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# Optimal siting and sizing of wind farms

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ABSTRACT

In this paper, we propose a novel technique to determine the optimal placement of wind farms, thereby taking into account wind characteristics and electrical grid constraints. We model the long-term variability of wind speed using a Weibull distribution according to wind direction intervals, and formulate the metrics that capture wind speed characteristics at a specific location, namely the arithmetic mean of wind speed, the theoretical wind power density and the capacity factor of a prospective wind power plant, to determine the feasibility of a wind power plant establishment. Furthermore, a linear optimization formulation is provided to determine the geographical locations and the installed capacities of wind farms, in order to maximize the expected annual wind power generation, while obeying the constraints from the electrical power grid and the transmission system operator. As a case study, the proposed wind speed model and the linear optimization formulation are used to evaluate the wind characteristics and the potential wind farm sites in Turkey.

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## 1. Introduction

Driven by the long-term goal to achieve a sustainable energy system, the utilization of renewable energy, especially wind power, is rising. Nevertheless, the intermittent nature of wind power challenges the stability and reliability of the power system. The wind speed characteristics at a wind farm determine the power generation from wind turbines. Therefore, for prospective investors and for power system analysts who carry out the reliability analyses, modelling the variation of wind speed is essential [1].

Many researchers have conducted case studies to investigate the distribution of wind speed for various purposes [2]: In risk analyses concerning extreme or maximum wind speeds, extreme value distributions [3] are typically used [4], whereas in grid integration studies, the Weibull distribution is widely used because of its flexibility and satisfactory results in fit tests [5,6]. This paper addresses the optimal wind power integration with a long-term focus. Consequently, the Weibull distribution is utilised to model the annual wind speed distributions.

Measurements indicate that there exists a relation between wind speed and wind direction [7–9]. Contrary to the modelling based on purely wind speed values [1,5,6], this paper models the variability of wind speed according to wind direction intervals [9], and derives the expressions for the long-term distributions of wind speed and the power output of a wind turbine. The paper further uses the metrics [10], average wind speed, wind power density, and the capacity factor of a wind power plant, to assess the wind characteristics at a geographical location, and to determine the feasible sites for the wind farms.

The main motivation for investing in a wind farm is the expected profit. On sites with strong wind, investors prefer to establish wind farms of substantial sizes. In order to avoid network congestion due to the power outputs of such wind farms, the grid operators are conservative about the integration of new power plants into the electrical grid and may impose limits on the maximum installed capacities in certain regions [11].

Various methodologies have been proposed to facilitate the determination of wind farm locations [12]. A number of these studies are based on the maximization of the profit of investors [13], which ignore the integration effects of wind farms into the electrical grid. Therefore, some studies propose methods for wind power integration according to the needs of the power grid (such as loss reduction, voltage regulation [14,15]). Nevertheless, those theoretical integration plans may fail to be realised as the proposed wind farm locations do not necessarily attract investors. To solve this wind farm placement problem, this paper presents a combined methodology for countrywide optimal wind power integration: Initially, the metrics that capture the quality of wind are utilised to

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assess the feasible locations for establishing wind farms from an investor's point of view. Subsequently, those feasible geographical sites are mapped on the electrical power grid, and the optimal siting and sizing of wind farms to maximize annual wind power generation, while obeying the constraints from the electrical power grid and the transmission system operator are determined. Therefore, the proposed placement of wind farms is of interest to investors, and the prospective integration of the power outputs of wind farms does not violate the transmission grid constraints.

The remainder of this paper is organised as follows: Section 2 explains the developed models for wind speed and the power output of a wind turbine. The proposed criteria for the evaluation of potential sites for wind farms are discussed in Section 3. Section 4 formulates the linear optimization problem to investigate the optimal integration of wind power plants, and the results of the optimization are presented in Section 5. Finally, Section 6 concludes the paper.

## 2. Probabilistic model for the power output of a wind turbine

This section proposes probabilistic models for wind speed and formulates the long-term variability of the power output of a wind turbine.

### 2.1. Wind speed characteristics

The annual variability of wind speed is used in assessing the integration of wind farms into the power grid [1], and typically, the Weibull distribution is used to represent the annual variation of wind speed [5,6]. The probability density function (pdf) $f_V(v)$ and the cumulative distribution function (cdf) $F_V(v)$ of the Weibull distribution are defined as

$$f_V(v) = ba^{-b}v^{b-1}e^{-\left(\frac{v}{a}\right)^b}$$
$$F_V(v) = 1 - e^{-\left(\frac{v}{a}\right)^b} \quad (1)$$

where $v$ denotes the Weibull random variable (wind speed), $a$ is a scale parameter and $b$ is a shape parameter [6].

Measurements show that wind speed characteristics depend on wind direction. In this paper, the dependence of wind speed on wind direction is incorporated into the probabilistic model of wind speed as follows: Annual wind measurement data (usually on an hourly basis) at a specific site are divided into $N_d$ intervals according to wind direction. Subsequently, the wind speed values clustered for each interval are represented by a fitted Weibull distribution and a frequency value that captures how often wind blows from this direction interval as compared to all intervals [9]. Fig. 1 illustrates a long-term wind speed model at an arbitrary site.

As a result, in the model, the probability density function of wind speed at a site is defined as

$$f_V(v) = \sum_{i=1}^{N_d} f_{V_i}(v)\omega_i \quad (2)$$

where $N_d$ is the total number of direction intervals, $f_{V_i}(v)$ is the Weibull probability density function of wind speed for the $i^{th}$ interval, and $\omega_i$ is the frequency of the $i^{th}$ interval.

### 2.2. The power output of a wind turbine

The power available in wind is converted to a useful form of energy by wind turbines. The power output of a wind turbine depends on wind speed and the characteristics of the wind turbine, such as efficiency, size and power curve. The power curve or the $p$-$v$ characteristic of a wind turbine defines how the power output of the wind turbine varies with wind speed [16]. In Fig. 2, a typical power curve of a wind turbine is illustrated.

The power curve of a wind turbine can be analysed in three regions: In order for the wind turbine to start generating power, wind speed must be greater than the cut-in speed $v_{in}$. Consequently, below the cut-in speed, in region I, the power output of a wind turbine is zero. Similarly, in region III, the wind turbine stops

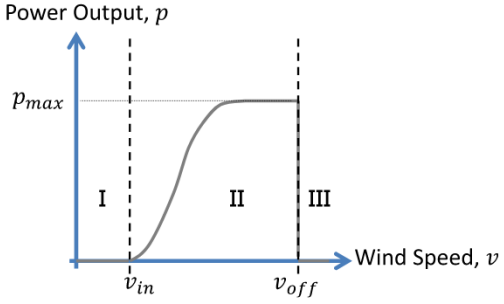



**Fig. 2.** A typical $p$-$v$ characteristic of a wind turbine.

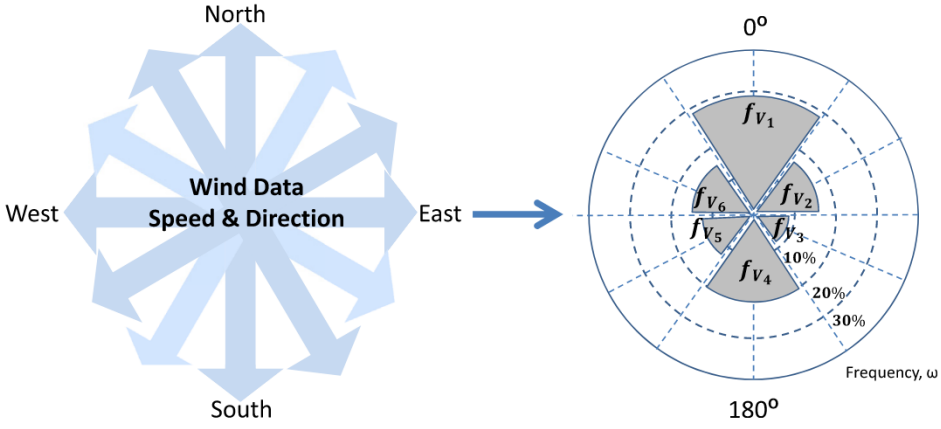



**Fig. 1.** Model for annual variability of wind speed ($N_d = 6$).

operating to prevent damage at higher speeds than the cut-off speed $v_{off}$ and does not generate power. Therefore, the wind turbine generates power when wind speed is between the cut-in and the cut-off speeds, in region II. In this region, the power output of a wind turbine increases with increasing wind speed till the rated speed at which the maximum power output of the wind turbine is reached and generated till the cut-off speed. Hence, the power output $p$ of a wind turbine can be expressed as

$$p = \begin{cases} 0 & \text{if } v < v_{in} \text{ or } v > v_{off}, \\ PC(v) & \text{if } v_{in} \le v \le v_{off}. \end{cases} \tag{3}$$

where $PC(.)$ represents the power curve of the wind turbine.

From (3), the probability of zero power output can be calculated as the sum of the probabilities that wind speed is smaller than the cut-in speed or larger than the cut-off speed:

$$\begin{aligned} F_P(0) &= \Pr(v < v_{in}) + \Pr\left(v > v_{off}\right) \\ &= F_V(v_{in} - \varepsilon) + 1 - F_V\left(v_{off} + \varepsilon\right) \end{aligned} \tag{4}$$

where $F_P(p)$ is the cdf of the power output of the wind turbine, and $\varepsilon$ is a small positive number.

The power curve of a wind turbine is a non-decreasing function in regions I and II. Therefore, from the change of variables technique in probability theory [17], the cdf of the power output can be expressed as the sum of the cdf of wind speed and the probability that power output is zero due to speeds higher than the cut-off speed:

$$F_P(p) = F_V(v) + 1 - F_V\left(v_{off} + \varepsilon\right) \tag{5}$$

where $p = PC(v)$, i.e., the power output of the wind turbine at wind speed $v$.

From the cdf of the power output, the expected annual power generation from the wind turbine $G_W$ can be found by integration:

$$G_W = \int_0^{p_{max}} (1 - F_P(p))dp \times 8760[\text{MWh}] \tag{6}$$

where $p_{max}$ is the size of the wind turbine in MW and 8760 represents the hours in a year.

## 3. The assesment of potential sites for wind farms

The location of a wind farm influences the power generated from wind turbines and the impact on the power grid. Therefore, the assessment of potential sites for wind farms is crucial to the wind power integration analyses. Based on the models for wind speed and the power output of a wind turbine presented in Section 2, this section develops the criteria for evaluating a potential site for wind farm construction.

### 3.1. The quality of wind

From an investor's point of view, the first important criterion to decide on the location of a wind power plant is the promise of strong wind. The main parameters that capture the wind power potential at a site are the average wind speed, the theoretical wind power density and the capacity factor of a prospective wind turbine [10].

#### 3.1.1. The arithmetic mean of wind speed

The arithmetic mean of wind speed $\bar{v}$ at a site is calculated using the expected value theorem [17] in the proposed model for wind speed in (2) as

$$\begin{aligned} \bar{v} &= \sum_{i=1}^{N_d} \int_0^{\infty} b_i a_i^{-b_i} v^{b_i} e^{-\left(\frac{v}{a_i}\right)^{b_i}} \, dv \, \omega_i \\ &= \sum_{i=1}^{N_d} a_i \Gamma\left(1 + \frac{1}{b_i}\right) \omega_i \end{aligned} \tag{7}$$

where $\Gamma(.)$ is the Gamma function, and $a_i$ and $b_i$ are the Weibull parameters of $f_{V_i}(v)$.

#### 3.1.2. The theoretical wind power density

The theoretical power available in wind $p_w$ at an instant of time [10] is calculated as

$$p_w = \frac{1}{2}\rho_a v^3 A_{\perp} \tag{8}$$

where $\rho_a$ is the air density and $A_{\perp}$ is the area perpendicular to wind, i.e., the blade sweep area of a wind turbine. As the calculation in (8) depends on the size of the wind turbine due to the cross-sectional area $A_{\perp}$, wind power density $\rho$ is defined [10] so that wind power potential can be captured regardless of the turbine size:

$$\rho = \frac{1}{2}\rho_a v^3. \tag{9}$$

The expected theoretical wind power density $\bar{\rho}$ at a site can be calculated by introducing the proposed model for wind speed in (2) and the expected value theorem into (9) as

$$\begin{aligned} \bar{\rho} &= \sum_{i=1}^{N_d} \frac{1}{2}\rho_a \int_0^{\infty} b_i a_i^{-b_i} v^{b_i+2} e^{-\left(\frac{v}{a_i}\right)^{b_i}} \, dv \, \omega_i \\ &= \frac{1}{2}\rho_a \sum_{i=1}^{N_d} a_i^3 \Gamma\left(1 + \frac{3}{b_i}\right) \omega_i \end{aligned} \tag{10}$$

#### 3.1.3. The capacity factor of a wind turbine

The capacity factor [10] of a wind turbine is defined as the ratio of actual power generation over a period of time, to the potential power generation if it were possible to operate at full capacity indefinitely:

$$\text{Capacity Factor} = \frac{\text{Total Generation}}{\text{Turbine Size} \times \text{Operating Hours}}. \tag{11}$$

For the proposed model, the annual capacity factor $\eta$ of a wind turbine at a site can be calculated using the expected annual power generation (6) in the definition of capacity factor (11) as

$$\eta = \frac{\int_0^{p_{max}} (1 - F_P(p))dp}{P_{max}}. \tag{12}$$

### 3.2. Economic and environmental criteria

Strong wind characteristics are good indicators of potential sites

for wind farms. However, it is not possible to construct a wind farm at every promising site. Additional criteria for site selection may be imposed due to economic or environmental concerns. Economic criteria could comprise the lack or difficulty of transportation to the site, the land cost or the distance to the electrical grid, whereas environmental factors could be the site being close to city centres, airports, or forested areas, having high altitude and so on. Indeed, in some countries, for wind power plants to have a generation license, investors have to submit an assessment showing that the wind farm to be established does not result in any harm to the nature and environment.[1] Therefore, these criteria should be included in the studies related to the determination of wind farm locations and the integration of wind farms into the power grid.

## 4. The optimal placement of wind farms under grid constraints

The integration of wind farms and their power outputs into the electrical grid affects the operation of the transmission system. Therefore, grid operators demand that newly integrated wind power plants into the electrical grid do not violate the transmission system constraints. Additionally, grid operators could declare regional upper bounds on wind farms due to economic and geographical concerns.[2] On the other hand, the expected profit from a wind farm is the main concern for power plant investors, and the revenue from a wind farm is closely related to the expected total generation. Therefore, on sites with strong wind, investors prefer to establish wind farms of substantial sizes whose power output integration into the electrical grid can cause problems. In Section 3, the criteria governing the potential sites for wind power plants are determined from an investor's point of view. In this section, an optimization problem is formulated to find the best solution to the wind farm placement problem: The expected total annual power generation from the selected sites is maximized, while adhering to the rules established by transmission system operator. Consequently, the objective function $f(S_k)$ to be maximized is defined as

$$f(S_k) = \sum_{k=1}^{N} \int_0^{S_k} (1 - F_{P_k}(p))dp \times 8760 \tag{13}$$

where $f(S_k)$ represents the total annual generation from the wind farms, $N$ is the total number of potential sites for wind farms satisfying the criteria in Section 3, $F_{P_k}(p)$ is the cdf of the wind farm power output at the $k^{\text{th}}$ site and $S_k$ represents the size of the wind farm at the $k^{\text{th}}$ site.

When wind power outputs are integrated into the power grid, power flow in the network changes, and the final state of the electrical grid must satisfy the power flow laws. We assume that a prospective wind farm at the $k^{\text{th}}$ site will be electrically connected to the power grid at a pre-defined network node (substation) $i$, and we use the linearised DC power flow [18] to represent the power flow behaviour in the grid. Consequently, the maximization of (13) is subjected to the linearised power flow equation in (14) for each network link $l$, and the power balance equation in (15) for each network node i:

$$P_l - \frac{\Theta_s - \Theta_r}{X_l} = 0 \tag{14}$$

$$P_{C_i} + P_{W_i} + P_{l_i} - L_i = 0 \tag{15}$$

where $P_l$ is the active power flow over the network link $l$, $\Theta_s$ is the voltage phase angle at the sending node of link $l$, $\Theta_r$ is the voltage phase angle at the receiving node of link $l$, $X_l$ is the reactance of link $l$, $P_{Ci}$ is the total power output of the existing generators at node $i$, $P_{Wi}$ is the total wind power output at node $i$, i.e., the sum of the power outputs of the wind farms that are electrically connected to node $i$, $P_{li}$ is the net link flow received at node $i$, and $L_i$ is the electrical load consumed at node i.

The power output of a wind turbine is shown to be a random variable in Section 2. Therefore, total wind power output $P_{Wi}$ integrated into the power grid from node $i$ at an instant is not known in advance. This paper deploys a diversity factor[3] $f_D$ to define the relation between the power output at an instant and the total installed capacity of wind farms connected to a node i:

$$f_{D_i} = \frac{P_{W_i}}{\sum_{k\to i}^{N} S_k} \tag{16}$$

where $\sum_{k\to i}^{N} S_k$ represents the sum of the installed capacities of wind farms electrically connected to node $i$, and $f_{D_i}$ is the diversity factor of node $i$. The diversity factor needs to be input to the optimization problem and reflects the level of conservatism wanted by the transmission system operator. The maximum diversity factor of a node is 1, demonstrating the instant at which all wind farms connected to node $i$ are producing their maximum power output. Consequently, assuming a diversity factor of 1 in the optimization formulation is the most conservative integration planning, as it is most confined by the power grid constraints.

As the power outputs of the wind farms are not known in advance, the balance between the generated power and the consumed electrical load in (15) is achieved by giving a margin of safety[4] to the selected existing reserve power plants, whereas for the other existing plants the generation output is fixed:

$$P_{C_{\text{imin}}} \le P_{C_i} \le P_{C_{\text{imin}}} \tag{17}$$

where $P_{C_{\text{imin}}}$ represents the minimum generation from existing generators at node $i$, and $P_{C_{\text{imax}}}$ represents the maximum generation from existing generators at node $i$.

Finally, the objective function in (13) is subjected to the constraints which are imposed by the transmission system operator: The final flows over each network link $l$ must be smaller than the maximum flow limit, and the total size of wind farms at each site $k$ and in each region $r$ should not exceed the limits set by the transmission system operator:

$$|P_l| \le P_{l,\max} \tag{18}$$

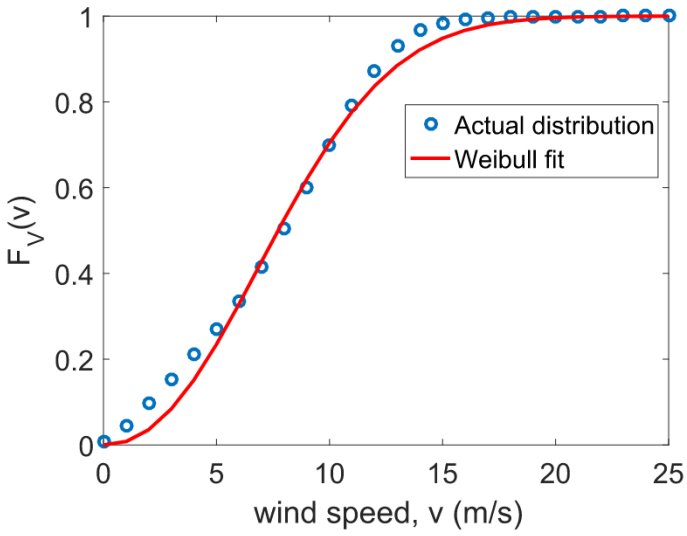
$$0 \le S_k \le S_{k,\max} \tag{19}$$

[1] An example of 'Environmental Impact Assessment' can be found at the website of the Republic of Turkey Ministry of Environment and Urbanisation: http://www.csb.gov.tr/gm/ced/.

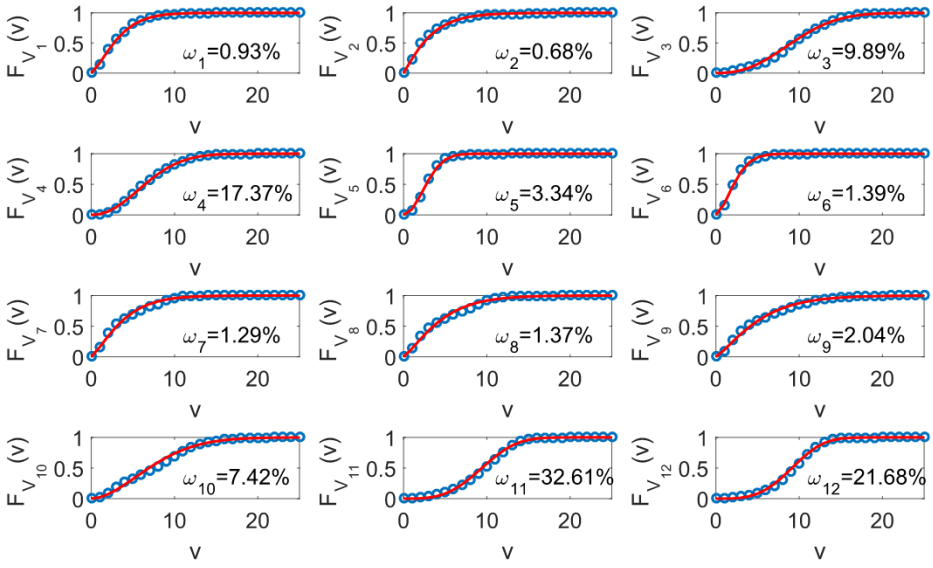
[2] An example of 'Regional Wind Power Plant Capacities' declared by the Turkish electricity transmission company can be found at: http://www.teias.gov.tr/duyurular/tablo.pdf.

[3] Demand diversity factor in power system analyses is defined as the ratio of the peak demand of the loads as a whole to the sum of the non-coincident peak demands of all individual units [19].

[4] Alternative techniques for dealing with uncertainty in data are presented in Ref. [20].




(a) Without direction intervals




(b) With direction intervals

**Fig. 3.** Comparison between the actual distribution and the Weibull model of wind speed variation at a specific location.

$$\sum_{k\in r}^{N} S_k \leq U_{r,\max} \tag{20}$$

where $P_{l,\max}$ is the maximum flow limit of the link $l$, $S_{k,\max}$ is the maximum total size of the wind farm at the $k^{\text{th}}$ site, $\sum_{k\in r}^{N} S_k$ represents the sum of the installed capacities of wind farms in region $r$ and $U_{r,\max}$ is the regional upper bound for the wind farms in region $r$.

## 5. Case study

This section presents a case study to demonstrate how the proposed methodology can be applied to assess the potential sites for wind farms in Turkey and to plan for the wind power integration into the power grid.

### 5.1. Wind characteristics

The countrywide wind data of Turkey that include historical annual hourly wind speed and direction values for every $6 \times 6$ km$^2$ geographical area have been utilised to define the probability density function of wind speed (2) for each site.[5] Fig. 3 illustrates a comparison between the traditional Weibull model and the model (2) according to wind direction intervals at a specific area. Total mean square error of the fit in Fig. 3b is 0.035% and is smaller[6] than

[5] The data are retrieved from TÜBITAK Marmara Research Center. The total number of geographical areas is 21,983. The total number of direction intervals $N_d$ in (2) is selected as 12, since the minimum error in the Weibull fitting is obtained for $N_d = 12$ for randomly chosen test areas, which is in line with [9].

[6] For this site, the mean square errors of Rayleigh and Gumbel fits are calculated as 0.093%, and 0.15% respectively.

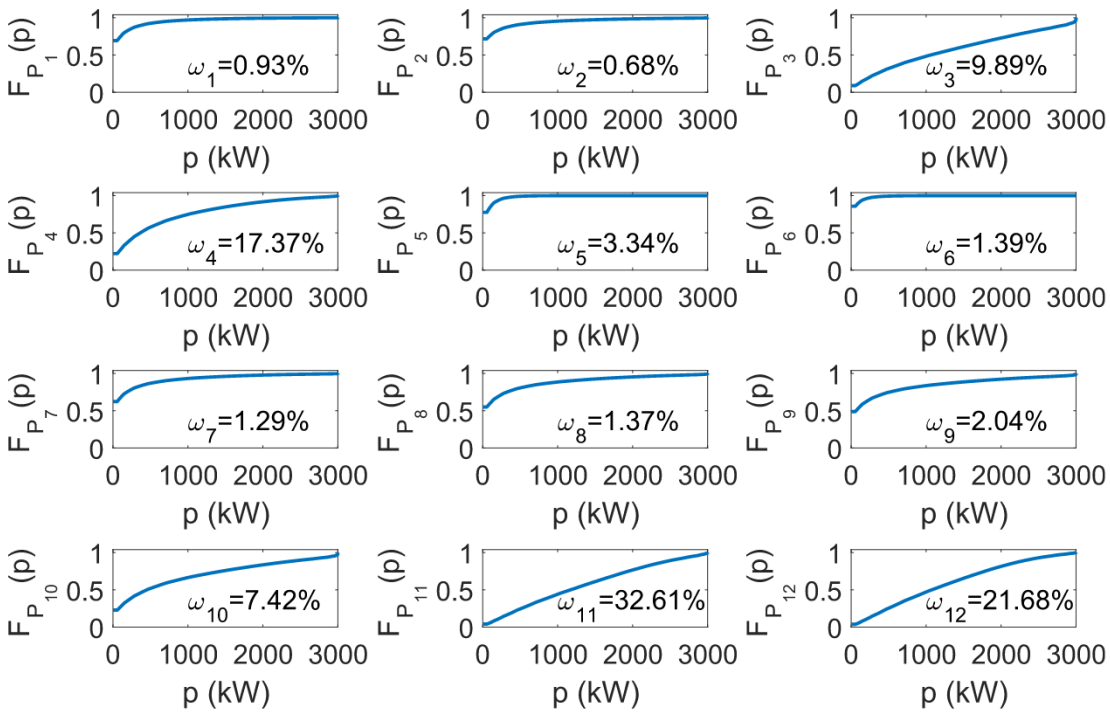



**Fig. 4.** The cdf of power output for direction intervals at a specific location.

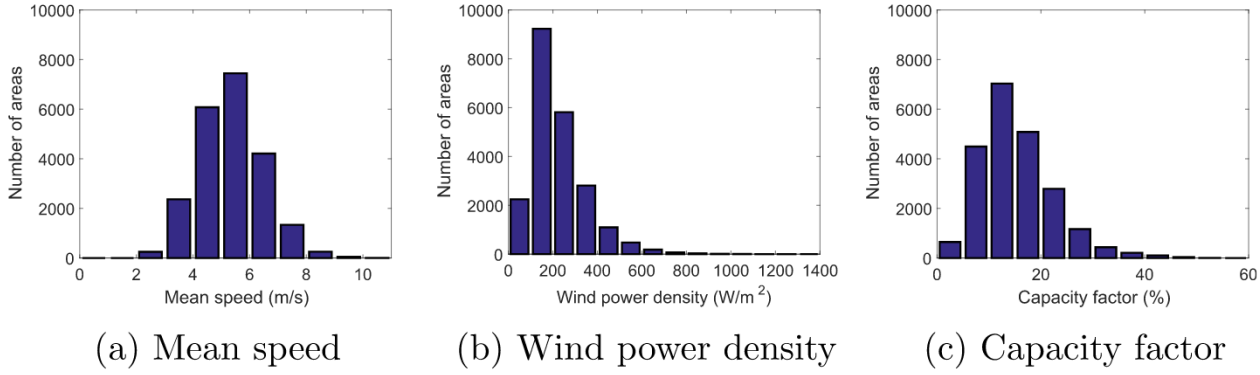



**Fig. 5.** The characteristics of wind in Turkey.

the mean square error of the fit 0.091% in Fig. 3a.

For the power curve model in (3), the power curve of a practical wind turbine[7] is used and the cdf of the power output of a prospective wind turbine at each geographical area in Turkey is calculated according to (5). As an example, Fig. 4 shows the cdf of the power output of a wind turbine at the specific area for the wind speed characteristics in Fig. 3b: The unpromising wind speed characteristics in the 1st, 2nd, 5th, 6th and 9th direction intervals result in higher probabilities of zero power output of the wind turbine.

### 5.2. Potential sites for wind farms

Following from Section 3, the arithmetic mean of wind speed (7), the theoretical wind power density (10), and the capacity factor of a prospective wind turbine (12) are calculated for each geographical area. In Fig. 5, the histograms of the countrywide calculated characteristics in Turkey are presented, and the thresholds for a potential wind farm site are set as 6 m/s for the arithmetic mean of wind speed, 200 W/m$^2$ for the theoretical wind power density, and 15% for the capacity factor of a prospective wind turbine. Under these constraints, more than three quarters of the geographical areas are not feasible for the establishment of wind farms.

The ineligible geographical areas for wind farms due to the economic and environmental criteria contain urban areas, natural parks, airports, etc. and areas at an altitude higher than 2000 m.[8] Those areas are also eliminated from the potential sites for wind farms. White coloured areas in Fig. 6 show the geographical areas in

[7] The power curve of the Vestas V112–3.0 MW wind turbine can be found at: http://www.vestas.com.

[8] The related lists of national parks, natural monuments, protected areas in Turkey can be found at the websites of Republic of Turkey, Energy Market Regulatory Authority, and Republic of Turkey, Ministry of Energy and Natural Resources: http://www.epdk.gov.tr, and http://www.eie.gov.tr, respectively.

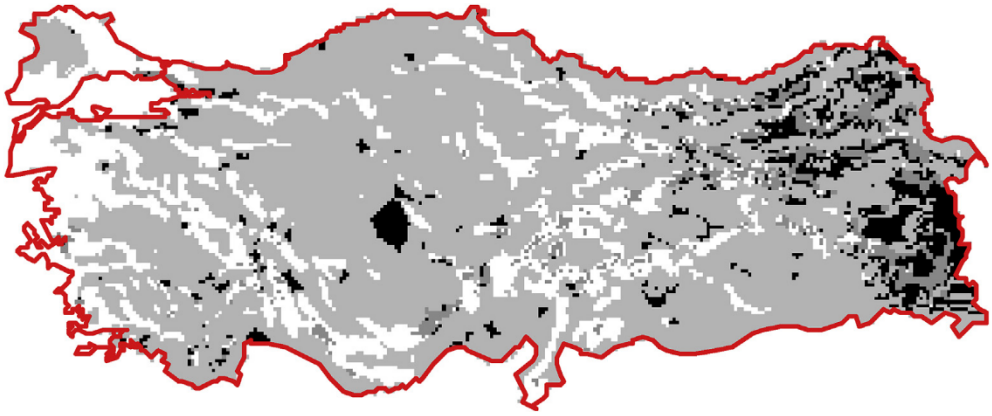


**Fig. 6.** The determination of potential sites for wind farms in Turkey. White areas satisfy the criteria in Section 3, whereas light grey, dark grey and black areas are ineligible for the establishment of wind power plants due to the quality of wind criterion, economic and environmental criteria, and both of the criteria, respectively.

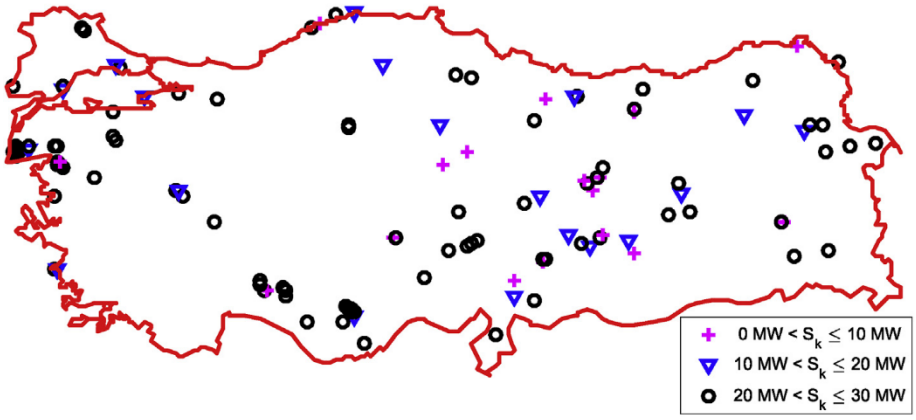



**Fig. 7.** The optimal wind farm placement when $f_D = 1$.

Turkey that satisfy the proposed criteria in Section 3 and each of them is treated as a potential site for the establishment of wind farms in the optimization formulation.

### 5.3. Optimal sites for wind farms

The optimal integration of wind farms into the Turkish electricity grid is investigated according to the proposed methodology in Section 4 for the declared 3 GW of the wind power integration[2]: Year 2017 forecasts for electricity generation, consumption and high voltage transmission grid are used in the model for the power grid[9] in (14) and (15) at the instant of peak load[10] conditions. Flexible generation is allowed for the largest 2 hydro-power plants of Turkey. The thermal ratings of transmission elements are used as the flow limits in (18). The geographic coordinates of grid substations are utilised to couple each feasible site $k$ with the closest electrical network node $i$, and with the region $r$. The maximum size of a wind farm in (19) is restricted to 30 MW and wind power capacities announced by the Turkish electricity transmission operator are used as the regional constraints in (20). Lastly, the optimization problem in Section 4 is solved by the linear programming solver (*linprog*) in MATLAB for the maximization of the objective function (13), subject to the linear equality constraints (14), (15), and the linear inequality constraints (17), (18), (19), (20) for different values of diversity factors in (16).

Fig. 7 illustrates the optimal siting and sizing[11] of wind farms for the diversity factor $f_D = 1$. The selected placement maximizes the annual wind power generation from the wind farms, while complying with the electrical grid constraints. The thermal ratings of 16 transmission elements are found to be binding[12] in the optimal solution, and the whereabouts of those transmission elements indicate the bottlenecks of the electrical network. The expected total annual wind power generation is calculated as 8.2796 TWh with the average capacity factor of 31.9%, which support the feasibility of the proposed placement plan from an investor's point of view.

[9] The substations are modeled as nodes, whereas the transmission lines and the transformers are modeled as links. The constructed model for electrical network has in total 1499 nodes and 2479 links.

[10] Peak load describes the period in which the power requirement of a power system is expected to be maximum.

[11] The selected sites and sizes of wind farms depend on the variations of wind speed at each site, in particular the expected annual power generation from wind turbine (6). Therefore, correct estimation of Weibull parameters is important. Reading errors due to inaccurate measurement devices may result in slightly different Weibull fit parameters that can affect the generation profile of a wind farm. However, conducted case studies have shown that errors do not alter the overall wind quality at a site significantly. Therefore, optimal sites are less sensitive to measurement errors than optimal sizes.

[12] A constraint is binding if changes in its value change the optimal solution. In other words, at the optimal solution, a binding inequality constraint is satisfied at its limit. Less severe constraints that do not affect the optimal solution are called non-binding.

**Table 1**
The value of the objective function in different power output scenarios.

| $f_D$ | $f(S_k)$ [TWh] |
|---|---|
| 0.1 | 8.4136 |
| 0.2 | 8.4136 |
| 0.3 | 8.4136 |
| 0.4 | 8.4136 |
| 0.5 | 8.4136 |
| 0.6 | 8.4136 |
| 0.7 | 8.4107 |
| 0.8 | 8.3953 |
| 0.9 | 8.3503 |
| 1.0 | 8.2796 |

Finally, the impact of the diversity factor on the optimal locations is analysed. Table 1 presents the results of the objective function for 10 different values of diversity factor (16) in the optimization problem. The same optimal sizes and locations, which produce 8.4136 TWh of annual wind generation, are selected in 6 out of 10 cases. The power grid inequality constraints (18) are observed to be non-binding for that solution, which means that the most contributing sites to the objective function (13) in each region are selected for the integration, and their simultaneous power output up to 60% of the total installed capacity does not violate the transmission grid constraints. However, in the remaining cases with diversity factor equal to or larger than 0.7, the power grid inequality constraints also become binding. Consequently, new sets of sizes and locations with less-favourable wind characteristics that comply with the power grid constraints are selected, which decreases the annual wind power generation. The diversity factor could be determined by transmission system operators depending on the planned degree of flexibility in the operation of transmission system. If minimum risk of overloads is desired by the transmission system operators, the solution of the optimization problem for maximum diversity factor, $f_D = 1$, could be used for the long-term integration plan of wind farms.

## 6. Conclusion

This paper has presented a comprehensive methodology to investigate the optimal wind power integration into the electrical grid. We calculated the probabilistic functions for the long-term variability of wind speed according to wind speed direction intervals and the power output of a wind turbine. The utilised model for the characteristic of wind speed has increased the accuracy of the modelling of the distribution of wind. Moreover, contrary to traditional generation placement studies that do not cover a detailed feasibility analysis, in this paper, the expressions for three metrics, arithmetic mean of wind speed, theoretical wind power density, and the capacity factor, which capture the quality of wind at a specific area, were derived and used in the determination of feasible locations for wind farm establishment. According to those metrics, more than 75% of the geographical areas in a case study for Turkey were infeasible for wind power plant establishment. We have further utilised a map-based approach to couple the feasible geographical locations with the nodes in the electrical grid and formulated an optimization problem to find the optimal sizing and siting of wind farms, such that the selected sites can generate maximum expected annual wind power generation, while satisfying the regional and the electrical grid constraints. The proposed methodology was tested for the integration scenario of wind power into the Turkish electricity grid, and a high average capacity factor, 31.9%, was obtained from the selected sites. The constructed optimization formulation facilitates the assessment of the placement of wind power plants, and could be used by transmission system operators as a long-term transmission network planning tool for the grid integration of wind power plants.

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