ESTIMATION OF RELIABILITY INDICES BY MONTE CARLO SIMULATION IN ELECTRIC POWER GENERATION SYSTEMS

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By

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-ABSTRACT-

ESTIMATION OF RELIABILITY INDICES BY MONTE CARLO SIMULATION IN ELECTRIC POWER GENERATION SYSTEMS

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An electric power network is a major example of a system where a rather high level of reliability is expected. Reliability indices have been introduced to facilitate the predictions such as loss of load probability and frequency of loss of load and unserved energy. Objective of this study is to calculate indices which describe the reliability of power generating systems by using Monte Carlo Simulation method .

Simulation methods provide the greatest capability for including operational considerations for such problems in which it is hard to model the system analytically and hence these methods are attractive from time and cost viewpoints. The reliability indices obtained from Monte Carlo Simulation Technique are compared with the ones obtained from analytical solution of a sample generating system containing 32 generators where the generators, functionally dependent on each other, fail with respect to a Multivariate Exponential Distribution structure. The

favorable comparison is observed between the two methods; analytical and simulation methods. Then the simulation model is to be generalized for larger systems and the Fortran coded version is adapted to a Pascal written version for faster convergence.

KEYWORDS: Monte Carlo Simulation, Multivariate Exponential Distribution, Electric Power System Reliability Indices.

-ÖZET-

MONTE CARLO BENZETIM YÖNTEMIYLE ELEKTRIK GÜÇ ÜRETIM SISTEMI GÜVENILIRLIK ENDEKSLERININ TAHMINI

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Elektrik enerji şebekesi, yüksek düzeyde güvenilirlik beklenen sistemlerin başında gelir. Güvenilirlik endeksleri yük kaybı olasılığı ve yük kaybının frekansı, verilemeyen enerji gibi parametrelerin önceden tahmini için önerilmişlerdir. Bu çalışmanın amacı, Monte Carlo benzetim yöntemini kullanarak, enerji üretim sistemlerinin güvenilirliğini tanımlayan endeksleri hesaplamaktır.

Benzetim yöntemleri, işletme gerçeklerini yansıtmak açısından yüksek bir beceriye sahip olup, zaman ve maliyet yönünden de caziptirler. "32 jeneratörlük bir örnek üretim sisteminde" Monte Carlo Benzetim yöntemiyle elde edilen endeksler yine aynı sistemden elde edilen analitik çözümlerle karşılaştırılmıştır. Bu jeneratörler birbirleriyle fonksiyonel olarak bağımlı olup Çok Değişkenli Üstel Dağılım kuralına göre arıza yapmaktadırlar. İki yöntem arasında yapılan karşılaştırma sonuçları olumludur. Daha sonra, aynı yöntem daha geniş sistemler için genelleştirilmiş ve Fortran kodlu anlatım Pascal kodlu anlatıma daha hızlı sonuçlandırma için uyarlanmıştır.
Anahtar Sözcükler: Monte Carlo Benzetimi, Çok Değişkenli Üstel Dağılım, Sistem Güvenilirlik Endeksleri.

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LIST OF TABLES

LIST OF FIGURES

LIST OF NOTATIONS AND ABBREVIATIONS

N : Number of units in electric generating power system

n_i : Number of times component i is observed to fail before the last failure

n_o : Number of times simultaneous failure of at least two component is observed

$n_o^{(c)}$: Number of times component i is observed to fail last, but not simultaneously with any other component

$n_o(i)$: Number of times component i is observed to have failed simultaneously with one or more of the other components

t_{ij} : Failure time of ith component of jth power station; $i=1,...,n$; $j=1,2,...,k$

$t_{(k)j}$: $\max(t_{1j},t_{2j},...,t_{kj})$; $j=1,2,...,n$

u : Uniform random number between 0 and 1

Ω_i : Failure rate of i^{th} unit

Ω_o : Dependence parameter

$\Omega_i^{(T)}$: INT-Estimator of i^{th} unit

$\Omega_o^{(T)}$: INT-Estimator of dependent unit

μ_i : Repair rate of i^{th} unit

$F(x)$: Cumulative distribution function

$\bar{F}(x)$: Survival function

$f(x)$: Density function

$t(x)$: Majorizing function

T_i : Failure times

U_i : First event in $Z_i(t)$

U_0 : First event in $Z_0(t)$

EFLOL : Expected value of Frequency of Loss of Load

ELOL : Expected value of Loss of Load

EUSE : Expected value of Unserved Energy

FLAG1 : Independent case(Ω_0 is small)

FLAG2 : Ω_0 = First order statistic

FLAG3 : Ω_0 = First quartile

FLAG4 : Ω_0 = Average rates of Ω_i ; $i=1,2,...,n$

FLOL : Frequency of Loss of Load

INT-Estimator : Intuitive Estimator

IT : Iteration number

LOL : Loss of Load

LOLP : Loss of Load Probability

MLE : Maximum Likelihood Estimator

MVE : Multivariate Exponential Distribution

MW : Megawatt

NIT : Number of Iteration

TOTCAP : Total Capacity

TTF_i : ith Time to Failure

TTR_i : ith Time to Repair

USE : Unserved Energy

VFLOL : Variance of Frequency of Loss of Load

VLOL : Variance of Loss of Load

VUSE : Variance of Unserved Energy

1. INTRODUCTION

1.1. General Perspective

System reliability problems arise in areas such as communication networks, electrical power systems, transportation systems or manufacturing systems. In recent years, the assessment of the reliability and availability of these complex systems has played an increasingly important role in the analysis, design and operation of these systems [2,5,11-13,16-19,22-25,28-30]. A very important element in the design and operation of a system is the estimation of the impact of the unreliability measure which must be quantitatively defined.

The reliability of an electric supply system has been defined as the probability of providing users with continuous service of satisfactory quality within prescribed tolerances for the time-period envisaged under the conditions encountered. In order for a reliability index to be more representative of real world, the interdependence of the generation and load must be recognized.

The objective of this study is to calculate the indices which describe the reliability of power generating systems by Monte Carlo Simulation Technique and then to compare it with those of the established analytical results which are described in [11,12]. Simulation methods provide the greatest capability for including operating considerations but also require the commitment of excessive time and cost. However, recent advances in simulation technology including fast random number generation techniques, more efficient file managements and advanced computer technology, have made simulation much more attractive from time and cost stand points.

Monte Carlo Technique is applied to the energy model to simulate random occurrences of outage which may result from

i- unit forced outages,

ii- demand through hourly load forecast,

iii- system operational constraints.

The model proposed, simulates the random events that occur and the operational decisions taken. Thus, the generating system is operated and planned through a model in a manner which closely simulates the reality. The actual system events are simulated hour after hour. If a digital computer is used, this simulation is accomplished at relatively high speed. The simulation model is performed for a sample system having 32 generators for study periods of 16 and 28 hours and then, it is to be generalized for larger systems.

1.2. Review of Literature

In the past decade the techniques developed for calculating the various measures of the reliability performance of the generating systems have assumed a deterministic outage data and thus the reliability index calculated is quoted as one number [11,12,13]. The variation of the reliability index around its mean has been investigated by employing various algebraic expansion techniques such as Taylor's series to approximate the expected value and variance of index without any statistical closed form representation. Later on, a statistical closed-form density function for the random variables of interest, Loss of Load (LOL) index in hours and Unserved Energy (USE) index in MW-Hour are developed [16,17]. The density functions are especially useful when the effects of LOL and USE are nonlinear [17].

In this part, some recent studies related to the modelling the system performance in terms of reliability indices will be presented.

In 1959, a mathematical model for use in the simulation of power generation outages was developed by Baldwin et al., [2]. They described outages as random variables and compared the past model with a stochastic model that they proposed by means of simulation. This paper was followed by many more others in which stochastic models for power systems were improved [11-13,16-19,22-25,28-30]. Wang in 1979 estimated the parameters of unit outage data from recorded outage data [18]. In 1981, Şahinoğlu developed statistically asymptotic closed-form density function for the random variable, Loss of Load index by using both classical and Bayesian approach after which Şahinoğlu et al. proposed an algorithm for major generating reliability indices by an analytical approach in 1983 [16,17] . Also, the paper of guidance in this study is presented by Patton et al. [12,13] in which operating considerations in reliability evaluation were presented through analytical and simulation methods respectively. The main analytical contribution in this thesis to the model is the assumption of natural dependency among units. Multivariate Exponential Distribution which is selected as hypothetical distribution for up and down times has been primarily introduced by Marshall et al. [10] in 1967. In 1976, Proschan et al. derived the Maximum Likelihood (MLE) and Intuitive(INT)-estimators of parameter vector Ω to represent the failure and repair rates [15].

1.3. Some Basic Concepts and Definitions

1.3.1.System Reliability Concept

Equipment or system reliability is often expressed as the probability that the device or system will perform its intended function in the required mode for the time period envisaged under the specific conditions encountered [25].

Generating unit capacity reliability has two basic forms; i) Static Reserve and ii) Operating(Spinning) Reserve

Static reserve studies are concerned with determining the installed reserve capacity sufficient to provide for unplanned and planned outages of generating units and is a capacity that must be available to meet the load changes and also capable of satisfying the loss of a certain portion of generating capacity. Operating(Spinning) reserve varies by the lapse of time during system operation according to need.

A forced outage describes the state of a component when it is not available to perform its intended function due to some chance event directly associated with a component, requiring that component will be taken out of service immediately or it may describe an outage caused by improper operation of equipment error. A scheduled outage is an outage that results when a component is deliberately taken out of service at a selected time usually for purposes of construction or repair [19]. A system comprising such individual generating components assigned to generate electric power to meet a given load forecast is the model under study.

The power reliability indices to be calculated are:

a. Loss of Load Probability (LOLP): It is the probability that system load exceeds the available generating capacity under assumption that peak load of each hour lasts all hour; that is, load level changes from hour to hour.

b. Frequency of Loss of Load (FLOL): It yields how often per hour the system is expected to experience a Loss of Load on the average.

c. Unserved Energy (USE): It is the expected magnitude of loss of energy in MW-hr for given period of study.

d. LOLP multiplied by the period of study gives the expected number of hours in which capacity deficiencies exist in a single area network, not interconnected with others. Loss of Load Probability is the major index which is most representative of the system reliability measures.

1.3.2. System Simulation Concept

The model developed here contains an application of Monte Carlo Simulation Techniques for calculating the reliability indices of an electricity power generating system. The simulation approach is consequently simple. In this probabilistic simulation approach, the stochastic elements of the system are represented via probability distributions and random observations are drawn from these distributions. Simulation analysis may be regarded as a natural and logical extension to the analytical and mathematical models [14]. There are many situations which cannot be represented mathematically due to the stochastic nature of the problem under study. For many situations defying a convenient mathematical formulation, Monte Carlo Simulation Method is the only tool that might be used to obtain relevant answers. This process is repeated in such a manner as to mimic the flow of time and hence, statistics on the simulated operation of the system are collected throughout this process for subsequent analyses [13,14].

Though this method may appear crude, it has been shown to be one of the most powerful and effective tools of system analysis. Currently 60% or more of the operations research systems studies in industry employ Monte Carlo Simulations as a method of analysis [8,9]. The primary criticisims of the method are that it is expensive both in modelling and computer costs, and that one must often be skillful in collecting and analyzing the resultant data in order to obtain valuable information [5,8,9,14].

In a digital computer simulation, representation of system is accomplished through the construction of a computer program which describes the system under study to the appropriate computer configuration. This representation will be in the form of à Fortran and Pascal digital programs. Especially in our study, Pascal is suggested for further usage since it has some good facilities such as speed and systematic programming in usage [5].

1.3.3. Analytical System Modelling

Along with the so far obtained historical outage data comprising times to failure and repair of the generating units, a number of unit and system operating considerations have important influence on system reliability. Hence, they must be modelled for accurate reliability index calculation. The most important of these operating considerations are:

1. Spinning Reserve Policy on real-time basis. Explicit modelling of unit start-up failures distinct from running failures.

2. Unit start-up and shutdown in the course of operation to satisfy operating reserve policies and unit commitment priorities and rules.

3. Interdependence of the generating units through what is called "duty cycle" of each generator. Proper treatment of unit duty cycles permits accurate consideration of unit exposure to unit failure due to running failures and starting failures. Relevant information on "duty cycle" is available in Reference [12] on p.2856 to detail.

The system model should include both elements of capacity and elements of load. Furthermore, it must be capable of developing on hourly basis the forecast system loads and it must provide for a hourly disposition of the generating capacity. The performance of a single generating unit may be described in terms of a sequence of periods of time i.e. up periods that alternate with down periods. The system goes from up state to down state with a constant rate Ω, from down to up with rate μ as in Figure.1.

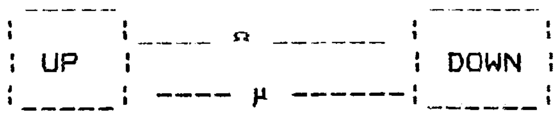


Figure.1. Two-state Model of a Generating Unit

During the up-period, the unit is available to meet the load, while during a down period the unit is to be repaired and hence it is unavailable. Units may be idle for economy reasons, but these economy outages need not be included in the forced outage model. A single unit has a capacity that is entirely available or entirely unavailable. Thus an available capacity history of the unit can be graphically expressed as a function of time "t" that has elapsed as of a certain initial instant as in Figure.2. A single outage history contains recursive occurrences of Time to Failures (TTF) and Time to Repairs (TTR). First Time to Failure (TTF_1) includes the time period from 0 to t_1. The component is in up-state and it works properly with capacity Cap_1 up to the time in which first outage occurs. Similiarly first Time to Repair (TTR_1) includes the time period from t_1 to t_2. In this interval, the component is in down-state and it should be repaired immediately. Therefore random occurrences of such TTF's and TTR's constitute an "outage history". The collection of all possible such histories is called a stochastic (chance or random) process.

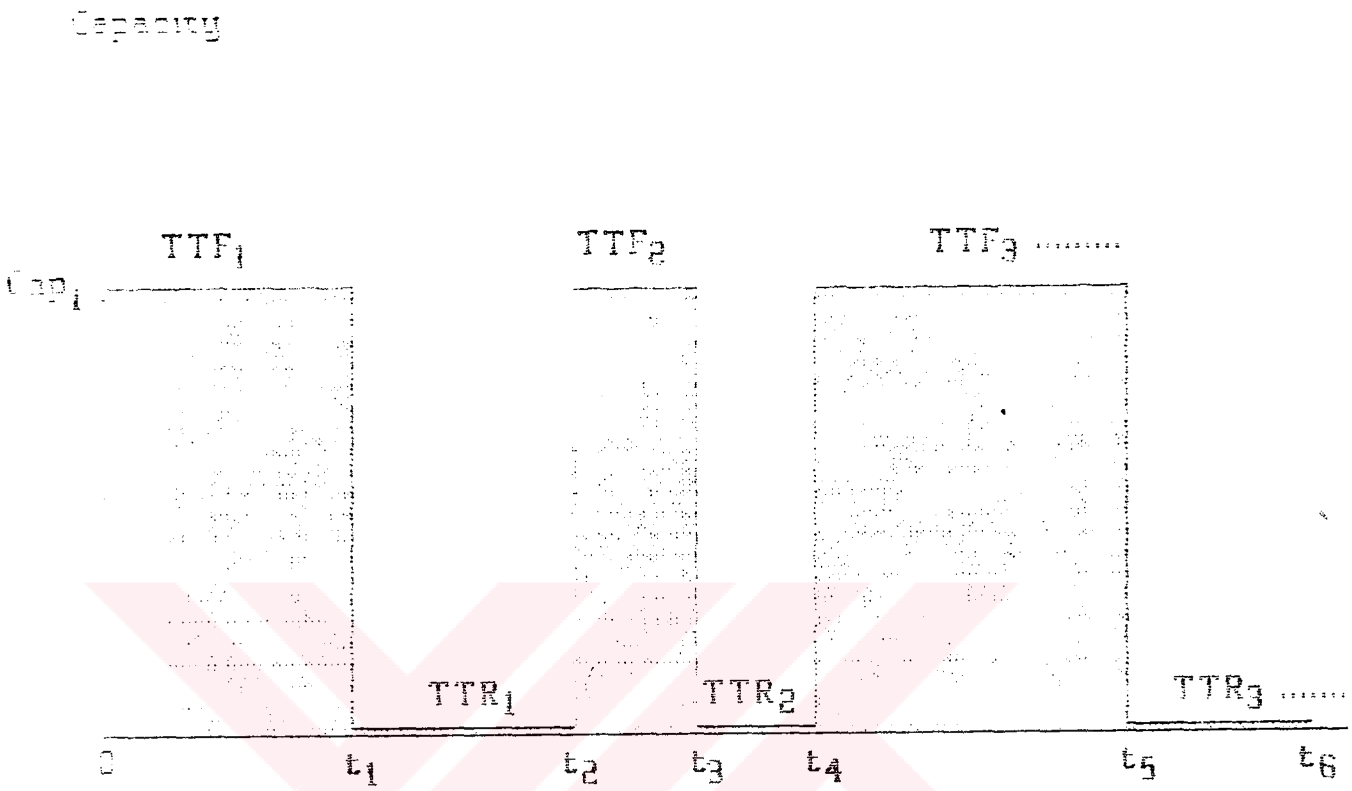


Figure.2. A Single Outage History

1.3.4. Choice of the Exponential Model for Outage Data:

In late fifties and early sixties (1959), C.J.Baldwin et al. for the first time have extensively studied the mathematical model for the failure and repair times in their pioneering paper series " Mathematical Models for Use in the Simulation of Power Generation Outages, I-Fundamental Considerations II-Power System Forced Outage Distributions" [2]. In this leading piece of work, the authors suggested the use of Exponential Distribution model for the up-period and down-period durations in Part I on p.1256 and in Part II on p. 1258, in addition to which they proposed and justified a number of "convenient checks of the suppositon that a set of data comes from an exponential population". A decade later in the late sixties in 1968, A.D. Patton, in his paper "Determination and Analysis of Data for Reliability Studies" [19] studied the "persistent-cause forced outage durations" of the various system components such as generators and transmission lines to suggest the use of Exponential model, in which he stated the upper and lower confidence limits of the Exponentially distributed outage durations, due to Epstein's (1959) paper entitled "Estimation from Life Test Data", Wayne University, Detroit, Michigan. Assumption of exponential model for up and down times was also considered by Gaver and Mazumdar in 1967 [23]. In late seventies and early eighties (1979), Wang conducted Statistical Goodness of Fit Tests in his paper "Estimation of Generation Unit Outage Parameters From Recorded Outage Data" [18] on p.3-5. As a result of his analyses, he concluded that "in-service" (TTF) data behaved unanimously with respect to Exponential Distribution whereas "forced-outage" (TTR) data indicated Exponential and Uniform Distributions as candidate models among which the former was preferred due to its convenience and constant rate. In 1981, Şahinoğlu in his Ph.D. dissertation [16] on p. 12-15, pointed out the Goodness of Fit Tests he carried out for twelve different generating stations provided by Public

Service Electric and Gas Company [21]. As a result, the Exponential density model proved most favorable as a result of employing Statistical Software (GOFT) of Goodness of Fit Tests in which Kolmogorov-Smirnov and Cramer-von-Mises nonparametric tests were employed to judge whether a hypothesized candidate distribution was a good fit or not. It is worth to mention that other candidate distributions were sometimes eligible such as LogNormal or Weibull along with the Exponential. However, due to its mathematical convenience and memoryless property, the Exponential failure model was preferred for the mentioned analytical studies. Moreover, in 1983 and 1984, a series of Goodness of Fit Tests were conducted by Şahinoğlu and Gebizlioğlu on several coal-operated power plants in Turkish Interconnected Power System using the same Software (GOFT) as a result of which most of the power plants displayed Exponential behaviour along with some other candidate models such as LogNormal and Weibull [22].

Hence, by the virtue of literature reviewed above, failure and repair times are reasonably assumed to vary according to a Negative Exponential Distribution with rates Ω and μ respectively. The following notation holds true:

Ω: failure rate

μ: repair rate

m: Mean Time To Failure(MTTF) :$1/\Omega$

r: Mean Time To Repair(MTTR) :$1/\mu$

Then the time dependent solution for up and down times are given [2,25] as in the below equations.

$$P(Up) = \frac{\mu}{\mu+\Omega} + \frac{\Omega}{\mu+\Omega} \exp\{-(\Omega+\mu)t\} \qquad (1)$$

$$P(Down) = \frac{\Omega}{\mu+\Omega} - \frac{\Omega}{\Omega+\mu} \exp\{-(\Omega+\mu)t\} \qquad (2)$$

In the long-run, as time t goes to infinity (∞),

$$P(Up) = \frac{\mu}{\mu+\Omega} = \text{Availability} \qquad (3)$$

$$P(Down) = \frac{\Omega}{\mu+\Omega} = \text{Unavailability} \qquad (4)$$

Each component can have different failure modes. Each failure mode follows a particular failure distribution function. Since, the failure and repair time density functions are Negative Exponential, then Ω and μ are taken constants so that the system can be modelled as Markov Chain with discrete states (up,down) and discrete index space (in discrete hours). Some properties of Exponential Distribution are outlined as follows:

1. Memoryless property of Exponential p.d.f. renders the failure time not to depend upon the length of the unit in service.
2. The average of any exponentially distributed random variable is equal to the reciprocal of rate parameter. Thus the failure rate is inverse of MTTF. The single parameter convenience is a recognized mathematical convenience.

It should be noted that the failure and repair rates of the generating units are the most important input quantities required in power system reliability analysis.

As it can be observed in Figure.3 the system model contains N many generators having corresponding failure and repair rates respectively. The following figure is the configuration of a set of components where each component has an individual outage history. But these outage histories are not independent and occurrences of individual outages will affect the whole system. Therefore vectors of TTF and TTR to represent failure and repair times for N many generating units are drawn from Multivariate Exponential Distribution (MVE) as a consequence of discussions in Section 1.3.4 where the individual units possess Exponential Density for their respective times to failure and repair.

Figure.3. Outage History for N-units

The units whose capacities are prescribed, start independently at time zero. Peak load at each epoch of time (hour) is known. The model reasonably assumes that load changes discretely on the hour and is constant throughout the hour [11,12]. Thus the system load is currently depicted in the model by specifying an hourly load cycle for the specific study period to be simulated. The system does not consider preventive maintenance in this study for simplicity although it can be incorporated in further research. Thus, each component is left in the service until it fails. Component repair is assumed to bring the component into a state as good as new according to "useful life period" characteristics described in the form of a bath-tub curve [25].

1.4. General Purpose of the Study

In this research, the aim is to estimate the reliability indices of an electrical energy producing system by using Monte Carlo Simulation Method. The system contains N generating units with known failure and repair rates and capacity values. Random occurrences for the generating units of consideration are the failure and repair times which are obtained from Multivariate Exponential Distribution (MVE) by means of simulation techniques. The MVE is the general multivariate law to present N many dependent (independence is special case) generating units with the presence of dependence among units. Also, some operational constraints such as spinning reserve level and start-up failure probability are introduced and the system is modelled according to these new constraints.

Main contributions conducted in this study are threefold:

i) to consider the dependence structure between generating units in the form of a Multivariate Exponential Distribution,

ii) to model the system by Monte Carlo Simulation Method as compared to an analytical method [11,12].

iii) to execute the simulation programs in Pascal programming language [5,7] for larger power systems with programming case and speed as compared to Fortran results.

An examplary and illustratory system is chosen to demonstrate the validity of the simulation technique and the existence of interdependence among units.

II. STATISTICAL METHODOLOGY

2.1. Random Number Generation and Monte Carlo Simulation

Simulation is a technique for using computers to imitate the operations of various kinds of real-world facilities or processes. The facility of interest is usually called a system, and in order to study it scientifically, we often have to make a set of assumptions about how it works. These assumptions, which usually take the form of mathematical or logical relationships, constitute a model which is used for trying to gain some understanding of how the corresponding system behaves. In Simulation, we use a digital computer to evaluate a model numerically over a time period of interest, and data are gathered to estimate the desired true characteristics of the model.

In other words, Simulation is a numerical technique for conducting experiments on a digital computer, which involves certain types of mathematical and logical relationships necessary to describe the behaviour and structure of a complex real-world system over the extended periods of time [9]. Simulation has often been described as the process of creating the essence of reality without ever actually attaining the reality itself. Simulation can serve as a "preservice test" to try out new policies and decision rules for operating a system, before running the risk of experimenting on the real system. It can be used to experiment with new situations about which we have little or no information, so as to prepare for what may happen [14].

Monte Carlo Methods comprise that branch of experimental mathematics which is concerned with experiments on random numbers. The simplest Monte Carlo approach is to observe random numbers, chosen in such a way that directly simulate the physical random processes of the original problem and refer to the desired solution from the behaviour of these random numbers.

The term random number or variate is used to mean a real-valued function defined over a sample space associated with the outcome of a conceptual chance experiment. Random numbers are stochastic variables which are uniformly distributed on the interval [0,1] and which show stochastic independence.

There exists many random number generators in the literature. In order to obtain a random sample which is generated without having any repetition or biasedness in the process of generation, it is important to use a reliable Uniform (0,1) random number generator with a very long cycle. The simulation model in this study employs a powerful random number generator called SUNIF [1]. It starts with a specified initial seed. Changing the value of initial seed restarts the generation of random numbers. Subroutine SUNIF is available in the Appendix.3.

Random aspects of simulation results in considering the generation of random variables from a statistical distribution. There exist some techniques for generating random numbers which depend on the distribution from which we want to generate. But for simplicity, the one that is used in this study will be explained [9]:

Inverse Transformation:

This method requires that the random variables are continuous and cumulative distribution function is invertible. This is simply relating the Uniform random variable "u" to the target distribution F(x) under question.

Then the random variate $X^{*}=F^{-1}(u)$ is calculated, where u is the number drawn from Uniform(0,1) distribution by simple inverse transformation.

Since the attention in this thesis is focused on Negative Exponential Distribution, we will concentrate on how to draw Exponential deviates [8]. Negative Exponential Distribution has density function

$$f(x)= \begin{cases} \Omega \exp(-\Omega x) & x\geq 0 \ , \ \Omega>0 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

Hence, the cumulative distribution for Exponential Density is given by,

$$F(x)=1-\exp(-\Omega x) \quad (6)$$

Applying inverse transformation method which means to relate F(x) to the random number u and solve for X^*, i.e.,

$$u = 1-\exp(-\Omega x)$$

$$X^*=-(1/\Omega)\ln(1-u)$$

where u is random variate generated from U(0,1) and X^* is the exponential random variate. Since u is a value between 0 and 1, then (1-u) will also have the same range: between 0 and 1. Therefore (1-u) has also Uniform(0,1) probability distribution and finally the desired target random variate:

$$X^*=-(1/\Omega)\ln(u) \quad (7)$$

The algorithm for Exponential Distribution is straightforward. Firstly, a uniform random number is drawn from Uniform(0,1). Taking the logarithm of this number and dividing by $-\Omega$ gives the Exponential random deviate.

2.2. Multivariate Exponential Distribution

In many reliability situations, it is more realistic to assume some form of positive dependence among components, although most commonly they are accepted to function as independent. This positive dependence among component life lengths arises from common environmental stresses and shocks due to components depending on common sources of power [3,10,15]. For example, let us consider an electrical power generating system where the units start independently.By the lapse of time, their failure times become influenced by each other because of interdependence. Another example is oil wells on a petroleum search area. The wells seem to produce oil without having any connection to the others, but it should be considered that they have dependency through the oil stream underground. Machines in an assembly of a manufacturing plant may well be functioning in an interrelated manner as far as failure times are concerned. Therefore, the system that we consider in this study is assumed to have interdependency among units. And then, failure and repair times are assumed to behave with respect to the Multivariate Exponential Density which contains a random vector of all units with their respective rates and additionally, a dependence parameter Ω_o defined as follows [3,10,15]:

DEFINITION.2.2.1

Let $T=(T_1,T_2,...,T_k)$ be a random vector with nonnegative components and s_k the set of vectors $\{s=(s_1,s_2,..,s_k)\}$ in which each $s_i=1$ or 0 but $s\neq(0,...,0)$. Let $\Omega=(\Omega_o: s\in S_k)$ be a (2^k-1)-dimensional parameter vector such that $0\leq\Omega_s<\infty$ for

$s \in S_k$, but, for each fixed i, i=1,...,k, $\Omega_s > 0$ for at least one s such that $s_i = 1$. Then T has the (2^k-1)-parameter Multivariate Exponential Distribution if the joint survival probability is:

$$P(T_1 > t_1, T_2 > t_2, ..., T_k > t_k) = \bar{F}(\underline{t}) \quad (8)$$

$$\bar{F}(\underline{t}) = \exp[-\sum_i \Omega_i t_i - \sum_{i<j} \Omega_{ij} \max(t_i, t_j) - \sum_{i<j<l} \Omega_{ijl} \max(t_i, t_j, t_l) -$$

$$.... -\Omega_{s} \max(t_1, t_2, .., t_k)] \qquad t_i \geq 0; \quad i=1,....,k$$

where $\underline{t} = (t_1, t_2, ..., t_k)$.

The k-dimensional distribution defined in Equation.8 contains (2^k-1)-parameters. But, (k+1)-parameter version is mainly considered since the estimation techniques can be more clearly presented and understood. Also, results are slightly more complete for the (k+1)-parameter case. Specifically, (k+1)-parameter MVE is defined as follows [15]:

DEFINITION.2.2.2

Let $T = (T_1, T_2, ..., T_k)$ be a random vector with nonnegative components. Then, T is said to have a (k+1)-parameter Multivariate Exponential Distribution if

$$P(T_1 > t_1, T_2 > t_2, ..., T_k > t_k) = \bar{F}(\underline{t}) \quad (9)$$

$$\bar{F}(\underline{t}) = \exp[-\sum_i \Omega_i t_i - \Omega_0 \max(t_1, t_2, \ldots, t_k)] \qquad t_i \geq 0; \quad i=1,\ldots,k$$

$$\Omega \in \Phi \ ; \ \Omega=(\Omega_1,\Omega_2,\ldots,\Omega_k,\Omega_0) \ ; \ \Phi=\{\Omega : 0 \leq \Omega_i \leq \infty, \quad \Omega_0+\Omega_i>0\}.$$

where $\bar{F}(\underline{t})$ is a joint survival probability function.

In more detail, suppose that the components of a k-component system die after receiving a shock which is always fatal. Consider the shock from source "i" as governed by Poisson Process $\{Zi(t), t\geq 0, i=1,2,\ldots,k\}$ with parameter Ωi, where the number of shocks from source "i" is experienced during $[0,t]$. A shock in $Z_i(t)$ which is a mutually independent Poisson process is selectively fatal to component i, while a shock in the $Z_0(t)$ process is simultaneously fatal to all k-components. If $U_0,U_1,\ldots,U_k$ denotes first event times in $Z_0(t),Z_1(t),\ldots,Z_k(t)$ respectively; then $T_i=\min(U_0,U_i)$, where U_0,U_i are independently distributed identically Exponential and T_i denotes the failure times. This property results in the Theorem.2.1.1 [15] to follow which is a representation of U_i's in terms of independent Exponential variables for (k+1)-parameters even though a (2^k-1) version has been derived in [3] but not used here.

THEOREM.2.2.1

T is distributed as MVE(k+1,Ω) iff there exists (k+1) mutually independent Exponential random variables $\{U_i\}_{i=0}^{k}$ with corresponding failure rates Ω_i such that $T_i=\min(U_0,U_i) \quad i=1,2,\ldots,k.$

This theorem reflects a very important property of MVE in the sense that the model is represented in terms of independent Exponential random variables. In particular, one-dimensional marginals are Exponential and two dimensional marginals are Bivariate Exponential where these facts are proved and derived by Barlow and Proschan in [3].

In the case of a series system (which functions only when all items function), system survival probability is greatest in the case of dependence. On the other hand, in the case of a parallel system (which fails only when all items fail), system survival probability is greatest in the case of independence. That is, the correlation coefficient which is an implication of dependency among units is also a function of dependency parameter Ω_0. Correlation coefficient, denoted by δ highly depends on dependency parameter Ω_0. That is,

$$P(U_0 < \min(U_1, U_2, U_3, \ldots, U_k)) = \delta = \frac{\Omega_0}{\Omega} \qquad (10)$$

where $\Omega = \Omega_1 + \Omega_2 + \Omega_3 + \ldots + \Omega_k + \Omega_0$

By the virtue of Equation.10, it can be observed that as "k" number of generating units goes to a large number, the sum Ω also goes large as a result of which the correlation coefficient $\delta > 0$ becomes smaller and smaller eventually converging to a bound; hence so does the relationship between k and δ . A theoretical development beside this heuristic relationship is beyond the scope of this thesis. On the other hand, as the number of the observations "n" collected on each of the k constituent units is increased, i.e. the more observation is sampled, the more information on Ω_0 and on Ω_1 will be acquired yielding more precise information on Ω and δ. Thus, the numerator and denominator terms of the δ will equally be affected in terms of increase or decrease, as a result of which the "n" does not bear any positive or negative responsibility on the magnitude of the correlation coefficient δ, whereas "k" does. In brief, dependence will reduce to an upper bound if the number of components in the system is increased, and so will the correlation coefficient converge to an upper bound. However, correlation coefficient will not be affected by the number of observations.

If the failure times of the system are available, parameters in the vector Ω can be estimated by using derived INT(Intuitive)-estimator formulas [15]. Also maximum likelihood estimators are available. But the solution of MLE's requires some iterative techniques since they cannot be expressed in closed-form. Alternatively, INT-estimators are developed from intuitive considerations for both types of Multivariate Distribution. The following is the formulas for (k+1)-parameter model where INT-estimator of "i" is denoted as $\Omega_1^{(T)}$ and INT-estimator of dependence factor is denoted as $\Omega_0^{(T)}$.

The sample values t_j where $j=1,\ldots,n$, as "n' k-tuples of failure times from n many k-component parallel systems on test are considered. For the statement "Component p fails due to an event in the $Z_i(t)$ process in the underlying fatal shock model", definitions in the estimators are as follow:

n_i= number of times component i observed to fail before the last failure.

n_o= number of times simultaneous failure of at least two components is observed.

$n_o(i)$= number of times component "i" is observed to have failed simultaneously with one or more of the other components.

$n_o^{(c)}$= number of times component "i" is observed to fail last, but not simultaneously with any other component .

Then INT-estimators derived by Proschan and Sullio [15] are

$$\Omega_i^{(T)} = \left[n_i + \frac{n_i}{n_i + n_o(i)} n_i^{(c)} \right] / \Sigma t_{ij}$$

$$= \frac{n_i}{n - n_i^{(c)}} n \; / \; \Sigma \, t_{ij} \qquad i=1,\ldots,k \tag{11}$$

$$\Omega_o^{(T)} = \left[n_o + \sum^{k} \frac{n_o(i)}{n_i + n_o(i)} n_i^{(c)} \right] / \; \Sigma \, t_{(k)j}$$

$$= \left[n - \sum_{i=1}^{k} \frac{n_i}{n_i - n_i^{(c)}} n_i^{(c)} \right] / \sum t_{(k)} \qquad (12)$$

In fact, INT coincides with MLE for the special case $n_1=0$. Regarding asymptotic properties, it can be shown that INT-estimators converge to Ω almost surely in the quadratic mean that is INT-Estimator is strongly and mean square consistent. This estimator has potential use as an alternative means of estimation and it is the first iterate in the iterative procedure. Also the limiting distribution of INT-estimators follow straightforward application of multivariate central limit theorem. Infact, INT(k+1) is not a special case of INT(2^k-1) for k>2. Calculation of the limiting distribution and efficiencies of INT relative to MLE seem impractible for MVE(2^k-1). As it is mentioned before, these estimators require recorded failure and repair times. But, a real-world example taken from IEEE Reliability Test System [6], is considered in this thesis where the inputs of the system are the failure and repair rates and capacity values of the generators. Available information on this data set is given in Appendix.4. Therefore, the estimation of parameters is out of scope of this study and the parameters which are given as failure and repair rates of the generators with an inherent dependency parameter are simply input. Hence, unknown dependency among units will be controlled by assigning a range of values to dependency parameter Ω_0 which is in the range from minimum value to maximum value of rates of the generating units. Otherwise, given certain system data containing "n" failure and repair times for "k" units, the individual rates and dependency parameter can be smoothly calculated by the INT-estimators.

After defining the multivariate distributional law used in the model, we will consider how to generate random sample from a target multivariate distribution.

2.3. Random Variable Generation from M.V.E.

A simulation having any random aspects at all must involve sampling or drawing random variables from one or more distributions. In this study we assume that the distribution has been already specified as Multivariate Exponential including the values of parameters and we address the issue of how we can generate random variables with this specific distribution in order to run our simulation model.

The basic ingredient needed for every method of drawing random variables from any distribution or random process is a source of i.i.d U(0,1) random variables. For this reason, it is very important that a statistically acceptable and reliable U(0,1) random number generator be available. Without an acceptible random number generator, it is impossible to generate random variables correctly from any distribution. In this study, the random numbers are obtained from a generator called SUNIF [1] in which it is required to start with an initial seed. For different seeds, the simulation is repeated to converge to a value and obtain a measure of variance for accuracy check. The choice of the seed value does not depend on any rule. Arbitrarily chosen seed values are considered to indicate the sensitivity of results with respect to different values of seed.

There are many techniques for generating random variables, and the particular algorithm used must, of course, depend on the distribution form which we wish to sample. Inverse transformation which is used most frequently when distribution function has a closed form and is invertible will be dealt with [9].

In this study, an algorithm for generating random deviates from MVE is developed. Since it has a dependence parameter and maximum term in the survival function it would be hard to derive the inverse of the distribution function. Then by Theorem.2.2.1 which chooses the minimum of independent random variables from Exponential Distribution (U_i) and the variable generated from dependence parameter Ω_0 (U_0) is used to have, for example, M variates from N-generating units with respective failure and repair rates. An algorithm for simulating MVE given by Theorem 2.2.1.

1. Set iteration number IT=1
2. Generate random deviate u from U(0,1)
3. Set $U_0=-\ln(u)/\Omega_0$

4. Set $U_i=-\ln(u)/\Omega_i$ for i=1,...,N=Number of units

5. $T_i=\min(U_o,U_i)$ for i=1,...,N=Number of units

6. IT=IT+1

7. Go to step 2 if IT<NIT=Number of iterations

This is a reasonably fast algorithm so long as depending on the value of Ω_o. If Ω_o is chosen as a very small number which implies the independence of generating units. Hence, $U_o=\Omega_o \exp(-\Omega_o x)$ is large and $\min(U_o,U_i)$ will always have the value obtained from U_i. Increase in the value of dependency parameter Ω_o will lead to increase in the dependency since $\min(U_o,U_i)$ will not always yield U_i but may also yield U_o.

2.4. Monte Carlo Simulation of the Electric Power System

The natural dependence of units on each other in meeting the load cycle, influences the units' forced outage rates during the real system operation. Operational constraints such as spinning reserve, unit start-up failure probability and unit interdependence are incorporated to the physical model of electric power generation. Also, Monte Carlo Simulation method offers an alternative means of reliability index calculation which may not require the simplifying rigid assumptions needed in analytical methods such as the exponentiality assumption of TTF and TTR.

The system consists of N different generating units with prescribed failure and repair rates. Demand occurs every hour and the level of demand, given in an historical array , is assumed to persist for the entire hour [11,12]. In the failure case the generator is shut-down because of a forced outage and then repair time is drawn. The residence times for down period may follow any desired p.d.f Gamma, Weibull, Erlang. However, Goodness-of-Fit tests performed to a set of collected data taken from generating systems has indicated that Exponential Distribution is the best choice for both repair and failure times as it is mentioned to detail before [16]. The problem is then one of multivariate situation by generating the sample from MVE when the whole system is considered. Then, one main contribution to the system reliability calculation is the assumption of a multivariate distribution which is not possible to incorporate in analytical formulae [12] as directly as facilitated, for example, by MVE in this study.

The sensitivity of generating system reliability indices to various operating considerations and constraints are investigated. A set of three cases is studied for the sample system [12].

Case1 contains the classical idealizing assumptions and is the basis for comparison of the other cases which systematically replenish other operating considerations. It assumes that all units run continuously except when in a state of total outage. In the simulation process, each one hour is divided into more refined increments. Indices are calculated by scanning the time period with these small increments. For obtaining more accurate results, the system is simulated through many iterations. More precisely, a multivariate exponential vector is generated for attaining the failure times. Similiarly repair time vector is drawn. Up to the time to failure(TTF) the available energy (in Mw) is equated to the capacity of each generator respectively. During the time to repair(TTR), available power is regarded as zero. This process is repeated until the period length is finished. The assigned study period is divided into mutually exclusive time increments. For each "1/200th" incremental hour, it is checked to see if there exists enough capacity available to meet the demand of the system customers, i.e. if "Total Capacity - Capacity on Forced Outage" at that incremental hour is still greater than the load demand. When number of increments in such defined situations are summed, Expected Loss of Load (ELOL) is obtained. Loss of Load Expected divided by the total period gives Loss of Load Probability. If the number of times that LOL occurs is counted, then Loss of Load Frequency is determined. Finally expected unserved or unsupplied energy(USE) in Mw-Hr yielding the magnitude of severity of LOL is calculated by calculating the area of deficiency in Mw x hour.

In Case2, the hourly peak load is increased by

X spinning Megawatts as well as interdependence of generating units is introduced. The spinning reserve is assumed to be distributed over all operating units, but the specific load carried by each individual unit is not defined. The effect of spinning reserve level is for the sample system to examine the relationships between spinning reserve level and long-term average system reliability. The simulation model is performed according to the new level of hourly peak load. Loss of load on the average, frequency of loss of load, and unserved energy on the average are calculated in the same manner as in Case1. Figure.4 is the pictorical representation of spinning reserve level onto the load. The original hourly peak load level is increased by X which represents the value of spinning reserve level MW units at all increments of hour shown as dotted line in Figure.4

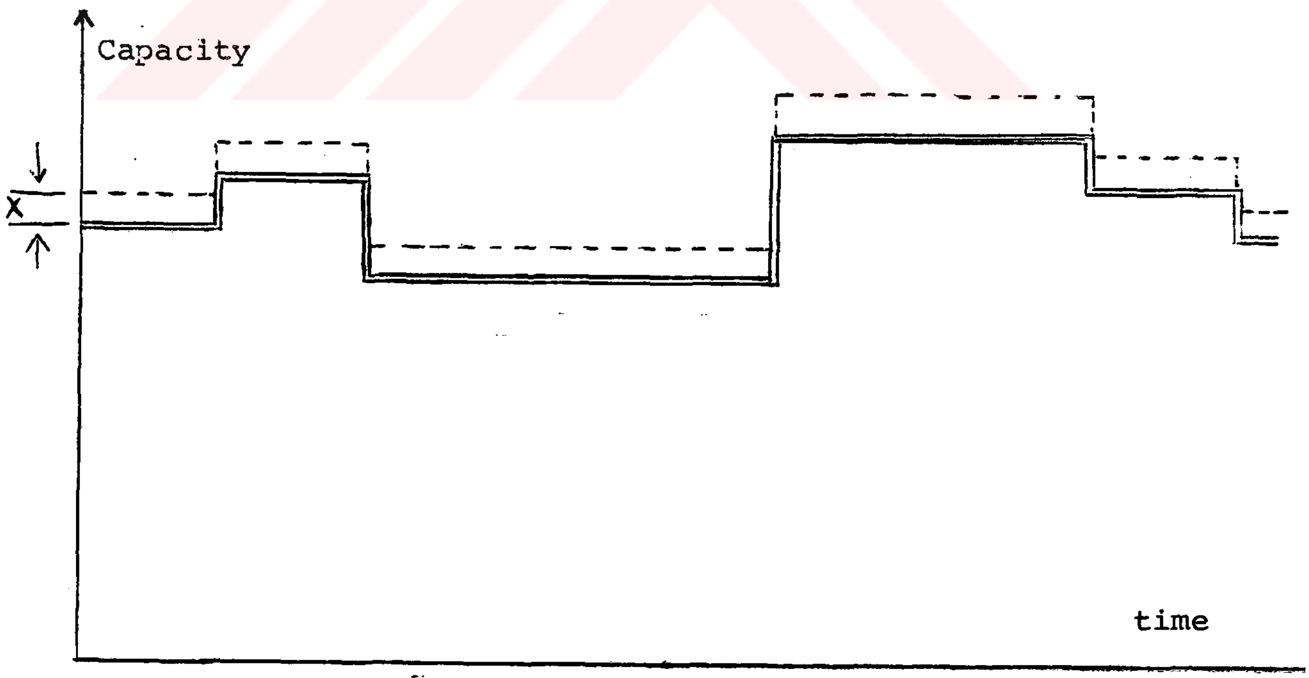


Figure.4 Spinning Reserve Level for Generating System

Case3 contains start-up failure probability and start-up failure delay due to each start-up repair. It is assumed that units may fail on start-up and further failure mechanism prevails for units already in service. Whenever a unit is started to replace a failed unit, a start-up failure delay may be imposed. In the simulation process, given the start-up failure probability, a uniform random number identifies that there exists a start-up failure. If the uniform random number drawn is less than the start-up failure probability, the repair time is prolonged as much as the start-up failure delay time which is an input data.

Another sensitivity study is included simultaneously on the degree of dependence from no-dependence(Ω_0=too small) to more dependence gradually enhanced; $\Omega_0=\Omega_{(1)}$, $\Omega_0=\Omega_{(n/4)}$, $\Omega_0=\Omega_{mean}$ to denote first order statistic or equivalently minimum value of rates, first quartile and the average of rates of Ω_i respectively, corresponding to the constituent generating units in the electric power system.

III. APPLICATION AND DISCUSSION

In this chapter, we present a real system application of the so far mentioned and suggested methodology for power system reliability indices of the operating system composed of 32 units and 28 hours of operating time with corresponding failure rates Ω_i, repair rates μ_i, capacities of each generator, start-up probabilities and start-up delay times of each generator. Spinning reserve load value is 50 Mw and start-up failure occurs with probability 0.05. System peak loads prescribed for 28 hours are considered for the sample system. The data, taken from IEEE Reliability Test System [6], is explained in more detail in Appendix.4.

Demand occurs every hour and the level of demand given in an hourly array is assumed to persist for the entire hour. Whenever a failure which denotes shutdown of the generator by a trouble occurs, repair is necessitated. Simulation model is applied to two examples. Examples depend on the same input data except that one contains the first 16 hourly peak load values of original data listed in Table 3.5 and the second has the total of 28 values. Main reason to present the same problem with different number of hourly peak load values is to show the improvement of simulation results when the period of study is prolonged.

By analytical method, this sample system has been modelled and the reliability indices have been calculated through a computer program which is programmed according to the analytical model presented in [12]. Analytical values written in the first columns of the tables are obtained along with the simulation study from a digital computer program written in Fortran-IV, programmed by Şahinoğlu et al. [11]. In this study, for the same system, favorable index values are obtained by Monte Carlo Simulation Method in which MVE distribution is utilized as a basis of dependency.

Two problems are considered. One contains the first 16 hourly loads of total 28 hours and another includes all 28 hourly peak loads. Convergence in the simulation results is then observed. Main comparison is based on the proximity of simulation results to analytical results for different combinations of seed values and degree of dependency. For Case1, Case2 and Case3 the values of reliability indices are tabulated. Each case is simulated for different values of dependency parameter Ω_o from least dependent to more averagely dependent case where Ω_o-average is taken as the mean of all rates. Tabulated simulation values are plotted and comparison is conducted according to observed trends.

Simulation technique for the sample system is programmed in Pascal language to save execution time and cost. Compared to Fortran-IV in the main frame, Pascal has superiority of speed (even more with Turbo-Pascal) on Fortran-IV and is more systematically programmable [5,7]. Then it will be possible to increase the number of iterations in the simulation to get a satisfactory convergence. Advantages of Pascal in the simulation model will be explained in more detail in the next section. Tables 3.2 to 3.4 and Tables 3.5 to 3.7 contain both simulation and analytical results. Figures 5 to 7 and Tables 8 to 10 are the pictorial representations of Case1,Case2, Case3 for 16 hours and 28 hours respectively. Notations in the tables are as follows:

EFLOL is for expected value of FLOL and SFLOL is for standard deviation of FLOL. Similarly, ELOL denotes expected value of LOL and SLOL is for standard deviation of LOL. EUSE denotes expected value of unserved energy and SUSE denotes the standard deviation of USE. Different levels of dependence are represented by keyword FLAG: Hence, FLAG1 means that there exists independence among units which is supplied by taking the value of dependency rate Ω_0 too small. FLAG2 represents minimum of the rates of generating units. FLAG3 is the first quartile value of these rates and FLAG4 denotes the average value of the same rates. Seed values in the table are related to the i.i.d. uniform U(0,1) random number generator. A subroutine SUNIF generates i.i.d uniform random numbers. SUNIF requires an initial ~~seed to start. It takes a maximum~~ of 2^{26} and minimum of 1 as initial seed value. To check the accuracy of simulation results to the analytical ones, a reasonable range of seed values are employed. The graphs of reliability indices versus seed values indicate the various fluctuations of convergence to analytical results schematically.

Example.3.1

In this example, the number of hourly peak loads is reduced to 16 hours listed in Table.3.1. Problem contains the first 16 hourly peak load values as the test system from an electric power generating system [6]. Inputs are listed in Table 3.1.

For Case1, Table 3.2 contains both analytical and simulation results. In the first case, like a default run, there is only one level of dependency parameter which represents the independent case, i.e. the dependency parameter is chosen a such small value that the random number drawn always will be the one obtained from corresponding rate Ω_1. Different values of seed are used to control the accuracy. All approximate values in Table 3.2 are averaged to the exact results although they tend to decrease when the value of seed is increased. Also, their variances should be considered since there may exist $\pm 3\sigma$ deviations from the analytical solution. Pascal results executed for seed value $2^{11}-1$ and for independent (FLAG1) are very close to results obtained by Fortran-IV.

Figure.5 shows how much fluctuations occur around analytical value for all indices.

All indices underestimate the analytical values, but there do not exist large deviations from analytical ones.

The results of Case2 are shown in Table 3.3 , which consist of both simulation and analytical results with different values of seed and different levels of dependence parameter which ranges from smallest to average value of rates. For different seeds, the simulated values of indices average out to the analytical solutions. That is, for any particular index obtained by simulation technique, if the approximate values are averaged out with respect to seed values, this average value will converge to the desired (analytical) value in a close order of magnitude as shown in the average row of the tables. When the dependency parameter is increased to the value of first quartile, the simulation results get closer to the analytical. This indicates that one can omit a certain degree of dependence among the units. But if the dependency parameter is increased more, the values get larger than the previous results obtained in FLAG1 & FLAG2. Table 3.3 also contains Pascal results obtained for Case2. It can be observed that the reliability indices have close approximations to Fortran-IV simulation results. Figure.6 contains the plotted values of simulation values for all indices and all flags. For EFLOL, FLAG2 and FLAG3 converge to analytical constant-line. The other flags have more fluctuations. For ELOL, FLAG2 and FLAG4 converge up to the seed value 121, but they deviate between 121 and $2^{11}-1$. Another seed value between these two can give a better fit. Same conclusion for the graph of EUSE is reached.

Similiar scheme is set up for Case3 in Table 3.4. which has the same argument as in Table 3.3. Again, this table consists of simulation results obtained for different combinations of seeds and flags. Although all combinations produce favorable results, dependency parameter having value around first quartile value will give relatively more accurate results indicating a subtle existence of dependence structure. For different values of the seed, Pascal version of Case3 gives the similar results with respect to the Fortran-IV version. Figure.7 is plotted for this case. In the figure, EFLOL has relevant fit to analytical result when dependence value is equal to mean value of rates i.e., FLAG4. Also FLAG3 has convergence to the desired line. For ELOL, FLAG1 and FLAG2 are close to analytical line. Also, FLAG3 gets closer to the said line.

For all cases the study period is divided into 200 incremental hours and the simulation is repeated 100 times for obtaining convergence to analytical. This can be enhanced for more accuracy if more computing resources such as time and storage space are available.

Example.3.2.

Now the problem contains the total of 28 hours instead of the first 16 hours as in Table 3.1. Different values of seed and different values of dependency parameter for Case1, Case2 and Case3 are simulated. Closer approximations than Example.3.1. are obtained for each case. Simulation is repeated 100 times and study period is scanned at "1/200 th" incremental hours.

Results for Case1 are listed in Table 3.5. This table contains both analytical and simulation results of the digital programs in Fortran-IV and Pascal like in the previous example and different values of seed are used in the simulation. Figure.8 shows the plotted values for ELOL,EFLOL,EUSE. For EFLOL and ELOL simulation line is very close to the analytical contant-line. EUSE has also a strong convergence to the analytical result compared with the previous example.

Similiarly, for Case2, Table 3.6 and for Case3, Table 3.7 are constructed. These tables contain the simulation results for different combinations of seed values and dependency parameters, such as the pair (Seed=15, FLAG2) etc., obtained in both, Fortran-IV and Pascal languages. Same conclusions as in Example 3.1 are valid for Case2 and Case3. That is, dependency should be taken into account and results of simulation for different seed values average out to yield satisfactorily close to exact results. For Case2, Figure.9 is plotted. FLAG2 and FLAG3 give better convergence to analytical constant-line for EFLOL. For ELOL, FLAG1 and FLAG3 are not good fits, but FLAG2 and FLAG4 have very close convergence to analytical result shown as constant line. Especially average value of FLAG4 is almost equal to analytical result. For EUSE, FLAG3, FLAG4 have good convergence. Also, average value of FLAG3 is very close to the analytical result.

For Case3, graphical representation is given in Figure.10. For EFLOL, FLAG2 and FLAG3 give good convergence. The other flags fluctuate much from the analytical line, this is because neither too much nor too little dependence is appropriate. Also, average value of FLAG2 is close to analytical result indicating subtle content of dependence. An interesting situation happens for ELOL. In the figure, as it is observed, all flags have scatterred around the analytical line. But, FLAG2, FLAG4 seem closest to the analytical line. Meanwhile, FLAG3 has average value being equal to the analytical result.

This two-step approach for the sample electric power generating system is performed to indicate how satisfactory the convergence to the exact values are realized when the period of study is increased. Comparison is exhibited by the way of tabulation and figuration. Tables and figures in the second example have shown that a better convergence will be achieved if any increase in the period of study is considered like that of one year (8760 hours), usually designated as the period of study in annual reliability studies.

Case1 differs from Case2 in the sense of reduced generator running time since in Case2 units are cycled on and off to maintain the spinning reserve rather than running continuously and being exposed to more failures. Also, start-up failures may have an adverse effect on reliability indices in Case3 different than Case1 and Case2. But the aim in this study is different, it is not one of comparing the effects of constraints on system performance but that of comparing analytical(theoretical) and simulation results. It is proposed to prove the reasonable accuracy of simulation with respect to analytical results for some beneficial reasons such as, simplicity in modelling when distribution is different from Exponential. So, in general, simulation results in three cases are relatively close to analytical results in "the order of magnitude" sense. Sensitivity of system performance is controlled by simulating system for various seed values and various values of dependence parameter. When the performance of the simulation results after implementing some sensitivity studies are compared with independent case, dependency phenomenon among the units seems to have gained more significance in the model. Especially Case2 and Case3 will definitely require to have a dependency structure among the units since it is studied in the analytical model [11,12].

Pascal is used as programming language for the simulation model as stated before. Several simulation languages such as GPSS, SIMSCRIPT,GASP IV, SIMAN etc. for modelling the reliability and availability of large scale systems are available. But they present disadvantages as the development of modelling is too time demanding and input - output formats are not suitable for reliability problems. Finally, the user must write some additional subroutines in another computer language to take into account some events that cannot be modelled with the simulator. The comparison of Fortran-IV and Pascal will not be in the sense of comparing them in a general case. Consideration will be focused on programming this specific simulation model presented in this study for the particular given data.

Although Fortran-IV is the oldest one, Pascal is currently developed and most widely used language for teaching programming. Its popularity is due to [5,7,8,9]:

i) The syntax of Pascal is relatively easy to learn

ii) Pascal facilitates writing structured programs that are relatively easy to read, understand and maintain and hence reduce the required programming time.

However, Fortran-IV version of the model is written and run to indicate the validation of Pascal in terms of speed and saving cost. It has been indicated that Pascal which is easy to be programmed and which supports structured programming techniques are faster than Fortran-IV even if main-frame is used. Pascal programming of the simulation model supports the results obtained from Fortran-IV.

TABLE3.1. Values Inputted into the Original Model [11]

Generator Number	Failure Rate (per hr)	Repair Rate (per hr)	Capacity Value (Mw)	Hour	Load Demand for 28 hours
1	0.02	0.98	12	1	2650.50
2	0.02	0.98	12	2	2850.00
3	0.02	0.98	12	3	2793.00
4	0.02	0.98	12	4	2736.00
5	0.02	0.98	12	5	2679.00
6	0.10	0.90	20	6	2194.00
7	0.10	0.90	20	7	2137.00
8	0.10	0.90	20	8	2523.28
9	0.10	0.90	20	9	2713.20
10	0.010	0.99	50	10	2658.94
11	0.010	0.99	50	11	2604.67
12	0.010	0.99	50	12	2550.41
13	0.010	0.99	50	13	2089.16
14	0.010	0.99	50	14	2034.90
15	0.010	0.99	50	15	2284.73
16	0.020	0.98	76	16	2456.70
17	0.020	0.98	76	17	2407.56
18	0.020	0.98	76	18	2358.43
19	0.020	0.98	76	19	2309.30
20	0.040	0.96	100	20	1891.66
21	0.040	0.96	100	21	1842.52
22	0.040	0.96	100	22	2385.45
23	0.040	0.96	155	23	2565.00
24	0.040	0.96	155	24	2513.70
25	0.040	0.96	155	25	2642.40
26	0.040	0.96	155	26	2411.16
27	0.050	0.95	197	27	1975.05
28	0.050	0.95	197	28	1923.75
29	0.050	0.95	197		
30	0.080	0.92	+357		
31	0.120	0.88	400		
32	0.120	0.88	400		

TABLE.3.2 Results for CASE1 with 16 hourly peak load values

THE RESULTS OF ANALYTICAL FORMULAE REPORTED IN [11]:

EFLOL (per hour)	ELOL(per hour)	EUSE(MW)
0.9102	0.4116	59.3169

SIMULATION RESULTS:

SEED VALUE	INDICES	PASCAL	FORTRAN-IV			
		FLAG1	FLAG1	FLAG2	FLAG3	FLAG4
15	EFLOL	0.72	0.87	NONE	NONE	NONE
	SFLOL	0.928	1.026			
	ELOL	0.279	0.390	NONE	NONE	NONE
	SLOL	0.496	0.52			
	EUSE	39.409	48.695	NONE	NONE	NONE
	SUSE	82.1	71.28			
25	EFLOL	0.50	0.76	NONE	NONE	NONE
	SFLOL	0.795	0.954			
	ELOL	0.392	0.3394	NONE	NONE	NONE
	SLOL	0.586	0.37			
	EUSE	50.265	49.409	NONE	NONE	NONE
	SUSE	50.63	80.31			
121	EFLOL	0.71	0.62	NONE	NONE	NONE
	SFLOL	0.862	0.87			
	ELOL	0.369	0.309	NONE	NONE	NONE
	SLOL	0.712	0.395			
	EUSE	57.69	45.507	NONE	NONE	NONE
	SUSE	53.51	57.89			
$2^{11}-1$	EFLOL	0.705	0.67	NONE	NONE	NONE
	SFLOL	1.215	0.813			
	ELOL	0.397	0.348	NONE	NONE	NONE
	SLOL	0.752	0.417			
	EUSE	50.167	48.8570	NONE	NONE	NONE
	SUSE	83.14	89.42			
AVERAGED VALUES	EFLOL	0.6587	0.73			
	ELOL	0.3592	0.3466			
	EUSE	49.3827	48.1169			

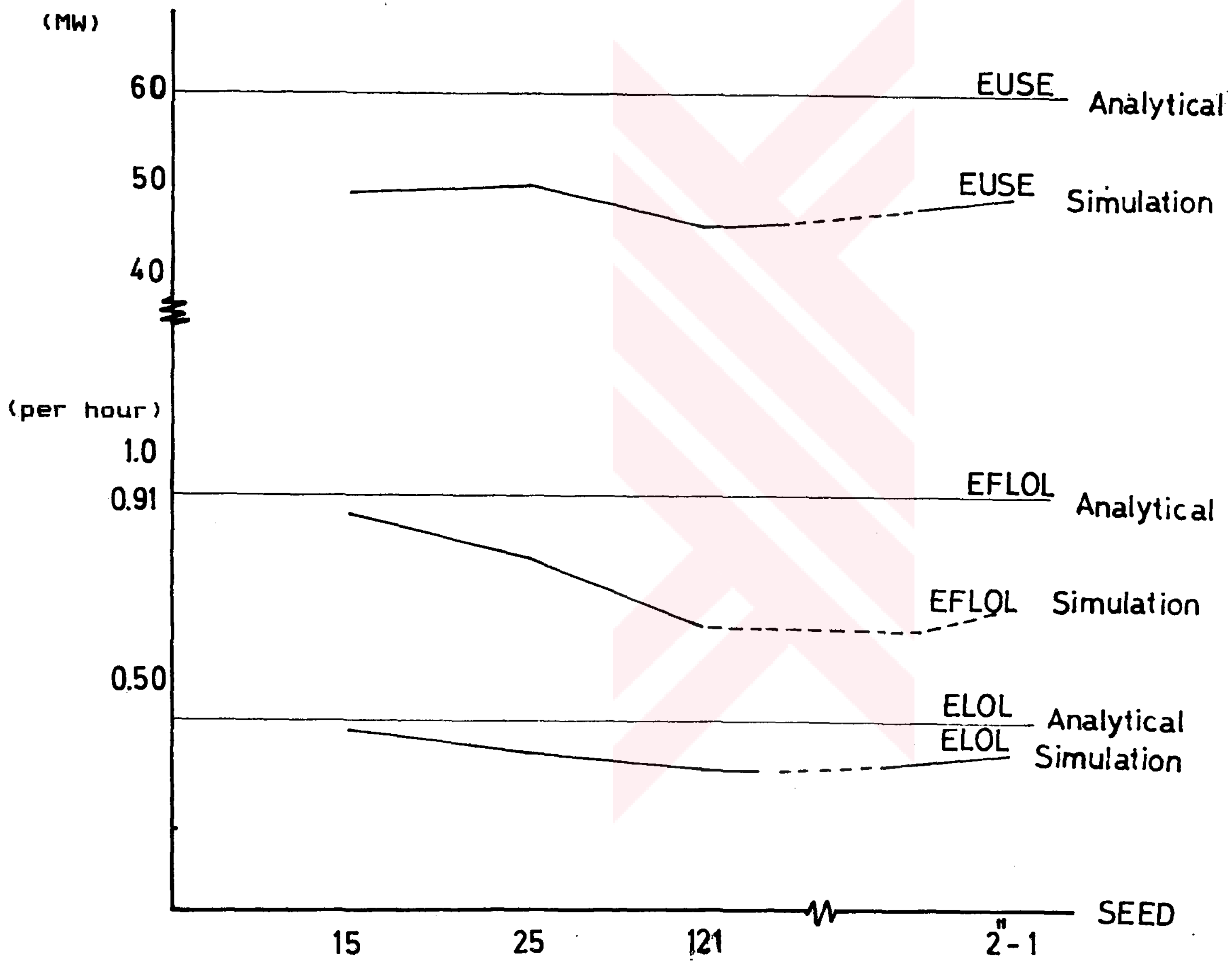


Figure.5. Graph of Case1 with 16 hours

TABLE.3.3 Results for CASE2 with 16 hourly peak load

THE RESULTS OF ANALYTICAL FORMULAE REPORTED IN [11]:

EFLOL	ELOL	USE
0.7609	0.3297	47.2894

SIMULATION RESULTS:

SEED VALUE	INDICES	PASCAL	FORTRAN-IV			
		FLAG1	FLAG1	FLAG2	FLAG3	FLAG4
15	EFLOL	0.865	0.89	0.71	0.98	1.08
	SFLOL	1.196	1.018	1.0	1.272	1.262
	ELOL	0.349	0.377	0.277	0.329	0.196
	SLOL	0.652	0.607	0.387	0.383	0.292
	EUSE	57.27	55.556	31.2824	33.5640	37.508
	SUSE	112.47	95.15	50.31	55.33	68.11
25	EFLOL	0.671	0.59	0.85	0.72	1.48
	SFLOL	1.117	0.873	1.130	0.895	1.411
	ELOL	0.361	0.290	0.280	0.142	0.29
	SLOL	0.764	0.4616	0.267	0.239	0.374
	EUSE	45.443	39.0128	32.7236	42.4177	39.516
	SUSE	36.2	94.66	50.35	51.66	69.17
121	EFLOL	0.886	0.85	1.23	1.103	1.48
	SFLOL	0.184	1.003	1.377	1.329	1.406
	ELOL	0.296	0.308	0.259	0.451	0.32
	SLOL	0.734	0.678	0.610	0.758	0.637
	EUSE	42.145	40.109	37.912	35.956	43.211
	SUSE	100.4	77.00	61.89	63.65	66.13
$2^{11}-1$	EFLOL	0.881	0.99	0.69	0.87	1.51
	SFLOL	0.926	1.18	0.8349	1.1658	0.3523
	ELOL	0.453	0.383	0.229	0.398	0.348
	SLOL	1.27	0.3201	0.0441	0.2587	0.1882
	EUSE	52.099	50.1639	35.77615	43.576	47.336
	SUSE	74.01	35.21	34.08	90.78	79.49
AVERAGED VALUES	EFLOL	0.8257	0.83	0.87	0.918	1.387
	ELOL	0.364	0.3399	0.2612	0.3995	0.2885
	EUSE	49.23	46.21	34.42	38.877	41.892

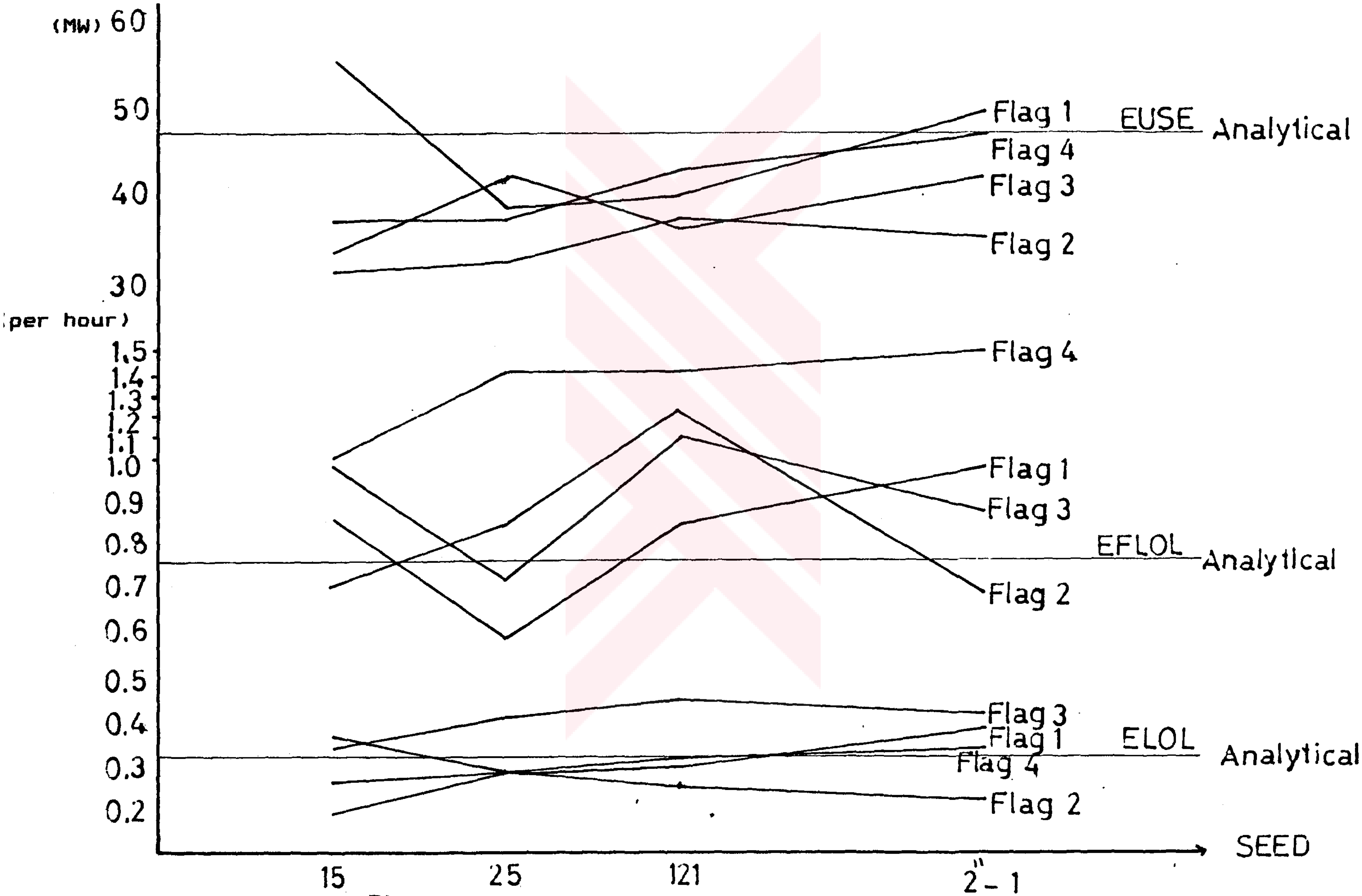


Figure.6. Graph of Case2 with 16 hours

TABLE.3.4 Results for CASE3 with 16 hourly peak load

THE RESULTS OF ANALYTICAL FORMULAE REPORTED IN [11]:

EFLOL	ELOL	EUSE
0.9878	0.44018	67.304

SIMULATION RESULTS:

SEED VALUE	INDICES	PASCAL	FORTRAN-IV			
		FLAG1	FLAG1	FLAG2	FLAG3	FLAG4
15	EFLOL	0.961	0.92	0.9	0.957	1.09
	SFLOL	0.353	1.0806	0.966	0.870	1.209
	ELOL	0.394	0.367	0.392	0.493	0.596
	SLOL	0.791	0.52	0.576	0.260	0.288
	EUSE	59.48	54.1857	48.301	37.14	56.537
	SUSE	164.8	106.16	58.23	52.74	32.27
25	EFLOL	0.841	0.83	0.773	0.876	1.07
	SFLOL	0.824	1.020	0.9687	1.00	0.908
	ELOL	0.3001	0.328	0.419	0.503	0.533
	SLOL	0.587	0.546	0.288	0.652	0.912
	EUSE	51.04	56.6243	53.990	35.91	47.539
	SUSE	43.86	38.01	36.91	46.38	65.20
121	EFLOL	0.835	0.85	0.94	0.899	1.09
	SFLOL	0.871	1.0621	0.831	0.181	0.928
	ELOL	0.394	0.360	0.421	0.3189	0.533
	SLOL	0.609	0.452	0.591	0.529	0.912
	EUSE	41.26	33.6144	42.372	50.395	49.4987
	SUSE	38.42	86.16	29.47	33.81	46.34
$2^{11}-1$	EFLOL	0.869	0.87	0.971	1.45	1.23
	SFLOL	0.344	0.813	0.858	0.104	1.377
	ELOL	0.4561	0.448	0.526	0.414	0.259
	SLOL	0.868	0.864	0.607	1.033	0.373
	EUSE	59.264	58.857	55.655	47.774	61.97
	SUSE	65.38	89.43	73.14	61.89	50.37
AVERAGED VALUES	EFLOL	0.876	0.867	0.896	1.045	1.12
	ELOL	0.3860	0.375	0.4396	0.4322	0.408
	EUSE	52.76	50.82	50.07	42.80	53.8861

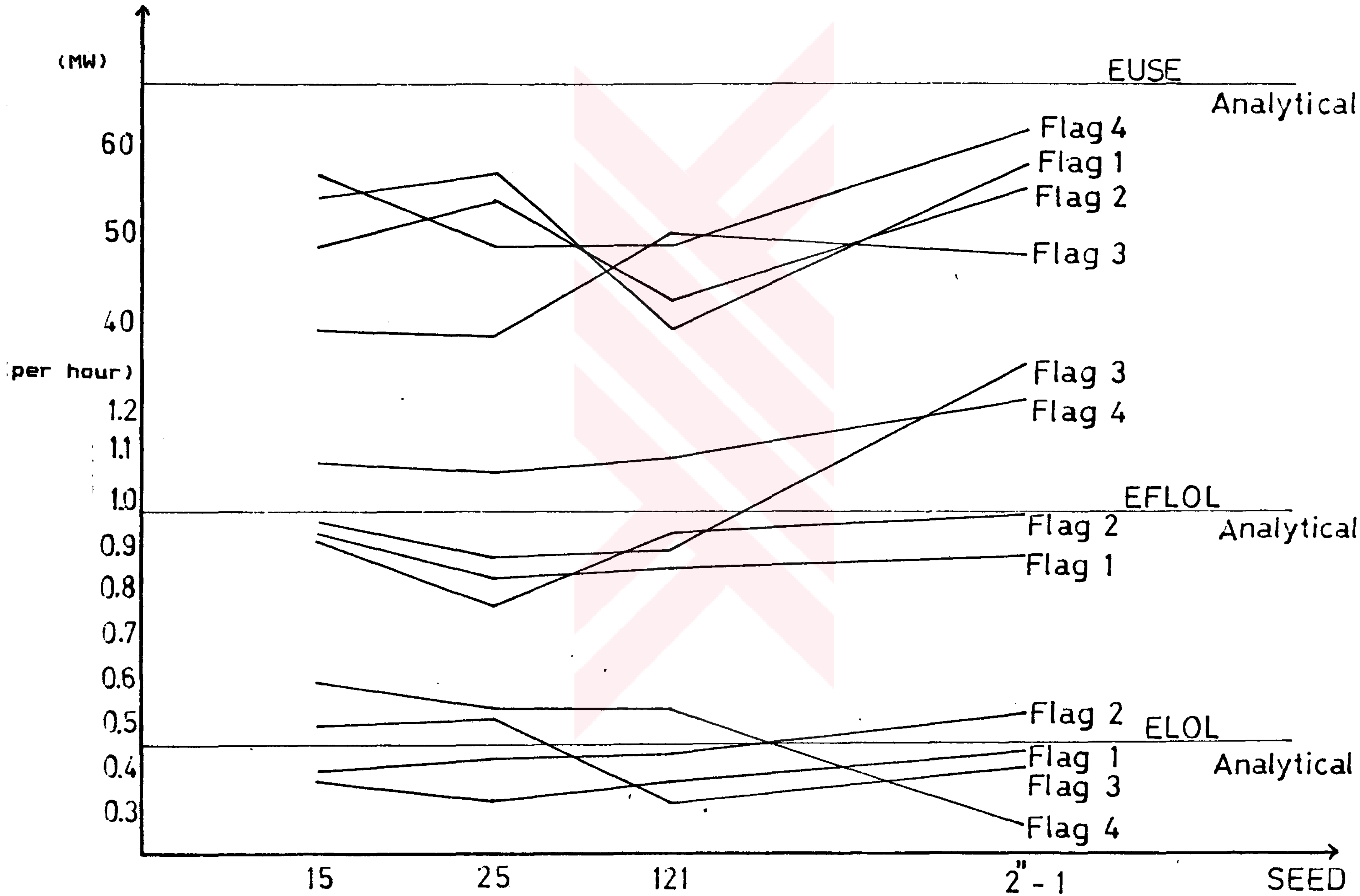


Figure.7. Graph of Case3 with 16 hours

TABLE.3.5 Results for CASE1 with 28 hourly peak load

THE RESULTS OF ANALYTICAL FORMULAE REPORTED IN [11]:

EFLOL	ELOL	EUSE
1.0852	0.454798	66.4498

SIMULATION RESULTS:

SEED VALUE	INDICES	PASCAL	FORTRAN-IV			
		FLAG1	FLAG1	FLAG2	FLAG3	FLAG4
15	EFLOL	1.07	1.15	NONE	NONE	NONE
	SFLOL	1.121	1.194			
	ELOL	0.477	0.413	NONE	NONE	NONE
	SLOL	0.619	0.582			
	EUSE	68.7954	57.6843	NONE	NONE	NONE
	SUSE	126.17	38.225			
25	EFLOL	0.85	1.27	NONE	NONE	NONE
	SFLOL	1.066	1.3408			
	ELOL	0.3614	0.527	NONE	NONE	NONE
	SLOL	0.601	0.719			
	EUSE	60.5587	60.6776	NONE	NONE	NONE
	SUSE	169.06	151.0			
121	EFLOL	0.97	1.14	NONE	NONE	NONE
	SFLOL	1.051	1.157			
	ELOL	0.3553	0.399	NONE	NONE	NONE
	SLOL	0.499	0.538			
	EUSE	49.4559	51.1951	NONE	NONE	NONE
	SUSE	100.7	91.09			
$2^{11}-1$	EFLOL	1.23	1.03	NONE	NONE	NONE
	SFLOL	1.369	1.409			
	ELOL	0.436	0.497	NONE	NONE	NONE
	SLOL	0.886	0.397			
	EUSE	58.6911	59.9882	NONE	NONE	NONE
	SUSE	126.13	80.89			
AVERAGED VALUES	EFLOL	1.03	1.1475	NONE	NONE	NONE
	ELOL	0.407	0.459			
	EUSE	59.375	57.3863			

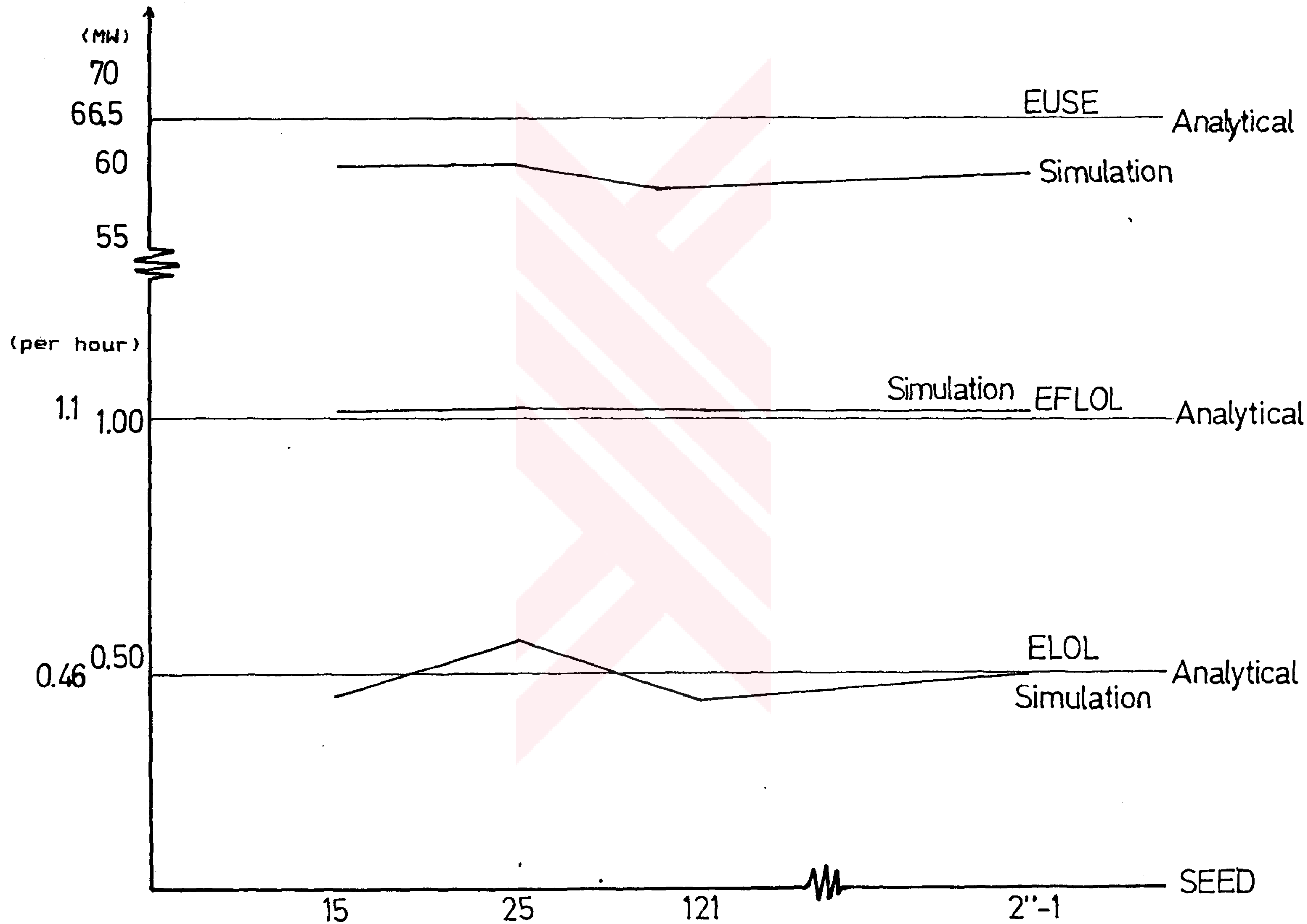


Figure.8. Graph of Case1 with 28 hours

TABLE.3.6 Results for CASE2 with 28 hourly peak load

THE RESULTS OF ANALYTICAL FORMULAE REPORTED IN [11]:

EFLOL	ELOL	EUSE
0.7929	0.330239	46.4588

SIMULATION RESULTS:

SEED VALUE	INDICES	PASCAL	FORTRAN-IV			
		FLAG1	FLAG1	FLAG2	FLAG3	FLAG4
15	EFLOL	1.33	1.5	0.94	0.73	0.91
	SFLOL	1.295	1.390	1.276	1.138	1.584
	ELOL	0.6362	0.588	0.487	0.304	0.361
	SLOL	0.7276	0.7067	0.297	0.298	0.5014
	EUSE	57.4815	52.666	42.051	39.025	54.069
	SUSE	156.85	147.14	42.56	51.24	90.56
25	EFLOL	1.12	1.01	0.976	0.865	1.16
	SFLOL	1.27	1.568	1.273	1.338	1.344
	ELOL	0.4438	0.580	0.350	0.320	0.312
	SLOL	0.639	0.852	0.387	0.482	0.331
	EUSE	50.9748	61.2495	38.1421	42.6743	41.0217
	SUSE	196.86	186.55	90.60	68.14	58.39
121	EFLOL	1.31	1.23	0.87	0.714	1.3024
	SFLOL	1.186	1.359	0.912	1.303	0.858
	ELOL	0.5033	0.552	0.284	0.4761	0.353
	SLOL	0.6422	0.713	0.268	0.5302	0.3745
	EUSE	67.9884	55.0535	41.1388	52.517	50.0391
	SUSE	123.83	118.76	38.62	42.47	34.55
$2^{11}-1$	EFLOL	0.8218	1.212	0.68	0.6631	1.1214
	SFLOL	0.372	1.344	0.747	0.352	0.9587
	ELOL	0.3915	0.4765	0.354	0.5276	0.2965
	SLOL	0.717	0.564	0.234	0.406	0.379
	EUSE	49.68	57.91	39.80	48.182	52.556
	SUSE	48.66	120.67	44.15	40.49	79.49
AVERAGED VALUES	EFLOL	1.145	1.238	0.866	0.743	1.12
	ELOL	0.4937	0.549	0.3687	0.4069	0.3306
	EUSE	56.53	56.71	40.289	45.599	49.421

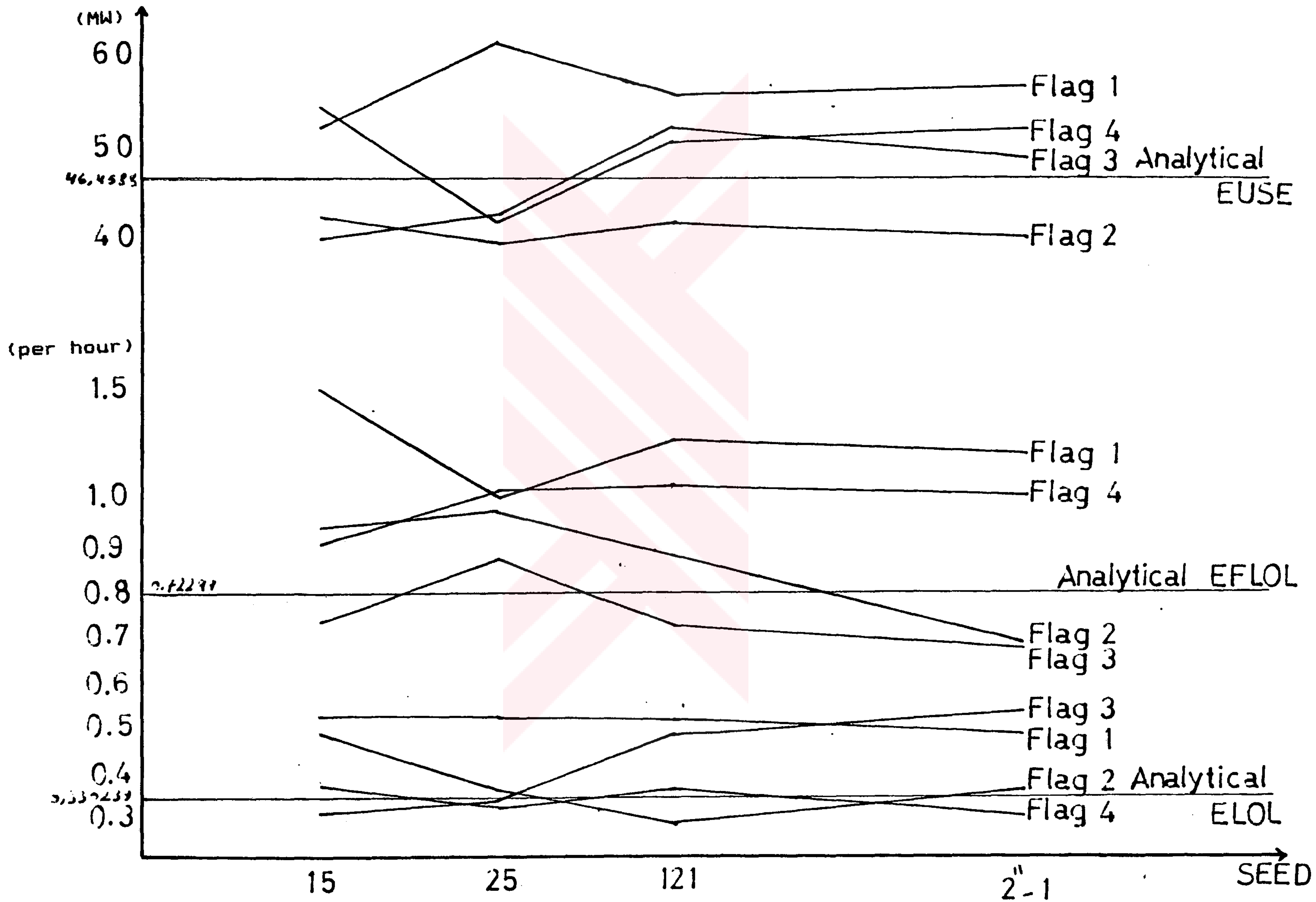


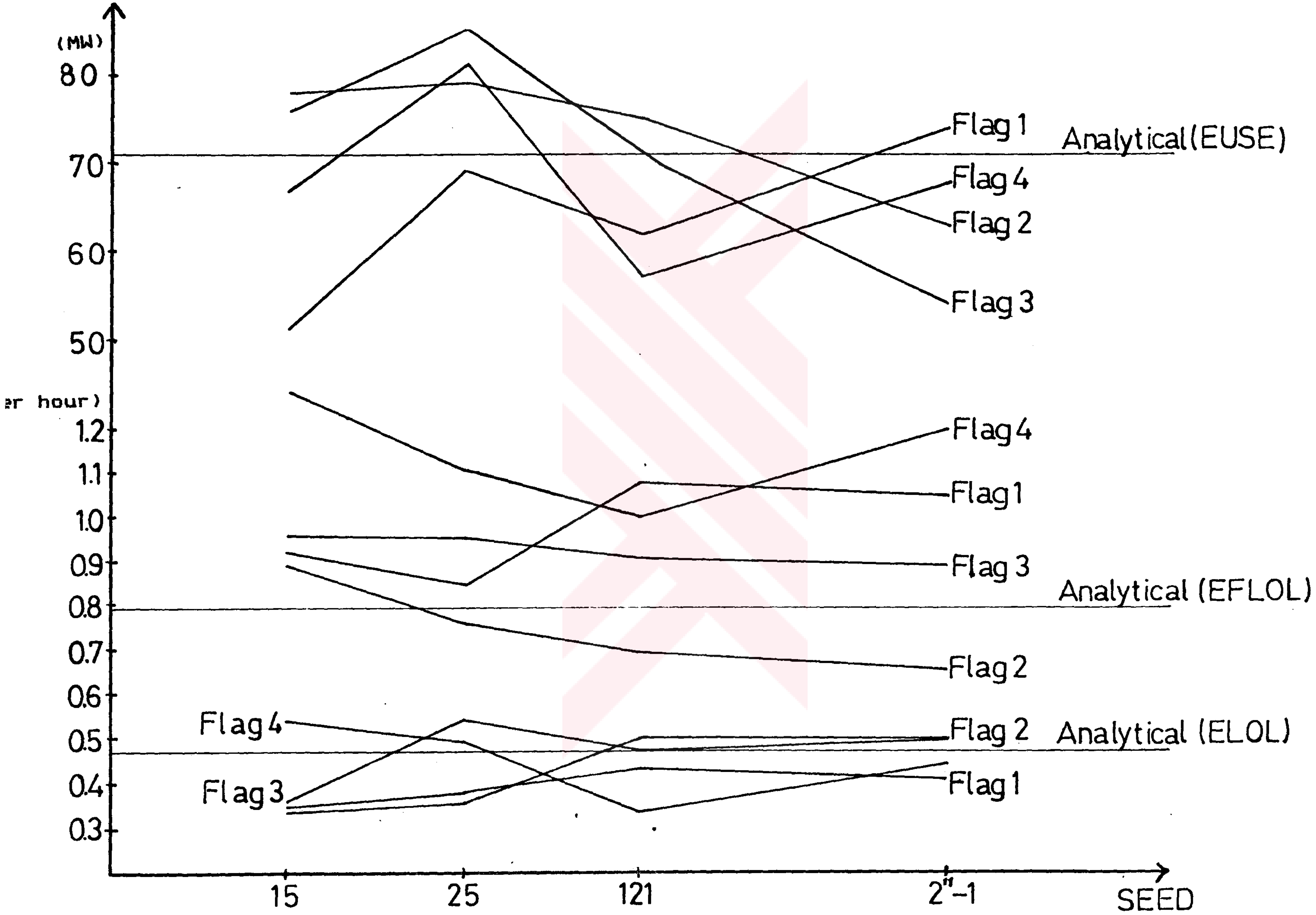
Figure.9. Graph of Case2 with 28 hours

TABLE.3.7 Results for CASE3 with 28 hourly peak load
THE RESULTS OF ANALYTICAL FORMULAE REPORTED IN [11]:

EFLOL	ELOL	EUSE
0.79299	0.4728	71.07633

SIMULATION RESULTS:

SEED VALUE	INDICES	PASCAL	FORTRAN-IV			
		FLAG1	FLAG1	FLAG2	FLAG3	FLAG4
	EFLOL	0.961	0.92	0.89	0.97	1.34
	SFLOL	1.227	1.156	0.8623	0.769	1.188
15	ELOL	0.514	0.357	0.140	0.161	0.24
	SLOL	0.659	0.6044	0.2853	0.3054	0.3377
	EUSE	65.49	51.602	78.1149	76.763	67.7416
	SUSE	111.68	109.34	37.675	35.37	36.040
	EFLOL	0.867	0.85	0.76	0.958	1.121
	SFLOL	1.215	1.411	1.078	1.302	1.182
25	ELOL	0.421	0.383	0.364	0.5417	0.4951
	SLOL	0.129	0.480	0.317	0.45	0.61
	EUSE	72.98	69.167	79.6979	87.65	81.27
	SUSE	160.09	49.015	44.89	47.90	48.123
	EFLOL	0.965	1.08	0.69	0.91	1.003
	SFLOL	0.259	1.5537	0.95	0.6134	1.4761
121	ELOL	0.496	0.436	0.531	0.476	0.345
	SLOL	0.671	0.601	0.225	0.492	0.321
	EUSE	65.43	62.06	75.30	71.611	57.614
	SUSE	145.65	156.0	37.488	50.146	67.178
	EFLOL	0.714	1.05	0.65	0.89	1.27
	SFLOL	1.063	1.215	0.8418	0.600	1.3848
$2^{11}-1$	ELOL	0.4925	0.419	0.502	0.504	0.441
	SLOL	0.9006	0.571	0.356	0.955	0.3212
	EUSE	74.2368	59.017	63.2289	54.183	68.963
	SUSE	40.3066	106.53	64.0922	463096	57.032
AVERAGED VALUES	EFLOL	0.8765	1.087	0.7475	0.932	1.463
	ELOL	0.4808	0.3987	0.3492	0.4706	0.456
	EUSE	69.53	60.46	74.085	72.551	68.897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er hour)
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Analytical(EUSE)
Flag 4
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Analytical (EFLOL)
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IV. CONCLUSION

This study presents a simulation model programmed in Turbo-Pascal capable of modelling large scale reliability systems in terms of reliability indices. The main idea is to investigate the sensitivity of generating system reliability indices to various operating considerations such as spinning reserve level and start-up failure , by using Monte Carlo Simulation Method which permits the testing and evaluation of current proposed systems without risk to current system performance or need for costly real-world experimentation. It also permits the study of complex problems where direct solution is not possible. Different than the simulation approach displayed in the paper by Patton et al. [13], it is assumed that the units have a natural dependency among themselves. Multivariate Exponential Distribution is used to generate the failure and repair times with a dependency factor Ω_0 which does not exist in the analytical model and in any study performed on such systems and it is an important contribution of this study. The reliability indices (ELOL,FLOL,USE) in terms of their respective mean and variances are calculated and tabulated. And the comparison of results is considered in the base of sample means and standard deviations. Additionally, the tables are presented schematically. Figures contain the plots of simulation values with respect to seed values for each of the cases and flags. These graphs are visual representations indicating how much deviation is observed from the analytical results.

The tables and the graphs performed to compare the simulation results with the analytical results indicate that the Monte Carlo Simulation Technique gives relatively close results to the analytical ones; especially if one focuses on a more representative reliability index as Loss of Load Expected. A sensitivity study is done by changing the dependence factor and the values of seeds. Tables show the influence of varying dependency factor among the units. Graphs support tables schematically and indicate the deviations from the desired constant-line, i.e. analytical result. Also, usage of Pascal speeds up the execution time rendering the simulation model prone to longer study periods. Different from other studies performed on this area, Pascal Language is firstly used in simulation, although there exists many efficient simulation languages.

The simulation model is also advantageous if we have failure and repair distributions different than Exponential Density. In analytical case it will be very tedious and difficult to formulate the model when the distribution, for example, is Lognormal or Weibull even if multivariate case is in consideration. As a matter of fact, the theory for other distributions is not yet clearly developed. Hovewer, by proposed method it is only required to change the subroutine that generates the failure and repair times.

Also, it is possible to increase the number of generators and hours of load to enlarge the system by latter method. Certain sensitivity studies by varying failure or repair rates of generators or spinning reserve or start-up failure probability and start-up failure delay can be incorporated.

As future research, some other constraints can be approximated by using Monte Carlo Simulation Method such as outage postponability and start-up delay (lead-time) which are also considered by Patton, Hogg and Blackstone in their corresponding paper [13]. Most unit outages are postponable to some degree. It supplies the system reliability safety margin and flexibility. The generator may be shutdown immediately or it may continue to run for a time less than or equal to a maximum postponement time which is treated as random variable with specified probability distribution. In the start-up delay case, each start-up is prolonged as much as start-up delay value as a preliminary warming-up or lead period and generator is accepted to be unavailable before it is fully running.

Also, maintenance of units, beyond the scope, can be considered for all cases. Beside the forced outages, maintenance outages such as preventive maintenance and reserve shutdowns can be incorporated into the model.

The model which has two-states, up and down in this study, can be assumed to have one additional state called derated state. The event in derated case is similiar to a failure except that the unit does not go out of service, but rather it continues to operate with some reduced capacity.

The above suggestions for future research can be modelled by the proposed Monte Carlo Simulation Method, where the same assumptions will also be operable simply by adding one or more special requirements in Pascal programming language.

As a result, a statistically feasible approach utilizing certain multivariate distributions to model the correlation of generator forced outage rates with each other and then the convolution of the generation with the load forecast data through Monte Carlo Simulation Method, will contribute to the proper representation of the practical real-world situation in power networks. The flexibility and versatility of the method both from statistical and engineering viewpoints has much to promise.

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APPENDIX.1.

ALGORITHM FOR MONTE CARLO SIMULATION MODEL:

In this appendix, the main algorithm for programming the model will be presented. An algorithm is defined as the step-by-step procedure for solving a problem. The following algorithm outlines the steps in the simulation model which is the base for both programming language proposed here. That is, algorithm remains the same for both languages. But structural differences in these languages may change the order or formulating steps without destroying the main structure in the algorithm.

Algorithm:

Step.1. Read inputs:
No.of generators, initial seed value, flag value, spinning reserve value, case value, period of time(in days), increment value, iteration value, failure rates, repair raates, start-up delay and start-up probability value.

Step.2. Initialize the value of demand to hourly peak load. If case equals to 2, add spin value to demand for each hour.

Step.3. Start iteration

3.1. Draw vector of failure times from MVE

3.2. Draw vector of repair times from MVE

3.2.1. If Case equals to 3, draw a random uniform number.

3.2.2. If this number is less than start-up probability prolonge repair time as start-up delay value.

3.3. Repeat the generation up to the end of total period length.

3.4. Go to Step 3.2 if number of generators< total number of generators

Step.4. Scan the time period by small increments

4.1. For one incremental hour sum up the capacity values of generators running at that hour.

4.2. Check if there exists a difference between total capacity and hourly peak load.

4.3. If the answer is positive sum up such increments to calculate LOL and multiply the absolute difference by the incremental hour for calculating USE.

Step.5. Count the number of times that the difference between total capacity and hourly peak load changes from positive to negative or vice versa to calculate FLOL.

Step.6. Go to Step 3 if number of iteration is less than total number of iteration.

Step.7. Calculate simple statistics: mean and variance of reliability indices.

APPENDIX.2

FLOWCHART FOR THE SIMULATION MODEL

1:

Read Input Data: Number of generators, initial seed value, flag value, spin value, case value, period of time(in hours), increment value, iteration value, failure rates, repair rates, start-up delay value, start-up failure probability.

↓

2:

Initialize value of demand to hourly peak load for each hour.Ifcase equals to 2, add spin value to value of demand

↓

3:

Start iteration: Draw failure and repair vectors from MVE up to end of time period for each generator. If case equals to 3, check if there exists start-up failure. For this draw a uniform random number. If it is less than start-up failure probability, prolonge repair time as much as start-up delay value.

↓

4:

Scan the period length by incremental hours: For each incremental hour, firstly sum up the capacity values of running generators at that hour. And check if this amount is less than hourly peak load. If the answer is Yes, count such increments for LOL and multiply absolute difference by the incremental hour to calculate USE.

↓

5:

Count the number of times that the difference changes from positive to negative or vice versa to calculate FLOL.
Add the values of LOL, FLOL, USE.
Is the number of iteration less than total number of iteration?

YES ← (return to step 3)

↓ NO

6:

Calculate mean and variance of reliability indices; LOL, FLOL, USE.

APPENDIX.3

FORTRAN-IV PROGRAM OF UNIFORM RANDOM NUMBER GENERATOR: SUNIF

```
      REAL FUNCTION SUNIF(R,IR)
      DOUBLE PRECISION R, FACTOR, TWO28
      DATA FACTOR/ 41475557.0D0/,TWO28/268435456.0D0/
C     IR=4*K1+1; R=IR/2**28; K1 IS INITIAL SEED
      IF (IR.GE.0) GO TO 1
      R=DMOD(R*FACTOR,1.0D0)
      SUNIF=SNGL(R)
      RETURN
    1 R=DFLOAT(IR)/TWO28
      SUNIF=SNGL(R)
      IR=-1
      RETURN
      END
```

APPENDIX.4.

GENERATION DATA OF THE IEEE RELIABILITY TEST SYSTEM [6]

The study conducted in this thesis on the IEEE Reliability Test systems demonstrate that both analytical and Monte Carlo Simulation results give reasonable agreement in the computed value of the mean and variance of the reliability indices. The generating system consists of 32 units with a total installed capacity of 3045 MW. The variance value for each unit forced outage rate was estimated using the procedure shown in Reference 6. The study was conducted over a period of 28 hours in which the annual system peak load is 2850 MW and minimum load is 1842.52 MW.

Unit Size (MW)	Number of Units	Installed Capacity	Ω	μ
12	5	60	0.02	0.98
20	4	80	0.10	0.90
50	6	300	0.01	0.99
76	4	304	0.02	0.98
100	3	300	0.04	0.96
155	4	620	0.04	0.96
197	3	591	0.05	0.95
350	1	350	0.08	0.92
400	2	800	0.12	0.88
TOTAL	32	3405 MW		

Load Data

The hourly peak loads for a period of 28 hours are as follows(in MW's):

2650.5	2850	2793	2736	2697	2194.5	2137.5
2523.28	2713.2	2658.94	2604.67	2550.41	2098.16	2034.9
2284.73	2456.7	2407.56	2358.43	2309.3	1891.66	1842.52
2385.45	2565	2513.7	2462.4	2411.1	1975.05	1923.75

Annual system peak load= 2850 MW

APPENDIX.5.

ANALYTICAL TREATMENT OF POWER GENERATION SYSTEMS

Based on the study of Patton, Singh, and Sahinoglu as given in [12], representation of analytical model will be explained briefly in this appendix.

In large electric power systems, reliability is defined as the probability of the power generation system supplying a desired load cycle within a prescribed period of time under the operating conditions encountered. Loss of Load Probability (LOLP) index conventionally calculated expresses the probability of the capacity on forced outage exceeding the reserve capacity in the generation system for a specified period of study. At any j^{th} hour , reserve R_j is defined as the total installed capacity "TOTCAP subtracted by the Load Forecast L_j" all expressed in Megawatt units. Hence, LOLP is the probability of the forced outage X (random variable) exceeding the reserve generation over entire period of system operation. LOLP multiplied by the length of study period yields the LOLE, the expected number of days(hours) of capacity deficiencies.

Oftentimes, however, the cumulated duration of loss of load which is directly related to the unavailability of the generation supply and consequently the loss of energy, is not sufficiently descriptive by itself. An annual outage time of ten hours would have a different impact on the residential customer, more so on an industrial customer and even more crucial on a nuclear power plant concerning off-site reliability, if it occured ten times a year each of one hour duration instead of five times a year each of two hours duration. Thus at least three reliability indices appear necessary to suitably describe the reliability of a generation system [24,25,12,28,29]. These indices are therefore the frequency of loss of load (f), the average duration of loss of load (d) and the product of these two indices (fxd) that yields the loss of load probability. Briefly, the loss of load event occurs at some j^{th} hour when the summed capacity of units on forced outage exceeds the reserve capacity or equally speaking, the capacity margin m_j defined as "Reserve Rj minus Capacity(C) on Forced Outage" assumes a negative value at hour j from a non-negative value at the preceeding hour j-1. The cumulation of such negative margin hours defines the loss of load expected. Then, the frequency of loss of load is simply the number of times the loss of load event will occur, that is, a zero-margin crossing will materialize from positive to negative margin within a given period of study. These said indices have been put forward purely in "expected" sense during earlier research [24,12] as in the following conventional formulae for which seperate tables of departure rates from capacity outage state X to states with more ($\sigma_{c+(x)}$) or less ($\sigma_{c+(x)}$) capacity levels are precalculated:

$$f(M)= \sum_{x} P_G(X) \{[\sigma_{c+}(X)-\sigma_{c-}(X)]P_L(C-X-M)\} + f_L(C-X-M) \quad (1)$$

$$P(M)= \sum_{x} P_G(X)\ P_L(C-X-M) \qquad (2)$$

where;

$f(M)$ = Frequency of margin less than or equal to M, (3)

$P(M)$ = Probability of margin less than or equal to M,

$P_L(C-X-M)$, $f_L(C-X-M)$ = Probability and frequency of load greater than or equal to (C-X-M),

$P_G(x)$= Exact probability of capacity outage equal to X=x

$\sigma_{c+}(x)$,$\sigma_{c-}(x)$=departure rates from capacity outage state x to states with more(+) or less(-) available capacity

$\sum_{x}$ = Summation over exact capacity outage states X.

In order to help clarify certain definitions of random variables, it will be useful to study the following exemplary illustration [30].

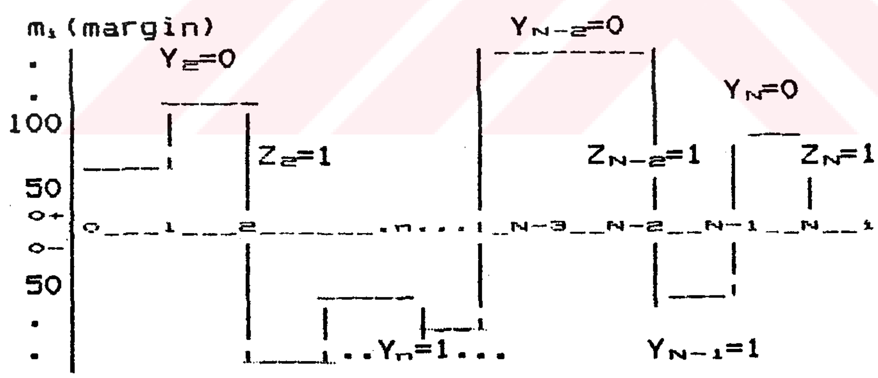

Figure.1. Illustration of random variables : m, Z, Y

we define $m_1, m_2, \ldots, m_N$ to be the marginal values at hourly steps where in a physical system, positive margin denotes operative (nondeficient) and negative margin denotes inoperative (deficient) state. Margin at a specific discretized hour is defined as the difference between total assets on one hand and consumption plus chance failures on the other. The physical system is an electrical power network where the total asset is the sum of the capacities of all electricity generating units; the consumption is the load forecast (daily/hourly constant peak load) demanded by the public and industry, and finally chance failures indicate the unplanned forced outages excluding the planned shutdowns.

$$m_i = TOTCAP - X - L_i \quad (5)$$

indicate the power balance at each discrete step "i" where the realization $\{m_i\}$ assumes either a positive margin (0^+) or negative margin (0^-), in a sequence of independent Bernoulli trials. Similarly, at each hourly transition, either a positive to negative margin crossing ($Z_i=1$) or else ($Z_i=0$) occurs as in a sequence of dependent Bernoulli trials.

Concerning the "level event", let the event space E_j be denoted for $j=1,2$ where $j=1$ corresponds to 0^- negative level for $m_i<0$ and $j=2$ corresponds to 0^+ positive level for $m_i\geq 0$. Hence $Y_i=1$ or $Y_i=0$ for $i=1,2,,n,..N$ form a sequence of Bernoulli r.v. in a two-state inhomogeneous chain. Thus, $\{Y_i\}$ are independent r.v.'s for $i=1,2,.,n,.N$ since m_i are taken independent due to assumed independence of L_i which are the load forecasts at each discrete hour [28,p.140].

Hence, the non-identical independent Bernoulli trials for $r=0$ order chain since at each step the probability of success is not identical. Therefore, $\{Y_i\}$ forms a nonstationary independent stochastic chain (process). Namely

$$Y_i = \begin{cases} 1 \text{ (success)}, & m_i<0 \\ 0 \text{ (failure)}, & m_i\geq 0 \end{cases} \qquad (6)$$

For the "crossing event", let the event space E_k be defined for $k=1,2$ where $k=1$ corresponds to positive-to-negative margin crossing ($Z=1$) when $m_i<0$ given $m_{i-1}\geq 0$, and $k=2$ corresponds to otherwise ($Z=0$). Hence $Z_i=1$ or $Z_i=0$ for $i=1,2,.n.,N$ constitutes a non-identical and non-independent sequence of Bernoulli trials. Namely, a first order ($r=1$) Markov Chain because the system can only go negative provided that it is residing at positive and an inhomogeneous Markov Chain due to nonstationary transition probabilities [26,27]. In notation :

$$Z_i = \begin{cases} 1 \text{ (success)}, & m_{i-1}\geq 0 \text{ and } m_i<0 \\ 0 \text{ (failure)}, & \text{elsewhere} \end{cases} \qquad (7)$$

We wish to compute the probabilities of success at each step for the Bernoulli r.v. Y_i and Z_i to also estimate the mean of the cumulated hours of unavailability $U_N=\Sigma Y_i$ and the mean of frequency of unavailability $F_N=\Sigma Z_i$.

APPENDIX.6.

OUTPUTS AND LISTING OF PROGRAMS

```
$RESET FREE
FILE  5(KIND=DISK,TITLE="D/FLAG",FILETYPE=7)
FILE  6(KIND=DISK,TITLE=" OUT/2324",PROTECTION=SAVE)
      DOUBLE PRECISION R
      DIMENSION LAM(32),CAP(32),DAY(30),TALEF(16000),ARZ(32,16000)
      DIMENSION TOTAL(16000),MU(32),DIF(16000),FAIL(32),REP(32)
      DIMENSION SVAL(32),DVAL(32),PVAL(32)
      REAL LAM,MU,LOSS,LAML
      READ(5,11)K1,PARL,PARM
      IR=4*K1+1;R=IR/2**23;NN=1
   11 FORMAT(I5,2F8.6)
    2 FORMAT(4I5)

      READ(5,44)CASE,SPIN,FLAG,IM
      READ(5,2)NUMGEN,NUMLOD,INCR,IT
      READ(5,55)(SVAL(I),I=1,NUMGEN)
      READ(5,55)(DVAL(I),I=1,NUMGEN)
      READ(5,55)(PVAL(I),I=1,NUMGEN)
      READ(5,1)(LAM(I),I=1,NUMGEN)
      READ(5,1)(MU(I),I=1,NUMGEN)
      READ(5,6)(CAP(I),I=1,NUMGEN)
      READ(5,3)(DAY(I),I=1,NUMLOD)
    1 FORMAT(8(F4.2,X))
    3 FORMAT(7(F7.2,X))
    6 FORMAT(16F3.0)
   44 FORMAT(I2,1X,I5,1X,I2,1X,I2)
   55 FORMAT(3(F5.3))

      WRITE(6,13)(LAM(I),I=1,NUMGEN)
   13 FORMAT("****** L A M D A   V A L U E S *****"/40("*")/8(F4.2,X))
      WRITE(6,9)(MU(I),I=1,NUMGEN)
    9 FORMAT("****** M U    V A L U E S *****"/40("*")/8(F4.2,X))
      WRITE(6,7)(CAP(I),I=1,NUMGEN)
    7 FORMAT("****** C A P A C I T Y   V A L U E S *****"/50("*")/
     *8(X,F4.0,X))
      WRITE(6,4)(DAY(I),I=1,NUMLOD)
    4 FORMAT("***** D E M A N D   V A L U E S *****"/63("*")/4(X,F7.2,
     *,/)
      WRITE(6,44)CASE,SPIN,FLAG,IM
      PRINT/,"********* RESULT OF SIMULATION ***********"
      PRINT/,"*******************************************"

      NINCR=INCR-1;N=1
      NUMIN=NUMLOD*INCR

           .
           .
           .

  123 DUM=0.
      SUM=SUM+AVER
      SKARE=SKARE+AVER**2
      SUM2=SUM2+LOSS ; SKARE2=SKARE2+LOSS**2
      SFREQ=SFREQ+FREQ
      SKARE3=SKARE3+FREQ**2
C     PRINT*/,MM,SUM,SUM2,SFREQ
  999 CONTINUE

      RMEAN1=SUM/MM  ; RMEAN2=SUM2/MM  ;RMEAN3=SFREQ/MM

      SVAR=(SKARE-((SUM)**2)/MM )/(MM-1)
      SVAR2=(SKARE2-(SUM2**2)/MM )/(MM-1)
      SVAR3=(SKARE3-(SFREQ**2)/MM )/(MM-1)

      WRITE(6,76) IT,INCR
 76   FORMAT(/"***FINAL RESULT OF ",I4,"MONTE CARLO SIMULATIONS ***"/,
     *"++++  WITH ",I4," MANY INCREMENTS PER STEP LENGTH ++++ "/)
      WRITE(6,75) SVAR,RMEAN1,SVAR2,RMEAN2,SVAR3,RMEAN3
   75 FORMAT(//"VAR.OF LOSS OF LOAD=",F10.4,/,"LOL.MEAN=",F6.3/55("*")
     *"VAR. OF LOSS OF ENERGY=",F15.4,/,"MEAN OF ENERGY LOSS=",F12.5,/
     *55("*")//"VAR.OF LOSS FREQUENCY=",F9.4/"MEAN OF LOSS FREQ=",F5.2

      STOP
      END
```

```
      SUBROUTINE MVE(R,IR,N,NN,I,PAR,PAR0,INCR,EXPO)
      REAL YF(5000),PAR(32),MIN(32),U(32),PAR0,U0,RND(5000),EXPO(32
      DOUBLE PRECISION R

      L=NN*N
      DO 40 K=1,L
 40   YF(K)=SUNIF(R,IR)

      DO 50 K=1,NN
 50   RND(K)=SUNIF(R,IR)
      K=1
      DO 20 M=1,NN
      U0=-ALOG(RND(M))/PAR0
      DO 30 J=1,N
      U(J)=-ALOG(YF(K))/PAR(J)
      MIN(J)=AMIN1(U0,U(J))
      EXPO(J)=MIN(J)*INCR
      K=K+1
30    CONTINUE
20    CONTINUE
      RETURN
      END

      SUBROUTINE CASE2(N,SPIN,DAY)
      DIMENSION DAY(N)
      DO 555 I=1,N
      DAY(I)=DAY(I)+SPIN
555   CONTINUE

      RETURN
      END

      SUBROUTINE CASE3(R,IR,I,SVAL,EXDEV)
      DOUBLE PRECISION R
      DIMENSION SVAL(32),EXDEV(32)

      RND=SUNIF(R,IR)
      IF(RND.GT.0.05) GO TO 500
      EXDEV(I)=EXDEV(I)+SVAL(I)
500   RETURN
      END

      REAL FUNCTION SUNIF(R,IR)
      DOUBLE PRECISION FACTOR,TWO28,R
      DATA FACTOR/41475557.0D0/,TWO28/268435456.0D0/
      IF(IR.GE.0) GO TO 100
      R=DMOD(R*FACTOR,1.0D0)
      SUNIF=SNGL(R)
      RETURN

 100  R=DFLOAT(IR)/TWO28
      R=DMOD(R*FACTOR,1.0D0)
      SUNIF=SNGL(R)
      IR=-1
      RETURN
      END
```

```
PROGRAM RELS(RANDF1:FILE<KIND=DISK,TITLE='RANDF1',FILETYPE=7>,
LST:FILE<KIND=PRINTER>);
TYPE
    X=ARRAY[1..32] OF REAL;
    X1= ARRAY [1..16000] OF REAL;
    X3=ARRAY [1..16] OF REAL;
    X4=ARRAY [1..32,1..16000] OF REAL;
    X5=ARRAY [1..5000] OF REAL;
    X6=ARRAY [1..32,1..7000] OF REAL;
VAR
    RANDF1, LST : TEXT;
    LAM,CAP,MU,DAY,REP,FAIL:X;
    FALEP,DIF,TOTAL: X1;
    CVAL,PVAL,DVAL:X;
    ART: X4;
    RND,YF:X5;
  KK,LL,RMEAN,RMEAN1,D,YFL,SPIN,EXDEV,FREQ,AVER,PROB,DUM,SUM,SUM2 :REAL
    PARM,PARL,RN,SEED,RMEAN2,RMEAN3,COUNT,LOSS,SVAR,SVAR2,SVAR3: REAL;
    SKARE,SKARE2,FLAGE,SKARE3,SFREQ, REMEAN1,REMEAN2, REMEAN3: REAL;
    SP,NM, MUMIN,MM,I,J,IR,NUMGEN,NUMLOD,INCR,IT,FL,V,FL1,NINCR: INTEGER
    LP, L,NJ,K,K1,CASE1,TDEV,NUMIN,IEXDEV,II,JJ,ITE,ITER,IM,N,D:INTEGER;
PROCEDURE MVE(VAR N,NN:INTEGER;VAR PAR:X;VAR PARO:REAL;
    VAR INCR:INTEGER;VAR EXPO:X;VAR YF,RND:X5);
 VAR
     MIN,U:ARRAY [1..32] OF REAL;
     PARO,UO,SEED: REAL;
     I,J,L,K :INTEGER;
   BEGIN
     L:=NN * N;
     K:=1;
     FOR I:=1 TO NN DO
     BEGIN
       UO:=-LN(RND[I])/PARO;
       FOR J:=1 TO N DO
       BEGIN
         U[J]:=-LN(YF[K])/PAR[J];
         IF UO<= U[J] THEN
          MIN[J]:=UO
         ELSE
          MIN[J]:=U[J];
         EXPO[J]:=MIN[J] *INCR;
        K:=K+1;
       END;
      END;
                 .
                 .
                 .
                 .
                 .
                 .
                 .
                 .
                 .
                 .
               END;
               BEGIN
               REWRITE(LST);
               RESET (RANDF1);
               SUM:=0;
               SUM2:=0;
               SKARE:=0;
               SKARE2:=0;
               SKARE3:=0;
               SFREQ:=0;
               READLN(RANDF1,CASE1,SPIN,FLAGE,IM);
               READLN(RANDF1,SEED,PARL,PARM);
               READLN(RANDF1,NUMGEN,NUMLOD,INCR,IT);
               WRITELN(LST,'CASE= ',CASE1,'    FLAGE= ',FLAGE);
               WRITELN(LST,'SEED= ',SEED,'  PARL= ',PARL,'   PARM =',PARM)
               WRITELN(LST,'NUMGEN= ',NUMGEN,'    NUMLOAD= ',NUMLOD);
```

```
FOR I:=1 TO NUMGEN DO
DVAL[I]:=0.001;
FOR I:=1 TO NUMGEN DO
DVAL[I]:=0.1;
FOR I:=1 TO NUMGEN DO
DVAL[I]:=0.05;
FOR I:=1 TO NUMGEN DO
READLN(RANDF1,LAM[I]);
FOR I:=1 TO NUMGEN DO
READLN(RANDF1,MU[I]);
FOR I:=1 TO NUMGEN DO
READLN(RANDF1, CAP[I]);
FOR I:=1 TO NUMLOD DO
READLN(RANDF1,DAY[I]);
WRITELN(LST,'******* RESULT OF SIMULATION ******');
MINCR:= INCR -1;
N:=1;
MM:=1;
NUMIN:=NUMLOD *INCR;
NUMIN:= NUMIN - 1;
IF CASE1=2 THEN
BEGIN
  FOR SP:=1 TO NUMLOD DO
  DAY[SP]:=DAY[SP]+SPIN
END;
K:=1;
FOR I:=1 TO NUMLOD DO
BEGIN
  .
  .
  .
  .
  .
  .
  .
  .
  .
  .
  .
  .
  .
  .
  .
  .
  .
  .
   RMEAN3 :=SFREQ/MM;
   SVAR := (SKARE - (SUM *SUM) / MM) / (MM-1);
   SVAR2 := (SKARE2 - (SUM2 * SUM2)/MM) / (MM-1);
   SVAR3 := (SKARE3 - (SFREQ * SFREQ) /MM) /(MM-1);
   WRITELN(LST,'FINAL RESULT OF ',IT, ' MONTECARLO SIMULATION');
   WRITELN(LST,'WITH ',INCR,' MANY INCREMENT PER STEP LENGTH');
   WRITELN(LST);
   WRITELN(LST);
   WRITELN(LST,'VAR. OF LOSS OF LOAD=',SVAR,'  LOL.MEAN=',RMEAN1);
   WRITELN(LST,'VAR. OF LOSS OF ENERGY=',SVAR2,'MEAN=',RMEAN2);
   WRITELN(LST,'VAR. OF LOSS OF FREQ.=',SVAR3,' MEAN=',RMEAN3);
  END;
  END;

  CLOSE(RANDF1)
END.
```

Output of Fortran-IV Program for Table 3.5

```
 1       50  1  33
********** RESULT OF SIMULATION ***********,
*****************************************,

***FINAL RESULT OF  100 MONTE CARLO SIMULATIONS ***
++++  WITH  200 MANY INCREMENTS PER STEP LENGTH ++++


MEAN OF LOSS OF LOAD=     0.4976
VAR. OF LOSS OF LOAD=    0.397
**************************************************

MEAN OF LOSS OF ENERGY=   59.9332
VAR. OF LOSS OF ENERGY=    30.39556

**************************************************

MEAN OF LOSS OF FREQUENCY=   1.03
VAR. OF LOSS OF FREQUENCY=     1.409
```

Output of Pascal Program for Table 3.5

```
CASE=     1        FLAGE=      1.0000000E+00
SEED=  2.0470000E+03          PARL=1.0000000E-06  PARM=1.0000000E-06
NUMGEN=    32     NUMLOAD=     23

*******   RESULT OF SIMULATION   *******

FINAL RESULT OF      100  MONTECARLO SIMULATION
WITH      200 MANY INCREMENT PER STEP LENGTH

MEAN OF LOSS OF LOAD= 4.3600000E-01    VAR. OF LOL= 3.3360000E-01
MEAN OF LOSS OF ENERGY= 5369.1100E-02  VAR. OF LOE= 1.2610000E+02
MEAN OF LOSS OF FREQUENCY= 1.2300000E+00  VAR. OF LOF= 1.3590000E+0
```