$b \to s\gamma$ decay in the two Higgs doublet model with flavor changing neutral currents

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Abstract

We study the $b \to s\gamma$ decay including the next to leading QCD corrections in the two Higgs doublet model with flavor changing neutral currents at the tree level. We find the constraints to the flavor changing parameters of the model, using the experimental results on the branching ratio of $B \to X_s\gamma$ decay, provided by the CLEO Collaboration, the restrictions coming from the $\Delta F = 2$ $(F = K, D, B)$ mixing and the ρ parameter.

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1 Introduction

Rare B meson decays are one of the most promising research area in particle physics and lie on the focus of theoretical and experimental physicists. In the Standard Model (SM), they are induced by flavor changing neutral currents (FCNC) at loop level and therefore sensitive to the gauge structure of the theory. From the experimental point of view, they play an outstanding role in the precise determination of the fundamental parameters of the SM, such as Cabbibo-Kobayashi-Maskawa (CKM) matrix elements, leptonic decay constants, etc. Furthermore, these decays provide a sensitive test to the new physics beyond the SM, such as two Higgs Doublet model (2HDM), Minimal Supersymmetric extension of the SM (MSSM) [1], etc. Among the rare B decays, $b \to s\gamma$ has received considerable interest since the branching ratios (Br) of the inclusive $B \to X_s\gamma$ [2] and exclusive $B \to K^*\gamma$ [3] have been already measured experimentally. Recently, the new experimental results for the inclusive $b \to s\gamma$ decay are announced by CLEO and ALEPH Collaborations [4]. Therefore, the $b \to s\gamma$ decay is under an extensive investigation in the framework of various extensions of the SM, in order to get information about the model parameters or improve the existing restrictions.

It is well known that the FCNC are forbidden at the tree level in the SM. This restriction is achieved in the extended model with the additional conditions. 2HDM is one of the simplest extensions of the SM, obtained by the addition of a new scalar $SU(2)$ doublet. The Yukawa lagrangian causes that the model possesses phenemologicaly dangerous FCNC's at the tree level. To protect the model from such terms, the ad hoc discrete symmetry [5] on the 2HDM scalar potential and the Yukawa interaction is proposed and there appear two different versions of the 2HDM depending on whether up and down quarks couple to the same or different scalar doublets. In model I, the up and down quarks get mass via vacuum expectation value (v.e.v.) of only one Higgs field. In model II, which coincides with the MSSM in the Higgs sector, the up and down quarks get mass via v.e.v. of the Higgs fields H_1 and H_2 respectively where $H_1(H_2)$ corresponds to first (second) Higgs doublet of 2HDM [6]. In the absence of the mentioned discrete symmetry, FCNC appears at the tree level and this model is called as model III in current literature [7, 8, 9]. A comprehensive phenemological analysis of the model III was done in series of works [7, 8, 10]. In particular, from a purely phenomenological point of view, low energy experiments involving $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$, $K_L \to \mu\bar{\mu}$, etc, place strong constraints on the existence of tree level flavor changing (FC) transitions, existing in the model III.

In the present work, we examine the $b \to s\gamma$ decay in the model III, taking the next to leading (NLO) QCD corrections into account, in a more detailed analysis compared to one existing in literature (see [7, 10]). Further, we obtain the constraints for the neutral couplings ξ^U_{Ntt}, ξ^D_{Nbb} and ξ^U_{Ntc} with the assumption that ξ^U_{Ncc}, ξ^D_{Nsb}, ξ^D_{Nss} and the other couplings which include the first generation indices are negligible compared to former ones (for the definition of $\xi_{N,ij}$ see section 2). Our predictions are based on the CLEO measurement $B \to X_s\gamma$ and the restrictions coming from the $\Delta F = 2$ ($F = K, D, B$) mixing and the ρ parameter [10]. Note that NLO QCD corrections to the $b \to s\gamma$ decay in 2HDM (for model I and II) were calculated in [11, 12].

The paper is organized as follows: In Section 2, we present the NLO QCD corrected Hamiltonian responsible for the $b \to s\gamma$ decay in the model III and discuss the effects of the additional $Left-Right$ flipped operators to the decay rate. Section 3 is devoted to the constraint analysis, more precisely to the the ratios $\frac{\xi^U_{Ntt}}{\xi^D_{Nbb}}$, $\frac{\xi^U_{Ntc}}{\xi^U_{Ntt}}$ and our conclusions.

2 Next to leading improved short-distance contributions in the model III for the decay $b \to s\gamma$

Before presenting the NLO QCD corrections to the $b \to s\gamma$ decay amplitude in the 2HDM (model III), we would like to remind briefly the main features of the 2HDM. The Yukawa interaction for the general case is

$$\mathcal{L}_Y = \eta^U_{ij}\bar{Q}_{iL}\tilde{\phi}_1 U_{jR} + \eta^D_{ij}\bar{Q}_{iL}\phi_1 D_{jR} + \xi^U_{ij}\bar{Q}_{iL}\tilde{\phi}_2 U_{jR} + \xi^D_{ij}\bar{Q}_{iL}\phi_2 D_{jR} + h.c. \tag{1}$$

where L and R denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, ϕ_i for $i = 1, 2$, are the two scalar doublets, $\eta^{U,D}_{ij}$ and $\xi^{U,D}_{ij}$ are the matrices of the Yukawa couplings. For convenience we choose ϕ_1 and ϕ_2 in the following basis:

$$\phi_1 = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2}\chi^+ \\ i\chi^0 \end{pmatrix}\right] ; \phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}H^+ \\ H_1 + iH_2 \end{pmatrix} , \tag{2}$$

where the vacuum expectation values are,

$$< \phi_1 >= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} ; < \phi_2 >= 0 \ . \tag{3}$$

This choice permits us to write the FC part of the interaction as

$$\mathcal{L}_{Y,FC} = \xi^U_{ij}\bar{Q}_{iL}\tilde{\phi}_2 U_{jR} + \xi^D_{ij}\bar{Q}_{iL}\phi_2 D_{jR} + h.c. \ , \tag{4}$$

with the following advantages:

- doublet ϕ_1 corresponds to the scalar doublet of the SM and H_0 to the SM Higgs field. This part of the Yukawa Lagrangian is responsible for the generation of the fermion masses with the couplings $\eta^{U,D}$.

- all new scalar fields belong to the ϕ_2 scalar doublet.

The couplings $\xi^{U,D}$ are the open window for the tree level FCNC's and can be expressed for the FC charged interactions as

$$\begin{aligned} \xi^U_{ch} &= \xi_{neutral}\, V_{CKM} \ , \\ \xi^D_{ch} &= V_{CKM}\, \xi_{neutral} \ , \end{aligned} \tag{5}$$

where $\xi^{U,D}_{neutral}$ [1] is defined by the expression

$$\xi^{U,D}_N = (V_L^{U,D})^{-1} \xi^{U,D} V_R^{U,D} \ . \tag{6}$$

Here the charged couplings appear as a linear combinations of neutral couplings multiplied by V_{CKM} matrix elements. This gives an important distinction between model III and model II (I).

After this preliminary remark, let us discuss the NLO QCD corrections to the $b \to s\gamma$ decay in the 2HDM for the general case. The appropriate framework is that of an effective theory obtained by integrating out the heavy degrees of freedom, which are, in this context, t quark, $W^\pm, H^\pm, H_1$, and H_2 bosons, where $H^\pm$ denote charged ,H_1 and H_2 denote neutral Higgs bosons. The LLog QCD corrections are done through matching the full theory with the effective low energy theory at the high scale $\mu = m_W$ and evaluating the Wilson coefficients from m_W down to the lower scale $\mu \sim O(m_b)$. Note that we choose the higher scale as $\mu = m_W$ since the evaluation from the scale $\mu = m_{H^\pm}$ to $\mu = m_W$ gives negligible contribution to the Wilson coefficients. Here we assume that the charged Higgs boson is heavy due to theoretical analysis of the $b \to s\gamma$ decay (see [11, 13]).

The effective Hamiltonian relevant for $b \to s\gamma$ decay is

$$\mathcal{H}_{eff} = -4 \frac{G_F}{\sqrt{2}} V_{tb} V^*_{ts} \sum_i C_i(\mu) O_i(\mu) \ , \tag{7}$$

where the O_i are operators given in eq. (8) and the C_i are Wilson coefficients renormalized at the scale μ. The coefficients are calculated perturbatively and expressed as functions of the heavy particle masses in the theory.

[1] In all next discussion we denote $\xi^{U,D}_{neutral}$ as $\xi^{U,D}_N$.

The operator basis depends on the model used and the conventional choice in the case of SM, 2HDM model II (I) and MSSM is

$$
\begin{aligned}
O_1 &= (\bar{s}_{L\alpha}\gamma_\mu c_{L\beta})(\bar{c}_{L\beta}\gamma^\mu b_{L\alpha}),\\
O_2 &= (\bar{s}_{L\alpha}\gamma_\mu c_{L\alpha})(\bar{c}_{L\beta}\gamma^\mu b_{L\beta}),\\
O_3 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha})\sum_{q=u,d,s,c,b}(\bar{q}_{L\beta}\gamma^\mu q_{L\beta}),\\
O_4 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\beta})\sum_{q=u,d,s,c,b}(\bar{q}_{L\beta}\gamma^\mu q_{L\alpha}),\\
O_5 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha})\sum_{q=u,d,s,c,b}(\bar{q}_{R\beta}\gamma^\mu q_{R\beta}),\\
O_6 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\beta})\sum_{q=u,d,s,c,b}(\bar{q}_{R\beta}\gamma^\mu q_{R\alpha}),\\
O_7 &= \frac{e}{16\pi^2}\bar{s}_\alpha\sigma_{\mu\nu}(m_b R + m_s L)b_\alpha\mathcal{F}^{\mu\nu},\\
O_8 &= \frac{g}{16\pi^2}\bar{s}_\alpha T^a_{\alpha\beta}\sigma_{\mu\nu}(m_b R + m_s L)b_\beta\mathcal{G}^{a\mu\nu},
\end{aligned}
\tag{8}
$$

where α and β are $SU(3)$ colour indices and $\mathcal{F}^{\mu\nu}$ and $\mathcal{G}^{\mu\nu}$ are the field strength tensors of the electromagnetic and strong interactions, respectively.

In our case, however, new operators with different chirality structures can arise since the general Yukawa lagrangian includes both L and R chiral interactions. The conventional operator set is extended first adding two new operators which are left-right analogues of O_1 and O_2, namely

$$
\begin{aligned}
O_9 &= (\bar{s}_{L\alpha}\gamma_\mu c_{L\beta})(\bar{c}_{R\beta}\gamma^\mu b_{R\alpha}),\\
O_{10} &= (\bar{s}_{L\alpha}\gamma_\mu c_{L\alpha})(\bar{c}_{R\beta}\gamma^\mu b_{R\beta}),
\end{aligned}
\tag{9}
$$

Further we need the second operator set $O'_1 - O'_{10}$ which are flipped chirality partners of $O_1 - O_{10}$:

$$
\begin{aligned}
O'_1 &= (\bar{s}_{R\alpha}\gamma_\mu c_{R\beta})(\bar{c}_{R\beta}\gamma^\mu b_{R\alpha}),\\
O'_2 &= (\bar{s}_{R\alpha}\gamma_\mu c_{R\alpha})(\bar{c}_{R\beta}\gamma^\mu b_{R\beta}),\\
O'_3 &= (\bar{s}_{R\alpha}\gamma_\mu b_{R\alpha})\sum_{q=u,d,s,c,b}(\bar{q}_{R\beta}\gamma^\mu q_{R\beta}),\\
O'_4 &= (\bar{s}_{R\alpha}\gamma_\mu b_{R\beta})\sum_{q=u,d,s,c,b}(\bar{q}_{R\beta}\gamma^\mu q_{R\alpha}),\\
O'_5 &= (\bar{s}_{R\alpha}\gamma_\mu b_{R\alpha})\sum_{q=u,d,s,c,b}(\bar{q}_{L\beta}\gamma^\mu q_{L\beta}),\\
O'_6 &= (\bar{s}_{R\alpha}\gamma_\mu b_{R\beta})\sum_{q=u,d,s,c,b}(\bar{q}_{L\beta}\gamma^\mu q_{L\alpha}),\\
O'_7 &= \frac{e}{16\pi^2}\bar{s}_\alpha\sigma_{\mu\nu}(m_b L + m_s R)b_\alpha\mathcal{F}^{\mu\nu},
\end{aligned}
$$

$$
\begin{aligned}
O'_8 &= \frac{g}{16\pi^2}\bar{s}_\alpha T^a_{\alpha\beta}\sigma_{\mu\nu}(m_b L + m_s R) b_\beta \mathcal{G}^{a\mu\nu}, \\
O'_9 &= (\bar{s}_{R\alpha}\gamma_\mu c_{R\beta})(\bar{c}_{L\beta}\gamma^\mu b_{L\alpha}) \, , \\
O'_{10} &= (\bar{s}_{R\alpha}\gamma_\mu c_{R\alpha})(\bar{c}_{L\beta}\gamma^\mu b_{L\beta}) \, . \qquad (10)
\end{aligned}
$$

This extended basis is the same as the basis for $SU(2)_L \times SU(2)_R \times U(1)$ extensions of SM [14]. Note that in the SM, model II (I) 2HDM and the MSSM, the absence of O'_7 and O'_8 are a consequence of assumption $m_s/m_b \sim 0$.

In the calculations, we take only the charged Higgs contributions into account and neglect the effects of neutral Higgs bosons for the reasons given below: The neutral bosons H_0, H_1 and H_2 are defined in terms of the mass eigenstates $\bar{H}_0$,h_0 and A_0 as

$$
\begin{aligned}
H_0 &= (\bar{H}_0 cos\alpha - h_0 sin\alpha) + v \, , \\
H_1 &= (h_0 cos\alpha + \bar{H}_0 sin\alpha) \, , \\
H_2 &= A_0 \, , \qquad (11)
\end{aligned}
$$

where α is the mixing angle and v is proportional to the vacuum expectation value of the doublet ϕ_1 (eq. (3)). Here we assume that the massess of neutral Higgs bosons h_0 and A_0 are heavy compared to the b-quark mass. The neutral Higgs scalar h_0 and pseduscalar A_0 give contribution only to C_7 for $b \to s\gamma$ decay. With the choice of $\alpha = 0$, $C_7^{h_0}$ and $C_7^{A_0}$ can be expressed at m_W level as

$$
\begin{aligned}
C_7^{h_0}(m_W) &= (V_{tb}V^*_{ts})^{-1} \sum_{i=d,s,b} \bar{\xi}^D_{N,bi}\, \bar{\xi}^D_{N,is} \frac{Q_i}{8\, m_i\, m_b} \, , \\
C_7^{A_0}(m_W) &= (V_{tb}V^*_{ts})^{-1} \sum_{i=d,s,b} \bar{\xi}^D_{N,bi}\, \bar{\xi}^D_{N,is} \frac{Q_i}{8\, m_i\, m_b} \, , \qquad (12)
\end{aligned}
$$

where m_i and Q_i are the masses and charges of the down quarks ($i = d,\, s,\, b$) respectively. Here we used the redefinition

$$
\xi^{U,D} = \sqrt{\frac{4G_F}{\sqrt{2}}}\, \bar{\xi}^{U,D} \, . \qquad (13)
$$

Eq. (12) shows that neutral Higgs bosons can give a large contribution to C_7, which does not respect the CLEO and ALEPH data [4]. At this stage we make an assumption that the couplings $\bar{\xi}^D_{N,is} (i = d, s, b)$ and $\bar{\xi}^D_{N,db}$ are negligible to be able to reach the conditions $\bar{\xi}^D_{N,bb}\, \bar{\xi}^D_{N,is} << 1$ and $\bar{\xi}^D_{N,db}\, \bar{\xi}^D_{N,ds} << 1$. These choices permit us to neglect the neutral Higgs effects.

Now, for the evaluation of Wilson coefficients, we need their initial values with standard matching computations. Denoting the Wilson coefficients for the additional charged Higgs contribution with $C_i^H(m_W)$, we have the initial values of the Wilson coefficients for the first set of operators (eqs.(8), (9))

$$
\begin{aligned}
C^H_{1,\ldots 6,9,10}(m_W) &= 0 \, , \\
C^H_7(m_W) &= \frac{1}{m_t^2}\,(\bar{\xi}^U_{N,tt}+\bar{\xi}^U_{N,tc}\frac{V^*_{cs}}{V^*_{ts}})\,(\bar{\xi}^U_{N,tt}+\bar{\xi}^U_{N,tc}\frac{V_{cb}}{V_{tb}})F_1(y) \, , \\
&+ \frac{1}{m_t m_b}\,(\bar{\xi}^U_{N,tt}+\bar{\xi}^U_{N,tc}\frac{V^*_{cs}}{V^*_{ts}})\,(\bar{\xi}^D_{N,bb}+\bar{\xi}^D_{N,sb}\frac{V_{ts}}{V_{tb}})F_2(y) \, , \\
C^H_8(m_W) &= \frac{1}{m_t^2}\,(\bar{\xi}^U_{N,tt}+\bar{\xi}^U_{N,tc}\frac{V^*_{cs}}{V^*_{ts}})\,(\bar{\xi}^U_{N,tt}+\bar{\xi}^U_{N,tc}\frac{V_{cb}}{V_{tb}})G_1(y) \, , \\
&+ \frac{1}{m_t m_b}\,(\bar{\xi}^U_{N,tt}+\bar{\xi}^U_{N,tc}\frac{V^*_{cs}}{V^*_{ts}})\,(\bar{\xi}^D_{N,bb}+\bar{\xi}^U_{N,sb}\frac{V_{ts}}{V_{tb}})G_2(y) \, . \qquad (14)
\end{aligned}
$$

The explicit forms of the Wilson coefficients in the SM ($C_i^{SM}(m_W)$) is presented in the literature [15]. For the primed Wison coefficients we get,

$$
C'^{SM}_{1,\ldots 6,9,10}(m_W) = 0 \, , \qquad (15)
$$

$$
\begin{aligned}
C'^H_{1,\ldots 6,9,10}(m_W) &= 0 \, , \\
C'^H_7(m_W) &= \frac{1}{m_t^2}\,(\bar{\xi}^D_{N,bs}\frac{V_{tb}}{V^*_{ts}}+\bar{\xi}^D_{N,ss})\,(\bar{\xi}^D_{N,bb}+\bar{\xi}^D_{N,sb}\frac{V_{ts}}{V_{tb}})F_1(y) \, , \\
&+ \frac{1}{m_t m_b}\,(\bar{\xi}^D_{N,bs}\frac{V_{tb}}{V^*_{ts}}+\bar{\xi}^D_{N,ss})\,(\bar{\xi}^U_{N,tt}+\bar{\xi}^U_{N,tc}\frac{V_{cb}}{V_{tb}})F_2(y) \, , \\
C'^H_8(m_W) &= \frac{1}{m_t^2}\,(\bar{\xi}^D_{N,bs}\frac{V_{tb}}{V^*_{ts}}+\bar{\xi}^D_{N,ss})\,(\bar{\xi}^D_{N,bb}+\bar{\xi}^D_{N,sb}\frac{V_{ts}}{V_{tb}})G_1(y) \, , \\
&+ \frac{1}{m_t m_b}\,(\bar{\xi}^D_{N,bs}\frac{V_{tb}}{V^*_{ts}}+\bar{\xi}^D_{N,ss})\,(\bar{\xi}^U_{N,tt}+\bar{\xi}^U_{N,tc}\frac{V_{cb}}{V_{tb}})G_2(y) \, , \qquad (16)
\end{aligned}
$$

where $x=m_t^2/m_W^2$ and $y=m_t^2/m^2_{H^\pm}$. The functions $F_1(y)$, $F_2(y)$, $G_1(y)$ and $G_2(y)$ are given as

$$
\begin{aligned}
F_1(y) &= \frac{y(7-5y-8y^2)}{72(y-1)^3}+\frac{y^2(3y-2)}{12(y-1)^4}\,lny \, , \\
F_2(y) &= \frac{y(5y-3)}{12(y-1)^2}+\frac{y(-3y+2)}{6(y-1)^3}\,lny \, , \\
G_1(y) &= \frac{y(-y^2+5y+2)}{24(y-1)^3}+\frac{-y^2}{4(y-1)^4}\,lny \, , \\
G_2(y) &= \frac{y(y-3)}{4(y-1)^2}+\frac{y}{2(y-1)^3}\,lny \, . \qquad (17)
\end{aligned}
$$

In calculations we neglect the small contributions of the internal u and c quarks compared to one due to the internal t quark.

For the initial values of the mentioned Wilson coefficients in the model III (eqs. (14), (15) and (16)), we have

$$C_i^{(\prime)2HDM}(m_W) \;=\; C_i^{(\prime)SM}(m_W) + C_i^{(\prime)H}(m_W)\,. \tag{18}$$

Using these initial values, we can calculate the coefficients C_i^{2HDM} and $C_i^{\prime 2HDM}$ at any lower scale with five quark effective theory where large logarithims can be summed using the renormalization group. Since the strong interactions preserve chirality, the operators in eqs. (8, 9) can not mix with their chirality flipped counterparts eq. (10) and the anomalous dimension matrices of two separate set of operators are the same and do not overlap. With the above choosen initial values of Wilson coefficients, their evaluations are similar to the SM case.

For completeness, note that, the operators O_5,O_6,O_9 and O_{10} (O'_5,O'_6, O'_9 and O'_{10}) give contributions to the matrix element of $b \to s\gamma$ and in the NDR scheme which we use here, the effective magnetic moment type Wilson coefficients are redefined as

$$\begin{aligned}
C_7^{eff}(\mu) &= C_7^{2HDM}(\mu) + Q_d\,(C_5^{2HDM}(\mu) + N_c\,C_6^{2HDM}(\mu))\;, \\
&+ Q_u\,(\frac{m_c}{m_b}\,C_{10}^{2HDM}(\mu) + N_c\,\frac{m_c}{m_b}\,C_9^{2HDM}(\mu))\;, \\
C_7^{\prime eff}(\mu) &= C_7^{\prime 2HDM}(\mu) + Q_d\,(C_5^{\prime 2HDM}(\mu) + N_c\,C_6^{\prime 2HDM}(\mu)) \\
&+ Q_u(\frac{m_c}{m_b}\,C_{10}^{\prime 2HDM}(\mu) + N_c\,\frac{m_c}{m_b}\,C_9^{\prime 2HDM}(\mu))\;,
\end{aligned} \tag{19}$$

where N_c is the color factor and Q_u (Q_d) is the charge of up (down) quarks. There is still another mixing in the operator set O_7, O_8, O_9, O_{10} ($O'_7, O'_8, O'_9, O'_{10}$) [14] and we do not take into account since the initial values of the Wilson coefficients C_{10} and C'_{10} are zero in our case.

The NLO corrected coefficients $C_7^{2HDM}(\mu)$ and $C_7^{\prime 2HDM}(\mu)$ are given as

$$\begin{aligned}
C_7^{2HDM}(\mu) &= C_7^{LO,2HDM}(\mu) + \frac{\alpha_s(\mu)}{4\pi}C_7^{(1)\,2HDM}(\mu)\;, \\
C_7^{\prime 2HDM}(\mu) &= C_7^{\prime LO,2HDM}(\mu) + \frac{\alpha_s(\mu)}{4\pi}C_7^{\prime(1)\,2HDM}(\mu)\;.
\end{aligned} \tag{20}$$

Here $\eta = \alpha_s(m_W)/\alpha_s(\mu)$, h_i and a_i are the numbers which appear during the evaluation [16]. The functions $C_7^{LO,2HDM}(\mu)$ [17] and $C_7^{\prime LO,2HDM}(\mu)$are the leading order QCD corrected Wilson coefficients:

$$C_7^{\prime LO,2HDM}(\mu) \;=\; \eta^{16/23}C_7^{\prime 2HDM}(m_W) + (8/3)(\eta^{14/23} - \eta^{16/23})C_8^{\prime 2HDM}(m_W) \tag{21}$$

and $C_7^{(1)\,2HDM}(\mu)$ is the α_s correction to the leading order result that its explicit form can be found in [11, 12]. $C_7^{\prime(1)\,2HDM}(\mu)$ can be obtained by replacing the Wilson coefficients in $C_7^{(1)\,2HDM}(\mu)$ with their primed counterparts. The NLO corrected coefficients $C_5^{2HDM}(\mu)$, $C_6^{2HDM}(\mu)$ and $C_5^{\prime 2HDM}(\mu)$, $C_6^{\prime 2HDM}(\mu)$ are numerically small at m_b scale therefore we neglect them in our calculations.

Finally, the NLO QCD corrected $b \to s\gamma$ decay rate in model III is obtained as

$$\Gamma(b \to s\gamma) = \frac{G_F^2 m_b^5}{32\pi^4} \alpha_{em} |V_{ts}^* V_{tb}|^2 (|C_7^{eff}(m_b)|^2 + |C_7^{\prime eff}(m_b)|^2) \, , \quad (22)$$

where α_{em} is the fine structure constant, and m_b is b-quark mass. $|C_7^{eff}(m_b)|^2$ is given in [11]

$$\begin{aligned} |C_7^{eff}(m_b)|^2 &= |D|^2 + A + \frac{\delta_\gamma^{NP}}{m_b^2} |C_7^{LO,2HDM}(m_b)|^2 \\ &+ \frac{\delta_c^{NP}}{m_b^2} Re\{\Big(C_7^{LO,2HDM}(m_b)\Big)^* \Big(C_2^{LO,2HDM}(m_b) - \frac{1}{6} C_1^{LO,2HDM}(m_b)\Big)\} \, . \quad (23) \end{aligned}$$

The functions D and A are [11]

$$\begin{aligned} D &= C_7^{LO,2HDM}(m_b) + \frac{\alpha_s}{4\pi}\Big(C_7^{(1)\,2HDM}(\mu) + \sum_i^8 C_i^{LO,2HDM}(m_b) r_i - \frac{16}{3} C_7^{LO,2HDM}(m_b)\Big) \, , \\ A &= \frac{\alpha_s(m_b)}{\pi} \sum_{i,j=1, i\le j}^{8} Re\{C_i^{LO,2HDM}(m_b)\Big(C_j^{LO,2HDM}(m_b)\Big)^* f_{ij}\} \, . \quad (24) \end{aligned}$$

The explicite expressions for f_{ij}, r_i, δ_γ^{NP} and δ_c^{NP} can be found in [11]. At this point we would like to note that the expressions for unprimed Wilson coefficients in our case can be obtained from the results in [11] by the following replacements:

$$\begin{aligned} |Y|^2 &\to \frac{1}{m_t^2}\, (\bar{\xi}^U_{N,tt} + \bar{\xi}^U_{N,tc} \frac{V_{cs}^*}{V_{ts}^*})\, (\bar{\xi}^U_{N,tt} + \bar{\xi}^U_{N,tc} \frac{V_{cb}}{V_{tb}}) \, , \\ XY &\to \frac{1}{m_t m_b}\, (\bar{\xi}^U_{N,tt} + \bar{\xi}^U_{N,tc} \frac{V_{cs}^*}{V_{ts}^*})\, (\bar{\xi}^D_{N,bb} + \bar{\xi}^D_{N,sb} \frac{V_{ts}}{V_{tb}}) \quad (25) \end{aligned}$$

To obtain $|C_7^{\prime eff}(m_b)|^2$, it is enough to use the primed Wilson coefficients at m_W level (eq. 16) since the evaluation of $C_7^{\prime eff}(\mu)$ from $\mu = m_W$ to $\mu = m_b$ is the same as that of $C_7^{eff}(\mu)$.

Note that, for model II (model I) Y and XY are

$$\begin{aligned} Y &= 1/tan\beta\ (1/tan\beta) \, , \\ XY &= 1\ (-1/tan^2\beta) \, . \quad (26) \end{aligned}$$

3 Constraint analysis

Now let us turn our attention to the constraint analysis. Restrictions to the free parameters, namely, the masses of the charged and neutral Higgs bosons and the ratio of the v.e.v. of the two Higgs fields, denoted by $\tan\beta$ in the framework of model I and II, have been predicted in series of works [18]. Recently the constraint which connects masses of the charged Higgs bosons, $m_{H^\pm}$ and $tan\beta$, is obtained by using the QCD corrected values in the LLO approximation and it is shown that the constraint region is sensitive to the renormalization scale, μ [13].

Usually, the stronger restrictions to the new couplings are obtained from the analysis of the $\Delta F = 2$ (here $F = K, B_d, D$) decays, the ρ parameter and the $B \to X_s \gamma$ decay. In [10], all these processes have been analysed and two possible scenarios are obtained depending on the choice whether the constraint from R_b^{exp} is enforced or not. Although the new experimental results are near the SM result, $R_b^{SM} = 0.2156$, the world average value for $R_b (= 0.21656 \pm 0.00074)$ is still almost one standard deviations higher than the SM one. This brings the possibility that an enhancement to the SM value is still necessary to get the correct experimental one. Such an enhancement is reached under the conditions $\xi_{bb}^D >> 1$ and $m_{H^\pm} \sim 400\, GeV$ [10], where ξ_{bb}^D is a model III parameter (see section 2) and v is the vacuum expectation value of the Higgs field responsible for the generation of fermion masses.

First, the constraints for the FC couplings from $\Delta F = 2$ processes for the model III were investigated withouth QCD corrections, under the following assumptions [10]

$$
\begin{aligned}
1.\ \lambda_{ij} &\sim \lambda \,, \\
2.\ \lambda_{uj} &= \lambda_{dj} << 1 \,,\ i,j = 1,2,3 \,,
\end{aligned}
$$

where $u(d)$ is up (down) quark and i, j are the generation numbers.

3. case 2 and further assumption

$$
\lambda_{bb}\ , \lambda_{sb} >> 1 \ and\ \lambda_{tt}\ , \lambda_{ct} << 1 \ . \tag{27}
$$

In the analysis, the ansatz

$$
\xi_{ij}^{UD} = \lambda_{ij} \sqrt{\frac{m_i m_j}{v}} \ , \tag{28}
$$

is used. Note that this ansatz coincides with the one proposed by Cheng and Sher.

Using the constraint coming from R_b^{exp}, the measurement $Br(B \to X_s \gamma)$, $\Delta F = 2$ mixing and the result coming from the analysis of the ρ parameter, the following restrictions are obtained [10]:

$$
\begin{aligned}
150\, GeV \leq m_{H^\pm} \leq 200\, GeV \ , \\
\lambda_{bb} >> 1 \ , \lambda_{tt} << 1 \ , \\
\lambda_{sb} >> 1 \ , \lambda_{ct} << 1 \ .
\end{aligned} \tag{29}
$$

Since the experimental results for R_b^{exp} are still unclear, we disregard the constraint coming from R_b^{exp} and we analyse the restrictions for the couplings $\bar{\xi}^U_{Ntt}$, $\bar{\xi}^D_{Nbb}$ and $\bar{\xi}^U_{Ntc}$ in the NLO aproximation, respecting the constraints due to the $\Delta F = 2$ mixing, the ρ parameter and using the measurement by the CLEO [4] Collaboration:

$$Br(B \to X_s \gamma) \;=\; (3.15 \pm 0.35 \pm 0.32) \cdot 10^{-4} \;.$$

Here, we explain why we use only the CLEO data in our analysis but not ALEPH one ($Br(B \to X_s\gamma) = (3.11 \pm 0.80 \pm 0.72) \cdot 10^{-4}$). The ALEPH data has a larger error compared to CLEO data and it leads to a wide restriction region for $|C_7^{eff}|$, which includes the one coming from the CLEO data. Therefore, the CLEO data allows us to get more stringest constraints on the model parameters.

The idea in this calculation is to take $\bar{\xi}_{Ntc} << \bar{\xi}^U_{Ntt}, \bar{\xi}^D_{Nbb}$ and $\bar{\xi}^D_{Nib} \sim 0\,, \bar{\xi}^D_{Nij} \sim 0$, where the indices i, j denote d and s quarks. This choice permit us to neglect the neutral Higgs contributions because the Yukawa vertices are the combinations of $\bar{\xi}^D_{Nib}$ and $\bar{\xi}^D_{Nij}$.

To reduce the b-quark mass dependence let us consider the ratio

$$\begin{aligned} R &= \frac{Br(B \to X_s\gamma)}{Br(B \to X_c e \bar{\nu}_e)} \\ &= \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha_{em}}{\pi g(z)\kappa(z)} |C_7^{eff}|^2 \,, \end{aligned} \tag{30}$$

where $g(z)$ is the phase space factor in semileptonic b-decay, $\kappa(z)$ is the QCD correction to the semileptonic decay width [19],

$$\begin{aligned} g(z) &= 1 - 8z^2 + 8z^6 - z^8 - 24z^4 ln\, z \,, \\ \kappa(z) &= 1 - \frac{2\alpha_s(m_b)}{3\pi}\{(\pi^2 - \frac{31}{4})(1-z) + \frac{3}{2}\} - (0.25 - 0.18(1 - 4\frac{(1-z^2)^4}{g(z)}) \,, \end{aligned} \tag{31}$$

and $z = m_c/m_b$.

Using the CLEO data and following the same procedure as given in [13], we reach the possible range for $|C_7^{eff}|$ as

$$0.257 \le |C_7^{eff}| \le 0.439 \,. \tag{32}$$

In fig. (1), we plot the parameter $\bar{\xi}^U_{N,tt}$ with respect to $\bar{\xi}^D_{N,bb}$ at $\mu = 4.8\ GeV$ and $m_{H^\pm} = 400\, GeV$. We see, that there are two different restriction regions, where the upper one corresponds to the positive C_7^{eff} value, however the lower one to the negative C_7^{eff} value. Increasing $\bar{\xi}^D_{N,bb}$ causes $|\bar{\xi}^U_{N,tt}|$ to decrease in both regions. With the given value of $\bar{\xi}^D_{N,bb} >> 1$, the condition $|r_{tb}| = |\frac{\bar{\xi}^U_{N,tt}}{\bar{\xi}^D_{N,bb}}| < 1$ is obtained. In the lower region it is possible that the ratio becomes negative, i.e. $r_{tb} = \frac{\bar{\xi}^U_{N,tt}}{\bar{\xi}^D_{N,bb}} < 0$. Further, increasing $m_{H^\pm}$ causes to increase $|r_{tb}|$ and the area of the restriction region.

Fig. (2) is devoted the same dependence as in fig. (1) and shows that the third region, which is almost a straight line, appears. In this region the ratio $r_{tb} >> 1$ and increases with increasing $m_{H^\pm}$ similar to the previous regions.

Finally, we consider $\bar{\xi}^U_{N,tt}$ dependence of $\bar{\xi}^U_{N,tc}$, which is a neutral FC coupling. In fig. (3) we plot the $\bar{\xi}^U_{N,tt}$ dependence of $\bar{\xi}^U_{N,tc}$ for fixed $\bar{\xi}^D_{N,bb} = 60\, m_b$, at $\mu = 4.8\, GeV$, and charged Higgs mass $m_{H^\pm} = 400\, GeV$. Here the selected region for $\bar{\xi}^U_{N,tt}$ is $40 \leq \bar{\xi}^U_{N,tt} \leq 48$. Increasing $\bar{\xi}^U_{N,tt}$ forces the ratio $r_{tc} = \frac{\bar{\xi}^U_{N,tc}}{\bar{\xi}^U_{N,tt}}$ to be negative. It is realized that the ratio $|r_{tc}|$ becomes smaller when $m_{H^\pm}$ is larger.

Still there is a region in which $\bar{\xi}^U_{N,tc}$ is constrained for the possible large value of $\bar{\xi}^U_{N,tt}$, namely $\bar{\xi}^U_{N,tt} = 8.0\, 10^4$ for $m_{H^\pm} = 400\, GeV$:

$$
\begin{aligned}
-0.24 < \bar{\xi}^U_{N,tc} < 0.24 \;, or \\
-3.26 < \bar{\xi}^U_{N,tc} < -3.19 \;,
\end{aligned}
\tag{33}
$$

In conclusion, we find the constraints for the Yukawa couplings $\bar{\xi}^U_{N,tt}$, $\bar{\xi}^D_{N,bb}$ and $\bar{\xi}^U_{N,tc}$ using the CLEO measurement $Br(B \to X_s \gamma)$ and respecting the restrictions due to the $\Delta F = 2$ mixing and the ρ parameter (see [10] for details). The constraints for the other parameters of the model III from the existing experimental results require more detailed new analysis.

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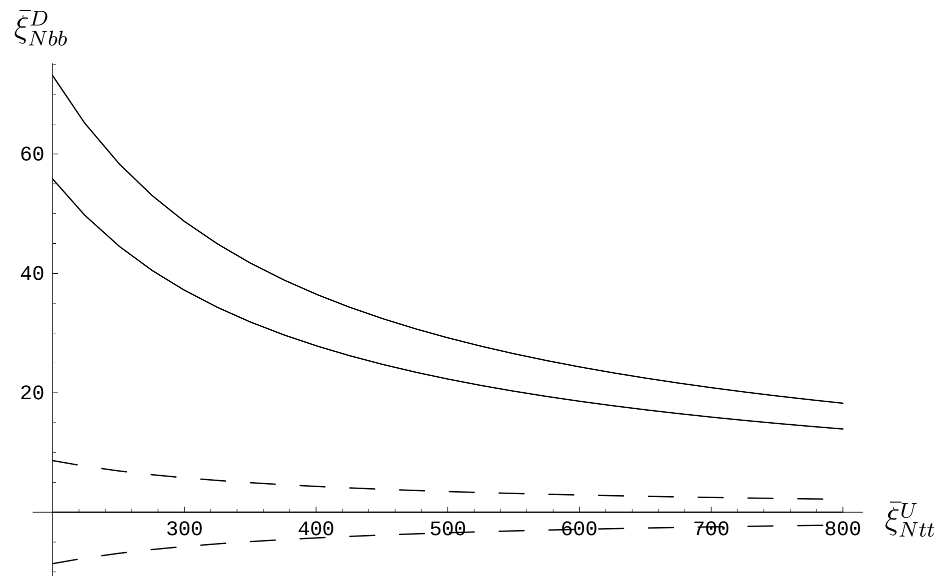

Figure 1: $\bar{\xi}^{U}_{Ntt}$ as a function of $\bar{\xi}^{D}_{Nbb}$ for the fixed value of the charged Higgs boson mass $m_{H^{\pm}} = 400\, GeV$ at $\mu = 4.8\, GeV$. Here the constraint region is lying in between solid (dashed) curves. The solid (dashed) curves are the boundaries of the constraint region corresponding to $C_7^{eff} > 0$ ($C_7^{eff} < 0$)

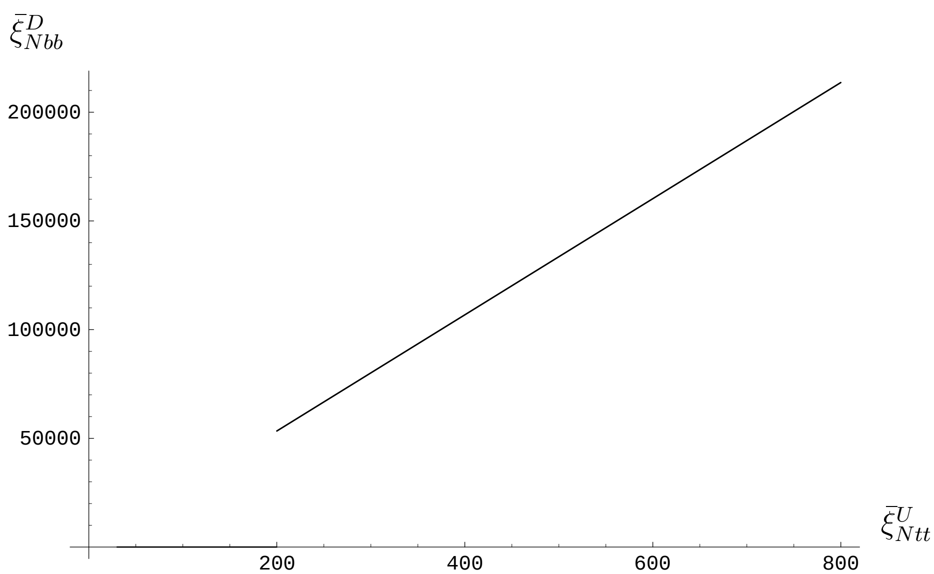

Figure 2: Same as fig 1, but the third possible constraint region.

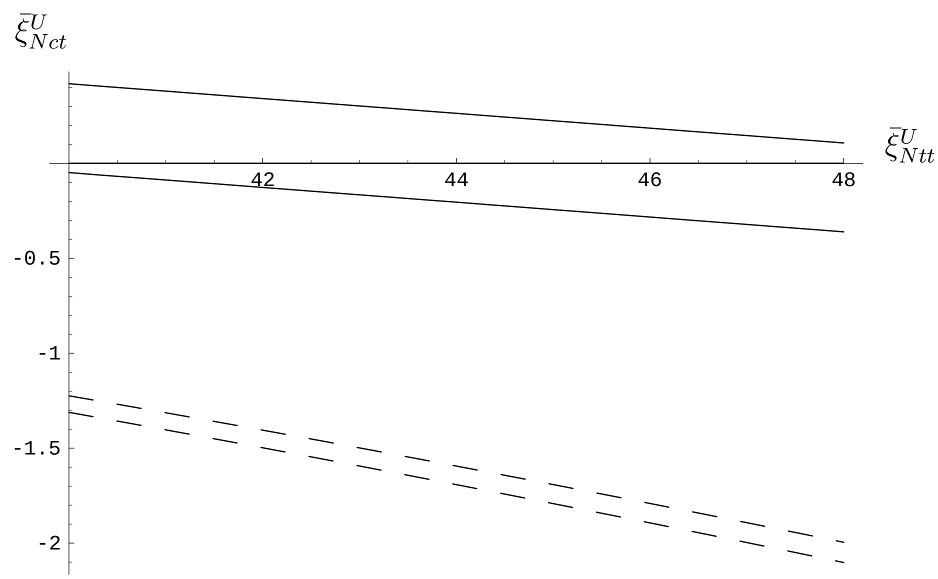

Figure 3: $\bar{\xi}^{U}_{Ntt}$ dependence of $\bar{\xi}^{U}_{Nct}$ for the fixed $\bar{\xi}^{U}_{Nbb} = 60\, m_b$, at $\mu = 4.8\, GeV$ and $m_{H^{\pm}} = 400\, GeV$. Here the constraint region is lying in between solid curves (dashed curves). The solid (dashed) curves are the boundaries of the constraint region corresponding to $C_7^{eff} > 0$ ($C_7^{eff} < 0$)