The magnetic moments of Λ_b and Λ_c baryons in light cone QCD sum rules

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Abstract

Using the most general form of the interpolating currents of heavy baryons, the magnetic moments of heavy baryons Λ_Q ($Q = b, c$) are calculated in framework of the light cone QCD sum rules. A comparison of our results on magnetic moments with the existing theoretical results calculated in various other frameworks are presented.

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1 Introduction

QCD sum rules [1] are very successful in determination of the masses and coupling constants of low–lying mesons and baryons. In this method a deep connection between hadron properties and QCD vacuum structure is established via few condensates. This approach is adopted and extended to many works (see for example [2] and the references therein). One of the important static characteristics of baryons is their magnetic moments. The magnetic moment of nucleon within QCD sum rules is obtained in [3, 4] using external field method. In [5] use is made of the QCD sum rules method in the presence of external electromagnetic field with field strength tensor $\mathcal{F}_{\mu\nu}$ to calculate the magnetic moments of the baryons Σ_c, Λ_c containing heavy quarks.

The goal of the present work is to calculate the magnetic moments of Λ_b and Λ_c in framework of an alternative approach to the traditional QCD sum rules, i.e., light cone QCD sum rules (LCQSR) (more about this method and its applications can be found in [6, 7] and references therein). Magnetic moments of the nucleons and decoupled baryons have been studied in LCQSR in [8] and [9, 10], respectively.

The paper is organized as follows. In section 2 the LCQSR for Λ_Q $(Q = b, c)$ magnetic moment is derived. In section 3 we present numerical results.

2 LCQSR for Λ_Q magnetic moment

Our starting point for determination of the Λ_Q magnetic moment is to consider the two–point correlator function

$$\Pi = i \int d^4x \, e^{ipx} \left\langle 0 | T\{ \eta_{\Lambda_Q}(x) \bar{\eta}_{\Lambda_Q}(0) \} | 0 \right\rangle_{\mathcal{F}_{\alpha\beta}} , \tag{1}$$

where $\mathcal{F}_{\alpha\beta}$ is the external electromagnetic field, η_{Λ_Q} is the interpolating current with Λ_Q quantum numbers. It is well known that there is a continuum of choices for the baryon interpolating currents. The general form of Λ_Q currents can be written as [11]

$$\eta_{\Lambda_Q} = 2\left(\eta_{\Lambda_1} + t\eta_{\Lambda_2}\right) , \tag{2}$$

where t is an arbitrary parameter and

$$\eta_{\Lambda_1} = \frac{1}{\sqrt{6}} \epsilon_{abc} \left[2(u_a^T C d_b)\gamma_5 Q_c + (u_a^T C Q_b)\gamma_5 d_c - (d_a^T C Q_b)\gamma_5 u_c \right] , \tag{3}$$

$$\eta_{\Lambda_2} = \frac{1}{\sqrt{6}} \epsilon_{abc} \left[2(u_a^T C\gamma_5 d_b) Q_c + (u_a^T C\gamma_5 Q_b) d_c - (d_a^T C\gamma_5 Q_b) u_c \right] , \tag{4}$$

where a, b, and c are color indices. Ioffe current corresponds to the choice $t = -1$.

Let us consider phenomenological part of the correlator (2). Saturating the correlator with the complete set of hadron states having the same quantum numbers with Λ_Q baryon, we get

$$\Pi = \sum_i \frac{\left\langle 0 | \eta_{\Lambda_Q} | B_i(p_1) \right\rangle}{p_1^2 - M^2} \left\langle B_i(p_1) | B_i(p_2) \right\rangle_{\mathcal{F}_{\alpha\beta}} \frac{\left\langle B_i(p_2) | \bar{\eta}_{\Lambda_Q} | 0 \right\rangle}{p_2^2 - M^2} , \tag{5}$$

where $p_2 = p_1 + q$, q is the photon momentum and B_i is the complete set of corresponding baryons having the same quantum numbers as B with masses M.

The interpolating current couples to the baryon states with amplitudes λ defined as

$$\left\langle 0|\eta_{\Lambda_Q}|\Lambda_Q\right\rangle = \lambda_{\Lambda_Q} u_{\Lambda_Q}(p) \ . \tag{6}$$

It follows from Eq. (5) that in order to calculate the phenomenological part of the correlator, an expression for the matrix element $\langle B(p_1)|B(p_2)\rangle_{\mathcal{F}_{\alpha\beta}}$ is needed. This matrix element can be parametrized as follows:

$$\begin{aligned}
\langle B(p_1)|B(p_2)\rangle_{\mathcal{F}_{\alpha\beta}} &= \bar{u}(p_1)\left[f_1\gamma_\mu + i\,\frac{\sigma_{\mu\alpha}q^\alpha}{2m_{\Lambda_Q}}f_2\right]u(p_2)\varepsilon^\mu \ , \\
&= \bar{u}(p_1)\left[(f_1+f_2)\gamma_\mu + \frac{(p_1+p_2)_\mu}{2m_{\Lambda_Q}}f_2\right]u(p_2)\varepsilon^\mu \ ,
\end{aligned} \tag{7}$$

where f_1 and f_2 are the form factors, which are functions of $q^2 = (p_2 - p_1)^2$ and ε^μ is the polarization four vector of the photon. In the present case, in order to calculate magnetic moment of Λ_Q, the values of the form factors at $q^2 = 0$ are needed. Using Eqs. (5)–(7), for the phenomenological part of the LCQSR we get:

$$\Pi = -\lambda_{\Lambda_Q}^2 \varepsilon^\mu \frac{\not{p}_1 + m_{\Lambda_Q}}{p_1^2 - m_{\Lambda_Q}^2}\left[(f_1+f_2)\gamma_\mu + \frac{(p_1+p_2)_\mu}{2m_{\Lambda_Q}}f_2\right]\frac{\not{p}_2 + m_{\Lambda_Q}}{p_2^2 - m_{\Lambda_Q}^2} \ . \tag{8}$$

Among a number of different structures present in Eq. (8), we choose $\not{p}_1\not{\varepsilon}\not{p}_2$ which contains the magnetic moment form factor $f_1 + f_2$. When calculated at $q^2 = 0$, this structure gives the magnetic moment of Λ_Q baryon in units of $e\hbar/2m_{\Lambda_Q}$. Isolating the phenomenological part of the correlator from this structure which describes the magnetic moment of the Λ_Q baryon, we get

$$\Pi = -\lambda_{\Lambda_Q}^2 \frac{1}{p_1^2 - m_{\Lambda_Q}^2}\mu_{\Lambda_Q}\frac{1}{p_2^2 - m_{\Lambda_Q}^2} \ , \tag{9}$$

where $\mu_{\Lambda_Q} = (f_1 + f_2)|_{q^2=0}$.

According to the QCD sum rules philosophy in order to construct sum rules we need to calculate the theoretical part of the correlator function Π. Calculating correlator (1) in QCD we get

$$\begin{aligned}
\Pi(p_1^2,p_2^2) = &-\frac{2}{3}\,\epsilon^{abc}\epsilon^{def}\int d^4x\, e^{ipx} \\
&\times \left\langle 0\right|\Big\{4\gamma_5 S_Q^{cf}\gamma_5 \text{Tr} S_d^{be} S_u^{\prime ad} + 4t\gamma_5 S_Q^{cf}\text{Tr} S_d^{be}\gamma_5 S_u^{\prime ad} \\
&+ 4t S_Q^{cf}\gamma_5 \text{Tr} S_d^{be} S_u^{\prime ad}\gamma_5 + 4t^2 S_Q^{cf}\text{Tr} S_d^{be}\gamma_5 S_u^{\prime ad}\gamma_5 \\
&+ 2\gamma_5 S_Q^{cf} S_u^{\prime ad} S_d^{be}\gamma_5 + 2t\gamma_5 S_Q^{cf}\gamma_5 S_u^{\prime ad} S_d^{be} \\
&+ 2t S_Q^{cf} S_u^{\prime ad}\gamma_5 S_d^{be}\gamma_5 + 2t^2 S_Q^{cf}\gamma_5 S_u^{\prime ad}\gamma_5 S_d^{be} \\
&+ 2\gamma_5 S_Q^{cf} S_d^{\prime be} S_u^{ad}\gamma_5 + 2t\gamma_5 S_Q^{cf}\gamma_5 S_d^{\prime be} S_u^{ad}
\end{aligned}$$

$$
\begin{aligned}
&+ 2tS_Q^{cf}S_d^{'be}\gamma_5 S_u^{ad}\gamma_5 + 2t^2 S_Q^{cf}\gamma_5 S_d^{'be}\gamma_5 S_u^{ad} \\
&+ 2\gamma_5 S_d^{be}S_u^{'ad}S_Q^{cf}\gamma_5 + 2t\gamma_5 S_d^{be}\gamma_5 S_u^{'ad}S_Q^{cf} \\
&\quad +2tS_d^{be}S_u^{'ad}\gamma_5 S_Q^{cf}\gamma_5 + 2t^2 S_d^{be}\gamma_5 S_u^{'ad}\gamma_5 S_Q^{cf} \\
&+ \gamma_5 S_d^{be}\gamma_5 \mathrm{Tr} S_Q^{cf}S_u^{'ad} + t\gamma_5 S_d^{be}\mathrm{Tr} S_Q^{cf}\gamma_5 S_u^{'ad} \\
&\quad +tS_d^{be}\gamma_5 \mathrm{Tr} S_Q^{cf}S_u^{'ad}\gamma_5 + t^2 S_d^{be}\mathrm{Tr} S_Q^{cf}\gamma_5 S_u^{'ad}\gamma_5 \\
&- \gamma_5 S_d^{be}S_Q^{'cf}S_u^{ad}\gamma_5 - t\gamma_5 S_d^{be}\gamma_5 S_Q^{'cf}S_u^{ad} \\
&- tS_d^{be}S_Q^{'cf}\gamma_5 S_u^{ad}\gamma_5 - t^2 S_d^{be}\gamma_5 S_Q^{'cf}\gamma_5 S_u^{ad} \\
&+ 2\gamma_5 S_u^{ad}S_d^{'be}S_Q^{cf}\gamma_5 + 2t\gamma_5 S_u^{ad}\gamma_5 S_d^{'be}S_Q^{cf} \\
&+ 2tS_u^{ad}S_d^{'be}\gamma_5 S_Q^{cf}\gamma_5 + 2t^2 S_u^{ad}\gamma_5 S_d^{'be}\gamma_5 S_Q^{cf} \\
&- \gamma_5 S_u^{ad}S_Q^{'cf}S_d^{be}\gamma_5 - t\gamma_5 S_u^{ad}\gamma_5 S_Q^{'cf}S_d^{be} \\
&- tS_u^{ad}S_Q^{'cf}\gamma_5 S_d^{be}\gamma_5 - t^2 S_u^{ad}\gamma_5 S_Q^{'cf}\gamma_5 S_d^{be} \\
&+ \gamma_5 S_u^{ad}\gamma_5 \mathrm{Tr} S_Q^{cf}S_d^{'be} + t\gamma_5 S_u^{ad}\mathrm{Tr} S_Q^{cf}\gamma_5 S_d^{'be} \\
&+ tS_u^{ad}\gamma_5 \mathrm{Tr} S_Q^{cf}S_d^{'be}\gamma_5 + t^2 S_u^{ad}\mathrm{Tr} S_Q^{cf}\gamma_5 S_d^{'be}\gamma_5 \ \Big\}\Big| 0\Big\rangle_{\mathcal{F}_{\alpha\beta}} \ , \qquad (10)
\end{aligned}
$$

where $S^{'} = CS^TC$, with C and T are being the charge conjugation and transpose of the operator, respectively.

The perturbative contribution (i.e., photon is radiated from the freely propagating quarks) can easily be obtained by making the following substitution in one of the propagators in Eq. (10)

$$
S_{\alpha\beta}^{ab} \rightarrow 2 \left(\int dy\, \mathcal{F}^{\mu\nu} y_\nu S^{free}(x-y)\gamma_\mu S^{free}(y) \right)_{\alpha\beta}^{ab} \ , \qquad (11)
$$

where the Fock–Schwinger gauge $x_\mu A^\mu(x) = 0$ and S^{free} is the free quark operator. In x–representation the propagator of the free massive quark is

$$
S_Q^{free} = \frac{m_Q^2}{4\pi^2} \frac{K_1\left(m_Q\sqrt{-x^2}\right)}{\sqrt{-x^2}} - i\, \frac{m_Q^2\ \not{x}}{4\pi^2 x^2} K_2\left(m_Q\sqrt{-x^2}\right) \ , \qquad (12)
$$

where m_Q is the heavy quark mass and K_i are the Bessel functions. Using the expansions for the Bessel functions

$$
\begin{aligned}
K_1(x) &\sim \frac{1}{x} + \mathcal{O}(x) \ , \\
K_2(x) &\sim \frac{2}{x^2} - \frac{1}{2} + \mathcal{O}(x^2) \ ,
\end{aligned}
$$

and formally setting $m_Q \rightarrow 0$, one can obtain the well known expression of the free propagator for massless quark in x representation:

$$
S_q^{free} = \frac{i\ \not{x}}{2\pi^2 x^4} \ . \qquad (13)
$$

The nonperturbative contributions can be obtained from Eq. (10) by replacing one of the propagators of light quarks with

$$S_{\alpha\beta}^{ab} \rightarrow -\frac{1}{4}\bar{q}^a A_j q^b \left(A_j\right)_{\alpha\beta} \ , \tag{14}$$

where $A_j = \left\{1,\ \gamma_5,\ \gamma_\alpha,\ i\gamma_5\gamma_\alpha,\ \sigma_{\alpha\beta}/\sqrt{2}\right\}$ and sum over A_j is implied.

The complete light cone expansion of the light quark propagator in external field is calculated in [12]. It receives contributions from the nonlocal operators $\bar{q}Gq$, $\bar{q}GGq$, $\bar{q}q\bar{q}q$, where G is the gluon field strength tensor. In this work we consider operators with only one gluon field and neglect terms with two gluons $\bar{q}GGq$, and four quarks $\bar{q}q\bar{q}q$, and this action can be justified on the basis of an expansion in conformal spin [13]. In this approximation heavy and massless light quark propagators read

$$\begin{aligned}
\left\langle 0 \left| T\left\{\bar{Q}(x)Q(0)\right\}\right| 0\right\rangle &= iS_Q^{free}(x) - ig_s \int \frac{d^4k}{(2\pi)^4}\, e^{-ikx} \int_0^1 dv \Bigg[\frac{\not{k}+m_Q}{(m_Q^2-k^2)^2}\, G^{\mu\nu}(vx)\sigma_{\mu\nu} \\
&+ \frac{1}{m_Q^2-k^2}\, vx_\mu G^{\mu\nu}\gamma_\nu\Bigg] \ ,
\end{aligned} \tag{15}$$

$$\begin{aligned}
\left\langle 0 \left| T\left\{\bar{q}(x)q(0)\right\}\right| 0\right\rangle &= i\, \frac{\not{x}}{2\pi^2x^4} - \frac{\langle\bar{q}q\rangle}{12}\left(1+\frac{x^2m_0^2}{16}\right) \\
&- i\, g_s \int dv \left[\frac{\not{x}}{16\pi^2x^2}\, G^{\mu\nu}(vx)\sigma_{\mu\nu} - \frac{i}{4\pi^2x^2}\, vx_\mu G^{\mu\nu}\gamma_\nu\right] \ ,
\end{aligned} \tag{16}$$

where m_0 is defined from the relation

$$\langle\bar{q}ig_sG_{\mu\nu}\sigma^{\mu\nu}q\rangle = m_0^2\langle\bar{q}q\rangle \ ,$$

and the operators in local part with dimension $d > 5$ are not taken into consideration since their contribution is negligible.

It follows from Eqs. (10)–(16) that, in order to calculate the theoretical part of the correlator function the matrix elements of non–local operators between photon and the vacuum state are needed. Up to twist–4, the photon wave functions are defined in the following way [13]–[15]:

$$\begin{aligned}
\langle\gamma(q)|\bar{q}\gamma_\alpha\gamma_5 q|0\rangle &= \frac{f}{4}e_q\epsilon_{\alpha\beta\rho\sigma}\varepsilon^\beta q^\rho x^\sigma \int_0^1 du\, e^{iuqx}\psi(u) \ , \\
\langle\gamma(q)|\bar{q}\sigma_{\alpha\beta}q|0\rangle &= ie_q\langle\bar{q}q\rangle \int_0^1 du\, e^{iuqx}\Bigg\{(\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha)\Big[\chi\phi(u) + x^2\Big(g_1(u)-g_2(u)\Big)\Big] \\
&+ \Big[qx(\varepsilon_\alpha x_\beta - \varepsilon_\beta x_\alpha) + \varepsilon x(x_\alpha q_\beta - x_\beta q_\alpha)\Big]g_2(u)\Bigg\} \ ,
\end{aligned} \tag{17}$$

where χ is the magnetic susceptibility of the quark condensate, e_q is the quark charge, the functions $\phi(u)$ and $\psi(u)$ are the leading twist–2 photon wave functions, while $g_1(u)$ and $g_2(u)$ are the twist–4 functions. Note that twist–3 photon wave functions are all neglected in further calculations since their contribution changes the results about 5%.

The theoretical part of the correlator can be obtained by substituting photon wave functions and expressions of light and heavy quark propagators into Eq. (10). The sum rules is obtained by equating the phenomenological and theoretical parts of the correlator. In order to suppress the contributions of higher states and of continuum (for more details, see [9, 10, 16, 17]) we perform double Borel transformations of the variables $p_1^2 = p^2$ and $p_2^2 = (p+q)^2$ on both sides of the correlator. It should be mentioned here that the Borel transformations for $K_\nu(x)$ functions, which appear in the propagator of a massive quark, were calculated in [18]. After lengthy calculations we get the following sum rules for the Λ_Q magnetic moment:

$$
\begin{aligned}
\mu_{\Lambda_Q}\lambda_{\Lambda_Q}^2\, e^{-m_{\Lambda_Q}^2/M^2} &= \frac{1}{32\pi^4}M^6\Big[(1-t)^2(e_u+e_d)\Psi(2,-1,m_Q^2/M^2) \\
&+ (13+10t+13t^2)e_Q\Psi(3,0,m_Q^2/M^2)\Big] \\
&+ \frac{1}{48\pi^2}(1-t)^2(e_u+e_d)M^4 f\psi(u_0)\Psi(1,-1,m_Q^2/M^2) \\
&+ \frac{m_Q}{24\pi^2}(-5+4t+t^2)M^4\langle\bar{q}q\rangle(e_u+e_d)\chi\varphi(u_0)\Psi(2,0,m_Q^2/M^2) \\
&- \frac{m_Q}{36}(-5+4t+t^2)\langle\bar{q}q\rangle(e_u+e_d)f\psi(u_0)F_5(m_Q^2/M^2) \\
&+ \frac{m_Q}{144M^2}m_0^2\langle\bar{q}q\rangle(e_u+e_d)f\psi(u_0)\Big\{(-5+4t+t^2)\Big[F_4(m_Q^2/M^2)-F_5(m_Q^2/M^2)\Big] \\
&\quad +3(-1+t^2)F_5(m_Q^2/M^2)\Big\} \\
&+ \frac{1}{18\pi^2}\langle\bar{q}q\rangle(e_u+e_d)\Big[g_1(u_0)-g_2(u_0)\Big]\Big[4\pi^2(1-2t+t^2)\langle\bar{q}q\rangle F_4(m_Q^2/M^2) \\
&- 3(-5+4t+t^2)m_Q M^2\Psi(1,0,m_Q^2/M^2)\Big] \\
&- \frac{m_0^2}{36M^2}\langle\bar{q}q\rangle^2(e_u+e_d)\Big[g_1(u_0)-g_2(u_0)\Big]\Big\{2(1-2t+t^2)F_1(m_Q^2/M^2) \\
&+ (-5+2t+3t^2)\Big[F_4(m_Q^2/M^2)-F_5(m_Q^2/M^2)\Big]\Big\} \\
&+ \frac{1}{144}\langle\bar{q}q\rangle^2(e_u+e_d)\chi\varphi(u_0)\Big[-8(1-2t+t^2)M^2F_5(m_Q^2/M^2) \\
&+ 2m_0^2(1-2t+t^2)F_4(m_Q^2/M^2)+m_0^2(-5+2t+3t^2)F_5(m_Q^2/M^2)\Big] \\
&+ \frac{m_0^2}{36M^2}e_Q\langle\bar{q}q\rangle^2(-11-2t+13t^2)\Big[F_4(m_Q^2/M^2)-F_5(m_Q^2/M^2)\Big] \\
&+ \frac{m_Q}{24\pi^2}M^2\langle\bar{q}q\rangle\Big[(e_u+e_d)(-5+4t+t^2)\Psi(1,0,m_Q^2/M^2) \\
&+ 2e_Q(-1-4t+5t^2)\Psi(2,1,m_Q^2/M^2)\Big] \\
&+ \frac{1}{18}e_Q\langle\bar{q}q\rangle^2(11+2t-13t^2)F_5(m_Q^2/M^2) \\
&- \frac{m_Q}{96\pi^2}m_0^2\langle\bar{q}q\rangle(e_u+e_d)\Big[(-5+4t+t^2)F_5(m_Q^2/M^2)+3(-1+t^2)\Psi(1,1,m_Q^2/M^2)\Big] \\
&+ \frac{m_Q}{288\pi^2}e_Q m_0^2\langle\bar{q}q\rangle(3+12t-16t^2)\Psi(1,1,m_Q^2/M^2)\ .
\end{aligned}
\tag{18}
$$

Here the functions $\Psi(\alpha,\beta,x)$ and $F_i(x)$ are defined as

$$
\begin{aligned}
\Psi(\alpha,\beta,x) &= \frac{1}{\Gamma(\alpha)}\int_1^\infty dt\, e^{-tx}\, t^{\beta-\alpha-1}(t-1)^{\alpha-1}~, \qquad (\alpha>0)~,\\
F_1(x) &= \Big(x^2-2x\Big)\, e^{-x}~,\\
F_2(x) &= \Big(x^2-4x+2\Big)\, e^{-x}~,\\
F_3(x) &= \Big(x^2-6x+6\Big)\, e^{-x}~,\\
F_4(x) &= xe^{-x}~,\\
F_5(x) &= e^{-x}~,
\end{aligned}
\tag{19}
$$

and

$$
u_0 = \frac{M_2^2}{M_1^2+M_2^2}~, \qquad M^2 = \frac{M_1^2 M_2^2}{M_1^2+M_2^2}~,
$$

where M_1^2 and M_2^2 are the Borel parameters. Since the initial and final baryons are the same, we will set $M_1^2=M_2^2=2M^2$, from which it follows that $u_0=1/2$.

It follows from Eq. (18) that the overlap amplitude λ_{Λ_Q} needs to be known in order to calculate the magnetic moment. This amplitude is determined from baryon mass sum rules. For the mass sum rules of baryons we get (see also [19])

$$
\begin{aligned}
m_{\Lambda_Q}\lambda_{\Lambda_Q}^2\, e^{-m_{\Lambda_Q}^2/M^2} &= \frac{m_Q}{32\pi^4}(-13+2b+11b^2)M^6\Psi(3,0,m_Q^2/M^2)\\
&+ \frac{\langle\bar{q}q\rangle}{12\pi^2}(1+4b-5b^2)M^4\Psi(1,-1,m_Q^2/M^2)\\
&- \frac{m_Q}{36M^2}m_0^2\langle\bar{q}q\rangle^2\Big\{3(5+2b+5b^2)\Big[F_4(m_Q^2/M^2)-F_5(m_Q^2/M^2)\Big]\\
&+ (-1+b)^2F_5(m_Q^2/M^2)\Big\}\\
&+ \frac{\langle\bar{q}q\rangle}{96\pi^2}m_0^2M^2\Big[(-1-4b+5b^2)F_5(m_Q^2/M^2)+(-5+4b+b^2)\Psi(1,0,m_Q^2/M^2)\Big]\\
&+ \frac{m_Q}{6}\langle\bar{q}q\rangle^2(5+2b+5b^2)F_5(m_Q^2/M^2)~,
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
\lambda_{\Lambda_Q}^2\, e^{-m_{\Lambda_Q}^2/M^2} &= \frac{3}{32\pi^4}(5+2b+5b^2)M^6\Psi(3,-1,m_Q^2/M^2)\\
&- \frac{m_0^2}{72M^2}\langle\bar{q}q\rangle^2\Big[(-26+4b+22b^2)F_4(m_Q^2/M^2)+(-1+b)^2F_5(m_Q^2/M^2)\Big]\\
&+ \frac{m_Q}{12\pi^2}\langle\bar{q}q\rangle(1+4b-5b^2)M^2\Psi(2,0,m_Q^2/M^2)\\
&+ \frac{\langle\bar{q}q\rangle^2}{18}(-13+2b+11b^2)F_5(m_Q^2/M^2)\\
&+ \frac{m_Q}{96\pi^2}m_0^2\langle\bar{q}q\rangle\Big[(-1-4b+5b^2)\Psi(1,0,m_Q^2/M^2)+6(-1+b^2)\Psi(2,1,m_Q^2/M^2)\Big]~.
\end{aligned}
\tag{21}
$$

Eqs. (20) and (21) correspond to the structures proportional to the unit operator and $\not{p}$, respectively. Subtraction of the continuum contribution in Eqs. (18), (20) and (21) can be performed by making the following substitution

$$M^{2n}\psi(\alpha,\beta,m_Q^2/M^2) \rightarrow \frac{1}{\Gamma(\alpha)}\frac{1}{\Gamma(n)}\int_{m_Q^2}^{s_0} ds e^{-s/M^2}\int_1^{s/m_Q^2} dt (s-tm_Q^2)^{n-1} t^{\beta-\alpha-1}(t-1)^{\alpha-1} \; , \qquad (22)$$

for $\alpha > 0$ and $n > 0$.

3 Numerical analysis

This section is devoted to the numerical analysis of the sum rules for the magnetic moment of Λ_Q baryon. It follows from Eq. (18) that the main input parameter of the LCQSR is the photon wave function. It was shown in [13, 14] that the leading twist–2 photon wave functions receive minor corrections from the higher conformal spin, so that to a good approximation they can be described by their asymptotic forms. In further numerical calculations we will use the following forms of the photon wave functions [13, 15]:

$$\begin{aligned}
\phi(u) &= 6u(1-u) \; , \qquad \psi(u) = 1 \; , \\
g_1(u) &= -\frac{1}{8}(1-u)(3-u) \; , \qquad g_2(u) = -\frac{1}{4}(1-u)^2 \; .
\end{aligned}$$

The values of the other input parameters which we need in the numerical calculations are: $f = 0.028 \; GeV^2$, $\chi = -4.4 \; GeV^{-2}$ [20] (in [21] this quantity is estimated to be $\chi = -3.3 \; GeV^{-2}$), $\langle \bar{q}q \rangle (1 \; GeV) = -(0.243)^3 \; GeV^3$, $m_0^2 = (0.8 \pm 0.2) \; GeV^2$ [22], $m_c = 1.3 \; GeV$ and $m_b = 4.8 \; GeV$.

In view of the fact that the magnetic moment is a physical quantity it must be independent of the auxiliary parameters t, Borel mass M^2, as well as the continuum threshold s_0. Therefore calculation of the magnetic moment requires simply finding a region where μ_{Λ_Q} is practically independent of the above–mentioned parameters.

Our strategy in trying to resolve this problem consists of three steps. In the first step, in order to find the working region, where μ_{Λ_Q} is supposed to be independent of the Borel parameter M^2, we study the dependence of the magnetic moment μ_{Λ_Q} on M^2 at several predetermined values of the threshold s_0 and at three different values of t. Along these lines, in Fig. (1) we present the dependence of μ_{Λ_c} on M^2 at $s_0 = 10 \; GeV^2$ and $s_0 = 15 \; GeV^2$, for different values of t. Similarly, Fig. (2) depicts the dependence of μ_{Λ_b} on M^2 at two different values of the threshold, $s_0 = 40 \; GeV^2$ and $s_0 = 45 \; GeV^2$. From Fig. (1) we observe that the working region of M^2 which is consistent with our requirements lies in the interval $3 \; GeV^2 \le M^2 \le 5 \; GeV^2$ for the Λ_c case and $15 \; GeV^2 \le M^2 \le 25 \; GeV^2$ for the Λ_b case. Obviously, from both these figures we observe that μ_{Λ_Q} is reasonably insensitive to the different choices of the continuum threshold, while it seems to be sensitive to the parameter t.

The next step, of course, is to explore the physical region for the parameter t. For this purpose we have used the mass sum rules given in Eqs. (20) and (21), both of which are required to be positive. Following this line of reasoning, we present in Figs. (3) and (4) the dependence of the mass sum rule (20) on $\cos\theta$, where θ is determined from the relation $\tan\theta = t$, for Λ_c and Λ_b, respectively. These figures depict that the relevant physical regions for the parameter t are given by $-0.78 \le \cos\theta \le 0.7$ for Λ_c and $-0.8 \le \cos\theta \le 0.7$

for Λ_b, respectively. The analysis of (21), clearly, leads to the conclusion that this sum rule is positive for arbitrary values of the parameter t. As a result of these observations, the physical region of t which guarantees the positiveness of the sum rules (20) and (21) separately, is confined to the interval $-0.78 \leq \cos\theta \leq 0.7$. Having this restriction on t, our final attempt is to determine the value of the magnetic moment μ_{Λ_Q}. For this purpose, we must find a region of t where μ_{Λ_Q} is independent of this parameter.

In Figs. (5), we present the dependence of the magnetic moment of heavy baryon Λ_c on $\cos\theta$ at the fixed value of the Borel parameter $M^2 = 4\ GeV^2$ for two different choices of the continuum threshold $s_0 = 3\ GeV^2$ and $s_0 = 4\ GeV^2$. Similarly, depicted in Fig. (6) is the dependence of the other heavy baryon Λ_b on $\cos\theta$ at $M^2 = 20\ GeV^2$ and $s_0 = 40\ GeV^2$; $s_0 = 45\ GeV^2$. We observe from these figures that the magnetic moment is quite stable in the region $-0.25 \leq \cos\theta \leq +0.5$, and practically seems to be independent of $\cos\theta$, t and the continuum threshold s_0. As a result of all these considerations we obtain for the magnetic moment

$$\begin{aligned} \mu_{\Lambda_c} &= (0.40 \pm 0.05)\, \mu_N \ , \\ \mu_{\Lambda_b} &= (-0.18 \pm 0.05)\, \mu_N \ , \end{aligned} \tag{23}$$

where μ_N is the nucleon magneton.

Finally we present a comparison of our result on μ_{Λ_Q} with the existing theoretical calculations in literature. For the magnetic moment μ_{Λ_c} the traditional QCD sum rules predicts $\mu_{\Lambda_c} = (0.15 \pm 0.05)\, \mu_N$ [5]. Our result on μ_{Λ_c} is close to the non–relativistic quark model prediction [23, 24], but there is substantial difference with the results predicted in [5]. In our opinion, this discrepancy can be attributed to the fact that the results presented in [5] were calculated for the choice $t = -1$, which is unphysical in our case. Magnetic moments of triplet and sextet heavy baryons have been calculated using heavy hadron chiral perturbation theory (HHChPT) in [25, 26, 27], and Λ_c and Λ_b baryons that we are interested belong to the triplet. It was shown in these works that in HHChPT the leading term in magnetic moments is proportional to e_Q/m_c, where Q is the heavy quark. The corrections to the leading term appear at the order of $\mathcal{O}(1/m_Q\Lambda_\chi^2)$, where $\Lambda_\chi \sim 1\ GeV$, and four arbitrary constants are required. If we set all four arbitrary constants to zero, for the magnetic moments of Λ_c and Λ_b baryons HHChPT predicts

$$\begin{aligned} \mu_{\Lambda_c} &\simeq 0.47\ \mu_N \ , \\ \mu_{\Lambda_b} &\simeq -0.23\ \mu_N \ . \end{aligned} \tag{24}$$

From Eqs. (23) and (24) we conclude that both approaches lead to close values for the magnetic moments of the heavy Λ_c and Λ_b baryons. The small difference between the predictions on the magnetic moments of heavy baryons of the two approaches is due to the subleading terms which are neglected in deriving Eq. (24).

A measurement of the magnetic moments of heavy baryons represents an experimental challenge. Few groups are contemplating the possibility of performing magnetic moments in the near future (BTeV and SELEX) [28].

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Figure captions

Fig. (1) The dependence of the magnetic moment μ_{Λ_c} on M^2 at two different values of the continuum threshold $s_0 = 10\ GeV^2$ and $s_0 = 15\ GeV^2$, for several fixed values of the parameter t. Here in this figure and in all following figures the magnetic moments of Λ_c and Λ_b baryons are given in units of the nucleon magneton μ_N.

Fig. (2) The dependence of the magnetic moment μ_{Λ_b} on M^2 at two different values of the continuum threshold $s_0 = 40\ GeV^2$ and $s_0 = 45\ GeV^2$, for several fixed values of the parameter t.

Fig. (3) The dependence of the mass sum rule m_{Λ_c} on $\cos\theta$, at two different values of the continuum threshold $s_0 = 10\ GeV^2$ and $s_0 = 15\ GeV^2$.

Fig. (4) The dependence of the mass sum rule m_{Λ_b} on $\cos\theta$, at two different values of the continuum threshold $s_0 = 40\ GeV^2$ and $s_0 = 45\ GeV^2$.

Fig. (5) The dependence of the magnetic moment μ_{Λ_c} on $\cos\theta$, at $M^2 = 4\ GeV^2$ and at two different values of the continuum threshold $s_0 = 3\ GeV^2$ and $s_0 = 4\ GeV^2$.

Fig. (6) The dependence of the magnetic moment μ_{Λ_b} on $\cos\theta$, at $M^2 = 20\ GeV^2$ and at two different values of the continuum threshold $s_0 = 40\ GeV^2$ and $s_0 = 45\ GeV^2$.

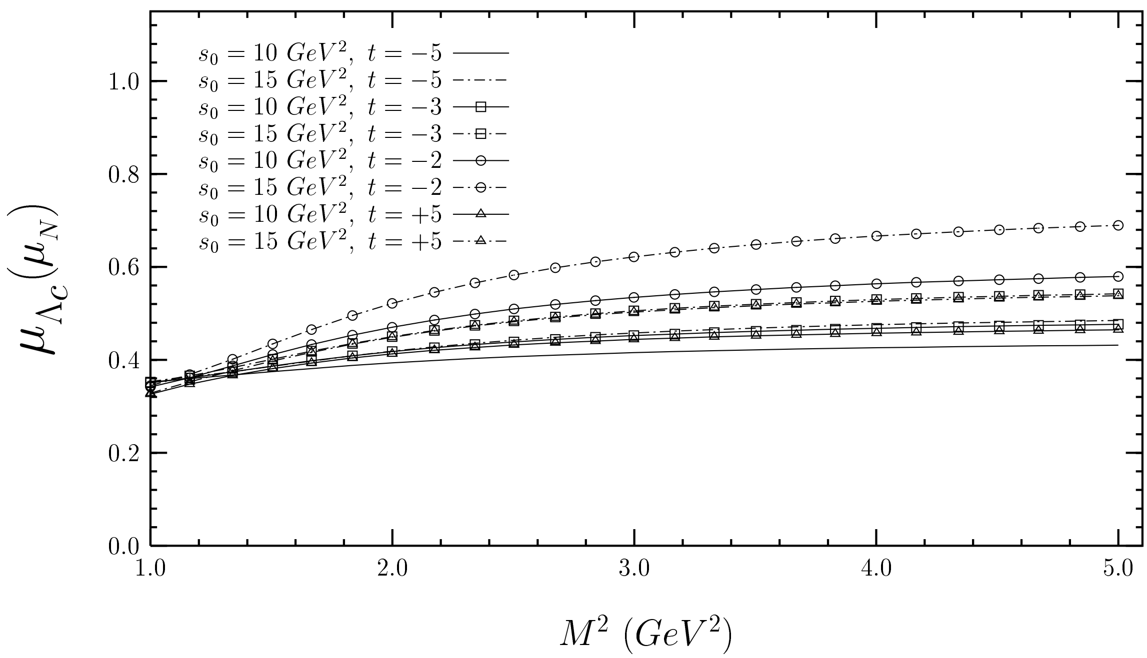

Figure 1:

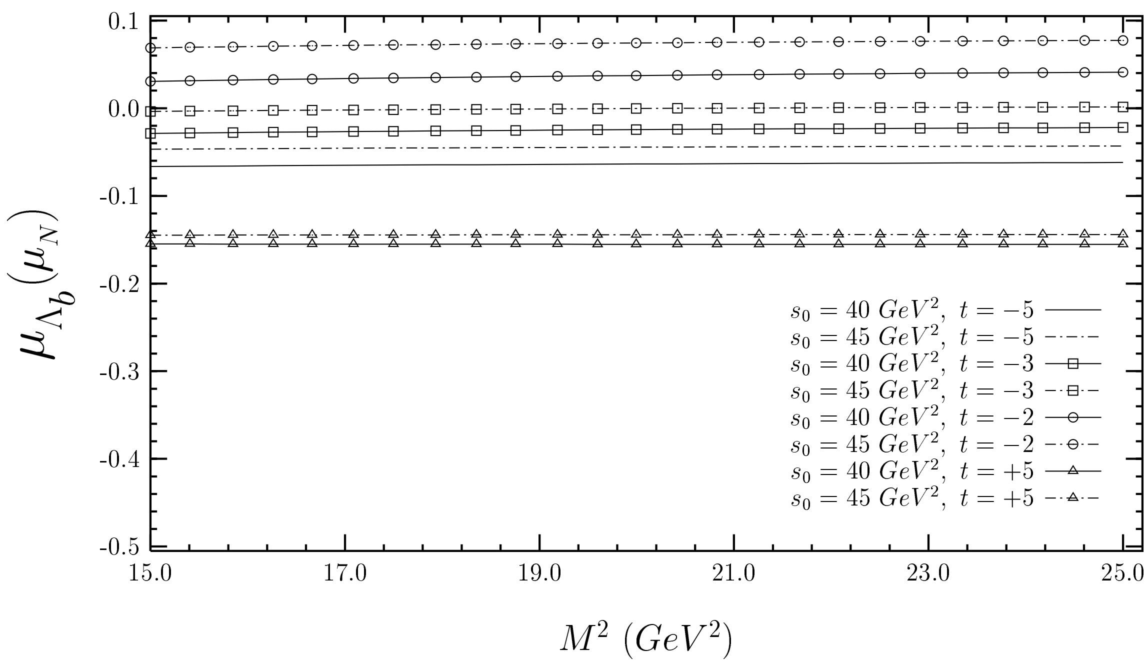

Figure 2:

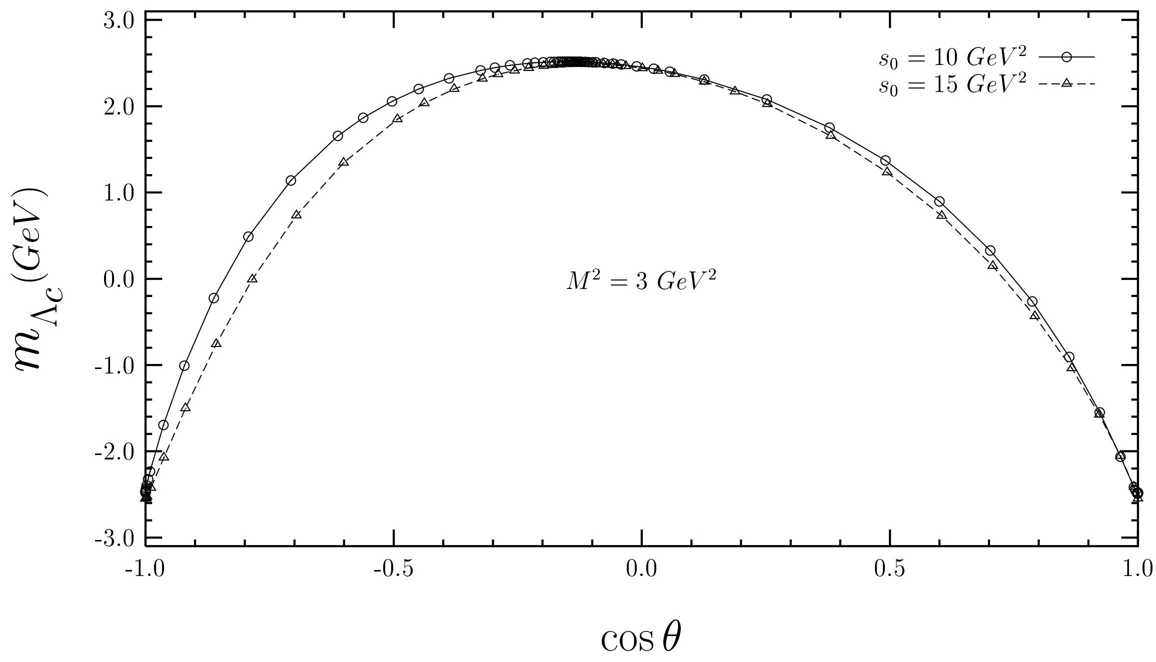

Figure 3:

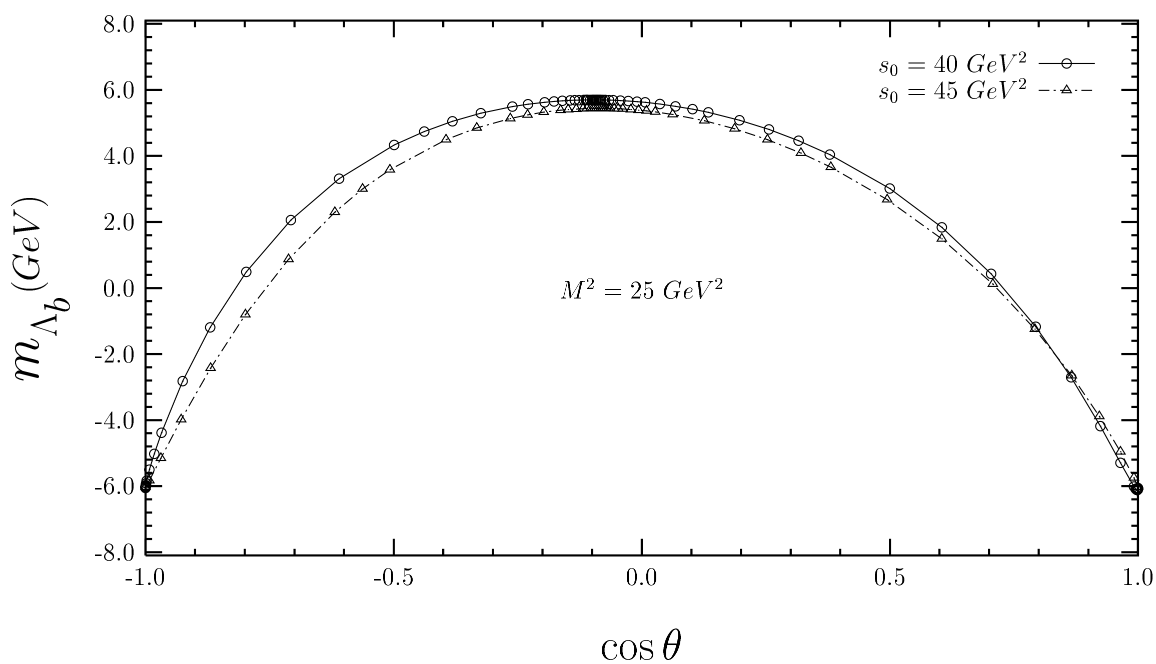

Figure 4:

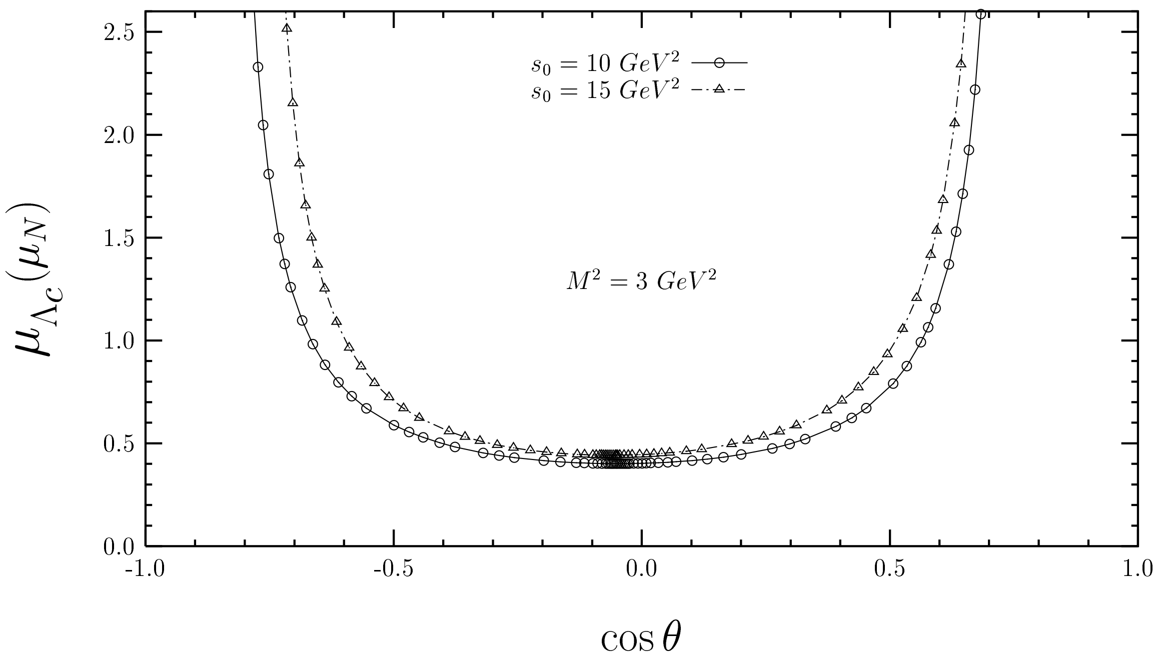

Figure 5:

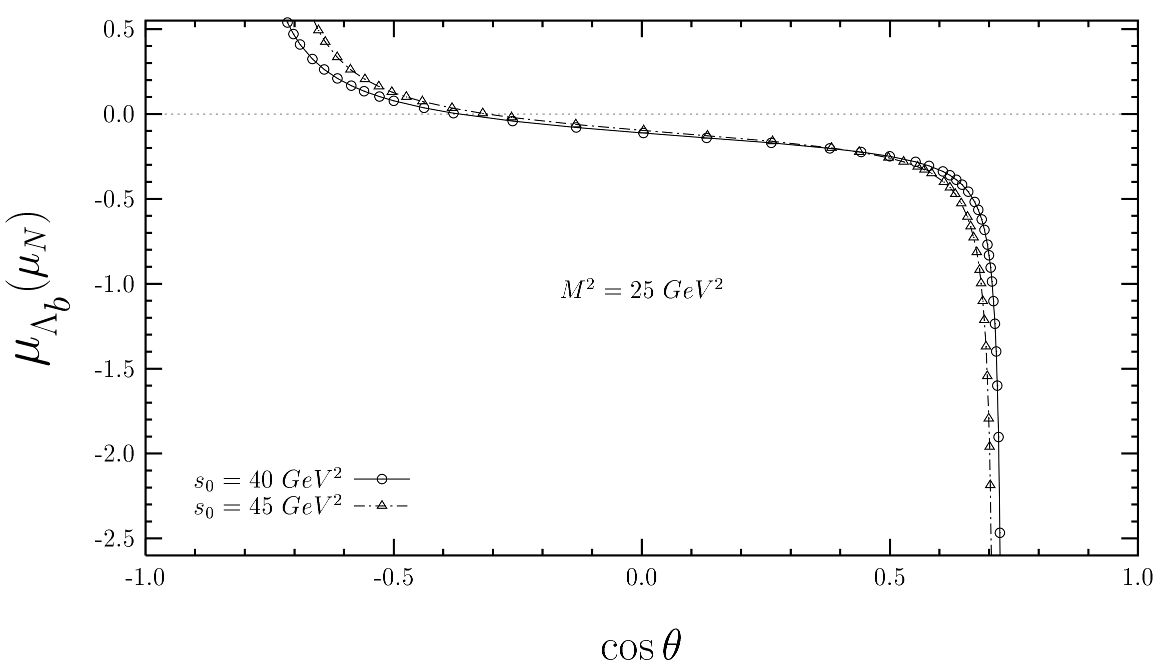

Figure 6: