

Diagonal and transition magnetic moments of negative parity heavy baryons in QCD sum rules

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Abstract

Diagonal and transition magnetic moments of the negative parity, spin-1/2 heavy baryons are studied in framework of the light cone QCD sum rules. By constructing the sum rules for different Lorentz structures, the unwanted contributions coming from negative (positive) to positive (negative) parity transitions are removed. It is obtained that the magnetic moments of all baryons, except Λ_b^0 , Σ_c^+ and $\Xi_c'^+$, are quite large. It is also found that the transition magnetic moments between neutral negative parity heavy Ξ_Q^0 and Ξ_Q^0 baryons are very small. Magnetic moments of the $\Sigma_Q \rightarrow \Lambda_Q$ and $\Xi_Q'^{\pm} \rightarrow \Xi_Q^{\pm}$ transitions are quite large and can be measured in further experiments.

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1 Introduction

Last decade was quite fruitful in the field of heavy baryon spectroscopy. At present, all baryons containing heavy charm and bottom quarks, except Ω_b^* baryon, as well as several heavy baryons with negative parity, have been observed by a number of collaborations (for a review see [1]). These progresses stimulated further theoretical studies, and experimental researches on heavy baryons at the existing facilities, especially at LHC.

Heavy baryons are well recognized to represent a rich “laboratory” for theoretical investigations. The properties of heavy baryons have been studied in framework of the various methods such as, relativistic quark model [2], variational approach [3], constituent quark model [4], lattice QCD model [5], QCD sum rules method [6]. Recent progress on this subject can be found in [7].

Study of the electromagnetic properties of baryons constitutes very important source of information in understanding their internal structure, and can provide valuable insight about the nonperturbative aspects of QCD. One of the most crucial quantities in this respect is the magnetic moments of baryons. Magnetic moments of the ground state $J^P = \frac{1}{2}^+$ heavy baryons have widely been studied in literature. Furthermore, magnetic moments of the heavy baryons have been investigated in the naive quark model in [8, 9], phenomenological quark model [10], relativistic three-quark model [11], variational approach [12], non-relativistic quark model with screening and effective quark mass [13, 14], nonrelativistic hypercentral model [15], chiral constituent quark model [16], chiral bag model [17], chiral perturbation theory [18], traditional QCD sum rules [19], and light cone QCD sum rules (LCSR) [20–22], respectively.

In the present work, we calculate the diagonal and transition magnetic moments of the negative parity, spin-1/2 heavy baryons in framework of the light cone QCD sum rules method.

The body of the paper is organized as follows. In section 2, we construct the light cone QCD sum rules for the diagonal and transition magnetic moments of the negative parity, heavy baryons. Section 3 is devoted to the numerical analysis of these sum rules for the magnetic moments; and contains discussion of the obtained results and conclusion.

2 Sum rules for the diagonal and transition magnetic moments of the negative parity heavy baryons

In this section the LCSR for the negative parity baryons are derived. In order to obtain the relevant sum rules, we consider the following correlator,

$$\Pi(p, q) = i \int d^4x e^{ipx} \langle 0 | T \{ \eta_{Q_2}(x) \bar{\eta}_{Q_1}(0) \} | 0 \rangle_\gamma, \quad (1)$$

where γ means the external electromagnetic field, $\eta_Q(x)$ is the interpolating current of the corresponding heavy baryon with spin-1/2. In order to obtain the sum rules for magnetic moments of the heavy baryons the correlation function function is calculated in two different ways: a) In terms of the hadrons; b) performing the operator product expansion (OPE) over the twists of operators, and using the photon distribution amplitudes (DAs) which

encode all nonperturbative effects. The sum rules for the magnetic moments of the heavy baryons are obtained by equating the coefficients of the appropriate Lorentz structures that survive in parts (a) and (b). Finally, in order to enhance the contributions coming from the ground states, and suppress the contributions of the continuum and higher states, Borel transformation with respect to the momentum squared of the initial and final baryon states, and continuum subtraction procedure are performed, successively (for more about the LCSR, see for example [23]).

To be able to calculate the correlator function in terms of the hadrons, we insert the complete set of baryon states that carry the same quantum numbers as the interpolating current $\eta(x)$. It should be noted here that the interpolating current can produce both positive and negative parity baryons from the vacuum state. Keeping this remark in mind, and isolating the contributions of the ground state baryons, we get

$$\begin{aligned}
\Pi(p, q) = & \frac{\langle 0 | \eta_{Q_2} | B_2^{(+)}(p, s) \rangle \langle B_2^{(+)}(p, s) \gamma(q) | B_1^{(+)}(p+q, s) \rangle \langle B_1^{(+)}(p+q, s) | \bar{\eta}_{Q_1}(0) \rangle}{p^2 - m_{B_2^{(+)}}^2} \\
& + \frac{\langle 0 | \eta_{Q_2} | B_2^{(-)}(p, s) \rangle \langle B_2^{(-)}(p, s) \gamma(q) | B_1^{(-)}(p+q, s) \rangle \langle B_1^{(-)}(p+q, s) | \bar{\eta}_{Q_1}(0) \rangle}{p^2 - m_{B_2^{(-)}}^2} \\
& + \frac{\langle 0 | \eta_{Q_2} | B_2^{(+)}(p, s) \rangle \langle B_2^{(+)}(p, s) \gamma(q) | B_1^{(-)}(p+q, s) \rangle \langle B_1^{(-)}(p+q, s) | \bar{\eta}_{Q_1}(0) \rangle}{p^2 - m_{B_2^{(+)}}^2} \\
& + \frac{\langle 0 | \eta_{Q_2} | B_2^{(-)}(p, s) \rangle \langle B_2^{(-)}(p, s) \gamma(q) | B_1^{(+)}(p+q, s) \rangle \langle B_1^{(+)}(p+q, s) | \bar{\eta}_{Q_1}(0) \rangle}{p^2 - m_{B_2^{(-)}}^2} + \dots \quad (2)
\end{aligned}$$

where $B^{(\pm)}$ and $m_{B^{(\pm)}}$ correspond to positive (negative) parity baryons and their masses, respectively; q is the photon momentum; and dots correspond to the higher states contributions.

The matrix elements in Eq. (2) are defined as,

$$\begin{aligned}
\langle 0 | \eta_{Q_2} | B^{(+)}(p) \rangle &= \lambda_{B^{(+)}} u^{(+)}(p) , \\
\langle 0 | \eta_{Q_2} | B^{(-)}(p) \rangle &= \lambda_{B^{(-)}} \gamma_5 u^{(-)}(p) , \\
\langle B_2^{(+)}(p) \gamma(q) | B_1^{(+)}(p+q) \rangle &= e \varepsilon^\mu \bar{u}^{(+)}(p) \left[\gamma_\mu f_1 - \frac{i \sigma_{\mu\nu} q^\nu}{m_{B_1^{(+)}} + m_{B_2^{(+)}}} f_2 \right] u^{(+)}(p+q) \\
&= e \varepsilon^\mu \bar{u}^{(+)}(p) \left[(f_1 + f_2) \gamma_\mu - \frac{(2p+q)_\mu}{m_{B_1^{(+)}} + m_{B_2^{(+)}}} f_2 \right] u^{(+)}(p+q) , \\
\langle B_2^{(-)}(p) \gamma(q) | B_1^{(+)}(p+q) \rangle &= e \varepsilon^\mu \bar{u}^{(-)}(p) \left[\gamma_\mu f_1^T - \frac{i \sigma_{\mu\nu} q^\nu}{m_{B_1^{(+)}} + m_{B_2^{(-)}}} f_2^T \right] \gamma_5 u^{(+)}(p+q) \\
&= e \varepsilon^\mu \bar{u}^{(-)}(p) \left[\left(f_1^T - \frac{m_{B_1^{(+)}} - m_{B_2^{(-)}}}{m_{B_1^{(+)}} + m_{B_2^{(-)}}} f_2^T \right) \gamma_\mu \right. \\
&\quad \left. - \frac{(2p+q)_\mu}{m_{B_1^{(+)}} + m_{B_2^{(-)}}} f_2^T \right] \gamma_5 u^{(+)}(p+q) ,
\end{aligned}$$

$$\begin{aligned}
\langle B_2^{(-)}(p)\gamma(q)|B_1^{(-)}(p+q)\rangle &= e\varepsilon^\mu\bar{u}^{(-)}(p)\left[\gamma_\mu f_1^* - \frac{i\sigma_{\mu\nu}q^\nu}{m_{B_1^{(-)}} + m_{B_2^{(-)}}}f_2^*\right]u^{(-)}(p+q) \\
&= e\varepsilon^\mu\bar{u}^{(-)}(p)\left[(f_1^* + f_2^*)\gamma_\mu - \frac{(2p+q)_\mu}{m_{B_1^{(-)}} + m_{B_2^{(-)}}}f_2^*\right]u^{(-)}(p+q) \quad (3)
\end{aligned}$$

where ε^μ is the four-polarization vector of the photon, and $\lambda_{B^{(+)}}$, $\lambda_{B^{(-)}}$ are the residues of the positive and negative parity baryons, respectively.

Performing summation over spins of the heavy baryons, for the correlation function from the hadronic side we get,

$$\begin{aligned}
\Pi(p, q) &= A(\not{p}_2 + m_{B_2^{(+)}})\not{\epsilon}(\not{p}_1 + m_{B_1^{(+)}}) \\
&\quad + B(\not{p}_2 - m_{B_2^{(-)}})\not{\epsilon}(\not{p}_1 - m_{B_1^{(-)}}) \\
&\quad + C(\not{p}_2 - m_{B_2^{(-)}})\not{\epsilon}\gamma_5(\not{p}_1 + m_{B_1^{(+)}}) \\
&\quad + D(\not{p}_2 + m_{B_2^{(+)}})\not{\epsilon}\gamma_5(\not{p}_1 - m_{B_1^{(-)}}), \quad (4)
\end{aligned}$$

where

$$\begin{aligned}
A &= \frac{\lambda_{B_1^{(+)}}\lambda_{B_2^{(+)}}(f_1 + f_2)}{(m_{B_1^{(+)}}^2 - p_1^2)(m_{B_2^{(+)}}^2 - p_2^2)} \\
B &= \frac{\lambda_{B_1^{(-)}}\lambda_{B_2^{(-)}}(f_1^* + f_2^*)}{(m_{B_1^{(-)}}^2 - p_1^2)(m_{B_2^{(-)}}^2 - p_2^2)} \\
C &= \frac{\lambda_{B_1^{(-)}}\lambda_{B_2^{(+)}}}{(m_{B_1^{(-)}}^2 - p_1^2)(m_{B_2^{(+)}}^2 - p_2^2)}\left[f_1^T + \frac{m_{B_1^{(-)}} - m_{B_2^{(+)}}}{m_{B_1^{(-)}} + m_{B_2^{(+)}}}f_2^T\right] \\
D &= \frac{\lambda_{B_1^{(+)}}\lambda_{B_2^{(-)}}}{(m_{B_1^{(+)}}^2 - p_1^2)(m_{B_2^{(-)}}^2 - p_2^2)}\left[f_1^T - \frac{m_{B_1^{(+)}} - m_{B_2^{(-)}}}{m_{B_1^{(+)}} + m_{B_2^{(-)}}}f_2^T\right], \quad (5)
\end{aligned}$$

where $p_1 = p + q$ and $p_2 = p$.

Among the terms in Eq. (4)

$$f_1 + f_2, \quad (f_1^* + f_2^*), \quad f_1^T + \frac{m_{B_1^{(-)}} - m_{B_2^{(+)}}}{m_{B_1^{(-)}} + m_{B_2^{(+)}}}f_2^T, \quad f_1^T - \frac{m_{B_1^{(+)}} - m_{B_2^{(-)}}}{m_{B_1^{(+)}} + m_{B_2^{(-)}}}f_2^T,$$

that are proportional to γ_μ , the first two correspond to the magnetic moments of the positive to positive, negative to negative transitions, respectively; and the third and the fourth ones describe the transition magnetic moments between positive and negative parity baryons at $q^2 = 0$. Our aim in the present work is to calculate the diagonal and the transition magnetic moment between the negative parity baryons, and therefore we should find a way to remove the other three contributions.

In order to determine the diagonal and the transition magnetic moments between negative parity baryons four equations are needed, for which we choose the following four Lorentz structures, $(\varepsilon \cdot p)I$, $(\varepsilon \cdot p)\not{p}$, $\not{p}\not{\epsilon}$ and $\not{\epsilon}$. Solving finally these four coupled equations, we obtain the unknown coefficient B which describes the negative to negative parity transition.

It follows from Eq. (1) that interpolating currents are needed in order to calculate the correlation function in terms of quarks and gluons. Here, it should be remembered that hadrons containing single heavy quark belong to either sextet or anti-triplet representations of $SU(3)$. Sextet (anti-triplet) representation is symmetric (antisymmetric) with respect to the exchange of light quarks. The heavy baryons Σ_Q , Ξ'_Q , and Ω_Q belonging to the sextet; and Ξ_Q and Λ_Q belonging to the triplet representations of the $SU(3)$ group. Using this fact the general form of interpolating currents belonging to sextet and anti-triplet representations can be written in the following form (see [24]),

$$\begin{aligned}\eta^{(s)} &= -\frac{1}{\sqrt{2}}\varepsilon^{abc}\left\{(q_1^{aT}CQ^b)\gamma_5q_2^c + t(q_1^{aT}C\gamma_5Q^b)q_2^c + (q_2^{aT}CQ^b)\gamma_5q_1^c + (q_2^{aT}C\gamma_5Q^b)q_1^c\right\}, \\ \eta^{(a)} &= -\frac{1}{\sqrt{6}}\varepsilon^{abc}\left\{2(q_1^{aT}Cq_2^b)\gamma_5Q^c + 2t(q_1^{aT}C\gamma_5q_2^b)Q^c + (q_1^{aT}CQ^b)\gamma_5q_2^c \right. \\ &\quad \left. + t(q_1^{aT}C\gamma_5Q^b)q_2^c - (q_2^{aT}CQ^b)\gamma_5q_1^c - t(q_2^{aT}C\gamma_5Q^b)q_1^c\right\}.\end{aligned}\quad (6)$$

where t is an arbitrary parameter ($t = -1$ corresponds to the so-called Ioffe current); a, b, c are the color indices; and C is the charge conjugation operator. The light quark contents of the sextet and antitriplet representations are given in Table 1.

	$\Sigma_{c(b)}^{(++)+}$	$\Sigma_{c(b)}^{+(0)}$	$\Sigma_{c(b)}^{0(-)}$	$\Xi_{c(b)}^{0(-)}$	$\Xi_{c(b)}^{\prime+(0)}$	$\Omega_{c(b)}^{0(-)}$	$\Lambda_{c(b)}^{+(0)}$	$\Xi_{c(b)}^{0(-)}$	$\Xi_{c(b)}^{\prime+(0)}$
q_1	u	u	d	d	u	s	u	d	u
q_2	u	d	d	s	s	s	d	s	s

Table 1: Quark contents of the heavy baryons belonging to the sextet and antitriplet representations.

Using the interpolating currents given in Eq. (6), one can easily calculate the theoretical part of the correlator function. As an example, we present the form of the correlation function for Σ_Q^+ in terms of the corresponding propagators,

$$\begin{aligned}\Pi^{\Sigma_Q^+} &= -2\varepsilon^{abc}\varepsilon^{a'b'c'}\int d^4x\langle 0|\sum_{\ell=1}^2\sum_{k=1}^2\left\{A_2^\ell S_u^{cc'}(x)A_2^k\text{Tr}S_Q^{bb'}(x)CA_1^k S_u^{aa'}(x)CA_1^\ell \right. \\ &\quad + A_2^\ell\left[S_u^{cc'}(x)(CA_1^k)^T S_Q^{bb'T}(x)(CA_1^\ell)^T S_u^{aa'}(x)A_2^k \right. \\ &\quad + S_u^{aa'}(x)(CA_1^k)^T S_Q^{bb'T}(x)(CA_1^\ell)^T S_u^{cc'}(x)A_2^k \\ &\quad \left. \left. + S_u^{aa'}(x)A_2^k\text{Tr}S_Q^{bb'T}(x)CA_1^k S_u^{cc'T}(x)CA_1^\ell\right\}|0\rangle_\gamma,\end{aligned}\quad (7)$$

where $A_1^1 = 1$, $A_1^2 = t\gamma_5$, $A_2^1 = \gamma_5$, $A_2^2 = t$, and $S_q(x)$ and $S_Q(x)$ are the full propagators of the light and heavy quarks.

The expressions of the correlator functions for the Σ_Q^- , Σ_Q^0 , Ξ_Q^0 , $\Xi_Q^{\prime-}$ and Ω_Q can be found by performing the following replacements,

$$\Pi^{\Sigma_Q^-} = \Pi^{\Sigma_Q^+}(u \rightarrow d),$$

$$\begin{aligned}
\Pi^{\Sigma_Q^0} &= \frac{1}{2} \left(\Pi^{\Sigma_Q^+} + \Pi^{\Sigma_Q^-} \right) , \\
\Pi^{\Xi_Q^0} &= \Pi^{\Sigma_Q^+}(u \rightarrow s) , \\
\Pi^{\Xi_Q^-} &= \Pi^{\Sigma_Q^+}(u \rightarrow s, s \rightarrow d) .
\end{aligned} \tag{8}$$

As has already been noted, in the present work we also calculate the magnetic moments of $\Sigma_Q \rightarrow \Lambda_Q$ and $\Xi'_Q \rightarrow \Xi_Q$ transitions. It follows from Eq. (7) in order to calculate the correlation function from the QCD side, the expressions of the light and heavy quark propagators in the presence of the external field are needed. The light cone expression of the light quark propagator in external field is calculated in [25] in which it is found that the contributions of the nonlocal operators such as $\bar{q}Gq$, $\bar{q}G^2q$, $\bar{q}q\bar{q}q$, are small. Neglecting these operators is justified in conformal spin expansion [26]. Note that in further analysis we retain only those terms that are linear in quark mass. The expression of the light quark operator in the presence of external field is given as,

$$\begin{aligned}
S_q(x) &= \frac{i\not{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2} - \frac{\langle \bar{q}q \rangle}{12} \left(1 - i\frac{m_q}{4}\not{x} \right) - \frac{x^2}{192} m_0^2 \langle \bar{q}q \rangle \left(1 - i\frac{m_q}{6}\not{x} \right) \\
&\quad - ig_s \int_0^1 du \left[\frac{\not{x}}{16\pi^2 x^2} G_{\mu\nu}(ux) \sigma_{\mu\nu} - \frac{i}{4\pi^2 x^2} u x^\mu G_{\mu\nu}(ux) \gamma^\nu \right. \\
&\quad \left. - i\frac{m_q}{32\pi^2} G_{\mu\nu} \sigma^{\mu\nu} \left(\ln \frac{-x^2 \Lambda^2}{4} + 2\gamma_E \right) \right] + \dots ,
\end{aligned} \tag{9}$$

where Λ is the cut-off energy separating perturbative and nonperturbative domains, and γ_E is the Euler constant.

In calculating the correlation function from the QCD side we also need the expression for the heavy quarks, whose explicit form in the coordinate space can be expressed as,

$$\begin{aligned}
S_Q(x) &= \frac{m_Q^2}{4\pi^2} \left\{ \frac{K_1(m_Q \sqrt{-x^2})}{\sqrt{-x^2}} + i\frac{\not{x}}{-x^2} K_2(m_Q \sqrt{-x^2}) \right\} \\
&\quad - \frac{g_s}{16\pi^2} \int_0^1 du G_{\mu\nu}(ux) \left[(\sigma^{\mu\nu} \not{x} + \not{x} \sigma^{\mu\nu}) \frac{K_1(m_Q \sqrt{-x^2})}{\sqrt{-x^2}} + 2\sigma^{\mu\nu} K_0(m_Q \sqrt{-x^2}) \right] + \dots \tag{10}
\end{aligned}$$

where $K_i(m_Q \sqrt{-x^2})$ are the modified Bessel functions. Taking into account the expressions of the light and heavy propagators, the correlation function given in Eq. (8) can be calculated from the QCD side. This correlation function contains three different type of contributions: a) Perturbative contributions, i.e., photon interacts with the quark propagators perturbatively. Technically this contribution can be calculated by replacing the one of the free quark operators (the first two terms in Eqs. (9) and (10)) by,

$$S^{free}(x) \rightarrow \int d^4 y S^{free}(x-y) \not{A}(y) S^{free}(x-y) , \tag{11}$$

and the remaining two propagators are the free ones. b) In the case when photon interacts with the heavy quark perturbatively, the free part must be removed at least in one of the light quark propagators, i.e., one (or both) light quark operators are replaced by the quark condensate. c) Nonperturbative contributions, i.e., photon interacts with the light quark

fields at large distance. This contribution can be calculated by replacing one of the light quark operators by,

$$S_{\alpha\beta}^{ab} \rightarrow -\frac{1}{4} (q^a \Gamma_i q^b) (\Gamma_i)_{\alpha\beta}, \quad (12)$$

and the remaining quarks constitute the full quark propagators. Here, Γ_j are the full set of Dirac matrices $\gamma_j = \{I, \gamma_5, \gamma_\mu, i\gamma_\mu\gamma_5, \sigma_{\mu\nu}/\sqrt{2}\}$. When Eq. (12) is used in calculation of the nonperturbative contributions, we see that matrix elements of the form $\langle \gamma(q) | \bar{q} \Gamma_i q | 0 \rangle$ are needed. These matrix elements are defined in terms of the photon distribution amplitudes [27], and are presented in Appendix A.

As has already been noted, in determination of the magnetic moment responsible for the negative to negative parity transition, four equations are needed, and for this purpose we choose the coefficients of the structures $(\varepsilon \cdot p)I$, $(\varepsilon \cdot p)\not{p}$, $\not{p}\not{\varepsilon}$ and $\not{\varepsilon}$. The sum rules for the negative to negative parity transition magnetic moments can be obtained by choosing the coefficients of the aforementioned Lorentz structures Π_i , and equate them to the corresponding coefficients in hadronic part. Solving then the linear equations for the coefficients describing the negative to negative transition magnetic moments, and performing Borel transformation over the variables $-p^2$ and $-(p+q)^2$ in order to suppress higher states and continuum contribution, we finally obtain the magnetic moment for the negative to negative parity baryon transitions as is given below,

$$\begin{aligned} \mu = & \frac{e^{m_{B_1^{(-)}}^2/2M^2} e^{m_{B_2^{(-)}}^2/2M^2}}{2\lambda_{B_1^{(-)}}\lambda_{B_2^{(-)}} (m_{B_1^{(+)}} + m_{B_1^{(-)}}) (m_{B_2^{(+)}} + m_{B_2^{(-)}})} \left\{ (m_{B_1^{(+)}} + m_{B_2^{(-)}}) (\Pi_1^B - m_{B_2^{(+)}}\Pi_2^B) \right. \\ & \left. + 2m_{B_2^{(+)}}\Pi_3^B - 2\Pi_4^B \right\}. \end{aligned} \quad (13)$$

In this expression we take $M_1^2 = M_2^2 = 2M^2$, since the masses of the diagonal transitions are same, and masses of the baryons that responsible for the $\Sigma_Q \rightarrow \Lambda_Q$ and $\Xi'_Q \rightarrow \Xi_Q$ transitions baryons are very close to each other. The expressions of Π_i^B for the Σ_b^0 baryon and $\Xi_b^0 \rightarrow \Xi_b^0$ transition are presented in Appendix B.

It follows from Eq. (13) that in determination of the diagonal and transition magnetic moments, the residues of the heavy negative parity, spin-1/2 baryons are needed. These residues can be determined from the analysis of the two-point correlation function

$$\Pi(q^2) = i \int d^4x e^{iqx} \langle 0 | T \{ \eta_Q(x) \bar{\eta}_Q(0) \} | 0 \rangle,$$

where η_Q is the interpolating current for the corresponding heavy baryon given by Eq. (6). This interpolating current interacts with both positive and negative parity heavy baryons. Saturating this correlation function with the ground states of positive and negative parity baryons we have,

$$\Pi(q^2) = \frac{|\lambda_{B^{(-)}}|^2 (\not{p} - m_{B^{(-)}})}{m_{B^{(-)}}^2 - p^2} + \frac{|\lambda_{B^{(+)}}|^2 (\not{p} + m_{B^{(+)}})}{m_{B^{(+)}}^2 - p^2}.$$

Eliminating the contributions coming from the positive parity baryons, the following sum rules for the residue and mass of the negative parity baryons are obtained,

$$|\lambda_{B^{(-)}}|^2 = \frac{1}{\pi} \frac{e^{m_{B^{(-)}}^2/M^2}}{m_{B^{(+)}} + m_{B^{(-)}}} \int ds e^{-s/M^2} [m_{B^{(+)}} \text{Im}\Pi_1^M(s) - \text{Im}\Pi_2^M(s)] ,$$

$$m_{B^{(-)}}^2 = \frac{\int_{m_Q^2}^{s_0} s ds e^{-s/M^2} [m_{B^{(+)}} \text{Im}\Pi_1^M(s) - \text{Im}\Pi_2^M(s)]}{\int_{m_Q^2}^{s_0} ds e^{-s/M^2} [m_{B^{(+)}} \text{Im}\Pi_1^M(s) - \text{Im}\Pi_2^M(s)]} .$$

Here Π_1^M and Π_2^M are the invariant functions corresponding to the structures \not{p} and I , respectively. The expressions of Π_1^M and Π_2^M for the Σ_Q^0 , Ξ'_Q and Ξ_Q baryons are presented in Appendix C.

3 Numerical analysis

This section is devoted to the numerical analysis of the sum rules obtained in the previous section for the magnetic moments of the diagonal and nondiagonal transitions of the $J^P = \frac{1}{2}^-$ heavy baryons. The main input parameters of the light cone QCD sum rules are the photon distribution amplitudes (DAs). The photon DAs are obtained in [27], and for completeness we present their expressions in Appendix A. Sum rules for the magnetic moment, together with the photon DAs, also contain the following input parameters: quark condensate $\langle \bar{q}q \rangle$, m_0^2 that appears in determination of the vacuum expectation value of the dimension-5 operator $\langle \bar{q}Gq \rangle = m_0^2 \langle \bar{q}q \rangle$, magnetic susceptibility χ of quarks, etc. In the present analysis we use $[\langle \bar{u}u \rangle = \langle \bar{d}d \rangle]_{\mu=1 \text{ GeV}} = -(0.243)^3 \text{ GeV}^3$ [28], $\langle \bar{s}s \rangle|_{\mu=1 \text{ GeV}} = 0.8 \langle \bar{u}u \rangle|_{\mu=1 \text{ GeV}}$, $m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$ [29]. The values of the magnetic susceptibilities can be found in numerous works (see for example [30–32]). In our numerical analysis we use $\chi(1 \text{ GeV}) = -2.85 \text{ GeV}^2$ obtained in [32], and $\Lambda = (0.5 \pm 0.1) \text{ GeV}$ [33].

Having determined input parameters, in this section we shall proceed with the analysis of the sum rules for the diagonal and transition magnetic moments of the $J^P = \frac{1}{2}^-$ heavy baryons. The sum rules contain three auxiliary parameters: continuum threshold s_0 , Borel mass square M^2 , and t appearing in the expression of the interpolating current. According to the QCD sum rules philosophy, any measurable quantity must be independent of these parameters. For this reason we try to find such regions of these parameters where magnetic moments are insensitive to their variations. This issue can be accomplished by the following three-step procedure. First, at fixed values of s_0 and t , we try to find region of M^2 where magnetic moment is independent of its variation. The upper bound of M^2 is determined by requiring that the contributions of higher states and continuum constitute, say, less than 40% of the contribution coming from the perturbative part. The lower bound of M^2 is obtained by demanding that higher twist contributions are less than the leading twist contributions. Analysis of our sum rules leads to the following regions of M^2 where magnetic moments are independent on its variation.

$$2.5 \text{ GeV}^2 \leq M^2 \leq 4.0 \text{ GeV}^2, \text{ for } \Sigma_c, \Xi'_c, \Lambda_c, \Xi_c ,$$

$$4.5 \text{ GeV}^2 \leq M^2 \leq 7.0 \text{ GeV}^2, \text{ for } \Sigma_b, \Xi'_b, \Lambda_b, \Xi_b .$$

Next, we try to find the domain of variation of the continuum threshold s_0 , which is the energy square where the continuum starts. The difference $\sqrt{s_0} - m$, m being the ground state mass, is the energy needed for the excitation of the particle to its first excited state, and usually this difference varies in the range between 0.3 GeV and 0.8 GeV . In our analysis we use the average value $\sqrt{s_0} - m = 0.5 \text{ GeV}$.

As an example, in Figs. (1) and (2) we present the dependence of the magnetic moments of Σ_c^0 baryon, at $s_0 = 12 \text{ GeV}^2$, and magnetic moment of the $\Sigma_b \rightarrow \Lambda_b$ transition at $s_0 = 40 \text{ GeV}^2$, on M^2 , respectively. It follows from these figures that, indeed, we have very good stability of the magnetic moment μ as M^2 varies in its above-mentioned working region. We also analyze these dependencies at $s_0 = 11 \text{ GeV}^2$ and $s_0 = 42.5 \text{ GeV}^2$; and find out that the discrepancy in the values of the magnetic moment is about 10%. In other words, the magnetic moments of Σ_c^0 baryon and $\Sigma_b \rightarrow \Lambda_b$ transition exhibit the expected insensitivity to the variations in s_0 and M^2 .

Having decided the working regions of M^2 and s_0 , the third and last step is to find the working region of the parameter t in which the predictions for the values magnetic moments of heavy baryons show good stability. For this aim, we study the dependence of the Σ_c^0 baryon and $\Sigma_b \rightarrow \Lambda_b$ transition magnetic moments on $\cos \theta$, where $t = \tan \theta$ which are presented in Figs. (3) and (4). We observe from Figs. (3) and (4) that, the magnetic moments of diagonal transitions are independent of the variation in $\cos \theta$ when it varies in the region $-0.7 \leq \cos \theta \leq -0.4$; and in the domain $-1.0 \leq \cos \theta \leq -0.7$ for the transition magnetic moments, respectively. Our numerical analysis predicts that the magnetic moment of the Σ_c^0 baryon is $\mu = (-2.0 \pm 0.1)\mu_N$, and for the $\Sigma_b \rightarrow \Lambda_b$ transition $\mu = (-0.3 \pm 0.05)\mu_N$, where μ_N is the nuclear magneton. Performing similar analysis we have calculated the diagonal and transition magnetic moments of the other $J^P = \frac{1}{2}^-$ heavy baryons whose results are presented in Table 2. We note here that, in many cases, the naive expectation that the relation between the negative and positive parity baryons, i.e.,

$$|\mu_B^-| = \frac{m_B^+}{m_B^-} |\mu_B^+| ,$$

is violated considerably. This violation can be attributed to the fact that in our analysis we take into account contributions coming from positive-to-positive and nondiagonal transitions.

As can easily be seen from Table 2, the transition magnetic moments between the neutral Ξ'_Q and Ξ_Q baryons are very close to zero. The magnetic moments of Λ_b^0 , Ξ_b^0 , Σ_b^+ and Ξ_c^+ are also small enough to be measured. However the diagonal and transition magnetic moments of the remaining baryons are quite large, and can be measured in future experiments.

In conclusion, the diagonal and nondiagonal transition magnetic moments of the negative parity spin-1/2 heavy baryons are calculated in framework of the LCSR. The contributions coming from the positive to positive, as well as positive to negative parity transitions are eliminated by constructing various sum rules. It is obtained that the magnetic moments of Λ_b^0 , Ξ_b^0 , Σ_c^+ and Ξ'_c baryons, as well as the transition magnetic moments between the neutral Ξ'_Q and Ξ_Q baryons are small, while all other magnetic moments are quite large and can be measured in future.

	μ		μ		μ
Σ_b^+	1.3 ± 0.3	Σ_c^{++}	2.2 ± 0.2	$\Sigma_c^+ \rightarrow \Lambda_c^+$	0.25 ± 0.05
Σ_b^0	0.5 ± 0.05	Σ_c^+	0.15 ± 0.02	$\Xi_c'^0 \rightarrow \Xi_c^0$	0.08 ± 0.01
Σ_b^-	-0.3 ± 0.1	Σ_c^0	-2.0 ± 0.1	$\Xi_c'^+ \rightarrow \Xi_c^+$	0.20 ± 0.05
$\Xi_b'^-$	-0.4 ± 0.1	$\Xi_c'^0$	-2.0 ± 0.2	$\Xi_b'^0 \rightarrow \Xi_b^0$	-0.008 ± 0.001
$\Xi_b'^0$	0.4 ± 0.1	$\Xi_c'^+$	0.15 ± 0.02	$\Xi_b'^- \rightarrow \Xi_b^-$	0.10 ± 0.01
Ω_b^-	-0.3 ± 0.1	Ω_c^0	-2.0 ± 0.2		
Λ_b^0	-0.11 ± 0.02	Λ_c^+	1.3 ± 0.2		
Ξ_b^-	-0.7 ± 0.1	Ξ_c^0	1.6 ± 0.2		
Ξ_b^0	-0.12 ± 0.02	Ξ_c^+	1.2 ± 0.2		

Table 2: Diagonal and transition magnetic moments of the negative parity, spin-1/2 baryons containing single heavy quark belonging to the sextet and antitriplet representations, in units of nuclear magneton μ_N

A comparison our results on the magnetic moments of the negative parity baryons with the predictions of other approaches, such as quark model, bag model, chiral perturbation theory, lattice QCD, etc., would be interesting.

Appendix A: Photon distribution amplitudes

In this Appendix we present the definitions of the matrix elements of the form $\langle \gamma(q) | \bar{q} \Gamma q | 0 \rangle$ in terms of the photon DAs, and the explicit expressions of the photon DAs entering into the matrix elements [27].

$$\begin{aligned}
\langle \gamma(q) | \bar{q}(x) \sigma_{\mu\nu} q(0) | 0 \rangle &= -ie_q \bar{q} q (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int_0^1 du e^{i\bar{u}qx} \left(\chi \varphi_\gamma(u) + \frac{x^2}{16} \mathbb{A}(u) \right) \\
&- \frac{i}{2(qx)} e_q \langle \bar{q} q \rangle \left[x_\nu \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) - x_\mu \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \right] \int_0^1 du e^{i\bar{u}qx} h_\gamma(u) \\
\langle \gamma(q) | \bar{q}(x) \gamma_\mu q(0) | 0 \rangle &= e_q f_{3\gamma} \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \int_0^1 du e^{i\bar{u}qx} \psi^v(u) \\
\langle \gamma(q) | \bar{q}(x) \gamma_\mu \gamma_5 q(0) | 0 \rangle &= -\frac{1}{4} e_q f_{3\gamma} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^\nu q^\alpha x^\beta \int_0^1 du e^{i\bar{u}qx} \psi^a(u) \\
\langle \gamma(q) | \bar{q}(x) g_s G_{\mu\nu}(vx) q(0) | 0 \rangle &= -ie_q \langle \bar{q} q \rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{S}(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu} i\gamma_5(vx) q(0) | 0 \rangle &= -ie_q \langle \bar{q} q \rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \tilde{\mathcal{S}}(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu}(vx) \gamma_\alpha \gamma_5 q(0) | 0 \rangle &= e_q f_{3\gamma} q_\alpha (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{A}(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) g_s G_{\mu\nu}(vx) i\gamma_\alpha q(0) | 0 \rangle &= e_q f_{3\gamma} q_\alpha (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{V}(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) \sigma_{\alpha\beta} g_s G_{\mu\nu}(vx) q(0) | 0 \rangle &= e_q \langle \bar{q} q \rangle \left\{ \left[\left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \left(g_{\alpha\nu} - \frac{1}{qx} (q_\alpha x_\nu + q_\nu x_\alpha) \right) q_\beta \right. \right. \\
&- \left(\varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \left(g_{\beta\nu} - \frac{1}{qx} (q_\beta x_\nu + q_\nu x_\beta) \right) q_\alpha \\
&- \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \left(g_{\alpha\mu} - \frac{1}{qx} (q_\alpha x_\mu + q_\mu x_\alpha) \right) q_\beta \\
&+ \left. \left(\varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \left(g_{\beta\mu} - \frac{1}{qx} (q_\beta x_\mu + q_\mu x_\beta) \right) q_\alpha \right] \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_1(\alpha_i) \\
&+ \left[\left(\varepsilon_\alpha - q_\alpha \frac{\varepsilon x}{qx} \right) \left(g_{\mu\beta} - \frac{1}{qx} (q_\mu x_\beta + q_\beta x_\mu) \right) q_\nu \right. \\
&- \left(\varepsilon_\alpha - q_\alpha \frac{\varepsilon x}{qx} \right) \left(g_{\nu\beta} - \frac{1}{qx} (q_\nu x_\beta + q_\beta x_\nu) \right) q_\mu \\
&- \left(\varepsilon_\beta - q_\beta \frac{\varepsilon x}{qx} \right) \left(g_{\mu\alpha} - \frac{1}{qx} (q_\mu x_\alpha + q_\alpha x_\mu) \right) q_\nu \\
&+ \left. \left(\varepsilon_\beta - q_\beta \frac{\varepsilon x}{qx} \right) \left(g_{\nu\alpha} - \frac{1}{qx} (q_\nu x_\alpha + q_\alpha x_\nu) \right) q_\mu \right] \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_2(\alpha_i) \\
&+ \frac{1}{qx} (q_\mu x_\nu - q_\nu x_\mu) (\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_3(\alpha_i)
\end{aligned}$$

$$+ \frac{1}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_4(\alpha_i) \Big\} , \quad (1)$$

where χ is the magnetic susceptibility, $\varphi_\gamma(u)$ is the leading twist-2, $\psi^v(u)$, $\psi^a(u)$, \mathcal{A} and \mathcal{V} are the twist-3, and $h_\gamma(u)$, \mathbb{A} , and \mathcal{T}_i ($i = 1, 2, 3, 4$) are the twist-4 photon DAs. The measure $\mathcal{D}\alpha_i$ is defined as

$$\int \mathcal{D}\alpha_i = \int_0^1 d\alpha_{\bar{q}} \int_0^1 d\alpha_q \int_0^1 d\alpha_g \delta(1 - \alpha_{\bar{q}} - \alpha_q - \alpha_g) .$$

The expressions of the DAs entering into the above matrix elements are defined as:

$$\begin{aligned} \varphi_\gamma(u) &= 6u\bar{u} \left[1 + \varphi_2(\mu) C_2^{\frac{3}{2}}(u - \bar{u}) \right] , \\ \psi^v(u) &= 3[3(2u - 1)^2 - 1] + \frac{3}{64} (15w_\gamma^V - 5w_\gamma^A) [3 - 30(2u - 1)^2 + 35(2u - 1)^4] , \\ \psi^a(u) &= [1 - (2u - 1)^2] [5(2u - 1)^2 - 1] \frac{5}{2} \left(1 + \frac{9}{16} w_\gamma^V - \frac{3}{16} w_\gamma^A \right) , \\ \mathcal{A}(\alpha_i) &= 360\alpha_q \alpha_{\bar{q}} \alpha_g^2 \left[1 + w_\gamma^A \frac{1}{2} (7\alpha_g - 3) \right] , \\ \mathcal{V}(\alpha_i) &= 540w_\gamma^V (\alpha_q - \alpha_{\bar{q}}) \alpha_q \alpha_{\bar{q}} \alpha_g^2 , \\ h_\gamma(u) &= -10(1 + 2\kappa^+) C_2^{\frac{1}{2}}(u - \bar{u}) , \\ \mathbb{A}(u) &= 40u^2 \bar{u}^2 (3\kappa - \kappa^+ + 1) + 8(\zeta_2^+ - 3\zeta_2) [u\bar{u}(2 + 13u\bar{u}) + 2u^3(10 - 15u + 6u^2) \ln(u) \\ &\quad + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2) \ln(\bar{u})] , \\ \mathcal{T}_1(\alpha_i) &= -120(3\zeta_2 + \zeta_2^+) (\alpha_{\bar{q}} - \alpha_q) \alpha_{\bar{q}} \alpha_q \alpha_g , \\ \mathcal{T}_2(\alpha_i) &= 30\alpha_g^2 (\alpha_{\bar{q}} - \alpha_q) [(\kappa - \kappa^+) + (\zeta_1 - \zeta_1^+) (1 - 2\alpha_g) + \zeta_2 (3 - 4\alpha_g)] , \\ \mathcal{T}_3(\alpha_i) &= -120(3\zeta_2 - \zeta_2^+) (\alpha_{\bar{q}} - \alpha_q) \alpha_{\bar{q}} \alpha_q \alpha_g , \\ \mathcal{T}_4(\alpha_i) &= 30\alpha_g^2 (\alpha_{\bar{q}} - \alpha_q) [(\kappa + \kappa^+) + (\zeta_1 + \zeta_1^+) (1 - 2\alpha_g) + \zeta_2 (3 - 4\alpha_g)] , \\ \mathcal{S}(\alpha_i) &= 30\alpha_g^2 \{ (\kappa + \kappa^+) (1 - \alpha_g) + (\zeta_1 + \zeta_1^+) (1 - \alpha_g) (1 - 2\alpha_g) \\ &\quad + \zeta_2 [3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g (1 - \alpha_g)] \} , \\ \tilde{\mathcal{S}}(\alpha_i) &= -30\alpha_g^2 \{ (\kappa - \kappa^+) (1 - \alpha_g) + (\zeta_1 - \zeta_1^+) (1 - \alpha_g) (1 - 2\alpha_g) \\ &\quad + \zeta_2 [3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g (1 - \alpha_g)] \} . \end{aligned}$$

The parameters entering the above DA's are borrowed from [27] whose values are $\varphi_2(1 \text{ GeV}) = 0$, $w_\gamma^V = 3.8 \pm 1.8$, $w_\gamma^A = -2.1 \pm 1.0$, $\kappa = 0.2$, $\kappa^+ = 0$, $\zeta_1 = 0.4$, $\zeta_2 = 0.3$, $\zeta_1^+ = 0$, and $\zeta_2^+ = 0$.

Appendix B

In this Appendix we present the expressions of the invariant functions Π_i appearing in the sum rules for the magnetic moments of Σ_b^0 baryon, and $\Xi_b^{\prime 0} \rightarrow \Xi_b^0$ transition .

FOR THE MAGNETIC MOMENT OF THE Σ_b^0 BARYON

1) Coefficient of the $(\varepsilon \cdot p)I$ structure

$$\begin{aligned}
\Pi_1^B = & -\frac{3}{128\pi^4}(-1+t)^2(e_b+e_d+e_u)m_b^3M^3(\mathcal{I}_2-2m_b^2\mathcal{I}_3+m_b^4\mathcal{I}_4) \\
& +\frac{1}{1536\pi^4}(-1+t)m_bM^4\left\{3[(e_d+e_u)(1-t)\langle g_s^2G^2\rangle+48(1+t)e_bm_b\pi^2(\langle\bar{d}d\rangle+\langle\bar{u}u\rangle)]\mathcal{I}_2\right. \\
& +4m_b^2[(e_d+e_u)(-1+t)\langle g_s^2G^2\rangle+72m_b(1+t)\pi^2(e_u\langle\bar{d}d\rangle+e_d\langle\bar{u}u\rangle)]\mathcal{I}_3\left.\right\} \\
& -\frac{3}{16\pi^2}(-1+t^2)m_b^4M^4(e_d\langle\bar{d}d\rangle+e_u\langle\bar{u}u\rangle)\tilde{j}(h_\gamma)\mathcal{I}_3 \\
& +\frac{1}{16\pi^2}(-1+t)^2(e_d+e_u)f_{3\gamma}m_b^3M^2(\mathcal{I}_2-m_b^2\mathcal{I}_3)\psi^v(u_0) \\
& +\frac{3}{32\pi^2}(-1+t^2)e_bm_b^2M^2(\langle\bar{d}d\rangle+\langle\bar{u}u\rangle)\mathcal{I}_1 \\
& +\frac{1}{768\pi^2}m_bM^2\mathcal{I}_2\left\{-3(-1+t^2)m_b[\langle\bar{d}d\rangle(7e_b-2e_u)m_0^2+24\langle\bar{d}d\rangle e_bm_b^2\right. \\
& +\left.(7e_b-2e_d)m_0^2\langle\bar{u}u\rangle+24e_bm_b^2\langle\bar{u}u\rangle]+2(-1+t)^2(e_d+e_u)f_{3\gamma}\langle g_s^2G^2\rangle\psi^v(u_0)\right\} \\
& -\frac{e^{-m_b^2/M^2}}{2304m_b\pi^2}(-1+t)M^2\left\{9(1+t)m_0^2m_b(7\langle\bar{d}d\rangle e_b+12e_u\langle\bar{d}d\rangle+7e_b\langle\bar{u}u\rangle+12e_d\langle\bar{u}u\rangle)\right. \\
& +\left.2f_{3\gamma}[(e_d+e_u)(-1+t)\langle g_s^2G^2\rangle+96(1+t)m_b\pi^2(e_d\langle\bar{u}u\rangle+e_u\langle\bar{d}d\rangle)]\psi^v(u_0)\right\} \\
& +\frac{e^{-m_b^2/M^2}}{48M^2}(-1+t^2)f_{3\gamma}m_0^2m_b^2(e_u\langle\bar{d}d\rangle+e_d\langle\bar{u}u\rangle)\psi^v(u_0) \\
& +\frac{e^{-m_b^2/M^2}}{13824M^4\pi^2}(-1+t^2)\langle g_s^2G^2\rangle m_b^2(e_u\langle\bar{d}d\rangle+e_d\langle\bar{u}u\rangle)[9m_0^2+16f_{3\gamma}\pi^2\psi^v(u_0)] \\
& +\frac{e^{-m_b^2/M^2}}{1728M^6}(-1+t^2)f_{3\gamma}\langle g_s^2G^2\rangle m_0^2m_b^2(e_u\langle\bar{d}d\rangle+e_d\langle\bar{u}u\rangle)\psi^v(u_0) \\
& -\frac{e^{-m_b^2/M^2}}{3456M^8}(-1+t^2)f_{3\gamma}\langle g_s^2G^2\rangle m_0^2m_b^4(e_u\langle\bar{d}d\rangle+e_d\langle\bar{u}u\rangle)\psi^v(u_0) \\
& +\frac{1}{768\pi^2}(-1+t^2)[-2\langle g_s^2G^2\rangle(e_u\langle\bar{d}d\rangle+e_d\langle\bar{u}u\rangle)-21e_bm_0^2m_b^2(\langle\bar{d}d\rangle+\langle\bar{u}u\rangle)]\mathcal{I}_1 \\
& +\frac{e^{-m_b^2/M^2}}{384\pi^2}(-1+t^2)\langle g_s^2G^2\rangle(e_d\langle\bar{d}d\rangle+e_u\langle\bar{u}u\rangle)\tilde{j}(h_\gamma) \\
& +\frac{e^{-m_b^2/M^2}}{96}(-1+t^2)f_{3\gamma}m_0^2(e_u\langle\bar{d}d\rangle+e_d\langle\bar{u}u\rangle)\psi^v(u_0) .
\end{aligned}$$

2) Coefficient of the $(\varepsilon \cdot p) \not{p}$ structure

$$\begin{aligned}
\Pi_2^B = & -\frac{1}{128\pi^4} m_b^2 M^6 \left\{ 3(5 + 2t + 5t^2)(e_d + e_u) m_b^2 (\mathcal{I}_3 - 2m_b^2 \mathcal{I}_4 + m_b^4 \mathcal{I}_5) \right. \\
& + e_b \left[(3 + 2t + 3t^2) \mathcal{I}_2 - 3(1 + t)^2 m_b^2 \mathcal{I}_3 - 3(-1 + t)^2 m_b^4 \mathcal{I}_4 + (3 - 2t + 3t^2) m_b^6 \mathcal{I}_5 \right] \left. \right\} \\
& + \frac{1}{128\pi^4} (3 + 2t + 3t^2) e_b m_b^2 M^4 (-\mathcal{I}_1 + 3m_b^2 \mathcal{I}_2 - 3m_b^4 \mathcal{I}_3 + m_b^6 \mathcal{I}_4) \\
& + \frac{1}{1536\pi^4} m_b^2 M^2 \left\{ (5 + 2t + 5t^2)(e_d + e_u) \langle g_s^2 G^2 \rangle \right. \\
& + 288(-1 + t^2) m_b \pi^2 \left[e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle + e_b (\langle \bar{d}d \rangle + \langle \bar{u}u \rangle) \right] \left. \right\} (\mathcal{I}_2 - m_b^2 \mathcal{I}_3) \\
& - \frac{e^{-m_b^2/M^2}}{768m_b M^2 \pi^2} (-1 + t^2) \langle g_s^2 G^2 \rangle m_0^2 (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \\
& + \frac{e^{-m_b^2/M^2}}{1536M^4 \pi^2} (-1 + t^2) \langle g_s^2 G^2 \rangle m_0^2 m_b (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \\
& - \frac{e^{-m_b^2/M^2}}{384m_b \pi^2} (-1 + t^2) \langle g_s^2 G^2 \rangle (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \\
& + \frac{1}{64\pi^2} 3(-1 + t^2) m_0^2 m_b (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \mathcal{I}_1 \\
& + \frac{1}{128\pi^2} (-1 + t^2) m_b \left\{ (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \langle g_s^2 G^2 \rangle \right. \\
& \left. - m_0^2 m_b^2 [7e_b (\langle \bar{d}d \rangle + \langle \bar{u}u \rangle) + 12(e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle)] \right\} \mathcal{I}_2 .
\end{aligned}$$

3) Coefficient of the $\not{p}\not{p}$ structure

$$\begin{aligned}
\Pi_3^B = & \frac{1}{128\pi^2} m_b^2 M^4 \left\{ (-1 + t^2) (e_d \langle \bar{d}d \rangle + e_u \langle \bar{u}u \rangle) \left[(\mathcal{I}_2 - 2m_b^2 \mathcal{I}_3) (i_1(\mathcal{S}) + i_1(\tilde{\mathcal{S}}, 1)) \right. \right. \\
& + i_1(\mathcal{T}_2, 1) - i_1(\mathcal{T}_4, 1) - 2i_1(\mathcal{T}_2, v) + 2i_1(\mathcal{T}_4, v) \left. \right] + 12m_b^2 \mathcal{I}_3 \tilde{j}(h_\gamma) \left. \right\} \\
& + (-1 + t)^2 (e_d + e_u) f_{3\gamma} m_b (-\mathcal{I}_2 + m_b^2 \mathcal{I}_3) \psi^{a'l}(u_0) \left. \right\} \\
& + \frac{e^{-m_b^2/M^2}}{9216m_b \pi^2} (-1 + t)^2 (e_d + e_u) f_{3\gamma} \langle g_s^2 G^2 \rangle M^2 \psi^{a'l}(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{96} (-1 + t^2) f_{3\gamma} M^2 (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \psi^{a'l}(u_0) \\
& - \frac{1}{3072\pi^2} (-1 + t)^2 (e_d + e_u) f_{3\gamma} \langle g_s^2 G^2 \rangle m_b M^2 \mathcal{I}_2 \psi^{a'l}(u_0)
\end{aligned}$$

$$\begin{aligned}
& - \frac{e^{-m_b^2/M^2}}{384M^2}(-1+t^2)f_{3\gamma}m_0^2m_b^2(e_u\langle\bar{d}d\rangle+e_d\langle\bar{u}u\rangle)\psi^{a'}(u_0) \\
& - \frac{e^{-m_b^2/M^2}}{6912M^4}(-1+t^2)f_{3\gamma}\langle g_s^2G^2\rangle m_b^2(e_u\langle\bar{d}d\rangle+e_d\langle\bar{u}u\rangle)\psi^{a'}(u_0) \\
& - \frac{e^{-m_b^2/M^2}}{13824M^6}(-1+t^2)f_{3\gamma}\langle g_s^2G^2\rangle m_0^2m_b^2(e_u\langle\bar{d}d\rangle+e_d\langle\bar{u}u\rangle)\psi^{a'}(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{27648M^8}(-1+t^2)f_{3\gamma}\langle g_s^2G^2\rangle m_0^2m_b^4(e_u\langle\bar{d}d\rangle+e_d\langle\bar{u}u\rangle)\psi^{a'}(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{9216\pi^2}(-1+t^2)\left\{\langle g_s^2G^2\rangle(e_d\langle\bar{d}d\rangle+e_u\langle\bar{u}u\rangle)\left[i_1(\mathcal{S})+i_1(\tilde{\mathcal{S}},1)+i_1(\mathcal{T}_2,1)-i_1(\mathcal{T}_4,1)\right.\right. \\
& \left.\left.-2i_1(\mathcal{T}_2,v)+2i_1(\mathcal{T}_4,v)-12\tilde{j}(h_\gamma)\right]-4f_{3\gamma}m_0^2\pi^2(e_u\langle\bar{d}d\rangle+e_d\langle\bar{u}u\rangle)\psi^{a'}(u_0)\right\}. \quad (1)
\end{aligned}$$

4) Coefficient of the \not{e} structure

$$\begin{aligned}
\Pi_4^B &= \frac{1}{256\pi^4}m_b^2M^8\left\{3(e_d+e_u)m_b^2\left[(1+t)^2\mathcal{I}_3+4(1+t^2)m_b^2\mathcal{I}_4-(5+2t+5t^2)m_b^4\mathcal{I}_5\right]\right. \\
& - e_b\left[(3+2t+3t^2)\mathcal{I}_2+3(1+t)^2m_b^2\mathcal{I}_3-3(3+2t+3t^2)m_b^4\mathcal{I}_4+(3-2t+3t^2)m_b^6\mathcal{I}_5\right]\left. \right\} \\
& + \frac{1}{256\pi^4}e_b m_b^2 M^6\left[4t\mathcal{I}_1+3m_b^2\mathcal{I}_2-6tm_b^2\mathcal{I}_2+3t^2m_b^2\mathcal{I}_2-6m_b^4\mathcal{I}_3-6t^2m_b^4\mathcal{I}_3+(3+2t+3t^2)m_b^6\mathcal{I}_4\right] \\
& + \frac{1}{128\pi^2}(e_d+e_u)f_{3\gamma}m_b^2M^6\left[(1+t)^2\mathcal{I}_2-2(1+4t+t^2)m_b^2\mathcal{I}_3\right]i_2(\mathcal{A},v) \\
& + \frac{1}{128\pi^2}(e_d+e_u)f_{3\gamma}m_b^2M^6\left[(1+t)^2\mathcal{I}_2-4(1+t+t^2)m_b^2\mathcal{I}_3\right]i_2(\mathcal{V},v) \\
& + \frac{1}{32\pi^2}(3+2t+3t^2)(e_d+e_u)f_{3\gamma}m_b^4M^6\mathcal{I}_3\psi^v(u_0) \\
& + \frac{1}{64\pi^2}(-1+t^2)m_b^3M^6(e_d\langle\bar{d}d\rangle+e_u\langle\bar{u}u\rangle)\chi(-\mathcal{I}_2+m_b^2\mathcal{I}_3)\varphi'_\gamma(u_0) \\
& - \frac{1}{128\pi^2}(3+2t+3t^2)(e_d+e_u)f_{3\gamma}m_b^4M^6\mathcal{I}_3\psi^{a'}(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{4608m_b\pi^2}(-1+t^2)\langle g_s^2G^2\rangle M^4(e_d\langle\bar{d}d\rangle+e_u\langle\bar{u}u\rangle)\chi\varphi'_\gamma(u_0) \\
& + \frac{1}{3072\pi^4}m_b^4M^4\left\{- (5+2t+5t^2)(e_d+e_u)\langle g_s^2G^2\rangle\right. \\
& \left.- 288(-1+t^2)m_b\pi^2\left[\langle\bar{d}d\rangle(e_b+e_u)+(e_b+e_d)\langle\bar{u}u\rangle\right]\right\}\mathcal{I}_3 \\
& + \frac{1}{128\pi^2}(-1+t^2)m_bM^4(e_d\langle\bar{d}d\rangle+e_u\langle\bar{u}u\rangle)\left[i_1(\mathcal{S},1)-i_1(\tilde{\mathcal{S}},1)+i_1(\mathcal{T}_1,1)\right. \\
& \left.- i_1(\mathcal{T}_2,1)+i_1(\mathcal{T}_3,1)-i_1(\mathcal{T}_4,1)\right]\mathcal{I}_1 \\
& + \frac{1}{1536\pi^4}m_bM^4\left\{m_b\left[(1+t^2)(e_d+e_u)\langle g_s^2G^2\rangle+144(-1+t^2)e_b m_b\pi^2(\langle\bar{d}d\rangle+\langle\bar{u}u\rangle)\right]\right\}
\end{aligned}$$

$$\begin{aligned}
& - (-1 + t^2)\pi^2 (e_d \langle \bar{d}d \rangle + e_u \langle \bar{u}u \rangle) \left[6m_b^2 \left(2i_1(\mathcal{S}, 1) - 2(i_1(\tilde{\mathcal{S}}, 1) - 2i_1(\mathcal{T}_1, 1) + i_1(\mathcal{T}_2, 1) + i_1(\mathcal{T}_4, 1)) \right. \right. \\
& - \left. \left. 6i_1(\mathcal{S}, v) - 2i_1(\mathcal{T}_3, v) + 2i_1(\mathcal{T}_4, v) \right) - \mathbb{A}'(u_0) \right] + \langle g_s^2 G^2 \rangle \chi \varphi'_\gamma(u_0) \Big] \mathcal{I}_2 \\
& - \frac{e^{-m_b^2/M^2}}{4608m_b\pi^2} (-1 + t^2) \langle g_s^2 G^2 \rangle M^2 (e_d \langle \bar{d}d \rangle + e_u \langle \bar{u}u \rangle) \left[i_1(\mathcal{T}_1, 1) - i_1(\mathcal{T}_3, 1) \right. \\
& + \left. 6i_1(\mathcal{S}, v) + 2i_1(\mathcal{T}_3, v) - 2i_1(\mathcal{T}_4, v) \right] \\
& + \frac{e^{-m_b^2/M^2}}{9216\pi^2} (e_d + e_u) f_{3\gamma} \langle g_s^2 G^2 \rangle M^2 \left[(1 + 6t + t^2) i_2(\mathcal{A}, v) + (3 + 2t + 3t^2) i_2(\mathcal{V}, v) \right] \\
& + \frac{e^{-m_b^2/M^2}}{2304\pi^2} f_{3\gamma} M^2 \left[- (3 + 2t + 3t^2) (e_d + e_u) \langle g_s^2 G^2 \rangle - 288(-1 + t^2) m_b \pi^2 (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \right] \psi^v(u_0) \\
& + \frac{1}{256\pi^2} (-1 + t^2) m_b M^2 \left\{ e_u \langle \bar{d}d \rangle \langle g_s^2 G^2 \rangle - \langle \bar{d}d \rangle (7e_b + 12e_u) m_0^2 m_b^2 \right. \\
& + \left. \left[e_d \langle g_s^2 G^2 \rangle - (7e_b + 12e_d) m_0^2 m_b^2 \right] \langle \bar{u}u \rangle \right] \mathcal{I}_2 \\
& + \frac{e^{-m_b^2/M^2}}{9216m_b\pi^2} (-1 + t^2) M^2 \left[- 36 (\langle g_s^2 G^2 \rangle - 6m_0^2 m_b^2) (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \right. \\
& + \left. \langle g_s^2 G^2 \rangle (e_d \langle \bar{d}d \rangle + e_u \langle \bar{u}u \rangle) \mathbb{A}'(u_0) \right] \\
& + \frac{e^{-m_b^2/M^2}}{9216\pi^2} f_{3\gamma} M^2 \left[(3 + 2t + 3t^2) (e_d + e_u) \langle g_s^2 G^2 \rangle + 288(-1 + t^2) m_b \pi^2 (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \right] \psi^{a'}(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{768M^2\pi^2} (-1 + t^2) m_b (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \left[\langle g_s^2 G^2 \rangle m_0^2 + \pi^2 f_{3\gamma} (\langle g_s^2 G^2 \rangle - 6m_0^2 m_b^2) \right. \\
& \times \left. (-4\psi^v(u_0) + \psi^{a'}(u_0)) \right] \\
& + \frac{e^{-m_b^2/M^2}}{9216M^4\pi^2} (-1 + t^2) \langle g_s^2 G^2 \rangle m_b (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \left[- 3m_0^2 m_b^2 \right. \\
& - \left. 2\pi^2 f_{3\gamma} (3m_0^2 - 2m_b^2) (4\psi^v(u_0) - \psi^{a'}(u_0)) \right] \\
& + \frac{e^{-m_b^2/M^2}}{1536M^6} (-1 + t^2) f_{3\gamma} \langle g_s^2 G^2 \rangle m_0^2 m_b^3 (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) (4\psi^v(u_0) - \psi^{a'}(u_0)) \\
& - \frac{e^{-m_b^2/M^2}}{9216M^8} (-1 + t^2) f_{3\gamma} \langle g_s^2 G^2 \rangle m_0^2 m_b^5 (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) (4\psi^v(u_0) - \psi^{a'}(u_0)) \\
& + \frac{e^{-m_b^2/M^2}}{18432\pi^2} (-1 + t^2) \langle g_s^2 G^2 \rangle m_b (\langle \bar{d}d \rangle e_d + e_u \langle \bar{u}u \rangle) \left[2i_1(\mathcal{T}_1, 1) - 2i_1(\mathcal{T}_3, 1) + 12i_1(\mathcal{S}, v) \right. \\
& + \left. 4i_1(\mathcal{T}_3, v) - 4i_1(\mathcal{T}_4, v) - \mathbb{A}'(u_0) \right] \\
& - \frac{e^{-m_b^2/M^2}}{1536m_b\pi^2} (-1 + t^2) (e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \left[\langle g_s^2 G^2 \rangle (m_0^2 - 2m_b^2) + 12f_{3\gamma} m_0^2 m_b^2 \pi^2 (4\psi^v(u_0) - \psi^{a'}(u_0)) \right].
\end{aligned}$$

FOR THE $\Xi_b^0 \rightarrow \Xi_b^0$ TRANSITION MAGNETIC MOMENT

1) Coefficient of the $(\varepsilon \cdot p)I$ structure

$$\begin{aligned}
\Pi_1^B = & -\frac{1}{32\sqrt{3}\pi^2}(-1+t)m_b^2M^4 \left\{ 4(2+t)m_b^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \mathcal{I}_3 \right. \\
& + e_b (\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) \left[(7+3t)\mathcal{I}_2 - 2(3+t)m_b^2\mathcal{I}_3 \right] \left. \right\} \\
& + \frac{\sqrt{3}}{8\pi^2}(-1+t)m_b^4M^4 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \mathcal{I}_3 \tilde{j}(h_\gamma) \\
& + \frac{1}{16\sqrt{3}\pi^2}(-1+t)(3+t)(e_s - e_u) f_{3\gamma} m_b^3 M^4 (\mathcal{I}_2 - m_b^2 \mathcal{I}_3) \psi^v(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{768\sqrt{3}\pi^2}(-1+t)M^2 \left\{ 12m_0^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left[4+t \left(2+m_b^2 e^{m_b^2/M^2} \mathcal{I}_2 \right) \right] \right. \\
& + e_b (\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) \left[24(7+3t)m_b^2 e^{m_b^2/M^2} (-\mathcal{I}_1 + m_b^2 \mathcal{I}_2) + m_0^2 \left(7(1+t) + (29+17t)m_b^2 e^{m_b^2/M^2} \mathcal{I}_2 \right) \right] \left. \right\} \\
& + \frac{e^{-m_b^2/M^2}}{1152\sqrt{3}m_b\pi^2}(-1+t) f_{3\gamma} M^2 \left\{ e_u \left[-96(1+t)m_b\pi^2 \langle \bar{s}s \rangle - (3+t) \langle g_s^2 G^2 \rangle \left(-1+3m_b^2 e^{m_b^2/M^2} \mathcal{I}_2 \right) \right] \right. \\
& + e_s \left[96(1+t)m_b\pi^2 \langle \bar{u}u \rangle + (3+t) \langle g_s^2 G^2 \rangle \left(-1+3m_b^2 e^{m_b^2/M^2} \mathcal{I}_2 \right) \right] \left. \right\} \psi^v(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{48\sqrt{3}M^2}(-1+t^2) f_{3\gamma} m_0^2 m_b^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \psi^v(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{6912\sqrt{3}M^4\pi^2}(-1+t) \langle g_s^2 G^2 \rangle m_b^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left[-3(2+t)m_0^2 + 8(1+t) f_{3\gamma} \pi^2 \psi^v(u_0) \right] \\
& + \frac{e^{-m_b^2/M^2}}{1728\sqrt{3}M^6}(-1+t^2) f_{3\gamma} \langle g_s^2 G^2 \rangle m_0^2 m_b^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \psi^v(u_0) \\
& - \frac{e^{-m_b^2/M^2}}{3456\sqrt{3}M^8}(-1+t^2) f_{3\gamma} \langle g_s^2 G^2 \rangle m_0^2 m_b^4 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \psi^v(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{2304\sqrt{3}\pi^2}(-1+t) \left[4(2+t) \langle g_s^2 G^2 \rangle (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) + 3(29+17t) e_b m_0^2 m_b^2 e^{m_b^2/M^2} (\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) \mathcal{I}_1 \right] \\
& - \frac{e^{-m_b^2/M^2}}{192\sqrt{3}\pi^2}(-1+t) \langle g_s^2 G^2 \rangle (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \tilde{j}(h_\gamma) \\
& - \frac{e^{-m_b^2/M^2}}{288\sqrt{3}}(-1+t^2) f_{3\gamma} m_0^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \psi^v(u_0) .
\end{aligned}$$

2) Coefficient of the $(\varepsilon \cdot p) \not{p}$ structure

$$\begin{aligned}
\Pi_2^B &= \frac{1}{8\sqrt{3}\pi^2}(-2+t+t^2)m_b^3M^2 \left[(e_b+e_u)\langle\bar{s}s\rangle - (e_b+e_s)\langle\bar{u}u\rangle \right] (\mathcal{I}_2 - m_b^2\mathcal{I}_3) \\
&\quad - \frac{e^{-m_b^2/M^2}}{1152\sqrt{3}m_bM^2\pi^2}(-2+t+t^2)\langle g_s^2G^2\rangle m_0^2 (e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle) \\
&\quad + \frac{e^{-m_b^2/M^2}}{2304\sqrt{3}M^4\pi^2}(-2+t+t^2)\langle g_s^2G^2\rangle m_0^2 m_b (e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle) \\
&\quad - \frac{e^{-m_b^2/M^2}}{576\sqrt{3}m_b\pi^2}(-1+t)(2+t)\langle g_s^2G^2\rangle (e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle) \\
&\quad + \frac{\sqrt{3}}{64\pi^2}(-1+t^2)m_0^2 m_b (e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle) \mathcal{I}_1 \\
&\quad + \frac{1}{192\sqrt{3}\pi^2}(-1+t)m_b \left\{ \left[(2+t)e_u\langle g_s^2G^2\rangle - 3(3+2t)e_b m_0^2 m_b^2 - 3(7+5t)e_u m_0^2 m_b^2 \right] \langle\bar{s}s\rangle \right. \\
&\quad \left. + \left[-(2+t)e_s\langle g_s^2G^2\rangle + 3(3+2t)e_b m_0^2 m_b^2 + 3(7+5t)e_s m_0^2 m_b^2 \right] \langle\bar{u}u\rangle \right\} \mathcal{I}_2 .
\end{aligned}$$

3) Coefficient of the $\not{p}\not{z}$ structure

$$\begin{aligned}
\Pi_3^B &= \frac{1}{256\sqrt{3}\pi^4}(-1+t)m_b^3M^6 \left[-3(3+t)(e_s-e_u) (\mathcal{I}_2 - 2m_b^2\mathcal{I}_3 + m_b^4\mathcal{I}_4) \right. \\
&\quad \left. + 8(-1+t)m_b\pi^2 (e_s\langle\bar{s}s\rangle - e_u\langle\bar{u}u\rangle) \chi (\mathcal{I}_3 - m_b^2\mathcal{I}_4) \varphi'_\gamma(u_0) \right] \\
&\quad - \frac{1}{768\sqrt{3}\pi^4}(-1+t)(3+t)m_b^3M^4 \left[\langle g_s^2G^2\rangle (e_u-e_s) + 24(e_b-e_u)m_b\pi^2\langle\bar{s}s\rangle \right. \\
&\quad \left. + 24(-e_b+e_s)m_b\pi^2\langle\bar{u}u\rangle \right] \mathcal{I}_3 \\
&\quad + \frac{1}{1024\sqrt{3}\pi^4}(e_s-e_u)m_bM^4 \left\{ -(-1+t)(3+t)\langle g_s^2G^2\rangle \mathcal{I}_2 + 16f_{3\gamma}m_b^2\pi^2 (-\mathcal{I}_2 + m_b^2\mathcal{I}_3) \right. \\
&\quad \left. \times \left[2(-1+t)(3+t)\psi^v(u_0) - (-1+t^2)\psi^{a'}(u_0) \right] \right\} \\
&\quad + \frac{1}{128\sqrt{3}\pi^2}(-1+t)m_b^2M^4 (e_s\langle\bar{s}s\rangle - e_u\langle\bar{u}u\rangle) \mathcal{I}_2 \left\{ (5+t)i_1(\mathcal{S}, 1) + (1+5t)i_1(\tilde{\mathcal{S}}, 1) \right. \\
&\quad + 2i_1(\mathcal{T}_1, 1) + i_1(\mathcal{T}_2, 1) + 2i_1(\mathcal{T}_3, 1) - 5i_1(\mathcal{T}_4, 1) - 6i_1(\mathcal{S}, v) - 2i_1(\tilde{\mathcal{S}}, v) \\
&\quad - t \left[2i_1(\mathcal{T}_1, 1) - 5i_1(\mathcal{T}_2, 1) + 2i_1(\mathcal{T}_3, 1) + i_1(\mathcal{T}_4, 1) + 2i_1(\mathcal{S}, v) + 6i_1(\tilde{\mathcal{S}}, v) \right. \\
&\quad \left. + 4i_1(\mathcal{T}_2, v) - 4i_1(\mathcal{T}_3, v) \right] - 4i_1(\mathcal{T}_3, v) + 4i_1(\mathcal{T}_4, v) \left. \right\} \\
&\quad - \frac{1}{128\sqrt{3}\pi^2}(-1+t)m_b^4M^4 (e_s\langle\bar{s}s\rangle - e_u\langle\bar{u}u\rangle) \mathcal{I}_3 \left\{ 4(2+t)i_1(\mathcal{S}, 1) + (4+8t)i_1(\tilde{\mathcal{S}}, 1) \right. \\
&\quad \left. - 4 \left[(-1+t)i_1(\mathcal{T}_1, 1) - i_1(\mathcal{T}_2, 1) + 2i_1(\mathcal{T}_4, 1) + 3i_1(\mathcal{S}, v) + i_1(\tilde{\mathcal{S}}, v) + i_1(\mathcal{T}_2, v) \right] \right\}
\end{aligned}$$

$$\begin{aligned}
& + t \left(-2i_1(\mathcal{T}_2, 1) + i_1(\mathcal{T}_4, 1) + i_1(\mathcal{S}, v) + 3i_1(\tilde{\mathcal{S}}, v) + i_1(\mathcal{T}_2, v) \right) \Big] \\
& + 4(1+t)i_1(\mathcal{T}_4, v) + 8(2+t)\tilde{j}(h_\gamma) + (-1+t)\mathbb{A}'(u_0) \Big\} \\
& + \frac{e^{-m_b^2/M^2}}{768\sqrt{3}\pi^2} (-1+t)M^2 \left\{ m_0^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left[-6(3+t) + (7+t)m_b^2 e^{m_b^2/M^2} \mathcal{I}_2 \right] \right. \\
& + e_b (\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) \left[(11+5t)m_0^2 - 24(3+t)m_b^2 e^{m_b^2/M^2} (-\mathcal{I}_1 + m_b^2 \mathcal{I}_2) \right] \Big\} \\
& - \frac{e^{-m_b^2/M^2}}{2304\sqrt{3}m_b\pi^2} (-1+t)(3+t)f_{3\gamma}M^2 \left[- (e_s - e_u) \langle g_s^2 G^2 \rangle - 96m_b\pi^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \right. \\
& + 3(e_s - e_u) \langle g_s^2 G^2 \rangle m_b^2 e^{m_b^2/M^2} \mathcal{I}_2 \Big] \psi^v(u_0) \\
& - \frac{1}{2304\sqrt{3}\pi^2} (-1+t)^2 \langle g_s^2 G^2 \rangle m_b^2 M^2 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \chi \mathcal{I}_2 \varphi'_\gamma(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{4608\sqrt{3}m_b\pi^2} f_{3\gamma}M^2 \left[96t(-1+t)m_b\pi^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \right. \\
& + (-1+t^2)(e_s - e_u) \langle g_s^2 G^2 \rangle \left(-1 + 3m_b^2 e^{m_b^2/M^2} \mathcal{I}_2 \right) \Big] \psi^{a'}(u_0) \\
& - \frac{e^{-m_b^2/M^2}}{192\sqrt{3}M^2} (-1+t)f_{3\gamma}m_0^2 m_b^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left[2(3+t)\psi^v(u_0) + t\psi^{a'}(u_0) \right] \\
& + \frac{e^{-m_b^2/M^2}}{27648\sqrt{3}M^4\pi^2} (-1+t) \langle g_s^2 G^2 \rangle m_b^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left\{ 3(3+t)m_0^2 \right. \\
& - 8f_{3\gamma}\pi^2 \left[2(3+t)\psi^v(u_0) + t\psi^{a'}(u_0) \right] \Big\} \\
& - \frac{e^{-m_b^2/M^2}}{6912\sqrt{3}M^6} (-1+t)f_{3\gamma} \langle g_s^2 G^2 \rangle m_0^2 m_b^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left[2(3+t)\psi^v(u_0) + t\psi^{a'}(u_0) \right] \\
& + \frac{e^{-m_b^2/M^2}}{13824\sqrt{3}M^8} (-1+t)f_{3\gamma} \langle g_s^2 G^2 \rangle m_0^2 m_b^4 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left[2(3+t)\psi^v(u_0) + t\psi^{a'}(u_0) \right] \\
& + \frac{e^{-m_b^2/M^2}}{9216\sqrt{3}\pi^2} (-1+t) \langle g_s^2 G^2 \rangle (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \left\{ 3(1+t)i_1(\mathcal{S}, 1) + 3(1+t)i_1(\tilde{\mathcal{S}}, 1) \right. \\
& + 2i_1(\mathcal{T}_1, 1) + 3i_1(\mathcal{T}_2, 1) - 2i_1(\mathcal{T}_3, 1) - 3i_1(\mathcal{T}_4, 1) - 6i_1(\mathcal{S}, v) - 2i_1(\tilde{\mathcal{S}}, v) - 4i_1(\mathcal{T}_2, v) \\
& + 4i_1(\mathcal{T}_3, v) + 16\tilde{j}(h_\gamma) - \mathbb{A}'(u_0) + t \left[-2i_1(\mathcal{T}_1, 1) + 3i_1(\mathcal{T}_2, 1) + 2i_1(\mathcal{T}_3, 1) - 3i_1(\mathcal{T}_4, 1) \right. \\
& - 2i_1(\mathcal{S}, v) - 6i_1(\tilde{\mathcal{S}}, v) - 4i_1(\mathcal{T}_3, v) + 4i_1(\mathcal{T}_4, v) + 8\tilde{j}(h_\gamma) + \mathbb{A}'(u_0) \Big] \Big\} \\
& - \frac{e^{-m_b^2/M^2}}{2304\sqrt{3}\pi^2} (-1+t) \left\{ (3+t) \langle g_s^2 G^2 \rangle (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) + 3(11+5t)e_b m_0^2 m_b^2 e^{m_b^2/M^2} (\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) \mathcal{I}_1 \right. \\
& + 2f_{3\gamma}m_0^2\pi^2 (e_u \langle \bar{s}s \rangle - e_s \langle \bar{u}u \rangle) \left[2(11+5t)\psi^v(u_0) + (2+5t)\psi^{a'}(u_0) \right] \Big\} .
\end{aligned}$$

4) Coefficient of the \not{e} structure

$$\begin{aligned}
\Pi_4^B = & \frac{\sqrt{3}}{32\pi^4}(1+t+t^2)(e_s - e_u)m_b^4 M^8 (-\mathcal{I}_3 + m_b^2 \mathcal{I}_4) \\
& + \frac{1}{128\sqrt{3}\pi^2}(e_s - e_u)f_{3\gamma}m_b^2 M^6 [-3(1+t)^2 \mathcal{I}_2 + 4(1+t+t^2)m_b^2 \mathcal{I}_3] i_2(\mathcal{A}, v) \\
& + \frac{1}{128\sqrt{3}\pi^2}(e_s - e_u)f_{3\gamma}m_b^2 M^6 [-3(1+t)^2 \mathcal{I}_2 + 2(1+4t+t^2)m_b^2 \mathcal{I}_3] i_2(\mathcal{V}, v) \\
& - \frac{1}{32\sqrt{3}\pi^2}(1+t+t^2)(e_s - e_u)f_{3\gamma}m_b^4 M^6 \mathcal{I}_3 [4\psi^v(u_0) - \psi^{a'}(u_0)] \\
& + \frac{\sqrt{3}}{64\pi^2}(-1+t^2)m_b^3 M^6 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \chi (\mathcal{I}_2 - m_b^2 \mathcal{I}_3) \varphi'_\gamma(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{1536\sqrt{3}m_b\pi^2}(-1+t^2)\langle g_s^2 G^2 \rangle M^4 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \chi \left(-1 + 3m_b^2 e^{m_b^2/M^2} \mathcal{I}_2 \right) \varphi'_\gamma(u_0) \\
& - \frac{1}{16\sqrt{3}\pi^2}(-2+t+t^2)m_b^5 M^4 [(e_b + e_u)\langle \bar{s}s \rangle - (e_b + e_s)\langle \bar{u}u \rangle] \mathcal{I}_3 \\
& - \frac{\sqrt{3}}{128\pi^2}(-1+t^2)m_b M^4 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \mathcal{I}_1 [i_1(\mathcal{T}_1, 1) + i_1(\mathcal{T}_3, 1)] \\
& - \frac{1}{128\sqrt{3}\pi^2}(-1+t)m_b M^4 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \mathcal{I}_1 \left\{ (5+t)i_1(\mathcal{S}, 1) \right. \\
& \left. - (1+5t) [i_1(\tilde{\mathcal{S}}, 1) + i_1(\mathcal{T}_2, 1)] - (5+t)i_1(\mathcal{T}_4, 1) \right\} \\
& - \frac{1}{768\sqrt{3}\pi^4}m_b^2 M^4 \mathcal{I}_2 \left\{ -(1+t+t^2)e_s \langle g_s^2 G^2 \rangle + (1+t+t^2)e_u \langle g_s^2 G^2 \rangle \right. \\
& \left. - 48(-2+t+t^2)e_b m_b \pi^2 (\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) + 3(-1+t)m_b \pi^2 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \right. \\
& \left. \times [2(-1+7t) (i_1(\tilde{\mathcal{S}}, 1) + i_1(\mathcal{T}_2, 1))] + 2(-7+t) (i_1(\mathcal{S}, 1) - i_1(\mathcal{T}_4, 1)) \right. \\
& \left. + 8(2+t)i_1(\mathcal{S}, v) - 4(-1+t)i_1(\tilde{\mathcal{S}}, v) + 8ti_1(\mathcal{T}_4, v) + 4(3+t) (i_1(\mathcal{T}_2, v) - 2\tilde{j}(h_\gamma)) \right. \\
& \left. + 3(1+t) (-4(i_1(\mathcal{T}_1, 1) + i_1(\mathcal{T}_3, v)) + \mathbb{A}'(u_0)) \right\} \\
& - \frac{e^{-m_b^2/M^2}}{9216\sqrt{3}\pi^2}(-1+t)^2(e_s - e_u)f_{3\gamma}\langle g_s^2 G^2 \rangle M^2 [i_2(\mathcal{A}, v) - i_2(\mathcal{V}, v)] \\
& - \frac{e^{-m_b^2/M^2}}{9216\sqrt{3}m_b\pi^2}(-1+t)\langle g_s^2 G^2 \rangle M^2 (e_s \langle \bar{s}s \rangle - e_u \langle \bar{u}u \rangle) \left\{ 4(-1+t)i_1(\mathcal{S}, 1) \right. \\
& + 2 \left[2(-1+t)i_1(\tilde{\mathcal{S}}, 1) - 3(1+t)i_1(\mathcal{T}_1, 1) - 2i_1(\mathcal{T}_2, 1) + 3i_1(\mathcal{T}_3, 1) \right. \\
& + 2(i_1(\mathcal{T}_4, 1) + 4i_1(\mathcal{S}, v) + i_1(\tilde{\mathcal{S}}, v) + 3i_1(\mathcal{T}_2, v) - 3i_1(\mathcal{T}_3, v) - 6\tilde{j}(h_\gamma)) \\
& + t(2i_1(\mathcal{T}_2, 1) + 3i_1(\mathcal{T}_3, 1) - 2i_1(\mathcal{T}_4, 1) + 4i_1(\mathcal{S}, v) - 2i_1(\tilde{\mathcal{S}}, v) + 2i_1(\mathcal{T}_2, v) \\
& \left. \left. - 6i_1(\mathcal{T}_3, v) + 4i_1(\mathcal{T}_4, v) - 4\tilde{j}(h_\gamma)) \right] + 3(1+t)\mathbb{A}'(u_0) \right\}
\end{aligned}$$

$$\begin{aligned}
& - \frac{1}{128\sqrt{3}\pi^2}(-1+t)(7+5t)m_0^2m_b^3M^2(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\mathcal{I}_2 \\
& - \frac{e^{-m_b^2/M^2}}{384\sqrt{3}m_b\pi^2}(-2+t+t^2)M^2(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\left[\langle g_s^2G^2\rangle\left(1 - e^{m_b^2/M^2}m_b^2\mathcal{I}_2\right)\right. \\
& \left. - 6m_0^2m_b^2 + 32f_{3\gamma}m_b^2\pi^2\psi^v(u_0)\right] \\
& + \frac{e^{-m_b^2/M^2}}{2304\sqrt{3}\pi^2}(1+t+t^2)(e_s - e_u)f_{3\gamma}\langle g_s^2G^2\rangle M^2[4\psi^v(u_0) - \psi^{a'}(u_0)] \\
& + \frac{e^{-m_b^2/M^2}}{48\sqrt{3}}(1+t-2t^2)e_sf_{3\gamma}m_bM^2\langle\bar{u}u\rangle\psi^{a'}(u_0) \\
& + \frac{e^{-m_b^2/M^2}}{384\sqrt{3}\pi^2}(-1+t)m_bM^2\left[-e^{m_b^2/M^2}3(3+2t)e_b m_0^2m_b^2(\langle\bar{s}s\rangle - \langle\bar{u}u\rangle)\mathcal{I}_2\right. \\
& \left. + 8(1+2t)e_uf_{3\gamma}\pi^2\langle\bar{s}s\rangle\psi^{a'}(u_0)\right] \\
& + \frac{e^{-m_b^2/M^2}}{1152\sqrt{3}M^2\pi^2}(-1+t)m_b(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\left\{(2+t)\langle g_s^2G^2\rangle m_0^2\right. \\
& \left. + f_{3\gamma}(\langle g_s^2G^2\rangle - 6m_0^2m_b^2)\pi^2\left[-4(2+t)\psi^v(u_0) + (1+2t)\psi^{a'}(u_0)\right]\right\} \\
& - \frac{e^{-m_b^2/M^2}}{13824\sqrt{3}M^4\pi^2}(-1+t)\langle g_s^2G^2\rangle m_b(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\left\{3(2+t)m_0^2m_b^2\right. \\
& \left. + 2f_{3\gamma}(3m_0^2 - 2m_b^2)\pi^2\left[4(2+t)\psi^v(u_0) - (1+2t)\psi^{a'}(u_0)\right]\right\} \\
& + \frac{e^{-m_b^2/M^2}}{2304\sqrt{3}M^6}(-1+t)f_{3\gamma}\langle g_s^2G^2\rangle m_0^2m_b^3(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\left[4(2+t)\psi^v(u_0) - (1+2t)\psi^{a'}(u_0)\right] \\
& - \frac{e^{-m_b^2/M^2}}{13824\sqrt{3}M^8}(-1+t)f_{3\gamma}\langle g_s^2G^2\rangle m_0^2m_b^5(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\left[4(2+t)\psi^v(u_0) - (1+2t)\psi^{a'}(u_0)\right] \\
& + \frac{e^{-m_b^2/M^2}}{18432\sqrt{3}\pi^2}(-1+t)\langle g_s^2G^2\rangle m_b(e_s\langle\bar{s}s\rangle - e_u\langle\bar{u}u\rangle)\left\{4(-1+t)i_1(\mathcal{S}, 1)\right. \\
& + 2\left[2(-1+t)i_1(\tilde{\mathcal{S}}, 1) - 3(1+t)i_1(\mathcal{T}_1, 1) - 2i_1(\mathcal{T}_2, 1) + 3i_1(\mathcal{T}_3, 1) + 2i_1(\mathcal{T}_4, 1) + 8i_1(\mathcal{S}, v)\right. \\
& + 2i_1(\tilde{\mathcal{S}}, v) + 6i_1(\mathcal{T}_2, v) - 6i_1(\mathcal{T}_3, v) - 12\tilde{j}(h_\gamma) + t\left(2i_1(\mathcal{T}_2, 1) + 3i_1(\mathcal{T}_3, 1) - 2i_1(\mathcal{T}_4, 1)\right. \\
& \left. + 4i_1(\mathcal{S}, v) - 2i_1(\tilde{\mathcal{S}}, v) + 2i_1(\mathcal{T}_2, v) - 6i_1(\mathcal{T}_3, v) + 4i_1(\mathcal{T}_4, v) - 4\tilde{j}(h_\gamma)\right]\left. + 3(1+t)\mathbb{A}'(u_0)\right\} \\
& + \frac{e^{-m_b^2/M^2}}{2304\sqrt{3}m_b\pi^2}(e_u\langle\bar{s}s\rangle - e_s\langle\bar{u}u\rangle)\left\{-(-2+t+t^2)\langle g_s^2G^2\rangle(m_0^2 - 2m_b^2)\right. \\
& \left. + 18(-1+t^2)f_{3\gamma}m_0^2m_b^2\pi^2\left[-4\psi^v(u_0) + \psi^{a'}(u_0)\right]\right\}.
\end{aligned}$$

The functions i_n ($n = 1, 2$), and $\tilde{j}_1(f(u))$ appearing in the invariant functions above are defined as:

$$i_1(\phi, f(v)) = \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) \delta'(k - u_0),$$

$$\begin{aligned}
i_2(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) \delta''(k - u_0) , \\
\tilde{j}(f(u)) &= \int_{u_0}^1 du f(u) , \\
\mathcal{I}_n &= \int_{m_b^2}^{\infty} ds \frac{e^{-s/M^2}}{s^n} ,
\end{aligned}$$

where

$$k = \alpha_q + \alpha_g \bar{v} , \quad u_0 = \frac{M_1^2}{M_1^2 + M_2^2} , \quad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2} .$$

Appendix C

In this appendix we give the expressions of the invariant amplitudes Π_1^M and Π_2^M entering into the mass sum rule for the Σ_b^0 and Ξ_b' or Ξ_b baryons. Here the masses of the light quarks are neglected.

A) FOR THE Σ_b^0 BARYON

$$\begin{aligned}
\Pi_1^M &= \frac{3}{256\pi^4} \left\{ -m_b^4 M^6 [5 + t(2 + 5t)] [m_b^4 \mathcal{I}_5 - 2m_b^2 \mathcal{I}_4 + \mathcal{I}_3] \right\} \\
&+ \frac{1}{192\pi^4} m_b^4 M^2 [\langle g_s^2 G^2 \rangle (1 + t + t^2) - 18m_b \pi^2 (-1 + t^2) (\langle \bar{d}d \rangle + \langle \bar{u}u \rangle)] \mathcal{I}_3 \\
&+ \frac{1}{3072\pi^4} m_b^2 M^2 [-\langle g_s^2 G^2 \rangle (13 + 10t + 13t^2) + 288m_b \pi^2 (-1 + t^2) (\langle \bar{d}d \rangle + \langle \bar{u}u \rangle)] \mathcal{I}_2 \\
&+ \frac{e^{-m_b^2/M^2}}{73728m_b M^2 \pi^4} \left\{ -\langle g_s^2 G^2 \rangle^2 m_b (1 + t)^2 + 768m_b m_0^2 \langle \bar{d}d \rangle \langle \bar{u}u \rangle \pi^4 (-1 + t)^2 \right. \\
&- 56 \langle g_s^2 G^2 \rangle m_0^2 \pi^2 (-1 + t^2) (\langle \bar{d}d \rangle + \langle \bar{u}u \rangle) \left. \right\} \\
&+ \frac{1}{768M^2 \pi^2} \langle g_s^2 G^2 \rangle m_b (-1 + t^2) (\langle \bar{d}d \rangle + \langle \bar{u}u \rangle) \mathcal{I}_1 \\
&+ \frac{e^{-m_b^2/M^2}}{18432M^4 \pi^2} m_b m_0^2 [\langle g_s^2 G^2 \rangle (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) (-1 + t^2) + 384m_b \langle \bar{d}d \rangle \langle \bar{u}u \rangle \pi^2 (-1 + t)^2] \\
&+ \frac{e^{-m_b^2/M^2}}{1728M^6} m_b^2 \langle g_s^2 G^2 \rangle (-1 + t)^2 \langle \bar{d}d \rangle \langle \bar{u}u \rangle \\
&+ \frac{e^{-m_b^2/M^2}}{1728M^8} m_b^2 m_0^2 \langle g_s^2 G^2 \rangle (-1 + t)^2 \langle \bar{d}d \rangle \langle \bar{u}u \rangle \\
&- \frac{e^{-m_b^2/M^2}}{3456M^{10}} m_b^4 m_0^2 \langle g_s^2 G^2 \rangle (-1 + t)^2 \langle \bar{d}d \rangle \langle \bar{u}u \rangle \\
&- \frac{e^{-m_b^2/M^2}}{768m_b \pi^2} [\langle g_s^2 G^2 \rangle (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) (-1 + t^2) + 32m_b \langle \bar{d}d \rangle \langle \bar{u}u \rangle \pi^2 (-1 + t)^2] \\
&+ \frac{1}{256\pi^2} m_b (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) (-1 + t^2) [(\langle g_s^2 G^2 \rangle - 13m_b^2 m_0^2) \mathcal{I}_2 + 6m_0^2 \mathcal{I}_1] ,
\end{aligned}$$

$$\begin{aligned}
\Pi_2^M &= -\frac{3}{256\pi^4} \left\{ -m_b^3 M^6 (-1 + t)^2 [m_b^4 \mathcal{I}_4 - 2m_b^2 \mathcal{I}_3 + \mathcal{I}_2] \right\} \\
&+ \frac{1}{3072\pi^4} m_b M^4 \left\{ 4m_b^2 [\langle g_s^2 G^2 \rangle (-1 + t)^2 + 72m_b (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) \pi^2 (-1 + t^2)] \mathcal{I}_3 - 3 \langle g_s^2 G^2 \rangle (-1 + t)^2 \mathcal{I}_2 \right\} \\
&- \frac{7e^{-m_b^2/M^2}}{256\pi^2} m_0^2 M^2 (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) (-1 + t^2) \\
&+ \frac{1}{1024\pi^4} m_b M^2 \left\{ m_b [3m_b \langle g_s^2 G^2 \rangle (-1 + t)^2 + 4m_0^2 (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) \pi^2 (-1 + t^2)] \mathcal{I}_2 - 2 \langle g_s^2 G^2 \rangle (-1 + t)^2 \mathcal{I}_1 \right\} \\
&- \frac{e^{-m_b^2/M^2}}{73728M^2 \pi^4} m_b [\langle g_s^2 G^2 \rangle^2 (-1 + t)^2 + 1536m_0^2 \langle \bar{d}d \rangle \langle \bar{u}u \rangle \pi^4 (3 + 2t + 3t^2)]
\end{aligned}$$

$$\begin{aligned}
& + \frac{e^{-m_b^2/M^2}}{18432M^4\pi^2} m_b [-11m_b m_0^2 \langle g_s^2 G^2 \rangle (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) (-1 + t^2) \\
& - 32 (\langle g_s^2 G^2 \rangle - 12m_0^2 m_b^2) \langle \bar{d}d \rangle \langle \bar{u}u \rangle (5 + 2t + 5t^2)] \\
& + \frac{e^{-m_b^2/M^2}}{1728M^6} m_b (m_b^2 - 3m_0^2) \langle g_s^2 G^2 \rangle \langle \bar{d}d \rangle \langle \bar{u}u \rangle (5 + 2t + 5t^2) \\
& + \frac{e^{-m_b^2/M^2}}{576M^8} m_b^3 m_0^2 \langle g_s^2 G^2 \rangle \langle \bar{d}d \rangle \langle \bar{u}u \rangle (5 + 2t + 5t^2) \\
& - \frac{e^{-m_b^2/M^2}}{3456M^{10}} m_b^5 m_0^2 \langle g_s^2 G^2 \rangle \langle \bar{d}d \rangle \langle \bar{u}u \rangle (5 + 2t + 5t^2) \\
& + \frac{e^{-m_b^2/M^2}}{36864m_b\pi^4} [\langle g_s^2 G^2 \rangle^2 (-1 + t)^2 - 1536m_b^2 \langle \bar{d}d \rangle \langle \bar{u}u \rangle \pi^4 (5 + 2t + 5t^2) \\
& + 96m_b \langle g_s^2 G^2 \rangle (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) \pi^2 (-1 + t^2)] . \tag{1}
\end{aligned}$$

B) FOR THE Ξ_b^0 BARYON

$$\begin{aligned}
\Pi_1^M & = \frac{3}{256\pi^4} \left\{ -m_b^4 M^6 [5 + t(2 + 5t)] [m_b^4 \mathcal{I}_5 - 2m_b^2 \mathcal{I}_4 + \mathcal{I}_3] \right\} \\
& + \frac{1}{192\pi^4} m_b^4 M^2 [\langle g_s^2 G^2 \rangle (1 + t + t^2) - 18m_b \pi^2 (-1 + t^2) (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle)] \mathcal{I}_3 \\
& + \frac{1}{3072\pi^4} m_b^2 M^2 [-\langle g_s^2 G^2 \rangle (13 + 10t + 13t^2) + 288m_b \pi^2 (-1 + t^2) (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle)] \mathcal{I}_2 \\
& + \frac{e^{-m_b^2/M^2}}{73728m_b M^2 \pi^4} \left\{ -\langle g_s^2 G^2 \rangle^2 m_b (1 + t)^2 + 768m_b m_0^2 \langle \bar{s}s \rangle \langle \bar{u}u \rangle \pi^4 (-1 + t)^2 \right. \\
& \left. - 56 \langle g_s^2 G^2 \rangle m_0^2 \pi^2 (-1 + t^2) (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle) \right\} \\
& + \frac{1}{768M^2 \pi^2} \langle g_s^2 G^2 \rangle m_b (-1 + t^2) (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle) \mathcal{E}_1 \\
& + \frac{e^{-m_b^2/M^2}}{18432M^4 \pi^2} m_b m_0^2 [\langle g_s^2 G^2 \rangle (\langle \bar{u}u \rangle + \langle \bar{s}s \rangle) (-1 + t^2) + 384m_b \langle \bar{s}s \rangle \langle \bar{u}u \rangle \pi^2 (-1 + t)^2] \\
& + \frac{e^{-m_b^2/M^2}}{1728M^6} m_b^2 \langle g_s^2 G^2 \rangle (-1 + t)^2 \langle \bar{s}s \rangle \langle \bar{u}u \rangle \\
& + \frac{e^{-m_b^2/M^2}}{1728M^8} m_b^2 m_0^2 \langle g_s^2 G^2 \rangle (-1 + t)^2 \langle \bar{s}s \rangle \langle \bar{u}u \rangle \\
& - \frac{e^{-m_b^2/M^2}}{3456M^{10}} m_b^4 m_0^2 \langle g_s^2 G^2 \rangle (-1 + t)^2 \langle \bar{s}s \rangle \langle \bar{u}u \rangle \\
& - \frac{e^{-m_b^2/M^2}}{768m_b \pi^2} [\langle g_s^2 G^2 \rangle (\langle \bar{u}u \rangle + \langle \bar{s}s \rangle) (-1 + t^2) + 32m_b \langle \bar{s}s \rangle \langle \bar{u}u \rangle \pi^2 (-1 + t)^2] \\
& + \frac{1}{256\pi^2} m_b (\langle \bar{u}u \rangle + \langle \bar{s}s \rangle) (-1 + t^2) [(\langle g_s^2 G^2 \rangle - 13m_b^2 m_0^2) \mathcal{I}_2 + 6m_0^2 \mathcal{I}_1] ,
\end{aligned}$$

$$\begin{aligned}
\Pi_2^M = & -\frac{3}{256\pi^4} \left\{ -m_b^3 M^6 (-1+t)^2 [m_b^4 \mathcal{I}_4 - 2m_b^2 \mathcal{I}_3 + \mathcal{I}_2] \right\} \\
& + \frac{1}{3072\pi^4} m_b M^4 \left\{ 4m_b^2 [\langle g_s^2 G^2 \rangle (-1+t)^2 + 72m_b (\langle \bar{u}u \rangle + \langle \bar{s}s \rangle) \pi^2 (-1+t^2)] \mathcal{I}_3 - 3\langle g_s^2 G^2 \rangle (-1+t)^2 \mathcal{I}_2 \right\} \\
& - \frac{7e^{-m_b^2/M^2}}{256\pi^2} m_0^2 M^2 (\langle \bar{u}u \rangle + \langle \bar{s}s \rangle) (-1+t^2) \\
& + \frac{1}{1024\pi^4} m_b M^2 \left\{ m_b [3m_b \langle g_s^2 G^2 \rangle (-1+t)^2 + 4m_0^2 (\langle \bar{u}u \rangle + \langle \bar{s}s \rangle) \pi^2 (-1+t^2)] \mathcal{I}_2 - 2\langle g_s^2 G^2 \rangle (-1+t)^2 \mathcal{I}_1 \right\} \\
& - \frac{e^{-m_b^2/M^2}}{73728M^2\pi^4} m_b [\langle g_s^2 G^2 \rangle^2 (-1+t)^2 + 1536m_0^2 \langle \bar{s}s \rangle \langle \bar{u}u \rangle \pi^4 (3+2t+3t^2)] \\
& + \frac{e^{-m_b^2/M^2}}{18432M^4\pi^2} m_b [-11m_b m_0^2 \langle g_s^2 G^2 \rangle (\langle \bar{u}u \rangle + \langle \bar{s}s \rangle) (-1+t^2) \\
& - 32 (\langle g_s^2 G^2 \rangle - 12m_0^2 m_b^2) \langle \bar{s}s \rangle \langle \bar{u}u \rangle (5+2t+5t^2)] \\
& + \frac{e^{-m_b^2/M^2}}{1728M^6} m_b (m_b^2 - 3m_0^2) \langle g_s^2 G^2 \rangle \langle \bar{s}s \rangle \langle \bar{u}u \rangle (5+2t+5t^2) \\
& + \frac{e^{-m_b^2/M^2}}{576M^8} m_b^3 m_0^2 \langle g_s^2 G^2 \rangle \langle \bar{s}s \rangle \langle \bar{u}u \rangle (5+2t+5t^2) \\
& - \frac{e^{-m_b^2/M^2}}{3456M^{10}} m_b^5 m_0^2 \langle g_s^2 G^2 \rangle \langle \bar{s}s \rangle \langle \bar{u}u \rangle (5+2t+5t^2) \\
& + \frac{e^{-m_b^2/M^2}}{36864m_b\pi^4} [\langle g_s^2 G^2 \rangle^2 (-1+t)^2 - 1536m_b^2 \langle \bar{s}s \rangle \langle \bar{u}u \rangle \pi^4 (5+2t+5t^2) \\
& + 96m_b \langle g_s^2 G^2 \rangle (\langle \bar{u}u \rangle + \langle \bar{s}s \rangle) \pi^2 (-1+t^2)] .
\end{aligned}$$

C) FOR THE Ξ_b^0 BARYON

$$\begin{aligned}
\Pi_1^M = & -\frac{1}{256\pi^4} (-3m_b^4 M^6 (5+2t+5t^2) (\mathcal{I}_3 - 2m_b^2 \mathcal{I}_4 + m_b^4 \mathcal{I}_5) \\
& + \frac{1}{3072\pi^4} m_b^2 M^2 [3\langle g_s^2 G^2 \rangle (1+t)^2 \mathcal{I}_2 - 16\langle g_s^2 G^2 \rangle m_b^2 (1+t+t^2) \mathcal{I}_3 \\
& - 32m_b \pi^2 (-1+t)(1+5t) (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle) (-\mathcal{I}_2 + m_b^2 \mathcal{I}_3)] \\
& + \frac{e^{-m_b^2/M^2}}{221184m_b M^2 \pi^4} \langle g_s^2 G^2 \rangle^2 m_b (13+10t+13t^2) + 768m_0^2 m_b \pi^4 (-1+t)(25+23t) \langle \bar{s}s \rangle \langle \bar{u}u \rangle \\
& - 8\langle g_s^2 G^2 \rangle \pi^2 (-1+t) (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle) [m_0^2 (1+5t) + 12m_b^2 e^{m_b^2/M^2} (5+t) \mathcal{I}_1] \\
& + \frac{e^{-m_b^2/M^2}}{55296M^4\pi^2} m_0^2 m_b (-1+t) [384m_b \pi^2 (13+11t) \langle \bar{s}s \rangle \langle \bar{u}u \rangle + \langle g_s^2 G^2 \rangle (31+11t) (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle)]
\end{aligned}$$

$$\begin{aligned}
& + \frac{e^{-m_b^2/M^2}}{5184M^6} \langle g_s^2 G^2 \rangle m_b^2 (-1+t)(13+11t) \langle \bar{s}s \rangle \langle \bar{u}u \rangle \\
& + \frac{e^{-m_b^2/M^2}}{5184M^8} \langle g_s^2 G^2 \rangle m_0^2 m_b^2 (-1+t)(13+11t) \langle \bar{s}s \rangle \langle \bar{u}u \rangle \\
& - \frac{e^{-m_b^2/M^2}}{10368M^{10}} \langle g_s^2 G^2 \rangle m_0^2 m_b^4 (-1+t)(13+11t) \langle \bar{s}s \rangle \langle \bar{u}u \rangle \\
& + \frac{e^{-m_b^2/M^2}}{6912m_b\pi^2} (-1+t) \left\{ \langle g_s^2 G^2 \rangle (1+5t) (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle) \left(-1 + 3m_b^2 e^{m_b^2/M^2} \mathcal{I}_2 \right) \right. \\
& \left. - 3m_b \left[32\pi^2 (13+11t) \langle \bar{s}s \rangle \langle \bar{u}u \rangle + 3m_0^2 m_b e^{m_b^2/M^2} (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle) [-6(1+t)\mathcal{I}_1 + m_b^2(7+11t)\mathcal{I}_2] \right] \right\} .
\end{aligned}$$

$$\begin{aligned}
\Pi_2^M & = -\frac{1}{256\pi^4} m_b^3 M^6 (-1+t)(13+11t) (\mathcal{I}_2 - 2m_b^2 \mathcal{I}_3 + m_b^4 \mathcal{I}_4) \\
& - \frac{1}{9216\pi^4} m_b M^4 (-1+t) \left[-96m_b^3 \pi^2 (1+5t) (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle) \mathcal{I}_3 + \langle g_s^2 G^2 \rangle (13+11t) (3\mathcal{I}_2 - 4m_b^2 \mathcal{I}_3) \right] \\
& - \frac{e^{-m_b^2/M^2}}{3072\pi^4} M^2 (-1+t) \left\{ 4m_0^2 \pi^2 (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle) \left[1 + 5t + m_b^2 e^{m_b^2/M^2} (5+t) \mathcal{I}_2 \right] \right. \\
& \left. + \langle g_s^2 G^2 \rangle m_b e^{m_b^2/M^2} [-2(-1+t)\mathcal{I}_1 + 3m_b^2(3+5t)\mathcal{I}_2] \right\} \\
& + \frac{e^{-m_b^2/M^2}}{221184M^2\pi^4} m_b (-1+t) \left[\langle g_s^2 G^2 \rangle^2 (11+13t) - 1536m_0^2 \pi^4 (-1+t) \langle \bar{s}s \rangle \langle \bar{u}u \rangle \right] \\
& + \frac{e^{-m_b^2/M^2}}{55296M^4\pi^2} m_b \left\{ 1152m_0^2 m_b^2 \pi^2 (5+2t+5t^2) \langle \bar{s}s \rangle \langle \bar{u}u \rangle \right. \\
& \left. - \langle g_s^2 G^2 \rangle \left[96\pi^2 (5+2t+5t^2) \langle \bar{s}s \rangle \langle \bar{u}u \rangle + m_0^2 m_b (-1+t)(29+t) (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle) \right] \right\} \\
& + \frac{e^{-m_b^2/M^2}}{1728M^6} \langle g_s^2 G^2 \rangle m_b (-3m_0^2 + m_b^2) (5+2t+5t^2) \langle \bar{s}s \rangle \langle \bar{u}u \rangle \\
& + \frac{e^{-m_b^2/M^2}}{576M^8} \langle g_s^2 G^2 \rangle m_0^2 m_b^3 (5+2t+5t^2) \langle \bar{s}s \rangle \langle \bar{u}u \rangle \\
& - \frac{e^{-m_b^2/M^2}}{3456M^{10}} \langle g_s^2 G^2 \rangle m_0^2 m_b^5 (5+2t+5t^2) \langle \bar{s}s \rangle \langle \bar{u}u \rangle \\
& + \frac{e^{-m_b^2/M^2}}{110592m_b\pi^4} \left[\langle g_s^2 G^2 \rangle^2 (11+2t-13t^2) - 4608m_b^2 \pi^4 (5+2t+5t^2) \langle \bar{s}s \rangle \langle \bar{u}u \rangle \right. \\
& \left. - 32 \langle g_s^2 G^2 \rangle m_b \pi^2 (-7+t)(-1+t) (\langle \bar{s}s \rangle + \langle \bar{u}u \rangle) \right] .
\end{aligned}$$

where

$$\mathcal{I}_n = \int_{m_b^2}^{\infty} ds \frac{e^{-s/M^2}}{s^n} .$$

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Figure captions

Fig. (1) The dependence of the magnetic moment of the Σ_c^0 baryon on M^2 , at several fixed values of t , and at $s_0 = 12.0 \text{ GeV}^2$, in units of nuclear magneton μ_N .

Fig. (2) The same as Fig. (1), but for the $\Sigma_b \rightarrow \Lambda_b$ transition at $s_0 = 40.0 \text{ GeV}^2$.

Fig. (3) The dependence of the magnetic moment of the Σ_c^0 baryon on $\cos\theta$, at several fixed values of M^2 , and at $s_0 = 12.0 \text{ GeV}^2$, in units of nuclear magneton μ_N .

Fig. (4) The same as Fig. (3), but for the $\Sigma_b \rightarrow \Lambda_b$ transition at $s_0 = 40.0 \text{ GeV}^2$.

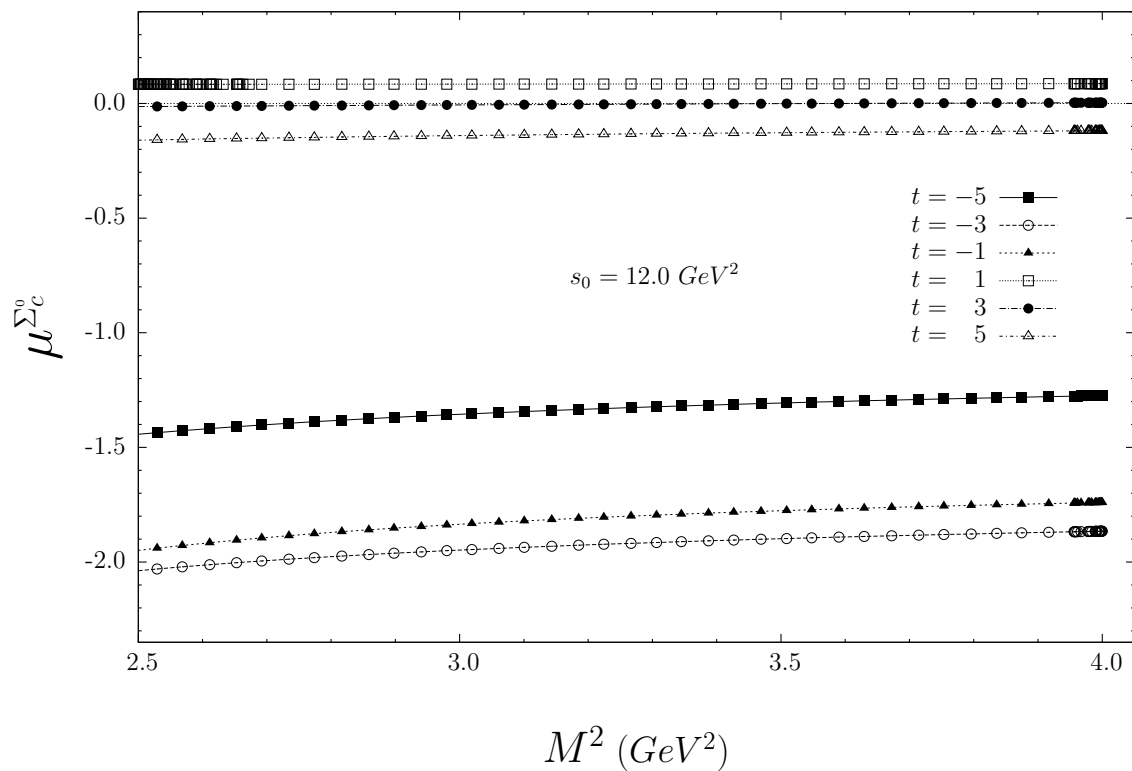


Figure 1:

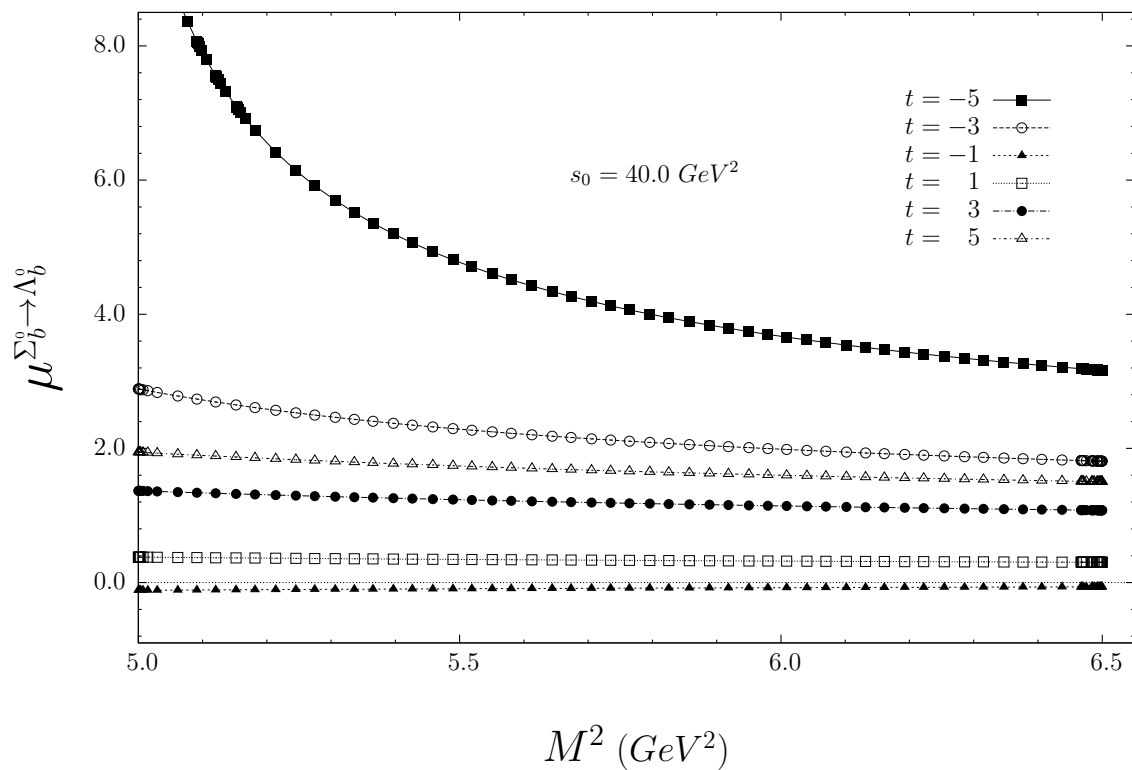


Figure 2:

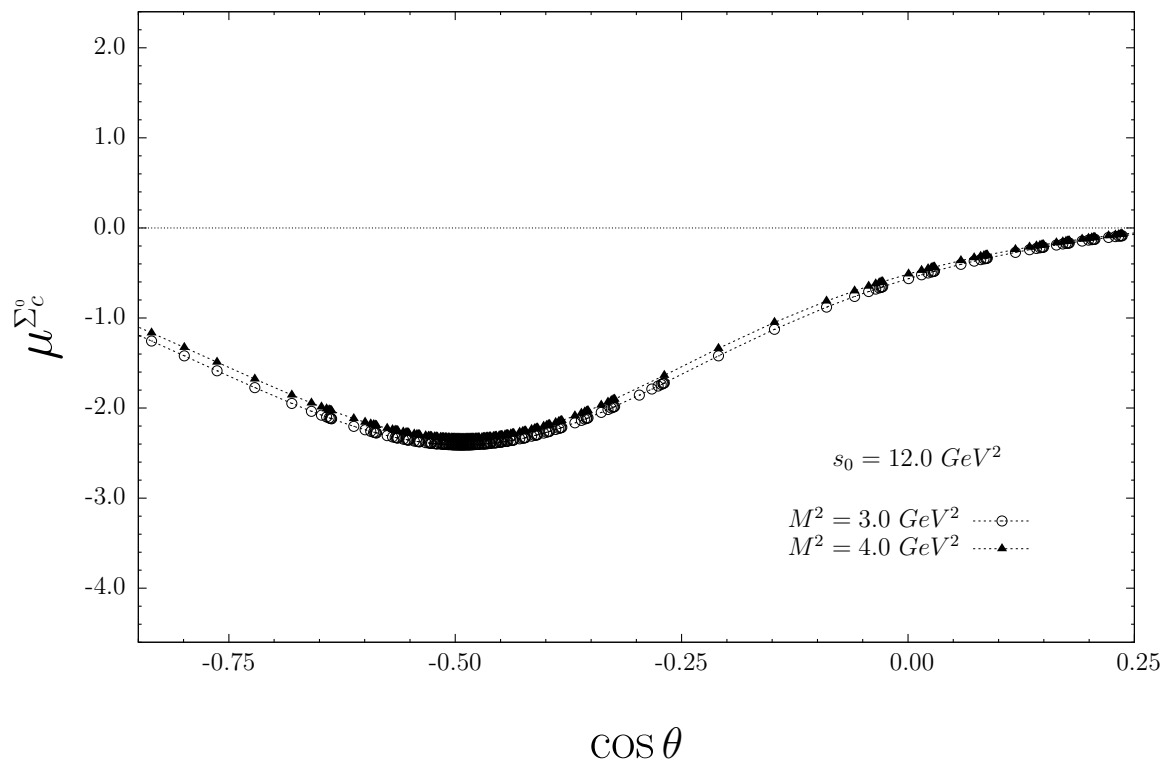


Figure 3:

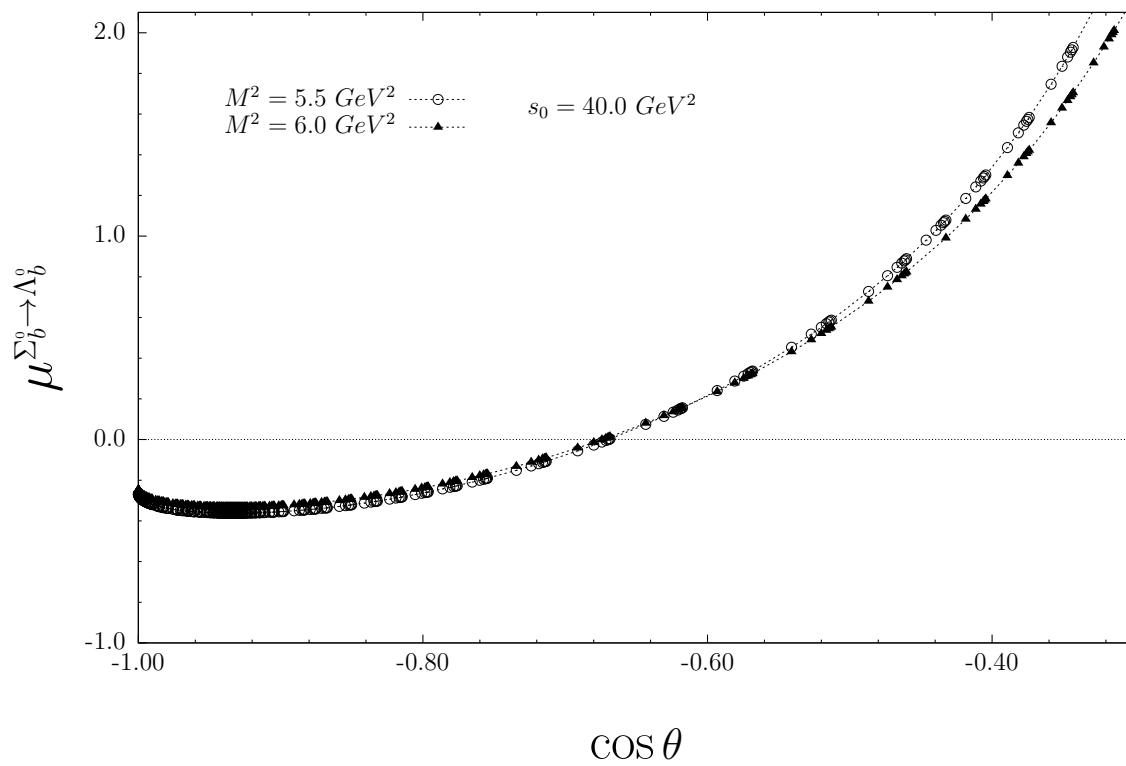


Figure 4: