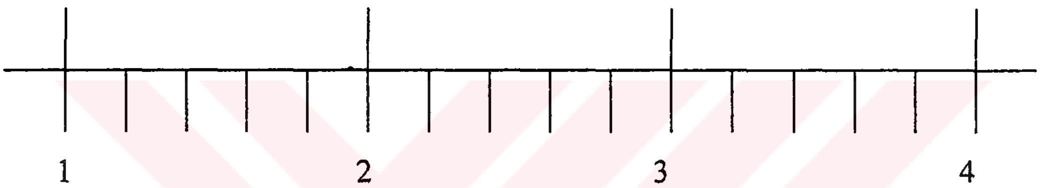
7) 3.6



8) 1.5

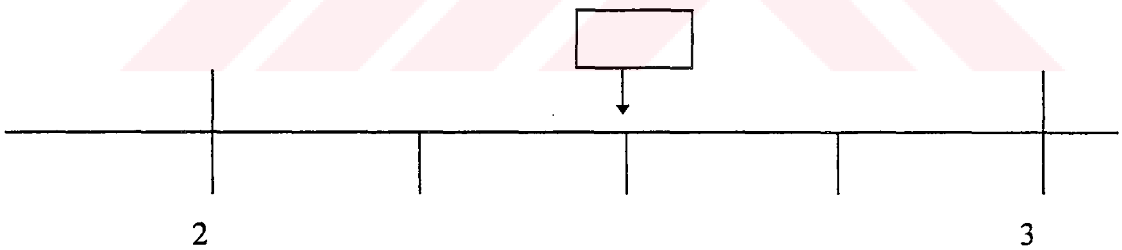


9) 2.04

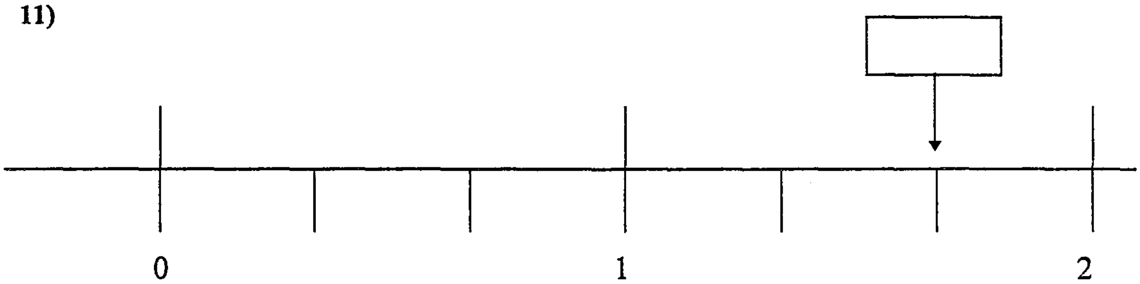


II. Aşağıdaki sayı doğruları üzerinde, okla gösterilen yerlerdeki sayıları verilen boş kutular içerisine ondalık sayı olarak yazınız.

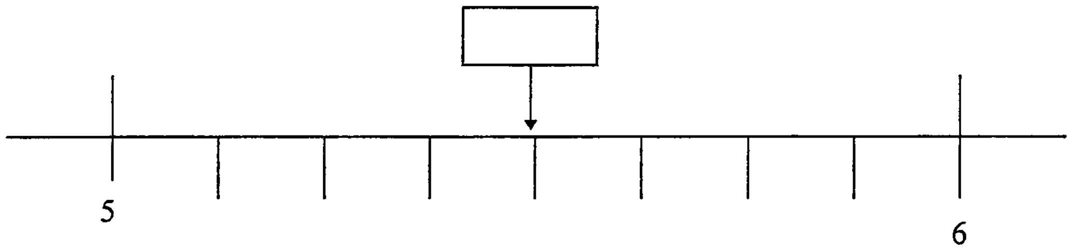
10)



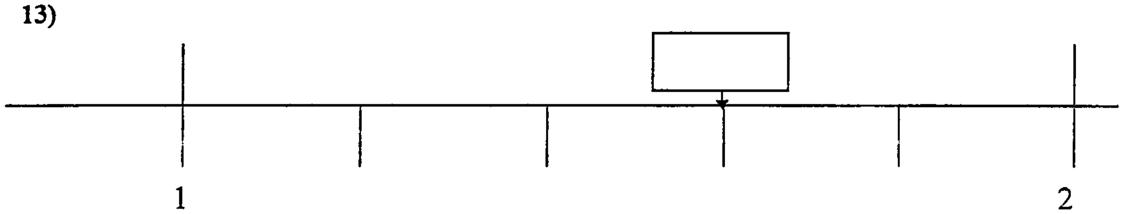
11)



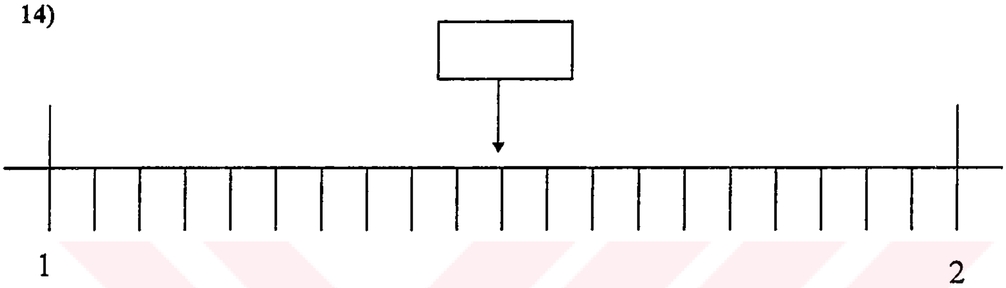
12)



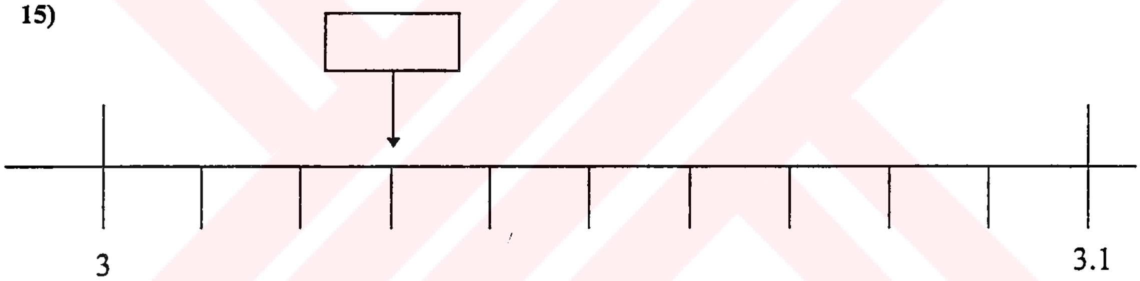
13)



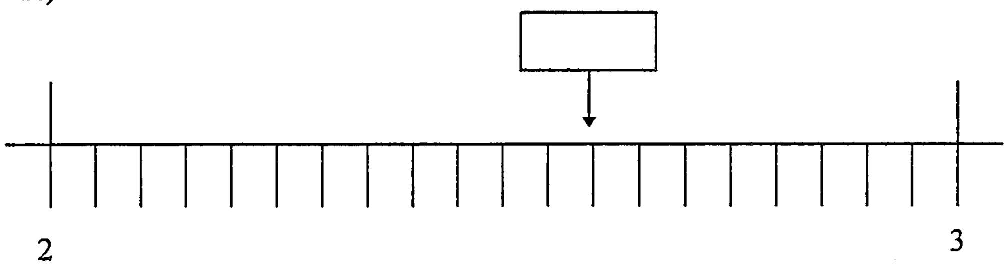
14)



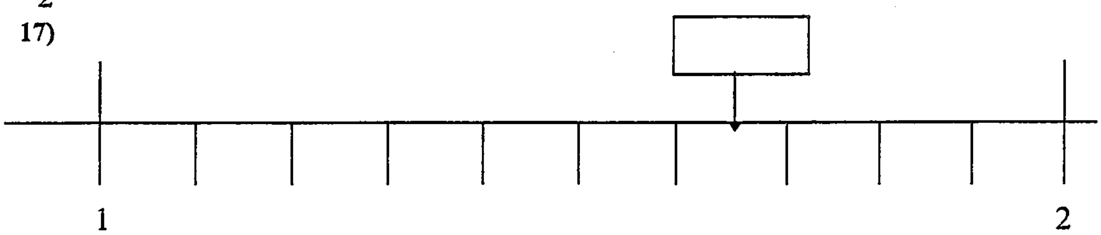
15)



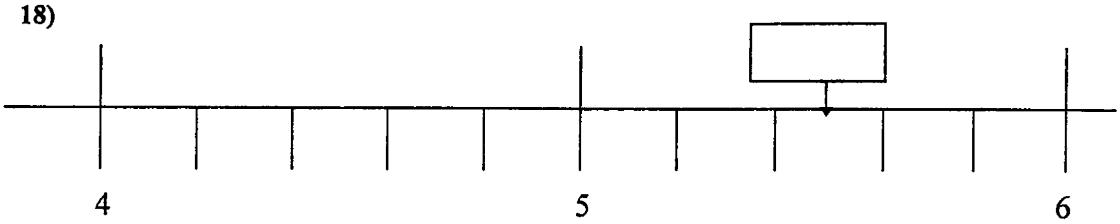
16)



17)



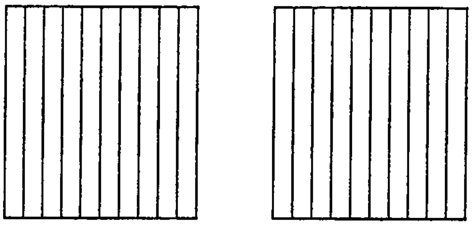
18)



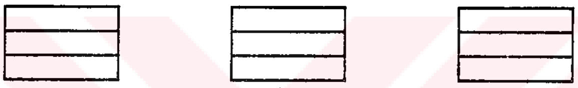
III. Aşağıdaki ondalık sayıları verilen karesel / dikdörtgensel modeller üzerinde tarama yaparak gösteriniz.

19) 0.4

20) 1.06

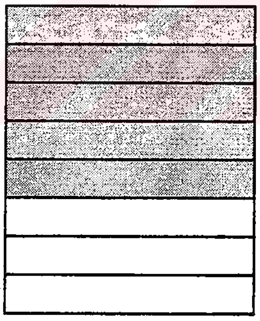


21) 2.25



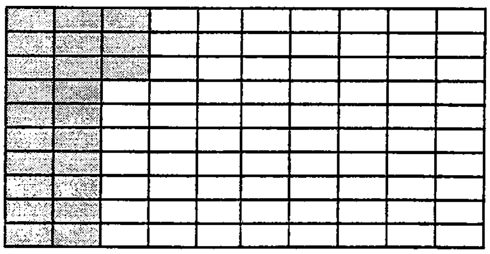
IV. Aşağıda taranmış olarak verilen bölgelerin ondalık sayı olarak karşılıklarını yazınız. (Her model 1-birimdir.)

22)



Cevap =

23)

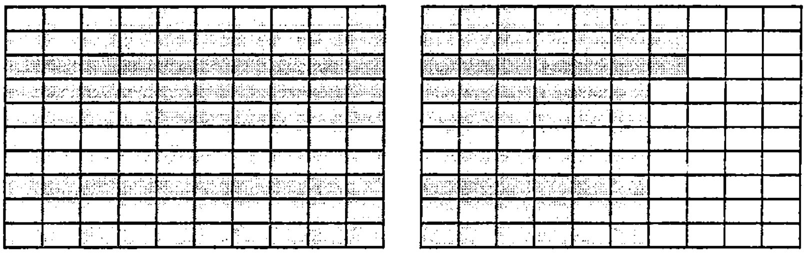


Cevap =

24)

Cevap =

25)



Cevap =

V. Aşağıda verilen sayı gruplarından en büyük sayıyı yuvarlak içerisine alınız.

26) 4.523 4.7 4.25

27) 17.3 17.57 17.427

28) 3.07 3.6 3.009

29) 0.7 0.48

30) 0.38 0.300

VI. Aşağıdaki sorular için uygun olan cevapları, her soru için verilen boş kutuların içine yazınız.

31) 4' den büyük fakat 4.1' den küçük bir sayı yazınız.

32) 0.63 ile 0.64 arasında olan kaç değişik sayı yazılabilir ?

33) 0.15 saat kaç dakikadır ?

34) 5.36 km kaç metredir ?

35) 2.45 saat kaç dakikadır ?

36) 7.45 metre kaç cm'dir ?

37) 8.56 kg kaç gr'dır ?

38) 7.3 yıl kaç aydır ?

VII. Aşağıdaki işlemlerle ilgili doğru seçeneği yuvarlak içerisine alınız. Bunu yaparken işlemlerin sonucunu sadece tahmin ediniz; uzun hesaplamalar yapmayınız.

39) 56.76 × 0.243

a) 56.76' dan küçük

b) 56.76' dan büyük

40) 67.37 ÷ 0.538

a) 67.37' den küçük

b) 67.37' den büyük

41) 0.24 × 0.657

a) 0.24' den küçük

b) 0.24' den büyük

42) 26.68 × 6.73

a) 26.68' den küçük

b) 26.68'den büyük

43) 0.576 ÷ 0.876

a) 0.576'dan küçük

b) 0.576'dan büyük

44) 83.05 ÷ 76.43

a) 83.05'den küçük

b) 83.05'den büyük

Appendix - E

Problems Test (For Post-Testing)

II. BÖLÜM - Aşağıdaki problemleri çözmek için sadece gerekli olan işlemi yazınız, hesaplama yoluna gitmeyiniz.

Örnek: 8 kardeş ortaklaşa 24 kilogram çikolata satın alıyor. Her birinin payına ne kadar çikolata düşer ?

İşlem : 24 ÷ 8

1) Bir araba dakikada 4 km yol alabiliyor. Bu arabanın hızının sabit kaldığı düşünülürse, 45 dakikada ne kadar yol gider ?

İşlem = [ ]

2) Bir doğum günü partisi için 6.82 litrelik kokteyl içecek hazırlanmıştır. 6.82 litrelik bu içecek 5 litrelik kaç adet kaba boşaltılabilir ?

İşlem = [ ]

3) Bir koşucu bir yarışı 0.67 saatte tamamlamıştır. Koşucunun ortalama hızı saatte 0.89 km ise, koşulan mesafe kaç km' dir ?

İşlem = [ ]

4) Aynı büyüklükteki 6 adet şişe toplam 7.83 litrelik bir kapasiteye sahiptir. Her şişenin kapasitesi ne kadardır ?

İşlem = [ ]

5) Kilosu 234000 TL olan, 7 kilo portakal almak için kaç paraya ihtiyaç vardır ?

İşlem = [ ]

6) Aynı büyüklükte 6 adet ceket dikmek için 6.32 metre kumaşa ihtiyaç duyulmaktadır. Yalnızca bir adet ceket dikmek için ne kadar kumaşa ihtiyaç vardır ?

İşlem = [ ]

7) Belediye kullanılan her ton su için 78.4 TL atık su parası almaktadır. 0.08 tonluk su harcayan bir aile ne kadar atık su parası verir ?

İşlem = [ ]

8) Ali'nin , Ahmet'in içeceğinden 2.3 litre daha fazla olma koşuluyla 5.8 litrelik içeceği vardır. Buna göre Ahmet'in içeceği kaç litredir ?

İşlem = [ ]

9) Bir ressam belirli bir rengi elde etmek için, sarı renge oranla 8.7 kat daha fazla kırmızı renk kullanmaktadır. Bu rengi elde etmek için 5.4 gr' lık sarı renk kullanılmışsa, ne kadar kırmızı renk kullanılmıştır ?

İşlem =

10) Bir kilogram buğday öğütüldüğü zaman 0.68 kg' lık un elde ediliyor. 13 kilogram buğday kullanarak ne kadar un elde edilebilir ?

İşlem =

11) Aynı büyüklükteki 8 adet kitap 14 kilo ağırlığındadır. Her kitabın ağırlığı ne kadardır ?

İşlem =

12) Bir kürek takımı 8 km' lik bir yolu 9.3 dakikada alabilmektedir. Bu kürek takımı 1 dakikada ne kadar yol almaktadır ?

İşlem =

13) Bir boyacı, bir otel odasını boyamak için, aynı büyüklükteki, 8 kutu boyaya ihtiyaç duymaktadır. Aynı büyüklükte 14 kutu boya ile kaç tane otel odası boyanabilir ?

İşlem =

14) Amerikada benzinin galonu 2.34 dolardan satılmaktadır. 0.68 galon kapasitesinde olan bir depoyu benzinle doldurmak için kaç dolara ihtiyaç vardır ?

İşlem =

15) Aynı büyüklükteki 6 parfüm şişesi toplam 0.54 litrelik bir kapasiteye sahiptir. Bu şişelerden bir tanesi ne kadarlık parfüm kapasitesine sahiptir ?

İşlem =

16) 0.54 kilogramlık bir miktar fıstığı kapasitesi 6 kg olan bir kutuya koyduğunuzu düşünün. Böyle bir durumda bu kutunun kaçta kaçı dolmuş olur ?

İşlem =

17) Aynı boyuttaki bir miktar kutuyu sarmak için 0.654 metre uzunlugunda iplere ihtiyaç duyulmaktadır. 0.78 metre uzunlugundaki bir halat kesilerek , ihtiyaç duyulan iplerden kaç tane elde edilebilir ?

İşlem =

18) Bir mahkum cezaevinden kaçmak için bir tünel kazmaya başlıyor. Birinci günün sonunda sadece 0.234 km kazabiliyor. Bu hızla 5 km ötedeki ormana ulaşması kaç gününü alır ?

İşlem =

19) 23.4 Avustralya şilini ile 1 adet Amerikan doları alınabilir. 0.8 Avustralya şilini ile ne kadar dolar alınabilir ?

İşlem =

20) Bir parça çikolata 12.35 gram ağırlığındaysa, aynı çikolatanın 13 parçası kaç gram yapar ?

İşlem =

21) Gerçek boyu 0.43 metre olan bir resim, 5.7 oranında büyültülmek isteniyor. Buna göre resmin yeni boyu ne olur ?

İşlem = [ ]

22) Yeni doğan bir çocuğun boyu 43.3 cm' dir. Bir ay sonra boyu 5.7 cm kadar uzuyor. Çocuğun yeni boyu ne olur ?

İşlem = [ ]

23) Aynı büyüklükteki 4 kutuyu sarmak için 6.35 metre ipe ihtiyaç vardır. Bu kutulardan sadece bir tanesini sarmak için ne kadar ip gerekir ?

İşlem = [ ]

24) Evinizdeki koltukları döşemek için 23.6 metrelik kumaşa ihtiyacınız vardır. Sadece 0.73 metrelik kumaşla işin ne kadarlık kısmı yapılabilir ?

İşlem = [ ]

25) Odanızdaki bir duvarı sıvamak için 4.34 litrelik bir karışım hazırladınız. Bu karışımı toplam kapasitesi 7 litre olan bir kaba boşaltırsanız, kabın kaçta kaçı dolmuş olur ?

İşlem = [ ]

26) Yakıt deposunda 4.7 litre benzin bulunan bir arabaya 6.8 litre daha benzin eklenirse, arabada ne kadar benzin olur ?

İşlem = [ ]

Appendix - F

Writing Word Problems Test (For Post-Testing)

III. BÖLÜM

Aşağıda verilen işlemler için, en uygun problemi verilen sayıları da göz önüne alarak yazınız.

1) $36 \div 6$

2) 4×0.76

3) 0.58×14

4) $0.38 \div 0.23$

5) 4×6

6) $6 \div 36$

7) $5.78 \div 34$

8) 23.07×0.78

9) $6.3 \div 58.7$

10) $0.67 \div 2.8$

Appendix - G

Interview Draft (For Pilot Testing)

FIRST DRAFT OF INTERVIEW SCHEDULE

About the Interview:

This interview follows a test, which aims to diagnose preservice elementary teachers' misconceptions in interpreting and applying decimals, and the ***purpose*** of the interview is to examine and describe preservice elementary teachers' performance in solving word problems in multiplication and division involving decimals. Each interviewee will be given at least one multiplication and one division problem. Average period for the interviews is 20-25 minutes.

Research Question: *What are some preservice elementary teachers' difficulties in solving word problems in multiplication and division involving decimals ?*

School: Class: Semester: Interviewer:

Date and Time (start-stop):

INTRODUCTION

Hello, this interview is devised to obtain more information about the conceptions you hold about multiplication and division and reasoning you use in solving word problems in multiplication and division involving decimals.

- What you say to me is completely confidential. We don't pass on anything people tell us, and we don't use names of individuals
- I'd like to tape our conversation, is it OK with you ?

TASK-1: The interviewee is given a problem similar / same to each of the problems s/he had [missed] on the written test and asked to write an expression that could be used to solve it.

1. Read this problem aloud for me and then tell me how you would solve it. You only need to tell me what operation you would use and what numbers. You don't have to do the actual computation.

< If the response is <u>correct</u> choose another problem, go to TASK-1 >

< If the response is <u>incorrect</u> go to TASK-2 >

TASK-2: S/he is asked to explain why s/he writes the expression s/he did and explain, and show how s/he would check his/her work.

2.1. Why did you write this expression ?

2.2. Could you explain in more detail ?

2.3. How can you check your work ?

TASK-3: After discovering or being shown that the written expression led to an incorrect answer, s/he was asked to verbalize what s/he believed led her/him to write the wrong expression.

3. What do you believe lead you to write the wrong expression ?

probes :

< If needed, go to TASK-1, otherwise STOP >

SECOND DRAFT OF INTERVIEW SCHEDULE

About the Interview:

This interview follows a test, which aims to diagnose preservice elementary teachers' misconceptions in interpreting and applying decimals, and the ***purpose*** of the interview is to examine and describe preservice elementary teachers' performance in solving word problems in multiplication and division involving decimals. Each interviewee will be given at least one multiplication and one division problem. Average period for the interviews is 20-25 minutes.

Research Question: *What are some preservice elementary teachers' difficulties in solving word problems in multiplication and division involving decimals ?*

School: Class: Semester: Interviewer:

Date and Time (start-stop):

INTRODUCTION

Hello, this interview is devised to obtain more information about the conceptions you hold about multiplication and division and reasoning you use in solving word problems in multiplication and division involving decimals.

- What you say to me is completely confidential. We don't pass on anything people tell us, and we don't use names of individuals
- I'd like to tape our conversation, is it OK with you ?

TASK-1: The interviewee is given a problem similar / same to each of the problems s/he had [missed] on the written test and asked to write an expression that could be used to solve it.

1. Read this problem aloud for me and then tell me how you would solve it. You only need to tell me what operation you would use and what numbers. You don't have to do the actual computation.

< If the response is <u>correct</u> choose another problem, go to TASK-1 >

< If the response is <u>incorrect</u> go to TASK-2 >

TASK-2: S/he is asked to explain why s/he writes the expression s/he did and explain, and show how s/he would check his/her work.

2.1. Why did you write this expression ?

ALT-2.1. If needed ask, what number do you (add, subtract, multiply, divide) to/from/by what number ?

2.2. Could you explain in more detail ?

2.3. How can you check your work ?

TASK-3: After discovering or being shown that the written expression led to an incorrect answer, s/he was asked to verbalize what s/he believed led her/him to write the wrong expression.

3. What do you believe lead you to write the wrong expression ?

probes :

3.1. ideas about operator and operand.
3.2. ideas about the meaning of divisor, dividend, and quotient.
3.3. ideas about the restrictions on numbers on these positions.
3.4. recognition of inconsistencies.
3.5. reflection on sources.

< If needed, go to TASK-1, otherwise STOP >

The documents in the dark areas shows the sections which have been added to the original schedule, to make the interview schedule more easily applicable

Appendix - H

Final Form of the Interview Schedule

First Part

How can you define decimal numbers ?
What is the basic difference between a decimal number and a whole number ?
What was the most difficult part or question for you in the tests ? Why ?
How can you define multiplication ? Give examples.
How can you define division ? Give examples.
Do decimal involvement in the problems or expressions makes the work harder ? Why ?
Did you learn decimals previously in detail when you consider your elementary, middle or high school education ?

Second Part

The preservice teacher first was given problems similar [or same] to those s/he missed on the written instruments and was asked to explain why s/he responded the way s/he had and how s/he could check his/her work.
The most frequently used questions were:

From Concepts Test: problems 4, 5, 11, 12, 15, 19, 20, 24, 26, 27, 33, 35, 37, 38, 40, 41

From Problems Test: problems 7, 9, 16, 17, 24

From Writing Word Problems Test: spontaneous

Each interviewee were asked approximately 10 questions in a period of 25-30 minutes.

MATERIALS

ONDALIK SAYILARA GİRİŞ

Giriş Dersi:

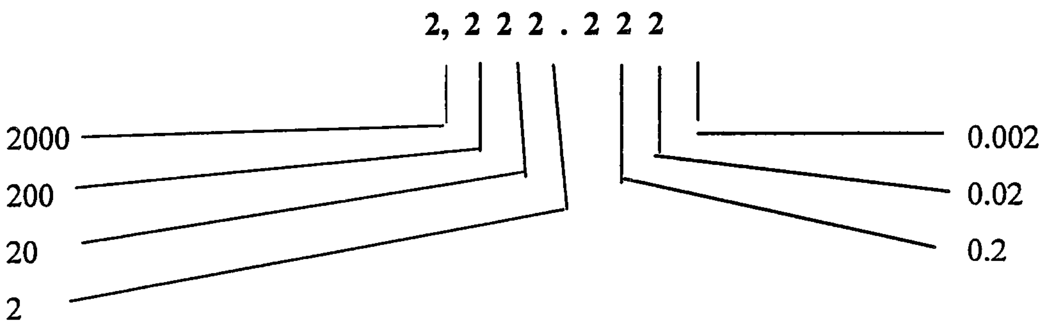
Bu bölümde öğretmen adaylarına bir ders saati süresince (45 dk.) ondalık sayıların genel yapısı üzerine bilgiler verilecektir.

$\frac{4}{10}, \frac{16}{100},$ ve $\frac{125}{1000}$ gibi sayıları bölme prensibinden yararlanarak , payı paydaya böldüğümüz zaman 0.4, 0.16, ve 0.125 gibi sayılar elde ederiz. Bu tür sayılara ondalık sayılar denir.

Ondalık sayı sisteminde göze çarpan en belirgin özellik birler basamağının sağ tarafına doğru bir genişlemenin olduğudur. Ancak bu durumda sağ taraftaki değerler 10'lar 100'ler şeklinde değil, 10'dalık, 100'delik olarak isimlendirilmektedir. Bu temel özellik aşağıdaki tabloda özetlenmiştir.

1000	100	10	1	0.1	0.01	0.001
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
Binler	yüzler	onlar	birler	ondalıklar	yüzdelikler	bindelikler

Örneğin: 2,222.222 sayısının basamak değerleri aşağıdaki gibi ifade edilebiler:



KATEGORİ - I

ONDALIK SAYILARI SAYI DOĞRULARI ÜZERİNDE YORUMLAMA

KAVRAM YANILGILARI

- Sayı doğruları üzerindeki her çizgi 1 birimi ifade eder.

- Herhangibir ondalık sayı bir sayı doğrusu üzerinde başlangıç noktası ile birinci çizgi arasında gösterilebilir .
- Bir sayı doğrusu üzerinde iki sayı arasında her biri 0.1 birim olacak şekilde 10 tane bölüm vardır .
- Bir sayı doğrusu üzerindeki her çizgi 0.5 bitime karşılık gelir.
- Ondalık sayılar 100'lük sisteme göre ifade edilir.
- Ondalık sayılar 10'luk sisteme göre ifade edilir.
- Bir sayı doğrusu üzerinde yer alan bir ondalık sayının yeri bölümlemeler değiştirildiği zaman değişir.
- Aynı model yada sayı doğrusu üzerinde aynı anda farklı alt birimler kullanılabilir.
- Sayı doğrusunda bölümlemelerin sayısını iki katına çıkarmak her alt birimin değerinin iki katına çıkmasına sebep olur.

DERS - 1/ Modül - 1

Amaç: Bu modülün amacı öğrencileri ondalık sayıları sayı doğruları üzerinde gösterirken kullanabilecekleri yöntemler açısından geliştirmek ve sahip oldukları kavram yanılgılarından arındırmaktır.

Davranışlar: Bu modülün sonunda öğrencilerde aşağıdaki davranışlar gözlenebilir:

1. Sayı doğrusu üzerinde işaretlenen/istenen yerin değerini hesaplayarak bulabilme.
2. Sayı doğrusu üzerinde işeratlenen/istenen yerin değerini tahmin yolu ile bulabilme.
3. Bir sayı doğrusu üzerinde farklı değerlerde alt bitimlerin kullanılamayacağını söyleme/yazma.

Kavram Yanılgıları:

- Sayı doğruları üzerindeki her çizgi 1 birimi ifade eder.
- Herhangibir ondalık sayı bir sayı doğrusu üzerinde başlangıç noktası ile birinci çizgi arasında gösterilebilir .
- Bir sayı doğrusu üzerinde iki sayı arasında her biri 0.1 birim olacak şekilde 10 tane bölüm vardır .
- Bir sayı doğrusu üzerindeki her çizgi 0.5 bitime karşılık gelir.
- Ondalık sayılar 100'lük sisteme göre ifade edilir.
- Ondalık sayılar 10'luk sisteme göre ifade edilir.
- Bir sayı doğrusu üzerinde yer alan bir ondalık sayının yeri bölümlemeler değiştirildiği zaman değişir.
- Aynı model yada sayı doğrusu üzerinde aynı anda farklı alt birimler kullanılabilir.

Araç-gereç: *Doğru mu ? Yanlış mı ?* ve Pekiştirme Etkinliği formları

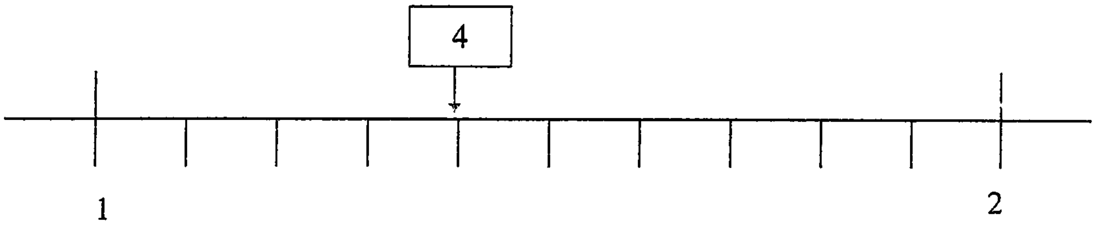
Giriş Etkinliği:

Sınıftaki her öğrenciye bir *Doğru mu ? Yanlış mı ?* formunu dağıtın ve yönergede belirtilenleri yapmalarını isteyin.

Doğru mu ? Yanlış mı ?

Aşağıdaki cevaplar ve çözüm yolları ondalık sayılarla ilgili sınava giren bir öğrenciye aittir. Bu öğrenciye verilen ondalık sayıları sayı doğrusu üzerinde göstermesi yada sayı doğrusunda işaretlenen bir noktanın ondalık sayı karşılığı sorulmuştur. Buna göre bu öğrencinin yaptıklarını gözden geçirip değerlendiriniz. Daha sonra yanınızdaki arkadaşınızın yaptıkları ile de karşılaştırınız.

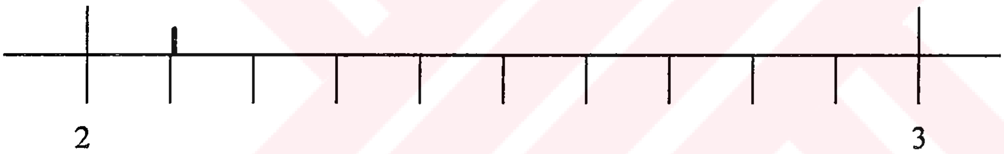
1)



Doğru yapılmış: ☐ , Neden ? :

Yanlış yapılmış: ☐ , Neden ? :

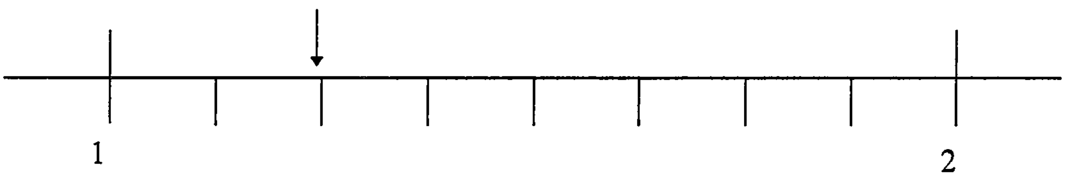
2) 2.4



Doğru yapılmış: ☐ , Neden ? :

Yanlış yapılmış: ☐ , Neden ? :

3) 1.3



Doğru yapılmış: ☐ , Neden ? :

Yanlış yapılmış: ☐ , Neden ? :

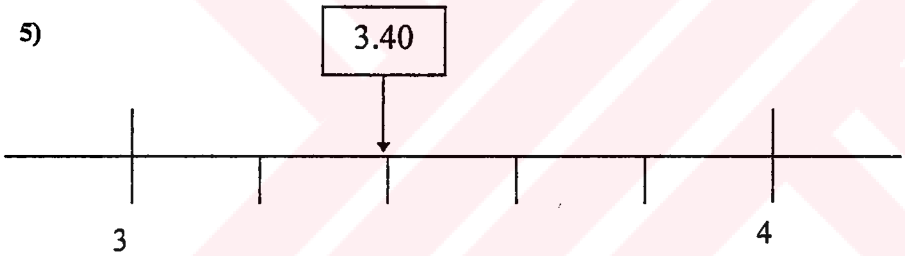
4)

Her çentik 0.5 değerinde olduğuna göre 2. çizgi ile 3.çizgi arasında olacaktır.

Doğru yapılmış: ☐ , Neden ? :

Yanlış yapılmış: ☐ , Neden ? :

5)

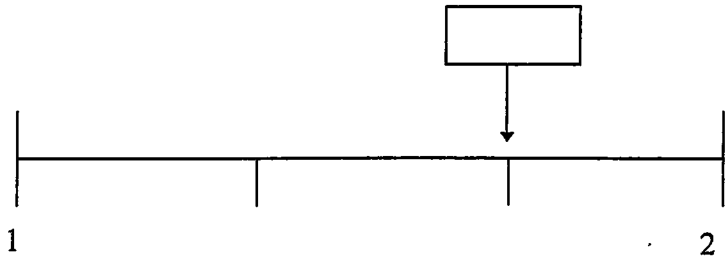


Herbir aralığı bulmak için ondalık sayılar için temel birim olan 100'ü çentik sayısına bölmek lazım, yani $\frac{100}{5} = 20$, *3 + 40 =3.40*

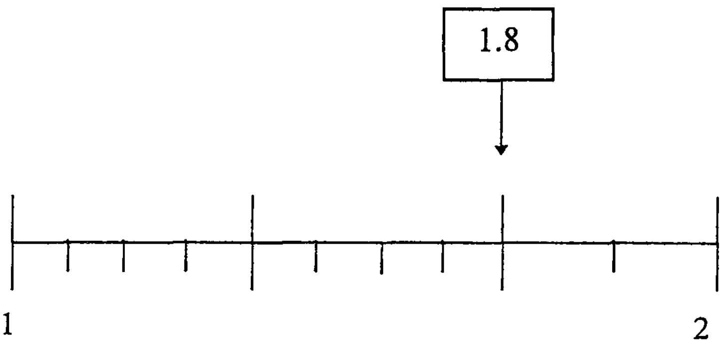
Doğru yapılmış: ☐ , Neden ? :

Yanlış yapılmış: ☐ , Neden ? :

6)



Burada sayı doğrusu 3'e bölünmüş fakat gerçekte 10'a bölünmesi gerekir, bu yüzden ilk iki bölümü kendi içerisinde 4'er parçadan toplam 8 parçaya son kısmıda 2 parçaya ayırırsam toplam 10 parçayı elde ederim ve okun 8'inci noktayı işaret ettiğini kolayca görürüm.



Doğru yapılmış: ☐ , Neden ? :

Yanlış yapılmış: ☐ , Neden ? :

<u>Sınıf Tartışması</u>:

İlk bölümü tamamladıktan sonra tartışma açarak kimin ne düşündüğünü ve niye öyle düşündüğünü sorun. Önemli gördüğünüz noktaları tahtaya yazın (Çatışma yaratanlara öncelki verin). Karşıt fikirleri savunan grupların kendilerini savunmalarına olanak verin ve ortaya çıkan mantıklı ifadeleri tahtaya yazın.

<u>Pekiştirme Etkinliği</u>:

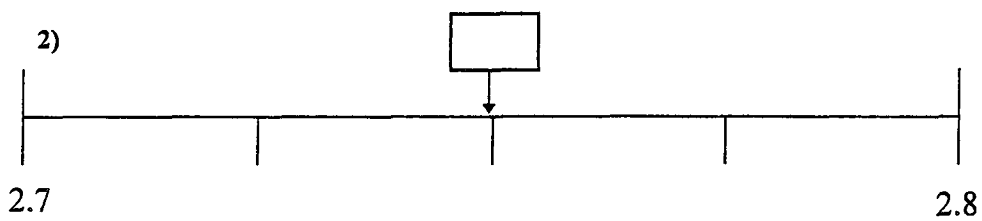
Önemli noktalara işaret ettikten sonra öğrencilerin bu kez aşağıdaki soruları cevaplandırmalarını ve yakınlarındaki arkadaşları ile kağıtlarını değiştirerek birbirlerinin yanıtlarını ve çözüm yollarını değerlendirmelerini isteyin.

<u>Pekiştirme Etkinliği:</u>

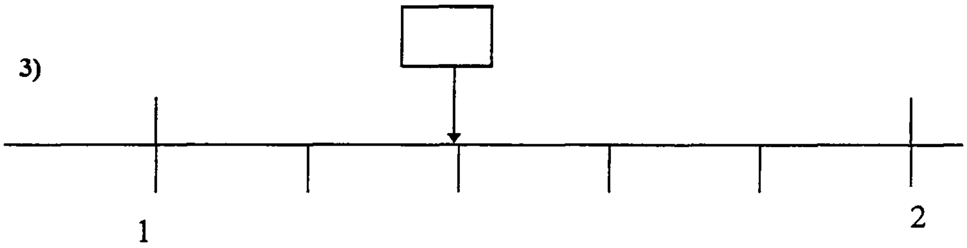
Aşağıdaki sayı doğruları üzerinde işaretlenmiş noktaların ondalık sayı olarak karşılıklarını verilen kutular içerisine yazınız.

1)

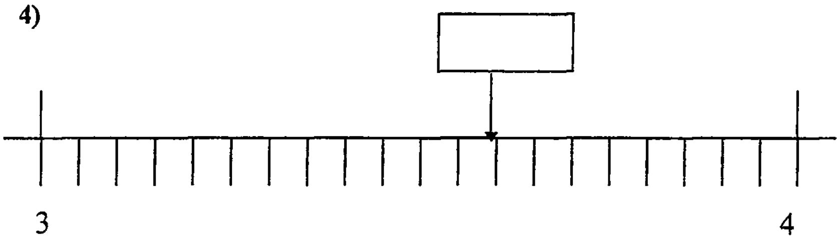
2)



3)



4)



Sınıf tartışması:

İkinci pekiştirme Etkinliğindan sonra tekrar bir tartışma açarak bu kez bazı noktalarda zayıf olan öğrencilere yönelik olarak kara tahta üzerine belirli sorular daha yazılarak *tahmin* yönteminden de yararlanılır ve sınıfça belirli sorular çözülür.

KATEGORİ -1 / DERS - 1/ Modül - 2

Amaç: Bu modülün amacı öğrencileri sayı doğrularındaki alt birim kullanımı açısından geliştirmek ve sahip oldukları kavram yanılgılarından arındırmaktır.

Davranışlar: Bu modülün sanunda öğrencilerde aşağıdaki davanışlar gözlenebilir:

1. Aynı sayıyı farklı alt birim sistemlerine sahip sayı doğrularında gösterebilme.
2. Sayı doğrularında alt birim sisteminde yapılan değişikliklerin daha önceden belirlenmiş bir noktanın yerini etkilemeyeceğini yazma/söyleme.

Kavram Yanılgıları:

- Bir sayı doğrusu üzerinde yer alan bir ondalık sayının yeri, bölümlemeler değiştirildiği zaman değişir.
- Sayı doğrusunda bölümlemelerin sayısını iki katına çıkarmak her alt birimin değerinin iki katına çıkmasına sebep olur.

Araç-gereç: Ejektörler isimli etkinlik formu.

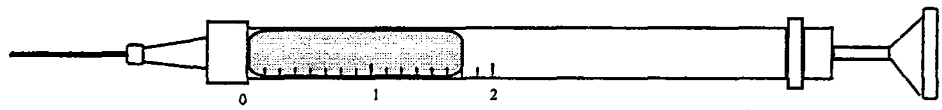
Giriş Çalışması:

Öğrencilere *Enjektörler* isimli etkinlik formunu dağıtın ve yönergede belirtilenleri yapmalarını isteyin.

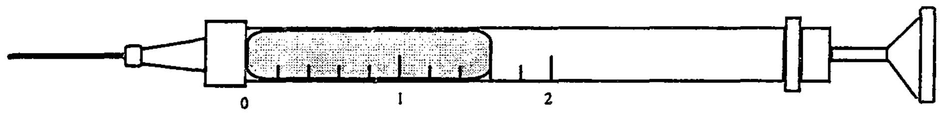
Enjektörler

Fareler üzerinde yeni bir ilacın denendiğini varsayın. Ancak daha önceki deneylerde, farelerin bu yeni ilaçtan 1.6 ml'den daha az veya daha çok almaları durumunda öldükleri ortaya çıkmıştır. Buna göre 3 ayrı fareye aşağıdaki A, B, ve C enjektörlerindeki ilaçların enjekte edildiğini düşünün, bu durumda hangi enjectördeki ilacı alan fare veya fareler ölür ?

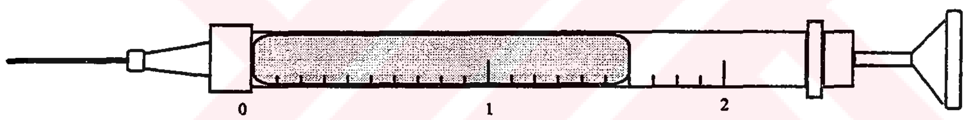
A



B



C

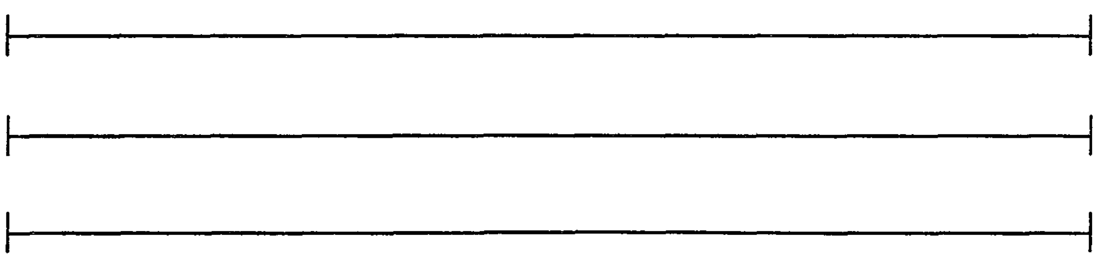


Sınıf Tartışması:

Farklı öğrencilerin yanıtlarından tahtaya yazarak kısa bir tartışma açın ve öğrencilerin doğru stratejileri bulmalarını sağlayın. Öğrencilerden birisinin konuyu toparlamasını isteyin.

Pekiştirme Etkinliği:

Aşağıdaki üç adet sayı doğrusunu sırası ile 20, 10 ve 5 bölmeye ayırarak 1.5 ondalık sayısını bu doğrular üzerinde gösteriniz.



KATEGORİ - 2

ONDALIK SAYILARI KARESEL/DİKDÖRTGENSEL MODELLER ÜZERİNDE YORUMLAMA

KAVRAM YANILGILARI:

- Ondalık sayılar 10'luk sisteme göre ifade edilir.
- Aynı model yada sayı doğrusu üzerinde aynı anda farklı alt birimler kullanılabilir.
- Bir birimlik bir model birden fazla bir birimden oluşan bir model olarak kullanılabilir.
- Ondalık sayılarda sıfırın değeri yoktur.
- Birden fazla bir birimlik modeller bir birimlik bir bütün olarak kullanılabilir.
- Bazı modellerin ondalık sayı karşılıkları yoktur.
- Bir modelin alt birim sistemi var olanlara yeni parçalar eklenerek artırılabilir.
- Bir modelin alt birim sistemi var oalnlardan istenenler atılarak azaltılabilir.

KATEGORİ - 2
DERS - 2/ Modül - 1:

Amaç: Bu modülün amacı öğrencileri ondalık sayıları karesel/dikdörtgensel modeller üzerinde ifade ederken kullanılabilecek yüntemler açısından geliştirne ve sahip oldukları kavram yanılgılarından arındırmaktır.

Davranışlar: Bu modülün sonunda öğrencilerde aşağıdaki davranışlar gözlenebilir:

1. Ondalık sayıların sadece 10'luk altbirim sistemine göre ifade edilmediğini yazma/söyleme.
2. Ondalık sayılarda özellikle ondalık noktadan hemen sonra kullanılan sıfırın bir değerinin olduğunu yazma/söyleme.
3. Bir adet ve birden fazla 1 birimlik modellerde ondalık sayıları gösterebilme.
4. Birden cok 1 birimden oluşan modeller sisteminin tek bir model gibi kullanılamayacağını yazma/söyleme.

Kavram Yanılgıları:

- Ondalık sayılar 10'luk sisteme göre ifade edilir.
- Bir birimlik bir model birden fazla bir birimden oluşan bir model olarak kullanılabilir.
- Ondalık sayılarda sıfırın değeri yoktur.
- Birden fazla bir birimlik modeller bir birimlik bir bütün olarak kullanılabilir.

Araç -Gereç: Tarama -1 isimli etkinlik formu

Giriş Etkinliği:

Tarama - 1

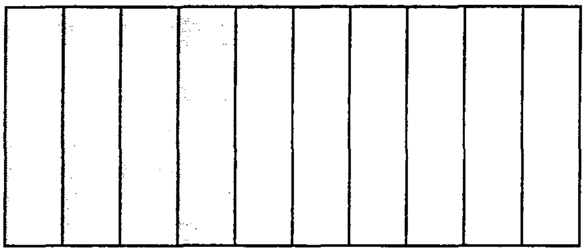
Aşağıdaki cevaplar ve çözüm yolları ondalık sayılarla ilgili sınava giren bir öğrenciye aittir. Bu öğrenciye verilen ondalık sayıları dikdörgensel/karesel modeller üzerinde göstermesi yada taralı bir bölgenin ondalık sayı karşılığı sorulmuştur. Buna göre bu öğrencinin yaptıklarını gözden geçirip değerlendiriniz. Daha sonra yanınızdaki arkadaşınızın yaptıkları ile de karşılaştırınız.

1) **0.3**

Doğru yapılmış: ☐ , Neden ? :

Yanlış yapılmış: ☐ , Neden ? :

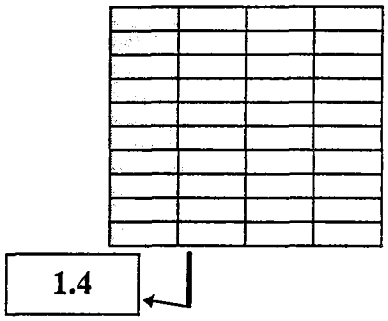
2) **0.04**



Doğru yapılmış: ☐ , Neden ? :

Yanlış yapılmış: ☐ , Neden ? :

3)



Doğru yapılmış: ☐ , Neden ?:

Yanlış yapılmış: ☐ , Neden ?:

4)

$$Taralı\ Bölge = \frac{7}{10} = 0.7$$

Doğru yapılmış: ☐ , Neden ? :

Yanlış yapılmış: ☐ , Neden ? :

<u>Sınıf Tartışması:</u>

Öğrencilerin değerlendirmeleri alınır ve önemli görülenler tahtaya yazılır. Bu arada sayı dogruları çalışmasında önerilen tahmin yönteminin ne ölçüde kullanıldığı ve yararlı olup olmadığı üzerinde durulur ve tekrar gündeme getirilir.

<u>Pekiştirme Çalışması:</u>

Sayı doğrusu çalışmasından farklı olarak bu savhada öğrencilerin kendilerinin bütün yaratıcılıklarını kullanarak soru yazmaları ve sınıfa yöneltmeleri istenir. Sorular tartışarak tahtada sınıfça çözülür. Günlük hayattan Örnekler verilmesi sağlanır.

DERS - 2/ Modül - 2:

<u>Amaç:</u> Bu modülün amacı öğrencileri modeller üzerinde altbirim sistemlerinin ne şekilde kullanılabileceği üzerinde yetirirmek ve sahip oldukları kavram yanılgılarından arındırmaktır.

<u>Davranışlar:</u> Bu modülün sonunda öğrencilerde aşağıdaki davranışlar gözlenebilir:

1. Bir modele yeni altbirimler ekleyerek altbirim sisteminin değiştirilemeyeceğini yazma/söyleme.
2. Bir modelin bazı altbirimlerini atarak altbirim sisteminin değiştirilemeyeceğini yazma/söyleme.

<u>Kavram Yanılgıları:</u>

- Bazı modellerin ondalık sayı karşılıkları yoktur.
- Bir modelin alt birim sistemi var olanlara yeni parçalar eklenerek artırılabilir.
- Bir modelin alt birim sistemi var oalnlardan istenenler atılarak azaltılabilir.

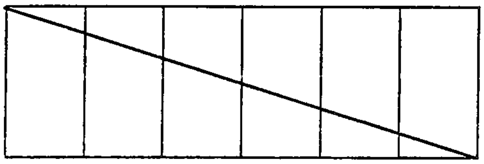
Araç-Gereç: Tarama - 2 isimli etkinlik formu

Giriş Etkinliği:

Aşağıdaki cevaplar ve çözüm yolları ondalık sayılarla ilgili sınava giren bir öğrenciye aittir. Bu öğrenciye verilen ondalık sayıları dikdörgensel/karesel modeller üzerinde göstermesi yada taralı bir bölgenin ondalık sayı karşılığı sorulmuştur. Buna göre bu öğrencinin yaptıklarını gözden geçirip değerlendiriniz. Daha sonra yanınızdaki arkadaşınızın yaptıkları ile de karşılaştırınız.

Tarama - 2

1)

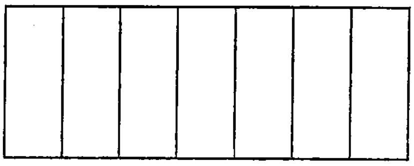


Ondalık sayı olarak karşılığı yoktur.

Doğru yapılmış: ☐ , Neden ? :

Yanlış yapılmış: ☐ , Neden ? :

2) **0.6 = ?**

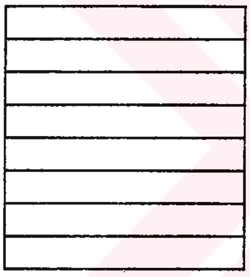


$0.6 = \frac{6}{10} = \frac{3}{5}$ *, yani 5'te 3'ü taranacak gerisi fazla 0.7 verilmiştir.*

Doğru yapılmış: ☐ , Neden ?

Yanlış yapılmış: ☐ , Neden ?

3) **0.8 = ?**



$0.8 = \frac{8}{10}$ *yani 10'da 8 isteniyor. Verilen modele iki dilim daha ekleyip 8'ini tarasak olur.*

Doğru yapılmış: ☐ , Neden ?

Yanlış yapılmış : ☐ , Neden ?

Grup Tartışması:

Bu kez önce ikişerli gruplar oluşturularak kağıtlar değiştirilir ve herkes bir diğer kişinin kağıdındaki yapılanları değerlendirir. Bir süre sonra tartısırlar ve yapılan hataları saptayarak daha sonra yapılacak Sınıf Tartışması için hazırlarlar ve bir sözcü saptanır.

Sınıf tartışması:

Dersin bu bölümünde grupların ortaklaşa saptadıkları hatalar üzerinde tartışılır ve daha uygun yöntemler bulunur.

Pekiştirme Çalışması:

İkişerli gruplar birbirlerine soru sorarak cavaplandırmaya çalışırlar.
Bu soruların aşağıdaki gibi olması önerilir:

a) 0.72 ondalık sayısını hem sayı doğrusu üzerinde hem de tarama yaparak modeller üzerinde gösteriniz.

b) Taralı olarak verilen bir bölgenin ondalık sayı olarak karşılığını bulunuz ve sayı doğrusu üzerinde gösteriniz.

KATEGORİ - 3

ONDALIK SAYILARIN KARŞILAŞTIRILMASI

KAVRAM YANILGILARI:

- Ondalık sayılarda sıfırın değeri yoktur.
- Bir ondalık sayının noktadan sonraki kısmının büyük olması onun büyük olmasına neden olur.
- Bir ondalık sayıda noktadan sonraki kısımda basamak sayısının artması onun küçülmesine neden olur.
- Bir ondalık sayının bir tamsayıya yuvarlanması için gerekli işlem sayısı az ise o ondalık sayı büyük olur.

KATEGORİ - 3
DERS -3/ Modül - 1:

Amaç: Bu modülün amacı öğrencilere ondalık sayılarda sıfırın hangi durumlarda değerinin olduğunu veya olamdığını kavratmak ve sahip oldukları kavram yanılgılarını ortadan kaldırmaktır.

Davranışlar: Bu modülün sonunda öğrencilerde aşağıdaki davranışlar gözlenebilir:

1. Verilen bir ondalık sayıdaki ondalık noktadan hemen önce ve hemen sonra sıfırın hangi değere sahip olduğunu yazma/söyleme.
2. Verilen bir ondalık sayıda sıfırın en başta ve en sonda yer alma durumlarında herhangibir değerinin olmadığını yazma/söyleme.

Kavram Yanılgısı:
- Ondalık sayılarda sıfırın değeri yoktur.

Araç - Gereç: *Sıfırın Değeri* isimli etkinlik formu, Hesap makinesi.

Giriş Etkinliği:

Her öğrenciye *Sıfırın Değeri* isimli etkinlik formu dağıtılır ve formda verilen diyalogları bireysel olarak değerlendirmeleri istenir.

- Aşağıdaki kutu içerisinde kaç tane değişik sayı vardır.

	03			0.003
3				
		0.3	3.000	
		30		0.03

- Yukarıdaki diyaloglara katılıyormusunuz.
- Hesap makinenize 00.03000 sayısını yazınız. Ne oldu ?
- Yukarıda verilen sayıları aynı sayılar ve farklı sayılar diye iki gruba ayırınız ve daha sonra yanınızdaki arkadaşınızla karsılaştırınız.

Grup Etkinkiği:

İkili gruplar kendi aralarında sonuçları karsılaştırdıktan sonra birlikte sıfırın bir sayı içerisindeki konumuna göre nasıl görev yaptığını ifade eden birtakım kurallar yazmalarını isteyiniz.

Sınıf Tartışması:

Grup etkinlikleri sona erdikten sonra her gruptan bir sözcünün oluşturdukları kuralları okumalarını isteyin ve gerek doğru ollanlar gerekse yanlış olanlar arasında bir kısmını beyaz tahtaya yazarak tartışmaya açın. Daha sonra öğrencilerle birlikte net bir liste hazırlayın.

Pekiştirme Etkinliği:

Sınıf tartışmasından sonra öğrencilere *Sıfırın Değeri* isimli etkinlikte verilen sayıları karesel modeller üzerinde tarama yaparak karşılaştırmalarını isteyin. Bu çalışma ikişerli veya üçerli gruplar halinde yapılabilir. Öğrenciler arasında dolaşarak gerekli uyarılarda bulunun ve onlara gerekli dönütleri verin.

KATEGORİ - 3

DERS - 3/ Modül - 2:

Amaç: Bu modülün amacı öğrencilere ondalık sayılarda karşılaştırmanın nasıl yapılacağını kavatmak ve sahip oldukları kavram yanılgılarını ortadan kaldırmaktır.

Davranışlar: Bu modülün sonunda öğrencilerde aşağıdaki davranışlar gözlenebilir:

1. Ondalık sayılarda basamak değerlerini doğru bir şekilde yazma/söyleme.
2. Ondalık sayılarda yapılan karşılaştırmaların doğal sayılarda yapılanlardan hangi yönleri ile farklı olduğunu yazma/söyleme.

Kavram Yanılgıları:

- Bir ondalık sayının noktadan sonraki kısmının büyük olması onun büyük olmasına neden olur.
- Bir ondalık sayıda noktadan sonraki kısımda basamak sayısının artması onun küçülmesine neden olur.
- Bir ondalık sayının bir tamsayıya yuvarlanması için gerekli işlem sayısı az ise o ondalık sayı büyük olur.

Araç - Gereç: *Sınav Sonuçları* isimli etkinlik formu.

Giriş Etkinliği:

Her öğrenciye *Sınav Sonuçları* isimli etkinlik formunu dağıtın ve yönergede belirtilenleri yapmalarını isteyin.

Sınav Sonuçları

Aşağıdaki forumda 4 sınava giren 7 grup öğrencinin ortalamaları verilmiştir. Buna her sınavdan alınan ortalamalara göre grupları en büyük ortalamadan en küçük ortalamaya kadar sıralayın ve daha sonra sonuçları arkadaşlarınızla karşılaştırın.

Temel Matematik sınavı sonuçları:

1. Grup	20.95
2. Grup	20.9
3. Grup	20.84
4. Grup	20.85
5. Grup	20.79
6. Grup	20.8
7. Grup	20.94

Müzik sınavı sonuçları

1. Grup	48.21
2. Grup	49
3. Grup	48.57
4. Grup	49.8
5. Grup	48.9
6. Grup	49.62
7. Grup	50

Coğrafya sınavı sonuçları:

1. Grup	10.9
2. Grup	10.23
3. Grup	10.64
4. Grup	10.03
5. Grup	10.4
6. Grup	10.19
7. Grup	10.69

İngilizce sınavı sonuçları

1. Grup	21.05
2. Grup	20.87
3. Grup	20.9
4. Grup	21.5
5. Grup	21
6. Grup	21.38
7. Grup	20.7

Sınıf Tartışması:

Her gruptan bir grup sözcüsü belirleyerek sonuçları tahtaya yazmalarını isteyin. Karşıt görüşü olanlara söz hakkı tanıyın ve kendilerini savunmalarını isteyin.

Pekiştirme Etkinliği:

Sınıf tartışmasında ortaya çıkan ve görüş ayrılığı saptanan ondalık sayıları belirli öğrencileri tahtaya çağırarak bu ondalık sayıları sayı doğruları oluşturarak bunlar üzerinde göstermelerini isteyiniz (tartışarak).

Pekiştirme Çalışması:

Sınıftan iki üç öğrencinin tahtaya gelişigüzel ondalık sayılar yazarak bunları sınıfla tartışarak modeller üzerinde tarama yaparak göstermelerini isteyiniz. Taranmış bölgelerle birlikte verilen ondalık sayıların nasıl karşılaştırıldığı üzerinde durun.

KATEGORİ - 4

ONDALIK SAYILARDA YOĞUNLUK

KATEGORİ - 4
DERS - 4:

Amaç: Bu dersin amacı öğrencilere sayılarda sınırlı olma özelliği yanında, iki sayı arasında sonsuz çoklukta sayı olduğunu kavratmak ve sahip oldukları kavram yanılgılarını ortadan kaldırmaktır.

Davranışlar: Bu dersin sonunda öğrencilerde aşağıdaki davranışlar gözlenebilir:

1. Verilen iki sayı arasında yer alan her hangibir sayı yazma/söyleme.
2. Verilen iki sayı arasında sonsuz çoklukta sayı olabileceğini yazma/söyleme.
3. Verilen ondalık sayıları aynı sayı doğrusu üzerinde gösterme.

Kavram Yanılgıları:

- Ondalık sayılar 10'luk sisteme göre ifade edilir.
- Ardışık iki ondalık sayı arasında başka sayı yoktur.
- Ardışık iki ondalık sayı arasında sınırlı düzeyde sayı vardır.
- Ardışık iki ondalık sayı arasındaki fark 1 birimdir.

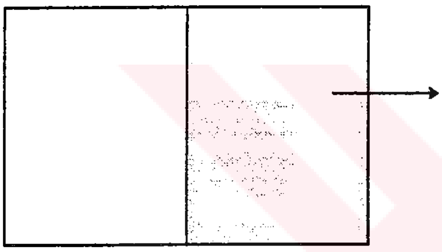
Araç - Gereç: *Sayıyı Bulun* isimli pekiştirme formu.

Giriş Etkinliği:

İlk olarak bütün sınıfa aklınızda 0 ile 1 arasında bir sayı olduğunu ve bunu belli ipuçları yardımıyla bulabileceklerini söyleyin. Aklınızda *0.253* sayısını tutun ve tahmin yürütmelerini isteyin. Yapacakları tahmine göre onlara *daha büyük* veya *daha küçük* gibi uyarılarda bulunun.

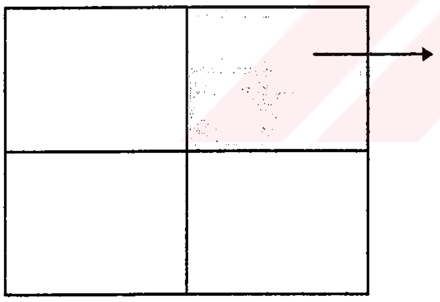
Sınıf tartışması:

Büyük bir olasılıkla sonuç karşısında hayrete düşen öğrenciler olacaktır. Bu durum da aşağıdaki gibi bir model uzerinde 0 ile 1 arasında bile sonsuz çoklukta (sayı) nokta olabileceğini tarama yaparak kanıtlayın.



$\frac{1}{2} = 0.5$

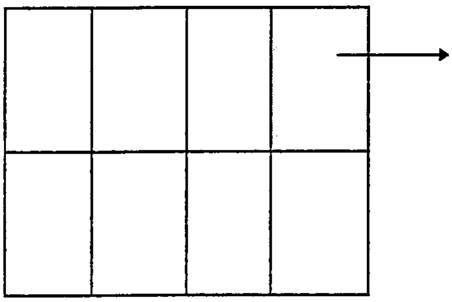
1 birimlik bir modelin iki eşit parçaya ayrılıp bir parcasının alınması ile aslında 0 ila 1 arasında olan 0.5 sayısının bulunduğunu belirtin.



$\frac{1}{4} = 0.25$

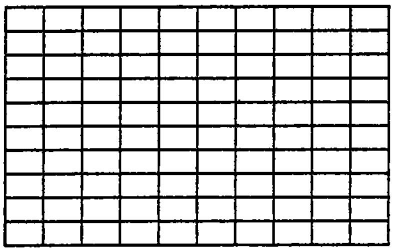
0 < 0.25 < 1

Gittikçe yoğunluğu artırarak genellemeye doğru gidişin kolaylaşmasını sağlayın.



$\frac{1}{8} = 0.125$

0 < 0.125 < 1



$$\frac{2}{100} = 0.02$$

0 < 0.02 < 1

Pekiştirme Etkinliği - 1:

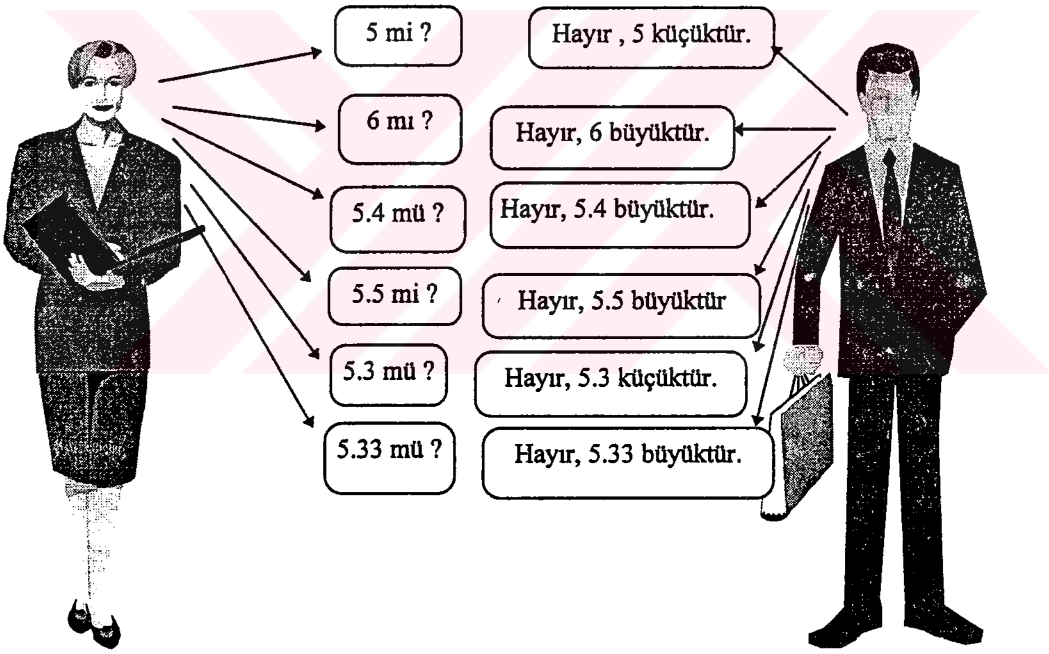
Öğrencilerden **0.5**, **0.125**, ve **0.02** sayılarını aynı sayı doğrusu üzerinde göstermelerini isteyin.

Pekiştirme Etkinliği - 2:

Bu kez öğrencilere *Sayıyı Bulun* isimli etkinlik formunu dağıtarak , yönergede belirtilenleri yerine getirmelerini isteyin.

Sayıyı Bulun

Aşağıda Ali ile Ayşenin aralarında geçen bir konuşması verilmiştir. Ali akıldan 0 ile 10 arasında bir sayı tutmuştur Ayşe ise tahminler yürüterek bu sayıyı bulmak istemektedir. Ali , Ayşe'nin tahminleri karşısında sadece "*büyük*" veya "*küçük*" demektedir.



- Tahminlerden hangisi size mantıksız geldi ? Neden ?
- Ali'nin aklında tuttuğu sayı ne olabilir ?
- Bütün olasılıkların bir listesini yapınız.
- Ayşe'nin daha tahmin yürütebileceği kaç sayı vardır ? Nelerdir ?

- Şimdi benzer bir oyunu yanınızdaki arkadaşınızla oynayın. 2 ile 4 arasında bir ondalık sayı tutun.

KATEGORİ - 5

ONDALIK SAYILARDAN OLUŞAN ÖLÇÜM BİRİMLERİ

KAVRAM YANILGILARI:

- Ondalık nokta bir ayıraçtır.
- Ondalık noktadan sonraki sayı üzerinde çalışılan ölçüm biriminin bir alt birimini temsil eder.

KATEGORİ - 5
DERS - 5/ Modül - 1:

Amaç: Bu modülün amacı öğrencilere ondalık noktanın altbirimleri birbirinden ayıran gelişigüzel bir ayıraç olmadığının kavratılması ve sahip oldukları kavram yanılgılarının ortadan kaldırılmasıdır.

Davranışlar: Bu modülün sonunda öğrencilerde aşağıdaki davranışlar gözlenebilir:

1. Saatle ilgili ondalık sayılarda ondalık noktadan sonraki basamakların 60'lık sisteme göre ifade edilmediğini yazama/söyleme.
2. Ondalık sayılarda ondalık noktadan sonraki basamakların sırası ile *onda bir*, *yüzde bir*, *binde bir* şeklinde ilerlediğini yazma/söyleme.

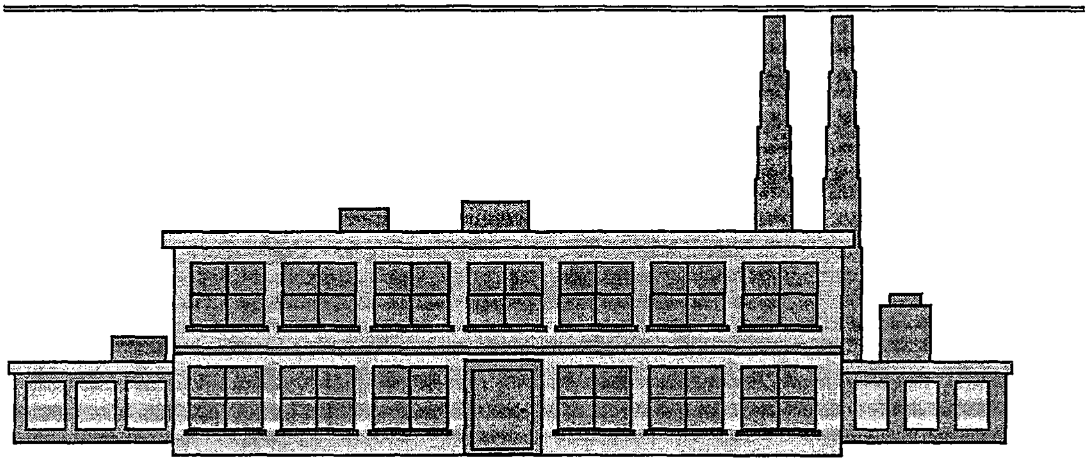
Kavram Yanılgıları:
- Ondalık nokta bir ayıraçtır.
- Ondalık noktadan sonraki sayı üzerinde çalışılan ölçüm biriminin bir alt birimini temsil eder.

Araç - Gereç: *Saatli Bomba* isimli etkinlik formu.

Giriş Etkinliği:
Her öğrenciye *Saatli Bomba* isimli etkinlik formu verilir ve yönergede belirtilenleri yapmaları istenir.

Saatli Bomba

Aşağıda gördüğünüz fabrikada toplam 500 işçi çalışmaktadır. Fabrika hergün akşam üzeri işçiler saat 5'te işi bırakır ve tam olarak saat 5'i 20 geçeyi gösterdiğinde fabrikadaki son işçi de kapıdan çıkmış olur. O gün fabrikaya bir saatli bomba yerleştirildiği konusunda bilgi aldığınızı düşünün. Size bombayı saat 4'te yerleştirdiklerini ve 1.25 saat sonra patlayacağını söylediklerini varsayın. Bu durumda işçilerin tümü fabrikayı terketmeden bomba patlar mı ?



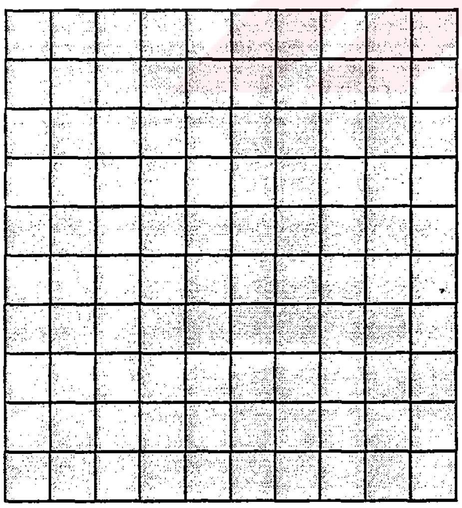
Sınıf Tartışması:

"Bomba işçilerin tümü fabrıkayı terketmeden önce patlar" ve "bomba işçiler ayrıldıktan sonra patlar" diyen iki grup öğrencinin kendilerini savunmasını isteyin. Hatalı olan grup yanlışını farkedene kadar devam edin. Daha sonra 1.25 saat lik süreyi dakika cinsinden aşağıdaki etkinlikte gösterin.

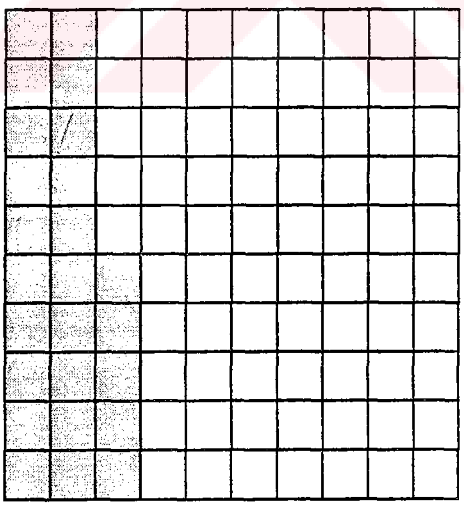
Pekiştirme Etkinliği - 1(*saydamda gösterilir*):

1.25 'in 1 tam 100'de 25 olduğuna işaret edin ve sizin bunun 60 ta kaç ettiğini bulmanız gerektiğini vurgulayın ve öğrencilerin de yardımı ile aşagıdaki modellerden yararlanın.

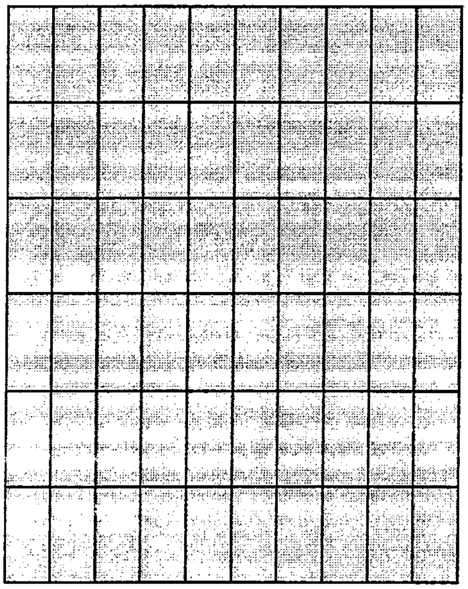
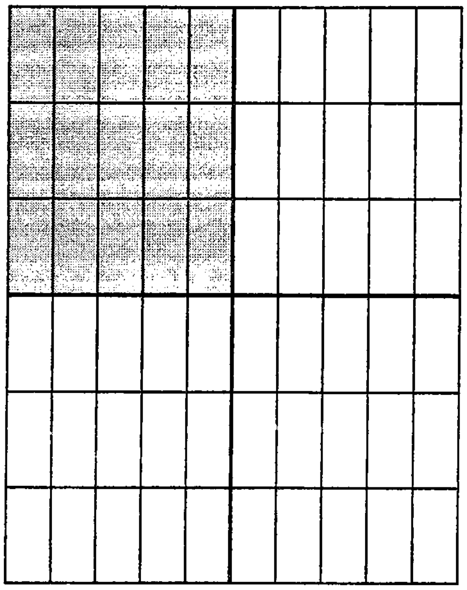
İlk model 1 tam yerine kullanılabilir ve tümü taranır.



İkinci model üzerinde bunun 100'de 25 lik kısmı taranır.



Daha sonra 25'in 60 bölüme ayrılmış bir bütün üzerinde kaça karşılık geldiğini görmek için ikinci model 60 eşit parçaya ayrılır.

Gerçekte 100'de 25 'in $\frac{1}{4}$ olduğu vurgulanır ve bunun ikinci model üzerinde 15'e karşılık geldiği gösterilir.

KATEGORİ - 5

DERS - 5/ Modül - 2:

Amaç: Bu modülün amacı öğrencilerin ondalık sayıları içeren ölçüm birimlerini farklı teknikler kullanarak (sayı, doğrusu, doğru orantı, karesel/dikdörtgensel model) gösterme becerilerini geliştirmek ve sahip oldukları kavram yanılgılarını ortadan kaldırmaktır.

Davranışlar: Bu modülün sonunda öğrencilerde aşağıdaki davranışlar gözlenebilir:

1. Ondalık sayı olarak verilen bir ölçüm birimini doğru orantı kullanarak istenilen bir altbirim cinsinden yazma.
2. Ondalık sayı olarak verilen bir ölçüm birimini sayı doğrusu kullanarak istenilen bir altbirim cinsinden yazma.
3. Ondalık sayı olarak verilen bir ölçüm birimini karesel/dikdörtgensel modeller kullanarak istenilen bir altbirim cinsinden yazma.

Kavram Yanılgıları:

- Ondalık nokta bir ayıraçtır.
- Ondalık noktadan sonraki sayı üzerinde çalışılan ölçüm biriminin bir alt birimini temsil eder.

Araç - Gereç: Pekiştirme formu - 2, Cetvel, Hesap makinesi

Pekiştirme Etkinliği - 2:

1) 5. 32 dakika ------------------------------ kaç saniyedir ?

2) 4.6 yıl ---------------------------------- kaç aydır ?

3) 5.20 ay ----------------------------------kaç gündür ?

KATEGORİ - 6

ONDALIK SAYILARI İÇEREN ÇARPMA VE BÖLME İŞLEMLERİ

KAVRAM YANILGILARI:
- Bölme küçültür.
- Çarma büyütür.
- Iki ondalık sayı aynı sayıda basamaktan oluşmuyorsa çarpılamaz.
- Ondalık nokta göz ardı edilebilir.

KATEGORİ - 6

DERS - 6/ Modül - 1:

Amaç: Bu modülün amacı öğrencilere birden küçük sayılarla çarpmanın ve bölmenin durumu nasıl değiştirdiğini kavratmak ve onları sahip oldukları kavram yanılgılarından arındırmaktır.

Davranışlar: Bu modülün sonunda öğrencilerde aşağıdaki davranışlar gözlenebilir:

1. Bir sayının 1'den küçük bir sayı ile çarpılması sonucu küçüleceğini yazma/söyleme.
2. Bir sayının 1'den küçük bir sayı ile bölünmesi sonucu büyüyeceğini yazma/söyleme.
3. Bir sayının çarpma ve bölme durumlarında neden büyüyüp küçüldüğünü açıklama.

Kavram yanılgıları:
- Bölme küçültür.
- Çarma büyütür.
- Iki ondalık sayı aynı sayıda basamaktan oluşmuyorsa çarpılamaz.
- Ondalık nokta göz ardı edilebilir.

Araç - Gereç: *Sınav Kağıdı* etkinlik formu, saydam, tepegöz.

Giriş Etkinliği:
Bir tepegöz kullanılarak bir saydam üzerinden tüm sınıfın görmesi için aşağıdaki sözde sınav kağıdı yansıtılır ve öğrencilerin bu kağıdı notlandırmaları istenir. Farklı görüşlere göre tartışma açılır.

Sınav Kağıdı

1) 5 × 0.234 = ?

Öğrencilerin Yanıtları =

1. Öğrenci :" 5'ten büyük çıkar."
2. Öğrenci :" 5'ten daha küçük bir sonuç çıkar."

2) **0.546 × 12 = ?**

Öğrencilerin Yanıtları =

1.Öğrenci:" 12'den küçük bir sonuç çıkar."
2. Öğrenci:" 12'den büyük bir sonuç çıkar."

3) **0.675 × 0.267 = ?**

Öğrencilerin Yanıtları =

1. Öğrenci:" 0.675'ten daha büyük çıkar."
2. Öğrenci:" 0.657'ted daha küçük çıkar."
3. Öğrenci:" 0.267'den daha büyük çıkar."
4. Öğrenci:" 0.267'den daha küçük çıkar."

4) **2 ÷ 0.265 = ?**

Öğrencilerin Yanıtları =

1. Öğrenci :" 2'den büyük çıkar."
2. Öğrenci :" 2'den daha küçük bir sonuç çıkar."

5) **0.456 ÷ 5 = ?**

Öğrencilerin Yanıtları =

1. Öğrenci :" 0.456'dan büyük çıkar."
2. Öğrenci :" 0.456'dan daha küçük bir sonuç çıkar."

6) **0.568 ÷ 0.765 = ?**

Öğrencilerin Yanıtları =

1. Öğrenci:" 0.568'den daha büyük çıkar."
2. Öğrenci:" 0.568'den daha küçük çıkar."

7) **0.876 ÷ 0.435 = ?**

Öğrencilerin Yanıtları =

1. Öğrenci:" 0.876'dan daha büyük çıkar."
2. Öğrenci:" 0.876'dan daha küçük çıkar."

Sınıf tartışması:

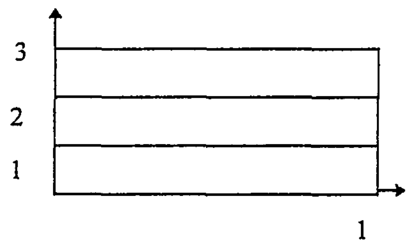
Öğrencilerden gelecek tepkilere göre tartışmanın şekli ve dozu ayarlanır. Hangi şartlarda bölme ve çarpmanın büyük hangi şartlarda küçük sonuçlar verdiği araştırılır.

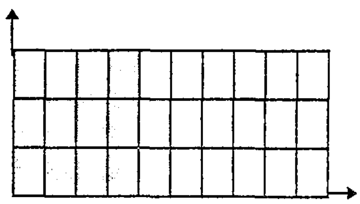
Pekiştirme Etkinliği :

Önce yine tepegöz yardımı ile sayı doğrularından oluşan koordinat eksenleri veya karesel modeller yardımı ile 1' den küçük sayı veya sayılardan oluşan çarpma işlemlerinin nasıl ifade edilebileceği üzerinde durulur.

1.DURUM: Sayılardan birinin 1' den küçük olması:

3 × 0.4 = ?





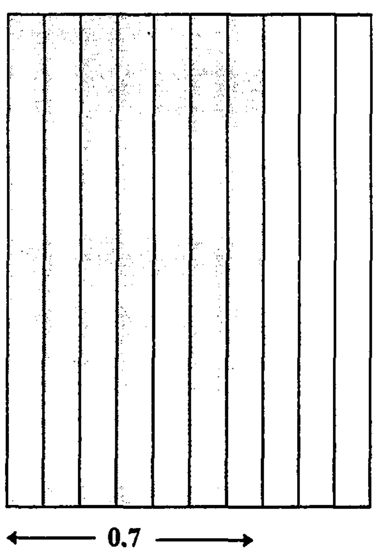
Başlangıçta her uzun blok 1 birim değerinde alınmıştır daha sonra her blok kendi içinde 10 eşit parçaya ayrılmışıtr. Yatay eksen üzerinden 0.4 karşılık gelen kısım tarandığı zaman **3 × 0.4** ifadesi bulunmuş olur yani **1.2.** Görüldüğü gibi 3'ten da küçük bir sonuç elde edilmiştir. Bu model üzerinde fiziksel olarak da gözlenmektedir.

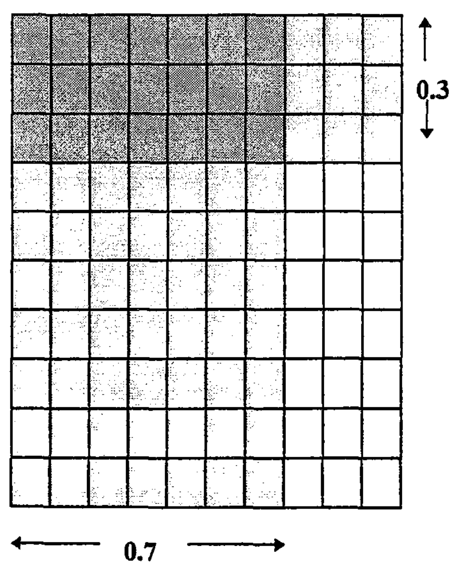
Bir başka anlatımla her yatay blok bir birim olduğuna göre taralı olan 12 parcanın 10 tanesinin 1 birim olduğu ve artakalan 2 birimin de 0.2 'yi temsil ettiği söylenirse sonuç 1 + 0.2 'den = 1.2 olur.

2. DURUM:(*saydam üzerinde gösterilir*) Her iki sayının da 1'den küçük olması:

0.7 × 0.3 = ?

Her model **1 - birim olarak kabul edilmektedir.**





Bu örnekte de bir öncekinde olduğu gibi 0.7 sayısının 0.3 sayısı ile çarpıldıktan sonra küçüldüğü gözlemlenmektedir. Sonuç **0.21** olmuştur.

KATEGORİ - 6
DERS - 6/ Modül - 2:

<u>Amaç</u>: Bu modülün amacı öğrencilere birden küçük sayılarla çarpmanın ve bölmenin durumu nasıl değiştirdiğini kavratmak ve onları sahip oldukları kavram yanılgılarından arındırmaktır.

<u>Davranışlar</u>: Bu modülün sonunda öğrencilerde aşağıdaki davranışlar gözlenebilir:
1. Bir sayının 1'den küçük bir sayı ile çarpılması sonucu küçüleceğini yazma/söyleme.
2. Bir sayının 1'den küçük bir sayı ile bölünmesi sonucu büyüyeceğini yazma/söyleme.
3. Bir sayının çarpma ve bölme durumlarında neden büyüyüp küçüldüğünü modeller yardımı ile açıklama.

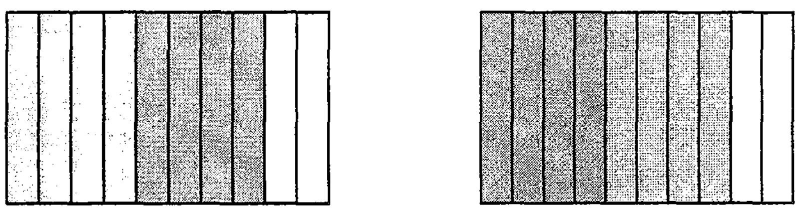
<u>Kavram Yanılgıları:</u>
- Bölme küçültür.
- Çarma büyütür.
- Iki ondalık sayı aynı sayıda basamaktan oluşmuyorsa çarpılamaz.
- Ondalık nokta göz ardı edilebilir.

<u>Araç - Gereç:</u> *Bölme modelleri* etkinlik formu

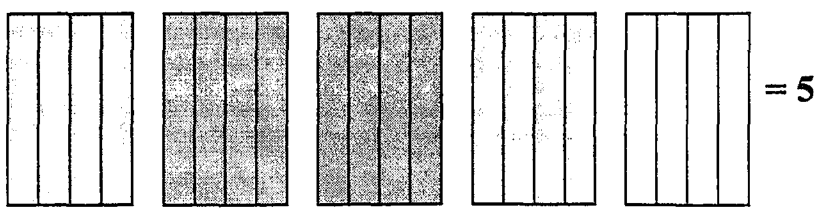
<u>Giriş Etkinliği:</u>
Sınıftaki her öğrenciye aşağıdaki bölme işlemleri verilip bunları bir önceki derste yaptıklarına benzer şekilde modeller üzerinde göstermeleri istenir.

<u>Örnek:</u>
$2 \div 0.4 = ?$

Her iki modeli önce 10 parçaya ayırıp daha sonra bunlar içerisindeki 0.4'lük parçaları bulmamız gerekir.



Bu durumda bu 0.4'lük parçalardan 5 adet olduğunu görürüz yani 2'den daha büyük bir değerle karşılaşırız.



Pekiştirme Etkinliği:

Aşağıdaki işlemleri modeller üzerinde ifade ediniz.

1) $1 \div 0.2 = ?$

2) $0.6 \times 0.8 = ?$

3) $4 \div 0.2 = ?$

KATEGORİ - 6

DERS - 7

Amaç: Bu dersin amacı öğrencilere birden küçük sayılarla çarpmanın ve bölmenin durumu nasıl değiştirdiğini kavratmak ve onları sahip oldukları kavram yanılgılarından arındırmaktır.

Davranışlar: Bu dersin sonunda öğrencilerde aşağıdaki davranışlar gözlenebilir:

1. Bir sayının 1'den küçük bir sayı ile çarpılması sonucu küçüleceğini yazma/söyleme.
2. Bir sayının 1'den küçük bir sayı ile bölünmesi sonucu büyüyeceğini yazma/söyleme.
3. Bir sayının çarpma ve bölme durumlarında neden büyüyüp küçüldüğünü açıklama.

Araç - Gereç: Tablolar ve değerler etkinlik formu.

Kavram Yanılgıları:

- Bölme küçültür.
- Çarma büyütür.
- Iki ondalık sayı aynı sayıda basamaktan oluşmuyorsa çarpılamaz.
- Ondalık nokta göz ardı edilebilir.

Giriş Etkinliği:

Sınıftaki her öğrenciye aşağıdaki tablo verilerek buradaki değerlerden oluşan bir grafik hazırlamaları istenir. Grafik kağıdı verilecektir.

A

x	y =0.5 x
0	0
2	1
4	2
6	3
8	4
10	5
12	6
14	7

B

x	y
0	0
2	2
4	4
6	6
8	8
10	10
12	12
14	14

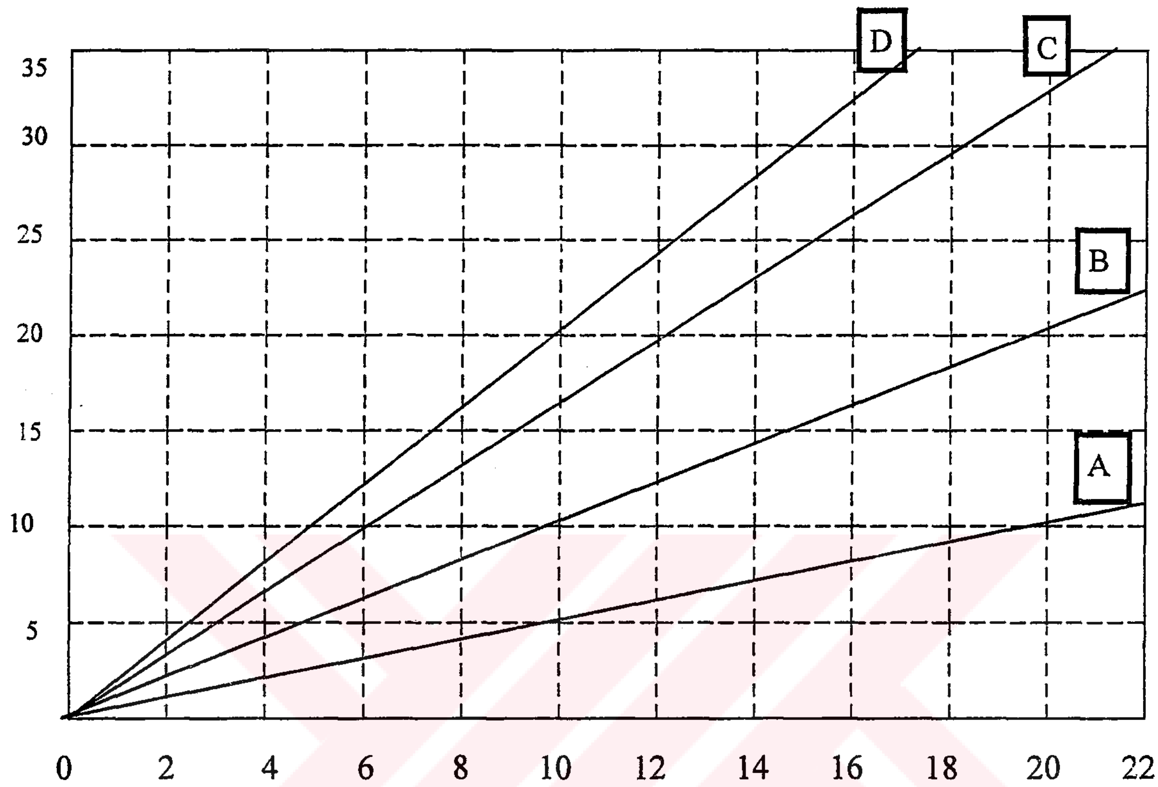
C

x	y = 1.6x
0	0
2	3.2
4	6.4
6	9.6
8	12.8
10	16
12	19.2
14	22.4

D

x	y = x/0.5
0	0
2	4
4	8
6	12
8	16
10	20
12	24
14	28

Grafik takriben aşağıdaki gibi olacak böylelikle öğrenciler 1'den büyük ve 1'den küçük sayılarla çarpmanın ve bölmenin durumu nasıl değiştirdiğini bir başka boyutta görme imkanına kavuşacaktır.



Burada özellikle A ve D grafiklerini karşılaştırarak çarpmanın nasıl azalan bölmenin ise nasıl artan bir karaktere dönüştüğü vurgulanabilir.

KATEGORİ - 7

ONDALIK SAYILARI İÇEREN SÖZEL PROBLEMLERDE İŞLEM SEÇİMİ

KAVRAM YANILGILARI

- Bir çarpma işleminde çarpan bir doğal sayı olmalıdır.
- Paylaştırma tipinde bir bölme modelinde bölen bölünenden küçük olmalıdır.
- Paylaştırma tipinde bir bölme modelinde bölen bir doğal sayı olmalıdır.
- Paylaştırma tipinde bir bölme modelinde bölünen sonçtan büyük olmalıdır.
- Bölümleme tipinde bir bölme modelinde bölünen bölenden büyük olmalıdır.

KATEGORİ - 7

DERS - 8/ Modül - 1:

Amaç: Bu modülün amacı öğrencilerin ondalık sayılar içeren çapma türündeki sözel problemleri çözme becerilerini artırmak ve sahip oldukları kavram yanılgılarını ortadan kaldırmaktır.

Davranışlar: Bu modülün sonunda öğrencilerde aşağıdaki davranışlar gözlenebilir:
1. Verilen çarpma problemi için işlem seçerken basit sayılardan yararlanma.
2. Verilen çarpma problemi için işlem seçerken tahmin yürütme stratejisinden yararlanma.
3. Verilen çarpma problemi için işlem seçerken şekillerden yararlanma.

Kavram Yanılgısı:
- Bir çarpma işleminde çarpan bir doğal sayı olmalıdır.

Araç - Gereç: Çarpma soru setleri, saydam, tepegöz.

Giriş Etkinliği:
Sınıftaki her öğrenciye aşağıdaki çarpma modellerinden olşan soru seti dağıtılır ve yan tarafa işlemi yazmaları istenir. Bunu yaparken uzun hesaplamalardan kaçınmaları önerilir.

---

1)Bir aşçı bir pasta için 0,62 kg un kullanıyor. 27 pasta yapmak için ne kadar una ihtiyacı vardır? [ ]

2)Bir araba bir litre benzinle 41,8 km yol alabiliyor. 8 litre benzinle ne kadar yol alabilir? [ ]

3) Bir koşucu bir yarışı 1,5 saatte tamamladı. Bu koşucunun ortalama hızı saatte 9 kilometreydi. Yarış kaç kilometreydi? [ ]

4) Amerikada yaşayan Ali'nin maket uçağı 0,53 litre benzin alıyor. Benzinin litresi 1,33 dolar ise Ali kaç para öder? [ ]

5) Bir vitamin tabletinde 0,15 gram C-vitamini vardır. Bu kaç ons eder?(1 gram 0,035 onstur.) [ ]

6) Bir elbisenin posterini bir dergi sayfasına yerleştirmek için, posterin gerçek büyüklüğünün 0,14'üne küçültülmesi gerekiyor. Orjinal posterde elbisenin boyu 2 metredir. Dergideki boyu kaç metre olacaktır? [ ]

Sınıf tartışması:
Her öğrenci ortalama 5 dakikalık süre içerisinde gerekli işlemleri yazdıktan sonra sorulan sorular tepegözden yansıtılarak sonuçlar üzerinde tartışılır. Doğru cevap verenlerle yanlış cevap verenlerin neler düşündükleri ve çözüm yolları üzerinde durulup hatalar vurgulanır.

Pekiştirme Etkinliği:

Daha önce sorulan çarpma sorularının aşağıdaki stratejiler kullanılarak çözülebileceği üzerinde durulur ve örnekler verilir.

Stratejiler (*basit sayılar kullanma, tahmin yürütme ve şekil çizme*):
1. Soruda işi zorlaştıran sayıların yerine orantılı olarak daha basit sayılar yerleştirilerek tekrar düşünülür (bunlar tamsayı olabilirler).

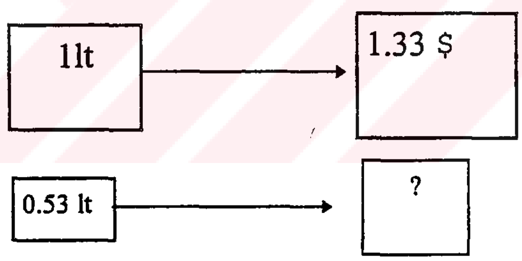
Örnek olarak aşağıdaki soruda 0.53 yerine *10* , 1.33 yerine ise *20* sayıları geçici olarak yerleştirilirse soru çok daha rahat algılanabilir.

Amerikada yaşayan Ali'nin maket uçağı 0,53 litre benzin alıyor. Benzinin litresi 1,33 dolar ise Ali kaç para öder?

2. Belirli şekiller yardımı ile soru daha rahat anlaşılır hale gelebilir:
Sayıların büyüklük küçüklüklerine göre belirli diagramlar veya sayı doğruları oluşturulabilir.

Örnek olarak aşağıdaki sorudaki işlemi seçmek için verilen sayıların büyüklükleri ile orantılı olarak belirli semboller kullanılabilir;

Amerikada yaşayan Ali'nin maket uçağı 0,53 litre benzin alıyor. Benzinin litresi 1,33 dolar ise Ali kaç para öder?



Buradaki bir litre bir birim olarak düşünüldüğü zaman buna karşılık gelen 1.33 olduğuna göra daha az bir birime 1.33'ten daha az bir karşılık bulunabileceği üzerinde durulur. Yani daha önce üzerinde çalışılan bir konu olan bir sayının 1'den küçük bir sayı ile *çarpılmasının* sayıyı küçülteceği hatırlatılır. Bir başka değişle 3'üncü bir strateji olarak *tahmin yürütmenin* işlem seçiminde nasıl işe yaradığı vurgulanır.

Pekiştirme Çalışması:

Öğrencilere bir set daha çarpma gerektiren (*daha çok kartezyen çarpım modeline uygun problemler verilir*)soru verilir ve her üç stratejiyi de gerekli hissettikleri durumlarda kullanarak bu sorular için işlem seçimi yapmalarını söyleyin. Öğrenciler daha sonra birbirlerinin kağıtlarını değerlendirme yoluna giderek bir grup tartışması yaparlar.

KATEGORİ - 7

DERS - 8/ Modül - 2:

Amaç: Bu modülün amacı öğrencilerin ondalık sayılar içeren çapma türündeki sözel problemleri çözme becerilerini artırmak ve sahip oldukları kavram yanılgılarını ortadan kaldırmaktır.

Davranış: Bu modülün sonunda öğrencilerde aşağıdaki davranış gözlenebilir:

Çarpma türünde problem türetme çalışmasında kısa sürede bir başkasının oluşturduğu sözçük veya işlem öbeklerine uygun tamamlamalarda bulunabilme.

Kavram Yanılgısı:

- Bir çarpma işleminde çarpan bir doğal sayı olmalıdır.

Araç - Gereç: Saydam, tepegöz.

Giriş Etkinliği:

Bu derste öğrencilere ancak çarpma ile çözülebilecek (*Kulanılacak sayılardan en az birinin ondalık sayı olması şartı belirtilir*) ve tek adımlık işlemler gerektiren sorular türetme çalışması yapılacağı belirtilir ve aşağıdaki örnek verilir.

Ali:	*Bir araba ...*
Oğuz:	*Bir litre benzinle...*
Seda:	*9.8 km yol alabiliyor....*
Ayşe:	*8 litre benzinle...*
Can:	*Ne kadar yol alabilir...*

Türetilen problem:

Bir araba bir litre benzinle 9,8 km yol alabiliyor. 8 litre benzinle ne kadar yol alabilir?

Her öğrencinin en fazla üç kelime ve/veya sayıdan oluşan bir ifade kullanabileceğini hatırlatın.

Pekiştirme Etkinliği:

Giriş etkinliğinde verilen örnekten yola çıkarak öğrencilere yaratıcılıklarını kullanarak soru türetmeleri istenir. Sınıftan rastgele bir öğrenci ile başlanır daha sonra her seferinde her sıradan bir öğrenci ortaya atılan ifadeyi mantıksal bir başka ifade ile tamamlamaya çalışır. Çalışmanın tümü saydamlar üzerinde yapılır. Bu çalışma dersin sonuna kadar devam eder. Duruma göre öğrenciler ikili olarak çalıştırılabilir.

Bu çalışmada öğrenciler tarafından türetilen sorular analiz edilerek daha sonra işlemler için soru yazma konusu içerisinde genelde kullanılan modeller tartışılırken örnek olarak kullanılacaktır.

KATEGORİ - 7
DERS - 9/ Modül - 1:

Amaç: Bu modülün amacı öğrencilerin ondalık sayılar içeren bölme türündeki sözel problemleri çözme becerilerini artırmak ve sahip oldukları kavram yanılgılarını ortadan kaldırmaktır.
Davranışlar: Bu modülün sonunda öğrencilerde aşağıdaki davranışlar gözlenebilir:

1. Verilen bölme problemi için işlem seçerken basit sayılardan yararlanma.
2. Verilen bölme problemi için işlem seçerken tahmin yürütme stratejisinden yararlanma.
3. Verilen bölme problemi için işlem seçerken şekillerden yararlanma.

Kavram Yanılgıları:

- Paylaştırma tipinde bir bölme modelinde bölen bölünenden küçük olmalıdır.
- Paylaştırma tipinde bir bölme modelinde bölen bir doğal sayı olmalıdır.
- Paylaştırma tipinde bir bölme modelinde bölünen sonçtan büyük olmalıdır.
- Bölümleme tipinde bir bölme modelinde bölünen bölenden büyük olmalıdır.

Araç - Gereç: Bölme soru setleri, saydam, tepegöz.

Giriş Etkinliği:

Sınıftaki her öğrenciye aşağıdaki *bölme* modellerinden olşan soru seti dağıtılır ve yan tarafa işlemi yazmaları istenir. Bunu yaparken uzun hesaplamalardan kaçınmaları önerilir.

Elinde 1400 metre elektrik kablosu olan bir elektrikçi 12 evin elektrik sistemini döşemek istiyor. Her evde ne kadar kablo kullanmalıdır?

25 arkadaş birlikte Spor-Toto oynayıp 22 milyon TL kazanıyor. Her biri kaç milyon kazanmış olur?

Elimde kutulara yerleştirilecek 600 kaset var. Her kutu 24 kaset alırsa kaç tane kutu dolar?

28,4 litrelik bir içeceği kapasitesi 3'er litre olan kaç adet kaba boşaltabiliriz ?

Bir mahkum cezaevinden kaçmak için bir tünel kazmaya başlıyor. Birinci günün sonunda sadece 0,174 kilometre kazabiliyor. Bu hızla 3 kilometre ötedeki ormana ulaşması kaç gününü alır?

Bir teknede 0,26 metre uzunluğundaki halatlara gerek duyulmaktadır. 0,78 metre uzunluğundaki bir halat kesilerek, ihtiyaç duyulan halatlardan kaç tane elde edilebilir?

Bir yastık kılıfı yapmak için 0,48 metre kumaş kullanılıyor. 2,4 metre kumaş ile kaç kılıf yapılır?

Bir kürek takımı 3 kilometrelik yolu 7,2 dakikada alabiliyor. Bu takım 1 dakikada ne kadar yol gider?

Amerikada etin kilosu 2,56 dolardır. Bir ev hanımı 2 dolarlık et satın alıyor. Bu et kaç kilodur?

Okulun düzenlediği koşu 4,8 kilometreydi. Yarışı kazanan öğrencenin hızı saatte ortalama 5,24 kilometreydi. Bu öğrenci yarışı kaç saatte bitirdi?

Bir masanın uzunluğu 92,3 santimetredir. Bu masa kaç inç uzunluğundadır? (1 inç yaklaşık olarak 2.54 santimetredir.)

Bir otomobilin benzin deposu 5,5 galon benzin alıyor. 1 litre = 0,22 galon olduğuna göre bu otomobilin benzin deposu kaç litreliktir?

Sınıf tartışması:

Her öğrenci ortalama 15 dakikalık süre içerisinde gerekli işlemleri yazdıktan sonra sorulan sorular tepegözden yansıtılarak sonuçlar üzerinde tartışılır. Doğru cevap verenlerle yanlış cevap verenlerin neler düşündükleri ve çözüm yolları üzerinde durulup hatalar vurgulanır.

Pekiştirme Etkinliği:

Daha önce sorulan çarpma sorularında olduğu gibi burada da *(1) basit sayı kullanma, (2) tahmin yürütme ve (3) şekil çizme* stratejilerinin kullanılarak işlem seçiminin daha rahat yapılabileceği hatırlatılır ve yeni bir set bölme (*daha çok bölümleme tipinde blöme sorularına yer verilir*) sorusu öğrencilere dağıtılarak belirtilen stratejileride kullanarak çözmeleri istenir.

Grup Tartışması:

Verilen soru setleri çözüldükten sonra her öğrenci yanındaki arkadaşının kağıdını değerlendirerek kullanılan yöntemler ve sonuçlar üzerinde tartışılır.

KATEGORİ - 7
DERS - 9/ Modül - 2:

Amaç: Bu modülün amacı öğrencilerin ondalık sayılar içeren bölme türündeki sözel problemleri çözme becerilerini artırmak ve sahip oldukları kavram yanılgılarını ortadan kaldırmaktır.

Davranış: Bu modülün sonunda öğrencilerde aşağıdaki davranış gözlenebilir:

Bölme türünde problem türetme çalışmasında kısa sürede bir başkasının oluşturduğu sözçük veya işlem öbeklerine uygun tamamlamalarda bulunabilme.

Kavram Yanılgıları:

- Paylaştırma tipinde bir bölme modelinde bölen bölünenden küçük olmalıdır.
- Paylaştırma tipinde bir bölme modelinde bölen bir doğal sayı olmalıdır.
- Paylaştırma tipinde bir bölme modelinde bölünen sonçtan büyük olmalıdır.
- Bölümleme tipinde bir bölme modelinde bölünen bölenden büyük olmalıdır.

Araç - Gereç: Saydam, tepegöz.

Giriş Etkinliği:

Bu derste öğrencilere ancak bölme ile çözülebilecek (*Kulanılacak sayılardan en az birinin ondalık sayı olması şartı belirtilir*) ve tek adımlık işlemler gerektiren sorular türetme çalışması yapılacağı belirtilir ve aşağıdaki örnek verilir.

Ali:	*Bir teknede...*
Oğuz:	*0.26 metre uzunluğunda...*
Seda:	*halatlara ihtiyaç vardır...*
Ayşe:	*0.78 metre uzunluğundaki ...*
Can:	*Halat kesilerek...*
Fatma:	*İhtiyaç duyulan halatlardan...*
Doğa:	*Kaç tane elde edilir...*

Türetilen Problem:

> Bir teknede 0,26 metre uzunluğundaki halatlara gerek duyulmaktadır. 0,78 metre uzunluğundaki bir halat kesilerek, ihtiyaç duyulan halatlardan kaç tane elde edilebilir?

Her öğrencinin en fazla üç kelime ve/veya sayıdan oluşan bir ifade kullanabileceğini hatırlatın.

Pekiştirme Etkinliği:

Giriş etkinliğinde verilen örnekten yola çıkarak öğrencilere yaratıcılıklarını kullanarak soru türetmeleri istenir. Sınıftan rastgele bir öğrenci ile başlanır daha sonra her seferinde her sıradan bir öğrenci ortaya atılan ifadeyi mantıksal bir başka ifade ile tamamlamaya çalışır. Çalışmanın tümü saydamlar üzerinde yapılır. Bu çalışma dersin sonuna kadar devam eder. Duruma göre öğrenciler ikili olarak çalıştırılabilir.

Bu çalışmada öğrenciler tarafından türetilen sorular analiz edilerek daha sonra işlemler için soru yazma konusu içerisinde genelde kullanılan modeller tartışılırken örnek olarak kullanılacaktır.

KATEGORİ - 8

ÇARPMA VE BÖLME İŞLEMLERİ İÇİN PROBLEM YAZMA

PROBLEM YAZMAYI ETKİLEYEBİLECEK KAVRAM YANILGILARI

- Bölme küçültür.
- Çarpma büyütür.
- Bir çarpma işleminde çarpan bir doğal sayı olmalıdır.
- Paylaştırma tipinde bir bölme modelinde bölen bölünenden küçük olmalıdır.
- Paylaştırma tipinde bir bölme modelinde bölen bir doğal sayı olmalıdır.
- Paylaştırma tipinde bir bölme modelinde bölünen sonçtan büyük olmalıdır.
- Bölümleme tipinde bir bölme modelinde bölünen bölenden büyük olmalıdır.

KATEGORİ - 8
DERS - 10/ Modül - 1:

Amaç: Bu modülün amacı öğrencilerin çarpma işlemleri için uygun problem yazma becerilerini geliştirmek ve sahip oldukları kavram yanılgılarını ortadan kaldırmaktır.

Davranışlar: Bu modülün sonunda öğrencilerde aşağıdaki davranişlar gözlenebilir:

1. Verilen bir çarpma işlemi için çözülebilir bir problem yazma.
2. Verilen bir çarpma işlemi için kapsam açısından kuvvetli bir problem yazma.
3. Verilen bir çarpma işlemi için özgün bir problem yazma.

Araç - Gereç: Çarpma İşlemleri Formu, saydam, tepegöz.

Kavram Yanılgıları:

- Çarpma büyütür.
- Bir çarpma işleminde çarpan bir doğal sayı olmalıdır.

Giriş Etkinliği:

Önce her öğrenciye aşağıdaki çarma işlemlerinden oluşan set verilir ve her işlem için kendilerine en uygun gelen problemi yazmaları istenir. Daha sonra her öğrenci yanındaki arkadaşına hazırladığı soruları vererek üste yazılan işlemi dikkate almadan istedikleri yöntemi kullanarak problemleri çözmeleri istenir.

5 × 0.68

0.63 × 22

12.05 × 0.93

Grup Tartışması:

Öğrenciler önce kendi aralarında karşılaştıkları zıtlıklar üzerinde bir süre tartışırlar.

Sınıf Tartışması:

İkili grupların kendi aralarında yaptıkları tartışmalardan sonra sınıftaki bir iki grubun yazdıkları sorular ve bunların yine kendi arkadaşları tarafından çözümleri tepegözde yansıtılır ve sonuçlar tüm sınıfça tartışılır.

KATEGORİ - 8

DERS - 10/ Modül - 2:

Amaç: Bu modülün amacı öğrencilerin çarpma işlemleri için uygun problem yazma becerilerini geliştirmek ve sahip oldukları kavram yanılgılarını ortadan kaldırmaktır.

Davranışlar: Bu modülün sonunda öğrencilerde aşağıdaki davranışlar gözlenebilir:

1. Verilen bir çarpma probleminin çözülebilir olup olmadığını yazma/söyleme.
2. Verilen bir çarpma probleminin kapsam açısından nedenleri ile birlikte kuvvetli olup olmadığını yazma/söyleme.
3. Verilen bir çarpma probleminin özgün olup olmadığını yazma/söyleme.
4. Çarpma işlemi için farklı tanımlamalarda bulunabilme.

Kavram Yanılgıları:
- Çarpma büyütür.
- Bir çarpma işleminde çarpan bir doğal sayı olmalıdır.

Araç-Gereç: Ders 8 / Modül 2'de türetilen çarpma problemleri seti.

Giriş Etkinliği:

Daha önceki dersteki öğrencilerin çarpma işlemlerine göre problem yazmadaki durumlarını göze alarak sınıftaki birçok öğrenciden çarpmayı tanımlamaları istenir ve bu tanımlardan çoğu tahtaya yazılarak öğrencilerin de yardımı ile bu tanımlar sınıflandırılmaya çalışılır. Eksik bırakılan noktaları tamamlamak amacı ile aşağıdaki pekiştirme çalışması yapılır.

Pekiştirme Etkinliği:

Burada çarpma değişik boyutlardan ele alınır ve öğrencilerle tartışılır.

1. Tekrarlı Toplam Çarpma Modeli:

Çarpma tekrarlı toplam durumlarının olduğu bir zamanda sonucu bulmak için kullanılabilecek bir yöntemdir: "Özgenin pullarını sıraladığı bir kitabı vardır. Kitabının toplam 9 sayfası vardır ve her sayfa 15 pul alabilmektedir. Bu durumda kitabında toplam kaç pul vardır ?" Böyle bir soru 15'i yanyana 9 kez yazıp toplamakla çözülebilir. Bir başka yol ise 9 ile 15'in çarpılmasıdır.

2. Kartezyen Çarpma Modeli:

Kartezyen çarpma modeli bir küme içerisindeki her elemanın diğer bir kümedeki bütün elemanlarla eşlenmesi prensibine dayanır. Alan hesabı, fiat x miktar, hız x zaman ve benzeri hesaplamalar kartezyen çarpım prensibi için verilebilecek temel örneklerdir.

Sınıf Tartışması:

Öğrencilerin belirtilen çarpma modellerinden örnekler vermeleri ve eleştirilerde bulunmaları istenir.

KATEGORİ - 8
DERS - 11/ Modül - 1:

Amaç: Bu modülün amacı öğrencilerin bölme işlemleri için uygun problem yazma becerilerini geliştirmek ve sahip oldukları kavram yanılgılarını ortadan kaldırmaktır.

Davranışlar: Bu modülün sonunda öğrencilerde aşağıdaki davranişlar gözlenebilir:

1. Verilen bir bölme işlemi için çözülebilir bir problem yazma.
2. Verilen bir bölme işlemi için kapsam açısından kuvvetli bir problem yazma.
3. Verilen bir bölme işlemi için özgün bir problem yazma.

Kavram Yanılgıları:
- Bölme küçültür.
- Paylaştırma tipinde bir bölme modelinde bölen bölünenden küçük olmalıdır.
- Paylaştırma tipinde bir bölme modelinde bölen bir doğal sayı olmalıdır.
- Paylaştırma tipinde bir bölme modelinde bölünen sonçtan büyük olmalıdır.
- Bölümleme tipinde bir bölme modelinde bölünen bölenden büyük olmalıdır.

Araç - Gereç: Bölme İşlemleri Formu, saydam, tepegöz.

Giriş Etkinliği:

Önce her öğrenciye aşağıdaki bölme işlemlerinden oluşan set verilir ve her işlem için kendilerine en uygun gelen problemi yazmaları istenir. Daha sonra her öğrenci yanındaki arkadaşına hazırladığı soruları vererek üste yazılan işlemi dikkate almadan istedikleri yöntemi kullanarak problemleri çözmeleri istenir.

0.34 ÷ 0.26

6.83 ÷ 36

7.3 ÷ 65.7

0.56 ÷ 5.4

Grup Tartışması:

Öğrenciler önce kendi aralarında karşılaştıkları zıtlıklar üzerinde bir süre tartışırlar.

Sınıf Tartışması:

İkili grupların kendi aralarında yaptıkları tartışmalardan sonra sınıftaki bir iki grubun yazdıkları sorular ve bunların yine kendi arkadaşları tarafından çözümleri tepegözde yansıtılır ve sonuçlar tüm sınıfça tartışılır.

KATEGORİ - 8
DERS - 11/ Modül - 2:

Amaç: Bu modülün amacı öğrencilerin bölme işlemleri için uygun problem yazma becerilerini geliştirmek ve sahip oldukları kavram yanılgılarını ortadan kaldırmaktır.

Davranışlar: Bu modülün sonunda öğrencilerde aşağıdaki davranışlar gözlenebilir:

1. Verilen bir bölme probleminin çözülebilir olup olmadığını yazma/söyleme.
2. Verilen bir bölme probleminin kapsam açısından nedenleri ile birlikte kuvvetli olup olmadığını yazma/söyleme.
3. Verilen bir bölme probleminin özgün olup olmadığını yazma/söyleme.
4. Bölme işlemi için farklı tanımlamalarda bulunabilme.

Kavram Yanılgıları:

- Bölme küçültür.
- Paylaştırma tipinde bir bölme modelinde bölen bölünenden küçük olmalıdır.

- Paylaştırma tipinde bir bölme modelinde bölen bir doğal sayı olmalıdır.
- Paylaştırma tipinde bir bölme modelinde bölünen sonçtan büyük olmalıdır.
- Bölümleme tipinde bir bölme modelinde bölünen bölenden büyük olmalıdır.

Araç-Gereç: Bölme durumları ek materyaller, tepegöz, ders-9/modül 2'de türetilen bölme soruları seti.

Giriş Etkinliği:

Daha önceki dersteki öğrencilerin bölme işlemlerine göre problem yazmadaki durumlarını göze alarak sınıftaki birçok öğrenciden bölmeyi tanımlamaları istenir ve bu tanımlardan çoğu tahtaya yazılarak öğrencilerin de yardımı ile bu tanımlar sınıflandırılmaya çalışılır. Eksik bırakılan noktaları tamamlamak amacı ile aşağıdaki pekiştirme çalışması yapılır.

Pekiştirme Etkinliği:

Burada bölme değişik boyutlardan ele alınır ve öğrencilerle tartışılır.

1. Paylaştırma Bölme Modeli:

Bu modelde önemli olan bir kümedeki her elemanın payına düşen miktarı bulmaktır.

Örnek: "8 kişilik bir grup arkadaş 24 bilyeyi paylaşmak istediği zaman her birinin payına 3'er bilye düşer"

2. Bölümleme Bölme Modeli:

Bu modelde amaç belirli bir miktarı paylaşacak olan eleman sayısını bulmaktır.

Örnek: "Elimizde bulunan 30 topu kapasitesi 3'er top olan kutulara dağıtmak istiyoruz, bu durumda kaç adet kutuya ihtiyaç vardır ?"

DERS 12: *Kapanış*

Bu derste daha önce görüşme yapılan öğrenclerin band kayıtları sınıfça izlenerek tartışılmıştır.

Appendix - J

Comparison of Mean Scores Gained from the Pre- Applications of the Tests and their Sub-Scales

Variable	Group	Mean	SD	2-Tail Sig.
	EG	25.7917	8.802	
ACT				0.114
	CG	21.6400	9.224	
	EG	15.9167	4.827	
APT				0.720
	CG	16.4400	5.300	
	EG	21.1250	6.842	
AWWPT				0.296
	CG	18.9200	7.729	
	EG	6.6667	2.444	
ACT1.1				0.151
	CG	5.5600	2.844	
	EG	5.2917	2.710	
ACT1.2				0.170
	CG	4.1600	2.968	
	EG	1.3550	1.135	
ACT2.1				0.090
	CG	0.8400	1.028	
	EG	1.5417	1.444	
ACT2.2				0.137
	CG	0.9600	1.241	
	EG	1.6250	1.209	
ACT3.1				0.764
	CG	1.5200	1.229	
	EG	1.3333	1.239	
ACT3.2				0.154
	CG	0.8400	1.143	
	EG	0.8750	0.338	
ACT4				0.584
	CG	0.8000	0.577	
	EG	3.9167	1.613	
ACT5				0.415
	CG	3.5200	1.759	
	EG	3.1667	1.633	
ACT6				0.846
	CG	3.0800	1.470	
	EG	7.7083	1.268	
APT1.1				0.372
	CG	7.2400	2.223	
	EG	5.6250	1.583	
APT1.2				0.907
	CG	5.6800	1.701	
	EG	1.4583	0.721	
APT1.3				0.622
	CG	1.5600	0.712	
	EG	10.6250	2.081	
APT2.1				0.091
	CG	9.2000	3.500	
	EG	5.7500	2.575	
APT2.2				0.856
	CG	5.8800	2.403	
	EG	3.0833	1.176	
APT2.3				0.542
	CG	3.3200	1.492	
	EG	3.6250	1.663	
APT2.4				0.549
	CG	3.9200	1.754	

Comparison of Mean Scores Gained from the Pre- Applications of the Tests and their Sub-Scales

	EG	8.4583	3.401	
AWWPT-MULT				0.751
	CG	8.1200	3.982	
	EG	12.6667	5.256	
AWWPT-DIV				0.222
	CG	10.8000	5.292	
	EG	5.8333	2.334	
AWWPT-DLTD				0.725
	CG	5.6000	2.273	
	EG	6.8333	4.082	
AWWPT-DGTD				0.168
	CG	5.2000	4.082	

Appendix - K

Overall Distribution of the Interviewees Holding the Observed Misconceptions Related to Decimals

	Decimals as points on number lines									Decimals on shaded areas									Comparing decimals				Denseness of decimals				Unit measures		Operations				Word problems					
Interviewees	1	2	3	4	5	6	7	8	9	3	10	11	12	8	13	14	6	15	16	17	18	11	19	20	21	3	22	23	24	25	26	27	28	29	30	31	32	N[1]
1		*			*		*				*	*							*			*	*				*	*	*			*	*			*		13
2	*	*			*	*		*	*	*	*	*							*					*			*	*	*				*					14
3	*	*								*	*		*						*					*			*	*	*	*			*	*			*	14
4		*		*						*		*												*			*	*	*	*					*	*	*	12
5																		*									*	*	*	*				*				6
6	*												*				*						*			*	*	*	*	*	*	*						11
7																				*									*	*								3
8						*														*		*					*	*		*	*		*	*				9
9											*					*			*					*	*					*				*		*		8
10										*		*		*	*								*						*	*	*		*	*				10
11	*																						*						*		*							4
12	*										*			*						*	*						*	*	*	*	*			*				11
13	*																				*						*	*						*				5
14			*				*	*	*							*	*							*	*		*	*	*					*	*			13
15	*									*									*				*						*	*								6
16				*																							*	*									*	4
17	*	*	*							*	*												*			*			*	*	*			*	*			12
18	*		*												*										*	*	*	*	*	*								9
19										*		*											*				*	*	*	*				*				8
20										*																	*	*	*									4
21																								*	*		*	*	*	*								6
22	*																												*	*	*							4
23																							*			*				*	*			*				5
24	*																						*												*			3
25																							*								*							2

Misconceptions:

1. On a number line between two numbers there are 10 divisions each being 0.1 unit.
2. Any decimal on a number line can be marked between the starting point and the first subunit mark.
3. Decimals are based on subunit 10.
4. Each subunit mark on a number line is 0.5 unit
5. Each division on a number line is 1 unit.
6. Decimals are based on subunit 100.
7. Place of a decimal number changes on a number line when the calibration changes.
8. Different subunits can be used on the same number line / model.
9. Doubling the calibration of a number line doubles the value of each subunit mark..
10. A one unit area model can be treated as multiple one unit models.
11. Zero in a decimal number is not a place holder.
12. Multiple one unit are models can be treated as a single one unit model.
13. Calibration of a model can be decreased without changing the size of each subunit area.
14. Calibration of a model can be increased without changing the size of each subunit area.
15. Some area models have no decimal representation.
16. A decimal is greater when the whole number after the decimal point is greater.
17. More digits after the decimal point makes a decimal smaller.
18. When the number of steps needed to round a decimal to a whole number is limited the number becomes larger.
19. There is a limited number of decimals between two consecutive decimals.
20. There is no any number between two consecutive decimals.
21. The difference between two consecutive decimals is one unit.
22. Decimal point is a separator.
23. The number after the decimal point may represent different units according to the subunit system of a given measure.
24. Division makes smaller.
25. Multiplication makes bigger.
26. Decimal point can be ignored.
27. Two decimals can not be multiplied if they have not the same number of digits.
28. In a multiplication expression the operator should be a whole number.
29. In quotitive division model the dividend should be greater than the divisor.
30. In a partitive division model the divisor should be a whole number.
31. In a partitive division model the dividend should be greater than the divisor.
32. In a partitive division model the dividend should be greater than the quotient.

(1): *N*, Shows the total number of misconceptions a related interviewee hold.

VITA

Osman Cankoy was born in Limasol / Cyprus, on December 25, 1964. He received his B. S. degree in Mathematics from the Middle East Technical University in July 1987. He worked in Ankara Yükseliş College as a mathematics teachers from 1987 to 1988. Later he worked in the department of science education, in the Middle East Technical University, as a research assistant form May 1989 to November 1989. He received his M. S. degree in Science Education from the Middle East Technical University in November 1989. He worked in the department of mathematics, in the Eastern Mediterranean University, as an instructor form September 1991 to September 1992. After that he worked in a government school, in Northern Cyprus, as a mathematics and physics teachers from 1992 to 1994. He has been working as an instructor in Atatürk Teacher Training College in Northern Cyprus since 1994. His main areas of interest are mathematics education, teacher education, and curriculum development.