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Mixing angle of doubly heavy baryons in QCD

T.M. Aliev [a,1], K. Azizi [b,*], M. Savcı [a]

[a] Physics Department, Middle East Technical University, 06531 Ankara, Turkey
[b] Physics Department, Faculty of Arts and Sciences, Doğuş University, Acıbadem-Kadıköy, 34722 Istanbul, Turkey

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ABSTRACT

We calculate the mixing angles between the spin-1/2, Ξ_{bc}–Ξ'_{bc} and Ω_{bc}–Ω'_{bc} states of doubly heavy baryons within the QCD sum rules method. It is found that the mixing angles are large and have the values $\varphi_{\Xi_{bc}} = 16^\circ \pm 5^\circ$ and $\varphi_{\Omega_{bc}} = 18^\circ \pm 6^\circ$, respectively. The mixing angles are slightly smaller compared to the predictions of the non-relativistic quark model, $\varphi_{\Xi_{bc}} = 25.5^\circ$ and $\varphi_{\Omega_{bc}} = 25.9^\circ$.

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1. Introduction

Baryons with two heavy quarks have been the subject of intensive theoretical studies. The study of these baryons can provide useful information for understanding the non-perturbative QCD effects. On the experimental side, only one Ξ_{cc}^{++} state is observed by the SELEX Collaboration. However, the quark model predicts the existence of other doubly heavy baryons, and their masses are estimated in this model (for a review on doubly heavy baryons, see for instance [1]).

Doubly heavy baryons also represent a very suitable framework for studying the consequences of heavy quark spin symmetry [2]. According to this symmetry, in the infinite heavy quark mass limit, the diquarks formed of two heavy quarks can possess total spin $s = 0$ or 1. Taking into account the spin of the light quark, the ground states of doubly heavy baryons can have total spin of 1/2 or 3/2.

Since the heavy quark mass is finite, the hyperfine interaction between one of the heavy quarks and light quark admix spin-0 and spin-1 components. Obviously, this mixing for the baryons with two identical heavy quarks should be very small, since the antisymmetry of the wave functions require radial or higher orbital angular momentum states. But for the heavy baryons with two different heavy quarks this mixing can be large in principle. It is shown in [3] that the hyperfine mixing can considerably change the decay widths of doubly heavy baryons. The mixing problem of doubly heavy baryons in semileptonic decays are discussed in many works [4–7].

As has been noted, the hyperfine mixing among the ground states of the doubly heavy baryons is studied in [3] within the framework of the quark model. The effects of this mixing for the electromagnetic decays of the doubly heavy baryons are investigated in [8]. Calculation of the mixing angle of baryons containing only one heavy quark within the QCD sum rules method [9] is given in [10].

In the present work, we generalize our previous study to the baryons containing double heavy quarks, i.e., we calculate the mixing angle between Ξ_{bc}–Ξ'_{bc} and Ω_{bc}–Ω'_{bc} states within the QCD sum rules approach.

2. Mixing angles between the Ξ_{bc}–Ξ'_{bc} and Ω_{bc}–Ω'_{bc} states

In order to calculate the mixing angles between Ξ_{bc}–Ξ'_{bc} and Ω_{bc}–Ω'_{bc} states within the QCD sum rules method, we consider the following correlation function:

$$\Pi = i \int d^4x\, e^{iqx} \langle 0| T\{\eta_1(x)\bar{\eta}_2(0)\}|0\rangle, \tag{1}$$

where η_1 and η_2 are the interpolating currents corresponding to the physical states. Obviously, these currents should be linear combinations of the interpolating currents of unmixed states η_1^0 and η_2^0, i.e.,

$$\eta_1 = \cos(\varphi)\eta_1^0 + \sin(\varphi)\eta_2^0,$$
$$\eta_2 = -\sin(\varphi)\eta_1^0 + \cos(\varphi)\eta_2^0. \tag{2}$$

According to the sum rules philosophy, the correlation function is calculated in two different ways, either in terms of hadronic parameters or quark–gluon degrees of freedom. Once this is accomplished, these two representations of the correlation function are equated, as a result of which we obtain the QCD sum rules for the corresponding physical quantities.

* Corresponding author.
E-mail addresses: taliev@metu.edu.tr (T.M. Aliev), kazizi@dogus.edu.tr (K. Azizi), savci@metu.edu.tr (M. Savcı).
[1] Permanent address: Institute of Physics, Baku, Azerbaijan.

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When we saturate the correlation function given in Eq. (1) with hadronic states we separate the ground state contributions, which should be equal to zero since the physical ground states described by the interpolating currents η_1 and η_2 are orthogonal. Here we would like to make the following two remarks. The mixing angles for the excited states are generally different from that of the ground states. For this reason, the physical part of the correlation function can get non-zero contributions from excited and continuum states. However, in the sum rules method, Borel transformation is performed in order to enhance the ground state contribution (see below). After this transformation, the contributions of the excited and continuum states are exponentially suppressed. Therefore, non-vanishing contributions to the physical part of the correlation function from the excited and continuum states should be very small.

Our second remark is related to the negative-parity baryon contributions to the correlation function. In principle, the negative-parity baryons can give contributions to the correlation function. These contributions disappear only if their mixing angles are the same as the one in the interpolating current. We assume that this is the case here, so we neglect the negative-parity baryon contributions in the present study.

Substituting Eq. (2) in Eq. (1) we get,

$$\tan(2\varphi) = \frac{2\Pi_{12}^{(0)}}{\Pi_{11}^{(0)} - \Pi_{22}^{(0)}}, \tag{3}$$

where $\Pi_{ij}^{(0)}$ correspond to the correlation function,

$$\Pi_{ij}^{(0)} = i\int d^4x\, e^{iqx}\langle 0|T\{\eta_i^{(0)}(x)\bar{\eta}_j^{(0)}(0)\}|0\rangle.$$

For interpolating currents η_1^0 and η_2^0 which correspond to the unmixed states, we choose,

$$\eta_1^0 = \frac{1}{\sqrt{2}}\epsilon^{abc}\{(b^{aT}Cq^b)\gamma_5 c^c + (c^{aT}Cq^b)\gamma_5 b^c + t(b^{cT}C\gamma_5 q^b)c^c + t(c^{aT}C\gamma_5 q^b)b^c\}, \tag{4}$$

$$\eta_2^0 = \frac{1}{\sqrt{6}}\epsilon^{abc}\{2(b^{aT}Cc^b)\gamma_5 q^c + (b^{aT}Cq^b)\gamma_5 c^c - (c^{aT}Cq^b)\gamma_5 b^c + 2t(b^{aT}C\gamma_5 c^b)q^c + t(b^{aT}C\gamma_5 q^b)c^c - t(c^{aT}C\gamma_5 q^b)b^c\}. \tag{5}$$

Considering the Lorentz invariance, the two-point correlation function can be written as:

$$\Pi_{ij}^{(0)} = \Pi_{ij}^{(1)}(q^2)\not{q} + \Pi_{ij}^{(2)}(q^2)I. \tag{6}$$

In further analysis of the mixing angle between the doubly heavy baryon states, we shall take into consideration both $\not{q}$ and I structures.

The invariant functions $\Pi_{ij}^{(1)}$ and $\Pi_{ij}^{(2)}$ can be related to their imaginary part with the help of the dispersion relation,

$$\Pi_{ij}^{(\alpha)} = \int_{(m_1+m_2)^2}^{\infty}\frac{\rho_{ij}^{(\alpha)}(s)\,ds}{s-q^2}, \tag{7}$$

where m_1 and m_2 are heavy quarks masses and $\rho_{ij}^{(\alpha)}$ are the spectral densities which are given as:

$$\rho_{ij}^{(\alpha)}(s) = \frac{1}{\pi}\,\mathrm{Im}\,\Pi_{ij}^{(\alpha)OPE}(s), \tag{8}$$

with the superscripts $\alpha = 1$ and 2 correspond to the structures $\not{q}$ and I, respectively. The expressions for the spectral densities are obtained as (see also [11]):

$$\begin{aligned}\rho_{11}^{(1)} = {} & \frac{3}{256\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}}\frac{d\alpha}{\alpha}\int_{\beta_{\min}}^{1-\alpha}\frac{d\beta}{\beta}(m_1^2\beta + m_2^2\alpha - s\alpha\beta) \\ & \times\{(m_1^2\beta + m_2^2\alpha - s\alpha\beta)(5+2t+5t^2) \\ & - 2(1-\alpha-\beta)(1-t)^2 m_1 m_2 \\ & + 6(1-t^2)m_q(m_1\beta + m_2\alpha)\} \\ & + \frac{\langle\bar{q}q\rangle}{32\pi^2}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha\{3(1-t^2)[(1-\alpha)m_1 + \alpha m_2] \\ & + m_q(1-\alpha)\alpha(5+2t+5t^2)\},\end{aligned} \tag{9}$$

$$\begin{aligned}\rho_{12}^{(1)} = {} & \frac{\sqrt{3}}{64\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}}\frac{d\alpha}{\alpha}\int_{\beta_{\min}}^{1-\alpha}\frac{d\beta}{\beta}(m_1^2\beta + m_2^2\alpha - s\alpha\beta) \\ & \times(-2+t+t^2)m_q(\beta m_1 - \alpha m_2) \\ & + \frac{\langle\bar{q}q\rangle}{16\sqrt{3}\pi^2}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha(-2+t+t^2)[(1-\alpha)m_1 - \alpha m_2],\end{aligned} \tag{10}$$

$$\begin{aligned}\rho_{22}^{(1)} = {} & \frac{1}{256\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}}\frac{d\alpha}{\alpha}\int_{\beta_{\min}}^{1-\alpha}\frac{d\beta}{\beta}(m_1^2\beta + m_2^2\alpha - s\alpha\beta) \\ & \times\{3(m_1^2\beta + m_2^2\alpha - s\alpha\beta)(5+2t+5t^2) \\ & + 2(1-t)[(1-\alpha-\beta)(13+11t)m_1 m_2 \\ & + (1+5t)m_q(\beta m_1 + \alpha m_2)]\} \\ & + \frac{\langle\bar{q}q\rangle}{96\pi^2}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha\{3m_q(1-\alpha)\alpha(5+2t+5t^2) \\ & + (1-t)(1+5t)[(1-\alpha)m_1 + \alpha m_2]\},\end{aligned} \tag{11}$$

$$\begin{aligned}\rho_{11}^{(2)} = {} & \frac{3}{256\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}}\frac{d\alpha}{\alpha}\int_{\beta_{\min}}^{1-\alpha}\frac{d\beta}{\beta}(m_1^2\beta + m_2^2\alpha - s\alpha\beta) \\ & \times\bigg\{-3\alpha(1-t^2)\frac{m_2}{\beta}(m_1^2\beta + m_2^2\alpha - s\alpha\beta) \\ & + m_1\bigg[-3\beta(1-t^2)\frac{1}{\alpha\beta}(m_2^2\beta + m_2^2\alpha - s\alpha\beta) \\ & - 2m_1 m_2(5+2t+5t^2)\bigg]\bigg\} \\ & + \frac{\langle\bar{q}q\rangle}{64\pi^2}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha\bigg\{(1-\alpha)\alpha(1-t)^2\bigg[3m_0^2 \\ & + \frac{4[m_1^2(1-\alpha) + m_2^2\alpha - s\alpha(1-\alpha)]}{\alpha(1-\alpha)} - 2s\bigg] \\ & - 2m_1 m_2(5+2t+5t^2) \\ & + 6(1-t^2)m_q[-(1-\alpha)m_1 - \alpha m_2]\bigg\},\end{aligned} \tag{12}$$

$$\rho_{12}^{(2)} = \frac{\sqrt{3}}{128\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}}\frac{d\alpha}{\alpha^2}\int_{\beta_{\min}}^{1-\alpha}\frac{d\beta}{\beta^2}\left(m_1^2\beta+m_2^2\alpha-s\alpha\beta\right)^2 \times\left(-2+t+t^2\right)(\beta m_1-\alpha m_2) + \frac{m_q\langle\bar{q}q\rangle}{16\sqrt{3}\pi^2}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha\left(-2+t+t^2\right)\left[(1-\alpha)m_1-\alpha m_2\right], \tag{13}$$

$$\rho_{22}^{(2)} = \frac{1}{256\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}}\frac{d\alpha}{\alpha}\int_{\beta_{\min}}^{1-\alpha}\frac{d\beta}{\beta}\left(m_1^2\beta+m_2^2\alpha-s\alpha\beta\right) \times\left[(m_2\alpha+m_1\beta)(-1+t)(1+5t)\frac{m_1^2\beta+m_2^2\alpha-s\alpha\beta}{\alpha\beta} - 6\left(5+2t+5t^2\right)m_1m_2m_q\right] + \frac{\langle\bar{q}q\rangle}{192\pi^2}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha\left\{(1-\alpha)\alpha(-1+t)(13+11t) \times\left[3m_0^2+\frac{4[m_1^2(1-\alpha)+m_2^2\alpha-s\alpha(1-\alpha)]}{\alpha(1-\alpha)}-2s\right] - 6\left(5+2t+5t^2\right)m_1m_2 + 2(1-t)(1+5t)m_q\left[-(1-\alpha)m_1-\alpha m_2\right]\right\}, \tag{14}$$

where,

$$\beta_{\min} = \frac{\alpha m_2^2}{s\alpha-m_1^2},$$

$$\alpha_{\min} = \frac{1}{2s}\left[s+m_1^2-m_2^2-\sqrt{\left(s+m_1^2-m_2^2\right)^2-4m_1^2 s}\right],$$

$$\alpha_{\max} = \frac{1}{2s}\left[s+m_1^2-m_2^2+\sqrt{\left(s+m_1^2-m_2^2\right)^2-4m_1^2 s}\right]. \tag{15}$$

Performing Borel transformation with respect to the variable $-q^2$ and assuming quark–hadron duality we get,

$$\Pi_{ij}^{(\alpha)} = \int_{(m_1+m_2)^2}^{s_0}\rho_{ij}^{(\alpha)}e^{-s/M^2}\,ds. \tag{16}$$

Substituting these expressions into Eq. (3), we obtain the expression for the mixing angle in the framework of the QCD sum rules method.

Now we are ready to perform numerical calculations. For the numerical values of the input parameters we use $\langle\bar{q}q\rangle(1\text{ GeV}) = -(246^{+28}_{-19}\text{ MeV})^3$ [12], $\bar{s}s = 0.8\langle\bar{q}q\rangle$, $m_0^2 = (0.8\pm0.2)\text{ GeV}^2$. For the masses of the heavy quarks we use their $\overline{\text{MS}}$ masses, which are given as $\bar{m}_b(\bar{m}_b) = 4.16\pm0.03$ GeV, $\bar{m}_c(\bar{m}_c) = 1.28\pm0.03$ GeV [13], and $m_s(2\text{ GeV}) = 102\pm8$ MeV [14]. The expressions of the invariant functions contain three auxiliary parameters, namely the Borel parameter M^2, continuum threshold s_0 and an arbitrary parameter t. For the working regions of the continuum threshold and Borel parameter we use the recent results obtained from analysis of the mass and residues of the doubly heavy baryons, i.e., $s_0 = (45\text{–}56)\text{ GeV}^2$, and $6\text{ GeV}^2 \leqslant M^2 \leqslant 16\text{ GeV}^2$ [11]. In the present study, the working regions of the parameter t are also taken to be $-0.72 \leqslant \cos\theta \leqslant -0.44$ and $0.44 \leqslant \cos\theta \leqslant 0.72$, where $t = \tan\theta$ (for details see [11]).

Considering these working regions for auxiliary parameters, we obtain $\varphi_{\Xi_{bc}} = 16^\circ\pm5^\circ$ for the Ξ_{bc}–Ξ'_{bc} case and $\varphi_{\Omega_{bc}} = 18^\circ\pm6^\circ$ corresponds to the Ω_{bc}–Ω'_{bc} mixing. These results have been obtained for the $\not{q}$ structure. Very close results are also obtained using the I structure. The same mixing angles are also evaluated in [4] within the non-relativistic quark model to have the values, $\varphi_{\Xi_{bc}} = 25.5^\circ$ and $\varphi_{\Omega_{bc}} = 25.9^\circ$. Comparing our results with these values, we see that the predictions of the QCD sum rules are slightly smaller compared to that of the non-relativistic quark model.

It should be noted here that, the consequence of mixing can considerably change the results of semileptonic and electromagnetic decays of heavy baryons firstly pointed out in [2].

In summary, we calculated the mixing angles between the doubly heavy Ξ_{bc}–Ξ'_{bc} and Ω_{bc}–Ω'_{bc} baryons using the QCD sum rules method, and obtained that the mixing angles are quite large. A comparison of our results on the mixing angles with the predictions of the non-relativistic quark model is also presented.

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