



ECF22 - Loading and Environmental effects on Structural Integrity

## Peridynamic Modelling of Delamination in DCB Specimen

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### Abstract

Peridynamics is a robust theory to capture failure initiation and failure propagation in both isotropic and orthotropic materials. This paper presents a method to model Mod I delamination failure in composite materials using PD approach implemented in FEA. A bilinear PD damage law is formulated for PD interface by modifying the original failure formulation of Silling. That bilinear PD interface behavior is inspired by the bilinear damage law of CZM models. A MATLAB code is generated to generate PD interactions and corresponding surface correction factors. Proposed formulation is adapted to FEA code ABAQUS. Using generated MATLAB pre-processing code, PD model of a DCB specimen is generated. In addition, the same problem is solved using Cohesive Zone Modelling approach in ABAQUS. PD and FEA results are compared with results from literature. Obtained results indicate that PD solutions correlate well with CZM and results from literature.

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*Keywords:* Peridynamic Theory; Failure; Composite Materials; DCB Test; Mode I

### Nomenclature

PD	Peridynamics
FEA	Finite Element Analysis
FEM	Finite Element Method
CZM	Cohesive Zone Modelling

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## 1. Introduction

In recent years, there has been an increase in the usage of composites materials. These materials are mostly used in aerospace, defence and automotive industries in order to take advantage of their high strength, stiffness and low weight. Failure of composite materials occurs due to breaking of fibers, development of micro cracks in matrix, debonding between fibers and matrix and delamination. One of the most common failure types is delamination-based failure. Delamination in composites and fracture of adhesive joints are two forms of the failure that have been the subject of research in the field of composite materials. The structural strength and stiffness decrease with the delamination. The catastrophic failure occurs because of the separation at the interface region (Fan et al., 2008).

In the experimental studies, delamination-based failure is generally examined within the framework of the fracture mechanics in which critical energy release rate ( $G_C$ ) has been commonly used for crack growth resistance.

Cohesive Zone Modeling (CZM) is a commonly used technique to model delamination failures. This concept firstly introduced by the Dugdale (Dugdale, 1960) with thin plastic zone that is generated in front of the notch. Following Dugdale's work, Barenblatt (Barenblatt, 1962) introduced cohesive forces on a molecular scale. Later, many researchers worked on to solve delamination problems (Hillerborg et al., 1976; Turon et al., 2007). Prediction of the delamination of complex structures can be made using CZM approach. This method is simple (Elices et al., 2002) and can be implemented easily using Finite Elements Method (FEM) (Alfano and Crisfield, 2001; Heidari-Rarani et al., 2013). Hence, CZM is widely used for analyzing the delamination failure in composite structures.

In CZM, material behaviour within the damage zone is explained with traction-separation law that is also known as the cohesive law. In this law, a damage zone is developed in the cohesive layer. The damage starts to develop when the stress limit is reached and the stress decreases as the damage grows. Finally, the stress becomes zero when the separation reaches a critical value. The relation between the stress ( $\sigma$ ) and displacement ( $\delta$ ) is governed by the cohesive law and the area under the  $\sigma - \delta$  gives the critical strain energy release rate ( $G_C$ ) (Blackman et al., 2003; Fan et al., 2008).

Peridynamics is a nonlocal theory of continuum mechanics which is also an alternative to conventional Finite Element models and CZM approach. Equations of motion in Classical Continuum Mechanics (CCM), contain spatial derivatives. Original equations of CCM are invalid when it comes to model a discontinuity in the structure such as a crack or void. Peridynamic theory resolves this discontinuity by replacing the spatial derivatives with integrals of interaction forces between grid points known as material points (Silling, 2000). That interactions between material points resemble to interactions in molecular dynamics (MD). In MD, a material point interacts with its neighbours within an infinite radius, whereas in PD formulation, those interactions are restricted within a finite region. Hu et al. used a PD model to predict Mod I and Mod II delamination failures and validated their results using FEA simulations (Hu et al., 2015). Macek and Silling (Macek and Silling, 2007) proposed an approach to generate PD models using finite element code ABAQUS with truss elements.

In this study, PD is implemented in FEA code (Macek and Silling, 2007). A PD damage law is proposed for the failure of the interface. PD results are compared with CZM solutions and the results of Turon et al. (Turon et al., 2007).

## 2. Cohesive Zone Model Approach

The cohesive elements represent the initiation and propagation of delamination using CZM approach. In this paper, the constitutive response of cohesive elements is defined by bilinear relation between the tractions and displacement jumps as shown in Fig. 1.

The initial response of the cohesive element is linear until the damage initiation. Here, the linear elastic part is defined using penalty stiffness which is suggested to set to  $10^6$  N/mm by (Turon et al., 2007). After failure initiation, the interface softens linearly.

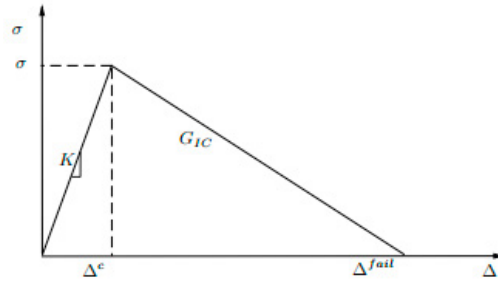


Fig. 1. Bilinear law for a cohesive zone model approach

### 3. Peridynamic Approach

Peridynamic theory is a nonlocal continuum theory with integro- differential formulation. Equation of motion in Peridynamic Theory is given as in Eq. 1.

$$\rho(x)\ddot{u}(x,t) = \int_H f(u' - u, x' - x)dH + b(x,t) \tag{1}$$

where  $\rho$  ,  $u$  ,  $f$  and  $b$  denote mass density, displacement, force density and body force respectively. In peridynamic theory, a material point  $x$  interacts with its neighbours in its horizon which has a finite radius as illustrated in Fig. 2.

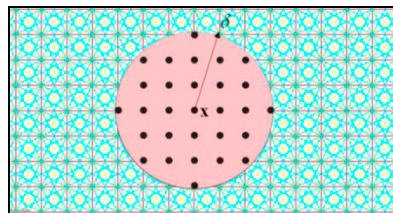


Fig. 2. Representation of peridynamic interactions and horizon

Peridynamic force density  $f$  is defined as (Silling, 2000):

$$f(\eta, \xi) = \frac{\xi + \eta}{|\xi + \eta|} c s \tag{2}$$

where the  $\xi$  ,  $\eta$  ,  $c$  and  $s$  denote the initial distance between material points, deformation, bond constant and stretch between material points respectively. The bond constant,  $c$  , is defined in terms of material constants as (Silling, 2000)

$$c = \frac{18K}{\pi\delta^4} \tag{3}$$

In Eq. 3. where  $K$  indicates the bulk modulus of the material and  $\delta$  is the horizon. In this study, bond based PD formulation is implemented in FEA code ABAQUS as mentioned in Macek and Sillings study (Macek and Silling, 2007). To implement PD in FEA, bonds are generated using a MATLAB pre-processing code with T3D2 truss elements. As indicated in (Macek and Silling, 2007), Elastic Modulus of trusses ( $E_t$ ) and cross sectional area ( $A_t$ ) are defined as

$$E_t = c\Delta x^4 \tag{4}$$

$$A_t = \Delta x^2 \tag{5}$$

where  $\Delta x$  indicates PD grid size. Surface correction factors are considered by multiplying the nominal stiffness of the trusses given in Eq.3. For brittle failure of materials, material failure is defined by a critical stretch value  $s_c$  given by (Silling, 2000).

$$s_c = \sqrt{\frac{10G_I}{\pi c\delta^5}} = \sqrt{\frac{5G_I}{9K\delta}} \tag{6}$$

where  $G_I$  is the Mode I fracture toughness,  $K$  is the Bulk Modulus of the material and  $\delta$  is the horizon.

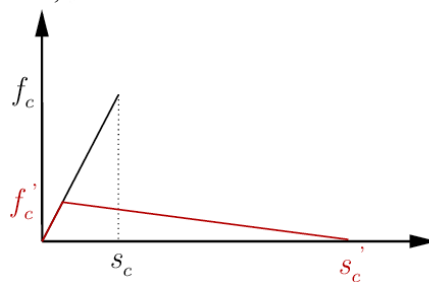


Fig. 3. PD material behaviour for the failure of bonds

The critical stretch  $s_c$  and the corresponding force density  $f_c$  are shown in Fig.3. The original material behaviour is brittle, and it is modified to a more ductile material behaviour for the interface. The original energy to break a bond is conserved and modified critical stretch  $s'_c$  and force density  $f'_c$  are displayed in Fig.3. Proposed material behaviour is a bilinear law with a linear softening behaviour which is similar to PD material behaviour of ductile metals given in (Yolum et al., 2016). The first linear part of the modified material behaviour represents the stiffness of the bonds given by Eq. 3. Then the interface starts to soften in a similar manner to CZM behaviour given by Fig.1.

#### 4. Solution of DCB Problem with CZM and PD Methods

In this study, the DCB specimen in (Turon et al., 2007) is modeled using PD and CZM solutions. Both numerical solutions are compared with the results of Turon et al. The geometry of the specimen is shown in Fig. 4. The length ( $L$ ), and the width ( $w$ ) of the specimen are 150 and 20 mm, respectively. Crack length ( $a_0$ ) is 35 mm and arm thickness ( $h$ ) is 1.55 mm. In PD solution, the bonds are modeled with T3D2 truss elements. In the FEA solution, the specimen is meshed with Four-noded plane stress elements (CPE4) and cohesive interface elements (COH2D4). Both numerical solutions are performed using the material properties shown in Table 1.

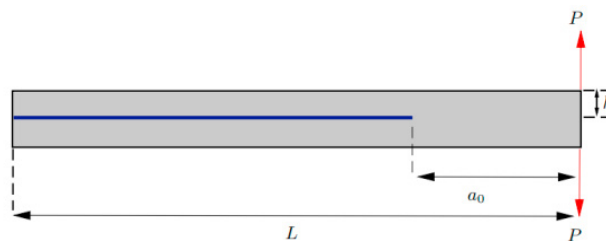


Fig. 4. The geometry of the DCB test specimen

Table 1. Material Properties of specimen (Turon et al., 2010)

$E_{11}$	$E_{22} = E_{33}$	$G_{12} = G_{13}$	$G_{23}$	$\nu_{12} = \nu_{13}$	$\nu_{23}$
120.0 GPa	10.5 GPa	5.25 GPa	3.48 GPa	0.3	0.5
$\sigma_n^{\max}$	$\sigma_s^{\max}$	$G_{IC}$	$G_{IIC}$	$K$	$\eta$
30 MPa	60 MPa	0.260 kJ/m <sup>2</sup>	1.002 kJ/m <sup>2</sup>	10 <sup>6</sup> N/mm	2.0

In the FEA and PD models displacement boundary conditions are applied at the two ends of the beam. FEA simulations are performed using 0.32 mm, 0.5 mm and 1 mm element sizes. PD mesh is displayed in Fig. 5 and PD mesh size is chosen as 0.31 mm. Stiffness of the bonds are calculated using Eq. 4. Surface corrections factors are considered by multiplying the bond constants with corresponding factors. A bilinear PD material law is used shown in Fig.3 and  $s_c'$  is chosen as 0.012. The load displacement results using several levels of mesh refinement are shown in Fig.6. Load displacement curve using the FEA with 0.32 mm mesh size is in good agreement with the results of Turon's study as shown in Fig 6a.

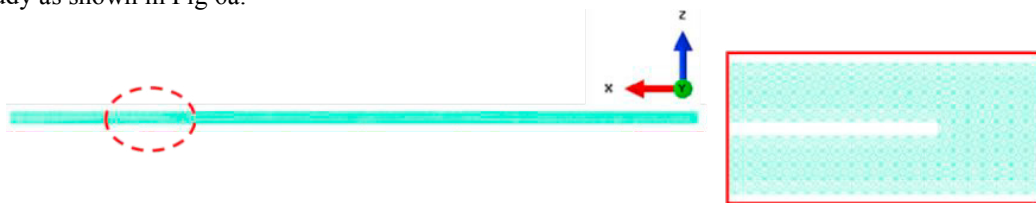


Fig. 5. PD mesh of DCB specimen

Comparison of PD solution and Turon's study in terms of load displacement results is given in Fig. 6b. In the linear region, both results are in good agreement. After failure initiation, there is a small change but the general failure behavior of the PD model is in good agreement in terms of failure load, pre and post failure stiffnesses with Turon's study.

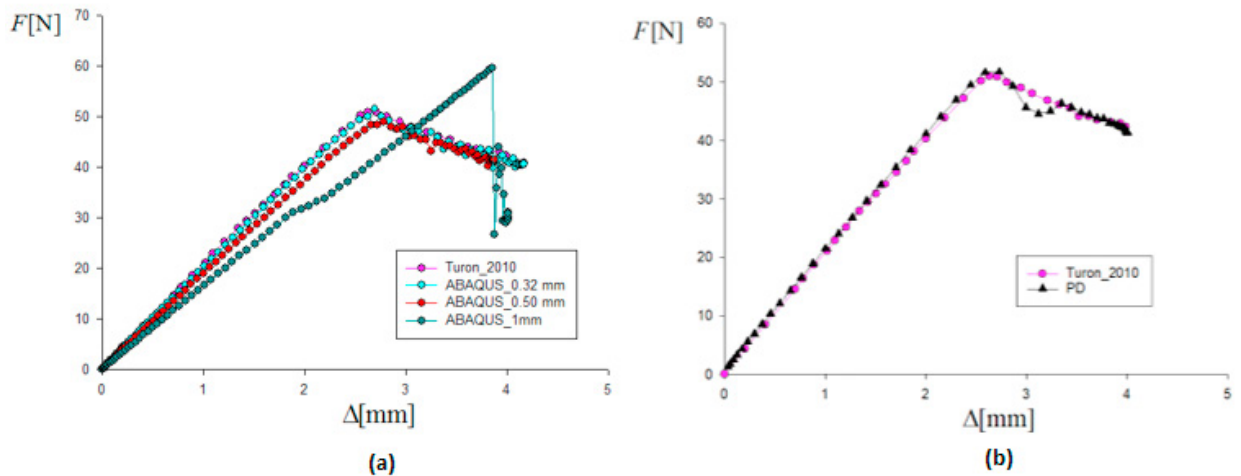


Fig. 6. Comparison of results of Turon et al., with different mesh sizes for different FEA (a), and PD (b)

## 5. Conclusions

In this study, a bilinear interface behavior is proposed to model Mod I delamination in composite materials. The proposed formulation is based on the conservation of the energy to break the bonds using original formulation of Silling (Silling, 2000). However, instead of using a material model with a sudden breakage, a bilinear law is proposed. PD solutions are correlated using CZM approach and results from the study of Turon (Turon et al., 2007). Results of PD solution is found to be in good agreement with both CZM and Turon's study.

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