Light vector meson octet–decuplet baryon vertices in QCD

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Abstract

The strong coupling constants between light vector mesons and octet–decuplet baryons are calculated in framework of the light cone QCD sum rules, taking into account $SU(3)$ flavor symmetry breaking effects. It is shown that all strong coupling constants can be represented in terms of a single universal function. Size of the $SU(3)$ symmetry breaking effects are estimated.

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1 Introduction

In the experiments performed at Jefferson Laboratory, MAMI, BNL, MIT on meson–nucleon, nucleon–hyperon, hyperon–hyperon reactions very rich data are accumulated. In order to describe the existing data, the coupling constants of pseudoscalar and vector mesons with baryons are needed. Calculation of these coupling constants within QCD, which is the fundamental theory of strong interactions, constitutes an important problem in studying the dynamics of the aforementioned reactions. At the hadronic scale QCD is nonperturbative and for this reason the calculation of the strong coupling constants of baryons with vector mesons becomes impossible using fundamental QCD Lagrangian. Therefore in order to calculate these constants some nonperturbative method is needed and for this aim we will use QCD sum rules method [1], which is more reliable and predictive in calculating the properties of hadrons. In the present work, the strong coupling constants of the decuplet–octet baryons with vector mesons g(DOV) are calculated in the framework of light cone sum rules method (LCSR) (more about this method and its applications can be found in [2]). Note that, the coupling constants of pseudoscalar meson octet baryon couplings, vector meson octet baryon couplings and pseudoscalar meson decuplet baryons are all studied within LCSR in [3], [4] and [5], respectively. In this work we extend our previous works to investigate the g(DOV) in the LCSR framework.

The plan of this article is as follows. In section 2 the strong coupling constants of DOV are calculated in LCSR method and relations between the above–mentioned strong coupling constants are obtained, when $SU(3)_f$ symmetry breaking effects are taken into account. Section 3 is devoted to the numerical analysis of the g(DOV).

2 Light cone QCD sum rules for the vector meson with decuplet–octet baryon coupling constants

In $SU(3)_f$ symmetry the coupling constants of all vector mesons with decuplet–octet baryons are described by the following interaction Lagrangian,

$$\mathcal{L}_{int} = g\varepsilon_{ijk}\bar{O}^j_\ell D^{mk\ell} V^i_m + h.c. \ , \tag{1}$$

where O D and V are the octet, decuplet baryons and V is the vector meson, respectively.

Octet baryons and octet vector mesons are given by,

$$O^\alpha_\beta = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{2}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix} ,$$

$$V^\alpha_\beta = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & \rho+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix} .$$

In this section we derive LCSR for the coupling constants of the vector meson with decuplet–octet baryons. In deriving these sum rules we start our analysis with the following correlation function,

$$\Pi_\mu^{D\to OV} = i\int d^4x e^{ipx}\left\langle V(q)\left| T\left\{\eta(x)\bar{\eta}_\mu(0)\right\}\right| 0\right\rangle ~, \tag{2}$$

where $V(q)$ is the vector meson with momentum q, D and O are the decuplet and octet baryons, η_μ and η are their interpolating currents respectively, and T is the time ordering operator.

The sum rules for the correlation function (2) are derived by calculating it in terms of the hadrons from one side (phenomenological part), and calculating it in the limit $-p^2 \to +\infty$ in the deep Euclidean region in terms of quarks and gluons and then equating these representations using the dispersion relations.

The phenomenological part can be obtained by inserting a complete set of intermediate hadronic states with the same quantum numbers as the corresponding interpolating currents. After isolating the ground state contributions of the decuplet and octet baryons, we get the following result,

$$\Pi_\mu^{D\to OV}(p,q) = \frac{\langle 0\left|\eta\right| O(p_2)\rangle \left\langle O(p_2)V(q)|D(p_1)\right\rangle}{p_2^2 - m_2^2}\, \frac{\langle D(p_1)\left|\bar{\eta}_\mu\right| 0\rangle}{p_1^2 - m_1^2} + \cdots ~, \tag{3}$$

where O and D denote the octet and decuplet baryons, $p_1 = p_2 + q$, p_2, m_1 and m_2 are their four–momentum and mass, respectively, and $\cdots$ represent the contributions of the higher states and continuum. The matrix elements in Eq. (3) are defined as (see [6])

$$\begin{aligned}\langle 0\left|\eta\right| O(p_2)\rangle &= \lambda_O u(p_2,s) ~,\\ \langle D(p_1)\left|\bar{\eta}_\mu\right| 0\rangle &= \lambda_D \bar{u}_\mu(p_1,s) ~.\end{aligned} \tag{4}$$

Using the Lorentz invariance, the matrix element $\langle O(p_2)V(q)|D(p_1)\rangle$ is parametrized in terms of three form factors g_1, g_2 and g_3 as follows [7]:

$$\begin{aligned}\langle O(p_2)V(q)|D(p_1)\rangle &= \bar{u}(p_2)\Big\{ g_1(q_\alpha \not\varepsilon - \varepsilon_\alpha \not q)\gamma_5 + g_2[(p\cdot\varepsilon)q_\alpha - (P\cdot q)\varepsilon_\alpha]\gamma_5 \\ &+ g_3[(q\cdot\varepsilon)q_\alpha - q^2\varepsilon_\alpha]\gamma_5 \Big\} u_\alpha(p_1) ~,\end{aligned} \tag{5}$$

where $P = (p_1 + p_2)/2$, and q and ε are the vector meson momentum and polarization vectors, respectively. In the present work we assume the vector meson to be on shell, that is $q\cdot\varepsilon = 0$ and $q^2 = m_V^2$.

In order to obtain the correlation function from the phenomenological side, summation over spins of octet and decuplet baryons are performed, i.e.,

$$\begin{aligned}\sum_s u(p_2,s)\bar{u}(p_2,s) &= \not p_2 + m_2 ~,\\ \sum_s u_\alpha(p_1,s)\bar{u}_\beta(p_1,s) &= -(\not p_1 + m_1)\left(g_{\alpha\beta} - \frac{\gamma_\alpha\gamma_\beta}{3m_1} - \frac{2p_{1\alpha}p_{1\beta}}{3m_1^2} + \frac{p_{1\alpha}\gamma_\beta - p_{1\beta}\gamma_\alpha}{3m_1}\right) ~.\end{aligned} \tag{6}$$

Using Eqs. (3–6), in principle, one can obtain the expression for the phenomenological part of the correlation function. In doing so, however, the following two problems appear. The first problem is related to the fact that the interpolating current η_μ couples not only to the $J^P = \frac{3}{2}^+$ states, but also to the $J^P = \frac{1}{2}^-$ states. The matrix element of the current η_μ between vacuum and $J^P = \frac{1}{2}^-$ states is determined by the following relation

$$\langle 0 \left| \eta_\mu \right| \frac{1}{2}^- (p_1) \rangle = A \left(\gamma_\mu - 4 \frac{p_{1\mu}}{m_{\frac{1}{2}^-}} \right) u(p_1, s) \ . \tag{7}$$

In deriving this relation the condition $\gamma^\mu p_\mu = 0$ has been used. Therefore the structures with γ_μ at the end, or the structures $\sim p_{1\mu}$ contain the contributions from the $J^P = \frac{1}{2}^-$ states which should be removed, and we will not consider them in further analysis.

The second problem is that not all Lorentz structures are independent. In order to overcome these problems we will use the ordering procedure of the Dirac matrices in such a way that it guarantees the independence of the Lorentz structures, as well as, it is free of the $J^P = \frac{1}{2}^-$ contributions. In this work we will choose the $\gamma_\mu \not\varepsilon \not q \not p \gamma_5$ ordering.

Taking into account this ordering and using Eqs. (3)–(6) for the phenomenological part of the correlation function we get

$$\begin{aligned} \Pi_\mu = & \frac{\lambda_0 \lambda_D}{[m_1 - (p+q)^2]} \frac{1}{(m_2^2 - p^2)} \Big[g_1 (m_1 + m_2) \not\varepsilon \not p \gamma_5 q_\mu - g_2 \not q \not p \gamma_5 (p \cdot \varepsilon) q_\mu + g_3 q^2 \not q \not p \gamma_5 \varepsilon_\mu \\ & + \text{other structures} \Big] \ . \end{aligned} \tag{8}$$

Obviously, the structures $\not\varepsilon \not p \gamma_5 q_\mu$, $\not q \not p \gamma_5 (p \cdot \varepsilon) q_\mu$ and $\not q \not p \gamma_5 \varepsilon_\mu$ do not contain contributions from $J^P = \frac{1}{2}^-$ states, because the contributions of $J^P = \frac{1}{2}^-$ states are proportional to γ_μ or $(p+q)_\mu$. Note that in Eq. (8) we make the replacements $p_2 = p$ and $p_1 = p + q$.

For calculation of the theoretical part of the correlation function from the QCD side the expressions of the interpolating currents of the decuplet and octet baryons are needed.

The general form of the interpolating currents for the decuplet baryons can be presented as [6]

$$\eta_\mu = A \epsilon^{abc} \left[\left(q_1^{aT} C \gamma_\mu q_2^b \right) q_3^c + \left(q_2^{aT} C \gamma_\mu q_3^b \right) q_1^c + \left(q_3^{aT} C \gamma_\mu q_1^b \right) q_2^c \right] , \tag{9}$$

where a, b, c are the color indices, C is the charge conjugation operator. The values of A and the quark flavors q_1, q_2 and q_3 for each member of the decuplet baryon are listed in Table 1.

The most general form of the interpolating currents of the octet baryons are of the following form [6, 8]:

$$\begin{aligned} \eta^{\Sigma^0} &= -\sqrt{\frac{1}{2}} \epsilon^{abc} \left[\left(u^{aT} C s^b \right) \gamma_5 d^c - \left(s^{aT} C d^b \right) \gamma_5 u^c + \beta \left(u^{aT} C \gamma_5 s^b \right) d^c - \beta \left(s^{aT} C \gamma_5 d^b \right) u^c \right] , \\ \eta^{\Sigma^+} &= -\sqrt{\frac{1}{2}} \eta^{\Sigma^0} \ (d \to u) \ , \\ \eta^{\Sigma^-} &= -\sqrt{\frac{1}{2}} \eta^{\Sigma^0} \ (u \to d) \ , \\ \eta^p &= -\eta^{\Sigma^+} \ (s \to d) \ , \end{aligned}$$

	A	q_1	q_2	q_3
Σ^{*0}	$\sqrt{2/3}$	u	d	s
Σ^{*+}	$\sqrt{1/3}$	u	u	s
Σ^{*-}	$\sqrt{1/3}$	d	d	s
Δ^{++}	$1/3$	u	u	u
Δ^{+}	$\sqrt{1/3}$	u	u	d
Δ^{0}	$\sqrt{1/3}$	d	d	u
Δ^{-}	$1/3$	d	d	d
Ξ^{*0}	$\sqrt{1/3}$	s	s	u
Ξ^{*-}	$\sqrt{1/3}$	s	s	d
Ω^{-}	$1/3$	s	s	s

Table 1: The values of A and the quark flavors q_1, q_2 and q_3

$$\begin{aligned}
\eta^n &= -\eta^{\Sigma^-}\ (s \to u)\ , \\
\eta^{\Xi^0} &= -\eta^n\ (d \to s)\ , \\
\eta^{\Xi^-} &= -\eta^p\ (u \to s)\ , \\
\eta^\Lambda &= \sqrt{\frac{1}{6}}\epsilon^{abc}\Big[2\Big(u^{aT}Cd^b\Big)\gamma_5 s^c + \Big(u^{aT}Cs^b\Big)\gamma_5 d^c + \Big(s^{aT}C\gamma_5 d^b\Big)u^c \\
&+ 2\beta\Big(u^{aT}C\gamma_5 d^b\Big)s^c + \beta\Big(u^{aT}C\gamma_5 s^b\Big)d^c + \Big(s^{aT}C\gamma_5 d^b\Big)u^c\Big]\ , \qquad (10)
\end{aligned}$$

where β is an auxiliary parameter and $\beta = -1$ case corresponds to the Ioffe current. There are the following relations between Λ and Σ^0 currents, which are shown in [9],

$$\begin{aligned}
2\eta^{\Sigma^0}\ (d \to s) + \eta^{\Sigma^0} &= -\sqrt{3}\eta^\Lambda\ , \\
2\eta^{\Sigma^0}\ (u \to s) + \eta^{\Sigma^0} &= \sqrt{3}\eta^\Lambda\ . \qquad (11)
\end{aligned}$$

In principle, having the explicit form of the interpolating currents, the correlation function can be calculated straightforwardly from the QCD side. Before calculating the correlation function from the QCD side we will try to find the relations among the invariant functions for the structures $\not\varepsilon\not p\gamma_5 q_\mu$, $\not q\not p\gamma_5(p\cdot\varepsilon)q_\mu$ and $\not q\not p\gamma_5\varepsilon_\mu$. The relations between the invariant functions are structure independent, while their explicit terms are structure dependent. In establishing the relations between invariant functions, we will follow the works of [3–5]. The main power of this approach, which we present below, is that it it takes into account $SU(3)_f$ symmetry violating effects. We will further show that all correlation functions which are needed for the determination of the coupling constants of the vector mesons with decuplet–octet baryons can be written in terms of only one invariant function for each structure.

Following the works [3]–[5], let us consider the correlation function describing the $\Sigma^{*0} \to \Sigma^0\rho^0$ transition and show that this transition is described by only one invariant function. The obtained result is enough to establish relations among $\Sigma^{*0} \to \Sigma^0\rho^0$ and $\Sigma^{*+} \to \Sigma^+\rho^0$

and $\Sigma^{*-} \to \Sigma^- \rho^0$ transitions. As has already been noted, the relations among invariant functions are structure independent, i.e., the relations are the same for all structures. The correlation function for the $\Sigma^{*0} \to \Sigma^0 \rho^0$ transition can formally be written as

$$\Pi^{\Sigma^{*0} \to \Sigma^0 \rho^0} = g_{\rho^0 uu} \Pi_1(u,d,s) + g_{\rho^0 dd} \Pi_1'(u,d,s) + g_{\rho^0 ss} \Pi_2(u,d,s) \ . \qquad (12)$$

The meson current is represented as

$$J_\mu = \sum_{u,d,s} g_{qq\rho} \bar{q} \gamma_\mu q \ ,$$

and for the ρ^0 meson $g_{\rho^0 uu} = -g_{\rho^0 dd} = 1/\sqrt{2}$ and $g_{\rho^0 ss} = 0$. The functions Π_1, Π_1' and Π_2 describe the emission of the ρ^0 meson from u, d and s quarks, respectively, and are defined formally as follows:

$$\begin{aligned}
\Pi_1(u,d,s) &= \left\langle \bar{u}u \left| \Sigma^0 \bar{\Sigma}^{*0} \right| 0 \right\rangle \ , \\
\Pi_2(u,d,s) &= \left\langle \bar{s}s \left| \Sigma^0 \bar{\Sigma}^{*0} \right| 0 \right\rangle \ , \qquad (13)
\end{aligned}$$

It follows from Eqs. (9) and (10) that, the interpolating currents of Σ^{*0} and Σ^0 are symmetric with respect to the exchange $u \leftrightarrow d$, and for this reason $\Pi_1'(u,d,s) = \Pi_1(d,u,s)$. Using this result Eq. (12) can be written as

$$\Pi^{\Sigma^{*0} \to \Sigma^0 \rho^0} = \frac{1}{\sqrt{2}} \Big[\Pi_1(u,d,s) - \Pi_1(d,u,s) \Big] \ . \qquad (14)$$

Obviously, in the exact isospin symmetry limit $\Pi^{\Sigma^{*0} \to \Sigma^0 \rho^0} = 0$.

The invariant functions responsible for $\Sigma^{*+} \to \Sigma^+ \rho^0$ transition can be obtained by replacing $d \to u$ in $\Pi_1(u,d,s)$ and using $\Sigma^{*0}(d \to u) = \sqrt{2} \Sigma^{*+}$ and $\Sigma^0(d \to u) = -\sqrt{2} \Sigma^+$, which gives

$$-2 \left\langle \bar{u}u \left| \Sigma^+ \bar{\Sigma}^{*+} \right| 0 \right\rangle = 4 \Pi_1(u,u,s) \ . \qquad (15)$$

The factor four appearing on the right–hand side of Eq. (15) is the result of the fact that there are four possible ways that the ρ^0 meson can be emitted from the u quarks. Using Eq. (15) we obtain the following relation for the invariant function Π_1, which is responsible for the transition $\Sigma^{*+} \to \Sigma^+ \rho^0$,

$$\begin{aligned}
\Pi^{\Sigma^{*+} \to \Sigma^+ \rho^0} &= g_{\rho^0 uu} \left\langle \bar{u}u \left| \Sigma^+ \bar{\Sigma}^{*+} \right| 0 \right\rangle + g_{\rho^0 ss} \left\langle \bar{s}s \left| \Sigma^+ \bar{\Sigma}^{*+} \right| 0 \right\rangle \\
&= -\sqrt{2} \Pi_1(u,u,s) \ . \qquad (16)
\end{aligned}$$

The relation for the $\Sigma^{*-} \to \Sigma^- \rho^0$ transition can be obtained by making the replacement $u \to d$ in Eq. (12) and using the fact that $\Sigma^{*0}(u \to d) = \sqrt{2} \Sigma^{*-}$ and $\Sigma^0(u \to d) = -\sqrt{2} \Sigma^-$, as a result of which we get,

$$\Pi^{\Sigma^{*-} \to \Sigma^- \rho^0} = -\sqrt{2} \Pi_1(d,d,s) \ . \qquad (17)$$

Under exact isospin symmetry limit, from Eqs. (16) and (17) we get,

$$\Pi^{\Sigma^{*+}\to\Sigma^+\rho^0} = \Pi^{\Sigma^{*-}\to\Sigma^-\rho^0} \ . \tag{18}$$

We proceed now to calculate the invariant functions involving Δ resonances. For this aim we consider $\Delta^+ \to p\rho^0$ transition. Using the fact that $\Delta^+ = \Sigma^{*+}(s\to d)$ and $p = -\Sigma^+(s\to d)$, we obtain from Eq. (16) that,

$$\begin{aligned}\Pi^{\Delta^+\to p\rho^0} &= -\Big(g_{\rho^0 uu}\left\langle \bar{u}u\left|\Sigma^+\bar{\Sigma}^{*+}\right|0\right\rangle\Big)(s\to d) - \Big(g_{\rho^0 ss}\left\langle \bar{s}s\left|\Sigma^+\bar{\Sigma}^{*+}\right|0\right\rangle\Big)(s\to d)\\ &= \sqrt{2}\Pi_1(u,u,d) - \frac{1}{\sqrt{2}}\Pi_2(u,u,d) \ .\end{aligned} \tag{19}$$

Obviously, one can easily see that

$$\Pi^{\Delta^0\to n\rho^0} = \sqrt{2}\Pi_1(d,d,u) - \frac{1}{\sqrt{2}}\Pi_2(d,d,u) \ . \tag{20}$$

Following similar lines of reasoning we obtain,

$$\begin{aligned}\Pi^{\Xi^{*0}\to\Xi^0\rho^0} &= \frac{1}{\sqrt{2}}\Pi_2(s,s,u) \ ,\\ \Pi^{\Xi^{*-}\to\Xi^-\rho^0} &= \frac{1}{\sqrt{2}}\Pi_2(s,s,d) \ .\end{aligned} \tag{21}$$

The remaining relations between the correlation functions involving ρ, ω and ϕ mesons are presented in Appendix A.

Up to this point the relations involving neutral ρ meson are obtained. We can now try to obtain similar relations among the invariant functions for the transitions involving charged ρ meson. For this purpose let us consider the matrix element $\left\langle \bar{d}d\left|\Sigma^0\bar{\Sigma}^{*0}\right|0\right\rangle$, where d quarks from Σ^0 and Σ^{*0} baryons form the final $\bar{d}d$ state, and the remaining u and s quarks are being the spectators. In the same manner, in the expression $\left\langle \bar{u}d\left|\Sigma^+\bar{\Sigma}^{*0}\right|0\right\rangle$, d quark from Σ^{*0} and u quark from Σ^+ form $\bar{u}d$ state and the remaining u and s quarks remain as spectators. Therefore, these matrix elements should be related and the explicit calculations indeed confirm this expectation. We find that,

$$\begin{aligned}\Pi^{\Sigma^{*0}\to\Sigma^+\rho^-} &= \left\langle \bar{u}d\left|\Sigma^+\bar{\Sigma}^{*0}\right|0\right\rangle = -\sqrt{2}\left\langle \bar{d}d\left|\Sigma^0\bar{\Sigma}^{*0}\right|0\right\rangle\\ &= -\sqrt{2}\Pi_1(d,u,s) \ .\end{aligned} \tag{22}$$

Making the exchange $u \leftrightarrow d$ in the above expression, we obtain,

$$\begin{aligned}\Pi^{\Sigma^{*0}\to\Sigma^-\rho^+} &= \left\langle \bar{d}u\left|\Sigma^-\bar{\Sigma}^{*0}\right|0\right\rangle = \sqrt{2}\left\langle \bar{u}u\left|\Sigma^0\bar{\Sigma}^{*0}\right|0\right\rangle\\ &= \sqrt{2}\Pi_1(u,d,s) \ .\end{aligned} \tag{23}$$

Using these arguments and performing similar calculations, we obtain the following relations among invariant functions involving charged ρ mesons:

$$\Pi^{\Sigma^{*-}\to\Sigma^0\rho^-} = \sqrt{2}\Pi_1(u,d,s) \ ,$$

$$
\begin{aligned}
\Pi^{\Xi^{*-}\to\Xi^0\rho^-} &= -2\Pi_1(d,s,s)\ ,\\
\Pi^{\Sigma^{*-}\to\Lambda\rho^-} &= -\sqrt{\frac{2}{3}}\Big[2\Pi_1(u,s,d)+\Pi_1(u,d,s)\Big]\ ,\\
\Pi^{\Delta^0\to p\rho^-} &= 2\Pi_1(u,u,d)\ ,\\
\Pi^{\Sigma^{*+}\to\Sigma^0\rho^+} &= \sqrt{2}\Pi_1(d,u,s)\ ,\\
\Pi^{\Xi^{*0}\to\Xi^-\rho^+} &= -\Pi_2(s,s,u)\ ,\\
\Pi^{\Sigma^{*+}\to\Lambda\rho^+} &= \sqrt{\frac{2}{3}}\Big[2\Pi_1(d,s,u)+\Pi_1(d,u,s)\Big]\ ,\\
\Pi^{\Delta^+\to n\rho^+} &= -2\Pi_1(d,d,u)\ . \qquad (24)
\end{aligned}
$$

The relations among the invariant functions involving K^* and ϕ mesons can be obtained easily using the similar arguments. These relations are presented in Appendix A for a given Lorentz structure.

All of the obtained results for the coupling constants of DOV can qualitatively be understood from a simple *diquark+quark* picture of the baryons in the following way. Let us consider the Hermitian conjugate channel, i.e., $O+V\to D$. Denote O as $O(q_1q_2,q_3)$ where q_1q_2 forms the diquark and q_3 is a single quark. In a reaction, for example $p\rho^+\to\Delta^{++}$, the single q_3 (in this case d) is the V–absorbing quark, and therefore, this reaction is described by Π_{q_3} (in our notation Π_2). In the reaction $n\rho^+\to\Delta^+$, the V–absorbing quark is from the diquark (here dd), and this reaction is described by, let us say, Π_1.

For the reactions of Δ decays with the participation of ρ^0, both functions Π_1 and Π_2 contribute. Obviously, only Π_1 and Π_2 contribute to the reactions $\Sigma^*\to\Sigma\rho^0$ and $\Xi^*\to\Xi\rho^0$, respectively. Similar situation take place for the reactions involving ω and ϕ mesons.

From calculations the following relation between Π_1 and Π_2 is obtained

$$
\Pi_2(u,d,s) = -\Pi_1(s,u,d) - \Pi_1(s,d,u)\ .
$$

This relation leads to the result that all coupling constants of the vector mesons with the decuplet–octet baryons can be written in terms of only one invariant function without using the $SU(3)_f$ flavor symmetry, which is the main result of this work.

We now concentrate on calculating the invariant function Π_1. For this aim, the correlation function which describes the transition $\Sigma^{*0}\to\Sigma^0\rho^0$ is enough. In deep Euclidean region, $-p_1^2\to\infty$, $-p_2^2\to\infty$, as we have already mentioned, the correlation function can be evaluated from the QCD side using OPE. In order to obtain the expressions of the correlation functions from QCD side, the propagator of the light quarks and the matrix elements of the nonlocal operators $\bar{q}(x_1)\Gamma q'(x_2)$ and $\bar{q}(x_1)G_{\mu\nu}q'(x_2)$ between the vacuum and the vector meson states are needed, where Γ represents the Dirac matrices relevant to the case under consideration, and $G_{\mu\nu}$ is the gluon field strength tensor.

Up to twist–4 accuracy, matrix elements $\langle V(q)\,|\bar{q}(x)\Gamma q(0)|\,0\rangle$ and $\langle V(q)\,|\bar{q}(x)G_{\mu\nu}q(0)|\,0\rangle$ are determined in terms of the distribution amplitudes (DA's) of the vector mesons [10–12]. These DA's are presented in Appendix B.

In further analysis, we use the following expression for the light quark propagator

$$
S_q(x) = \frac{i\not{x}}{2\pi^2x^4} - \frac{m_q}{4\pi^2x^2} - \frac{\langle\bar{q}q\rangle}{12}\left(1-\frac{im_q}{4}\not{x}\right) - \frac{x^2}{192}m_0^2\,\langle\bar{q}q\rangle\left(1-\frac{im_q}{6}\not{x}\right)
$$

$$- ig_s \int_0^1 du \Big\{ \frac{\not x}{16\pi^2 x^2} G_{\mu\nu}(ux)\sigma^{\mu\nu} - ux^\mu G_{\mu\nu}(ux)\gamma^\nu \frac{i}{4\pi^2 x^2}$$
$$- \frac{i m_q}{32\pi^2} G_{\mu\nu}(ux)\sigma^{\mu\nu} \left[\ln\left(\frac{-x^2\Lambda^2}{4} \right) + 2\gamma_E \right] \Big\} , \quad (25)$$

where γ_E is the Euler constant, Λ is a scale parameter, and we will choose it as a factorization scale, i.e., $\Lambda = 1.0~GeV$ (for more detail, see [13]). In the calculations, $SU(3)_f$ symmetry violation effects are included in the nonzero strange quark mass and strange quark condensate. These effects are also taken into account in calculation of the DA's [10–12].

Having the expressions of the light quark propagator and those of the DA's, the theoretical part of the correlation functions can be calculated. Equating both representations of correlation function and separating coefficients of Lorentz structures $\not\varepsilon \not p \gamma_5 q_\mu$, $\not q \not p \gamma_5 (p\cdot\varepsilon) q_\mu$ and $\not q \not p \gamma_5 \varepsilon_\mu$, and applying Borel transformation to both side of the correlation functions on the variables p^2 and $(p+q)^2$ in order to suppress the contributions of the higher states and continuum (see [14]), we get the sum rules for the corresponding vector meson decuplet–octet baryon couplings. The contributions of higher states and the continuum are subtracted using quark-hadron duality. After standard calculations, for each Lorentz structure the expressions for the invariant functions $\Pi_1^{(\alpha)}$ are obtained and their expressions are presented in Appendix C. Here superscript α refers to the invariant functions $\Pi^{(\alpha)}$ relevant to the coupling constants g_1, g_2 and g_3, correspondingly.

For a given transition $D \to OV$, once the Borel transformed and continuum subtracted coefficient functions $\Pi^{(1)}$, $\Pi^{(2)}$ and $\Pi^{(3)}$ are obtained, the coupling constants can be written as

$$\begin{aligned}
g_1 &= \frac{1}{m_1+m_2} \frac{1}{\lambda_O \lambda_D} e^{\frac{m_1^2}{M_1^2} + \frac{m_2^2}{M_2^2} + \frac{m_V^2}{M_1^2+M_2^2}} \Pi_1^{(1)} ~, \\
g_2 &= \frac{1}{\lambda_O \lambda_D} e^{\frac{m_1^2}{M_1^2} + \frac{m_2^2}{M_2^2} + \frac{m_V^2}{M_1^2+M_2^2}} \Pi_1^{(2)} ~, \\
g_3 &= \frac{1}{m_V^2} \frac{1}{\lambda_O \lambda_D} e^{\frac{m_1^2}{M_1^2} + \frac{m_2^2}{M_2^2} + \frac{m_V^2}{M_1^2+M_2^2}} \Pi_1^{(3)} ~.
\end{aligned} \quad (26)$$

It follows from these expressions that for the vector meson decuplet–octet baryon strong coupling constants, the residues of baryons are needed. The residues of baryons are obtained from the analysis of two–point correlation function as are given in [6, 8, 14]. The currents of the other baryons can be obtained from Σ^0 current by making appropriate substitutions of quarks. For this reason, for determination of the residues, we give the sum rule only for Σ^0 and Σ^{*0}.

$$\begin{aligned}
\lambda_{\Sigma^0}^2 e^{-m_{\Sigma^0}^2/M^2} &= \frac{M^6}{1024\pi^2}(5+2\beta+5\beta^2) E_2(x) - \frac{m_0^2}{96M^2}(-1+\beta)^2 \langle \bar u u\rangle \langle \bar d d\rangle \\
&- \frac{m_0^2}{16M^2}(-1+\beta^2)\langle \bar s s\rangle \Big(\langle \bar u u\rangle + \langle \bar d d\rangle \Big) \\
&+ \frac{3m_0^2}{128}(-1+\beta^2)\Big[m_s \Big(\langle \bar u u\rangle + \langle \bar d d\rangle \Big) + (m_u+m_d)\langle \bar s s\rangle \Big]
\end{aligned}$$

$$
\begin{aligned}
&- \frac{1}{64\pi^2}(-1+\beta)^2 M^2 \Big(m_d \langle \bar{u}u \rangle + m_u \langle \bar{d}d \rangle \Big) E_0(x) \\
&- \frac{3M^2}{64\pi^2}(-1+\beta^2)\Big[m_s \Big(\langle \bar{u}u \rangle + \langle \bar{d}d \rangle \Big) + (m_u + m_d) \langle \bar{s}s \rangle \Big] E_0(x) \\
&+ \frac{1}{128\pi^2}(5+2\beta+5\beta^2)\Big(m_u \langle \bar{u}u \rangle + m_d \langle \bar{d}d \rangle + m_s \langle \bar{s}s \rangle \Big) \\
&+ \frac{1}{24}\Big[3(-1+\beta^2) \langle \bar{s}s \rangle \Big(\langle \bar{u}u \rangle + \langle \bar{d}d \rangle \Big) + (-1+\beta^2) \langle \bar{u}u \rangle \langle \bar{d}d \rangle \Big] \\
&+ \frac{m_0^2}{256\pi^2}(-1+\beta)^2 \Big(m_u \langle \bar{d}d \rangle + m_d \langle \bar{u}u \rangle \Big) \\
&+ \frac{m_0^2}{26\pi^2}(-1+\beta^2)\Big[13 m_s \Big(\langle \bar{u}u \rangle + \langle \bar{d}d \rangle \Big) + 11(m_u + m_d) \langle \bar{s}s \rangle \Big] \\
&- \frac{m_0^2}{192\pi^2}(1+\beta+\beta^2)\Big(m_u \langle \bar{u}u \rangle + m_d \langle \bar{d}d \rangle - 2m_s \langle \bar{s}s \rangle \Big) ,
\end{aligned}
$$

$$
\begin{aligned}
m_{\Sigma^{*0}} \lambda_{\Sigma^{*0}}^2 e^{-\frac{m_\Sigma^2}{M^2}} &= \big(\langle \bar{u}u \rangle + \langle \bar{d}d \rangle + \langle \bar{s}s \rangle \big) \frac{M^4}{9\pi^2} E_1(x) - (m_u + m_d + m_s) \frac{M^6}{32\pi^4} E_2(x) \\
&- \big(\langle \bar{u}u \rangle + \langle \bar{d}d \rangle + \langle \bar{s}s \rangle \big) m_0^2 \frac{M^2}{18\pi^2} E_0(x) \\
&- \frac{2}{3} \left(1 + \frac{5m_0^2}{72M^2} \right) \big(m_u \langle \bar{d}d \rangle \langle \bar{s}s \rangle + m_d \langle \bar{s}s \rangle \langle \bar{u}u \rangle + m_s \langle \bar{d}d \rangle \langle \bar{u}u \rangle \big) \\
&+ \big(m_s \langle \bar{d}d \rangle \langle \bar{s}s \rangle + m_u \langle \bar{d}d \rangle \langle \bar{u}u \rangle + m_d \langle \bar{s}s \rangle \langle \bar{u}u \rangle \big) \frac{m_0^2}{12M^2} , \qquad (27)
\end{aligned}
$$

where $x = s_0/M^2$, and

$$
E_n(x) = 1 - e^{-x} \sum_{i=0}^{n} \frac{x^i}{i!} .
$$

The contribution of the higher states and continuum in the invariant functions are subtracted by taking into account the following replacements

$$
\begin{aligned}
e^{-m_V^2/4M^2} M^2 \left(\ln \frac{M^2}{\Lambda^2} - \gamma_E \right) &\rightarrow \int_{m_V^2/4}^{s_0} ds e^{-s/M^2} \ln \frac{s - m_V^2/4}{\Lambda^2} \\
e^{-m_V^2/4M^2} \left(\ln \frac{M^2}{\Lambda^2} - \gamma_E \right) &\rightarrow \ln \frac{s_0 - m_V^2/4}{\Lambda^2} e^{-s_0/M^2} + \frac{1}{M^2} \int_{m_V^2/4}^{s_0} ds e^{-s/M^2} \ln \frac{s - m_V^2/4}{\Lambda^2} \\
e^{-m_V^2/4M^2} \frac{1}{M^2} \left(\ln \frac{M^2}{\Lambda^2} - \gamma_E \right) &\rightarrow \frac{1}{M^2} \ln \frac{s_0 - m_V^2/4}{\Lambda^2} e^{-s_0/M^2} + \frac{1}{s_0 - m_V^2/4} e^{-s_0/M^2} \\
&+ \frac{1}{M^4} \int_{m_V^2/4}^{s_0} ds e^{-s/M^2} \ln \frac{s - m_V^2/4}{\Lambda^2} \\
e^{-m_V^2/4M^2} M^{2n} &\rightarrow \frac{1}{\Gamma(n)} \int_{m_V^2/4}^{s_0} ds e^{-s/M^2} \left(s - m_V^2/4 \right)^{n-1} . \qquad (28)
\end{aligned}
$$

3 Numerical analysis and discussion

This section is devoted to the numerical analysis of the sum rules for the vector meson octet–decuplet baryon coupling constants. The main input parameters of the light cone sum rules in our case are the vector meson DA's. The DA's of the vector mesons are given in [10–12]. The values of the leptonic constants f_V and f_V^T, and of the twist–2 and twist–3 parameters $a_i^\parallel$, $a_i^\perp$,$\zeta_{3V}^\parallel$, $\tilde{\lambda}_{3V}^\parallel$, $\tilde{\omega}_{3V}^\parallel$, $\kappa_{3V}^\parallel$, $\omega_{3V}^\parallel$, $\lambda_{3V}^\parallel$, $\kappa_{3V}^\perp$, $\omega_{3V}^\perp$, $\lambda_{3V}^\perp$, as well as twist–4 parameters $\zeta_4^\parallel$, $\tilde{\omega}_4^\parallel$, $\zeta_4^\perp$, $\tilde{\zeta}_4^\perp$, $\kappa_{4V}^\parallel$, $\kappa_{4V}^\perp$ are given in Table (1) and Table (2), respectively, in [12]. The value of the other input parameters which are needed in the sum rule are $\langle \bar{q} q \rangle = -(0.243\ GeV)^3$, $m_0^2 = 0.8$ [8], $\langle g_s^2 G^2 \rangle = 0.47\ GeV^4$ [1].

It should be noted here that, the masses of initial and final baryons are close to each other. Therefore we can choose $M_1^2 = M_2^2 = 2M^2$, and consequently $u_0 = 1/2$. Hence, in further numerical analysis, the values of the DA's only at $u_0 = 1/2$ are needed.

It follows from the explicit expressions of the sum rules for the vector meson decuplet–octet baryon coupling constants that, in addition to the DA's, they also contain three auxiliary parameters, namely, Borel mass parameter, continuum threshold s_0, and the parameter β in the interpolating current. Therefore, we need to find the working regions of those parameters where the results of the vector meson decuplet–octet baryon coupling constants are practically independent of them.

The upper bound of the Borel parameter M^2 can be found by requiring that the higher states and continuum contributions to a correlation function should be less than, say 40% of the total value of the same correlation function. The lower bound of M^2 can be found by requiring that the contribution of the highest term with the power of $1/M^2$ be 20–25% less than that of the highest power of M^2. Using these restrictions, we obtain the working region for the Borel parameters. The continuum threshold is varied in the regions $s_0 = (m_B + 0.5)^2$ and $s_0 = (m_B + 0.7)^2$.

In Fig. (1)–(3) we present the dependence of the couplings g_1, g_2 and g_3 for the $\Sigma^{*+} \to \Sigma^+ \rho^0$ transition at five different values of the parameter β at $s_0 = 4\ GeV^2$. We observe from these figures that these couplings have good stability in the "working" region of M^2. Obviously, the coupling constants are also expected to be independent of the auxiliary parameter β. In order to find the working region of β where these couplings, we depict in Figs. (4)–(5) the dependence of g_1, g_2 and g_3 on $\cos\theta$ for the $\Sigma^{*+} \to \Sigma^+ \rho^0$ transition, where θ is defined as $\tan\theta = \beta$. It follows from these figures that the common working region of β for the coupling constants is $-0.5 < \cos\theta < 0.3$, where these constants exhibit weak dependence on β. Up to now we apply the standard procedure in analysis of the sum rules, i.e., s_0 is chosen to be independent of M^2 and q^2. However, in general the continuum threshold should depend on M^2 and momentum transfer squared q^2 [15]. This dependence leads to, so called, systematic uncertainties, and for this reason, the usual criteria for the stability of the results on M^2 does not provide ordering of the realistic errors. We consider these systematic errors to be about 15% (see [15]). Under these conditions, the result of our analysis predict that $g_1 = 6 \pm 2$, $g_2 = 2 \pm 0.5$ and $g_3 = 20 \pm 4$.

The same coupling constants for the other transitions are presented in Table 1 (here we present results only for the absolute values of the coupling constants). Only those results which could not be obtained by the $SU(2)$ symmetry rotations are presented in this table. The errors presented in Table–1 take into account the uncertainties coming from the variation of M^2, s_0 and β, as well as uncertainties inherit in the input parameters and systematic errors.

From Tables 1–3 we get the following results:

- As far as the coupling constant g_1 is concerned, for the channels presented in Table–1, we observe that there is good agreement between the predictions of the general current and the Ioffe current for the octet baryons. All of the results presented within the limit of errors satisfy the relations among the coupling constants and invariant functions.

- In the case of the coupling constant g_2, there are considerable discrepancies between the predictions of the general current and Ioffe current for the octet baryons for the $\Sigma^{*+} \to \Xi^0 K^{*+}$, $\Omega^- \to \Xi^0 K^{*-}$, $\Delta^+ \to \Sigma^0 K^{*+}$, $\Delta^{++} \to \Sigma^+ K^{*+}$, $\Xi^{*0} \to \Lambda \bar{K}^{*0}$, $\Delta^- \to \Sigma^- \bar{K}^{*0}$ and $\Xi^{*0} \to \Sigma^+ K^{*-}$ channels. The characteristic property of all these channels is that they all involve the K^* meson. Note that, for these channels, the relations among the coupling constants and invariant functions are also strongly violated. In the general current case all these discrepancies can be attributed to not having stable region for $\cos\theta$. In other words the results for the coupling constants of these transitions are not reliable. For the remaining transitions the results for both cases are close to each other within the error limits.

- For the coupling constant g_3 our results can be summarized as follows: The relations between the coupling constant g_3 and the invariant function Π_1 are strongly violated in the channels $\Sigma^{*-} \to \Lambda \rho^-$, $\Xi^{*0} \to \Xi^0 \rho^0$, as well as in the channels involving ϕ meson. These discrepancies, as is the case for the g_2 coupling constant, can be attributed to the absence of the stability region g_3 with respect to the $\cos\theta$. For this reason, for the above–mentioned transitions, the predictions for the coupling constant g_3 are not reliable.

The gap between the results of the two currents can be attributed to the fact that, for many transitions the value $\beta = -1$ lies outside the stability region of β, causing considerable discrepancies between the predictions of the two currents.

In conclusion, the strong coupling constant of the vector mesons with decuplet–octet baryons are studied within LQSR. It is shown that all coupling constants can be calculated in terms of a single universal function.

g_1 channel	General current		Ioffe current	
	Result	$SU(3)_f$	Result	$SU(3)_f$
$g_1^{\Sigma^{*+}\to\Sigma^+\rho^0}$	-5.6 ± 1.6	-5.7	-7.8 ± 0.5	-7.8
$g_1^{\Delta^0\to p\rho^-}$	9.1 ± 2.9	8	12.2 ± 0.8	11
$g_1^{\Xi^{*0}\to\Xi^0\rho^0}$	-5.5 ± 1.5	-5.7	-7.2 ± 0.3	-7.8
$g_1^{\Sigma^{*-}\to\Lambda\rho^-}$	-10 ± 3	-9.8	-13.5 ± 0.7	-13.5
$g_1^{\Delta^+\to\Sigma^0K^{*+}}$	-13.1 ± 4.1	-11.3	-18.4 ± 1.4	-15.6
$g_1^{\Sigma^{*+}\to\Xi^0K^{*+}}$	8.9 ± 2.5	8	12 ± 0.5	11
$g_1^{\Sigma^{*+}\to p\bar{K}^{*0}}$	-9.5 ± 2.7	-8	-12.4 ± 0.9	-11
$g_1^{\Omega^-\to\Xi^0K^{*-}}$	-14 ± 4	-13.9	-18.8 ± 1.4	-19.1
$g_1^{\Xi^{*0}\to\Sigma^+K^{*-}}$	-8.6 ± 2.5	-8	-11.6 ± 0.9	-11
$g_1^{\Xi^{*0}\to\Lambda\bar{K}^{*0}}$	10.8 ± 3.0	9.8	14.1 ± 0.9	13.5
$g_1^{\Sigma^{*+}\to\Sigma^+\omega}$	-5.1 ± 1.4	-5.7	-6.9 ± 0.4	-7.8
$g_1^{\Xi^{*0}\to\Xi^0\omega}$	-4.9 ± 1.3	-5.7	-6.5 ± 0.5	-7.8
$g_1^{\Sigma^{*+}\to\Sigma^+\phi}$	8.0 ± 2.8	8	11.6 ± 0.7	11
$g_1^{\Xi^{*0}\to\Xi^0\phi}$	7.1 ± 1.9	8	10.9 ± 0.8	11

Table 2: The values of the coupling constant g_1 for various channels.

g_2 channel	General current		Ioffe current	
	Result	$SU(3)_f$	Result	$SU(3)_f$
$g_2^{\Sigma^{*+}\to\Sigma^+\rho^0}$	-2.3 ± 1	-2.3	-3.6 ± 0.6	-3.8
$g_2^{\Delta^0\to p\rho^-}$	3.3 ± 1.5	3.2	5.3 ± 1.0	5.4
$g_2^{\Xi^{*0}\to\Xi^0\rho^0}$	-2.2 ± 1	-2.3	-3.4 ± 0.5	-3.8
$g_2^{\Sigma^{*-}\to\Lambda\rho^-}$	-4 ± 1.7	-3.9	-6.3 ± 1.2	-6.6
$g_2^{\Delta^+\to\Sigma^0K^{*+}}$	-16.4 ± 6	-4.5	-27.3 ± 7.3	-7.6
$g_2^{\Sigma^{*+}\to\Xi^0K^{*+}}$	11.1 ± 3	3.2	16.9 ± 3.5	5.4
$g_2^{\Sigma^{*+}\to p\bar{K}^{*0}}$	-10.8 ± 4.8	-3.2	-17.9 ± 4.7	-5.4
$g_2^{\Omega^-\to\Xi^0K^{*-}}$	-17.3 ± 6	-5.5	-27.7 ± 6.0	-9.4
$g_2^{\Xi^{*0}\to\Sigma^+K^{*-}}$	-6.3 ± 4.0	-3.2	-16.1 ± 3.4	-5.4
$g_2^{\Xi^{*0}\to\Lambda\bar{K}^{*0}}$	12.4 ± 4.8	3.9	19.2 ± 3.9	6.6
$g_2^{\Sigma^{*+}\to\Sigma^+\omega}$	-2.0 ± 0.7	-2.3	-3.2 ± 0.6	-3.8
$g_2^{\Xi^{*0}\to\Xi^0\omega}$	-1.9 ± 0.6	-2.3	-3.0 ± 0.4	-3.8
$g_2^{\Sigma^{*+}\to\Sigma^+\phi}$	4.4 ± 1.8	3.2	7.2 ± 1.5	5.4
$g_2^{\Xi^{*0}\to\Xi^0\phi}$	4.2 ± 2	3.2	6.7 ± 1.1	5.4

Table 3: The values of the coupling constant g_2 for various channels.

g_3 channel	General current		Ioffe current	
	Result	$SU(3)_f$	Result	$SU(3)_f$
$g_3^{\Sigma^{*+}\to\Sigma^+\rho^0}$	31.4±3.5	12.9	27.4±1.8	18.7
$g_3^{\Delta^0\to p\rho^-}$	−33.5±3.1	−18.2	−35.2±2.6	−26.4
$g_3^{\Xi^{*0}\to\Xi^0\rho^0}$	16.2±3.8	12.9	27.9±1.6	18.7
$g_3^{\Sigma^{*-}\to\Lambda\rho^-}$	36.5±8.5	22.3	45.7±2.6	32.3
$g_3^{\Delta^+\to\Sigma^0K^{*+}}$	22.6±2.4	25.7	19.6±1.9	37.3
$g_3^{\Sigma^{*+}\to\Xi^0K^{*+}}$	−17.5±2.3	−18.2	−17.3±1.8	−26.4
$g_3^{\Sigma^{*+}\to p\bar{K}^{*0}}$	14.6±1.7	18.2	12.6±2.6	26.4
$g_3^{\Omega^-\to\Xi^0K^{*-}}$	33.5±3	31.5	31.4±2.4	45.7
$g_3^{\Xi^{*0}\to\Sigma^+K^{*-}}$	17.3±0.7	18.2	16.5±1.4	26.4
$g_3^{\Xi^{*0}\to\Lambda\bar{K}^{*0}}$	−36.5±8.2	−22.3	−45.5±1.7	−32.3
$g_3^{\Sigma^{*+}\to\Sigma^+\omega}$	23.9±6.5	12.9	27.9±2.1	18.7
$g_3^{\Xi^{*0}\to\Xi^0\omega}$	19.1±4.3	12.9	23.9±1.3	18.7
$g_3^{\Sigma^{*+}\to\Sigma^+\phi}$	−24.2±6.2	−18.2	−31.7±2.1	−26.4
$g_3^{\Xi^{*0}\to\Xi^0\phi}$	−16.2±3.8	−18.2	−33.2±1.7	−26.4

Table 4: The values of the coupling constant g_3 for various channels.

Appendix A :

In this appendix we present the relations among the correlation functions involving ρ, ω, ϕ and K^* mesons.

- The vertices involving ρ meson.

$$
\begin{aligned}
\Pi^{\Sigma^{*0}\to\Lambda\rho^0} &= -\frac{1}{\sqrt{6}}[2\Pi_1(u,s,d)+\Pi_1(u,d,s)-\Pi_1(d,u,s)-2\Pi_1(d,s,u)] \ , \\
\Pi^{\Delta^-\to n\rho^-} &= 2\sqrt{3}\Pi_1(d,d,d) \ , \\
\Pi^{\Delta^{++}\to p\rho^+} &= -2\sqrt{3}\Pi_1(u,u,u) \ . \qquad \text{(A.1)}
\end{aligned}
$$

- The vertices involving K^* meson.

$$
\begin{aligned}
\Pi^{\Delta^+\to\Sigma^0K^{*+}} &= -\sqrt{2}[\Pi_1(s,u,d)-\Pi_1(s,d,u)] \ , \\
\Pi^{\Delta^+\to\Lambda K^{*+}} &= \frac{\sqrt{2}}{\sqrt{3}}[\Pi_1(s,d,u)-\Pi_1(s,u,d)] \ , \\
\Pi^{\Delta^0\to\Sigma^-K^{*+}} &= -2\Pi_1(s,d,d) \ , \\
\Pi^{\Sigma^{*+}\to\Xi^0K^{*+}} &= 2\Pi_1(u,s,u) \ , \\
\Pi^{\Sigma^{*0}\to\Xi^-K^{*+}} &= \sqrt{2}\Pi_1(u,s,d) \ , \\
\Pi^{\Delta^{++}\to\Sigma^+K^{*+}} &= -2\sqrt{3}\Pi_1(s,u,u) \ , \\
\Pi^{\Sigma^{*0}\to pK^{*-}} &= \sqrt{2}\Pi_1(u,u,d) \ , \\
\Pi^{\Omega^-\to\Xi^0K^{*-}} &= -\sqrt{2}\Pi_1(u,d,s) \ , \\
\Pi^{\Sigma^{*-}\to nK^{*-}} &= 2\sqrt{2}\Pi_1(s,d,d) \ , \\
\Pi^{\Xi^{*0}\to\Sigma^+K^{*-}} &= -2\Pi_1(s,d,d) \ , \\
\Pi^{\Xi^{*-}\to\Sigma^0K^{*-}} &= \sqrt{2}\Pi_1(u,d,s) \ , \\
\Pi^{\Xi^{*-}\to\Lambda K^{*-}} &= -\frac{\sqrt{2}}{\sqrt{3}}[2\Pi_1(u,s,d)+\Pi_1(u,d,s)] \ , \\
\Pi^{\Xi^{*0}\to\Sigma^0K^{*0}} &= \sqrt{2}\Pi_1(d,u,s) \ , \\
\Pi^{\Xi^{*0}\to\Lambda K^{*0}} &= \frac{\sqrt{2}}{\sqrt{3}}[2\Pi_1(d,s,u)+\Pi_1(d,u,s)] \ , \\
\Pi^{\Xi^{*-}\to\Sigma^-K^{*0}} &= 2\Pi_1(d,d,s) \ , \\
\Pi^{\Sigma^{*0}\to nK^{*0}} &= -\sqrt{2}\Pi_1(d,d,u) \ , \\
\Pi^{\Omega^-\to\Xi^-K^{*0}} &= 2\sqrt{3}\Pi_1(s,s,s) \ , \\
\Pi^{\Sigma^{*+}\to pK^{*0}} &= -2\Pi_1(s,u,u) \ , \\
\Pi^{\Sigma^{*0}\to\Xi^0\bar{K}^{*0}} &= \sqrt{2}\Pi_1(d,s,u) \ ,
\end{aligned}
$$

$$
\begin{aligned}
\Pi^{\Delta^- \to \Sigma^- \bar{K}^{*0}} &= -2\sqrt{3}\Pi_1(d,d,d) \ , \\
\Pi^{\Sigma^{*-} \to \Xi^- \bar{K}^{*0}} &= -2\Pi_1(d,s,d) \ , \\
\Pi^{\Delta^0 \to \Sigma^0 \bar{K}^{*0}} &= -\sqrt{2}[\Pi_1(s,d,u) + \Pi_1(s,u,d)] \ , \\
\Pi^{\Delta^0 \to \Lambda \bar{K}^{*0}} &= \frac{\sqrt{2}}{\sqrt{3}}[\Pi_1(s,d,u) - \Pi_1(s,u,d)] \ , \\
\Pi^{\Delta^+ \to \Sigma^+ \bar{K}^{*0}} &= 2\Pi_1(s,u,u) \ .
\end{aligned} \tag{A.2}
$$

- Finally, we present the expressions of the vertices involving ω and ϕ mesons.

$$
\begin{aligned}
\Pi^{\Sigma^{*0} \to \Sigma^0 \omega} &= \frac{1}{\sqrt{2}}[\Pi_1(u,d,s) + \Pi_1(d,u,s)] \ , \\
\Pi^{\Sigma^{*+} \to \Sigma^+ \omega} &= -\sqrt{2}\Pi_1(u,u,s) \ , \\
\Pi^{\Sigma^{*-} \to \Sigma^- \omega} &= \sqrt{2}\Pi_1(d,d,s) \ , \\
\Pi^{\Delta^+ \to p\omega} &= -\sqrt{2}[\Pi_1(d,u,u) - \Pi_1(u,u,d)] \ , \\
\Pi^{\Delta^0 \to n\omega} &= \sqrt{2}[\Pi_1(u,d,d) - \Pi_1(d,d,u)] \ , \\
\Pi^{\Xi^{*0} \to \Xi^0 \omega} &= -\sqrt{2}\Pi_1(u,s,s) \ , \\
\Pi^{\Xi^{*-} \to \Xi^- \omega} &= \sqrt{2}\Pi_1(d,s,s) \ , \\
\Pi^{\Sigma^{*0} \to \Lambda \omega} &= -\frac{1}{\sqrt{6}}[2\Pi_1(u,s,d) - 2\Pi_1(d,s,u) - \Pi_1(d,u,s) + \Pi_1(u,d,s)] \ , \\
\Pi^{\Sigma^{*0} \to \Sigma^0 \phi} &= -[\Pi_1(s,d,u) + \Pi_1(s,u,d)] \ , \\
\Pi^{\Sigma^{*+} \to \Sigma^+ \phi} &= 2\Pi_1(s,u,u) \ , \\
\Pi^{\Sigma^{*-} \to \Sigma^- \phi} &= -2\Pi_1(s,d,d) \ , \\
\Pi^{\Delta^+ \to p\phi} &= \Pi^{\Delta^0 \to n\phi} = 0 \ , \\
\Pi^{\Xi^{*0} \to \Xi^0 \phi} &= 2\Pi_1(s,s,u) \ , \\
\Pi^{\Xi^{*-} \to \Xi^- \phi} &= -2\Pi_1(s,s,d) \ , \\
\Pi^{\Sigma^{*0} \to \Lambda \phi} &= -\frac{1}{\sqrt{3}}[\Pi_1(s,u,d) - \Pi_1(s,d,u)] \ .
\end{aligned} \tag{A.3}
$$

Obviously, in the $SU(2)$ symmetry limit, $\Pi^{\Delta^+ \to \Lambda K^{*+}}$, $\Pi^{\Delta^0 \to \Lambda \bar{K}^{*0}}$, $\Pi^{\Delta^+ \to p\omega}$, $\Pi^{\Delta^0 \to n\omega}$, $\Pi^{\Sigma^{*0} \to \Lambda\omega}$ and $\Pi^{\Sigma^{*0} \to \Lambda\phi}$ are all equal to zero. Note that, in calculating $\Pi^{D \to OV}$ using Eqs. (A.1)–(A.3), one should use the wave functions of the corresponding vector meson in Π_1.

Appendix B :

In this appendix we present the DA's of the vector mesons appearing in the matrix elements $\langle V(q)\,|\bar{q}(x)\Gamma q(0)|\,0\rangle$ and $\langle V(q)\,|\bar{q}(x)G_{\mu\nu}q(0)|\,0\rangle$, up to twist–4 accuracy [10–12]:

$$
\begin{aligned}
\langle V(q,\lambda)\,|\bar{q}_1(x)\gamma_\mu q_2(0)|\,0\rangle &= f_V m_V \Bigg\{ \frac{\varepsilon^\lambda\cdot x}{q\cdot x} q_\mu \int_0^1 du e^{i\bar{u}qx} \left[\phi_\parallel(u) + \frac{m_V^2 x^2}{16} A_\parallel(u)\right] \\
&+ \left(\varepsilon_\mu^\lambda - q_\mu \frac{\varepsilon^\lambda\cdot x}{q\cdot x}\right) \int_0^1 du e^{i\bar{u}qx} g_\perp^v(u) \\
&- \frac{1}{2} x_\mu \frac{\varepsilon^\lambda\cdot x}{(q\cdot x)^2} m_V^2 \int_0^1 du e^{i\bar{u}qx} \left[g_3(u) + \phi_\parallel(u) - 2 g_\perp^v(u)\right] \Bigg\}~, \\
\langle V(q,\lambda)\,|\bar{q}_1(x)\gamma_\mu\gamma_5 q_2(0)|\,0\rangle &= -\frac{1}{4} \epsilon_\mu^{\nu\alpha\beta} \varepsilon^\lambda q_\alpha x_\beta f_V m_V \int_0^1 du e^{i\bar{u}qx} g_\perp^a(u)~, \\
\langle V(q,\lambda)\,|\bar{q}_1(x)\sigma_{\mu\nu} q_2(0)|\,0\rangle &= -i f_V^T \Bigg\{ (\varepsilon_\mu^\lambda q_\nu - \varepsilon_\nu^\lambda q_\mu) \int_0^1 du e^{i\bar{u}qx} \left[\phi_\perp(u) + \frac{m_V^2 x^2}{16} A_\perp(u)\right] \\
&+ \frac{\varepsilon^\lambda\cdot x}{(q\cdot x)^2} (q_\mu x_\nu - q_\nu x_\mu) \int_0^1 du e^{i\bar{u}qx} \left[h_\parallel^t - \frac{1}{2}\phi_\perp - \frac{1}{2} h_3(u)\right] \\
&+ \frac{1}{2} (\varepsilon_\mu^\lambda x_\nu - \varepsilon_\nu^\lambda x_\mu) \frac{m_V^2}{q\cdot x} \int_0^1 du e^{i\bar{u}qx} \left[h_3(u) - \phi_\perp(u)\right] \Bigg\}~, \\
\langle V(q,\lambda)\,|\bar{q}_1(x)\sigma_{\alpha\beta} g G_{\mu\nu}(ux) q_2(0)|\,0\rangle &= f_V^T m_V^2 \frac{\varepsilon^\lambda\cdot x}{2 q\cdot x} \left[q_\alpha q_\mu g_{\beta\nu}^\perp - q_\beta q_\mu g_{\alpha\nu}^\perp - q_\alpha q_\nu g_{\beta\mu}^\perp + q_\beta q_\nu g_{\alpha\mu}^\perp\right] \\
&\times \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + u\alpha_g) qx} \mathcal{T}(\alpha_i) \\
&+ f_V^T m_V^2 \left[q_\alpha \varepsilon_\mu^\lambda g_{\beta\nu}^\perp - q_\beta \varepsilon_\mu^\lambda g_{\alpha\nu}^\perp - q_\alpha \varepsilon_\nu^\lambda g_{\beta\mu}^\perp + q_\beta \varepsilon_\nu^\lambda g_{\alpha\mu}^\perp\right] \\
&\times \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + u\alpha_g) qx} \mathcal{T}_1^{(4)}(\alpha_i) \\
&+ f_V^T m_V^2 \left[q_\mu \varepsilon_\alpha^\lambda g_{\beta\nu}^\perp - q_\mu \varepsilon_\beta^\lambda g_{\alpha\nu}^\perp - q_\nu \varepsilon_\alpha^\lambda g_{\beta\mu}^\perp + q_\nu \varepsilon_\beta^\lambda g_{\alpha\mu}^\perp\right] \\
&\times \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + u\alpha_g) qx} \mathcal{T}_2^{(4)}(\alpha_i) \\
&+ \frac{f_V^T m_V^2}{q\cdot x} \Big[q_\alpha q_\mu \varepsilon_\beta^\lambda x_\nu - q_\beta q_\mu \varepsilon_\alpha^\lambda x_\nu - q_\alpha q_\nu \varepsilon_\beta^\lambda x_\mu + q_\beta q_\nu \varepsilon_\alpha^\lambda x_\mu \\
&\times \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + u\alpha_g) qx} \mathcal{T}_3^{(4)}(\alpha_i) \\
&+ \frac{f_V^T m_V^2}{q\cdot x} \Big[q_\alpha q_\mu \varepsilon_\nu^\lambda x_\beta - q_\beta q_\mu \varepsilon_\nu^\lambda x_\alpha - q_\alpha q_\nu \varepsilon_\mu^\lambda x_\beta + q_\beta q_\nu \varepsilon_\mu^\lambda x_\alpha \\
&\times \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + u\alpha_g) qx} \mathcal{T}_4^{(4)}(\alpha_i)~,
\end{aligned}
$$

$$
\begin{aligned}
\langle V(q,\lambda)\,|\bar{q}_1(x) g_s G_{\mu\nu}(ux) q_2(0)|\,0\rangle &= -i f_V^T m_V (\varepsilon_\mu^\lambda q_\nu - \varepsilon_\nu^\lambda q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + u\alpha_g)qx} \mathcal{S}(\alpha_i)\ , \\
\left\langle V(q,\lambda) \left| \bar{q}_1(x) g_s \widetilde{G}_{\mu\nu}(ux) \gamma_5 q_2(0) \right| 0 \right\rangle &= -i f_V^T m_V (\varepsilon_\mu^\lambda q_\nu - \varepsilon_\nu^\lambda q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + u\alpha_g)qx} \widetilde{\mathcal{S}}(\alpha_i)\ , \\
\left\langle V(q,\lambda) \left| \bar{q}_1(x) g_s \widetilde{G}_{\mu\nu}(ux) \gamma_\alpha \gamma_5 q_2(0) \right| 0 \right\rangle &= f_V m_V q_\alpha (\varepsilon_\mu^\lambda q_\nu - \varepsilon_\nu^\lambda q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + u\alpha_g)qx} \mathcal{A}(\alpha_i)\ , \\
\langle V(q,\lambda)\,|\bar{q}_1(x) g_s G_{\mu\nu}(ux) i\gamma_\alpha q_2(0)|\,0\rangle &= f_V m_V q_\alpha (\varepsilon_\mu^\lambda q_\nu - \varepsilon_\nu^\lambda q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + u\alpha_g)qx} \mathcal{V}(\alpha_i)\ , \qquad \text{(B.1)}
\end{aligned}
$$

where $\widetilde{G}_{\mu\nu} = (1/2)\epsilon_{\mu\nu\alpha\beta} G^{\alpha\beta}$ is the dual gluon field strength tensor, and $\int \mathcal{D}\alpha_i = \int d\alpha_q d\alpha_{\bar{q}} d\alpha_g \delta(1 - \alpha_q - \alpha_{\bar{q}} - \alpha_g)$.

Appendix C :

$$
\begin{aligned}
\Pi_1^{(1)} &= \frac{M^4}{192\sqrt{3}\pi^2}\Big\{-12f_V^\perp[m_s(1+\beta)+2m_d\beta]\phi_V^\perp(u_0) \\
&+ f_V^\parallel m_V(1-\beta)[8i_3(\mathcal{A},1-v)+12\psi_{3;V}^\perp(u_0)-\psi_{3;V}^{\perp\prime}(u_0)-2\mathbb{B}(u_0)]\Big\} \\
&+ \left(\gamma_E-\ln\frac{M^2}{\Lambda^2}\right)\Big\{\frac{f_V^\perp m_V^2 M^2}{24\sqrt{3}\pi^2}\Big([m_s(1+3\beta)-2m_d\beta]i_2(\mathcal{S},1) \\
&+ [m_s(3+\beta)-2m_d]i_2(\widetilde{\mathcal{S}},1)+2[m_d(1-\beta)+m_s(1+2\beta)]i_2(\mathcal{T}_1,1) \\
&+ 2[m_d(1-\beta)-m_s(2+\beta)]i_2(\mathcal{T}_2,1)+[2m_d\beta+m_s(1+\beta)]i_2(\mathcal{T}_3,1) \\
&+ [2m_d\beta-m_s(1+3\beta)]i_2(\mathcal{T}_4,1)\Big) \\
&- \frac{f_V^\perp m_V^4}{24\sqrt{3}\pi^2}\Big([2m_d\beta+m_s(1+\beta)]i_1(\mathcal{T}_3,1)+[2m_d\beta-m_s(1+3\beta)]i_1(\mathcal{T}_4,1)\Big) \\
&- \frac{f_V^\perp\langle g_s^2G^2\rangle}{384\sqrt{3}\pi^2M^2}\Big(m_V^2[2m_d-m_s(3+\beta)]\widetilde{i}_4(\mathbb{C}_T)-2M^2[2m_d\beta+m_s(1+\beta)]\phi_V^\perp(u_0)\Big)\Big\} \\
&+ \frac{f_V^\parallel m_0^2 m_V\langle g_s^2G^2\rangle}{144\sqrt{3}M^6}\beta[m_d\langle\bar{s}s\rangle+m_s\langle\bar{d}d\rangle]\psi_{3;V}^\perp(u_0) \\
&- \frac{m_V}{1728\sqrt{3}\pi^2M^4}\Big\{3f_V^\perp m_V^3\langle g_s^2G^2\rangle[2m_d\beta-m_s(1+3\beta)]i_1(\mathcal{T}_4,1) \\
&+ 4\pi^2 f_V^\parallel\Big(\langle\bar{s}s\rangle[m_0^2m_sm_V^2(1-\beta)-6\langle g_s^2G^2\rangle m_d\beta]+\langle\bar{d}d\rangle[m_0^2m_dm_V^2(1-\beta) \\
&- 6\langle g_s^2G^2\rangle m_s\beta]+m_0^2m_V^2(1-\beta)[m_d\langle\bar{d}d\rangle+m_s\langle\bar{s}s\rangle]\Big)\psi_{3;V}^\perp(u_0)\Big\} \\
&+ \frac{f_V^\perp m_V^2\langle g_s^2G^2\rangle}{576\sqrt{3}\pi^2M^2}\Big\{2\beta(2m_d-m_s)i_2(\mathcal{S},1)+2[m_d(1+\beta)-m_s]i_2(\widetilde{\mathcal{S}},1-2v) \\
&- 2\beta m_d i_2(\mathcal{T}_1,1+2v)-2m_s(1+\beta)i_2(\mathcal{T}_1,v)+m_s(1+3\beta)i_2(\mathcal{T}_1,1) \\
&+ [m_s(3+\beta)-2m_d]i_2(\mathcal{T}_2,1)+4[m_d(1+\beta)-m_s]i_2(\mathcal{T}_2,v) \\
&+ [m_s(1+\beta)+2\beta m_d]i_2(\mathcal{T}_3,1-2v)+4\beta m_d i_2(\mathcal{T}_4,v) \\
&- 2m_s[i_2(\mathcal{T}_4,1-v)+\beta i_2(\mathcal{T}_4,2-v)]\Big\} \\
&- \frac{1}{432\sqrt{3}M^2}\Big\{24f_V^\perp m_V^4[2\beta\langle\bar{d}d\rangle-(1+3\beta)\langle\bar{s}s\rangle]i_1(\mathcal{T}_4,1) \\
&+ 6f_V^\parallel m_V^3(1-\beta)\Big([m_d\langle\bar{d}d\rangle-m_s\langle\bar{s}s\rangle]i_2(\mathcal{V},5-4v)-[m_d\langle\bar{d}d\rangle+m_s\langle\bar{s}s\rangle]i_2(\mathcal{A},5-6v)\Big) \\
&- f_V^\parallel m_0^2m_V(1-\beta)[m_d\langle\bar{d}d\rangle+m_s\langle\bar{s}s\rangle][2\mathbb{B}(u_0)+\psi_{3;V}^{\perp\prime}(u_0)] \\
&+ 2f_V^\parallel m_V\Big(2m_0^2(1+\beta)[m_d\langle\bar{s}s\rangle+m_s\langle\bar{d}d\rangle]-9m_V^2(1-\beta)(m_d\langle\bar{d}d\rangle+m_s\langle\bar{s}s\rangle)\Big)\psi_{3;V}^\perp(u_0)\Big\} \\
&- \frac{f_V^\perp m_V^2}{9216\sqrt{3}\pi^2M^2}\Big\{16\Big(\langle g_s^2G^2\rangle[2m_d-m_s(3+\beta)] \\
&- 2\pi^2m_0^2[\langle\bar{d}d\rangle(5-\beta)-2\langle\bar{s}s\rangle(5+2\beta)]\Big)\widetilde{i}_4(\mathbb{C}_T)+9\langle g_s^2G^2\rangle[2\beta m_d+m_s(1+\beta)]\mathbb{A}_T(u_0)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{M^2}{96\sqrt{3}\pi^2}\Big\{ f_V^\perp m_V^2 \Big(-3[2\beta m_d + m_s(1+\beta)]\mathbb{A}_T(u_0) + 4[6\beta m_d - m_s(1+5\beta)]i_2(\mathcal{S},1) \\
&+ 4[2m_d\beta + m_s(1+\beta)]i_2(\widetilde{\mathcal{S}},1) - 16[m_d(1+\beta) - m_s]i_2(\widetilde{\mathcal{S}},v) \\
&+ 4[2m_d + m_s(3+7\beta)]i_2(\mathcal{T}_1,1) - 16m_d\beta i_2(\mathcal{T}_1,1+v) - 8m_s(1+\beta)i_2(\mathcal{T}_1,v) \\
&- 4[2m_d\beta + m_s(1+\beta)]i_2(\mathcal{T}_2,1) - 16[m_d(1+\beta) - m_s]i_2(\mathcal{T}_2,v) \\
&+ 8[2m_d\beta + m_s(1+\beta)]i_2(\mathcal{T}_3,1-v) + 4[2m_d\beta - m_s(3+7\beta)]i_2(\mathcal{T}_4,1) \\
&+ 8[2m_d\beta + m_s(1+\beta)]i_2(\mathcal{T}_4,v) - 3[2m_d - m_s(3+\beta)]\widetilde{i}_4(\mathbb{C}_T)\Big) \\
&- 16\pi^2 f_V^\perp[2\beta\langle\bar{d}d\rangle + \langle\bar{s}s\rangle(1+\beta)]\phi_V^\perp(u_0) - f_V^\parallel m_V^3(1-\beta)[2i_2(\mathcal{A},5-6v) + \psi_{3;V}^\perp(u_0)]\Big\} \\
&- \frac{m_V f_V^\parallel}{144\sqrt{3}}\Big\{ 2(1-\beta)[m_d\langle\bar{d}d\rangle + m_s\langle\bar{s}s\rangle][3\mathbb{B}(u_0) - 2i_3(\mathcal{A},1-v) + \psi_{3;V}^{\perp\prime}(u_0)] \\
&+ 4(1-\beta)[m_d\langle\bar{d}d\rangle - m_s\langle\bar{s}s\rangle]i_3(\mathcal{V},1-v) \\
&- 12\Big([4\beta m_d + (1-\beta)m_s]\langle\bar{s}s\rangle + [(1-\beta)m_d + 4\beta m_s]\langle\bar{d}d\rangle\Big)\psi_{3;V}^\perp(u_0)\Big\} \\
&- \frac{f_V^\perp m_V^2}{72\sqrt{3}}\Big\{[2\beta\langle\bar{d}d\rangle + (1+\beta)\langle\bar{s}s\rangle][3\mathbb{A}_T(u_0) + 8i_2(\mathcal{T}_1,v) - 4i_2(\mathcal{T}_3,1-2v) - 8i_2(\mathcal{T}_4,v)] \\
&- 8\beta[2\langle\bar{d}d\rangle - \langle\bar{s}s\rangle]i_2(\mathcal{S},1) - 8[(1+\beta)\langle\bar{d}d\rangle - \langle\bar{s}s\rangle]i_2(\widetilde{\mathcal{S}},1-2v) \\
&+ 4[2\beta\langle\bar{d}d\rangle - (1+3\beta)\langle\bar{s}s\rangle]i_2(\mathcal{T}_1,1) + 4[2\langle\bar{d}d\rangle - (3+\beta)\langle\bar{s}s\rangle]i_2(\mathcal{T}_2,1) \\
&- 16[(1+\beta)\langle\bar{d}d\rangle - \langle\bar{s}s\rangle]i_2(\mathcal{T}_2,v) + 8(1+2\beta)\langle\bar{s}s\rangle i_2(\mathcal{T}_4,1) + 6[2\langle\bar{d}d\rangle - (3+\beta)\langle\bar{s}s\rangle]\widetilde{i}_4(\mathbb{C}_T) \\
&- \frac{3m_V^2}{\pi^2}\Big([2\beta m_d + (1+\beta)m_s]i_1(\mathcal{T}_3,1) + 2[2\beta m_d - (1+3\beta)m_s]i_1(\mathcal{T}_4,1)\Big)\Big\} \\
&+ \frac{f_V^\perp}{864\sqrt{3}\pi^2}\Big\{3\langle g_s^2G^2\rangle[2\beta m_d + (1+\beta)m_s] - 40\pi^2 m_0^2[2\beta\langle\bar{d}d\rangle + (1+\beta)\langle\bar{s}s\rangle]\Big\}\phi_V^\perp(u_0)\ ,
\end{aligned}
\tag{C.1}
$$

$$
\begin{aligned}
\Pi_1^{(2)} &= \Big(\gamma_E - \ln\frac{M^2}{\Lambda^2}\Big)\Big\{ -\frac{f_V^\perp m_V^2}{12\sqrt{3}\pi^2}(1-\beta)[(2m_d - m_s)i_1(\mathcal{T},1) - 2m_d i_1(\mathcal{T}_3,1) \\
&- 2(m_d - m_s)i_1(\mathcal{T}_4,1)] - \frac{f_V^\perp m_V^2\langle g_s^2G^2\rangle}{96\sqrt{3}\pi^2 M^4}[2m_d - m_s(3+\beta)]\widetilde{\widetilde{i}}_4(\mathbb{B}_T)\Big\} \\
&+ \frac{f_V^\parallel m_V^3 m_0^2}{18\sqrt{3}M^6}\Big\{[m_d\langle\bar{s}s\rangle(3+\beta) - m_s\langle\bar{d}d\rangle(1+\beta)]i_0(\Psi,1) \\
&+ [m_d\langle\bar{s}s\rangle(1+3\beta) - m_s\langle\bar{d}d\rangle(1+\beta)]i_0(\widetilde{\Psi},1)\Big\} \\
&- \frac{f_V^\perp m_V^2\langle g_s^2G^2\rangle}{288\sqrt{3}\pi^2 M^4}\Big\{(m_d - 2m_s)(1+\beta)[i_1(\mathcal{T},1) - 2i_1(\mathcal{T}_4,1)] \\
&- [2m_d - m_s(3+\beta)](3+\beta)[i_1(\mathcal{T},v) - 2i_1(\mathcal{T}_4,v)]\Big\} \\
&+ \frac{2f_V^\parallel m_V^3}{9\sqrt{3}M^4}[m_d\langle\bar{s}s\rangle(3+\beta) - m_s\langle\bar{d}d\rangle(1+\beta)]i_0(\Psi,1)
\end{aligned}
$$

$$
\begin{aligned}
&+ \frac{f_V^{\parallel} m_V^3}{144\sqrt{3}M^4}(1-\beta)\Big\{4[m_d\langle\bar{d}d\rangle - m_s\langle\bar{s}s\rangle][i_1(\Phi,1) + i_1(\widetilde{\Psi},1)] \\
&- [m_d\langle\bar{d}d\rangle + m_s\langle\bar{s}s\rangle][4i_1(\widetilde{\Phi},1) + 4i_1(\Psi,1-2v) + 3\widetilde{i}_4(\mathbb{A})] \\
&+ 8[2m_d\langle\bar{d}d\rangle - m_s\langle\bar{s}s\rangle]i_1(\Phi,v) + 8[m_d\langle\bar{d}d\rangle + 2m_s\langle\bar{s}s\rangle]i_1(\widetilde{\Phi},v)\Big\} \\
&+ \frac{f_V^{\parallel} m_V m_0^2}{216\sqrt{3}M^4}[m_d\langle\bar{d}d\rangle + m_s\langle\bar{s}s\rangle](1-\beta)[2\widetilde{i}_4(\mathbb{B}) - 2\widetilde{i}_4(\phi_V^{\parallel}) + \psi_{3;V}^{\perp}(u_0)] \\
&+ \frac{f_V^{\perp} m_V^2}{432\sqrt{3}\pi^2 M^4}\Big\{\langle g_s^2 G^2\rangle[6m_d - 3m_s(3+\beta)] - 8\pi^2 m_0^2[\langle\bar{d}d\rangle(5-\beta) - 2\langle\bar{s}s\rangle(5+2\beta)]\Big\}\widetilde{\widetilde{i}}_4(\mathbb{B}_T) \\
&+ \frac{f_V^{\parallel} m_V}{72\sqrt{3}M^2}(1-\beta)\Big\{4[m_d\langle\bar{d}d\rangle i_2(\mathcal{A},v) - m_s\langle\bar{s}s\rangle i_2(\mathcal{A},1)] \\
&- 4[m_d\langle\bar{d}d\rangle - 2m_s\langle\bar{s}s\rangle]i_2(\mathcal{V},v) - 4m_s\langle\bar{s}s\rangle i_2(\mathcal{V},1) \\
&- 3[m_d\langle\bar{d}d\rangle + m_s\langle\bar{s}s\rangle][2\widetilde{i}_4(\mathbb{B}) - 2\widetilde{i}_4(\phi_V^{\parallel}) + \psi_{3;V}^{\perp}(u_0)]\Big\} \\
&- \frac{f_V^{\perp} m_V^2}{9\sqrt{3}M^2}\Big\{(1+\beta)[\langle\bar{d}d\rangle - 2\langle\bar{s}s\rangle][i_1(\mathcal{T},1) - i_1(\mathcal{T}_4,1)] \\
&- [2\langle\bar{d}d\rangle - \langle\bar{s}s\rangle(3+\beta)][i_1(\mathcal{T},v) - i_1(\mathcal{T}_4,v) + 3\widetilde{\widetilde{i}}_4(\mathbb{B}_T)]\Big\} \\
&- \frac{f_V^{\parallel} M_V M^2}{96\sqrt{3}\pi^2}(1-\beta)[4i_2(\mathcal{A},1-v) + 4i_2(\mathcal{V},1-v) + 2\widetilde{i}_4(\mathbb{B}) - 2\widetilde{i}_4(\phi_V^{\parallel}) + \psi_{3;V}^{\perp}(u_0)] \\
&- \frac{f_V^{\perp} m_V^2}{24\sqrt{3}\pi^2}\Big\{2[m_d(1-3\beta) + m_s(1+3\beta)]i_1(\mathcal{T},1) + 4[2m_d\beta - m_s(1+3\beta)]i_1(\mathcal{T}_4,1) \\
&+ [2m_d - m_s(3+\beta)][2i_1(\mathcal{T},v) - 4i_1(\mathcal{T}_4,v) + 3\widetilde{\widetilde{i}}_4(\mathbb{B}_T)] - 4m_d(1-\beta)i_1(\mathcal{T}_3,1)\Big\} \\
&+ \frac{f_V^{\parallel} m_V^3}{192\sqrt{3}\pi^2}(1-\beta)\Big\{8[4i_0(\Psi,1-2v) + 4i_0(\widetilde{\Psi},1) + i_1(\Phi,v) - i_1(\widetilde{\Phi},1-3v) \\
&- i_1(\Psi,1-2v)] - 3\widetilde{i}_4(\mathbb{A})\Big\}\ , \qquad \text{(C.2)}
\end{aligned}
$$

$$
\begin{aligned}
\Pi_1^{(3)} &= -\frac{M^4}{96\sqrt{3}\pi^2}\Big\{f_V^{\parallel} m_V(1-\beta)[2i_3(\mathcal{A},1-v) - 2i_3(\mathcal{V},1-v) - 2\widetilde{i}_4(\mathbb{B}) + 2\widetilde{i}_4(\phi_V^{\parallel}) \\
&+ 5\psi_{3;V}^{\perp}(u_0)] - 6f_V^{\perp}[2\beta m_d + (1+\beta)m_s]\phi_V^{\perp}(u_0)\Big\} \\
&+ \left(\gamma_E - \ln\frac{M^2}{\Lambda^2}\right)\bigg\{-\frac{f_V^{\perp} m_V^2 M^2}{24\sqrt{3}\pi^2}\Big([m_s(1+3\beta) - 2m_d\beta]i_2(\mathcal{S},1) \\
&- [m_s(3+\beta) - 2m_d][i_2(\widetilde{\mathcal{S}},1) - i_2(\mathcal{T}_4,1)] + 2[m_d(1-\beta) - m_s(2+\beta)]i_2(\mathcal{T}_1,1) \\
&+ 2[m_d(1-\beta) + m_s(1+2\beta)]i_2(\mathcal{T}_2,1) - [2m_d + m_s(1+\beta)]i_2(\mathcal{T}_3,1)\Big) \\
&+ \frac{f_V^{\perp}\langle g_s^2 G^2\rangle}{192\sqrt{3}\pi^2 M^2}\Big(2m_V^2[2m_d - m_s(3+\beta)]\widetilde{\widetilde{i}}_4(\mathbb{B}_T) + M^2[2m_d\beta + m_s(1+\beta)]\phi_V^{\perp}(u_0)\Big)\bigg\}
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{f_V^{\parallel} m_V \langle g_s^2 G^2\rangle}{144\sqrt{3}M^6}\beta[m_d\langle\bar{s}s\rangle + m_s\langle\bar{d}d\rangle](m_0^2+2M^2)\psi_{3;V}^{\perp}(u_0)\\
&+ \frac{f_V^{\perp} m_V^2 \langle g_s^2 G^2\rangle}{2304\sqrt{3}\pi^2 M^2}\Big\{[2m_d\beta + m_s(1+\beta)][3\mathbb{A}_T(u_0) - 4i_2(\mathcal{T}_3, 1-2v)]\\
&- 8[m_d(1+\beta)-m_s][i_2(\widetilde{\mathcal{S}}, 1-2v) - i_2(\mathcal{T}_4, 1-2v)] - 8(2m_d - m_s)i_2(\mathcal{S},1)\\
&+ 4[m_s(3+\beta) - 2m_d]i_2(\mathcal{T}_1, 1) + 16[m_d(1+\beta) - m_s]i_2(\mathcal{T}_1, v)\\
&- 4[2m_d\beta - m_s(1+3\beta)]i_2(\mathcal{T}_2,1) - 8[2\beta m_d + m_s(1+\beta)]i_2(\mathcal{T}_2, v)\Big\}\\
&- \frac{f_V^{\parallel} m_V^3}{288\sqrt{3}M^2}(1-\beta)\Big\{[m_d\langle\bar{d}d\rangle - m_s\langle\bar{s}s\rangle][8i_1(\Phi,1) + 9i_1(\widetilde{\Psi},1)]\\
&- 2[m_d\langle\bar{d}d\rangle + m_s\langle\bar{s}s\rangle][4i_1(\widetilde{\Phi},1) + 4i_1(\Psi, 1-2v) + 3\widetilde{i}_4(\mathbb{A})] + 16[2m_d\langle\bar{d}d\rangle - m_s\langle\bar{s}s\rangle]i_1(\Phi, v)\\
&+ 16[m_d\langle\bar{d}d\rangle + 2m_s\langle\bar{s}s\rangle]i_1(\widetilde{\Phi}, v) + 16[m_d\langle\bar{d}d\rangle i_2(\mathcal{A},1) - m_s\langle\bar{s}s\rangle i_2(\mathcal{A}, v)]\\
&- 16[m_d\langle\bar{d}d\rangle i_2(\mathcal{V},1) - m_s\langle\bar{s}s\rangle i_2(\mathcal{V}, v)]\Big\}\\
&- \frac{f_V^{\parallel} m_V m_0^2}{216\sqrt{3}M^2}\Big\{2(1-\beta)[m_d\langle\bar{d}d\rangle + m_s\langle\bar{s}s\rangle][\widetilde{i}_4(\mathbb{B}) - \widetilde{i}_4(\phi_V^{\parallel})]\\
&- \Big(\langle\bar{s}s\rangle[2m_d(1+\beta) - m_s(1-\beta)] - \langle\bar{d}d\rangle[m_d(1-\beta) - 2m_s(1+\beta)]\Big)\psi_{3;V}^{\perp}(u_0)\Big\}\\
&- \frac{f_V^{\perp} m_V^2}{432\sqrt{3}\pi^2 M^2}\Big\{\langle g_s^2 G^2\rangle[6m_d - 3m_s(3+\beta)] - 8\pi^2 m_0^2[\langle\bar{d}d\rangle(5-\beta) - 2\langle\bar{s}s\rangle(5+2\beta)]\Big\}\widetilde{\widetilde{i}}_4(\mathbb{B}_T)\\
&- \frac{f_V^{\perp} m_V^2 M^2}{96\sqrt{3}\pi^2}\Big\{[2m_d\beta + m_s(1+\beta)][3\mathbb{A}_T(u_0) - 4i_2(\widetilde{\mathcal{S}},1) - 4i_2(\mathcal{T},1)\\
&+ 8i_2(\mathcal{T}_3, v) + 4i_2(\mathcal{T}_4, 1)] + 16[m_d(1+\beta) - m_s][i_2(\widetilde{\mathcal{S}}, v) + i_2(\mathcal{T}_1, v) - i_2(\mathcal{T}_4, v)]\\
&- 4[6m_d\beta - m_s(1+5\beta)]i_2(\mathcal{S},1) - 8(m_d + m_s)(1+\beta)i_2(\mathcal{T}_3, 1)\Big\}\\
&- \frac{f_V^{\parallel} m_V^3 M^2}{192\sqrt{3}\pi^2}(1-\beta)\Big\{8[i_1(\Phi, v) - i_1(\widetilde{\Phi}, 1-3v) - i_1(\Psi, 1-2v)\\
&- i_2(\mathcal{V}, 1-v) + i_2(\mathcal{A}, 1-v)] - 3\widetilde{i}_4(\mathbb{A})\Big\}\\
&+ \frac{f_V^{\perp} M^2}{24\sqrt{3}\pi^2}\Big\{3m_V^2[2m_d - m_s(3+\beta)]\widetilde{\widetilde{i}}_4(\mathbb{B}_T) - 4\pi^2[2\langle\bar{d}d\rangle\beta + \langle\bar{s}s\rangle(1+\beta)]\phi_V^{\perp}(u_0)\Big\}\\
&- \frac{f_V^{\parallel} m_V}{36\sqrt{3}}(1-\beta)\Big\{[m_d\langle\bar{d}d\rangle i_3(\mathcal{A},1) - m_s\langle\bar{s}s\rangle i_3(\mathcal{A}, v)] - [m_d\langle\bar{d}d\rangle i_3(\mathcal{V},1) - m_s\langle\bar{s}s\rangle i_3(\mathcal{V}, v)]\\
&- 3[m_d\langle\bar{d}d\rangle + m_s\langle\bar{s}s\rangle][\widetilde{i}_4(\mathbb{B}) - \widetilde{i}_4(\phi_V^{\parallel})]\Big\}\\
&- \frac{f_V^{\parallel} m_V}{24\sqrt{3}\pi^2}\Big\{[m_d(1-\beta) + 8m_s\beta]\langle\bar{d}d\rangle + [8\beta m_d + m_s(1-\beta)]\langle\bar{s}s\rangle\Big\}\psi_{3;V}^{\perp}(u_0)\\
&- \frac{f_V^{\perp} m_V^2}{72\sqrt{3}}\Big\{[2\beta\langle\bar{d}d\rangle + (1+\beta)\langle\bar{s}s\rangle][4i_2(\mathcal{T}_3, 1-2v) - 3\mathbb{A}_T(u_0)] + 24[2\langle\bar{d}d\rangle - \langle\bar{s}s\rangle(3+\beta)]\widetilde{\widetilde{i}}_4(\mathbb{B}_T)\\
&+ 8[\langle\bar{d}d\rangle(1+\beta) - \langle\bar{s}s\rangle][i_2(\widetilde{\mathcal{S}}, 1-2v) - i_2(\mathcal{T}_4, 1-2v)] + 8\beta[2\langle\bar{d}d\rangle - \langle\bar{s}s\rangle]i_2(\mathcal{S},1)\Big\}
\end{aligned}
$$

$$
- \frac{f_V^\perp m_V^2}{18\sqrt{3}} \Big\{ [2\langle \bar{d}d \rangle - (3+\beta)\langle \bar{s}s \rangle] i_2(\mathcal{T}_1, 1) - 4[\langle \bar{d}d \rangle (1+\beta) - \langle \bar{s}s \rangle] i_2(\mathcal{T}_1, v)
$$

$$
+ [2\beta\langle \bar{d}d \rangle - (1+3\beta)\langle \bar{s}s \rangle] i_2(\mathcal{T}_2, 1) + 2[2\langle \bar{d}d \rangle \beta + (1+\beta)\langle \bar{s}s \rangle] i_2(\mathcal{T}_2, v) \Big\}
$$

$$
- \frac{f_V^\perp}{864\sqrt{3}\pi^2} \Big\{ 3\langle g_s^2 G^2 \rangle [2m_d\beta + m_s(1+\beta)] - 40\pi^2 m_0^2 [2\langle \bar{d}d \rangle \beta + \langle \bar{s}s \rangle (1+\beta)] \Big\} \phi_V^\perp(u_0) \ . \qquad \text{(C.3)}
$$

The functions i_n, $\widetilde{i}_4$ and $\widetilde{\widetilde{i}}_4$ are defined as

$$
\begin{aligned}
i_0(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v)(k - u_0)\theta(k - u_0) \ , \\
i_1(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v)\theta(k - u_0) \ , \\
i_2(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v)\delta(k - u_0) \ , \\
i_3(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v)\delta'(k - u_0) \ , \\
\widetilde{i}_4(f(u)) &= \int_{u_0}^1 du f(u) \ , \\
\widetilde{\widetilde{i}}_4(f(u)) &= \int_{u_0}^1 du (u - u_0) f(u) \ ,
\end{aligned}
$$

where

$$
k = \alpha_q + \alpha_g \bar{v} \ , \qquad u_0 = \frac{M_1^2}{M_1^2 + M_2^2} \ , \qquad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2} \ .
$$

Note that $\Pi^{(i)}(u, d, s)$, $(i = u, d, s)$ do not depend explicitly on the u quark properties m_u and $\langle \bar{u}u \rangle$. The dependence is implicit in the meson distribution amplitudes and leptonic constants.

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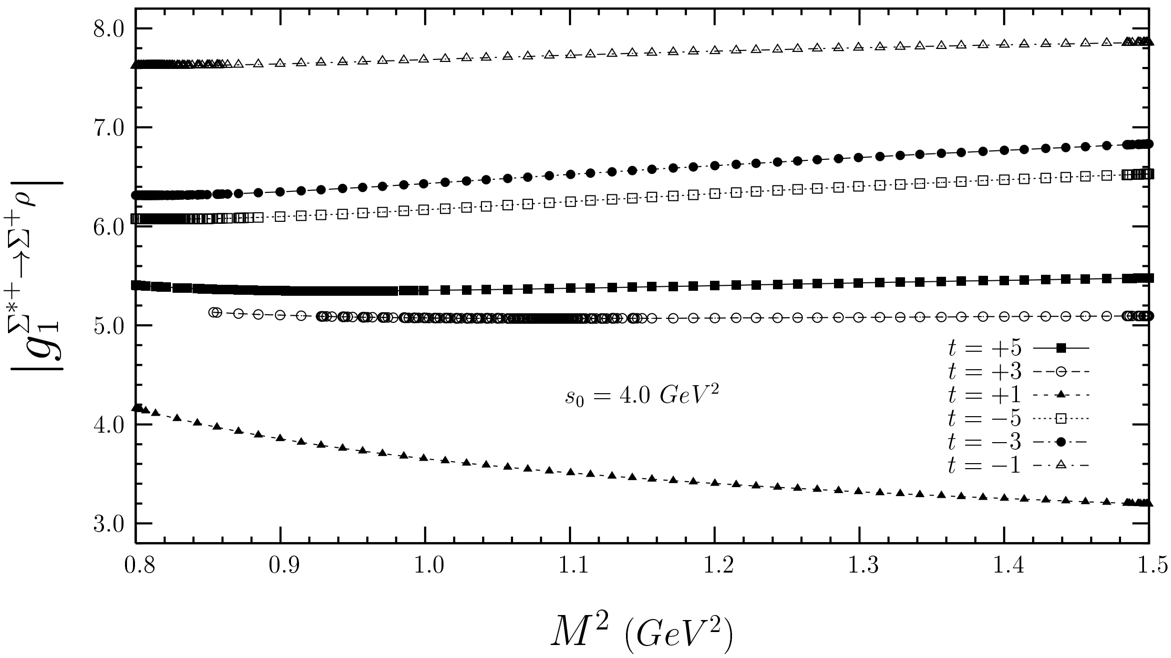


Figure 1:

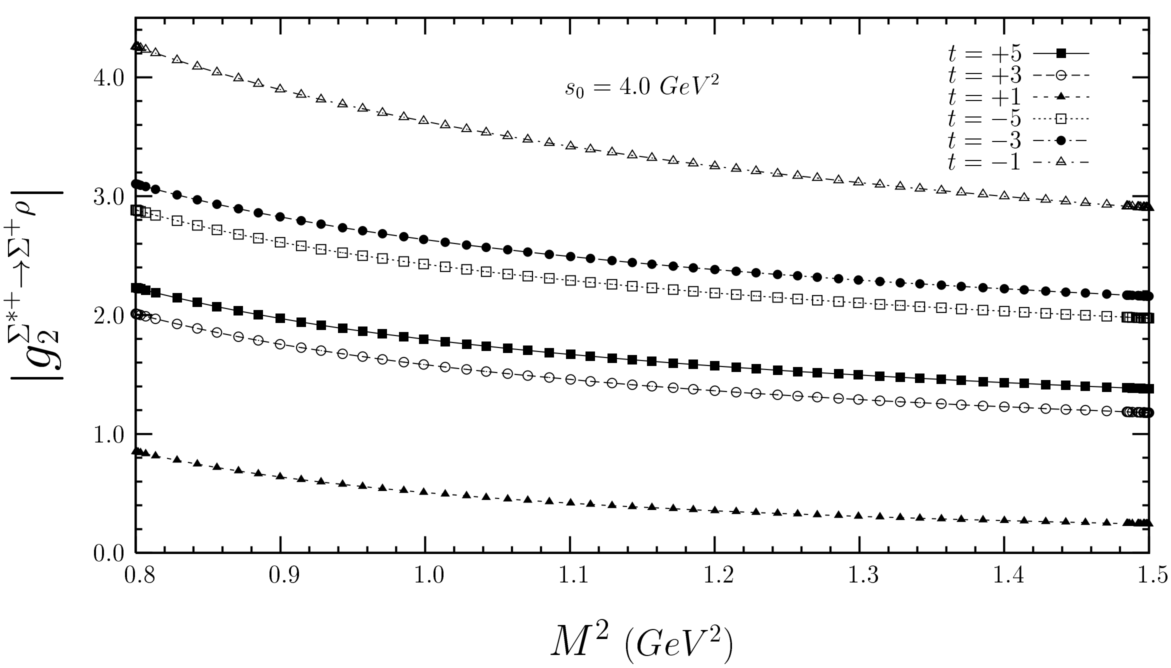


Figure 2:

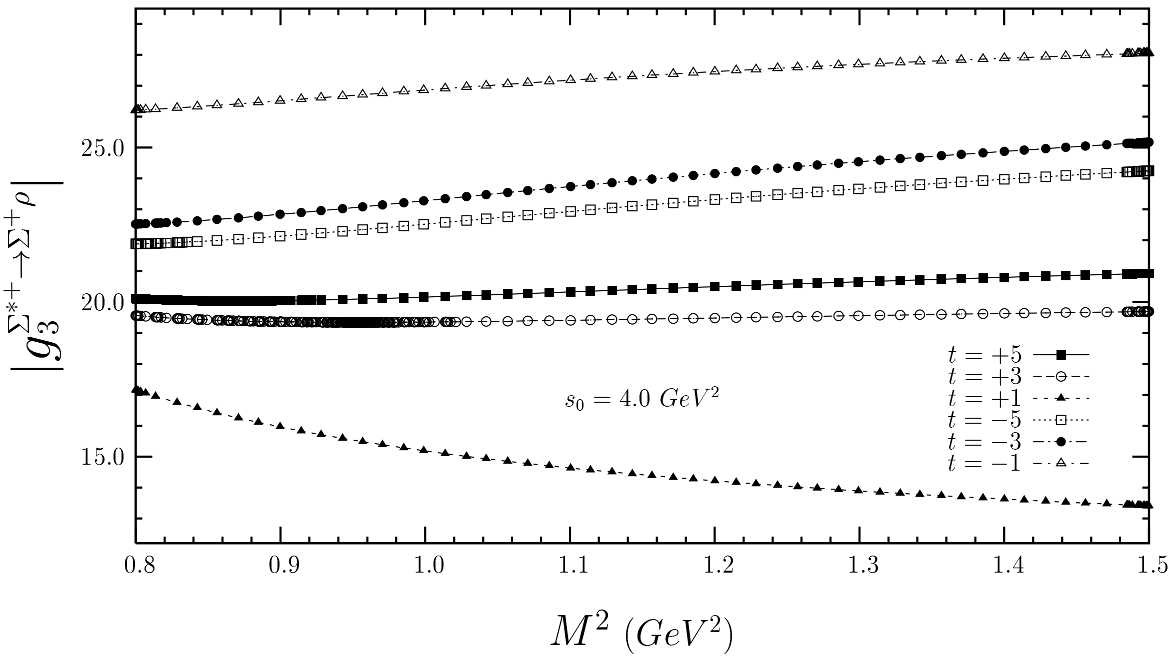
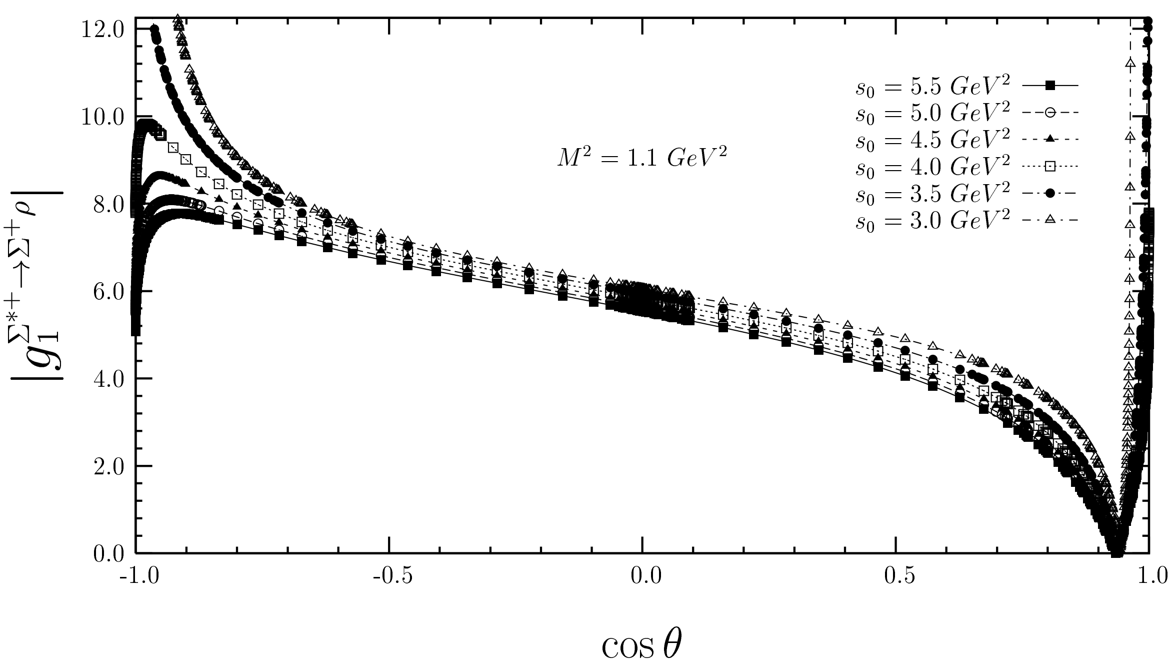


Figure 3:



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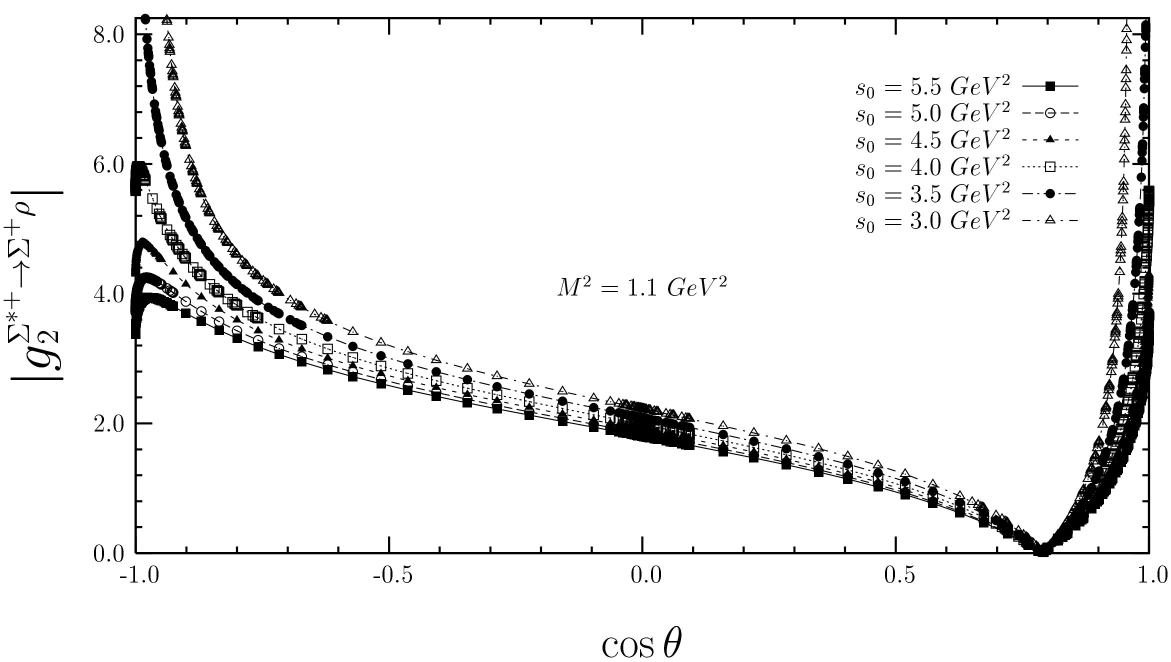

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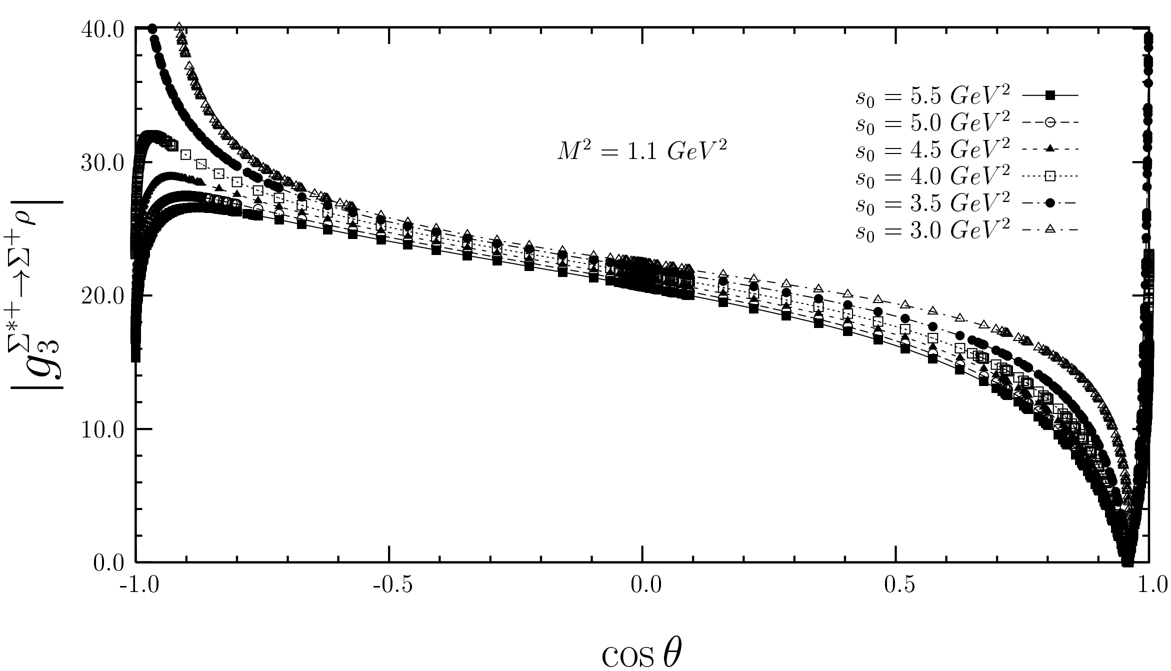

Figure 6: