$\Sigma_Q\Lambda_Q\pi$ Coupling Constant in Light Cone QCD Sum Rules

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Abstract

The strong coupling constants $g_{\Sigma_Q\Lambda_Q\pi}$ ($Q = b$ and c) are studied in the framework of the light cone QCD sum rules using the most general form of the baryonic currents. The predicted coupling constants are used to estimate the decay widths for the $\Sigma_Q \longrightarrow \Lambda_Q\pi$ decays which are compared with the predictions of the other approaches and existing experimental data.

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1 Introduction

Recent years have witnessed advances in the heavy baryon spectroscopy, with the discoveries of the heavy baryons involving the b and c quarks. Since the spin of the baryon carries information on the spin of the heavy quark, the study of the heavy baryons might also lead us to study the spin effects at the loop level in the standard model.

To study the meson-baryon couplings, a non-perturbative method is needed. Among all non-perturbative approaches, the QCD sum rules approach [1]-[3] has received special attention to study the properties of hadrons. In the case of the light baryons, this method has been successfully applied for calculation of the meson-baryon coupling constants. The pion-nucleon coupling constant has been studied in traditional three-point QCD sum rules [4]-[12]. The kaon-baryon coupling constants have also been calculated in the same framework in [13]-[16]. The latter has also been studied in light cone QCD sum rules (LCQSR) in [17]. The coupling constant for K meson-octet baryons and π meson-octet baryons have also been calculated in [18] in LCQSR.

The QCD sum rules is also applied to the study of the heavy hadron mass spectrum (see e.g. [19]). The masses are also studied in QCD string model [20] and using quark model in [21, 22]. In [22], sum rules between the masses of the heavy baryons derived using the quark model has been analyzed and experimental tests of sum rules for heavy baryon masses have been discussed in [23]. In the present work, using the general form of the current for Σ_Q and Λ_Q baryons, we calculate the $g_{\Sigma_Q \Lambda_Q \pi}$ ($Q = b$ and c) coupling constants in the framework of the LCQSR approach. Having computed the coupling constants, we also evaluate the total decay widths for strong $\Sigma_Q \longrightarrow \Lambda_Q \pi$ decays and compare our results with the predictions of the relativistic three-quark model (RTQM) [24], light-front quark model (LFQM) [25] and existing experimental data. The paper encompasses three sections: in the next section, we calculate the LCQSR for the coupling constant $g_{\Sigma_Q \Lambda_Q \pi}$. Section III is devoted to the numerical analysis of the coupling constant $g_{\Sigma_Q \Lambda_Q \pi}$, our prediction for the total decay rates and discussion.

2 Light cone QCD sum rules for the coupling constant $g_{\Sigma_Q \Lambda_Q \pi}$

To calculate the coupling constant $g_{\Sigma_Q \Lambda_Q \pi}$ in LCQSR, one starts with a suitably chosen correlation function. In this work, the following correlation functions is chosen:

$$\Pi = i \int d^4 x e^{ipx} \langle \pi(q) \mid \mathcal{T}\{\eta_{\Lambda_Q}(x) \bar{\eta}_{\Sigma_Q}(0)\} \mid 0 \rangle, \tag{1}$$

where η_{Σ_Q} and η_{Λ_Q} are the interpolating currents of the heavy baryons Σ_Q and Λ_Q. In this correlator, the hadrons are represented by their interpolating quark currents. This correlation function can be calculated in two different ways: on the one hand, inserting complete sets of hadronic states into the correlation function, it can be expressed in terms of hadronic parameters such as the masses, residues and the coupling constants. On the other hand, it can be calculated in terms of quark-gluon parameters in the deep Euclidean region when $p^2 \to -\infty$ and $(p+q)^2 \to -\infty$. The coupling constant is determined by matching these two different representations of the correlation function and applying double Borel transformation with respect to the momentum of both hadrons to suppress the contributions of the higher states and continuum.

The derivation of the physical (or phenomenological) representation of the correlation function follows the same lines as in the case of light hadrons (see e.g. [18]). For completeness, we repeat the derivation below. First, one inserts two complete sets of states between the interpolating currents in (1) with quantum numbers of the Σ_Q and Λ_Q baryons.

$$\Pi = \frac{\langle 0 \mid \eta_{\Lambda_Q} \mid \Lambda_Q(p_2)\rangle}{p_2^2 - m_{\Lambda_Q}^2} \langle \Lambda_Q(p_2)\pi(q) \mid \Sigma_Q(p_1)\rangle \frac{\langle \Sigma_Q(p_1) \mid \eta_{\Sigma_Q} \mid 0\rangle}{p_1^2 - m_{\Sigma_Q}^2} + ..., \tag{2}$$

where $p_1 = p + q$, $p_2 = p$, and ... stands for the contributions of higher states and continuum. The vacuum to baryon matrix element of the interpolating currents are defined as

$$\langle 0 \mid \eta_B \mid B(p,s)\rangle = \lambda_B u_B(p,s), \tag{3}$$

where $B = \Sigma_Q$ or Λ_Q, $u_B(p,s)$ is a spinor describing the baryon B and λ_B is the residue of the B baryon. The last ingredient is the matrix element $\langle \Lambda_Q(p_2)\pi(q) \mid \Sigma_Q(p_1)\rangle$ which can be parameterized in terms of the coupling constant $g_{\Sigma_Q\Lambda_Q\pi}$ as

$$\langle \Lambda_Q(p_2)\pi(q) \mid \Sigma_Q(p_1)\rangle = g_{\Sigma_Q\Lambda_Q\pi}\overline{u}(p_2)i\gamma_5 u(p_1). \tag{4}$$

Using Eqs. (2-4) and summing over the spin of the baryons, the following representation of the correlator for the phenomenological side is obtained:

$$\begin{aligned} \Pi &= i\frac{g_{\Sigma_Q\Lambda_Q\pi}\lambda_{\Lambda_Q}\lambda_{\Sigma_Q}}{(p_1^2 - m_{\Sigma_Q}^2)(p_2^2 - m_{\Lambda_Q}^2)}\left[- \not{p}\not{q}\gamma_5 - m_{\Sigma_Q}\not{q}\gamma_5\right. \\ &+ \left.(m_{\Lambda_Q} - m_{\Sigma_Q})\not{p}\gamma_5 + (m_{\Sigma_Q}m_{\Lambda_Q} - p^2)\gamma_5\right]. \end{aligned} \tag{5}$$

Note that, the structures $\not{p}\gamma_5$ and γ_5 have very small coefficient due to the fact that $m_{\Sigma_Q} \simeq m_{\Lambda_Q}$, hence they will not yield reliable sum rules.

To calculate the representation of the correlation function, Eq. (1), from QCD side, we need the explicit expressions of the interpolating currents for Σ_Q and Λ_Q baryons. In principal, any operator having the same quantum numbers as the corresponding baryon can be used. It is well known that there is a continuum of choices for the heavy spin-$\frac{1}{2}$ baryons interpolating currents that does not contain any derivatives. The general form of the Σ_Q and Λ_Q currents can be written as (see also [26])

$$\begin{aligned}
\eta_{\Sigma_Q} &= -\frac{1}{\sqrt{2}}\epsilon_{abc}\Big\{(u^{aT}CQ^b)\gamma_5 d^c + \beta(u^{aT}C\gamma_5 Q^b)d^c \\
&- [(Q^{aT}Cd^b)\gamma_5 u^c + \beta(Q^{aT}C\gamma_5 d^b)u^c]\Big\}\,, \\
\eta_{\Lambda_Q} &= \frac{1}{\sqrt{6}}\epsilon_{abc}\Big\{2[(u^{aT}Cd^b)\gamma_5 Q^c + \beta'(u^{aT}C\gamma_5 d^b)Q^c] + (u^{aT}CQ^b)\gamma_5 d^c \\
&+ \beta'(u^{aT}C\gamma_5 Q^b)d^c + (Q^{aT}Cd^b)\gamma_5 u^c + \beta'(Q^{aT}C\gamma_5 d^b)u^c\Big\}\,,
\end{aligned} \tag{6}$$

where β and β' are arbitrary parameters. For simplicity, we assume $\beta = \beta'$. The $\beta = -1$ corresponds to the Ioffe current and C is the charge conjugation operator and a, b and c are color indices.

After contracting out all quark pairs in Eq. (1), the following expression for the correlation function in terms of the quark propagators is obtained

$$\begin{aligned}
\Pi &= \frac{i}{\sqrt{3}}\epsilon_{abc}\epsilon_{a'b'c'}\int d^4x e^{ipx}\langle\pi(q)\mid\Big\{\gamma_5 S_Q^{ca'} S_u'^{ab'} S_d^{bc'}\gamma_5 \\
&- \gamma_5 S_Q^{cb'} S_d'^{ba'} S_u^{ac'}\gamma_5 - 1/2(\gamma_5 S_d^{ca'} S_Q'^{bb'} S_u^{ac'}\gamma_5 - \gamma_5 S_u^{cb'} S_Q'^{aa'} S_d^{bc'}\gamma_5 \\
&+ Tr[S_Q^{ba'} S_u'^{ab'}]\gamma_5 S_d^{cc'}\gamma_5 - Tr[S_d^{ba'} S_Q'^{ab'}]\gamma_5 S_u^{cc'}\gamma_5) \\
&+ \beta\Big[\gamma_5 S_Q^{ca'}\gamma_5 S_u'^{ab'} S_d^{bc'} - \gamma_5 S_Q^{cb'}\gamma_5 S_d'^{ba'} S_u^{ac'} + S_Q^{ca'} S_u'^{ab'}\gamma_5 S_d^{bc'}\gamma_5 \\
&- S_Q^{cb'} S_d'^{ba'}\gamma_5 S_u^{ac'}\gamma_5 + 1/2(\gamma_5 S_u^{cb'}\gamma_5 S_Q'^{aa'} S_d^{bc'} - \gamma_5 S_d^{ca'}\gamma_5 S_Q'^{bb'} S_u^{ac'} \\
&- S_d^{ca'} S_Q'^{bb'}\gamma_5 S_u^{ac'}\gamma_5 + S_u^{cb'} S_Q'^{aa'}\gamma_5 S_d^{bc'}\gamma_5 - S_d^{cc'}\gamma_5 Tr[\gamma_5 S_Q^{ba'} S_u'^{ab'}] \\
&+ S_u^{cc'}\gamma_5 Tr[\gamma_5 S_d^{ba'} S_Q'^{ab'}] - \gamma_5 S_d^{cc'} Tr[S_Q^{ba'}\gamma_5 S_u'^{ab'}] + \gamma_5 S_u^{cc'} Tr[S_d^{ba'}\gamma_5 S_Q'^{ab'}])\Big] \\
&+ \beta^2\Big[S_Q^{ca'}\gamma_5 S_u'^{ab'}\gamma_5 S_d^{bc'} + S_Q^{cb'}\gamma_5 S_d'^{ba'}\gamma_5 S_u^{ac'} + 1/2(S_u^{cb'}\gamma_5 S_Q'^{aa'}\gamma_5 S_d^{bc'}
\end{aligned}$$

$$- \quad S_d^{ca'}\gamma_5 S_Q'^{bb'}\gamma_5 S_u^{ac'} - Tr[\gamma_5 S_u^{ab'}\gamma_5 S_Q'^{ba'}]S_d^{cc'} + Tr[\gamma_5 S_Q^{ab'}\gamma_5 S_d'^{ba'}]S_u^{cc'})\Big]\Big\} \mid 0\rangle, \tag{7}$$

where $S' = CS^TC$ and $S_{Q(q)} \quad (q = u,d)$ is the full heavy (light) quark propagator. Note that, the Eq. (7) is a schematical representation for the full expression. To obtain the full expression from the Eq. (7), one should replace S_u by $u(0)\bar{u}(x)$ to calculate the emission from the u quark, and then add to this the result obtained by replacing S_d by $d(0)\bar{d}(x)$. From Eq. (7), it follows that the expression of the light and heavy quark propagators are needed.

The light cone expansion of the quark propagator in the external field is calculated in [27]. The propagator receives contributions from higher Fock states proportional to the condensates of the operators $\bar{q}Gq$, $\bar{q}GGq$ and $\bar{q}q\bar{q}q$, where G is the gluon field strength tensor. In this work, we neglect contributions with two gluons as well as four quark operators due to the fact that their contributions are small [28]. In this approximation, the heavy and light quark propagators have the following expressions:

$$\begin{aligned}
S_Q(x) &= S_Q^{free}(x) - ig_s\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\int_0^1 dv\Bigg[\frac{\not{k}+m_Q}{(m_Q^2-k^2)^2}G^{\mu\nu}(vx)\sigma_{\mu\nu} \\
&+ \frac{1}{m_Q^2-k^2}vx_\mu G^{\mu\nu}\gamma_\nu\Bigg], \\
S_q(x) &= S_q^{free}(x) - \frac{\langle\bar{q}q\rangle}{12} - \frac{x^2}{192}m_0^2\langle\bar{q}q\rangle \\
&\quad -ig_s\int_0^1 du\left[\frac{\not{x}}{16\pi^2x^2}G_{\mu\nu}(ux)\sigma_{\mu\nu} - ux^\mu G_{\mu\nu}(ux)\gamma^\nu\frac{i}{4\pi^2x^2}\right].
\end{aligned} \tag{8}$$

The expression of the free light and heavy quark propagators in the x representation are:

$$S_q^{free} = \frac{i\not{x}}{2\pi^2x^4},$$

$$S_Q^{free} = \frac{m_Q^2}{4\pi^2}\frac{K_1(m_Q\sqrt{-x^2})}{\sqrt{-x^2}} - i\frac{m_Q^2 \not{x}}{4\pi^2 x^2}K_2(m_Q\sqrt{-x^2}), \tag{9}$$

where K_i are the Bessel functions.

In order to calculate the contributions of the pion emission, the matrix elements $\langle \pi(q) \mid \bar{q}\Gamma_i q \mid 0\rangle$ are needed. Here, Γ_i is any member of the complete set of Dirac matrices $\{1, \gamma_5, \gamma_\alpha, i\gamma_5\gamma_\alpha, \sigma_{\alpha\beta}/\sqrt{2}\}$. These matrix elements are determined in terms of the pion distribution amplitudes (DA's) as follows [29, 30].

$$\begin{aligned}
\langle\pi(p)|\bar{q}(x)\gamma_\mu\gamma_5 q(0)|0\rangle &= -if_\pi p_\mu \int_0^1 du e^{i\bar{u}px}\left(\varphi_\pi(u) + \frac{1}{16}m_\pi^2 x^2 \mathbb{A}(u)\right) \\
&- \frac{i}{2}f_\pi m_\pi^2 \frac{x_\mu}{px}\int_0^1 du e^{i\bar{u}px}\mathbb{B}(u), \\
\langle\pi(p)|\bar{q}(x)i\gamma_5 q(0)|0\rangle &= \mu_\pi \int_0^1 du e^{i\bar{u}px}\varphi_P(u), \\
\langle\pi(p)|\bar{q}(x)\sigma_{\alpha\beta}\gamma_5 q(0)|0\rangle &= \frac{i}{6}\mu_\pi\left(1-\tilde{\mu}_\pi^2\right)(p_\alpha x_\beta - p_\beta x_\alpha)\int_0^1 du e^{i\bar{u}px}\varphi_\sigma(u), \\
\langle\pi(p)|\bar{q}(x)\sigma_{\mu\nu}\gamma_5 g_s G_{\alpha\beta}(vx)q(0)|0\rangle &= i\mu_\pi\left[p_\alpha p_\mu\left(g_{\nu\beta} - \frac{1}{px}(p_\nu x_\beta + p_\beta x_\nu)\right)\right. \\
&- p_\alpha p_\nu\left(g_{\mu\beta} - \frac{1}{px}(p_\mu x_\beta + p_\beta x_\mu)\right) \\
&- p_\beta p_\mu\left(g_{\nu\alpha} - \frac{1}{px}(p_\nu x_\alpha + p_\alpha x_\nu)\right) \\
&+ \left. p_\beta p_\nu\left(g_{\mu\alpha} - \frac{1}{px}(p_\mu x_\alpha + p_\alpha x_\mu)\right)\right] \\
&\times \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}} + v\alpha_g)px}\mathcal{T}(\alpha_i), \\
\langle\pi(p)|\bar{q}(x)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(vx)q(0)|0\rangle &= p_\mu(p_\alpha x_\beta - p_\beta x_\alpha)\frac{1}{px}f_\pi m_\pi^2 \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}} + v\alpha_g)px}\mathcal{A}_\parallel(\alpha_i) \\
&+ \left[p_\beta\left(g_{\mu\alpha} - \frac{1}{px}(p_\mu x_\alpha + p_\alpha x_\mu)\right)\right.
\end{aligned}$$

$$
\begin{aligned}
& - \quad p_\alpha \left(g_{\mu\beta} - \frac{1}{px}(p_\mu x_\beta + p_\beta x_\mu) \right) \Big] f_\pi m_\pi^2 \\
& \times \quad \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}} + v\alpha_g)px} \mathcal{A}_\perp(\alpha_i), \\
\langle \pi(p) | \bar{q}(x) \gamma_\mu i g_s G_{\alpha\beta}(vx) q(0) | 0 \rangle &= p_\mu (p_\alpha x_\beta - p_\beta x_\alpha) \frac{1}{px} f_\pi m_\pi^2 \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}} + v\alpha_g)px} \mathcal{V}_\parallel(\alpha_i) \\
& + \quad \Big[p_\beta \left(g_{\mu\alpha} - \frac{1}{px}(p_\mu x_\alpha + p_\alpha x_\mu) \right) \\
& - \quad p_\alpha \left(g_{\mu\beta} - \frac{1}{px}(p_\mu x_\beta + p_\beta x_\mu) \right) \Big] f_\pi m_\pi^2 \\
& \times \quad \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}} + v\alpha_g)px} \mathcal{V}_\perp(\alpha_i), \qquad (10)
\end{aligned}
$$

where $\mu_\pi = f_\pi \frac{m_\pi^2}{m_u + m_d}$, $\tilde{\mu}_\pi = \frac{m_u + m_d}{m_\pi}$ and the functions $\varphi_\pi(u)$, $\mathbb{A}(u)$, $\mathbb{B}(u)$, $\varphi_P(u)$, $\varphi_\sigma(u)$, $\mathcal{T}(\alpha_i)$, $\mathcal{A}_\perp(\alpha_i)$, $\mathcal{A}_\parallel(\alpha_i)$, $\mathcal{V}_\perp(\alpha_i)$ and $\mathcal{V}_\parallel(\alpha_i)$ are functions of definite twist and their expressions will be given in the numerical analysis section. The measure $\mathcal{D}\alpha$ is defined as

$$
\int \mathcal{D}\alpha = \int_0^1 d\alpha_{\bar{q}} \int_0^1 d\alpha_q \int_0^1 d\alpha_g \delta(1 - \alpha_{\bar{q}} - \alpha_q - \alpha_g). \qquad (11)
$$

Note that, in the approximation of this work where we neglect the light quark masses, $m_\pi^2 = 0$, $\tilde{\mu}_\pi = 0$, $\mu_\pi = -\langle \bar{u}u \rangle / f_\pi = -\langle \bar{d}d \rangle / f_\pi$.

Using the expressions of the light and heavy full propagators and the pion DA's, the correlation function Eq. (1) can be calculated in terms of QCD parameters. Separating the coefficient of the structure $\not{p}\not{q}\gamma_5$ in both representations, and equating them, sum rules for the coupling constant $g_{\Sigma_Q \Lambda_Q \pi}$ is obtained. The contribution of the higher states is subtracted using quark hadron duality, and in order to further suppress their contribution, Borel transformation with respect to the variables $p_2^2 = p^2$ and $p_1^2 = (p+q)^2$ is applied. Here, we should mention that we have also studied the other structure in Eq. (5), i.e., $\not{q}\gamma_5$ but its result for coupling constant is not stable and only the $\not{p}\not{q}\gamma_5$ structure leads to reliable prediction on the coupling constant

$g_{\Sigma_Q \Lambda_Q \pi}$.

The sum rules for the coupling constant is obtained as

$$\lambda_{\Sigma_Q} \lambda_{\Lambda_Q} e^{-\frac{m^2_{\Lambda_Q}+m^2_{\Sigma_Q}}{2M^2}} g_{\Sigma_Q \Lambda_Q \pi} = \Pi, \quad (12)$$

where the function Π is

$$\Pi = \int_{m_Q^2}^{s_0} e^{\frac{-s}{M^2}} \rho(s) ds + e^{\frac{-m_Q^2}{M^2}} \Gamma, \quad (13)$$

with

$$\begin{aligned}
\rho(s) &= (<\overline{d}d> + <\overline{u}u>) \frac{1}{12\sqrt{6}} f_\pi (\beta - 1) \beta \varphi_\pi(u_0) \psi_{00} \\
&- \frac{1}{96\sqrt{6}\pi^2} \Bigg[m_Q(\beta-1) \Bigg\{ -6 \Bigg[-2(\psi_{20} - \psi_{31}) m_Q \mu_\pi [-\zeta_5(1+2\beta) + \zeta_6(1+\beta)] \\
&- \psi_{10} m_Q \mu_\pi [3\zeta_5(1+\beta) - 4\zeta_6] - m_Q \mu_\pi (3\zeta_5(1+\beta) - 4\zeta_6) ln(\frac{m_Q^2}{s}) \Bigg] \\
&+ 6 f_\pi m_Q^2 (1+\beta)[2\psi_{10} - \psi_{20} + \psi_{31} + 2ln(\frac{m_Q^2}{s}) \varphi_\pi(u_0) \\
&+ 2(\psi_{20} - \psi_{31}) m_Q \mu_\pi (1+2\beta) \varphi_\sigma(u_0) \Bigg\} \Bigg], \quad (14)
\end{aligned}$$

and

$$\begin{aligned}
\Gamma &= \frac{m_0^2}{192\sqrt{6}} (<\overline{d}d> + <\overline{u}u>) \Bigg[\frac{2f_\pi}{9} [-11 - 17\beta + (7+\beta)](\beta-1) \varphi_\pi(u_0) \\
&- \frac{8m_Q^2}{3M^2} m_Q \mu_\pi (\beta^2 + \beta + 1) \varphi_\sigma(u_0) \\
&- \frac{4m_Q}{9M^2} \Bigg\{ 9 f_\pi m_Q (\beta-1) \beta \varphi_\pi(u_0) - \mu_\pi (3\beta^2 + 2\beta + 3) \varphi_\sigma(u_0) \Bigg\} \Bigg] \\
&+ \frac{1}{6\sqrt{6}} (<\overline{d}d> + <\overline{u}u>) m_Q \mu_\pi (\beta^2 + \beta + 1) \varphi_\sigma(u_0). \quad (15)
\end{aligned}$$

The other functions entering Eqs. (14-15) are given as

$$\zeta_j = \int \mathcal{D}\alpha_i \int_0^1 dv f_j(\alpha_i) \delta(\alpha_q + v\alpha_g - u_0),$$

$$\zeta_j' = \int \mathcal{D}\alpha_i \int_0^1 dv g_j(\alpha_i)\delta'(\alpha_q + v\alpha_g - u_0),$$
$$\psi_{nm} = \frac{(s-{m_Q}^2)^n}{s^m (m_Q^2)^{n-m}}, \qquad (16)$$

and $f_1(\alpha_i) = \mathcal{V}_{\parallel}(\alpha_i),\ f_2(\alpha_i) = v\mathcal{V}_{\parallel}(\alpha_i),\ f_3(\alpha_i) = \mathcal{V}_{\perp}(\alpha_i),\ f_4(\alpha_i) = v\mathcal{V}_{\perp}(\alpha_i)$, $g_1(\alpha_i) = \mathcal{T}(\alpha_i)$ and $g_2(\alpha_i) = v\mathcal{T}(\alpha_i)$ are the pion distribution amplitudes. Note that, in the above equations, the Borel parameter M^2 is defined as $M^2 = \frac{M_1^2 M_2^2}{M_1^2+M_2^2}$ and $u_0 = \frac{M_1^2}{M_1^2+M_2^2}$. Since the mass of the initial and final baryons are close to each other, we can set $M_1^2 = M_2^2 = 2M^2$ and $u_0 = \frac{1}{2}$. The contributions of the terms $\sim < G^2 >$ are also calculated, but their numerical values are very small and therefore for customary in the expressions these terms are omitted.

For calculation of the coupling constants of the considered baryons, their residues, $\lambda_{\Sigma_Q(\Lambda_Q)}$ are needed. Their expressions are obtained as:

$$-\lambda^2_{\Sigma_Q(\Lambda_Q)} e^{-m^2_{\Sigma_Q(\Lambda_Q)}/M^2} = \int_{m_Q^2}^{s_0} e^{\frac{-s}{M^2}} \rho_{1(2)}(s) ds + e^{\frac{-m_Q^2}{M^2}} \Gamma_{1(2)}, \qquad (17)$$

with

$$\begin{aligned}
\rho_1(s) &= (<\bar{d}d> + <\bar{u}u>)\frac{(\beta^2-1)}{64\pi^2}\Big\{\frac{m_0^2}{4m_Q}(6\psi_{00} - 13\psi_{02} - 6\psi_{11}) \\
&+ 3m_Q(2\psi_{10} - \psi_{11} - \psi_{12} + 2\psi_{21})\Big\} \\
&+ \frac{m_Q^4}{2048\pi^4}[5 + \beta(2+5\beta)][12\psi_{10} - 6\psi_{20} + 2\psi_{30} - 4\psi_{41} + \psi_{42} - 12ln(\frac{s}{m_Q^2})],
\end{aligned} \qquad (18)$$

$$\begin{aligned}
\rho_2(s) &= (<\bar{d}d> + <\bar{u}u>)\frac{(\beta-1)}{192\pi^2}\Big\{\frac{m_0^2}{4m_Q}[6(1+\beta)\psi_{00} - (7+11\beta)\psi_{02} \\
&- 6(1+\beta)\psi_{11}] + (1+5\beta)m_Q(2\psi_{10} - \psi_{11} - \psi_{12} + 2\psi_{21})\Big\}
\end{aligned}$$

$$+ \quad \frac{m_Q^4}{2048\pi^4}[5+\beta(2+5\beta)][12\psi_{10} - 6\psi_{20} + 2\psi_{30} - 4\psi_{41} + \psi_{42} - 12ln(\frac{s}{m_Q^2})], \tag{19}$$

$$\begin{aligned}
\Gamma_1 &= \frac{(\beta-1)^2}{24} < \overline{d}d >< \overline{u}u > \left[\frac{m_Q^2 m_0^2}{2M^4} + \frac{m_0^2}{4M^2} - 1 \right], \\
\Gamma_2 &= \frac{(\beta-1)}{72} < \overline{d}d >< \overline{u}u > \left[\frac{m_Q^2 m_0^2}{2M^4}(13+11\beta) \right. \\
&+ \left. \frac{m_0^2}{4M^2}(25+23\beta) - (13+11\beta) \right].
\end{aligned} \tag{20}$$

3 Numerical analysis

This section is devoted to the numerical analysis for the coupling constant $g_{\Sigma_Q \Lambda_Q \pi}$ and calculation of the total decay width for $\Sigma_Q \longrightarrow \Lambda_Q \pi$. The input parameters used in the analysis of the sum rules are $\langle \bar{u}u \rangle(1\ GeV) = \langle \bar{d}d \rangle(1\, GeV) = -(0.243)^3\ GeV^3$, ß$(1\, GeV) = 0.8\langle \bar{u}u \rangle(1\, GeV)$, $m_b = 4.7\, GeV$, $m_c = 1.23\ GeV$, $m_{\Sigma_b} = 5.805\ GeV$, $m_{\Sigma_c} = 2.439\ GeV$, $m_{\Lambda_b} = 5.622\ GeV$, $m_{\Lambda_c} = 2.297\ GeV$, and $m_0^2(1\ GeV) = (0.8 \pm 0.2)\ GeV^2$ [31]. From the sum rules for coupling constant, it is clear that the π-meson wave functions are needed. These wave functions are given as [29, 30]

$$\begin{aligned}
\phi_\pi(u) &= 6u\bar{u}\left(1 + a_1^\pi C_1(2u-1) + a_2^\pi C_2^{\frac{3}{2}}(2u-1)\right), \\
\mathcal{T}(\alpha_i) &= 360\eta_3 \alpha_{\bar{q}} \alpha_q \alpha_g^2 \left(1 + w_3 \frac{1}{2}(7\alpha_g - 3)\right), \\
\phi_P(u) &= 1 + \left(30\eta_3 - \frac{5}{2}\frac{1}{\mu_\pi^2}\right) C_2^{\frac{1}{2}}(2u-1) \\
&+ \left(-3\eta_3 w_3 - \frac{27}{20}\frac{1}{\mu_\pi^2} - \frac{81}{10}\frac{1}{\mu_\pi^2} a_2^\pi\right) C_4^{\frac{1}{2}}(2u-1), \\
\phi_\sigma(u) &= 6u\bar{u}\left[1 + \left(5\eta_3 - \frac{1}{2}\eta_3 w_3 - \frac{7}{20}\mu_\pi^2 - \frac{3}{5}\mu_\pi^2 a_2^\pi\right) C_2^{\frac{3}{2}}(2u-1)\right], \\
\mathcal{V}_\parallel(\alpha_i) &= 120\alpha_q \alpha_{\bar{q}} \alpha_g \left(v_{00} + v_{10}(3\alpha_g - 1)\right),
\end{aligned}$$

$$
\begin{aligned}
\mathcal{A}_{\parallel}(\alpha_i) &= 120\alpha_q\alpha_{\bar{q}}\alpha_g\left(0 + a_{10}(\alpha_q - \alpha_{\bar{q}})\right), \\
\mathcal{V}_{\perp}(\alpha_i) &= -30\alpha_g^2\left[h_{00}(1-\alpha_g) + h_{01}(\alpha_g(1-\alpha_g) - 6\alpha_q\alpha_{\bar{q}}) + h_{10}(\alpha_g(1-\alpha_g) - \frac{3}{2}(\alpha_{\bar{q}}^2 + \alpha_q^2))\right], \\
\mathcal{A}_{\perp}(\alpha_i) &= 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q)\left[h_{00} + h_{01}\alpha_g + \frac{1}{2}h_{10}(5\alpha_g - 3)\right], \\
B(u) &= g_\pi(u) - \phi_\pi(u), \\
g_\pi(u) &= g_0 C_0^{\frac{1}{2}}(2u-1) + g_2 C_2^{\frac{1}{2}}(2u-1) + g_4 C_4^{\frac{1}{2}}(2u-1), \\
\mathbb{A}(u) &= 6u\bar{u}\left[\frac{16}{15} + \frac{24}{35}a_2^\pi + 20\eta_3 + \frac{20}{9}\eta_4 + \left(-\frac{1}{15} + \frac{1}{16} - \frac{7}{27}\eta_3 w_3 - \frac{10}{27}\eta_4\right)C_2^{\frac{3}{2}}(2u-1)\right. \\
&+ \left(-\frac{11}{210}a_2^\pi - \frac{4}{135}\eta_3 w_3\right)C_4^{\frac{3}{2}}(2u-1)\Bigg] \\
&+ \left(-\frac{18}{5}a_2^\pi + 21\eta_4 w_4\right)\left[2u^3(10 - 15u + 6u^2)\ln u\right. \\
&+ \left.2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln\bar{u} + u\bar{u}(2 + 13u\bar{u})\right], \qquad (21)
\end{aligned}
$$

where $C_n^k(x)$ are the Gegenbauer polynomials,

$$
\begin{aligned}
h_{00} &= v_{00} = -\frac{1}{3}\eta_4, \\
a_{10} &= \frac{21}{8}\eta_4 w_4 - \frac{9}{20}a_2^\pi, \\
v_{10} &= \frac{21}{8}\eta_4 w_4, \\
h_{01} &= \frac{7}{4}\eta_4 w_4 - \frac{3}{20}a_2^\pi, \\
h_{10} &= \frac{7}{4}\eta_4 w_4 + \frac{3}{20}a_2^\pi, \\
g_0 &= 1, \\
g_2 &= 1 + \frac{18}{7}a_2^\pi + 60\eta_3 + \frac{20}{3}\eta_4, \\
g_4 &= -\frac{9}{28}a_2^\pi - 6\eta_3 w_3. \qquad (22)
\end{aligned}
$$

The constants appearing in the wave functions are calculated at the renormalization scale $\mu = 1 \quad GeV^2$ and they are given as $a_1^\pi = 0$, $a_2^\pi = 0.44$, $\eta_3 = 0.015$, $\eta_4 = 10$, $w_3 = -3$ and $w_4 = 0.2$.

The sum rules for the coupling constant also contains three auxiliary parameters: Borel mass parameter M^2, continuum threshold s_0 and general parameter β enters the expressions of the interpolating currents. In principal, M^2 and β are completely arbitrary and hence the coupling constant, which is a physical observable, should be independent of their exact values. In practice, though, due to the approximations made in the calculations, there is a residual dependence of the predictions on these unphysical parameters. Hence, a range for these parameter should be found where the predictions are practically insensitive to variations of these parameters. To find the working region for M^2, we proceed as follows. The upper bound is obtained requiring that the contribution of the higher states and continuum should be less than that of the ground state. The lower bound of M^2 is determined from condition that the highest power of $1/M^2$ be less than say $30^0/_0$ of the highest power of M^2. These two conditions are both satisfied in the region $15\ GeV^2 \leq M^2 \leq 30\ GeV^2$ and $4\ GeV^2 \leq M^2 \leq 10\ GeV^2$ for baryons containing b and c-quark, respectively. The third parameter, s_0 has a physical meaning, and it should have a value near the first excited state. The value of the continuum threshold is calculated from the two-point sum rules. We choose the interval $s_0 = (6.0^2 - 6.2^2)\ \ GeV^2$ and $s_0 = (2.5^2 - 2.7^2)\ \ GeV^2$ for baryons containing the b and c quark, respectively.

In Figs. 1 and 2, we present the dependence of the coupling constants $g_{\Sigma_b\Lambda_b\pi}$ and $g_{\Sigma_c\Lambda_c\pi}$, at fixed values of the continuum threshold s_0 and the general parameter β. From these figures, we see a good stability for coupling constants $g_{\Sigma_b\Lambda_b\pi}$ and $g_{\Sigma_c\Lambda_c\pi}$ with respect to the Borel mass square M^2 in the working region. The next step is to determine the working region for auxiliary parameter β. For this aim, in Figs. 3 and 4, we depict the dependence of the coupling constants $g_{\Sigma_b\Lambda_b\pi}$ and $g_{\Sigma_c\Lambda_c\pi}$ on $cos\theta$ where $tan\theta = \beta$, at two fixed values of M^2. From these Figures, we see that the best stability for the coupling constants $g_{\Sigma_b \Lambda_b \pi}$ and $g_{\Sigma_c \Lambda_c \pi}$ is in the region $-0.5 \leq cos\theta \leq 0.2$.

Our final results on coupling constants $g_{\Sigma_b \Lambda_b \pi}$ and $g_{\Sigma_c \Lambda_c \pi}$ are:

$$
\begin{aligned}
g_{\Sigma_b \Lambda_b \pi} &= 23.5 \pm 4.9, \\
g_{\Sigma_c \Lambda_c \pi} &= 10.8 \pm 2.2.
\end{aligned}
\tag{23}
$$

The quoted errors are due to the uncertainties in the input parameters as well as variation of the Borel parameter M^2, continuum threshold s_0 and general parameter β.

Having computed the coupling constant $g_{\Sigma_Q \Lambda_Q \pi}$, the next step is to calculate the total decay width for $\Sigma_b \longrightarrow \Lambda_b \pi$ and $\Sigma_c \longrightarrow \Lambda_c \pi$ decays. From Eq. (4) the transition amplitude is $M = g_{\Sigma_Q \Lambda_Q \pi} \overline{u} i \gamma_5 u$ and the differential decay width is found in terms of the coupling constant as:

$$
\Gamma = \frac{|g_{\Sigma_Q \Lambda_Q \pi}|^2}{8\pi m_{\Sigma_Q}^2} (m_{\Sigma_Q} - m_{\Lambda_Q})^2 |\overrightarrow{q}|,
\tag{24}
$$

where $|\overrightarrow{q}| = (m_{\Sigma_Q}^2 - m_{\Lambda_Q}^2)/2m_{\Sigma_Q}$. The numerical values of the decay rates are given in Table 1. In order to compare with the predictions of other methods, in the same table, we present the predictions of the relativistic three-quark model (RTQM) [24], light-front quark model (LFQM) [25] and existing experimental data [32]. This table depicts a good consistency among the methods and the experimental data in order of magnitudes for charm case. Note that, due to the isospin symmetry the decays of different charge $\Sigma_c^{++,+} \longrightarrow \Lambda_c^+ \pi^{+,0,-} (\Sigma_b^{+,0,-} \longrightarrow \Lambda_b \pi^{+,0,-})$ have the same decay widths. Experimentally, only the widths for $\Sigma_c^{++,0} \longrightarrow \Lambda_c^+ \pi^{+,-}$ are measured and the value in the table is their average. Only the upper bound for the

$\Sigma_c^+ \longrightarrow \Lambda_c^+ \pi^0$ is known and it is consistent with other decay modes. Our prediction for the decay rate of the bottom case can be tested in the future experiments.

	$\Gamma(\Sigma_c \longrightarrow \Lambda_c \pi)$	$\Gamma(\Sigma_b \longrightarrow \Lambda_b \pi)$
Present work	2.16 ± 0.85	3.93 ± 1.5
RTQM [24]	3.63 ± 0.27	-
LFQM [25]	1.555 ± 0.165	-
Exp. [32]	2.21 ± 0.40	-

Table 1: Results for the decay rates of $\Sigma_Q \longrightarrow \Lambda_Q \pi$ in different approaches in MeV.

In summary, we calculated the $g_{\Sigma_b \Lambda_b \pi}$ and $g_{\Sigma_c \Lambda_c \pi}$ coupling constants in the light cone QCD sum rules approach. Using these coupling constants, we also evaluated the total decay width for the strong $\Sigma_b \longrightarrow \Lambda_b \pi$ and $\Sigma_c \longrightarrow \Lambda_c \pi$ decays and compared with the predictions of the other approaches and existing experimental data.

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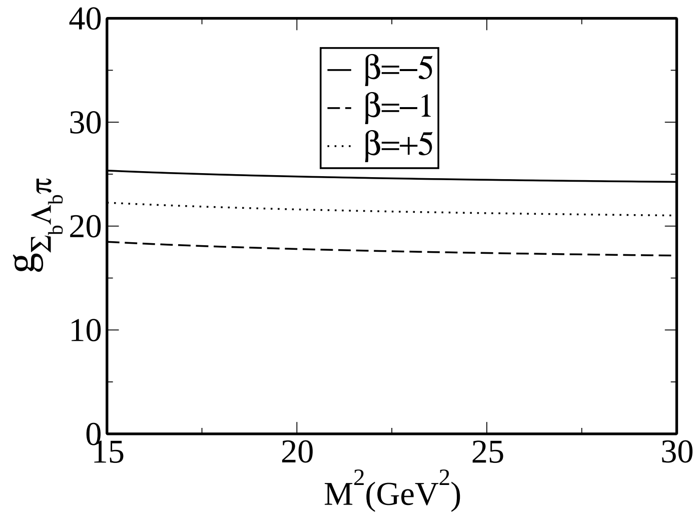

Figure 1: The dependence of the $g_{\Sigma_b \Lambda_b \pi}$ on the Borel parameter M^2 at fixed value of the continuum threshold $s_0 = 6.0^2$.

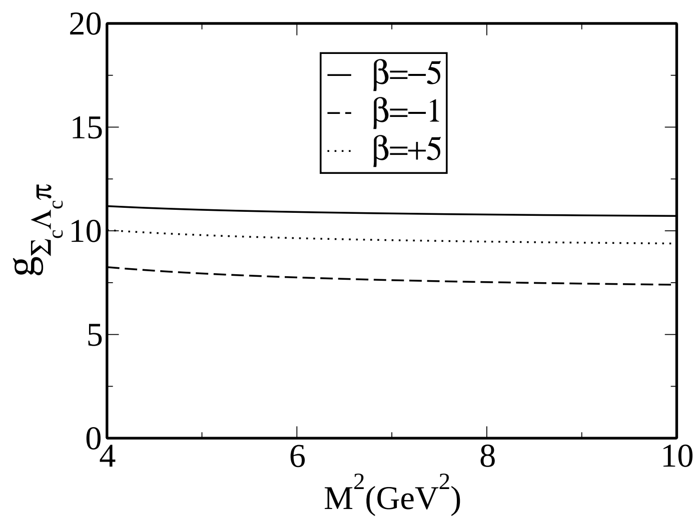

Figure 2: The same as Fig. 1, but for $g_{\Sigma_c \Lambda_c \pi}$ and fixed value of the continuum threshold $s_0 = 2.5^2$.

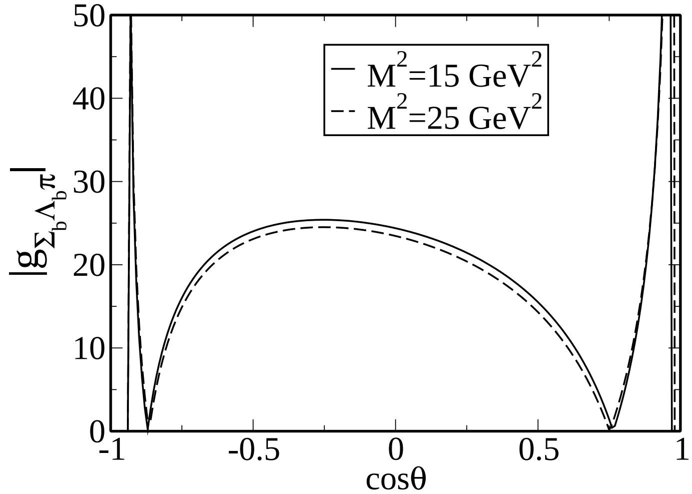

Figure 3: The dependence of $|g_{\Sigma_b \Lambda_b \pi}|$ on $cos\theta$ at fixed value of the continuum threshold $s_0 = 6.0^2$.

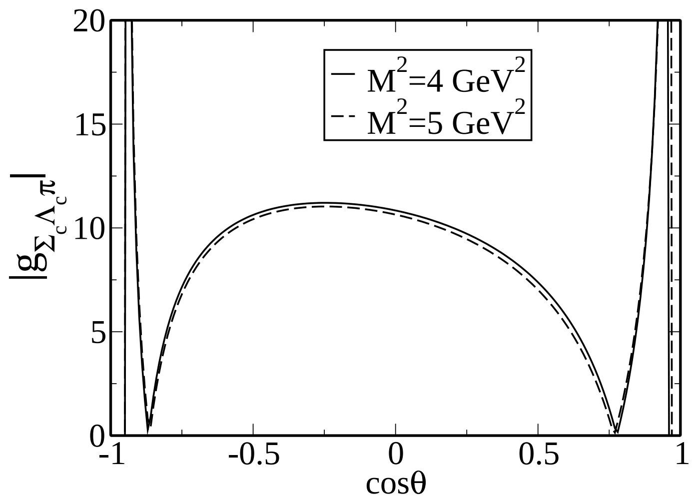

Figure 4: The same as Fig. 3, but for $|g_{\Sigma_c \Lambda_c \pi}|$ and fixed value of the continuum threshold $s_0 = 2.5^2$.