General analysis of lepton polarizations in rare $B \to K^* \ell^+ \ell^-$ decay beyond the standard model

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Abstract

The general analysis of lepton polarization asymmetries in rare $B \to K^* \ell^+ \ell^-$ decay is investigated. Using the most general, model independent effective Hamiltonian, the general expressions of the longitudinal, normal and transversal polarization asymmetries for ℓ^- and ℓ^+ and combinations of them are presented. The dependence of lepton polarizations and their combinations on new Wilson coefficients are studied in detail. Our analysis shows that the lepton polarization asymmetries are very sensitive to the scalar and tensor type interactions, which will be very useful in looking for new physics beyond the standard model.

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1 Introduction

Rare B meson decays, induced by flavor–changing neutral current (FCNC) $b \to s(d)$ transitions, are quite promising for establishing new physics beyond the standard model (SM). In particular the flavor changing inclusive $b \to s(d)\ell^+\ell^-$ decay, which takes place in the SM at loop level, is very sensitive to the gauge structure of the SM. Moreover $b \to s(d)\ell^+\ell^-$ decay is known to be very sensitive to the various extensions of the SM. New physics effects manifest themselves in rare B meson decays in two different ways, either through new contributions to the Wilson coefficients existing in the SM or through the new structures in the effective Hamiltonian which are absent in the SM. Note that $b \to s(d)\ell^+\ell^-$ transition has been extensively studied in framework of the SM and its various extensions [1]–[15]. One of the efficient ways in establishing new physics beyond the SM is the measurement of the lepton polarization [15]–[24]. All previous studies for the lepton polarization have been limited to SM and its minimal extensions, except the work [19]. In [19] the analysis of the τ lepton polarization for the inclusive $b \to s\tau^+\tau^-$ decay was presented in a model independent way. In the same work, it was also shown that the investigation of the τ lepton polarization can give unambiguous information about the existence of the scalar and tensor type interactions.

It is well known that the theoretical study of the inclusive decays is rather easy but their experimental investigation is difficult. However for the exclusive decays the situation is contrary to the inclusive case, i.e., their experimental detection is very easy but theoretical investigation has its own drawbacks. This is due to the fact that for description of the exclusive decay form factors, i.e., the matrix elements of the effective Hamiltonian between initial and final meson states, are needed. This problem is related to the nonperturbative sector of the QCD and and it can only be solved in framework of the nonperturbative approaches.

These matrix elements have been studied in framework of different approaches, such as chiral theory [25], three–point QCD sum rules [26] and light cone QCD sum rules [27, 28]. In this work we will use the weak decay form factors calculated using light cone QCD sum rules method [27, 28].

The aim of the present work is to present a rigorous study of the lepton polarizations in the exclusive $B \to K^*\ell^+\ell^-$ ($\ell = \mu,\ \tau$) decay for a general form of the effective Hamiltonian including tensor type interactions as well. In the present work we extend results of previous studies on lepton polarization [20]–[23] and then perform a general analysis (in a model independent way in the sense without forcing concrete values for the Wilson coefficients corresponding to any specific model) including all possible form of interactions. Our analysis shows that the so–called tensor type interactions give dominant contribution to the lepton polarization asymmetries. The paper is organized as follows. In section 2, using a general form of four–Fermi interaction we derive the model independent expressions for the longitudinal, transversal and normal polarizations of leptons. In section 3 we investigate the dependence of the above–mentioned polarizations on the four–Fermi interactions. We also present the combined analysis of the ℓ^- and ℓ^+ asymmetries and our results.

2 Lepton polarizations

We start this section by computing the lepton polarization asymmetries, using the most general, model independent four–Fermi interactions. Following [19], we write the effective Hamiltonian for the $b \to s\ell^+\ell^-$ transition in terms of twelve model independent four–Fermi interactions.

$$
\begin{aligned}
\mathcal{H}_{eff} &= \frac{G\alpha}{\sqrt{2}\pi} V_{ts}V_{tb}^\ast \Bigg\{ C_{SL}\, \bar{s} i \sigma_{\mu\nu} \frac{q^\nu}{q^2}\, L\, b\, \bar{\ell}\gamma^\mu \ell + C_{BR}\, \bar{s} i \sigma_{\mu\nu} \frac{q^\nu}{q^2}\, R\, b\, \bar{\ell}\gamma^\mu \ell \\
&+ C_{LL}^{tot}\, \bar{s}_L \gamma_\mu b_L\, \bar{\ell}_L \gamma^\mu \ell_L + C_{LR}^{tot}\, \bar{s}_L \gamma_\mu b_L\, \bar{\ell}_R \gamma^\mu \ell_R + C_{RL}\, \bar{s}_R \gamma_\mu b_R\, \bar{\ell}_L \gamma^\mu \ell_L \\
&+ C_{RR}\, \bar{s}_R \gamma_\mu b_R\, \bar{\ell}_R \gamma^\mu \ell_R + C_{LRLR}\, \bar{s}_L b_R\, \bar{\ell}_L \ell_R + C_{RLLR}\, \bar{s}_R b_L\, \bar{\ell}_L \ell_R \\
&+ C_{LRRL}\, \bar{s}_L b_R\, \bar{\ell}_R \ell_L + C_{RLRL}\, \bar{s}_R b_L\, \bar{\ell}_R \ell_L + C_T\, \bar{s}\sigma_{\mu\nu} b\, \bar{\ell}\sigma^{\mu\nu}\ell \\
&+ i C_{TE}\, \epsilon^{\mu\nu\alpha\beta} \bar{s}\sigma_{\mu\nu} b\, \bar{\ell}\sigma_{\alpha\beta}\ell \Bigg\}~,
\end{aligned} \tag{1}
$$

where the chiral projection operators L and R in (1) are defined as

$$
L = \frac{1-\gamma_5}{2}~, \qquad R = \frac{1+\gamma_5}{2}~,
$$

and C_X are the coefficients of the four–Fermi interactions. The first two of these coefficients, C_{SL} and C_{BR}, are the nonlocal Fermi interactions which correspond to $-2m_s C_7^{eff}$ and $-2m_b C_7^{eff}$ in the SM, respectively. The following four terms in this expression are the vector type interactions with coefficients C_{LL}, C_{LR}, C_{RL} and C_{RR}. Two of these vector interactions containing C_{LL}^{tot} and C_{LR}^{tot} do already exist in the SM in combinations of the form $(C_9^{eff} - C_{10})$ and $(C_9^{eff} + C_{10})$. Therefore by writing

$$
\begin{aligned}
C_{LL}^{tot} &= C_9^{eff} - C_{10} + C_{LL}~, \\
C_{LR}^{tot} &= C_9^{eff} + C_{10} + C_{LR}~,
\end{aligned}
$$

one concludes that C_{LL}^{tot} and C_{LR}^{tot} describe the sum of the contributions from SM and the new physics. The terms with coefficients C_{LRLR}, C_{RLLR}, C_{LRRL} and C_{RLRL} describe the scalar type interactions. The remaining two terms leaded by the coefficients C_T and C_{TE}, obviously, describe the tensor type interactions.

Having the general form of four–Fermi interaction for the $b \to s\ell^+\ell^-$ transition, our next problem is calculation of the matrix element for the $B \to K^\ast\ell^+\ell^-$ decay. In other words, we need the matrix elements

$$
\begin{aligned}
&\left\langle K^\ast \left| \bar{s}\gamma_\mu (1\pm\gamma_5) b \right| B \right\rangle~, \\
&\left\langle K^\ast \left| \bar{s} i \sigma_{\mu\nu} q^\nu (1\pm\gamma_5) b \right| B \right\rangle~, \\
&\left\langle K^\ast \left| \bar{s} (1\pm\gamma_5) b \right| B \right\rangle~, \\
&\left\langle K^\ast \left| \bar{s}\sigma_{\mu\nu} b \right| B \right\rangle~,
\end{aligned}
$$

in order to calculate the decay amplitude for the $B \to K^\ast\ell^+\ell^-$ decay. These matrix elements are defined as follows:

$$
\left\langle K^\ast(p_{K^\ast},\varepsilon) \left| \bar{s}\gamma_\mu (1\pm\gamma_5) b \right| B(p_B) \right\rangle =
$$

$$-\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p_{K^*}^{\lambda}q^{\sigma}\frac{2V(q^2)}{m_B+m_{K^*}} \pm i\varepsilon_{\mu}^{*}(m_B+m_{K^*})A_1(q^2) \qquad (2)$$
$$\mp i(p_B+p_{K^*})_{\mu}(\varepsilon^* q)\frac{A_2(q^2)}{m_B+m_{K^*}} \mp iq_{\mu}\frac{2m_{K^*}}{q^2}(\varepsilon^* q)\left[A_3(q^2)-A_0(q^2)\right] ,$$

$$\langle K^*(p_{K^*},\varepsilon)\,|\bar{s}i\sigma_{\mu\nu}q^{\nu}(1\pm\gamma_5)b|\,B(p_B)\rangle =$$
$$4\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p_{K^*}^{\lambda}q^{\sigma}T_1(q^2) \pm 2i\left[\varepsilon_{\mu}^{*}(m_B^2-m_{K^*}^2)-(p_B+p_{K^*})_{\mu}(\varepsilon^* q)\right]T_2(q^2) \qquad (3)$$
$$\pm 2i(\varepsilon^* q)\left[q_{\mu}-(p_B+p_{K^*})_{\mu}\frac{q^2}{m_B^2-m_{K^*}^2}\right]T_3(q^2) ,$$

$$\langle K^*(p_{K^*},\varepsilon)\,|\bar{s}\sigma_{\mu\nu}b|\,B(p_B)\rangle =$$
$$i\epsilon_{\mu\nu\lambda\sigma}\Bigg[-2T_1(q^2)\varepsilon^{*\lambda}(p_B+p_{K^*})^{\sigma}+\frac{2}{q^2}(m_B^2-m_{K^*}^2)\varepsilon^{*\lambda}q^{\sigma} \qquad (4)$$
$$-\frac{4}{q^2}\left(T_1(q^2)-T_2(q^2)-\frac{q^2}{m_B^2-m_{K^*}^2}T_3(q^2)\right)(\varepsilon^* q)p_{K^*}^{\lambda}q^{\sigma}\Bigg] .$$

where $q=p_B-p_{K^*}$ is the momentum transfer and ε is the polarization vector of K^* meson. In order to ensure finiteness of (2) at $q^2=0$, we assume that $A_3(q^2=0)=A_0(q^2=0)$ and $T_1(q^2=0)=T_2(q^2=0)$. The matrix element $\langle K^*\,|\bar{s}(1\pm\gamma_5)b|\,B\rangle$ can be calculated from Eq. (2) by contracting both sides of Eq. (2) with q^{μ} and using equation of motion. Neglecting the mass of the strange quark we get

$$\langle K^*(p_{K^*},\varepsilon)\,|\bar{s}(1\pm\gamma_5)b|\,B(p_B)\rangle = \frac{1}{m_b}\Big[\mp 2im_{K^*}(\varepsilon^* q)A_0(q^2)\Big] . \qquad (5)$$

In deriving Eq. (5) we have used the relation (see [15, 26])

$$2m_{K^*}A_3(q^2)=(m_B+m_{K^*})A_1(q^2)-(m_B-m_{K^*})A_2(q^2) .$$

Taking into account Eqs. (1–5), the matrix element of the $B\to K^*\ell^+\ell^-$ decay can be written as

$$\mathcal{M}(B\to K^*\ell^+\ell^-)=\frac{G\alpha}{4\sqrt{2}\pi}V_{tb}V_{ts}^*$$
$$\times\Bigg\{\bar{\ell}\gamma^{\mu}(1-\gamma_5)\ell\Big[-2A_1\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p_{K^*}^{\lambda}q^{\sigma}-iB_1\varepsilon_{\mu}^{*}+iB_2(\varepsilon^* q)(p_B+p_{K^*})_{\mu}+iB_3(\varepsilon^* q)q_{\mu}\Big]$$
$$+\bar{\ell}\gamma^{\mu}(1+\gamma_5)\ell\Big[-2C_1\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p_{K^*}^{\lambda}q^{\sigma}-iD_1\varepsilon_{\mu}^{*}+iD_2(\varepsilon^* q)(p_B+p_{K^*})_{\mu}+iD_3(\varepsilon^* q)q_{\mu}\Big]$$
$$+\bar{\ell}(1-\gamma_5)\ell\Big[iB_4(\varepsilon^* q)\Big]+\bar{\ell}(1+\gamma_5)\ell\Big[iB_5(\varepsilon^* q)\Big]$$
$$+4\bar{\ell}\sigma^{\mu\nu}\ell\Big(iC_T\epsilon_{\mu\nu\lambda\sigma}\Big)\Big[-2T_1\varepsilon^{*\lambda}(p_B+p_{K^*})^{\sigma}+B_6\varepsilon^{*\lambda}q^{\sigma}-B_7(\varepsilon^* q)p_{K^*}{}^{\lambda}q^{\sigma}\Big]$$
$$+16C_{TE}\bar{\ell}\sigma_{\mu\nu}\ell\Big[-2T_1\varepsilon^{*\mu}(p_B+p_{K^*})^{\nu}+B_6\varepsilon^{*\mu}q^{\nu}-B_7(\varepsilon^* q)p_{K^*}{}^{\mu}q^{\nu}\Big]\Bigg\} , \qquad (6)$$

where

$$A_1 = (C_{LL}^{tot}+C_{RL})\frac{V}{m_B+m_{K^*}}-2(C_{BR}+C_{SL})\frac{T_1}{q^2} ,$$

$$
\begin{aligned}
B_1 &= (C_{LL}^{tot} - C_{RL})(m_B + m_{K^*})A_1 - 2(C_{BR} - C_{SL})(m_B^2 - m_{K^*}^2)\frac{T_2}{q^2} \;, \\
B_2 &= \frac{C_{LL}^{tot} - C_{RL}}{m_B + m_{K^*}}A_2 - 2(C_{BR} - C_{SL})\frac{1}{q^2}\left[T_2 + \frac{q^2}{m_B^2 - m_{K^*}^2}T_3\right] \;, \\
B_3 &= 2(C_{LL}^{tot} - C_{RL})m_{K^*}\frac{A_3 - A_0}{q^2} + 2(C_{BR} - C_{SL})\frac{T_3}{q^2} \;, \\
C_1 &= A_1(C_{LL}^{tot} \to C_{LR}^{tot} \;, \quad C_{RL} \to C_{RR}) \;, \\
D_1 &= B_1(C_{LL}^{tot} \to C_{LR}^{tot} \;, \quad C_{RL} \to C_{RR}) \;, \\
D_2 &= B_2(C_{LL}^{tot} \to C_{LR}^{tot} \;, \quad C_{RL} \to C_{RR}) \;, \\
D_3 &= B_3(C_{LL}^{tot} \to C_{LR}^{tot} \;, \quad C_{RL} \to C_{RR}) \;, \\
B_4 &= -2(C_{LRRL} - C_{RLRL})\frac{m_{K^*}}{m_b}A_0 \;, \\
B_5 &= -2(C_{LRLR} - C_{RLLR})\frac{m_{K^*}}{m_b}A_0 \;, \\
B_6 &= 2(m_B^2 - m_{K^*}^2)\frac{T_1 - T_2}{q^2} \;, \\
B_7 &= \frac{4}{q^2}\left(T_1 - T_2 - \frac{q^2}{m_B^2 - m_{K^*}^2}T_3\right) \;. \qquad (7)
\end{aligned}
$$

The form of Eq. (6) reflects the fact that its difference from the SM case is due to the last four terms only, namely, scalar and tensor type interactions. The next task to be considered is calculation of the final lepton polarizations with the help of the matrix element for the $B \to K^* \ell^+ \ell^-$ decay. For this purpose we we define the following orthogonal unit vectors, $S_L^{-\mu}$ in the rest frame of ℓ^- and $S_L^{+\mu}$ in the rest frame of ℓ^+, for the polarization of the leptons along the longitudinal (L), transversal (T) and normal (N) directions:

$$
\begin{aligned}
S_L^{-\mu} &\equiv (0, \vec{e}_L^{\,-}) = \left(0, \frac{\vec{p}_-}{|\vec{p}_-|}\right) \;, \\
S_N^{-\mu} &\equiv (0, \vec{e}_N^{\,-}) = \left(0, \frac{\vec{p} \times \vec{p}_-}{|\vec{p} \times \vec{p}_-|}\right) \;, \\
S_T^{-\mu} &\equiv (0, \vec{e}_T^{\,-}) = \left(0, \vec{e}_N^{\,-} \times \vec{e}_L^{\,-}\right) \;, \qquad (8) \\
S_L^{+\mu} &\equiv (0, \vec{e}_L^{\,+}) = \left(0, \frac{\vec{p}_+}{|\vec{p}_+|}\right) \;, \\
S_N^{+\mu} &\equiv (0, \vec{e}_N^{\,+}) = \left(0, \frac{\vec{p} \times \vec{p}_+}{|\vec{p} \times \vec{p}_+|}\right) \;, \\
S_T^{+\mu} &\equiv (0, \vec{e}_T^{\,+}) = \left(0, \vec{e}_N^{\,+} \times \vec{e}_L^{\,+}\right) \;,
\end{aligned}
$$

where $\vec{p}_\pm$ and $\vec{p}$ are the three momenta of $\ell^\pm$ and K^* meson in the center of mass (CM) frame of the $\ell^+\ell^-$ system, respectively. The longitudinal unit vectors S_L^- and S_L^+ are boosted to CM frame of $\ell^+\ell^-$ by Lorentz transformation,

$$
S_{L,CM}^{-\mu} = \left(\frac{|\vec{p}_-|}{m_\ell}, \frac{E_\ell\, \vec{p}_-}{m_\ell\, |\vec{p}_-|}\right) \;,
$$

$$S^{+\mu}_{L,CM} = \left(\frac{|\vec{p}_-|}{m_\ell}, -\frac{E_\ell \, \vec{p}_-}{m_\ell \, |\vec{p}_-|} \right) , \tag{9}$$

while vectors of perpendicular directions are not changed by boost.

The differential decay rate of the $B \to K^* \ell^+ \ell^-$ decay for any spin direction $\vec{n}^{(\pm)}$ of the $\ell^{(\pm)}$, where $\vec{n}^{(\pm)}$ is the unit vector in the $\ell^{(\pm)}$ rest frame, can be written in the following form

$$\frac{d\Gamma(\vec{n}^{(\pm)})}{dq^2} = \frac{1}{2} \left(\frac{d\Gamma}{dq^2} \right)_0 \left[1 + \left(P_L^{(\pm)} \vec{e}_L^{(\pm)} + P_N^{(\pm)} \vec{e}_N^{(\pm)} + P_T^{(\pm)} \vec{e}_T^{(\pm)} \right) \cdot \vec{n}^{(\pm)} \right] , \tag{10}$$

where the superscripts $^+$ and $^-$ correspond to ℓ^+ and ℓ^- cases, the subscript $_0$ corresponds to the unpolarized decay rate, whose explicit form will be presented below and P_L, P_N and P_T represent the longitudinal, normal and transversal polarizations, respectively. The explicit form of the unpolarized decay rate in Eq. (10) is

$$\begin{aligned}
\left(\frac{d\Gamma}{dq^2} \right)_0 &= \frac{G^2 \alpha^2}{2^{14} \pi^5 m_B} \left| V_{tb} V_{ts}^* \right|^2 \lambda^{1/2} v \\
&\times \Bigg\{ \frac{32}{3} m_B^4 \lambda \Big[(m_B^2 s - m_\ell^2) \left(|A_1|^2 + |C_1|^2 \right) + 6 m_\ell^2 \, \mathrm{Re}(A_1 C_1^*) \Big] \\
&+ 96 m_\ell^2 \, \mathrm{Re}(B_1 D_1^*) - \frac{4}{r} m_B^2 m_\ell \lambda \, \mathrm{Re}[(B_1 - D_1)(B_4^* - B_5^*)] \\
&+ \frac{8}{r} m_B^2 m_\ell^2 \lambda \Big(\mathrm{Re}[B_1(-B_3^* + D_2^* + D_3^*)] + \mathrm{Re}[D_1(B_2^* + B_3^* - D_3^*)] - \mathrm{Re}(B_4 B_5^*) \Big] \Big) \\
&+ \frac{4}{r} m_B^4 m_\ell (1-r) \lambda \Big(\mathrm{Re}[(B_2 - D_2)(B_4^* - B_5^*)] + 2 m_\ell \, \mathrm{Re}[(B_2 - D_2)(B_3^* - D_3^*)] \Big) \\
&- \frac{8}{r} m_B^4 m_\ell^2 \lambda (2 + 2r - s) \, \mathrm{Re}(B_2 D_2^*) + \frac{4}{r} m_B^4 m_\ell s \lambda \, \mathrm{Re}[(B_3 - D_3)(B_4^* - B_5^*)] \\
&+ \frac{4}{r} m_B^4 m_\ell^2 s \lambda \left| B_3 - D_3 \right|^2 + \frac{2}{r} m_B^2 (m_B^2 s - 2 m_\ell^2) \lambda \left(|B_4|^2 + |B_5|^2 \right) \\
&- \frac{8}{3rs} m_B^2 \lambda \Big[m_\ell^2 (2 - 2r + s) + m_B^2 s (1 - r - s) \Big] \Big[\mathrm{Re}(B_1 B_2^*) + \mathrm{Re}(D_1 D_2^*) \Big] \\
&+ \frac{4}{3rs} \Big[2 m_\ell^2 (\lambda - 6rs) + m_B^2 s (\lambda + 12 rs) \Big] \left(|B_1|^2 + |D_1|^2 \right) \\
&+ \frac{4}{3rs} m_B^4 \lambda \Big(m_B^2 s \lambda + m_\ell^2 [2\lambda + 3s(2 + 2r - s)] \Big) \left(|B_2|^2 + |D_2|^2 \right) \\
&+ \frac{32}{r} m_B^6 m_\ell \lambda^2 \, \mathrm{Re}[(B_2 + D_2)(B_7 C_{TE})^*] \\
&- \frac{32}{r} m_B^4 m_\ell \lambda (1 - r - s) \Big(\mathrm{Re}[(B_1 + D_1)(B_7 C_{TE})^*] + 2 \, \mathrm{Re}[(B_2 + D_2)(B_6 C_{TE})^*] \Big) \\
&+ \frac{64}{r} (\lambda + 12 rs) m_B^2 m_\ell \, \mathrm{Re}[(B_1 + D_1)(B_6 C_{TE})^*] \\
&+ \frac{256}{3rs} |t_1|^2 |C_T|^2 m_B^2 \Big(4 m_\ell^2 [\lambda(8r - s) - 12 rs (2 + 2r - s)] \\
&+ m_B^2 s [\lambda(16 r + s) + 12 rs (2 + 2r - s)] \Big)
\end{aligned} \tag{11}$$

$$
\begin{aligned}
&+ \frac{1024}{3rs}\left|t_1\right|^2 \left|C_{TE}\right|^2 m_B^2 \Big(8 m_\ell^2 \left[\lambda(4r+s) + 12rs(2+2r-s)\right] \\
&+ m_B^2 s \left[\lambda(16r+s) + 12rs(2+2r-s)\right]\Big) \\
&- \frac{128}{r} m_B^2 m_\ell \left[\lambda + 12r(1-r)\right] \mathrm{Re}[(B_1+D_1)(t_1 C_{TE})^*] \\
&+ \frac{128}{r} m_B^4 m_\ell \lambda (1+3r-s)\, \mathrm{Re}[(B_2+D_2)(t_1 C_{TE})^*] + 512 m_B^4 m_\ell \lambda\, \mathrm{Re}[(A_1+C_1)(t_1 C_T)^*] \\
&+ \frac{16}{3r} m_B^2 \left(4(m_B^2 s + 8 m_\ell^2) \left|C_{TE}\right|^2 + m_B^2 s v^2 \left|C_T\right|^2 \right) \times \Big(4(\lambda + 12rs) \left|B_6\right|^2 \\
&+ m_B^4 \lambda^2 \left|B_7\right|^2 - 4 m_B^2 (1-r-s) \lambda\, \mathrm{Re}(B_6 B_7^*) - 16 \left[\lambda + 12r(1-r)\right] \mathrm{Re}(t_1 B_6^*) \\
&+ 8 m_B^2 (1+3r-s) \lambda\, \mathrm{Re}(t_1 B_7^*) \Big) \Bigg\} ~,
\end{aligned}
$$

where $s = q^2/m_B^2$, $r = m_{K^*}^2/m_B^2$ and $v = \sqrt{1 - \frac{4m_\ell^2}{q^2}}$ is the lepton velocity.

The polarizations P_L, P_N and P_T are defined as:

$$
P_i^{(\pm)}(q^2) = \frac{\dfrac{d\Gamma}{dq^2}(\vec{n}^{(\pm)} = \vec{e}_i^{\,(\pm)}) - \dfrac{d\Gamma}{dq^2}(\vec{n}^{(\pm)} = -\vec{e}_i^{\,(\pm)})}{\dfrac{d\Gamma}{dq^2}(\vec{n}^{(\pm)} = \vec{e}_i^{\,(\pm)}) + \dfrac{d\Gamma}{dq^2}(\vec{n}^{(\pm)} = -\vec{e}_i^{\,(\pm)})} ~,
$$

where $P^{(\pm)}$ represents the charged lepton $\ell^{(\pm)}$ polarization asymmetry for $i = L,~N,~T$, i.e., P_L and P_T are the longitudinal and transversal asymmetries in the decay plane, respectively, and P_N is the normal component to both of them. With respect to the direction of the lepton polarization, P_L and P_T are P–odd, T–even, while P_N is P–even, T–odd and CP–odd. After lengthy calculations for the longitudinal polarization of the $\ell^{(\pm)}$, we get

$$
\begin{aligned}
P_L^- &= \frac{4}{\Delta} m_B^2 v \Bigg\{ \frac{1}{3r} \lambda^2 m_B^4 \Big[\left|B_2\right|^2 - \left|D_2\right|^2 \Big] + \frac{1}{r} \lambda m_\ell\, \mathrm{Re}[(B_1 - D_1)(B_4^* + B_5^*)] \\
&- \frac{1}{r} \lambda m_B^2 m_\ell (1-r)\, \mathrm{Re}[(B_2 - D_2)(B_4^* + B_5^*)] + \frac{8}{3} \lambda m_B^4 s \Big[\left|A_1\right|^2 - \left|C_1\right|^2 \Big] \\
&- \frac{1}{2r} \lambda m_B^2 s \Big[\left|B_4\right|^2 - \left|B_5\right|^2 \Big] - \frac{1}{r} \lambda m_B^2 m_\ell s\, \mathrm{Re}[(B_3 - D_3)(B_4^* + B_5^*)] \\
&- \frac{2}{3r} \lambda m_B^2 (1-r-s) \Big[\mathrm{Re}(B_1 B_2^*) - \mathrm{Re}(D_1 D_2^*) \Big] + \frac{1}{3r} (\lambda + 12rs) \Big[\left|B_1\right|^2 - \left|D_1\right|^2 \Big] \\
&+ \frac{256}{3} \lambda m_B^2 m_\ell \Big(\mathrm{Re}[A_1^* (C_T + C_{TE}) t_1] - \mathrm{Re}[C_1^* (C_T - C_{TE}) t_1] \Big) \\
&+ \frac{4}{3r} \lambda^2 m_B^4 m_\ell \Big(\mathrm{Re}[B_2^* (C_T + 4C_{TE}) B_7] + \mathrm{Re}[D_2^* (C_T - 4C_{TE}) B_7] \Big) \\
&- \frac{8}{3r} \lambda m_B^2 m_\ell (1-r-s) \Big(\mathrm{Re}[B_2^* (C_T + 4C_{TE}) B_6] + \mathrm{Re}[D_2^* (C_T - 4C_{TE}) B_6] \Big) \\
&- \frac{4}{3r} \lambda m_B^2 m_\ell (1-r-s) \Big(\mathrm{Re}[B_1^* (C_T + 4C_{TE}) B_7] + \mathrm{Re}[D_1^* (C_T - 4C_{TE}) B_7] \Big) \\
&+ \frac{8}{3r} (\lambda + 12rs) m_\ell \Big(\mathrm{Re}[B_1^* (C_T + 4C_{TE}) B_6] + \mathrm{Re}[D_1^* (C_T - 4C_{TE}) B_6] \Big)
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{16}{3r} m_\ell [\lambda + 12r(1-r)] \Big(\mathrm{Re}[B_1^*(C_T + 4C_{TE}) t_1] + \mathrm{Re}[D_1^*(C_T - 4C_{TE}) t_1] \Big) \qquad (12) \\
&+ \frac{16}{3r} \lambda m_B^2 m_\ell (1+3r-s) \Big(\mathrm{Re}[B_2^*(C_T + 4C_{TE}) t_1] + \mathrm{Re}[D_2^*(C_T - 4C_{TE}) t_1] \Big) \\
&+ \frac{16}{3r} \lambda^2 m_B^6 s \left| B_7 \right|^2 \mathrm{Re}(C_T C_{TE}^*) \\
&+ \frac{64}{3r} (\lambda + 12rs) m_B^2 s \left| B_6 \right|^2 \mathrm{Re}(C_T C_{TE}^*) \\
&- \frac{64}{3r} \lambda m_B^4 s (1-r-s)\, \mathrm{Re}(B_6 B_7^*) \mathrm{Re}(C_T C_{TE}^*) \\
&+ \frac{128}{3r} \lambda m_B^4 s (1+3r-s)\, \mathrm{Re}(B_7 t_1^*) \mathrm{Re}(C_T C_{TE}^*) \\
&- \frac{256}{3r} m_B^2 s [\lambda + 12r(1-r)]\, \mathrm{Re}(B_6 t_1^*) \mathrm{Re}(C_T C_{TE}^*) \\
&+ \frac{256}{3r} m_B^2 [\lambda(4r+s) + 12r(1-r)^2] \left| t_1 \right|^2 \, \mathrm{Re}(C_T C_{TE}^*) \Bigg\} ,
\end{aligned}
$$

$$
\begin{aligned}
P_L^+ &= \frac{4}{\Delta} m_B^2 v \Bigg\{ - \frac{1}{3r} \lambda^2 m_B^4 \Big[\left| B_2 \right|^2 - \left| D_2 \right|^2 \Big] + \frac{1}{r} \lambda m_\ell \, \mathrm{Re}[(B_1 - D_1)(B_4^* + B_5^*)] \\
&- \frac{1}{r} \lambda m_B^2 m_\ell (1-r)\, \mathrm{Re}[(B_2 - D_2)(B_4^* + B_5^*)] - \frac{8}{3} \lambda m_B^4 s \Big[\left| A_1 \right|^2 - \left| C_1 \right|^2 \Big] \\
&- \frac{1}{2r} \lambda m_B^2 s \Big[\left| B_4 \right|^2 - \left| B_5 \right|^2 \Big] - \frac{1}{r} \lambda m_B^2 m_\ell s \, \mathrm{Re}[(B_3 - D_3)(B_4^* + B_5^*)] \\
&+ \frac{2}{3r} \lambda m_B^2 (1-r-s) \Big[\mathrm{Re}(B_1 B_2^*) - \mathrm{Re}(D_1 D_2^*) \Big] - \frac{1}{3r} (\lambda + 12rs) \Big[\left| B_1 \right|^2 - \left| D_1 \right|^2 \Big] \\
&- \frac{256}{3} \lambda m_B^2 m_\ell \Big(\mathrm{Re}[A_1^*(C_T - C_{TE}) t_1] - \mathrm{Re}[C_1^*(C_T + C_{TE}) t_1] \Big) \\
&+ \frac{4}{3r} \lambda^2 m_B^4 m_\ell \Big(\mathrm{Re}[B_2^*(C_T - 4C_{TE}) B_7] + \mathrm{Re}[D_2^*(C_T + 4C_{TE}) B_7] \Big) \\
&- \frac{8}{3r} \lambda m_B^2 m_\ell (1-r-s) \Big(\mathrm{Re}[B_2^*(C_T - 4C_{TE}) B_6] + \mathrm{Re}[D_2^*(C_T + 4C_{TE}) B_6] \Big) \\
&- \frac{4}{3r} \lambda m_B^2 m_\ell (1-r-s) \Big(\mathrm{Re}[B_1^*(C_T - 4C_{TE}) B_7] + \mathrm{Re}[D_1^*(C_T + 4C_{TE}) B_7] \Big) \\
&+ \frac{8}{3r} (\lambda + 12rs) m_\ell \Big(\mathrm{Re}[B_1^*(C_T - 4C_{TE}) B_6] + \mathrm{Re}[D_1^*(C_T + 4C_{TE}) B_6] \Big) \\
&- \frac{16}{3r} m_\ell [\lambda + 12r(1-r)] \Big(\mathrm{Re}[B_1^*(C_T - 4C_{TE}) t_1] + \mathrm{Re}[D_1^*(C_T + 4C_{TE}) t_1] \Big) \qquad (13) \\
&+ \frac{16}{3r} \lambda m_B^2 m_\ell (1+3r-s) \Big(\mathrm{Re}[B_2^*(C_T - 4C_{TE}) t_1] + \mathrm{Re}[D_2^*(C_T + 4C_{TE}) t_1] \Big) \\
&+ \frac{16}{3r} \lambda^2 m_B^6 s \left| B_7 \right|^2 \mathrm{Re}(C_T C_{TE}^*) \\
&+ \frac{64}{3r} (\lambda + 12rs) m_B^2 s \left| B_6 \right|^2 \mathrm{Re}(C_T C_{TE}^*) \\
&- \frac{64}{3r} \lambda m_B^4 s (1-r-s)\, \mathrm{Re}(B_6 B_7^*) \mathrm{Re}(C_T C_{TE}^*)
\end{aligned}
$$

$$
\begin{aligned}
&+ \frac{128}{3r}\lambda m_B^4 s(1+3r-s)\,\mathrm{Re}(B_7 t_1^*)\mathrm{Re}(C_T C_{TE}^*)\\
&- \frac{256}{3r} m_B^2 s[\lambda + 12r(1-r)]\,\mathrm{Re}(B_6 t_1^*)\mathrm{Re}(C_T C_{TE}^*)\\
&+ \left.\frac{256}{3r} m_B^2[\lambda(4r+s) + 12r(1-r)^2]\,\left|t_1\right|^2\,\mathrm{Re}(C_T C_{TE}^*)\right\}\,,
\end{aligned}
$$

where Δ is the term inside curly brackets of Eq. (11). From Eqs. (12) and (13) we observe that the terms containing "pure" SM contribution, i.e., the terms containing C_{BR}, C_{SL}, C_{LL}^{tot} and C_{LR}^{tot} are the same for both lepton and antilepton but with opposite sign. However for the terms containing new physics effects this does not hold. In other words, such terms may have same or different signs for lepton and antilepton. In due course this difference can be a useful tool for looking new physics effects.

Further calculations lead the following expressions for the transverse polarization $P_T^{(\pm)}$:

$$
\begin{aligned}
P_T^- &= \frac{\pi}{\Delta} m_B \sqrt{s\lambda}\Bigg\{ - 8 m_B^2 m_\ell\,\mathrm{Re}[(A_1+C_1)(B_1^*+D_1^*)]\\
&+ \frac{1}{r} m_B^2 m_\ell (1+3r-s)\Big[\mathrm{Re}(B_1 D_2^*) - \mathrm{Re}(B_2 D_1^*)\Big]\\
&+ \frac{1}{rs} m_\ell (1-r-s)\Big[\left|B_1\right|^2 - \left|D_1\right|^2\Big]\\
&+ \frac{2}{rs} m_\ell^2 (1-r-s)\Big[\mathrm{Re}(B_1 B_5^*) - \mathrm{Re}(D_1 B_4^*)\Big]\\
&- \frac{1}{r} m_B^2 m_\ell (1-r-s)\,\mathrm{Re}[(B_1+D_1)(B_3^*-D_3^*)]\\
&- \frac{2}{rs} m_B^2 m_\ell^2 \lambda\Big[\mathrm{Re}(B_2 B_5^*) - \mathrm{Re}(D_2 B_4^*)\Big]\\
&+ \frac{1}{rs} m_B^4 m_\ell (1-r)\lambda\Big[\left|B_2\right|^2 - \left|D_2\right|^2\Big] + \frac{1}{r} m_B^4 m_\ell \lambda\,\mathrm{Re}[(B_2+D_2)(B_3^*-D_3^*)]\\
&- \frac{1}{rs} m_B^2 m_\ell [\lambda + (1-r-s)(1-r)]\Big[\mathrm{Re}(B_1 B_2^*) - \mathrm{Re}(D_1 D_2^*)\Big]\\
&+ \frac{1}{rs} (1-r-s)(2m_\ell^2 - m_B^2 s)\Big[\mathrm{Re}(B_1 B_4^*) - \mathrm{Re}(D_1 B_5^*)\Big]\\
&+ \frac{1}{rs} m_B^2 \lambda (2m_\ell^2 - m_B^2 s)\Big[\mathrm{Re}(D_2 B_5^*) - \mathrm{Re}(B_2 B_4^*)\Big]\\
&- \frac{16}{rs} \lambda m_B^2 m_\ell^2\,\mathrm{Re}[(B_1-D_1)(B_7 C_{TE})^*]\\
&+ \frac{16}{rs} \lambda m_B^4 m_\ell^2 (1-r)\,\mathrm{Re}[(B_2-D_2)(B_7 C_{TE})^*]\\
&+ \frac{8}{r} \lambda m_B^4 m_\ell\,\mathrm{Re}[(B_4-B_5)(B_7 C_{TE})^*]\\
&+ \frac{16}{r} \lambda m_B^4 m_\ell^2\,\mathrm{Re}[(B_3-D_3)(B_7 C_{TE})^*]\\
&+ \frac{32}{rs} m_\ell^2 (1-r-s)\,\mathrm{Re}[(B_1-D_1)(B_6 C_{TE})^*] \qquad (14)\\
&- \frac{32}{rs} m_B^2 m_\ell^2 (1-r)(1-r-s)\,\mathrm{Re}[(B_2-D_2)(B_6 C_{TE})^*]
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{16}{r} m_B^2 m_\ell (1-r-s)\,\mathrm{Re}[(B_4-B_5)(B_6 C_{TE})^*] \\
&- \frac{32}{r} m_B^2 m_\ell^2 (1-r-s)\,\mathrm{Re}[(B_3-D_3)(B_6 C_{TE})^*] \\
&- 16 m_B^2 \Big(4 m_\ell^2\,\mathrm{Re}[A_1^*(C_T+2C_{TE})B_6] - m_B^2 s\,\mathrm{Re}[A_1^*(C_T-2C_{TE})B_6]\Big) \\
&+ 16 m_B^2 \Big(4 m_\ell^2\,\mathrm{Re}[C_1^*(C_T-2C_{TE})B_6] - m_B^2 s\,\mathrm{Re}[C_1^*(C_T+2C_{TE})B_6]\Big) \\
&+ \frac{32}{s} m_B^2 (1-r) \Big(4 m_\ell^2\,\mathrm{Re}[A_1^*(C_T+2C_{TE})t_1] - m_B^2 s\,\mathrm{Re}[A_1^*(C_T-2C_{TE})t_1]\Big) \\
&- \frac{32}{s} m_B^2 (1-r) \Big(4 m_\ell^2\,\mathrm{Re}[C_1^*(C_T-2C_{TE})t_1] - m_B^2 s\,\mathrm{Re}[C_1^*(C_T+2C_{TE})t_1]\Big) \\
&+ \frac{64}{rs} m_B^2 m_\ell^2 (1-r)(1+3r-s)\,\mathrm{Re}[(B_2-D_2)(t_1 C_{TE})^*] \\
&+ \frac{64}{r} m_B^2 m_\ell^2 (1+3r-s)\,\mathrm{Re}[(B_3-D_3)(t_1 C_{TE})^*] \\
&+ \frac{32}{r} m_B^2 m_\ell (1+3r-s)\,\mathrm{Re}[(B_4-B_5)(t_1 C_{TE})^*] \\
&+ \frac{64}{rs} [m_B^2 r s - m_\ell^2 (1+7r-s)]\,\mathrm{Re}[(B_1-D_1)(t_1 C_{TE})^*] \\
&- \frac{32}{s} (4 m_\ell^2 + m_B^2 s)\,\mathrm{Re}[(B_1+D_1)(t_1 C_T)^*] \\
&- 2048 m_B^2 m_\ell\,\mathrm{Re}[(C_T t_1)(B_6 C_{TE})^*] \\
&+ \frac{4096}{s} m_B^2 m_\ell (1-r)\,|t_1|^2\ \mathrm{Re}(C_T C_{TE}^*)\Bigg\}\ ,
\end{aligned}
$$

$$
\begin{aligned}
P_T^+ &= \frac{\pi}{\Delta} m_B \sqrt{s}\lambda \Bigg\{ -8 m_B^2 m_\ell\,\mathrm{Re}[(A_1+C_1)(B_1^*+D_1^*)] \\
&- \frac{1}{r} m_B^2 m_\ell (1+3r-s)\,\Big[\mathrm{Re}(B_1 D_2^*) - \mathrm{Re}(B_2 D_1^*)\Big] \\
&- \frac{1}{rs} m_\ell (1-r-s)\Big[\,|B_1|^2 - |D_1|^2\,\Big] \\
&+ \frac{1}{rs} (2m_\ell^2 - m_B^2 s)(1-r-s)\Big[\mathrm{Re}(B_1 B_5^*) - \mathrm{Re}(D_1 B_4^*)\Big] \\
&+ \frac{1}{r} m_B^2 m_\ell (1-r-s)\,\mathrm{Re}[(B_1+D_1)(B_3^*-D_3^*)] \\
&- \frac{1}{rs} m_B^2 \lambda (2m_\ell^2 - m_B^2 s)\Big[\mathrm{Re}(B_2 B_5^*) - \mathrm{Re}(D_2 B_4^*)\Big] \\
&- \frac{1}{rs} m_B^4 m_\ell (1-r)\lambda\Big[\,|B_2|^2 - |D_2|^2\,\Big] - \frac{1}{r} m_B^4 m_\ell \lambda\,\mathrm{Re}[(B_2+D_2)(B_3^*-D_3^*)] \\
&+ \frac{1}{rs} m_B^2 m_\ell [\lambda + (1-r-s)(1-r)]\Big[\mathrm{Re}(B_1 B_2^*) - \mathrm{Re}(D_1 D_2^*)\Big] \\
&+ \frac{2}{rs} m_\ell^2 (1-r-s)\Big[\mathrm{Re}(B_1 B_4^*) - \mathrm{Re}(D_1 B_5^*)\Big] \\
&+ \frac{2}{rs} m_B^2 m_\ell^2 \lambda\Big[\mathrm{Re}(D_2 B_5^*) - \mathrm{Re}(B_2 B_4^*)\Big]
\end{aligned}
$$

$$
\begin{aligned}
&+ \frac{16}{rs}\lambda m_B^2 m_\ell^2 \,\mathrm{Re}[(B_1 - D_1)(B_7 C_{TE})^*] \\
&- \frac{16}{rs}\lambda m_B^4 m_\ell^2 (1-r)\,\mathrm{Re}[(B_2 - D_2)(B_7 C_{TE})^*] \\
&- \frac{8}{r}\lambda m_B^4 m_\ell \,\mathrm{Re}[(B_4 - B_5)(B_7 C_{TE})^*] \\
&- \frac{16}{r}\lambda m_B^4 m_\ell^2 \,\mathrm{Re}[(B_3 - D_3)(B_7 C_{TE})^*] \\
&- \frac{32}{rs} m_\ell^2 (1-r-s)\,\mathrm{Re}[(B_1 - D_1)(B_6 C_{TE})^*] \qquad (15) \\
&+ \frac{32}{rs} m_B^2 m_\ell^2 (1-r)(1-r-s)\,\mathrm{Re}[(B_2 - D_2)(B_6 C_{TE})^*] \\
&+ \frac{16}{r} m_B^2 m_\ell (1-r-s)\,\mathrm{Re}[(B_4 - B_5)(B_6 C_{TE})^*] \\
&+ \frac{32}{r} m_B^2 m_\ell^2 (1-r-s)\,\mathrm{Re}[(B_3 - D_3)(B_6 C_{TE})^*] \\
&+ 16 m_B^2 \Big(4 m_\ell^2 \,\mathrm{Re}[A_1^*(C_T - 2C_{TE})B_6] - m_B^2 s \,\mathrm{Re}[A_1^*(C_T + 2C_{TE})B_6]\Big) \\
&- 16 m_B^2 \Big(4 m_\ell^2 \,\mathrm{Re}[C_1^*(C_T + 2C_{TE})B_6] - m_B^2 s \,\mathrm{Re}[C_1^*(C_T - 2C_{TE})B_6]\Big) \\
&- \frac{32}{s} m_B^2 (1-r)\Big(4 m_\ell^2 \,\mathrm{Re}[A_1^*(C_T - 2C_{TE})t_1] - m_B^2 s \,\mathrm{Re}[A_1^*(C_T + 2C_{TE})t_1]\Big) \\
&+ \frac{32}{s} m_B^2 (1-r)\Big(4 m_\ell^2 \,\mathrm{Re}[C_1^*(C_T + 2C_{TE})t_1] - m_B^2 s \,\mathrm{Re}[C_1^*(C_T - 2C_{TE})t_1]\Big) \\
&- \frac{64}{rs} m_B^2 m_\ell^2 (1-r)(1+3r-s)\,\mathrm{Re}[(B_2 - D_2)(t_1 C_{TE})^*] \\
&- \frac{64}{r} m_B^2 m_\ell^2 (1+3r-s)\,\mathrm{Re}[(B_3 - D_3)(t_1 C_{TE})^*] \\
&- \frac{32}{r} m_B^2 m_\ell (1+3r-s)\,\mathrm{Re}[(B_4 - B_5)(t_1 C_{TE})^*] \\
&- \frac{64}{rs}[m_B^2 rs - m_\ell^2 (1+7r-s)]\,\mathrm{Re}[(B_1 - D_1)(t_1 C_{TE})^*] \\
&- \frac{32}{s}(4 m_\ell^2 + m_B^2 s)\,\mathrm{Re}[(B_1 + D_1)(t_1 C_T)^*] \\
&- 2048 m_B^2 m_\ell \,\mathrm{Re}[(C_T t_1)(B_6 C_{TE})^*] \\
&+ \frac{4096}{s} m_B^2 m_\ell (1-r)\,|t_1|^2 \;\mathrm{Re}(C_T C_{TE}^*)\Bigg\}\;.
\end{aligned}
$$

Finally for normal asymmetries we get

$$
\begin{aligned}
P_N^- &= \frac{1}{\Delta}\pi v m_B^3 \sqrt{s\lambda}\Bigg\{ 8 m_\ell \,\mathrm{Im}[(B_1^* C_1) + (A_1^* D_1)] \\
&- \frac{1}{r} m_B^2 \lambda \,\mathrm{Im}[(B_2^* B_4) + (D_2^* B_5)] \\
&+ \frac{1}{r} m_B^2 m_\ell \lambda \,\mathrm{Im}[(B_2 - D_2)(B_3^* - D_3^*)]
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{1}{r} m_\ell \,(1+3r-s)\,\mathrm{Im}[(B_1-D_1)(B_2^*-D_2^*)] \\
&+ \frac{1}{r}(1-r-s)\,\mathrm{Im}[(B_1^* B_4)+(D_1^* B_5)] \\
&- \frac{1}{r} m_\ell \,(1-r-s)\,\mathrm{Im}[(B_1-D_1)(B_3^*-D_3^*)] \\
&- \frac{8}{r} m_B^2 m_\ell \lambda \,\mathrm{Im}[(B_4+B_5)(B_7 C_{TE})^*] \qquad (16)\\
&+ \frac{16}{r} m_\ell \,(1-r-s)\,\mathrm{Im}[(B_4+B_5)(B_6 C_{TE})^*] \\
&- \frac{32}{r} m_\ell \,(1+3r-s)\,\mathrm{Im}[(B_4+B_5)(t_1 C_{TE})^*] \\
&- 16 m_B^2 s \Big(\mathrm{Im}[A_1^*(C_T-2C_{TE})B_6] + \mathrm{Im}[C_1^*(C_T+2C_{TE})B_6] \Big) \\
&+ 32 m_B^2 (1-r) \Big(\mathrm{Im}[A_1^*(C_T-2C_{TE})t_1] + \mathrm{Im}[C_1^*(C_T+2C_{TE})t_1] \Big) \\
&+ 32 \Big(\mathrm{Im}[B_1^*(C_T-2C_{TE})t_1] - \mathrm{Im}[D_1^*(C_T+2C_{TE})t_1] \Big) \\
&+ 512 m_\ell \,\Big(\left|C_T\right|^2 - 4\left|C_{TE}\right|^2 \Big)\, \mathrm{Im}(B_6^* t_1) \Big\}\,,
\end{aligned}
$$

$$
\begin{aligned}
P_N^+ \;=\; & \frac{1}{\Delta} \pi v m_B^3 \sqrt{s}\lambda \Big\{ -8 m_\ell \,\mathrm{Im}[(B_1^* C_1)+(A_1^* D_1)] \\
&+ \frac{1}{r} m_B^2 \lambda \,\mathrm{Im}[(B_2^* B_5)+(D_2^* B_4)] \\
&+ \frac{1}{r} m_B^2 m_\ell \lambda \,\mathrm{Im}[(B_2-D_2)(B_3^*-D_3^*)] \\
&- \frac{1}{r} m_\ell \,(1+3r-s)\,\mathrm{Im}[(B_1-D_1)(B_2^*-D_2^*)] \\
&- \frac{1}{r}(1-r-s)\,\mathrm{Im}[(B_1^* B_5)+(D_1^* B_4)] \\
&- \frac{1}{r} m_\ell \,(1-r-s)\,\mathrm{Im}[(B_1-D_1)(B_3^*-D_3^*)] \\
&+ \frac{8}{r} m_B^2 m_\ell \lambda \,\mathrm{Im}[(B_4+B_5)(B_7 C_{TE})^*] \qquad (17)\\
&- \frac{16}{r} m_\ell \,(1-r-s)\,\mathrm{Im}[(B_4+B_5)(B_6 C_{TE})^*] \\
&+ \frac{32}{r} m_\ell \,(1+3r-s)\,\mathrm{Im}[(B_4+B_5)(t_1 C_{TE})^*] \\
&- 16 m_B^2 s \Big(\mathrm{Im}[A_1^*(C_T+2C_{TE})B_6] + \mathrm{Im}[C_1^*(C_T-2C_{TE})B_6] \Big) \\
&+ 32 m_B^2 (1-r) \Big(\mathrm{Im}[A_1^*(C_T+2C_{TE})t_1] + \mathrm{Im}[C_1^*(C_T-2C_{TE})t_1] \Big) \\
&- 32 \Big(\mathrm{Im}[B_1^*(C_T+2C_{TE})t_1] - \mathrm{Im}[D_1^*(C_T-2C_{TE})t_1] \Big) \\
&+ 512 m_\ell \,\Big(\left|C_T\right|^2 - 4\left|C_{TE}\right|^2 \Big)\, \mathrm{Im}(B_6^* t_1) \Big\}\,.
\end{aligned}
$$

Concerning expressions $P_L^{(\pm)}$, $P_T^{(\pm)}$ and $P_N^{(\pm)}$ few remarks are in order. The difference between P_L^- and P_L^+ results from the scalar and tensor type interactions. Similar situation takes place for the normal polarization $P_N^{(\pm)}$ of leptons and antileptons. In the $m_\ell \to 0$ limit, the difference between P_T^- and P_T^+ is due to again existence of new physics, i.e., scalar and tensor type interactions. For these reasons the experimental study of $P_L^{(\pm)}$ and $P_T^{(\pm)}$ can give essential information about new physics. Note that similar situation takes place for the inclusive channel $b \to s\ell^+\ell^-$ (see [19]).

Combined analysis of the lepton and antilepton polarizations can also give very useful hints in search of new physics, since in the SM $P_L^- + P_L^+ = 0$, $P_N^- + P_N^+ = 0$ and $P_T^- - P_T^+ \approx 0$.

Using Eqs. (12), (13) we get

$$
\begin{aligned}
P_L^- + P_L^+ &= \frac{4}{\Delta} m_B^2 v \Bigg\{ \frac{2}{r} m_\ell \lambda \,\mathrm{Re}[(B_1 - D_1)(B_4^* + B_5^*)] \\
&- \frac{2}{r} m_B^2 m_\ell \lambda (1-r) \,\mathrm{Re}[(B_2 - D_2)(B_4^* + B_5^*)] \\
&- \frac{1}{r} m_B^2 s \lambda \Big(|B_4|^2 - |B_5|^2 \Big) - \frac{2}{r} m_B^2 m_\ell s \lambda \,\mathrm{Re}[(B_3 - D_3)(B_4^* + B_5^*)] \\
&+ \frac{8}{3r} m_B^4 m_\ell \lambda^2 \,\mathrm{Re}[(B_2 + D_2)(B_7 C_T)^*] \\
&+ \frac{32}{3r} m_B^6 s \lambda^2 |B_7|^2 \,\mathrm{Re}(C_T C_{TE}^*) \\
&- \frac{8}{3r} m_B^2 m_\ell \lambda (1-r-s) \,\mathrm{Re}[(B_1 + D_1)(B_7 C_T)^*] \\
&- \frac{16}{3r} m_B^2 m_\ell \lambda (1-r-s) \,\mathrm{Re}[(B_2 + D_2)(B_6 C_T)^*] \\
&- \frac{128}{3r} m_B^4 s \lambda (1-r-s) \,\mathrm{Re}(B_6 B_7^*) \,\mathrm{Re}(C_T C_{TE}^*) \\
&+ \frac{16}{3r} m_\ell (\lambda + 12 r s) \,\mathrm{Re}[(B_1 + D_1)(B_6 C_T)^*] \\
&+ \frac{128}{3r} m_B^2 s (\lambda + 12 r s) \; |B_6|^2 \,\mathrm{Re}(C_T C_{TE}^*) \\
&+ \frac{512}{3r} m_B^2 \left[\lambda(4r + s) + 12 r (1-r)^2\right] \; |t_1|^2 \; \mathrm{Re}(C_T C_{TE}^*) \\
&- \frac{512}{3r} m_B^2 s \left[\lambda + 12 r (1-r)\right] \mathrm{Re}(t_1 B_6^*) \,\mathrm{Re}(C_T C_{TE}^*) \\
&+ \frac{256}{3r} m_B^4 s \lambda (1 + 3r - s) \,\mathrm{Re}(t_1 B_7^*) \,\mathrm{Re}(C_T C_{TE}^*) \\
&+ \frac{512}{3} m_B^2 m_\ell \lambda \,\mathrm{Re}[(A_1 + C_1)(t_1 C_{TE})^*] \\
&- \frac{32}{3r} m_\ell \left[\lambda + 12 r (1-r)\right] \mathrm{Re}[(B_1 + D_1)(t_1 C_T)^*] \\
&+ \frac{32}{3r} m_B^2 m_\ell \lambda (1 + 3r - s) \,\mathrm{Re}[(B_2 + D_2)(t_1 C_T)^*] \Bigg\} ~.
\end{aligned} \tag{18}
$$

For the case of transversal polarization, it is the difference of the lepton and antilepton polarizations that is relevant and it can be calculated from Eqs. (14) and (15)

$$
\begin{aligned}
P_T^- - P_T^+ &= \frac{\pi}{\Delta} m_B \sqrt{s\lambda} \Bigg\{ \frac{2}{rs} m_B^4 m_\ell (1-r)\lambda \Big[|B_2|^2 - |D_2|^2 \Big] \\
&+ \frac{1}{r} m_B^4 \lambda \, \mathrm{Re}[(B_2+D_2)(B_4^* - B_5^*)] \\
&+ \frac{2}{r} m_B^4 m_\ell \lambda \, \mathrm{Re}[(B_2+D_2)(B_3^* - D_3^*)] \\
&+ \frac{2}{r} m_B^2 m_\ell (1+3r-s) \Big[\mathrm{Re}(B_1 D_2^*) - \mathrm{Re}(B_2 D_1^*) \Big] \\
&+ \frac{2}{rs} m_\ell (1-r-s) \Big[|B_1|^2 - |D_1|^2 \Big] \\
&- \frac{1}{r} m_B^2 (1-r-s) \mathrm{Re}[(B_1+D_1)(B_4^* - B_5^*)] \\
&- \frac{2}{r} m_B^2 m_\ell (1-r-s) \mathrm{Re}[(B_1+D_1)(B_3^* - D_3^*)] \\
&- \frac{2}{rs} m_B^2 m_\ell [\lambda + (1-r)(1-r-s)] \Big[\mathrm{Re}(B_1 B_2^*) - \mathrm{Re}(D_1 D_2^*) \Big] \\
&- \frac{32}{rs} m_B^2 m_\ell^2 \lambda \, \mathrm{Re}[(B_1 - D_1)(B_7 C_{TE})^*] \\
&+ \frac{32}{rs} m_B^4 m_\ell^2 \lambda (1-r) \, \mathrm{Re}[(B_2 - D_2)(B_7 C_{TE})^*] \\
&+ \frac{16}{r} m_B^4 m_\ell \lambda \, \mathrm{Re}[(B_4 - B_5)(B_7 C_{TE})^*] \\
&+ \frac{32}{r} m_B^4 m_\ell^2 \lambda \, \mathrm{Re}[(B_3 - D_3)(B_7 C_{TE})^*] \\
&+ \frac{64}{rs} m_\ell^2 (1-r-s) \, \mathrm{Re}[(B_1 - D_1)(B_6 C_{TE})^*] \\
&- \frac{64}{rs} m_B^2 m_\ell^2 (1-r)(1-r-s) \, \mathrm{Re}[(B_2 - D_2)(B_6 C_{TE})^*] \\
&- \frac{32}{r} m_B^2 m_\ell (1-r-s) \, \mathrm{Re}[(B_4 - B_5)(B_6 C_{TE})^*] \\
&- \frac{64}{r} m_B^2 m_\ell^2 (1-r-s) \, \mathrm{Re}[(B_3 - D_3)(B_6 C_{TE})^*] \\
&+ 32 m_B^4 s v^2 \, \mathrm{Re}[(A_1 - C_1)(B_6 C_T)^*] \\
&+ \frac{64}{r} m_B^2 m_\ell (1+3r-s) \, \mathrm{Re}[(B_4 - B_5)(t_1 C_{TE})^*] \\
&- 64 m_B^4 (1-r) v^2 \, \mathrm{Re}[(A_1 - C_1)(t_1 C_T)^*] \\
&+ \frac{128}{rs} [m_B^2 rs - m_\ell^2 (1+7r-s)] \, \mathrm{Re}[(B_1 - D_1)(t_1 C_{TE})^*] \\
&+ \frac{128}{rs} m_B^2 m_\ell^2 (1-r)(1+3r-s) \mathrm{Re}[(B_2 - D_2)(t_1 C_{TE})^*] \\
&+ \frac{128}{r} m_B^2 m_\ell^2 (1+3r-s) \mathrm{Re}[(B_3 - D_3)(t_1 C_{TE})^*] \Bigg\} .
\end{aligned} \tag{19}
$$

In the same manner it follows from Eqs. (16) and (17)

$$
\begin{aligned}
P_N^- + P_N^+ &= \frac{1}{\Delta}\pi v m_B^3 \sqrt{s\lambda}\Bigg\{ -\frac{2}{r} m_\ell (1+3r-s)\,\mathrm{Im}[(B_1-D_1)(B_2^*-D_2^*)] \\
&- \frac{2}{r} m_\ell (1-r-s)\,\mathrm{Im}[(B_1-D_1)(B_3^*-D_3^*)] \\
&- \frac{1}{r}(1-r-s)\,\mathrm{Im}[(B_1-D_1)(B_4^*-B_5^*)] \\
&+ \frac{2}{r} m_B^2 m_\ell \lambda\,\mathrm{Im}[(B_2-D_2)(B_3^*-D_3^*)] \\
&+ \frac{1}{r} m_B^2 \lambda\,\mathrm{Im}[(B_2-D_2)(B_4^*-B_5^*)] \\
&+ 32 m_B^2 s\,\mathrm{Im}[(A_1+C_1)(B_6 C_T)^*] \\
&+ 1024 m_\ell \Big(|C_T|^2 - |4C_{TE}|^2 \Big)\mathrm{Im}(B_6^* t_1) \\
&- 64 m_B^2 (1-r)\,\mathrm{Im}[(A_1+C_1)(t_1 C_T)^*] \\
&+ 128\,\mathrm{Im}[(B_1+D_1)(t_1 C_{TE})^*]\Bigg\}\ .
\end{aligned} \tag{20}
$$

It is evident from Eq. (18) that the "pure" SM contribution to the $P_L^- + P_L^+$ completely disappears. Therefore a measurement of the nonzero value of $P_L^- + P_L^+$ in future experiments, is an indication of the discovery of new physics beyond SM.

3 Numerical analysis

The input parameters we used in our analysis are: $|V_{tb}V_{ts}^*| = 0.0385$, $\alpha^{-1} = 129$, $G_F = 1.17 \times 10^{-5}\ GeV^{-2}$, $\Gamma_B = 4.22 \times 10^{-13}\ GeV$, $C_9^{eff} = 4.344$, $C_{10} = -4.669$. It should be noted here that the above–value of the Wilson coefficient C_9^{eff} we have used in our numerical calculations corresponds only to short distance contribution. In addition to the short distance contribution C_9^{eff} also receives long distance contributions associated with the real $\bar{c}c$ intermediate states, i.e., with the J/ψ family. In this work we restricted ourselves only to short distance contributions. As far as C_7^{eff} is concerned, experimental results fixes only the modulo of it. For this reason throughout our analysis we have considered both possibilities, i.e., $C_7^{eff} = \mp 0.313$, where the upper sign corresponds to the SM prediction. The values of the input parameters which are summarized above, have been fixed by their central values.

For the values of the form factors, we have used the results of [28], where the radiative corrections to the leading twist contribution and $SU(3)$ breaking effects are also taken into account. The q^2 dependence of the form factors can be represented in terms of three parameters as

$$
F(q^2) = \frac{F(0)}{1 - a_F\,\dfrac{q^2}{m_B^2} + b_F \left(\dfrac{q^2}{m_B^2}\right)^2}\ ,
$$

where the values of parameters $F(0)$, a_F and b_F for the $B \to K^*$ decay are listed in Table 1. Note that in the present analysis the final state Coulomb interactions of the leptons with the other charged particles are neglected since this effect is known to be much smaller than the averaged values of the SM (see [24]). Furthermore the final state interaction of the lepton polarization for the $K_L \to \pi^+ \mu^- \bar{\nu}_\mu$ or $K^+ \to \pi^+ \mu^- \mu^+$ decays is estimated to be of the order of $\alpha(m_\mu/m_K) \approx 10^{-3}$ [29]. For this reason the final state interaction effect is neglected as well.

	$F(0)$	a_F	b_F
$A_1^{B\to K^*}$	0.34 ± 0.05	0.60	-0.023
$A_2^{B\to K^*}$	0.28 ± 0.04	1.18	0.281
$V^{B\to K^*}$	0.46 ± 0.07	1.55	0.575
$T_1^{B\to K^*}$	0.19 ± 0.03	1.59	0.615
$T_2^{B\to K^*}$	0.19 ± 0.03	0.49	-0.241
$T_3^{B\to K^*}$	0.13 ± 0.02	1.20	0.098

Table 1: B meson decay form factors in a three-parameter fit, where the radiative corrections to the leading twist contribution and SU(3) breaking effects are taken into account.

We observe from the explicit form of the expressions of the lepton polarizations that they all depend on q^2 and the new Wilson coefficients. Therefore it may be experimentally difficult to study the dependence of the the polarizations of each lepton on all $\ell^+\ell^-$ center of mass energies and on new Wilson coefficients. So we eliminate the dependence of the lepton polarizations on one of the variables, namely q^2, by performing integration over q^2 in the allowed kinematical region, so that the lepton polarizations are averaged. The averaged lepton polarizations are defined as

$$\langle P_i \rangle = \frac{\displaystyle\int_{4m_\ell^2}^{(m_b - m_{K^*})^2} P_i \frac{d\mathcal{B}}{dq^2} dq^2}{\displaystyle\int_{4m_\ell^2}^{(m_b - m_{K^*})^2} \frac{d\mathcal{B}}{dq^2} dq^2} \ . \tag{21}$$

We present our analysis in a series Figures. Figs. (1) and (2) depict the dependence of the averaged longitudinal polarization $\left\langle P_L^- \right\rangle$ of ℓ^- and the combination $\left\langle P_L^- + P_L^+ \right\rangle$ on new Wilson coefficients, at $C_7^{eff} = -0.313$ for $B \to K^* \mu^+ \mu^-$ decay. From these figures we observe that $\left\langle P_L^- \right\rangle$ is more sensitive to the existence of the tensor interaction, while the combined average $\left\langle P_L^- + P_L^+ \right\rangle$ is to both scalar and tensor type interactions. As has already been noted, this is an expected result since vector type interactions are canceled when the combined longitudinal polarization asymmetry of the lepton and antilepton is considered. From Fig. (2) we see that $\left\langle P_L^- + P_L^+ \right\rangle = 0$ at $C_X = 0$, which confirms the SM result as expected. For the other choice of C_7^{eff}, i.e., $C_7^{eff} = 0.313$ the situation is similar to the previous case, but the magnitude of $\left\langle P_L^- + P_L^+ \right\rangle$ is smaller. Figs. (3) and (4) are the same as Figs.(1) and (2) but for the $B \to K^* \tau^+ \tau^-$ decay. Similar to the muon longitudinal polarization, $\left\langle P_L^- \right\rangle$ is strongly dependent on the tensor interaction coefficients C_T and C_{TE}. It is very interesting to observe that for $C_{TE} > 0.5$ $\left\langle P_L^- \right\rangle$ changes sign, but for all other cases $\left\langle P_L^- \right\rangle$ is negative.

From Fig. (4) we see that the dependence of $\left\langle P_L^- + P_L^+ \right\rangle$ on C_T is stronger. Furthermore if the values of the new Wilson coefficients C_{LRRL}, C_{LRLR} and C_T are negative (positive) so is $\left\langle P_L^- \right\rangle$ negative (positive). The situation is to the contrary for the coefficients C_{RLRL}, C_{RLLR}, i.e., $\left\langle P_L^- + P_L^+ \right\rangle$ is positive (negative) when the corresponding Wilson coefficients are negative (positive). Absolutely similar situation takes place for $C_7^{eff} > 0$. For these reasons determination of the sign and of course magnitude of $\left\langle P_L^- \right\rangle$ and $\left\langle P_L^- + P_L^+ \right\rangle$ can give promising information about new physics.

In Figs. (5) and (6) the dependence of the average transversal polarization $\left\langle P_T^- \right\rangle$ and the combination $\left\langle P_T^- - P_T^+ \right\rangle$ on the new Wilson coefficients, respectively, for the $B \to K^* \mu^+ \mu^-$ decay and at $C_7^{eff} = -0.313$. We observe from Fig. (5) that the average transversal polarization is strongly dependent on C_T, C_{TE}, C_{LRRL} and C_{RLRL} and quite weakly to remaining Wilson coefficients. It is also interesting to note that for the negative (positive) values of the coefficients C_{TE} and C_{LRRL}, $\left\langle P_T^- \right\rangle$ is negative (positive) while it follows the opposite path for the coefficients C_T and C_{RLRL}. For the $\left\langle P_T^- - P_T^+ \right\rangle$ case, there appears strong dependence on the tensor interactions C_T and C_{TE}, as well as all four scalar interactions with coefficients C_{LRRL}, C_{RLLR}, C_{LRLR}, C_{RLRL}. The behavior of this combined average is identical for the coefficients C_{LRLR}, C_{RLRL} and C_{LRRL}, C_{RLLR} in pairs, so that four lines responsible for these interactions appear only to be two. Moreover $\left\langle P_T^- - P_T^+ \right\rangle$ is negative (positive) for the negative (positive) values of the new Wilson coefficients C_{TE}, C_{LRRL} and C_{RLLR}. The situation is the other way around for the coefficients C_T, C_{LRLR} and C_{RLRL}. Remembering that in SM, in massless lepton case $\left\langle P_T^- \right\rangle \approx 0$ and $\left\langle P_T^- - P_T^+ \right\rangle \approx 0$, determination of the signs of the $\left\langle P_T^- \right\rangle$ and $\left\langle P_T^- - P_T^+ \right\rangle$ can give quite a useful information about the existence of new physics. For the choice of $C_7^{eff} = 0.313$, apart from the minor differences in their magnitudes, the behaviors of $\left\langle P_T^- \right\rangle$ and $\left\langle P_T^- - P_T^+ \right\rangle$ are similar as in the previous case.

As is obvious from Figs. (7) and (8), $\left\langle P_T^- \right\rangle$ shows stronger dependence on C_T and $\left\langle P_T^- - P_T^+ \right\rangle$ on C_T and C_{TE}, respectively, at $C_7^{eff} = -0.313$ for the $B \to K^* \tau^+ \tau^-$ decay. Again change in signs of $\left\langle P_T^- \right\rangle$ and $\left\langle P_T^- - P_T^+ \right\rangle$ are observed depending on the change in the tensor interaction coefficients. As has already been noted, determination of the sign and magnitude of $\left\langle P_T^- \right\rangle$ and $\left\langle P_T^- - P_T^+ \right\rangle$ are useful tools in looking for new physics.

Note that for simplicity all new Wilson coefficients in this work are assumed to be real. Under this condition $\left\langle P_N^- \right\rangle$ and $\left\langle P_N^- + P_N^+ \right\rangle$ have non-vanishing values coming from the imaginary part of SM, i.e., from C_9^{eff}. From Fig. (9) we see that $\left\langle P_N^- \right\rangle$ is strongly dependent on all tensor and scalar type interactions. On the other hand Fig. (10) depicts that the behavior $\left\langle P_N^- + P_N^+ \right\rangle$ is determined by only the tensor interactions, for $B \to K^* \mu^+ \mu^-$ decay.

Similar behavior takes place for the $B \to K^* \tau^+ \tau^-$ decay as well, as can easily be seen in Figs. (11) and (12). The change in sign and magnitude of both $\left\langle P_N^- \right\rangle$ and $\left\langle P_N^- + P_N^+ \right\rangle$ that are observed in these figures is an indication of the fact that an experimental verification of them can give unambiguous information about new physics.

In Figs. (13), (14) and (15) we present parametric plot of the correlations between the integrated branching ratio and averaged lepton polarization asymmetries of τ^- and τ^+ as a function of the new Wilson coefficients. In Fig. (13) we present the flows in the $\left(\mathcal{B},\ \left\langle P_L^- + P_L^+ \right\rangle\right)$ plane by varying the coefficients of the tensor and scalar type interactions. Fig. (14) shows the flows in $\left(\mathcal{B},\ \left\langle P_T^- - P_T^+ \right\rangle\right)$ plane by varying the coefficients of vector, scalar and tensor type interactions. Finally, Fig. (15) depicts the flows in $\left(\mathcal{B}, \left\langle P_N^- + P_N^+ \right\rangle\right)$ plane by changing the coefficients of the tensor type interactions only.

It should be noted that the influence of the variation of various coefficients confirms our previous results, i.e., the influence of the tensor interactions is quite large. The ranges of variation of the new Wilson coefficients are determined by assuming that the value of the branching ratio is about the SM prediction. For example if branching ratio is restricted to have the values in the range $10^{-7} \le \mathcal{B}(B \to K^* \tau^+ \tau^-) \le 5 \times 10^{-7}$, then it follows from Fig. (13) that the new Wilson coefficients of the tensor interactions lie in the region $-2.6 \le C_T \le 1.55$ or $-0.35 \le C_{TE} \le 1.15$, while all scalar interaction coefficients vary in the range between -4 and 4 (in the present work we vary all coefficients in the range -4 and 4).

Finally we would like to discuss briefly the detectibilty of the lepton polarization asymmetries. Experimentally, to be able to measure an asymmetry $\langle P_i \rangle$ of a decay with the branching ratio B at the $n\sigma$, the required number of events are $N = n^2/(\mathcal{B} \langle P_i \rangle^2$. As an example for detecting $\langle P_T \rangle \simeq 0.3$ the number of events expected is $N \simeq 6 \times 10^7 n^2$ events. Therefore at B factories detection of polarization asymmetries for τ could be accessible.

4 Summary and Conclusions

In this work we present the most general analysis of the lepton polarization asymmetries in the rare $B \to K^* \ell^+ \ell^-$ ($\ell = \mu,\ \tau$) using the general, model independent form of the effective Hamiltonian. The dependence of the longitudinal, transversal and normal polarization asymmetries of ℓ^+ and ℓ^- and their combined asymmetries on the new Wilson coefficients are studied. It is found that the lepton polarization asymmetries are very sensitive to the existence tensor and scalar type interactions. Moreover, $\langle P_T \rangle$ and $\langle P_N \rangle$ change their signs for the $B \to K^* \mu^+ \mu^-$ and $\langle P_L \rangle$ and $\langle P_T \rangle$ change their signs for the $B \to K^* \tau^+ \tau^-$ decays, respectively, as the new Wilson coefficients vary in the region of interest. This conclusion is valid also for the combined polarization effects $\left\langle P_L^- + P_L^+ \right\rangle$, $\left\langle P_T^- - P_T^+ \right\rangle$ and $\left\langle P_N^- + P_N^+ \right\rangle$ for the same decay channel. It is well known that in the SM, $\left\langle P_L^- + P_L^+ \right\rangle = \left\langle P_T^- - P_T^+ \right\rangle = \left\langle P_N^- + P_N^+ \right\rangle \simeq 0$ in the limit $m_\ell \to 0$. Therefore any deviation from this relation and determination of the sign of polarization is decisive and effective tool in looking for new physics beyond SM.

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Figure captions

Fig. (1) The dependence of the average longitudinal polarization asymmetry $\left\langle P_L^- \right\rangle$ of ℓ^- on the new Wilson coefficients at $C_7^{eff} = -0.313$ for the $B \to K^* \mu^- \mu^+$ decay.

Fig. (2) The dependence of the combined average longitudinal polarization asymmetry $\left\langle P_L^- + P_L^+ \right\rangle$ of ℓ^- and ℓ^+ on the new Wilson coefficients at $C_7^{eff} = -0.313$ for the $B \to K^* \mu^- \mu^+$ decay.

Fig. (3) The same as in Fig. (1), but for the $B \to K^* \tau^- \tau^+$ decay.

Fig. (4) The same as in Fig. (2), but for the $B \to K^* \tau^- \tau^+$ decay.

Fig. (5) The same as in Fig. (1), but for the average transversal polarization asymmetry $\left\langle P_T^- \right\rangle$ of ℓ^-.

Fig. (6) The same as in Fig. (2), but for the transversal polarization asymmetry $\left\langle P_T^- - P_T^+ \right\rangle$.

Fig. (7) The same as in Fig. (5), but for the $B \to K^* \tau^- \tau^+$ decay.

Fig. (8) The same as in Fig. (6), but for the $B \to K^* \tau^- \tau^+$ decay.

Fig. (9) The dependence of the average normal asymmetry $\left\langle P_N^- \right\rangle$ of ℓ^- on the new Wilson coefficients at $C_7^{eff} = -0.313$ for the $B \to K^* \mu^- \mu^+$ decay.

Fig. (10) The dependence of the combined average normal polarization asymmetry $\left\langle P_N^- + P_N^+ \right\rangle$ of ℓ^- and ℓ^+ on the new Wilson coefficients at $C_7^{eff} = -0.313$ for the $B \to K^* \mu^- \mu^+$ decay.

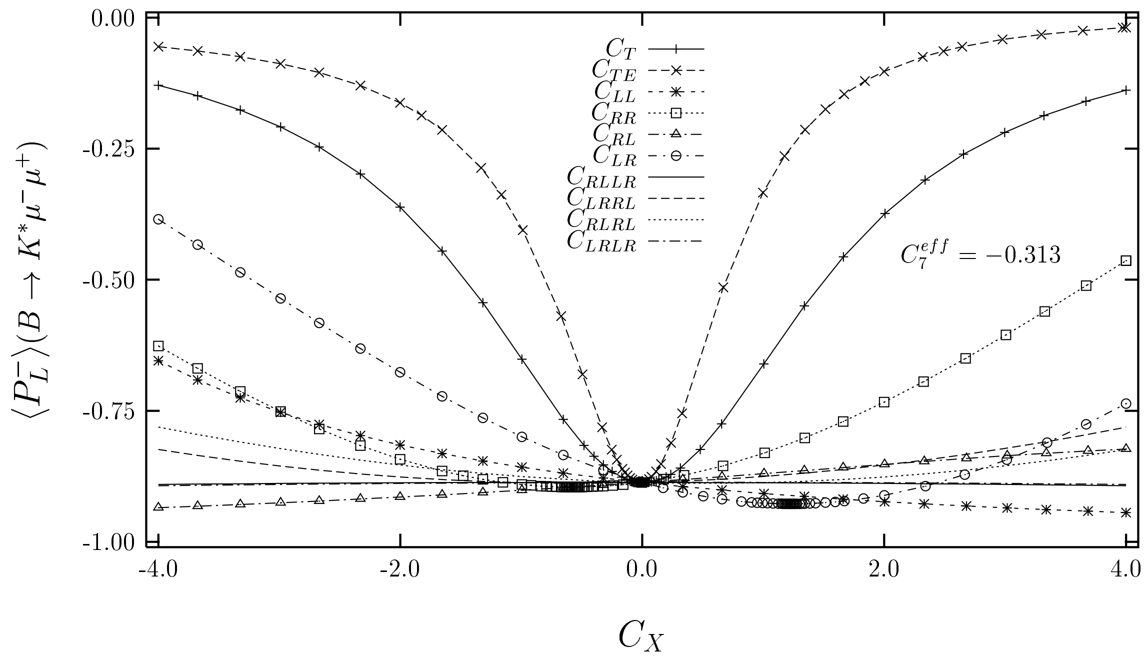
Fig. (11) The same as in Fig. (9), but for the $B \to K^* \tau^- \tau^+$ decay.

Fig. (12) The same as in Fig. (10), but for the $B \to K^* \tau^- \tau^+$ decay.

Fig. (13) Parametric plot of the correlation between the integrated branching ratio $\mathcal{B}$ (in units of 10^{-7}) and the combined average longitudinal lepton polarization asymmetry $\left\langle P_L^- + P_L^+ \right\rangle$ at $C_7^{eff} = -0.313$ as function of the new Wilson coefficients as indicated in the figure, for the $B \to K^* \tau^- \tau^+$ decay.

Fig. (14) The same as in Fig. (13), but for the combined average transversal lepton polarization asymmetry $\left\langle P_T^- - P_T^+ \right\rangle$.

Fig. (15) The same as in Fig. (13), but for the combined average normal lepton polarization asymmetry $\left\langle P_N^- + P_N^+ \right\rangle$.



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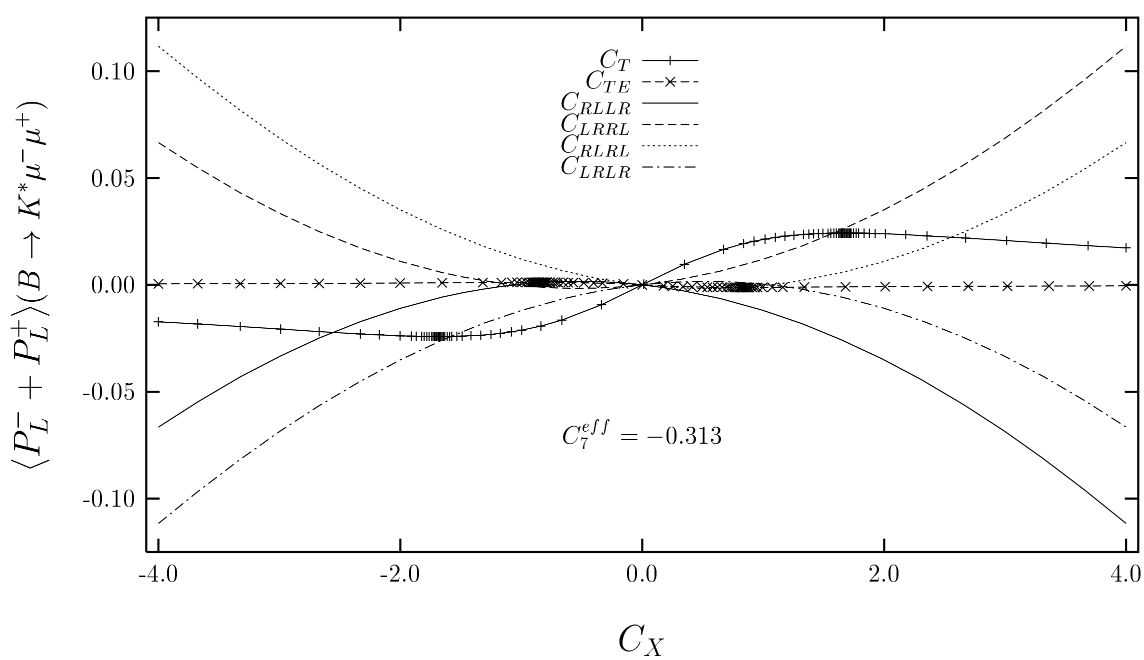

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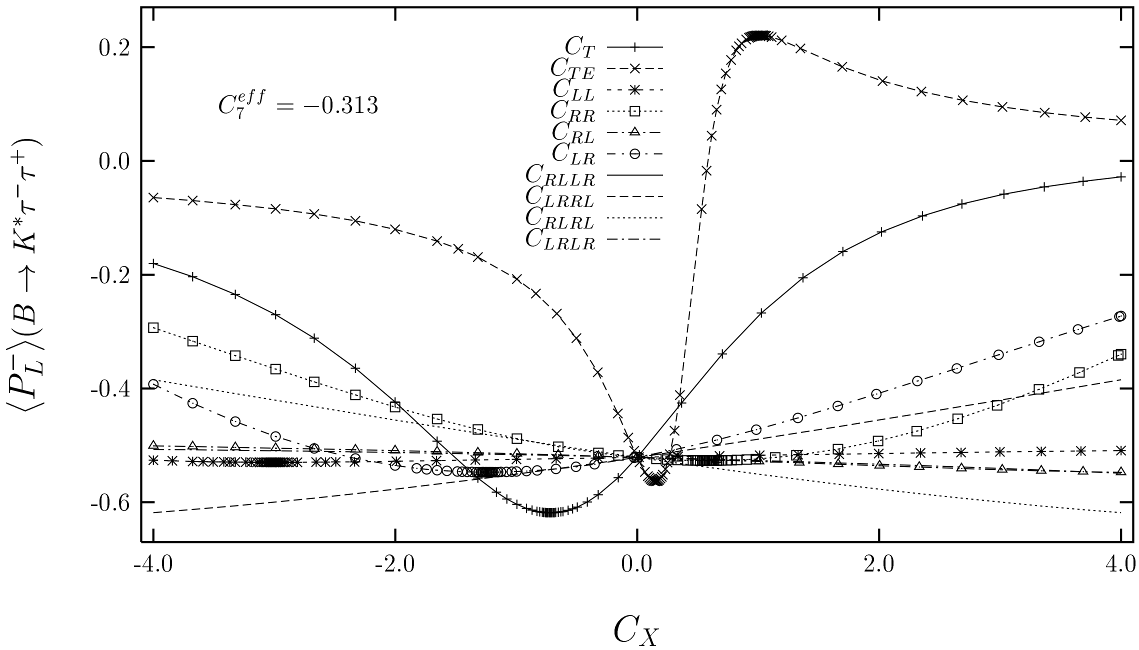

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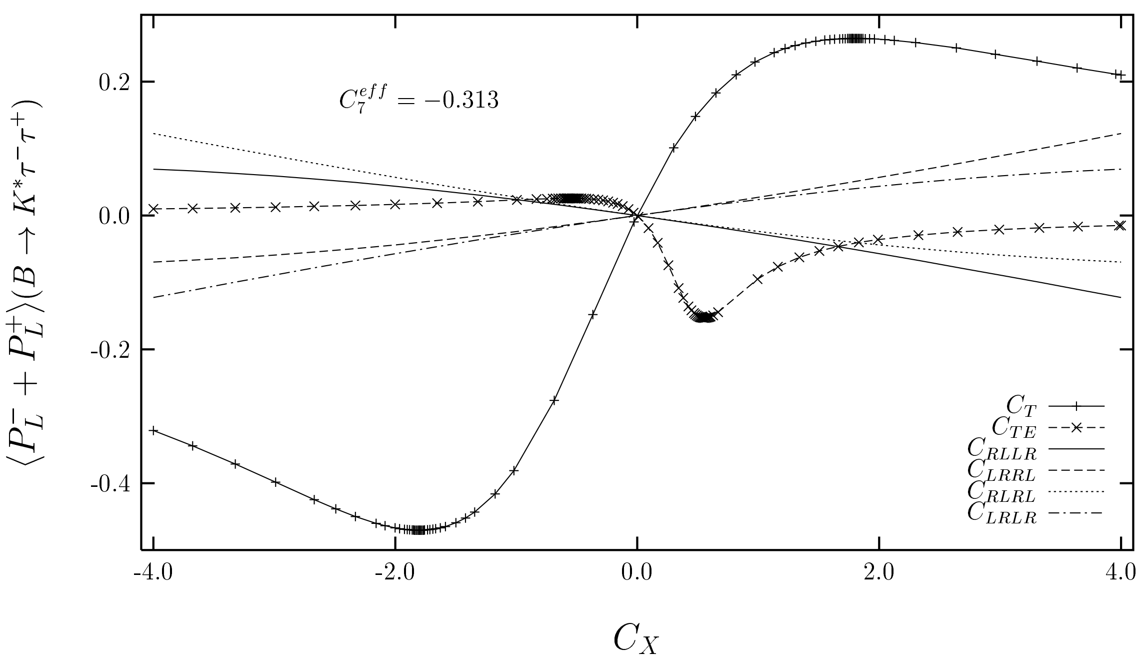

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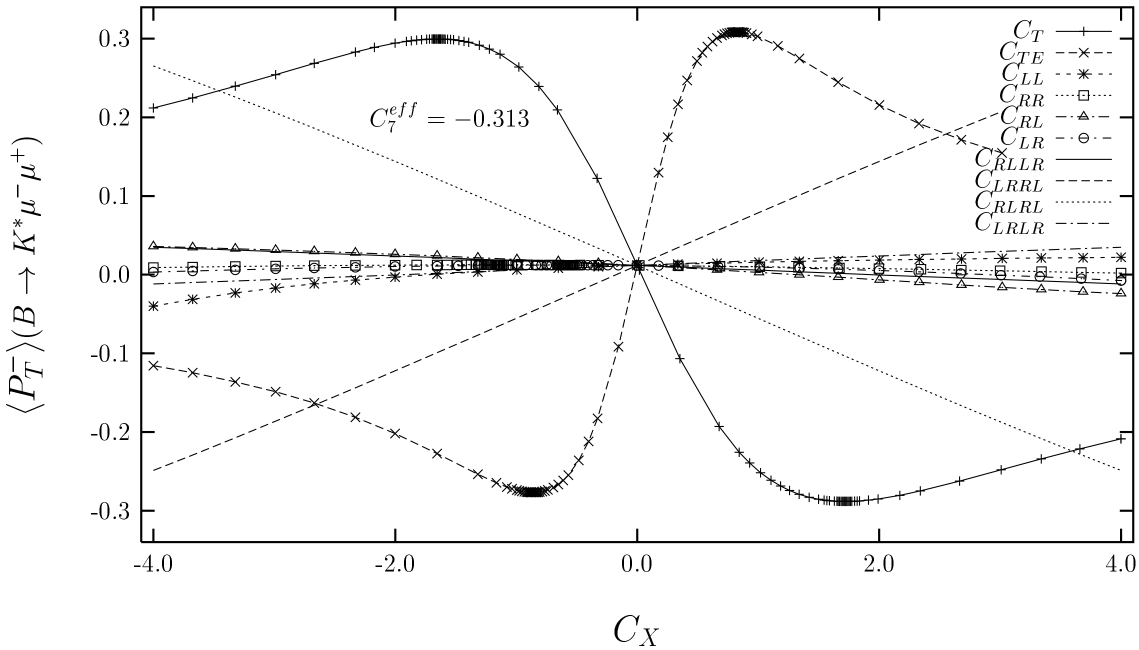

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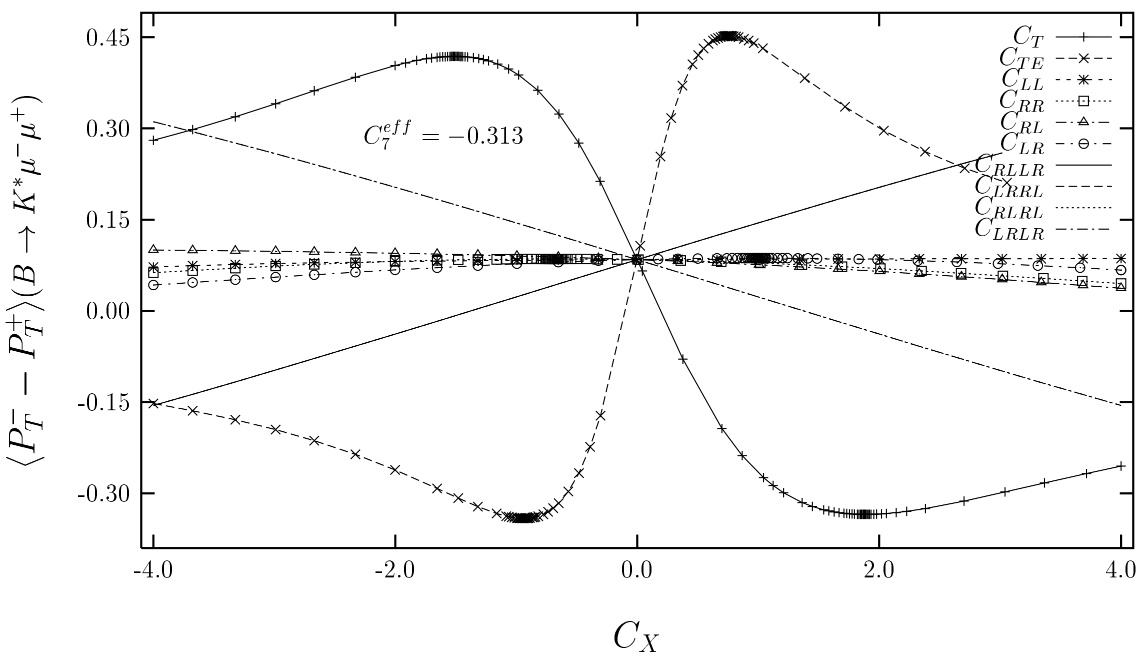

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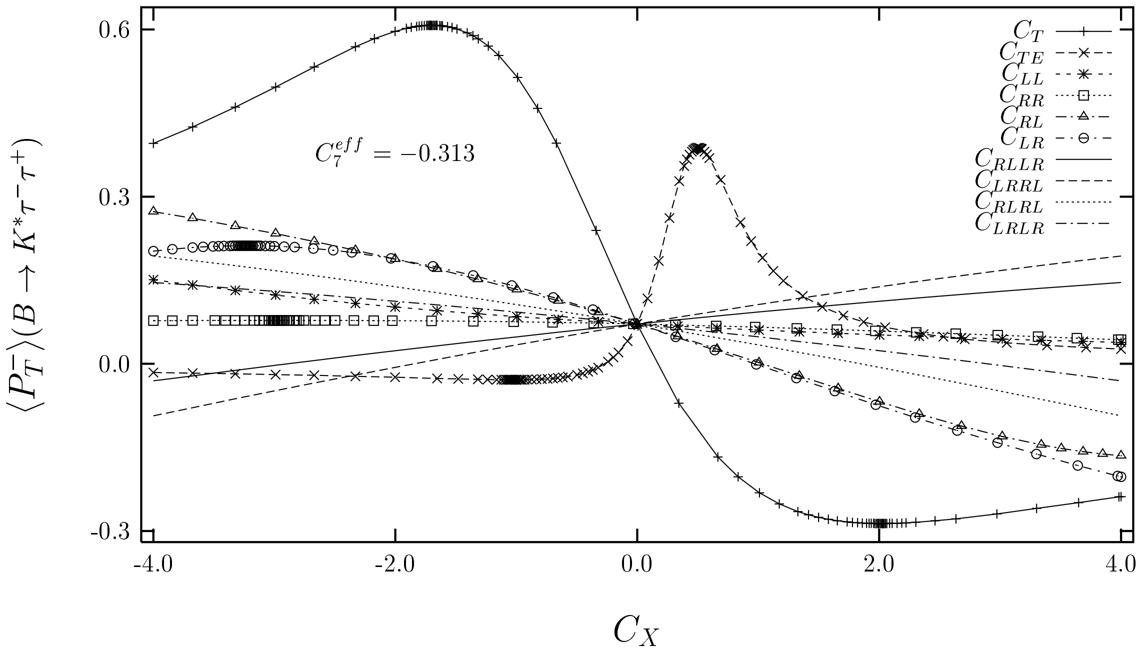

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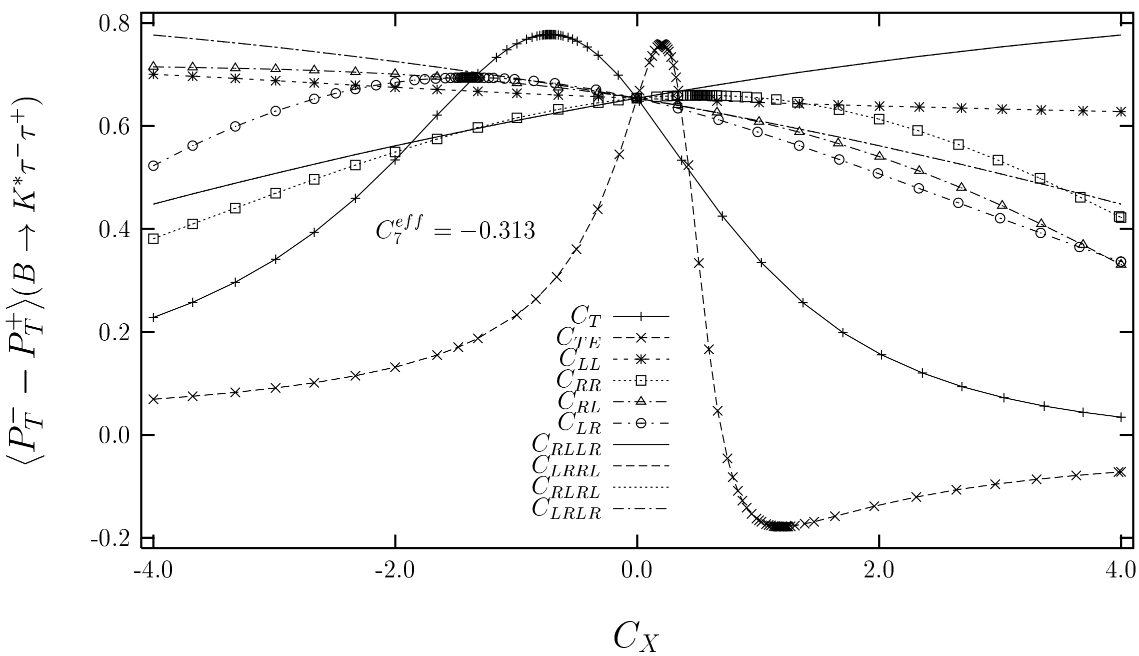

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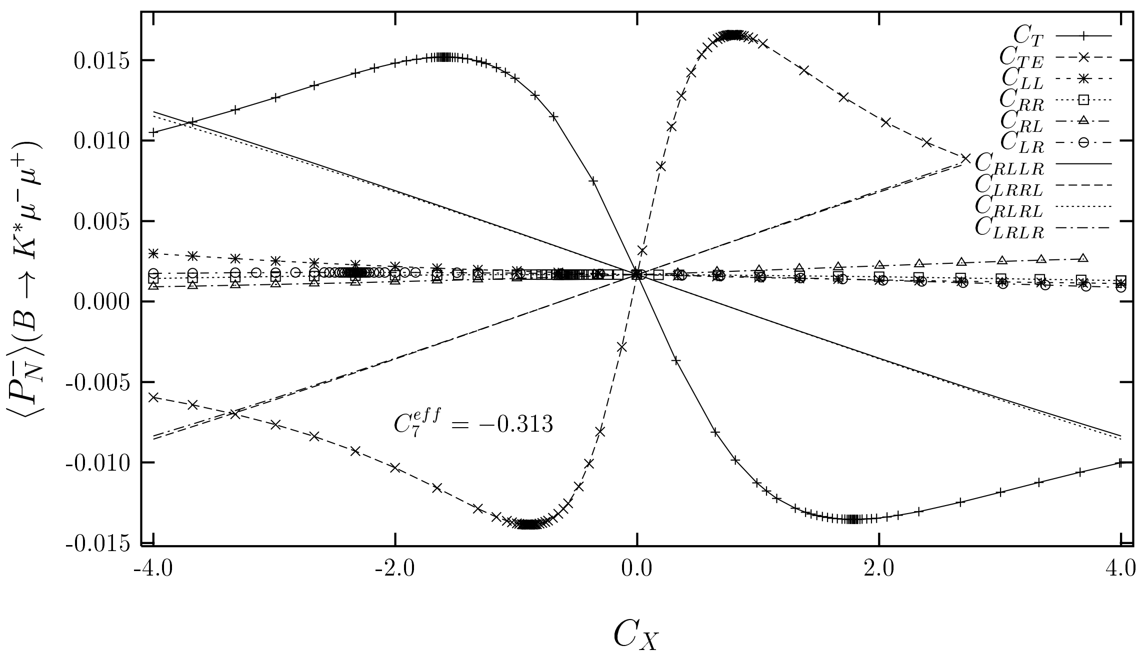

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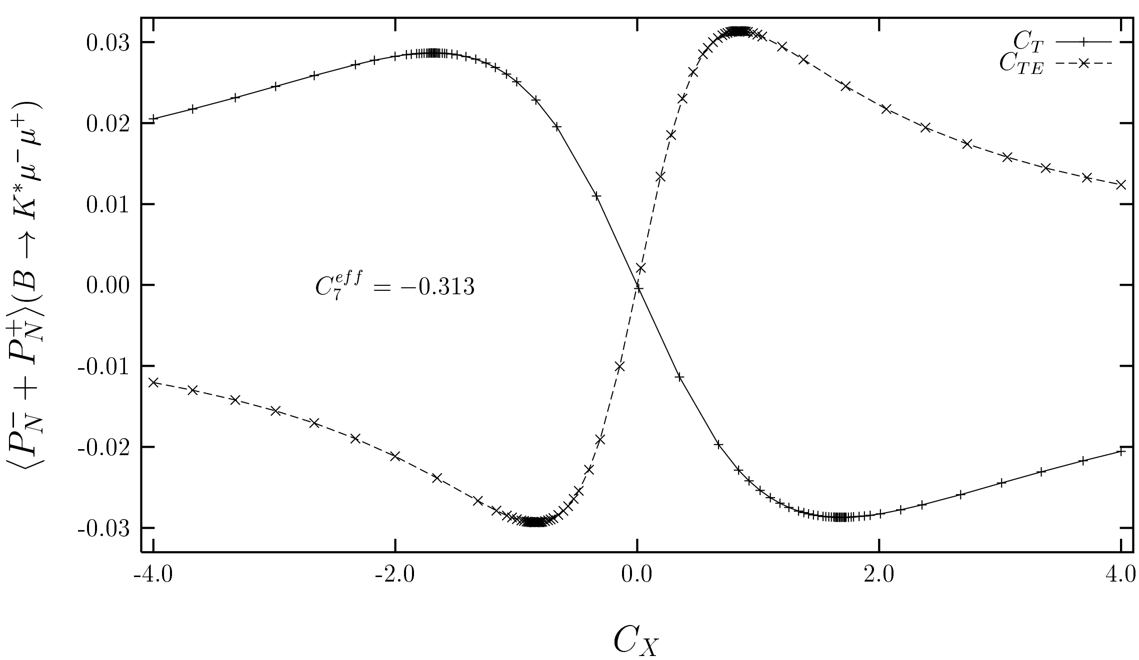
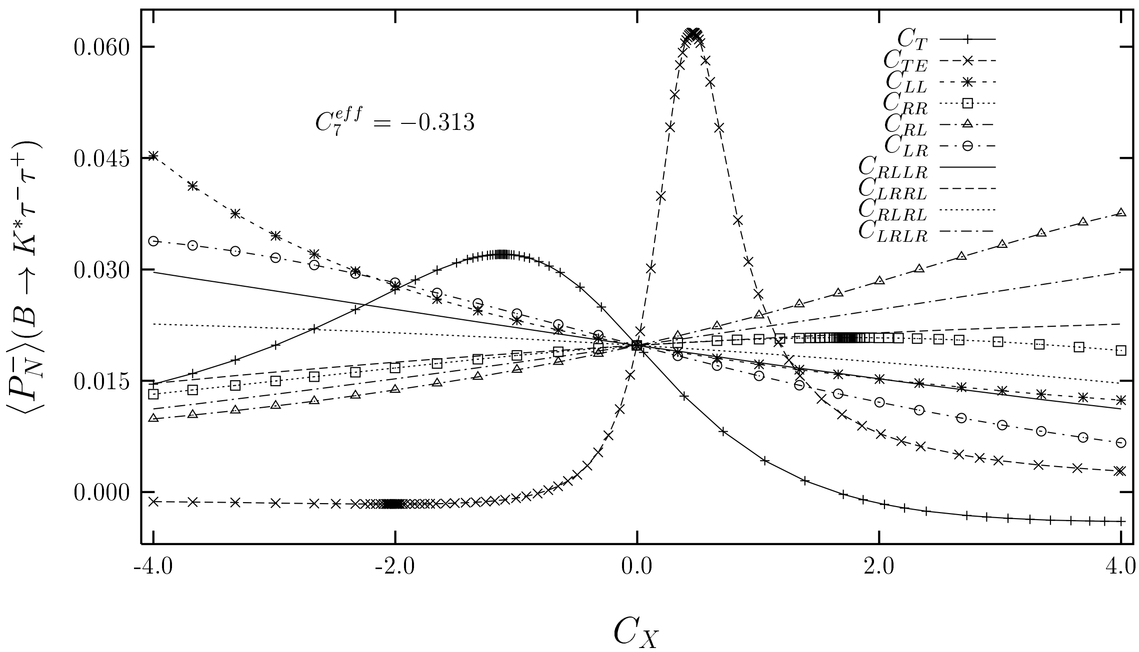


Figure 10:



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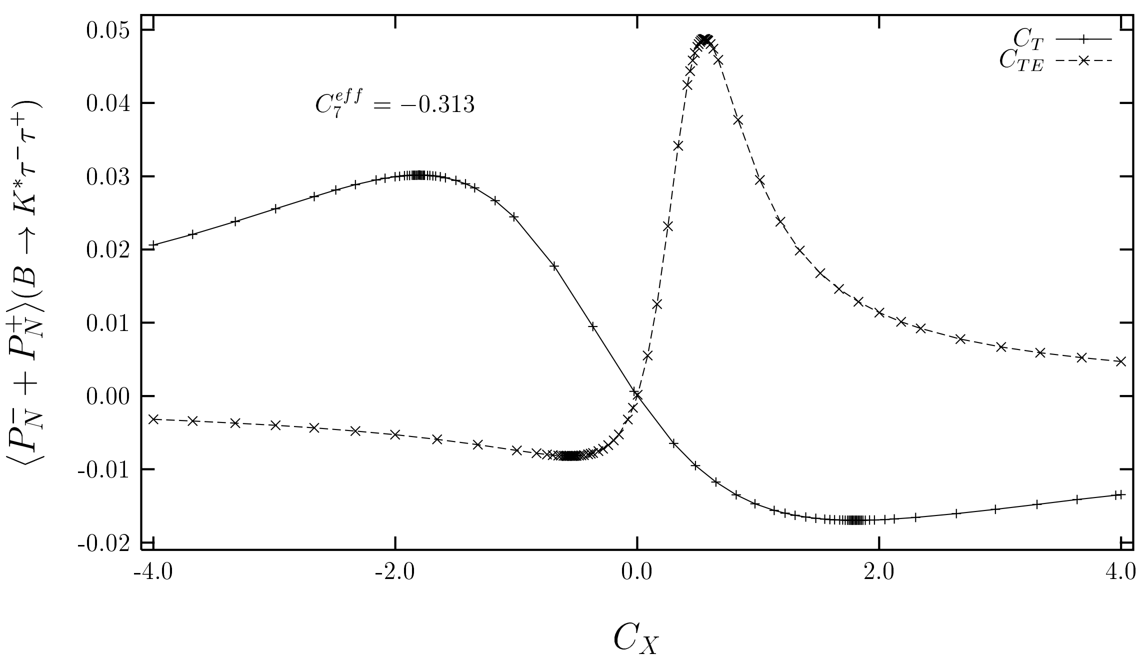

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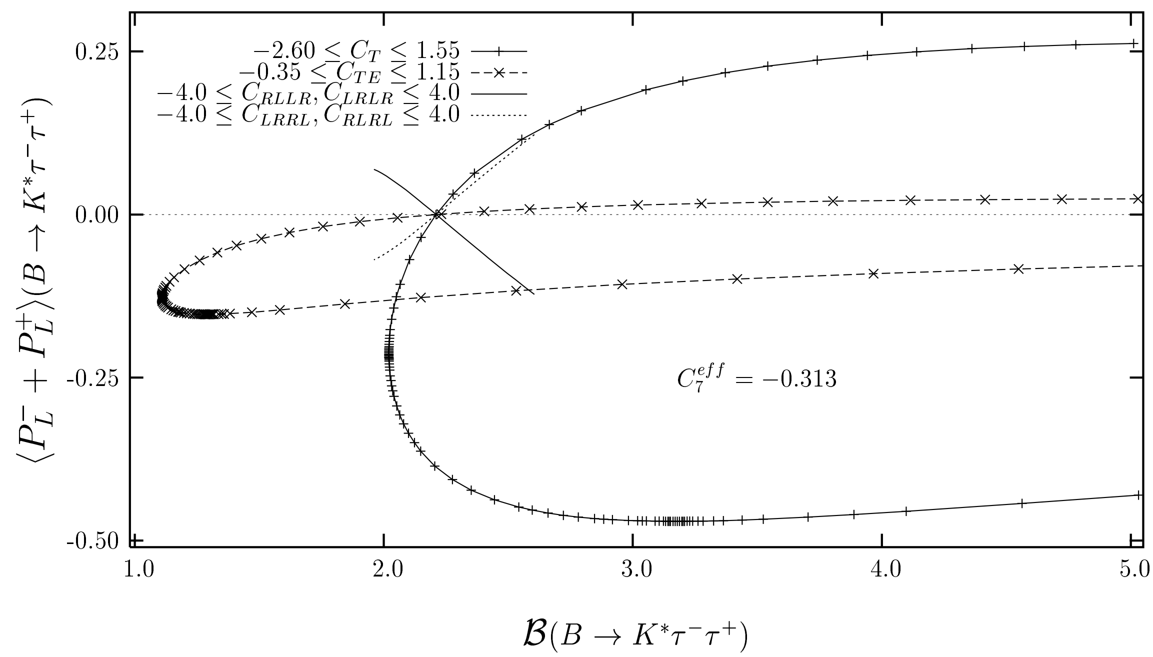

Figure 13:

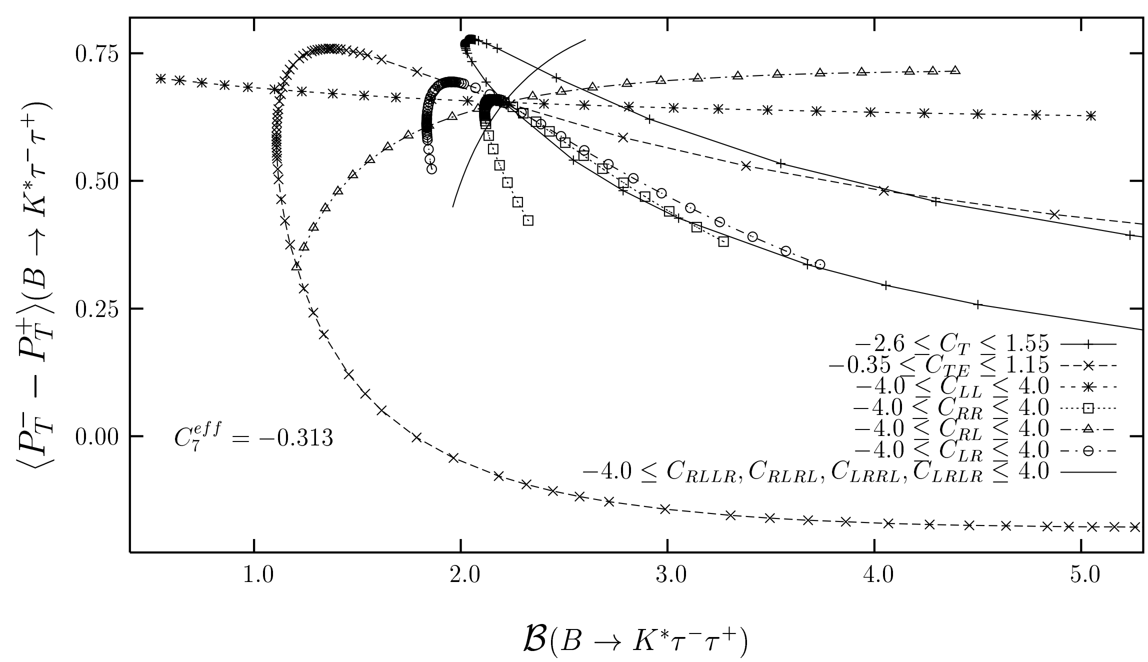

Figure 14:

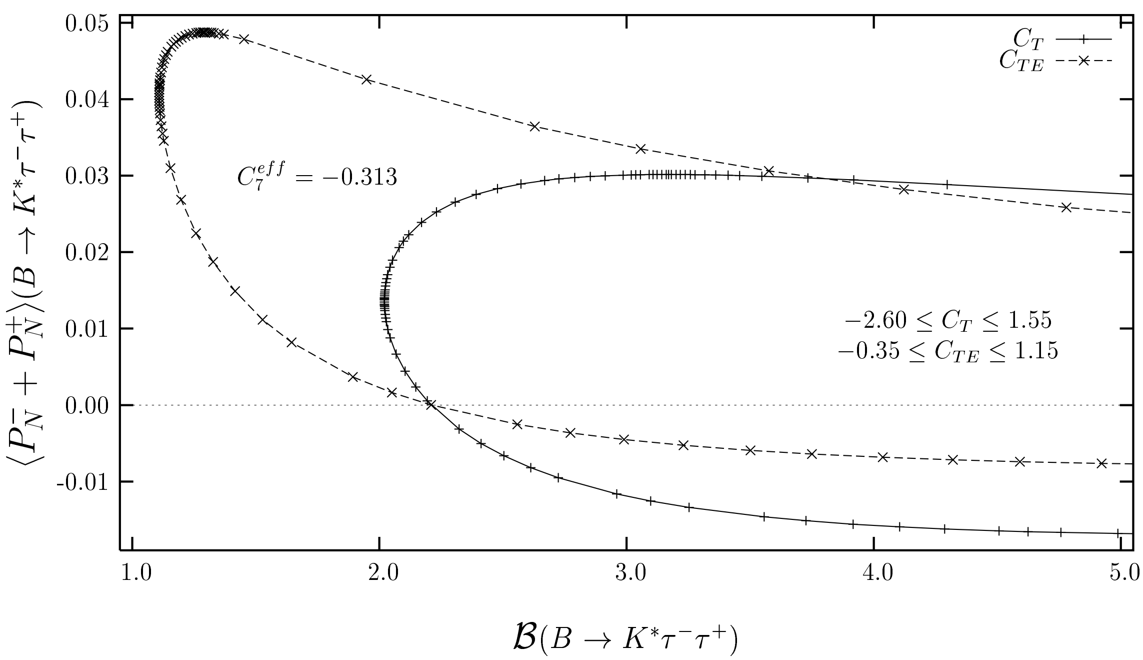
0.05
0.04
0.03
0.02
0.01
0.00
-0.01
1.0
2.0
3.0
4.0
5.0
$\langle P_N^- + P_N^+ \rangle (B \to K^* \tau^- \tau^+)$
$\mathcal{B}(B \to K^* \tau^- \tau^+)$
C_T
C_{TE}
$C_7^{eff} = -0.313$
$-2.60 \leq C_T \leq 1.55$
$-0.35 \leq C_{TE} \leq 1.15$

Figure 15: