New physics effects in the rare $B_s \to \gamma\, \ell^+ \ell^-$ decays with polarized photon

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Abstract

Using the most general model independent form of the effective Hamiltonian, the rare $B_s \to \gamma\, \ell^+ \ell^-$ decays are studied by taking into account the polarization of the photon. The total and the differential branching ratios for these decays, when photon is in the positive and negative helicity states, are presented. Dependence of these observables on the new Wilson coefficients are studied. It is also investigated the sensitivity of "photon polarization asymmetry" in $B_s \to \gamma\, \ell^+ \ell^-$ decays to the new Wilson coefficients. It has been shown that all these physical observables are very sensitive to the existence of new physics beyond SM and their experimental measurements can give valuable information about it.

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1 Introduction

The rare B-meson decays induced by the flavor–changing neutral currents (FCNC) have always been important channels for obtaining information about the fundamental parameters of the standard model (SM), testing its predictions at loop level and probing possible new physics.

The observation of radiative penguin mediated processes, in both the exclusive $B \to K^*\gamma$[2] and inclusive $B \to X_s\gamma$ [3] channels have prompted the investigation of the radiative rare B meson decays with a new momentum. Among these, the semileptonic $B_s \to \gamma\,\ell^+\ell^-$ ($\ell = e, \mu, \tau$) decays have received a special interest due to their relative cleanliness and sensitivity to new physics as well as ongoing experiments at the two B-factories [4, 5]. It is well known that corresponding pure leptonic processes $B_s \to \ell^+\ell^-$ have helicity suppression so that their decay width are too small to be measured for the light lepton modes. In SM the branching ratio of the $BR(B_s \to e^+e^-,\ \mu^+\mu^-) \simeq 4.2\times10^{-14}$ and 1.8×10^{-9}, respectively. Although τ channel is free from this suppression, its experimental detection is quite hard due to the low efficiency. In $B_s \to \tau^+\tau^-\gamma$ decay, helicity suppression is overcome by the photon emission in addition to the lepton pair. Therefore, it is expected for $B_s \to \gamma\ell^+\ell^-$ decay to have a larger branching ratio and this makes its investigation interesting. Indeed, $B_s \to \gamma\ell^+\ell^-$ decays have been widely investigated in the framework of the SM for light and heavy lepton modes [6]-[9], and reported $BR(B_s \to \gamma\,e^+e^-,\ \gamma\,\mu^+\mu^-,\ \gamma\,\tau^+\tau^-) = 2.35\times10^{-9}$, 1.9×10^{-9} and 9.54×10^{-9}, respectively. The new physics effects in these decays have also been studied in some models, like MSSM [10]-[12] and the two Higgs doublet model [14]-[17], and shown that different observables, like branching ratio, forward-backward asymmetry, etc., are very sensitive to the physics beyond the SM. Investigation of the polarization effects may provide another efficient way in establishing the new physics. Along this line, the polarization asymmetries of the final state lepton in $B_s \to \gamma\ell^+\ell^-$ decays have been studied in MSSM in [13] and concluded that they can be very useful for accurate determination of various Wilson coefficients.

In a radiative decay mode like ours, the final state photon can also emerge with a definite polarization and provide another kinematical variable to study the new physics effects [11]. In this paper, we will study the rare $B_s \to \gamma\,\ell^+\ell^-$ decay by taking into account the photon polarization. Although experimental measurement of this variable would be much more difficult than that of e.g., polarization of the final leptons in $B_s \to \gamma\,\ell^+\ell^-$ decay, this is still another kinematical variable for studying radiative decays. In our work we will investigate sensitivity of such "photon polarization asymmetry" in $B_s \to \gamma\,\ell^+\ell^-$ decay to the new Wilson coefficients, in addition to the study of total and differential branching ratios with polarized final state photon. Doing this we use a most general model independent effective Hamiltonian, which contains the scalar and tensor type interactions as well as the vector types (See Eq.(1) below). We note that in a recent work [18] we have studied a related mode $B_s \to \gamma\,\nu\bar{\nu}$ with a polarized photon in a similar way and showed that the spectrum is sensitive to the types of the interactions so that it is useful to discriminate the various new physics effects.

The paper is organized as follows: In section 2, we present the most general, model independent form of the effective Hamiltonian and the parametrization of the hadronic matrix elements in terms of appropriate form factors. We then calculate the differential decay width and the photon polarization asymmetry for the $B \to \gamma\,\ell^+\ell^-$ decay when the photon is in positive and negative helicity states . Section 3 is devoted to the numerical analysis and discussion of our results.

2 Matrix element for the $B_s \to \gamma\,\ell^+\ell^-$ decay

The matrix element for the process $B_s \to \gamma\,\ell^+\ell^-$ can be obtained from that of the purely leptonic $B \to \ell^+\ell^-$ decay. Therefore, we start with the effective Hamiltonian for $b \to s\ell^+\ell^-$ transition written in terms of twelve model independent four-Fermi interactions as follows [19]:

$$
\begin{aligned}
\mathcal{H}_{eff} &= \frac{G\alpha}{\sqrt{2}\pi} V_{ts} V_{tb}^* \Bigg\{ C_{SL}\, \bar{s} i \sigma_{\mu\nu} \frac{q^\nu}{q^2}\, L\, b\, \bar{\ell}\gamma^\mu \ell + C_{BR}\, \bar{s} i \sigma_{\mu\nu} \frac{q^\nu}{q^2}\, R\, b\, \bar{\ell}\gamma^\mu \ell \\
&+ C_{LL}^{tot}\, \bar{s}_L \gamma_\mu b_L\, \bar{\ell}_L \gamma^\mu \ell_L + C_{LR}^{tot}\, \bar{s}_L \gamma_\mu b_L\, \bar{\ell}_R \gamma^\mu \ell_R + C_{RL}\, \bar{s}_R \gamma_\mu b_R\, \bar{\ell}_L \gamma^\mu \ell_L \\
&+ C_{RR}\, \bar{s}_R \gamma_\mu b_R\, \bar{\ell}_R \gamma^\mu \ell_R + C_{LRLR}\, \bar{s}_L b_R\, \bar{\ell}_L \ell_R + C_{RLLR}\, \bar{s}_R b_L\, \bar{\ell}_L \ell_R \\
&+ C_{LRRL}\, \bar{s}_L b_R\, \bar{\ell}_R \ell_L + C_{RLRL}\, \bar{s}_R b_L\, \bar{\ell}_R \ell_L + C_T\, \bar{s}\sigma_{\mu\nu} b\, \bar{\ell}\sigma^{\mu\nu}\ell \\
&+ i C_{TE}\, \epsilon^{\mu\nu\alpha\beta} \bar{s}\sigma_{\mu\nu} b\, \bar{\ell}\sigma_{\alpha\beta}\ell \Bigg\} ,
\end{aligned} \tag{1}
$$

where L and R are the chiral projection operators defined as $(1 \pm \gamma_5)/2$, respectively. In (1), C_X are the coefficients of the four–Fermi interactions with $X = LL, LR, RL, RR$ describing vector, $X = LRLR, RLLR, LRRL, RLRL$ scalar and $X = T, TE$ tensor type interactions. We note that several of the Wilson coefficients in Eq. (1) do already exist in the SM: in the SM, C_{LL} and C_{LR} are in the form $C_9^{eff} - C_{10}$ and $C_9^{eff} + C_{10}$ for the $b \to s\ell^+\ell^-$ decay, while the coefficients C_{SL} and C_{BR} correspond to $-2m_s C_7^{eff}$ and $-2m_b C_7^{eff}$, respectively. Therefore, writing

$$
\begin{aligned}
C_{LL}^{tot} &= C_9^{eff} - C_{10} + C_{LL} ~, \\
C_{LR}^{tot} &= C_9^{eff} + C_{10} + C_{LR} ~,
\end{aligned}
$$

we see that C_{LL}^{tot} and C_{LR}^{tot} contain the contributions from the SM and also from the new physics.

Having established the general form of the effective Hamiltonian, we proceed to calculate the matrix element of the $B_s \to \gamma\,\ell^+\ell^-$ decay. This exclusive decay can receive short-distance contributions from the box, Z, and photon penguin diagrams for $b \to s$ transition by attaching an additional photon line to any internal or external lines. As pointed out before [7, 8], contributions coming from the release of the free photon from any charged internal line will be suppressed by a factor of m_b^2/M_W^2 and we neglect them in the following analysis. When a photon is released from the initial quark lines it contributes to the so-called "structure dependent" (SD) part of the amplitude, $\mathcal{M}_{SD}$. Then, it follows from Eq. (1) that, in order to calculate $\mathcal{M}_{SD}$, the matrix elements needed and their definitions in term of the various form factors are as follows [7, 20]:

$$
\langle \gamma(k) \left| \bar{s}\gamma_\mu (1 \mp \gamma_5) b \right| B(p_B) \rangle = \frac{e}{m_B^2} \Big\{ \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} q^\lambda k^\sigma g(q^2)
$$

$$\pm i\left[\varepsilon^{*\mu}(kq) - (\varepsilon^* q)k^\mu\right] f(q^2)\Big\} ~, \qquad (2)$$

$$\left<\gamma(k)\left|\bar s \sigma_{\mu\nu} b\right| B(p_B)\right> = \frac{e}{m_B^2}\epsilon_{\mu\nu\lambda\sigma}\Big[G\varepsilon^{*\lambda}k^\sigma + H\varepsilon^{*\lambda}q^\sigma + N(\varepsilon^* q)q^\lambda k^\sigma\Big] ~, \qquad (3)$$

$$\left<\gamma(k)\left|\bar s(1\mp\gamma_5) b\right| B(p_B)\right> = 0 ~, \qquad (4)$$

$$\left<\gamma\left|\bar s i \sigma_{\mu\nu} q^\nu b\right| B(p_B)\right> = \frac{e}{m_B^2} i\,\epsilon_{\mu\nu\alpha\beta} q^\nu \varepsilon^{\alpha *} k^\beta G ~, \qquad (5)$$

and

$$\left<\gamma(k)\left|\bar s i \sigma_{\mu\nu} q^\nu (1+\gamma_5) b\right| B(p_B)\right> = \frac{e}{m_B^2}\Big\{\epsilon_{\mu\alpha\beta\sigma}\,\varepsilon^{\alpha *} q^\beta k^\sigma g_1(q^2) + i\left[\varepsilon_\mu^*(qk) - (\varepsilon^* q)k_\mu\right] f_1(q^2)\Big\} ~, \qquad (6)$$

where ε_μ^* and k_μ are the four vector polarization and four momentum of the photon, respectively, q is the momentum transfer, p_B is the momentum of the B meson, and G, H and N can be expressed in terms of the form factors g_1 and f_1 by using Eqs. (3), (5) and (6). The matrix element describing the structure–dependent part can be obtained from Eqs. (2)–(6) as

$$\begin{aligned}
{\cal M}_{SD} &= \frac{\alpha G_F}{4\sqrt{2}\,\pi} V_{tb}V_{ts}^* \frac{e}{m_B^2}\bigg\{\bar\ell\gamma^\mu(1-\gamma_5)\ell\left[A_1\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}q^\alpha k^\beta + i\,A_2\Big(\varepsilon_\mu^*(kq) - (\varepsilon^* q)k_\mu\Big)\right] \\
&+ \bar\ell\gamma^\mu(1+\gamma_5)\ell\left[B_1\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}q^\alpha k^\beta + i\,B_2\Big(\varepsilon_\mu^*(kq) - (\varepsilon^* q)k_\mu\Big)\right] \\
&+ i\,\epsilon_{\mu\nu\alpha\beta}\bar\ell\sigma^{\mu\nu}\ell\left[G\varepsilon^{*\alpha}k^\beta + H\varepsilon^{*\alpha}q^\beta + N(\varepsilon^* q)q^\alpha k^\beta\right] \qquad (7) \\
&+ i\,\bar\ell\sigma_{\mu\nu}\ell\left[G_1(\varepsilon^{*\mu}k^\nu - \varepsilon^{*\nu}k^\mu) + H_1(\varepsilon^{*\mu}q^\nu - \varepsilon^{*\nu}q^\mu) + N_1(\varepsilon^* q)(q^\mu k^\nu - q^\nu k^\mu)\right]\bigg\} ~,
\end{aligned}$$

where

$$\begin{aligned}
A_1 &= \frac{1}{q^2}\Big(C_{BR}+C_{SL}\Big)g_1 + \Big(C_{LL}^{tot}+C_{RL}\Big)g ~, \\
A_2 &= \frac{1}{q^2}\Big(C_{BR}-C_{SL}\Big)f_1 + \Big(C_{LL}^{tot}-C_{RL}\Big)f ~, \\
B_1 &= \frac{1}{q^2}\Big(C_{BR}+C_{SL}\Big)g_1 + \Big(C_{LR}^{tot}+C_{RR}\Big)g ~, \\
B_2 &= \frac{1}{q^2}\Big(C_{BR}-C_{SL}\Big)f_1 + \Big(C_{LR}^{tot}-C_{RR}\Big)f ~, \\
G &= 4C_T g_1 \quad , \quad N = -4C_T\frac{1}{q^2}(f_1+g_1) ~, \\
H &= N(qk) \quad , \quad G_1 = -8C_{TE}g_1 ~, \\
N_1 &= 8C_{TE}\frac{1}{q^2}(f_1+g_1) \quad , \quad H_1 = N_1(qk) ~.
\end{aligned}$$

When photon is radiated from the lepton line we get the the so-called "internal Bremsstrahlung" (IB) contribution, ${\cal M}_{IB}$. Using the expressions

$$\begin{aligned}
\left<0\left|\bar s\gamma_\mu\gamma_5 b\right|B(p_B)\right> &= -\,i f_B p_{B\mu} ~, \\
\left<0\left|\bar s\sigma_{\mu\nu}(1+\gamma_5) b\right|B(p_B)\right> &= 0 ~,
\end{aligned}$$

and conservation of the vector current, we get

$$\mathcal{M}_{IB} = \frac{\alpha G_F}{4\sqrt{2}\,\pi} V_{tb}V_{ts}^* e f_B i \Bigg\{ F\,\bar{\ell}\Bigg(\frac{\not\varepsilon^*\not p_B}{2p_1k} - \frac{\not p_B\not\varepsilon^*}{2p_2k}\Bigg)\gamma_5\ell + F_1\,\bar{\ell}\Bigg[\frac{\not\varepsilon^*\not p_B}{2p_1k} - \frac{\not p_B\not\varepsilon^*}{2p_2k} + 2m_\ell\Bigg(\frac{1}{2p_1k}+\frac{1}{2p_2k}\Bigg)\not\varepsilon^*\Bigg]\ell\Bigg\}\ , \tag{8}$$

where

$$F = 2m_\ell\Big(C_{LR}^{tot} - C_{LL}^{tot} + C_{RL} - C_{RR}\Big) + \frac{m_B^2}{m_b}\Big(C_{LRLR} - C_{RLLR} - C_{LRRL} + C_{RLRL}\Big)\ ,$$
$$F_1 = \frac{m_B^2}{m_b}\Big(C_{LRLR} - C_{RLLR} + C_{LRRL} - C_{RLRL}\Big)\ . \tag{9}$$

Finally, the total matrix element for the $B_s \to \gamma\,\ell^+\ell^-$ decay is obtained as a sum of the $\mathcal{M}_{SD}$ and $\mathcal{M}_{IB}$ terms,

$$\mathcal{M} = \mathcal{M}_{SD} + \mathcal{M}_{IB}. \tag{10}$$

The next task is the calculation of the differential decay rate of $B_s \to \gamma\,\ell^+\ell^-$ decay as a function of dimensionless parameter $x = 2E_\gamma/m_B$, where E_γ is the photon energy. In the center of mass (c.m.) frame of the dileptons $\ell^+\ell^-$, where we take $z = \cos\theta$ and θ is the angle between the momentum of the B_s-meson and that of ℓ^-, double differential decay width is found to be

$$\frac{d\Gamma}{dx\,dz} = \frac{1}{(2\pi)^3 64}\, x\, v\, m_B\, |\mathcal{M}|^2\ , \tag{11}$$

with

$$|\mathcal{M}|^2 = |\mathcal{M}_{SD}|^2 + |\mathcal{M}_{IB}|^2 + 2Re(\mathcal{M}_{SD}\mathcal{M}_{IB}^*) \tag{12}$$

where $v = \sqrt{1 - \frac{4r}{1-x}}$ and $r = m_\ell^2/m_B^2$. We note that $|\mathcal{M}_{IB}|^2$ term has infrared singularity due to the emission of soft photon. In order to obtain a finite result, we follow the approach described in [8] and impose a cut on the photon energy, i.e., we require $E_\gamma \geq 25$ MeV, which corresponds to detect only hard photons experimentally. This cut requires that $E_\gamma \geq \delta\, m_B/2$ with $\delta = 0.01$.

In such a radiative decay, the final state photon can emerge with a definite polarization and there follows the question of how sensitive the branching ratio is to the new Wilson coefficients when the photon is in the positive or negative helicity states. To find an answer to this question, we evaluate $\frac{d\Gamma(\varepsilon^*=\varepsilon_1)}{dx}$ and $\frac{d\Gamma(\varepsilon^*=\varepsilon_2)}{dx}$ for $B_s \to \gamma\,\ell^+\ell^-$ decay, in the c.m. frame of $\ell^+\ell^-$, in which four-momenta and polarization vectors , ε_1 and ε_2, are as follows:

$$P_B = (E_B, 0, 0, E_k)\ ,\ \ k = (E_k, 0, 0, E_k)\ ,\ \ p_1 = (p, 0, p\sqrt{1-z^2}, -pz)\ ,$$
$$p_2 = (p, 0, -p\sqrt{1-z^2}, pz)\ ,\ \ \varepsilon_1 = (0, 1, i, 0)/\sqrt{2}\ ,\ \ \varepsilon_2 = (0, 1, -i, 0)/\sqrt{2}\ , \tag{13}$$

where $E_B = m_B(2-x)/2\sqrt{1-x}$, $E_k = m_B x/2\sqrt{1-x}$, and $p = m_B\sqrt{1-x}/2$. Using the above forms, we obtain

$$\frac{d\Gamma(\varepsilon^* = \varepsilon_i)}{dx} = \left|\frac{\alpha G_F}{4\sqrt{2}\,\pi} V_{tb}V_{ts}^*\right|^2 \frac{\alpha}{(2\,\pi)^3}\,\frac{\pi}{4}\, m_B\, \Delta(\varepsilon_i) \tag{14}$$

where

$$
\begin{aligned}
\Delta(\varepsilon_i) = \frac{vx}{3}\Big\{ & 4\,x\Big((8r+x)\,|H_1|^2 - (4r-x)\,|H|^2\Big) - 6m_\ell(1-x)^2\mathrm{Im}[(A_2 \pm A_1 + B_2 \pm B_1)G_1^*] \\
& + \frac{2}{x}(1-x)^2(2r+x)\Big(\,|G_1|^2 + |G|^2 \pm 2\mathrm{Im}[-G_1G^*]\Big)) - 12m_\ell(1-x)x\mathrm{Im}[(A_2 \pm A_1 + B_2 \pm B_1)H_1^*] \\
& \pm 4(1-x)\Big((8r+x)\mathrm{Im}[GH_1^*] + (4r-x)\mathrm{Im}[G_1H^*]\Big) + 6m_\ell^2(1-x)^2\mathrm{Re}[(A_1 \pm A_2)(B_1 \pm B_2)] \\
& + m_B^2(1-x)^2(x-r)\Big(\,|A_1|^2 + |A_2|^2 + |B_1|^2 + |B_2|^2 \pm 2\mathrm{Re}[A_1A_2^* + B_1B_2^*]\Big) \\
& - 6m_\ell(1-x)^2\mathrm{Re}[(A_2 \pm A_1 + B_2 \pm B_1)G^*] + 4(1-x)\Big((8r+x)\mathrm{Re}[G_1H_1^*] - (4r-x)\mathrm{Re}[GH^*]\Big)\Big\} \\
& + \frac{2x}{(1-x)^2}f_B^2\Big\{(-2vx + (1-4r+x^2)\ln[u])\,|F|^2 \pm 2(1-x)(2vx - (1-4r+x)\ln[u])\mathrm{Re}[FF_1^*] \\
& + \Big(2vx(4r-1) + (1+16r^2+x^2-4r(1+2x))\ln[u]\,|F_1|^2\Big)\Big\} \\
& + 2xf_B\Big\{ \pm (vx + 2r\ln[u])\mathrm{Im}[-FH_1^*] \pm m_\ell(1-x)\ln[u]\mathrm{Re}[(A_2 \pm A_1 + B_2 \pm B_1)F^*] \\
& - m_\ell(2vx + (1-4r-x)\ln[u])\mathrm{Re}[(A_2 \pm A_1 + B_2 \pm B_1)F_1^*] \\
& - 2(v - 2r\ln[u])\mathrm{Im}[(-F_1 \pm F)(G_1^* \pm G^*)] \pm 2(vx - 2r\ln[u])\mathrm{Re}[F_1H^*] \\
& + \frac{2}{(1-x)}\Big((vx(1+x) + 2r(1-3x)\ln[u])\mathrm{Im}[F_1H_1^*] - (1+x)(vx - 2r\ln[u])\mathrm{Re}[F_1H^*]\Big)\Big\} \qquad (15)
\end{aligned}
$$

where $+(-)$ is for $i = 1(2)$ and $u = 1 + v/1 - v$.

The effects of polarized photon can be also studied through a variable "photon polarization asymmetry" [11]:

$$
H(x) = \frac{\frac{d\Gamma(\varepsilon^*=\varepsilon_1)}{dx} - \frac{d\Gamma(\varepsilon^*=\varepsilon_2)}{dx}}{\frac{d\Gamma(\varepsilon^*=\varepsilon_1)}{dx} + \frac{d\Gamma(\varepsilon^*=\varepsilon_2)}{dx}} = \frac{\Delta(\varepsilon_1) - \Delta(\varepsilon_2)}{\Delta_0}\,, \qquad (16)
$$

where

$$
\begin{aligned}
\Delta(\varepsilon_1) - \Delta(\varepsilon_2) = \frac{4}{3}x^2v\Big\{ & \frac{2x(1+2r-x)}{(-1+x)}\mathrm{Im}[G_1G^*] - 3m_\ell x\Big(\mathrm{Im}[(A_1+B_1)G_1^*] + \mathrm{Re}[(A_2+B_2)G^*]\Big) \\
- \quad & 6m_\ell(1-x)((\mathrm{Im}[(A_1+B_1)H_1^*]) + 2\Big((1+8r-x)\mathrm{Im}[GH_1^*] - (1-4r-x)\mathrm{Im}[G_1H^*]\Big) \\
+ \quad & m_B^2x\Big(3r(\mathrm{Re}[A_2B_1^* + A_1B_2^*] + (1-r-x)\mathrm{Re}[B_1B_2^* + A_1A_2^*]\Big)\Big\} \\
+ \quad & 8f_B^2\Big(2v(1-x) - (2-4r-x)\ln[u]\Big) + 4f_Bx\Big\{2(v(x-1) - 2r\ln[u])\mathrm{Im}[FH_1^*] \\
+ \quad & m_\ell x\ln[u]\mathrm{Re}[(A_2+B_2)F^*] + m_\ell\Big(2v(x-1) + (4r-x)\ln[u]\Big)\mathrm{Re}[(A_1+B_1)F_1^*] \\
+ \quad & 2(v - 2r\ln[u])\mathrm{Re}[F_1G^*] - \mathrm{Im}[FG_1^*] + 2(v(1-x) - 2r\ln[u])\mathrm{Re}[F_1H^*]\Big\} \qquad (17)
\end{aligned}
$$

and

$$
\begin{aligned}
\Delta_0 = \Bigg\{ & x^3 v \left(4 m_\ell \,\mathrm{Re}\Big([A_1+B_1]G^*\Big) - \, 4 m_B^2 r \,\mathrm{Re}\Big(A_1 B_1^* + A_2 B_2^*\Big)\right.\\
& -4\Big[\, |H_1|^2\,(1-x) + \mathrm{Re}\Big(G_1 H_1^*\Big) x\Big] \frac{(1+8r-x)}{x^2} - 4\Big[\, |H|^2\,(1-x) + \mathrm{Re}\Big(G H^*\Big) x\Big] \frac{(1-4r-x)}{x^2}\\
& + \frac{1}{3} m_B^2 \Big[2\,\mathrm{Re}\Big(G N^*\Big) + m_B^2\,|N|^2\,(1-x)\Big](1-4r-x)\\
& + \frac{1}{3} m_B^2 \Big[2\,\mathrm{Re}\Big(G_1 N_1^*\Big) + m_B^2\,|N_1|^2\,(1-x)\Big](1+8r-x)\\
& - \frac{2}{3} m_B^2 \Big(\, |A_1|^2 + |A_2|^2 + |B_1|^2 + |B_2|^2\,\Big)(1-r-x) - \frac{4}{3}\Big(\, |G|^2 + |G_1|^2\,\Big) \frac{(1+2r-x)}{(1-x)}\\
& \left. + 2 m_\ell \,\mathrm{Im}\Big([A_2+B_2][6H_1^*(1-x) + 2G_1^* x - m_B^2\, N_1^* x(1-x)]\Big)\frac{1}{x}\right)\\
& + 4 f_B \left(2v\left[\mathrm{Re}\Big(F G^*\Big)\frac{1}{(1-x)} - \mathrm{Re}\Big(F H^*\Big) + \, m_B^2\,\mathrm{Re}\Big(F N^*\Big) + m_\ell\,\mathrm{Re}\Big([A_2+B_2]F_1^*\Big)\right] x(1-x)\right.\\
& + \ln[u]\Bigg[m_\ell\,\mathrm{Re}\Big([A_2+B_2]F_1^*\Big)\, x(x-4r) + 2\,\mathrm{Re}\Big(F H^*\Big)\Big[1-x+2r(x-2)\Big]\\
& \left. - 4 r x\,\mathrm{Re}\Big(F G^*\Big) + m_B^2\,\mathrm{Re}\Big(F N^*\Big)\, x(x-1) - m_\ell\,\mathrm{Re}\Big([A_1+B_1]F^*\Big)\, x^2\Bigg]\right)\\
& + 4 f_B^2 \left(2v\,\Big(\, |F|^2 + (1-4r)\,|F_1|^2\,\Big)\frac{(1-x)}{x}\right.\\
& \left. + \ln[u]\left[\, |F|^2\left(2 + \frac{4r}{x} - \frac{2}{x} - x\right) + |F_1|^2\left(2(1-4r) - \frac{2\,(1-6r+8r^2)}{x} - x\right)\right]\right)\Bigg\}\,. \qquad (18)
\end{aligned}
$$

The expression in Eq.(17) agrees with [11] for the SM case with neutral Higgs contributions.

3 Numerical analysis and discussion

We present here our numerical analysis about the branching ratios (BR) and the photon polarization asymmetries (H) for the $B_s \to \gamma \ell^+ \ell^-$ decays with $\ell = \mu, \tau$. We first give the input parameters used in our numerical analysis :

$$
\begin{aligned}
& m_B = 5.28\,GeV\,,\; m_b = 4.8\,GeV\,,\; m_\mu = 0.105\,GeV\,,\; m_\tau = 1.78\,GeV\,,\\
& f_B = 0.2\,GeV\,,\;\; |V_{tb}V_{ts}^*| = 0.045\;,\;\; \alpha^{-1} = 137\;, G_F = 1.17\times 10^{-5}\,GeV^{-2}\\
& \tau_{B_s} = 1.54\times 10^{-12}\,s\;,\;\; C_9^{eff} = 4.344\;,\;\; C_{10} = -4.669. \qquad (19)
\end{aligned}
$$

Furthermore we assume in this work that all new Wilson coefficients are real and vary in the region $-4 \le C_X \le 4$. We note that such a choice for the range of the new Wilson coefficients follows from the experimental bounds on the branching ratios of the $B \to K^* \mu^+ \mu^-$ [21] and $B_s \to \mu^+ \mu^-$ decays [22]. It should be noted here that the value of the Wilson coefficient C_9^{eff} above corresponds only to the short-distance contributions. C_9^{eff} also receives long-distance contributions associated with the real $\bar{c}c$ intermediate states; but in this work we consider only the short distance effects.

To make some numerical predictions, we also need the explicit forms of the form factors $g,\ f,\ g_1$ and f_1. They are calculated in framework of light–cone QCD sum rules in [20, 7], and also in [23] in terms of two parameters $F(0)$ and m_F. In our work we have used the results of [7], in which q^2 dependencies of the form factors are given as

$$g(q^2) \;=\; \frac{1\,GeV}{\left(1-\frac{q^2}{5.6^2}\right)^2}\;,\; f(q^2)=\frac{0.8\,GeV}{\left(1-\frac{q^2}{6.5^2}\right)^2}\;,\; g_1(q^2)=\frac{3.74\,GeV^2}{\left(1-\frac{q^2}{40.5}\right)^2}\;,\; f_1(q^2)=\frac{0.68\,GeV^2}{\left(1-\frac{q^2}{30}\right)^2}\;.$$

We present the results of our analysis in a series of figures. Before their discussion we give our SM predictions for the unpolarized BRs, for reference:

$$\begin{aligned} BR(B_s \to \gamma\mu^+\mu^-) &= 1.52\times 10^{-8}\;, \\ BR(B_s \to \gamma\tau^+\tau^-) &= 1.19\times 10^{-8}\;, \end{aligned}$$

which are in good agreement with the results of ref. [15].

In Figs. (1) and (2), we present the dependence of the $BR^{(1)}$ and $BR^{(2)}$ for $B_s \to \gamma\mu^+\mu^-$ decay on the new Wilson coefficients, where the superscripts (1) and (2) correspond to the positive and negative helicity states of photon, respectively. From these figures we see that $BR^{(1)}$ and $BR^{(2)}$ are more sensitive to all type of the scalar interactions as compared to the vector and tensor types; receiving the maximum contribution from the one with coefficient C_{RLRL} and C_{LRLR}, respectively. From Fig. (2), we also observe that dependence of $BR^{(2)}$ on all the new Wilson coefficients is symmetric with respect to the zero point, while for $BR^{(1)}$, this symmetry is slightly lifted for the vector type interactions (Fig.(1)). It follows that $BR^{(2)}$ decreases in the region $-4 \le C_X \le 0$ and tends to increase in between $0 \le C_X \le +4$. $BR^{(1)}$ exhibits a similar behavior, except for the vector interactions with coefficients C_{LL}, C_{RL} and C_{LR}: it is almost insensitive to the existence of vector C_{LR} type interactions and slightly increases with the increasing values of C_{LL} and C_{RL}, receiving a value lower than the SM one between -4 and 0.

Differential branching ratio can also give useful information about new physics effects. Therefore, in Figs. (3)- (8) we present the dependence of the differential branching ratio with a polarized photon for the $B_s \to \gamma\,\mu^+\mu^-$ decay on the dimensionless variable $x = 2E_\gamma/m_B$ at different values of vector, tensor and scalar interactions with coefficients C_{LL}, C_{TE} and C_{RLRL}. We observe that tensor (scalar) type interactions change the spectrum near the large (small)-recoil limit, $x \to 1$ ($x \to 0$), as seen from Figs.(5,6) (Figs.(7,8)). However, the vector type interactions increase the spectrum in the center of the phase space and do not change it at the large or small-recoil limit (Figs.(3,4)). We also see from Figs.(3) and (4) that when $C_{LL} > 0$, the related vector interaction gives constructive contribution to the SM result, but for the negative values of C_{LL} the contribution is destructive. Therefore, it is possible to get the information about the sign of new Wilson coefficients from measurement of the differential branching ratio.

From Figs.(1-8), we also see that the branching ratios with a positive helicity photon are greater than those with a negative helicity one. To see this we rewrite Eq.(15) for the SM in the limit $m_\ell \to 0$,

$$
\begin{aligned}
\Delta(\varepsilon_i) &= \frac{m_B^2}{3} x^2(-1+x)^2 \left\{ \left| (C_9^{eff} - C_{10})(g \pm f) - \frac{2C_7}{(1-x)m_B^2} m_b (g_1 \pm f_1) \right|^2 \right. \\
&+ \left. \left| (C_9^{eff} + C_{10})(g \pm f) - \frac{2C_7}{(1-x)m_B^2} m_b (g_1 \pm f_1) \right|^2 \right\}
\end{aligned} \tag{20}
$$

where $+(-)$ is for $i = 1(2)$. It obviously follows that $BR^{(1)} > BR^{(2)}$. We note that this fact can be seen more clearly from the comparison of the differential BRs for (1) and (2) cases for the vector interactions with the coefficient C_{LL}, given in Figs.(3) and (4), where $dBR^{(1)}/dx$ is larger about four times compared to $dBR^{(2)}/dx$.

In addition to the total and differential branching ratios, for radiative decays like ours, studying the effects of polarized photon may provide useful information about new Wilson coefficients. For this purpose, we present the dependence of the integrated photon polarization asymmetry H for $B_s \to \gamma\, \mu^+\mu^-$ decay on the new Wilson coefficients in Figs.(9) and (10). We see from Fig.(9) that spectrum of H is almost symmetrical with respect to the zero point for all the new Wilson coefficients, except the C_{RL}. The coefficient C_{RL}, when it is between -2 and 0, is also the only one which gives the constructive contribution to the SM prediction of H, which we find $H(B_s \to \gamma\, \mu^+\mu^-) = 0.64$. This behavior is also seen from Fig.(10), in which we plot the differential photon polarization asymmetry $H(x)$ for the same decay as a function of x for the different values of the vector interaction with coefficients C_{RL}. From these two figures, we can conclude that performing measurement of H at different photon energies can give information about the signs of the new Wilson coefficients, as well as their magnitudes.

Note that the results presented in this work can easily be applied to the $B_s \to \gamma\, \tau^+\tau^-$ decay. For example, in Figs.(11) and (12), we present the dependence of the $BR^{(1)}$ and $BR^{(2)}$ for $B_s \to \gamma\tau^+\tau^-$ decay on the new Wilson coefficients. We observe that in contrary to the $\mu^+\mu^-$ final state, spectrum of $BR^{(1)}$ and $BR^{(2)}$ for $\tau^+\tau^-$ final state is not symmetrical with respect to zero point, except for the coefficient C_{TE}. Otherwise, we observe three types of behavior for $BR^{(2)}$ from Fig.(12): as the new Wilson coefficients C_{LRRL}, C_{RLLR}, C_{LL} and C_{RR} increase, $BR^{(2)}$ also increases. This behavior is reversed for coefficients C_{LRLR}, C_{RLRL}, C_{LR} and C_{RL}, i.e., $BR^{(2)}$ decreases with the increasing values of these coefficients. However, situation is different for the tensor type interactions : $BR^{(2)}$ decreases when C_T and C_{TE} increase from -4 to 0 and then increases in the positive half of the range. We also observe from Fig.(11) that spectrum of $BR^{(1)}$ is identical to that of $BR^{(2)}$ for the coefficients C_{LRLR}, C_{LRRL}, C_{RLLR}, C_{LL}, C_{RR} and C_{TE} in between $-4 \leq C_X \leq +4$. For the rest of the coefficients, namely C_{RLRL}, C_{LR}, C_T, it stand slightly below and almost parallel to the SM prediction in the positive half of the range, although its behavior is the same as $BR^{(2)}$ when $-4 \leq C_X \leq 0$.

Finally we present two more figures related to the photon polarization asymmetry H for $B_s \to \gamma\, \tau^+\tau^-$ decay. Fig. (13) shows the dependence of the integrated photon polarization asymmetry H on the new Wilson coefficients. We present the differential photon polarization asymmetry $H(x)$ for the same decay as a function of x for the different values of the scaler interactions with coefficients C_{LRRL} in (14). We see from Fig. (13) that in contrary to the $\mu^+\mu^-$ final state, spectrum of H for $\tau^+\tau^-$ final state is not symmetrical with respect to zero point. It also follows that when $0 \leq C_X \leq 4$ the dominant contribution to H for $B_s \to \gamma \tau^+ \tau^-$ decay comes from C_{RLRL} and C_{LR}. However, for the negative part of the range H receives constructive contributions mostly from C_{LRRL}, as clearly seen also from Fig.(14).

In conclusion, we have studied the total and the differential branching ratios of the rare $B_s \to \gamma \ell^+ \ell^-$ decay by taking into account the polarization effects of the photon. Doing this we use a most general model independent effective Hamiltonian, which contains the scalar and tensor type interactions as well as the vector types. We have also investigated the sensitivity of "photon polarization asymmetry" in this radiative decay to the new Wilson coefficients. It has been shown that all these physical observables are very sensitive to the existence of new physics beyond SM and their experimental measurements can give valuable information about it.

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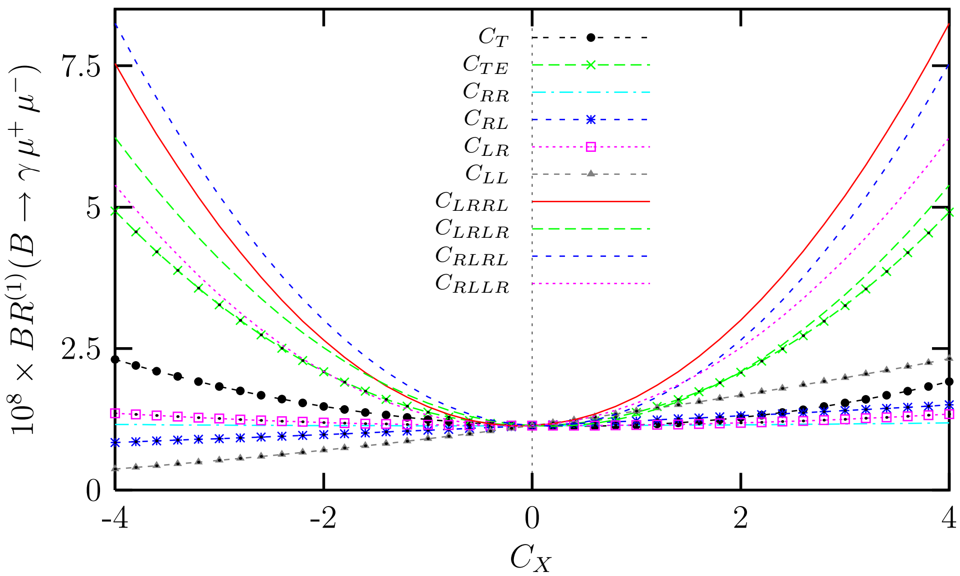

Figure 1: The dependence of the integrated branching ratio for the $B_s \to \gamma\, \mu^+\mu^-$ decay with photon in the positive helicity state on the new Wilson coefficients

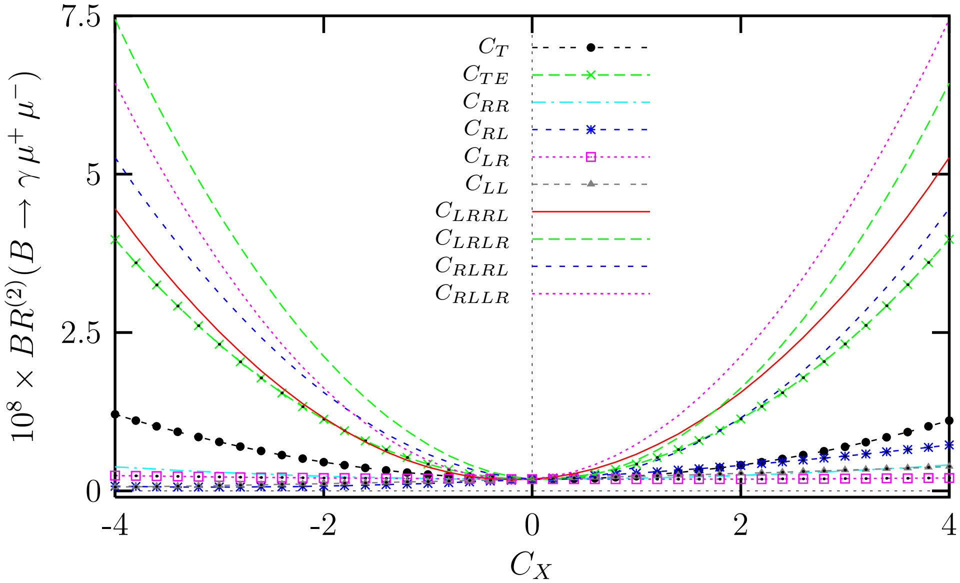

Figure 2: The same as Fig.(1), but with photon in negative helicity state.

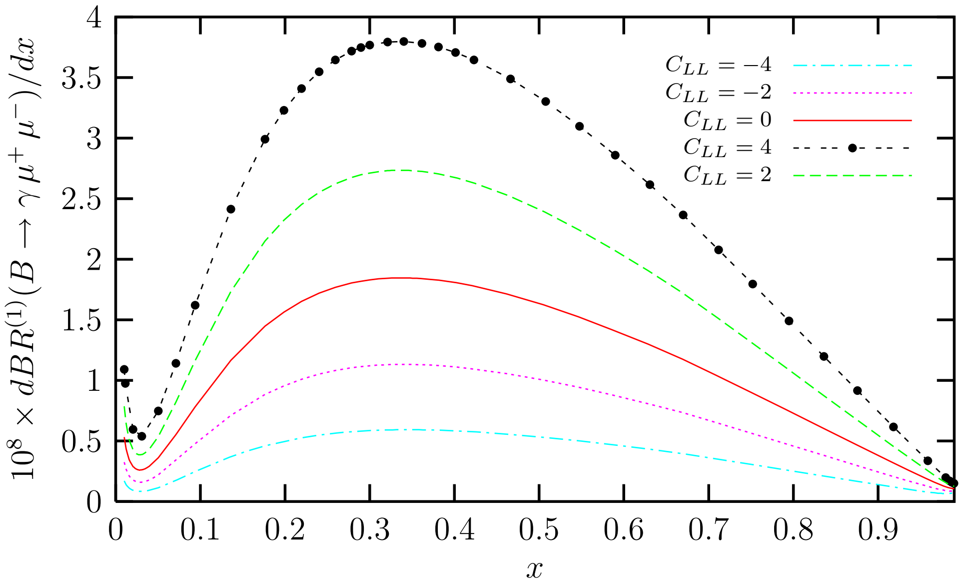

Figure 3: The dependence of the differential branching ratio for the $B_s \to \gamma\,\mu^+\mu^-$ decay with photon in the positive helicity state on the dimensionless variable $x = 2E_\gamma/m_B$ at different values of vector interaction with coefficient C_{LL}

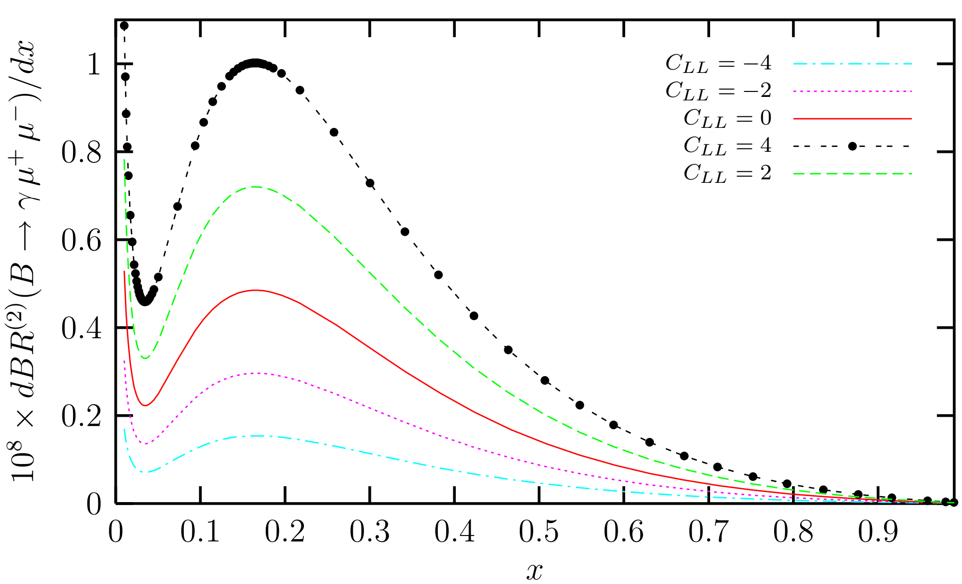

Figure 4: The same as Fig.(3), but with photon in the negative helicity state.

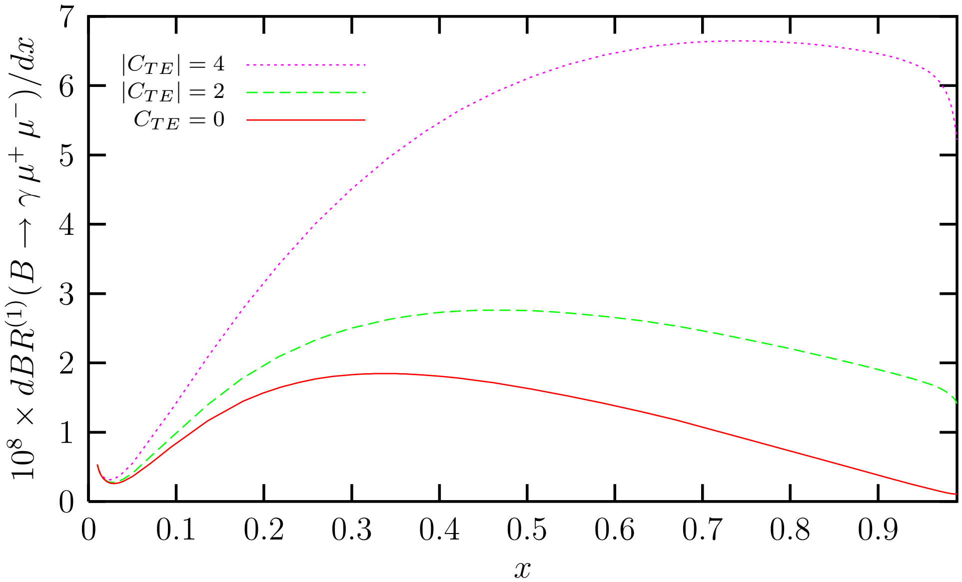

Figure 5: The dependence of the differential branching ratio for the $B_s \to \gamma\,\mu^+\mu^-$ decay with photon in the positive helicity state on the dimensionless variable $x = 2E_\gamma/m_B$ at different values of tensor interaction with coefficient C_{TE}

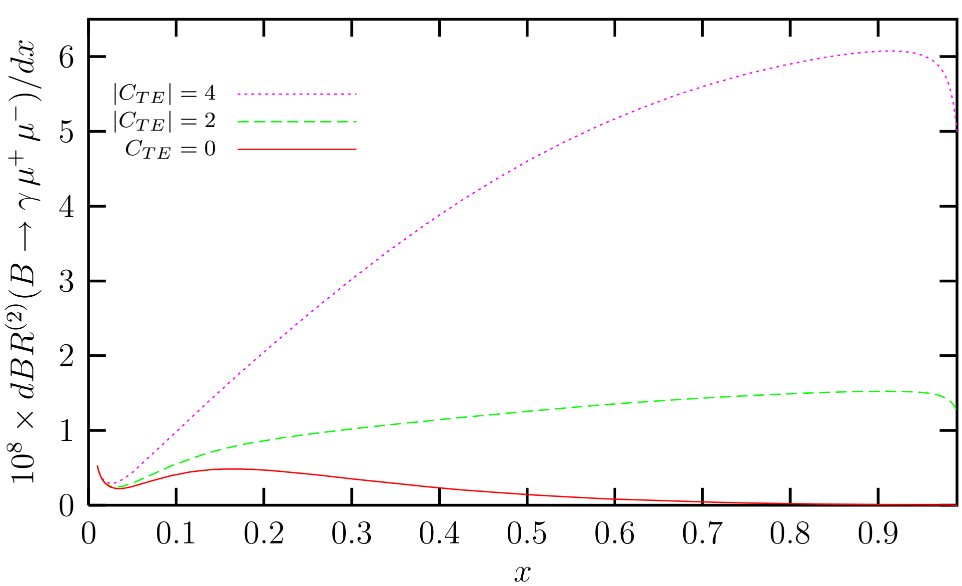

Figure 6: The same as Fig.(5), but with photon in the negative helicity state.

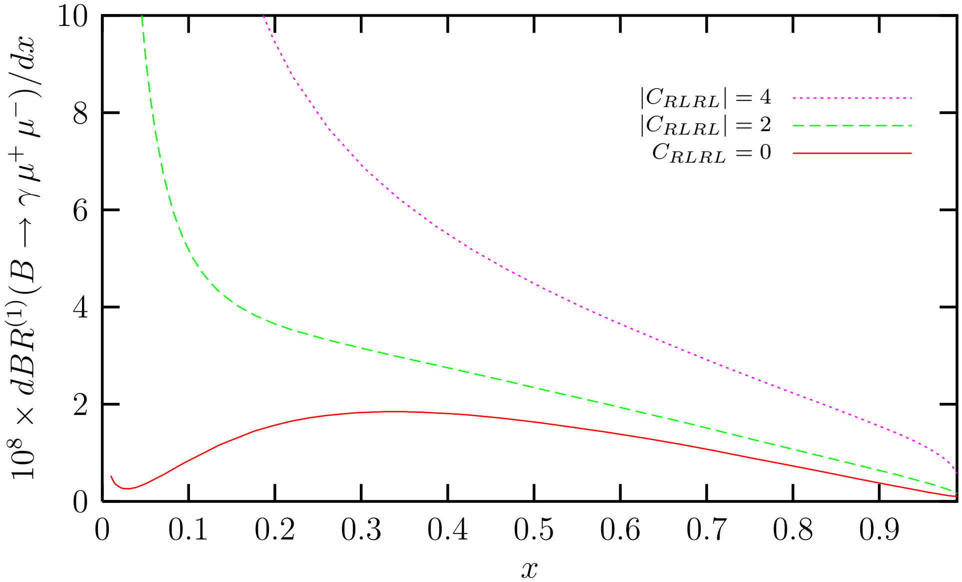

Figure 7: The dependence of the differential branching ratio for the $B_s \to \gamma\,\mu^+\mu^-$ decay with photon in the positive helicity state on the dimensionless variable $x = 2E_\gamma/m_B$ at different values of scalar interaction with coefficient C_{RLRL}

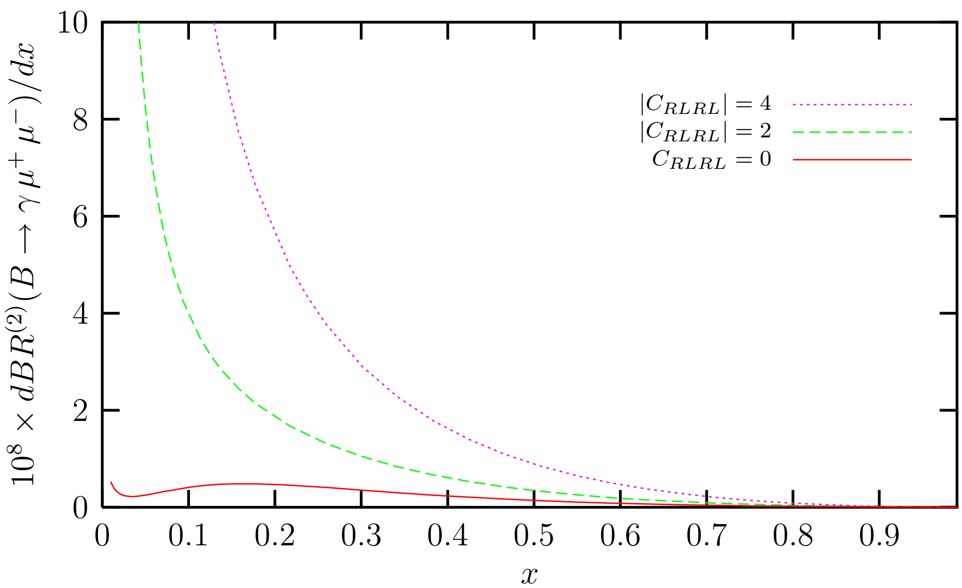

Figure 8: The same as Fig.(7), but with photon in the negative helicity state.

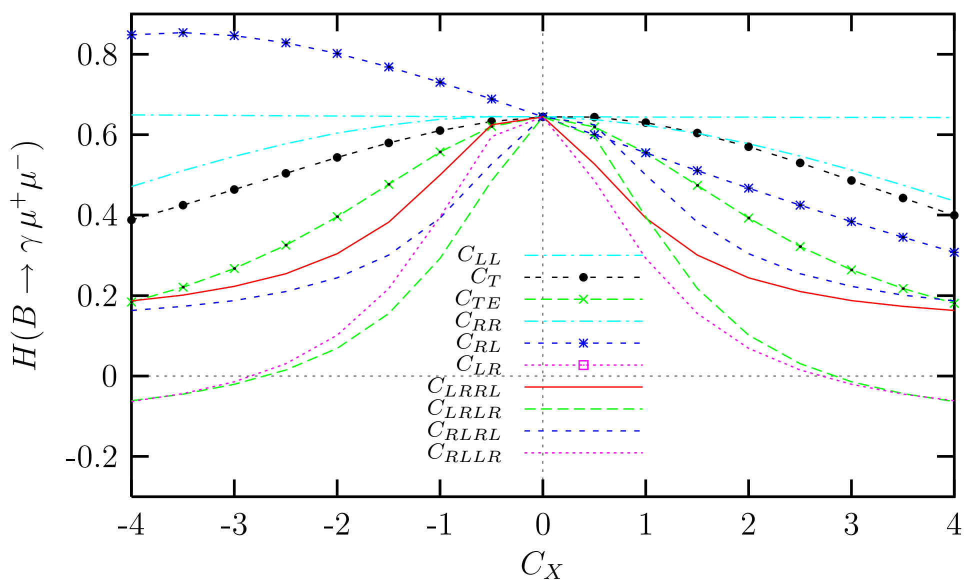

Figure 9: The dependence of the integrated photon polarization asymmetry for the $B_s \to \gamma\,\mu^+\mu^-$ decay on the new Wilson coefficients.

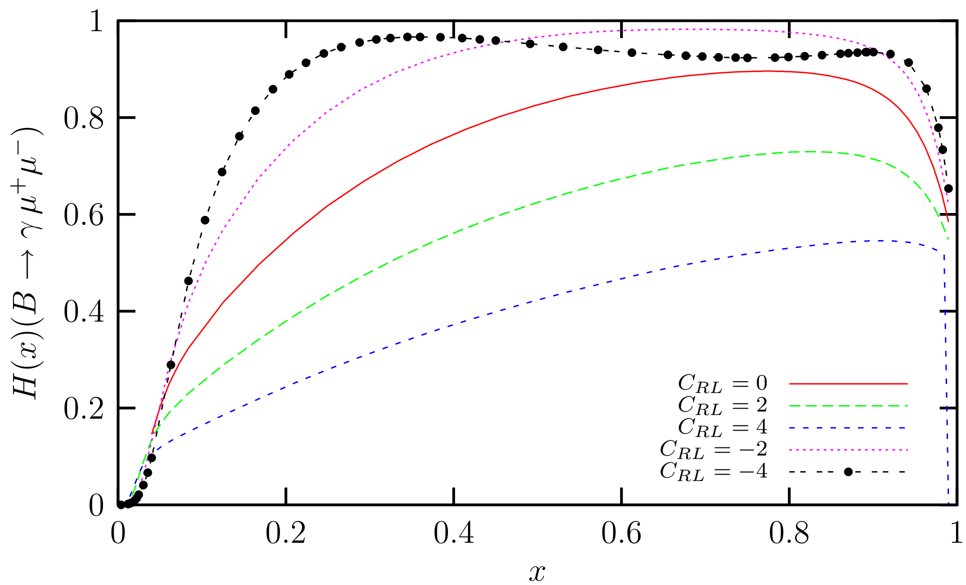

Figure 10: The dependence of the differential photon polarization asymmetry for the $B_s \to \gamma\,\mu^+\mu^-$ decay on the dimensionless variable $x = 2E_\gamma/m_B$ for different values of C_{RL}.

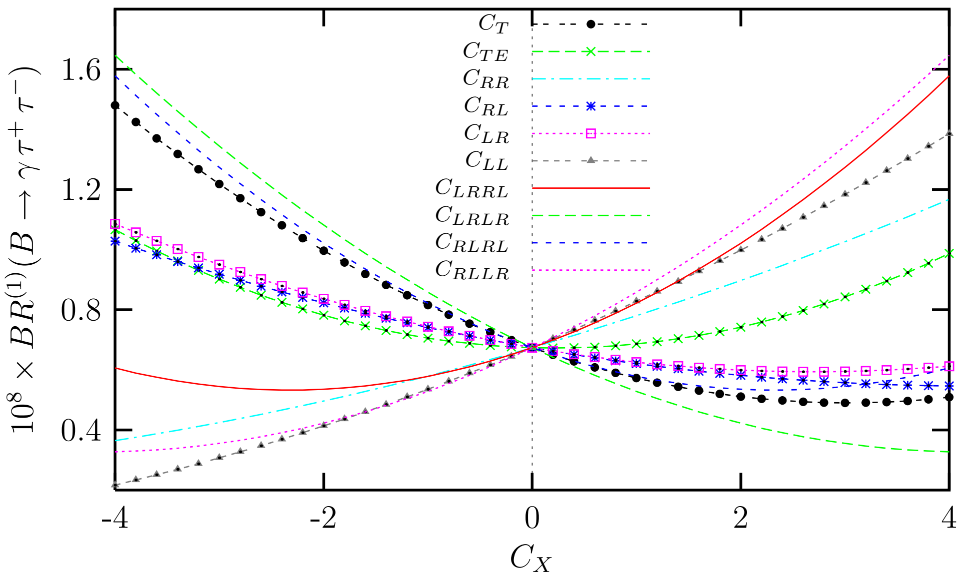

Figure 11: The dependence of the integrated branching ratio for the $B_s \to \gamma \tau^+ \tau^-$ decay with photon in the positive helicity state on the new Wilson coefficients

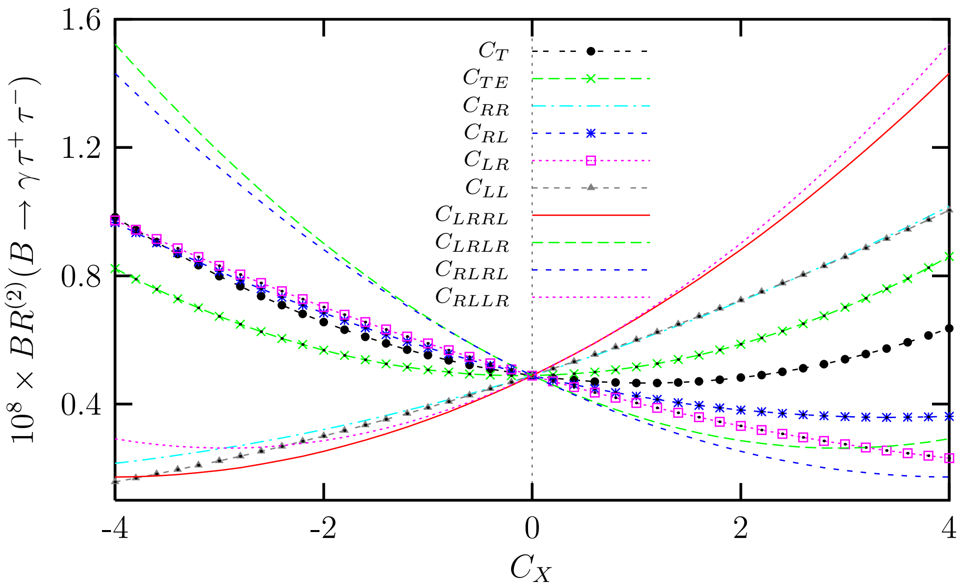

Figure 12: The same as Fig.(11), but with photon in negative helicity state.

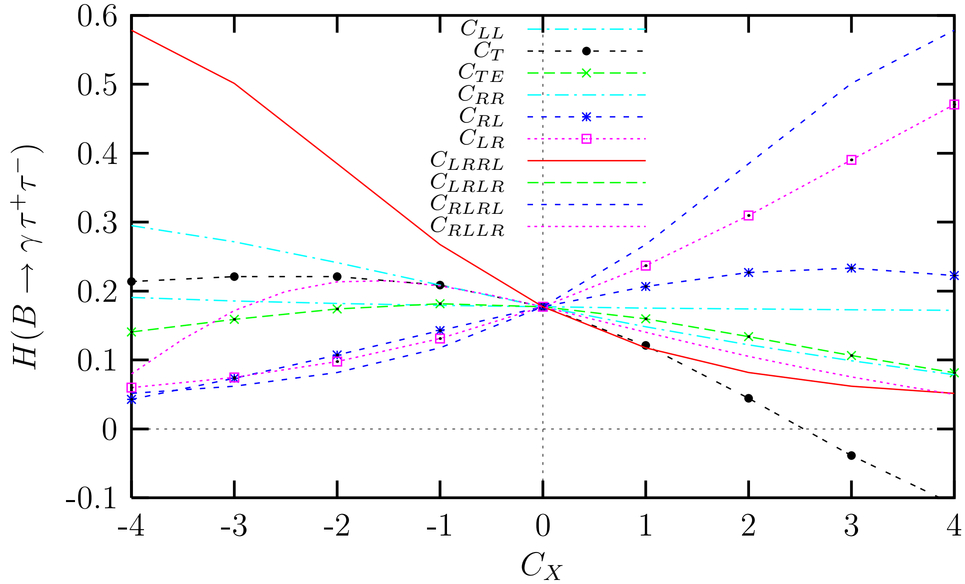

Figure 13: The dependence of the integrated photon polarization asymmetry for the $B_s \to \gamma\,\tau^+\tau^-$ decay on the new Wilson coefficients.

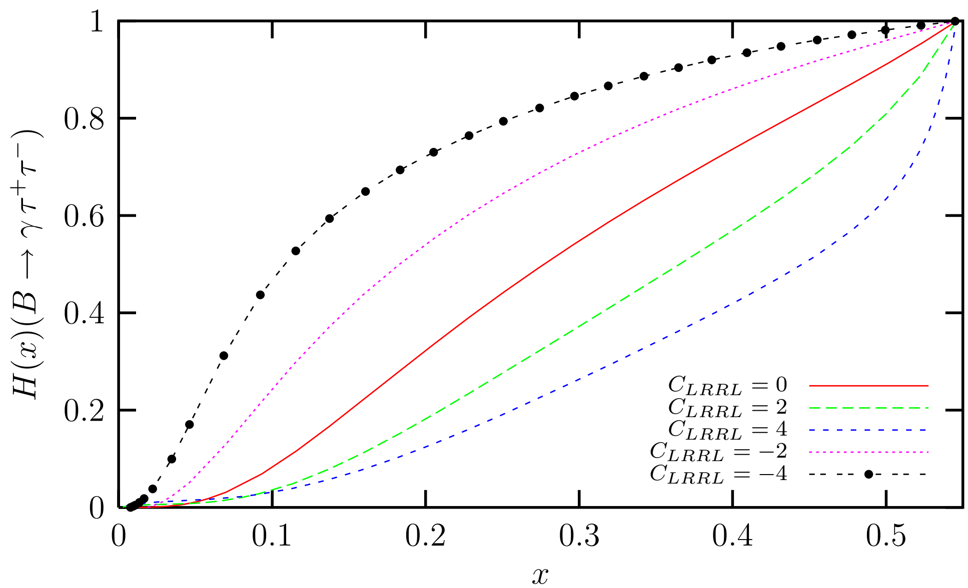

Figure 14: The dependence of the differential photon polarization asymmetry for the $B_s \to \gamma\,\tau^+\tau^-$ decay on the dimensionless variable $x = 2E_\gamma/m_B$ for different values of C_{LRRL}.