Probing the sources of CP violation via $B \to K^\star \ell^+ \ell^-$ decay

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Abstract

The $B \to K^* \ell^- \ell^+$ ($\ell = \mu,\ \tau$) is analyzed in a minimally extended Standard Model in which the Wilson coefficients have new CP–odd phases. The sensitivity of the CP asymmetry and lepton polarization asymmetries to the new phases is discussed. It is found that the CP asymmetry is sensitive to the new phase in the Wilson coefficient C_7 whereas the normal lepton polarization asymmetry is sensitive to the phase in the Wilson coefficient C_{10}. Additionally, the correlation between the CP and normal lepton polarization asymmetries is studied. A simultaneous measurement of these two asymmetries can be useful in search for the existence of the new sources of CP violation beyond the Standard Model.
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I. INTRODUCTION

Violation of the CP symmetry has now become a well–established fact in Kaon system [1]. Once the era of B–factories start with the operation of KEK-B, B-TeV, LHC-B and SLAC's Asymmetric B factory, it will be possible to test the standard model (SM) at one–loop accuracy. In general, possible incompatibility of the experimental data with the SM predictions will mark the existence of 'new physics' contributions. Among all, an experimental determination of the CP–violating quantities and their comparison with the SM predictions will be particularly useful in search for the new physics effects.

From the experimental perspective the exclusive decay modes (such as $B \to K^* \gamma$ [2], $B \to K^* \ell^+ \ell^-$, $B \to K \ell^+ \ell^-$) are easy to measure. From the theoretical view point, however, the corresponding inclusive modes ($b \to s\gamma$ and $b \to s \ell^+ \ell^-$) can be cleanly estimated as the only machinery needed are the Wilson coefficients describing the short–distance physics. A proper description of the exclusive decay modes, on the other hand, depends on both Wilson coefficients (short–distance physics) and the hadronic form factors (long–distance physics). This causes a relative increase of the uncertainties due to hadronization effects.

For the purpose of studying the sources of CP violation, it is convenient to concentrate on those observables which are sensitive to the possible CP phases. Among these, for example, CP asymmetries and lepton polarization asymmetries are such ones [3]– [5]. Recently, a detailed study of the lepton polarization asymmetries in $B \to X_s \ell^+ \ell^-$ decay has been performed in a rather general model by including nine additional Wilson coefficients not found in the SM [6]. Keeping this kind of short–distance generality it is convenient to discuss the exclusive decay modes such as $B \to K^* \ell^+ \ell^-$ [7,8]. Such an analysis will be useful for a first–hand comparison with the experiment as the inclusive modes are generally hard to measure.

In Sec. II we start with a general non–standard description of the short–distance physics as in [6]. Then we parametrize the long–distance quantities by appropriate form factors and obtain the hadronic transition amplitude. In Sec. III we derive general analytic expressions for asymmetries will be give. In doing this all sources of CP violation will be ascribed to short–distance physics. In Sec. IV the asymmetries and their relation to the Wilson coefficients will be analyzed numerically. In Sec. V results are discussed and the conclusion is stated.

II. THE DECAY AMPLITUDE

The exclusive B decays $B \to K^* \ell^+ \ell^-$ are conveniently described by the partonic decay $b \to s\ell^+\ell^-$ at distances $\mathcal{O}(M_W^{-1})$. The effective Hamiltonian describing this rare b decay at the scale $\mu \sim M_W$ should, however, be evolved down to mesonic mass scale $\mu \sim m_b$ using the QCD evolution equations. Then the decay amplitude describing $b \to s\ell^+\ell^-$ takes the form [6,7]

$$\begin{aligned}
\mathrm{M}(b \to s\ell^+\ell^-) = \frac{G\alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \Big\{ & C_{SL}\, \bar{s} i\sigma_{\mu\nu} \frac{q^\nu}{q^2}(m_s L) b\, \bar{\ell}\gamma_\mu \ell + C_{BR}\, \bar{s} i\sigma_{\mu\nu}\frac{q^\nu}{q^2}(m_b R) b\, \bar{\ell}\gamma_\mu \ell \\
& + C_{LL}\, \bar{s}_L \gamma_\mu b_L\, \bar{\ell}_L \gamma^\mu \ell_L + C_{LR}\, \bar{s}_L \gamma_\mu b_L\, \bar{\ell}_R \gamma^\mu \ell_R + C_{RL}\, \bar{s}_R \gamma_\mu b_R\, \bar{\ell}_L \gamma^\mu \ell_L \\
& + C_{RR}\, \bar{s}_R \gamma_\mu b_R\, \bar{\ell}_R \gamma^\mu \ell_R + C_{LRLR}\, \bar{s}_L b_R\, \bar{\ell}_L \ell_R + C_{RLLR}\, \bar{s}_R b_L\, \bar{\ell}_L \ell_R \\
& + C_{LRRL}\, \bar{s}_L b_R\, \bar{\ell}_R \ell_L + C_{RLRL}\, \bar{s}_R b_L\, \bar{\ell}_R \ell_L + C_T\, \bar{s}\sigma_{\mu\nu} b\, \bar{\ell}\sigma^{\mu\nu}\ell \\
& + i C_{TE}\, \epsilon^{\mu\nu\alpha\beta} \bar{s}\sigma_{\mu\nu} b\, \bar{\ell}\sigma_{\alpha\beta}\ell \Big\}\,,
\end{aligned} \tag{1}$$

where each of the Wilson coefficients $C_{SL}, \cdots, C_{TE}$ is evaluated at the B-meson mass scale, $\mu \sim m_b$. In this expression, $L(R) = (1-(+)\gamma_5)/2$ are the left (right) projection operators, V_{ij} are the elements of the CKM matrix, and $q = p_B - p_{K^*} = p_+ + p_-$ is the momentum transfer to the dilepton channel. This decay amplitude has a rather general form as it includes nine additional operators not found in the minimal standard model. The only simplifying assumption about this decay amplitude will be twofold: (1) Neglect of the strange quark mass everywhere in the analysis, (2) Neglect of the tensor operators having the coefficients C_T and C_{TE}. The former is justified by the smallness of the ratio m_s/m_b and the latter is justified by the previous analyzes which show that their contributions are much smaller than other operators (for details, see [9]).

The quark level decay amplitude (1) controls the semileptonic decays $B \to (K, K^*)\, \ell^+\ell^-$. The amplitudes for these exclusive decays can be obtained after evaluating the matrix elements of the quark operators in (1) between the $|B(p_B)\rangle$ and $\langle K^*(p_{K^*})|$ states. In particular, explicit expressions for $\langle K^* |\bar{s}\gamma_\mu(1 \pm \gamma_5) b| B\rangle$, $\langle K^* |\bar{s} i\sigma_{\mu\nu} q^\nu (1+\gamma_5) b| B\rangle$ and $\langle K^* |\bar{s}(1 \pm \gamma_5) b| B\rangle$ are needed. Computation of such hadronic matrix elements is bound to parametrizations of the form factors depending only on the momentum transfer square, or equivalently, the dilepton invariant mass $m_{\ell\ell}^2 = (p_B - p_{K^*})^2 = (p_+ + p_-)^2 \equiv q^2$. Introducing appropriate from factors one obtains

$$\begin{aligned}
& \langle K^*(p_{K^*}, \varepsilon)\, |\bar{s}\gamma_\mu (1 \pm \gamma_5) b|\, B(p_B)\rangle = \\
& -\epsilon_{\mu\nu\rho\sigma} \varepsilon^{*\nu} p_{K^*}^\rho q^\sigma \frac{2V(q^2)}{m_B + m_{K^*}} \pm i\varepsilon_\mu^* (m_B + m_{K^*}) A_1(q^2) \mp i (p_B + p_{K^*})_\mu (\varepsilon^* q) \frac{A_2(q^2)}{m_B + m_{K^*}}
\end{aligned}$$

$$\mp i q_\mu \frac{2m_{K^*}}{q^2} (\varepsilon^* q) \Big[A_3(q^2) - A_0(q^2)\Big] \ , \tag{2}$$

$$\begin{aligned}
&\langle K^*(p_{K^*},\varepsilon) \left| \bar s i \sigma_{\mu\nu} q^\nu (1+\gamma_5) b \right| B(p_B) \rangle = \\
&4\epsilon_{\mu\nu\rho\sigma} \varepsilon^{*\nu} p_{K^*}^\rho q^\sigma T_1(q^2) + 2i \Big[\varepsilon_\mu^* (m_B^2 - m_{K^*}^2) - (p_B + p_{K^*})_\mu (\varepsilon^* q)\Big] T_2(q^2) \\
&+2i(\varepsilon^* q) \left[q_\mu - (p_B + p_{K^*})_\mu \frac{q^2}{m_B^2 - m_{K^*}^2}\right] T_3(q^2) \ ,
\end{aligned} \tag{3}$$

where the explicit expressions for $V(q^2)$, $A_{0,1,2,3}(q^2)$ and $T_{1,2,3}(q^2)$ will be given below.

To ensure the finiteness of (2) as $q^2 \to 0$, it is usually assumed that $A_3(q^2=0) = A_0(q^2=0)$. Besides, to calculate the matrix elements of the scalar operators, $\langle K^* \left| \bar s (1 \pm \gamma_5) b \right| B \rangle$, it is necessary to contract (2) with q_μ and use the equation of motion, giving

$$\begin{aligned}
\langle K^*(p_{K^*},\varepsilon) \left| \bar s (1 \pm \gamma_5) b \right| B(p_B) \rangle &= \frac{1}{m_b} \Big\{ \mp i (\varepsilon^* q)(m_B + m_{K^*}) A_1(q^2) \\
&\pm i (m_B - m_{K^*})(\varepsilon^* q) A_2(q^2) \pm 2i m_{K^*} (\varepsilon^* q) \Big[A_3(q^2) - A_0(q^2)\Big] \ \Big\}.
\end{aligned} \tag{4}$$

Additionally, again using the equation of motion, the form factor A_3 can be expressed as a linear combination of the form factors A_1 and A_2 (see [10])

$$A_3(q^2) = \frac{m_B + m_{K^*}}{2m_{K^*}} A_1(q^2) - \frac{m_B - m_{K^*}}{2m_{K^*}} A_2(q^2) \ . \tag{5}$$

Having this relation at hand, one finally obtains

$$\langle K^*(p_{K^*},\varepsilon) \left| \bar s (1 \pm \gamma_5) b \right| B(p_B) \rangle = \frac{1}{m_b} \Big\{ \mp 2i m_{K^*} (\varepsilon^* q) A_0(q^2) \Big\} \ . \tag{6}$$

This completes the evaluation of the necessary transition matrix elements.

As mentioned before the form factors entering (2)-(6) represent the hadronization process which lacks a Lagrangian description. They are thus generally computed in framework of certain nonperturbative approaches such as chiral theory [11], three point QCD sum rules method [10], relativistic quark model by the light–front formalism [12], effective heavy quark theory [13] and light cone QCD sum rules [14–16]. In what follows we will use the results of the work [15] in which the form factors are described by a three–parameter fit where the radiative corrections up to leading twist contribution and SU(3)–breaking effects are taken into account. Letting $F(q^2) \in \{V(q^2), A_0(q^2), A_1(q^2), A_2(q^2), A_3(q^2), T_1(q^2), T_2(q^2), T_3(q^2)\}$, the q^2–dependence of any of these form factors could be parametrized as

$$F(s) = \frac{F(0)}{1 - a_F\, s + b_F\, s^2} \ ,$$

	$F(0)$	a_F	b_F
$A_0^{B\to K^*}$	0.47	1.64	0.94
$A_1^{B\to K^*}$	0.34 ± 0.05	0.60	-0.023
$A_2^{B\to K^*}$	0.28 ± 0.04	1.18	0.281
$V^{B\to K^*}$	0.46 ± 0.07	1.55	0.575
$T_1^{B\to K^*}$	0.19 ± 0.03	1.59	0.615
$T_2^{B\to K^*}$	0.19 ± 0.03	0.49	-0.241
$T_3^{B\to K^*}$	0.13 ± 0.02	1.20	0.098

TABLE I. The form factors for $B \to K^* \ell^+ \ell^-$ in a three–parameter fit.

where the parameters $F(0)$, a_F and b_F are listed in Table 1 for each form factor. Here $s = q^2/m_B^2$ is the dilepton invariant mass in units of B–meson mass (See, [15,16]).

Making use of the hadronic matrix elements (2)-(6) of the basic quark current structures in (1), it is straightforward to determine the decay amplitude for $B \to K^* \ell^+ \ell^-$ decay:

$$
\begin{aligned}
&\mathrm{M}(B \to K^* \ell^+ \ell^-) = \frac{G\alpha}{4\sqrt{2}\pi} V_{tb} V_{ts}^* \\
&\times \Bigg\{ \bar{\ell} \gamma_\mu (1-\gamma_5) \ell \Big[-2\mathcal{V}_{L_1} \epsilon_{\mu\nu\rho\sigma} \varepsilon^{*\nu} p_{K^*}^\rho q^\sigma - i\mathcal{V}_{L_2} \varepsilon_\mu^* + i\mathcal{V}_{L_3} (\varepsilon^* q)(p_B + p_{K^*})_\mu + i\mathcal{V}_{L_4} (\varepsilon^* q) q_\mu \Big] \\
&+ \bar{\ell} \gamma_\mu (1+\gamma_5) \ell \Big[-2\mathcal{V}_{R_1} \epsilon_{\mu\nu\rho\sigma} \varepsilon^{*\nu} p_{K^*}^\rho q^\sigma - i\mathcal{V}_{R_2} \varepsilon_\mu^* + i\mathcal{V}_{R_3} (\varepsilon^* q)(p_B + p_{K^*})_\mu + i\mathcal{V}_{R_4} (\varepsilon^* q) q_\mu \Big] \\
&+ \bar{\ell}(1-\gamma_5)\ell \Big[i\mathcal{S}_L (\varepsilon^* q) \Big] + \bar{\ell}(1+\gamma_5)\ell \Big[i\mathcal{S}_R (\varepsilon^* q) \Big] \Bigg\} ,
\end{aligned}
\tag{7}
$$

where $\mathcal{V}_{L_i}$ and $\mathcal{V}_{R_i}$ are the coefficients of left– and right–handed leptonic currents with vector structure, respectively. Clearly, $\mathcal{S}_{L,R}$ are the weights of scalar leptonic currents with respective chirality. These new coefficients are functions of the Wilson coefficients in the partonic decay amplitude (1) and the form factors introduced in defining the hadronic matrix elements above. Their explicit expressions are given by

$$
\begin{aligned}
\mathcal{V}_{L_1} &= (C_{LL} + C_{RL}) \frac{V(q^2)}{m_B + m_{K^*}} - 2C_{BR} \frac{m_b}{q^2} T_1, \\
\mathcal{V}_{L_2} &= (C_{LL} - C_{RL})(m_B + m_{K^*}) A_1 - 2C_{BR} \frac{m_b}{q^2} (m_B^2 - m_{K^*}^2) T_2 , \\
\mathcal{V}_{L_3} &= \frac{C_{LL} - C_{RL}}{m_B + m_{K^*}} A_2 - 2C_{BR} \frac{m_b}{q^2} \left[T_2 + \frac{q^2}{m_B^2 - m_{K^*}^2} T_3 \right] , \\
\mathcal{V}_{L_4} &= (C_{LL} - C_{RL}) \frac{2m_{K^*}}{q^2} (A_3 - A_0) - 2C_{BR} \frac{m_b}{q^2} T_3 ,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{V}_{R_1} &= \mathcal{V}_{L_1}(C_{LL} \to C_{LR}\ ,\quad C_{RL} \to C_{RR})\ ,\\
\mathcal{V}_{R_2} &= \mathcal{V}_{L_2}(C_{LL} \to C_{LR}\ ,\quad C_{RL} \to C_{RR})\ ,\\
\mathcal{V}_{R_3} &= \mathcal{V}_{L_3}(C_{LL} \to C_{LR}\ ,\quad C_{RL} \to C_{RR})\ ,\\
\mathcal{V}_{R_4} &= \mathcal{V}_{L_4}(C_{LL} \to C_{LR}\ ,\quad C_{RL} \to C_{RR})\ ,\\
\mathcal{S}_L &= -(C_{LRRL} - C_{RLRL})\left(\frac{2m_{K^*}}{m_b} A_0\right)\ ,\\
\mathcal{S}_R &= -(C_{LRLR} - C_{RLLR})\left(\frac{2m_{K^*}}{m_b} A_0\right)\ ,
\end{aligned}
$$

where q^2 dependencies are implied. It is clear that all these effective form factors are functions of the specific form factors (2–6) and the Wilson coefficients in (1). Therefore, they carry information about both short– and long–distance physics.

The hadronic matrix element $\mathrm{M}(B \to K^* \ell^+ \ell^-)$ is the basic machinery for the computation of all physical quantities pertaining to this decay. In particular, the computation of the energetic distributions, total rate and various asymmetries follow from $(B \to K^* \ell^+ \ell^-)$ using the usual methods. In the next section, necessary asymmetries and other relevant quantities will be computed.

III. ASYMMETRIES

For an analysis of the asymmetries it is necessary to compute the differential decay rate for $B \to K^* \ell^+ \ell^-$ decay. For unpolarized leptons at the final state, using the decay amplitude in (7), the differential decay rate is found to be

$$
\begin{aligned}
\left(\frac{d\Gamma}{dq^2}\right)_0 &= \frac{G^2 \alpha^2}{2^{14} \pi^5 m_B} \left| V_{tb} V_{ts}^* \right|^2 \lambda^{1/2} v\\
&\times \Bigg\{ 32 \lambda m_B^4 \Big[\frac{1}{3} (m_B^2 s - m_\ell^2)(|\mathcal{V}_{L_1}|^2 + |\mathcal{V}_{R_1}|^2) + 2 m_\ell^2 \,\mathrm{Re}(\mathcal{V}_{L_1} \mathcal{V}_{R_1}^*) \Big]\\
&+ 96 m_\ell^2 \,\mathrm{Re}(\mathcal{V}_{L_2} \mathcal{V}_{R_2}^*) - \frac{4}{r} m_B^2 m_\ell \lambda \,\mathrm{Re}[(\mathcal{V}_{L_2} - \mathcal{V}_{R_2})(\mathcal{S}_L^* - \mathcal{S}_R^*)]\\
&+ \frac{8}{r} m_B^2 m_\ell^2 \lambda \,\mathrm{Re}\Big[\mathcal{V}_{L_2}^* (\mathcal{V}_{L_4} + \mathcal{V}_{R_3} - \mathcal{V}_{R_4}) + \mathcal{V}_{R_2}^* (\mathcal{V}_{L_3} - \mathcal{V}_{L_4} + \mathcal{V}_{R_4}) - (\mathcal{S}_L \mathcal{S}_R^*) \Big]\\
&+ \frac{4}{r} m_B^4 m_\ell (1-r) \lambda \Big\{ \mathrm{Re}[(\mathcal{V}_{L_3} - \mathcal{V}_{R_3})(\mathcal{S}_L^* - \mathcal{S}_R^*)] \Big\}\\
&+ \frac{8}{r} m_B^4 m_\ell^2 (1-r) \lambda \Big\{ \mathrm{Re}[-(\mathcal{V}_{L_3} - \mathcal{V}_{R_3})(\mathcal{V}_{L_4}^* - \mathcal{V}_{R_4}^*)] \Big\}\\
&- \frac{8}{r} m_B^4 m_\ell^2 \lambda (2 + 2r - s) \,\mathrm{Re}(\mathcal{V}_{L_3} \mathcal{V}_{R_3}^*) - \frac{4}{r} m_B^4 m_\ell s \lambda \,\mathrm{Re}[(\mathcal{V}_{L_4} - \mathcal{V}_{R_4})(\mathcal{S}_L^* - \mathcal{S}_R^*)]\\
&- \frac{4}{r} m_B^4 m_\ell^2 s \lambda \Big[|\mathcal{V}_{L_4}|^2 + |\mathcal{V}_{R_4}|^2 - 2 \,\mathrm{Re}(\mathcal{V}_{L_4} \mathcal{V}_{R_4}^*) \Big] + \frac{2}{r} m_B^2 (m_B^2 - 2 m_\ell^2) \lambda \Big[|\mathcal{S}_L|^2 + |\mathcal{S}_R|^2 \Big]
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{8}{3rs} m_B^2 \lambda \Big[m_\ell^2 (2-2r+s) + m_B^2 s (1-r-s) \Big] \Big[\mathrm{Re}(\mathcal{V}_{L_2} \mathcal{V}_{L_3}^*) + \mathrm{Re}(\mathcal{V}_{R_2} \mathcal{V}_{R_3}^*) \Big] \\
&+ \frac{4}{rs} \Big[2 m_\ell^2 (\lambda - 6rs) + m_B^2 s (\lambda + 12rs) \Big] \Big[\left| \mathcal{V}_{L_2} \right|^2 + \left| \mathcal{V}_{R_2} \right|^2 \Big] \\
&+ \frac{4}{3rs} m_B^4 \lambda \Big\{ m_B^2 s \lambda + m_\ell^2 \Big[2\lambda + 3s(2+2r-s) \Big] \Big\} \Big[\left| \mathcal{V}_{L_3} \right|^2 + \left| \mathcal{V}_{R_3} \right|^2 \Big] \Bigg\} ~. \qquad (8)
\end{aligned}
$$

where the subscript "0" is intended for the unpolarized decay rate. In this expression $s = q^2/m_B^2$, $r = m_{K^*}^2/m_B^2$, $v^2 = 1-(4m_\ell^2)/q^2$, and finally $\lambda(1,r,s) = 1+s^2+r^2-2r-2s-2rs$ is the familiar triangle function.

Our next task is the calculation of the lepton polarization asymmetries. For the computation of these asymmetries the unpolarized decay rate (8) is not sufficient. The measurement of these asymmetries require the specification of the total number of leptons of a given kind (for example, negatively-charged) in a given direction. Therefore, it is necessary to take into account the polarization of the lepton beam in a given direction. Considering, for example, the negatively-charged lepton, one can introduce the following three polarization vectors in the rest frame of ℓ^-:

$$
\begin{aligned}
\vec{e}_L &= \frac{\vec{p}_-}{|\vec{p}_-|} ~, \\
\vec{e}_N &= \frac{\vec{p}_{K^*} \times \vec{p}_-}{|\vec{p}_{K^*} \times \vec{p}_-|} ~, \\
\vec{e}_T &= \frac{(\vec{p}_{K^*} \times \vec{p}_-) \times \vec{p}_-}{|(\vec{p}_{K^*} \times \vec{p}_-) \times \vec{p}_-|} ~, \qquad (9)
\end{aligned}
$$

where $\vec{e}_i \cdot \vec{e}_j = \delta_{i,j}$, $p_- \cdot \vec{e}_i = 0$, $i,j = L,T,N$. Here, $\vec{e}_L$, $\vec{e}_T$ and $\vec{e}_N$ correspond, respectively, to the 'longitudinal', 'transversal' and 'normal' polarization directions of ℓ^- with respect to its direction of motion, $\vec{e}_L$. One notices that $\vec{e}_L$ and $\vec{e}_T$ are co–planar, and $\vec{e}_N$ is perpendicular to this plane. In the rest frame of ℓ^-, the temporal components of the corresponding four–vectors vanish. However, in the dilepton rest frame (that is, $\vec{q}=0$), the four–vector corresponding to $\vec{e}_L$ is boosted to $(\vec{p}_-/m_\ell, (E_\ell/m_\ell)\vec{e}_L)$ leaving $\vec{e}_T$ and $\vec{e}_N$ unchanged. In the following, all results will be conveniently given in the dilepton rest frame.

The differential decay rate for any spin direction $\vec{n}$ of the ℓ^-, where $\vec{n}$ is a unit vector in the ℓ^- rest frame satisfying $\vec{n}\cdot\vec{n} = 1, \vec{n}\cdot\vec{p}_- = 0$, can be expressed in the following form

$$
\frac{d\Gamma(\vec{n})}{dq^2} = \frac{1}{2} \left(\frac{d\Gamma}{dq^2} \right)_0 \Big[1 + \Big(P_L \vec{e}_L + P_N \vec{e}_N + P_T \vec{e}_T \Big) \cdot \vec{n} \Big] ~, \qquad (10)
$$

where the coefficients of unit vectors, P_L, P_N and P_T, are recognized as the 'longitudinal', 'normal' and 'transversal' polarization asymmetries. A simple formula for extracting these asymmetries from the polarized decay rate follows from (10) itself:

$$P_i(q^2) = \frac{\dfrac{d\Gamma}{dq^2}(\vec{n} = \vec{e}_i) - \dfrac{d\Gamma}{dq^2}(\vec{n} = -\vec{e}_i)}{\dfrac{d\Gamma}{dq^2}(\vec{n} = \vec{e}_i) + \dfrac{d\Gamma}{dq^2}(\vec{n} = -\vec{e}_i)} \ , \tag{11}$$

where $i = L, T, N$. One notes that the denominator in this expression is identical to the unpolarized decay rate (8). On the other hand, the numerator depends on the spin direction of the lepton under consideration. In essence what $P_i(q^2)$ measures is the difference between the rates for a particular direction and its opposite for a given dilepton invariant mass $m_{\ell\ell} = \sqrt{q^2}$.

Using the hadronic decay amplitude (7) in the general polarization asymmetry formulae (11), after a lengthy calculation the polarization asymmetries P_L, P_N and P_T are found to have the following explicit expressions:

$$\begin{aligned}
P_L = \frac{1}{\Delta} v \Bigg\{ & \frac{4}{3r}\lambda^2 m_B^6 \Big[|\mathcal{V}_{L_3}|^2 - |\mathcal{V}_{R_3}|^2 \Big] + \frac{4}{r}\lambda m_B^2 m_\ell \,\mathrm{Re}[(\mathcal{V}_{L_2} - \mathcal{V}_{R_2})(\mathcal{S}_L^* + \mathcal{S}_R^*)] \\
& - \frac{4}{r}\lambda m_B^4 m_\ell (1-r) \,\mathrm{Re}[(\mathcal{V}_{L_3} - \mathcal{V}_{R_3})(\mathcal{S}_L^* + \mathcal{S}_R^*)] + \frac{32}{3}\lambda m_B^6 s \Big[|\mathcal{V}_{L_1}|^2 - |\mathcal{V}_{R_1}|^2 \Big] \\
& - \frac{2}{r}\lambda m_B^4 s \Big[|\mathcal{S}_L|^2 - |\mathcal{S}_R|^2 \Big] + \frac{4}{r}\lambda m_B^4 m_\ell s \,\mathrm{Re}[(\mathcal{V}_{L_4} - \mathcal{V}_{R_4})(\mathcal{S}_L^* + \mathcal{S}_R^*)] \\
& - \frac{8}{3r}\lambda m_B^4 (1-r-s) \Big[\mathrm{Re}(\mathcal{V}_{L_2}\mathcal{V}_{L_3}^*) - \mathrm{Re}(\mathcal{V}_{R_2}\mathcal{V}_{R_3}^*) \Big] \\
& + \frac{4}{3r}\lambda m_B^2 (\lambda + 12rs) \Big[|\mathcal{V}_{L_2}|^2 - |\mathcal{V}_{R_2}|^2 \Big] \Bigg\} ,
\end{aligned}$$

$$\begin{aligned}
P_T = \frac{1}{\Delta}\sqrt{\lambda}\pi \Bigg\{ & - 8 m_B^3 m_\ell \sqrt{s} \,\mathrm{Re}[(\mathcal{V}_{L_1} + \mathcal{V}_{R_1})(\mathcal{V}_{L_2}^* + \mathcal{V}_{R_2}^*)] \\
& + \frac{1}{r} m_B^3 m_\ell (1 + 3r + s)\sqrt{s} \Big[\mathrm{Re}(\mathcal{V}_{L_2}\mathcal{V}_{R_3}^*) - \mathrm{Re}(\mathcal{V}_{L_3}\mathcal{V}_{R_2}^*) \Big] \\
& + \frac{1}{r\sqrt{s}} m_B m_\ell (1-r-s) \Big[|\mathcal{V}_{L_2}|^2 - |\mathcal{V}_{R_2}|^2 \Big] \\
& + \frac{2}{r\sqrt{s}} m_B m_\ell^2 (1-r-s) \Big[\mathrm{Re}(\mathcal{V}_{L_2}\mathcal{S}_R^*) - \mathrm{Re}(\mathcal{V}_{R_2}\mathcal{S}_L^*) \Big] \\
& + \frac{1}{r} m_B^3 m_\ell (1-r-s)\sqrt{s} \,\mathrm{Re}[(\mathcal{V}_{L_2} + \mathcal{V}_{R_2})(\mathcal{V}_{L_4} - \mathcal{V}_{R_4})] \\
& + \frac{2}{r\sqrt{s}} m_B^3 m_\ell^2 \lambda \Big[- \mathrm{Re}(\mathcal{V}_{L_3}\mathcal{S}_R^*) + \mathrm{Re}(\mathcal{V}_{R_3}\mathcal{S}_L^*) \Big] \\
& + \frac{1}{r\sqrt{s}} m_B^5 m_\ell (1-r)\lambda \Big[|\mathcal{V}_{L_3}|^2 - |\mathcal{V}_{R_3}|^2 \Big] \\
& + \frac{1}{r} m_B^5 m_\ell \lambda \sqrt{s} \,\mathrm{Re}[-(\mathcal{V}_{L_3} + \mathcal{V}_{R_3})(\mathcal{V}_{L_4}^* - \mathcal{V}_{R_4}^*)]
\end{aligned}$$

$$
\begin{aligned}
&+ \frac{1}{r\sqrt{s}} m_B^3 m_\ell [(1-r-s)(1-r)+\lambda]\Big[- \mathrm{Re}(\mathcal{V}_{L_2}\mathcal{V}_{L_3}^*) + \mathrm{Re}(\mathcal{V}_{R_2}\mathcal{V}_{R_3}^*)\Big] \\
&+ \frac{1}{r\sqrt{s}} m_B (1-r-s)(-2m_\ell^2 + m_B^2 s)\Big[\mathrm{Re}(\mathcal{V}_{R_2}\mathcal{S}_R^*) - \mathrm{Re}(\mathcal{V}_{L_2}\mathcal{S}_L^*)\Big] \\
&+ \frac{1}{r\sqrt{s}} m_B^3 \lambda (-2m_\ell^2 + m_B^2 s)\Big[- \mathrm{Re}(\mathcal{V}_{R_3}\mathcal{S}_R^*) + \mathrm{Re}(\mathcal{V}_{L_3}\mathcal{S}_L^*)\Big]\Bigg\} ,
\end{aligned}
$$

$$
\begin{aligned}
P_N = {} & \frac{1}{\Delta}\pi v m_B^3 \sqrt{\lambda}\sqrt{s}\Bigg\{ 8 m_\ell \,\mathrm{Im}(\mathcal{V}_{L_2}^*\mathcal{V}_{R_1} + \mathcal{V}_{L_1}^*\mathcal{V}_{R_2}) \\
&+ \frac{1}{r} m_\ell (1+3r-s)\,\mathrm{Im}\Big[(\mathcal{V}_{L_2}+\mathcal{V}_{R_2})(\mathcal{V}_{L_3}^* - \mathcal{V}_{R_3}^*)\Big] \\
&+ \frac{1}{r} m_B^2 \lambda \mathrm{Im}\Big[(m_\ell \mathcal{V}_{L_4} - m_\ell \mathcal{V}_{R_4} - \mathcal{S}_L)\mathcal{V}_{L_3}^* - (m_\ell \mathcal{V}_{R_4} - m_\ell \mathcal{V}_{L_4} - \mathcal{S}_R)\mathcal{V}_{R_3}^*\Big] \\
&+ \frac{1}{r}(1-r-s)\mathrm{Im}\Big[(\mathcal{S}_L - m_\ell \mathcal{V}_{L_4} + m_\ell \mathcal{V}_{R_4})\mathcal{V}_{L_2}^* - (\mathcal{S}_R - m_\ell \mathcal{V}_{R_4} + m_\ell \mathcal{V}_{L_4})\mathcal{V}_{R_2}^*\Big]\Bigg\} , \qquad (12)
\end{aligned}
$$

where Δ is the expression within the curly parenthesis in the unpolarized differential decay rate in (8). These expressions for the polarization asymmetries are quite general except for the neglect of strange quark mass and the tensor operators (the last two operators in (1)) as mentioned before.

Before proceeding, it is convenient to make a few useful observations on the lepton asymmetries. Particularly interesting one is the massless (light) lepton limit: $m_\ell \to 0$. In this case, P_L depends only on the bilinears of $\mathcal{V}_{L_i}$ and $\mathcal{V}_{R_i}$, that is, the effects of the scalar operators in (1) completely decouple. On the other hand, P_T and P_N happen to depend only on the interference terms between the coefficients of the vector operators ($\mathcal{V}_{L_i}$ and $\mathcal{V}_{R_i}$) and those of the scalar operators ($\mathcal{S}_{L,R}$). However, one notices that the scalar operators in (1) can be induced by an exchange of the scalar particle (such as two Higgs doublet models [17]) in which case the coefficients $\mathcal{S}_{L,R}$ are necessarily proportional to the lepton mass. Therefore, in the limit of massless (light) leptons only the longitudinal asymmetry P_L can remain non–vanishing. Conversely, in near future, if experiment yields non–vanishing P_N and P_T for $B \to K^* e^+ e^-$ decay this would imply the generation of scalar operators by mechanisms beyond the Higgs model where the fermion scalar coupling is always proportional to the fermion mass.

In general, the lepton polarization asymmetries (12) are able to probe real as well as imaginary parts of the effective form factors $\mathcal{V}_{L_i}$, $\mathcal{V}_{R_i}$ and $\mathcal{S}_{L,R}$. As the parametrization (7) shows the hadronic form factors are inherently real, and thus the imaginary parts of $\mathcal{V}_{L_i}$, $\mathcal{V}_{R_i}$ and $\mathcal{S}_{L,R}$ in (7) can come only from the Wilson coefficients in (1). Below we will keep this picture, that is, we will be dealing only with the CP violation effects due to short–distance physics parametrized by the Wilson coefficients. At this point it is useful to distinguish between CP properties of the phases in the Wilson coefficients. In principle, $C_{BR}, \cdots C_{TE}$ all can have finite phases; however, these phases can have *strong* and *weak* subparts. Here by *strong* and *weak* we mean *even* and *odd* phases under CP conjugation. To be able to distinguish such distinct components of the phases it is not sufficient to analyze the polarization asymmetries alone. One, in particular, has to consider the CP asymmetry of the decay which is inherently sensitive to CP character of the phases of the Wilson coefficients. Using the unpolarized decay rate (8), the CP asymmetry for $B \to K^* \ell^+ \ell^-$ decay is defined by:

$$A_{CP}(q^2) = \frac{\left(\dfrac{d\Gamma}{dq^2}\right)_0 (B \to K^* \ell^+ \ell^-) - \left(\dfrac{d\Gamma}{dq^2}\right)_0 (\overline{B} \to \overline{K^*} \ell^+ \ell^-)}{\left(\dfrac{d\Gamma}{dq^2}\right)_0 (B \to K^* \ell^+ \ell^-) + \left(\dfrac{d\Gamma}{dq^2}\right)_0 (\overline{B} \to \overline{K^*} \ell^+ \ell^-)} , \tag{13}$$

where the processes to which $d\Gamma/dq^2$ refers are explicitly shown in the arguments. Making use of the explicit expression for the unpolarized decay rate (8) one can determine the detailed dependence of $A_{CP}(q^2)$ on the model parameters. For this purpose it is useful to introduce the following parametrization for the quantities $\mathcal{V}_{L_i}$, $\mathcal{V}_{R_i}$ and $\mathcal{S}_{L,R}$ (7):

$$\mathcal{V}_{L_i} = |\mathcal{V}_{L_i}| e^{i\phi_w^{L_i} + i\phi_s^{L_i}} \ , \ \ \mathcal{V}_{R_i} = |\mathcal{V}_{R_i}| e^{i\phi_w^{R_i} + i\phi_s^{R_i}} \ , \ \ \mathcal{S}_{L,R} = |\mathcal{V}_{L,R}| e^{i\phi_w^{L,R} + i\phi_s^{L,R}} \ , \tag{14}$$

where $i = 1, \cdots, 4$. In this expression subscript "s" ("w") stands for *strong* (*weak*) phases mentioned above. By definition, $\mathcal{V}$'s and $\mathcal{S}$'s are combinations of hadronic form factors and Wilson coefficients so that the phases $\phi_{w,s}$ defined by (14) are explicit functions of the dilepton invariant mass. With this definition of the from factors it is possible to find a suggestive form for the CP asymmetry:

$$\begin{aligned}
A_{CP}(q^2) = \frac{1}{\Sigma} \Big\{ & - 64 \lambda m_B^4 m_\ell^2 \, |\mathcal{V}_{L_1}| \, |\mathcal{V}_{R_1}| \sin \Delta\phi_s^{L_1,R_1} \sin \Delta\phi_w^{L_1,R_1} \\
& - 96 m_\ell^2 \, |\mathcal{V}_{L_2}| \, |\mathcal{V}_{R_2}| \sin \Delta\phi_s^{L_2,R_2} \sin \Delta\phi_w^{L_2,R_2} + \frac{4}{r} m_B^2 m_\ell \lambda \Big[|\mathcal{V}_{L_2}| \, |\mathcal{S}_L| \sin \Delta\phi_s^{L_2,L} \sin \Delta\phi_w^{L_2,L} \\
& + |\mathcal{V}_{R_2}| \, |\mathcal{S}_R| \sin \Delta\phi_s^{R_2,R} \sin \Delta\phi_w^{R_2,R} - |\mathcal{V}_{L_2}| \, |\mathcal{S}_R| \sin \Delta\phi_s^{L_2,R} \sin \Delta\phi_w^{L_2,R} \\
& - |\mathcal{V}_{R_2}| \, |\mathcal{S}_L| \sin \Delta\phi_s^{R_2,L} \sin \Delta\phi_w^{R_2,L} \Big] - \frac{4}{r} m_B^4 m_\ell (1-r) \lambda \Big[|\mathcal{V}_{L_3}| \, |\mathcal{S}_L| \sin \Delta\phi_s^{L_3,L} \sin \Delta\phi_w^{L_3,L} \\
& + |\mathcal{V}_{R_3}| \, |\mathcal{S}_R| \sin \Delta\phi_s^{R_3,R} \sin \Delta\phi_w^{R_3,R} - |\mathcal{V}_{L_3}| \, |\mathcal{S}_R| \sin \Delta\phi_s^{L_3,R} \sin \Delta\phi_w^{L_3,R} \\
& - |\mathcal{V}_{R_3}| \, |\mathcal{S}_L| \sin \Delta\phi_s^{R_3,L} \sin \Delta\phi_w^{R_3,L} \Big] + \frac{4}{r} m_B^4 m_\ell s \lambda \Big[|\mathcal{V}_{L_4}| \, |\mathcal{S}_L| \sin \Delta\phi_s^{L_4,L} \sin \Delta\phi_w^{L_4,L} \\
& + |\mathcal{V}_{R_4}| \, |\mathcal{S}_R| \sin \Delta\phi_s^{R_4,R} \sin \Delta\phi_w^{R_4,R} - |\mathcal{V}_{L_4}| \, |\mathcal{S}_R| \sin \Delta\phi_s^{L_4,R} \sin \Delta\phi_w^{L_4,R}
\end{aligned}$$

$$
\begin{aligned}
&- |\mathcal{V}_{R_4}|\, |\mathcal{S}_L| \sin\Delta\phi_s^{R_4,L} \sin\Delta\phi_w^{R_4,L}\Big] + \frac{8}{r} m_B^4 m_\ell^2 (1-r)\lambda \Big[|\mathcal{V}_{L_3}|\, |\mathcal{V}_{L_4}| \sin\Delta\phi_s^{L_3,L_4} \sin\Delta\phi_w^{L_3,L_4} \\
&+ |\mathcal{V}_{R_3}|\, |\mathcal{V}_{R_4}| \sin\Delta\phi_s^{R_3,R_4} \sin\Delta\phi_w^{R_3,R_4} - |\mathcal{V}_{L_3}|\, |\mathcal{V}_{R_4}| \sin\Delta\phi_s^{L_3,R_4} \sin\Delta\phi_w^{L_3,R_4} \\
&- |\mathcal{V}_{R_3}|\, |\mathcal{V}_{L_4}| \sin\Delta\phi_s^{R_3,L_4} \sin\Delta\phi_w^{R_3,L_4}\Big] \\
&+ \frac{8}{r} m_B^4 m_\ell^2 \lambda (2+2r-s)\; |\mathcal{V}_{L_3}|\, |\mathcal{V}_{R_3}| \sin\Delta\phi_s^{L_3,R_3} \sin\Delta\phi_w^{L_3,R_3} \\
&- \frac{8}{r} m_B^4 m_\ell^2 s\lambda\; |\mathcal{V}_{L_4}|\, |\mathcal{V}_{R_4}| \sin\Delta\phi_s^{L_4,R_4} \sin\Delta\phi_w^{L_4,R_4} \\
&+ \frac{8}{3rs} m_B^2 \lambda \Big[|\mathcal{V}_{L_2}|\, |\mathcal{V}_{L_3}| \sin\Delta\phi_s^{L_2,L_3} \sin\Delta\phi_w^{L_2,L_3} + |\mathcal{V}_{R_2}|\, |\mathcal{V}_{R_3}| \sin\Delta\phi_s^{R_2,R_3} \sin\Delta\phi_w^{R_2,R_3}\Big] \\
&- \frac{8}{r} m_B^2 m_\ell^2 \lambda \Big[|\mathcal{V}_{L_2}|\, |\mathcal{V}_{L_4}| \sin\Delta\phi_s^{L_2,L_4} \sin\Delta\phi_w^{L_2,L_4} + |\mathcal{V}_{L_2}|\, |\mathcal{V}_{R_3}| \sin\Delta\phi_s^{L_2,R_3} \sin\Delta\phi_w^{L_2,R_3} \\
&- |\mathcal{V}_{L_2}|\, |\mathcal{V}_{R_4}| \sin\Delta\phi_s^{L_2,R_4} \sin\Delta\phi_w^{L_2,R_4} + |\mathcal{V}_{R_2}|\, |\mathcal{V}_{R_4}| \sin\Delta\phi_s^{R_2,R_4} \sin\Delta\phi_w^{R_2,R_4} \\
&+ |\mathcal{V}_{R_2}|\, |\mathcal{V}_{L_3}| \sin\Delta\phi_s^{R_2,L_3} \sin\Delta\phi_w^{R_2,L_3} - |\mathcal{V}_{R_2}|\, |\mathcal{V}_{L_4}| \sin\Delta\phi_s^{R_2,L_4} \sin\Delta\phi_w^{R_2,L_4} \\
&- |\mathcal{S}_L|\, |\mathcal{S}_R| \sin\Delta\phi_s^{L,R} \sin\Delta\phi_w^{L,R}\Big] \Bigg\}\ , \qquad (15)
\end{aligned}
$$

where $\Delta\phi_x^{a,b} \equiv \phi_x^a - \phi_x^b$. The quantity Σ in the denominator is even under all these phases, and has the expression

$$
\begin{aligned}
\Sigma &= \text{Numerator of } A_{CP}\Big(\sin\Delta\phi_s^{a,b} \sin\Delta\phi_w^{a,b} \longrightarrow -\cos\Delta\phi_s^{a,b} \cos\Delta\phi_w^{a,b}\Big) \\
&+ \Bigg\{ \frac{32}{3} \lambda m_B^4 (m_B^2 s - m_\ell^2) \Big[|\mathcal{V}_{L_1}|^2 + |\mathcal{V}_{R_1}|^2 \Big] - \frac{4}{r} m_B^4 m_\ell^2 s\lambda \Big[|\mathcal{V}_{L_4}|^2 + |\mathcal{V}_{R_4}|^2 \Big] \\
&+ \frac{2}{r} m_B^2 (m_B^2 - 2m_\ell^2)\lambda \Big[|\mathcal{S}_L|^2 + |\mathcal{S}_R|^2 \Big] \\
&+ \frac{4}{rs} \Big[2m_\ell^2 (\lambda - 6rs) + m_B^2 s(\lambda + 12rs)\Big] \Big[|\mathcal{V}_{L_2}|^2 + |\mathcal{V}_{R_2}|^2 \Big] \\
&+ \frac{4}{3rs} m_B^4 \lambda \Big\{ m_B^2 s\lambda + m_\ell^2 \Big[2\lambda + 3s(2+2r-s)\Big]\Big\} \Big[|\mathcal{V}_{L_3}|^2 + |\mathcal{V}_{R_3}|^2 \Big] \Bigg\}\ . \qquad (16)
\end{aligned}
$$

Until now the decay rate (8), the lepton polarization asymmetries (12) and CP asymmetry (13) have been computed by adopting a rather general quark level amplitude (1) for $B \to K^* \ell^+ \ell^-$ decay. Presently this exclusive decay has a direct bound coming from recent CDF measurement [18]: $\mathrm{BR}(B \to K^* \mu^+ \mu^-) < 4.0 \times 10^{-6}$. In addition to this direct bound, existing CLEO result [2] for $\mathrm{BR}(B \to K^* \gamma)$ imposes another important, albeit partial, constraint on the parameter space. Indeed, using the notation of (1) and appropriate form factors derived in (2)-(6) the total decay rate for $B \to K^* \gamma$ can be written as

$$
\Gamma(B \to K^* \gamma) = \frac{G^2 \alpha m_B^3 m_b^2}{128 \pi^4} |V_{ts} V_{tb}^*|^2 \left(1 - \frac{m_K^2}{m_B^2} \right)^3 |C_{BR} T_1(0)|^2\ , \qquad (17)
$$

which constrains directly $|C_{BR}\, T_1(0)|$. Therefore, the norm of the Wilson coefficient for the dipole operator is fixed by $\mathrm{BR}(B \to K^* \gamma)$. This constraint, however, does not say anything about the CP violation potential of C_{BR}.

IV. LARGE CP–PHASES WITHIN AN SM–LIKE MODEL

As as been emphasized before, the underlying model for the partonic decay amplitude (1) is rather general one. In principle, all the Wilson coefficients can be non-zero and may have arbitrary phases. However, a realistic model should meet the requirements of the SM to leading order, and especially, should not cause unacceptable deviations from the existing experimental data confirmed already by the SM. For this reason it is convenient to establish the discussion on a general model with close reference to the SM predictions.

Therefore, here we follow a minimal prescription such that the general Wilson coefficients in (1) are

- endowed with new phases beyond the SM,
- identical to SM ones when these phases vanish.

Such an approach obviously neglects the new physics contributions to the norms of the Wilson coefficients; however, at the aim of determining the information content of the polarization and CP asymmetries on the sources of CP violation, it suffices. This is kind of a minimal approach for parametrizing the new physics CP violation for asymmetry measurements in $B \to K^* \ell^+ \ell^-$ decay.

Adopting this approach, one can make the following assignments for the general Wilson coefficients in (1). First, the coefficients describing the scalar–scalar type interactions vanish identically

$$C_{LRRL} = C_{RLLR} = C_{LRLR} = C_{RLRL} = 0. \tag{18}$$

It is known that such coefficients exist, for example, in the two Higgs doublet models (2HDM) which have an extended Higgs sector compared to the SM. In such models these scalar–scalar interactions are induced by the Higgs exchange, and the resulting Wilson coefficients are proportional to $m_b m_\ell / m_h^2$ which is maximal for $\ell = \tau$. However, to the extent one neglects m_s/m_b, $m_b m_\ell / m_h^2$ is, too, negligible in the light of recent LEP limit on the Higgs boson mass m_h [19] is taken into account.

The Wilson coefficient for the dipole operator C_{BR} obeys

$$C_{BR} = -2C_7^{eff}(m_b)\ e^{i\phi_7} \equiv -2\left(C_7(m_b) - \frac{1}{3}C_5(m_b) - C_6(m_b)\right)\ e^{i\phi_7} \tag{19}$$

where ϕ_7 is an arbitrary phase, and it is not constrained by BR($B \to K^*\gamma$) at all.

Finally the coefficients of the vector–vector interactions in (1) are given by

$$C_{LL(LR)} = C_9^{eff}(m_b)\ e^{i\phi_9} - (+)C_{10}(m_b)\ e^{i\phi_{10}}\ ,\quad C_{RL} = C_{RR} = 0 \tag{20}$$

where the coefficient C_{10} is known to be scale independent: $C_{10}(m_b) = C_{10}(M_W)$.

In the SM the Wilson coefficients $C_7^{eff}(m_b)$ and $C_{10}(m_b)$ are strictly real as can be read off from Table II. Moreover, the SM prediction for $C_7^{eff}(m_b)$ is already consistent with the CLEO determination of BR($B \to K^*\gamma$) in (17). Therefore, through the choice of C_{BR} in (19) the experimental constraint is already taken into account. Although individual Wilson coefficients at $\mu \sim m_b$ level are all real (see Table II) the effective Wilson coefficient $C_9^{eff}(m_b, q^2)$ has a finite phase, and is an explicit function of the dilepton invariant mass, q^2. To see its phase content it is useful to reproduce its explicit expression here:

$$C_9^{eff}(m_b) = C_9(m_b)\left\{1 + \frac{\alpha_s\left(\mu\right)}{\pi}\omega\left(\hat{s}\right)\right\} + Y_{SD}\left(m_b, \hat{s}\right) + Y_{LD}\left(m_b, \hat{s}\right) \tag{21}$$

where $C_9(m_b)$ is read off from Table II. Here $\omega\left(\hat{s}\right)$ represents the $\mathcal{O}(\alpha_s)$ corrections coming from one–gluon exchange in the matrix element of the corresponding operator $\mathcal{O}_9$ [20]:

$$\begin{aligned}\omega\left(\hat{s}\right) &= -\frac{2}{9}\pi^2 - \frac{4}{3}Li_2\left(\hat{s}\right) - \frac{2}{3}\ln\left(\hat{s}\right)\,\ln\left(1-\hat{s}\right) - \frac{5+4\hat{s}}{3\left(1+2\hat{s}\right)}\ln\left(1-\hat{s}\right)\\ &- \frac{2\hat{s}\left(1+\hat{s}\right)\left(1-2\hat{s}\right)}{3\left(1-\hat{s}\right)^2\left(1+2\hat{s}\right)}\ln\left(\hat{s}\right) + \frac{5+9\hat{s}-6\hat{s}^2}{3\left(1-\hat{s}\right)\left(1+2\hat{s}\right)}\ .\end{aligned} \tag{22}$$

In (21) Y_{SD} and Y_{LD} represent, respectively, the short– and long–distance contributions of the four–quark operators $\mathcal{O}_{i=1,\cdots,6}$ [20,21]. Here Y_{SD} can be obtained by a perturbative calculation

$$\begin{aligned}Y_{SD}\left(m_b, \hat{s}\right) &= g\left(\hat{m}_c, \hat{s}\right)\left[3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6\right]\\ &- \frac{1}{2}g\left(1, \hat{s}\right)\left[4C_3 + 4C_4 + 3C_5 + C_6\right]\\ &- \frac{1}{2}g\left(0, \hat{s}\right)\left[C_3 + 3C_4\right] + \frac{2}{9}\left[3C_3 + C_4 + 3C_5 + C_6\right]\\ &- \frac{V_{us}^* V_{ub}}{V_{ts}^* V_{tb}}\left[3C_1 + C_2\right]\left[g\left(0, \hat{s}\right) - g\left(\hat{m}_c, \hat{s}\right)\right]\ ,\end{aligned} \tag{23}$$

where the loop function $g\left(m_q, s\right)$ stands for the loops of quarks with mass m_q at the dilepton invariant mass s. This function develops absorbtive parts for dilepton energies $s = 4m_q^2$:

$$g(\hat{m}_q, \hat{s}) = -\frac{8}{9}\ln\hat{m}_q + \frac{8}{27} + \frac{4}{9}y_q - \frac{2}{9}(2+y_q)\sqrt{|1-y_q|}$$
$$\times\Big\{\Theta(1-y_q)\left(\ln\frac{1+\sqrt{1-y_q}}{1-\sqrt{1-y_q}} - i\pi\right) + \Theta(y_q-1)2\arctan\frac{1}{\sqrt{y_q-1}}\Big\}, \tag{24}$$

where $\hat{m}_q = m_q/m_b$ and $y_q = 4\hat{m}_q^2/\hat{s}$. Hence, due to the absorbtive parts of $g(\hat{m}_q, \hat{s})$, there are strong phases coming from Y_{SD}. One, in particular, notices the terms proportional to $g(0, \hat{s})$ which have a non–vanishing imaginary parts independent of the dilepton invariant mass.

In addition to these perturbative contributions, the $\bar{c}c$ loops can excite low–lying charmonium states $\psi(1s), \cdots, \psi(6s)$ whose contributions are represented by Y_{LD} [22]:

$$\begin{aligned} Y_{LD}(m_b, \hat{s}) &= \frac{3}{\alpha^2}\Big\{ -\frac{V_{cs}^* V_{cb}}{V_{ts}^* V_{tb}} C^{(0)} - \frac{V_{us}^* V_{ub}}{V_{ts}^* V_{tb}}\left[3C_3 + C_4 + 3C_5 + C_6\right] \Big\} \\ &\times \sum_{V_i=\psi(1s),\cdots,\psi(6s)} \frac{\pi\kappa_i\Gamma\left(V_i \to \ell^+\ell^-\right) M_{V_i}}{\left(M_{V_i}^2 - \hat{s}m_b^2 - iM_{V_i}\Gamma_{V_i}\right)} , \end{aligned} \tag{25}$$

where κ_i are the Fudge factors for $B \to K^* V_i \to K^* \ell^+\ell^-$ transition, and $C^{(0)} \equiv 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6$. Here the sum runs over all all charmonium resonances with mass m_{V_i} and total decay rate Γ_{V_i}. Contrary to Y_{SD}, the long–distance contribution Y_{LD} has both weak and strong phases. The weak phases follow from the CKM elements whereas the strong phases come from the $\hat{s}$ values for which i-th charmonium state is on shell. Therefore, the Wilson coefficient $C_9^{eff}(m_b)$ has both weak and strong phases already in the SM.

$C_1(m_b)$	$C_2(m_b)$	$C_3(m_b)$	$C_4(m_b)$	$C_5(m_b)$	$C_6(m_b)$	$C_7^{eff}(m_b)$	$C_9(m_b)$	$C_{10}(m_b)$
-0.248	1.107	0.011	-0.026	0.007	-0.031	-0.313	4.344	-4.669

TABLE II. The numerical values of the Wilson coefficients at $\mu \sim m_b$ scale within the SM. The corresponding numerical value of C^0 is 0.362.

In this sense, the Wilson coefficients $C_7^{eff}(m_b)$ and $C_{10}(m_b)$ cannot develop any strong phase, and thus, ϕ_7 and ϕ_{10} should necessarily originate from physics beyond the SM. A few observations on the asymmetries help much in simplifying the analysis: (i) Due to the dependencies of the asymmetries on the Wilson coefficients, it is clear that one can re–phase one of the Wilson coefficients. For instance, one can choose $\phi_9 \equiv 0$ leaving $C_9^{eff}(m_b)$ with its SM phases only. (ii) As mentioned above, the Wilson coefficients $C_7^{eff}(m_b)$ and $C_{10}(m_b)$ cannot develop strong phases from light quark loops so that ϕ_7 and ϕ_{10} can be chosen to have purely *weak* character.

In the light of analytic derivations as well as particular observations mentioned above, one can investigate the dependence of the asymmetries on these *new weak* phases ϕ_7 and ϕ_{10} to have an estimate of their information content. It is conceivable that such an analysis will provide a tool to mark possible sources of CP violation beyond the CKM matrix.

V. NUMERICAL ESTIMATES

In this section we present our numerical estimates for the asymmetries $A_{CP},\ P_L,\ P_T$ and P_N for $B \to K^* \mu^+ \mu^-$ and $B \to K^* \tau^+ \tau^-$ decays separately. We take hadronic form factors from Table I and the Wilson coefficients from Table II. For the remaining parameters we take $m_b = 4.8\ GeV,\ m_c = 1.35\ GeV,\ m_B = 5.28\ GeV,\ m_{K^*} = 0.892\ GeV$.

The dilepton invariant mass has the kinematical interval $4m_\ell^2 \le q^2 \le (m_B - m_{K^*})^2$ in which the charmonium resonances can be excited. The dominant contribution comes from the three low–lying resonances $J/\psi, \psi^{'}, \psi^{''}$ in the interval $8\ GeV^2 \lesssim q^2 \lesssim 14.5\ GeV^2$. In order to minimize the hadronic uncertainties we will discard this subinterval in the analysis below by dividing the q^2 region to *low* and *high* dilepton mass intervals

$$
\begin{aligned}
&\text{Region I}: \quad 4m_\ell^2 \le q^2 \le 8\ GeV^2, \\
&\text{Region II}: \quad 14.5\ GeV^2 \le q^2 \le (m_B - m_{K^*})^2,
\end{aligned}
\tag{26}
$$

where the contribution of the higher resonances do still exists in the second region.

Due to $1/q^2$ factor in front of C_{BR}, in Region I the contribution of the dipole type operator dominates. Therefore, asymmetries which involve the differences of the decay rates are suppressed in Region I compared to ones in Region II. This property will be illustrated in Figs. 1 – 2 and the remaining analysis for the asymmetries will be performed only for Region II where the asymmetries are larger.

As mentioned previously, in the model under concern, there are two weak phases: ϕ_7 and ϕ_{10}. However, a close inspection of the CP asymmetry shows that, it is independent of ϕ_{10}. This follows from the fact that CP asymmetry can exist only when interference terms involve *strong* and *weak* phases. In this model, similar to SM, there is no interference terms involving C_9^{eff} and C_{10}. For this reason A_{CP} is independent of ϕ_{10}.

First, we illustrate A_{CP} in ϕ_7–q^2 plane for $B \to K^* \mu^+ \mu^-$ decay for Region I and Region II in Fig. 1 and Fig. 2, respectively. In both figures we take $\phi_{10} = 0$, and as noted above A_{CP} is already independent of this phase. In Region I the CP asymmetry is practically independent of q^2, and becomes maximal for marginal CP violation, $\phi_7 = \pi/2$. In Region II, however, the q^2 dependence is comparatively enhanced as the dominance of dipole coefficient is now reduced. Besides, as figures suggest the CP asymmetry in Region II is one order of magnitude larger than in Region I, and this confirms our expectation above.

One notes that the average asymmetries could be measured more easily in experiments. Therefore, from now on we will discuss averaged CP and lepton polarization asymmetries in Region II. The averaging procedure is defined by

$$\langle Q \rangle = \frac{\int_{14.5\ GeV^2}^{(m_B - m_{K^*})^2} Q \frac{d\Gamma}{dq^2} dq^2}{\int_{14.5\ GeV^2}^{(m_B - m_{K^*})^2} \frac{d\Gamma}{dq^2} dq^2} \tag{27}$$

where $Q = P_L, P_N, P_T$ or A_{CP}.

Depicted in Fig. 3 (Fig. 4) is the ϕ_7 dependence of the averaged asymmetries $\langle P_L \rangle$, $\langle P_T \rangle$, $\langle P_N \rangle$ and $\langle A_{CP} \rangle$ at $\phi_{10} = 0$ ($\phi_{10} = \pi/2$) for $B \to K^* \mu^+ \mu^-$ decay. Similarly, Figs. 5 and 6 show the ϕ_7 dependence of the same quantities for $B \to K^* \tau^+ \tau^-$. As noted before, the CP asymmetry depends only on ϕ_7; however, as these figures show clearly among all asymmetries P_N is very sensitive to ϕ_{10}: For $\phi_{10} = 0$ ($\phi_{10} = \pi/2$) P_N is purely positive (negative). In addition to this, P_N at $\phi_{10} = \pi/2$ is one order of magnitude larger than that at $\phi_{10} = 0$. This property is valid for both $\mu^+\mu^-$ and $\tau^+\tau^-$ final states. Besides, since P_N is proportional to the lepton mass, the $B \to K^* \tau^+ \tau^-$ decay is much more relevant for its measurement. This sensitivity of P_N on ϕ_{10} can be explained as follows: P_N depends on the imaginary part of the bilinear combinations of the Wilson coefficients, such as Im $\left[C_9^{eff} C_{10}^*\right]$. When $\phi_{10} = \pi/2$ ($\phi_{10} = 0$) C_{10} is pure imaginary (real) and therefore $\mathrm{Im}[C_9^{eff} C_{10}^*] = |C_{10}| \mathrm{Re}[C_9^{eff}]$ ($\mathrm{Im}[C_9^{eff} C_{10}^*] = |C_{10}| \mathrm{Im}[C_9^{eff}]$). Since $|\mathrm{Re}[C_9^{eff}]| >> |\mathrm{Im}[C_9^{eff}]|$, P_N at $\phi_{10} = \pi/2$ is roughly one order of magnitude larger than its value at $\phi_{10} = 0$. Remaining two asymmetries, P_L and P_T, are less sensitive to ϕ_{10}.

In Fig.7 and 8 we present the correlation between $\langle A_{CP} \rangle$ and $\langle P_N \rangle$ for $B \to K^* \tau^+ \tau^-$ decay by varying ϕ_7 from 0 to 2π at $\phi_{10} = 0$ and $\phi_{10} = \pi/2$, respectively. For $\mu^+\mu^-$ channel P_N is much smaller so we do not analyze this case. The SM predictions are given by the intersections of $\langle A_{CP} \rangle = 0$ line and the curves themselves. Due to the sign ambiguity of C_7 there are two solutions. All other points on the curves are generated by the new physics phases. If a simultaneous measurement of $\langle A_{CP} \rangle$ and $\langle P_N \rangle$ gives a point on the curve and if this point is distinct from the SM prediction then this will be an indication of the new physics contribution. Moreover, such a simultaneous measurement enables us to determine the sign of the new phases unambiguously.

VI. CONCLUSION

In this work we have adopted a model–independent approach in studying the sensitivity of the CP and lepton polarization asymmetries to new CP phases. In parcticular, we have taken the Wilson coefficients being identical to the SM ones except for their phases. The main result of the present study is that the CP asymmetry and normal lepton polarization asymmetry are the most sensitive quantities to new sources of weak phases beyond the SM. While A_{CP} is sensitive to ϕ_7 only, P_N is more sensitive to ϕ_{10}. Therefore, measurement of these two asymmetries can establish the existence or absence of the new sources of CP violation beyond the SM. Moreover, a simultaneous measurement of the averaged CP and normal polarization asymmetries will unambiguously determine the sign of the new phases.

In a specific model such as 2HDM or supersymmetry the Wilson coefficients possess new CP phases not found in the SM. The question of how infromative the asymmetries in $B \to K^* \ell^+ \ell^-$ decay about new sources of CP violation in 2HDM model or supersymmetry will be discussed elsewhere.

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FIGURE CAPTIONS

Fig. 1 The dependence of the CP asymmetry A_{CP} for $B \to K^* \mu^- \mu^+$ on q^2 and ϕ_7 at $\phi_{10} = 0$ for Region I.

Fig. 2 The same as in Fig. 1 but for Region II.

Fig. 3 The dependence of $\langle A_{CP} \rangle$, $\langle P_L \rangle$, $\langle P_T \rangle$ and $\langle P_N \rangle$ for $B \to K^* \mu^- \mu^+$ on ϕ_7 at $\phi_{10} = 0$ for Region II.

Fig. 4 The same as in Fig. 3 but for $\phi_{10} = \pi/2$.

Fig. 5 The same as in Fig. 3 but for $B \to K^* \tau^- \tau^+$ decay.

Fig. 6 The same as in Fig. 5 but for $\phi_{10} = \pi/2$.

Fig. 7 The correlation between the averaged CP and normal lepton polarization asymmetry at $\phi_{10} = 0$ for $B \to K^* \tau^- \tau^+$ decay.

Fig. 8 The same as Fig. 7 but for $\phi_{10} = \pi/2$.

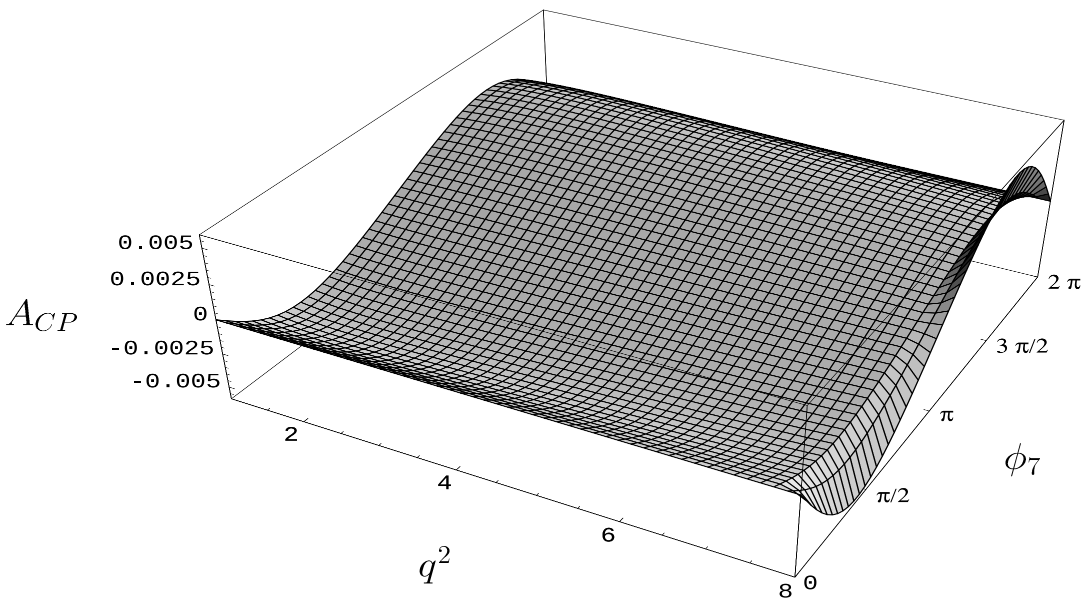

0.005
0.0025
0
-0.0025
-0.005
A_{CP}
2
4
6
8
q^2
0
π/2
π
3 π/2
2 π
ϕ_7

FIG. 1.

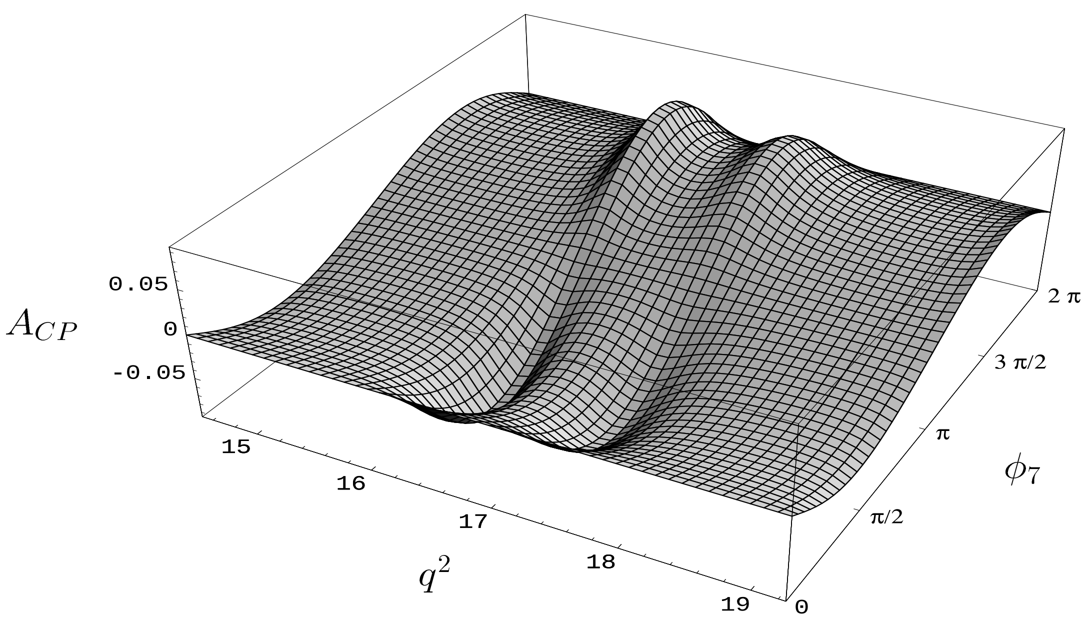
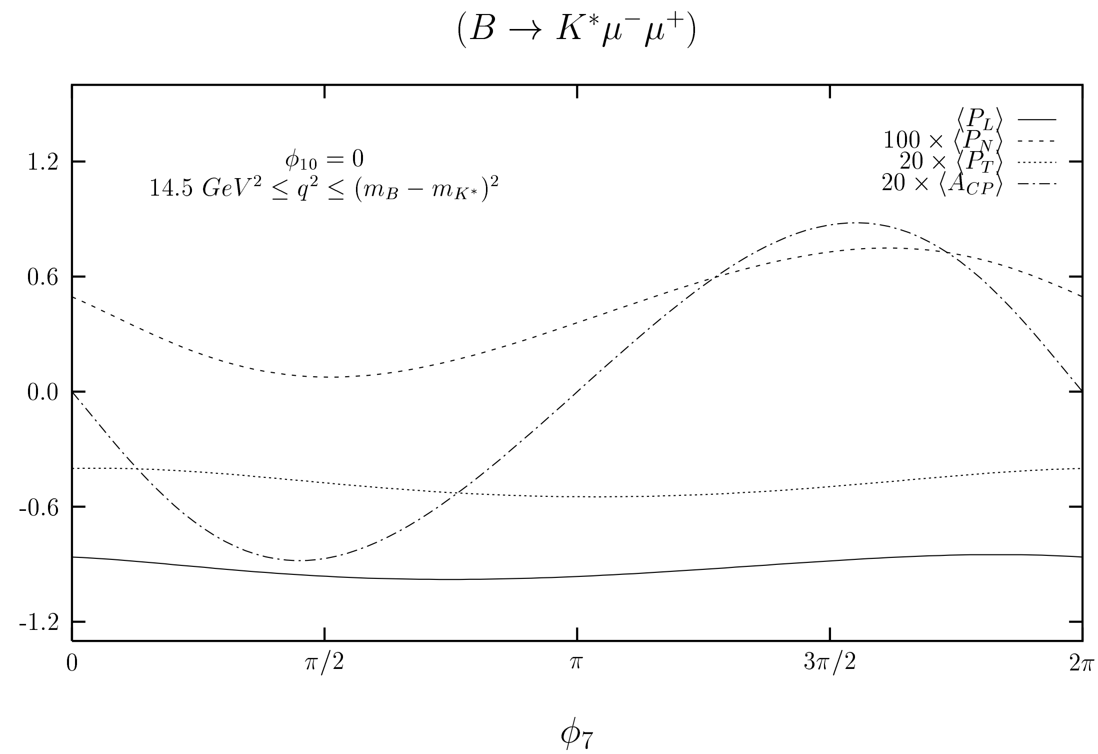
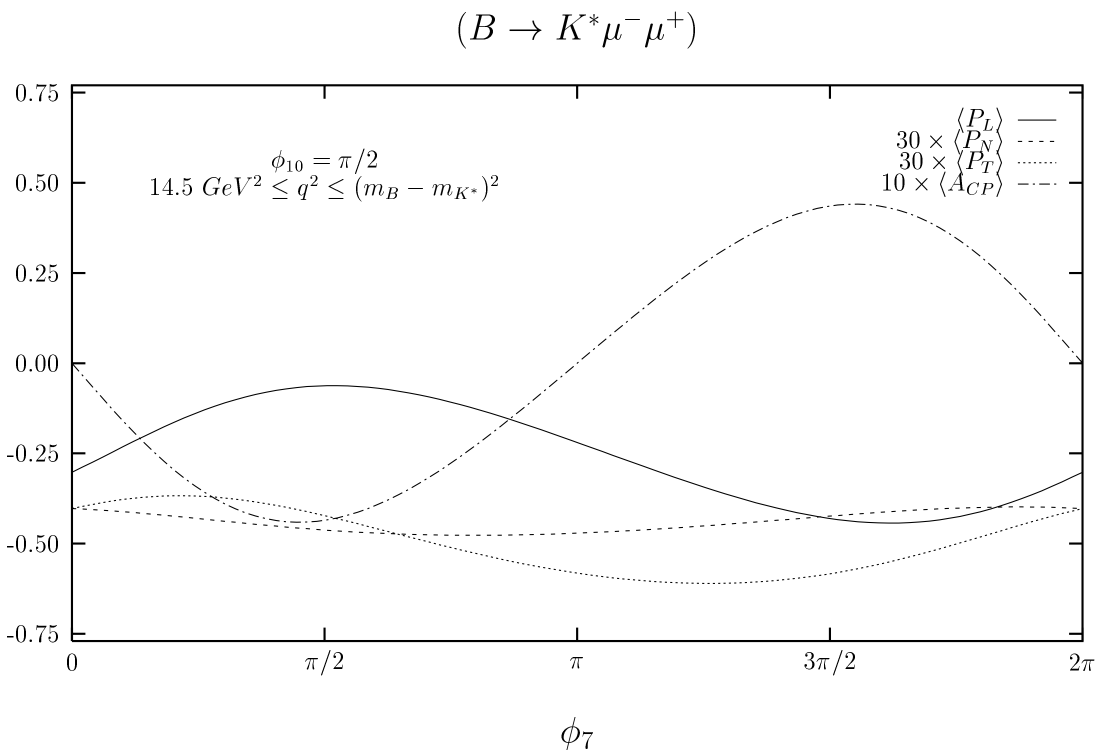

0.05
0
-0.05
A_{CP}
15
16
17
18
19
q^2
0
π/2
π
3 π/2
2 π
ϕ_7

FIG. 2.



$(B \to K^* \mu^- \mu^+)$
$\phi_{10} = 0$
$14.5\ GeV^2 \le q^2 \le (m_B - m_{K^*})^2$
$\langle P_L \rangle$
$100 \times \langle P_N \rangle$
$20 \times \langle P_T \rangle$
$20 \times \langle A_{CP} \rangle$
1.2
0.6
0.0
-0.6
-1.2
0
$\pi/2$
π
$3\pi/2$
2π
ϕ_7

FIG. 3.



$(B \to K^* \mu^- \mu^+)$
$\phi_{10} = \pi/2$
$14.5\ GeV^2 \le q^2 \le (m_B - m_{K^*})^2$
$\langle P_L \rangle$
$30 \times \langle P_N \rangle$
$30 \times \langle P_T \rangle$
$10 \times \langle A_{CP} \rangle$
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
0
$\pi/2$
π
$3\pi/2$
2π
ϕ_7

FIG. 4.

$(B \to K^* \tau^- \tau^+)$

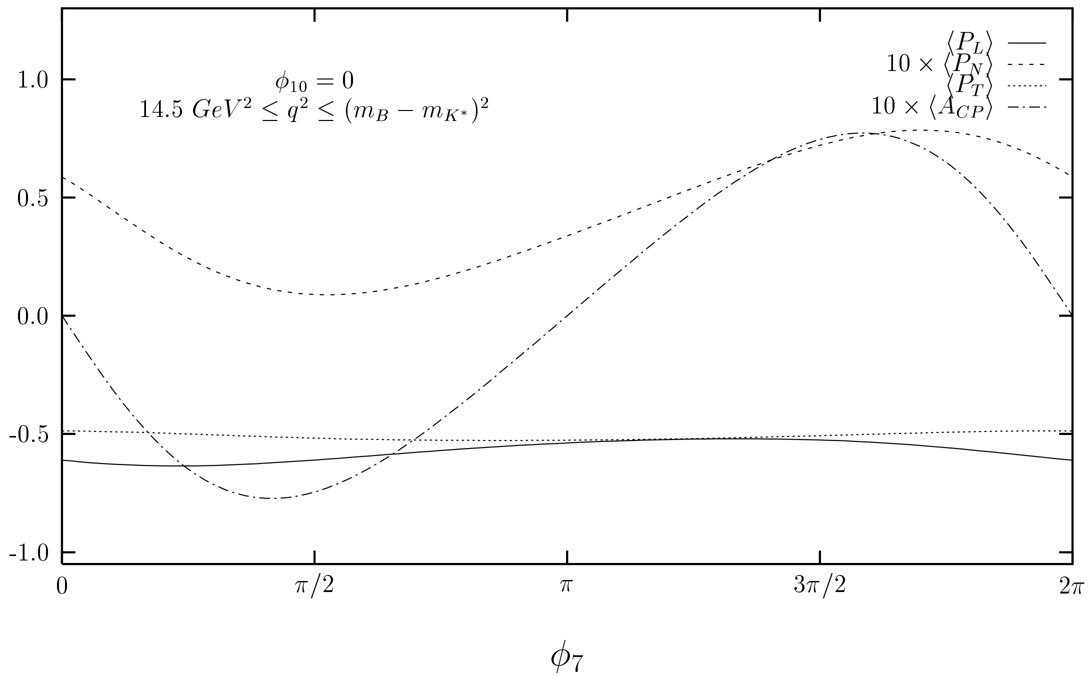


FIG. 5.

$(B \to K^* \tau^- \tau^+)$

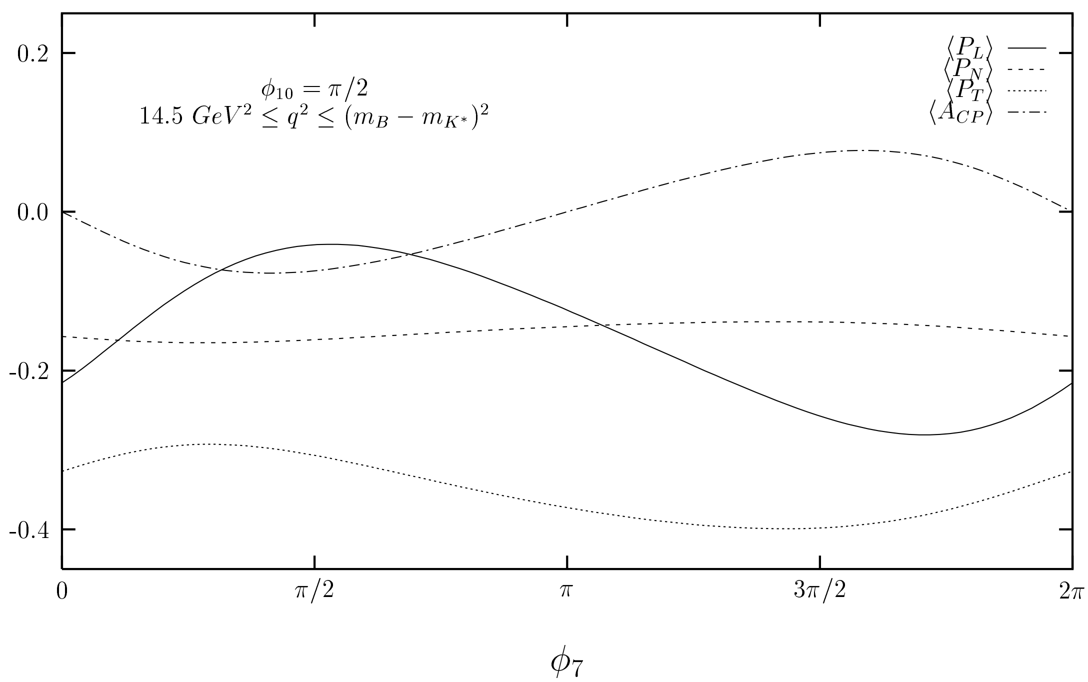


FIG. 6.

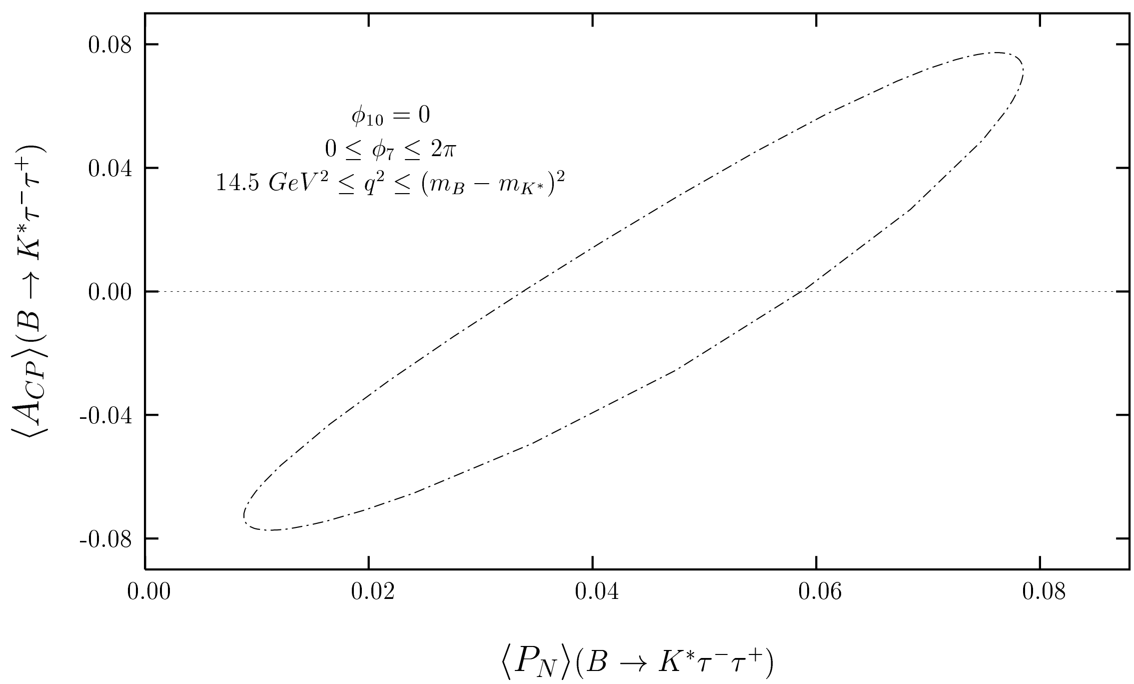

$\phi_{10} = 0$
$0 \leq \phi_7 \leq 2\pi$
$14.5\ GeV^2 \leq q^2 \leq (m_B - m_{K^*})^2$
0.08
0.04
0.00
-0.04
-0.08
0.00
0.02
0.04
0.06
0.08
$\langle A_{CP} \rangle (B \to K^* \tau^- \tau^+)$
$\langle P_N \rangle (B \to K^* \tau^- \tau^+)$

FIG. 7.

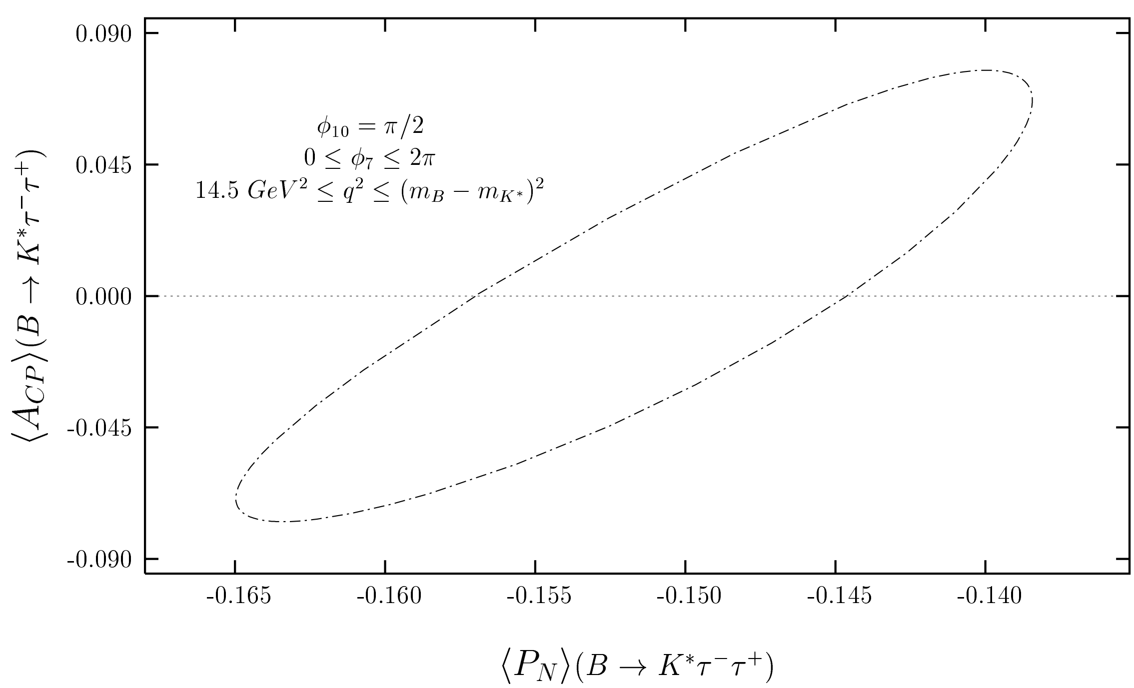

$\phi_{10} = \pi/2$
$0 \leq \phi_7 \leq 2\pi$
$14.5\ GeV^2 \leq q^2 \leq (m_B - m_{K^*})^2$
0.090
0.045
0.000
-0.045
-0.090
-0.165
-0.160
-0.155
-0.150
-0.145
-0.140
$\langle A_{CP} \rangle (B \to K^* \tau^- \tau^+)$
$\langle P_N \rangle (B \to K^* \tau^- \tau^+)$

FIG. 8.