

RARE $B \rightarrow \nu\bar{\nu}\gamma$ DECAY IN LIGHT CONE QCD SUM RULE

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April 30, 2018

Abstract

Using the light cone QCD Sum Rules Method, we study the rare $B \rightarrow \nu\bar{\nu}\gamma$ decay and find that the branching ratios are, $B(B_s \rightarrow \nu\bar{\nu}\gamma) \simeq 7.5 \times 10^{-8}$, $B(B_d \rightarrow \nu\bar{\nu}\gamma) \simeq 4.2 \times 10^{-9}$. A comparison of our results, on branching ratio, with constituent quark and pole dominance model predictions are presented.

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1 Introduction

The Flavour Changing Neutral Current (FCNC) process is one of the most promising field for testing the Standard Model (SM) predictions at loop level and for establishing new physics beyond that (for a review see [1] and references therein). The rare decays provide a direct and reliable tool for extracting an information about the fundamental parameters of the Standard Model (SM), such as, the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V_{td} , V_{ts} , V_{td} and V_{ub} [2].

Right after the experimental observation of the $b \rightarrow s\gamma$ [3] and $B \rightarrow X_s\gamma$ [4] processes, the interest is focused on the other possible rare B -meson decays, that are expected to get observed at future B -meson factories and fixed target machines. Besides measuring the CKM matrix elements, the role played by the rare B -meson decays could be very important for extracting more information about some hadronic parameters, such as, the leptonic decays f_{B_s} and f_{B_d} . Pure leptonic decays of the form, $B_s \rightarrow \mu^+\mu^-$ and $B_s \rightarrow l^+l^-$ are not useful for this purpose, since their helicities are suppressed and they have branching ratios $B(B_s \rightarrow \mu^+\mu^-) \simeq 1.8 \times 10^{-9}$ and $B(B_s \rightarrow l^+l^-) \simeq 4.2 \times 10^{-14}$ [5]. For B_d meson case the situations gets worse due to the smaller CKM angle. Although the process $B_s \rightarrow \tau^+\tau^-$, whose branching ratio in the SM is $B(B_s \rightarrow \tau^+\tau^-) = 8 \times 10^{-7}$ [6], is free of this suppression, its observability expected to be compatible with the branching ratio of the $B_s \rightarrow \mu^+\mu^-$ decay, only when its efficiency is larger(better) than 10^{-2} .

Larger branching ratio is expected when a photon is emitted in addition to the lepton pair, with no helicity suppression. For that reason, the investigation of the $B_{s(d)} \rightarrow l^+l^-\gamma$ becomes interesting. Note that in the SM, the decay $B_s \rightarrow \nu\bar{\nu}$ is forbidden by the helicity conservation. However, similar to the $B_s \rightarrow \tau^+\tau^-$ case, the photon radiation process $B_s \rightarrow \nu\bar{\nu}\gamma$ takes place, without any helicity suppression. This decay is investigated in the SM, using the constituent quark and pole models as the alternative approaches, for the determination of the leptonic decay constants f_{B_s} and f_{B_d} in [7]. It was shown in that work that the diagrams with photon radiation from light quarks give the dominant contribution to the decay amplitude that is inversely proportional to the constituent light quark mass. But the "constituent quark mass" itself is poorly understood. Therefore, any prediction in the framework of the above mentioned approaches on the branching ratios is strongly model dependent. Note that, similar obstacle exists for the $B_{s(d)} \rightarrow l^+l^-\gamma$ decays as well [8].

In this work we investigate the $B_s \rightarrow \nu\bar{\nu}\gamma$ process, practically in a model independent way, namely, within the framework of the light cone QCD sum rules method (more about the method and its applications can be found in a recent review [9]). The paper is organized as follows: In sect.2 we give the relevant effective Hamiltonian for the $b \rightarrow s\nu\bar{\nu}$ decay. In sect.3 we derive the sum rules for the transition formfactors. Sect.4 is devoted to the numerical analysis for the formfactors, where we calculate the differential and total decay width for the $B \rightarrow \nu\bar{\nu}\gamma$ and confront our results with those of [7]. Our calculations show that the constituent quark model and sum rules predictions are equal for the constituent quark mass $m_s(m_d) = (250) MeV$.

2 Effective Hamiltonian

We start by considering the quark level process $b \rightarrow q\nu\bar{\nu}$ ($q = s, d$). This process is described by the box and Z-mediated penguin diagrams. The effective Hamiltonian for this process was calculated in [6, 10] to yield

$$\mathcal{H}_{eff} = C\bar{q}\gamma_\mu(1 - \gamma_5)b\bar{\nu}\gamma_\mu(1 - \gamma_5)\nu \quad (1)$$

where,

$$C = \frac{G_F\sqrt{\alpha}}{2\sqrt{2}\pi\sin^2(\theta_W)}V_{tb}V_{ts}^*\frac{x}{8}\left[\frac{x+2}{x-1} + \frac{3(x-2)}{(x-1)^2}\ln(x)\right],$$

with,

$$x = \frac{m_t^2}{m_W^2}.$$

In our calculations we shall neglect the QCD corrections to the coefficient A, since they are negligible (see for example [6]).

At quark level the process $B_{s(d)} \rightarrow \nu\bar{\nu}\gamma$ is described by the same diagrams as $b \rightarrow q\nu\bar{\nu}$, in which photon is emitted from any charged particle. Incidentally, we should note the following peculiarities of this process:

a) when photon is emitted from internal charged particles (W and top quark), the above mentioned process will be suppressed by a factor $\frac{m_b^2}{m_W^2}$ (see [7]), in comparison to the process $b \rightarrow q\nu\bar{\nu}$, so that one can neglect the contributions of such diagrams.

b) The Wilson coefficient C is the same for the processes $b \rightarrow q\nu\bar{\nu}\gamma$ and $b \rightarrow q\nu\bar{\nu}$ as a consequence of the extension of the Low's low energy theorem (for more detail see [11]).

So we have two types of diagram that give contributions to the process $b \rightarrow q\nu\bar{\nu}\gamma$, when photon is emitted from initial b and light quark lines. The corresponding matrix element for the process $B_{s(d)} \rightarrow \nu\bar{\nu}\gamma$ is given as,

$$\langle \gamma | \mathcal{H}_{eff} | B \rangle = C\bar{\nu}\gamma_\mu(1 - \gamma_5)\nu \langle \gamma | \bar{q}\gamma_\mu(1 - \gamma_5)b | B \rangle . \quad (2)$$

The matrix element $\langle \gamma | \bar{q}\gamma_\mu(1 - \gamma_5)b | B \rangle$ can be written in terms of the two gauge invariant and independent structures, namely,

$$\begin{aligned} \langle \gamma(q) | \bar{q}\gamma_\mu(1 - \gamma_5)b | B(p+q) \rangle &= \sqrt{4\pi\alpha} \left[\epsilon_{\mu\alpha\beta\sigma} e_\alpha^* p_\beta q_\sigma \frac{g(p^2)}{m_B^2} + \right. \\ &\quad \left. + i \left(e_\mu^*(pq) - (e^*p)q_\mu \right) \frac{f(p^2)}{m_B^2} \right] . \end{aligned} \quad (3)$$

Here, e_μ and q_μ stand for the polarization vector and momentum of the photon, $p+q$ is the momentum of the B meson, $g(p^2)$ and $f(p^2)$ correspond to parity conserved and parity violated formfactors for the $B \rightarrow \nu\bar{\nu}\gamma$ decay. The main problem then, is to calculate the formfactors $g(p^2)$ and $f(p^2)$. For this aim we will utilize the light cone QCD sum rules method.

3 QCD Sum rules for the transition formfactors

Derivation of the effective Hamiltonian eq.(1) is one of the basic steps in the analysis of the $B_q \rightarrow \nu\bar{\nu}\gamma$ decay. We need to carry the calculation at the hadronic level, in another words, we must calculate the transition formfactors within the framework of some reliable theoretical scheme. We shall use QCD sum rules, more precisely, the light cone QCD sum rules method, to achieve this aim.

According to the QCD sum rules ideology, one starts with the calculation of the transition amplitude for the $B_q \rightarrow \nu\bar{\nu}\gamma$ decay, by writing the representation of a suitable correlator function in terms of hadron and quark-gluon parameters. So, to start with, we consider the following correlator:

$$\Pi_\mu(p, q) = i \int d^4x e^{ipx} \langle \gamma(q) | T \left[\bar{q}\gamma_\mu(1 - \gamma_5)b(x)\bar{b}i\gamma_5q \right] | 0 \rangle . \quad (4)$$

The general Lorentz decomposition of the above correlator is,

$$\Pi_\mu(p, q) = \sqrt{4\pi\alpha} \left\{ \epsilon_{\mu\nu\alpha\beta} e_\nu^* p_\alpha q_\beta \Pi_1 + i \left[e_\mu^*(pq) - (e^*p)q_\mu \right] \Pi_2 \right\} , \quad (5)$$

with, Π_1 and Π_2 corresponding to the parity conserving and parity violating components of the correlator, e_μ and q_β are the four-vector polarization and momentum of the photon, respectively.

The formidable task here, is to calculate Π_1 and Π_2 . This problem can be solved in the deep Euclidean region, where p^2 and $(p+q)^2$ are negative and large. The correlator function (4) in the framework of the light cone sum rules method was calculated in this deep region in [12] (see also [13]-[15]). We have recalculated this correlator and our final answer is in confirmation with the results of [12]. Omitting the details of the calculation, which can be found in [15], and after performing the Borel transformation for the formfactors g and f , the QCD sum rules method gives us:

$$\begin{aligned}
g &= \frac{m_b}{f_B} \int_\delta^1 \frac{du}{u} \exp\left(\frac{m_B^2}{M^2} - \frac{m_b^2 - \bar{u}p^2}{uM^2}\right) \times \\
&\times \left\{ e_q \langle \bar{q}q \rangle \left[\chi \Phi(u) - 4 \left(g^{(1)}(u) - g^{(2)}(u) \right) \frac{m_b^2 + uM^2}{u^2 M^4} \right] + \frac{m_b f}{2uM^2} g_1(u) + \right. \\
&\left. + \frac{3m_b}{4\pi^2} \left[(e_q - e_b) \bar{u} \frac{m_b^2 - p^2}{m_b^2 - \bar{u}p^2} + e_b \ln\left(\frac{m_b^2 - \bar{u}p^2}{um_b^2}\right) \right] \right\}, \tag{6}
\end{aligned}$$

$$\begin{aligned}
f &= \frac{m_b}{f_B} \int_\delta^1 \frac{du}{u} \exp\left(\frac{m_B^2}{M^2} - \frac{m_b^2 - \bar{u}p^2}{uM^2}\right) \left\{ e_q \langle \bar{q}q \rangle \left[\chi \Phi(u) - 4g^{(1)}(u) \frac{m_b^2 + uM^2}{u^2 M^4} \right] + \right. \\
&+ \frac{3m_b^3}{4\pi^2(m_b^2 - p^2)} \left[(e_q - e_b) \left(2u - 1 + \frac{p^2}{m_b^2} - \frac{p^2 u^2}{m_b^2 - \bar{u}p^2} \right) \frac{\bar{u}(m_b^2 - p^2)}{m_b^2 - \bar{u}p^2} - \right. \\
&\left. \left. - (e_q + e_b) u \frac{p^2}{m_b^2} \frac{\bar{u}(m_b^2 - p^2)}{m_b^2 - \bar{u}p^2} + e_b \left(2u - 1 + \frac{p^2}{m_b^2} \right) \ln\left(\frac{m_b^2 - \bar{u}p^2}{um_b^2}\right) \right] \right\}. \tag{7}
\end{aligned}$$

Here $\Phi(u)$ and $g_1(u)$ are the leading twist-2, while $g^{(1)}$ and $g^{(2)}$ are the twist-4 photon wave functions, χ is the magnetic susceptibility, $f = \frac{e_q}{g_\rho} f_\rho m_\rho$, with $f_\rho = 200 \text{ MeV}$ [12], $\bar{u} = 1 - u$, e_q and e_b are the charges of the light and beauty quarks, f_B is the leptonic decay constant, and $\delta = (m_b^2 - p^2)/(s_0 - p^2)$. The terms without the photon wave functions correspond to the perturbative contributions, when photon is emitted from heavy and light quark lines in the loop diagrams. The asymptotic form of the wave function $\Phi(u)$ is well known [16]-[19]:

$$\Phi(u) = 6u\bar{u}.$$

The twist-4 wave functions entering in eqs.(6) and (7) are given by [13],

$$\begin{aligned} g^{(1)}(u) &= -\frac{1}{8}\bar{u}(2+\bar{u}) , \\ g^{(2)}(u) &= -\frac{1}{4}\bar{u}^2 . \end{aligned}$$

4 Numerical Analysis

The main issue concerning eqs.(6) and (7), are the determination of the $g(p^2)$ and $f(p^2)$. We first give a list of the parameters entering in eqs.(6) and (7):

$$\begin{aligned} \langle \bar{q}q \rangle_d &= -(0.24 \text{ GeV})^3 [19], \quad \langle \bar{q}q \rangle_s = 0.8 \langle \bar{q}q \rangle_d [20], \\ f_B &= 0.14 \text{ GeV} [21], \quad s_0 = 35 \text{ GeV}^2, \quad m_b = 4.7 \text{ GeV}, \quad g_\rho = 5.5 [12]. \\ |V_{tb}V_{ts}^*| &= 0.045, \quad |V_{td}V_{ts}^*| = 0.010 [22]. \end{aligned} \tag{8}$$

The value of χ in the presence of external field was determined in [23, 24]:

$$\chi(\mu^2 = 1 \text{ GeV}^2) = -4.4 \text{ GeV}^{-2} .$$

If we include the anomalous dimension of the current $\bar{q}\sigma_{\alpha\beta}q$, that is equal to $-\frac{4}{27}$ at $\mu = m_b$, we get, $\chi(\mu^2 = m_b^2) = -3.4 \text{ GeV}^2$. Following [12], we shall take $g_1(u) = 1$, to the leading twist accuracy. The Borel parameter M^2 has been varied in the region from $8 \text{ GeV}^2 < M^2 < 20 \text{ GeV}^2$. We have found that, within the variation limits of M^2 in this region, the results change by less than 8%. The sum rules for $g(p^2)$ and $f(p^2)$ are meaningful in the region $m_b^2 - p^2 \sim (\text{few GeV}^2)$, which is smaller than the maximal available value $p^2 = m_b^2$. For an extension of the results to whole region of p^2 , we use the extrapolation formula. The best agreement is achieved with the dipole formulas (for more detail, see [12] and [25]).

$$g(p^2) \simeq \frac{h_1}{\left(11 - \frac{p^2}{m_1^2}\right)}, \quad f(p^2) \simeq \frac{h_2}{\left(1 - \frac{p^2}{m_2^2}\right)^2},$$

with,

$$\begin{aligned} h_1 &\simeq 1.0 \text{ GeV}, \quad m_1 \simeq 5.6 \text{ GeV}, \\ h_2 &\simeq 0.8 \text{ GeV}, \quad m_2 \simeq 6.5 \text{ GeV}. \end{aligned}$$

Using eq.(2) and eq.(3) for the total decay rate, we get

$$\Gamma = \frac{\alpha C^2 m_B^5}{256\pi^2} I, \tag{9}$$

where,

$$I = \frac{1}{m_B^2} \int_0^1 dx (1-x)^3 x \{f^2(x) + g^2(x)\} .$$

Here $x = 1 - \frac{2E_\gamma}{m_B}$ is the normalized photon energy. Let us compare our results with the ones that are obtained within the framework of the constituent quark and pole dominance models [7] (Note that eqs.(6), (7), (15) and (16) in [7], are all misprinted and all these equations must be multiplied by the factor 3). The correct results are as follows:

$$\begin{aligned} \frac{d\Gamma}{dx} &= \frac{2m_B^5 C^2 \alpha f_{B_q}^2}{m_q^2 (48\pi)^2} x(1-x) , \\ \Gamma &= \frac{3C^2 \alpha f_{B_q}^2 m_B^5}{(144\pi)^2 m_q^2} , \quad (\text{Constituent Quark Model}) \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{d\Gamma}{dx} &= \frac{C^2 \alpha g^2 f_{B^*}^2 m_{B^*}^2 m_B^7 (1-x)^3 x}{128\pi^2 (m_{B^*}^2 - x m_B^2)^2} , \\ \Gamma &= \frac{C^2 \alpha f_{B_q}^2 m_{B^*}^8 g^2}{768\pi^2 m_{B_q}^3} f \left(\frac{m_{B_q}^2}{m_{B^*}^2} \right) , \quad (\text{Pole Dominance Model}) \end{aligned} \quad (11)$$

where,

$$f(y) = -17y^3 + 42y^2 - 24y - 6(4-y)(1-y)^2 \ln(1-y) .$$

The coupling constant for $B_q B_q^* \gamma$ transition in the constituent quark model is given by [26],

$$g = +\frac{e_q}{m_q} . \quad (12)$$

This coupling constant in the light cone QCD sum rules was calculated in [15] to give:

$$g = -\frac{0.1}{f_B f_{B^*} m_B} . \quad (13)$$

Using the values of the input parameters and the lifetimes $\tau(B_s) = 1.34 \times 10^{-12} s$, $\tau(B_d) = 1.50 \times 10^{-12} s$ [24], we calculated the branching ratios of the decays, $B_s \rightarrow \nu \bar{\nu} \gamma$ and $B_d \rightarrow \nu \bar{\nu} \gamma$. The results are presented in Table 1. The results in the Table for the third and fourth columns are obtained using the values for the coupling constant g given by eqs.(12) and (13), respectively. Note that, for the constituent masses, we used $m_d \simeq 0.35 GeV$ and $m_s \simeq 0.51 GeV$. We find out that, eqs.(10) and (11) yield results that are numerically close to eq.(9), with the constituent quark masses $m_q \simeq f_q \sqrt{2}$. If we set $f_q \simeq 200 MeV$ we get

$m_d \sim 250 \text{ MeV}$. If we use this value of the constituent quark mass, the branching ratios for $B_s \rightarrow \nu\bar{\nu}\gamma$ and $B_d \rightarrow \nu\bar{\nu}\gamma$, increase by a factor of 4 and 2.5, respectively.

Also, for a comparison, we have calculated the photon spectra using the constituent quark, pole dominance and QCD sum rules models, and found that the photon spectra for the constituent quark and pole dominance models are fully symmetrical. But, as a result of the balance between a typical highly asymmetric resonance-type behaviour given by the non-perturbative contributions and a perturbative photon emission, the sum rules model yields a slightly asymmetrical prediction.

In conclusion, we calculate the branching ratios for the processes $B_s \rightarrow \nu\bar{\nu}\gamma$ and $B_d \rightarrow \nu\bar{\nu}\gamma$, in SM within the framework of the light QCD sum rules and obtained that $B(B_s \rightarrow \nu\bar{\nu}\gamma) \simeq 7.5 \times 10^{-7}$ and $B(B_d \rightarrow \nu\bar{\nu}\gamma) \simeq 4.2 \times 10^{-9}$. Within this range of branching ratios, it is possible to detect these processes in the future B factories and LHC.

	Sum rules	Constituent quark model	Pole dominance	Pole dominance
$B(B_s)$	7.50×10^{-8}	$1.93 \times 10^{-8} (\frac{f_B}{0.2})^2$	$1.79 \times 10^{-8} (\frac{f_{B^*}}{0.2})^2$	$0.94 \times 10^{-8} (\frac{0.2}{f_B})^2$
$B(B_d)$	0.42×10^{-8}	$2.26 \times 10^{-9} (\frac{f_B}{0.2})^2$	$2.10 \times 10^{-9} (\frac{f_{B^*}}{0.2})^2$	$0.52 \times 10^{-9} (\frac{0.2}{f_B})^2$

Table

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