

Modeling of Asymmetric Shear Wall-Frame Building Structures

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Abstract

Based on the conventional wide column analogy, two different three-dimensional shear wall models for open and closed sections are proposed. These approximate models are verified in comparison to not only the results available in the literature but also the ones obtained by using models containing shell elements. With the help of these new models five different groups of shear wall-frame structures with different floor plans and different heights are analyzed. The first three natural vibration periods are determined and time history analyses are performed. The results of these computations are observed to be in good agreement with those obtained by detailed models containing shell elements.

Keywords: shear wall; shear wall-frame structures; wide column analogy; natural vibration periods; time history analysis

1. Introduction

A reinforced concrete shear wall-frame building structure is a multistorey structural system that consists of reinforced concrete frames interacting with reinforced concrete shear walls. Modeling and analysis of such systems are more complicated than frame systems. In particular, for an asymmetric shear wall-frame building system that contains nonplanar shear walls, conventional two-dimensional methods may not give accurate results and a three-dimensional modeling and analysis should be made. The plan views of typical nonplanar (open and closed section) shear walls of such buildings are given in Fig.1. In the analysis of building structures that contain nonplanar shear walls; the main problem is modeling these assemblies by using an accurate and feasible method. This study is based on the modeling of these structures by using three dimensional frame elements.

One of the most common planar shear wall models is the two dimensional equivalent frame model (wide column analogy), which was developed by Clough *et al.* (1964), Candy (1964) and Macleod (1967). In this model, as seen in Fig.2., the planar shear wall is replaced by an idealized frame structure consisting of a column and rigid beams at the floor levels. The column is placed at the wall's centroidal axis and assigned to have the wall's inertia and axial area. The rigid beams

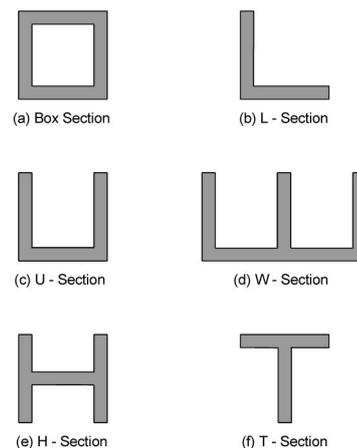


Fig.1. Plan View of Typical Shear Wall Sections

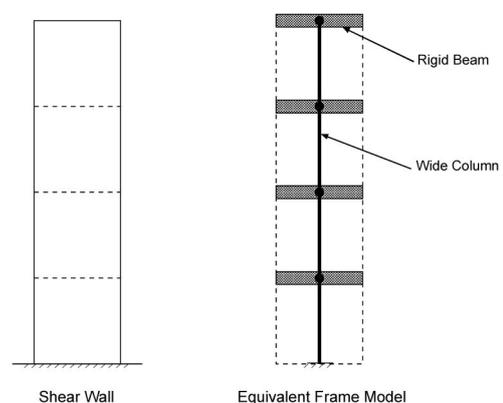


Fig.2. Equivalent Frame Model of a Planar Shear Wall

that join the wide column to the connecting beams are located at each floor level. The equivalent frame model was improved by Macleod (1976, 1977), Macleod and

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Hosny (1977) for the analysis of nonplanar shear walls. The analogous frame method is an alternative method proposed by Smith and Girgis (1984) in which a shear wall is modeled by a wide column, rigid horizontal beams, and diagonal braces. Kwan (1993a) proposed a model that overcomes the artificial flexure problem of the equivalent frame method in three-dimensional analyses. In addition to these, different shear wall modeling techniques such as the finite element method (Girijavallabhan, 1969; MacLeod, 1969; Al-Mahaidi and Nilson, 1975; Cook, 1975; and Kwan 1993b), the finite strip method (Cheung and Swaddiwudhipong, 1975), the higher order finite elements developed for modeling shear walls (Chan and Cheung, 1973), and the continuum method (MacLeod, 1990) are the other well known methods. On the other hand, a significant number of studies on the investigation of shear wall-frame structures have been made (Ghali and Neville, 1978; Rutenberg and Eisenberger, 1986; Swaddiwudhipong *et al.*, 1986; Thambiratnam and Irvine, 1989; Smith and Cruvellier, 1990; Syngellakis and Younes, 1991; and Hoenderkamp, 2001).

In this study, two different three-dimensional models for open and closed section shear wall assemblies are developed, which are based on the conventional equivalent frame model and consist of typical three dimensional frame elements. In the modeling studies, it is assumed that (a) the deformations are in the elastic range, (b) no shear deformations take place, and (c) frame elements (beams and columns) and shear walls have uniform cross-sections throughout their length. In the verification studies, SAP2000 (Wilson and Habibullah, 1995) software is used. However, the proposed models can be implemented in any three-dimensional frame analysis program. The verification studies consist of two parts. In the first part, a number of shear wall and shear wall-frame structures, which have been modeled and analyzed by several researchers in the past, are taken into consideration. The results given in these studies are compared with the results obtained by using the proposed models. In the second part, five groups of shear wall-frame building structures having different floor plans are considered. Each building structure type is considered to have 3, 6, 9, 12 and 15 storeys. The performances of the suggested shear wall models in finding the first three natural vibration periods and time history analysis are determined by performing dynamic analysis on these sample building structures, in which the results obtained using the proposed models and the models containing shell elements of SAP2000 are compared. It is assumed that the models with the shell elements give correct results.

2. Shear Wall Models

A. Open Section Shear Walls

In the model suggested for the open section shear walls, each planar wall in the assembly is replaced with

a column having the same mechanical properties of the wall as in the equivalent frame method. In order to ensure the vertical compatibility of the displacements, the rigid beams at floor level are rigidly connected to each other at the corners. In addition, the ends of the rigid beams are released (disconnected) from the connection joint only for torsional moments. In other words, the transfer of the torsional moments between the rigid beams is prevented. In Figs.3.(a) to 3.(e) typical open section shear wall assemblies and the corresponding models are given.

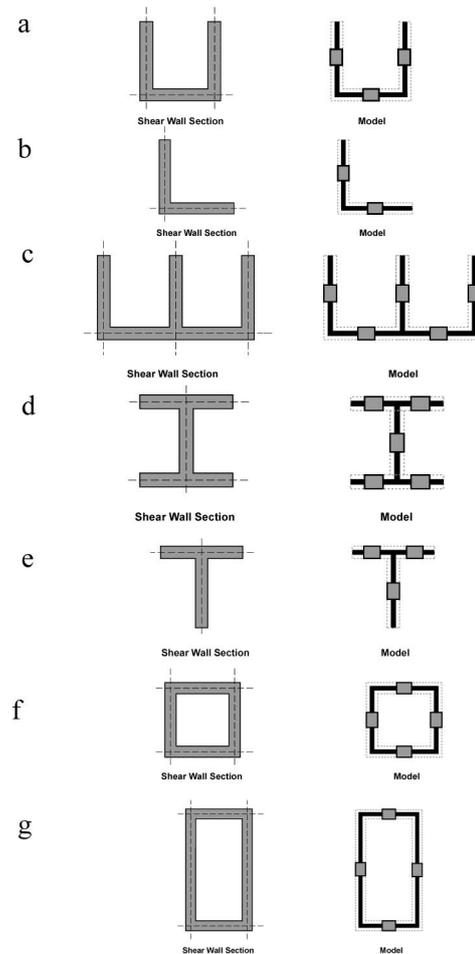


Fig.3. Plans and Proposed Models of (a) U, (b) L, (c) W, (d) H, (e) T, (f) Square, and (g) Rectangular Shear Wall Assemblies

In the three dimensional analyses of open section shear walls modeled by the conventional equivalent frame model, it is observed that serious errors occur especially in the analysis of these structures subjected to torsion. The system becomes stiffer compared to the case with finite element modeling. By releasing the ends of the rigid beams from the connection joints only for torsional moments, a more realistic model can be obtained. In order to clarify this situation, a four-storey W-shaped shear wall assembly, which is shown in Fig.3.(c), is considered. The structure is assumed to have rigid diaphragms at the floor levels. The heights of all stories are considered to be 3.0 meters and the

length of each planar wall is taken as 3.0 meters. The assembly is subjected to torsional load of 300 t-m at each floor level. The modulus of elasticity and Poisson's ratio of concrete are taken as 2.531×10^7 kN/m² and 0.20, respectively. The shell elements of SAP2000 are also used for modeling the considered assembly. After several analyses, the optimum number of shell elements to be used in modeling a planar wall module between two floor levels is found as 16 (4x4). The rotations obtained by the conventional wide column model, proposed model and the model that contains shell elements are given in Fig.4. As seen from the figure, good agreement is obtained between the proposed model and the model that contains shell elements. On the other hand, the behavior of the conventional wide column model is much stiffer than the other models.

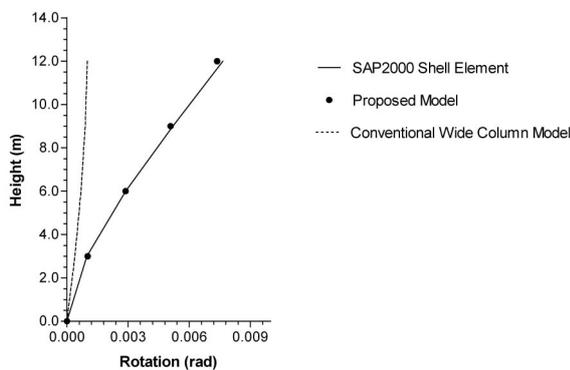


Fig.4. Floor Rotation Graph of W-shaped Shear Wall Assembly

B. Closed Section Shear Walls

The proposed model for the closed section shear wall assemblies is similar to the model developed for the open section shear wall assemblies. The columns are placed at the walls' centroidal axes and assigned to have the same mechanical properties of the walls. Rigid beams are located at the floor levels and make rigid connections with each other. Similar to the model suggested for the open sections, the ends of the rigid beams are released at the connections only for torsional moments. In the case of pure torsion applied at the shear wall sections, it is observed that the rigid beams behave independently from the wide columns and make closed loops at the floor level. For this reason, the torsional stiffness of the model becomes much smaller than the torsional stiffness of the actual closed assembly, as it is a summation of the torsional stiffness of disconnected wide columns in the model. Smith and Girgis (1986) also stated this problem and reported that the closed section shear walls modeled by the conventional equivalent frame method become less stiff compared to the model using the finite element method. The proposed model solves this problem by modifying the torsional constant of the shear wall section considered. This procedure has three steps: (1) Calculation of the torsional constant of the closed section (J_c), (2) calculation of the torsional constant

of the wide columns (J_i), and (3) calculation of the modified torsional constants of the wide columns using the following equation:

$$\bar{J}_i = \frac{J_c}{n} \cdot J_i B_i \quad (1)$$

In the above equation, \bar{J}_i is the modified torsional constant of the wide column i , B_i is a constant depending on the horizontal distance between the centroid of the wide column and the centroid of the closed section, and n is the total number of planar shear walls in the model. The torsional constants (J_c) of square and rectangular solid sections can be obtained by the following equations (Ghali and Neville, 1978):

$$J_c = 0.1406b^4 \quad (\text{Square}) \quad (2)$$

$$J_c = bt^3 \left[\frac{1}{3} - 0.21 \frac{t}{b} \left(1 - \frac{t^4}{12b^4} \right) \right] \quad (\text{Rectangle}) \quad (3)$$

For Eq. (2), b is the dimension of one side of the square section and for Eq. (3) b is the larger and t is the smaller dimension of the rectangular section. For a rectangular cross-section, the following equation should be used for determining B_i :

$$B_i = \frac{4J_i a_i^2}{\sum_{k=1}^n J_k a_k^2} \quad (4)$$

In the above equation, a_i is the distance between the centroid of the i -th wide column and the centroid of the closed shear wall in the plan. For the square section shear wall, the value of B_i for all wide columns is obtained as 1.0. Examples for the closed section shear wall assemblies and the corresponding models are given in Figs.3.(f) and 3.(g).

3. Verification Studies

A. Comparison with Past Studies

A.1 Variable Thickness Core Assembly

In the studies of Kwan (1992) and Nadjai and Johnson (1998), a 100-meter-high variable thickness closed core wall is analyzed. The core consists of two layers with different thicknesses. The thickness of the first part (between 0 and 50 meters) is 1.0 meter and the thickness of the second part (between 50 and 100 meters) is 0.5 meter. The structure is subjected to a torsional load of 100 t-m at the top. The dimensions of the core are taken as 10x10 meters. Kwan (1992) derived a solid wall element and used it in the analysis of the assembly. In his study, the structure is divided into 20 storeys, each 5.0 meters in height. Each storey is modeled by four solid wall elements interconnected to form a hollow section. The exact theoretical values based on the theory of Bredt-Batho were also given in that study. Nadjai and Johnson (1998) studied the same structure using the discrete force method. The

proposed model consists of four wide columns located at the middle of each planar shear wall in the assembly and rigid beams at floor levels. Similar to the studies mentioned above, the core is divided into 20 storeys each having a height of 5.0 meters. The rotation values that were obtained by Kwan (1992), Nadjai and Johnson (1998), Bredt-Batho theory, and the proposed model are compared in Fig.5.

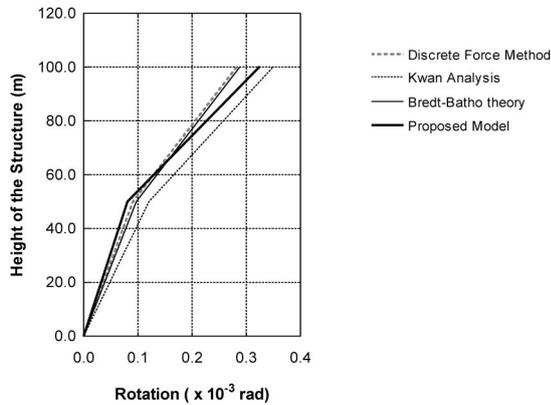


Fig.5. Comparison of the Rotations of the Core Assembly

A.2 Coupled Nonplanar Shear Wall Assembly

A six-storey Plexiglas structure consists of three planar walls interconnected to form a U-channel shaped shear wall assembly. The central wall has a row of openings at the middle and the assembly is considered to have four planar wall units and a row of coupling beams. The total height of the structure is 48 inches (1.219 m), and it is subjected to a lateral load of 25 pounds (111.2 N) at the top. The connecting beams have a depth of 1.5 inches (0.038 m). The other dimensions of the assembly are given in Fig.6. The material properties of the model are taken to be $E = 0.4 \times 10^6$ psi (2.76 GPa) and $G = 0.148 \times 10^6$ psi (1.02 GPa). This problem was first analyzed by Tso and Biswas (1973). The theoretical solution and experimental results are given in their study.

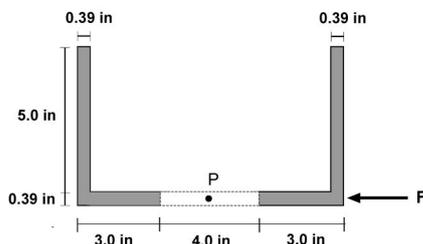


Fig.6. Plan of the Open Section Shear Wall Assembly

Ho and Liu (1985) studied the same structure and analyzed it by using a method that is a combination of the finite strip method and the continuum method. Kwan (1993a) also studied the assembly using an improved wide column analogy. The proposed model consists of four wide columns and connecting beams. In Figs.7.(a) and 7.(b), the horizontal displacements

and rotations of point P on the floor levels of the assembly are compared. Good agreement is obtained between the proposed model and the experimental results (a relative difference of 9.83 % in top floor displacement and 2.96 % in top floor rotation).

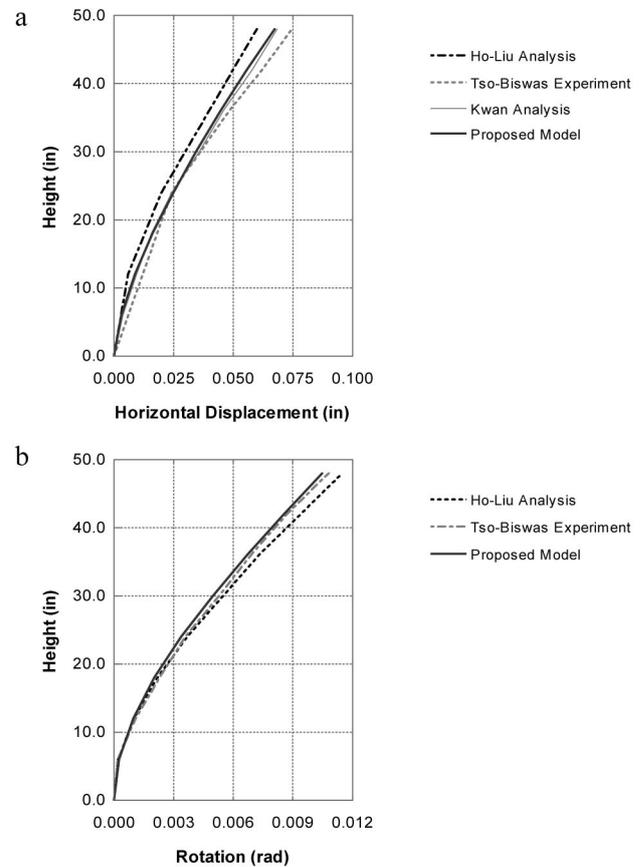


Fig.7. (a) Horizontal Displacements, (b) Rotations of the Open Section Shear Wall Assembly

A.3 Shear Wall-Frame Building Structure

Hoenderkamp (2002) studied an asymmetric shear wall-frame building structure as shown in Fig.8. The building consists of a core with lintel beams, four single shear walls and five identical rigid frames. It has 16 storeys with a total height of 48 meters. A horizontal distributed load of 40 kN/m acts at the center of the structure as shown in the figure. The modulus of elasticity is taken as $E = 20 \times 10^6$ kN/m². The core has a wall thickness of 0.2 meter and the lintel beams measure 0.2 meter by 0.5 meter. All other dimensions are given in Fig.8. Hoenderkamp used a three dimensional analytical method based on the continuum approach in the analysis of the structure. In modeling the building structure according to the proposed model, planar wall units are replaced by the wide columns and the rigid beams and it is assumed that floors are infinitely rigid. This building is also modeled by using shell elements of SAP2000. The results of the deflections and rotations of the core assembly are compared in Figs.9.(a) and 9.(b), respectively. The difference between the proposed model and

Hoenderkamp's analytical method is 4.71 % at the locations of maximum deformation and 4.76 % at the locations of maximum rotation. Finally, in Fig.9.(c) the bending moments acting on the core assembly are compared. The maximum difference between the two models is obtained at the foundation level (a relative difference of 11.30 %).

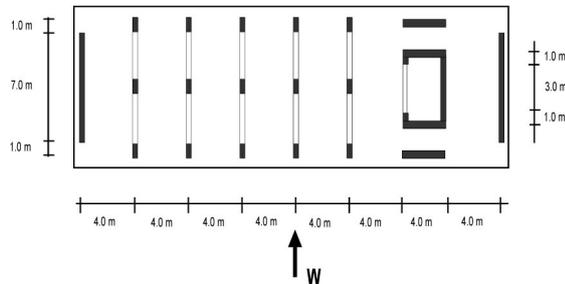


Fig.8. Plan View of the Shear Wall-frame Structure

B. Comparisons on Sample Building Structures

In this part of the study, dynamic analyses are performed on several different shear wall-frame building structures in which the nonplanar shear wall assemblies of these buildings are modeled by the proposed models and shell elements of SAP2000. The results of the analyses of sample building structures obtained by using SAP2000 shell elements are assumed to give correct results. After a number of analyses on the sample buildings, the optimum number of shell elements to be used in modeling a planar wall module located between two floor levels is found to be 16 (4x4). Five different groups of shear wall-frame building structures with different heights are considered. Their floor plans are given in Figs.10.(a) to 10(e). The analyses are performed on 3, 6, 9, 12 and 15 storey sample-building structures. The code BSi-j is used to represent the type of the building structure and the total number of storeys. For example, BS2-12 corresponds to the second type of building structure with 12 storeys. For all building structures, the height of a typical storey is taken as 3.0 m, the dimensions of all columns and beams are taken as 30x30 cm, and the thickness of the shear walls is assumed to be 25 cm. In the analyses, the rigid beams at the floor levels are assumed to be massless. The modulus of elasticity and Poisson's ratio of concrete are taken as 2.531×10^7 kN/m² and 0.20, respectively, and the mass density of concrete is taken as 0.255 t/m³. The values of the floor masses and moments of inertia of the building structures used in dynamic analyses are given in Table 1. They are assumed to be concentrated at the centroid of the floors.

In addition, the rigid diaphragm floor assumption is taken into consideration in the modeling of the sample building structures. The results of the verification studies based on dynamic analysis are given in two parts. In the first part, the computations of the natural vibration periods of the sample building structures are

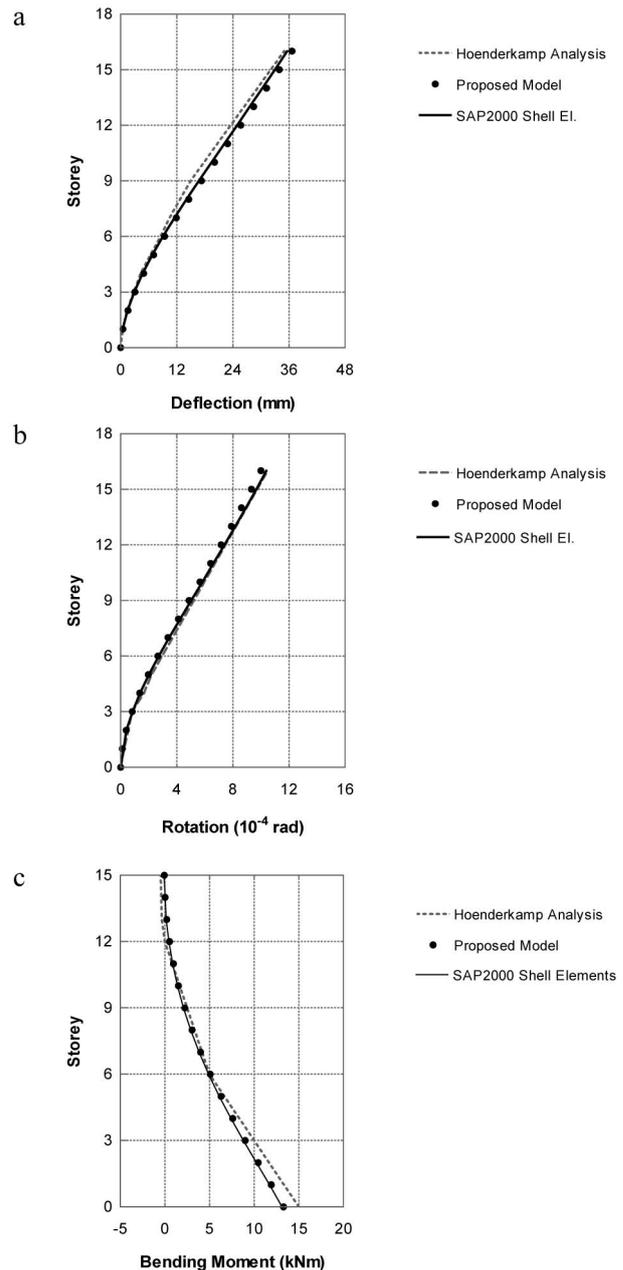


Fig.9. Comparison of (a) Storey Deflections, (b) Storey Rotations, and (c) Bending Moments on the Core

performed and in the second part, the results obtained by time history analyses are compared.

B.1 Natural Vibration Periods

The evaluation of the natural vibration periods of the building structures is quite important in the dynamic analysis of structures. Moreover, in most of the building codes that suggest the equivalent lateral load analysis, the first natural vibration period of a building structure should be obtained in order to determine the lateral loads acting at the floor levels.

The validity of the suggested models in calculating the natural vibration periods of the sample building structures is determined by comparing the values obtained using SAP2000 shell elements. The maximum relative differences in the three natural vibration

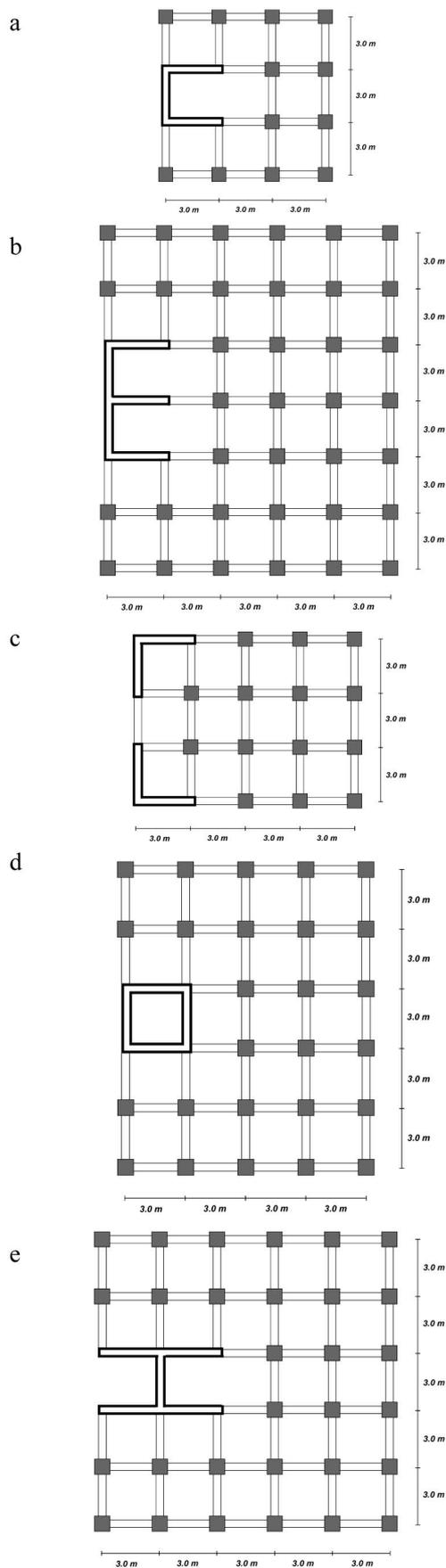


Fig.10. Floor Plan of the Building Structures (a) BS1, (b) BS2, (c) BS3, (d) BS4, and (e) BS5

Table 1. Floor Masses and Corresponding Moment of Inertias of the Sample Building Structures

Building Type	Floor Mass (kg)	Moment of Inertia (kg/m ²)
BS1	3.67×10^3	5.0×10^3
BS2	1.29×10^4	5.4×10^5
BS3	5.51×10^3	1.0×10^5
BS4	8.72×10^3	2.5×10^5
BS5	1.06×10^4	3.4×10^5

periods of the sample building structures are obtained as 2.31 % for the first natural vibration periods, 7.06 % for the second natural vibration periods, and 6.48 % for the third natural vibration periods. As an example, the first natural vibration periods of BS1, BS2, BS3, BS4 and BS5 type building structures are tabulated in Table 2. The corresponding mode shapes obtained by using the two modeling methods are similar according to the results of the analyses.

B.2 Time History Analysis

The performances of the proposed shear wall models in time history analysis are checked against the models in which shell elements of SAP2000 are used for modeling shear wall assemblies. The results of the time history analyses of the three sample building structures (BS2-6, BS4-12 and BS5-15), in which the two modeling techniques are used, are presented in this part. The acceleration-time record of the El Centro Earthquake (Fig.11.) is applied directly to the base of the sample building structures. The record of the first 20 seconds of the earthquake, with a step size of 0.02 second, is considered and a damping ratio of 5 % is used in the analyses. The analyses are performed on both x and y directions of the sample buildings in elastic range and the following parameters are computed for both models: (a) Maximum base shear, (b) maximum displacement of the top storey, and (c) maximum resultant shear at the base of the shear wall assembly. Fig.12. gives the base shear history graphs obtained by time history analysis in x-direction for the sample building BS2-6, in which the proposed model is used. The maximum base shear forces of the two modeling techniques are compared in Table 3 for the sample structures. The maximum base shear in x direction (F_x) is obtained by time history analysis of the sample building structures in x-direction and u_y is obtained by the time history analysis in y-direction.

The maximum relative differences in maximum base shear forces between the two modeling techniques are obtained as 9.31 % for the base shear in x-direction (F_x) and 6.43 % for the base shear in y-direction (F_y). In Fig.13., the top storey displacement history graph obtained by time history analyses in x-direction using the proposed modeling technique for the sample building BS4-12 is given. Table 4. provides a comparison of the two techniques regarding the maximum top storey displacements for the three structures. u_x is obtained by the time history analysis of the sample building structures in x-direction and u_y

Table 2. Comparison of the First Natural Vibration Periods of the Sample Building Structures

Building Type	SAP2000 Shell Elements (s)	Proposed Model (s)
BS1-3	0.307110	0.309649
BS1-6	0.675322	0.678735
BS1-9	1.074864	1.079450
BS1-12	1.499999	1.506029
BS1-15	1.953667	1.961320
BS2-3	0.381536	0.383313
BS2-6	0.803812	0.803481
BS2-9	1.246663	1.244240
BS2-12	1.700775	1.696627
BS2-15	2.164530	2.159201
BS3-3	0.270471	0.269953
BS3-6	0.656740	0.648689
BS3-9	1.083998	1.066807
BS3-12	1.541342	1.505687
BS3-15	2.023151	1.981266
BS4-3	0.223960	0.220323
BS4-6	0.476262	0.472876
BS4-9	0.799621	0.796315
BS4-12	1.176619	1.171179
BS4-15	1.607246	1.609430
BS5-3	0.223960	0.220323
BS5-6	0.476262	0.472876
BS5-9	0.799621	0.796315
BS5-12	1.176619	1.171179
BS5-15	1.607246	1.609430

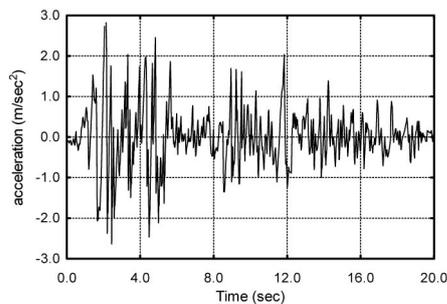


Fig. 11. El Centro Earthquake Record

is obtained by time history analysis in y-direction. The maximum relative difference between the two models is 1.23 % for maximum top storey displacements in x-direction and 2.31 % for maximum top storey displacements in y-direction. Finally, the comparison of the two techniques in terms of the maximum resultant shear forces at the base of the shear walls of the three sample structures is given in Table 5. The maximum relative differences in the resultant shear forces at the base of the shear wall assemblies obtained for the two models are 5.82 % for the time history analysis in x-direction and 4.78 % for the time history

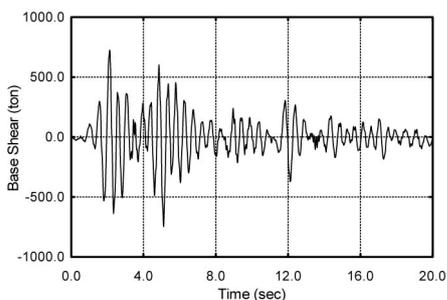


Fig. 12. Base Shear Force History for BS2-6 Type Building Structure

Table 3. Comparison of Maximum Base Shear Forces in the Sample Building Structures

Building Type	SAP2000 Shell El. (ton)	Proposed Model (ton)
BS2-6		
Max. Base Shear- F_x	806.4	804.0
Max. Base Shear- F_y	539.4	589.6
BS4-12		
Max. Base Shear- F_x	539.0	534.2
Max. Base Shear- F_y	480.4	500.2
BS5-15		
Max. Base Shear- F_x	485.0	516.2
Max. Base Shear- F_y	464.8	453.7

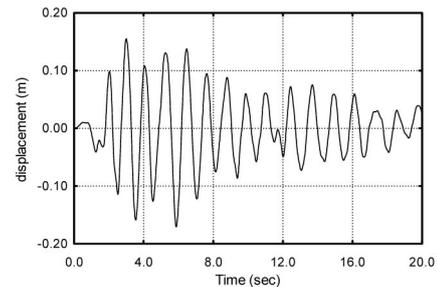


Fig. 13. Top Storey Displacement Histories for BS4-12 Type Building Structure

Table 4. Comparison of Maximum Top Storey Displacements in the Sample Building Structures

Building Type	SAP2000 Shell El. (m)	Proposed Model (m)
BS2-6		
Max. Top Storey Disp.- u_x	0.06454	0.06480
Max. Top Storey Disp.- u_y	0.07219	0.07386
BS4-12		
Max. Top Storey Disp.- u_x	0.17030	0.17240
Max. Top Storey Disp.- u_y	0.16500	0.16500
BS5-15		
Max. Top Storey Disp.- u_x	0.12720	0.12790
Max. Top Storey Disp.- u_y	0.17440	0.17100

Table 5. Comparison of Maximum Resultant Shear Forces on the Base of the Shear Wall Assembly in the Sample Building Structures

Building Type	SAP2000 Shell El. (ton)	Proposed Model (ton)
BS2-6		
Max. Resultant Shear Force- F_x	746.3	739.3
Max. Resultant Shear Force- F_y	401.3	410.2
BS4-12		
Max. Resultant Shear Force- F_x	507.4	498.1
Max. Resultant Shear Force- F_y	403.9	423.2
BS5-15		
Max. Resultant Shear Force- F_x	467.2	494.4
Max. Resultant Shear Force- F_y	393.2	391.2

analysis in y-direction.

The total number of structural elements used for modeling the sample building structures, the total running time, and the size of stiffness files produced in time history analysis are given in Table 6. An IBM ThinkPad 600X computer was used in the analyses. As seen from the table, the building models for which the

Table 6. Comparison of the Two Shear Wall Models in Time History Analysis

Building Type	Total Number of Elements	Size of Stiffness File (bytes)	Run Time (s)
BS2-6			
With SAP Shell El.	600 frame el., 480 shell el.	7,970,056	50
With Proposed Model	690 frame el.	767,368	18
BS4-12			
With SAP Shell El.	852 frame el., 768 shell el.	7,943,000	62
With Proposed Model	996 frame el.	916,552	22
BS5-15			
With SAP Shell El.	1245 frame el., 1200 shell el.	19,725,600	143
With Proposed Model	1470 frame el.	1,608,436	31

proposed models are used have significant advantages regarding the total running time and stiffness file size.

4. Conclusions

Two approximate shear wall models, for open and closed sections, are proposed based on the conventional wide column analogy. The validity of these models was first investigated by considering a number of structures studied by several researchers in the past. In the second part, five groups of sample shear wall-frame building structures with different floor plans and heights are considered. The results obtained by using the suggested models agree well with those in the literature and with the results of the analyses of the sample building structures in which shell elements are used in the models.

One of the main advantages of the proposed models is that they can be used in any stiffness based three-dimensional frame analysis software having a joint release option. In addition, the time spent on forming a complete building model and total running time of the analyses are less when the proposed models are used. For the shear wall-frame buildings having a large number of elements and degrees of freedom, the improvement is more significant.

References

- Al-Mahaidi, R.S. and Nilson, A.H. (1975) Coupled shear wall analysis by Lagrange multipliers. *Journal of Structural Div., ASCE*, 101 (11).
- Candy, C.F. (1964) Analysis of shear wall-frames by computer. *New Zealand Engrg.*, 19 (9), pp.342-47.
- Chan, H.C. and Cheung, Y.K. (1973) Analysis of shear walls using higher order elements. *Building and Environment*, 14 (3).
- Cheung, Y. K. and Swaddiwudhipong, S. (1975) Analysis of frame shear wall structures using finite strip elements. *Proc. Inst. Civ. Engrg. Part 2*, 65, pp.517-535.
- Clough, R.W., King, I.P. and Wilson, E.L. (1964) Structural analysis of multistorey buildings. *Journal of Structural Div., ASCE*, 90 (19).
- Cook, R.D. (1975) Avoidance of parasitic shear in plane element. *Journal of Structural Div., ASCE*, 101 (6).
- Ghali, A. and Neville, A.M. (1978) *Structural analysis -a unified classical and matrix approach*. New York, Chapman and Hall Ltd.
- Girijavallabhan, C.V. (1969) Analysis of shear walls with openings. *Journal of Structural Div., ASCE*, 95 (10).
- Ho, D. and Liu, C.H. (1985) Analysis of shear wall and shear core assembly subjected to lateral and torsional loading. *Proc. Instn. Civ. Engrs, Part 2*, 79, pp.119-133.
- Hoenderkamp, J.C.D. (2001) Elastic analysis of asymmetric tall building structures. *The Structural Design of Tall Buildings*, 10, pp.245-261.
- Hoenderkamp, J.C.D. (2002) Simplified analysis of asymmetric high-rise structures with cores. *The Structural Design of Tall Buildings*, 11, pp.93-107.
- Kwan, A.K.H. (1992) Reformulation of the frame method. *Proc. Inst. Civ., Engrs.*, 94, pp.103-116.
- Kwan, A.K.H. (1993a) Improved wide-column-frame analogy for shear/core wall analysis. *Journal of Structural Div., ASCE*, 119 (2).
- Kwan, A.K.H. (1993b) Mixed finite element method for analysis of coupled shear/core walls. *Journal of Structural Div., ASCE*, 119 (5).
- MacLeod, I.A. (1967) Lateral stiffness analysis of shear walls with openings. *Proc. Symp. of Tall Buildings*, London, England, Pergamon Press, pp.223-52.
- MacLeod, I.A. (1969) New rectangular finite element for shear wall analysis. *Journal of Structural Div., ASCE*, 95 (3).
- MacLeod, I.A. (1976) General frame element for shear wall analysis. *Proc. Instn. Civ. Engrs.*, Part 2, 61, pp.785-790.
- MacLeod, I.A. (1977) Structural analysis of wall systems. *The Structural Engineer*, November, pp.487-495.
- MacLeod, I.A. (1990) *Analytical modeling of structural systems*. Ellis Horwood Limited.
- MacLeod, I.A. and Hosny, H.M. (1977) Frame analysis of shear wall cores. *Journal of Structural Div., ASCE*, 103(10).
- Nadjai, A. and Johnson, D. (1998) Torsion in tall buildings by a discrete force method. *The Structural Design of Tall Buildings*, 7, pp.217-231.
- Rutenberg, A. and Eisenberger M. (1986) Simple planar modeling of asymmetric shear buildings for lateral forces. *Computers and Structures*, 24 (6), pp.885-891.
- Smith, B.S. and Cruvellier, M. (1990) Planar modeling techniques for asymmetric building structures. *Proc. Instn Civ. Engrs, Part 2*, 89, pp.1-14.
- Smith, B.S. and Girgis, A. (1984) Simple analogous frames for shear wall analysis. *Journal of Structural Div., ASCE*, 110(11).
- Smith, B.S. and Girgis, A. (1986) Deficiencies in the wide column analogy for shear wall core analysis. *Concrete International*, April.
- Swaddiwudhipong S., Balendra T., Quek, S.T. and Lee, S.L. (1986) Computer program for the analysis of asymmetric frame-shear wall structures. *Computers and Structures*, 22 (3), pp.343-362.
- Syngellakis, S. and Younes, I. (1991) The transfer matrix method applied to frame-shear wall systems. *Computers and Structures*, 41 (2), pp.197-206.
- Thambiratnam D.P. and Irvine, H.M. (1989) Microcomputer analysis of torsionally coupled multistorey buildings-I. Shear beam model. *Computers and Structures*, 32 (5), pp.1175-1182.
- Tso, W.K. and Biswas, J.K. (1973) Analysis of core wall structures subjected to applied torque. *Building Sciences*, 8, pp.251-257.
- Wilson, E.L. and Habibullah, A. (1995) *SAP2000, Structural Analysis Program*. Computers and Structures Inc., Berkeley, California, USA.