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RARE RADIATIVE $B \to \tau^+\tau^-\gamma$ DECAY

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Abstract

The radiative $B \to \tau^+\tau^-\gamma$ decay is investigated in the framework of the Standard Model. When only short (short and long together) distance contributions are taken into account, the Branching Ratio is found as 9.54×10^{-9} (1.52×10^{-8}), for the value of the cut $\delta = 0.01$ imposed on the photon energy.

1 Introduction

Experimental discovery of the inclusive and exclusive $B \to X_s \gamma$ and $B \to K^* \gamma$ [1] decays stimulated the study of the radiative rare B meson decays with a new momentum. From theoretical point of view this is due to the fact that they are very sensitive to the flavor structure of the electroweak interactions, as well as QCD radiative corrections and the new physics beyond the Standard Model (SM) [2]. From experimental point of view studying radiative B–meson decays allows more precise determination of the parameters of the SM, such as the elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix, the leptonic decay constants etc., which are yet poorly known (see for example [3]).

Currently there is impressive effort in this direction, and many new facilities are under construction for studying the rare B–meson decays, namely, symmetric and asymmetric B–meson factories at Cornell, KEK and SLAC. Progress is also being made in hadronic environment at HERA–B and there are some plans for TeV–B and LHC–B. These machines will serve to measure the processes, for which SM predicts very small Branching Ratios. Among the rare decays, the flavor changing decays of the B–meson which proceed via electroweak penguins, are of special interest due to their relative cleanliness and their sensitivity to the new physics. The rare $B \to \tau^+ \tau^- \gamma$ decay belongs to this category.

From helicity arguments it is clear that the matrix element of $B \to \ell^+ \ell^-$ $(\ell = e,\ \mu)$ decay will be proportional to the lepton mass and therefore the corresponding Branching Ratios will be strongly suppressed. Note that in SM, $\mathcal{B}(B \to e^+ e^-) \simeq 4.2 \times 10^{-14}$ and $\mathcal{B}(B \to \mu^+ \mu^-) \simeq 1.8 \times 10^{-9}$. It is well known however that the the $\mathcal{B}(B \to \tau^+ \tau^-) \simeq 8 \times 10^{-7}$ in SM [4], and thus this decay can be measured in future B–factories with high enough efficiency.

When photon is emitted in addition to the lepton pair, no helicity suppression exists, and a "large" Branching Ratio is expected. Indeed in [5] it was shown that the $\mathcal{B}(B \to e^+ e^- \gamma) \simeq 2.35 \times 10^{-9}$. For $B \to \ell^+ \ell^- \gamma$ $(\ell = e,\ \mu)$, the contributions of the diagrams, where photon is radiated from any charged internal line, can safely be neglected, as they are strongly suppressed by a factor m_b^2/m_W^2 in the Wilson coefficients (see [5]). Moreover, it follows from helicity arguments that, the contributions of the diagrams where a photon is emitted from the final charged lepton lines, must be proportional to the lepton mass m_ℓ $(\ell = e,\ \mu)$, and hence they are negligible as well. Therefore in $B \to \ell^+ \ell^- \gamma$ $(\ell = e,\ \mu)$, the main contribution should come from the diagrams, where photon is emitted from the initial quarks.

In $B \to \tau^+ \tau^- \gamma$ decay, the situation becomes very different. In this case, we cannot neglect the contribution of the diagrams, where photon is radiated from the final τ–leptons, since the mass of the τ–lepton is not so much smaller than that of the B–meson. So, in $B \to \tau^+ \tau^- \gamma$ decay comparable contributions come from diagrams where photon is radiated both from initial and final fermions. These contributions can give essential information about the relative roles of the strong and electroweak interactions.

In this work we investigate the $B \to \tau^+ \tau^- \gamma$ decay, and the paper is organized as follows. In Section 2 we give the necessary theoretical framework for the $B \to \tau^+ \tau^- \gamma$ decay. Section 3 is devoted to the numerical analysis and the discussion of the results. In Appendix the detailed description of the cancellation of the infrared (IR) singularities, is given.

2 Theoretical framework for the $B \to \tau^+\tau^-\gamma$ decay.

The matrix element for the $b \to s\tau^+\tau^-\gamma$ decay can be obtained from that of the $b \to s\tau^+\tau^-$. It is well known that the short distance contributions to $b \to s\tau^+\tau^-$ decay comes from the box, Z– and photon–mediated penguins. Thus, in the SM, QCD–corrected amplitude for $b \to s\tau^+\tau^-$ can be written as [6, 7].

$$\begin{aligned} \mathcal{M} &= \frac{\alpha G_F}{2\sqrt{2}\,\pi} V_{tb}V_{ts}^* \Bigg\{ C_9^{eff}\, \bar{s}\gamma_\mu(1-\gamma_5)b\, \bar{\tau}\gamma_\mu\tau + C_{10}\bar{s}\gamma_\mu(1-\gamma_5)b\,\bar{\tau}\gamma_\mu\gamma_5\tau \\ &- 2C_7\frac{m_b}{p^2}\bar{s}i\sigma_{\mu\nu}p_\nu(1+\gamma_5)b\,\bar{\tau}\gamma_\mu\tau \Bigg\} . \end{aligned} \tag{1}$$

In Eq. (1) p is the momentum transfer, and the mass of the strange quark is neglected, V_{ij}'s are the corresponding elements of the CKM matrix. The analytical expression of all Wilson coefficients C_9^{eff}, C_{10} and C_7 can be found in [6, 7]. In order to obtain the matrix element for $b \to s\tau^+\tau^-\gamma$, it is necessary to attach photon to any charged internal, as well as external line. Contributions of the diagrams with photon attached to the any charged internal line, are strongly suppressed and therefore we shall neglect these in the following discussions. Thus, as explained previously the main contributions to the $b \to s\tau^+\tau^-\gamma$ decay comes from diagrams, when photon is radiated from initial and final fermions.

When a photon is attached to the initial quark lines, the corresponding matrix element for the $B \to \tau^+\tau^-\gamma$ decay can be written as

$$\begin{aligned} \mathcal{M}_1 &= \langle\gamma|\mathcal{M}|B\rangle = \\ & \frac{\alpha G_F}{2\sqrt{2}\,\pi} V_{tb}V_{ts}^* \Bigg\{ C_9^{eff}\bar{\tau}\gamma_\mu\tau\langle\gamma|\bar{s}\gamma_\mu(1-\gamma_5)b|B\rangle \\ &+ C_{10}\bar{\tau}\gamma_\mu\gamma_5\tau\langle\gamma|\bar{s}\gamma_\mu(1-\gamma_5)b|B\rangle \\ &- 2C_7\frac{m_b}{p^2}\langle\gamma|\bar{s}i\sigma_{\mu\nu}p_\nu(1+\gamma_5)b|B\rangle\bar{\tau}\gamma_\mu\tau \Bigg\} . \end{aligned} \tag{2}$$

These matrix elements can be written in terms of the two independent, gauge invariant, parity conserving and parity violating form factors [5, 8]:

$$\begin{aligned} \langle\gamma|\bar{s}\gamma_\mu(1-\gamma_5)b|B\rangle &= \frac{e}{m_B^2}\Bigg\{\epsilon_{\mu\alpha\beta\sigma}\epsilon_\alpha^* p_\beta q_\sigma\, g(p^2) \\ &+ i\left[\epsilon_\mu^*(pq) - (\epsilon^* p)q_\mu\right]\, f(p^2)\Bigg\} , \\ \langle\gamma|\bar{s}i\sigma_{\mu\nu}p_\nu(1+\gamma_5)b|B\rangle &= \frac{e}{m_B^2}\Bigg\{\epsilon_{\mu\alpha\beta\sigma}\epsilon_\alpha^* p_\beta q_\sigma\, g_1(p^2) \\ &+ i\left[\epsilon_\mu^*(pq) - (\epsilon^* p)q_\mu\right]\, f_1(p^2)\Bigg\} . \end{aligned} \tag{3}$$

Here ϵ_μ and q_μ are the four vector polarization and four momentum of the photon, respectively, and p is the momentum transfer. Substituting Eq. (3) in (4), for the matrix element $\mathcal{M}_1$ (structure dependent part) we get

$$\begin{aligned} \mathcal{M}_1 &= \frac{\alpha G_F}{2\sqrt{2}\,\pi} V_{tb} V_{ts}^* e \Bigg\{ \epsilon_{\mu\alpha\beta\sigma} \epsilon_\alpha^* p_\beta q_\sigma \left[A\, \bar\tau \gamma_\mu \tau + C\, \bar\tau \gamma_\mu \gamma_5 \tau \right] \\ &+ i \Big[\epsilon_\mu^* (pq) - (\epsilon^* p) q_\mu \Big] \left[B\, \bar\tau \gamma_\mu \tau + D\, \bar\tau \gamma_\mu \gamma_5 \tau \right] \Bigg\} , \end{aligned} \tag{4}$$

where

$$\begin{aligned} A &= \frac{1}{m_B^2} \left[C_9^{eff}\, g(p^2) - 2C_7 \frac{m_b}{p^2}\, g_1(p^2) \right] , \\ B &= \frac{1}{m_B^2} \left[C_9^{eff}\, f(p^2) - 2C_7 \frac{m_b}{p^2}\, f_1(p^2) \right] , \\ C &= \frac{C_{10}}{m_B^2}\, g(p^2) , \\ D &= \frac{C_{10}}{m_B^2}\, f(p^2) . \end{aligned} \tag{5}$$

When a photon is radiated from the final τ–leptons, the corresponding matrix element is (Bremstrahlung part)

$$\mathcal{M}_2 = \frac{\alpha G_F}{2\sqrt{2}\,\pi} V_{tb} V_{ts}^* e i f_B C_{10} 2 m_\tau \left[\bar\tau \left(\frac{\not\epsilon \not P_B}{2 p_1 q} - \frac{\not P_B \not\epsilon}{2 p_2 q} \right) \gamma_5 \tau \right] , \tag{6}$$

where P_B is the momentum of the B–meson. In obtaining this expression we have used

$$\begin{aligned} \langle 0 | \bar s \gamma_\mu \gamma_5 b | B \rangle &= -\, i f_B P_{B\mu} , \\ \langle 0 | \bar s \sigma_{\mu\nu} (1+\gamma_5) b | B \rangle &= 0 , \end{aligned} \tag{7}$$

and the conservation of the vector current.

Finally, the total matrix element for the $B \to \tau^+\tau^-\gamma$ decay is obtained as a sum of the $\mathcal{M}_1$ and $\mathcal{M}_2$:

$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2 . \tag{8}$$

The square of the matrix element, summed over the spins of the τ–leptons and the polarization of the photon, can be written as

$$|\mathcal{M}|^2 = |\mathcal{M}_1|^2 + |\mathcal{M}_2|^2 + 2\,\mathrm{Re}\,(\mathcal{M}_1 \mathcal{M}_2^*) , \tag{9}$$

where

$$
\begin{aligned}
|\mathcal{M}_1|^2 &= \left|\frac{\alpha G_F}{2\sqrt{2}\,\pi} V_{tb} V_{ts}^*\right|^2 4\pi\alpha \Bigg\{ 8\,\mathrm{Re}\,(B^* C + A^* D)\, p^2 \,(p_1 q - p_2 q)\,(p_1 q + p_2 q) \\
&+ 4\left[|C|^2 + |D|^2\right] \left[\left(p^2 - 2m_\tau^2\right)\left((p_1 q)^2 + (p_2 q)^2\right) - 4m_\tau^2\,(p_1 q)\,(p_2 q)\right] \\
&+ 4\left[|A|^2 + |B|^2\right] \Big[\left(p^2 + 2m_\tau^2\right)\left((p_1 q)^2 + (p_2 q)^2\right) \\
&+ 4m_\tau^2\,(p_1 q)\,(p_2 q)\Big]\Bigg\}\ , \qquad (10)
\end{aligned}
$$

$$
\begin{aligned}
2\,\mathrm{Re}\,(\mathcal{M}_1 \mathcal{M}_2^*) &= -\left|\frac{\alpha G_F}{2\sqrt{2}\,\pi} V_{tb} V_{ts}^*\right|^2 4\pi\alpha \Bigg\{ 16\, C_{10} f_B m_\tau^2 \Bigg[\mathrm{Re}(A)\, \frac{(p_1 q + p_2 q)^3}{(p_1 q)\,(p_2 q)} \\
&+ \mathrm{Re}(D)\, \frac{(p_1 q + p_2 q)^2\,(p_2 q - p_1 q)}{(p_1 q)\,(p_2 q)}\Bigg]\Bigg\}\ , \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
|\mathcal{M}_2|^2 &= -\left|\frac{\alpha G_F}{2\sqrt{2}\,\pi} V_{tb} V_{ts}^*\right|^2 4\pi\alpha \Bigg\{ -32 + 8\,\frac{m_\tau^2}{(p_1 q)^2}\left(p^2 + 2p_2 q\right) \\
&+ \frac{16}{p_1 q}\left[3m_\tau^2 - p^2 - p_2 q\right] + 8\,\frac{m_\tau^2}{(p_2 q)^2}\left(p^2 + 2p_1 q\right) \\
&+ \frac{16}{p_2 q}\left[3m_\tau^2 - p^2 - p_1 q\right] + 8\,\frac{p^2}{(p_1 q)\,(p_2 q)}\left[2m_\tau^2 - p^2\right]\Bigg\}\ . \qquad (12)
\end{aligned}
$$

Here $p_1,\ p_2$ are momenta of the final τ–leptons, and q is the photon momentum. The quantity $|\mathcal{M}|^2$ depends only on the scalar products of the momenta of the external particles. In the rest frame of the B–meson, all these scalar products are fixed, if the photon energy E_γ and the lepton energy E_1 are specified. The Dalitz boundary is given as

$$
0 \le E_\gamma \le \frac{m_B^2 - 4m_\tau^2}{2m_B}\ , \qquad (13)
$$

$$
\frac{m_B - E_\gamma}{2} - \frac{E_\gamma}{2}\sqrt{1 - \frac{4m_\tau^2}{m_B^2 - 2m_B E_\gamma}} \le E_1 \le \frac{m_B - E_\gamma}{2} + \frac{E_\gamma}{2}\sqrt{1 - \frac{4m_\tau^2}{m_B^2 - 2m_B E_\gamma}}\ . \qquad (14)
$$

The $|\mathcal{M}_1|^2$ term is completely infrared–free; the interference term has an integrable infrared singularity. Thus infrared divergence appears only in $|\mathcal{M}_2|^2$. The infrared singularity originates in the Bremstrahlung processes, when photon is soft. It is clear that in this limit, the $B \to \tau^+\tau^-\gamma$ decay cannot be distinguished from $B \to \tau^+\tau^-$. Therefore both processes must be considered together in order to obtain finite result for the decay rate. In the Appendix we show that IR singular terms in $|\mathcal{M}_2|^2$ exactly cancel the $O(\alpha)$ virtual correction in $B \to \tau^+\tau^-$ amplitude.

In this work, our point of view is slightly different from the standard description. Namely, we consider the Bremstrahlung process as a different process but not as the α

correction to the $B \to \tau^+\tau^-$ decay. In other words, we consider the photon in $B \to \tau^+\tau^-\gamma$ as a hard photon. Therefore, in order to obtain the decay width of the $B \to \tau^+\tau^- +$ (hard photon), we must impose a cut on the photon energy, which will correspond to the experimental cut imposed on the minimum energy for detectable photon. We require the energy of the photon to be larger than 50 MeV, i.e., $E_\gamma \geq a\, m_B$, where $a \geq 0.01$.

After integrating over the phase space, and taking into account the cut for the photon energy, for the decay rate we get,

$$\begin{aligned}
\Gamma &= \left| \frac{\alpha G_F}{2\sqrt{2}\,\pi} V_{tb} V_{ts}^* \right|^2 \frac{\alpha}{(2\,\pi)^3}\, m_B^5 \pi \\
&\times \Bigg\{ \frac{1}{12} \int_\delta^{1-4r} x^3\, dx \sqrt{1 - \frac{4r}{1-x}}\, m_B^2 \Big[\left(|A|^2 + |B|^2 \right) (1 - x + 2r) \\
&+ \left(|C|^2 + |D|^2 \right) (1 - x - 4r) \Big] \\
&- 2 C_{10} f_B r \int_\delta^{1-4r} x^2\, dx\, \mathrm{Re}\,(A)\, \ln \frac{1 + \sqrt{1 - \dfrac{4r}{1-x}}}{1 - \sqrt{1 - \dfrac{4r}{1-x}}} \\
&- 4 \left| f_B C_{10} \right|^2 r \frac{1}{m_B^2} \int_\delta^{1-4r} dx \Bigg[\left(2 + \frac{4r}{x} - \frac{2}{x} - x \right) \ln \frac{1 + \sqrt{1 - \dfrac{4r}{1-x}}}{1 - \sqrt{1 - \dfrac{4r}{1-x}}} \\
&+ \frac{2}{x} (1 - x) \sqrt{1 - \frac{4r}{1-x}} \Bigg] \Bigg\}~, \qquad (15)
\end{aligned}$$

where $x = \dfrac{2E_\gamma}{m_B}$ is the dimensionless photon energy, $r = \dfrac{m_\tau^2}{m_B^2}$ and $\delta = 2a$, satisfying

$$\delta \leq x \leq 1 - \frac{4 m_\tau^2}{m_B^2}~.$$

From Eq. (15) it follows that for calculating the decay width we need explicit forms of the form factors g, f, g_1 and f_1. These form factors are calculated in the framework of light–cone QCD sum rules in [4] (see also [8]), and their p^2 dependences, to a very good accuracy, can be represented in the following dipole forms,

$$\begin{aligned}
g_(p^2) &= \frac{1\ GeV}{(1 - \dfrac{p^2}{5.6^2})^2}~, \qquad f_(p^2) = \frac{0.8\ GeV}{(1 - \dfrac{p^2}{6.5^2})^2}~, \\
g_1(p^2) &= \frac{3.74\ GeV^2}{(1 - \dfrac{p^2}{40.5})^2}~, \qquad f_1(p^2) = \frac{0.68\ GeV^2}{(1 - \dfrac{p^2}{30})^2}~, \qquad (16)
\end{aligned}$$

which we will use in the numerical analysis.

3 Numerical analysis and discussion

For the input parameters, which enter into the expression for the decay width we use the following values: $m_b = 4.8\,GeV,\ m_c = 1.35\,GeV,\ m_\tau = 1.78\,GeV,\ m_B = 5.28\,GeV,\ |V_{tb}V_{ts^*}| = 0.045$. We use the pole form of the form factors given in Eq. (16). For B–meson life time we take $\tau\,(B_s) = 1.64 \times 10^{-12}\ s$ [9]. The value of the Wilson coefficients $C_7\,(m_b)$ and $C_{10}\,(m_b)$, to the leading logarithmic approximation, are (see for example [6, 7]):

$$C_7\,(m_b) = -\,0.315, \qquad C_{10}\,(m_b) = -\,4.6242\ . \tag{17}$$

The expression $C_9^{eff}\,(m_b)$ for the $b \to s$ transition, in the next–to–leading–order approximation, is given as

$$\begin{aligned} C_9^{eff}(m_b) &= C_9(m_b) + 0.124w(\hat{s}) + g(\hat{m}_c, \hat{s})(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \\ &\quad -\frac{1}{2}g(\hat{m}_q, \hat{s})(C_3 + 3C_4) - \frac{1}{2}g(\hat{m}_b, \hat{s})(4C_3 + 4C_4 + 3C_5 + C_6) \\ &\quad +\frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6)\ , \end{aligned} \tag{18}$$

with

$$\begin{aligned} &C_1 = -\,0.249, \qquad C_2 = 1.108, \qquad C_3 = 1.112 \times 10^{-2}, C_4 = -\,2.569 \times 10^{-2}, \\ &C_5 = 7.4 \times 10^{-3}, \qquad C_6 = -\,3.144 \times 10^{-2}, \qquad C_9 = 4.227\ , \end{aligned} \tag{19}$$

where $\hat{m}_q = m_q/m_b,\ \hat{s} = p^2/m_b^2$. The value of C_9^{eff} for the $b \to d$ transition can be obtained by adding to Eq. (18) the term

$$\lambda_u \left[g\,(\hat{m}_c, \hat{s}) - g\,(\hat{m}_d, \hat{s}) \right] (3C_1 + C_2)\ ,$$

where

$$\lambda_u = \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}\ ,$$

and replacing $V_{tb}V_{ts}^*$ in Eq. (15) by $V_{tb}V_{td}^*$. In Eq. (18), $w(\hat{s})$ represents the one gluon correction to the matrix element of operator O_9, and its explicit form can be found in [6, 7], while the function $g(\hat{m}_q, \hat{s})$ arises from the one–loop contributions of the four quark operators O_1–O_6, i.e.,

$$g(\hat{m}_q, \hat{s}') = -\frac{8}{9}\ln(\hat{m}_q) + \frac{8}{27} + \frac{4}{9}y_q - \frac{2}{9}(2 + y_q)\sqrt{|1 - y_q|}$$

$$\times \left\{ \Theta(1 - y_q) \left(\ln\frac{1 + \sqrt{1 - y_q}}{1 - \sqrt{1 - y_q}} - i\pi \right) + \Theta(y_q - 1)\mathrm{arctg}\frac{1}{\sqrt{y_q - 1}} \right\}\ ,$$

where $y_q = 4\hat{m}_q^2/\hat{s}'$, and $\hat{s}' = p^2/m_b^2$.

For a more complete analysis of the $B \to \tau^+\tau^-\gamma$ decay, one has to take into account the long distance contributions. For this aim it is necessary to make the following replacement

$$g(\hat{m}_c, \hat{s}') \to g(\hat{m}_c, \hat{s}') - \frac{3\pi}{\alpha^2} \sum_{V = J/\psi, \psi'} \frac{\hat{m}_V \mathrm{B}(V \to \tau^+\tau^-)\hat{\Gamma}_{tot}^V}{\hat{s}' - \hat{m}_V^2 + i\hat{m}_V\hat{\Gamma}_{tot}^V}\ , \tag{20}$$

where $\hat{m}_V = m_V/m_b$, $\hat{\Gamma}_{tot} = \Gamma/m_b$.

Our results for the Branching Ratio $\mathcal{B}(B \to \tau^+\tau^-\gamma)$ for two different values of the cut ($\delta = 0.01$ and $\delta = 0.02$) are presented in Table 1.

	Short Distance Contributions to the Branching Ratio		**Short and Long Distance Contributions to the Branching Ratio**	
	$\delta = 0.01$	$\delta = 0.02$	$\delta = 0.01$	$\delta = 0.02$
Structure dependent part	4.19×10^{-9}	4.19×10^{-9}	9.95×10^{-9}	7.68×10^{-9}
Bremstrahlung part	4.11×10^{-9}	3.16×10^{-9}	4.11×10^{-9}	3.16×10^{-9}
Interference part	1.24×10^{-9}	1.23×10^{-9}	1.16×10^{-9}	1.16×10^{-9}
Total	9.54×10^{-9}	8.59×10^{-9}	1.52×10^{-8}	1.20×10^{-8}

Table 1:

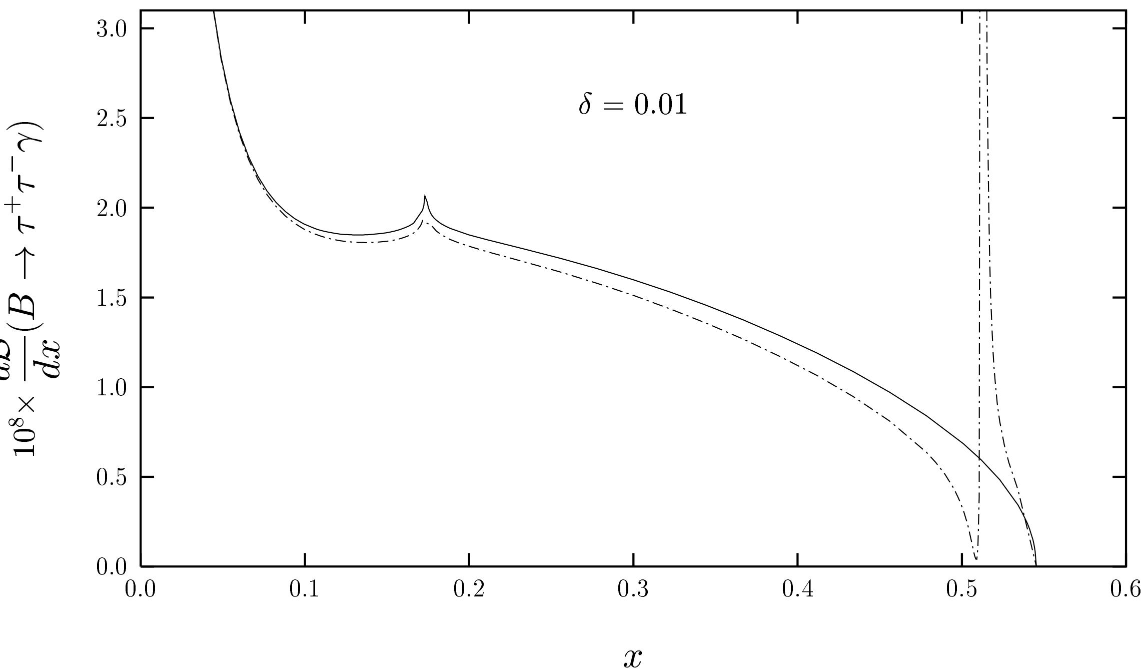
Note that, when only short distance effects are taken into consideration, the structure–dependent and the Bremstrahlung parts give, more or less, comparable contributions. However, if long distance contributions are also considered in addition to the short ones, the structure–dependent part contributes more as compared to that of the Bremstrahlung part. This is due to the fact that, the structure dependent part contains $(J/\psi, \psi')$ resonance contributions in the latter case (see Eq. (20)).

From these results it follows that there is a good chance for detecting τ lepton decay in the future B–meson factories, provided that the efficiency is $\sim 1/3$.

Figure Captions

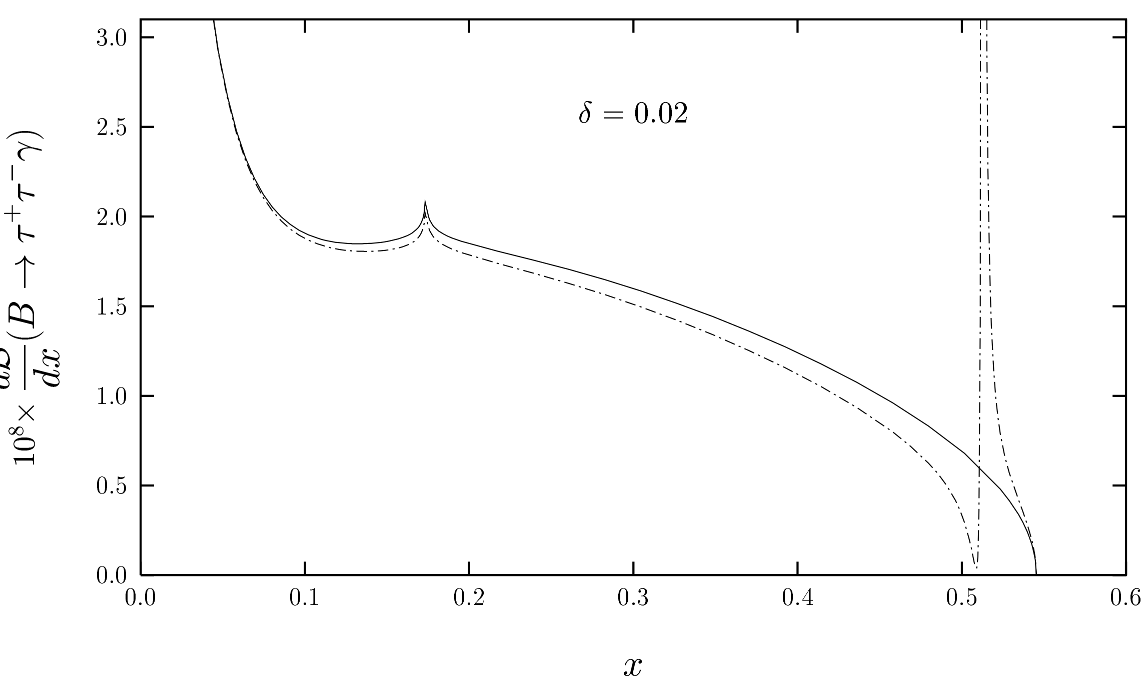
1. Dependence of the Differential Branching Ratio for the $B \to \tau^+\tau^-\gamma$ decay on the dimensionless variable $x = \dfrac{2E_\gamma}{m_B}$, for the value of the cut $\delta = 0.01$ imposed on the photon energy. In this figure, the curve with the sharp peak represents the long distance contributions.

2. Same as Fig. 1, but for the cut value $\delta = 0.02$.



$\delta = 0.01$
$10^8 \times \frac{d\mathcal{B}}{dx}(B \to \tau^+\tau^-\gamma)$
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
x

Fig. 1



$\delta = 0.02$
$10^8 \times \frac{d\mathcal{B}}{dx}(B \to \tau^+\tau^-\gamma)$
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
x

Fig. 2

Appendix : **Cancellation of the infrared divergences**

In this section we will show the explicit cancellation between the infrared singularities arising from the Bremstrahlung of a soft photon in the $B \to \tau^+\tau^-\gamma$ rate and in the $O(\alpha)$ virtual corrections to the $B \to \tau^+\tau^-$ amplitude.

As we have explained in Section 2, we are not going to include $O(\alpha)$ virtual corrections to the $B \to \tau^+\tau^-$ amplitude in the calculation of the rate of $B \to \tau^+\tau^-\gamma$. The Bremstrahlung process here is not considered as an $O(\alpha)$ correction to the $B \to \tau^+\tau^-$ amplitude, but as a different process, namely the decay of a B–meson into τ–lepton pair plus a hard photon. In order to calculate the physical rate of interest, we just have to impose a cut on the energy of photon, which will correspond to the experimental cut imposed on the minimum energy for detectable photon.

In this sense our approach is very similar to the one given in [11], in studying $B_s \to X_s\gamma\gamma$ decay. Therefore, in the present Appendix we will consider only those aspects of the discussion which are necessary to show the cancellation of the infrared singularities. For removing the infrared singularities we will use the method presented in [12], where the main idea is to use the dimensional regularization method, i.e., to replace

$$\frac{d^3q}{2\,E_\gamma\;(2\pi)^3} \to \frac{d^{(n-1)}q}{2\,|q|\,(2\pi)^{(n-1)}}\;, \tag{A.1}$$

where $q = (q^0,\; q_i)$ is an n dimensional light–like vector:

$$q^0 = |q| = \Big(|q_1|^2 + \cdots + |q_{n-1}|^2\Big)^{1/2}\;.$$

All calculations are performed in the rest frame of the B–meson. We are interested in the situation when charged lepton is observed in the energy interval $E_m - \Delta E \le E \le E_m$, where E_m is that of the charged lepton energy in the two–body decay $B \to \tau^+\tau^-$ and $\Delta E \ll E_m$. We will retain terms of logarithmic and zeroth order in ΔE. It is clear that in this limit, only soft photons give contributions.

Let us first consider the Bremstrahlung part. From $|\mathcal{M}_2|^2$ it follows that only terms proportional to $1/\left(p_1q\right)^2,\; 1/\left(p_2q\right)^2$ and $1/\left[\left(p_1q\right)\left(p_2q\right)\right]$ give infrared singularities (we will omit the terms which give finite contributions to the to the decay rate), i.e.,

$$\begin{aligned}
|\mathcal{M}_2|^2_{IR} = &- \left|\frac{\alpha G_F V_{tb}V_{ts}^* f_B C_{10}}{2\sqrt{2}\,\pi}\right|^2 m_\tau^2 4\pi\alpha \\
&\times \left\{ 8m_\tau^2 \frac{p^2}{\left(p_1q\right)^2} + 8m_\tau^2 \frac{p^2}{\left(p_2q\right)^2} + 16m_\tau^2 \left[\frac{\left(p_1q\right)}{\left(p_2q\right)^2} + \frac{\left(p_2q\right)}{\left(p_1q\right)^2}\right]\right. \\
&\left. + 16m_\tau^2 \frac{p^2}{\left(p_1q\right)\left(p_2q\right)} - 8\frac{p^4}{\left(p_1q\right)\left(p_2q\right)} \right\}\;.
\end{aligned} \tag{A.2}$$

Let us consider the first term in the bracket

$$\begin{aligned}
I &= \int \frac{d^3\vec{p_1}}{2E_1}\,\frac{d^3\vec{p_2}}{2E_2}\,\frac{d^{(n-1)}q}{2\,|q|\,(2\pi)^{n-1}}\,\frac{p^2}{\left(p_1q\right)^2}\,\frac{1}{(2\pi)^2}\,\delta(p_B - p_1 - p_2 - q) \\
&= \int \frac{d^3\vec{p_1}}{2E_1}\,\frac{d^{(n-1)}q}{2\,|q|\,(2\pi)^{n-1}}\,\frac{p^2}{\left(p_1q\right)^2}\,\frac{1}{(2\pi)^2}\,\delta\Big(m_B^2 - 2m_BE_1 - 2m_Bq^0 + 2E_1q\left(1 - \beta z\right)\Big)\;,
\end{aligned} \tag{A.3}$$

where $\beta = |\vec{p_1}|/E_1$, and we choose the first axis along the direction of $\vec{p_1}$. The integrand depends only on the angle $\theta_1 = \theta$, so that we can immediately perform integration over the other angles. Integration over all of the angular variables leads to the following result

$$\int d^{(n-1)}q \to \frac{2\pi^{n/2-1}}{\Gamma(n/2-1)} \int d|q|\, q^{(n-2)} \int_{-1}^{1} dz \left(1-z^2\right)^{n/2-2} , \tag{A.4}$$

where $z = cos\theta$. Using (A.4), and performing integration over radial $d|q|$ we obtain

$$\begin{aligned} I = & \frac{1}{2(2\pi)^{n-1}} \frac{1}{(2\pi)^2} \frac{2\pi^{n/2-1}}{\Gamma(n/2-1)} \int_{E_m-\Delta E}^{E_m} \frac{4\pi\beta\, dE_1}{4E_1} \int dz \frac{\left(1-z^2\right)^{n/2-1}}{(1-\beta z)^2} \\ & \times \left\{ m_B^2 \frac{\left(m_B^2 - 2m_B E_1\right)^{n-5}}{\left[2m_B - 2E_1(1-\beta z)\right]^{n-4}} - 2m_B \frac{\left(m_B^2 - 2m_B E_1\right)^{n-4}}{\left[2m_B - 2E_1(1-\beta z)\right]^{n-3}} \right\} . \end{aligned} \tag{A.5}$$

Second term gives finite contribution and therefore we will omit it. In our case $E_m = m_B/2$.

Introducing next a new variable $t = E_m - E_1$ and expanding all terms in Taylor series about $E_1 = E_m$, we get

$$\begin{aligned} I = & \frac{1}{2^{n+2}\pi^{n/2+1}} \frac{1}{\Gamma(n/2-1)} \int dz \frac{\left(1-z^2\right)^{n/2-2}}{(1-\beta_m z)^2} \frac{\beta_m}{E_m} \\ & \times \int_0^{\Delta E} dt \frac{m_B^{n+3} t^{n-5}}{2E_m^{n-4}(1+\beta_m z)^{n-4}} . \end{aligned} \tag{A.6}$$

After integrating over t we have

$$\begin{aligned} I = & \frac{1}{2^{n+2}\pi^{n/2+1}} \frac{1}{\Gamma(n/2-1)} \frac{\beta_m}{2} \\ & \times \left\{ \left(\frac{1}{n-4} + \ln\Delta E \right) \int_{-1}^{1} dz \frac{\left(1-z^2\right)^{n/2-2}}{(1-\beta_m z)^2} \frac{1}{(1+\beta_m z)^{n-4}} \right\} . \end{aligned} \tag{A.7}$$

The last step is to expand (A.7) in a Laurent series about $n = 4$ and perform trivial integrations over dx (retaining only $1/(n-4)$ and $\ln\Delta E$ terms). Calculating in a similar manner all the other terms, we get (we retain only infrared divergent terms and those that are proportional to $\ln\Delta E$)

$$\begin{aligned} \Gamma_{\rm IR} = & - \left| \frac{\alpha G_F V_{tb} V_{ts}^* f_B C_{10}}{2\sqrt{2}\,\pi} \right|^2 m_\tau^2 m_B^2 \frac{\alpha}{4} \beta_m \pi^{-n/2} \frac{1}{2m_B} \\ & \times \left\{ \left(\frac{1}{n-4} + \ln\Delta E \right) \left[8 + 4\left(1+\beta_m^2\right) \right] \frac{1}{\beta_m} \ln\frac{1-\beta_m}{1+\beta_m} + \cdots \right\} . \end{aligned} \tag{A.8}$$

Now let us calculate $O(\alpha)$ virtual corrections to the $B \to \tau^+\tau^-$ decay. The matrix element for the $B \to \tau^+\tau^-$ decay with virtual corrections can be represented as

$$\mathcal{M} = \mathcal{M}_0 \left\{ 1 + 4\pi\alpha \mathrm{K} + (Z_m - 1) \right\} , \tag{A.9}$$

where

$$\mathcal{M}_0 = \frac{\alpha G_F}{2\sqrt{2}\,\pi} f_B C_{10} V_{tb} V_{ts}^* 2m_\tau \, \bar{\tau}\gamma_5\tau \ ,$$

is the matrix element for $B \to \tau^+\tau^-$ decay without virtual corrections, K denotes $O(\alpha)$ corrections due to the photon exchange (vertex) to the $\mathcal{M}_0$, and the last term corresponds to the wave function renormalization. Note that all the calculations were performed in the Landau gauge. The matrix element square with summation over spins of the final particles is given as

$$|\mathcal{M}|^2 = |\mathcal{M}_0|^2 \left\{ 1 + 4\pi\alpha 2\mathrm{Re}(\mathrm{K}) + 2\left(Z_m - 1\right) \right\} . \tag{A.10}$$

In the Landau gauge, fermion wave function renormalization constant is given as

$$Z_m - 1 = \frac{\alpha}{4\pi}\left(\frac{6}{n-4} - 4\right) . \tag{A.11}$$

After standard calculation for the infrared singular part for the virtual corrections, we finally get

$$\begin{aligned} \Gamma_{\mathrm{IR}} = & \left| \frac{\alpha G_F V_{tb} V_{ts}^* f_B C_{10}}{2\sqrt{2}\,\pi} \right|^2 m_\tau^2 m_B \frac{\alpha\beta}{4\pi^2} \\ & \times \left\{ \frac{1}{n-4}\left[4 + \frac{2}{\beta}\left(1+\beta\right)^2 \right] \ln\frac{1-\beta}{1+\beta} + \cdots \right\} . \end{aligned} \tag{A.12}$$

From Eqs. (A.8) and (A.12) we see that the infrared singular terms from Bremstrahlung and vertex corrections exactly cancel each other, and the decay rate is infrared–free.

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