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Massive, Topologically Massive, Models

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Abstract

In three dimensions, there are two distinct mass-generating mechanisms for gauge fields: adding the usual Proca/Pauli-Fierz, or the more esoteric Chern-Simons (CS), terms. Here, we analyze the three-term models where both types are present, and their various limits. Surprisingly, in the tensor case, these seemingly innocuous systems are physically unacceptable. If the sign of the Einstein term is "wrong", as is in fact required in the CS theory, then the excitation masses are always complex; with the usual sign, there is a (known) region of the two mass parameters where reality is restored, but instead we show that a ghost problem arises, while, for the "pure mass " two-term system without an Einstein action, complex masses are unavoidable. This contrasts with the smooth behavior of the corresponding vector models. Separately, we show that the "partial masslessness" exhibited by (plain) massive spin-2 models in de Sitter backgrounds is shared by the three-term system: it also enjoys a reduced local gauge invariance when this mass parameter is tuned to the cosmological constant.

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Topologically massive tensor (TMG) gauge theories in $D = 3$ are well-understood models, whose linearized versions describe single massive but gauge-invariant excitations [1]. We analyze here the augmented, 3-term, system (MTMG) that breaks the invariance through an explicit mass term; the vector analogs are briefly reviewed for contrast.

We are motivated by two quite separate developments: In the first, it was shown from the propagator's poles that the mass eigenvalues of a particular class of MTMG systems are solutions of a cubic equation, two of whose roots become complex in a range of the underlying two "mass" parameter space [2]. In contrast, the vector system's masses solve a quadratic equation with everywhere real, positive roots [3]. We will analyze the excitations and mass counts for generic signs and values of these parameters and for both permitted sign choices of the Einstein action, as well as in its absence. We will uncover not only the objectionable complex masses, but also exhibit the unavoidable presence of ghosts and tachyon excitations in the underlying two-term models.

Our second topic is the study of MTMG in a constant curvature, rather than flat, background. Here we follow recent results which discuss the "partial masslessness" of ordinary massive gravity at a value of the mass tuned to the cosmological constant, where a residual gauge invariance eliminates the helicity zero mode but also leads to non-unitary regions in the (m^2, Λ) plane [4]. It is natural to ask whether this phenomenon persists for MTMG (it has no vector analog), given the common gauge covariance of the two systems' kinetic terms, and we will see that it does.

The action we consider here is the sum of Einstein, (third derivative order) Chern-Simons, and standard Pauli-Fierz mass, terms.

$$I = \int_M d^3x \left\{ a\sqrt{g}R - \frac{1}{2\mu}\epsilon^{\lambda\mu\nu}\Gamma^\rho{}_{\lambda\sigma}(\partial_\mu\Gamma^\sigma{}_{\rho\nu} + \frac{2}{3}\Gamma^\sigma{}_{\mu\beta}\Gamma^\beta{}_{\nu\rho}) - \frac{m^2}{4}(h_{\mu\nu}h^{\mu\nu} - h^2)\right\}, \qquad (1)$$

at quadratic order in $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$, $h \equiv \eta^{\mu\nu}h_{\mu\nu}$; our signature is $(-, +, +)$. Here the sign of μ is arbitrary but effectively irrelevant, that of m^2 is *a priori* free, while a allows for choosing the Einstein term's sign ($a = +1$ is the usual one) or even removing it, so this is the most general such model. All operations are with respect to the flat background $\eta_{\mu\nu}$. Note that $\mu = \infty$ represents massive gravity with 2 excitations (massive spin 2 in $3D$

has as many modes as massless spin 2 in $4D$); $m = 0$ is TMG with 1 mode. Pure Einstein theory, $\mu^{-1} = m^2 = 0$, has no excitations in D=3.

At this point, one can already see one insurmountable discontinuity latent in (1): As was shown in [1], the sign of the Einstein term in pure TMG must be $a = -1$, opposite to that in the usual Einstein gravity, in order for the energy to be positive, independent of the sign of μ. However since the usual massive spin-2 system does have excitations, both the relative and overall signs of the Einstein and mass terms are forced to be the *usual* Einstein and m^2 signs to avoid ghosts and tachyons: there is an unavoidable conflict in the choice of Einstein action sign a in the two cases.

For comparison, we first describe the generic vector case with a Proca mass term added to the TME action,

$$I = \int d^3x \left\{ \frac{-a}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\kappa}{2} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho - \frac{1}{2} m^2 A_\mu A^\mu \right\}. \tag{2}$$

To begin with, one may set $a = +1$ as it must be positive both in the Proca ($\kappa = 0$) and TME ($m = 0$) limits; $a = -1$ would also introduce tachyons. [As was noted long ago [5], setting $a = 0$ yields just another version of TME and hence it is equivalent to setting $m^2 = 0$; this also becomes clear in our canonical analysis of the theory.] Making a "2+1" decomposition of the conjugate variables,

$$F_{0i} = \epsilon^{ij} \hat{\partial}_j \omega + \hat{\partial}_i f, \qquad A_i = \epsilon^{ij} \hat{\partial}_j \chi + \hat{\partial}_i \varphi, \qquad \hat{\partial}_i \equiv \frac{\partial_i}{\sqrt{-\nabla^2}}, \tag{3}$$

reduces (2) to

$$\begin{aligned} I = \int d^3x \Big\{ (\omega &+ \kappa\varphi)\dot{\chi} + f\dot{\varphi} - \frac{1}{2}[\chi(m^2 - \nabla^2)\chi + \omega^2 + f^2 + m^2\varphi^2] \\ &+ \frac{m^2}{2} A_0^2 + A_0 \sqrt{-\nabla^2}(\kappa\chi - f) \Big\}, \end{aligned} \tag{4}$$

which, after eliminating A_0, can be diagonalized to represent two massive degrees of freedom with masses $m_\pm = \frac{1}{2} \left\{ \sqrt{\kappa^2 + 4m^2} \pm |\kappa| \right\}$ in agreement with those found in the propagator poles [3]. It is easily seen that the actions of various limiting theories are smoothly reached, including the equivalence of the $a = 0$ and the "self-dual" model of [5].

To analyze MTMG for generic values of (μ, m), and the sign factor a in (1), we first decompose $h_{\mu\nu}$,

$$h_{ij} = (\delta_{ij} + \hat{\partial}_i \hat{\partial}_j)\, \phi - \hat{\partial}_i \hat{\partial}_j \chi + (\epsilon_{ik} \hat{\partial}_k \hat{\partial}_j + \epsilon_{jk} \hat{\partial}_k \hat{\partial}_i\,)\xi, \qquad h_{0i} = -\epsilon_{ij} \partial_j \eta + \partial_i N_L, \qquad h_{00} \equiv N. \quad (5)$$

The various components of (1) are

$$I_E = \frac{a}{2} \int d^3x \left\{ \phi \ddot{\chi} + \phi \nabla^2 \left(N + 2\dot{N}_L \right) + (\nabla^2 \eta + \dot{\xi})^2 \right\}, \tag{6}$$

$$I_{CS} = \frac{1}{2\mu} \int d^3x \left\{ (\nabla^2 \eta + \dot{\xi}) \left[\nabla^2 (N + 2\dot{N}_L) + \ddot{\chi} + \Box \phi \right] \right\}, \tag{7}$$

$$I_{PF} = \frac{m^2}{2} \int d^3x \left\{ [N_L \nabla^2 N_L + \eta \nabla^2 \eta + \xi^2 - \phi \chi + N(\phi + \chi)] \right\}. \tag{8}$$

Unlike the vector model, this generically represents three (rather than two) massive excitations: three of the six $h_{\mu\nu}$ can be eliminated by constraints. We can determine the mass spectrum without having to diagonalize the fields, by forming the (cubic) eigenvalue equation,

$$(\Box^3 - \mu^2 a^2 \Box^2 + 2am^2 \mu^2 \Box - \mu^2 m^4) = 0. \tag{9}$$

This equation, for $a = 1$, was obtained from the pole of the propagator in [2], where it was also noted that the roots are complex unless the mass parameter ratio $\mu^2/m^2 \equiv \lambda \geq 27/4$. The explicit form of the three roots is not particularly illuminating, but we note, for example, that at $\lambda = 27/4$ (for $a = 1$) hitherto complex roots coalesce and the masses are simply

$$m_1 = m_2 = 2m_3 = \frac{2}{3}\mu = m\sqrt{3} \tag{10}$$

The limit $a = 0$, corresponds to keeping only the CS and mass terms, while dropping the Einstein part. Here the eigenvalue equation says that the 3 roots are just $(|\mu| m^2)^{1/3}$ times the cube roots of unity, whose two imaginary values are unavoidable: this model is never viable. [There is no tensor analog of "self-dual"-TME equivalence.]

The analysis so far has dealt with $a = +1$. However, the viability of the theory requires two correct signs: the first one to avoid tachyons ,the second to avoid ghosts, *i.e.* one needs both the relative sign in $(\Box - m^2)$ as well as the overall sign in the action, $+\int \phi(\Box - m^2)\phi$. In [1], it was shown that TMG required $a = -1$ for ghost-freedom (no tachyons arise for either sign choice.) Thus we conclude that at least the small m^2 limit to a two-term theory is unphysical, despite having a real mass μ (the seemingly massless other two modes are non-propagating). [We have not pursued in detail the diagonalization required to check the finite region for ghost signs.] Indeed, for $a = -1$, there are two complex mass roots of (9) for *any* finite μ^2/m^2 value. In summary, for our three-term models, $a = +1$ has acceptable mass ranges but faces ghost problems, and both $a = -1$ and $a = 0$ are always forbidden. None of these obstacles are present in the vector models, thanks to their lower derivative order and quadratic mass roots.

Consider now the different issue of the behavior of our models in de Sitter ($\Lambda > 0$) backgrounds. In any dimension, massive gravity (or any other higher spin) acquires gauge invariance at a non-zero mass parameter tuned to Λ [4]. The existence, in $D = 3$, of the gauge invariant Cotton tensor, which gives mass to spin-2 fields while keeping gauge invariance, warrants a separate discussion of MTMG in a cosmological background $g_{\mu\nu}$. The latter is defined by

$$R_{\mu\rho\nu\sigma} = \Lambda(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma}), \qquad R_{\mu\nu} = 2\Lambda g_{\mu\nu}, \qquad R = 6\Lambda. \tag{11}$$

The linearized Ricci tensor in this background reads

$$R^L_{\mu\nu} = \frac{1}{2}\left\{-\Box h_{\mu\nu} - \nabla_\mu\nabla_\nu h + \nabla^\sigma\nabla_\nu h_{\sigma\mu} + \nabla^\sigma\nabla_\mu h_{\sigma\nu}\right\}. \tag{12}$$

The unique, conserved, "Einstein" tensor is

$$G^L{}_{\mu\nu} \equiv R^L{}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R_L - 2\Lambda(h_{\mu\nu} - \frac{1}{2}g_{\mu\nu}h), \qquad \nabla_\mu\, G_L{}^{\mu\nu} \equiv 0. \tag{13}$$

Let us recall [4] that massive gravity,

$$G^L{}_{\mu\nu} + m^2(h_{\mu\nu} - g_{\mu\nu}h) = 0, \tag{14}$$

together with the Bianchi identities, leads (at $D = 3$) to the on-shell restriction

$$m^2(\Lambda - 2m^2)h = 0. \tag{15}$$

For arbitrary m^2 and Λ, this just constrains the massive field to be traceless. For $m^2 = \Lambda/2$, a "partial masslessness" arises from the novel scalar gauge invariance at this point.

The unique linearized version of the Cotton tensor of TMG [1]

$$C_L{}^{\mu\nu} = \frac{1}{\sqrt{g}}\epsilon^{\mu\alpha\beta}\ g_{\beta\sigma}\nabla_\alpha \left\{ R_L{}^{\sigma\nu} - 2\Lambda h^{\sigma\nu} - \frac{1}{4}g^{\sigma\nu}(R_L - 2\Lambda h) \right\}, \tag{16}$$

is still symmetric, traceless and conserved with respect to the background,

$$\epsilon^{\sigma\mu\nu}C^L_{\mu\nu} = 0, \qquad g^{\mu\nu}C^L{}_{\mu\nu} = 0, \qquad \nabla_\mu C^{L\,\mu\nu} = 0. \tag{17}$$

Hence MTMG in de Sitter space reads

$$G^L{}_{\mu\nu} + \frac{1}{\mu}C^L{}_{\mu\nu} + m^2(h_{\mu\nu} - g_{\mu\nu}h) = 0. \tag{18}$$

Since it is traceless, the presence of $C_{\mu\nu}{}^L$ does not affect the the condition (15); it merely adds one degree of freedom to those of the massive theory. The (m^2, Λ) plane is divided into two regions by the same (μ-independent) $m^2 = \Lambda/2$ line for both { MTMG ,MG}. In the $m^2 > \Lambda/2$ region there are respectively $\{3, 2\}$ excitations. On the line, the helicity zero one vanishes leaving $\{2, 1\}$ excitations. The $m^2 < \Lambda/2$ region is non-unitary due to the return of the helicity zero excitation with a non-unitary sign. Finally at $m^2 = 0$, we recover the full linearized diffeomorphism invariance and therefore there are $\{1, 0\}$ modes: this is just the {TMG , free Einstein } point.

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