CONSTRAINING FOUR GENERATION SM WITH $b \to s\gamma$ AND $b \to sg$ DECAYS

T. M. Aliev[a], D. A. Demir[b], N. K. Pak[a]

[a] *Middle East Technical University, Department of Physics, 06531, Ankara, Turkey*

[b] *Department of Physics, University of Pennsylvania, Philadelphia, PA 19104-6396*

Abstract

Using the experimental result on $b \to s\gamma$ and theoretical result on $b \to sg$, a four -generation SM is analysed to constrain the rephasing-invariant combinations of the CKM matrix and masses of the fourth generation quarks.

1 Introduction

Until now no experimental or theoretical proof for the absence of the extra generations of leptons and quarks has been given. The direct search for extra generations and tests of their indirect effects will be one of the major goals of the existing and next generation accelerators. For example, CDF [1] gives a lower bound of 139 GeV for the fourth generation quark masses assuming they are stable enough when leaving the detector. If the interaction with the detector is taken into account this lower bound falls by 30-40 GeV. Thus, we can barely assume that the lower bound for the masses of the fourth generation quarks is somewhere around M_Z.

The fourth generation quarks and leptons have been considered in current literature due to various theoretical motivations. In [2] a four generation SM is proposed to solve α_s, R_b and R_c crises of SM at Z pole. As another solution to these problems, in [3] a four generation supersymmetric SM with $m_t \approx M_W$ is proposed. In addition to these, four generation SM has been used in calculating the neutron Electric Dipole Moment (EDM) (in the framework of the Kobayashi-Maskawa Model) to break the persisting $10^{-32}e - cm$ theoretical boundary which is seven orders of magnitude less than the experimental upper bound.

Note that theoretically the smallness of the electroweak radiative corrections to the LEP1 observables, following Veltman's arguments, enables us to conclude that the masses of the fourth generation quarks and leptons must be almost degenerate. This is the case because large mass splitting in the fourth generation would induce unacceptably large loop corrections to LEP1 observables.

In this work we shall base our analysis on a four generation SM. We have two basic aims: 1) Determination of those rephasing invariant combinations of CKM matrix relevant to the calculation of the neutron EDM. 2) Foundation of some likelihood bounds for the fourth generation quark masses.

Denoting the 'top' and 'bottom' quarks of the fourth generation by t' and b' respectively, one can find three independent rephasing invariant combinations of the elements of the CKM matrix: $Im[V_{td}^* V_{tb} V_{cb}^* V_{cd}]$, $Im[V_{td}^* V_{tb} V_{t'b}^* V_{t'd}]$ and $Im[V_{ts}^* V_{tb} V_{t'b}^* V_{t's}]$. Other combinations can be expressed in terms of these and the moduli of the elements of the CKM matrix. As it was shown already in [4], strange quark dominates in the neutron EDM, and thus a calculation of d_s yields at least an order of magnitude prediction for the neutron EDM.

In this sense our analysis is focused on the calculation of $Im[V_{ts}^* V_{tb} V_{t'b}^* V_{t's}]$.

For purposes mentioned above, in our analysis we shall make use of the experimental and theoretical results on the branching ratios of the FCNC decays $b \to s\gamma$ and $b \to sg$.

We will first outline the derivations of the basic expressions for the quantities of interest.

Last section is devoted to numerical analysis and discussions.

2 Derivations

$b \to s\gamma$ and $b \to sg$, being FCNC decays, start occuring at the loop level of perturbation theory. These decays involve consecutive $b \to u, c, t, t'$ and $u, c, t, t' \to s$ transitions. Thus decay amplitude has the form

$$A = \sum_{f=u,c,t,t'} \lambda_f R(m_f^2/M_W^2) \tag{1}$$

where $\lambda_f = V_{fs}^* V_{fb}$ and the function R represents the combination of the results of the loop integrations and the relevant operators responsible for $b \to s\gamma$ and $b \to sg$ decays. The unitarity of the 4x4 CKM matrix gives

$$\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0 \tag{2}$$

It is known that λ_u is much smaller than others, so from (2) it follows that

$$\lambda_c \approx -(\lambda_t + \lambda_{t'}) \tag{3}$$

Neglecting m_u^2 and m_c^2 compared to M_W^2, combining R's of different flavors with the help of (3), and taking the relevant operator structure for the decay under concern, one obtains the following expressions for the $b \to s\gamma$ and $b \to sg$ decay amplitudes:

$$A_{b\to s\gamma} = \frac{G_F}{\sqrt{2}} [\lambda_t C_7 + \lambda_{t'} C_7'] O_7 \tag{4}$$

$$A_{b\to sg} = \frac{G_F}{\sqrt{2}} [\lambda_t C_8 + \lambda_{t'} C_8'] O_8. \tag{5}$$

where O_7 and O_8 are magnetic and chromo- magnetic penguin operators

$$O_7 = \frac{e}{4\pi^2} m_b \bar{s} \sigma . F \, P_R b \tag{6}$$

$$O_8 = \frac{g_s}{4\pi^2} m_b \bar{s} \sigma . G \, P_R b \tag{7}$$

Here $P_R = (1+\gamma_5)/2$ and $F_{\mu\nu}$ and $G_{\mu\nu}$ are photon and gluon field strength tensors, respectively. The Wilson coefficients C_7, C_7', C_8 and C_8' in (4) and (5) are evaluated at the mass scale $\mu \approx m_b$ using renorm group equations with five active flavours [5,6].

$$C_7(\mu) = -\frac{1}{2}\eta^{16/23}A(x_t) - \frac{4}{3}(\eta^{14/23} - \eta^{16/23})B(x_t) + \sum_{i=1}^{8} h_i \eta^{a_i} \tag{8}$$

$$\begin{aligned} C_8(\mu) &= -\frac{1}{2}(B(x_t) - 1.725)\eta^{14/23} - 0.9135\eta^{0.4086} + 0.0873\eta^{-0.4230} \\ &\quad -0.0571\eta^{-0.8994} + 0.0209\eta^{0.1456} \end{aligned} \tag{9}$$

where $x_t = m_t^2/M_W^2$, $\eta = \frac{\alpha_s(M_W)}{\alpha_s(m_b)}$, and

$$h_i = (2.2996,\ -1.088,\ -3/7,\ -1/14,\ -0.6494,\ -0.038,\ -0.0186,\ -0.0057) \tag{10}$$

$$a_i = (14/23,\ 16/23,\ 6/23,\ -12/23,\ 0.4086,\ -0.423,\ -0.8994,\ 0.1456) \tag{11}$$

$A(x)$ and $B(x)$ in (8) and (9) are given by

$$A(x) = \frac{x(x-1)(8x^2+5x-7) + 6x^2(2-3x)ln(x)}{12(x-1)^4)} \tag{12}$$

$$B(x) = \frac{x(x-1)(x^2-5x-2) + 6x^2 ln(x)}{4(x-1)^4)} \tag{13}$$

C_7' and C_8' could, respectively, be obtained from (8) and (9) by replacing x_t by $x_{t'} = m_{t'}^2/M_W^2$.

In order to minimize the b -quark mass dependence, which leads to uncertainities, it is preferable to define braching ratios $BR(b \to s\gamma)$ and $BR(b \to sg)$ by $BR^{exp}(b \to ce\nu)/\Gamma^{theor}(b \to ce\nu)$ times the decay rates calculated from the amplitudes in (4) and (5) where $\Gamma^{theor}(b \to ce\nu)$ is given by [6,7]

$$\Gamma^{theor}(b \to ce\nu) = \frac{G_F^2 \mid \lambda_c \mid^2}{192\pi^3 \mid V_{cs} \mid^2} m_b^5 g(m_c/m_b)\kappa(m_c/m_b) \tag{14}$$

with the phase space factor

$$g(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 ln(x). \tag{15}$$

The factor $\kappa(x)$ describes the leading order (LO) QCD corrections and given by

$$\kappa(x) = 1 - \frac{2\alpha_s}{3\pi}((\pi^2 - 31/4)(1-x)^2 + 3/2). \tag{16}$$

Finally, upto two -loop accuracy where $\alpha_s(\mu)$ is given by

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 L}(1 - \frac{\beta_1 ln(L)}{\beta_0^2 L}) \tag{17}$$

where $L = ln(\frac{\mu^2}{\Lambda^2_{QCD}})$, $\beta_0 = 23/3$ and $\beta_1 = 116/3$, for five active flavours. Then $BR(b \to s\gamma)$ and $BR(b \to sg)$ can be written as

$$BR(b \to s\gamma) = P_\gamma (C_7' - C_7)^2[(R + R_7)^2 + I^2] \tag{18}$$

$$BR(b \to sg) = P_g (C_8' - C_8)^2[(R + R_8)^2 + I^2] \tag{19}$$

where the various symbols introduced for convenience have the meaning

$$P_\gamma = \frac{6\alpha_e \mid V_{cs} \mid^2 BR^{exp}(b \to ce\nu)}{\pi g(m_c/m_b)\kappa(m_c/m_b)} \tag{20}$$

$$P_g = \frac{8\alpha_s \mid V_{cs} \mid^2 BR^{exp}(b \to ce\nu)}{\pi g(m_c/m_b)\kappa(m_c/m_b)} \tag{21}$$

$$R_7 = \frac{C_7}{C_7' - C_7} \tag{22}$$

$$R_8 = \frac{C_8}{C_8' - C_8} \tag{23}$$

$$R = Re[\frac{\lambda_{t'}}{\lambda_t + \lambda_{t'}}] \tag{24}$$

$$I = Im[\frac{\lambda_{t'}}{\lambda_t + \lambda_{t'}}] \tag{25}$$

It is clear that $BR(b \to s\gamma)$ and $BR(b \to sg)$ depend on the combination $\frac{\lambda_{t'}}{\lambda_t + \lambda_{t'}}$. Introducing a polar representatinon for λ_t and $\lambda_{t'}$ as

$$\lambda_t = se^{i\theta} \tag{26}$$

$$\lambda_{t'} = s'e^{i\theta'} \tag{27}$$

a straightforward calculation yields

$$\frac{s'}{s} = \frac{\sqrt{R^2+I^2}}{\sqrt{(1-R)^2+I^2}} \tag{28}$$

$$Sin\beta = \frac{I}{\sqrt{(R^2+I^2)((1-R)^2+I^2)}} \tag{29}$$

where $\beta = \theta - \theta'$.

A representation of λ_t and $\lambda_{t'}$ as in (26) and (27) is independent of particular parametrisation of the CKM matrix [8]. $BR(b \to s\gamma)$ in (18) contains theoretical uncertainities coming from V_{cb}, $\alpha_s(\mu)$, $BR^{exp}(b \to ce\nu)$ and quark masses. To reduce these uncertainities one can construct the ratio

$$r = \frac{BR(b \to s\gamma)}{BR(b \to sg)}. \tag{30}$$

First solving (18) and (30) simultaneously for R and I and then using (28) and (29) one can express s'/s and $Sin\beta$ in terms of r and the parameters defined in equatinons (20-23).

The ratio r in (30) is useful in reducing the theoretical uncertainities. Furthermore, since (18) and (19) are similar equations in their dependence on the parameters with uncertainities (except for $\alpha_e \to \alpha_s$ interchage) their theoretical calculation in any model cannot produce too different error bounds. It is known already that theoretical and experimental results for $BR(b \to s\gamma)$ are close to each other assuming specific values for CKM elements [7]. Therefore, one can safely regard r to be free of theoretical and experimental uncertainities and it can be indentified with the ratio of the central value of $BR^{exp}(b \to s\gamma)$ to that of $BR(b \to sg)$. The latter is unknown, except for the SM prediction of $BR^{SM}(b \to sg) \sim (2.3 \pm 0.6) \times 10^{-3}$ [9]. Thus throughout the analysis r will be regarded a free parameter.

¿From these observations one can also predict the uncertainities in s' and β. ¿From (28) one observes that the uncertainity in s' is determined mainly by s, namely the uncertainities coming from $BR^{exp}(b \to s\gamma)$ and other parameters in (18) contribute negligeably. However, as is seen from (29) the uncertainity in β is of the same order as that in I.

3 Numerical Analysis and Discussions

Table 1 shows the values of the parameters used in the numerical analysis. As is seen from this table $BR^{exp}(b \to s\gamma)$ is uncertain by $\sim 30\%$ which is much larger than the errors in other quantities. However, as is shown in [7] the theoretical result for the branching ratio of the hadronic decay $BR(B \to X_s\gamma)$ is uncertain by a similar amount. Thus, as long as $BR(B \to X_s\gamma)$ is represented by $BR(b \to s\gamma)$ with the LO Wilson coefficients in the framework of the spectator model, the main source of uncertainity comes from the experimental result. This causes, in particular, the quantities I and R, defined in (24) and (25), be uncertain by a similar amount as $BR^{exp}(b \to s\gamma)$. As it was argued at the end of the last section, r in (30) depends mainly on the central values of the branching ratios. After analyzing s' and $Sin\beta$ for the central values of the quantities in Table 1, the percentage error in their values will be set to that of s and I, respectively.

¿From the Wofenstein parametrisation of the CKM matrix, $\mid \lambda_t \mid \sim \lambda^2$, where $\lambda \approx 0.22$ is the Cabibbo angle. In [10] it is argued that $Im[\lambda_t \lambda_{t'}^*] \sim \lambda^5$ and $\mathrm{Im}[V_{td}^* V_{tb} V_{cb}^* V_{cd}]$, $\mathrm{Im}[V_{td}^* V_{tb} V_{t'b}^* V_{t'd}] \sim \lambda^7$, which makes $Im[\lambda_t \lambda_{t'}^*]$ more important than the other two. Thus, to make comparison explicit, in the final graphical results we shall present the r dependence of the function $g(r)$ defined by

$$g(r) = Im[\lambda_t \lambda_{t'}^*]/\lambda^5. \tag{31}$$

As it will be seen soon, for a fixed value of $m_{t'}$, Sinβ is very sensitive to the variations in r; Sinβ is meaningful (real and in [-1, 1] interval) only for a limited range of r values.

If the addition of a fourth generation is an extension of SM, the allowed range of r must include the SM prediction of ~ 0.1. This, in particular, follows from the sequental character of the families in case of which one does not expect the occurence of new operators in the associated decay amplitudes as summarized by (4) and (5). In this sense, one can discard some range of $m_{t'}$ values that drives the SM prediction outside the window of the allowed values of r.

3.1 Light Top Case

The most important non-oblique correction to Z decays arises in the $Zb\bar{b}$ vertex which depends quadratically on m_t. Therefore, naively one expects R_b to move toward the SM prediction for a lower value of m_t. In this context, in [3] a supersymmetric SM with $m_t \sim M_W$ and $m_{t'} \sim m_t^{CDF}$ is proposed. Here supersymmetry is necessary to satisfy the requirements of the CDF signal.

In this section we shall analyse $g(r)$ and Sinβ for $m_t \sim M_W$ and $m_{t'} \sim m_t^{CDF}$. In particular, we shall set $m_t = 85GeV$ and vary $m_{t'}$ in the CDF range. This m_t and $m_{t'}$ values are away from representing the CDF data. In particular, dominance of the decay mode $t' \to bW^+$, and width of t -quark must be explained in some appropriate model, such as supersymmetry. We leave aside this problem and analyze the light top quark case assuming its results might shed light on the construction of some extension of SM.

In Table 2 we summarize ,as $m_{t'}$ wanders in the CDF range, the minimum (r_{min}) and maximum (r_{max}) values of r between which all quantities are meaningful. We observe from this table that as $m_{t'}$ increases from 158 GeV to 194 GeV the allowed range of r gets narrower. Thus, as $m_{t'}$ approaches to the upper CDF limit, r is forced to take values closer to the SM prediction of $r \sim 0.1$. If $m_{t'}$ is further increased r_{max} approaches SM prediction, in particular, $r \sim 0.1$ is just left outside the allowed interval of r values for $m_{t'} \approx 1.5TeV$.

Fig.1 shows the variation of Im$[Sin\beta]$ with r for $m_t = 85\,$GeV and $m_{t'} =$ 176 GeV. Sinβ becomes imaginary for $r < 0.027560$ and $r > 0.16766$ leaving a window around $r \sim 0.1$ as the appropriate region.

Fig. 2 shows the variation of $g(r)$ in (31) with r. It takes the value 3.5 at $r = 0.027560$ and vanishes at $r = 0.16766$. In the rather narrow interval $0.027560 < r < 0.030$ $g(r)$ falls rapidly from 3.5 to unity. Starting from unity at $r \sim 0.030$, it descents gradually to zero at $r = 0.16766$. For $r \sim 0.1$, $g(r) \sim 0.2$ making Im$[\lambda_t \lambda_{t'}^*] \sim \lambda^6$ which is less than the assumtion made in [10]. However, for $r <\sim 0.030$, Im$[\lambda_t \lambda_{t'}^*]$ is well above λ^5, approaching λ^4 at the lower end.

The errors in the input parameters effect the value of Im$[\lambda_t \lambda_{t'}^*]$ for a particular value of r. The allowed range of r , The allowed range of r , however, is not sensitive to uncertainities in the parameters in Table 1, as it varies with the central value of $BR(b \to sg)$ in a particular model. This analysis is performed for the central values of the input parameters . As mentioned before one expects, Sinβ be uncertain by $\sim 30\%$, so the numerical results for Im$[\lambda_t \lambda_{t'}^*]$ have an uncertainity of the same order.

3.2 Heavy Top Case

In this section we shall analyse $g(r)$ and Sinβ for $m_t = 176\,GeV$ and $m_{t'} \geq M_Z$.

In Table 3 we summarize ,as $m_{t'}$ increases from M_Z to 300 GeV, the minimum (r_{min}) and maximum (r_{max}) values of r between which all quantities are meaningful. We observe from this table that allowed range for r is narrower than that in the light top quark case. Infact, as $m_{t'}$ exceeds 200 GeV SM prediction for r is just left outside the allowed range of r. As $m_{t'}$ increases further the window of the allowed values of r forgets completely the the SM expectation about r. One can therefore fairly say that for heavy top quark consistent with CDF results, mass of the top quark of the fourth generation must be somewhere in between M_Z and $\sim 200\,GeV$.

Fig.3 shows the variation of Im$[Sin\beta]$ with r for $m_t = 176\,$GeV and $m_{t'} = 150$ GeV. Sinβ becomes imaginary for $r < 0.02716$ and $r > 0.11284$ which permits a very narrow interval in which r may take a value.

Fig. 4 shows the variation of $g(r)$ with r. It takes the value ~ 1.2 at $r = 0.027116$, then falls first rapidly to ~ 0.2 aroud $r \sim 0.030$, and gradually to zero at $r \sim 0.11284$. As we observe from this figure, $g(r)$ is smaller than that in the light top case. Next, one concludes that Im$[\lambda_t \lambda_{t'}^*]$ is less than λ^5 almost throughout the interval. Especially, around $r \sim 0.1$, Im$[\lambda_t \lambda_{t'}^*]$ is around ~ 0.04, making it $\sim \lambda^7$.

For higher values of $m_{t'}$ the SM prediction of $r \sim 0.1$ is far outside the allowed region. This calculation is performed for the central values of the parameters in Table 1. As mentioned before one expects, Sinβ be uncertain by $\sim 30\%$, so the numerical results for Im$[\lambda_t \lambda_{t'}^*]$ has an uncertainity of the same order.

3.3 Comparison of Two Cases and Conclusions

If one assumes that r does not depart from its SM value, rather clear expectations follow. In particular, in both cases, it is not possible to draw Im$[\lambda_t \lambda_{t'}^*]$

to $\sim \lambda^5$. As is seen from Fig.2, for $r = 0.1$, $\mathrm{Im}[\lambda_t \lambda_{t'}^*] \sim \lambda^6$. In the heavy top quark case it is even smaller; as Fig.4 suggests, for $r = 0.1$, $\mathrm{Im}[\lambda_t \lambda_{t'}^*] \sim \lambda^{7.4}$.

In both cases, as $m_{t'}$ increases, allowed range for r squeezed to the lower end. In particular, for the light top case $r = 0.1$ point is thrown to "no solution" solution for $m_{t'} \sim 1.5TeV$ which is much larger than the $m_{t'} \sim 200GeV$ -point of the heavy top quark case. However, choosing large values for $m_{t'}$ does not guarantee that $g(r)$ takes values of the order of unity. Except for r values around r_{min}, for any $m_{t'}$, however large, generally $g(r)$ is much smaller than unity, forcing $\mathrm{Im}[\lambda_t \lambda_{t'}^*] << \lambda^5$. Thus, consistency of large $m_{t'}$ and $\mathrm{Im}[\lambda_t \lambda_{t'}^*] \sim \lambda^5$ assumption of [10] is not generally satisfied except for the special point of $r \approx r_{min}$. This, in particular, requires a very large value for $BR(b \to sg)$ which is unlikely to occur under four -generation SM diagrammatics of the $b \to sg$ decay. One thus concludes that, including the 30% uncertainity in $g(r)$, it is unlikely to have $\mathrm{Im}[\lambda_t \lambda_{t'}^*] \sim \lambda^5$ at any $m_{t'}$ in both cases.

In the light top quark case the window of the allowed range of r values is wide. Thus, it permits more deviations from the SM expectation. However, the SM prediction of $r \sim 0.1$ is well included in the window. In this sense, since $r \sim 0.1$ is a subset of the allowed r values one has some extension of SM. Despite these results, the light top quark case (in this four generation SM form) is not acceptable phenomenologically since it is not consistent with CDF signal.

In the heavy top case, the allowed window of r values is narrower than that of the light top case. Thus, deviations from the SM prediction is not large as in the case of light top case. Width of the window changes rapidly with $m_{t'}$. Moreover, since for $m_{t'} >\sim 200\, GeV$ $r \sim 0.1$ point is thrown to "no solution" region of r values, one can boldly say that $m_{t'} >\sim 200\, GeV$ is unlikely to be acceptable. It is in this sense that one is able to propose some upper bound for $m_{t'}$. This analysis gives results only on $m_{t'}$, so fourth generation lepton masses and $m_{b'}$ are not restricted at all. There is no phenomenological difficulty with this case as long as the "almost- degeneracy" condition explained in the introduction is satisfied. If such a fourth sequental family of leptons and quarks do indeed exist LEP1.5 or LEP2 will be able to detect them.

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Parameter	Range
μ(GeV)	4.8
$m_t^{CDF}(GeV)$	176$\pm$ 18
$\mid V_{cb} \mid$	0.9743$\pm$ 0.0007
$BR^{exp}(b \to ce\nu)$	(10.4$\pm$0.4)%
$BR^{exp}(b \to s\gamma)$	(2.32$\pm$0.67) 10^{-4}
Λ_{QCD}(GeV)	0.195$\pm$ 0.005
M_W(GeV)	80.33
m_c/m_b	0.30$\pm$0.02

Table 1: Values of the input parameters used in the numerical analysis

$m_{t'}(GeV)$	r_{min}	r_{max}
158	0.027523	0.18000
176	0.027560	0.16766
194	0.027590	0.15765

Table 2: Boundaries of the allowed range of r values for $m_t = 85 GeV$.

$m_{t'}(GeV)$	r_{min}	r_{max}
91	0.0274400	0.15903
150	0.0271600	0.11284
200	0.0269943	0.09644
300	0.0269500	0.08080

Table 3: Boundaries of the allowed range of r values for $m_t = 176 GeV$.