Model independent analysis of Λ baryon polarizations in $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay

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Abstract

We present the model independent analysis of Λ baryon polarizations in the $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay. The sensitivity of the averaged Λ polarizations to the new Wilson coefficients is studied. It is observed that there exist certain regions of the new Wilson coefficients where the branching ratio coincides with the standard model prediction, while the Λ baryon polarizations deviate from the standard model results remarkably.

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1 Introduction

Flavor-changing neutral current (FCNC) $b \to s(d)\ell^+\ell^-$ transitions provide potentially the most sensitive and stringiest test for the standard model (SM) in the flavor sector at loop level, since FCNC transitions are forbidden in the SM in the Born approximation. At the same time these decays are very sensitive to the new physics beyond the SM. New physics appear in rare decays through the Wilson coefficients which can take values different from their SM counterpart or through the new operator structures in an effective Hamiltonian [1].

First measurements of the $B \to X_s \gamma$ decay were reported by CLEO Collaboration [2] and at present more precise measurements are currently being carried out in the experiments at B factories [3]. Exclusive decay involving the $b \to s\gamma$ transition has been measured in [4]–[6]. After these measurements of the radiative decay induced by the $b \to s\gamma$ transition, main interest has been focused on the rare decays induced by the $b \to s\ell^+\ell^-$ transition, which have relatively large branching ratio in the SM. These decays have been extensively studied in the SM and its various extensions [7]–[22].

The exclusive $B \to K^*(K)\ell^+\ell^-$ decays, which are described by $b \to s\ell^+\ell^-$ transition at inclusive level, have been widely studied in literature (see [22]–[26] and references therein). Recently BaBar Collaboration announced evidence of the $B \to K\ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$ decays with the branching ratios $\mathcal{B}(B \to K\ell^+\ell^-) = (0.78^{+0.24+0.11}_{-0.20-0.18}) \times 10^{-6}$, $\mathcal{B}(B \to K^*\ell^+\ell^-) =$ $(1.68^{+0.68}_{-0.58} \pm 0.28) \times 10^{-6}$ [27]. The $B \to K\ell^+\ell^-$ decay has been also observed at BELLE detector [28] with the branching ratio $\mathcal{B}(B \to K\ell^+\ell^-) = (0.75^{+0.25}_{-0.21} \pm 0.09) \times 10^{-6}$. Another exclusive decay which is described at inclusive level by the $b \to s\ell^+\ell^-$ transition is the baryonic $\Lambda_b \to \Lambda\ell^+\ell^-$ decay. Interest to the baryonic decays can be attributed to the fact that, unlike mesonic decays, they could maintain the helicity structure of the effective Hamiltonian for the $b \to s$ transition. Note that, new physics effects in the $\Lambda_b \to \Lambda\gamma$ decay were studied in [29].

In this work we analyze the possibility of searching for new physics in the heavy baryon $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay by measuring the polarization of Λ baryon, using the most general model independent form of the effective Hamiltonian. It should be mentioned here that the sensitivity of the lepton polarization to the new Wilson coefficients, which are responsible for the existence of new physics beyond the SM in the $B \to K \ell^+ \ell^-$ and $B \to K^* \ell^+ \ell^-$ and $\Lambda_b \to \Lambda \ell^+ \ell^-$ decays, is investigated in [30], [31] and [32], respectively, using the most general form of the effective Hamiltonian. It is shown in these works that the lepton polarizations are really very sensitive to the new physics effects.

The paper is organized as follows. In section 2, using the most general form of the effective Hamiltonian, the general expressions for the longitudinal and normal polarizations of the Λ baryon are derived. Section 3 is devoted to the study of the dependence of the Λ polarizations on the new Wilson coefficients.

2 Lepton polarizations

At quark level, the matrix element of the $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay is described by the $b \to s \ell^+ \ell^-$ transition. The effective Hamiltonian responsible for the $b \to s \ell^+ \ell^-$ transition can be

written in terms of twelve model independent four-Fermi interactions as [24, 33]

$$\mathcal{M} = \frac{G\alpha}{\sqrt{2\pi}} V_{tb} V_{ts}^* \left\{ C_{SL} \bar{s}_R i \sigma_{\mu\nu} \frac{q^{\nu}}{q^2} b_L \bar{\ell} \gamma^{\mu} \ell + C_{BR} \bar{s}_L i \sigma_{\mu\nu} \frac{q^{\nu}}{q^2} b_R \bar{\ell} \gamma^{\mu} \ell + C_{LL}^{tot} \bar{s}_L \gamma_{\mu} b_L \bar{\ell}_L \gamma^{\mu} \ell_L \right. \\ \left. + C_{LR}^{tot} \bar{s}_L \gamma_{\mu} b_L \bar{\ell}_R \gamma^{\mu} \ell_R + C_{RL} \bar{s}_R \gamma_{\mu} b_R \bar{\ell}_L \gamma^{\mu} \ell_L + C_{RR} \bar{s}_R \gamma_{\mu} b_R \bar{\ell}_R \gamma^{\mu} \ell_R \right. \\ \left. + C_{LRLR} \bar{s}_L b_R \bar{\ell}_L \ell_R + C_{RLLR} \bar{s}_R b_L \bar{\ell}_L \ell_R + C_{LRRL} \bar{s}_L b_R \bar{\ell}_R \ell_L + C_{RLRL} \bar{s}_R b_L \bar{\ell}_R \ell_L \right.$$

$$\left. + C_T \bar{s} \sigma_{\mu\nu} b \bar{\ell} \sigma^{\mu\nu} \ell + i C_{TE} \epsilon_{\mu\nu\alpha\beta} \bar{s} \sigma^{\mu\nu} b \bar{\ell} \sigma^{\alpha\beta} \ell \right\}, \qquad (1)$$

where the subindices L and R stand for the chiral operators $L = (1 - \gamma_5)/2$ and $R = (1 + \gamma_5)/2$. The coefficients of the first two terms, C_{SL} and C_{BR} describe the penguin contributions, which correspond to $-2m_sC_7^{eff}$ and $-2m_bC_7^{eff}$ in the SM, respectively. The next four terms with coefficients C_{LL}^{tot} , C_{LR}^{tot} , C_{RL} and C_{RR} in Eq. (1) describe vector type interactions. Two of these coefficients C_{LL}^{tot} and C_{LR}^{tot} contain SM results in the form $C_9^{eff} - C_{10}$ and $C_9^{eff} - C_{10}$, respectively. For this reason we can write

$$C_{LL}^{tot} = C_9^{eff} - C_{10} + C_{LL} ,$$

$$C_{LR}^{tot} = C_9^{eff} + C_{10} + C_{LR} ,$$
(2)

where C_{LL} and C_{LR} describe the contributions of new physics. The next four terms in Eq. (1) with coefficients C_{LRLR} , C_{RLLR} , C_{LRRL} and C_{RLRL} represent the scalar type interactions. The remaining last two terms led by the coefficients C_T and C_{TE} are the tensor type interactions.

The amplitude of the exclusive $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay is obtained by calculating the matrix element of \mathcal{H}_{eff} for the $b \to s \ell^+ \ell^-$ transition between initial and final baryon states $\langle \Lambda | \mathcal{H}_{eff} | \Lambda_b \rangle$. We see from Eq. (1) that for calculating the $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay amplitude, the following matrix elements are needed

$$\begin{split} & \langle \Lambda \left| \bar{s} \gamma_{\mu} (1 \mp \gamma_5) b \right| \Lambda_b \rangle , \\ & \langle \Lambda \left| \bar{s} \sigma_{\mu\nu} (1 \mp \gamma_5) b \right| \Lambda_b \rangle , \\ & \langle \Lambda \left| \bar{s} (1 \mp \gamma_5) b \right| \Lambda_b \rangle . \end{split}$$

The relevant matrix elements parametrized in terms of the form factors are as follows (see [34, 35])

$$\langle \Lambda \left| \bar{s} \gamma_{\mu} b \right| \Lambda_{b} \rangle = \bar{u}_{\Lambda} \Big[f_{1} \gamma_{\mu} + i f_{2} \sigma_{\mu\nu} q^{\nu} + f_{3} q_{\mu} \Big] u_{\Lambda_{b}} , \qquad (3)$$

$$\langle \Lambda \left| \bar{s} \gamma_{\mu} \gamma_{5} b \right| \Lambda_{b} \rangle = \bar{u}_{\Lambda} \Big[g_{1} \gamma_{\mu} \gamma_{5} + i g_{2} \sigma_{\mu\nu} \gamma_{5} q^{\nu} + g_{3} q_{\mu} \gamma_{5} \Big] u_{\Lambda_{b}} , \qquad (4)$$

$$\langle \Lambda \left| \bar{s}\sigma_{\mu\nu}b \right| \Lambda_b \rangle = \bar{u}_{\Lambda} \Big[f_T \sigma_{\mu\nu} - i f_T^V \left(\gamma_{\mu}q^{\nu} - \gamma_{\nu}q^{\mu} \right) - i f_T^S \left(P_{\mu}q^{\nu} - P_{\nu}q^{\mu} \right) \Big] u_{\Lambda_b} , \qquad (5)$$

$$\langle \Lambda \left| \bar{s}\sigma_{\mu\nu}\gamma_5 b \right| \Lambda_b \rangle = \bar{u}_\Lambda \Big| g_T \sigma_{\mu\nu} - ig_T^V \left(\gamma_\mu q^\nu - \gamma_\nu q^\mu \right) - ig_T^S \left(P_\mu q^\nu - P_\nu q^\mu \right) \Big| \gamma_5 u_{\Lambda_b} , \qquad (6)$$

where $P = P_{\Lambda_b} + P_{\Lambda}$ and $q = P_{\Lambda_b} - P_{\Lambda}$.

The form factors of the magnetic dipole operators are defined as

$$\langle \Lambda \left| \bar{s}i\sigma_{\mu\nu}q^{\nu}b \right| \Lambda_b \rangle = \bar{u}_{\Lambda} \Big[f_1^T \gamma_{\mu} + i f_2^T \sigma_{\mu\nu}q^{\nu} + f_3^T q_{\mu} \Big] u_{\Lambda_b} , \langle \Lambda \left| \bar{s}i\sigma_{\mu\nu}\gamma_5 q^{\nu}b \right| \Lambda_b \rangle = \bar{u}_{\Lambda} \Big[g_1^T \gamma_{\mu}\gamma_5 + i g_2^T \sigma_{\mu\nu}\gamma_5 q^{\nu} + g_3^T q_{\mu}\gamma_5 \Big] u_{\Lambda_b} .$$

$$(7)$$

Note that, using the identity

$$\sigma_{\mu\nu}\gamma_5 = -\frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}$$

the second expression in Eq. (7) can be written as

$$\langle \Lambda | \bar{s} i \sigma_{\mu\nu} \gamma_5 q^{\nu} b | \Lambda_b \rangle = \bar{u}_{\Lambda} \Big[f_T i \sigma_{\mu\nu} \gamma_5 q^{\nu} \Big] u_{\Lambda_b} \; .$$

Multiplying (5) and (6) by iq^{ν} and comparing with (7), one can easily obtain the following relations between the form factors

$$f_{2}^{T} = f_{T} + f_{T}^{S} q^{2} ,$$

$$f_{1}^{T} = \left[f_{T}^{V} + f_{T}^{S} \left(m_{\Lambda_{b}} + m_{\Lambda} \right) \right] q^{2} = -\frac{q^{2}}{m_{\Lambda_{b}} - m_{\Lambda}} f_{3}^{T} ,$$

$$g_{2}^{T} = g_{T} + g_{T}^{S} q^{2} ,$$

$$g_{1}^{T} = \left[g_{T}^{V} - g_{T}^{S} \left(m_{\Lambda_{b}} - m_{\Lambda} \right) \right] q^{2} = \frac{q^{2}}{m_{\Lambda_{b}} + m_{\Lambda}} g_{3}^{T} .$$
(8)

The matrix element of the scalar (pseudoscalar) operators $\bar{s}b$ and $\bar{s}\gamma_5 b$ can be obtained from (3) and (4) by multiplying both sides to q^{μ} and using equation of motion. Neglecting the mass of the strange quark, we get

$$\langle \Lambda |\bar{s}b| \Lambda_b \rangle = \frac{1}{m_b} \bar{u}_\Lambda \Big[f_1 \left(m_{\Lambda_b} - m_\Lambda \right) + f_3 q^2 \Big] u_{\Lambda_b} , \qquad (9)$$

$$\langle \Lambda | \bar{s} \gamma_5 b | \Lambda_b \rangle = \frac{1}{m_b} \bar{u}_\Lambda \Big[g_1 \left(m_{\Lambda_b} + m_\Lambda \right) \gamma_5 - g_3 q^2 \gamma_5 \Big] u_{\Lambda_b} . \tag{10}$$

Using these definitions of the form factors, for the matrix element of the $\Lambda_b \to \Lambda \ell^+ \ell^$ we get [35, 36]

$$\mathcal{M} = \frac{G\alpha}{4\sqrt{2\pi}} V_{tb} V_{ts}^{*} \left\{ \bar{\ell}\gamma^{\mu} \ell \,\bar{u}_{\Lambda} \Big[A_{1}\gamma_{\mu} (1+\gamma_{5}) + B_{1}\gamma_{\mu} (1-\gamma_{5}) + B_{1}(1-\gamma_{5}) + i\sigma_{\mu\nu} q^{\nu} [A_{2}(1+\gamma_{5}) + B_{2}(1-\gamma_{5})] + q_{\mu} [A_{3}(1+\gamma_{5}) + B_{3}(1-\gamma_{5})] \Big] u_{\Lambda_{b}} \right. \\ \left. + \bar{\ell}\gamma^{\mu}\gamma_{5}\ell \,\bar{u}_{\Lambda} \Big[D_{1}\gamma_{\mu} (1+\gamma_{5}) + E_{1}\gamma_{\mu} (1-\gamma_{5}) + i\sigma_{\mu\nu} q^{\nu} [D_{2}(1+\gamma_{5}) + E_{2}(1-\gamma_{5})] \right] \\ \left. + q_{\mu} [D_{3}(1+\gamma_{5}) + E_{3}(1-\gamma_{5})] \Big] u_{\Lambda_{b}} + \bar{\ell}\ell \,\bar{u}_{\Lambda} (N_{1} + H_{1}\gamma_{5}) u_{\Lambda_{b}} + \bar{\ell}\gamma_{5}\ell \,\bar{u}_{\Lambda} (N_{2} + H_{2}\gamma_{5}) u_{\Lambda_{b}} \right. \\ \left. + 4C_{T}\bar{\ell}\sigma^{\mu\nu}\ell \,\bar{u}_{\Lambda} \Big[f_{T}\sigma_{\mu\nu} - if_{T}^{V} (q_{\nu}\gamma_{\mu} - q_{\mu}\gamma_{\nu}) - if_{T}^{S} (P_{\mu}q_{\nu} - P_{\nu}q_{\mu}) \Big] u_{\Lambda_{b}} \right\} , \qquad (11)$$

where the explicit forms of the functions A_i , B_i , D_i , E_i , H_j and N_j (i = 1, 2, 3 and j = 1, 2) are as follows [35]

$$\begin{split} A_{1} &= \frac{1}{q^{2}} \left(f_{1}^{T} - g_{1}^{T} \right) C_{SL} + \frac{1}{q^{2}} \left(f_{1}^{T} + g_{1}^{T} \right) C_{BR} + \frac{1}{2} \left(f_{1} - g_{1} \right) \left(C_{LL}^{tot} + C_{LR}^{tot} \right) \\ &+ \frac{1}{2} \left(f_{1} + g_{1} \right) \left(C_{RL} + C_{RR} \right) , \\ A_{2} &= A_{1} \left(1 \rightarrow 2 \right) , \\ A_{3} &= A_{1} \left(1 \rightarrow 2 \right) , \\ B_{1} &= A_{1} \left(g_{1} \rightarrow -g_{1} \right) g_{1}^{T} \rightarrow -g_{1}^{T} \right) , \\ B_{2} &= B_{1} \left(1 \rightarrow 2 \right) , \\ B_{3} &= B_{1} \left(1 \rightarrow 3 \right) , \\ D_{1} &= \frac{1}{2} \left(C_{RR} - C_{RL} \right) \left(f_{1} + g_{1} \right) + \frac{1}{2} \left(C_{LR}^{tot} - C_{LL}^{tot} \right) \left(f_{1} - g_{1} \right) , \\ D_{2} &= D_{1} \left(1 \rightarrow 2 \right) , \\ D_{3} &= D_{1} \left(1 \rightarrow 3 \right) , \\ E_{1} &= D_{1} \left(g_{1} \rightarrow -g_{1} \right) , \\ E_{2} &= E_{1} \left(1 \rightarrow 2 \right) , \\ N_{1} &= \frac{1}{m_{b}} \left(f_{1} \left(m_{\Lambda_{b}} - m_{\Lambda} \right) + f_{3}q^{2} \right) \left(C_{LRLR} + C_{RLR} + C_{LRRL} + C_{RLRL} \right) , \\ N_{2} &= N_{1} \left(C_{LRRL} \rightarrow -C_{LRRL} \right) , \\ H_{1} &= \frac{1}{m_{b}} \left(g_{1} \left(m_{\Lambda_{b}} + m_{\Lambda} \right) - g_{3}q^{2} \right) \left(C_{LRLR} - C_{RLRL} \right) . \end{split}$$

From the expressions of the above-mentioned matrix elements we observe that $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay is described in terms of many form factors. It is shown in [37] that when HQET is applied the number of independent form factors reduces to two (F_1 and F_2) irrelevant of the Dirac structure of the corresponding operators, i.e.,

$$\langle \Lambda(p_{\Lambda}) | \bar{s} \Gamma b | \Lambda(p_{\Lambda_b}) \rangle = \bar{u}_{\Lambda} \Big[F_1(q^2) + \not \!\!/ F_2(q^2) \Big] \Gamma u_{\Lambda_b} , \qquad (13)$$

where Γ is an arbitrary Dirac structure, $v^{\mu} = p^{\mu}_{\Lambda_b}/m_{\Lambda_b}$ is the four-velocity of Λ_b , and $q = p_{\Lambda_b} - p_{\Lambda}$ is the momentum transfer. Comparing the general form of the form factors given in Eqs. (3)–(9) with (13), one can easily obtain the following relations among them (see also [34])

$$g_{1} = f_{1} = f_{2}^{T} = g_{2}^{T} = F_{1} + \sqrt{r}F_{2} ,$$

$$g_{2} = f_{2} = g_{3} = f_{3} = g_{T}^{V} = f_{T}^{V} = \frac{F_{2}}{m_{\Lambda_{b}}} ,$$

$$g_{T}^{S} = f_{T}^{S} = 0 ,$$

$$g_{1}^{T} = f_{1}^{T} = \frac{F_{2}}{m_{\Lambda_{b}}}q^{2} ,$$

$$g_3^T = \frac{F_2}{m_{\Lambda_b}} (m_{\Lambda_b} + m_{\Lambda}) ,$$

$$f_3^T = -\frac{F_2}{m_{\Lambda_b}} (m_{\Lambda_b} - m_{\Lambda}) ,$$
(14)

where $r = m_{\Lambda}^2 / m_{\Lambda_b}^2$.

Having obtained the matrix element for the $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay, our next aim is the calculation of Λ baryon polarizations using this matrix element. For this purpose we write the Λ baryon spin four-vector in terms of a unit vector $\vec{\xi}$ along the Λ baryon spin in its rest frame as

$$s_{\mu} = \left(\frac{\vec{p}_{\Lambda} \cdot \vec{\xi}}{m_{\Lambda}}, \vec{\xi} + \frac{\vec{p}_{\Lambda}(\vec{p}_{\Lambda} \cdot \vec{\xi})}{E_{\Lambda} + m_{\Lambda}}\right) , \qquad (15)$$

and choose the unit vectors along the longitudinal, transversal and normal components of the Λ polarization to be

$$\vec{e}_L = \frac{\vec{p}_\Lambda}{|\vec{p}_\Lambda|} , \quad \vec{e}_T = \frac{\vec{p}_\ell \times \vec{p}_\Lambda}{|\vec{p}_\ell \times \vec{p}_\Lambda|} , \quad \vec{e}_N = \vec{e}_T \times \vec{e}_L , \qquad (16)$$

respectively, where \vec{p}_{ℓ} and \vec{p}_{Λ} are the three momenta of ℓ and Λ , in the center of mass frame of the $\ell^+\ell^-$ system.

The differential decay rate of the $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay for any spin direction $\vec{\xi}$ along the Λ baryon can be written as

$$\frac{d\Gamma(\vec{\xi})}{ds} = \frac{1}{2} \left(\frac{d\Gamma}{ds} \right)_0 \left[1 + \left(P_L \vec{e}_L + P_N \vec{e}_N + P_T \vec{e}_T \right) \cdot \vec{\xi} \right], \tag{17}$$

where $(d\Gamma/ds)_0$ corresponds to the unpolarized differential decay rate, $s = q^2/m_{\Lambda_b}^2$ and P_L , P_N and P_T represent the longitudinal, normal and transversal polarizations of the Λ baryon, respectively. The unpolarized decay width in Eq. (17) can be written as

$$\left(\frac{d\Gamma}{ds}\right)_{0} = \frac{G^{2}\alpha^{2}}{8192\pi^{5}} \left|V_{tb}V_{ts}^{*}\right|^{2} \lambda^{1/2}(1,r,s)v\left[\mathcal{T}_{0}(s) + \frac{1}{3}\mathcal{T}_{2}(s)\right],$$
(18)

where $\lambda(1, r, s) = 1 + r^2 + s^2 - 2r - 2s - 2rs$ is the triangle function, $r = m_{\Lambda}^2/m_{\Lambda_b}^2$ and $v = \sqrt{1 - 4m_{\ell}^2/q^2}$ is the lepton velocity. The explicit expressions for \mathcal{T}_0 and \mathcal{T}_2 can be found in [35].

The polarizations P_L , P_N and P_T are defined as:

$$P_i(q^2) = \frac{\frac{d\Gamma}{ds}(\vec{\xi} = \vec{e}_i) - \frac{d\Gamma}{ds}(\vec{\xi} = -\vec{e}_i)}{\frac{d\Gamma}{ds}(\vec{\xi} = \vec{e}_i) + \frac{d\Gamma}{ds}(\vec{\xi} = -\vec{e}_i)},$$
(19)

where i = L, N, T. P_L and P_N are P-odd, T-even, while P_T is P-even, T-odd and CP-odd. Note that transversal polarization of the Λ baryon has already been studied in [36]. In the massless lepton limit the explicit expressions of the P_L and P_N for the Λ baryon are:

$$P_{L} = \frac{16m_{\Lambda_{b}}^{4}\sqrt{\lambda}}{\mathcal{T}_{0}(s) + \frac{1}{3}\mathcal{T}_{2}(s)} \left\{ -4m_{\Lambda_{b}}s \operatorname{Re}[A_{1}^{*}B_{2} - A_{2}^{*}B_{1}] \right. \\ \left. - s \left(v^{2}\operatorname{Re}[F_{1}^{*}H_{1}] + \operatorname{Re}[F_{2}^{*}H_{2}]\right) \right. \\ \left. - \frac{4}{3}m_{\Lambda_{b}}sv^{2} \left(\sqrt{r}\operatorname{Re}[A_{1}^{*}A_{2} - B_{1}^{*}B_{2}] + 3\operatorname{Re}[D_{1}^{*}E_{2} - D_{2}^{*}E_{1}] + \sqrt{r}\operatorname{Re}[D_{1}^{*}D_{2} - E_{1}^{*}E_{2}]\right) \right. \\ \left. + \frac{1}{3}\left\{ \left[3(1 - r + s) - v^{2}(1 - r - s) \right] (|A_{1}|^{2} - |B_{1}|^{2} + |D_{1}|^{2} - |E_{1}|^{2}) \right\} \right. \\ \left. - \frac{1}{3}m_{\Lambda_{b}}^{2}s[3(1 - r + s) + v^{2}(1 - r - s)](|A_{2}|^{2} - |B_{2}|^{2}) \right. \\ \left. - \frac{2}{3}m_{\Lambda_{b}}^{2}sv^{2}(2 - 2r + s) \left(|D_{2}|^{2} - |E_{2}|^{2}\right) \right. \\ \left. - \frac{256}{3}m_{\Lambda_{b}}^{2}sv^{2}[(1 + \sqrt{r})^{2} - s]\operatorname{Re}[C_{T}^{*}C_{TE}]\operatorname{Re}[f_{T}^{*}f_{T}^{S}] \right. \\ \left. - \frac{256}{3}m_{\Lambda_{b}}s[3 - \sqrt{r}(3 - 2v^{2})]\operatorname{Re}[C_{T}^{*}C_{TE}]\operatorname{Re}[f_{T}^{*}f_{T}^{S}] \right. \\ \left. + \frac{256}{3}[3 - 3r - (1 - r - s)v^{2}]\operatorname{Re}[C_{T}^{*}C_{TE}]|f_{T}|^{2} \right\},$$

$$(20)$$

$$P_{N} = \frac{8\pi m_{\Lambda_{b}}^{4} v \sqrt{s}}{\mathcal{T}_{0}(s) + \frac{1}{3} \mathcal{T}_{2}(s)} \Biggl\{ -2(1 - r + s) \sqrt{r} \operatorname{Re}[A_{1}^{*}D_{1} + B_{1}^{*}E_{1}] \\ + 4(1 + \sqrt{r})[(1 - \sqrt{r})^{2} - s] \left(\operatorname{Re}[(C_{T}f_{T})^{*}H_{1}] - 2\operatorname{Re}[(C_{TE}f_{T})^{*}H_{2}] \right) \\ + 4(1 - \sqrt{r})[(1 + \sqrt{r})^{2} - s] \left(2\operatorname{Re}[(C_{TE}f_{T})^{*}F_{1}] - \operatorname{Re}[(C_{T}f_{T})^{*}F_{2}] \right) \\ + 4m_{\Lambda_{b}}s\sqrt{r} \operatorname{Re}[A_{1}^{*}E_{2} + A_{2}^{*}E_{1} + B_{1}^{*}D_{2} + B_{2}^{*}D_{1}] \\ - 2m_{\Lambda_{b}}^{2}s\sqrt{r}(1 - r + s) \operatorname{Re}[A_{2}^{*}D_{2} + B_{2}^{*}E_{2}^{*}] \\ - 4m_{\Lambda_{b}}[(1 - \sqrt{r})^{2} - s]s \left(\operatorname{Re}[(C_{T}f_{T}^{V})^{*}H_{1}] - 2\operatorname{Re}[(C_{TE}f_{T}^{V})^{*}H_{2}] \right) \\ + 2(1 - r - s) \left(\operatorname{Re}[A_{1}^{*}E_{1} + B_{1}^{*}D_{1}] + m_{\Lambda_{b}}^{2}\operatorname{sRe}[A_{2}^{*}E_{2} + B_{2}^{*}D_{2}] \right) \\ - m_{\Lambda_{b}}[(1 - r)^{2} - s^{2}] \operatorname{Re}[A_{1}^{*}D_{2} + A_{2}^{*}D_{1} + B_{1}^{*}E_{2} + B_{2}^{*}E_{1}] \Biggr\},$$

$$(21)$$

respectively. For the massive lepton case, expressions for the longitudinal and normal polarizations are quite lengthy, and for this reason, they are not presented in the text. The explicit form of these expressions can be found in [38].

3 Numerical analysis

In this section we will study the dependence of the lepton polarizations, as well as combined lepton polarization to the new Wilson coefficients. The main input parameters in the calculations are the form factors. Since the literature lacks exact calculations for the form factors of the $\Lambda_b \to \Lambda$ transition, we will use the results from QCD sum rules approach in combination with HQET [37, 39], which reduces the number of quite many form factors into two. The s dependence of these form factors can be represented in the following way

$$F(q^2) = \frac{F(0)}{1 - a_F s + b_F s^2}$$

where parameters $F_i(0)$, a and b are listed in table 1.

| | F(0) | a_F | b_F |
|-------|--------|---------|-----------|
| F_1 | 0.462 | -0.0182 | -0.000176 |
| F_2 | -0.077 | -0.0685 | 0.00146 |

Table 1: Transition form factors for $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay in the QCD sum rules method.

We use the next-to-leading order logarithmic approximation for the resulting values of the Wilson coefficients C_9^{eff} , C_7 and C_{10} in the SM [40, 41] at the renormalization point $\mu = m_b$. It should be noted that, in addition to short distance short distance contribution, C_9^{eff} receives also long distance contributions from the real $\bar{c}c$ resonant states of the J/ψ family. The Wilson coefficient C_9^{eff} is given by

$$C_9^{eff} = C_9(\mu) + Y_{pert} + \frac{3\pi}{\alpha^2} \tilde{C}^{(0)} \sum_{V=J/\psi,\psi',\dots} \kappa_i \frac{\Gamma(V_i \to \ell^+ \ell^-) m_{V_i}}{m_{V_i}^2 - q^2 - i m_{V_i} \Gamma_{V_i}} , \qquad (22)$$

where $C_9(\mu = m_b) = 4.214$ in the next to leading logarithmic order (NLL) [40], $Y_{pert}(q^2/m_b^2)$ arises from the one-loop matrix elements of the four-quark operators and its explicit expression can be found in [41], and $\tilde{C}^{(0)} = 3\tilde{C}_1 + \tilde{C}_2 + 3\tilde{C}_3 + \tilde{C}_4 + 3\tilde{C}_5 + \tilde{C}_6$. The values of these Wilson coefficients in NLL order can be found in [42]. In Eq. (22), m_{V_i} and Γ_{V_i} are the masses and widths of the J/ψ family. The fudge factor κ_i for the lowest resonances are chosen as $\kappa_{J/\psi} = 1.65$ and $\kappa_{\psi'} = 2.36$ [43], and for higher resonances the average of $\kappa_{J/\psi}$ and $\kappa_{\psi'}$ is used. In the present work we neglect the long distance contributions to the C_9^{eff} , i.e., we restrict ourselves by considering only short distance effects. We will discuss the influence of the long distance effects in one of our future works.

It follows from Eq. (18) that in performing the numerical analysis of the Λ baryon polarizations, the values of the new Wilson coefficients, which are responsible for the new physics beyond the SM, are needed. In further numerical analysis we vary all new Wilson coefficients in the range $-|C_{10}| \leq C_X \leq |C_{10}|$. The experimental bound on the branching ratio of the $B \to K^* \mu^+ \mu^-$ [27, 28] and $B \to \mu^+ \mu^-$ [44] decays suggest that this is the right order of magnitude for the vector and scalar interaction coefficients. As has been mentioned in the introduction section, BaBar and BELLE Collaborations have presented their preliminary results on the branching ratios of the $B \to K^* \ell^+ \ell^-$ and $B \to K \ell^+ \ell^$ decays. when one uses the results of both Collaborations on these branching ratios, stronger restrictions are imposed on some of the new Wilson coefficients. For example, $-2 \leq C_{LL} \leq$ $0, 0 \leq C_{RL} \leq 2.3, -1.5 \leq C_T \leq 1.5$ and $-3.3 \leq C_{TE} \leq 2.6$, and all of the remaining coefficients vary in the region $-4 \leq C_X \leq 4$. The experimental results on the $B \to K^* \ell^+ \ell^-$ and $B \to K\ell^+\ell^-$ decays are preliminary and for this reason we did not take into account the above-mentioned restrictions. Moreover, we assume that all new Wilson coefficients are real, as well as all of the form factors that we use in the present work. As a result the normal polarization of Λ is equal to zero, since it is proportional to the imaginary parts of the combinations of the new Wilson coefficients and of the form factors.

Before proceeding further with the numerical analysis, few words about lepton polarizations are in order. It follows from explicit expressions of the Λ baryon polarizations that they depend on both s and the new Wilson coefficients. Therefore it may experimentally be difficult to study their dependence on both of these variables simultaneously. For this reason it is better if we eliminate the dependence of the Λ baryon polarization on one of the variables. We choose to eliminate the variable s by performing integration over s in the allowed kinematical region, so that Λ baryon polarizations are averaged over. The averaged Λ baryon polarizations are defined as

$$\langle P_i \rangle = \frac{\int P_i \frac{d\mathcal{B}}{ds} ds}{\int \frac{d\mathcal{B}}{ds} ds} \,. \tag{23}$$

The dependence of the averaged lepton polarizations $\langle P_L \rangle$ and $\langle P_N \rangle$ on the new Wilson coefficients are shown in Figs (1)–(2). From these figures we obtain the following results.

- $\langle P_L \rangle$ is strongly dependent to the tensor interaction and quite sensitive to the Wilson coefficients C_{RR} and C_{RL} , for both $\Lambda_b \to \Lambda \mu^+ \mu^-$ and $\Lambda_b \to \Lambda \tau^+ \tau^-$ channels. We observe from Fig. (1) that the value of $\langle P_L \rangle$ is negative for all values of the new Wilson coefficients for $\Lambda_b \to \Lambda \mu^+ \mu^-$ decay, while, as can easily be seen from Fig. (2), it is positive when $C_T \leq -1.7$ and $C_{TE} \geq 0.5$ for the $\Lambda_b \to \Lambda \tau^+ \tau^-$ channel. The $C_X = 0$ point corresponds to the SM case. It follows from Figs. (1) and (2) that the departure from the SM becomes substantial when $C_X \neq 0$. This result confirms that the measurement of the longitudinal Λ baryon polarization can be a very decisive tool in looking for new physics beyond the SM.
- The situation for $\langle P_N \rangle$ is drastically different compared to that for $\langle P_L \rangle$. $\langle P_N \rangle$ is strongly dependent to the vector interaction coefficient C_{RL} for both channels. The τ channel is also sensitive to the tensor interaction coefficient C_{TE} when $C_{TE} > 0$. It follows from Figs. (3) and (4) that $\langle P_N \rangle$ is positive (negative) when $C_{RL} < 0$ $(C_{RL} > 0)$. In the τ channel $\langle P_N \rangle$ is negative when C_{TE} positive.

From these discussions we can conclude that change in sign and magnitude of the Λ baryon polarization is an indication of the new physics beyond the SM. Determining the sign of the Λ baryon polarization determines the sign of the new Wilson coefficients and type of the new interactions.

Finally we would like to mention about the branching ratio, whose measurement is easier compared to that of the measurement of the Λ baryon polarization, as well as being an efficient tool in establishing the new physics beyond the SM. In this connection, there follows the question: can one establish new physics by concentrating on the lepton polarization only. In other words, are there are certain regions of the new Wilson coefficients in which the value of the branching ratio coincides with that of the SM prediction, while Λ baryon polarization does not? In order to answer this we study the correlation between the branching ratio and the averaged Λ baryon polarizations by varying the new Wilson coefficients. In further analysis the values of the branching ratio ranges between $10^{-6} \leq \mathcal{B}(\Lambda_b \to \Lambda \mu^+ \mu^-) \leq$ 6×10^{-6} and $10^{-7} \leq \mathcal{B}(\Lambda_b \to \Lambda \tau^+ \tau^-) \leq 6 \times 10^{-7}$, which are of the same order of magnitude with the SM predictions. A first glance to the analysis depicted in Figs. (5)–(9) yields the following results.

- The numerical analysis for the $\Lambda_b \to \Lambda \mu^+ \mu^-$ decay for the longitudinal Λ baryon polarization yields that such regions of C_X , in which branching ratio does agree with the SM prediction while the averaged longitudinal Λ baryon polarization does not, are absent. However, as can easily be seen from Figs. (5) and (6), such regions of C_X exist (C_{LR} and C_{RL}) for the averaged normal polarization of the Λ baryon.
- For the Λ_b → Λτ⁺τ⁻ decay, we observe the existence of a very narrow region for the vector interaction coefficient C_{RL}, in which the branching ratio coincides with the SM prediction but the averaged longitudinal polarization does not. However, a study of the correlation between the averaged normal polarization of the Λ baryon and branching ratio, depicted in Figs. (7), (8) and (9), leads to more promising expectations. In other words, for the new Wilson coefficients C_{LR}, C_{RLLR} and C_{RLRL} there indeed exist such regions where branching ratio coincides with the SM prediction, but ⟨P_N⟩ deviates substantially from that of the SM prediction.

In conclusion, we have studied the sensitivity of the Λ baryon polarizations to the new Wilson coefficients. It is shown that there exist certain regions of various new Wilson coefficients for which the branching ratio of the $\Lambda_b \to \Lambda \ell^+ \ell^-$ decays coincide with the SM prediction, while Λ baryon polarizations deviate substantially from its counterparts predicted by the SM.

References

- [1] S. Fukae, C. S. Kim, T. Morozumi and T. Yoshikawa, *Phys. Rev.* D59, 074013 (1999).
- [2] M. S. Alam at. al, CLEO Collaboration, Phys. Rev. Lett. 74 (1995) 2885.
- [3] K. Abe at. al, Belle Collaboration, Phys. Lett. **B511** (2001) 151.
- [4] S. Chen at. al, CLEO Collaboration, Phys. Rev. Lett. 87 (2001) 251807.
- [5] H. Tajima, Belle Collaboration, Int. J. Mod. Phys. A17 (2002) 2967.
- [6] B. Aubert at. al, BaBar Collaboration, Phys. Rev. Lett. 88 (2002) 101805.
- [7] W. S. Hou, R. S. Willey and A. Soni, *Phys. Rev. Lett.* 58, 1608 (1987).
- [8] N. G. Deshpande and J. Trampetic, *Phys. Rev. Lett.* **60**, 2583 (1988).
- [9] C. S. Kim, T. Morozumi and A. I. Sanda, *Phys. Lett.* B218, 343 (1989).
- [10] B. Grinstein, M. J. Savage and M. B. Wise, Nucl. Phys. **B319**, 271 (1989).
- [11] C. Dominguez, N. Paver and Riazuddin, Phys. Lett. B214, 459 (1988).
- [12] N. G. Deshpande, J. Trampetic and K. Ponose, *Phys. Rev.* D39, 1461 (1989).
- [13] P. J. O'Donnell and H. K. Tung, *Phys. Rev.* **D43**, 2067 (1991).
- [14] N. Paver and Riazuddin, *Phys. Rev.* **D45**, 978 (1992).
- [15] A. Ali, T. Mannel and T. Morozumi, *Phys. Lett.* **B273**, 505 (1991).
- [16] A. Ali, G. F. Giudice and T. Mannel, Z. Phys. C67, 417 (1995).
- [17] C. Greub, A. Ioannissian and D. Wyler, *Phys. Lett.* B346, 145 (1995);
 D. Liu, *Phys. Lett.* B346, 355 (1995);
 G. Burdman, *Phys. Rev.* D52, 6400 (1995);
 Y. Okada, Y. Shimizu and M. Tanaka, *Phys.Lett.* B405, 297 (1997).
- [18] A. J. Buras and M. Münz, *Phys. Rev.* **D52**, 186 (1995).
- [19] N. G. Deshpande, X. -G. He and J. Trampetic, *Phys. Lett.* B367, 362 (1996).
- [20] S. Bertolini, F. Borzumati, A. Masiero, G. Ridolfi, *Nucl. Phys.* B353, 591 (1991).
- [21] F. Krüger and L. M. Sehgal *Phys. Rev.* **D55**,2799 (1997).
- [22] F. Krüger and L. M. Sehgal *Phys. Rev.* **D56**, 5452 (1997).
- [23] T. M. Aliev, A. Ozpineci, M. Savcı, *Phys. Rev.* D56 (1997) 4260.
- [24] T. M. Aliev, C. S. Kim, Y. G. Kim, *Phys. Rev.* D62 (2000) 014026.

- [25] T. M. Aliev, D. A. Demir, M. Savcı, *Phys. Rev.* D62 (2000) 074016.
- [26] P. Ball, V. M. Braun, *Phys. Rev.* **D58** (1998) 094016.
- [27] K. Abe at. al, Belle Collaboration, Phys. Rev. Lett. 88 (2002) 021801.
- [28] B. Aubert et. al, BaBar Collaboration, prep. hep-ex/0207082 (2002); Phys. Rev. Lett.
 88 (2002) 241801.
- [29] T. Mannel and S. Recksiegel, J. Phys. G24 (1998) 979;
 G. Hiller and A. Kagan, Phys. Rev. D60 (2002) 074038.
- [30] T. M. Aliev, M. K. Çakmak, M. Savcı, Nucl. Phys. B607 (2001) 305.
- [31] T. M. Aliev, A. Ozpineci, M. K. Çakmak, M. Savci, *Phys. Rev.* D64 (2001) 055007.
- [32] T. M. Aliev, A. Ozpineci, M. Savci, *Phys. Rev.* D65 (2002) 115002.
- [33] S. Fukae, C. S. Kim, T. Yoshikawa, *Phys. Rev.* D61 (2000) 074015.
- [34] Chuan–Hung Chen and C. Q. Geng, *Phys. Rev.* D64 (2001) 074001.
- [35] T. M. Aliev, A. Ozpineci, M. Savcı, Nucl. Phys. B (in press); prep. hep-ph/0202129 (2002).
- [36] T. M. Aliev, A. Ozpineci, M. Savcı, C. Yüce, *Phys. Lett.* **B542** (2002) 229.
- [37] T. Mannel, W. Roberts and Z. Ryzak, Nucl. Phys. B355 (1991) 38.
- [38] T. M. Aliev, A. Ozpineci, M. Savcı, prep. hep-ph/0301019 (2002).
- [39] C.-S. Huang, H.-G. Yan, *Phys. Rev.* **D59** (1999) 114022.
- [40] C. Bobeth, M. Misiak, and J. Urban, Nucl. Phys. **B574** (2000) 291.
- [41] M. Beneke, T. Feldman, D. Seidel, Nucl. Phys. B612 (2001) 25.
- [42] A. Ali and A. Safir, Eur. Phys. J. C25 (2002) 583.
- [43] A. Ali, P. Ball, L. T. Handoko and G. Hiller, *Phys. Rev.* D61 (2000) 074024.
- [44] V. Halyo, prep. hep-ex/0207010 (2002).

Figure captions

Fig. (1) The dependence of the averaged longitudinal Λ baryon polarization $\langle P_L \rangle$ on the new Wilson coefficients for the $\Lambda_b \to \Lambda \mu^+ \mu^-$ decay.

Fig. (2) The same as in Fig. (1), but for the $\Lambda_b \to \Lambda \tau^+ \tau^-$ decay.

Fig. (3) The same as in Fig. (1), but for the averaged normal Λ baryon polarization $\langle P_N \rangle$.

Fig. (4) The same as in Fig. (3), but for the $\Lambda_b \to \Lambda \tau^+ \tau^-$ decay.

Fig. (5) Parametric plot of the correlation between the branching ratio \mathcal{B} and the averaged normal polarization $\langle P_N \rangle$ as a function of the new vector C_{LL} and C_{LR} Wilson coefficients for the $\Lambda_b \to \Lambda \mu^+ \mu^-$ decay.

Fig. (6) The same as in Fig. (5), but for the new vector C_{RR} and C_{RL} Wilson coefficients.

Fig. (7) The same as in Fig. (5), but for the $\Lambda_b \to \Lambda \tau^+ \tau^-$ decay.

Fig. (8) The same as in Fig. (6), but for the $\Lambda_b \to \Lambda \tau^+ \tau^-$ decay.

Fig. (9) Parametric plot of the correlation between the branching ratio \mathcal{B} and the averaged normal polarization $\langle P_N \rangle$ as a function of the new scalar C_{LRRL} , C_{LRLR} , C_{RLLR} and C_{RLRL} and C_{LR} Wilson coefficients for the $\Lambda_b \to \Lambda \tau^+ \tau^-$ decay.



Figure 1:



Figure 2:



Figure 3:



Figure 4:



Figure 5:



Figure 6:



Figure 7:



Figure 8:



Figure 9: