

# Kaluza-Klein Monopole in AdS Spacetime

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## Abstract

We construct analogs of the flat space Kaluza-Klein (KK) monopoles in locally Anti-de Sitter (AdS) spaces for  $D \geq 5 + 1$ . We show that, unlike the flat space KK monopole, there is no five dimensional static KK monopole in AdS that smoothly reduces to the flat space one as the cosmological constant goes to zero. Thus, one needs at least two extra dimensions, one of which is compact, to get a static KK monopole in cosmological backgrounds.

Keywords: Kaluza-Klein Monopole, Solitons, AdS

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# 1 Introduction

Gravitational solitons are topologically stable, everywhere smooth, particle-like solutions of pure gravity theories. Not all dimensions accommodate these spacetimes. For example,  $D = 4$  General Relativity without a cosmological constant does not have solitons and thus, globally flat (Lorentzian)  $R^{(3,1)}$  is the unique singularity free solution. This no-go result in four dimensions persists even if a cosmological constant ( $\Lambda$ ) is added to the Einstein's theory. As shown in [1], if the soliton is required to be asymptotically AdS ( $\Lambda < 0$ ), then globally AdS spacetime is the unique, static, non-singular solution. For de-Sitter ( $\Lambda > 0$ ) case, the cosmological horizon complicates a rigorous proof, but a similar no-soliton result is expected to hold [1].

Non-trivial soliton solutions can be found if either one or both of the following requirements are relaxed:

- a) the spacetime is four dimensional,
- b) the solutions are asymptotically flat or asymptotically AdS.

In fact, adding an extra compact spatial dimension leads to many solitons one of which is the well-known Kaluza-Klein monopole of Sorkin, Gross and Perry [2, 3]. This is a  $4 + 1$   $D$ , static, Ricci-flat space-time that looks exactly like a Dirac monopole to a  $3 + 1$   $D$  'effective' observer who cannot see the compact extra dimension. Magnetic field of the KK monopole spreads radially in the  $3D$  space. Once the size of the extra dimension is fixed, to get the proper  $3 + 1$   $D$  gravity plus Maxwell's Electromagnetic theory, monopole's *inertial mass* is determined to be about three times the Planck mass ( $6 \times 10^{-5}$  g) [3]. In spite of its finite mass, the monopole does not exert any gravitational force on massive neutral test particles. It only interacts with the moving charged particles, as a monopole should do. Also, in asymptotically *locally* AdS spacetimes (as opposed to condition (b)), a non-trivial solution (the AdS soliton) was found more recently by Horowitz and Myers [4]. The AdS soliton, which exists even in four dimensions, is conjectured to have the minimum (negative) energy among all the asymptotically locally AdS spacetimes. The uniqueness of the AdS soliton as the lowest energy configuration provided support for a new 'positive energy conjecture' in gravity [5].

Although the Ricci flat KK monopoles are well studied both in pure gravity theories and in string/M-theory [6], analogous solutions in cosmological backgrounds have not been constructed before. In this paper, we show that in the  $D = 4 + 1$  cosmological Einstein theory, such 'AdS

KK monopole' solutions that smoothly goes to the flat space KK monopole as  $\Lambda \rightarrow 0$  do not exist. [Here, we insist on recovering the flat space solutions as  $\Lambda \rightarrow 0$ . Since we are specifically looking for the 'KK monopole' in a background with negative cosmological constant, we should obtain the flat monopole metric in this limit. ] However, for  $D \geq 5 + 1$ , we find static KK monopole-like solitons with asymptotically locally AdS geometries. Given the recent interest in the AdS spacetime both in the context of AdS/CFT duality and the brane world scenarios, it seems proper to study monopoles in this background.

The outline of the paper is as follows: In Sec. 2, we review the flat space KK monopole and show that there is no static monopole solution in  $5D$  cosmological backgrounds. In Sec. 3, we construct a KK monopole in  $D = 6$  asymptotically locally AdS background; find another new soliton in this dimension and interpret it as a brane-world. We conclude with Sec 4.

## 2 KK monopole and the no-go result in $5D$ AdS

First, let us review the essence of the Ricci-flat KK monopole construction [2, 3], and try to formulate a recipe for the AdS case. Our conventions are:  $(-, +, +, \dots +)$  for the signature,  $[\nabla_M, \nabla_N]V_L = R_{MNL}{}^S V_S$ ,  $R_{MN} \equiv R_{MLN}{}^L$  for the Riemann and Ricci curvatures, respectively. The flat KK monopole was obtained by trivially adding a time direction to the four dimensional Taub-NUT (TN) space:

$$ds^2 = -dt^2 + ds_{\text{TN}}^2, \quad (1)$$

where

$$ds_{\text{TN}}^2 = V^{-1}(r) \left( dx^5 + \vec{A} \cdot d\vec{r} \right)^2 + V(r) d\vec{r} \cdot d\vec{r}. \quad (2)$$

TN metric is a gravitational instanton: a Euclidean, Ricci-flat metric that solves the  $D = 4$  self-dual Einstein's equations which reduce to a first order BPS-like equation:

$$\vec{\nabla} \times \vec{A} = \pm \vec{\nabla} V(r). \quad (3)$$

Since the time direction is flat, the  $5D$  metric is also Ricci flat. The metric has a time-like Killing vector, along with  $g_{0i} = 0$ . The BPS equation implies that  $V(r)$  satisfies the harmonic equation on  $R^3$ . Charge-one monopole solution of (3) with a  $\delta$ -function singularity is:

$$V = 1 + \frac{4M}{r}. \quad (4)$$

Thus, the vector potential  $\vec{A}$  and the magnetic field  $\vec{B}(r)$  are:

$$\vec{A}(r, \theta) = \frac{4M(1 - \cos \theta)}{r \sin \theta} \hat{\phi}, \quad \vec{B}(r) = \frac{4M\hat{r}}{r^2}. \quad (5)$$

Metric (2) becomes regular everywhere if  $x^5$  is compactified on a circle with radius  $8M$ . This solution cannot decay to the trivial vacuum (zero charge,  $M = 0$  case) because of its non-trivial topological structure with 1 unit of Euler character ( $\chi_E$ ). For our purposes, it is important to notice that in the zero magnetic charge limit, as  $M \rightarrow 0$ ,  $V(r) \rightarrow 1$  and  $\vec{A}(r) \rightarrow 0$ . Thus, flat KK monopole metric (1) becomes the  $5D$  Minkowski metric:

$$ds^2 = -dt^2 + dx_4^2 + d\vec{r} \cdot d\vec{r}. \quad (6)$$

Therefore, the interpretation is that, when the charge  $M$  is turned on in a  $5D$  Minkowski space-time, the monopole curves the background so that the metric takes the form of (1).

Multi-monopole solutions with equal charges (both in sign and in magnitude) located at various points can also be obtained, since there are no forces between these BPS objects [7]. In [8], it was shown that ‘KK vortices’, with magnetic fields trapped effectively in  $2D$ , can be constructed by summing up infinitely many KK monopoles on a line. Also, by adding 5 or 6 more spatial flat directions to the metric, one gets a  $D5$  or  $D6$  brane which is a BPS solution to string or M theory, respectively [6].

How can one generalize the above construction to the spaces with constant non-zero curvature? First, there is the issue of the sign of the cosmological constant. Since the spatial hypersurfaces of de-Sitter space (dS) are compact (with no boundary), one should not expect a single monopole solution in dS. Rather, magnetic dipole-like totally neutral solutions might exist. [Related is the issue of the non-existence of a global time-like Killing vector in the relevant solutions in dS.] On the other hand, in AdS which has open hypersurfaces, and hence a boundary to define flux, one expects monopole solutions. However, this expectation will turn out to be wrong in  $D = 5$ . Unlike the flat space case, for negative cosmological backgrounds,  $D = 6$  is the minimal number of dimensions where a monopole-like solution exists for pure gravity equations.

Now, let us adapt the essential ingredients of the above Ricci flat construction to AdS. The first thing we need is a proper background metric of a spacetime with a negative cosmological constant  $\Lambda$  (i.e. the zero charge monopole). Needless to say that, in this case, the time direction

cannot simply be added to a four dimensional gravitational instanton such as the Taub-NUT or the AdS-Taub-NUT. It has to be a warped product metric. For  $\Lambda := -2L^2 < 0$ , Einstein spaces are solutions of the equations of motion:

$$R_{MN} = -2L^2 g_{MN} . \quad (7)$$

The usual maximally symmetric solution of (7) in  $5D$  is  $AdS_{4+1}$  spacetime:

$$ds^2 = -\cosh^2(Lr/\sqrt{2})dt^2 + dr^2 + (2/L^2)\sinh^2(Lr/\sqrt{2})d\Omega_3 , \quad (8)$$

where the metric on  $S^3$  can be (up to gauge invariance) taken as the usual  $S^2$  foliation form  $d\Omega_3 = d\psi^2 + \sin^2\psi(d\theta^2 + \sin^2\theta d\phi^2)$  or the Clifford torus ( $S^1 \times S^1$ ) foliation form  $d\Omega_3 = d\psi^2 + \cos^2\psi d\theta^2 + \sin^2\psi d\phi^2$ . Here also notice that, when the cosmological constant is turned off, the above metric smoothly becomes the  $5D$  Minkowski metric. But, it is clear that the metric (8) cannot be deformed to an effective  $3+1D$  spacetime whose spatial part is  $AdS_3$  that supports the KK monopole with spherically symmetric magnetic field. In fact, the proper background should be of the  $AdS_2 \times AdS_3$  form with the line element:

$$ds^2 = -\frac{1}{2}\exp(-2\sqrt{2}Lx^5)dt^2 + dx_5^2 + dr^2 + \frac{1}{L^2}\sinh^2(Lr)d\Omega_2 , \quad (9)$$

where  $d\Omega_2 = d\theta^2 + \sin^2\theta d\phi^2$ . To make our choice of background more transparent, let us recall that the background (as well as the soliton) has to be an Einstein space. The first candidate ( the maximally symmetric one (8) ), as argued above, does not work: one cannot compactify one of the  $\psi$ ,  $\theta$  or  $\phi$  coordinates on an *arbitrary* circle, which is needed for the KK interpretation of the metric. For the metric (8) to be Einstein,  $\psi$ ,  $\theta$ ,  $\phi$  coordinates all take values on *fixed* radii circles, rendering the metric too 'rigid' to accommodate a KK monopole. One, then, has to find another Einstein space. For several other reasons laid out below, the metric (9) is the suitable one.

Analogous to the flat KK monopole, the sought-after soliton should yield the above metric in the limit of zero charge (as  $M \rightarrow 0$ ). From (9) one can immediately anticipate the form of the magnetic field  $\vec{B}$ . Due to the flux conservation, the magnetic field seen by an observer who lives in the effective  $3+1D$  spacetime, should be

$$\vec{B} = \frac{4ML^2\hat{r}}{\sinh^2(Lr)} . \quad (10)$$

This is the field of a 3D Abelian ‘hyperbolic monopole’ [9, 10] (instead of an  $R^3$  monopole of the flat KK monopole metric). Hence the total magnetic flux  $\Phi =: \int \vec{B} \cdot d\vec{s}$  of the hyperbolic monopole (10) on a large sphere (for fixed  $t, x^5$ ) is the same as the flat space case:  $\Phi = 16\pi M$ . With the definition,  $B := F = dA$ , the gauge potential 1-form retains its flat space form:  $A = 4M(1 - \cos\theta) d\phi$ . We also require that as  $\Lambda \rightarrow 0$ , the AdS monopole reduces to the flat KK monopole. This is because, the solution we are looking for is the ‘KK-monopole’ in a background with negative cosmological constant, as we turn  $\Lambda$  off, we should obtain flat KK monopole metric (1).

Bearing these constraints in mind, we write down the generic, static ansatz which might solve the equations of motion:

$$ds^2 = -\frac{1}{2}a^2(r, x^5) \exp(-2\sqrt{2}Lx^5) dt^2 + b^2(r, x^5) [dx^5 + 4M(1 - \cos\theta) d\phi]^2 + v^2(r, x^5) [dr^2 + (1/L^2) \sinh^2(Lr) d\Omega_2] . \quad (11)$$

For the zero magnetic charge limit ( $M = 0$ ), we require  $a(r, x^5) = b(r, x^5) = v(r, x^5) = 1$ . For  $L = 0$ , the solution should yield:

$$a^2(r, x^5) = 2 \quad \text{and} \quad b^{-1}(r, x^5) = v(r, x^5) = 1 + \frac{4M}{r} . \quad (12)$$

We claim that there are no non-trivial monopole-like solutions of the form (11) with the desired properties laid out above. The easiest way to show this, is to work directly with the equations of motion. We only need to look at the simplest components of the Ricci tensor, the ones that are supposed to vanish. Let us start with  $R_{\theta x^5}$ :

$$R_{\theta x^5} = \frac{8M^2 L^2 b^2(r, x^5)}{v^2(r, x^5) \sinh^2(Lr)} \frac{(\cos\theta - 1)}{\sin\theta} \left[ \sqrt{2}L - 3 \frac{b'(r, x^5)}{b(r, x^5)} - \frac{a'(r, x^5)}{a(r, x^5)} + \frac{v'(r, x^5)}{v(r, x^5)} \right] , \quad (13)$$

where  $'$  denotes  $\partial_{x^5}$ . For non-vanishing  $M$  and  $L$ , setting  $R_{\theta x^5} = 0$ , we find:

$$b(r, x^5) = \left[ \frac{v(r, x^5)}{a(r, x^5)} \right]^{1/3} f(r) \exp(\sqrt{2}Lx^5/3) , \quad (14)$$

where  $f(r)$  is an arbitrary function. Plugging (14) into  $R_{\theta\phi}$  yields:

$$R_{\theta\phi} = \frac{4M}{3} \frac{(\cos\theta - 1)^2}{\sin\theta} \left[ \frac{a'(r, x^5)}{a(r, x^5)} + 2 \frac{v'(r, x^5)}{v(r, x^5)} - \sqrt{2}L \right] . \quad (15)$$

Then, the equation  $R_{\theta\phi} = 0$  is solved by:

$$v^2(r, x^5) = \frac{g(r)}{a(r, x^5)} \exp(\sqrt{2}Lx^5), \quad (16)$$

where again  $g(r)$  is arbitrary. Using (14, 16),  $R_{rx^5}$  is simplified as:

$$R_{rx^5} = \frac{3}{2} \frac{\partial_r a(r, x^5)}{a^2(r, x^5)} \left[ \sqrt{2}La(r, x^5) - a'(r, x^5) \right]. \quad (17)$$

For the metric to be Einstein we set  $R_{rx^5} = 0$ , leading to:

$$a(r, x^5) = h(r) \exp(\sqrt{2}Lx^5) \quad \text{or}, \quad (18)$$

$$a(r, x^5) = a(x^5). \quad (19)$$

Hence, collecting everything, one can write the final metric for the case (18) as:

$$ds^2 = -\frac{1}{2}h^2(r) dt^2 + \frac{g(r)}{h(r)} [dr^2 + (1/L^2) \sinh^2(Lr) d\Omega_2] + \frac{f^2(r)g^{1/3}(r)}{h(r)} [dx^5 + 4M(1 - \cos\theta) d\phi]^2. \quad (20)$$

Observe that the  $x^5$  dependence is completely dropped out. Therefore, it is clear that  $M = 0$  does not yield the background metric (9). The remaining case (19) also fails in recovering the background metric in the certain limit. Thus, there is no  $5D$  KK monopole in AdS space that limits to the flat space KK monopole as  $\Lambda \rightarrow 0$ . Here, although we have shown the non-existence of KK monopoles in AdS spacetime under some physical requirements in the corresponding limits, we have not ruled out the existence of any other type of solitons that might live in cosmological backgrounds.

### 3 6D AdS monopoles

In the previous section, we have seen that we cannot embed the  $3D$  hyperbolic monopole in a  $5D$  cosmological space. The main problem was that the time direction could not simply be added to the spatial part. On the other hand, as we show below, in  $D \geq 5 + 1$  cosmological spacetimes, one can construct solitons which resemble the flat space KK monopoles.

For monopole solutions, the need for at least two extra dimensions in the cosmological case should not be surprising. In fact, the only other non-trivial soliton we know for  $\Lambda < 0$  -the

AdS soliton, has to be at least 6-dimensional if 3-dimensional spherical symmetry is imposed [4]. Indeed, the KK monopole we are searching for, is also spherically symmetric. [ Note that in the metric, one breaks the spherical symmetry by choosing a gauge for the gauge potential  $\vec{A}$ . But, physically, the spacetime is spherically symmetric. ]

To construct a  $D = 6$  soliton with a  $3D$  magnetic field of the form (10), for the reasons discussed in the previous section, we take the following  $AdS_2 \times AdS_4$  background:

$$ds^2 = -\exp(-2Lx^6)dt^2 + dx_6^2 + \frac{3}{2} [dr^2 + (1/L^2) \sinh^2(rL)d\Omega_2 + \cosh^2(rL)dx_5^2] , \quad (21)$$

and deform it to:

$$ds^2 = -\exp(-2Lx^6)dt^2 + dx_6^2 + \frac{3}{2} H^2(r) \left\{ V(r) (dr^2 + (1/L^2) \sinh^2(rL)d\Omega_2) + \frac{\cosh^2(rL)}{V(r)} [dx^5 + 4M(1 - \cos\theta) d\phi]^2 \right\} . \quad (22)$$

This metric is the most general one that meets our requirements for various limits. The non-trivial  $4D$  part of the metric, with  $S^3$  foliations instead of  $AdS_3$ , was studied before by Pedersen [11]. Staticity, ‘spherical symmetry’ and diffeomorphism invariance leave two independent functions in the metric, best parametrized as above. Spherical symmetry is obvious in the curvature invariants of the metric (the only physical quantities), since the non vanishing ones depend only on  $r$ . For various limits, we should get:

$$M = 0 \rightarrow H(r) = V(r) = 1 , \quad \Lambda = 0 \rightarrow H(r) = 1 , \quad V(r) = 1 + \frac{4M}{r} . \quad (23)$$

Equations of motion can be simplified by observing that for non-zero  $\Lambda$ ,  $r \rightarrow \sinh(rL)/L$  in the magnetic field. Therefore, one can take the ansatz:

$$V(r) = 1 + \frac{4ML}{\sinh(rL)} . \quad (24)$$

Then the remaining function  $H(r)$  can be determined from the equations of motion:

$$H(r) = \frac{1}{1 - 4ML \sinh(rL)} . \quad (25)$$

Thus, the  $D = 6$  AdS KK monopole metric is:

$$\begin{aligned}
ds^2 = & -\exp(-2Lx^6)dt^2 + dx_6^2 \\
& + \frac{3/2}{(1 - 4ML \sinh(rL))^2} \left\{ \left[ 1 + \frac{4ML}{\sinh(rL)} \right] (dr^2 + (1/L^2) \sinh^2(rL) d\Omega_2^2) \right. \\
& \left. + \cosh^2(rL) \left[ 1 + \frac{4ML}{\sinh(rL)} \right]^{-1} [dx^5 + 4M(1 - \cos \theta) d\phi]^2 \right\}. \tag{26}
\end{aligned}$$

The monopole is located at  $r = 0$ . Note that as  $r \rightarrow 0$ , the  $4D$  part of the metric reduces to the singular flat TN space given in (2). Again, to get rid of the singularity at the origin,  $x^5$  should be compactified on a circle with radius  $8M$ .

Because of its direct product structure, there are two disconnected boundaries of the above metric: One at  $x^6 = -\infty$  and the other at  $Lr = \sinh^{-1}(1/4ML)$ . [ $x^6 = +\infty$  is a Killing horizon.] The first boundary is the usual-trivial boundary of  $AdS_2$ . In the  $r$ -coordinate, the latter boundary is at a finite distance but can be moved to ‘infinity’ by transforming the metric into new coordinates  $\tilde{r} := \int dr H(r) \sqrt{V(r)}$ . For  $r \in [0, (1/L) \sinh^{-1}(1/4ML)]$ ,  $\tilde{r} \in [0, \infty]$ . Only ‘light’ can travel to the boundary at a finite time. Strictly speaking, for  $4D$  part alone, there is of course no light-like geodesic. But to understand the boundary let us set  $\phi = \theta = \text{const.}$  and set  $ds_{4D}^2 = 0$  to find the Euclidean time:  $ix^5 = \int_{\epsilon}^{(1/L) \sinh^{-1}(1/4ML)} dr V(r) / \cosh(rL) = \text{finite}$ , where  $\epsilon > 0$ . [Note that if the light starts its journey at  $r = 0$ , it will take an infinite time to reach the boundary, since  $r = 0$  is a Killing horizon.] Thus, the metric (26) describes a non-singular geodesically complete space. Another way to see the smoothness of the metric is to check whether curvature invariants are regular or not. It suffices to look at the  $4D$  part of the metric. Among the Riemann invariants [12, 13], apart from the obvious ones like the Ricci scalar,  $R_{\mu\nu}R^{\mu\nu}$  etc., the only non-vanishing invariants are the Weyl scalars:

$$\begin{aligned}
W_1 := & \frac{1}{8} C_{\mu\nu\sigma\rho} C^{\mu\nu\sigma\rho} = \frac{1}{8} C_{\mu\nu\sigma\rho}^* C^{\mu\nu\sigma\rho} \\
= & \frac{64M^2 L^6}{3} \left( \frac{1 - 4ML \sinh(Lr)}{4ML + \sinh(Lr)} \right)^6, \tag{27}
\end{aligned}$$

and

$$\begin{aligned}
W_2 := & -\frac{1}{16} C_{\mu\nu}{}^{\sigma\rho} C_{\sigma\rho}{}^{\lambda\eta} C_{\lambda\eta}{}^{\mu\nu} = -\frac{1}{16} C_{\mu\nu}^*{}^{\sigma\rho} C_{\sigma\rho}{}^{\tau\eta} C_{\tau\eta}{}^{\mu\nu} \\
= & \frac{512 M^3 L^9}{9} \left( \frac{-1 + 4ML \sinh(Lr)}{4ML + \sinh(Lr)} \right)^9, \tag{28}
\end{aligned}$$

where  $C_{\mu\nu\sigma\rho}$  is the Weyl tensor and  $C_{\mu\nu\sigma\rho}^* := \frac{1}{2}\epsilon_{\mu\nu\lambda\eta}C^{\lambda\eta}{}_{\sigma\rho}$  is its dual. None of the invariants are singular. Weyl scalars vanish at the infinity where  $Lr = \sinh^{-1}(1/4ML)$  in our coordinates. As a check of the calculation of the above invariants, let us note that  $\Lambda \rightarrow 0$  limit of the 4D part of (26) is the flat TN metric multiplied by 3/2 and has the following Weyl scalars:

$$W_1 = \frac{64}{3} \frac{M^2}{(4M+r)^6}, \quad (29)$$

$$W_2 = -\frac{512}{9} \frac{M^3}{(4M+r)^9}, \quad (30)$$

which are nothing but the  $\Lambda \rightarrow 0$  limits of (27) and (28) respectively.

Finally, let us say a few words about the energy of the metric (26). The gravitational energy of the flat space KK monopole was studied in [14, 15]. As is often the case for the gravitational systems with compact dimensions, a useful energy definition is rather tricky because of the complications in choosing a proper background metric with respect to which the energy should be defined. The total gravitational energy of a given metric makes sense, if it can be considered as a ‘particle’ or a ‘perturbation’ in a given background or vacuum. The background, by definition, solves the equations of motion everywhere and has zero energy. The topology of the background metric ought to be the same as the metric whose energy is measured. The flat space KK monopole is a perfectly smooth solution, and there is no other one with the same topology whose energy can be compared with the monopole. Therefore the KK monopole, considered as *the* vacuum, has trivially zero energy. We should note that in [14, 15], the chosen background metrics are *not* solutions everywhere. They are *only* asymptotic solutions. Our metric (26), being a smooth solution everywhere, has also zero energy. This being a rather boring result, one might, like [14, 15], relax the condition on the background and take an asymptotic solution as a background. Here, we do not go into the details but heuristically argue that the AdS KK monopole’s energy expression is similar to the flat space one. Following Abbott and Deser (AD) [16]<sup>3</sup>, let us choose  $\bar{g}_{\mu\nu}$  as the background for which  $V(r) = 1$  in (26), and  $h_{\mu\nu}$  as the perturbation part, defined as  $h_{\mu\nu} := g_{\mu\nu} - \bar{g}_{\mu\nu}$ . We do not drop the  $g_{x^5\phi}$  term: namely, the vector potential is taken to be *non-zero*, to keep the topology of the background the same as the KK monopole. Then,  $O(h)$  terms in the equations of motion constitute the background-conserved

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<sup>3</sup>See also [17]

energy momentum tensor

$$R_{\mu\nu}^L - \frac{1}{2}\bar{g}_{\mu\nu}R^L - 2\Lambda h_{\mu\nu} := T_{\mu\nu}(h), \quad (31)$$

where  $L$  denotes linearization. Given  $\bar{\xi}^\mu$  as a background Killing vector, an ordinarily conserved current can be constructed:  $\bar{\nabla}_\mu(\sqrt{-\bar{g}}T^{\mu\nu}\bar{\xi}_\nu) = \partial_\mu(\sqrt{-\bar{g}}T^{\mu\nu}\bar{\xi}_\nu)$ . Using this, we write the AD Killing energy, up to trivial numerical factors, as  $E = \int d^5x\sqrt{-\bar{g}}T^{00}\bar{\xi}_0$ . For the metric (26),  $\bar{\xi}^\nu = (-1, \vec{0})$ , thus  $E = \int d^5x\sqrt{-\bar{g}}T^{00}\bar{\xi}_0$ . Since the AdS KK monopole is an Einstein space,  $R^L = 0 \rightarrow R_{\mu\nu}^L = 2\Lambda h_{\mu\nu}$  and hence  $T^{00} = 0$  everywhere, except at  $r = 0$  where the derivatives give  $\delta$ -function singularities. Therefore, all the contributions to the energy volume integral come from the origin. We already know that the non-trivial part of the metric (26) reduces to the flat-space KK monopole metric around the origin. Thus, the AdS KK monopole has the same energy expression as the flat one, which was obtained to be  $M$  in [14, 15]. The above argument is heuristic and depended on the ‘volume’ integral of the energy expression. One could, in principle, turn the volume integrals for conserved charges (like energy) into surface integrals at infinity. This method should yield, albeit after a lengthy calculation, the same result as above.

### A 6D KK Monopole Brane-World

Having six dimensions at our disposal, we can construct more than one kind of solitons. Here, we relax our earlier condition that the low energy spacetime has a negative cosmological constant. Using the five dimensional flat Kaluza-Klein monopole, a six dimensional soliton can be given by the following line element:

$$ds^2 = \exp(2kLx^6) \{-dt^2 + ds_{\text{TN}}^2\} + dx_6^2, \quad (32)$$

which satisfies  $R_{MN} = -5k^2L^2g_{MN}$ . Multi-monopoles with equal charges, located at different spatial points can be obtained as in the flat space case.  $x^5$  direction has to be compact for the solution to be smooth but the  $x^6$  direction has an infinite extent. Generalization to  $D$  dimensions is straightforward: one can add more flat directions as long as they are multiplied by  $\exp(2kLx^6)$ .

Let us choose  $k = -1$  for definiteness. Then,  $x^6 = -\infty$  is the boundary of the space and  $x^6 = +\infty$  is the Killing horizon. In this form, all dimensions (except the compact  $x^5$  directions) are accessible to the low energy observers. To localize the gravity, and get an effective  $3 + 1D$  brane-world, we can cut and paste (32) into the Randall-Sundrum  $Z_2$  invariant form [18]:

$$ds^2 = \exp(-2L|x^6|) \{-dt^2 + ds_{\text{TN}}^2\} + dx_6^2. \quad (33)$$

This metric is not differentiable at the ‘origin’, where the brane ( $D4$  Taub-NUT brane ) is located. Physically,  $3 + 1D$  observers see a flat-space KK monopole. The metric (33) is a six dimensional brane world with gravity localized on a Ricci-flat brane.

## 4 Conclusions

We have shown that the five dimensional cosmological Einstein gravity (with a Lorentzian signature and a negative cosmological constant ) does not have Kaluza-Klein monopole type static soliton solutions. On the other hand, in  $D \geq 6$ , we have constructed analogs of the flat space KK monopoles which are asymptotically locally AdS space-times. It would be interesting to find out if these solutions preserve some supersymmetry and if they can be embedded in string or M-theory. In the presence of anti-symmetric tensor fields, in addition to the usual translational collective degrees of freedom, the flat space KK monopole ( $D5$  or  $D6$  brane ) can have dyonic deformations [19]. An extension of these ideas to the AdS KK monopole would also be worth studying.

In this paper, we have insisted the solitons to be static. If we relax this condition, we can find time-dependent  $4 + 1D$  or Euclidean  $5D$  solutions. One such (Euclidean) example with the Ricci-flat 4D slices would be:

$$ds^2 = dt^2 + \exp(-2tL) ds_{\text{TN}}^2 . \quad (34)$$

Another example, with negatively curved  $4D$  slices is:

$$ds^2 = -dt^2 + \frac{\sin^2(tL)}{(1 - 4ML \sinh(rL))^2} \left\{ \left[ 1 + \frac{4ML}{\sinh(rL)} \right] (dr^2 + (1/L^2) \sinh^2(rL) d\Omega_2^2) \right. \\ \left. + \cosh^2(rL) \left[ 1 + \frac{4ML}{\sinh(rL)} \right]^{-1} [dx^5 + 4M(1 - \cos \theta) d\phi]^2 \right\} . \quad (35)$$

Construction of multi-monopoles in AdS, which does not seem to be a straightforward task, is under current investigation.

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