Comment on the new AdS universe

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We show that Bonnor's new Anti-de Sitter (AdS) universe and its D-dimensional generalization is the previously studied AdS soliton.

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Recently, Bonnor [1] noted a second non-singular solution to cosmological Einstein equations $R_{\mu\nu} = \Lambda g_{\mu\nu}$ in *four* dimensions, with $\Lambda < 0$; the first obviously being the maximally symmetric AdS spacetime. In this comment, we show that Bonnor's "non-uniform AdS universe" is nothing but the AdS soliton, whose higher dimensional generalizations also exist [2].

The metric introduced in [1] belongs to a family of cylindrically symmetric, static spacetimes that are solutions to the four dimensional Einstein equations with a negative cosmological constant, and that were independently found a long time ago by Linet [3] and Tian [4]. Various properties of these metrics were studied in [5], where non-singular sheet sources of them were also found and their relation to the black-string solutions were examined. Roughly speaking, these can be given the interpretation of infinite rods whose mass per unit length is defined through a parameter σ with a specific range. The spacetime that Bonnor considered is the one obtained by setting this parameter $\sigma = 0$ (see [5] and [1] for details).

The generalization of this solution to $D = n + 1$ dimensions follows as

$$ds^2 = d\rho^2 + \cosh^{4/n}\left(\frac{n\rho}{2\ell}\right)\Big(- dt^2 + \frac{4\ell^2}{n^2}\,\tanh^2\left(\frac{n\rho}{2\ell}\right) d\phi^2 + \sum_{i=1}^{n-2} dx_i^2\Big)\,, \tag{1}$$

which solves

$$R_{\mu\nu} = -\frac{n}{\ell^2} g_{\mu\nu}\,,$$

with the ranges $t, x_i \in \mathbb{R}$ (where $i = 1, \ldots, n-2$), $\rho \in \mathbb{R}^+$ and $\phi \in [0, 2\pi)$. When $n = 3$, this reduces to the metric in [1]. For $n = 2$, one simply gets the AdS solution in $2 + 1$ dimensions. (1) has no singularities or horizons. Any invariant that involves only the curvature scalar or the Ricci tensor is identical to its counterpart belonging to the usual maximally symmetric AdS. However, the regularity and the non-uniformity of the solution is apparent from the Kretschmann scalar

$$R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma} = \frac{1}{\ell^4\cosh^4\left(\frac{n\rho}{2\ell}\right)}\Big(n(n-2)(n-1)^2 + \frac{n(n+1)}{2}\Big[1 + \cosh\left(\frac{n\rho}{\ell}\right)\Big]^2\Big)\,. \tag{2}$$

Let us show that (1) is a special case of the AdS soliton [2], which reads

$$ds^2 = \frac{r^2}{\ell^2}\left[\left(1 - \frac{r_0^n}{r^n}\right)d\tau^2 + \sum_{i=1}^{n-2}(dx^i)^2 - dt^2\right] + \left(1 - \frac{r_0^n}{r^n}\right)^{-1}\frac{\ell^2}{r^2}\,dr^2\,. \tag{3}$$

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(3) was obtained by the double analytic continuation of a near extremal $(n-1)$-brane solution. x^i and the t denote the coordinates on the "brane" and $r \geq r_0$, where r_0 is a free parameter. A conical singularity is avoided if τ has a period $\beta = 4\pi\ell^2/(nr_0)$ [2].

Consider the coordinate transformation $r = r_0 \cosh^{2/n}(n\rho/2\ell)$, which respects the proper ranges of r and ρ [7]. Then (3) turns into

$$ds^2 = d\rho^2 + \frac{r_0^2}{\ell^2}\cosh^{4/n}\left(\frac{n\rho}{2\ell}\right)\Big(-dt^2 + \tanh^2\left(\frac{n\rho}{2\ell}\right)d\tau^2 + \sum_{i=1}^{n-2} dx_i^2\Big). \tag{4}$$

Choosing $r_0 = \ell$ and $\tau = 2\ell\phi/n$, one obtains (1).

Finally, it is obvious that (1) does not make sense in $D = 1+1$ dimensions. However, one can separately consider the $D = 2$ Euclidean space. There, the cigar soliton of Witten [6]

$$ds^2 = d\rho^2 + \tanh^2\rho\, d\phi^2 \tag{5}$$

can be thought of as the $D = 2$ analog of (1). Note that, the cigar is also singularity free with a curvature scalar $R = 4/\cosh^2\rho$ but, of course, it is asymptotically flat unlike (1).

To conclude, we have shown that Bonnor's solution [1] and its D-dimensional generalization is the AdS soliton [2].

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