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Plastic slip patterning through rate dependent non-convex gradient-enhanced plasticity

Tuncay Yalçınkaya[1,2,*], **W. A. M. Brekelmans**[2], and **M.G.D. Geers**[2]

[1] European Commission, JRC, Institute for Energy, P.O. Box 2, 1755 ZG Petten, The Netherlands

[2] Eindhoven University of Technology, Department of Mechanical Engineering, Section of Materials Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands

The paper presents a non-convex rate dependent strain gradient plasticity framework for the description of plastic slip patterning in metal crystals. The non-convexity is treated as an intrinsic property of the free energy of the material. Departing from explicit expressions for the free energy, the non-convex strain gradient crystal plasticity model is derived in a thermodynamically consistent manner, including the accompanying slip law. For the numerical solution of the problem, the displacement and the plastic slip fields are considered as primary variables. These fields are determined on a global level by solving simultaneously the linear momentum balance and the resulting slip evolution equation. The slip law differs from classical ones in the sense that it naturally includes a contribution from the non-convex free energy term, which enables patterning of the deformation field. The formulation of the computational framework is partially dual to a Ginzburg Landau type of phase field modeling approach.

1 Introduction

The mechanical response of many engineering materials is often influenced by an existing or emerging microstructure (dislocation sub-structures, shear bands etc.). There have been various approaches to model the formation and evolution of such microstructures which involve coupled models. The difficulty is the localization of the corresponding field and strain hardening-softening elastoplastic behavior which yields numerical instabilities. In order to remedy the ill-posedness of post critical results during the formation of microstructures several models have been proposed including methods using calculus of variations, non-local methods, viscous regularization techniques and Cosserat theories. In order to contribute to this, inspired by the success of phase field models, we propose an approach to illustrate the ability of non-convex field models to predict the emergence and evolution of dislocation slip microstructures in a rate dependent strain gradient plasticity framework [1]. The framework studies the viscous relaxation of plastic slip patterning in a system with energetic hardening.

2 Non-convex strain gradient plasticity

For conceptual simplicity, the formulations are casted in a 1D setting, not limiting their extension to 2D or 3D. In the geometrically linear small strain context the strain is assumed to be decomposed additively, $\varepsilon = \varepsilon^e + \varepsilon^p$, into an elastic part ε^e and a plastic part ε^p. In the 1D case, the total amount of plastic slip γ and the plastic strain are identical. The free energy ψ is assumed to be a function of the state variables, ε^e, γ and $\nabla\gamma$ where $\nabla\gamma = \partial\gamma/\partial x$ is the gradient of the plastic slip.

The force systems are characterized through their work-conjugated nature with respect to the state variables by postulation of the principle of virtual power. Considering a generalized virtual velocity without slip, the virtual velocity field can be chosen arbitrarily and this leads to the classical macroscopic force (stress) balance $\partial\sigma/\partial x = 0$ while considering the virtual slip field arbitrarily without generalized virtual velocity leads to the microscopic force balance, $\partial\xi/\partial x + \sigma - \pi = 0$, where σ, π and ξ are the thermodynamical forces conjugate to the internal state variables ε^e, γ and $\nabla\gamma$ respectively.

The local internal power expression can simply be written as, $P_i = \sigma\dot{\varepsilon}^e + \pi\dot{\gamma} + \xi\nabla\dot{\gamma}$ and the local dissipation inequality can be expressed as $D = P_i - \dot{\psi} \geq 0$. The free energy is assumed to take the additive structure $\psi = \psi_e + \psi_\gamma + \psi_{\nabla\gamma}$ where ψ_e is the elastically stored energy, ψ_γ is the microstructurally stored energy due to plastic slip and $\psi_{\nabla\gamma}$ is the energy associated to the gradients of slip. The dissipation can be written as,

$$D = \sigma\dot{\varepsilon}^e + \pi\dot{\gamma} + \xi\nabla\dot{\gamma} - \frac{\partial\psi}{\partial\varepsilon^e}\dot{\varepsilon}^e - \frac{\partial\psi}{\partial\gamma}\dot{\gamma} - \frac{\partial\psi}{\partial\nabla\gamma}\nabla\dot{\gamma} = \underbrace{\left(\sigma - \frac{d\psi_e}{d\varepsilon^e}\right)}_{0}\dot{\varepsilon}^e + \left(\pi - \frac{d\psi_\gamma}{d\gamma}\right)\dot{\gamma} + \underbrace{\left(\xi - \frac{d\psi_{\nabla\gamma}}{d\nabla\gamma}\right)}_{0}\nabla\dot{\gamma} \geq 0 \quad (1)$$

* Corresponding author: Email tuncay.yalcinkaya@ec.europa.eu, phone +31 224 565483 fax +31 224 565641

Additional to the stress σ, the microforce ξ conjugate to the slip gradient $\nabla\gamma$ is also assumed to be energetic: $\sigma = d\psi_e/d\varepsilon^e$ and $\xi = d\psi_{\nabla\gamma}/d\nabla\gamma$. The remaining term in the dissipation expression (1) reads,

$$D = \underbrace{\left(\pi - \frac{d\psi_\gamma}{d\gamma}\right)}_{\sigma^{\text{dis}}} \dot{\gamma} \geq 0 \tag{2}$$

The term dissipatively conjugated to the slip rate is identified as the dissipative stress: $\sigma^{\text{dis}} = \pi - d\psi_\gamma/d\gamma$. Then a constitutive equation is proposed satisfying the inequality (2): $\sigma^{\text{dis}} = \text{sign}(\dot{\gamma})\varphi$, where φ represents the actual slip resistance, $\varphi = s\left(|\dot{\gamma}|/\dot{\gamma}_0\right)^m$ where $\dot{\gamma}_0$ and m are the reference slip rate and the rate sensitivity exponent respectively. Furthermore, s is the resistance to dislocation slip which is assumed to be constant for simplicity. Extracting the slip rates yields the slip law,

$$\dot{\gamma} = \dot{\gamma}_0 \left(|\underbrace{\frac{\partial \xi}{\partial x} + \sigma}_{\pi} - \frac{d\psi_\gamma}{d\gamma}|/s \right)^{\frac{1}{m}} \text{sign}(\pi - \frac{d\psi_\gamma}{d\gamma}) \tag{3}$$

Each of the stress contributions in the slip law is derived from the free energy, i.e. $\sigma = \partial\psi/\partial\varepsilon^e$, $\xi = \partial\psi/\partial\nabla\gamma$. The non-convexity is obtained by adding a non-convex contribution to the free energy. The additional term is assumed to be a polynomial function of the plastic slip

$$\psi = \psi_e + \psi_{\nabla\gamma} + \psi_\gamma = \frac{1}{2}E\varepsilon^{e2} + \frac{1}{2}A(\nabla\gamma)^2 + (C_1\gamma^4 + C_2\gamma^3 + C_3\gamma^2 + C_4\gamma + C_5) \tag{4}$$

The governing system of equations is given by the linear momentum balance and the plastic slip evolution (3). Then the Galerkin procedure is followed to get the weak forms of the equations which are solved within a finite element context.

3 Numerical examples

Two numerical examples are presented in this section where a fictitious 1D bar with length L is deformed under tensile loading. The bar is constrained at the ends (suppressed displacement at $x = 0$ and prescribed displacement at $x = L$). Hard boundary and soft boundary cases correspond to $\gamma = 0$ and $\partial\gamma/\partial x = 0$ respectively. The stress vs. strain curves are presented for the

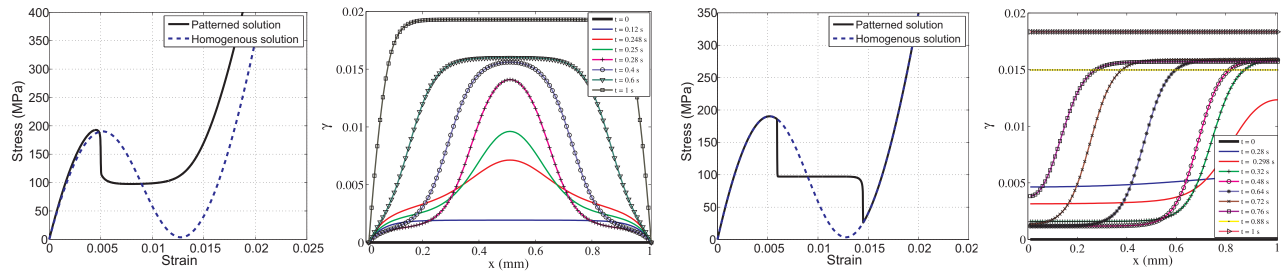

Fig. 1 Stress vs. strain response and plastic slip evolution for a low monotonic loading rate with hard boundary conditions (first two sets of curves) and soft boundary conditions (last two sets of curves).

patterned solution together with the homogenous solution which are recovered for the high loading rates. The stress response shows a typical plateau for the values of the strain where plastic slip is observed.

4 Conclusion

The non-convexity in the presented model is incorporated by a double-well function, not addressing particular materials yet. We present a generic formulation in order to demonstrate the microstructure evolution in a thermodynamical mathematical rigorous setting. A more practical elaboration applied to a particular material is obviously needed in future work, provided reliable experimental data on the non-convex term is available.

References

[1] T. Yalcinkaya, W. A. M. Brekelmans, and M. G. D. Geers, J. Mech. Phys. Solids **59**,1-17 (2011).